Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/26-1.1.1.6

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3.186 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx$ . . . 1776
3.187 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}}\,dx$ . . . 1784
3.188 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx$ . . . 1792
3.189 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{11/2}}\,dx$ . . . 1800
3.190 $\int (a+bx)^m(A+Bx)(c+dx)^n\,dx$ . . . 1808
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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**192**]. This is test number [26].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (192)	0.00 (0)
Mathematica	100.00 (192)	0.00 (0)
Maple	92.19 (177)	7.81 (15)
Giac	92.19 (177)	7.81 (15)
Fricas	86.46 (166)	13.54 (26)
Reduce	72.92 (140)	27.08 (52)
Maxima	48.96 (94)	51.04 (98)
Sympy	44.79 (86)	55.21 (106)
Mupad	13.54 (26)	86.46 (166)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

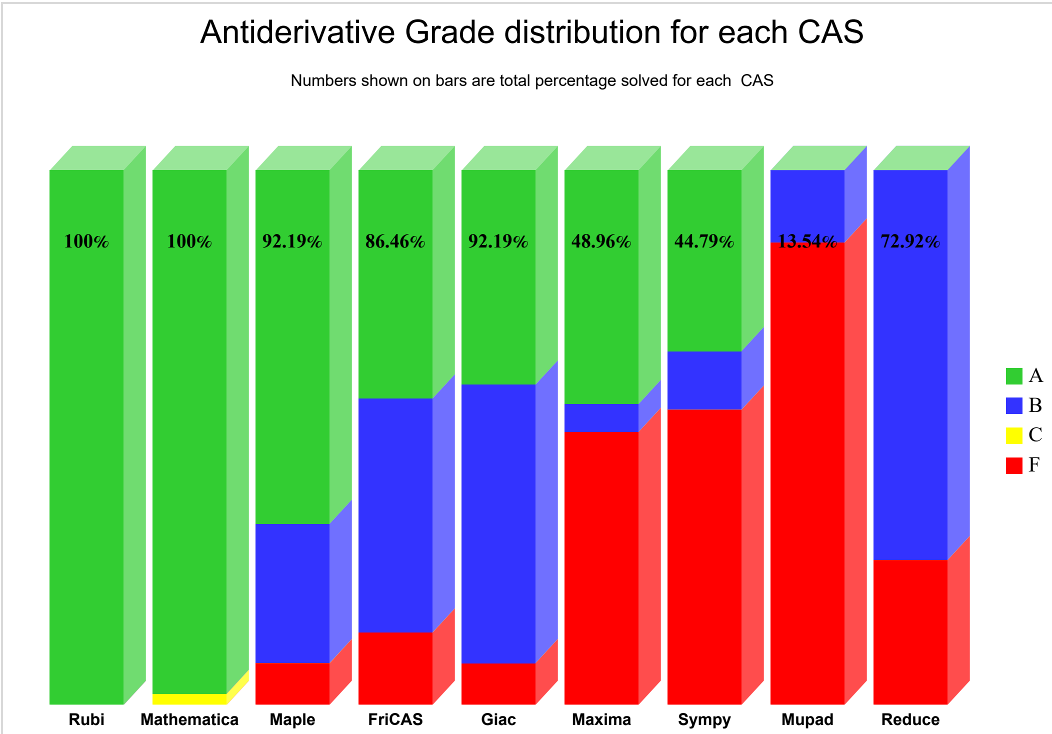
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

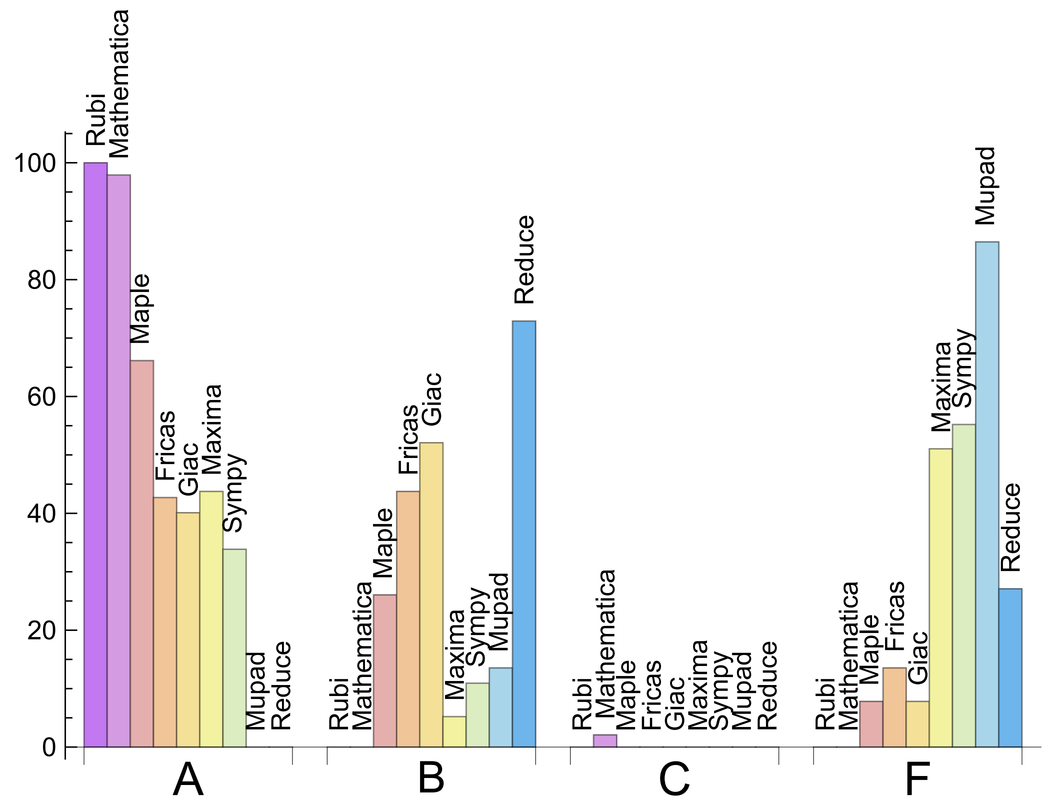
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	97.917	0.000	2.083	0.000
Maple	66.146	26.042	0.000	7.812
Maxima	43.750	5.208	0.000	51.042
Fricas	42.708	43.750	0.000	13.542
Giac	40.104	52.083	0.000	7.812
Sympy	33.854	10.938	0.000	55.208
Mupad	0.000	13.542	0.000	86.458
Reduce	0.000	72.917	0.000	27.083

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	15	100.00	0.00	0.00
Giac	15	100.00	0.00	0.00
Fricas	26	57.69	42.31	0.00
Reduce	52	100.00	0.00	0.00
Maxima	98	15.31	0.00	84.69
Sympy	106	51.89	40.57	7.55
Mupad	166	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Reduce	0.16
Giac	0.29
Maple	0.62
Rubi	0.69
Mathematica	0.97
Mupad	4.00
Fricas	5.86
Sympy	19.92

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	319.77	1.03	326.00	1.00
Mathematica	320.17	1.00	295.00	0.98
Mupad	334.15	1.58	346.00	1.63
Maxima	389.03	1.41	383.00	1.30
Reduce	682.68	2.01	447.00	1.54
Maple	886.11	2.51	427.00	1.15
Giac	1004.67	2.87	654.00	1.97
Fricas	1069.60	3.21	765.00	2.46
Sympy	1886.51	5.86	426.00	1.56

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

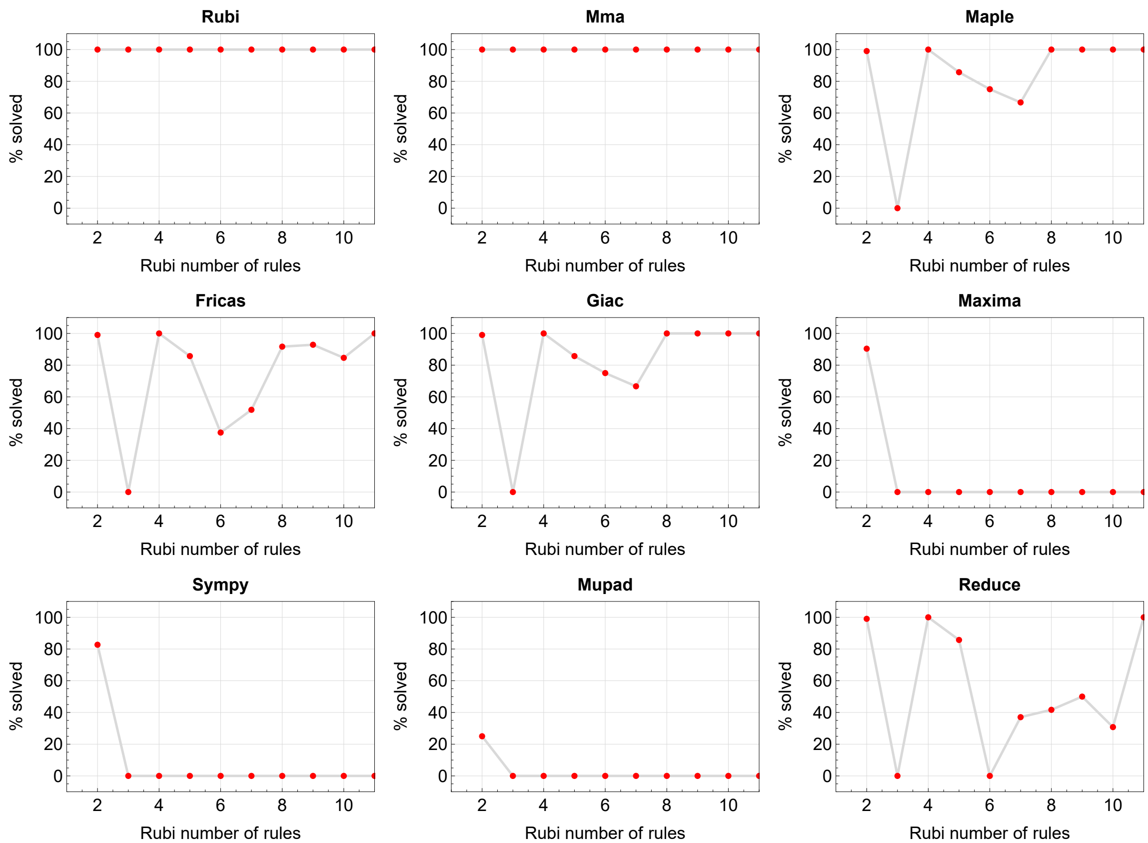


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

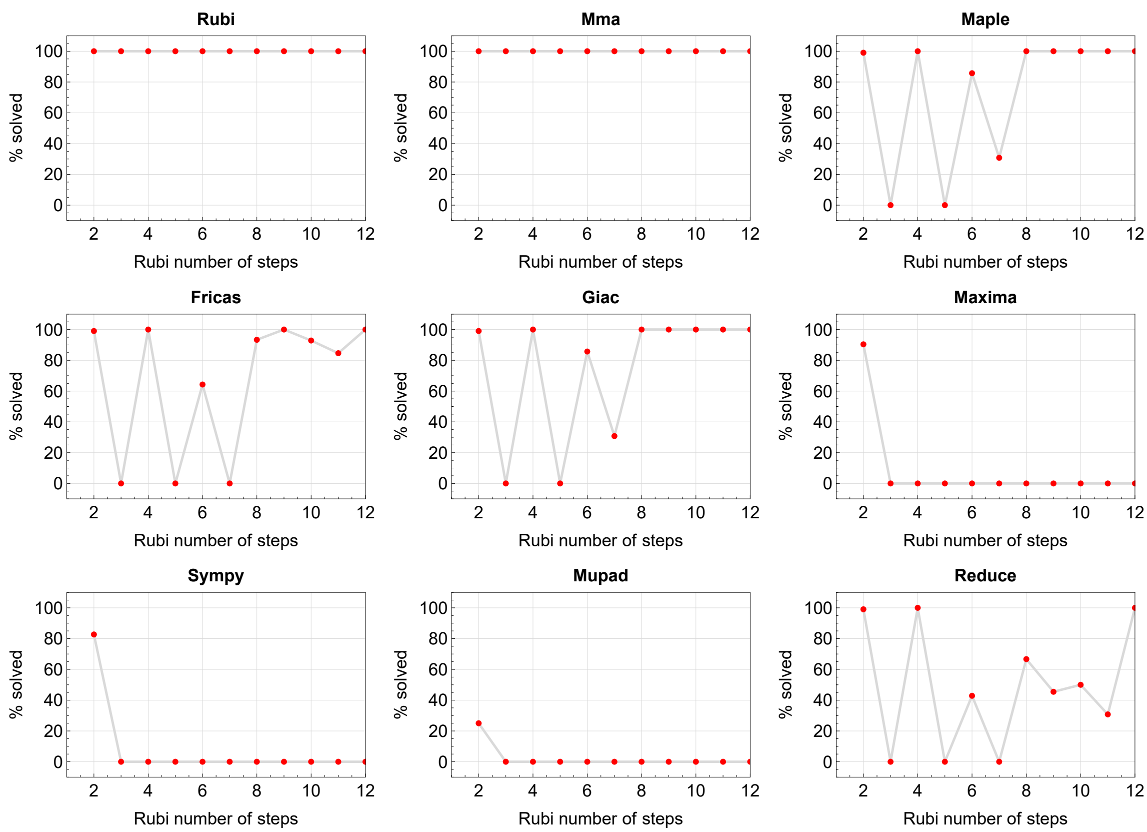


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

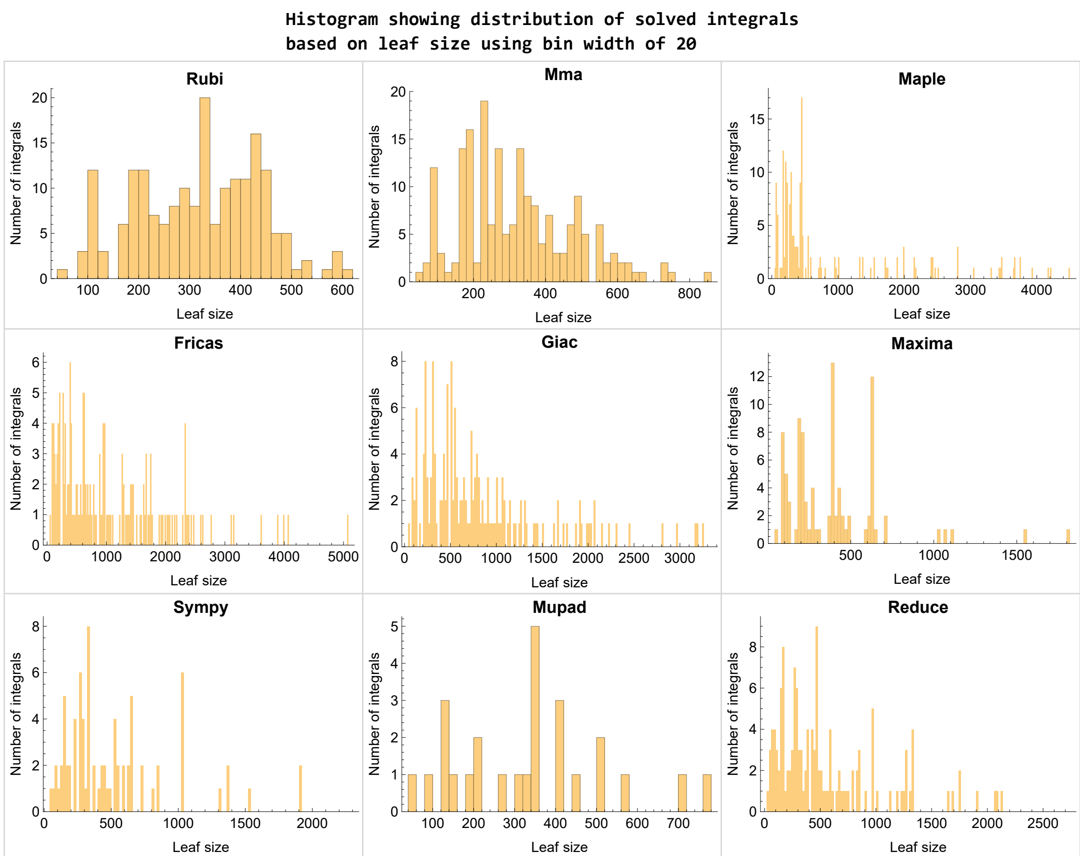


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

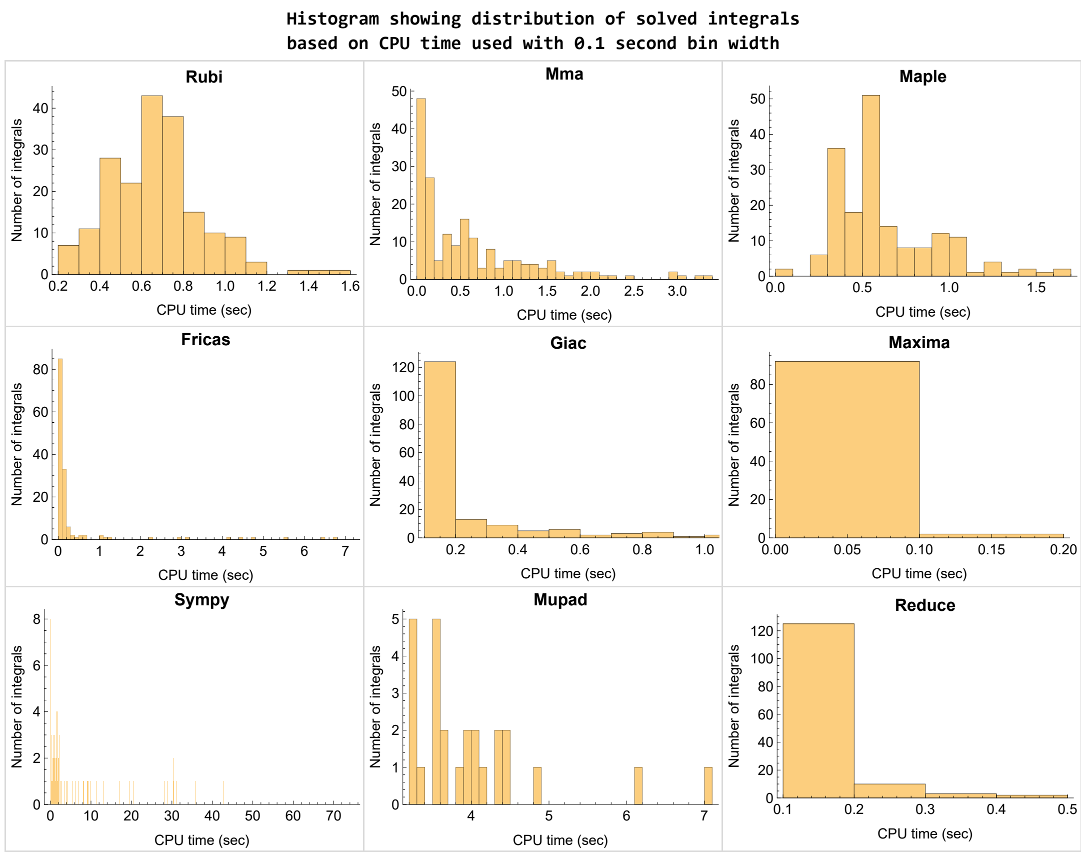


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

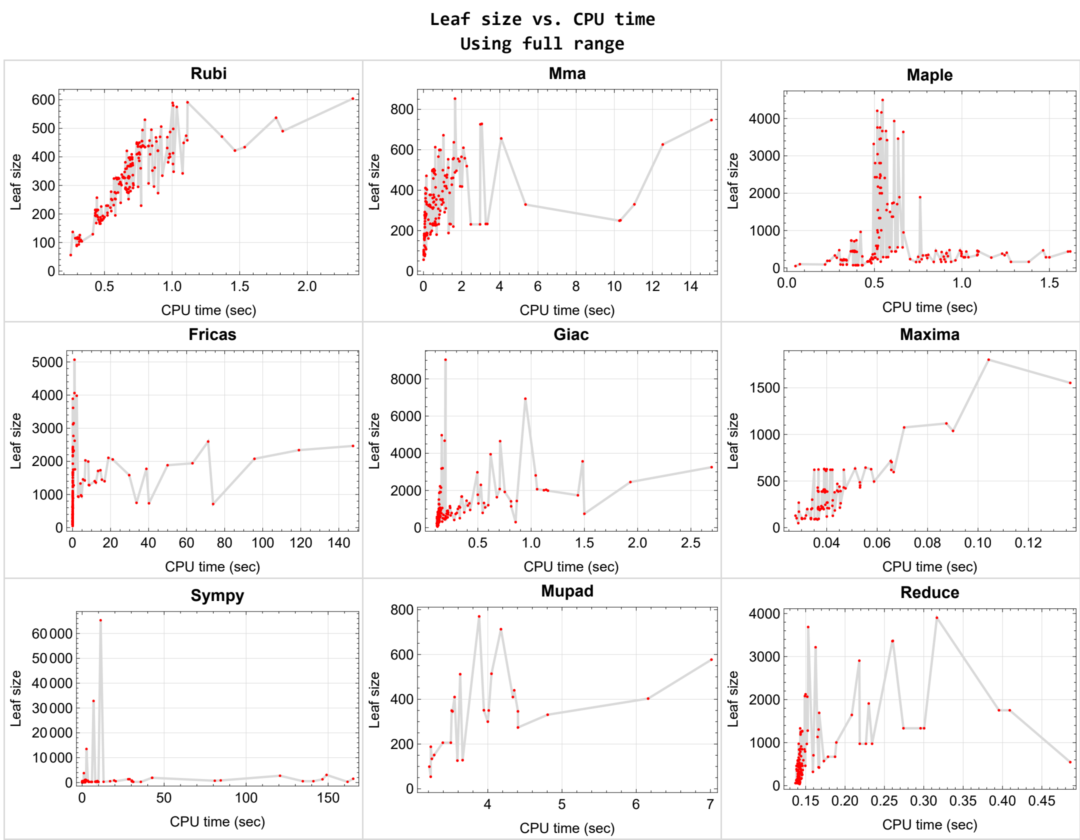


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{186, 187, 188, 189, 191, 192\}$

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

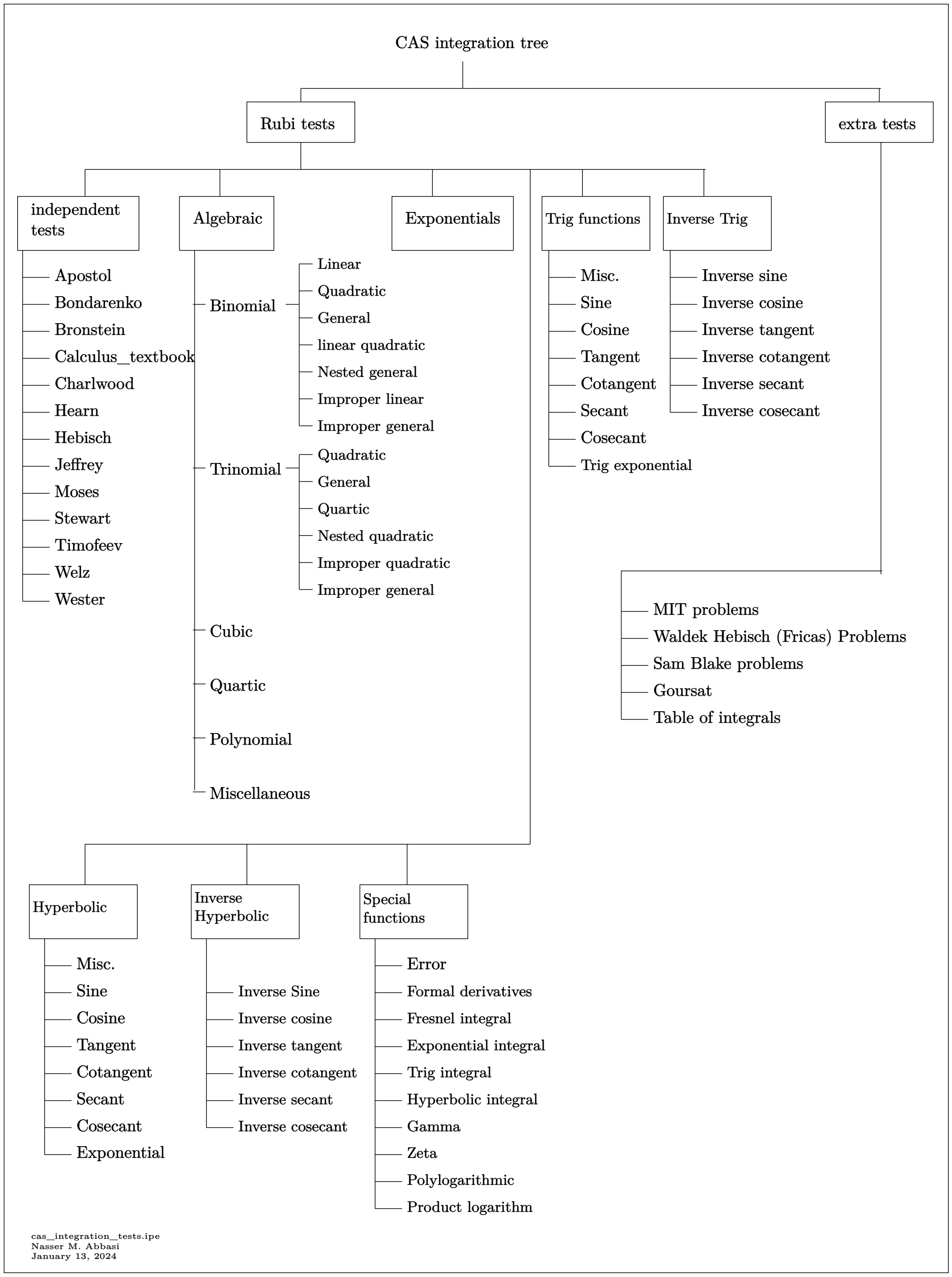
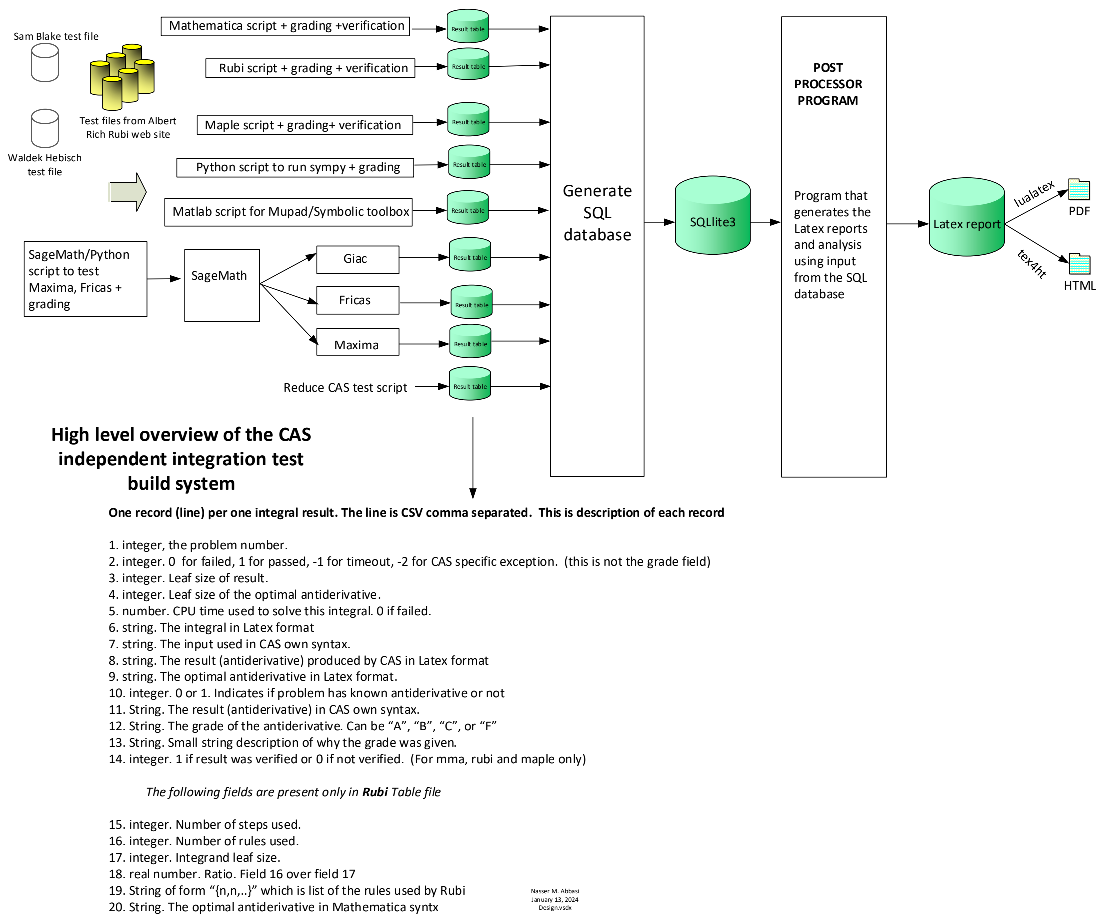


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

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Maple . . . 30
Fricas . . . 31
Maxima . . . 31
Giac . . . 32
Mupad . . . 32
Sympy . . . 33
Reduce . . . 33

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192 }

B grade { }

C grade { 113, 123, 139, 145 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 118, 128, 137, 144, 150, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173 }

B grade { 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 151, 152, 174, 175, 176, 177 }

C grade { }

F normal fail { 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 36, 37, 43, 44, 50, 51, 55, 56, 57, 58, 59, 65, 66, 67, 68, 79, 80, 81, 82, 83, 87, 88, 89, 90, 94, 95, 96, 97, 101, 102, 103, 104, 110, 111, 112, 113, 114, 120, 121, 122, 131, 132, 133, 138, 139, 140, 153, 154, 155, 156, 157, 160, 161, 162, 163, 167, 168, 169, 170 }

B grade { 6, 7, 8, 18, 19, 20, 21, 31, 32, 33, 34, 35, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 52, 53, 54, 60, 61, 62, 63, 64, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 84, 85, 86, 91, 92, 93, 98, 99, 100, 105, 106, 107, 115, 116, 123, 124, 125, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 152, 158, 159, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177 }

C grade { }

F normal fail { 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192 }

F(-1) timedout fail { 108, 109, 117, 118, 119, 126, 127, 128, 129, 130, 151 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 48, 49, 50, 51, 55, 56, 57, 58, 63, 64, 65, 66, 71, 72, 73, 74, 79, 80, 81, 82, 87, 88, 89, 90, 94, 95, 96, 97, 153, 154, 155, 156, 160, 161, 162, 163, 167, 168, 169, 170, 177 }

B grade { 33, 40, 46, 47, 52, 53, 54, 174, 175, 176 }

C grade { }

F normal fail { 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192 }

F(-1) timedout fail { }

F(-2) exception fail { 59, 60, 61, 62, 67, 68, 69, 70, 75, 76, 77, 78, 83, 84, 85, 86, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 164, 165, 166, 171, 172, 173 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 59, 60, 80, 81, 82, 83, 84, 89, 90, 91, 92, 96, 97, 98, 99, 103, 104, 112, 113, 121, 133, 139, 140, 141, 158, 159, 161, 162, 163, 164, 165, 166, 169, 170, 171 }

B grade { 9, 18, 21, 26, 33, 34, 35, 37, 47, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 85, 86, 87, 88, 93, 94, 95, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 167, 168, 172, 173, 174, 175, 176, 177 }

C grade { }

F normal fail { 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 43, 44, 50, 51, 55, 56, 57, 58, 79, 80, 81, 82 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 52, 53, 54, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 26, 27, 28, 29, 34, 35, 36, 37, 42, 43, 44, 49, 50, 51, 56, 57, 58, 59, 64, 65, 66, 67, 72, 73, 75, 80, 81, 82, 83, 87, 88, 89, 90, 91, 94, 95, 96, 98, 154, 155, 156, 161, 162, 163, 167, 168, 169, 170 }

B grade { 32, 33, 39, 40, 41, 46, 47, 52, 53, 55, 63, 71, 74, 79, 97, 153, 160, 174, 175, 176, 177 }

C grade { }

F normal fail { 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 164, 171, 178, 180, 181, 182, 183, 184 }

F(-1) timedout fail { 10, 11, 12, 21, 22, 23, 24, 25, 30, 31, 38, 45, 48, 54, 60, 61, 62, 68, 69, 70, 76, 77, 78, 84, 85, 86, 92, 93, 99, 100, 118, 119, 127, 128, 129, 130, 158, 159, 165, 166, 172, 173, 187 }

F(-2) exception fail { 179, 185, 186, 188, 189, 190, 191, 192 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 110, 111, 112, 120, 121, 131, 132, 133, 134, 141, 142, 149, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177 }

C grade { }

F normal fail { 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 147, 148, 150, 151, 152, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	246	272	268	268	333	331	231	331
N.S.	1	1.00	1.26	1.39	1.37	1.37	1.71	1.70	1.18	1.70
time (sec)	N/A	0.580	0.078	0.285	0.029	0.066	0.034	0.122	0.140	4.807

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	189	184	184	228	229	166	206
N.S.	1	1.00	0.90	0.97	0.94	0.94	1.17	1.17	0.85	1.06
time (sec)	N/A	0.522	0.034	0.244	0.043	0.063	0.029	0.117	0.141	3.394

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	99	100	100	124	126	101	126
N.S.	1	1.00	1.00	0.95	0.96	0.96	1.19	1.21	0.97	1.21
time (sec)	N/A	0.332	0.011	0.074	0.034	0.064	0.022	0.119	0.140	3.590

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	48	48	53	54	48	54
N.S.	1	1.00	1.00	0.88	0.86	0.86	0.95	0.96	0.86	0.96
time (sec)	N/A	0.250	0.006	0.049	0.029	0.064	0.018	0.121	0.142	3.230

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	154	202	214	217	185	236	167	0
N.S.	1	1.00	0.92	1.20	1.27	1.29	1.10	1.40	0.99	0.00
time (sec)	N/A	0.439	0.052	0.317	0.045	0.073	0.329	0.123	0.142	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	169	219	221	341	238	308	278	0
N.S.	1	1.00	0.98	1.27	1.28	1.98	1.38	1.79	1.62	0.00
time (sec)	N/A	0.462	0.056	0.308	0.037	0.069	0.818	0.120	0.143	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	169	211	231	369	260	223	363	346
N.S.	1	1.00	0.96	1.20	1.31	2.10	1.48	1.27	2.06	1.97
time (sec)	N/A	0.474	0.056	0.323	0.038	0.082	5.448	0.124	0.146	4.406

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	171	212	245	358	275	220	305	410
N.S.	1	1.00	0.95	1.18	1.36	1.99	1.53	1.22	1.69	2.28
time (sec)	N/A	0.456	0.062	0.311	0.039	0.077	31.205	0.121	0.143	4.338

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	176	209	261	305	292	389	272	440
N.S.	1	1.00	0.93	1.11	1.38	1.61	1.54	2.06	1.44	2.33
time (sec)	N/A	0.451	0.100	0.318	0.046	0.075	161.849	0.123	0.141	4.356

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	170	216	269	269	0	240	190	274
N.S.	1	1.00	0.88	1.12	1.39	1.39	0.00	1.24	0.98	1.42
time (sec)	N/A	0.434	0.075	0.334	0.040	0.077	0.000	0.122	0.141	4.406

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	170	224	268	268	0	236	235	300
N.S.	1	1.00	0.87	1.15	1.37	1.37	0.00	1.21	1.21	1.54
time (sec)	N/A	0.444	0.069	0.325	0.040	0.076	0.000	0.123	0.140	4.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	178	225	293	293	0	240	246	350
N.S.	1	1.00	0.91	1.15	1.50	1.50	0.00	1.23	1.26	1.79
time (sec)	N/A	0.432	0.076	0.327	0.042	0.081	0.000	0.118	0.145	4.012

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	407	475	429	429	581	577	364	577
N.S.	1	1.00	1.33	1.55	1.40	1.40	1.89	1.88	1.19	1.88
time (sec)	N/A	0.826	0.135	0.299	0.036	0.068	0.045	0.117	0.140	7.014

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	292	332	300	300	403	403	265	403
N.S.	1	1.00	0.95	1.08	0.98	0.98	1.31	1.31	0.86	1.31
time (sec)	N/A	0.712	0.068	0.274	0.034	0.067	0.039	0.121	0.140	6.160

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	184	189	171	171	228	229	166	206
N.S.	1	1.00	0.90	0.93	0.84	0.84	1.12	1.12	0.81	1.01
time (sec)	N/A	0.522	0.035	0.231	0.029	0.068	0.029	0.117	0.140	3.502

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	96	91	90	90	99	101	83	99
N.S.	1	1.00	0.88	0.83	0.83	0.83	0.91	0.93	0.76	0.91
time (sec)	N/A	0.325	0.010	0.218	0.035	0.067	0.026	0.120	0.143	3.212

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	271	429	395	398	379	500	274	0
N.S.	1	1.00	0.92	1.46	1.34	1.35	1.29	1.70	0.93	0.00
time (sec)	N/A	0.687	0.111	0.399	0.036	0.070	0.626	0.117	0.141	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	275	454	408	616	454	579	471	0
N.S.	1	1.00	0.98	1.62	1.46	2.20	1.62	2.07	1.68	0.00
time (sec)	N/A	0.693	0.063	0.377	0.039	0.077	1.982	0.122	0.143	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	276	446	421	681	530	468	607	0
N.S.	1	1.00	0.97	1.57	1.48	2.40	1.87	1.65	2.14	0.00
time (sec)	N/A	0.663	0.066	0.366	0.047	0.081	20.491	0.125	0.142	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	278	442	431	675	546	461	580	0
N.S.	1	1.00	0.97	1.53	1.50	2.34	1.90	1.60	2.01	0.00
time (sec)	N/A	0.655	0.079	0.365	0.053	0.083	134.582	0.125	0.140	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	278	437	440	610	0	726	492	0
N.S.	1	1.00	0.95	1.50	1.51	2.09	0.00	2.49	1.68	0.00
time (sec)	N/A	0.656	0.140	0.377	0.045	0.084	0.000	0.126	0.141	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	283	450	470	537	0	460	428	0
N.S.	1	1.00	0.94	1.50	1.56	1.78	0.00	1.53	1.42	0.00
time (sec)	N/A	0.626	0.144	0.382	0.044	0.083	0.000	0.126	0.143	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	330	435	453	453	0	500	333	0
N.S.	1	1.00	1.08	1.43	1.49	1.49	0.00	1.64	1.09	0.00
time (sec)	N/A	0.608	0.169	0.390	0.053	0.078	0.000	0.122	0.144	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	341	457	486	486	0	504	386	0
N.S.	1	1.00	1.11	1.49	1.58	1.58	0.00	1.64	1.26	0.00
time (sec)	N/A	0.620	0.155	0.387	0.053	0.082	0.000	0.126	0.144	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	340	457	495	495	0	504	397	0
N.S.	1	1.00	1.11	1.49	1.61	1.61	0.00	1.64	1.29	0.00
time (sec)	N/A	0.663	0.159	0.385	0.059	0.086	0.000	0.121	0.143	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	403	713	609	611	653	835	254	0
N.S.	1	1.00	0.98	1.73	1.47	1.48	1.58	2.02	0.62	0.00
time (sec)	N/A	1.007	0.178	0.385	0.040	0.074	0.828	0.123	0.146	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	266	427	378	379	379	500	164	0
N.S.	1	1.00	0.88	1.42	1.26	1.26	1.26	1.66	0.54	0.00
time (sec)	N/A	0.715	0.089	0.387	0.039	0.071	0.588	0.118	0.141	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	159	201	189	191	185	235	91	0
N.S.	1	1.00	0.89	1.13	1.06	1.07	1.04	1.32	0.51	0.00
time (sec)	N/A	0.484	0.039	0.341	0.037	0.070	0.368	0.121	0.143	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	80	83	87	88	78	90	38	0
N.S.	1	1.00	0.91	0.94	0.99	1.00	0.89	1.02	0.43	0.00
time (sec)	N/A	0.295	0.013	0.320	0.036	0.067	0.171	0.121	0.142	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	126	125	130	171	0	132	46	0
N.S.	1	1.00	0.98	0.97	1.01	1.33	0.00	1.02	0.36	0.00
time (sec)	N/A	0.414	0.047	0.483	0.032	0.100	0.000	0.121	0.142	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	162	167	210	414	0	216	163	0
N.S.	1	1.00	0.98	1.01	1.27	2.49	0.00	1.30	0.98	0.00
time (sec)	N/A	0.470	0.052	0.536	0.037	0.110	0.000	0.115	0.144	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	218	233	392	695	1306	420	448	0
N.S.	1	1.00	0.95	1.02	1.71	3.03	5.70	1.83	1.96	0.00
time (sec)	N/A	0.526	0.074	0.491	0.046	0.120	146.574	0.127	0.143	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	260	275	627	1031	1360	669	469	0
N.S.	1	1.00	0.95	1.01	2.30	3.78	4.98	2.45	1.72	0.00
time (sec)	N/A	0.585	0.083	0.503	0.058	0.101	28.975	0.130	0.141	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	411	740	624	892	728	912	685	0
N.S.	1	1.00	1.00	1.80	1.52	2.18	1.78	2.22	1.67	0.00
time (sec)	N/A	0.982	0.101	0.397	0.037	0.083	3.522	0.133	0.144	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	278	452	387	567	454	580	451	0
N.S.	1	1.00	0.97	1.57	1.35	1.98	1.58	2.02	1.57	0.00
time (sec)	N/A	0.677	0.062	0.373	0.040	0.077	2.181	0.127	0.144	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	174	219	199	295	238	310	251	0
N.S.	1	1.00	0.96	1.21	1.10	1.63	1.31	1.71	1.39	0.00
time (sec)	N/A	0.476	0.077	0.337	0.032	0.068	1.022	0.121	0.141	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	88	87	91	137	87	178	108	0
N.S.	1	1.00	0.97	0.96	1.00	1.51	0.96	1.96	1.19	0.00
time (sec)	N/A	0.295	0.022	0.342	0.034	0.068	0.317	0.119	0.141	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	165	167	210	416	0	217	160	0
N.S.	1	1.00	0.99	1.01	1.27	2.51	0.00	1.31	0.96	0.00
time (sec)	N/A	0.465	0.051	0.478	0.040	0.117	0.000	0.122	0.141	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	216	225	431	814	1525	314	744	0
N.S.	1	1.00	0.97	1.01	1.94	3.67	6.87	1.41	3.35	0.00
time (sec)	N/A	0.544	0.072	0.477	0.047	0.131	165.303	0.129	0.142	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	273	278	703	1420	1912	451	1223	0
N.S.	1	1.00	0.99	1.01	2.55	5.14	6.93	1.63	4.43	0.00
time (sec)	N/A	0.668	0.082	0.484	0.066	0.117	42.773	0.124	0.144	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	383	733	634	978	814	781	794	0
N.S.	1	1.00	0.96	1.85	1.60	2.46	2.05	1.97	2.00	0.00
time (sec)	N/A	0.976	0.106	0.367	0.051	0.088	30.355	0.129	0.149	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	279	444	397	620	530	468	516	0
N.S.	1	1.00	0.96	1.53	1.36	2.13	1.82	1.61	1.77	0.00
time (sec)	N/A	0.711	0.072	0.368	0.044	0.080	17.053	0.125	0.140	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	174	211	207	318	260	225	276	346
N.S.	1	1.00	0.95	1.15	1.12	1.73	1.41	1.22	1.50	1.88
time (sec)	N/A	0.494	0.076	0.329	0.041	0.076	6.020	0.118	0.140	3.524

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	90	101	153	102	88	108	151
N.S.	1	1.00	1.00	0.97	1.09	1.65	1.10	0.95	1.16	1.62
time (sec)	N/A	0.310	0.022	0.306	0.028	0.071	0.793	0.121	0.140	3.277

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	233	233	379	658	0	420	317	0
N.S.	1	1.00	1.01	1.01	1.64	2.85	0.00	1.82	1.37	0.00
time (sec)	N/A	0.568	0.067	0.518	0.043	0.128	0.000	0.129	0.142	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	270	277	715	1351	1912	452	1252	0
N.S.	1	1.00	0.98	1.00	2.59	4.89	6.93	1.64	4.54	0.00
time (sec)	N/A	0.678	0.090	0.506	0.065	0.112	42.633	0.129	0.144	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	360	377	1075	2183	3074	1012	2124	0
N.S.	1	1.00	0.99	1.04	2.97	6.03	8.49	2.80	5.87	0.00
time (sec)	N/A	0.872	0.109	0.502	0.071	0.151	149.067	0.133	0.150	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	384	731	644	1003	0	764	844	0
N.S.	1	1.00	0.96	1.83	1.61	2.51	0.00	1.91	2.11	0.00
time (sec)	N/A	0.985	0.121	0.369	0.055	0.088	0.000	0.129	0.148	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	280	442	409	618	546	461	534	0
N.S.	1	1.00	0.95	1.50	1.39	2.09	1.85	1.56	1.81	0.00
time (sec)	N/A	0.749	0.087	0.392	0.039	0.077	141.038	0.122	0.142	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	183	212	218	315	275	217	265	410
N.S.	1	1.00	0.97	1.12	1.15	1.67	1.46	1.15	1.40	2.17
time (sec)	N/A	0.500	0.080	0.339	0.038	0.076	35.799	0.126	0.140	3.552

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	86	92	119	147	121	95	126	188
N.S.	1	1.00	0.83	0.89	1.16	1.43	1.17	0.92	1.22	1.83
time (sec)	N/A	0.314	0.018	0.342	0.040	0.075	1.853	0.117	0.138	3.232

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	277	275	619	909	1360	664	479	0
N.S.	1	1.00	1.01	1.00	2.26	3.32	4.96	2.42	1.75	0.00
time (sec)	N/A	0.676	0.081	0.501	0.066	0.101	28.119	0.130	0.143	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	345	361	1038	1929	2739	772	2067	0
N.S.	1	1.00	0.98	1.03	2.95	5.48	7.78	2.19	5.87	0.00
time (sec)	N/A	0.855	0.111	0.490	0.090	0.150	120.677	0.140	0.152	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	471	510	1553	3119	0	1509	3217	0
N.S.	1	1.00	0.99	1.08	3.28	6.58	0.00	3.18	6.79	0.00
time (sec)	N/A	1.104	0.154	0.516	0.137	0.261	0.000	0.134	0.163	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	494	437	621	782	1028	1913	590	770
N.S.	1	1.00	1.13	1.00	1.42	1.79	2.35	4.37	1.35	1.76
time (sec)	N/A	0.789	0.530	1.025	0.047	0.078	2.191	0.144	0.140	3.883

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	324	282	387	500	641	1269	382	512
N.S.	1	1.00	0.99	0.87	1.19	1.53	1.97	3.89	1.17	1.57
time (sec)	N/A	0.603	0.284	1.013	0.037	0.077	1.731	0.135	0.138	3.628

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	183	161	198	265	321	721	208	349
N.S.	1	1.00	0.86	0.75	0.93	1.24	1.50	3.37	0.97	1.63
time (sec)	N/A	0.440	0.163	0.839	0.037	0.075	1.422	0.126	0.140	3.513

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	71	94	129	143	302	90	134
N.S.	1	1.00	0.68	0.61	0.80	1.10	1.22	2.58	0.77	1.15
time (sec)	N/A	0.313	0.065	0.432	0.037	0.072	0.954	0.124	0.139	3.246

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	234	235	0	579	328	357	292	0
N.S.	1	1.00	1.02	1.03	0.00	2.53	1.43	1.56	1.28	0.00
time (sec)	N/A	0.514	0.354	0.704	0.000	0.117	8.023	0.134	0.143	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	296	245	231	0	1045	0	299	446	0
N.S.	1	1.30	1.07	1.01	0.00	4.58	0.00	1.31	1.96	0.00
time (sec)	N/A	0.866	0.605	0.770	0.000	0.112	0.000	0.133	0.140	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	342	329	313	0	1621	0	542	837	0
N.S.	1	1.17	1.12	1.07	0.00	5.53	0.00	1.85	2.86	0.00
time (sec)	N/A	1.078	0.959	0.920	0.000	0.122	0.000	0.137	0.142	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	449	427	398	0	2354	0	938	1279	0
N.S.	1	1.18	1.12	1.05	0.00	6.19	0.00	2.47	3.37	0.00
time (sec)	N/A	1.086	1.485	0.912	0.000	0.165	0.000	0.152	0.153	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	499	440	621	944	1028	3198	723	0
N.S.	1	1.00	1.14	1.00	1.42	2.16	2.35	7.30	1.65	0.00
time (sec)	N/A	0.826	0.517	1.095	0.039	0.081	2.091	0.169	0.142	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	326	285	387	616	641	2150	481	0
N.S.	1	1.00	1.00	0.87	1.19	1.89	1.97	6.60	1.48	0.00
time (sec)	N/A	0.641	0.335	1.018	0.045	0.077	1.713	0.155	0.141	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	188	162	198	337	321	1246	273	0
N.S.	1	1.00	0.88	0.76	0.93	1.57	1.50	5.82	1.28	0.00
time (sec)	N/A	0.445	0.177	0.907	0.039	0.074	1.409	0.138	0.140	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	83	72	94	172	143	536	125	0
N.S.	1	1.00	0.71	0.62	0.80	1.47	1.22	4.58	1.07	0.00
time (sec)	N/A	0.298	0.069	0.378	0.030	0.071	0.927	0.129	0.140	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	343	333	0	941	427	566	461	0
N.S.	1	1.00	1.23	1.20	0.00	3.38	1.54	2.04	1.66	0.00
time (sec)	N/A	0.549	0.550	0.777	0.000	0.091	8.275	0.140	0.143	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	375	354	349	0	1066	0	556	790	0
N.S.	1	1.28	1.20	1.19	0.00	3.63	0.00	1.89	2.69	0.00
time (sec)	N/A	1.006	0.675	0.808	0.000	0.099	0.000	0.145	0.143	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	434	350	336	0	1675	0	609	838	0
N.S.	1	1.31	1.06	1.02	0.00	5.06	0.00	1.84	2.53	0.00
time (sec)	N/A	1.537	0.982	0.796	0.000	0.117	0.000	0.148	0.142	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	490	461	425	0	2362	0	981	1272	0
N.S.	1	1.22	1.15	1.06	0.00	5.89	0.00	2.45	3.17	0.00
time (sec)	N/A	1.818	1.550	0.884	0.000	0.155	0.000	0.154	0.146	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	504	440	621	1105	1028	4675	856	0
N.S.	1	1.00	1.15	1.00	1.42	2.52	2.35	10.67	1.95	0.00
time (sec)	N/A	0.825	0.541	1.092	0.035	0.087	2.405	0.188	0.143	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	328	285	387	733	641	3175	580	0
N.S.	1	1.00	1.01	0.87	1.19	2.25	1.97	9.74	1.78	0.00
time (sec)	N/A	0.586	0.308	1.010	0.040	0.079	2.111	0.159	0.141	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	186	162	198	407	321	1867	338	0
N.S.	1	1.00	0.87	0.76	0.93	1.90	1.50	8.72	1.58	0.00
time (sec)	N/A	0.452	0.179	0.938	0.034	0.079	1.692	0.138	0.141	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	83	72	94	213	483	817	160	0
N.S.	1	1.00	0.71	0.62	0.80	1.82	4.13	6.98	1.37	0.00
time (sec)	N/A	0.302	0.076	0.394	0.036	0.069	0.447	0.135	0.140	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	478	458	0	1397	592	863	653	0
N.S.	1	1.00	1.45	1.39	0.00	4.25	1.80	2.62	1.98	0.00
time (sec)	N/A	0.635	0.626	0.851	0.000	0.100	9.442	0.148	0.141	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	458	499	476	0	1626	0	914	1188	0
N.S.	1	1.23	1.35	1.28	0.00	4.38	0.00	2.46	3.20	0.00
time (sec)	N/A	1.113	1.017	0.929	0.000	0.110	0.000	0.145	0.144	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	537	511	466	0	1663	0	1002	1332	0
N.S.	1	1.28	1.21	1.11	0.00	3.95	0.00	2.38	3.16	0.00
time (sec)	N/A	1.769	1.308	0.989	0.000	0.123	0.000	0.151	0.143	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	604	488	432	0	2398	0	1057	1262	0
N.S.	1	1.33	1.07	0.95	0.00	5.28	0.00	2.33	2.78	0.00
time (sec)	N/A	2.337	1.649	0.997	0.000	0.144	0.000	0.166	0.146	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	495	440	621	624	1027	854	461	713
N.S.	1	1.00	1.14	1.01	1.42	1.43	2.36	1.96	1.06	1.64
time (sec)	N/A	0.765	0.462	1.092	0.041	0.077	1.775	0.133	0.138	4.177

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	323	285	387	387	639	558	287	514
N.S.	1	1.00	1.00	0.88	1.19	1.19	1.97	1.72	0.89	1.59
time (sec)	N/A	0.570	0.269	1.037	0.038	0.086	1.464	0.126	0.140	4.049

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	184	162	198	197	320	309	147	351
N.S.	1	1.00	0.87	0.76	0.93	0.93	1.51	1.46	0.69	1.66
time (sec)	N/A	0.444	0.141	0.882	0.032	0.070	1.171	0.129	0.142	3.947

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	82	72	128	90	163	128	59	128
N.S.	1	1.00	0.71	0.63	1.11	0.78	1.42	1.11	0.51	1.11
time (sec)	N/A	0.304	0.060	0.408	0.028	0.068	0.631	0.119	0.140	3.660

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	161	157	0	551	275	248	153	0
N.S.	1	1.00	0.86	0.84	0.00	2.93	1.46	1.32	0.81	0.00
time (sec)	N/A	0.470	0.258	0.738	0.000	0.086	4.287	0.125	0.143	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	229	231	226	0	969	0	271	421	0
N.S.	1	1.12	1.13	1.10	0.00	4.73	0.00	1.32	2.05	0.00
time (sec)	N/A	0.771	0.582	0.878	0.000	0.105	0.000	0.129	0.141	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	334	298	301	0	1577	0	529	847	0
N.S.	1	1.20	1.07	1.08	0.00	5.65	0.00	1.90	3.04	0.00
time (sec)	N/A	0.928	0.876	0.896	0.000	0.126	0.000	0.139	0.145	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	431	409	382	0	2338	0	976	1305	0
N.S.	1	1.15	1.09	1.02	0.00	6.23	0.00	2.60	3.48	0.00
time (sec)	N/A	0.964	1.215	0.911	0.000	0.174	0.000	0.144	0.166	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	500	440	629	633	853	1067	463	0
N.S.	1	1.00	1.15	1.01	1.45	1.46	1.97	2.46	1.07	0.00
time (sec)	N/A	0.771	0.475	0.623	0.039	0.083	84.520	0.141	0.140	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	326	289	395	396	534	651	289	0
N.S.	1	1.00	1.01	0.90	1.23	1.23	1.66	2.02	0.90	0.00
time (sec)	N/A	0.593	0.334	0.578	0.039	0.077	30.445	0.141	0.144	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	188	173	206	206	280	323	149	0
N.S.	1	1.00	0.90	0.82	0.98	0.98	1.33	1.54	0.71	0.00
time (sec)	N/A	0.440	0.163	0.467	0.034	0.073	8.121	0.137	0.139	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	82	74	102	100	144	127	61	0
N.S.	1	1.00	0.73	0.65	0.90	0.88	1.27	1.12	0.54	0.00
time (sec)	N/A	0.309	0.062	0.403	0.034	0.070	2.005	0.129	0.143	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	193	191	161	0	834	262	200	172	0
N.S.	1	1.11	1.10	0.93	0.00	4.79	1.51	1.15	0.99	0.00
time (sec)	N/A	0.487	0.375	0.758	0.000	0.095	9.978	0.126	0.145	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	273	259	228	0	1505	0	388	513	0
N.S.	1	1.24	1.18	1.04	0.00	6.84	0.00	1.76	2.33	0.00
time (sec)	N/A	0.896	0.646	0.900	0.000	0.123	0.000	0.138	0.145	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	381	387	379	0	2436	0	617	970	0
N.S.	1	1.23	1.25	1.23	0.00	7.88	0.00	2.00	3.14	0.00
time (sec)	N/A	0.971	1.372	0.998	0.000	0.181	0.000	0.149	0.151	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	485	439	627	645	729	1030	470	0
N.S.	1	1.00	1.12	1.01	1.44	1.49	1.68	2.37	1.08	0.00
time (sec)	N/A	0.800	0.557	0.619	0.040	0.090	80.899	0.156	0.143	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	317	288	393	408	476	622	296	0
N.S.	1	1.00	0.98	0.89	1.22	1.27	1.48	1.93	0.92	0.00
time (sec)	N/A	0.596	0.368	0.550	0.039	0.089	30.303	0.140	0.145	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	177	168	204	218	282	302	156	0
N.S.	1	1.00	0.84	0.80	0.97	1.04	1.34	1.44	0.74	0.00
time (sec)	N/A	0.439	0.180	0.460	0.037	0.077	9.157	0.130	0.144	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	75	72	98	110	425	115	67	0
N.S.	1	1.00	0.66	0.64	0.87	0.97	3.76	1.02	0.59	0.00
time (sec)	N/A	0.299	0.070	0.401	0.031	0.079	0.408	0.124	0.142	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	230	202	0	1227	306	281	302	0
N.S.	1	1.00	1.10	0.96	0.00	5.84	1.46	1.34	1.44	0.00
time (sec)	N/A	0.551	0.471	0.963	0.000	0.117	13.083	0.136	0.142	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	348	354	275	0	2281	0	439	913	0
N.S.	1	1.27	1.30	1.01	0.00	8.36	0.00	1.61	3.34	0.00
time (sec)	N/A	1.011	0.892	1.168	0.000	0.165	0.000	0.137	0.144	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	471	554	459	0	3614	0	767	1692	0
N.S.	1	1.26	1.49	1.23	0.00	9.69	0.00	2.06	4.54	0.00
time (sec)	N/A	1.369	1.547	1.088	0.000	0.283	0.000	0.159	0.167	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	639	409	558	2806	0	1750	0	3946	1334	0
N.S.	1	0.64	0.87	4.39	0.00	2.74	0.00	6.18	2.09	0.00
time (sec)	N/A	0.739	2.166	0.536	0.000	0.335	0.000	0.620	0.274	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	368	419	1998	0	1266	0	1668	974	0
N.S.	1	0.71	0.81	3.87	0.00	2.45	0.00	3.23	1.89	0.00
time (sec)	N/A	0.676	1.924	0.531	0.000	0.234	0.000	0.347	0.219	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	327	329	1334	0	886	0	509	672	0
N.S.	1	0.83	0.84	3.39	0.00	2.25	0.00	1.30	1.71	0.00
time (sec)	N/A	0.673	5.348	0.527	0.000	0.260	0.000	0.171	0.178	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	325	262	1548	0	948	0	506	32	0
N.S.	1	1.02	0.82	4.87	0.00	2.98	0.00	1.59	0.10	0.00
time (sec)	N/A	0.680	0.809	0.569	0.000	4.771	0.000	0.203	200.013	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	275	310	2145	0	1360	0	654	32	0
N.S.	1	1.04	1.17	8.12	0.00	5.15	0.00	2.48	0.12	0.00
time (sec)	N/A	0.600	1.172	0.533	0.000	12.258	0.000	0.244	200.016	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	312	328	2464	0	1774	0	790	32	0
N.S.	1	1.20	1.27	9.51	0.00	6.85	0.00	3.05	0.12	0.00
time (sec)	N/A	0.617	0.648	0.535	0.000	38.838	0.000	0.303	200.013	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	359	458	2501	0	2077	0	951	32	0
N.S.	1	1.12	1.43	7.79	0.00	6.47	0.00	2.96	0.10	0.00
time (sec)	N/A	0.663	0.517	0.533	0.000	95.648	0.000	0.428	200.026	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	398	377	550	0	0	0	1086	32	0
N.S.	1	1.06	1.01	1.47	0.00	0.00	0.00	2.90	0.09	0.00
time (sec)	N/A	0.687	0.381	0.517	0.000	0.000	0.000	0.569	200.013	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	444	613	950	0	0	0	1636	32	0
N.S.	1	0.90	1.24	1.92	0.00	0.00	0.00	3.31	0.06	0.00
time (sec)	N/A	0.747	0.617	0.519	0.000	0.000	0.000	0.678	200.025	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	762	450	728	3758	0	2320	0	6936	1752	0
N.S.	1	0.59	0.96	4.93	0.00	3.04	0.00	9.10	2.30	0.00
time (sec)	N/A	0.758	3.055	0.532	0.000	0.616	0.000	0.946	0.395	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	639	409	555	2806	0	1750	0	2977	1334	0
N.S.	1	0.64	0.87	4.39	0.00	2.74	0.00	4.66	2.09	0.00
time (sec)	N/A	0.762	1.815	0.529	0.000	0.372	0.000	0.496	0.300	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	368	420	1998	0	1274	0	756	974	0
N.S.	1	0.71	0.81	3.87	0.00	2.47	0.00	1.47	1.89	0.00
time (sec)	N/A	0.698	1.181	0.542	0.000	0.604	0.000	0.207	0.234	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	367	747	2428	0	1424	0	785	32	0
N.S.	1	0.84	1.70	5.53	0.00	3.24	0.00	1.79	0.07	0.00
time (sec)	N/A	0.688	15.090	0.503	0.000	6.417	0.000	0.254	200.014	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	411	339	2408	0	1400	0	1006	32	0
N.S.	1	1.03	0.85	6.04	0.00	3.51	0.00	2.52	0.08	0.00
time (sec)	N/A	0.746	1.041	0.531	0.000	16.847	0.000	0.312	200.018	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	358	395	3045	0	1884	0	1190	32	0
N.S.	1	1.06	1.17	9.01	0.00	5.57	0.00	3.52	0.09	0.00
time (sec)	N/A	0.656	0.996	0.518	0.000	49.951	0.000	0.406	200.012	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	347	472	3446	0	2339	0	1325	32	0
N.S.	1	1.16	1.57	11.49	0.00	7.80	0.00	4.42	0.11	0.00
time (sec)	N/A	0.649	1.133	0.530	0.000	119.025	0.000	0.557	200.021	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	390	566	3318	0	0	0	1433	32	0
N.S.	1	1.12	1.63	9.53	0.00	0.00	0.00	4.12	0.09	0.00
time (sec)	N/A	0.689	0.780	0.496	0.000	0.000	0.000	0.866	200.019	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	398	365	591	0	0	0	1415	32	0
N.S.	1	1.06	0.97	1.58	0.00	0.00	0.00	3.77	0.09	0.00
time (sec)	N/A	0.706	0.562	0.510	0.000	0.000	0.000	0.811	200.014	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	446	599	1006	0	0	0	2037	32	0
N.S.	1	0.90	1.21	2.03	0.00	0.00	0.00	4.11	0.06	0.00
time (sec)	N/A	0.738	0.855	0.517	0.000	0.000	0.000	1.141	200.020	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	762	450	726	3758	0	2320	0	4652	1752	0
N.S.	1	0.59	0.95	4.93	0.00	3.04	0.00	6.10	2.30	0.00
time (sec)	N/A	0.739	2.982	0.517	0.000	0.587	0.000	0.709	0.409	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	639	409	558	2806	0	1758	0	1064	1334	0
N.S.	1	0.64	0.87	4.39	0.00	2.75	0.00	1.67	2.09	0.00
time (sec)	N/A	0.702	1.582	0.504	0.000	1.265	0.000	0.240	0.296	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	567	414	519	3460	0	1994	0	1136	32	0
N.S.	1	0.73	0.92	6.10	0.00	3.52	0.00	2.00	0.06	0.00
time (sec)	N/A	0.757	2.268	0.635	0.000	8.278	0.000	0.324	200.012	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	455	626	3664	0	2058	0	1449	32	0
N.S.	1	0.87	1.20	7.02	0.00	3.94	0.00	2.78	0.06	0.00
time (sec)	N/A	0.778	12.537	0.545	0.000	21.136	0.000	0.394	200.017	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	530	459	3421	0	1942	0	1769	32	0
N.S.	1	1.05	0.91	6.75	0.00	3.83	0.00	3.49	0.06	0.00
time (sec)	N/A	0.799	1.471	0.572	0.000	63.062	0.000	0.499	200.014	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	392	553	4206	0	2465	0	1920	32	0
N.S.	1	0.95	1.35	10.23	0.00	6.00	0.00	4.67	0.08	0.00
time (sec)	N/A	0.705	1.295	0.516	0.000	147.453	0.000	0.753	200.018	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	381	637	4496	0	0	0	1988	32	0
N.S.	1	1.11	1.86	13.11	0.00	0.00	0.00	5.80	0.09	0.00
time (sec)	N/A	0.655	1.591	0.547	0.000	0.000	0.000	1.158	200.017	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	421	672	4163	0	0	0	2012	32	0
N.S.	1	1.12	1.78	11.04	0.00	0.00	0.00	5.34	0.08	0.00
time (sec)	N/A	0.666	1.045	0.540	0.000	0.000	0.000	1.120	200.020	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	398	377	591	0	0	0	1739	32	0
N.S.	1	1.06	1.01	1.58	0.00	0.00	0.00	4.64	0.09	0.00
time (sec)	N/A	0.700	0.528	0.520	0.000	0.000	0.000	1.439	200.015	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	446	599	1006	0	0	0	2452	32	0
N.S.	1	0.90	1.21	2.03	0.00	0.00	0.00	4.94	0.06	0.00
time (sec)	N/A	0.741	0.894	0.539	0.000	0.000	0.000	1.932	200.021	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	494	853	1499	0	0	0	3251	32	0
N.S.	1	0.80	1.38	2.42	0.00	0.00	0.00	5.25	0.05	0.00
time (sec)	N/A	0.781	1.652	0.589	0.000	0.000	0.000	2.694	200.021	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	516	367	420	1998	0	1278	0	1672	974	0
N.S.	1	0.71	0.81	3.87	0.00	2.48	0.00	3.24	1.89	0.00
time (sec)	N/A	0.671	1.116	0.542	0.000	0.249	0.000	0.350	0.227	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	326	330	1334	0	888	0	710	672	0
N.S.	1	0.83	0.84	3.39	0.00	2.26	0.00	1.81	1.71	0.00
time (sec)	N/A	0.651	11.052	0.533	0.000	0.162	0.000	0.220	0.187	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	286	226	812	0	590	0	323	428	0
N.S.	1	1.05	0.83	2.99	0.00	2.17	0.00	1.19	1.57	0.00
time (sec)	N/A	0.666	0.619	0.527	0.000	0.201	0.000	0.166	0.166	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	253	237	1370	0	930	0	437	573	0
N.S.	1	1.13	1.06	6.14	0.00	4.17	0.00	1.96	2.57	0.00
time (sec)	N/A	0.577	0.625	0.606	0.000	3.124	0.000	0.192	0.173	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	252	227	1707	0	1280	0	550	32	0
N.S.	1	1.09	0.98	7.36	0.00	5.52	0.00	2.37	0.14	0.00
time (sec)	N/A	0.551	0.524	0.574	0.000	8.796	0.000	0.216	200.023	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	330	313	1729	0	1584	0	746	32	0
N.S.	1	1.11	1.06	5.84	0.00	5.35	0.00	2.52	0.11	0.00
time (sec)	N/A	0.638	0.465	0.602	0.000	29.801	0.000	1.500	200.018	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	397	377	550	0	712	0	1086	32	0
N.S.	1	1.06	1.01	1.47	0.00	1.90	0.00	2.90	0.09	0.00
time (sec)	N/A	0.680	0.328	0.587	0.000	73.914	0.000	0.333	200.013	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	415	543	3460	0	2028	0	1146	32	0
N.S.	1	0.74	0.97	6.16	0.00	3.61	0.00	2.04	0.06	0.00
time (sec)	N/A	0.757	1.881	0.536	0.000	6.759	0.000	0.813	200.014	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	374	249	2428	0	1454	0	793	32	0
N.S.	1	0.85	0.56	5.51	0.00	3.30	0.00	1.80	0.07	0.00
time (sec)	N/A	0.694	10.270	0.513	0.000	5.515	0.000	0.545	200.012	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	332	279	1548	0	976	0	507	32	0
N.S.	1	1.04	0.87	4.84	0.00	3.05	0.00	1.58	0.10	0.00
time (sec)	N/A	0.678	0.840	0.519	0.000	4.440	0.000	0.331	200.015	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	249	228	1370	0	940	0	295	321	0
N.S.	1	1.12	1.02	6.14	0.00	4.22	0.00	1.32	1.44	0.00
time (sec)	N/A	0.556	0.688	0.600	0.000	2.906	0.000	0.855	0.159	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	239	227	1745	0	1332	0	412	547	0
N.S.	1	1.20	1.14	8.72	0.00	6.66	0.00	2.06	2.74	0.00
time (sec)	N/A	0.620	0.495	0.610	0.000	4.169	0.000	0.278	0.486	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	330	314	1893	0	1732	0	817	32	0
N.S.	1	1.20	1.14	6.86	0.00	6.28	0.00	2.96	0.12	0.00
time (sec)	N/A	0.684	0.519	0.643	0.000	14.665	0.000	0.372	200.017	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	396	377	550	0	748	0	2300	32	0
N.S.	1	1.12	1.06	1.55	0.00	2.11	0.00	6.48	0.09	0.00
time (sec)	N/A	0.753	0.360	0.633	0.000	33.649	0.000	0.528	200.017	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	457	251	3664	0	2106	0	2808	32	0
N.S.	1	0.88	0.48	7.05	0.00	4.05	0.00	5.40	0.06	0.00
time (sec)	N/A	0.834	10.321	0.566	0.000	18.857	0.000	1.043	200.013	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	412	355	2408	0	1446	0	2075	32	0
N.S.	1	1.05	0.91	6.14	0.00	3.69	0.00	5.29	0.08	0.00
time (sec)	N/A	0.754	1.001	0.559	0.000	15.429	0.000	0.705	200.015	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	276	291	2145	0	1402	0	1305	32	0
N.S.	1	1.05	1.11	8.16	0.00	5.33	0.00	4.96	0.12	0.00
time (sec)	N/A	0.576	1.109	0.537	0.000	11.666	0.000	0.421	200.015	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	252	235	1707	0	1286	0	736	32	0
N.S.	1	1.11	1.03	7.49	0.00	5.64	0.00	3.23	0.14	0.00
time (sec)	N/A	0.582	0.498	0.630	0.000	8.575	0.000	0.291	200.017	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	336	309	1893	0	1710	0	1208	1004	0
N.S.	1	1.37	1.26	7.70	0.00	6.95	0.00	4.91	4.08	0.00
time (sec)	N/A	0.710	0.486	0.761	0.000	13.410	0.000	0.598	0.189	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	386	361	548	0	734	0	2066	32	0
N.S.	1	1.19	1.11	1.69	0.00	2.27	0.00	6.38	0.10	0.00
time (sec)	N/A	0.758	0.484	0.661	0.000	40.181	0.000	1.056	200.021	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	440	599	950	0	0	0	3566	32	0
N.S.	1	1.01	1.37	2.17	0.00	0.00	0.00	8.16	0.07	0.00
time (sec)	N/A	0.783	0.827	0.666	0.000	0.000	0.000	1.483	200.013	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	422	440	3640	0	2598	0	1300	37	0
N.S.	1	1.16	1.21	10.03	0.00	7.16	0.00	3.58	0.10	0.00
time (sec)	N/A	1.465	1.068	0.665	0.000	71.296	0.000	0.505	200.020	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	501	437	621	782	1028	1913	591	0
N.S.	1	1.00	1.14	1.00	1.42	1.79	2.35	4.37	1.35	0.00
time (sec)	N/A	0.786	0.516	1.606	0.042	0.082	1.946	0.151	0.147	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	325	283	387	500	641	1269	383	0
N.S.	1	1.00	1.00	0.87	1.19	1.53	1.97	3.89	1.17	0.00
time (sec)	N/A	0.593	0.304	1.500	0.038	0.082	1.559	0.135	0.144	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	183	161	198	265	321	721	209	0
N.S.	1	1.00	0.86	0.75	0.93	1.24	1.50	3.37	0.98	0.00
time (sec)	N/A	0.432	0.156	1.280	0.037	0.076	1.283	0.128	0.142	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	82	72	94	129	143	303	91	0
N.S.	1	1.00	0.70	0.62	0.80	1.10	1.22	2.59	0.78	0.00
time (sec)	N/A	0.306	0.065	0.391	0.030	0.073	0.955	0.135	0.141	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	414	340	0	462	0	685	1132	0
N.S.	1	1.00	1.12	0.92	0.00	1.25	0.00	1.85	3.05	0.00
time (sec)	N/A	0.710	0.634	1.244	0.000	0.088	0.000	0.177	0.165	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	487	476	338	0	2767	0	622	2905	0
N.S.	1	1.17	1.14	0.81	0.00	6.62	0.00	1.49	6.95	0.00
time (sec)	N/A	0.986	1.231	1.087	0.000	0.444	0.000	0.198	0.218	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	530	439	610	406	0	4061	0	891	3359	0
N.S.	1	0.83	1.15	0.77	0.00	7.66	0.00	1.68	6.34	0.00
time (sec)	N/A	0.853	2.092	1.256	0.000	1.020	0.000	0.211	0.260	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	495	440	621	624	1027	854	462	0
N.S.	1	1.00	1.14	1.01	1.42	1.43	2.36	1.96	1.06	0.00
time (sec)	N/A	0.771	0.501	1.619	0.042	0.076	1.735	0.130	0.146	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	324	285	387	387	639	558	288	0
N.S.	1	1.00	1.00	0.88	1.19	1.19	1.97	1.72	0.89	0.00
time (sec)	N/A	0.607	0.290	1.480	0.037	0.074	1.416	0.132	0.144	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	185	162	198	197	320	310	148	0
N.S.	1	1.00	0.87	0.76	0.93	0.93	1.51	1.46	0.70	0.00
time (sec)	N/A	0.443	0.149	1.382	0.037	0.070	1.157	0.125	0.140	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	82	72	127	90	165	127	60	0
N.S.	1	1.00	0.71	0.63	1.10	0.78	1.43	1.10	0.52	0.00
time (sec)	N/A	0.307	0.060	0.402	0.042	0.071	0.654	0.121	0.139	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	340	264	0	1656	0	476	706	0
N.S.	1	1.00	1.03	0.80	0.00	5.02	0.00	1.44	2.14	0.00
time (sec)	N/A	0.640	0.524	0.802	0.000	0.118	0.000	0.183	0.160	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	422	463	317	0	2621	0	552	1644	0
N.S.	1	1.07	1.17	0.80	0.00	6.64	0.00	1.40	4.16	0.00
time (sec)	N/A	0.895	1.221	0.997	0.000	1.105	0.000	0.198	0.209	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	409	565	381	0	3981	0	911	3364	0
N.S.	1	0.80	1.10	0.74	0.00	7.76	0.00	1.78	6.56	0.00
time (sec)	N/A	0.838	1.989	1.230	0.000	2.213	0.000	0.217	0.261	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	497	440	629	633	855	1067	462	0
N.S.	1	1.00	1.14	1.01	1.44	1.45	1.96	2.45	1.06	0.00
time (sec)	N/A	0.747	0.592	0.598	0.039	0.086	19.541	0.147	0.150	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	325	285	395	397	536	651	288	0
N.S.	1	1.00	1.00	0.88	1.22	1.23	1.65	2.01	0.89	0.00
time (sec)	N/A	0.603	0.348	0.568	0.040	0.081	9.083	0.142	0.141	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	186	174	206	206	282	323	148	0
N.S.	1	1.00	0.88	0.82	0.97	0.97	1.33	1.52	0.70	0.00
time (sec)	N/A	0.435	0.193	0.462	0.038	0.078	3.858	0.141	0.141	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	82	74	102	100	146	127	60	0
N.S.	1	1.00	0.71	0.64	0.89	0.87	1.27	1.10	0.52	0.00
time (sec)	N/A	0.284	0.069	0.429	0.030	0.072	1.564	0.125	0.137	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	378	372	295	0	1671	0	461	420	0
N.S.	1	1.19	1.17	0.93	0.00	5.27	0.00	1.45	1.32	0.00
time (sec)	N/A	0.708	0.797	0.750	0.000	0.126	0.000	0.185	0.167	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	498	496	355	0	3152	0	813	1910	0
N.S.	1	1.20	1.20	0.86	0.00	7.61	0.00	1.96	4.61	0.00
time (sec)	N/A	1.009	1.727	1.090	0.000	0.583	0.000	0.222	0.230	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	541	495	656	469	0	5068	0	1154	3902	0
N.S.	1	0.91	1.21	0.87	0.00	9.37	0.00	2.13	7.21	0.00
time (sec)	N/A	0.847	4.074	1.464	0.000	1.026	0.000	0.238	0.317	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	455	418	3932	1802	3889	65321	9032	3687	0
N.S.	1	1.00	0.92	8.64	3.96	8.55	143.56	19.85	8.10	0.00
time (sec)	N/A	0.776	2.016	0.613	0.104	0.139	11.291	0.197	0.153	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	308	2175	1118	2156	32849	4972	2082	0
N.S.	1	1.00	0.91	6.43	3.31	6.38	97.19	14.71	6.16	0.00
time (sec)	N/A	0.628	0.776	0.520	0.087	0.117	6.973	0.161	0.149	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	199	964	596	964	13522	2224	973	0
N.S.	1	1.00	0.88	4.27	2.64	4.27	59.83	9.84	4.31	0.00
time (sec)	N/A	0.476	0.438	0.421	0.067	0.098	2.661	0.145	0.141	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	108	308	234	394	3798	728	345	0
N.S.	1	1.00	0.86	2.44	1.86	3.13	30.14	5.78	2.74	0.00
time (sec)	N/A	0.318	0.137	0.431	0.042	0.085	1.069	0.128	0.139	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	181	0	0	0	0	0	1114	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	5.49	0.00
time (sec)	N/A	0.432	0.361	0.000	0.000	0.000	0.000	0.000	0.143	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	215	252	180	0	0	0	0	0	0	0
N.S.	1	1.17	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.678	0.687	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	360	188	0	0	0	0	0	0	0
N.S.	1	1.33	0.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	1.311	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	506	220	0	0	0	0	0	0	0
N.S.	1	1.52	0.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.919	1.564	0.000	0.000	0.000	0.000	0.000	3.949	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	483	469	233	0	0	0	0	0	0	0
N.S.	1	0.97	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.965	1.374	0.000	0.000	0.000	0.000	0.000	2.544	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	484	470	232	0	0	0	0	0	0	0
N.S.	1	0.97	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.911	0.800	0.000	0.000	0.000	0.000	0.000	0.956	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	482	468	231	0	0	0	0	0	0	0
N.S.	1	0.97	0.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.881	0.546	0.000	0.000	0.000	0.000	0.000	0.658	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	413	421	232	0	0	0	0	0	0	0
N.S.	1	1.02	0.56	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.895	1.423	0.000	0.000	0.000	0.000	0.000	1.023	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	419	454	231	0	0	0	0	0	0	0
N.S.	1	1.08	0.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.885	2.480	0.000	0.000	0.000	0.000	0.000	1.218	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	404	575	231	0	0	0	0	0	0	0
N.S.	1	1.42	0.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.035	2.964	0.000	0.000	0.000	0.000	0.000	107.135	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	411	580	233	0	0	0	0	0	32	0
N.S.	1	1.41	0.57	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.008	3.266	0.000	0.000	0.000	0.000	0.000	200.020	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	415	589	233	0	0	0	0	0	32	0
N.S.	1	1.42	0.56	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.004	3.344	0.000	0.000	0.000	0.000	0.000	200.025	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	137	117	0	0	0	0	0	0	0
N.S.	1	1.05	0.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.265	0.103	0.000	0.000	0.000	0.000	0.000	0.213	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	256	257	187	0	0	0	0	0	27	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.444	0.195	0.000	0.000	0.000	0.000	0.000	200.021	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	505	591	254	0	0	0	0	0	32	0
N.S.	1	1.17	0.50	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.114	0.254	0.000	0.000	0.000	0.000	0.000	200.018	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [110] had the largest ratio of [.3235290000000000011]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	28	0.071
2	A	2	2	1.00	28	0.071
3	A	2	2	1.00	26	0.077
4	A	2	2	1.00	21	0.095
5	A	2	2	1.00	28	0.071
6	A	2	2	1.00	28	0.071
7	A	2	2	1.00	28	0.071
8	A	2	2	1.00	28	0.071
9	A	2	2	1.00	28	0.071
10	A	2	2	1.00	28	0.071
11	A	2	2	1.00	28	0.071
12	A	2	2	1.00	28	0.071
13	A	2	2	1.00	30	0.067
14	A	2	2	1.00	30	0.067
15	A	2	2	1.00	28	0.071
16	A	2	2	1.00	23	0.087
17	A	2	2	1.00	30	0.067
18	A	2	2	1.00	30	0.067
19	A	2	2	1.00	30	0.067
20	A	2	2	1.00	30	0.067
21	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	30	0.067
23	A	2	2	1.00	30	0.067
24	A	2	2	1.00	30	0.067
25	A	2	2	1.00	30	0.067
26	A	2	2	1.00	30	0.067
27	A	2	2	1.00	30	0.067
28	A	2	2	1.00	28	0.071
29	A	2	2	1.00	23	0.087
30	A	2	2	1.00	30	0.067
31	A	2	2	1.00	30	0.067
32	A	2	2	1.00	30	0.067
33	A	2	2	1.00	30	0.067
34	A	2	2	1.00	30	0.067
35	A	2	2	1.00	30	0.067
36	A	2	2	1.00	28	0.071
37	A	2	2	1.00	23	0.087
38	A	2	2	1.00	30	0.067
39	A	2	2	1.00	30	0.067
40	A	2	2	1.00	30	0.067
41	A	2	2	1.00	30	0.067
42	A	2	2	1.00	30	0.067
43	A	2	2	1.00	28	0.071
44	A	2	2	1.00	23	0.087
45	A	2	2	1.00	30	0.067
46	A	2	2	1.00	30	0.067
47	A	2	2	1.00	30	0.067
48	A	2	2	1.00	30	0.067
49	A	2	2	1.00	30	0.067
50	A	2	2	1.00	28	0.071
51	A	2	2	1.00	23	0.087
52	A	2	2	1.00	30	0.067
53	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	30	0.067
55	A	2	2	1.00	32	0.062
56	A	2	2	1.00	32	0.062
57	A	2	2	1.00	30	0.067
58	A	2	2	1.00	25	0.080
59	A	2	2	1.00	32	0.062
60	A	6	5	1.30	32	0.156
61	A	8	7	1.17	32	0.219
62	A	8	7	1.18	32	0.219
63	A	2	2	1.00	32	0.062
64	A	2	2	1.00	32	0.062
65	A	2	2	1.00	30	0.067
66	A	2	2	1.00	25	0.080
67	A	2	2	1.00	32	0.062
68	A	6	5	1.28	32	0.156
69	A	9	8	1.31	32	0.250
70	A	10	9	1.22	32	0.281
71	A	2	2	1.00	32	0.062
72	A	2	2	1.00	32	0.062
73	A	2	2	1.00	30	0.067
74	A	2	2	1.00	25	0.080
75	A	2	2	1.00	32	0.062
76	A	6	5	1.23	32	0.156
77	A	8	7	1.28	32	0.219
78	A	8	7	1.33	32	0.219
79	A	2	2	1.00	32	0.062
80	A	2	2	1.00	32	0.062
81	A	2	2	1.00	30	0.067
82	A	2	2	1.00	25	0.080
83	A	2	2	1.00	32	0.062
84	A	6	5	1.12	32	0.156
85	A	10	9	1.20	32	0.281

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	8	7	1.15	32	0.219
87	A	2	2	1.00	32	0.062
88	A	2	2	1.00	32	0.062
89	A	2	2	1.00	30	0.067
90	A	2	2	1.00	25	0.080
91	A	2	2	1.11	32	0.062
92	A	6	5	1.24	32	0.156
93	A	9	8	1.23	32	0.250
94	A	2	2	1.00	32	0.062
95	A	2	2	1.00	32	0.062
96	A	2	2	1.00	30	0.067
97	A	2	2	1.00	25	0.080
98	A	2	2	1.00	32	0.062
99	A	6	5	1.27	32	0.156
100	A	9	8	1.26	32	0.250
101	A	11	10	0.64	34	0.294
102	A	10	9	0.71	34	0.265
103	A	9	8	0.83	34	0.235
104	A	9	8	1.02	34	0.235
105	A	9	8	1.04	34	0.235
106	A	9	8	1.20	34	0.235
107	A	9	8	1.12	34	0.235
108	A	6	6	1.06	34	0.176
109	A	7	7	0.90	34	0.206
110	A	12	11	0.59	34	0.324
111	A	11	10	0.64	34	0.294
112	A	10	9	0.71	34	0.265
113	A	10	9	0.84	34	0.265
114	A	10	9	1.03	34	0.265
115	A	10	9	1.06	34	0.265
116	A	10	9	1.16	34	0.265
117	A	10	9	1.12	34	0.265

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	6	1.06	34	0.176
119	A	7	7	0.90	34	0.206
120	A	12	11	0.59	34	0.324
121	A	11	10	0.64	34	0.294
122	A	11	10	0.73	34	0.294
123	A	11	10	0.87	34	0.294
124	A	11	10	1.05	34	0.294
125	A	11	10	0.95	34	0.294
126	A	11	10	1.11	34	0.294
127	A	11	10	1.12	34	0.294
128	A	6	6	1.06	34	0.176
129	A	7	7	0.90	34	0.206
130	A	8	8	0.80	34	0.235
131	A	10	9	0.71	34	0.265
132	A	9	8	0.83	34	0.235
133	A	8	7	1.05	34	0.206
134	A	8	7	1.13	34	0.206
135	A	8	7	1.09	34	0.206
136	A	8	7	1.11	34	0.206
137	A	6	6	1.06	34	0.176
138	A	11	10	0.74	34	0.294
139	A	10	9	0.85	34	0.265
140	A	9	8	1.04	34	0.235
141	A	8	7	1.12	34	0.206
142	A	8	7	1.20	34	0.206
143	A	8	7	1.20	34	0.206
144	A	6	6	1.12	34	0.176
145	A	11	10	0.88	34	0.294
146	A	10	9	1.05	34	0.265
147	A	9	8	1.05	34	0.235
148	A	8	7	1.11	34	0.206
149	A	8	7	1.37	34	0.206

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	6	6	1.19	34	0.176
151	A	7	7	1.01	34	0.206
152	A	11	10	1.16	39	0.256
153	A	2	2	1.00	32	0.062
154	A	2	2	1.00	32	0.062
155	A	2	2	1.00	30	0.067
156	A	2	2	1.00	25	0.080
157	A	2	2	1.00	32	0.062
158	A	4	4	1.17	32	0.125
159	A	11	10	0.83	32	0.312
160	A	2	2	1.00	32	0.062
161	A	2	2	1.00	32	0.062
162	A	2	2	1.00	30	0.067
163	A	2	2	1.00	25	0.080
164	A	2	2	1.00	32	0.062
165	A	4	4	1.07	32	0.125
166	A	10	9	0.80	32	0.281
167	A	2	2	1.00	32	0.062
168	A	2	2	1.00	32	0.062
169	A	2	2	1.00	30	0.067
170	A	2	2	1.00	25	0.080
171	A	2	2	1.19	32	0.062
172	A	4	4	1.20	32	0.125
173	A	10	9	0.91	32	0.281
174	A	2	2	1.00	30	0.067
175	A	2	2	1.00	30	0.067
176	A	2	2	1.00	28	0.071
177	A	2	2	1.00	23	0.087
178	A	2	2	1.00	30	0.067
179	A	3	3	1.17	30	0.100
180	A	7	7	1.33	30	0.233
181	A	6	6	1.52	30	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	7	7	0.97	32	0.219
183	A	7	7	0.97	32	0.219
184	A	7	7	0.97	32	0.219
185	A	7	7	1.02	32	0.219
186	A	7	7	1.08	32	0.219
187	A	7	7	1.42	32	0.219
188	A	7	7	1.41	32	0.219
189	A	7	7	1.42	32	0.219
190	A	3	3	1.05	20	0.150
191	A	5	5	1.00	25	0.200
192	A	6	6	1.17	30	0.200

CHAPTER 3

Listing of integrals

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3.29 $\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx$ 310
3.30 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx$ 316
3.31 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx$ 322
3.32 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx$ 328
3.33 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4(c+dx)}\,dx$ 336
3.34 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$ 345
3.35 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$ 355
3.36 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$ 363
3.37 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2}\,dx$ 370
3.38 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2}\,dx$ 376
3.39 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx$ 382
3.40 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx$ 390
3.41 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$ 399
3.42 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$ 408
3.43 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$ 416
3.44 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3}\,dx$ 423
3.45 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx$ 429
3.46 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx$ 436
3.47 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^3}\,dx$ 445
3.48 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$ 454
3.49 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$ 463
3.50 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$ 471
3.51 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx$ 478
3.52 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx$ 484
3.53 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^4}\,dx$ 493
3.54 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx$ 502
3.55 $\int (a+bx)^3\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ 511

3.56 $\int (a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ 522
3.57 $\int (a+bx)\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ 531
3.58 $\int \sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ 539
3.59 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$ 545
3.60 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$ 553
3.61 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$ 562
3.62 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$ 572
3.63 $\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 582
3.64 $\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 592
3.65 $\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 600
3.66 $\int (c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 608
3.67 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$ 615
3.68 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$ 623
3.69 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$ 632
3.70 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$ 642
3.71 $\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 653
3.72 $\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 663
3.73 $\int (a+bx)(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 672
3.74 $\int (c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ 680
3.75 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$ 687
3.76 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$ 696
3.77 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$ 706
3.78 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$ 716
3.79 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$ 727
3.80 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$ 738
3.81 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$ 747
3.82 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$ 755
3.83 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx$ 761
3.84 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx$ 768
3.85 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx$ 777
3.86 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4\sqrt{c+dx}}\,dx$ 787
3.87 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$ 797
3.88 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$ 806

3.89 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$ . . . 814
3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx$ . . . 821
3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx$ . . . 827
3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx$ . . . 834
3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{3/2}}\,dx$ . . . 843
3.94 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$ . . . 854
3.95 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$ . . . 864
3.96 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$ . . . 872
3.97 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx$ . . . 879
3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{5/2}}\,dx$ . . . 885
3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx$ . . . 893
3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{5/2}}\,dx$ . . . 902
3.101 $\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ . . . 912
3.102 $\int \sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ . . . 924
3.103 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$ . . . 936
3.104 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$ . . . 947
3.105 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$ . . . 957
3.106 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx$ . . . 968
3.107 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx$ . . . 979
3.108 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx$ . . . 990
3.109 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx$ . . . 999
3.110 $\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$ . . . 1009
3.111 $\int (a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1024
3.112 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$ . . . 1036
3.113 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$ . . . 1047
3.114 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$ . . . 1058
3.115 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx$ . . . 1069
3.116 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx$ . . . 1080
3.117 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx$ . . . 1092
3.118 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx$ . . . 1102
3.119 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{15/2}}\,dx$ . . . 1111

3.120 $\int (a+bx)^{5/2}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1121
3.121 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$ . . . 1135
3.122 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$ . . . 1149
3.123 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$ . . . 1161
3.124 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx$ . . . 1176
3.125 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx$ . . . 1189
3.126 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx$ . . . 1201
3.127 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx$ . . . 1214
3.128 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{15/2}}\,dx$ . . . 1225
3.129 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{17/2}}\,dx$ . . . 1234
3.130 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{19/2}}\,dx$ . . . 1244
3.131 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$ . . . 1255
3.132 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$ . . . 1266
3.133 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$ . . . 1277
3.134 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx$ . . . 1286
3.135 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx$ . . . 1296
3.136 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx$ . . . 1305
3.137 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{9/2}}\,dx$ . . . 1315
3.138 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx$ . . . 1324
3.139 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx$ . . . 1337
3.140 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx$ . . . 1349
3.141 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx$ . . . 1359
3.142 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx$ . . . 1369
3.143 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$ . . . 1379
3.144 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx$ . . . 1389
3.145 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx$ . . . 1398
3.146 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx$ . . . 1412
3.147 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx$ . . . 1423
3.148 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx$ . . . 1434
3.149 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx$ . . . 1444

3.150 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx$ . . . 1454
3.151 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx$ . . . 1463
3.152 $\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$ . . . 1473
3.153 $\int (a+bx)^3\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1484
3.154 $\int (a+bx)^2\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1494
3.155 $\int (a+bx)\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1502
3.156 $\int \sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1510
3.157 $\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$ . . . 1516
3.158 $\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$ . . . 1525
3.159 $\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$ . . . 1536
3.160 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx$ . . . 1549
3.161 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx$ . . . 1559
3.162 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx$ . . . 1567
3.163 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx$ . . . 1574
3.164 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx$ . . . 1580
3.165 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{2/3}}\,dx$ . . . 1589
3.166 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{2/3}}\,dx$ . . . 1600
3.167 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx$ . . . 1612
3.168 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx$ . . . 1622
3.169 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx$ . . . 1630
3.170 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx$ . . . 1637
3.171 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx$ . . . 1643
3.172 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx$ . . . 1651
3.173 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{4/3}}\,dx$ . . . 1660
3.174 $\int (a+bx)^3(c+dx)^n\,(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1674
3.175 $\int (a+bx)^2(c+dx)^n\,(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1684
3.176 $\int (a+bx)(c+dx)^n\,(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1694
3.177 $\int (c+dx)^n\,(A+Bx+Cx^2+Dx^3)\,dx$ . . . 1704
3.178 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$ . . . 1711
3.179 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$ . . . 1717
3.180 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$ . . . 1724
3.181 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$ . . . 1732

3.182 $\int (a+bx)^{3/2}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$ 1740
3.183 $\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$ 1749
3.184 $\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}} dx$ 1758
3.185 $\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}} dx$ 1767
3.186 $\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}} dx$ 1776
3.187 $\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{7/2}} dx$ 1784
3.188 $\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{9/2}} dx$ 1792
3.189 $\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{11/2}} dx$ 1800
3.190 $\int (a+bx)^m(A+Bx)(c+dx)^n dx$ 1808
3.191 $\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2\right) dx$ 1814
3.192 $\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$ 1821

3.1 $\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result . . . 97
Mathematica [A] (verified) . . . 98
Rubi [A] (verified) . . . 98
Maple [A] (verified) . . . 100
Fricas [A] (verification not implemented) . . . 100
Sympy [A] (verification not implemented) . . . 101
Maxima [A] (verification not implemented) . . . 102
Giac [A] (verification not implemented) . . . 103
Mupad [B] (verification not implemented) . . . 104
Reduce [B] (verification not implemented) . . . 105

Optimal result

Integrand size = 28, antiderivative size = 195

$$\begin{aligned}\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)(a+bx)^4}{4b^5}\\ &+\frac{\left(b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)\right)(a+bx)^5}{5b^5}\\ &+\frac{\left(b^2(cC+Bd)+6a^2dD-3ab(Cd+cD)\right)(a+bx)^6}{6b^5}\\ &+\frac{(bCd+bcD-4adD)(a+bx)^7}{7b^5}+\frac{dD(a+bx)^8}{8b^5}\end{aligned}$$

output
```
1/4*(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x+a)^4/b^5+1/5*(b^3*(A*d+B
*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))*(b*x+a)^5/b^5+1/6*(b^2*
(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))*(b*x+a)^6/b^5+1/7*(C*b*d-4*D*a*d+D*b*
c)*(b*x+a)^7/b^5+1/8*d*D*(b*x+a)^8/b^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.26

$$\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= a^3Acx+\frac{1}{2}a^2(3Abc+aBc+aAd)x^2+\frac{1}{3}a(3Ab(bc+ad)+a(3bBc+acC+aBd))x^3$$
$$+\frac{1}{4}\left(Ab^2(bc+3ad)+a\left(3b^2Bc+3ab(cC+Bd)+a^2(Cd+cD)\right)\right)x^4$$
$$+\frac{1}{5}\left(b^3(Bc+Ad)+3ab^2(cC+Bd)+a^3dD+3a^2b(Cd+cD)\right)x^5$$
$$+\frac{1}{6}b\left(b^2(cC+Bd)+3a^2dD+3ab(Cd+cD)\right)x^6$$
$$+\frac{1}{7}b^2(bCd+bcD+3adD)x^7+\frac{1}{8}b^3dDx^8$$

input
```
Integrate[(a + b*x)^3*(c + d*x)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*c*x + (a^2*(3*A*b*c + a*B*c + a*A*d)*x^2)/2 + (a*(3*A*b*(b*c + a*d)
+ a*(3*b*B*c + a*c*C + a*B*d))*x^3)/3 + ((A*b^2*(b*c + 3*a*d) + a*(3*b^2*B
*c + 3*a*b*(c*C + B*d) + a^2*(C*d + c*D)))*x^4)/4 + ((b^3*(B*c + A*d) + 3*
a*b^2*(c*C + B*d) + a^3*d*D + 3*a^2*b*(C*d + c*D))*x^5)/5 + (b*(b^2*(c*C +
 B*d) + 3*a^2*d*D + 3*a*b*(C*d + c*D))*x^6)/6 + (b^2*(b*C*d + b*c*D + 3*a*
d*D)*x^7)/7 + (b^3*d*D*x^8)/8
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{(a+bx)^3(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4} + \frac{(a+bx)^5\left(6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)\right)}{b^4} + \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^4(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4b^5} + \frac{(a+bx)^6\left(6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)\right)}{6b^5} + \frac{(a+bx)^5\left(-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)\right)}{5b^5} + \frac{(a+bx)^7(-4adD+bcD+bCd)}{7b^5} + \frac{dD(a+bx)^8}{8b^5}$$

input
```
Int[(a + b*x)^3*(c + d*x)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(a + b*x)^4)/(4*b^5) + ((
b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D))*(
a + b*x)^5)/(5*b^5) + ((b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D))*(
a + b*x)^6)/(6*b^5) + ((b*C*d + b*c*D - 4*a*d*D)*(a + b*x)^7)/(7*b^5) + (d
*D*(a + b*x)^8)/(8*b^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.39

method	result
norman	$\frac{b^3dDx^8}{8}+\left(\frac{1}{7}b^3dC+\frac{3}{7}Da\,b^2d+\frac{1}{7}Db^3c\right)x^7+\left(\frac{1}{6}b^3Bd+\frac{1}{2}Ca\,b^2d+\frac{1}{6}C\,b^3c+\frac{1}{2}Da^2bd+\frac{1}{2}Da$
default	$\frac{b^3dDx^8}{8}+\frac{\left(\left(3a\,b^2d+b^3c\right)D+b^3dC\right)x^7}{7}+\frac{\left(\left(3a^2bd+3a\,b^2c\right)D+\left(3a\,b^2d+b^3c\right)C+b^3Bd\right)x^6}{6}+\frac{\left(\left(a^3d+3a^2bc\right)D+\left(3a^2bd+3\right.\right.}{}$
gosper	$\frac{1}{7}x^7b^3dC+x^3B\,a^2bc+\frac{3}{2}x^2A\,a^2bc+\frac{3}{4}x^4B\,a^2bd+\frac{3}{4}x^4Ba\,b^2c+\frac{3}{4}x^4C\,a^2bc+x^3A\,a^2bd+x^3$
parallelrisch	$\frac{1}{7}x^7b^3dC+x^3B\,a^2bc+\frac{3}{2}x^2A\,a^2bc+\frac{3}{4}x^4B\,a^2bd+\frac{3}{4}x^4Ba\,b^2c+\frac{3}{4}x^4C\,a^2bc+x^3A\,a^2bd+x^3$
orering	$\frac{x\left(105b^3dDx^7+120C\,b^3d\,x^6+360Da\,b^2d\,x^6+120Db^3c\,x^6+140B\,b^3d\,x^5+420Ca\,b^2d\,x^5+140C\,b^3c\,x^5+420Da^2bd\,x^5+420Da\,b\right.}{}$

input
```
int((b*x+a)^3*(d*x+c)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^3*d*D*x^8+(1/7*b^3*d*C+3/7*D*a*b^2*d+1/7*D*b^3*c)*x^7+(1/6*b^3*B*d+1
/2*C*a*b^2*d+1/6*C*b^3*c+1/2*D*a^2*b*d+1/2*D*a*b^2*c)*x^6+(1/5*b^3*d*A+3/5
*B*a*b^2*d+1/5*B*b^3*c+3/5*C*a^2*b*d+3/5*a*b^2*c*C+1/5*a^3*d*D+3/5*a^2*b*c
*D)*x^5+(3/4*A*a*b^2*d+1/4*A*b^3*c+3/4*B*a^2*b*d+3/4*B*a*b^2*c+1/4*C*a^3*d
+3/4*C*a^2*b*c+1/4*a^3*c*D)*x^4+(A*a^2*b*d+A*a*b^2*c+1/3*B*a^3*d+B*a^2*b*c
+1/3*a^3*c*C)*x^3+(1/2*A*a^3*d+3/2*A*a^2*b*c+1/2*a^3*c*B)*x^2+a^3*c*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

$$\begin{aligned}
&\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{8}Db^3dx^8+\frac{1}{7}\left(Db^3c+\left(3\,Dab^2+Cb^3\right)d\right)x^7 \\
&+\frac{1}{6}\left(\left(3\,Dab^2+Cb^3\right)c+\left(3\,Da^2b+3\,Cab^2+Bb^3\right)d\right)x^6+Aa^3cx \\
&+\frac{1}{5}\left(\left(3\,Da^2b+3\,Cab^2+Bb^3\right)c+\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)d\right)x^5 \\
&+\frac{1}{4}\left(\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)c+\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)d\right)x^4 \\
&+\frac{1}{3}\left(\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)c+\left(Ba^3+3\,Aa^2b\right)d\right)x^3 \\
&+\frac{1}{2}\left(Aa^3d+\left(Ba^3+3\,Aa^2b\right)c\right)x^2
\end{aligned}$$

input
```
integrate((b*x+a)^3*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b^3*d*x^8 + 1/7*(D*b^3*c + (3*D*a*b^2 + C*b^3)*d)*x^7 + 1/6*((3*D*a*
b^2 + C*b^3)*c + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d)*x^6 + A*a^3*c*x + 1/5*
((3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c + (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^
3)*d)*x^5 + 1/4*((D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c + (C*a^3 + 3*B*
a^2*b + 3*A*a*b^2)*d)*x^4 + 1/3*((C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c + (B*a^
3 + 3*A*a^2*b)*d)*x^3 + 1/2*(A*a^3*d + (B*a^3 + 3*A*a^2*b)*c)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.71

$$\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = Aa^3cx+\frac{Db^3dx^8}{8}+x^7\left(\frac{Cb^3d}{7}+\frac{3Dab^2d}{7}+\frac{Db^3c}{7}\right) + x^6\left(\frac{Bb^3d}{6}+\frac{Cab^2d}{2}+\frac{Cb^3c}{6}+\frac{Da^2bd}{2}+\frac{Dab^2c}{2}\right) + x^5\left(\frac{Ab^3d}{5}+\frac{3Bab^2d}{5}+\frac{Bb^3c}{5}+\frac{3Ca^2bd}{5}+\frac{3Cab^2c}{5}+\frac{Da^3d}{5}+\frac{3Da^2bc}{5}\right) + x^4\cdot\left(\frac{3Aab^2d}{4}+\frac{Ab^3c}{4}+\frac{3Ba^2bd}{4}+\frac{3Bab^2c}{4}+\frac{Ca^3d}{4}+\frac{3Ca^2bc}{4}+\frac{Da^3c}{4}\right) + x^3\left(Aa^2bd+Aab^2c+\frac{Ba^3d}{3}+Ba^2bc+\frac{Ca^3c}{3}\right)+x^2\left(\frac{Aa^3d}{2}+\frac{3Aa^2bc}{2}+\frac{Ba^3c}{2}\right)$$

input
```
integrate((b*x+a)**3*(d*x+c)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*c*x + D*b**3*d*x**8/8 + x**7*(C*b**3*d/7 + 3*D*a*b**2*d/7 + D*b**3*
c/7) + x**6*(B*b**3*d/6 + C*a*b**2*d/2 + C*b**3*c/6 + D*a**2*b*d/2 + D*a*b
**2*c/2) + x**5*(A*b**3*d/5 + 3*B*a*b**2*d/5 + B*b**3*c/5 + 3*C*a**2*b*d/5
 + 3*C*a*b**2*c/5 + D*a**3*d/5 + 3*D*a**2*b*c/5) + x**4*(3*A*a*b**2*d/4 +
A*b**3*c/4 + 3*B*a**2*b*d/4 + 3*B*a*b**2*c/4 + C*a**3*d/4 + 3*C*a**2*b*c/4
 + D*a**3*c/4) + x**3*(A*a**2*b*d + A*a*b**2*c + B*a**3*d/3 + B*a**2*b*c +
 C*a**3*c/3) + x**2*(A*a**3*d/2 + 3*A*a**2*b*c/2 + B*a**3*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

$$\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{8}Db^3dx^8+\frac{1}{7}\left(Db^3c+\left(3\,Dab^2+Cb^3\right)d\right)x^7$$
$$+\frac{1}{6}\left(\left(3\,Dab^2+Cb^3\right)c+\left(3\,Da^2b+3\,Cab^2+Bb^3\right)d\right)x^6+Aa^3cx$$
$$+\frac{1}{5}\left(\left(3\,Da^2b+3\,Cab^2+Bb^3\right)c+\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)d\right)x^5$$
$$+\frac{1}{4}\left(\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)c+\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)d\right)x^4$$
$$+\frac{1}{3}\left(\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)c+\left(Ba^3+3\,Aa^2b\right)d\right)x^3$$
$$+\frac{1}{2}\left(Aa^3d+\left(Ba^3+3\,Aa^2b\right)c\right)x^2$$

input
```
integrate((b*x+a)^3*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b^3*d*x^8 + 1/7*(D*b^3*c + (3*D*a*b^2 + C*b^3)*d)*x^7 + 1/6*((3*D*a*
b^2 + C*b^3)*c + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d)*x^6 + A*a^3*c*x + 1/5*
((3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c + (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^
3)*d)*x^5 + 1/4*((D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c + (C*a^3 + 3*B*
a^2*b + 3*A*a*b^2)*d)*x^4 + 1/3*((C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c + (B*a^
3 + 3*A*a^2*b)*d)*x^3 + 1/2*(A*a^3*d + (B*a^3 + 3*A*a^2*b)*c)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.70

$$\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{8}Db^3dx^8+\frac{1}{7}Db^3cx^7+\frac{3}{7}Dab^2dx^7+\frac{1}{7}Cb^3dx^7+\frac{1}{2}Dab^2cx^6+\frac{1}{6}Cb^3cx^6+\frac{1}{2}Da^2bdx^6+\frac{1}{2}Cab^2dx^6+\frac{1}{6}Bb^3dx^6+\frac{3}{5}Da^2bcx^5+\frac{3}{5}Cab^2cx^5+\frac{1}{5}Bb^3cx^5+\frac{1}{5}Da^3dx^5+\frac{3}{5}Ca^2bdx^5+\frac{3}{5}Bab^2dx^5+\frac{1}{5}Ab^3dx^5+\frac{1}{4}Da^3cx^4+\frac{3}{4}Ca^2bcx^4+\frac{3}{4}Bab^2cx^4+\frac{1}{4}Ab^3cx^4+\frac{1}{4}Ca^3dx^4+\frac{3}{4}Ba^2bdx^4+\frac{3}{4}Aab^2dx^4+\frac{1}{3}Ca^3cx^3+Ba^2bcx^3+Aab^2cx^3+\frac{1}{3}Ba^3dx^3+Aa^2bdx^3+\frac{1}{2}Ba^3cx^2+\frac{3}{2}Aa^2bcx^2+\frac{1}{2}Aa^3dx^2+Aa^3cx$$

input
```
integrate((b*x+a)^3*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b^3*d*x^8 + 1/7*D*b^3*c*x^7 + 3/7*D*a*b^2*d*x^7 + 1/7*C*b^3*d*x^7 +
1/2*D*a*b^2*c*x^6 + 1/6*C*b^3*c*x^6 + 1/2*D*a^2*b*d*x^6 + 1/2*C*a*b^2*d*x^
6 + 1/6*B*b^3*d*x^6 + 3/5*D*a^2*b*c*x^5 + 3/5*C*a*b^2*c*x^5 + 1/5*B*b^3*c*
x^5 + 1/5*D*a^3*d*x^5 + 3/5*C*a^2*b*d*x^5 + 3/5*B*a*b^2*d*x^5 + 1/5*A*b^3*
d*x^5 + 1/4*D*a^3*c*x^4 + 3/4*C*a^2*b*c*x^4 + 3/4*B*a*b^2*c*x^4 + 1/4*A*b^
3*c*x^4 + 1/4*C*a^3*d*x^4 + 3/4*B*a^2*b*d*x^4 + 3/4*A*a*b^2*d*x^4 + 1/3*C*
a^3*c*x^3 + B*a^2*b*c*x^3 + A*a*b^2*c*x^3 + 1/3*B*a^3*d*x^3 + A*a^2*b*d*x^
3 + 1/2*B*a^3*c*x^2 + 3/2*A*a^2*b*c*x^2 + 1/2*A*a^3*d*x^2 + A*a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.70

$$\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{a^3\,c\,x^4\,D}{4}+\frac{a^3\,d\,x^5\,D}{5}+\frac{b^3\,c\,x^7\,D}{7}+\frac{b^3\,d\,x^8\,D}{8}+A\,a^3\,c\,x+\frac{A\,a^3\,d\,x^2}{2}+\frac{B\,a^3\,c\,x^2}{2}+\frac{A\,b^3\,c\,x^4}{4}+\frac{B\,a^3\,d\,x^3}{3}+\frac{C\,a^3\,c\,x^3}{3}+\frac{A\,b^3\,d\,x^5}{5}+\frac{B\,b^3\,c\,x^5}{5}+\frac{C\,a^3\,d\,x^4}{4}+\frac{B\,b^3\,d\,x^6}{6}+\frac{C\,b^3\,c\,x^6}{6}+\frac{C\,b^3\,d\,x^7}{7}+\frac{3\,C\,a^2\,b\,d\,x^5}{5}+\frac{C\,a\,b^2\,d\,x^6}{2}+\frac{3\,a^2\,b\,c\,x^5\,D}{5}+\frac{a\,b^2\,c\,x^6\,D}{2}+\frac{a^2\,b\,d\,x^6\,D}{2}+\frac{3\,a\,b^2\,d\,x^7\,D}{7}+\frac{3\,A\,a^2\,b\,c\,x^2}{2}+A\,a\,b^2\,c\,x^3+A\,a^2\,b\,d\,x^3+B\,a^2\,b\,c\,x^3+\frac{3\,A\,a\,b^2\,d\,x^4}{4}+\frac{3\,B\,a\,b^2\,c\,x^4}{4}+\frac{3\,B\,a^2\,b\,d\,x^4}{4}+\frac{3\,C\,a^2\,b\,c\,x^4}{4}+\frac{3\,B\,a\,b^2\,d\,x^5}{5}+\frac{3\,C\,a\,b^2\,c\,x^5}{5}$$

input
```
int((a + b*x)^3*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^3*c*x^4*D)/4 + (a^3*d*x^5*D)/5 + (b^3*c*x^7*D)/7 + (b^3*d*x^8*D)/8 + A*
a^3*c*x + (A*a^3*d*x^2)/2 + (B*a^3*c*x^2)/2 + (A*b^3*c*x^4)/4 + (B*a^3*d*x
^3)/3 + (C*a^3*c*x^3)/3 + (A*b^3*d*x^5)/5 + (B*b^3*c*x^5)/5 + (C*a^3*d*x^4
)/4 + (B*b^3*d*x^6)/6 + (C*b^3*c*x^6)/6 + (C*b^3*d*x^7)/7 + (3*C*a^2*b*d*x
^5)/5 + (C*a*b^2*d*x^6)/2 + (3*a^2*b*c*x^5*D)/5 + (a*b^2*c*x^6*D)/2 + (a^2
*b*d*x^6*D)/2 + (3*a*b^2*d*x^7*D)/7 + (3*A*a^2*b*c*x^2)/2 + A*a*b^2*c*x^3
+ A*a^2*b*d*x^3 + B*a^2*b*c*x^3 + (3*A*a*b^2*d*x^4)/4 + (3*B*a*b^2*c*x^4)/
4 + (3*B*a^2*b*d*x^4)/4 + (3*C*a^2*b*c*x^4)/4 + (3*B*a*b^2*d*x^5)/5 + (3*C
*a*b^2*c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.18

$$\int (a+bx)^3(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{x(105b^3d^2x^7+360a\,b^2d^2x^6+240b^3cd\,x^6+420a^2b\,d^2x^5+840a\,b^2cd\,x^5+140b^4d\,x^5+140b^3c^2x^5+168a^3\ldots}{}$$

input
```
int((b*x+a)^3*(d*x+c)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(840*a**4*c + 420*a**4*d*x + 1680*a**3*b*c*x + 1120*a**3*b*d*x**2 + 280
*a**3*c**2*x**2 + 420*a**3*c*d*x**3 + 168*a**3*d**2*x**4 + 1680*a**2*b**2*
c*x**2 + 1260*a**2*b**2*d*x**3 + 630*a**2*b*c**2*x**3 + 1008*a**2*b*c*d*x*
*4 + 420*a**2*b*d**2*x**5 + 840*a*b**3*c*x**3 + 672*a*b**3*d*x**4 + 504*a*
b**2*c**2*x**4 + 840*a*b**2*c*d*x**5 + 360*a*b**2*d**2*x**6 + 168*b**4*c*x
**4 + 140*b**4*d*x**5 + 140*b**3*c**2*x**5 + 240*b**3*c*d*x**6 + 105*b**3*
d**2*x**7))/840
```

3.2 $\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 195

$$\begin{aligned}
&\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)(a+bx)^3}{3b^5} \\
&+ \frac{\left(b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)\right)(a+bx)^4}{4b^5} \\
&+ \frac{\left(b^2(cC+Bd)+6a^2dD-3ab(Cd+cD)\right)(a+bx)^5}{5b^5} \\
&+ \frac{(bCd+bcD-4adD)(a+bx)^6}{6b^5} + \frac{dD(a+bx)^7}{7b^5}
\end{aligned}$$

output
```
1/3*(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x+a)^3/b^5+1/4*(b^3*(A*d+B
*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))*(b*x+a)^4/b^5+1/5*(b^2*
(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))*(b*x+a)^5/b^5+1/6*(C*b*d-4*D*a*d+D*b*
c)*(b*x+a)^6/b^5+1/7*d*D*(b*x+a)^7/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

$$\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= a^2Acx+\frac{1}{2}a(2Abc+aBc+aAd)x^2+\frac{1}{3}(Ab(bc+2ad)+a(2bBc+acC+aBd))x^3$$
$$+\frac{1}{4}\left(b^2(Bc+Ad)+2ab(cC+Bd)+a^2(Cd+cD)\right)x^4$$
$$+\frac{1}{5}\left(b^2(cC+Bd)+a^2dD+2ab(Cd+cD)\right)x^5+\frac{1}{6}b(bCd+bcD+2adD)x^6+\frac{1}{7}b^2dDx^7$$

input
```
Integrate[(a + b*x)^2*(c + d*x)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*c*x + (a*(2*A*b*c + a*B*c + a*A*d)*x^2)/2 + ((A*b*(b*c + 2*a*d) + a*
(2*b*B*c + a*c*C + a*B*d))*x^3)/3 + ((b^2*(B*c + A*d) + 2*a*b*(c*C + B*d)
+ a^2*(C*d + c*D))*x^4)/4 + ((b^2*(c*C + B*d) + a^2*d*D + 2*a*b*(C*d + c*D
))*x^5)/5 + (b*(b*C*d + b*c*D + 2*a*d*D)*x^6)/6 + (b^2*d*D*x^7)/7
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(a+bx)^2(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4}+\frac{(a+bx)^4\left(6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)\right)}{b^4}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^3(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^5}+\frac{(a+bx)^5\left(6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)\right)}{5b^5}+\frac{(a+bx)^4\left(-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)\right)}{4b^5}+\frac{(a+bx)^6(-4adD+bcD+bCd)}{6b^5}+\frac{dD(a+bx)^7}{7b^5}$$

input
```
Int[(a + b*x)^2*(c + d*x)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(a + b*x)^3)/(3*b^5) + ((
b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D))*(
a + b*x)^4)/(4*b^5) + ((b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D))*(
a + b*x)^5)/(5*b^5) + ((b*C*d + b*c*D - 4*a*d*D)*(a + b*x)^6)/(6*b^5) + (d
*D*(a + b*x)^7)/(7*b^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.97

method	result
default	$\frac{d\,b^2 D x^7}{7}+\frac{\left(\left(2abd+b^2c\right)D+d\,b^2C\right)x^6}{6}+\frac{\left(\left(d\,a^2+2abc\right)D+\left(2abd+b^2c\right)C+b^2Bd\right)x^5}{5}+\frac{\left(a^2cD+\left(d\,a^2+2abc\right)C+\left(2abd+b^2\right.\right.}{4}$
norman	$\frac{d\,b^2 D x^7}{7}+\left(\frac{1}{6}d\,b^2C+\frac{1}{3}Dabd+\frac{1}{6}Db^2c\right)x^6+\left(\frac{1}{5}b^2Bd+\frac{2}{5}Cabd+\frac{1}{5}C\,b^2c+\frac{1}{5}a^2dD+\frac{2}{5}Dabc\right)$
orering	$\frac{x\left(60d\,b^2Dx^6+70C\,b^2d\,x^5+140Dabd\,x^5+70Db^2c\,x^5+84B\,b^2d\,x^4+168Cabd\,x^4+84C\,b^2c\,x^4+84Da^2d\,x^4+168Dabc\,x^4+105A\right.}{}$
gosper	$\frac{1}{7}d\,b^2Dx^7+\frac{1}{6}x^6d\,b^2C+\frac{1}{3}x^6Dabd+\frac{1}{6}x^6Db^2c+\frac{1}{5}x^5b^2Bd+\frac{2}{5}x^5Cabd+\frac{1}{5}x^5C\,b^2c+\frac{1}{5}x^5a^2d$
parallelrisch	$\frac{1}{7}d\,b^2Dx^7+\frac{1}{6}x^6d\,b^2C+\frac{1}{3}x^6Dabd+\frac{1}{6}x^6Db^2c+\frac{1}{5}x^5b^2Bd+\frac{2}{5}x^5Cabd+\frac{1}{5}x^5C\,b^2c+\frac{1}{5}x^5a^2d$

input
```
int((b*x+a)^2*(d*x+c)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*d*b^2*D*x^7+1/6*((2*a*b*d+b^2*c)*D+d*b^2*C)*x^6+1/5*((a^2*d+2*a*b*c)*D
+(2*a*b*d+b^2*c)*C+b^2*B*d)*x^5+1/4*(a^2*c*D+(a^2*d+2*a*b*c)*C+(2*a*b*d+b^
2*c)*B+d*b^2*A)*x^4+1/3*(a^2*c*C+(a^2*d+2*a*b*c)*B+(2*a*b*d+b^2*c)*A)*x^3+
1/2*(a^2*c*B+(a^2*d+2*a*b*c)*A)*x^2+a^2*c*A*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.94

$$\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{7}Db^2dx^7+\frac{1}{6}\left(Db^2c+\left(2\,Dab+Cb^2\right)d\right)x^6$$
$$+\frac{1}{5}\left(\left(2\,Dab+Cb^2\right)c+\left(Da^2+2\,Cab+Bb^2\right)d\right)x^5+Aa^2cx$$
$$+\frac{1}{4}\left(\left(Da^2+2\,Cab+Bb^2\right)c+\left(Ca^2+2\,Bab+Ab^2\right)d\right)x^4$$
$$+\frac{1}{3}\left(\left(Ca^2+2\,Bab+Ab^2\right)c+\left(Ba^2+2\,Aab\right)d\right)x^3+\frac{1}{2}\left(Aa^2d+\left(Ba^2+2\,Aab\right)c\right)x^2$$

input
```
integrate((b*x+a)^2*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/7*D*b^2*d*x^7 + 1/6*(D*b^2*c + (2*D*a*b + C*b^2)*d)*x^6 + 1/5*((2*D*a*b
+ C*b^2)*c + (D*a^2 + 2*C*a*b + B*b^2)*d)*x^5 + A*a^2*c*x + 1/4*((D*a^2 +
2*C*a*b + B*b^2)*c + (C*a^2 + 2*B*a*b + A*b^2)*d)*x^4 + 1/3*((C*a^2 + 2*B*
a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*x^3 + 1/2*(A*a^2*d + (B*a^2 + 2*A*a*
b)*c)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.17

$$\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = Aa^2cx+\frac{Db^2dx^7}{7}+x^6\left(\frac{Cb^2d}{6}+\frac{Dabd}{3}+\frac{Db^2c}{6}\right) + x^5\left(\frac{Bb^2d}{5}+\frac{2Cabd}{5}+\frac{Cb^2c}{5}+\frac{Da^2d}{5}+\frac{2Dabc}{5}\right) + x^4\left(\frac{Ab^2d}{4}+\frac{Babd}{2}+\frac{Bb^2c}{4}+\frac{Ca^2d}{4}+\frac{Cabc}{2}+\frac{Da^2c}{4}\right)+x^3 \cdot\left(\frac{2Aabd}{3}+\frac{Ab^2c}{3}+\frac{Ba^2d}{3}+\frac{2Babc}{3}+\frac{Ca^2c}{3}\right)+x^2\left(\frac{Aa^2d}{2}+Aabc+\frac{Ba^2c}{2}\right)$$

input
```
integrate((b*x+a)**2*(d*x+c)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*c*x + D*b**2*d*x**7/7 + x**6*(C*b**2*d/6 + D*a*b*d/3 + D*b**2*c/6)
+ x**5*(B*b**2*d/5 + 2*C*a*b*d/5 + C*b**2*c/5 + D*a**2*d/5 + 2*D*a*b*c/5)
+ x**4*(A*b**2*d/4 + B*a*b*d/2 + B*b**2*c/4 + C*a**2*d/4 + C*a*b*c/2 + D*a
**2*c/4) + x**3*(2*A*a*b*d/3 + A*b**2*c/3 + B*a**2*d/3 + 2*B*a*b*c/3 + C*a
**2*c/3) + x**2*(A*a**2*d/2 + A*a*b*c + B*a**2*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.94

$$\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{7}Db^2dx^7+\frac{1}{6}\left(Db^2c+\left(2\,Dab+Cb^2\right)d\right)x^6 + \frac{1}{5}\left(\left(2\,Dab+Cb^2\right)c+\left(Da^2+2\,Cab+Bb^2\right)d\right)x^5+Aa^2cx + \frac{1}{4}\left(\left(Da^2+2\,Cab+Bb^2\right)c+\left(Ca^2+2\,Bab+Ab^2\right)d\right)x^4 + \frac{1}{3}\left(\left(Ca^2+2\,Bab+Ab^2\right)c+\left(Ba^2+2\,Aab\right)d\right)x^3+\frac{1}{2}\left(Aa^2d+\left(Ba^2+2\,Aab\right)c\right)x^2$$

input

```
integrate((b*x+a)^2*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output

```
1/7*D*b^2*d*x^7 + 1/6*(D*b^2*c + (2*D*a*b + C*b^2)*d)*x^6 + 1/5*((2*D*a*b
+ C*b^2)*c + (D*a^2 + 2*C*a*b + B*b^2)*d)*x^5 + A*a^2*c*x + 1/4*((D*a^2 +
2*C*a*b + B*b^2)*c + (C*a^2 + 2*B*a*b + A*b^2)*d)*x^4 + 1/3*((C*a^2 + 2*B*
a*b + A*b^2)*c + (B*a^2 + 2*A*a*b)*d)*x^3 + 1/2*(A*a^2*d + (B*a^2 + 2*A*a*
b)*c)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.17

$$\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{7}Db^2dx^7+\frac{1}{6}Db^2cx^6+\frac{1}{3}Dabdx^6+\frac{1}{6}Cb^2dx^6+\frac{2}{5}Dabcx^5+\frac{1}{5}Cb^2cx^5 + \frac{1}{5}Da^2dx^5+\frac{2}{5}Cabdx^5+\frac{1}{5}Bb^2dx^5+\frac{1}{4}Da^2cx^4+\frac{1}{2}Cabcx^4+\frac{1}{4}Bb^2cx^4 + \frac{1}{4}Ca^2dx^4+\frac{1}{2}Babdx^4+\frac{1}{4}Ab^2dx^4+\frac{1}{3}Ca^2cx^3+\frac{2}{3}Babcx^3+\frac{1}{3}Ab^2cx^3 + \frac{1}{3}Ba^2dx^3+\frac{2}{3}Aabdx^3+\frac{1}{2}Ba^2cx^2+Aabcx^2+\frac{1}{2}Aa^2dx^2+Aa^2cx$$

input

```
integrate((b*x+a)^2*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/7*D*b^2*d*x^7 + 1/6*D*b^2*c*x^6 + 1/3*D*a*b*d*x^6 + 1/6*C*b^2*d*x^6 + 2/
5*D*a*b*c*x^5 + 1/5*C*b^2*c*x^5 + 1/5*D*a^2*d*x^5 + 2/5*C*a*b*d*x^5 + 1/5*
B*b^2*d*x^5 + 1/4*D*a^2*c*x^4 + 1/2*C*a*b*c*x^4 + 1/4*B*b^2*c*x^4 + 1/4*C*
a^2*d*x^4 + 1/2*B*a*b*d*x^4 + 1/4*A*b^2*d*x^4 + 1/3*C*a^2*c*x^3 + 2/3*B*a*
b*c*x^3 + 1/3*A*b^2*c*x^3 + 1/3*B*a^2*d*x^3 + 2/3*A*a*b*d*x^3 + 1/2*B*a^2*
c*x^2 + A*a*b*c*x^2 + 1/2*A*a^2*d*x^2 + A*a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06

$$\begin{aligned}\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{c\,x^4\,D\,(15\,a^2+24\,a\,b\,x+10\,b^2\,x^2)}{60}+\frac{d\,x^5\,D\,(21\,a^2+35\,a\,b\,x+15\,b^2\,x^2)}{105}\\&\quad+\frac{A\,c\,x\,(3\,a^2+3\,a\,b\,x+b^2\,x^2)}{3}+\frac{A\,d\,x^2\,(6\,a^2+8\,a\,b\,x+3\,b^2\,x^2)}{12}\\&\quad+\frac{B\,c\,x^2\,(6\,a^2+8\,a\,b\,x+3\,b^2\,x^2)}{12}+\frac{B\,d\,x^3\,(10\,a^2+15\,a\,b\,x+6\,b^2\,x^2)}{30}\\&\quad+\frac{C\,c\,x^3\,(10\,a^2+15\,a\,b\,x+6\,b^2\,x^2)}{30}+\frac{C\,d\,x^4\,(15\,a^2+24\,a\,b\,x+10\,b^2\,x^2)}{60}\end{aligned}$$

input
```
int((a + b*x)^2*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(c*x^4*D*(15*a^2 + 10*b^2*x^2 + 24*a*b*x))/60 + (d*x^5*D*(21*a^2 + 15*b^2*
x^2 + 35*a*b*x))/105 + (A*c*x*(3*a^2 + b^2*x^2 + 3*a*b*x))/3 + (A*d*x^2*(6
*a^2 + 3*b^2*x^2 + 8*a*b*x))/12 + (B*c*x^2*(6*a^2 + 3*b^2*x^2 + 8*a*b*x))/
12 + (B*d*x^3*(10*a^2 + 6*b^2*x^2 + 15*a*b*x))/30 + (C*c*x^3*(10*a^2 + 6*b
^2*x^2 + 15*a*b*x))/30 + (C*d*x^4*(15*a^2 + 10*b^2*x^2 + 24*a*b*x))/60
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85

$$\int (a+bx)^2(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{x(60b^2d^2x^6+140ab\,d^2x^5+140b^2cd\,x^5+84a^2d^2x^4+336abcd\,x^4+84b^3d\,x^4+84b^2c^2x^4+210a^2cd\,x^3+3}{}$$

input
```
int((b*x+a)^2*(d*x+c)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(420*a**3*c + 210*a**3*d*x + 630*a**2*b*c*x + 420*a**2*b*d*x**2 + 140*a
**2*c**2*x**2 + 210*a**2*c*d*x**3 + 84*a**2*d**2*x**4 + 420*a*b**2*c*x**2
+ 315*a*b**2*d*x**3 + 210*a*b*c**2*x**3 + 336*a*b*c*d*x**4 + 140*a*b*d**2*
x**5 + 105*b**3*c*x**3 + 84*b**3*d*x**4 + 84*b**2*c**2*x**4 + 140*b**2*c*d
*x**5 + 60*b**2*d**2*x**6))/420
```

3.3 $\int (a+bx)(c+dx) \left(A+Bx+Cx^2+Dx^3\right) dx$

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Optimal result

Integrand size = 26, antiderivative size = 104

$$\begin{aligned}\int (a+bx)(c+dx) \left(A+Bx+Cx^2+Dx^3\right) dx &= aAcx+\frac{1}{2}(Abc+aBc+aAd)x^2+\frac{1}{3}(bBc+acC+Abd+aBd)x^3\\ &\quad+\frac{1}{4}(bcC+bBd+aCd+acD)x^4+\frac{1}{5}(bCd+bcD+adD)x^5+\frac{1}{6}bdDx^6\end{aligned}$$

output
```
a*A*c*x+1/2*(A*a*d+A*b*c+B*a*c)*x^2+1/3*(A*b*d+B*a*d+B*b*c+C*a*c)*x^3+1/4*
(B*b*d+C*a*d+C*b*c+D*a*c)*x^4+1/5*(C*b*d+D*a*d+D*b*c)*x^5+1/6*b*d*D*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\begin{aligned}\int (a+bx)(c+dx) \left(A+Bx+Cx^2+Dx^3\right) dx &= aAcx+\frac{1}{2}(Abc+aBc+aAd)x^2+\frac{1}{3}(bBc+acC+Abd+aBd)x^3\\ &\quad+\frac{1}{4}(bcC+bBd+aCd+acD)x^4+\frac{1}{5}(bCd+bcD+adD)x^5+\frac{1}{6}bdDx^6\end{aligned}$$

input
```
Integrate[(a + b*x)*(c + d*x)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*c*x + ((A*b*c + a*B*c + a*A*d)*x^2)/2 + ((b*B*c + a*c*C + A*b*d + a*B*
d)*x^3)/3 + ((b*c*C + b*B*d + a*C*d + a*c*D)*x^4)/4 + ((b*C*d + b*c*D + a*
d*D)*x^5)/5 + (b*d*D*x^6)/6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(x^2(aBd+acC+Abd+bBc)+x(aAd+aBc+Abc)+aAc+x^3(acD+aCd+bBd+bcC)+x^4(adD+bc\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aBd+acC+Abd+bBc)+\frac{1}{2}x^2(aAd+aBc+Abc)+aAcx+\frac{1}{4}x^4(acD+aCd+bBd+bcC)+\frac{1}{5}x^5(adD+bcD+bCd)+\frac{1}{6}bdDx^6$$

input
```
Int[(a + b*x)*(c + d*x)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*c*x + ((A*b*c + a*B*c + a*A*d)*x^2)/2 + ((b*B*c + a*c*C + A*b*d + a*B*
d)*x^3)/3 + ((b*c*C + b*B*d + a*C*d + a*c*D)*x^4)/4 + ((b*C*d + b*c*D + a*
d*D)*x^5)/5 + (b*d*D*x^6)/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

method	result
default	$\frac{bdDx^6}{6}+\frac{((ad+bc)D+Cbd)x^5}{5}+\frac{(Dac+(ad+bc)C+Bbd)x^4}{4}+\frac{(Cac+(ad+bc)B+Abd)x^3}{3}+\frac{(Bac+(ad+bc)A)x^2}{2}+$
norman	$\frac{bdDx^6}{6}+\left(\frac{1}{5}Cbd+\frac{1}{5}Dad+\frac{1}{5}Dbc\right)x^5+\left(\frac{1}{4}Bbd+\frac{1}{4}Cad+\frac{1}{4}Cbc+\frac{1}{4}Dac\right)x^4+\left(\frac{1}{3}Abd+\frac{1}{3}B\right.$
orering	$\frac{x\left(10Dbd\,x^5+12Cbd\,x^4+12Dad\,x^4+12Dbc\,x^4+15Bbd\,x^3+15Cad\,x^3+15Cbc\,x^3+15Dac\,x^3+20Abd\,x^2+20Bad\,x^2+20Bbc\,x^2\right.}{60}$
gosper	$\frac{1}{6}bdDx^6+\frac{1}{5}x^5Cbd+\frac{1}{5}x^5Dad+\frac{1}{5}x^5Dbc+\frac{1}{4}x^4Bbd+\frac{1}{4}x^4Cad+\frac{1}{4}x^4Cbc+\frac{1}{4}x^4Dac+\frac{1}{3}x^3$
parallelrisch	$\frac{1}{6}bdDx^6+\frac{1}{5}x^5Cbd+\frac{1}{5}x^5Dad+\frac{1}{5}x^5Dbc+\frac{1}{4}x^4Bbd+\frac{1}{4}x^4Cad+\frac{1}{4}x^4Cbc+\frac{1}{4}x^4Dac+\frac{1}{3}x^3$

input
```
int((b*x+a)*(d*x+c)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*b*d*D*x^6+1/5*((a*d+b*c)*D+C*b*d)*x^5+1/4*(D*a*c+(a*d+b*c)*C+B*b*d)*x^
4+1/3*(C*a*c+(a*d+b*c)*B+A*b*d)*x^3+1/2*(B*a*c+(a*d+b*c)*A)*x^2+a*A*c*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int (a+bx)(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{6}Dbdx^6+\frac{1}{5}(Dbc+(Da+Cb)d)x^5+\frac{1}{4}((Da+Cb)c+(Ca+Bb)d)x^4$$
$$+Aacx+\frac{1}{3}((Ca+Bb)c+(Ba+Ab)d)x^3+\frac{1}{2}(Aad+(Ba+Ab)c)x^2$$

input
```
integrate((b*x+a)*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/6*D*b*d*x^6 + 1/5*(D*b*c + (D*a + C*b)*d)*x^5 + 1/4*((D*a + C*b)*c + (C*
a + B*b)*d)*x^4 + A*a*c*x + 1/3*((C*a + B*b)*c + (B*a + A*b)*d)*x^3 + 1/2*
(A*a*d + (B*a + A*b)*c)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.19

$$\int (a+bx)(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = Aacx+\frac{Dbdx^6}{6}+x^5\left(\frac{Cbd}{5}+\frac{Dad}{5}+\frac{Dbc}{5}\right)+x^4\left(\frac{Bbd}{4}+\frac{Cad}{4}+\frac{Cbc}{4}+\frac{Dac}{4}\right) + x^3\left(\frac{Abd}{3}+\frac{Bad}{3}+\frac{Bbc}{3}+\frac{Cac}{3}\right)+x^2\left(\frac{Aad}{2}+\frac{Abc}{2}+\frac{Bac}{2}\right)$$

input
```
integrate((b*x+a)*(d*x+c)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*c*x + D*b*d*x**6/6 + x**5*(C*b*d/5 + D*a*d/5 + D*b*c/5) + x**4*(B*b*d/
4 + C*a*d/4 + C*b*c/4 + D*a*c/4) + x**3*(A*b*d/3 + B*a*d/3 + B*b*c/3 + C*a
*c/3) + x**2*(A*a*d/2 + A*b*c/2 + B*a*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int (a+bx)(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{6}\,Dbdx^6+\frac{1}{5}\,(Dbc+(Da+Cb)d)x^5+\frac{1}{4}\,((Da+Cb)c+(Ca+Bb)d)x^4 + Aacx+\frac{1}{3}\,((Ca+Bb)c+(Ba+Ab)d)x^3+\frac{1}{2}\,(Aad+(Ba+Ab)c)x^2$$

input
```
integrate((b*x+a)*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*D*b*d*x^6 + 1/5*(D*b*c + (D*a + C*b)*d)*x^5 + 1/4*((D*a + C*b)*c + (C*
a + B*b)*d)*x^4 + A*a*c*x + 1/3*((C*a + B*b)*c + (B*a + A*b)*d)*x^3 + 1/2*
(A*a*d + (B*a + A*b)*c)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21

$$\int (a+bx)(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{6}Dbdx^6+\frac{1}{5}Dbcx^5+\frac{1}{5}Dadx^5+\frac{1}{5}Cbdx^5+\frac{1}{4}Dacx^4+\frac{1}{4}Cbcx^4+\frac{1}{4}Cadx^4+\frac{1}{4}Bbdx^4+\frac{1}{3}Cacx^3+\frac{1}{3}Bbcx^3+\frac{1}{3}Badx^3+\frac{1}{3}Abdx^3+\frac{1}{2}Bacx^2+\frac{1}{2}Abcx^2+\frac{1}{2}Aadx^2+Aacx$$

input
```
integrate((b*x+a)*(d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/6*D*b*d*x^6 + 1/5*D*b*c*x^5 + 1/5*D*a*d*x^5 + 1/5*C*b*d*x^5 + 1/4*D*a*c*
x^4 + 1/4*C*b*c*x^4 + 1/4*C*a*d*x^4 + 1/4*B*b*d*x^4 + 1/3*C*a*c*x^3 + 1/3*
B*b*c*x^3 + 1/3*B*a*d*x^3 + 1/3*A*b*d*x^3 + 1/2*B*a*c*x^2 + 1/2*A*b*c*x^2
+ 1/2*A*a*d*x^2 + A*a*c*x
```

Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21

$$\int (a+bx)(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{A\,a\,d\,x^2}{2}+\frac{A\,b\,c\,x^2}{2}+\frac{B\,a\,c\,x^2}{2}+\frac{A\,b\,d\,x^3}{3}+\frac{B\,a\,d\,x^3}{3}+\frac{B\,b\,c\,x^3}{3}+\frac{C\,a\,c\,x^3}{3}+\frac{B\,b\,d\,x^4}{4}+\frac{C\,a\,d\,x^4}{4}+\frac{C\,b\,c\,x^4}{4}+\frac{a\,c\,x^4\,D}{4}+\frac{C\,b\,d\,x^5}{5}+\frac{a\,d\,x^5\,D}{5}+\frac{b\,c\,x^5\,D}{5}+\frac{b\,d\,x^6\,D}{6}+A\,a\,c\,x$$

input
```
int((a + b*x)*(c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*a*d*x^2)/2 + (A*b*c*x^2)/2 + (B*a*c*x^2)/2 + (A*b*d*x^3)/3 + (B*a*d*x^3
)/3 + (B*b*c*x^3)/3 + (C*a*c*x^3)/3 + (B*b*d*x^4)/4 + (C*a*d*x^4)/4 + (C*b
*c*x^4)/4 + (a*c*x^4*D)/4 + (C*b*d*x^5)/5 + (a*d*x^5*D)/5 + (b*c*x^5*D)/5
+ (b*d*x^6*D)/6 + A*a*c*x
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int (a+bx)(c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{x(10b\,d^2x^5+12a\,d^2x^4+24bcd\,x^4+30acd\,x^3+15b^2d\,x^3+15b\,c^2x^3+40abd\,x^2+20a\,c^2x^2+20b^2c\,x^2+3}{60}$$

input
```
int((b*x+a)*(d*x+c)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(60*a**2*c + 30*a**2*d*x + 60*a*b*c*x + 40*a*b*d*x**2 + 20*a*c**2*x**2
+ 30*a*c*d*x**3 + 12*a*d**2*x**4 + 20*b**2*c*x**2 + 15*b**2*d*x**3 + 15*b*
c**2*x**3 + 24*b*c*d*x**4 + 10*b*d**2*x**5))/60
```

3.4 $\int (c+dx)\left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result 120
Mathematica [A] (verified) 120
Rubi [A] (verified) 121
Maple [A] (verified) 122
Fricas [A] (verification not implemented) 122
Sympy [A] (verification not implemented) 123
Maxima [A] (verification not implemented) 123
Giac [A] (verification not implemented) 124
Mupad [B] (verification not implemented) 124
Reduce [B] (verification not implemented) 125

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int (c+dx)\left(A+Bx+Cx^2+Dx^3\right) dx = Acx+\frac{1}{2}(Bc+Ad)x^2+\frac{1}{3}(cC+Bd)x^3 + \frac{1}{4}(Cd+cD)x^4+\frac{1}{5}dDx^5$$

output
```
A*c*x+1/2*(A*d+B*c)*x^2+1/3*(B*d+C*c)*x^3+1/4*(C*d+D*c)*x^4+1/5*d*D*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (c+dx)\left(A+Bx+Cx^2+Dx^3\right) dx = Acx+\frac{1}{2}(Bc+Ad)x^2+\frac{1}{3}(cC+Bd)x^3 + \frac{1}{4}(Cd+cD)x^4+\frac{1}{5}dDx^5$$

input
```
Integrate[(c + d*x)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
A*c*x + ((B*c + A*d)*x^2)/2 + ((c*C + B*d)*x^3)/3 + ((C*d + c*D)*x^4)/4 +
(d*D*x^5)/5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx) \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(x(Ad+Bc)+Ac+x^2(Bd+cC)+x^3(cD+Cd)+dDx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(Ad+Bc)+Acx+\frac{1}{3}x^3(Bd+cC)+\frac{1}{4}x^4(cD+Cd)+\frac{1}{5}dDx^5$$

input
```
Int[(c + d*x)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
A*c*x + ((B*c + A*d)*x^2)/2 + ((c*C + B*d)*x^3)/3 + ((C*d + c*D)*x^4)/4 +
(d*D*x^5)/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

method	result	size
default	$Acx+\frac{(Ad+Bc)x^2}{2}+\frac{(Bd+Cc)x^3}{3}+\frac{(Cd+Dc)x^4}{4}+\frac{dDx^5}{5}$	49
norman	$\frac{dDx^5}{5}+\left(\frac{Cd}{4}+\frac{Dc}{4}\right)x^4+\left(\frac{Bd}{3}+\frac{Cc}{3}\right)x^3+\left(\frac{Ad}{2}+\frac{Bc}{2}\right)x^2+Acx$	52
orering	$\frac{x(12Dd\,x^4+15Cd\,x^3+15Dc\,x^3+20Bd\,x^2+20Cc\,x^2+30Adx+30Bcx+60Ac)}{60}$	54
gosper	$\frac{1}{5}dDx^5+\frac{1}{4}x^4Cd+\frac{1}{4}x^4Dc+\frac{1}{3}x^3Bd+\frac{1}{3}x^3Cc+\frac{1}{2}x^2Ad+\frac{1}{2}x^2Bc+Acx$	55
parallelrisch	$\frac{1}{5}dDx^5+\frac{1}{4}x^4Cd+\frac{1}{4}x^4Dc+\frac{1}{3}x^3Bd+\frac{1}{3}x^3Cc+\frac{1}{2}x^2Ad+\frac{1}{2}x^2Bc+Acx$	55

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c*x+1/2*(A*d+B*c)*x^2+1/3*(B*d+C*c)*x^3+1/4*(C*d+D*c)*x^4+1/5*d*D*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{5}\,Ddx^5+\frac{1}{4}\,(Dc+Cd)x^4+\frac{1}{3}\,(Cc+Bd)x^3+Acx+\frac{1}{2}\,(Bc+Ad)x^2$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/5*D*d*x^5 + 1/4*(D*c + C*d)*x^4 + 1/3*(C*c + B*d)*x^3 + A*c*x + 1/2*(B*c
 + A*d)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int (c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = Acx+\frac{Ddx^5}{5}+x^4\left(\frac{Cd}{4}+\frac{Dc}{4}\right)+x^3\left(\frac{Bd}{3}+\frac{Cc}{3}\right)+x^2\left(\frac{Ad}{2}+\frac{Bc}{2}\right)$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*c*x + D*d*x**5/5 + x**4*(C*d/4 + D*c/4) + x**3*(B*d/3 + C*c/3) + x**2*(A
*d/2 + B*c/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{5}\,Ddx^5+\frac{1}{4}\,(Dc+Cd)x^4+\frac{1}{3}\,(Cc+Bd)x^3+Acx+\frac{1}{2}\,(Bc+Ad)x^2$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/5*D*d*x^5 + 1/4*(D*c + C*d)*x^4 + 1/3*(C*c + B*d)*x^3 + A*c*x + 1/2*(B*c
 + A*d)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{5}\,Ddx^5+\frac{1}{4}\,Dcx^4+\frac{1}{4}\,Cdx^4+\frac{1}{3}\,Ccx^3+\frac{1}{3}\,Bdx^3+\frac{1}{2}\,Bcx^2+\frac{1}{2}\,Adx^2+Acx$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/5*D*d*x^5 + 1/4*D*c*x^4 + 1/4*C*d*x^4 + 1/3*C*c*x^3 + 1/3*B*d*x^3 + 1/2*
B*c*x^2 + 1/2*A*d*x^2 + A*c*x
```

Mupad [B] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (c+dx)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{c\,x^4\,D}{4}+\frac{d\,x^5\,D}{5}+A\,c\,x+\frac{A\,d\,x^2}{2}+\frac{B\,c\,x^2}{2}+\frac{B\,d\,x^3}{3}+\frac{C\,c\,x^3}{3}+\frac{C\,d\,x^4}{4}$$

input
```
int((c + d*x)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(c*x^4*D)/4 + (d*x^5*D)/5 + A*c*x + (A*d*x^2)/2 + (B*c*x^2)/2 + (B*d*x^3)/
3 + (C*c*x^3)/3 + (C*d*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (c+dx) \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{x(6d^2x^4+15cd\,x^3+10bd\,x^2+10c^2x^2+15adx+15bcx+30ac)}{30}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(30*a*c + 15*a*d*x + 15*b*c*x + 10*b*d*x**2 + 10*c**2*x**2 + 15*c*d*x**
3 + 6*d**2*x**4))/30
```

3.5 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{a+bx} \, dx$

Optimal result 126
Mathematica [A] (verified) 127
Rubi [A] (verified) 127
Maple [A] (verified) 128
Fricas [A] (verification not implemented) 129
Sympy [A] (verification not implemented) 129
Maxima [A] (verification not implemented) 130
Giac [A] (verification not implemented) 130
Mupad [F(-1)] 131
Reduce [B] (verification not implemented) 131

Optimal result

Integrand size = 28, antiderivative size = 168

$$\begin{aligned}
\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{a+bx} \, dx
&= \frac{(b^3(Bc+Ad)-ab^2(cC+Bd)-a^3dD+a^2b(Cd+cD))\,x}{b^4} \\
&\quad + \frac{(b^2(cC+Bd)+a^2dD-ab(Cd+cD))\,x^2}{2b^3} + \frac{(bCd+bcD-adD)x^3}{3b^2} \\
&\quad + \frac{dDx^4}{4b} + \frac{(bc-ad)\,(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{b^5}
\end{aligned}$$

output
```
(b^3*(A*d+B*c)-a*b^2*(B*d+C*c)-a^3*d*D+a^2*b*(C*d+D*c))*x/b^4+1/2*(b^2*(B*
d+C*c)+a^2*d*D-a*b*(C*d+D*c))*x^2/b^3+1/3*(C*b*d-D*a*d+D*b*c)*x^3/b^2+1/4*
d*D*x^4/b+(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx$$
$$= \frac{bx(-12a^3dD+6a^2b(2Cd+2cD+dDx)-2ab^2(6cC+6Bd+3Cdx+3cDx+2dDx^2)+b^3(12Ad+6\ldots}{1\ldots}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(b*x*(-12*a^3*d*D + 6*a^2*b*(2*C*d + 2*c*D + d*D*x) - 2*a*b^2*(6*c*C + 6*B
*d + 3*C*d*x + 3*c*D*x + 2*d*D*x^2) + b^3*(12*A*d + 6*B*(2*c + d*x) + x*(6
*c*C + 4*C*d*x + 4*c*D*x + 3*d*D*x^2))) + 12*(b*c - a*d)*(A*b^3 - a*(b^2*B
 - a*b*C + a^2*D))*Log[a + b*x])/(12*b^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4(a+bx)}+\frac{x\left(a^2dD-ab(cD+Cd)+b^2(Bd+cC)\right)}{b^3}+\frac{a^3(-d)D+a^2b(cD\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)\log(a+bx)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^5}+\frac{x^2\left(a^2dD-ab(cD+Cd)+b^2(Bd+cC)\right)}{2b^3}+\frac{x\left(a^3(-d)D+a^2b(cD+Cd)-ab^2(Bd+cC)+b^3(Ad+Bc)\right)}{b^4}+\frac{x^3(-adD+bcD+bCd)}{3b^2}+\frac{dDx^4}{4b}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
((b^3*(B*c + A*d) - a*b^2*(c*C + B*d) - a^3*d*D + a^2*b*(C*d + c*D))*x)/b^
4 + ((b^2*(c*C + B*d) + a^2*d*D - a*b*(C*d + c*D))*x^2)/(2*b^3) + ((b*C*d
+ b*c*D - a*d*D)*x^3)/(3*b^2) + (d*D*x^4)/(4*b) + ((b*c - a*d)*(A*b^3 - a*
(b^2*B - a*b*C + a^2*D))*Log[a + b*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\left(b^3dA-Ba\,b^2d+B\,b^3c+C\,a^2bd-a\,b^2cC-a^3dD+a^2bcD\right)x}{b^4}+\frac{\left(b^2Bd-Cabd+C\,b^2c+a^2dD-Dabc\right)x^2}{2b^3}+\frac{(Cbd-Dad+Dbc)}{3b^2}$
default	$\frac{\frac{1}{4}Dd\,x^4b^3+\frac{1}{3}C\,b^3d\,x^3-\frac{1}{3}Da\,b^2d\,x^3+\frac{1}{3}Db^3c\,x^3+\frac{1}{2}B\,b^3d\,x^2-\frac{1}{2}Ca\,b^2d\,x^2+\frac{1}{2}C\,b^3c\,x^2+\frac{1}{2}Da^2bd\,x^2-\frac{1}{2}Da\,b^2c\,x^2+b^3dAx-Ba}{b^4}$
parallelrisch	$-\frac{12D\ln(bx+a)a^3bc+4Dx^3a\,b^3d+6C\,x^2a\,b^3d-6Dx^2a^2b^2d+6Dx^2a\,b^3c+12A\ln(bx+a)a\,b^3d-12B\ln(bx+a)a^2b^2d+12Bxa\,b}{}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(A*b^3*d-B*a*b^2*d+B*b^3*c+C*a^2*b*d-C*a*b^2*c-D*a^3*d+D*a^2*b*c)/b^4*x+1/
2/b^3*(B*b^2*d-C*a*b*d+C*b^2*c+D*a^2*d-D*a*b*c)*x^2+1/3*(C*b*d-D*a*d+D*b*c
)*x^3/b^2+1/4*d*D*x^4/b-(A*a*b^3*d-A*b^4*c-B*a^2*b^2*d+B*a*b^3*c+C*a^3*b*d
-C*a^2*b^2*c-D*a^4*d+D*a^3*b*c)/b^5*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx$$
$$= \frac{3\,Db^4dx^4+4\,(Db^4c-(Dab^3-Cb^4)d)x^3-6\,((Dab^3-Cb^4)c-(Da^2b^2-Cab^3+Bb^4)d)x^2+12\,((Da^2b$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
1/12*(3*D*b^4*d*x^4 + 4*(D*b^4*c - (D*a*b^3 - C*b^4)*d)*x^3 - 6*((D*a*b^3
- C*b^4)*c - (D*a^2*b^2 - C*a*b^3 + B*b^4)*d)*x^2 + 12*((D*a^2*b^2 - C*a*b
^3 + B*b^4)*c - (D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*d)*x - 12*((D*a^3*
b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c - (D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^
3)*d)*log(b*x + a))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.10

$$\begin{aligned}\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx &= \frac{Ddx^4}{4b}+x^3\left(\frac{Cd}{3b}-\frac{Dad}{3b^2}+\frac{Dc}{3b}\right)+x^2\left(\frac{Bd}{2b}-\frac{Cad}{2b^2}+\frac{Cc}{2b}+\frac{Da^2d}{2b^3}-\frac{Dac}{2b^2}\right)\\&\quad+x\left(\frac{Ad}{b}-\frac{Bad}{b^2}+\frac{Bc}{b}+\frac{Ca^2d}{b^3}-\frac{Cac}{b^2}-\frac{Da^3d}{b^4}+\frac{Da^2c}{b^3}\right)\\&\quad+\frac{(ad-bc)\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\log(a+bx)}{b^5}\end{aligned}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x+a),x)
```

output
```
D*d*x**4/(4*b) + x**3*(C*d/(3*b) - D*a*d/(3*b**2) + D*c/(3*b)) + x**2*(B*d
/(2*b) - C*a*d/(2*b**2) + C*c/(2*b) + D*a**2*d/(2*b**3) - D*a*c/(2*b**2))
+ x*(A*d/b - B*a*d/b**2 + B*c/b + C*a**2*d/b**3 - C*a*c/b**2 - D*a**3*d/b*
*4 + D*a**2*c/b**3) + (a*d - b*c)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)
*log(a + b*x)/b**5
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx$$
$$= \frac{3\,Db^3dx^4 + 4\,(Db^3c - (Dab^2 - Cb^3)d)x^3 - 6\,((Dab^2 - Cb^3)c - (Da^2b - Cab^2 + Bb^3)d)x^2 + 12\,((Da^2b}{12\,b^4}$$
$$- \frac{((Da^3b - Ca^2b^2 + Bab^3 - Ab^4)c - (Da^4 - Ca^3b + Ba^2b^2 - Aab^3)d)\log\left(bx+a\right)}{b^5}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(3*D*b^3*d*x^4 + 4*(D*b^3*c - (D*a*b^2 - C*b^3)*d)*x^3 - 6*((D*a*b^2
- C*b^3)*c - (D*a^2*b - C*a*b^2 + B*b^3)*d)*x^2 + 12*((D*a^2*b - C*a*b^2 +
 B*b^3)*c - (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d)*x)/b^4 - ((D*a^3*b - C*
a^2*b^2 + B*a*b^3 - A*b^4)*c - (D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*d)*
log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx$$
$$= \frac{3\,Db^3dx^4 + 4\,Db^3cx^3 - 4\,Dab^2dx^3 + 4\,Cb^3dx^3 - 6\,Dab^2cx^2 + 6\,Cb^3cx^2 + 6\,Da^2bdx^2 - 6\,Cab^2dx^2 + 6\,B}{12\,b^4}$$
$$- \frac{(Da^3bc - Ca^2b^2c + Bab^3c - Ab^4c - Da^4d + Ca^3bd - Ba^2b^2d + Aab^3d)\log\left(|bx+a|\right)}{b^5}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
1/12*(3*D*b^3*d*x^4 + 4*D*b^3*c*x^3 - 4*D*a*b^2*d*x^3 + 4*C*b^3*d*x^3 - 6*
D*a*b^2*c*x^2 + 6*C*b^3*c*x^2 + 6*D*a^2*b*d*x^2 - 6*C*a*b^2*d*x^2 + 6*B*b^
3*d*x^2 + 12*D*a^2*b*c*x - 12*C*a*b^2*c*x + 12*B*b^3*c*x - 12*D*a^3*d*x +
12*C*a^2*b*d*x - 12*B*a*b^2*d*x + 12*A*b^3*d*x)/b^4 - (D*a^3*b*c - C*a^2*b
^2*c + B*a*b^3*c - A*b^4*c - D*a^4*d + C*a^3*b*d - B*a^2*b^2*d + A*a*b^3*d
)*log(abs(b*x + a))/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx=\int \frac{(c+d\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{a+b\,x}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx = \frac{12\log(bx+a)\,a^4d^2-24\log(bx+a)\,a^3bcd+12\log(bx+a)\,a^2b^2c^2-12a^3b\,d^2x+24a^2b^2cdx+6a^2b^2d^2x^2}{12b^5}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

output
```
(12*log(a + b*x)*a**4*d**2 - 24*log(a + b*x)*a**3*b*c*d + 12*log(a + b*x)*
a**2*b**2*c**2 - 12*a**3*b*d**2*x + 24*a**2*b**2*c*d*x + 6*a**2*b**2*d**2*
x**2 - 12*a*b**3*c**2*x - 12*a*b**3*c*d*x**2 - 4*a*b**3*d**2*x**3 + 12*b**
5*c*x + 6*b**5*d*x**2 + 6*b**4*c**2*x**2 + 8*b**4*c*d*x**3 + 3*b**4*d**2*x
**4)/(12*b**5)
```

3.6 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 172

$$\begin{aligned}\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx &= \frac{\left(b^2(cC+Bd)+3a^2dD-2ab(Cd+cD)\right)x}{b^4}+\frac{(bCd+bcD-2adD)x^2}{2b^3}\\ &\quad+\frac{dDx^3}{3b^2}-\frac{(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)}{b^5(a+bx)}\\ &\quad+\frac{\left(b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)\right)\log(a+bx)}{b^5}\end{aligned}$$

output
```
(b^2*(B*d+C*c)+3*a^2*d*D-2*a*b*(C*d+D*c))*x/b^4+1/2*(C*b*d-2*D*a*d+D*b*c)*
x^2/b^3+1/3*d*D*x^3/b^2-(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^5/(b*x+
a)+(b^3*(A*d+B*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))*ln(b*x+a)
/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$=\frac{6b(b^2(cC+Bd)+3a^2dD-2ab(Cd+cD))\,x+3b^2(bCd+bcD-2adD)x^2+2b^3dDx^3-\frac{6(bc-ad)(Ab^3-a(\cdots}{a+\cdots}}{6b^5}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
(6*b*(b^2*(c*C + B*d) + 3*a^2*d*D - 2*a*b*(C*d + c*D))*x + 3*b^2*(b*C*d +
b*c*D - 2*a*d*D)*x^2 + 2*b^3*d*D*x^3 - (6*(b*c - a*d)*(A*b^3 - a*(b^2*B -
a*b*C + a^2*D)))/(a + b*x) + 6*(b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*
a^3*d*D + 3*a^2*b*(C*d + c*D))*Log[a + b*x])/(6*b^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \ 2123$$
$$\int \left(\frac{(bc-ad)(Ab^3-a(a^2D-abC+b^2B))}{b^4(a+bx)^2}+\frac{3a^2dD-2ab(cD+Cd)+b^2(Bd+cC)}{b^4}+\frac{-4a^3dD+3a^2b(cD-\cdots}{\cdots}\right.$$
$$\Big\downarrow \ 2009$$

$$-\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^5(a+bx)}+\frac{x\left(3a^2dD-2ab(cD+Cd)+b^2(Bd+cC)\right)}{b^4}+\frac{\log(a+bx)\left(-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)\right)}{b^5}+\frac{x^2(-2adD+bcD+bCd)}{2b^3}+\frac{dDx^3}{3b^2}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
((b^2*(c*C + B*d) + 3*a^2*d*D - 2*a*b*(C*d + c*D))*x)/b^4 + ((b*C*d + b*c*
D - 2*a*d*D)*x^2)/(2*b^3) + (d*D*x^3)/(3*b^2) - ((b*c - a*d)*(A*b^3 - a*(b
^2*B - a*b*C + a^2*D)))/(b^5*(a + b*x)) + ((b^3*(B*c + A*d) - 2*a*b^2*(c*C
 + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D))*Log[a + b*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.27

method	result
default	$\frac{\frac{1}{3}dDx^3b^2+\frac{1}{2}Cb^2dx^2-Dabdx^2+\frac{1}{2}Db^2cx^2+b^2Bdx-2Cabdx+Cb^2cx+3a^2dDx-2Dabcx}{b^4}-\frac{-Aab^3d+Ab^4c+Ba^2b^2d-B}{}$
norman	$\frac{\frac{(3Cbd-4Dad+3Dbc)x^3}{6b^2}+\frac{\left(2b^2Bd-3Cabd+2Cb^2c+4a^2dD-3Dabc\right)x^2}{2b^3}+\frac{dDx^4}{3b}-\frac{\left(Aab^3d-Ab^4c-2Ba^2b^2d+Bab^3c+3Ca^3bd-2Ca^2b^2\right.}{ab^4}}{bx+a}$
parallelrisch	$\frac{18D\ln(bx+a)a^3bc-4Dx^3ab^3d-9Cx^2ab^3d+12Dx^2a^2b^2d-9Dx^2ab^3c+6A\ln(bx+a)xb^4d+6B\ln(bx+a)xb^4c-24Da^4d+6A}{}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/3*d*D*x^3*b^2+1/2*C*b^2*d*x^2-D*a*b*d*x^2+1/2*D*b^2*c*x^2+b^2*B*d
*x-2*C*a*b*d*x+C*b^2*c*x+3*a^2*d*D*x-2*D*a*b*c*x)-1/b^5*(-A*a*b^3*d+A*b^4*
c+B*a^2*b^2*d-B*a*b^3*c-C*a^3*b*d+C*a^2*b^2*c+D*a^4*d-D*a^3*b*c)/(b*x+a)+1
/b^5*(A*b^3*d-2*B*a*b^2*d+B*b^3*c+3*C*a^2*b*d-2*C*a*b^2*c-4*D*a^3*d+3*D*a^
2*b*c)*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(170) = 340$.

Time = 0.07 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.98

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx = \frac{2\,Db^4dx^4+(3\,Db^4c-(4\,Dab^3-3\,Cb^4)d)x^3-3\,((3\,Dab^3-2\,Cb^4)c-(4\,Da^2b^2-3\,Cab^3+2\,Bb^4)d)x^2 \cdots}{\cdots}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/6*(2*D*b^4*d*x^4 + (3*D*b^4*c - (4*D*a*b^3 - 3*C*b^4)*d)*x^3 - 3*((3*D*a
*b^3 - 2*C*b^4)*c - (4*D*a^2*b^2 - 3*C*a*b^3 + 2*B*b^4)*d)*x^2 + 6*(D*a^3*
b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c - 6*(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*
b^3)*d - 6*((2*D*a^2*b^2 - C*a*b^3)*c - (3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3
)*d)*x + 6*((3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c - (4*D*a^4 - 3*C*a^3*b +
 2*B*a^2*b^2 - A*a*b^3)*d + ((3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c - (4*D*a^
3*b - 3*C*a^2*b^2 + 2*B*a*b^3 - A*b^4)*d)*x)*log(b*x + a))/(b^6*x + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.38

$$\begin{aligned}\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx &= \frac{Ddx^3}{3b^2}+x^2\left(\frac{Cd}{2b^2}-\frac{Dad}{b^3}+\frac{Dc}{2b^2}\right)+x\left(\frac{Bd}{b^2}-\frac{2Cad}{b^3}+\frac{Cc}{b^2}+\frac{3Da^2d}{b^4}-\frac{2Dac}{b^3}\right)\\ &\quad+\frac{Aab^3d-Ab^4c-Ba^2b^2d+Bab^3c+Ca^3bd-Ca^2b^2c-Da^4d+Da^3bc}{ab^5+b^6x}\\ &\quad-\frac{(-Ab^3d+2Bab^2d-Bb^3c-3Ca^2bd+2Cab^2c+4Da^3d-3Da^2bc)\log{(a+bx)}}{b^5}\end{aligned}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**2,x)
```

output
```
D*d*x**3/(3*b**2) + x**2*(C*d/(2*b**2) - D*a*d/b**3 + D*c/(2*b**2)) + x*(B
*d/b**2 - 2*C*a*d/b**3 + C*c/b**2 + 3*D*a**2*d/b**4 - 2*D*a*c/b**3) + (A*a
*b**3*d - A*b**4*c - B*a**2*b**2*d + B*a*b**3*c + C*a**3*b*d - C*a**2*b**2
*c - D*a**4*d + D*a**3*b*c)/(a*b**5 + b**6*x) - (-A*b**3*d + 2*B*a*b**2*d
- B*b**3*c - 3*C*a**2*b*d + 2*C*a*b**2*c + 4*D*a**3*d - 3*D*a**2*b*c)*log(
a + b*x)/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$
$$= \frac{(Da^3b - Ca^2b^2 + Bab^3 - Ab^4)c - (Da^4 - Ca^3b + Ba^2b^2 - Aab^3)d}{b^6x + ab^5}$$
$$+ \frac{2\,Db^2dx^3 + 3\,(Db^2c - (2\,Dab - Cb^2)d)x^2 - 6\,((2\,Dab - Cb^2)c - (3\,Da^2 - 2\,Cab + Bb^2)d)x}{6\,b^4}$$
$$+ \frac{((3\,Da^2b - 2\,Cab^2 + Bb^3)c - (4\,Da^3 - 3\,Ca^2b + 2\,Bab^2 - Ab^3)d)\log(bx + a)}{b^5}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
((D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c - (D*a^4 - C*a^3*b + B*a^2*b^2
- A*a*b^3)*d)/(b^6*x + a*b^5) + 1/6*(2*D*b^2*d*x^3 + 3*(D*b^2*c - (2*D*a*b
 - C*b^2)*d)*x^2 - 6*((2*D*a*b - C*b^2)*c - (3*D*a^2 - 2*C*a*b + B*b^2)*d)
*x)/b^4 + ((3*D*a^2*b - 2*C*a*b^2 + B*b^3)*c - (4*D*a^3 - 3*C*a^2*b + 2*B*
a*b^2 - A*b^3)*d)*log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$= \frac{\left(2\,Dd + \frac{3\,(Db^2c-4\,Dabd+Cb^2d)}{(bx+a)b} - \frac{6\,(3\,Dab^3c-Cb^4c-6\,Da^2b^2d+3\,Cab^3d-Bb^4d)}{(bx+a)^2b^2}\right)(bx+a)^3}{6\,b^5}$$
$$- \frac{(3\,Da^2bc-2\,Cab^2c+Bb^3c-4\,Da^3d+3\,Ca^2bd-2\,Bab^2d+Ab^3d)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^5}$$
$$+ \frac{\frac{Da^3b^4c}{bx+a} - \frac{Ca^2b^5c}{bx+a} + \frac{Bab^6c}{bx+a} - \frac{Ab^7c}{bx+a} - \frac{Da^4b^3d}{bx+a} + \frac{Ca^3b^4d}{bx+a} - \frac{Ba^2b^5d}{bx+a} + \frac{Aab^6d}{bx+a}}{b^8}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*(2*D*d + 3*(D*b^2*c - 4*D*a*b*d + C*b^2*d)/((b*x + a)*b) - 6*(3*D*a*b^
3*c - C*b^4*c - 6*D*a^2*b^2*d + 3*C*a*b^3*d - B*b^4*d)/((b*x + a)^2*b^2))*
(b*x + a)^3/b^5 - (3*D*a^2*b*c - 2*C*a*b^2*c + B*b^3*c - 4*D*a^3*d + 3*C*a
^2*b*d - 2*B*a*b^2*d + A*b^3*d)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^5
 + (D*a^3*b^4*c/(b*x + a) - C*a^2*b^5*c/(b*x + a) + B*a*b^6*c/(b*x + a) -
A*b^7*c/(b*x + a) - D*a^4*b^3*d/(b*x + a) + C*a^3*b^4*d/(b*x + a) - B*a^2*
b^5*d/(b*x + a) + A*a*b^6*d/(b*x + a))/b^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \int \frac{(c+d\,x)\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^2}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2,x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$
$$= \frac{-12\log(bx+a)\,a^4d^2+18\log(bx+a)\,a^3bcd-12\log(bx+a)\,a^3b\,d^2x-3\log(bx+a)\,a^2b^3d-6\log(bx+}{}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

output
```
( - 12*log(a + b*x)*a**4*d**2 + 18*log(a + b*x)*a**3*b*c*d - 12*log(a + b*
x)*a**3*b*d**2*x - 3*log(a + b*x)*a**2*b**3*d - 6*log(a + b*x)*a**2*b**2*c
**2 + 18*log(a + b*x)*a**2*b**2*c*d*x + 3*log(a + b*x)*a*b**4*c - 3*log(a
+ b*x)*a*b**4*d*x - 6*log(a + b*x)*a*b**3*c**2*x + 3*log(a + b*x)*b**5*c*x
 + 12*a**3*b*d**2*x - 18*a**2*b**2*c*d*x + 6*a**2*b**2*d**2*x**2 + 3*a*b**
4*d*x + 6*a*b**3*c**2*x - 9*a*b**3*c*d*x**2 - 2*a*b**3*d**2*x**3 + 3*b**5*
d*x**2 + 3*b**4*c**2*x**2 + 3*b**4*c*d*x**3 + b**4*d**2*x**4)/(3*b**5*(a +
 b*x))
```

3.7 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^3} \, dx$

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Optimal result

Integrand size = 28, antiderivative size = 176

$$\begin{aligned}
&\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3} \, dx \\
&= \frac{(bCd+bcD-3adD)x}{b^4} + \frac{dDx^2}{2b^3} - \frac{(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)}{2b^5(a+bx)^2} \\
&\quad - \frac{b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)}{b^5(a+bx)} \\
&\quad + \frac{(b^2(cC+Bd)+6a^2dD-3ab(Cd+cD))\log(a+bx)}{b^5}
\end{aligned}$$

output
```
(C*b*d-3*D*a*d+D*b*c)*x/b^4+1/2*d*D*x^2/b^3-1/2*(-a*d+b*c)*(A*b^3-a*(B*b^2
-C*a*b+D*a^2))/b^5/(b*x+a)^2-(b^3*(A*d+B*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*
a^2*b*(C*d+D*c))/b^5/(b*x+a)+(b^2*(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))*ln(
b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$$
$$= \frac{2b(bCd+bcD-3adD)x+b^2dDx^2-\frac{(bc-ad)(Ab^3-a(b^2B-abC+a^2D))}{(a+bx)^2}+\frac{-2b^3(Bc+Ad)+4ab^2(cC+Bd)+8a^3dD-6a^2b(Cd}{a+bx}}{2b^5}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
(2*b*(b*C*d + b*c*D - 3*a*d*D)*x + b^2*d*D*x^2 - ((b*c - a*d)*(A*b^3 - a*(
b^2*B - a*b*C + a^2*D)))/(a + b*x)^2 + (-2*b^3*(B*c + A*d) + 4*a*b^2*(c*C
+ B*d) + 8*a^3*d*D - 6*a^2*b*(C*d + c*D))/(a + b*x) + 2*(b^2*(c*C + B*d) +
 6*a^2*d*D - 3*a*b*(C*d + c*D))*Log[a + b*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{(bc-ad)(Ab^3-a(a^2D-abC+b^2B))}{b^4(a+bx)^3}+\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{b^4(a+bx)}+\frac{-4a^3dD+3a^2b(cD}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^5(a+bx)^2}+\frac{\log(a+bx)\left(6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)\right)}{b^5}-\frac{-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)}{b^5(a+bx)}+\frac{x(-3adD+bcD+bCd)}{b^4}+\frac{dDx^2}{2b^3}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
((b*C*d + b*c*D - 3*a*d*D)*x)/b^4 + (d*D*x^2)/(2*b^3) - ((b*c - a*d)*(A*b^
3 - a*(b^2*B - a*b*C + a^2*D)))/(2*b^5*(a + b*x)^2) - (b^3*(B*c + A*d) - 2
*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D))/(b^5*(a + b*x)) + ((
b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D))*Log[a + b*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\frac{(Cbd-2Dad+Dbc)x^3}{b^2}-\frac{Aab^3d+Ab^4c-3Ba^2b^2d+Bab^3c+9Ca^3bd-3Ca^2b^2c-18Da^4d+9Da^3bc}{2b^5}-\frac{\left(b^3dA-2Bab^2d+Bb^3c+6Ca^2bd-2c\right.}{b^4}}{(bx+a)^2}$
default	$\frac{\frac{1}{2}dDx^2b+Cbdx-3Dadx+Dbcx}{b^4}-\frac{-Aab^3d+Ab^4c+Ba^2b^2d-Bab^3c-Ca^3bd+Ca^2b^2c+Da^4d-Da^3bc}{2b^5(bx+a)^2}-\frac{b^3dA-2Bab^2c}{}$
parallelrisch	$-\frac{6D\ln(bx+a)a^3bc+4Dx^3ab^3d-18Da^4d-2B\ln(bx+a)a^2b^2d-4Bxab^3d+12Cxa^2b^2d-4Cxab^3c-24Dxa^3bd+12Dxa^2b^2c}{}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
((C*b*d-2*D*a*d+D*b*c)/b^2*x^3-1/2*(A*a*b^3*d+A*b^4*c-3*B*a^2*b^2*d+B*a*b^
3*c+9*C*a^3*b*d-3*C*a^2*b^2*c-18*D*a^4*d+9*D*a^3*b*c)/b^5-(A*b^3*d-2*B*a*b
^2*d+B*b^3*c+6*C*a^2*b*d-2*C*a*b^2*c-12*D*a^3*d+6*D*a^2*b*c)/b^4*x+1/2*d*D
*x^4/b)/(b*x+a)^2+1/b^5*(B*b^2*d-3*C*a*b*d+C*b^2*c+6*D*a^2*d-3*D*a*b*c)*ln
(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(172) = 344$.

Time = 0.08 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.10

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$$
$$= \frac{Db^4dx^4 + 2\,(Db^4c - (2\,Dab^3 - Cb^4)d)x^3 + (4\,Dab^3c - (11\,Da^2b^2 - 4\,Cab^3)d)x^2 - (5\,Da^3b - 3\,Ca^2b^2 +}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(D*b^4*d*x^4 + 2*(D*b^4*c - (2*D*a*b^3 - C*b^4)*d)*x^3 + (4*D*a*b^3*c
- (11*D*a^2*b^2 - 4*C*a*b^3)*d)*x^2 - (5*D*a^3*b - 3*C*a^2*b^2 + B*a*b^3 +
 A*b^4)*c + (7*D*a^4 - 5*C*a^3*b + 3*B*a^2*b^2 - A*a*b^3)*d - 2*((2*D*a^2*
b^2 - 2*C*a*b^3 + B*b^4)*c - (D*a^3*b - 2*C*a^2*b^2 + 2*B*a*b^3 - A*b^4)*d
)*x - 2*(((3*D*a*b^3 - C*b^4)*c - (6*D*a^2*b^2 - 3*C*a*b^3 + B*b^4)*d)*x^2
 + (3*D*a^3*b - C*a^2*b^2)*c - (6*D*a^4 - 3*C*a^3*b + B*a^2*b^2)*d + 2*((3
*D*a^2*b^2 - C*a*b^3)*c - (6*D*a^3*b - 3*C*a^2*b^2 + B*a*b^3)*d)*x)*log(b*
x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [A] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx = \frac{Ddx^2}{2b^3}+x\left(\frac{Cd}{b^3}-\frac{3Dad}{b^4}+\frac{Dc}{b^3}\right) + \frac{-Aab^3d-Ab^4c+3Ba^2b^2d-Bab^3c-5Ca^3bd+3Ca^2b^2c+7Da^4d-5Da^3bc+x(-2Ab^4d+4Bab^3d}{2a^2b^5+4ab^6x+2b^7x^2} + \frac{(Bb^2d-3Cabd+Cb^2c+6Da^2d-3Dabc)\log{(a+bx)}}{b^5}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**3,x)
```

output
```
D*d*x**2/(2*b**3) + x*(C*d/b**3 - 3*D*a*d/b**4 + D*c/b**3) + (-A*a*b**3*d
- A*b**4*c + 3*B*a**2*b**2*d - B*a*b**3*c - 5*C*a**3*b*d + 3*C*a**2*b**2*c
 + 7*D*a**4*d - 5*D*a**3*b*c + x*(-2*A*b**4*d + 4*B*a*b**3*d - 2*B*b**4*c
- 6*C*a**2*b**2*d + 4*C*a*b**3*c + 8*D*a**3*b*d - 6*D*a**2*b**2*c))/(2*a**
2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + (B*b**2*d - 3*C*a*b*d + C*b**2*c + 6*
D*a**2*d - 3*D*a*b*c)*log(a + b*x)/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx = -\frac{(5\,Da^3b-3\,Ca^2b^2+Bab^3+Ab^4)c-(7\,Da^4-5\,Ca^3b+3\,Ba^2b^2-Aab^3)d+2\,((3\,Da^2b^2-2\,Cab^3+}{2\,(b^7x^2+2\,ab^6x+a^2b^5)} + \frac{Dbdx^2+2\,(Dbc-(3\,Da-Cb)d)x}{2\,b^4} - \frac{((3\,Dab-Cb^2)c-(6\,Da^2-3\,Cab+Bb^2)d)\log{(bx+a)}}{b^5}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((5*D*a^3*b - 3*C*a^2*b^2 + B*a*b^3 + A*b^4)*c - (7*D*a^4 - 5*C*a^3*b
 + 3*B*a^2*b^2 - A*a*b^3)*d + 2*((3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c - (4*
D*a^3*b - 3*C*a^2*b^2 + 2*B*a*b^3 - A*b^4)*d)*x)/(b^7*x^2 + 2*a*b^6*x + a^
2*b^5) + 1/2*(D*b*d*x^2 + 2*(D*b*c - (3*D*a - C*b)*d)*x)/b^4 - ((3*D*a*b -
 C*b^2)*c - (6*D*a^2 - 3*C*a*b + B*b^2)*d)*log(b*x + a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$
$$= -\frac{(3\,Dabc - Cb^2c - 6\,Da^2d + 3\,Cabd - Bb^2d)\log\left(|bx+a|\right)}{b^5} + \frac{Db^3dx^2 + 2\,Db^3cx - 6\,Dab^2dx + 2\,Cb^3dx}{2\,b^6} - \frac{5\,Da^3bc - 3\,Ca^2b^2c + Bab^3c + Ab^4c - 7\,Da^4d + 5\,Ca^3bd - 3\,Ba^2b^2d + Aab^3d + 2\,(3\,Da^2b^2c - 2\,Cab^3}{2\,(bx+a)^2b^5}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-(3*D*a*b*c - C*b^2*c - 6*D*a^2*d + 3*C*a*b*d - B*b^2*d)*log(abs(b*x + a))
/b^5 + 1/2*(D*b^3*d*x^2 + 2*D*b^3*c*x - 6*D*a*b^2*d*x + 2*C*b^3*d*x)/b^6 -
 1/2*(5*D*a^3*b*c - 3*C*a^2*b^2*c + B*a*b^3*c + A*b^4*c - 7*D*a^4*d + 5*C*
a^3*b*d - 3*B*a^2*b^2*d + A*a*b^3*d + 2*(3*D*a^2*b^2*c - 2*C*a*b^3*c + B*b
^4*c - 4*D*a^3*b*d + 3*C*a^2*b^2*d - 2*B*a*b^3*d + A*b^4*d)*x)/((b*x + a)^
2*b^5)
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.97

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$
$$= \frac{\frac{3\,B\,a^2\,d}{2\,b^3}+\frac{2\,B\,a\,d\,x}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2} - \frac{\frac{B\,a\,c}{2\,b^2}+\frac{B\,c\,x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2} - \frac{\frac{A\,a\,d}{2\,b^2}+\frac{A\,d\,x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$
$$+ \frac{\frac{3\,C\,a^2\,c}{2\,b^3}+\frac{2\,C\,a\,c\,x}{b^2}}{a^2+2\,a\,b\,x+b^2\,x^2} + \frac{B\,d\,\ln\left(a+b\,x\right)}{b^3} + \frac{C\,c\,\ln\left(a+b\,x\right)}{b^3}$$
$$+ \frac{d\,D\left(\frac{(a+b\,x)^2}{2}+\frac{4\,a^3}{a+b\,x}-\frac{a^4}{2\,(a+b\,x)^2}+6\,a^2\,\ln\left(a+b\,x\right)-4\,a\,b\,x\right)}{b^5}$$
$$- \frac{C\,d\left(3\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{3\,a^2}{a+b\,x}-\frac{a^3}{2\,(a+b\,x)^2}\right)}{b^4}$$
$$- \frac{A\,c}{2\,b\,\left(a^2+2\,a\,b\,x+b^2\,x^2\right)} - \frac{c\,D\left(3\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{3\,a^2}{a+b\,x}-\frac{a^3}{2\,(a+b\,x)^2}\right)}{b^4}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3,x)
```

output
```
((3*B*a^2*d)/(2*b^3) + (2*B*a*d*x)/b^2)/(a^2 + b^2*x^2 + 2*a*b*x) - ((B*a*
c)/(2*b^2) + (B*c*x)/b)/(a^2 + b^2*x^2 + 2*a*b*x) - ((A*a*d)/(2*b^2) + (A*
d*x)/b)/(a^2 + b^2*x^2 + 2*a*b*x) + ((3*C*a^2*c)/(2*b^3) + (2*C*a*c*x)/b^2
)/(a^2 + b^2*x^2 + 2*a*b*x) + (B*d*log(a + b*x))/b^3 + (C*c*log(a + b*x))/
b^3 + (d*D*((a + b*x)^2/2 + (4*a^3)/(a + b*x) - a^4/(2*(a + b*x)^2) + 6*a^
2*log(a + b*x) - 4*a*b*x))/b^5 - (C*d*(3*a*log(a + b*x) - b*x + (3*a^2)/(a
 + b*x) - a^3/(2*(a + b*x)^2)))/b^4 - (A*c)/(2*b*(a^2 + b^2*x^2 + 2*a*b*x)
) - (c*D*(3*a*log(a + b*x) - b*x + (3*a^2)/(a + b*x) - a^3/(2*(a + b*x)^2)
))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.06

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \frac{12\log(bx+a)\,a^5d^2+6a^5d^2-12\log(bx+a)\,a^4bcd+24\log(bx+a)\,a^4b\,d^2x+12\log(bx+a)\,a^3b^2d^2x^2+}{}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
(12*log(a + b*x)*a**5*d**2 - 12*log(a + b*x)*a**4*b*c*d + 24*log(a + b*x)*
a**4*b*d**2*x + 2*log(a + b*x)*a**3*b**3*d + 2*log(a + b*x)*a**3*b**2*c**2
 - 24*log(a + b*x)*a**3*b**2*c*d*x + 12*log(a + b*x)*a**3*b**2*d**2*x**2 +
 4*log(a + b*x)*a**2*b**4*d*x + 4*log(a + b*x)*a**2*b**3*c**2*x - 12*log(a
 + b*x)*a**2*b**3*c*d*x**2 + 2*log(a + b*x)*a*b**5*d*x**2 + 2*log(a + b*x)
*a*b**4*c**2*x**2 + 6*a**5*d**2 - 6*a**4*b*c*d + a**3*b**3*d + a**3*b**2*c
**2 - 12*a**3*b**2*d**2*x**2 - a**2*b**4*c + 12*a**2*b**3*c*d*x**2 - 4*a**
2*b**3*d**2*x**3 - a*b**5*d*x**2 - 2*a*b**4*c**2*x**2 + 4*a*b**4*c*d*x**3
+ a*b**4*d**2*x**4 + b**6*c*x**2)/(2*a*b**5*(a**2 + 2*a*b*x + b**2*x**2))
```

3.8 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^4} dx$

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Optimal result

Integrand size = 28, antiderivative size = 180

$$\begin{aligned}\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx &= \frac{dDx}{b^4}-\frac{(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)}{3b^5(a+bx)^3}\\ &\quad-\frac{b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)}{2b^5(a+bx)^2}\\ &\quad-\frac{b^2(cC+Bd)+6a^2dD-3ab(Cd+cD)}{b^5(a+bx)}+\frac{(bCd+bcD-4adD)\log(a+bx)}{b^5}\end{aligned}$$

output

```
d*D*x/b^4-1/3*(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^5/(b*x+a)^3-1/2*(
b^3*(A*d+B*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))/b^5/(b*x+a)^2
-(b^2*(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))/b^5/(b*x+a)+(C*b*d-4*D*a*d+D*b*
c)*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx$$
$$= \frac{6bdDx-\frac{2(bc-ad)\left(Ab^3-a\left(b^2B-abC+a^2D\right)\right)}{(a+bx)^3}+\frac{-3b^3(Bc+Ad)+6ab^2(cC+Bd)+12a^3dD-9a^2b(Cd+cD)}{(a+bx)^2}-\frac{6\left(b^2(cC+Bd)+6a^2dD-3a\right.}{a+bx}}{6b^5}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
(6*b*d*D*x - (2*(b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(a + b*x)
^3 + (-3*b^3*(B*c + A*d) + 6*a*b^2*(c*C + B*d) + 12*a^3*d*D - 9*a^2*b*(C*d
 + c*D))/(a + b*x)^2 - (6*(b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D)
))/(a + b*x) + 6*(b*C*d + b*c*D - 4*a*d*D)*Log[a + b*x])/(6*b^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4(a+bx)^4}+\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{b^4(a+bx)^2}+\frac{-4a^3dD+3a^2b(cD\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^5(a+bx)^3}-\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{b^5(a+bx)}-\frac{-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)}{2b^5(a+bx)^2}+\frac{\log(a+bx)(-4adD+bcD+bCd)}{b^5}+\frac{dDx}{b^4}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
(d*D*x)/b^4 - ((b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(3*b^5*(a
+ b*x)^3) - (b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(
C*d + c*D))/(2*b^5*(a + b*x)^2) - (b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*
d + c*D))/(b^5*(a + b*x)) + ((b*C*d + b*c*D - 4*a*d*D)*Log[a + b*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.18

method	result
norman	$\frac{\frac{dDx^4}{b}-\frac{Aa\,b^3d+2A\,b^4c+2B\,a^2b^2d+Ba\,b^3c-11C\,a^3bd+2C\,a^2b^2c+44Da^4d-11Da^3bc}{6b^5}-\frac{\left(b^2Bd-3Cabd+C\,b^2c+12a^2dD-3Dabc\right)x^2}{b^3}-(b^3}{(bx+a)^3}$
default	$\frac{dDx}{b^4}-\frac{b^3dA-2Ba\,b^2d+B\,b^3c+3C\,a^2bd-2a\,b^2cC-4a^3dD+3a^2bcD}{2b^5(bx+a)^2}-\frac{b^2Bd-3Cabd+C\,b^2c+6a^2dD-3Dabc}{b^5(bx+a)}+(Cbd-$
parallelrisch	$-\frac{-6D\ln(bx+a)a^3bc-18C\,x^2a\,b^3d+72Dx^2a^2b^2d-18Dx^2a\,b^3c+44Da^4d+6Bxa\,b^3d-27Cx\,a^2b^2d+6Cxa\,b^3c+108Dx\,a^3bd-}{}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
(d*D*x^4/b-1/6*(A*a*b^3*d+2*A*b^4*c+2*B*a^2*b^2*d+B*a*b^3*c-11*C*a^3*b*d+2
*C*a^2*b^2*c+44*D*a^4*d-11*D*a^3*b*c)/b^5-(B*b^2*d-3*C*a*b*d+C*b^2*c+12*D*
a^2*d-3*D*a*b*c)/b^3*x^2-1/2*(A*b^3*d+2*B*a*b^2*d+B*b^3*c-9*C*a^2*b*d+2*C*
a*b^2*c+36*D*a^3*d-9*D*a^2*b*c)/b^4*x)/(b*x+a)^3+(C*b*d-4*D*a*d+D*b*c)*ln(
b*x+a)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(177) = 354$.

Time = 0.08 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \frac{6\,Db^4dx^4 + 18\,Dab^3dx^3 + 6\,((3\,Dab^3 - Cb^4)c - (3\,Da^2b^2 - 3\,Cab^3 + Bb^4)d)x^2 + (11\,Da^3b - 2\,Ca^2b^2 -}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(6*D*b^4*d*x^4 + 18*D*a*b^3*d*x^3 + 6*((3*D*a*b^3 - C*b^4)*c - (3*D*a^
2*b^2 - 3*C*a*b^3 + B*b^4)*d)*x^2 + (11*D*a^3*b - 2*C*a^2*b^2 - B*a*b^3 -
2*A*b^4)*c - (26*D*a^4 - 11*C*a^3*b + 2*B*a^2*b^2 + A*a*b^3)*d + 3*((9*D*a
^2*b^2 - 2*C*a*b^3 - B*b^4)*c - (18*D*a^3*b - 9*C*a^2*b^2 + 2*B*a*b^3 + A*
b^4)*d)*x + 6*(D*a^3*b*c + (D*b^4*c - (4*D*a*b^3 - C*b^4)*d)*x^3 + 3*(D*a*
b^3*c - (4*D*a^2*b^2 - C*a*b^3)*d)*x^2 - (4*D*a^4 - C*a^3*b)*d + 3*(D*a^2*
b^2*c - (4*D*a^3*b - C*a^2*b^2)*d)*x)*log(b*x + a))/(b^8*x^3 + 3*a*b^7*x^2
 + 3*a^2*b^6*x + a^3*b^5)
```

Sympy [A] (verification not implemented)

Time = 31.20 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \frac{Ddx}{b^4} + \frac{-Aab^3d - 2Ab^4c - 2Ba^2b^2d - Bab^3c + 11Ca^3bd - 2Ca^2b^2c - 26Da^4d + 11Da^3bc + x^2(-6Bb^4d + 18}{6a^3b^5 +} - \frac{(-Cbd + 4Dad - Dbc)\log(a+bx)}{b^5}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**4,x)
```

output
```
D*d*x/b**4 + (-A*a*b**3*d - 2*A*b**4*c - 2*B*a**2*b**2*d - B*a*b**3*c + 11
*C*a**3*b*d - 2*C*a**2*b**2*c - 26*D*a**4*d + 11*D*a**3*b*c + x**2*(-6*B*b
**4*d + 18*C*a*b**3*d - 6*C*b**4*c - 36*D*a**2*b**2*d + 18*D*a*b**3*c) + x
*(-3*A*b**4*d - 6*B*a*b**3*d - 3*B*b**4*c + 27*C*a**2*b**2*d - 6*C*a*b**3*
c - 60*D*a**3*b*d + 27*D*a**2*b**2*c))/(6*a**3*b**5 + 18*a**2*b**6*x + 18*
a*b**7*x**2 + 6*b**8*x**3) - (-C*b*d + 4*D*a*d - D*b*c)*log(a + b*x)/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx = \frac{6\left((3\,Dab^3-Cb^4)c-(6\,Da^2b^2-3\,Cab^3+Bb^4)d\right)x^2+(11\,Da^3b-2\,Ca^2b^2-Bab^3-2\,Ab^4)c-(26\,Da^4}{6\left(b^8x^3+3\,ab^7x\right.} + \frac{Ddx}{b^4} + \frac{(Dbc-(4\,Da-Cb)d)\log\left(bx+a\right)}{b^5}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/6*(6*((3*D*a*b^3 - C*b^4)*c - (6*D*a^2*b^2 - 3*C*a*b^3 + B*b^4)*d)*x^2 +
 (11*D*a^3*b - 2*C*a^2*b^2 - B*a*b^3 - 2*A*b^4)*c - (26*D*a^4 - 11*C*a^3*b
 + 2*B*a^2*b^2 + A*a*b^3)*d + 3*((9*D*a^2*b^2 - 2*C*a*b^3 - B*b^4)*c - (20
*D*a^3*b - 9*C*a^2*b^2 + 2*B*a*b^3 + A*b^4)*d)*x)/(b^8*x^3 + 3*a*b^7*x^2 +
 3*a^2*b^6*x + a^3*b^5) + D*d*x/b^4 + (D*b*c - (4*D*a - C*b)*d)*log(b*x +
a)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx = \frac{Ddx}{b^4}+\frac{(Dbc-4\,Dad+Cbd)\log\left(|bx+a|\right)}{b^5} + \frac{11\,Da^3bc-2\,Ca^2b^2c-Bab^3c-2\,Ab^4c-26\,Da^4d+11\,Ca^3bd-2\,Ba^2b^2d-Aab^3d+6\,(3\,Dab^3c-Cb^4c\cdots}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="giac")
```

output
```
D*d*x/b^4 + (D*b*c - 4*D*a*d + C*b*d)*log(abs(b*x + a))/b^5 + 1/6*(11*D*a^
3*b*c - 2*C*a^2*b^2*c - B*a*b^3*c - 2*A*b^4*c - 26*D*a^4*d + 11*C*a^3*b*d
- 2*B*a^2*b^2*d - A*a*b^3*d + 6*(3*D*a*b^3*c - C*b^4*c - 6*D*a^2*b^2*d + 3
*C*a*b^3*d - B*b^4*d)*x^2 + 3*(9*D*a^2*b^2*c - 2*C*a*b^3*c - B*b^4*c - 20*
D*a^3*b*d + 9*C*a^2*b^2*d - 2*B*a*b^3*d - A*b^4*d)*x)/((b*x + a)^3*b^5)
```

Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.28

$$\begin{aligned}
&\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx \\
&= \frac{c\,D\left(\ln\left(a+b\,x\right)+\frac{3\,a}{a+b\,x}-\frac{3\,a^2}{2\,{\left(a+b\,x\right)}^2}+\frac{a^3}{3\,{\left(a+b\,x\right)}^3}\right)}{b^4} \\
&\quad - \frac{C\,c\,a^2+3\,C\,c\,a\,b\,x+3\,C\,c\,b^2\,x^2}{3\,a^3\,b^3+9\,a^2\,b^4\,x+9\,a\,b^5\,x^2+3\,b^6\,x^3} - \frac{\frac{A\,a\,d}{6\,b^2}+\frac{A\,d\,x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3} \\
&\quad - \frac{\frac{B\,a\,c}{6\,b^2}+\frac{B\,c\,x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3} - \frac{B\,d\,a^2+3\,B\,d\,a\,b\,x+3\,B\,d\,b^2\,x^2}{3\,a^3\,b^3+9\,a^2\,b^4\,x+9\,a\,b^5\,x^2+3\,b^6\,x^3} \\
&\quad - \frac{d\,D\left(4\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{6\,a^2}{a+b\,x}-\frac{2\,a^3}{{\left(a+b\,x\right)}^2}+\frac{a^4}{3\,{\left(a+b\,x\right)}^3}\right)}{b^5} \\
&\quad - \frac{A\,c}{3\,b\,\left(a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3\right)} \\
&\quad + \frac{C\,d\left(\ln\left(a+b\,x\right)+\frac{3\,a}{a+b\,x}-\frac{3\,a^2}{2\,{\left(a+b\,x\right)}^2}+\frac{a^3}{3\,{\left(a+b\,x\right)}^3}\right)}{b^4}
\end{aligned}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4,x)
```

output
```
(c*D*(log(a + b*x) + (3*a)/(a + b*x) - (3*a^2)/(2*(a + b*x)^2) + a^3/(3*(a
 + b*x)^3)))/b^4 - (C*a^2*c + 3*C*b^2*c*x^2 + 3*C*a*b*c*x)/(3*a^3*b^3 + 3*
b^6*x^3 + 9*a^2*b^4*x + 9*a*b^5*x^2) - ((A*a*d)/(6*b^2) + (A*d*x)/(2*b))/(
a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x) - ((B*a*c)/(6*b^2) + (B*c*x)/(2*b
))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x) - (B*a^2*d + 3*B*b^2*d*x^2 +
3*B*a*b*d*x)/(3*a^3*b^3 + 3*b^6*x^3 + 9*a^2*b^4*x + 9*a*b^5*x^2) - (d*D*(4
*a*log(a + b*x) - b*x + (6*a^2)/(a + b*x) - (2*a^3)/(a + b*x)^2 + a^4/(3*(
a + b*x)^3)))/b^5 - (A*c)/(3*b*(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x))
+ (C*d*(log(a + b*x) + (3*a)/(a + b*x) - (3*a^2)/(2*(a + b*x)^2) + a^3/(3*
(a + b*x)^3)))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \frac{-24\log(bx+a)\,a^5d^2+12\log(bx+a)\,a^4bcd-72\log(bx+a)\,a^4b\,d^2x+36\log(bx+a)\,a^3b^2cdx-72\log(b$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x)
```

output
```
( - 24*log(a + b*x)*a**5*d**2 + 12*log(a + b*x)*a**4*b*c*d - 72*log(a + b*
x)*a**4*b*d**2*x + 36*log(a + b*x)*a**3*b**2*c*d*x - 72*log(a + b*x)*a**3*
b**2*d**2*x**2 + 36*log(a + b*x)*a**2*b**3*c*d*x**2 - 24*log(a + b*x)*a**2
*b**3*d**2*x**3 + 12*log(a + b*x)*a*b**4*c*d*x**3 - 20*a**5*d**2 + 10*a**4
*b*c*d - 36*a**4*b*d**2*x - a**3*b**3*d + 18*a**3*b**2*c*d*x - 3*a**2*b**4
*c - 3*a**2*b**4*d*x + 24*a**2*b**3*d**2*x**3 - 3*a*b**5*c*x - 12*a*b**4*c
*d*x**3 + 6*a*b**4*d**2*x**4 + 2*b**6*d*x**3 + 2*b**5*c**2*x**3)/(6*a*b**5
*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.9 $\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 189

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx = -\frac{(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)}{4b^5(a+bx)^4} - \frac{b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)}{3b^5(a+bx)^3} - \frac{b^2(cC+Bd)+6a^2dD-3ab(Cd+cD)}{2b^5(a+bx)^2} - \frac{bCd+bcD-4adD}{b^5(a+bx)} + \frac{dD\log(a+bx)}{b^5}$$

output
```
-1/4*(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^5/(b*x+a)^4-1/3*(b^3*(A*d+
B*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))/b^5/(b*x+a)^3-1/2*(b^2
*(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))/b^5/(b*x+a)^2-(C*b*d-4*D*a*d+D*b*c)/
b^5/(b*x+a)+d*D*ln(b*x+a)/b^5
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx = -\frac{\frac{3(bc-ad)\left(Ab^3-a\left(b^2B-abC+a^2D\right)\right)}{(a+bx)^4}+\frac{4\left(b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)\right)}{(a+bx)^3}+\frac{6\left(b^2(cC+Bd)+6a^2dD-3ab(Cd+cD\right.}{(a+bx)^2}}{12b^5}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^5,x]
```

output
```
-1/12*((3*(b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(a + b*x)^4 + (
4*(b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D)
))/(a + b*x)^3 + (6*(b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D)))/(a
+ b*x)^2 + (12*(b*C*d + b*c*D - 4*a*d*D))/(a + b*x) - 12*d*D*Log[a + b*x])
/b^5
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4(a+bx)^5}+\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{b^4(a+bx)^3}+\frac{-4a^3dD+3a^2b(cD-\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4b^5(a+bx)^4}-\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{2b^5(a+bx)^2}-\frac{-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)}{3b^5(a+bx)^3}-\frac{-4adD+bcD+bCd}{b^5(a+bx)}+\frac{dD\log(a+bx)}{b^5}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^5,x]
```

output
```
-1/4*((b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^5*(a + b*x)^4) -
 (b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D))
/(3*b^5*(a + b*x)^3) - (b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D))/(
2*b^5*(a + b*x)^2) - (b*C*d + b*c*D - 4*a*d*D)/(b^5*(a + b*x)) + (d*D*Log[
a + b*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.11

method	result
norman	$\frac{-\frac{Aa\,b^3d+3A\,b^4c+B\,a^2b^2d+Ba\,b^3c+3C\,a^3bd+C\,a^2b^2c-25Da^4d+3Da^3bc}{12b^5}-\frac{(Cbd-4Dad+Dbc)x^3}{b^2}-\frac{(b^2Bd+3Cabd+C\,b^2c-18a^2dD+3D}{2b^3}}{(bx+a)^4}$
default	$-\frac{-Aa\,b^3d+A\,b^4c+B\,a^2b^2d-Ba\,b^3c-C\,a^3bd+C\,a^2b^2c+Da^4d-Da^3bc}{4b^5(bx+a)^4}-\frac{b^2Bd-3Cabd+C\,b^2c+6a^2dD-3Dabc}{2b^5(bx+a)^2}-\frac{Cbd-}{b^5}$
parallelrisch	$-\frac{-48Dx^3a\,b^3d+18C\,x^2a\,b^3d-108Dx^2a^2b^2d+18Dx^2a\,b^3c-25Da^4d+4Bxa\,b^3d+12Cx\,a^2b^2d+4Cxa\,b^3c-88Dx\,a^3bd+12D}{}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/12*(A*a*b^3*d+3*A*b^4*c+B*a^2*b^2*d+B*a*b^3*c+3*C*a^3*b*d+C*a^2*b^2*c-
25*D*a^4*d+3*D*a^3*b*c)/b^5-(C*b*d-4*D*a*d+D*b*c)/b^2*x^3-1/2*(B*b^2*d+3*C
*a*b*d+C*b^2*c-18*D*a^2*d+3*D*a*b*c)/b^3*x^2-1/3*(A*b^3*d+B*a*b^2*d+B*b^3*
c+3*C*a^2*b*d+C*a*b^2*c-22*D*a^3*d+3*D*a^2*b*c)/b^4*x)/(b*x+a)^4+d*D*ln(b*
x+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx = -\frac{12\left(Db^4c-(4\,Dab^3-Cb^4)d\right)x^3+6\left((3\,Dab^3+Cb^4)c-(18\,Da^2b^2-3\,Cab^3-Bb^4)d\right)x^2+(3\,Da^3b+}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
-1/12*(12*(D*b^4*c - (4*D*a*b^3 - C*b^4)*d)*x^3 + 6*((3*D*a*b^3 + C*b^4)*c
 - (18*D*a^2*b^2 - 3*C*a*b^3 - B*b^4)*d)*x^2 + (3*D*a^3*b + C*a^2*b^2 + B*
a*b^3 + 3*A*b^4)*c - (25*D*a^4 - 3*C*a^3*b - B*a^2*b^2 - A*a*b^3)*d + 4*((
3*D*a^2*b^2 + C*a*b^3 + B*b^4)*c - (22*D*a^3*b - 3*C*a^2*b^2 - B*a*b^3 - A
*b^4)*d)*x - 12*(D*b^4*d*x^4 + 4*D*a*b^3*d*x^3 + 6*D*a^2*b^2*d*x^2 + 4*D*a
^3*b*d*x + D*a^4*d)*log(b*x + a))/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 +
 4*a^3*b^6*x + a^4*b^5)
```

Sympy [A] (verification not implemented)

Time = 161.85 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx = \frac{Dd\log(a+bx)}{b^5} + \frac{-Aab^3d-3Ab^4c-Ba^2b^2d-Bab^3c-3Ca^3bd-Ca^2b^2c+25Da^4d-3Da^3bc+x^3(-12Cb^4d+48Da}{}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**5,x)
```

output
```
D*d*log(a + b*x)/b**5 + (-A*a*b**3*d - 3*A*b**4*c - B*a**2*b**2*d - B*a*b*
*3*c - 3*C*a**3*b*d - C*a**2*b**2*c + 25*D*a**4*d - 3*D*a**3*b*c + x**3*(-
12*C*b**4*d + 48*D*a*b**3*d - 12*D*b**4*c) + x**2*(-6*B*b**4*d - 18*C*a*b*
*3*d - 6*C*b**4*c + 108*D*a**2*b**2*d - 18*D*a*b**3*c) + x*(-4*A*b**4*d -
4*B*a*b**3*d - 4*B*b**4*c - 12*C*a**2*b**2*d - 4*C*a*b**3*c + 88*D*a**3*b*
d - 12*D*a**2*b**2*c))/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2
+ 48*a*b**8*x**3 + 12*b**9*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^5}\,dx = -\frac{12\,(Db^4c-(4\,Dab^3-Cb^4)d)x^3+6\,((3\,Dab^3+Cb^4)c-(18\,Da^2b^2-3\,Cab^3-Bb^4)d)x^2+(3\,Da^3b+}{12\,(b^9}+\frac{Dd\log(bx+a)}{b^5}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/12*(12*(D*b^4*c - (4*D*a*b^3 - C*b^4)*d)*x^3 + 6*((3*D*a*b^3 + C*b^4)*c
 - (18*D*a^2*b^2 - 3*C*a*b^3 - B*b^4)*d)*x^2 + (3*D*a^3*b + C*a^2*b^2 + B*
a*b^3 + 3*A*b^4)*c - (25*D*a^4 - 3*C*a^3*b - B*a^2*b^2 - A*a*b^3)*d + 4*((
3*D*a^2*b^2 + C*a*b^3 + B*b^4)*c - (22*D*a^3*b - 3*C*a^2*b^2 - B*a*b^3 - A
*b^4)*d)*x)/(b^9*x^4 + 4*a*b^8*x^3 + 6*a^2*b^7*x^2 + 4*a^3*b^6*x + a^4*b^5
) + D*d*log(b*x + a)/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(184) = 368$.

Time = 0.12 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.06

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx = -\frac{Dd\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^5} - \frac{\frac{12\,Db^{16}c}{bx+a} - \frac{18\,Dab^{16}c}{(bx+a)^2} + \frac{12\,Da^2b^{16}c}{(bx+a)^3} - \frac{3\,Da^3b^{16}c}{(bx+a)^4} + \frac{6\,Cb^{17}c}{(bx+a)^2} - \frac{8\,Cab^{17}c}{(bx+a)^3} + \frac{3\,Ca^2b^{17}c}{(bx+a)^4} + \frac{4\,Bb^{18}c}{(bx+a)^3} - \frac{3\,Bab^{18}c}{(bx+a)^4} + \frac{3\,Ab^{19}c}{(bx+a)^4} - }{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x, algorithm="giac")
```

output
```
-D*d*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^5 - 1/12*(12*D*b^16*c/(b*x +
 a) - 18*D*a*b^16*c/(b*x + a)^2 + 12*D*a^2*b^16*c/(b*x + a)^3 - 3*D*a^3*b^
16*c/(b*x + a)^4 + 6*C*b^17*c/(b*x + a)^2 - 8*C*a*b^17*c/(b*x + a)^3 + 3*C
*a^2*b^17*c/(b*x + a)^4 + 4*B*b^18*c/(b*x + a)^3 - 3*B*a*b^18*c/(b*x + a)^
4 + 3*A*b^19*c/(b*x + a)^4 - 48*D*a*b^15*d/(b*x + a) + 36*D*a^2*b^15*d/(b*
x + a)^2 - 16*D*a^3*b^15*d/(b*x + a)^3 + 3*D*a^4*b^15*d/(b*x + a)^4 + 12*C
*b^16*d/(b*x + a) - 18*C*a*b^16*d/(b*x + a)^2 + 12*C*a^2*b^16*d/(b*x + a)^
3 - 3*C*a^3*b^16*d/(b*x + a)^4 + 6*B*b^17*d/(b*x + a)^2 - 8*B*a*b^17*d/(b*
x + a)^3 + 3*B*a^2*b^17*d/(b*x + a)^4 + 4*A*b^18*d/(b*x + a)^3 - 3*A*a*b^1
8*d/(b*x + a)^4)/b^20
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.33

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx$$
$$= \frac{c\,D\left(\frac{3\,a}{2\,(a+b\,x)^2}-\frac{1}{a+b\,x}-\frac{a^2}{(a+b\,x)^3}+\frac{a^3}{4\,(a+b\,x)^4}\right)}{b^4} - \frac{C\,c\,a^2+4\,C\,c\,a\,b\,x+6\,C\,c\,b^2\,x^2}{12\,a^4\,b^3+48\,a^3\,b^4\,x+72\,a^2\,b^5\,x^2+48\,a\,b^6\,x^3+12\,b^7\,x^4} - \frac{\frac{A\,a\,d}{12\,b^2}+\frac{A\,d\,x}{3\,b}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4} - \frac{\frac{B\,a\,c}{12\,b^2}+\frac{B\,c\,x}{3\,b}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4} - \frac{A\,c}{4\,b\,\left(a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4\right)} - \frac{B\,d\,a^2+4\,B\,d\,a\,b\,x+6\,B\,d\,b^2\,x^2}{12\,a^4\,b^3+48\,a^3\,b^4\,x+72\,a^2\,b^5\,x^2+48\,a\,b^6\,x^3+12\,b^7\,x^4} + \frac{d\,D\left(\ln(a+b\,x)+\frac{4\,a}{a+b\,x}-\frac{3\,a^2}{(a+b\,x)^2}+\frac{4\,a^3}{3\,(a+b\,x)^3}-\frac{a^4}{4\,(a+b\,x)^4}\right)}{b^5} + \frac{C\,d\,x^4}{4\,a\,(a+b\,x)^4}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^5,x)
```

output
```
(c*D*((3*a)/(2*(a + b*x)^2) - 1/(a + b*x) - a^2/(a + b*x)^3 + a^3/(4*(a +
b*x)^4)))/b^4 - (C*a^2*c + 6*C*b^2*c*x^2 + 4*C*a*b*c*x)/(12*a^4*b^3 + 12*b
^7*x^4 + 48*a^3*b^4*x + 48*a*b^6*x^3 + 72*a^2*b^5*x^2) - ((A*a*d)/(12*b^2)
 + (A*d*x)/(3*b))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x
) - ((B*a*c)/(12*b^2) + (B*c*x)/(3*b))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^
2*b^2*x^2 + 4*a^3*b*x) - (A*c)/(4*b*(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b
^2*x^2 + 4*a^3*b*x)) - (B*a^2*d + 6*B*b^2*d*x^2 + 4*B*a*b*d*x)/(12*a^4*b^3
 + 12*b^7*x^4 + 48*a^3*b^4*x + 48*a*b^6*x^3 + 72*a^2*b^5*x^2) + (d*D*(log(
a + b*x) + (4*a)/(a + b*x) - (3*a^2)/(a + b*x)^2 + (4*a^3)/(3*(a + b*x)^3)
 - a^4/(4*(a + b*x)^4)))/b^5 + (C*d*x^4)/(4*a*(a + b*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx$$
$$= \frac{12\log(bx+a)\,a^5d^2+48\log(bx+a)\,a^4b\,d^2x+72\log(bx+a)\,a^3b^2d^2x^2+48\log(bx+a)\,a^2b^3d^2x^3+12\log}{}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x)
```

output
```
(12*log(a + b*x)*a**5*d**2 + 48*log(a + b*x)*a**4*b*d**2*x + 72*log(a + b*
x)*a**3*b**2*d**2*x**2 + 48*log(a + b*x)*a**2*b**3*d**2*x**3 + 12*log(a +
b*x)*a*b**4*d**2*x**4 + 13*a**5*d**2 + 40*a**4*b*d**2*x - 2*a**3*b**3*d -
a**3*b**2*c**2 + 36*a**3*b**2*d**2*x**2 - 4*a**2*b**4*c - 8*a**2*b**4*d*x
- 4*a**2*b**3*c**2*x - 4*a*b**5*c*x - 6*a*b**5*d*x**2 - 6*a*b**4*c**2*x**2
 - 12*a*b**4*d**2*x**4 + 6*b**5*c*d*x**4)/(12*a*b**5*(a**4 + 4*a**3*b*x +
6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4))
```

3.10 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^6} \, dx$

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Optimal result

Integrand size = 28, antiderivative size = 193

$$\begin{aligned}\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6} \, dx &= -\frac{(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)}{5b^5(a+bx)^5} \\ &\quad -\frac{b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)}{4b^5(a+bx)^4} \\ &\quad -\frac{b^2(cC+Bd)+6a^2dD-3ab(Cd+cD)}{3b^5(a+bx)^3}-\frac{bCd+bcD-4adD}{2b^5(a+bx)^2}-\frac{dD}{b^5(a+bx)}\end{aligned}$$

output
```
-1/5*(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^5/(b*x+a)^5-1/4*(b^3*(A*d+
B*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))/b^5/(b*x+a)^4-1/3*(b^2
*(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))/b^5/(b*x+a)^3-1/2*(C*b*d-4*D*a*d+D*b
*c)/b^5/(b*x+a)^2-d*D/b^5/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx = -\frac{12a^4dD+3Ab^3(4bc+ad+5bdx)+3a^3b(Cd+D(c+20dx))+a^2b^2(2Bd+15dx(C+8Dx)+c(2C$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^6,x]
```

output
```
-1/60*(12*a^4*d*D + 3*A*b^3*(4*b*c + a*d + 5*b*d*x) + 3*a^3*b*(C*d + D*(c
+ 20*d*x)) + a^2*b^2*(2*B*d + 15*d*x*(C + 8*D*x) + c*(2*C + 15*D*x)) + 5*b
^4*x*(B*(3*c + 4*d*x) + 2*x*(3*d*x*(C + 2*D*x) + c*(2*C + 3*D*x))) + a*b^3
*(B*(3*c + 10*d*x) + 10*x*(c*(C + 3*D*x) + 3*d*x*(C + 4*D*x))))/(b^5*(a +
b*x)^5)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4(a+bx)^6}+\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{b^4(a+bx)^4}+\frac{-4a^3dD+3a^2b(cD-$$

$$\Big\downarrow \; 2009$$

$$-\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^5(a+bx)^5}-\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{3b^5(a+bx)^3}-\frac{-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)}{4b^5(a+bx)^4}-\frac{-4adD+bcD+bCd}{2b^5(a+bx)^2}-\frac{dD}{b^5(a+bx)}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^6,x]
```

output
```
-1/5*((b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^5*(a + b*x)^5) -
 (b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D))
/(4*b^5*(a + b*x)^4) - (b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D))/(
3*b^5*(a + b*x)^3) - (b*C*d + b*c*D - 4*a*d*D)/(2*b^5*(a + b*x)^2) - (d*D)
/(b^5*(a + b*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.12

method	result
norman	$\frac{-\frac{dDx^4}{b}-\frac{(Cbd+4Dad+Dbc)x^3}{2b^2}-\frac{\left(2b^2Bd+3Cabd+2Cb^2c+12a^2dD+3Dabc\right)x^2}{6b^3}-\frac{\left(3b^3dA+2Bab^2d+3Bb^3c+3Ca^2bd+2ab^2cC+12a^3dD\right.}{12b^4}}{(bx+a)^5}$
default	$-\frac{-Aab^3d+Ab^4c+Ba^2b^2d-Bab^3c-Ca^3bd+Ca^2b^2c+Da^4d-Da^3bc}{5b^5(bx+a)^5}-\frac{b^3dA-2Bab^2d+Bb^3c+3Ca^2bd-2ab^2cC-4a^3c}{4b^5(bx+a)^4}$
gosper	$-\frac{60dDx^4b^4+30Cx^3b^4d+120Dx^3ab^3d+30Dx^3b^4c+20Bx^2b^4d+30Cx^2ab^3d+20Cx^2b^4c+120Dx^2a^2b^2d+30Dx^2ab^3c+15}{}$
parallelrisch	$-\frac{60dDx^4b^4+30Cx^3b^4d+120Dx^3ab^3d+30Dx^3b^4c+20Bx^2b^4d+30Cx^2ab^3d+20Cx^2b^4c+120Dx^2a^2b^2d+30Dx^2ab^3c+15}{}$
orering	$-\frac{60dDx^4b^4+30Cx^3b^4d+120Dx^3ab^3d+30Dx^3b^4c+20Bx^2b^4d+30Cx^2ab^3d+20Cx^2b^4c+120Dx^2a^2b^2d+30Dx^2ab^3c+15}{}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/(b*x+a)^5*(-d*D*x^4/b-1/2*(C*b*d+4*D*a*d+D*b*c)/b^2*x^3-1/6*(2*B*b^2*d+3
*C*a*b*d+2*C*b^2*c+12*D*a^2*d+3*D*a*b*c)/b^3*x^2-1/12*(3*A*b^3*d+2*B*a*b^2
*d+3*B*b^3*c+3*C*a^2*b*d+2*C*a*b^2*c+12*D*a^3*d+3*D*a^2*b*c)/b^4*x-1/60*(3
*A*a*b^3*d+12*A*b^4*c+2*B*a^2*b^2*d+3*B*a*b^3*c+3*C*a^3*b*d+2*C*a^2*b^2*c+
12*D*a^4*d+3*D*a^3*b*c)/b^5)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx =$$
$$-\frac{60\,Db^4dx^4+30\,(Db^4c+(4\,Dab^3+Cb^4)d)x^3+10\,((3\,Dab^3+2\,Cb^4)c+(12\,Da^2b^2+3\,Cab^3+2\,Bb^4)d}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x, algorithm="fricas")
```

output
```
-1/60*(60*D*b^4*d*x^4 + 30*(D*b^4*c + (4*D*a*b^3 + C*b^4)*d)*x^3 + 10*((3*
D*a*b^3 + 2*C*b^4)*c + (12*D*a^2*b^2 + 3*C*a*b^3 + 2*B*b^4)*d)*x^2 + (3*D*
a^3*b + 2*C*a^2*b^2 + 3*B*a*b^3 + 12*A*b^4)*c + (12*D*a^4 + 3*C*a^3*b + 2*
B*a^2*b^2 + 3*A*a*b^3)*d + 5*((3*D*a^2*b^2 + 2*C*a*b^3 + 3*B*b^4)*c + (12*
D*a^3*b + 3*C*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*d)*x)/(b^10*x^5 + 5*a*b^9*x^4
 + 10*a^2*b^8*x^3 + 10*a^3*b^7*x^2 + 5*a^4*b^6*x + a^5*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^6}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^6}\,dx = -\frac{60\,Db^4dx^4+30\,(Db^4c+(4\,Dab^3+Cb^4)d)x^3+10\,((3\,Dab^3+2\,Cb^4)c+(12\,Da^2b^2+3\,Cab^3+2\,Bb^4)d}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/60*(60*D*b^4*d*x^4 + 30*(D*b^4*c + (4*D*a*b^3 + C*b^4)*d)*x^3 + 10*((3*
D*a*b^3 + 2*C*b^4)*c + (12*D*a^2*b^2 + 3*C*a*b^3 + 2*B*b^4)*d)*x^2 + (3*D*
a^3*b + 2*C*a^2*b^2 + 3*B*a*b^3 + 12*A*b^4)*c + (12*D*a^4 + 3*C*a^3*b + 2*
B*a^2*b^2 + 3*A*a*b^3)*d + 5*((3*D*a^2*b^2 + 2*C*a*b^3 + 3*B*b^4)*c + (12*
D*a^3*b + 3*C*a^2*b^2 + 2*B*a*b^3 + 3*A*b^4)*d)*x)/(b^10*x^5 + 5*a*b^9*x^4
 + 10*a^2*b^8*x^3 + 10*a^3*b^7*x^2 + 5*a^4*b^6*x + a^5*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx =$$
$$-\frac{60\,Db^4dx^4+30\,Db^4cx^3+120\,Dab^3dx^3+30\,Cb^4dx^3+30\,Dab^3cx^2+20\,Cb^4cx^2+120\,Da^2b^2dx^2+30}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x, algorithm="giac")
```

output
```
-1/60*(60*D*b^4*d*x^4 + 30*D*b^4*c*x^3 + 120*D*a*b^3*d*x^3 + 30*C*b^4*d*x^
3 + 30*D*a*b^3*c*x^2 + 20*C*b^4*c*x^2 + 120*D*a^2*b^2*d*x^2 + 30*C*a*b^3*d
*x^2 + 20*B*b^4*d*x^2 + 15*D*a^2*b^2*c*x + 10*C*a*b^3*c*x + 15*B*b^4*c*x +
 60*D*a^3*b*d*x + 15*C*a^2*b^2*d*x + 10*B*a*b^3*d*x + 15*A*b^4*d*x + 3*D*a
^3*b*c + 2*C*a^2*b^2*c + 3*B*a*b^3*c + 12*A*b^4*c + 12*D*a^4*d + 3*C*a^3*b
*d + 2*B*a^2*b^2*d + 3*A*a*b^3*d)/((b*x + a)^5*b^5)
```

Mupad [B] (verification not implemented)

Time = 4.41 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx$$
$$= \frac{C\,d\,x^4\,(5\,a+b\,x)}{20\,a^2\,{(a+b\,x)}^5}-\frac{d\,D\,\left(\frac{1}{a+b\,x}-\frac{2\,a}{{(a+b\,x)}^2}+\frac{2\,a^2}{{(a+b\,x)}^3}-\frac{a^3}{{(a+b\,x)}^4}+\frac{a^4}{5\,{(a+b\,x)}^5}\right)}{b^5}$$
$$-\frac{A\,d\,(a+5\,b\,x)}{20\,b^2\,{(a+b\,x)}^5}-\frac{B\,c\,(a+5\,b\,x)}{20\,b^2\,{(a+b\,x)}^5}$$
$$-\frac{B\,d\,(8\,a^2+40\,a\,b\,x+80\,b^2\,x^2)}{240\,b^3\,{(a+b\,x)}^5}-\frac{C\,c\,(8\,a^2+40\,a\,b\,x+80\,b^2\,x^2)}{240\,b^3\,{(a+b\,x)}^5}$$
$$-\frac{A\,c}{5\,b\,\left(a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5\right)}+\frac{c\,x^4\,(5\,a+b\,x)\,D}{20\,a^2\,{(a+b\,x)}^5}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^6,x)
```

output
```
(C*d*x^4*(5*a + b*x))/(20*a^2*(a + b*x)^5) - (d*D*(1/(a + b*x) - (2*a)/(a
+ b*x)^2 + (2*a^2)/(a + b*x)^3 - a^3/(a + b*x)^4 + a^4/(5*(a + b*x)^5)))/b
^5 - (A*d*(a + 5*b*x))/(20*b^2*(a + b*x)^5) - (B*c*(a + 5*b*x))/(20*b^2*(a
 + b*x)^5) - (B*d*(8*a^2 + 80*b^2*x^2 + 40*a*b*x))/(240*b^3*(a + b*x)^5) -
 (C*c*(8*a^2 + 80*b^2*x^2 + 40*a*b*x))/(240*b^3*(a + b*x)^5) - (A*c)/(5*b*
(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + 5*a^4*b*x
)) + (c*x^4*(5*a + b*x)*D)/(20*a^2*(a + b*x)^5)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx$$
$$= \frac{12b^4d^2x^5 - 60a\,b^3cd\,x^3 - 60a^2b^2cd\,x^2 - 20a\,b^4d\,x^2 - 20a\,b^3c^2x^2 - 30a^3bcdx - 25a^2b^3dx - 10a^2b^2c^2x - 1}{60a\,b^4\left(b^5x^5 + 5a\,b^4x^4 + 10a^2b^3x^3 + 10a^3b^2x^2 + 5a^4bx + a^5\right.}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x)
```

output
```
( - 6*a**4*c*d - 5*a**3*b**2*d - 2*a**3*b*c**2 - 30*a**3*b*c*d*x - 15*a**2
*b**3*c - 25*a**2*b**3*d*x - 10*a**2*b**2*c**2*x - 60*a**2*b**2*c*d*x**2 -
 15*a*b**4*c*x - 20*a*b**4*d*x**2 - 20*a*b**3*c**2*x**2 - 60*a*b**3*c*d*x*
*3 + 12*b**4*d**2*x**5)/(60*a*b**4*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2
+ 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))
```

3.11 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^7} dx$

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Optimal result

Integrand size = 28, antiderivative size = 195

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^7}\,dx = -\frac{(bc-ad)(Ab^3-a(b^2B-abC+a^2D))}{6b^5(a+bx)^6} - \frac{b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)}{5b^5(a+bx)^5} - \frac{b^2(cC+Bd)+6a^2dD-3ab(Cd+cD)}{4b^5(a+bx)^4} - \frac{bCd+bcD-4adD}{3b^5(a+bx)^3} - \frac{dD}{2b^5(a+bx)^2}$$

output
```
-1/6*(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^5/(b*x+a)^6-1/5*(b^3*(A*d+
B*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))/b^5/(b*x+a)^5-1/4*(b^2
*(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))/b^5/(b*x+a)^4-1/3*(C*b*d-4*D*a*d+D*b
*c)/b^5/(b*x+a)^3-1/2*d*D/b^5/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^7}\,dx = -\frac{2a^4dD+2Ab^3(5bc+ad+6bdx)+a^3b(Cd+D(c+12dx))+b^4x(3B(4c+5dx)+5x(3cC+4Cdx+$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^7,x]
```

output
```
-1/60*(2*a^4*d*D + 2*A*b^3*(5*b*c + a*d + 6*b*d*x) + a^3*b*(C*d + D*(c + 1
2*d*x)) + b^4*x*(3*B*(4*c + 5*d*x) + 5*x*(3*c*C + 4*C*d*x + 4*c*D*x + 6*d*
D*x^2)) + a^2*b^2*(c*(C + 6*D*x) + d*(B + 6*x*(C + 5*D*x))) + a*b^3*(2*B*(
c + 3*d*x) + x*(3*c*(2*C + 5*D*x) + 5*d*x*(3*C + 8*D*x))))/(b^5*(a + b*x)^
6)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^7}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4(a+bx)^7}+\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{b^4(a+bx)^5}+\frac{-4a^3dD+3a^2b(cD-$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{6b^5(a+bx)^6}-\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{4b^5(a+bx)^4}-\frac{-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)}{5b^5(a+bx)^5}-\frac{-4adD+bcD+bCd}{3b^5(a+bx)^3}-\frac{dD}{2b^5(a+bx)^2}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^7,x]
```

output
```
-1/6*((b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^5*(a + b*x)^6) -
 (b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D))
/(5*b^5*(a + b*x)^5) - (b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D))/(
4*b^5*(a + b*x)^4) - (b*C*d + b*c*D - 4*a*d*D)/(3*b^5*(a + b*x)^3) - (d*D)
/(2*b^5*(a + b*x)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.15

method	result
norman	$\frac{-\frac{dDx^4}{2b}-\frac{\left(d b^2 C+2Dabd+Db^2 c\right)x^3}{3b^3}-\frac{\left(b^3 Bd+Ca b^2 d+C b^3 c+2Da^2 bd+Da b^2 c\right)x^2}{4b^4}-\frac{\left(2Ad b^4+Ba b^3 d+2Bc b^4+C a^2 b^2 d+Ca b^3 c+2Da^3\right.}{10b^5}}{(bx+a)^6}$
default	$-\frac{b^3 dA-2Ba b^2 d+B b^3 c+3C a^2 bd-2a b^2 cC-4a^3 dD+3a^2 bcD}{5b^5 (bx+a)^5}-\frac{b^2 Bd-3Cabd+C b^2 c+6a^2 dD-3Dabc}{4b^5 (bx+a)^4}-\frac{dD}{2b^5 (bx+a)^2}-$
gosper	$-\frac{30dDx^4 b^4+20C x^3 b^4 d+40Dx^3 a b^3 d+20Dx^3 b^4 c+15B x^2 b^4 d+15C x^2 a b^3 d+15C x^2 b^4 c+30Dx^2 a^2 b^2 d+15Dx^2 a b^3 c+12Ax}{}$
orering	$-\frac{30dDx^4 b^4+20C x^3 b^4 d+40Dx^3 a b^3 d+20Dx^3 b^4 c+15B x^2 b^4 d+15C x^2 a b^3 d+15C x^2 b^4 c+30Dx^2 a^2 b^2 d+15Dx^2 a b^3 c+12Ax}{}$
parallelrisch	$-\frac{30Dd x^4 b^5+20C b^5 d x^3+40Da b^4 d x^3+20Db^5 c x^3+15B b^5 d x^2+15Ca b^4 d x^2+15C b^5 c x^2+30Da^2 b^3 d x^2+15Da b^4 c x^2+12}{}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/(b*x+a)^6*(-1/2*d*D*x^4/b-1/3*(C*b^2*d+2*D*a*b*d+D*b^2*c)/b^3*x^3-1/4*(B
*b^3*d+C*a*b^2*d+C*b^3*c+2*D*a^2*b*d+D*a*b^2*c)/b^4*x^2-1/10*(2*A*b^4*d+B*
a*b^3*d+2*B*b^4*c+C*a^2*b^2*d+C*a*b^3*c+2*D*a^3*b*d+D*a^2*b^2*c)/b^5*x-1/6
0*(2*A*a*b^4*d+10*A*b^5*c+B*a^2*b^3*d+2*B*a*b^4*c+C*a^3*b^2*d+C*a^2*b^3*c+
2*D*a^4*b*d+D*a^3*b^2*c)/b^6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^7}\,dx =$$
$$-\frac{30\,Db^4dx^4+20\,(Db^4c+(2\,Dab^3+Cb^4)d)x^3+15\,((Dab^3+Cb^4)c+(2\,Da^2b^2+Cab^3+Bb^4)d)x^2+(}{60\,(b^{11}x^6+6\,ab}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/60*(30*D*b^4*d*x^4 + 20*(D*b^4*c + (2*D*a*b^3 + C*b^4)*d)*x^3 + 15*((D*
a*b^3 + C*b^4)*c + (2*D*a^2*b^2 + C*a*b^3 + B*b^4)*d)*x^2 + (D*a^3*b + C*a
^2*b^2 + 2*B*a*b^3 + 10*A*b^4)*c + (2*D*a^4 + C*a^3*b + B*a^2*b^2 + 2*A*a*
b^3)*d + 6*((D*a^2*b^2 + C*a*b^3 + 2*B*b^4)*c + (2*D*a^3*b + C*a^2*b^2 + B
*a*b^3 + 2*A*b^4)*d)*x)/(b^11*x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3
*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a^5*b^6*x + a^6*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^7}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^7}\,dx = -\frac{30\,Db^4dx^4+20\,(Db^4c+(2\,Dab^3+Cb^4)d)x^3+15\,((Dab^3+Cb^4)c+(2\,Da^2b^2+Cab^3+Bb^4)d)x^2+(}{60\,(b^{11}x^6+6\,ab}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/60*(30*D*b^4*d*x^4 + 20*(D*b^4*c + (2*D*a*b^3 + C*b^4)*d)*x^3 + 15*((D*
a*b^3 + C*b^4)*c + (2*D*a^2*b^2 + C*a*b^3 + B*b^4)*d)*x^2 + (D*a^3*b + C*a
^2*b^2 + 2*B*a*b^3 + 10*A*b^4)*c + (2*D*a^4 + C*a^3*b + B*a^2*b^2 + 2*A*a*
b^3)*d + 6*((D*a^2*b^2 + C*a*b^3 + 2*B*b^4)*c + (2*D*a^3*b + C*a^2*b^2 + B
*a*b^3 + 2*A*b^4)*d)*x)/(b^11*x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3
*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a^5*b^6*x + a^6*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^7}\,dx = -\frac{30\,Db^4dx^4+20\,Db^4cx^3+40\,Dab^3dx^3+20\,Cb^4dx^3+15\,Dab^3cx^2+15\,Cb^4cx^2+30\,Da^2b^2dx^2+15\,C$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/60*(30*D*b^4*d*x^4 + 20*D*b^4*c*x^3 + 40*D*a*b^3*d*x^3 + 20*C*b^4*d*x^3
 + 15*D*a*b^3*c*x^2 + 15*C*b^4*c*x^2 + 30*D*a^2*b^2*d*x^2 + 15*C*a*b^3*d*x
^2 + 15*B*b^4*d*x^2 + 6*D*a^2*b^2*c*x + 6*C*a*b^3*c*x + 12*B*b^4*c*x + 12*
D*a^3*b*d*x + 6*C*a^2*b^2*d*x + 6*B*a*b^3*d*x + 12*A*b^4*d*x + D*a^3*b*c +
 C*a^2*b^2*c + 2*B*a*b^3*c + 10*A*b^4*c + 2*D*a^4*d + C*a^3*b*d + B*a^2*b^
2*d + 2*A*a*b^3*d)/((b*x + a)^6*b^5)
```

Mupad [B] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^7}\,dx &= \frac{c\,D\left(\frac{3\,a}{4\,(a+b\,x)^4}-\frac{1}{3\,(a+b\,x)^3}-\frac{3\,a^2}{5\,(a+b\,x)^5}+\frac{a^3}{6\,(a+b\,x)^6}\right)}{b^4} \\ &\quad - \frac{A\,c}{6\,b\,\left(a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6\right)} \\ &\quad + \frac{C\,d\left(\frac{3\,a}{4\,(a+b\,x)^4}-\frac{1}{3\,(a+b\,x)^3}-\frac{3\,a^2}{5\,(a+b\,x)^5}+\frac{a^3}{6\,(a+b\,x)^6}\right)}{b^4} - \frac{A\,d\,(a+6\,b\,x)}{30\,b^2\,{(a+b\,x)}^6} \\ &\quad - \frac{B\,c\,(a+6\,b\,x)}{30\,b^2\,{(a+b\,x)}^6} - \frac{B\,d\,\left(8\,a^2+48\,a\,b\,x+120\,b^2\,x^2\right)}{480\,b^3\,{(a+b\,x)}^6} \\ &\quad - \frac{C\,c\,\left(8\,a^2+48\,a\,b\,x+120\,b^2\,x^2\right)}{480\,b^3\,{(a+b\,x)}^6} + \frac{d\,x^5\,\left(6\,a+b\,x\right)\,D}{30\,a^2\,{(a+b\,x)}^6}\end{aligned}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^7,x)
```

output
```
(c*D*((3*a)/(4*(a + b*x)^4) - 1/(3*(a + b*x)^3) - (3*a^2)/(5*(a + b*x)^5)
+ a^3/(6*(a + b*x)^6)))/b^4 - (A*c)/(6*b*(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15
*a^4*b^2*x^2 + 20*a^3*b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)) + (C*d*((3*a)
/(4*(a + b*x)^4) - 1/(3*(a + b*x)^3) - (3*a^2)/(5*(a + b*x)^5) + a^3/(6*(a
 + b*x)^6)))/b^4 - (A*d*(a + 6*b*x))/(30*b^2*(a + b*x)^6) - (B*c*(a + 6*b*
x))/(30*b^2*(a + b*x)^6) - (B*d*(8*a^2 + 120*b^2*x^2 + 48*a*b*x))/(480*b^3
*(a + b*x)^6) - (C*c*(8*a^2 + 120*b^2*x^2 + 48*a*b*x))/(480*b^3*(a + b*x)^
6) + (d*x^5*(6*a + b*x)*D)/(30*a^2*(a + b*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^7}\,dx = \frac{-30b^4d^2x^4-40a\,b^3d^2x^3-40b^4cd\,x^3-30a^2b^2d^2x^2-30a\,b^3cd\,x^2-15b^5d\,x^2-15b^4c^2x^2-12a^3b\,d^2x-1\cdots}{60b^5\,(b^6x^6+6a\,b^5x^5+15a^2b^4x^4+20a^3b^3x^3\cdots}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x)
```

output
```
( - 2*a**4*d**2 - 2*a**3*b*c*d - 12*a**3*b*d**2*x - 3*a**2*b**3*d - a**2*b
**2*c**2 - 12*a**2*b**2*c*d*x - 30*a**2*b**2*d**2*x**2 - 12*a*b**4*c - 18*
a*b**4*d*x - 6*a*b**3*c**2*x - 30*a*b**3*c*d*x**2 - 40*a*b**3*d**2*x**3 -
12*b**5*c*x - 15*b**5*d*x**2 - 15*b**4*c**2*x**2 - 40*b**4*c*d*x**3 - 30*b
**4*d**2*x**4)/(60*b**5*(a**6 + 6*a**5*b*x + 15*a**4*b**2*x**2 + 20*a**3*b
**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*x**6))
```

3.12 $\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^8} dx$

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Optimal result

Integrand size = 28, antiderivative size = 195

$$\begin{aligned}\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^8}\,dx &= -\frac{(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)}{7b^5(a+bx)^7} \\ &\quad -\frac{b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD)}{6b^5(a+bx)^6} \\ &\quad -\frac{b^2(cC+Bd)+6a^2dD-3ab(Cd+cD)}{5b^5(a+bx)^5}-\frac{bCd+bcD-4adD}{4b^5(a+bx)^4}-\frac{dD}{3b^5(a+bx)^3}\end{aligned}$$

output

```
-1/7*(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^5/(b*x+a)^7-1/6*(b^3*(A*d+
B*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))/b^5/(b*x+a)^6-1/5*(b^2
*(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))/b^5/(b*x+a)^5-1/4*(C*b*d-4*D*a*d+D*b
*c)/b^5/(b*x+a)^4-1/3*d*D/b^5/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx =$$
$$-\frac{4a^4dD+10Ab^3(6bc+ad+7bdx)+a^3b(3Cd+3cD+28dDx)+a^2b^2(4Bd+21dx(C+4Dx)+c(4C$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^8,x]
```

output
```
-1/420*(4*a^4*d*D + 10*A*b^3*(6*b*c + a*d + 7*b*d*x) + a^3*b*(3*C*d + 3*c*
D + 28*d*D*x) + a^2*b^2*(4*B*d + 21*d*x*(C + 4*D*x) + c*(4*C + 21*D*x)) +
7*b^4*x*(2*B*(5*c + 6*d*x) + x*(5*d*x*(3*C + 4*D*x) + 3*c*(4*C + 5*D*x)))
+ a*b^3*(2*B*(5*c + 14*d*x) + 7*x*(c*(4*C + 9*D*x) + d*x*(9*C + 20*D*x))))
/(b^5*(a + b*x)^7)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx$$
$$\Big\downarrow \ 2123$$
$$\int \left(\frac{(bc-ad)(Ab^3-a(a^2D-abC+b^2B))}{b^4(a+bx)^8}+\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{b^4(a+bx)^6}+\frac{-4a^3dD+3a^2b(cD-$$
$$\Big\downarrow \ 2009$$

$$-\frac{(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^5(a+bx)^7}-\frac{6a^2dD-3ab(cD+Cd)+b^2(Bd+cC)}{5b^5(a+bx)^5}-\frac{-4a^3dD+3a^2b(cD+Cd)-2ab^2(Bd+cC)+b^3(Ad+Bc)}{6b^5(a+bx)^6}-\frac{-4adD+bcD+bCd}{4b^5(a+bx)^4}-\frac{dD}{3b^5(a+bx)^3}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^8,x]
```

output
```
-1/7*((b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^5*(a + b*x)^7) -
 (b^3*(B*c + A*d) - 2*a*b^2*(c*C + B*d) - 4*a^3*d*D + 3*a^2*b*(C*d + c*D))
/(6*b^5*(a + b*x)^6) - (b^2*(c*C + B*d) + 6*a^2*d*D - 3*a*b*(C*d + c*D))/(
5*b^5*(a + b*x)^5) - (b*C*d + b*c*D - 4*a*d*D)/(4*b^5*(a + b*x)^4) - (d*D)
/(3*b^5*(a + b*x)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.15

method	result
default	$-\frac{b^{2}Bd-3Cabd+C\,b^{2}c+6a^{2}dD-3Dabc}{5b^{5}\left(bx+a\right)^{5}}-\frac{Cbd-4Dad+Dbc}{4b^{5}\left(bx+a\right)^{4}}-\frac{-Aa\,b^{3}d+A\,b^{4}c+B\,a^{2}b^{2}d-Ba\,b^{3}c-C\,a^{3}bd+C\,a^{2}b^{2}c+D}{7b^{5}\left(bx+a\right)^{7}}$
gosper	$-\frac{140dDx^{4}b^{4}+105C\,x^{3}b^{4}d+140Dx^{3}a\,b^{3}d+105Dx^{3}b^{4}c+84B\,x^{2}b^{4}d+63C\,x^{2}a\,b^{3}d+84C\,x^{2}b^{4}c+84Dx^{2}a^{2}b^{2}d+63Dx^{2}a\,b^{3}c+}{}$
orering	$-\frac{140dDx^{4}b^{4}+105C\,x^{3}b^{4}d+140Dx^{3}a\,b^{3}d+105Dx^{3}b^{4}c+84B\,x^{2}b^{4}d+63C\,x^{2}a\,b^{3}d+84C\,x^{2}b^{4}c+84Dx^{2}a^{2}b^{2}d+63Dx^{2}a\,b^{3}c+}{}$
norman	$\frac{-\frac{dD\,x^{4}}{3b}-\frac{\left(3b^{3}dC+4Da\,b^{2}d+3Db^{3}c\right)x^{3}}{12b^{4}}-\frac{\left(4Bd\,b^{4}+3Ca\,b^{3}d+4Cc\,b^{4}+4Da^{2}b^{2}d+3Da\,b^{3}c\right)x^{2}}{20b^{5}}-\frac{\left(10Ad\,b^{5}+4Ba\,b^{4}d+10Bc\,b^{5}+3C\,a^{2}b^{3}d+\right.}{60b^{6}}}{\left(bx+a\right)^{7}}$
parallelrisch	$-\frac{140Dd\,x^{4}b^{6}+105C\,b^{6}d\,x^{3}+140Da\,b^{5}d\,x^{3}+105Db^{6}c\,x^{3}+84B\,b^{6}d\,x^{2}+63Ca\,b^{5}d\,x^{2}+84C\,b^{6}c\,x^{2}+84Da^{2}b^{4}d\,x^{2}+63Da\,b^{5}c\,x}{}$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b^5*(B*b^2*d-3*C*a*b*d+C*b^2*c+6*D*a^2*d-3*D*a*b*c)/(b*x+a)^5-1/4*(C*
b*d-4*D*a*d+D*b*c)/b^5/(b*x+a)^4-1/7*(-A*a*b^3*d+A*b^4*c+B*a^2*b^2*d-B*a*b
^3*c-C*a^3*b*d+C*a^2*b^2*c+D*a^4*d-D*a^3*b*c)/b^5/(b*x+a)^7-1/3*d*D/b^5/(b
*x+a)^3-1/6/b^5*(A*b^3*d-2*B*a*b^2*d+B*b^3*c+3*C*a^2*b*d-2*C*a*b^2*c-4*D*a
^3*d+3*D*a^2*b*c)/(b*x+a)^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^8}\,dx =$$
$$-\frac{140\,Db^4dx^4+35\,(3\,Db^4c+(4\,Dab^3+3\,Cb^4)d)x^3+21\,((3\,Dab^3+4\,Cb^4)c+(4\,Da^2b^2+3\,Cab^3+4\,B}{420\,(b^{12}x}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x, algorithm="fricas")
```

output
```
-1/420*(140*D*b^4*d*x^4 + 35*(3*D*b^4*c + (4*D*a*b^3 + 3*C*b^4)*d)*x^3 + 2
1*((3*D*a*b^3 + 4*C*b^4)*c + (4*D*a^2*b^2 + 3*C*a*b^3 + 4*B*b^4)*d)*x^2 +
(3*D*a^3*b + 4*C*a^2*b^2 + 10*B*a*b^3 + 60*A*b^4)*c + (4*D*a^4 + 3*C*a^3*b
 + 4*B*a^2*b^2 + 10*A*a*b^3)*d + 7*((3*D*a^2*b^2 + 4*C*a*b^3 + 10*B*b^4)*c
 + (4*D*a^3*b + 3*C*a^2*b^2 + 4*B*a*b^3 + 10*A*b^4)*d)*x)/(b^12*x^7 + 7*a*
b^11*x^6 + 21*a^2*b^10*x^5 + 35*a^3*b^9*x^4 + 35*a^4*b^8*x^3 + 21*a^5*b^7*
x^2 + 7*a^6*b^6*x + a^7*b^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^8}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^8}\,dx = -\frac{140\,Db^4dx^4+35\,(3\,Db^4c+(4\,Dab^3+3\,Cb^4)d)x^3+21\,((3\,Dab^3+4\,Cb^4)c+(4\,Da^2b^2+3\,Cab^3+4\,B}{420\,(b^{12}x}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x, algorithm="maxima")
```

output
```
-1/420*(140*D*b^4*d*x^4 + 35*(3*D*b^4*c + (4*D*a*b^3 + 3*C*b^4)*d)*x^3 + 2
1*((3*D*a*b^3 + 4*C*b^4)*c + (4*D*a^2*b^2 + 3*C*a*b^3 + 4*B*b^4)*d)*x^2 +
(3*D*a^3*b + 4*C*a^2*b^2 + 10*B*a*b^3 + 60*A*b^4)*c + (4*D*a^4 + 3*C*a^3*b
 + 4*B*a^2*b^2 + 10*A*a*b^3)*d + 7*((3*D*a^2*b^2 + 4*C*a*b^3 + 10*B*b^4)*c
 + (4*D*a^3*b + 3*C*a^2*b^2 + 4*B*a*b^3 + 10*A*b^4)*d)*x)/(b^12*x^7 + 7*a*
b^11*x^6 + 21*a^2*b^10*x^5 + 35*a^3*b^9*x^4 + 35*a^4*b^8*x^3 + 21*a^5*b^7*
x^2 + 7*a^6*b^6*x + a^7*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx = -\frac{140\,Db^4dx^4+105\,Db^4cx^3+140\,Dab^3dx^3+105\,Cb^4dx^3+63\,Dab^3cx^2+84\,Cb^4cx^2+84\,Da^2b^2dx^2+6}{}$$

input
```
integrate((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x, algorithm="giac")
```

output
```
-1/420*(140*D*b^4*d*x^4 + 105*D*b^4*c*x^3 + 140*D*a*b^3*d*x^3 + 105*C*b^4*
d*x^3 + 63*D*a*b^3*c*x^2 + 84*C*b^4*c*x^2 + 84*D*a^2*b^2*d*x^2 + 63*C*a*b^
3*d*x^2 + 84*B*b^4*d*x^2 + 21*D*a^2*b^2*c*x + 28*C*a*b^3*c*x + 70*B*b^4*c*
x + 28*D*a^3*b*d*x + 21*C*a^2*b^2*d*x + 28*B*a*b^3*d*x + 70*A*b^4*d*x + 3*
D*a^3*b*c + 4*C*a^2*b^2*c + 10*B*a*b^3*c + 60*A*b^4*c + 4*D*a^4*d + 3*C*a^
3*b*d + 4*B*a^2*b^2*d + 10*A*a*b^3*d)/((b*x + a)^7*b^5)
```

Mupad [B] (verification not implemented)

Time = 4.01 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx)(A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx$$
$$= \frac{c\,D\left(\frac{3\,a}{5\,(a+b\,x)^5}-\frac{1}{4\,(a+b\,x)^4}-\frac{a^2}{2\,(a+b\,x)^6}+\frac{a^3}{7\,(a+b\,x)^7}\right)}{b^4}$$
$$-\frac{A\,c}{7\,b\,\left(a^7+7\,a^6\,b\,x+21\,a^5\,b^2\,x^2+35\,a^4\,b^3\,x^3+35\,a^3\,b^4\,x^4+21\,a^2\,b^5\,x^5+7\,a\,b^6\,x^6+b^7\,x^7\right)}$$
$$-\frac{d\,D\left(\frac{1}{3\,(a+b\,x)^3}-\frac{a}{(a+b\,x)^4}+\frac{6\,a^2}{5\,(a+b\,x)^5}-\frac{2\,a^3}{3\,(a+b\,x)^6}+\frac{a^4}{7\,(a+b\,x)^7}\right)}{b^5}$$
$$+\frac{C\,d\left(\frac{3\,a}{5\,(a+b\,x)^5}-\frac{1}{4\,(a+b\,x)^4}-\frac{a^2}{2\,(a+b\,x)^6}+\frac{a^3}{7\,(a+b\,x)^7}\right)}{b^4}-\frac{A\,d\,(a+7\,b\,x)}{42\,b^2\,(a+b\,x)^7}$$
$$-\frac{B\,c\,(a+7\,b\,x)}{42\,b^2\,(a+b\,x)^7}-\frac{B\,d\,(8\,a^2+56\,a\,b\,x+168\,b^2\,x^2)}{840\,b^3\,(a+b\,x)^7}-\frac{C\,c\,(8\,a^2+56\,a\,b\,x+168\,b^2\,x^2)}{840\,b^3\,(a+b\,x)^7}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^8,x)
```

output
```
(c*D*((3*a)/(5*(a + b*x)^5) - 1/(4*(a + b*x)^4) - a^2/(2*(a + b*x)^6) + a^
3/(7*(a + b*x)^7)))/b^4 - (A*c)/(7*b*(a^7 + b^7*x^7 + 7*a*b^6*x^6 + 21*a^5
*b^2*x^2 + 35*a^4*b^3*x^3 + 35*a^3*b^4*x^4 + 21*a^2*b^5*x^5 + 7*a^6*b*x))
- (d*D*(1/(3*(a + b*x)^3) - a/(a + b*x)^4 + (6*a^2)/(5*(a + b*x)^5) - (2*a
^3)/(3*(a + b*x)^6) + a^4/(7*(a + b*x)^7)))/b^5 + (C*d*((3*a)/(5*(a + b*x)
^5) - 1/(4*(a + b*x)^4) - a^2/(2*(a + b*x)^6) + a^3/(7*(a + b*x)^7)))/b^4
- (A*d*(a + 7*b*x))/(42*b^2*(a + b*x)^7) - (B*c*(a + 7*b*x))/(42*b^2*(a +
b*x)^7) - (B*d*(8*a^2 + 168*b^2*x^2 + 56*a*b*x))/(840*b^3*(a + b*x)^7) - (
C*c*(8*a^2 + 168*b^2*x^2 + 56*a*b*x))/(840*b^3*(a + b*x)^7)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^8}\,dx$$
$$= \frac{-70b^4d^2x^4-70a\,b^3d^2x^3-105b^4cd\,x^3-42a^2b^2d^2x^2-63a\,b^3cd\,x^2-42b^5d\,x^2-42b^4c^2x^2-14a^3b\,d^2x-}{210b^5\left(b^7x^7+7a\,b^6x^6+21a^2b^5x^5+35a^3b^4x^4+\right.}$$

input
```
int((d*x+c)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x)
```

output
```
( - 2*a**4*d**2 - 3*a**3*b*c*d - 14*a**3*b*d**2*x - 7*a**2*b**3*d - 2*a**2
*b**2*c**2 - 21*a**2*b**2*c*d*x - 42*a**2*b**2*d**2*x**2 - 35*a*b**4*c - 4
9*a*b**4*d*x - 14*a*b**3*c**2*x - 63*a*b**3*c*d*x**2 - 70*a*b**3*d**2*x**3
 - 35*b**5*c*x - 42*b**5*d*x**2 - 42*b**4*c**2*x**2 - 105*b**4*c*d*x**3 -
70*b**4*d**2*x**4)/(210*b**5*(a**7 + 7*a**6*b*x + 21*a**5*b**2*x**2 + 35*a
**4*b**3*x**3 + 35*a**3*b**4*x**4 + 21*a**2*b**5*x**5 + 7*a*b**6*x**6 + b*
*7*x**7))
```

3.13 $\int (a+bx)^3(c+dx)^2 (A+Bx+Cx^2+Dx^3) \, dx$

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Optimal result

Integrand size = 30, antiderivative size = 307

$$
\begin{aligned}
&\int (a+bx)^3(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= \frac{(bc-ad)^2 \left(Ab^3-a(b^2B-abC+a^2D)\right)(a+bx)^4}{4b^6} \\
&\quad + \frac{(bc-ad)\left(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD)\right)(a+bx)^5}{5b^6} \\
&\quad + \frac{\left(b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D)\right)(a+bx)^6}{6b^6} \\
&\quad + \frac{\left(10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D)\right)(a+bx)^7}{7b^6} \\
&\quad + \frac{d(bCd+2bcD-5adD)(a+bx)^8}{8b^6} + \frac{d^2D(a+bx)^9}{9b^6}
\end{aligned}
$$

output

```
1/4*(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x+a)^4/b^6+1/5*(-a*d+b*c
)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))*(b*x
+a)^5/b^6+1/6*(b^3*(A*d^2+2*B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*c
)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))*(b*x+a)^6/b^6+1/7*(10*a^2*d^2*D-4*a*b*d*(
C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))*(b*x+a)^7/b^6+1/8*d*(C*b*d-5*D*a*d+2
*D*b*c)*(b*x+a)^8/b^6+1/9*d^2*D*(b*x+a)^9/b^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.33

$$\int (a+bx)^3(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= a^3Ac^2x+\frac{1}{2}a^2c(3Abc+aBc+2aAd)x^2$$
$$+\frac{1}{3}a\left(ac(3bBc+acC+2aBd)+A\left(3b^2c^2+6abcd+a^2d^2\right)\right)x^3$$
$$+\frac{1}{4}\left(Ab\left(b^2c^2+6abcd+3a^2d^2\right)\right.$$
$$\left.+a\left(3b^2Bc^2+3abc(cC+2Bd)+a^2\left(2cCd+Bd^2+c^2D\right)\right)\right)x^4+\frac{1}{5}\left(b^3c(Bc+2Ad)\right.$$
$$\left.+3ab^2\left(c^2C+2Bcd+Ad^2\right)+a^3d(Cd+2cD)+3a^2b\left(2cCd+Bd^2+c^2D\right)\right)x^5$$
$$+\frac{1}{6}\left(b^3\left(c^2C+2Bcd+Ad^2\right)+a^3d^2D+3a^2bd(Cd+2cD)+3ab^2\left(2cCd+Bd^2+c^2D\right)\right)x^6$$
$$+\frac{1}{7}b\left(3a^2d^2D+3abd(Cd+2cD)+b^2\left(2cCd+Bd^2+c^2D\right)\right)x^7$$
$$+\frac{1}{8}b^2d(bCd+2bcD+3adD)x^8+\frac{1}{9}b^3d^2Dx^9$$

input
```
Integrate[(a + b*x)^3*(c + d*x)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*c^2*x + (a^2*c*(3*A*b*c + a*B*c + 2*a*A*d)*x^2)/2 + (a*(a*c*(3*b*B*c
 + a*c*C + 2*a*B*d) + A*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2))*x^3)/3 + ((A*b*
(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2) + a*(3*b^2*B*c^2 + 3*a*b*c*(c*C + 2*B*d)
 + a^2*(2*c*C*d + B*d^2 + c^2*D)))*x^4)/4 + ((b^3*c*(B*c + 2*A*d) + 3*a*b^
2*(c^2*C + 2*B*c*d + A*d^2) + a^3*d*(C*d + 2*c*D) + 3*a^2*b*(2*c*C*d + B*d
^2 + c^2*D))*x^5)/5 + ((b^3*(c^2*C + 2*B*c*d + A*d^2) + a^3*d^2*D + 3*a^2*
b*d*(C*d + 2*c*D) + 3*a*b^2*(2*c*C*d + B*d^2 + c^2*D))*x^6)/6 + (b*(3*a^2*
d^2*D + 3*a*b*d*(C*d + 2*c*D) + b^2*(2*c*C*d + B*d^2 + c^2*D))*x^7)/7 + (b
^2*d*(b*C*d + 2*b*c*D + 3*a*d*D)*x^8)/8 + (b^3*d^2*D*x^9)/9
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(a+bx)^3(bc-ad)^2 \left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^5}+\frac{(a+bx)^6 \left(10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2\right.\right.}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^4(bc-ad)^2 \left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4b^6}+\frac{(a+bx)^7 \left(10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)\right)}{7b^6}+\frac{(a+bx)^6 \left(-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)\right)}{6b^6}+\frac{(a+bx)^5(bc-ad) \left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{5b^6}+\frac{d(a+bx)^8(-5adD+2bcD+bCd)}{8b^6}+\frac{d^2D(a+bx)^9}{9b^6}$$

input

```
Int[(a + b*x)^3*(c + d*x)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output

```
((b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(a + b*x)^4)/(4*b^6) +
((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a^2*
b*(4*C*d + 3*c*D))*(a + b*x)^5)/(5*b^6) + ((b^3*(c^2*C + 2*B*c*d + A*d^2)
- 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^2*
D))*(a + b*x)^6)/(6*b^6) + ((10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2
*c*C*d + B*d^2 + c^2*D))*(a + b*x)^7)/(7*b^6) + (d*(b*C*d + 2*b*c*D - 5*a*
d*D)*(a + b*x)^8)/(8*b^6) + (d^2*D*(a + b*x)^9)/(9*b^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.55

method	result
norman	$\frac{b^3 d^2 D x^9}{9}+\left(\frac{1}{8} b^3 d^2 C+\frac{3}{8} D a\, b^2 d^2+\frac{1}{4} D b^3 c d\right) x^8+\left(\frac{1}{7} b^3 B\, d^2+\frac{3}{7} C a\, b^2 d^2+\frac{2}{7} C\, b^3 c d+\frac{3}{7} D a^2 b\, d\right.$
default	$\frac{b^3 d^2 D x^9}{9}+\frac{\left(\left(3 a\, b^2 d^2+2 d\, b^3 c\right) D+b^3 d^2 C\right) x^8}{8}+\frac{\left(\left(3 a^2 b\, d^2+6 a\, b^2 c d+b^3 c^2\right) D+\left(3 a\, b^2 d^2+2 d\, b^3 c\right) C+b^3 B\, d^2\right) x^7}{7}+\frac{\left(\left(a^3 d^2\right.\right.}{}$
gosper	$\frac{1}{2} x^6 B a\, b^2 d^2+\frac{1}{3} x^6 B\, b^3 c d+\frac{1}{3} x^3 a^3 c^2 C+x^6 a^2 b c d D+\frac{6}{5} x^5 B a\, b^2 c d+\frac{6}{5} x^5 C\, a^2 b c d+\frac{1}{9} b^3 d^2 D x$
parallelrisch	$\frac{1}{2} x^6 B a\, b^2 d^2+\frac{1}{3} x^6 B\, b^3 c d+\frac{1}{3} x^3 a^3 c^2 C+x^6 a^2 b c d D+\frac{6}{5} x^5 B a\, b^2 c d+\frac{6}{5} x^5 C\, a^2 b c d+\frac{1}{9} b^3 d^2 D x$
orering	$x\left(280 b^3 d^2 D x^8+315 C\, b^3 d^2 x^7+945 D a\, b^2 d^2 x^7+630 D b^3 c d\, x^7+360 B\, b^3 d^2 x^6+1080 C a\, b^2 d^2 x^6+720 C\, b^3 c d\, x^6+1080 D a^2 b\, d^2 x^6\right.$

input
```
int((b*x+a)^3*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^3*d^2*D*x^9+(1/8*b^3*d^2*C+3/8*D*a*b^2*d^2+1/4*D*b^3*c*d)*x^8+(1/7*b
^3*B*d^2+3/7*C*a*b^2*d^2+2/7*C*b^3*c*d+3/7*D*a^2*b*d^2+6/7*D*a*b^2*c*d+1/7
*D*b^3*c^2)*x^7+(1/6*b^3*d^2*A+1/2*B*a*b^2*d^2+1/3*B*b^3*c*d+1/2*C*a^2*b*d
^2+C*a*b^2*c*d+1/6*C*b^3*c^2+1/6*a^3*d^2*D+a^2*b*c*d*D+1/2*a*b^2*c^2*D)*x^
6+(3/5*A*a*b^2*d^2+2/5*A*b^3*c*d+3/5*B*a^2*b*d^2+6/5*B*a*b^2*c*d+1/5*B*b^3
*c^2+1/5*C*a^3*d^2+6/5*C*a^2*b*c*d+3/5*C*a*b^2*c^2+2/5*D*a^3*c*d+3/5*D*a^2
*b*c^2)*x^5+(3/4*A*a^2*b*d^2+3/2*A*a*b^2*c*d+1/4*A*b^3*c^2+1/4*B*a^3*d^2+3
/2*B*a^2*b*c*d+3/4*B*a*b^2*c^2+1/2*C*a^3*c*d+3/4*C*a^2*b*c^2+1/4*a^3*c^2*D
)*x^4+(1/3*A*a^3*d^2+2*A*a^2*b*c*d+A*a*b^2*c^2+2/3*B*a^3*c*d+B*a^2*b*c^2+1
/3*a^3*c^2*C)*x^3+(A*a^3*c*d+3/2*A*a^2*b*c^2+1/2*a^3*c^2*B)*x^2+a^3*c^2*A*
x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.40

$$
\begin{aligned}
&\int (a+bx)^3(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx\\
&=\frac{1}{9}\,Db^3d^2x^9+\frac{1}{8}\left(2\,Db^3cd+\left(3\,Dab^2+Cb^3\right)d^2\right)x^8\\
&\quad+\frac{1}{7}\left(Db^3c^2+2\left(3\,Dab^2+Cb^3\right)cd+\left(3\,Da^2b+3\,Cab^2+Bb^3\right)d^2\right)x^7+Aa^3c^2x\\
&\quad+\frac{1}{6}\left(\left(3\,Dab^2+Cb^3\right)c^2+2\left(3\,Da^2b+3\,Cab^2+Bb^3\right)cd+\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)d^2\right)x^6\\
&\quad+\frac{1}{5}\left(\left(3\,Da^2b+3\,Cab^2+Bb^3\right)c^2+2\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)cd+\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)d^2\right)x\\
&\quad+\frac{1}{4}\left(\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)c^2+2\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)cd+\left(Ba^3+3\,Aa^2b\right)d^2\right)x^4\\
&\quad+\frac{1}{3}\left(Aa^3d^2+\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)c^2+2\left(Ba^3+3\,Aa^2b\right)cd\right)x^3\\
&\quad+\frac{1}{2}\left(2\,Aa^3cd+\left(Ba^3+3\,Aa^2b\right)c^2\right)x^2
\end{aligned}
$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*D*b^3*d^2*x^9 + 1/8*(2*D*b^3*c*d + (3*D*a*b^2 + C*b^3)*d^2)*x^8 + 1/7*
(D*b^3*c^2 + 2*(3*D*a*b^2 + C*b^3)*c*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d
^2)*x^7 + A*a^3*c^2*x + 1/6*((3*D*a*b^2 + C*b^3)*c^2 + 2*(3*D*a^2*b + 3*C*
a*b^2 + B*b^3)*c*d + (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^2)*x^6 + 1/
5*((3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2 + 2*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2
+ A*b^3)*c*d + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^2)*x^5 + 1/4*((D*a^3 + 3*
C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d +
 (B*a^3 + 3*A*a^2*b)*d^2)*x^4 + 1/3*(A*a^3*d^2 + (C*a^3 + 3*B*a^2*b + 3*A*
a*b^2)*c^2 + 2*(B*a^3 + 3*A*a^2*b)*c*d)*x^3 + 1/2*(2*A*a^3*c*d + (B*a^3 +
3*A*a^2*b)*c^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.89

$$\int (a+bx)^3(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= Aa^3c^2x + \frac{Db^3d^2x^9}{9} + x^8\left(\frac{Cb^3d^2}{8} + \frac{3Dab^2d^2}{8} + \frac{Db^3cd}{4}\right) + x^7\left(\frac{Bb^3d^2}{7} + \frac{3Cab^2d^2}{7} + \frac{2Cb^3cd}{7} + \frac{3Da^2bd^2}{7} + \frac{6Dab^2cd}{7} + \frac{Db^3c^2}{7}\right) + x^6\left(\frac{Ab^3d^2}{6} + \frac{Bab^2d^2}{2} + \frac{Bb^3cd}{3} + \frac{Ca^2bd^2}{2} + Cab^2cd + \frac{Cb^3c^2}{6} + \frac{Da^3d^2}{6} + Da^2bcd + \frac{Dab^2c^2}{2}\right) + x^5\cdot\left(\frac{3Aab^2d^2}{5} + \frac{2Ab^3cd}{5} + \frac{3Ba^2bd^2}{5} + \frac{6Bab^2cd}{5} + \frac{Bb^3c^2}{5} + \frac{Ca^3d^2}{5} + \frac{6Ca^2bcd}{5} + \frac{3Cab^2c^2}{5} + \frac{2Da^3cd}{5} + \frac{3Da^2bc^2}{5}\right) + x^4\cdot\left(\frac{3Aa^2bd^2}{4} + \frac{3Aab^2cd}{2} + \frac{Ab^3c^2}{4} + \frac{Ba^3d^2}{4} + \frac{3Ba^2bcd}{2} + \frac{3Bab^2c^2}{4} + \frac{Ca^3cd}{2} + \frac{3Ca^2bc^2}{4} + \frac{Da^3c^2}{4}\right) + x^3\left(\frac{Aa^3d^2}{3} + 2Aa^2bcd + Aab^2c^2 + \frac{2Ba^3cd}{3} + Ba^2bc^2 + \frac{Ca^3c^2}{3}\right) + x^2\left(Aa^3cd + \frac{3Aa^2bc^2}{2} + \frac{Ba^3c^2}{2}\right)$$

input
```
integrate((b*x+a)**3*(d*x+c)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*c**2*x + D*b**3*d**2*x**9/9 + x**8*(C*b**3*d**2/8 + 3*D*a*b**2*d**2
/8 + D*b**3*c*d/4) + x**7*(B*b**3*d**2/7 + 3*C*a*b**2*d**2/7 + 2*C*b**3*c*
d/7 + 3*D*a**2*b*d**2/7 + 6*D*a*b**2*c*d/7 + D*b**3*c**2/7) + x**6*(A*b**3
*d**2/6 + B*a*b**2*d**2/2 + B*b**3*c*d/3 + C*a**2*b*d**2/2 + C*a*b**2*c*d
+ C*b**3*c**2/6 + D*a**3*d**2/6 + D*a**2*b*c*d + D*a*b**2*c**2/2) + x**5*(
3*A*a*b**2*d**2/5 + 2*A*b**3*c*d/5 + 3*B*a**2*b*d**2/5 + 6*B*a*b**2*c*d/5
+ B*b**3*c**2/5 + C*a**3*d**2/5 + 6*C*a**2*b*c*d/5 + 3*C*a*b**2*c**2/5 + 2
*D*a**3*c*d/5 + 3*D*a**2*b*c**2/5) + x**4*(3*A*a**2*b*d**2/4 + 3*A*a*b**2*
c*d/2 + A*b**3*c**2/4 + B*a**3*d**2/4 + 3*B*a**2*b*c*d/2 + 3*B*a*b**2*c**2
/4 + C*a**3*c*d/2 + 3*C*a**2*b*c**2/4 + D*a**3*c**2/4) + x**3*(A*a**3*d**2
/3 + 2*A*a**2*b*c*d + A*a*b**2*c**2 + 2*B*a**3*c*d/3 + B*a**2*b*c**2 + C*a
**3*c**2/3) + x**2*(A*a**3*c*d + 3*A*a**2*b*c**2/2 + B*a**3*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.40

$$
\begin{aligned}
&\int (a+bx)^3(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{9}\,Db^3d^2x^9+\frac{1}{8}\left(2\,Db^3cd+\left(3\,Dab^2+Cb^3\right)d^2\right)x^8 \\
&\quad+\frac{1}{7}\left(Db^3c^2+2\left(3\,Dab^2+Cb^3\right)cd+\left(3\,Da^2b+3\,Cab^2+Bb^3\right)d^2\right)x^7+Aa^3c^2x \\
&\quad+\frac{1}{6}\left(\left(3\,Dab^2+Cb^3\right)c^2+2\left(3\,Da^2b+3\,Cab^2+Bb^3\right)cd+\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)d^2\right)x^6 \\
&\quad+\frac{1}{5}\left(\left(3\,Da^2b+3\,Cab^2+Bb^3\right)c^2+2\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)cd+\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)d^2\right)x \\
&\quad+\frac{1}{4}\left(\left(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3\right)c^2+2\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)cd+\left(Ba^3+3\,Aa^2b\right)d^2\right)x^4 \\
&\quad+\frac{1}{3}\left(Aa^3d^2+\left(Ca^3+3\,Ba^2b+3\,Aab^2\right)c^2+2\left(Ba^3+3\,Aa^2b\right)cd\right)x^3 \\
&\quad+\frac{1}{2}\left(2\,Aa^3cd+\left(Ba^3+3\,Aa^2b\right)c^2\right)x^2
\end{aligned}
$$

input

```
integrate((b*x+a)^3*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output

```
1/9*D*b^3*d^2*x^9 + 1/8*(2*D*b^3*c*d + (3*D*a*b^2 + C*b^3)*d^2)*x^8 + 1/7*
(D*b^3*c^2 + 2*(3*D*a*b^2 + C*b^3)*c*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d
^2)*x^7 + A*a^3*c^2*x + 1/6*((3*D*a*b^2 + C*b^3)*c^2 + 2*(3*D*a^2*b + 3*C*
a*b^2 + B*b^3)*c*d + (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^2)*x^6 + 1/
5*((3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2 + 2*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2
+ A*b^3)*c*d + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^2)*x^5 + 1/4*((D*a^3 + 3*
C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d +
 (B*a^3 + 3*A*a^2*b)*d^2)*x^4 + 1/3*(A*a^3*d^2 + (C*a^3 + 3*B*a^2*b + 3*A*
a*b^2)*c^2 + 2*(B*a^3 + 3*A*a^2*b)*c*d)*x^3 + 1/2*(2*A*a^3*c*d + (B*a^3 +
3*A*a^2*b)*c^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.88

$$\begin{aligned}&\int (a+bx)^3(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx\\&=\frac{1}{9}Db^3d^2x^9+\frac{1}{4}Db^3cdx^8+\frac{3}{8}Dab^2d^2x^8+\frac{1}{8}Cb^3d^2x^8+\frac{1}{7}Db^3c^2x^7+\frac{6}{7}Dab^2cdx^7\\&+\frac{2}{7}Cb^3cdx^7+\frac{3}{7}Da^2bd^2x^7+\frac{3}{7}Cab^2d^2x^7+\frac{1}{7}Bb^3d^2x^7+\frac{1}{2}Dab^2c^2x^6+\frac{1}{6}Cb^3c^2x^6\\&+Da^2bcdx^6+Cab^2cdx^6+\frac{1}{3}Bb^3cdx^6+\frac{1}{6}Da^3d^2x^6+\frac{1}{2}Ca^2bd^2x^6+\frac{1}{2}Bab^2d^2x^6\\&+\frac{1}{6}Ab^3d^2x^6+\frac{3}{5}Da^2bc^2x^5+\frac{3}{5}Cab^2c^2x^5+\frac{1}{5}Bb^3c^2x^5+\frac{2}{5}Da^3cdx^5+\frac{6}{5}Ca^2bcdx^5\\&+\frac{6}{5}Bab^2cdx^5+\frac{2}{5}Ab^3cdx^5+\frac{1}{5}Ca^3d^2x^5+\frac{3}{5}Ba^2bd^2x^5+\frac{3}{5}Aab^2d^2x^5+\frac{1}{4}Da^3c^2x^4\\&+\frac{3}{4}Ca^2bc^2x^4+\frac{3}{4}Bab^2c^2x^4+\frac{1}{4}Ab^3c^2x^4+\frac{1}{2}Ca^3cdx^4+\frac{3}{2}Ba^2bcdx^4+\frac{3}{2}Aab^2cdx^4\\&+\frac{1}{4}Ba^3d^2x^4+\frac{3}{4}Aa^2bd^2x^4+\frac{1}{3}Ca^3c^2x^3+Ba^2bc^2x^3+Aab^2c^2x^3+\frac{2}{3}Ba^3cdx^3\\&+2\,Aa^2bcdx^3+\frac{1}{3}Aa^3d^2x^3+\frac{1}{2}Ba^3c^2x^2+\frac{3}{2}Aa^2bc^2x^2+Aa^3cdx^2+Aa^3c^2x\end{aligned}$$

input
```
integrate((b*x+a)^3*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*D*b^3*d^2*x^9 + 1/4*D*b^3*c*d*x^8 + 3/8*D*a*b^2*d^2*x^8 + 1/8*C*b^3*d^
2*x^8 + 1/7*D*b^3*c^2*x^7 + 6/7*D*a*b^2*c*d*x^7 + 2/7*C*b^3*c*d*x^7 + 3/7*
D*a^2*b*d^2*x^7 + 3/7*C*a*b^2*d^2*x^7 + 1/7*B*b^3*d^2*x^7 + 1/2*D*a*b^2*c^
2*x^6 + 1/6*C*b^3*c^2*x^6 + D*a^2*b*c*d*x^6 + C*a*b^2*c*d*x^6 + 1/3*B*b^3*
c*d*x^6 + 1/6*D*a^3*d^2*x^6 + 1/2*C*a^2*b*d^2*x^6 + 1/2*B*a*b^2*d^2*x^6 +
1/6*A*b^3*d^2*x^6 + 3/5*D*a^2*b*c^2*x^5 + 3/5*C*a*b^2*c^2*x^5 + 1/5*B*b^3*
c^2*x^5 + 2/5*D*a^3*c*d*x^5 + 6/5*C*a^2*b*c*d*x^5 + 6/5*B*a*b^2*c*d*x^5 +
2/5*A*b^3*c*d*x^5 + 1/5*C*a^3*d^2*x^5 + 3/5*B*a^2*b*d^2*x^5 + 3/5*A*a*b^2*
d^2*x^5 + 1/4*D*a^3*c^2*x^4 + 3/4*C*a^2*b*c^2*x^4 + 3/4*B*a*b^2*c^2*x^4 +
1/4*A*b^3*c^2*x^4 + 1/2*C*a^3*c*d*x^4 + 3/2*B*a^2*b*c*d*x^4 + 3/2*A*a*b^2*
c*d*x^4 + 1/4*B*a^3*d^2*x^4 + 3/4*A*a^2*b*d^2*x^4 + 1/3*C*a^3*c^2*x^3 + B*
a^2*b*c^2*x^3 + A*a*b^2*c^2*x^3 + 2/3*B*a^3*c*d*x^3 + 2*A*a^2*b*c*d*x^3 +
1/3*A*a^3*d^2*x^3 + 1/2*B*a^3*c^2*x^2 + 3/2*A*a^2*b*c^2*x^2 + A*a^3*c*d*x^
2 + A*a^3*c^2*x
```

Mupad [B] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 577, normalized size of antiderivative = 1.88

$$\int (a+bx)^3(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{a^3\,c^2\,x^4\,D}{4}+\frac{a^3\,d^2\,x^6\,D}{6}+\frac{b^3\,c^2\,x^7\,D}{7}+\frac{b^3\,d^2\,x^9\,D}{9}+A\,a^3\,c^2\,x+\frac{B\,a^3\,c^2\,x^2}{2}$$
$$+\frac{A\,a^3\,d^2\,x^3}{3}+\frac{A\,b^3\,c^2\,x^4}{4}+\frac{C\,a^3\,c^2\,x^3}{3}+\frac{B\,a^3\,d^2\,x^4}{4}+\frac{B\,b^3\,c^2\,x^5}{5}+\frac{A\,b^3\,d^2\,x^6}{6}$$
$$+\frac{C\,a^3\,d^2\,x^5}{5}+\frac{C\,b^3\,c^2\,x^6}{6}+\frac{B\,b^3\,d^2\,x^7}{7}+\frac{C\,b^3\,d^2\,x^8}{8}+\frac{2\,B\,a^3\,c\,d\,x^3}{3}+\frac{2\,A\,b^3\,c\,d\,x^5}{5}$$
$$+\frac{C\,a^3\,c\,d\,x^4}{2}+\frac{B\,b^3\,c\,d\,x^6}{3}+\frac{2\,C\,b^3\,c\,d\,x^7}{7}+\frac{2\,a^3\,c\,d\,x^5\,D}{5}+\frac{b^3\,c\,d\,x^8\,D}{4}$$
$$+\frac{3\,A\,a^2\,b\,c^2\,x^2}{2}+A\,a\,b^2\,c^2\,x^3+B\,a^2\,b\,c^2\,x^3+\frac{3\,A\,a^2\,b\,d^2\,x^4}{4}+\frac{3\,B\,a\,b^2\,c^2\,x^4}{4}$$
$$+\frac{3\,A\,a\,b^2\,d^2\,x^5}{5}+\frac{3\,C\,a^2\,b\,c^2\,x^4}{4}+\frac{3\,B\,a^2\,b\,d^2\,x^5}{5}+\frac{3\,C\,a\,b^2\,c^2\,x^5}{5}+\frac{B\,a\,b^2\,d^2\,x^6}{2}$$
$$+\frac{C\,a^2\,b\,d^2\,x^6}{2}+\frac{3\,C\,a\,b^2\,d^2\,x^7}{7}+\frac{3\,a^2\,b\,c^2\,x^5\,D}{5}+\frac{a\,b^2\,c^2\,x^6\,D}{2}+\frac{3\,a^2\,b\,d^2\,x^7\,D}{7}$$
$$+\frac{3\,a\,b^2\,d^2\,x^8\,D}{8}+A\,a^3\,c\,d\,x^2+a^2\,b\,c\,d\,x^6\,D+\frac{6\,a\,b^2\,c\,d\,x^7\,D}{7}+2\,A\,a^2\,b\,c\,d\,x^3$$
$$+\frac{3\,A\,a\,b^2\,c\,d\,x^4}{2}+\frac{3\,B\,a^2\,b\,c\,d\,x^4}{2}+\frac{6\,B\,a\,b^2\,c\,d\,x^5}{5}+\frac{6\,C\,a^2\,b\,c\,d\,x^5}{5}+C\,a\,b^2\,c\,d\,x^6$$

input
```
int((a + b*x)^3*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^3*c^2*x^4*D)/4 + (a^3*d^2*x^6*D)/6 + (b^3*c^2*x^7*D)/7 + (b^3*d^2*x^9*D
)/9 + A*a^3*c^2*x + (B*a^3*c^2*x^2)/2 + (A*a^3*d^2*x^3)/3 + (A*b^3*c^2*x^4
)/4 + (C*a^3*c^2*x^3)/3 + (B*a^3*d^2*x^4)/4 + (B*b^3*c^2*x^5)/5 + (A*b^3*d
^2*x^6)/6 + (C*a^3*d^2*x^5)/5 + (C*b^3*c^2*x^6)/6 + (B*b^3*d^2*x^7)/7 + (C
*b^3*d^2*x^8)/8 + (2*B*a^3*c*d*x^3)/3 + (2*A*b^3*c*d*x^5)/5 + (C*a^3*c*d*x
^4)/2 + (B*b^3*c*d*x^6)/3 + (2*C*b^3*c*d*x^7)/7 + (2*a^3*c*d*x^5*D)/5 + (b
^3*c*d*x^8*D)/4 + (3*A*a^2*b*c^2*x^2)/2 + A*a*b^2*c^2*x^3 + B*a^2*b*c^2*x^
3 + (3*A*a^2*b*d^2*x^4)/4 + (3*B*a*b^2*c^2*x^4)/4 + (3*A*a*b^2*d^2*x^5)/5
+ (3*C*a^2*b*c^2*x^4)/4 + (3*B*a^2*b*d^2*x^5)/5 + (3*C*a*b^2*c^2*x^5)/5 +
(B*a*b^2*d^2*x^6)/2 + (C*a^2*b*d^2*x^6)/2 + (3*C*a*b^2*d^2*x^7)/7 + (3*a^2
*b*c^2*x^5*D)/5 + (a*b^2*c^2*x^6*D)/2 + (3*a^2*b*d^2*x^7*D)/7 + (3*a*b^2*d
^2*x^8*D)/8 + A*a^3*c*d*x^2 + a^2*b*c*d*x^6*D + (6*a*b^2*c*d*x^7*D)/7 + 2*
A*a^2*b*c*d*x^3 + (3*A*a*b^2*c*d*x^4)/2 + (3*B*a^2*b*c*d*x^4)/2 + (6*B*a*b
^2*c*d*x^5)/5 + (6*C*a^2*b*c*d*x^5)/5 + C*a*b^2*c*d*x^6
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.19

$$\int (a+bx)^3(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{x(280b^3d^3x^8 + 945a\,b^2d^3x^7 + 945b^3c\,d^2x^7 + 1080a^2b\,d^3x^6 + 3240a\,b^2c\,d^2x^6 + 360b^4d^2x^6 + 1080b^3c^2d\,x^6 \cdots}{}$$

input
```
int((b*x+a)^3*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(2520*a**4*c**2 + 2520*a**4*c*d*x + 840*a**4*d**2*x**2 + 5040*a**3*b*c*
*2*x + 6720*a**3*b*c*d*x**2 + 2520*a**3*b*d**2*x**3 + 840*a**3*c**3*x**2 +
 1890*a**3*c**2*d*x**3 + 1512*a**3*c*d**2*x**4 + 420*a**3*d**3*x**5 + 5040
*a**2*b**2*c**2*x**2 + 7560*a**2*b**2*c*d*x**3 + 3024*a**2*b**2*d**2*x**4
+ 1890*a**2*b*c**3*x**3 + 4536*a**2*b*c**2*d*x**4 + 3780*a**2*b*c*d**2*x**
5 + 1080*a**2*b*d**3*x**6 + 2520*a*b**3*c**2*x**3 + 4032*a*b**3*c*d*x**4 +
 1680*a*b**3*d**2*x**5 + 1512*a*b**2*c**3*x**4 + 3780*a*b**2*c**2*d*x**5 +
 3240*a*b**2*c*d**2*x**6 + 945*a*b**2*d**3*x**7 + 504*b**4*c**2*x**4 + 840
*b**4*c*d*x**5 + 360*b**4*d**2*x**6 + 420*b**3*c**3*x**5 + 1080*b**3*c**2*
d*x**6 + 945*b**3*c*d**2*x**7 + 280*b**3*d**3*x**8))/2520
```

3.14 $\int (a+bx)^2(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

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Optimal result

Integrand size = 30, antiderivative size = 307

$$\begin{aligned}
&\int (a+bx)^2(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx \\
&= \frac{(bc-ad)^2 \left(Ab^3-a(b^2B-abC+a^2D)\right)(a+bx)^3}{3b^6} \\
&\quad + \frac{(bc-ad)\left(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD)\right)(a+bx)^4}{4b^6} \\
&\quad + \frac{\left(b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D)\right)(a+bx)^5}{5b^6} \\
&\quad + \frac{\left(10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D)\right)(a+bx)^6}{6b^6} \\
&\quad + \frac{d(bCd+2bcD-5adD)(a+bx)^7}{7b^6} + \frac{d^2D(a+bx)^8}{8b^6}
\end{aligned}$$

output
```
1/3*(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(b*x+a)^3/b^6+1/4*(-a*d+b*c
)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))*(b*x
+a)^4/b^6+1/5*(b^3*(A*d^2+2*B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*c
)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))*(b*x+a)^5/b^6+1/6*(10*a^2*d^2*D-4*a*b*d*(
C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))*(b*x+a)^6/b^6+1/7*d*(C*b*d-5*D*a*d+2
*D*b*c)*(b*x+a)^7/b^6+1/8*d^2*D*(b*x+a)^8/b^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.95

$$\begin{aligned}
&\int (a+bx)^2(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= a^2Ac^2x+\frac{1}{2}ac(aBc+2A(bc+ad))x^2 \\
&\quad+\frac{1}{3}\left(ac(2bBc+acC+2aBd)+A\left(b^2c^2+4abcd+a^2d^2\right)\right)x^3 \\
&\quad+\frac{1}{4}\left(b^2c(Bc+2Ad)+2ab\left(c^2C+2Bcd+Ad^2\right)+a^2\left(2cCd+Bd^2+c^2D\right)\right)x^4 \\
&\quad+\frac{1}{5}\left(b^2\left(c^2C+2Bcd+Ad^2\right)+a^2d(Cd+2cD)+2ab\left(2cCd+Bd^2+c^2D\right)\right)x^5 \\
&\quad+\frac{1}{6}\left(a^2d^2D+2abd(Cd+2cD)+b^2\left(2cCd+Bd^2+c^2D\right)\right)x^6 \\
&\quad+\frac{1}{7}bd(bCd+2bcD+2adD)x^7+\frac{1}{8}b^2d^2Dx^8
\end{aligned}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*c^2*x + (a*c*(a*B*c + 2*A*(b*c + a*d))*x^2)/2 + ((a*c*(2*b*B*c + a*c
*C + 2*a*B*d) + A*(b^2*c^2 + 4*a*b*c*d + a^2*d^2))*x^3)/3 + ((b^2*c*(B*c +
 2*A*d) + 2*a*b*(c^2*C + 2*B*c*d + A*d^2) + a^2*(2*c*C*d + B*d^2 + c^2*D))
*x^4)/4 + ((b^2*(c^2*C + 2*B*c*d + A*d^2) + a^2*d*(C*d + 2*c*D) + 2*a*b*(2
*c*C*d + B*d^2 + c^2*D))*x^5)/5 + ((a^2*d^2*D + 2*a*b*d*(C*d + 2*c*D) + b^
2*(2*c*C*d + B*d^2 + c^2*D))*x^6)/6 + (b*d*(b*C*d + 2*b*c*D + 2*a*d*D)*x^7
)/7 + (b^2*d^2*D*x^8)/8
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{(a+bx)^2(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^5}+\frac{(a+bx)^5\left(10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2\right.\right.}{b^5}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+bx)^3(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^6}+\frac{(a+bx)^6\left(10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)\right)}{6b^6}+\frac{(a+bx)^5\left(-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)\right)}{5b^6}+\frac{(a+bx)^4(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{4b^6}+\frac{d(a+bx)^7(-5adD+2bcD+bCd)}{7b^6}+\frac{d^2D(a+bx)^8}{8b^6}$$

input
```
Int[(a + b*x)^2*(c + d*x)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(a + b*x)^3)/(3*b^6) +
((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a^2*
b*(4*C*d + 3*c*D))*(a + b*x)^4)/(4*b^6) + ((b^3*(c^2*C + 2*B*c*d + A*d^2)
- 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^2*
D))*(a + b*x)^5)/(5*b^6) + ((10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2
*c*C*d + B*d^2 + c^2*D))*(a + b*x)^6)/(6*b^6) + (d*(b*C*d + 2*b*c*D - 5*a*
d*D)*(a + b*x)^7)/(7*b^6) + (d^2*D*(a + b*x)^8)/(8*b^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.08

method	result
norman	$\frac{b^2 d^2 D x^8}{8} + \left(\frac{1}{7} b^2 d^2 C + \frac{2}{7} D a b\, d^2 + \frac{2}{7} D b^2 c d\right) x^7 + \left(\frac{1}{6} b^2 B\, d^2 + \frac{1}{3} C a b\, d^2 + \frac{1}{3} C\, b^2 c d + \frac{1}{6} a^2 d^2 D +\right.$
default	$\frac{b^2 d^2 D x^8}{8} + \frac{\left(\left(2ab\, d^2+2b^2 cd\right) D+b^2 d^2 C\right) x^7}{7} + \frac{\left(\left(a^2 d^2+4abcd+b^2 c^2\right) D+\left(2ab\, d^2+2b^2 cd\right) C+b^2 B\, d^2\right) x^6}{6} + \frac{\left(\left(2a^2 cd+2b\, c^2\right.\right.}{}$
gosper	$\frac{2}{7} x^7 D b^2 c d + \frac{1}{4} x^4 a^2 c^2 D + \frac{1}{3} x^3 A\, a^2 d^2 + \frac{1}{3} x^3 A\, b^2 c^2 + \frac{1}{7} x^7 b^2 d^2 C + \frac{1}{6} x^6 b^2 B\, d^2 + a^2 c^2 A x + \frac{1}{8} b^2$
parallelrisch	$\frac{2}{7} x^7 D b^2 c d + \frac{1}{4} x^4 a^2 c^2 D + \frac{1}{3} x^3 A\, a^2 d^2 + \frac{1}{3} x^3 A\, b^2 c^2 + \frac{1}{7} x^7 b^2 d^2 C + \frac{1}{6} x^6 b^2 B\, d^2 + a^2 c^2 A x + \frac{1}{8} b^2$
orering	$\frac{x\left(105 b^2 d^2 D x^7+120 C\, b^2 d^2 x^6+240 D a b\, d^2 x^6+240 D b^2 c d\, x^6+140 B\, b^2 d^2 x^5+280 C a b\, d^2 x^5+280 C\, b^2 c d\, x^5+140 D a^2 d^2 x^5+560\right.}{}$

input
```
int((b*x+a)^2*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*d^2*D*x^8+(1/7*b^2*d^2*C+2/7*D*a*b*d^2+2/7*D*b^2*c*d)*x^7+(1/6*b^2
*B*d^2+1/3*C*a*b*d^2+1/3*C*b^2*c*d+1/6*a^2*d^2*D+2/3*D*a*b*c*d+1/6*D*b^2*c
^2)*x^6+(1/5*A*b^2*d^2+2/5*B*a*b*d^2+2/5*B*b^2*c*d+1/5*C*a^2*d^2+4/5*C*a*b
*c*d+1/5*C*b^2*c^2+2/5*D*a^2*c*d+2/5*D*a*b*c^2)*x^5+(1/2*A*a*b*d^2+1/2*A*b
^2*c*d+1/4*B*a^2*d^2+B*a*b*c*d+1/4*B*b^2*c^2+1/2*C*a^2*c*d+1/2*C*a*b*c^2+1
/4*a^2*c^2*D)*x^4+(1/3*A*a^2*d^2+4/3*A*a*b*c*d+1/3*A*b^2*c^2+2/3*B*a^2*c*d
+2/3*B*a*b*c^2+1/3*a^2*c^2*C)*x^3+(A*a^2*c*d+A*a*b*c^2+1/2*a^2*c^2*B)*x^2+
a^2*c^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.98

$$\int (a+bx)^2(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{8}Db^2d^2x^8+\frac{1}{7}\left(2Db^2cd+\left(2Dab+Cb^2\right)d^2\right)x^7$$
$$+\frac{1}{6}\left(Db^2c^2+2\left(2Dab+Cb^2\right)cd+\left(Da^2+2Cab+Bb^2\right)d^2\right)x^6+Aa^2c^2x$$
$$+\frac{1}{5}\left(\left(2Dab+Cb^2\right)c^2+2\left(Da^2+2Cab+Bb^2\right)cd+\left(Ca^2+2Bab+Ab^2\right)d^2\right)x^5$$
$$+\frac{1}{4}\left(\left(Da^2+2Cab+Bb^2\right)c^2+2\left(Ca^2+2Bab+Ab^2\right)cd+\left(Ba^2+2Aab\right)d^2\right)x^4$$
$$+\frac{1}{3}\left(Aa^2d^2+\left(Ca^2+2Bab+Ab^2\right)c^2+2\left(Ba^2+2Aab\right)cd\right)x^3$$
$$+\frac{1}{2}\left(2Aa^2cd+\left(Ba^2+2Aab\right)c^2\right)x^2$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b^2*d^2*x^8 + 1/7*(2*D*b^2*c*d + (2*D*a*b + C*b^2)*d^2)*x^7 + 1/6*(D
*b^2*c^2 + 2*(2*D*a*b + C*b^2)*c*d + (D*a^2 + 2*C*a*b + B*b^2)*d^2)*x^6 +
A*a^2*c^2*x + 1/5*((2*D*a*b + C*b^2)*c^2 + 2*(D*a^2 + 2*C*a*b + B*b^2)*c*d
 + (C*a^2 + 2*B*a*b + A*b^2)*d^2)*x^5 + 1/4*((D*a^2 + 2*C*a*b + B*b^2)*c^2
 + 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d + (B*a^2 + 2*A*a*b)*d^2)*x^4 + 1/3*(A*a
^2*d^2 + (C*a^2 + 2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A*a*b)*c*d)*x^3 + 1/
2*(2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.31

$$\int (a+bx)^2(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= Aa^2c^2x+\frac{Db^2d^2x^8}{8}+x^7\left(\frac{Cb^2d^2}{7}+\frac{2Dabd^2}{7}+\frac{2Db^2cd}{7}\right)$$
$$+x^6\left(\frac{Bb^2d^2}{6}+\frac{Cabd^2}{3}+\frac{Cb^2cd}{3}+\frac{Da^2d^2}{6}+\frac{2Dabcd}{3}+\frac{Db^2c^2}{6}\right)$$
$$+x^5\left(\frac{Ab^2d^2}{5}+\frac{2Babd^2}{5}+\frac{2Bb^2cd}{5}+\frac{Ca^2d^2}{5}+\frac{4Cabcd}{5}+\frac{Cb^2c^2}{5}+\frac{2Da^2cd}{5}+\frac{2Dabc^2}{5}\right)$$
$$+x^4\left(\frac{Aabd^2}{2}+\frac{Ab^2cd}{2}+\frac{Ba^2d^2}{4}+Babcd+\frac{Bb^2c^2}{4}+\frac{Ca^2cd}{2}+\frac{Cabc^2}{2}+\frac{Da^2c^2}{4}\right)$$
$$+x^3\left(\frac{Aa^2d^2}{3}+\frac{4Aabcd}{3}+\frac{Ab^2c^2}{3}+\frac{2Ba^2cd}{3}+\frac{2Babc^2}{3}+\frac{Ca^2c^2}{3}\right)$$
$$+x^2\left(Aa^2cd+Aabc^2+\frac{Ba^2c^2}{2}\right)$$

input
```
integrate((b*x+a)**2*(d*x+c)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*c**2*x + D*b**2*d**2*x**8/8 + x**7*(C*b**2*d**2/7 + 2*D*a*b*d**2/7
+ 2*D*b**2*c*d/7) + x**6*(B*b**2*d**2/6 + C*a*b*d**2/3 + C*b**2*c*d/3 + D*
a**2*d**2/6 + 2*D*a*b*c*d/3 + D*b**2*c**2/6) + x**5*(A*b**2*d**2/5 + 2*B*a
*b*d**2/5 + 2*B*b**2*c*d/5 + C*a**2*d**2/5 + 4*C*a*b*c*d/5 + C*b**2*c**2/5
 + 2*D*a**2*c*d/5 + 2*D*a*b*c**2/5) + x**4*(A*a*b*d**2/2 + A*b**2*c*d/2 +
B*a**2*d**2/4 + B*a*b*c*d + B*b**2*c**2/4 + C*a**2*c*d/2 + C*a*b*c**2/2 +
D*a**2*c**2/4) + x**3*(A*a**2*d**2/3 + 4*A*a*b*c*d/3 + A*b**2*c**2/3 + 2*B
*a**2*c*d/3 + 2*B*a*b*c**2/3 + C*a**2*c**2/3) + x**2*(A*a**2*c*d + A*a*b*c
**2 + B*a**2*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int (a+bx)^2(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= \frac{1}{8}Db^2d^2x^8+\frac{1}{7}\left(2\,Db^2cd+\left(2\,Dab+Cb^2\right)d^2\right)x^7 \\
&\quad+\frac{1}{6}\left(Db^2c^2+2\left(2\,Dab+Cb^2\right)cd+\left(Da^2+2\,Cab+Bb^2\right)d^2\right)x^6+Aa^2c^2x \\
&\quad+\frac{1}{5}\left(\left(2\,Dab+Cb^2\right)c^2+2\left(Da^2+2\,Cab+Bb^2\right)cd+\left(Ca^2+2\,Bab+Ab^2\right)d^2\right)x^5 \\
&\quad+\frac{1}{4}\left(\left(Da^2+2\,Cab+Bb^2\right)c^2+2\left(Ca^2+2\,Bab+Ab^2\right)cd+\left(Ba^2+2\,Aab\right)d^2\right)x^4 \\
&\quad+\frac{1}{3}\left(Aa^2d^2+\left(Ca^2+2\,Bab+Ab^2\right)c^2+2\left(Ba^2+2\,Aab\right)cd\right)x^3 \\
&\quad+\frac{1}{2}\left(2\,Aa^2cd+\left(Ba^2+2\,Aab\right)c^2\right)x^2
\end{aligned}$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b^2*d^2*x^8 + 1/7*(2*D*b^2*c*d + (2*D*a*b + C*b^2)*d^2)*x^7 + 1/6*(D
*b^2*c^2 + 2*(2*D*a*b + C*b^2)*c*d + (D*a^2 + 2*C*a*b + B*b^2)*d^2)*x^6 +
A*a^2*c^2*x + 1/5*((2*D*a*b + C*b^2)*c^2 + 2*(D*a^2 + 2*C*a*b + B*b^2)*c*d
 + (C*a^2 + 2*B*a*b + A*b^2)*d^2)*x^5 + 1/4*((D*a^2 + 2*C*a*b + B*b^2)*c^2
 + 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d + (B*a^2 + 2*A*a*b)*d^2)*x^4 + 1/3*(A*a
^2*d^2 + (C*a^2 + 2*B*a*b + A*b^2)*c^2 + 2*(B*a^2 + 2*A*a*b)*c*d)*x^3 + 1/
2*(2*A*a^2*c*d + (B*a^2 + 2*A*a*b)*c^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.31

$$\int (a+bx)^2(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{1}{8}Db^2d^2x^8+\frac{2}{7}Db^2cdx^7+\frac{2}{7}Dabd^2x^7+\frac{1}{7}Cb^2d^2x^7+\frac{1}{6}Db^2c^2x^6+\frac{2}{3}Dabcdx^6+\frac{1}{3}Cb^2cdx^6+\frac{1}{6}Da^2d^2x^6+\frac{1}{3}Cabd^2x^6+\frac{1}{6}Bb^2d^2x^6+\frac{2}{5}Dabc^2x^5+\frac{1}{5}Cb^2c^2x^5+\frac{2}{5}Da^2cdx^5+\frac{4}{5}Cabcdx^5+\frac{2}{5}Bb^2cdx^5+\frac{1}{5}Ca^2d^2x^5+\frac{2}{5}Babd^2x^5+\frac{1}{5}Ab^2d^2x^5+\frac{1}{4}Da^2c^2x^4+\frac{1}{2}Cabc^2x^4+\frac{1}{4}Bb^2c^2x^4+\frac{1}{2}Ca^2cdx^4+Babcdx^4+\frac{1}{2}Ab^2cdx^4+\frac{1}{4}Ba^2d^2x^4+\frac{1}{2}Aabd^2x^4+\frac{1}{3}Ca^2c^2x^3+\frac{2}{3}Babc^2x^3+\frac{1}{3}Ab^2c^2x^3+\frac{2}{3}Ba^2cdx^3+\frac{4}{3}Aabcdx^3+\frac{1}{3}Aa^2d^2x^3+\frac{1}{2}Ba^2c^2x^2+Aabc^2x^2+Aa^2cdx^2+Aa^2c^2x$$

input
```
integrate((b*x+a)^2*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b^2*d^2*x^8 + 2/7*D*b^2*c*d*x^7 + 2/7*D*a*b*d^2*x^7 + 1/7*C*b^2*d^2*
x^7 + 1/6*D*b^2*c^2*x^6 + 2/3*D*a*b*c*d*x^6 + 1/3*C*b^2*c*d*x^6 + 1/6*D*a^
2*d^2*x^6 + 1/3*C*a*b*d^2*x^6 + 1/6*B*b^2*d^2*x^6 + 2/5*D*a*b*c^2*x^5 + 1/
5*C*b^2*c^2*x^5 + 2/5*D*a^2*c*d*x^5 + 4/5*C*a*b*c*d*x^5 + 2/5*B*b^2*c*d*x^
5 + 1/5*C*a^2*d^2*x^5 + 2/5*B*a*b*d^2*x^5 + 1/5*A*b^2*d^2*x^5 + 1/4*D*a^2*
c^2*x^4 + 1/2*C*a*b*c^2*x^4 + 1/4*B*b^2*c^2*x^4 + 1/2*C*a^2*c*d*x^4 + B*a*
b*c*d*x^4 + 1/2*A*b^2*c*d*x^4 + 1/4*B*a^2*d^2*x^4 + 1/2*A*a*b*d^2*x^4 + 1/
3*C*a^2*c^2*x^3 + 2/3*B*a*b*c^2*x^3 + 1/3*A*b^2*c^2*x^3 + 2/3*B*a^2*c*d*x^
3 + 4/3*A*a*b*c*d*x^3 + 1/3*A*a^2*d^2*x^3 + 1/2*B*a^2*c^2*x^2 + A*a*b*c^2*
x^2 + A*a^2*c*d*x^2 + A*a^2*c^2*x
```

Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.31

$$\int (a+bx)^2(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{a^2\,c^2\,x^4\,D}{4}+\frac{a^2\,d^2\,x^6\,D}{6}+\frac{b^2\,c^2\,x^6\,D}{6}+\frac{b^2\,d^2\,x^8\,D}{8}+A\,a^2\,c^2\,x+\frac{B\,a^2\,c^2\,x^2}{2}+\frac{A\,a^2\,d^2\,x^3}{3}+\frac{A\,b^2\,c^2\,x^3}{3}+\frac{C\,a^2\,c^2\,x^3}{3}+\frac{B\,a^2\,d^2\,x^4}{4}+\frac{B\,b^2\,c^2\,x^4}{4}+\frac{A\,b^2\,d^2\,x^5}{5}+\frac{C\,a^2\,d^2\,x^5}{5}+\frac{C\,b^2\,c^2\,x^5}{5}+\frac{B\,b^2\,d^2\,x^6}{6}+\frac{C\,b^2\,d^2\,x^7}{7}+\frac{2\,B\,a^2\,c\,d\,x^3}{3}+\frac{A\,b^2\,c\,d\,x^4}{2}+\frac{C\,a\,b\,d^2\,x^6}{3}+\frac{C\,a^2\,c\,d\,x^4}{2}+\frac{2\,B\,b^2\,c\,d\,x^5}{5}+\frac{C\,b^2\,c\,d\,x^6}{3}+\frac{2\,a\,b\,c^2\,x^5\,D}{5}+\frac{2\,a\,b\,d^2\,x^7\,D}{7}+\frac{2\,a^2\,c\,d\,x^5\,D}{5}+\frac{2\,b^2\,c\,d\,x^7\,D}{7}+A\,a\,b\,c^2\,x^2+\frac{2\,B\,a\,b\,c^2\,x^3}{3}+\frac{A\,a\,b\,d^2\,x^4}{2}+A\,a^2\,c\,d\,x^2+\frac{C\,a\,b\,c^2\,x^4}{2}+\frac{2\,B\,a\,b\,d^2\,x^5}{5}+\frac{4\,A\,a\,b\,c\,d\,x^3}{3}+B\,a\,b\,c\,d\,x^4+\frac{4\,C\,a\,b\,c\,d\,x^5}{5}+\frac{2\,a\,b\,c\,d\,x^6\,D}{3}$$

input
```
int((a + b*x)^2*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*c^2*x^4*D)/4 + (a^2*d^2*x^6*D)/6 + (b^2*c^2*x^6*D)/6 + (b^2*d^2*x^8*D
)/8 + A*a^2*c^2*x + (B*a^2*c^2*x^2)/2 + (A*a^2*d^2*x^3)/3 + (A*b^2*c^2*x^3
)/3 + (C*a^2*c^2*x^3)/3 + (B*a^2*d^2*x^4)/4 + (B*b^2*c^2*x^4)/4 + (A*b^2*d
^2*x^5)/5 + (C*a^2*d^2*x^5)/5 + (C*b^2*c^2*x^5)/5 + (B*b^2*d^2*x^6)/6 + (C
*b^2*d^2*x^7)/7 + (2*B*a^2*c*d*x^3)/3 + (A*b^2*c*d*x^4)/2 + (C*a*b*d^2*x^6
)/3 + (C*a^2*c*d*x^4)/2 + (2*B*b^2*c*d*x^5)/5 + (C*b^2*c*d*x^6)/3 + (2*a*b
*c^2*x^5*D)/5 + (2*a*b*d^2*x^7*D)/7 + (2*a^2*c*d*x^5*D)/5 + (2*b^2*c*d*x^7
*D)/7 + A*a*b*c^2*x^2 + (2*B*a*b*c^2*x^3)/3 + (A*a*b*d^2*x^4)/2 + A*a^2*c*
d*x^2 + (C*a*b*c^2*x^4)/2 + (2*B*a*b*d^2*x^5)/5 + (4*A*a*b*c*d*x^3)/3 + B*
a*b*c*d*x^4 + (4*C*a*b*c*d*x^5)/5 + (2*a*b*c*d*x^6*D)/3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.86

$$\int (a+bx)^2(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{x(105b^2d^3x^7+240ab\,d^3x^6+360b^2c\,d^2x^6+140a^2d^3x^5+840abc\,d^2x^5+140b^3d^2x^5+420b^2c^2d\,x^5+504a^2}{}$$

input
```
int((b*x+a)^2*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(840*a**3*c**2 + 840*a**3*c*d*x + 280*a**3*d**2*x**2 + 1260*a**2*b*c**2
*x + 1680*a**2*b*c*d*x**2 + 630*a**2*b*d**2*x**3 + 280*a**2*c**3*x**2 + 63
0*a**2*c**2*d*x**3 + 504*a**2*c*d**2*x**4 + 140*a**2*d**3*x**5 + 840*a*b**
2*c**2*x**2 + 1260*a*b**2*c*d*x**3 + 504*a*b**2*d**2*x**4 + 420*a*b*c**3*x
**3 + 1008*a*b*c**2*d*x**4 + 840*a*b*c*d**2*x**5 + 240*a*b*d**3*x**6 + 210
*b**3*c**2*x**3 + 336*b**3*c*d*x**4 + 140*b**3*d**2*x**5 + 168*b**2*c**3*x
**4 + 420*b**2*c**2*d*x**5 + 360*b**2*c*d**2*x**6 + 105*b**2*d**3*x**7))/8
40
```

3.15 $\int (a+bx)(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result 204
Mathematica [A] (verified) 205
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Optimal result

Integrand size = 28, antiderivative size = 204

$$\int (a+bx)(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = -\frac{(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^3}{3d^5} - \frac{\left(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D)\right)(c+dx)^4}{4d^5} + \frac{\left(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D)\right)(c+dx)^5}{5d^5} + \frac{(bCd-4bcD+adD)(c+dx)^6}{6d^5} + \frac{bD(c+dx)^7}{7d^5}$$

output
```
-1/3*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^3/d^5-1/4*(a*d*(-B*d
^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^4/d^5+1
/5*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^5/d^5+1/6*(C*b*d+D
*a*d-4*D*b*c)*(d*x+c)^6/d^5+1/7*b*D*(d*x+c)^7/d^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

$$\int (a+bx)(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= aAc^2x+\frac{1}{2}c(Abc+aBc+2aAd)x^2+\frac{1}{3}\left(bBc^2+ac^2C+2Abcd+2aBcd+aAd^2\right)x^3 + \frac{1}{4}\left(bc^2C+2bBcd+2acCd+Abd^2+aBd^2+ac^2D\right)x^4 + \frac{1}{5}\left(2bcCd+bBd^2+aCd^2+bc^2D+2acdD\right)x^5+\frac{1}{6}d(bCd+2bcD+adD)x^6+\frac{1}{7}bd^2Dx^7$$

input
```
Integrate[(a + b*x)*(c + d*x)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*c^2*x + (c*(A*b*c + a*B*c + 2*a*A*d)*x^2)/2 + ((b*B*c^2 + a*c^2*C + 2*
A*b*c*d + 2*a*B*c*d + a*A*d^2)*x^3)/3 + ((b*c^2*C + 2*b*B*c*d + 2*a*c*C*d
+ A*b*d^2 + a*B*d^2 + a*c^2*D)*x^4)/4 + ((2*b*c*C*d + b*B*d^2 + a*C*d^2 +
b*c^2*D + 2*a*c*d*D)*x^5)/5 + (d*(b*C*d + 2*b*c*D + a*d*D)*x^6)/6 + (b*d^2
*D*x^7)/7
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^3 \left(b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)}{d^4}+\frac{(c+dx)^2(ad-bc)\left(Ad^3-B\right.}{d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(c+dx)^4 \left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{4d^5}-\frac{(c+dx)^3(bc-ad) \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^5}+\frac{(c+dx)^5 \left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{5d^5}+\frac{(c+dx)^6(adD-4bcD+bCd)}{6d^5}+\frac{bD(c+dx)^7}{7d^5}$$

input
```
Int[(a + b*x)*(c + d*x)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
-1/3*((b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^3)/d^5 - (
(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^
3*D))*(c + d*x)^4)/(4*d^5) + ((a*d*(C*d - 3*c*D) - b*(3*c*C*d - B*d^2 - 6*
c^2*D))*(c + d*x)^5)/(5*d^5) + ((b*C*d - 4*b*c*D + a*d*D)*(c + d*x)^6)/(6*
d^5) + (b*D*(c + d*x)^7)/(7*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

method	result
default	$\frac{b\,d^2 D x^7}{7}+\frac{\left(\left(a\,d^2+2bcd\right)D+b\,d^2 C\right)x^6}{6}+\frac{\left(\left(2acd+b\,c^2\right)D+\left(a\,d^2+2bcd\right)C+bB\,d^2\right)x^5}{5}+\frac{\left(a\,c^2 D+\left(2acd+b\,c^2\right)C+\left(a\,d^2+2\right.\right.}{4}$
norman	$\frac{b\,d^2 D x^7}{7}+\left(\frac{1}{6}b\,d^2 C+\frac{1}{6}Da\,d^2+\frac{1}{3}Dbcd\right)x^6+\left(\frac{1}{5}bB\,d^2+\frac{1}{5}Ca\,d^2+\frac{2}{5}Cbcd+\frac{2}{5}Dacd+\frac{1}{5}Db\,c^2\right.$
orering	$\frac{x\left(60b\,d^2 D x^6+70Cb\,d^2 x^5+70Da\,d^2 x^5+140Dbcd\,x^5+84Bb\,d^2 x^4+84Ca\,d^2 x^4+168Cbcd\,x^4+168Dacd\,x^4+84Db\,c^2 x^4+105Ab\right.}{}$
gosper	$\frac{1}{7}b\,d^2 D x^7+\frac{1}{6}x^6 b\,d^2 C+\frac{1}{6}x^6 Da\,d^2+\frac{1}{3}x^6 Dbcd+\frac{1}{5}x^5 bB\,d^2+\frac{1}{5}x^5 Ca\,d^2+\frac{2}{5}x^5 Cbcd+\frac{2}{5}x^5 D$
parallelrisch	$\frac{1}{7}b\,d^2 D x^7+\frac{1}{6}x^6 b\,d^2 C+\frac{1}{6}x^6 Da\,d^2+\frac{1}{3}x^6 Dbcd+\frac{1}{5}x^5 bB\,d^2+\frac{1}{5}x^5 Ca\,d^2+\frac{2}{5}x^5 Cbcd+\frac{2}{5}x^5 D$

input
```
int((b*x+a)*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/7*b*d^2*D*x^7+1/6*((a*d^2+2*b*c*d)*D+b*d^2*C)*x^6+1/5*((2*a*c*d+b*c^2)*D
+(a*d^2+2*b*c*d)*C+b*B*d^2)*x^5+1/4*(a*c^2*D+(2*a*c*d+b*c^2)*C+(a*d^2+2*b*
c*d)*B+b*d^2*A)*x^4+1/3*(a*c^2*C+(2*a*c*d+b*c^2)*B+(a*d^2+2*b*c*d)*A)*x^3+
1/2*(a*c^2*B+(2*a*c*d+b*c^2)*A)*x^2+a*c^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

$$\begin{aligned}
\int (a+bx)(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx
&= \frac{1}{7}\,Dbd^2x^7+\frac{1}{6}\left(2\,Dbcd+(Da+Cb)d^2\right)x^6 \\
&\quad+\frac{1}{5}\left(Dbc^2+2\,(Da+Cb)cd+(Ca+Bb)d^2\right)x^5+Aac^2x \\
&\quad+\frac{1}{4}\left((Da+Cb)c^2+2\,(Ca+Bb)cd+(Ba+Ab)d^2\right)x^4 \\
&\quad+\frac{1}{3}\left(Aad^2+(Ca+Bb)c^2+2\,(Ba+Ab)cd\right)x^3+\frac{1}{2}\left(2\,Aacd+(Ba+Ab)c^2\right)x^2
\end{aligned}$$

input
```
integrate((b*x+a)*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/7*D*b*d^2*x^7 + 1/6*(2*D*b*c*d + (D*a + C*b)*d^2)*x^6 + 1/5*(D*b*c^2 + 2
*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*x^5 + A*a*c^2*x + 1/4*((D*a + C*b)*c^2
 + 2*(C*a + B*b)*c*d + (B*a + A*b)*d^2)*x^4 + 1/3*(A*a*d^2 + (C*a + B*b)*c
^2 + 2*(B*a + A*b)*c*d)*x^3 + 1/2*(2*A*a*c*d + (B*a + A*b)*c^2)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.12

$$\int (a+bx)(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= Aac^2x+\frac{Dbd^2x^7}{7}+x^6\left(\frac{Cbd^2}{6}+\frac{Dad^2}{6}+\frac{Dbcd}{3}\right)+x^5\left(\frac{Bbd^2}{5}+\frac{Cad^2}{5}+\frac{2Cbcd}{5}+\frac{2Dacd}{5}+\frac{Dbc^2}{5}\right)+x^4\left(\frac{Abd^2}{4}+\frac{Bad^2}{4}+\frac{Bbcd}{2}+\frac{Cacd}{2}+\frac{Cbc^2}{4}+\frac{Dac^2}{4}\right)+x^3\left(\frac{Aad^2}{3}+\frac{2Abcd}{3}+\frac{2Bacd}{3}+\frac{Bbc^2}{3}+\frac{Cac^2}{3}\right)+x^2\left(Aacd+\frac{Abc^2}{2}+\frac{Bac^2}{2}\right)$$

input
```
integrate((b*x+a)*(d*x+c)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*c**2*x + D*b*d**2*x**7/7 + x**6*(C*b*d**2/6 + D*a*d**2/6 + D*b*c*d/3)
+ x**5*(B*b*d**2/5 + C*a*d**2/5 + 2*C*b*c*d/5 + 2*D*a*c*d/5 + D*b*c**2/5)
+ x**4*(A*b*d**2/4 + B*a*d**2/4 + B*b*c*d/2 + C*a*c*d/2 + C*b*c**2/4 + D*a
*c**2/4) + x**3*(A*a*d**2/3 + 2*A*b*c*d/3 + 2*B*a*c*d/3 + B*b*c**2/3 + C*a
*c**2/3) + x**2*(A*a*c*d + A*b*c**2/2 + B*a*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

$$\int (a+bx)(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{1}{7}Dbd^2x^7 + \frac{1}{6}\left(2\,Dbcd + (Da+Cb)d^2\right)x^6 + \frac{1}{5}\left(Dbc^2 + 2\,(Da+Cb)cd + (Ca+Bb)d^2\right)x^5 + Aac^2x + \frac{1}{4}\left((Da+Cb)c^2 + 2\,(Ca+Bb)cd + (Ba+Ab)d^2\right)x^4 + \frac{1}{3}\left(Aad^2 + (Ca+Bb)c^2 + 2\,(Ba+Ab)cd\right)x^3 + \frac{1}{2}\left(2\,Aacd + (Ba+Ab)c^2\right)x^2$$

input
```
integrate((b*x+a)*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*D*b*d^2*x^7 + 1/6*(2*D*b*c*d + (D*a + C*b)*d^2)*x^6 + 1/5*(D*b*c^2 + 2
*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*x^5 + A*a*c^2*x + 1/4*((D*a + C*b)*c^2
 + 2*(C*a + B*b)*c*d + (B*a + A*b)*d^2)*x^4 + 1/3*(A*a*d^2 + (C*a + B*b)*c
^2 + 2*(B*a + A*b)*c*d)*x^3 + 1/2*(2*A*a*c*d + (B*a + A*b)*c^2)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.12

$$\int (a+bx)(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{1}{7}Dbd^2x^7 + \frac{1}{3}Dbcdx^6 + \frac{1}{6}Dad^2x^6 + \frac{1}{6}Cbd^2x^6 + \frac{1}{5}Dbc^2x^5 + \frac{2}{5}Dacdx^5 + \frac{2}{5}Cbcdx^5 + \frac{1}{5}Cad^2x^5 + \frac{1}{5}Bbd^2x^5 + \frac{1}{4}Dac^2x^4 + \frac{1}{4}Cbc^2x^4 + \frac{1}{2}Cacdx^4 + \frac{1}{2}Bbcdx^4 + \frac{1}{4}Bad^2x^4 + \frac{1}{4}Abd^2x^4 + \frac{1}{3}Cac^2x^3 + \frac{1}{3}Bbc^2x^3 + \frac{2}{3}Bacdx^3 + \frac{2}{3}Abcdx^3 + \frac{1}{3}Aad^2x^3 + \frac{1}{2}Bac^2x^2 + \frac{1}{2}Abc^2x^2 + Aacdx^2 + Aac^2x$$

input
```
integrate((b*x+a)*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/7*D*b*d^2*x^7 + 1/3*D*b*c*d*x^6 + 1/6*D*a*d^2*x^6 + 1/6*C*b*d^2*x^6 + 1/
5*D*b*c^2*x^5 + 2/5*D*a*c*d*x^5 + 2/5*C*b*c*d*x^5 + 1/5*C*a*d^2*x^5 + 1/5*
B*b*d^2*x^5 + 1/4*D*a*c^2*x^4 + 1/4*C*b*c^2*x^4 + 1/2*C*a*c*d*x^4 + 1/2*B*
b*c*d*x^4 + 1/4*B*a*d^2*x^4 + 1/4*A*b*d^2*x^4 + 1/3*C*a*c^2*x^3 + 1/3*B*b*
c^2*x^3 + 2/3*B*a*c*d*x^3 + 2/3*A*b*c*d*x^3 + 1/3*A*a*d^2*x^3 + 1/2*B*a*c^
2*x^2 + 1/2*A*b*c^2*x^2 + A*a*c*d*x^2 + A*a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.01

$$\int (a+bx)(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{a\,x^4\,D\,(15\,c^2+24\,c\,d\,x+10\,d^2\,x^2)}{60}+\frac{b\,x^5\,D\,(21\,c^2+35\,c\,d\,x+15\,d^2\,x^2)}{105}$$
$$+\frac{A\,a\,x\,(3\,c^2+3\,c\,d\,x+d^2\,x^2)}{3}+\frac{A\,b\,x^2\,(6\,c^2+8\,c\,d\,x+3\,d^2\,x^2)}{12}$$
$$+\frac{B\,a\,x^2\,(6\,c^2+8\,c\,d\,x+3\,d^2\,x^2)}{12}+\frac{B\,b\,x^3\,(10\,c^2+15\,c\,d\,x+6\,d^2\,x^2)}{30}$$
$$+\frac{C\,a\,x^3\,(10\,c^2+15\,c\,d\,x+6\,d^2\,x^2)}{30}+\frac{C\,b\,x^4\,(15\,c^2+24\,c\,d\,x+10\,d^2\,x^2)}{60}$$

input
```
int((a + b*x)*(c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^4*D*(15*c^2 + 10*d^2*x^2 + 24*c*d*x))/60 + (b*x^5*D*(21*c^2 + 15*d^2*
x^2 + 35*c*d*x))/105 + (A*a*x*(3*c^2 + d^2*x^2 + 3*c*d*x))/3 + (A*b*x^2*(6
*c^2 + 3*d^2*x^2 + 8*c*d*x))/12 + (B*a*x^2*(6*c^2 + 3*d^2*x^2 + 8*c*d*x))/
12 + (B*b*x^3*(10*c^2 + 6*d^2*x^2 + 15*c*d*x))/30 + (C*a*x^3*(10*c^2 + 6*d
^2*x^2 + 15*c*d*x))/30 + (C*b*x^4*(15*c^2 + 10*d^2*x^2 + 24*c*d*x))/60
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

$$\int (a+bx)(c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= \frac{x(60b\,d^3x^6+70a\,d^3x^5+210bc\,d^2x^5+252ac\,d^2x^4+84b^2d^2x^4+252b\,c^2d\,x^4+210ab\,d^2x^3+315a\,c^2d\,x^3+$$

input
```
int((b*x+a)*(d*x+c)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(420*a**2*c**2 + 420*a**2*c*d*x + 140*a**2*d**2*x**2 + 420*a*b*c**2*x +
 560*a*b*c*d*x**2 + 210*a*b*d**2*x**3 + 140*a*c**3*x**2 + 315*a*c**2*d*x**
3 + 252*a*c*d**2*x**4 + 70*a*d**3*x**5 + 140*b**2*c**2*x**2 + 210*b**2*c*d
*x**3 + 84*b**2*d**2*x**4 + 105*b*c**3*x**3 + 252*b*c**2*d*x**4 + 210*b*c*
d**2*x**5 + 60*b*d**3*x**6))/420
```

3.16 $\int (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx$

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Optimal result

Integrand size = 23, antiderivative size = 109

$$\int (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^3}{3d^4} - \frac{\left(2cCd-Bd^2-3c^2D\right)(c+dx)^4}{4d^4} + \frac{(Cd-3cD)(c+dx)^5}{5d^4} + \frac{D(c+dx)^6}{6d^4}$$

output
```
1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^3/d^4-1/4*(-B*d^2+2*C*c*d-3*D*c^
2)*(d*x+c)^4/d^4+1/5*(C*d-3*D*c)*(d*x+c)^5/d^4+1/6*D*(d*x+c)^6/d^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) dx = Ac^2x+\frac{1}{2}c(Bc+2Ad)x^2+\frac{1}{3}\left(c^2C+2Bcd+Ad^2\right)x^3 + \frac{1}{4}\left(2cCd+Bd^2+c^2D\right)x^4 + \frac{1}{5}d(Cd+2cD)x^5+\frac{1}{6}d^2Dx^6$$

input
```
Integrate[(c + d*x)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
A*c^2*x + (c*(B*c + 2*A*d)*x^2)/2 + ((c^2*C + 2*B*c*d + A*d^2)*x^3)/3 + ((
2*c*C*d + B*d^2 + c^2*D)*x^4)/4 + (d*(C*d + 2*c*D)*x^5)/5 + (d^2*D*x^6)/6
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{(c+dx)^2 \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3}+\frac{(c+dx)^3 \left(Bd^2+3c^2D-2cCd\right)}{d^3}+\frac{(c+dx)^4(Cd-3cD)}{d^3}+\frac{D}{\,}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^3 \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^4}-\frac{(c+dx)^4 \left(-Bd^2-3c^2D+2cCd\right)}{4d^4}+\frac{(c+dx)^5(Cd-3cD)}{5d^4}+\frac{D(c+dx)^6}{6d^4}$$

input
```
Int[(c + d*x)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^3)/(3*d^4) - ((2*c*C*d - B*
d^2 - 3*c^2*D)*(c + d*x)^4)/(4*d^4) + ((C*d - 3*c*D)*(c + d*x)^5)/(5*d^4)
+ (D*(c + d*x)^6)/(6*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
default	$\frac{d^2 D x^6}{6}+\frac{(d^2 C+2cdD)x^5}{5}+\frac{(B d^2+2Ccd+Dc^2)x^4}{4}+\frac{(d^2 A+2cdB+C c^2)x^3}{3}+\frac{(2cdA+c^2 B)x^2}{2}+c^2 Ax$
norman	$\frac{d^2 D x^6}{6}+\left(\frac{1}{5}d^2 C+\frac{2}{5}cdD\right)x^5+\left(\frac{1}{4}B d^2+\frac{1}{2}Ccd+\frac{1}{4}Dc^2\right)x^4+\left(\frac{1}{3}d^2 A+\frac{2}{3}cdB+\frac{1}{3}C c^2\right)x^3+$
gosper	$\frac{1}{6}d^2 D x^6+\frac{1}{5}x^5 d^2 C+\frac{2}{5}x^5 cdD+\frac{1}{4}x^4 B d^2+\frac{1}{2}x^4 Ccd+\frac{1}{4}x^4 Dc^2+\frac{1}{3}x^3 d^2 A+\frac{2}{3}x^3 cdB+\frac{1}{3}x^3$
parallelrisch	$\frac{1}{6}d^2 D x^6+\frac{1}{5}x^5 d^2 C+\frac{2}{5}x^5 cdD+\frac{1}{4}x^4 B d^2+\frac{1}{2}x^4 Ccd+\frac{1}{4}x^4 Dc^2+\frac{1}{3}x^3 d^2 A+\frac{2}{3}x^3 cdB+\frac{1}{3}x^3$
orering	$\frac{x(10d^2 D x^5+12C d^2 x^4+24Dcd x^4+15B d^2 x^3+30Ccd x^3+15Dc^2 x^3+20A d^2 x^2+40Bcd x^2+20C c^2 x^2+60Acdx+30B c^2 x+6}{60}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/6*d^2*D*x^6+1/5*(C*d^2+2*D*c*d)*x^5+1/4*(B*d^2+2*C*c*d+D*c^2)*x^4+1/3*(A
*d^2+2*B*c*d+C*c^2)*x^3+1/2*(2*A*c*d+B*c^2)*x^2+c^2*A*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int (c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{6}\,Dd^2x^6+\frac{1}{5}\,(2\,Dcd+Cd^2)x^5+\frac{1}{4}\,(Dc^2+2\,Ccd+Bd^2)x^4+Ac^2x+\frac{1}{3}\,(Cc^2+2\,Bcd+Ad^2)x^3+\frac{1}{2}\,(Bc^2+2\,Acd)x^2$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/6*D*d^2*x^6 + 1/5*(2*D*c*d + C*d^2)*x^5 + 1/4*(D*c^2 + 2*C*c*d + B*d^2)*
x^4 + A*c^2*x + 1/3*(C*c^2 + 2*B*c*d + A*d^2)*x^3 + 1/2*(B*c^2 + 2*A*c*d)*
x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int (c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx = Ac^2x+\frac{Dd^2x^6}{6}+x^5\left(\frac{Cd^2}{5}+\frac{2Dcd}{5}\right)+x^4\left(\frac{Bd^2}{4}+\frac{Ccd}{2}+\frac{Dc^2}{4}\right)+x^3\left(\frac{Ad^2}{3}+\frac{2Bcd}{3}+\frac{Cc^2}{3}\right)+x^2\left(Acd+\frac{Bc^2}{2}\right)$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*c**2*x + D*d**2*x**6/6 + x**5*(C*d**2/5 + 2*D*c*d/5) + x**4*(B*d**2/4 +
C*c*d/2 + D*c**2/4) + x**3*(A*d**2/3 + 2*B*c*d/3 + C*c**2/3) + x**2*(A*c*d
 + B*c**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int (c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{6}\,Dd^2x^6+\frac{1}{5}\,(2\,Dcd+Cd^2)x^5+\frac{1}{4}\,(Dc^2+2\,Ccd+Bd^2)x^4+Ac^2x+\frac{1}{3}\,(Cc^2+2\,Bcd+Ad^2)x^3+\frac{1}{2}\,(Bc^2+2\,Acd)x^2$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*D*d^2*x^6 + 1/5*(2*D*c*d + C*d^2)*x^5 + 1/4*(D*c^2 + 2*C*c*d + B*d^2)*
x^4 + A*c^2*x + 1/3*(C*c^2 + 2*B*c*d + A*d^2)*x^3 + 1/2*(B*c^2 + 2*A*c*d)*
x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int (c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{6}\,Dd^2x^6+\frac{2}{5}\,Dcdx^5+\frac{1}{5}\,Cd^2x^5+\frac{1}{4}\,Dc^2x^4+\frac{1}{2}\,Ccdx^4+\frac{1}{4}\,Bd^2x^4+\frac{1}{3}\,Cc^2x^3+\frac{2}{3}\,Bcdx^3+\frac{1}{3}\,Ad^2x^3+\frac{1}{2}\,Bc^2x^2+Acdx^2+Ac^2x$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/6*D*d^2*x^6 + 2/5*D*c*d*x^5 + 1/5*C*d^2*x^5 + 1/4*D*c^2*x^4 + 1/2*C*c*d*
x^4 + 1/4*B*d^2*x^4 + 1/3*C*c^2*x^3 + 2/3*B*c*d*x^3 + 1/3*A*d^2*x^3 + 1/2*
B*c^2*x^2 + A*c*d*x^2 + A*c^2*x
```

Mupad [B] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int (c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A\,x\,\left(3\,c^2+3\,c\,d\,x+d^2\,x^2\right)}{3}+\frac{B\,x^2\,\left(6\,c^2+8\,c\,d\,x+3\,d^2\,x^2\right)}{12}+\frac{C\,x^3\,\left(10\,c^2+15\,c\,d\,x+6\,d^2\,x^2\right)}{30}+\frac{c^2\,x^4\,D}{4}+\frac{d^2\,x^6\,D}{6}+\frac{2\,c\,d\,x^5\,D}{5}$$

input
```
int((c + d*x)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*x*(3*c^2 + d^2*x^2 + 3*c*d*x))/3 + (B*x^2*(6*c^2 + 3*d^2*x^2 + 8*c*d*x)
)/12 + (C*x^3*(10*c^2 + 6*d^2*x^2 + 15*c*d*x))/30 + (c^2*x^4*D)/4 + (d^2*x
^6*D)/6 + (2*c*d*x^5*D)/5
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int (c+dx)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{x(10d^3x^5 + 36c\,d^2x^4 + 15b\,d^2x^3 + 45c^2d\,x^3 + 20a\,d^2x^2 + 40bcd\,x^2 + 20c^3x^2 + 60acdx + 30b\,c^2x + 60a\,c^2}{60}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(x*(60*a*c**2 + 60*a*c*d*x + 20*a*d**2*x**2 + 30*b*c**2*x + 40*b*c*d*x**2
+ 15*b*d**2*x**3 + 20*c**3*x**2 + 45*c**2*d*x**3 + 36*c*d**2*x**4 + 10*d**
3*x**5))/60
```

3.17 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 294

$$\begin{aligned}
&\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx \\
&= \frac{(bc-ad)\,(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))\,x}{b^5} \\
&\quad+\frac{(b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D))\,(a+bx)^2}{2b^6} \\
&\quad+\frac{(10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D))\,(a+bx)^3}{3b^6} \\
&\quad+\frac{d(bCd+2bcD-5adD)(a+bx)^4}{4b^6}+\frac{d^2D(a+bx)^5}{5b^6} \\
&\quad+\frac{(bc-ad)^2\,(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{b^6}
\end{aligned}$$

output
```
(-a*d+b*c)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D
*c))*x/b^5+1/2*(b^3*(A*d^2+2*B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*
c)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))*(b*x+a)^2/b^6+1/3*(10*a^2*d^2*D-4*a*b*d*
(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))*(b*x+a)^3/b^6+1/4*d*(C*b*d-5*D*a*d+
2*D*b*c)*(b*x+a)^4/b^6+1/5*d^2*D*(b*x+a)^5/b^6+(-a*d+b*c)^2*(A*b^3-a*(B*b^
2-C*a*b+D*a^2))*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$
$$= \frac{bx(60a^4d^2D-30a^3bd(2Cd+4cD+dDx)+10a^2b^2(6c^2D+6cd(2C+Dx)+d^2(6B+x(3C+2Dx)))}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(b*x*(60*a^4*d^2*D - 30*a^3*b*d*(2*C*d + 4*c*D + d*D*x) + 10*a^2*b^2*(6*c^
2*D + 6*c*d*(2*C + D*x) + d^2*(6*B + x*(3*C + 2*D*x))) + b^4*(30*A*d*(4*c
+ d*x) + 20*B*(3*c^2 + 3*c*d*x + d^2*x^2) + x*(10*c^2*(3*C + 2*D*x) + 10*c
*d*x*(4*C + 3*D*x) + 3*d^2*x^2*(5*C + 4*D*x))) - 5*a*b^3*(6*c^2*(2*C + D*x
) + 4*c*d*(6*B + x*(3*C + 2*D*x)) + d^2*(12*A + x*(6*B + 4*C*x + 3*D*x^2))
)) + 60*(b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Log[a + b*x])/(6
0*b^6)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)^2 (Ab^3-a(a^2D-abC+b^2B))}{b^5(a+bx)} + \frac{(a+bx)^2 (10a^2d^2D-4abd(2cD+Cd)+b^2(Bd^2+c^2D+2cC}{b^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)^2\log(a+bx)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^6}+\frac{(a+bx)^3\left(10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)\right)}{3b^6}+\frac{(a+bx)^2\left(-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)\right)}{2b^6}+\frac{x(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{b^5}+\frac{d(a+bx)^4(-5adD+2bcD+bCd)}{4b^6}+\frac{d^2D(a+bx)^5}{5b^6}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a^2*
b*(4*C*d + 3*c*D))*x)/b^5 + ((b^3*(c^2*C + 2*B*c*d + A*d^2) - 10*a^3*d^2*D
 + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^2*D))*(a + b*x)^
2)/(2*b^6) + ((10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2*c*C*d + B*d^2
 + c^2*D))*(a + b*x)^3)/(3*b^6) + (d*(b*C*d + 2*b*c*D - 5*a*d*D)*(a + b*x)
^4)/(4*b^6) + (d^2*D*(a + b*x)^5)/(5*b^6) + ((b*c - a*d)^2*(A*b^3 - a*(b^2
*B - a*b*C + a^2*D))*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.46

method	result
norman	$\frac{(b^3 d^2 A - Ba b^2 d^2 + 2B b^3 cd + C a^2 b d^2 - 2Ca b^2 cd + C b^3 c^2 - a^3 d^2 D + 2a^2 bcdD - a b^2 c^2 D) x^2}{2b^4} + \frac{(b^2 B d^2 - Cab d^2 + 2C b^2 cd + a^2}{3b^3}$
default	$-\frac{-\frac{1}{3} Db^4 c^2 x^3 - \frac{1}{2} A b^4 d^2 x^2 + \frac{1}{3} Ca b^3 d^2 x^3 + 2Ba b^3 cdx - Da^2 b^2 cd x^2 - 2C a^2 b^2 cdx + 2Da^3 bcdx + \frac{2}{3} Da b^3 cd x^3 + \frac{1}{2} Ba b^3 d^2 x^2 -}{}$
parallelrisch	$\frac{40C x^3 b^5 cd + 20Dx^3 a^2 b^3 d^2 - 30B x^2 a b^4 d^2 + 60Dx^2 a^2 b^3 cd - 120Bxa b^4 cd - 120A \ln(bx+a) a b^4 cd + 120B \ln(bx+a) a^2 b^3 cd - 120}{}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^4*(A*b^3*d^2-B*a*b^2*d^2+2*B*b^3*c*d+C*a^2*b*d^2-2*C*a*b^2*c*d+C*b^3
*c^2-D*a^3*d^2+2*D*a^2*b*c*d-D*a*b^2*c^2)*x^2+1/3/b^3*(B*b^2*d^2-C*a*b*d^2
+2*C*b^2*c*d+D*a^2*d^2-2*D*a*b*c*d+D*b^2*c^2)*x^3-(A*a*b^3*d^2-2*A*b^4*c*d
-B*a^2*b^2*d^2+2*B*a*b^3*c*d-B*b^4*c^2+C*a^3*b*d^2-2*C*a^2*b^2*c*d+C*a*b^3
*c^2-D*a^4*d^2+2*D*a^3*b*c*d-D*a^2*b^2*c^2)/b^5*x+1/5*D/b*d^2*x^5+1/4/b^2*
d*(C*b*d-D*a*d+2*D*b*c)*x^4+(A*a^2*b^3*d^2-2*A*a*b^4*c*d+A*b^5*c^2-B*a^3*b
^2*d^2+2*B*a^2*b^3*c*d-B*a*b^4*c^2+C*a^4*b*d^2-2*C*a^3*b^2*c*d+C*a^2*b^3*c
^2-D*a^5*d^2+2*D*a^4*b*c*d-D*a^3*b^2*c^2)/b^6*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx} \, dx$$
$$= \frac{12\, Db^5 d^2 x^5 + 15\,(2\, Db^5 cd - (Dab^4 - Cb^5) d^2) x^4 + 20\,(Db^5 c^2 - 2\,(Dab^4 - Cb^5) cd + (Da^2 b^3 - Cab^4 + B}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
1/60*(12*D*b^5*d^2*x^5 + 15*(2*D*b^5*c*d - (D*a*b^4 - C*b^5)*d^2)*x^4 + 20
*(D*b^5*c^2 - 2*(D*a*b^4 - C*b^5)*c*d + (D*a^2*b^3 - C*a*b^4 + B*b^5)*d^2)
*x^3 - 30*((D*a*b^4 - C*b^5)*c^2 - 2*(D*a^2*b^3 - C*a*b^4 + B*b^5)*c*d + (
D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*d^2)*x^2 + 60*((D*a^2*b^3 - C*a*b
^4 + B*b^5)*c^2 - 2*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*c*d + (D*a^4
*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*d^2)*x - 60*((D*a^3*b^2 - C*a^2*b^3
+ B*a*b^4 - A*b^5)*c^2 - 2*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*c*d
 + (D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*d^2)*log(b*x + a))/b^6
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx$$
$$= \frac{Dd^2x^5}{5b} + x^4\left(\frac{Cd^2}{4b} - \frac{Dad^2}{4b^2} + \frac{Dcd}{2b}\right) + x^3\left(\frac{Bd^2}{3b} - \frac{Cad^2}{3b^2} + \frac{2Ccd}{3b} + \frac{Da^2d^2}{3b^3} - \frac{2Dacd}{3b^2} + \frac{Dc^2}{3b}\right) + x^2\left(\frac{Ad^2}{2b} - \frac{Bad^2}{2b^2} + \frac{Bcd}{b} + \frac{Ca^2d^2}{2b^3} - \frac{Cacd}{b^2} + \frac{Cc^2}{2b} - \frac{Da^3d^2}{2b^4} + \frac{Da^2cd}{b^3} - \frac{Dac^2}{2b^2}\right) + x\left(-\frac{Aad^2}{b^2} + \frac{2Acd}{b} + \frac{Ba^2d^2}{b^3} - \frac{2Bacd}{b^2} + \frac{Bc^2}{b} - \frac{Ca^3d^2}{b^4} + \frac{2Ca^2cd}{b^3} - \frac{Cac^2}{b^2} + \frac{Da^4d^2}{b^5} - \frac{2Da^3cd}{b^4} + \frac{Da^2c^2}{b^3}\right) - \frac{(ad-bc)^2\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\log(a+bx)}{b^6}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a),x)
```

output
```
D*d**2*x**5/(5*b) + x**4*(C*d**2/(4*b) - D*a*d**2/(4*b**2) + D*c*d/(2*b))
+ x**3*(B*d**2/(3*b) - C*a*d**2/(3*b**2) + 2*C*c*d/(3*b) + D*a**2*d**2/(3*
b**3) - 2*D*a*c*d/(3*b**2) + D*c**2/(3*b)) + x**2*(A*d**2/(2*b) - B*a*d**2
/(2*b**2) + B*c*d/b + C*a**2*d**2/(2*b**3) - C*a*c*d/b**2 + C*c**2/(2*b) -
 D*a**3*d**2/(2*b**4) + D*a**2*c*d/b**3 - D*a*c**2/(2*b**2)) + x*(-A*a*d**
2/b**2 + 2*A*c*d/b + B*a**2*d**2/b**3 - 2*B*a*c*d/b**2 + B*c**2/b - C*a**3
*d**2/b**4 + 2*C*a**2*c*d/b**3 - C*a*c**2/b**2 + D*a**4*d**2/b**5 - 2*D*a*
*3*c*d/b**4 + D*a**2*c**2/b**3) - (a*d - b*c)**2*(-A*b**3 + B*a*b**2 - C*a
**2*b + D*a**3)*log(a + b*x)/b**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$
$$= \frac{12\,Db^4d^2x^5+15\,(2\,Db^4cd-(Dab^3-Cb^4)d^2)x^4+20\,(Db^4c^2-2\,(Dab^3-Cb^4)cd+(Da^2b^2-Cab^3+B}{}$$
$$-\frac{((Da^3b^2-Ca^2b^3+Bab^4-Ab^5)c^2-2\,(Da^4b-Ca^3b^2+Ba^2b^3-Aab^4)cd+(Da^5-Ca^4b+Ba^3b^2-}{b^6}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
1/60*(12*D*b^4*d^2*x^5 + 15*(2*D*b^4*c*d - (D*a*b^3 - C*b^4)*d^2)*x^4 + 20
*(D*b^4*c^2 - 2*(D*a*b^3 - C*b^4)*c*d + (D*a^2*b^2 - C*a*b^3 + B*b^4)*d^2)
*x^3 - 30*((D*a*b^3 - C*b^4)*c^2 - 2*(D*a^2*b^2 - C*a*b^3 + B*b^4)*c*d + (
D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*d^2)*x^2 + 60*((D*a^2*b^2 - C*a*b^3
 + B*b^4)*c^2 - 2*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d + (D*a^4 - C
*a^3*b + B*a^2*b^2 - A*a*b^3)*d^2)*x)/b^5 - ((D*a^3*b^2 - C*a^2*b^3 + B*a*
b^4 - A*b^5)*c^2 - 2*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*c*d + (D*
a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*d^2)*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.70

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$
$$= \frac{12\,Db^4d^2x^5+30\,Db^4cdx^4-15\,Dab^3d^2x^4+15\,Cb^4d^2x^4+20\,Db^4c^2x^3-40\,Dab^3cdx^3+40\,Cb^4cdx^3+20}{}$$
$$-\frac{(Da^3b^2c^2-Ca^2b^3c^2+Bab^4c^2-Ab^5c^2-2\,Da^4bcd+2\,Ca^3b^2cd-2\,Ba^2b^3cd+2\,Aab^4cd+Da^5d^2-C}{b^6}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
1/60*(12*D*b^4*d^2*x^5 + 30*D*b^4*c*d*x^4 - 15*D*a*b^3*d^2*x^4 + 15*C*b^4*
d^2*x^4 + 20*D*b^4*c^2*x^3 - 40*D*a*b^3*c*d*x^3 + 40*C*b^4*c*d*x^3 + 20*D*
a^2*b^2*d^2*x^3 - 20*C*a*b^3*d^2*x^3 + 20*B*b^4*d^2*x^3 - 30*D*a*b^3*c^2*x
^2 + 30*C*b^4*c^2*x^2 + 60*D*a^2*b^2*c*d*x^2 - 60*C*a*b^3*c*d*x^2 + 60*B*b
^4*c*d*x^2 - 30*D*a^3*b*d^2*x^2 + 30*C*a^2*b^2*d^2*x^2 - 30*B*a*b^3*d^2*x^
2 + 30*A*b^4*d^2*x^2 + 60*D*a^2*b^2*c^2*x - 60*C*a*b^3*c^2*x + 60*B*b^4*c^
2*x - 120*D*a^3*b*c*d*x + 120*C*a^2*b^2*c*d*x - 120*B*a*b^3*c*d*x + 120*A*
b^4*c*d*x + 60*D*a^4*d^2*x - 60*C*a^3*b*d^2*x + 60*B*a^2*b^2*d^2*x - 60*A*
a*b^3*d^2*x)/b^5 - (D*a^3*b^2*c^2 - C*a^2*b^3*c^2 + B*a*b^4*c^2 - A*b^5*c^
2 - 2*D*a^4*b*c*d + 2*C*a^3*b^2*c*d - 2*B*a^2*b^3*c*d + 2*A*a*b^4*c*d + D*
a^5*d^2 - C*a^4*b*d^2 + B*a^3*b^2*d^2 - A*a^2*b^3*d^2)*log(abs(b*x + a))/b
^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx} \, dx = \int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{a+b\,x} \, dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{a+bx} \, dx = \frac{-60\log(bx+a)\,a^5d^3+180\log(bx+a)\,a^4bc\,d^2-180\log(bx+a)\,a^3b^2c^2d+60\log(bx+a)\,a^2b^3c^3+60a^4}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

output
```
( - 60*log(a + b*x)*a**5*d**3 + 180*log(a + b*x)*a**4*b*c*d**2 - 180*log(a
 + b*x)*a**3*b**2*c**2*d + 60*log(a + b*x)*a**2*b**3*c**3 + 60*a**4*b*d**3
*x - 180*a**3*b**2*c*d**2*x - 30*a**3*b**2*d**3*x**2 + 180*a**2*b**3*c**2*
d*x + 90*a**2*b**3*c*d**2*x**2 + 20*a**2*b**3*d**3*x**3 - 60*a*b**4*c**3*x
 - 90*a*b**4*c**2*d*x**2 - 60*a*b**4*c*d**2*x**3 - 15*a*b**4*d**3*x**4 + 6
0*b**6*c**2*x + 60*b**6*c*d*x**2 + 20*b**6*d**2*x**3 + 30*b**5*c**3*x**2 +
 60*b**5*c**2*d*x**3 + 45*b**5*c*d**2*x**4 + 12*b**5*d**3*x**5)/(60*b**6)
```

3.18 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 280

$$\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \frac{(b^3(c^2C+2Bcd+Ad^2)-4a^3d^2D+3a^2bd(Cd+2cD)-2ab^2(2cCd+Bd^2+c^2D))\,x}{b^5} + \frac{(3a^2d^2D-2abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D))\,x^2}{2b^4} + \frac{d(bCd+2bcD-2adD)x^3}{3b^3} + \frac{d^2Dx^4}{4b^2} - \frac{(bc-ad)^2\,(Ab^3-a(b^2B-abC+a^2D))}{b^6(a+bx)} + \frac{(bc-ad)\,(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))\log(a+bx)}{b^6}$$

output
```
(b^3*(A*d^2+2*B*c*d+C*c^2)-4*a^3*d^2*D+3*a^2*b*d*(C*d+2*D*c)-2*a*b^2*(B*d^
2+2*C*c*d+D*c^2))*x/b^5+1/2*(3*a^2*d^2*D-2*a*b*d*(C*d+2*D*c)+b^2*(B*d^2+2*
C*c*d+D*c^2))*x^2/b^4+1/3*d*(C*b*d-2*D*a*d+2*D*b*c)*x^3/b^3+1/4*d^2*D*x^4/
b^2-(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^6/(b*x+a)+(-a*d+b*c)*(b^3
*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))*ln(b*x+a)/
b^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} dx$$
$$= \frac{12b(b^3(c^2C+2Bcd+Ad^2)-4a^3d^2D+3a^2bd(Cd+2cD)-2ab^2(2cCd+Bd^2+c^2D))x+6b^2(3a^2d^2D}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
(12*b*(b^3*(c^2*C + 2*B*c*d + A*d^2) - 4*a^3*d^2*D + 3*a^2*b*d*(C*d + 2*c*
D) - 2*a*b^2*(2*c*C*d + B*d^2 + c^2*D))*x + 6*b^2*(3*a^2*d^2*D - 2*a*b*d*(
C*d + 2*c*D) + b^2*(2*c*C*d + B*d^2 + c^2*D))*x^2 + 4*b^3*d*(b*C*d + 2*b*c
*D - 2*a*d*D)*x^3 + 3*b^4*d^2*D*x^4 - (12*(b*c - a*d)^2*(A*b^3 - a*(b^2*B
- a*b*C + a^2*D)))/(a + b*x) + 12*(b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(
2*c*C + 3*B*d) - 5*a^3*d*D + a^2*b*(4*C*d + 3*c*D))*Log[a + b*x])/(12*b^6)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)^2 (Ab^3-a(a^2D-abC+b^2B))}{b^5(a+bx)^2}+\frac{x(3a^2d^2D-2abd(2cD+Cd)+b^2(Bd^2+c^2D+2cCd))}{b^4}+\frac{-4}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^6(a+bx)}+
\frac{x^2\left(3a^2d^2D-2abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)\right)}{2b^4}+
\frac{(bc-ad)\log(a+bx)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{b^6}+
\frac{x\left(-4a^3d^2D+3a^2bd(2cD+Cd)-2ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)\right)}{b^5}+
\frac{dx^3(-2adD+2bcD+bCd)}{3b^3}+\frac{d^2Dx^4}{4b^2}
$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
((b^3*(c^2*C + 2*B*c*d + A*d^2) - 4*a^3*d^2*D + 3*a^2*b*d*(C*d + 2*c*D) -
2*a*b^2*(2*c*C*d + B*d^2 + c^2*D))*x)/b^5 + ((3*a^2*d^2*D - 2*a*b*d*(C*d +
 2*c*D) + b^2*(2*c*C*d + B*d^2 + c^2*D))*x^2)/(2*b^4) + (d*(b*C*d + 2*b*c*
D - 2*a*d*D)*x^3)/(3*b^3) + (d^2*D*x^4)/(4*b^2) - ((b*c - a*d)^2*(A*b^3 -
a*(b^2*B - a*b*C + a^2*D)))/(b^6*(a + b*x)) + ((b*c - a*d)*(b^3*(B*c + 2*A
*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a^2*b*(4*C*d + 3*c*D))*Log[a + b
*x])/b^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.62

method	result
norman	$\frac{\left(2A\,a^{2}b^{3}d^{2}-2Aa\,b^{4}cd+A\,b^{5}c^{2}-3B\,a^{3}b^{2}d^{2}+4B\,a^{2}b^{3}cd-Ba\,b^{4}c^{2}+4C\,a^{4}b\,d^{2}-6C\,a^{3}b^{2}cd+2C\,a^{2}b^{3}c^{2}-5Da^{5}d^{2}+8Da^{4}bcd-3Da^{3}b^{2}c^{2}\right)x}{b^{5}a}$
default	$\frac{\frac{1}{4}d^{2}Dx^{4}b^{3}+\frac{1}{3}C\,b^{3}d^{2}x^{3}-\frac{2}{3}Da\,b^{2}d^{2}x^{3}+\frac{2}{3}Db^{3}cd\,x^{3}+\frac{1}{2}B\,b^{3}d^{2}x^{2}-Ca\,b^{2}d^{2}x^{2}+C\,b^{3}cd\,x^{2}+\frac{3}{2}Da^{2}b\,d^{2}x^{2}-2Da\,b^{2}cd\,x^{2}+\frac{1}{2}Db^{3}c}{b^{5}}$
parallelrisch	$-\frac{-12C\,x^{3}b^{5}cd-10Dx^{3}a^{2}b^{3}d^{2}+18B\,x^{2}a\,b^{4}d^{2}-48Dx^{2}a^{2}b^{3}cd-24A\ln(bx+a)a\,b^{4}cd+48B\ln(bx+a)a^{2}b^{3}cd-72C\ln(bx+a)a^{3}}{}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((2*A*a^2*b^3*d^2-2*A*a*b^4*c*d+A*b^5*c^2-3*B*a^3*b^2*d^2+4*B*a^2*b^3*c*d-
B*a*b^4*c^2+4*C*a^4*b*d^2-6*C*a^3*b^2*c*d+2*C*a^2*b^3*c^2-5*D*a^5*d^2+8*D*
a^4*b*c*d-3*D*a^3*b^2*c^2)/b^5/a*x+1/6*(3*B*b^2*d^2-4*C*a*b*d^2+6*C*b^2*c*
d+5*D*a^2*d^2-8*D*a*b*c*d+3*D*b^2*c^2)/b^3*x^3+1/2*(2*A*b^3*d^2-3*B*a*b^2*
d^2+4*B*b^3*c*d+4*C*a^2*b*d^2-6*C*a*b^2*c*d+2*C*b^3*c^2-5*D*a^3*d^2+8*D*a^
2*b*c*d-3*D*a*b^2*c^2)/b^4*x^2+1/4*D/b*d^2*x^5+1/12*d*(4*C*b*d-5*D*a*d+8*D
*b*c)/b^2*x^4)/(b*x+a)-(2*A*a*b^3*d^2-2*A*b^4*c*d-3*B*a^2*b^2*d^2+4*B*a*b^
3*c*d-B*b^4*c^2+4*C*a^3*b*d^2-6*C*a^2*b^2*c*d+2*C*a*b^3*c^2-5*D*a^4*d^2+8*
D*a^3*b*c*d-3*D*a^2*b^2*c^2)/b^6*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(278) = 556.

Time = 0.08 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.20

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$
$$= \frac{3\,Db^5d^2x^5+(8\,Db^5cd-(5\,Dab^4-4\,Cb^5)d^2)x^4+2\,(3\,Db^5c^2-2\,(4\,Dab^4-3\,Cb^5)cd+(5\,Da^2b^3-4\,Cab}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(3*D*b^5*d^2*x^5 + (8*D*b^5*c*d - (5*D*a*b^4 - 4*C*b^5)*d^2)*x^4 + 2*
(3*D*b^5*c^2 - 2*(4*D*a*b^4 - 3*C*b^5)*c*d + (5*D*a^2*b^3 - 4*C*a*b^4 + 3*
B*b^5)*d^2)*x^3 + 12*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*c^2 - 24*(D
*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*c*d + 12*(D*a^5 - C*a^4*b + B*a^
3*b^2 - A*a^2*b^3)*d^2 - 6*((3*D*a*b^4 - 2*C*b^5)*c^2 - 2*(4*D*a^2*b^3 - 3
*C*a*b^4 + 2*B*b^5)*c*d + (5*D*a^3*b^2 - 4*C*a^2*b^3 + 3*B*a*b^4 - 2*A*b^5
)*d^2)*x^2 - 12*((2*D*a^2*b^3 - C*a*b^4)*c^2 - 2*(3*D*a^3*b^2 - 2*C*a^2*b^
3 + B*a*b^4)*c*d + (4*D*a^4*b - 3*C*a^3*b^2 + 2*B*a^2*b^3 - A*a*b^4)*d^2)*
x + 12*((3*D*a^3*b^2 - 2*C*a^2*b^3 + B*a*b^4)*c^2 - 2*(4*D*a^4*b - 3*C*a^3
*b^2 + 2*B*a^2*b^3 - A*a*b^4)*c*d + (5*D*a^5 - 4*C*a^4*b + 3*B*a^3*b^2 - 2
*A*a^2*b^3)*d^2 + ((3*D*a^2*b^3 - 2*C*a*b^4 + B*b^5)*c^2 - 2*(4*D*a^3*b^2
- 3*C*a^2*b^3 + 2*B*a*b^4 - A*b^5)*c*d + (5*D*a^4*b - 4*C*a^3*b^2 + 3*B*a^
2*b^3 - 2*A*a*b^4)*d^2)*x)*log(b*x + a))/(b^7*x + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \frac{Dd^2x^4}{4b^2} + x^3\left(\frac{Cd^2}{3b^2} - \frac{2Dad^2}{3b^3} + \frac{2Dcd}{3b^2}\right)$$
$$+ x^2\left(\frac{Bd^2}{2b^2} - \frac{Cad^2}{b^3} + \frac{Ccd}{b^2} + \frac{3Da^2d^2}{2b^4} - \frac{2Dacd}{b^3} + \frac{Dc^2}{2b^2}\right)$$
$$+ x\left(\frac{Ad^2}{b^2} - \frac{2Bad^2}{b^3} + \frac{2Bcd}{b^2} + \frac{3Ca^2d^2}{b^4} - \frac{4Cacd}{b^3} + \frac{Cc^2}{b^2} - \frac{4Da^3d^2}{b^5} + \frac{6Da^2cd}{b^4} - \frac{2Dac^2}{b^3}\right)$$
$$+ \frac{-Aa^2b^3d^2 + 2Aab^4cd - Ab^5c^2 + Ba^3b^2d^2 - 2Ba^2b^3cd + Bab^4c^2 - Ca^4bd^2 + 2Ca^3b^2cd - Ca^2b^3c^2 + D}{ab^6 + b^7x}$$
$$+ \frac{(ad-bc)\,(-2Ab^3d + 3Bab^2d - Bb^3c - 4Ca^2bd + 2Cab^2c + 5Da^3d - 3Da^2bc)\log(a+bx)}{b^6}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a)**2,x)
```

output
```
D*d**2*x**4/(4*b**2) + x**3*(C*d**2/(3*b**2) - 2*D*a*d**2/(3*b**3) + 2*D*c
*d/(3*b**2)) + x**2*(B*d**2/(2*b**2) - C*a*d**2/b**3 + C*c*d/b**2 + 3*D*a*
*2*d**2/(2*b**4) - 2*D*a*c*d/b**3 + D*c**2/(2*b**2)) + x*(A*d**2/b**2 - 2*
B*a*d**2/b**3 + 2*B*c*d/b**2 + 3*C*a**2*d**2/b**4 - 4*C*a*c*d/b**3 + C*c**
2/b**2 - 4*D*a**3*d**2/b**5 + 6*D*a**2*c*d/b**4 - 2*D*a*c**2/b**3) + (-A*a
**2*b**3*d**2 + 2*A*a*b**4*c*d - A*b**5*c**2 + B*a**3*b**2*d**2 - 2*B*a**2
*b**3*c*d + B*a*b**4*c**2 - C*a**4*b*d**2 + 2*C*a**3*b**2*c*d - C*a**2*b**
3*c**2 + D*a**5*d**2 - 2*D*a**4*b*c*d + D*a**3*b**2*c**2)/(a*b**6 + b**7*x
) + (a*d - b*c)*(-2*A*b**3*d + 3*B*a*b**2*d - B*b**3*c - 4*C*a**2*b*d + 2*
C*a*b**2*c + 5*D*a**3*d - 3*D*a**2*b*c)*log(a + b*x)/b**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$
$$= \frac{(Da^3b^2 - Ca^2b^3 + Bab^4 - Ab^5)c^2 - 2\,(Da^4b - Ca^3b^2 + Ba^2b^3 - Aab^4)cd + (Da^5 - Ca^4b + Ba^3b^2 - Aa}{b^7x + ab^6}$$
$$+ \frac{3\,Db^3d^2x^4 + 4\,(2\,Db^3cd - (2\,Dab^2 - Cb^3)d^2)x^3 + 6\,(Db^3c^2 - 2\,(2\,Dab^2 - Cb^3)cd + (3\,Da^2b - 2\,Cab^2}{}$$
$$+ \frac{((3\,Da^2b^2 - 2\,Cab^3 + Bb^4)c^2 - 2\,(4\,Da^3b - 3\,Ca^2b^2 + 2\,Bab^3 - Ab^4)cd + (5\,Da^4 - 4\,Ca^3b + 3\,Ba^2b^2}{b^6}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
((D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*c^2 - 2*(D*a^4*b - C*a^3*b^2 +
B*a^2*b^3 - A*a*b^4)*c*d + (D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*d^2)/
(b^7*x + a*b^6) + 1/12*(3*D*b^3*d^2*x^4 + 4*(2*D*b^3*c*d - (2*D*a*b^2 - C*
b^3)*d^2)*x^3 + 6*(D*b^3*c^2 - 2*(2*D*a*b^2 - C*b^3)*c*d + (3*D*a^2*b - 2*
C*a*b^2 + B*b^3)*d^2)*x^2 - 12*((2*D*a*b^2 - C*b^3)*c^2 - 2*(3*D*a^2*b - 2
*C*a*b^2 + B*b^3)*c*d + (4*D*a^3 - 3*C*a^2*b + 2*B*a*b^2 - A*b^3)*d^2)*x)/
b^5 + ((3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c^2 - 2*(4*D*a^3*b - 3*C*a^2*b^2
+ 2*B*a*b^3 - A*b^4)*c*d + (5*D*a^4 - 4*C*a^3*b + 3*B*a^2*b^2 - 2*A*a*b^3)
*d^2)*log(b*x + a)/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(278) = 556$.

Time = 0.12 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.07

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$= \frac{\left(3\,Dd^2 + \frac{4\,(2\,Db^2cd-5\,Dabd^2+Cb^2d^2)}{(bx+a)b} + \frac{6\,(Db^4c^2-8\,Dab^3cd+2\,Cb^4cd+10\,Da^2b^2d^2-4\,Cab^3d^2+Bb^4d^2)}{(bx+a)^2b^2} - \frac{12\,(3\,Dab^5c^2-Cb^6c^2-12}{}\right.}{12\,b^6}$$
$$-\frac{(3\,Da^2b^2c^2 - 2\,Cab^3c^2 + Bb^4c^2 - 8\,Da^3bcd + 6\,Ca^2b^2cd - 4\,Bab^3cd + 2\,Ab^4cd + 5\,Da^4d^2 - 4\,Ca^3bd^2}{b^6}$$
$$+\frac{\frac{Da^3b^6c^2}{bx+a} - \frac{Ca^2b^7c^2}{bx+a} + \frac{Bab^8c^2}{bx+a} - \frac{Ab^9c^2}{bx+a} - \frac{2\,Da^4b^5cd}{bx+a} + \frac{2\,Ca^3b^6cd}{bx+a} - \frac{2\,Ba^2b^7cd}{bx+a} + \frac{2\,Aab^8cd}{bx+a} + \frac{Da^5b^4d^2}{bx+a} - \frac{Ca^4b^5d^2}{bx+a} + \frac{Ba^3}{bx}}{b^{10}}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/12*(3*D*d^2 + 4*(2*D*b^2*c*d - 5*D*a*b*d^2 + C*b^2*d^2)/((b*x + a)*b) +
6*(D*b^4*c^2 - 8*D*a*b^3*c*d + 2*C*b^4*c*d + 10*D*a^2*b^2*d^2 - 4*C*a*b^3*
d^2 + B*b^4*d^2)/((b*x + a)^2*b^2) - 12*(3*D*a*b^5*c^2 - C*b^6*c^2 - 12*D*
a^2*b^4*c*d + 6*C*a*b^5*c*d - 2*B*b^6*c*d + 10*D*a^3*b^3*d^2 - 6*C*a^2*b^4
*d^2 + 3*B*a*b^5*d^2 - A*b^6*d^2)/((b*x + a)^3*b^3))*(b*x + a)^4/b^6 - (3*
D*a^2*b^2*c^2 - 2*C*a*b^3*c^2 + B*b^4*c^2 - 8*D*a^3*b*c*d + 6*C*a^2*b^2*c*
d - 4*B*a*b^3*c*d + 2*A*b^4*c*d + 5*D*a^4*d^2 - 4*C*a^3*b*d^2 + 3*B*a^2*b^
2*d^2 - 2*A*a*b^3*d^2)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^6 + (D*a^3
*b^6*c^2/(b*x + a) - C*a^2*b^7*c^2/(b*x + a) + B*a*b^8*c^2/(b*x + a) - A*b
^9*c^2/(b*x + a) - 2*D*a^4*b^5*c*d/(b*x + a) + 2*C*a^3*b^6*c*d/(b*x + a) -
 2*B*a^2*b^7*c*d/(b*x + a) + 2*A*a*b^8*c*d/(b*x + a) + D*a^5*b^4*d^2/(b*x
+ a) - C*a^4*b^5*d^2/(b*x + a) + B*a^3*b^6*d^2/(b*x + a) - A*a^2*b^7*d^2/(
b*x + a))/b^10
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx=\int \frac{(c+d\,x)^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{(a+b\,x)^2}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.68

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx = \frac{60\log(bx+a)\,a^5d^3+6b^6d^2x^3+12b^5c^3x^2+3b^5d^3x^5-24\log(bx+a)\,a^2b^3c^3-60a^4b\,d^3x-30a^3b^2d^3x^2+\cdots}{\cdots}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

output
```
(60*log(a + b*x)*a**5*d**3 - 144*log(a + b*x)*a**4*b*c*d**2 + 60*log(a + b
*x)*a**4*b*d**3*x + 12*log(a + b*x)*a**3*b**3*d**2 + 108*log(a + b*x)*a**3
*b**2*c**2*d - 144*log(a + b*x)*a**3*b**2*c*d**2*x - 24*log(a + b*x)*a**2*
b**4*c*d + 12*log(a + b*x)*a**2*b**4*d**2*x - 24*log(a + b*x)*a**2*b**3*c*
*3 + 108*log(a + b*x)*a**2*b**3*c**2*d*x + 12*log(a + b*x)*a*b**5*c**2 - 2
4*log(a + b*x)*a*b**5*c*d*x - 24*log(a + b*x)*a*b**4*c**3*x + 12*log(a + b
*x)*b**6*c**2*x - 60*a**4*b*d**3*x + 144*a**3*b**2*c*d**2*x - 30*a**3*b**2
*d**3*x**2 - 12*a**2*b**4*d**2*x - 108*a**2*b**3*c**2*d*x + 72*a**2*b**3*c
*d**2*x**2 + 10*a**2*b**3*d**3*x**3 + 24*a*b**5*c*d*x - 6*a*b**5*d**2*x**2
 + 24*a*b**4*c**3*x - 54*a*b**4*c**2*d*x**2 - 24*a*b**4*c*d**2*x**3 - 5*a*
b**4*d**3*x**4 + 24*b**6*c*d*x**2 + 6*b**6*d**2*x**3 + 12*b**5*c**3*x**2 +
 18*b**5*c**2*d*x**3 + 12*b**5*c*d**2*x**4 + 3*b**5*d**3*x**5)/(12*b**6*(a
 + b*x))
```

3.19 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^3} dx$

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Optimal result

Integrand size = 30, antiderivative size = 284

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx &= \frac{(6a^2d^2D-3abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D))\,x}{b^5} \\ &+ \frac{d(bCd+2bcD-3adD)x^2}{2b^4}+\frac{d^2Dx^3}{3b^3}-\frac{(bc-ad)^2\,(Ab^3-a(b^2B-abC+a^2D))}{2b^6(a+bx)^2} \\ &- \frac{(bc-ad)\,(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))}{b^6(a+bx)} \\ &+ \frac{(b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D))\log(a+bx)}{b^6}\end{aligned}$$

output
```
(6*a^2*d^2*D-3*a*b*d*(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))*x/b^5+1/2*d*(C
*b*d-3*D*a*d+2*D*b*c)*x^2/b^4+1/3*d^2*D*x^3/b^3-1/2*(-a*d+b*c)^2*(A*b^3-a*
(B*b^2-C*a*b+D*a^2))/b^6/(b*x+a)^2-(-a*d+b*c)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*
d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))/b^6/(b*x+a)+(b^3*(A*d^2+2*B*c*d+C*
c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*c)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))*ln(
b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^3} dx$$
$$= \frac{6b(6a^2d^2D-3abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D))x+3b^2d(bCd+2bcD-3adD)x^2+2b^3d^2Dx^3}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
(6*b*(6*a^2*d^2*D - 3*a*b*d*(C*d + 2*c*D) + b^2*(2*c*C*d + B*d^2 + c^2*D))
*x + 3*b^2*d*(b*C*d + 2*b*c*D - 3*a*d*D)*x^2 + 2*b^3*d^2*D*x^3 - (3*(b*c -
 a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(a + b*x)^2 - (6*(b*c - a*d)*
(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a^2*b*(4*C*d + 3*
c*D)))/(a + b*x) + 6*(b^3*(c^2*C + 2*B*c*d + A*d^2) - 10*a^3*d^2*D + 6*a^2
*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^2*D))*Log[a + b*x])/(6*b
^6)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^3} dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)^2 (Ab^3-a(a^2D-abC+b^2B))}{b^5(a+bx)^3}+\frac{6a^2d^2D-3abd(2cD+Cd)+b^2(Bd^2+c^2D+2cCd)}{b^5}+\frac{-10a^3}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^6(a+bx)^2}+\frac{x\left(6a^2d^2D-3abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)\right)}{b^5}+\frac{\log(a+bx)\left(-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)\right)}{b^6}-\frac{(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{b^6(a+bx)}+\frac{dx^2(-3adD+2bcD+bCd)}{2b^4}+\frac{d^2Dx^3}{3b^3}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
((6*a^2*d^2*D - 3*a*b*d*(C*d + 2*c*D) + b^2*(2*c*C*d + B*d^2 + c^2*D))*x)/
b^5 + (d*(b*C*d + 2*b*c*D - 3*a*d*D)*x^2)/(2*b^4) + (d^2*D*x^3)/(3*b^3) -
((b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(2*b^6*(a + b*x)^2) -
((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a^2*
b*(4*C*d + 3*c*D)))/(b^6*(a + b*x)) + ((b^3*(c^2*C + 2*B*c*d + A*d^2) - 10
*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^2*D))*
Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.57

method	result
norman	$\frac{\left(2Aa\,b^{3}d^{2}-2A\,b^{4}cd-6B\,a^{2}b^{2}d^{2}+4Ba\,b^{3}cd-B\,b^{4}c^{2}+12C\,a^{3}b\,d^{2}-12C\,a^{2}b^{2}cd+2Ca\,b^{3}c^{2}-20Da^{4}d^{2}+24Da^{3}bcd-6Da^{2}b^{2}c^{2}\right)x}{b^{5}}+\frac{3A\,a^{2}b^{3}\cdots}{}$
default	$\frac{\frac{1}{3}d^{2}Dx^{3}b^{2}+\frac{1}{2}C\,b^{2}d^{2}x^{2}-\frac{3}{2}Dab\,d^{2}x^{2}+Db^{2}cd\,x^{2}+b^{2}B\,d^{2}x-3Cab\,d^{2}x+2C\,b^{2}cdx+6a^{2}d^{2}Dx-6Dabcdx+Db^{2}c^{2}x}{b^{5}}-\frac{A\,a^{2}b^{3}d\cdots}{}$
parallelrisch	$\frac{12C\,x^{3}b^{5}cd+20Dx^{3}a^{2}b^{3}d^{2}+24Bxa\,b^{4}cd+12B\ln(bx+a)a^{2}b^{3}cd-36C\ln(bx+a)a^{3}b^{2}cd-24Dx^{3}a\,b^{4}cd+12A\ln(bx+a)xa\,b^{4}d^{2}\cdots}{}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
((2*A*a*b^3*d^2-2*A*b^4*c*d-6*B*a^2*b^2*d^2+4*B*a*b^3*c*d-B*b^4*c^2+12*C*a
^3*b*d^2-12*C*a^2*b^2*c*d+2*C*a*b^3*c^2-20*D*a^4*d^2+24*D*a^3*b*c*d-6*D*a^
2*b^2*c^2)/b^5*x+1/2*(3*A*a^2*b^3*d^2-2*A*a*b^4*c*d-A*b^5*c^2-9*B*a^3*b^2*
d^2+6*B*a^2*b^3*c*d-B*a*b^4*c^2+18*C*a^4*b*d^2-18*C*a^3*b^2*c*d+3*C*a^2*b^
3*c^2-30*D*a^5*d^2+36*D*a^4*b*c*d-9*D*a^3*b^2*c^2)/b^6+1/3*(3*B*b^2*d^2-6*
C*a*b*d^2+6*C*b^2*c*d+10*D*a^2*d^2-12*D*a*b*c*d+3*D*b^2*c^2)/b^3*x^3+1/3*D
/b*d^2*x^5+1/6*d*(3*C*b*d-5*D*a*d+6*D*b*c)/b^2*x^4)/(b*x+a)^2+1/b^6*(A*b^3
*d^2-3*B*a*b^2*d^2+2*B*b^3*c*d+6*C*a^2*b*d^2-6*C*a*b^2*c*d+C*b^3*c^2-10*D*
a^3*d^2+12*D*a^2*b*c*d-3*D*a*b^2*c^2)*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(280) = 560$.

Time = 0.08 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.40

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$
$$= \frac{2\,Db^5d^2x^5+(6\,Db^5cd-(5\,Dab^4-3\,Cb^5)d^2)x^4+2\,(3\,Db^5c^2-6\,(2\,Dab^4-Cb^5)cd+(10\,Da^2b^3-6\,Cab\cdots}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/6*(2*D*b^5*d^2*x^5 + (6*D*b^5*c*d - (5*D*a*b^4 - 3*C*b^5)*d^2)*x^4 + 2*(
3*D*b^5*c^2 - 6*(2*D*a*b^4 - C*b^5)*c*d + (10*D*a^2*b^3 - 6*C*a*b^4 + 3*B*
b^5)*d^2)*x^3 - 3*(5*D*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4 + A*b^5)*c^2 + 6*(7
*D*a^4*b - 5*C*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c*d - 3*(9*D*a^5 - 7*C*a^4
*b + 5*B*a^3*b^2 - 3*A*a^2*b^3)*d^2 + 3*(4*D*a*b^4*c^2 - 2*(11*D*a^2*b^3 -
 4*C*a*b^4)*c*d + (21*D*a^3*b^2 - 11*C*a^2*b^3 + 4*B*a*b^4)*d^2)*x^2 - 6*(
(2*D*a^2*b^3 - 2*C*a*b^4 + B*b^5)*c^2 - 2*(D*a^3*b^2 - 2*C*a^2*b^3 + 2*B*a
*b^4 - A*b^5)*c*d - (D*a^4*b + C*a^3*b^2 - 2*B*a^2*b^3 + 2*A*a*b^4)*d^2)*x
 - 6*((3*D*a^3*b^2 - C*a^2*b^3)*c^2 - 2*(6*D*a^4*b - 3*C*a^3*b^2 + B*a^2*b
^3)*c*d + (10*D*a^5 - 6*C*a^4*b + 3*B*a^3*b^2 - A*a^2*b^3)*d^2 + ((3*D*a*b
^4 - C*b^5)*c^2 - 2*(6*D*a^2*b^3 - 3*C*a*b^4 + B*b^5)*c*d + (10*D*a^3*b^2
- 6*C*a^2*b^3 + 3*B*a*b^4 - A*b^5)*d^2)*x^2 + 2*((3*D*a^2*b^3 - C*a*b^4)*c
^2 - 2*(6*D*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4)*c*d + (10*D*a^4*b - 6*C*a^3*b
^2 + 3*B*a^2*b^3 - A*a*b^4)*d^2)*x)*log(b*x + a))/(b^8*x^2 + 2*a*b^7*x + a
^2*b^6)
```

Sympy [A] (verification not implemented)

Time = 20.49 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.87

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \frac{Dd^2x^3}{3b^3} + x^2\left(\frac{Cd^2}{2b^3} - \frac{3Dad^2}{2b^4} + \frac{Dcd}{b^3}\right) + x\left(\frac{Bd^2}{b^3} - \frac{3Cad^2}{b^4} + \frac{2Ccd}{b^3} + \frac{6Da^2d^2}{b^5} - \frac{6Dacd}{b^4} + \frac{Dc^2}{b^3}\right) + \frac{3Aa^2b^3d^2 - 2Aab^4cd - Ab^5c^2 - 5Ba^3b^2d^2 + 6Ba^2b^3cd - Bab^4c^2 + 7Ca^4bd^2 - 10Ca^3b^2cd + 3Ca^2b^3c^2}{} - \frac{(-Ab^3d^2 + 3Bab^2d^2 - 2Bb^3cd - 6Ca^2bd^2 + 6Cab^2cd - Cb^3c^2 + 10Da^3d^2 - 12Da^2bcd + 3Dab^2c^2)\log}{b^6}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a)**3,x)
```

output
```
D*d**2*x**3/(3*b**3) + x**2*(C*d**2/(2*b**3) - 3*D*a*d**2/(2*b**4) + D*c*d
/b**3) + x*(B*d**2/b**3 - 3*C*a*d**2/b**4 + 2*C*c*d/b**3 + 6*D*a**2*d**2/b
**5 - 6*D*a*c*d/b**4 + D*c**2/b**3) + (3*A*a**2*b**3*d**2 - 2*A*a*b**4*c*d
 - A*b**5*c**2 - 5*B*a**3*b**2*d**2 + 6*B*a**2*b**3*c*d - B*a*b**4*c**2 +
7*C*a**4*b*d**2 - 10*C*a**3*b**2*c*d + 3*C*a**2*b**3*c**2 - 9*D*a**5*d**2
+ 14*D*a**4*b*c*d - 5*D*a**3*b**2*c**2 + x*(4*A*a*b**4*d**2 - 4*A*b**5*c*d
 - 6*B*a**2*b**3*d**2 + 8*B*a*b**4*c*d - 2*B*b**5*c**2 + 8*C*a**3*b**2*d**
2 - 12*C*a**2*b**3*c*d + 4*C*a*b**4*c**2 - 10*D*a**4*b*d**2 + 16*D*a**3*b*
*2*c*d - 6*D*a**2*b**3*c**2))/(2*a**2*b**6 + 4*a*b**7*x + 2*b**8*x**2) - (
-A*b**3*d**2 + 3*B*a*b**2*d**2 - 2*B*b**3*c*d - 6*C*a**2*b*d**2 + 6*C*a*b*
*2*c*d - C*b**3*c**2 + 10*D*a**3*d**2 - 12*D*a**2*b*c*d + 3*D*a*b**2*c**2)
*log(a + b*x)/b**6
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx =$$

$$-\frac{(5\,Da^3b^2 - 3\,Ca^2b^3 + Bab^4 + Ab^5)c^2 - 2\,(7\,Da^4b - 5\,Ca^3b^2 + 3\,Ba^2b^3 - Aab^4)cd + (9\,Da^5 - 7\,Ca^4b}{}$$

$$+\frac{2\,Db^2d^2x^3 + 3\,(2\,Db^2cd - (3\,Dab - Cb^2)d^2)x^2 + 6\,(Db^2c^2 - 2\,(3\,Dab - Cb^2)cd + (6\,Da^2 - 3\,Cab + B}{6\,b^5}$$

$$-\frac{((3\,Dab^2 - Cb^3)c^2 - 2\,(6\,Da^2b - 3\,Cab^2 + Bb^3)cd + (10\,Da^3 - 6\,Ca^2b + 3\,Bab^2 - Ab^3)d^2)\log(bx + a}{b^6}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*((5*D*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4 + A*b^5)*c^2 - 2*(7*D*a^4*b - 5
*C*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*c*d + (9*D*a^5 - 7*C*a^4*b + 5*B*a^3*b
^2 - 3*A*a^2*b^3)*d^2 + 2*((3*D*a^2*b^3 - 2*C*a*b^4 + B*b^5)*c^2 - 2*(4*D*
a^3*b^2 - 3*C*a^2*b^3 + 2*B*a*b^4 - A*b^5)*c*d + (5*D*a^4*b - 4*C*a^3*b^2
+ 3*B*a^2*b^3 - 2*A*a*b^4)*d^2)*x)/(b^8*x^2 + 2*a*b^7*x + a^2*b^6) + 1/6*(
2*D*b^2*d^2*x^3 + 3*(2*D*b^2*c*d - (3*D*a*b - C*b^2)*d^2)*x^2 + 6*(D*b^2*c
^2 - 2*(3*D*a*b - C*b^2)*c*d + (6*D*a^2 - 3*C*a*b + B*b^2)*d^2)*x)/b^5 - (
(3*D*a*b^2 - C*b^3)*c^2 - 2*(6*D*a^2*b - 3*C*a*b^2 + B*b^3)*c*d + (10*D*a^
3 - 6*C*a^2*b + 3*B*a*b^2 - A*b^3)*d^2)*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^3} \, dx =$$
$$-\frac{(3\,Dab^2c^2-Cb^3c^2-12\,Da^2bcd+6\,Cab^2cd-2\,Bb^3cd+10\,Da^3d^2-6\,Ca^2bd^2+3\,Bab^2d^2-Ab^3d^2)\log}{b^6}$$
$$-\frac{5\,Da^3b^2c^2-3\,Ca^2b^3c^2+Bab^4c^2+Ab^5c^2-14\,Da^4bcd+10\,Ca^3b^2cd-6\,Ba^2b^3cd+2\,Aab^4cd+9\,Da^5}{}$$
$$+\frac{2\,Db^6d^2x^3+6\,Db^6cdx^2-9\,Dab^5d^2x^2+3\,Cb^6d^2x^2+6\,Db^6c^2x-36\,Dab^5cdx+12\,Cb^6cdx+36\,Da^2b^4}{6\,b^9}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-(3*D*a*b^2*c^2 - C*b^3*c^2 - 12*D*a^2*b*c*d + 6*C*a*b^2*c*d - 2*B*b^3*c*d
 + 10*D*a^3*d^2 - 6*C*a^2*b*d^2 + 3*B*a*b^2*d^2 - A*b^3*d^2)*log(abs(b*x +
 a))/b^6 - 1/2*(5*D*a^3*b^2*c^2 - 3*C*a^2*b^3*c^2 + B*a*b^4*c^2 + A*b^5*c^
2 - 14*D*a^4*b*c*d + 10*C*a^3*b^2*c*d - 6*B*a^2*b^3*c*d + 2*A*a*b^4*c*d +
9*D*a^5*d^2 - 7*C*a^4*b*d^2 + 5*B*a^3*b^2*d^2 - 3*A*a^2*b^3*d^2 + 2*(3*D*a
^2*b^3*c^2 - 2*C*a*b^4*c^2 + B*b^5*c^2 - 8*D*a^3*b^2*c*d + 6*C*a^2*b^3*c*d
 - 4*B*a*b^4*c*d + 2*A*b^5*c*d + 5*D*a^4*b*d^2 - 4*C*a^3*b^2*d^2 + 3*B*a^2
*b^3*d^2 - 2*A*a*b^4*d^2)*x)/((b*x + a)^2*b^6) + 1/6*(2*D*b^6*d^2*x^3 + 6*
D*b^6*c*d*x^2 - 9*D*a*b^5*d^2*x^2 + 3*C*b^6*d^2*x^2 + 6*D*b^6*c^2*x - 36*D
*a*b^5*c*d*x + 12*C*b^6*c*d*x + 36*D*a^2*b^4*d^2*x - 18*C*a*b^5*d^2*x + 6*
B*b^6*d^2*x)/b^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^3} \, dx = \int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^3} \,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.14

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^3} \, dx$$
$$= \frac{216 \log(bx+a) a^4 b^2 c d^2 x - 108 \log(bx+a) a^3 b^3 c^2 d x + 108 \log(bx+a) a^3 b^3 c d^2 x^2 + 24 \log(bx+a) a^2 b^5 c}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
( - 60*log(a + b*x)*a**6*d**3 + 108*log(a + b*x)*a**5*b*c*d**2 - 120*log(a
 + b*x)*a**5*b*d**3*x - 12*log(a + b*x)*a**4*b**3*d**2 - 54*log(a + b*x)*a
**4*b**2*c**2*d + 216*log(a + b*x)*a**4*b**2*c*d**2*x - 60*log(a + b*x)*a*
*4*b**2*d**3*x**2 + 12*log(a + b*x)*a**3*b**4*c*d - 24*log(a + b*x)*a**3*b
**4*d**2*x + 6*log(a + b*x)*a**3*b**3*c**3 - 108*log(a + b*x)*a**3*b**3*c*
*2*d*x + 108*log(a + b*x)*a**3*b**3*c*d**2*x**2 + 24*log(a + b*x)*a**2*b**
5*c*d*x - 12*log(a + b*x)*a**2*b**5*d**2*x**2 + 12*log(a + b*x)*a**2*b**4*
c**3*x - 54*log(a + b*x)*a**2*b**4*c**2*d*x**2 + 12*log(a + b*x)*a*b**6*c*
d*x**2 + 6*log(a + b*x)*a*b**5*c**3*x**2 - 30*a**6*d**3 + 54*a**5*b*c*d**2
 - 6*a**4*b**3*d**2 - 27*a**4*b**2*c**2*d + 60*a**4*b**2*d**3*x**2 + 6*a**
3*b**4*c*d + 3*a**3*b**3*c**3 - 108*a**3*b**3*c*d**2*x**2 + 20*a**3*b**3*d
**3*x**3 - 3*a**2*b**5*c**2 + 12*a**2*b**5*d**2*x**2 + 54*a**2*b**4*c**2*d
*x**2 - 36*a**2*b**4*c*d**2*x**3 - 5*a**2*b**4*d**3*x**4 - 6*a*b**6*c*d*x*
*2 + 6*a*b**6*d**2*x**3 - 6*a*b**5*c**3*x**2 + 18*a*b**5*c**2*d*x**3 + 9*a
*b**5*c*d**2*x**4 + 2*a*b**5*d**3*x**5 + 3*b**7*c**2*x**2)/(6*a*b**6*(a**2
 + 2*a*b*x + b**2*x**2))
```

3.20 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 288

$$\begin{aligned}
&\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx \\
&= \frac{d(bCd+2bcD-4adD)x}{b^5}+\frac{d^2Dx^2}{2b^4}-\frac{(bc-ad)^2(Ab^3-a(b^2B-abC+a^2D))}{3b^6(a+bx)^3} \\
&\quad -\frac{(bc-ad)(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))}{2b^6(a+bx)^2} \\
&\quad -\frac{b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D)}{b^6(a+bx)} \\
&\quad +\frac{(10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D))\log(a+bx)}{b^6}
\end{aligned}$$

output

```
d*(C*b*d-4*D*a*d+2*D*b*c)*x/b^5+1/2*d^2*D*x^2/b^4-1/3*(-a*d+b*c)^2*(A*b^3-
a*(B*b^2-C*a*b+D*a^2))/b^6/(b*x+a)^3-1/2*(-a*d+b*c)*(b^3*(2*A*d+B*c)-a*b^2
*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))/b^6/(b*x+a)^2-(b^3*(A*d^2+2*
B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*c)-3*a*b^2*(B*d^2+2*C*c*d+D*c
^2))/b^6/(b*x+a)+(10*a^2*d^2*D-4*a*b*d*(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^
2))*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx$$
$$= \frac{6bd(bCd+2bcD-4adD)x+3b^2d^2Dx^2-\frac{2(bc-ad)^2\left(Ab^3-a\left(b^2B-abC+a^2D\right)\right)}{(a+bx)^3}-\frac{3(bc-ad)\left(b^3(Bc+2Ad)-ab^2(2cC+3Bd)\right.}{(a+bx)^2}}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
(6*b*d*(b*C*d + 2*b*c*D - 4*a*d*D)*x + 3*b^2*d^2*D*x^2 - (2*(b*c - a*d)^2*
(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(a + b*x)^3 - (3*(b*c - a*d)*(b^3*(B*
c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a^2*b*(4*C*d + 3*c*D)))/(
a + b*x)^2 + (6*(-(b^3*(c^2*C + 2*B*c*d + A*d^2)) + 10*a^3*d^2*D - 6*a^2*b
*d*(C*d + 2*c*D) + 3*a*b^2*(2*c*C*d + B*d^2 + c^2*D)))/(a + b*x) + 6*(10*a
^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2*c*C*d + B*d^2 + c^2*D))*Log[a +
b*x])/(6*b^6)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx$$
$$\Big\downarrow \; 2123$$
$$\int \left(\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^5(a+bx)^4}+\frac{10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)}{b^5(a+bx)}+\frac{-10a\ldots}{}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^6(a+bx)^3}+\frac{\log(a+bx)\left(10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)\right)}{b^6}-\frac{-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)}{b^6(a+bx)}-\frac{(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{2b^6(a+bx)^2}+\frac{dx(-4adD+2bcD+bCd)}{b^5}+\frac{d^2Dx^2}{2b^4}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
(d*(b*C*d + 2*b*c*D - 4*a*d*D)*x)/b^5 + (d^2*D*x^2)/(2*b^4) - ((b*c - a*d)
^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(3*b^6*(a + b*x)^3) - ((b*c - a*d)
*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a^2*b*(4*C*d + 3
*c*D)))/(2*b^6*(a + b*x)^2) - (b^3*(c^2*C + 2*B*c*d + A*d^2) - 10*a^3*d^2*
D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^2*D))/(b^6*(a +
 b*x)) + ((10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2*c*C*d + B*d^2 + c
^2*D))*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.53

method	result
default	$\frac{d\left(\frac{1}{2}dDx^2b+Cbdx-4Dadx+2Dbcx\right)}{b^5}-\frac{-2Aa\,b^3d^2+2A\,b^4cd+3B\,a^2b^2d^2-4Ba\,b^3cd+B\,b^4c^2-4C\,a^3b\,d^2+6C\,a^2b^2cd-2Ca\,b}{2b^6(bx+a)^2}$
norman	$-\frac{2A\,a^2b^3d^2+2Aa\,b^4cd+2A\,b^5c^2-11B\,a^3b^2d^2+4B\,a^2b^3cd+Ba\,b^4c^2+44C\,a^4b\,d^2-22C\,a^3b^2cd+2C\,a^2b^3c^2-110Da^5d^2+88Da^4bcd-11Da}{6b^6}$
parallelrisch	$-\frac{-18B\,x^2a\,b^4d^2+144Dx^2a^2b^3cd+12Bxa\,b^4cd-12C\ln(bx+a)a^3b^2cd-18B\ln(bx+a)x\,a^2b^3d^2+72C\ln(bx+a)x\,a^3b^2d^2-180}{}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
d/b^5*(1/2*d*D*x^2*b+C*b*d*x-4*D*a*d*x+2*D*b*c*x)-1/2*(-2*A*a*b^3*d^2+2*A*
b^4*c*d+3*B*a^2*b^2*d^2-4*B*a*b^3*c*d+B*b^4*c^2-4*C*a^3*b*d^2+6*C*a^2*b^2*
c*d-2*C*a*b^3*c^2+5*D*a^4*d^2-8*D*a^3*b*c*d+3*D*a^2*b^2*c^2)/b^6/(b*x+a)^2
-1/b^6*(A*b^3*d^2-3*B*a*b^2*d^2+2*B*b^3*c*d+6*C*a^2*b*d^2-6*C*a*b^2*c*d+C*
b^3*c^2-10*D*a^3*d^2+12*D*a^2*b*c*d-3*D*a*b^2*c^2)/(b*x+a)+1/b^6*(B*b^2*d^
2-4*C*a*b*d^2+2*C*b^2*c*d+10*D*a^2*d^2-8*D*a*b*c*d+D*b^2*c^2)*ln(b*x+a)-1/
3*(A*a^2*b^3*d^2-2*A*a*b^4*c*d+A*b^5*c^2-B*a^3*b^2*d^2+2*B*a^2*b^3*c*d-B*a
*b^4*c^2+C*a^4*b*d^2-2*C*a^3*b^2*c*d+C*a^2*b^3*c^2-D*a^5*d^2+2*D*a^4*b*c*d
-D*a^3*b^2*c^2)/b^6/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(283) = 566$.

Time = 0.08 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.34

$$\int\frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx$$
$$=\frac{3\,Db^5d^2x^5+3\,(4\,Db^5cd-(5\,Dab^4-2\,Cb^5)d^2)x^4+9\,(4\,Dab^4cd-(7\,Da^2b^3-2\,Cab^4)d^2)x^3+(11\,Da^3b^2}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/6*(3*D*b^5*d^2*x^5 + 3*(4*D*b^5*c*d - (5*D*a*b^4 - 2*C*b^5)*d^2)*x^4 + 9
*(4*D*a*b^4*c*d - (7*D*a^2*b^3 - 2*C*a*b^4)*d^2)*x^3 + (11*D*a^3*b^2 - 2*C
*a^2*b^3 - B*a*b^4 - 2*A*b^5)*c^2 - 2*(26*D*a^4*b - 11*C*a^3*b^2 + 2*B*a^2
*b^3 + A*a*b^4)*c*d + (47*D*a^5 - 26*C*a^4*b + 11*B*a^3*b^2 - 2*A*a^2*b^3)
*d^2 + 3*(2*(3*D*a*b^4 - C*b^5)*c^2 - 4*(3*D*a^2*b^3 - 3*C*a*b^4 + B*b^5)*
c*d - (3*D*a^3*b^2 + 6*C*a^2*b^3 - 6*B*a*b^4 + 2*A*b^5)*d^2)*x^2 + 3*((9*D
*a^2*b^3 - 2*C*a*b^4 - B*b^5)*c^2 - 2*(18*D*a^3*b^2 - 9*C*a^2*b^3 + 2*B*a*
b^4 + A*b^5)*c*d + (27*D*a^4*b - 18*C*a^3*b^2 + 9*B*a^2*b^3 - 2*A*a*b^4)*d
^2)*x + 6*(D*a^3*b^2*c^2 + (D*b^5*c^2 - 2*(4*D*a*b^4 - C*b^5)*c*d + (10*D*
a^2*b^3 - 4*C*a*b^4 + B*b^5)*d^2)*x^3 - 2*(4*D*a^4*b - C*a^3*b^2)*c*d + (1
0*D*a^5 - 4*C*a^4*b + B*a^3*b^2)*d^2 + 3*(D*a*b^4*c^2 - 2*(4*D*a^2*b^3 - C
*a*b^4)*c*d + (10*D*a^3*b^2 - 4*C*a^2*b^3 + B*a*b^4)*d^2)*x^2 + 3*(D*a^2*b
^3*c^2 - 2*(4*D*a^3*b^2 - C*a^2*b^3)*c*d + (10*D*a^4*b - 4*C*a^3*b^2 + B*a
^2*b^3)*d^2)*x)*log(b*x + a))/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b
^6)
```

Sympy [A] (verification not implemented)

Time = 134.58 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx = \frac{Dd^2x^2}{2b^4} + x\left(\frac{Cd^2}{b^4} - \frac{4Dad^2}{b^5} + \frac{2Dcd}{b^4}\right) + \frac{-2Aa^2b^3d^2 - 2Aab^4cd - 2Ab^5c^2 + 11Ba^3b^2d^2 - 4Ba^2b^3cd - Bab^4c^2 - 26Ca^4bd^2 + 22Ca^3b^2cd - 2Ca}{} + \frac{\left(Bb^2d^2 - 4Cabd^2 + 2Cb^2cd + 10Da^2d^2 - 8Dabcd + Db^2c^2\right)\log{(a+bx)}}{b^6}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a)**4,x)
```

output

```
D*d**2*x**2/(2*b**4) + x*(C*d**2/b**4 - 4*D*a*d**2/b**5 + 2*D*c*d/b**4) +
(-2*A*a**2*b**3*d**2 - 2*A*a*b**4*c*d - 2*A*b**5*c**2 + 11*B*a**3*b**2*d**
2 - 4*B*a**2*b**3*c*d - B*a*b**4*c**2 - 26*C*a**4*b*d**2 + 22*C*a**3*b**2*
c*d - 2*C*a**2*b**3*c**2 + 47*D*a**5*d**2 - 52*D*a**4*b*c*d + 11*D*a**3*b*
*2*c**2 + x**2*(-6*A*b**5*d**2 + 18*B*a*b**4*d**2 - 12*B*b**5*c*d - 36*C*a
**2*b**3*d**2 + 36*C*a*b**4*c*d - 6*C*b**5*c**2 + 60*D*a**3*b**2*d**2 - 72
*D*a**2*b**3*c*d + 18*D*a*b**4*c**2) + x*(-6*A*a*b**4*d**2 - 6*A*b**5*c*d
+ 27*B*a**2*b**3*d**2 - 12*B*a*b**4*c*d - 3*B*b**5*c**2 - 60*C*a**3*b**2*d
**2 + 54*C*a**2*b**3*c*d - 6*C*a*b**4*c**2 + 105*D*a**4*b*d**2 - 120*D*a**
3*b**2*c*d + 27*D*a**2*b**3*c**2))/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b*
*8*x**2 + 6*b**9*x**3) + (B*b**2*d**2 - 4*C*a*b*d**2 + 2*C*b**2*c*d + 10*D
*a**2*d**2 - 8*D*a*b*c*d + D*b**2*c**2)*log(a + b*x)/b**6
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$$
$$= \frac{(11\,Da^3b^2 - 2\,Ca^2b^3 - Bab^4 - 2\,Ab^5)c^2 - 2\,(26\,Da^4b - 11\,Ca^3b^2 + 2\,Ba^2b^3 + Aab^4)cd + (47\,Da^5 - 26\,C}{}$$
$$+ \frac{Dbd^2x^2 + 2\,(2\,Dbcd - (4\,Da - Cb)d^2)x}{2\,b^5}$$
$$+ \frac{(Db^2c^2 - 2\,(4\,Dab - Cb^2)cd + (10\,Da^2 - 4\,Cab + Bb^2)d^2)\log(bx+a)}{b^6}$$

input

```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="maxima")
```

output

```
1/6*((11*D*a^3*b^2 - 2*C*a^2*b^3 - B*a*b^4 - 2*A*b^5)*c^2 - 2*(26*D*a^4*b
- 11*C*a^3*b^2 + 2*B*a^2*b^3 + A*a*b^4)*c*d + (47*D*a^5 - 26*C*a^4*b + 11*
B*a^3*b^2 - 2*A*a^2*b^3)*d^2 + 6*((3*D*a*b^4 - C*b^5)*c^2 - 2*(6*D*a^2*b^3
 - 3*C*a*b^4 + B*b^5)*c*d + (10*D*a^3*b^2 - 6*C*a^2*b^3 + 3*B*a*b^4 - A*b^
5)*d^2)*x^2 + 3*((9*D*a^2*b^3 - 2*C*a*b^4 - B*b^5)*c^2 - 2*(20*D*a^3*b^2 -
 9*C*a^2*b^3 + 2*B*a*b^4 + A*b^5)*c*d + (35*D*a^4*b - 20*C*a^3*b^2 + 9*B*a
^2*b^3 - 2*A*a*b^4)*d^2)*x)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6
) + 1/2*(D*b*d^2*x^2 + 2*(2*D*b*c*d - (4*D*a - C*b)*d^2)*x)/b^5 + (D*b^2*c
^2 - 2*(4*D*a*b - C*b^2)*c*d + (10*D*a^2 - 4*C*a*b + B*b^2)*d^2)*log(b*x +
 a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$$
$$= \frac{(Db^2c^2 - 8\,Dabcd + 2\,Cb^2cd + 10\,Da^2d^2 - 4\,Cabd^2 + Bb^2d^2)\log(|bx+a|)}{b^6}$$
$$+ \frac{Db^4d^2x^2 + 4\,Db^4cdx - 8\,Dab^3d^2x + 2\,Cb^4d^2x}{2\,b^8}$$
$$+ \frac{11\,Da^3b^2c^2 - 2\,Ca^2b^3c^2 - Bab^4c^2 - 2\,Ab^5c^2 - 52\,Da^4bcd + 22\,Ca^3b^2cd - 4\,Ba^2b^3cd - 2\,Aab^4cd + 47\,}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="giac")
```

output
```
(D*b^2*c^2 - 8*D*a*b*c*d + 2*C*b^2*c*d + 10*D*a^2*d^2 - 4*C*a*b*d^2 + B*b^
2*d^2)*log(abs(b*x + a))/b^6 + 1/2*(D*b^4*d^2*x^2 + 4*D*b^4*c*d*x - 8*D*a*
b^3*d^2*x + 2*C*b^4*d^2*x)/b^8 + 1/6*(11*D*a^3*b^2*c^2 - 2*C*a^2*b^3*c^2 -
 B*a*b^4*c^2 - 2*A*b^5*c^2 - 52*D*a^4*b*c*d + 22*C*a^3*b^2*c*d - 4*B*a^2*b
^3*c*d - 2*A*a*b^4*c*d + 47*D*a^5*d^2 - 26*C*a^4*b*d^2 + 11*B*a^3*b^2*d^2
- 2*A*a^2*b^3*d^2 + 6*(3*D*a*b^4*c^2 - C*b^5*c^2 - 12*D*a^2*b^3*c*d + 6*C*
a*b^4*c*d - 2*B*b^5*c*d + 10*D*a^3*b^2*d^2 - 6*C*a^2*b^3*d^2 + 3*B*a*b^4*d
^2 - A*b^5*d^2)*x^2 + 3*(9*D*a^2*b^3*c^2 - 2*C*a*b^4*c^2 - B*b^5*c^2 - 40*
D*a^3*b^2*c*d + 18*C*a^2*b^3*c*d - 4*B*a*b^4*c*d - 2*A*b^5*c*d + 35*D*a^4*
b*d^2 - 20*C*a^3*b^2*d^2 + 9*B*a^2*b^3*d^2 - 2*A*a*b^4*d^2)*x)/((b*x + a)^
3*b^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^4}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$$
$$= \frac{-216\log(bx+a)\,a^4b^2c\,d^2x+54\log(bx+a)\,a^3b^3c^2dx-216\log(bx+a)\,a^3b^3c\,d^2x^2+54\log(bx+a)\,a^2b^4c}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x)
```

output
```
(60*log(a + b*x)*a**6*d**3 - 72*log(a + b*x)*a**5*b*c*d**2 + 180*log(a + b
*x)*a**5*b*d**3*x + 6*log(a + b*x)*a**4*b**3*d**2 + 18*log(a + b*x)*a**4*b
**2*c**2*d - 216*log(a + b*x)*a**4*b**2*c*d**2*x + 180*log(a + b*x)*a**4*b
**2*d**3*x**2 + 18*log(a + b*x)*a**3*b**4*d**2*x + 54*log(a + b*x)*a**3*b*
*3*c**2*d*x - 216*log(a + b*x)*a**3*b**3*c*d**2*x**2 + 60*log(a + b*x)*a**
3*b**3*d**3*x**3 + 18*log(a + b*x)*a**2*b**5*d**2*x**2 + 54*log(a + b*x)*a
**2*b**4*c**2*d*x**2 - 72*log(a + b*x)*a**2*b**4*c*d**2*x**3 + 6*log(a + b
*x)*a*b**6*d**2*x**3 + 18*log(a + b*x)*a*b**5*c**2*d*x**3 + 50*a**6*d**3 -
 60*a**5*b*c*d**2 + 90*a**5*b*d**3*x + 5*a**4*b**3*d**2 + 15*a**4*b**2*c**
2*d - 108*a**4*b**2*c*d**2*x - 2*a**3*b**4*c*d + 9*a**3*b**4*d**2*x + 27*a
**3*b**3*c**2*d*x - 60*a**3*b**3*d**3*x**3 - 3*a**2*b**5*c**2 - 6*a**2*b**
5*c*d*x + 72*a**2*b**4*c*d**2*x**3 - 15*a**2*b**4*d**3*x**4 - 3*a*b**6*c**
2*x - 4*a*b**6*d**2*x**3 - 18*a*b**5*c**2*d*x**3 + 18*a*b**5*c*d**2*x**4 +
 3*a*b**5*d**3*x**5 + 4*b**7*c*d*x**3 + 2*b**6*c**3*x**3)/(6*a*b**6*(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.21 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^5}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 292

$$\begin{aligned}
&\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^5}\,dx \\
&= \frac{d^2Dx}{b^5} - \frac{(bc-ad)^2\left(Ab^3-a(b^2B-abC+a^2D)\right)}{4b^6(a+bx)^4} \\
&\quad - \frac{(bc-ad)\left(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD)\right)}{3b^6(a+bx)^3} \\
&\quad - \frac{b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D)}{2b^6(a+bx)^2} \\
&\quad - \frac{10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D)}{b^6(a+bx)} \\
&\quad + \frac{d(bCd+2bcD-5adD)\log(a+bx)}{b^6}
\end{aligned}$$

output
```
d^2*D*x/b^5-1/4*(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^6/(b*x+a)^4-1
/3*(-a*d+b*c)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+
3*D*c))/b^6/(b*x+a)^3-1/2*(b^3*(A*d^2+2*B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*
d*(C*d+2*D*c)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^2-(10*a^2*d^2*D-4
*a*b*d*(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)+d*(C*b*d-5*D*a*d
+2*D*b*c)*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^5}\,dx =$$
$$-\frac{-12bd^2Dx+\frac{3(bc-ad)^2(Ab^3-a(b^2B-abC+a^2D))}{(a+bx)^4}+\frac{4(bc-ad)(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))}{(a+bx)^3}+\frac{6(b^3(}{}}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^5,x]
```

output
```
-1/12*(-12*b*d^2*D*x + (3*(b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)
))/(a + b*x)^4 + (4*(b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d)
 - 5*a^3*d*D + a^2*b*(4*C*d + 3*c*D)))/(a + b*x)^3 + (6*(b^3*(c^2*C + 2*B*
c*d + A*d^2) - 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d +
 B*d^2 + c^2*D)))/(a + b*x)^2 + (12*(10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D)
+ b^2*(2*c*C*d + B*d^2 + c^2*D)))/(a + b*x) - 12*d*(b*C*d + 2*b*c*D - 5*a*
d*D)*Log[a + b*x])/b^6
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^5}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)^2(Ab^3-a(a^2D-abC+b^2B))}{b^5(a+bx)^5}+\frac{10a^2d^2D-4abd(2cD+Cd)+b^2(Bd^2+c^2D+2cCd)}{b^5(a+bx)^2}+\frac{-10a}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{4b^6(a+bx)^4}-\frac{10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)}{b^6(a+bx)}-\frac{-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)}{2b^6(a+bx)^2}-\frac{(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{3b^6(a+bx)^3}+\frac{d\log(a+bx)(-5adD+2bcD+bCd)}{b^6}+\frac{d^2Dx}{b^5}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^5,x]
```

output
```
(d^2*D*x)/b^5 - ((b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(4*b^6
*(a + b*x)^4) - ((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) -
5*a^3*d*D + a^2*b*(4*C*d + 3*c*D)))/(3*b^6*(a + b*x)^3) - (b^3*(c^2*C + 2*
B*c*d + A*d^2) - 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d
 + B*d^2 + c^2*D))/(2*b^6*(a + b*x)^2) - (10*a^2*d^2*D - 4*a*b*d*(C*d + 2*
c*D) + b^2*(2*c*C*d + B*d^2 + c^2*D))/(b^6*(a + b*x)) + (d*(b*C*d + 2*b*c*
D - 5*a*d*D)*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.50

method	result
norman	$\frac{\frac{Dd^2x^5}{b}-\frac{A\,a^2b^3d^2+2Aa\,b^4cd+3A\,b^5c^2+3B\,a^3b^2d^2+2B\,a^2b^3cd+Ba\,b^4c^2-25C\,a^4b\,d^2+6C\,a^3b^2cd+C\,a^2b^3c^2+125Da^5d^2-50Da^4bcd+3D}{12b^6}}{}$
default	$\frac{d^2Dx}{b^5}-\frac{A\,a^2b^3d^2-2Aa\,b^4cd+A\,b^5c^2-B\,a^3b^2d^2+2B\,a^2b^3cd-Ba\,b^4c^2+C\,a^4b\,d^2-2C\,a^3b^2cd+C\,a^2b^3c^2-Da^5d^2+2Da^4bcd}{4b^6(bx+a)^4}$
parallelrisch	$-\frac{24C\,x^3b^5cd+240Dx^3a^2b^3d^2+18B\,x^2a\,b^4d^2-216Dx^2a^2b^3cd+8Bxa\,b^4cd-96Dx^3a\,b^4cd-48C\ln(bx+a)x\,a^3b^2d^2+240D\ln}{}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
(D/b*d^2*x^5-1/12*(A*a^2*b^3*d^2+2*A*a*b^4*c*d+3*A*b^5*c^2+3*B*a^3*b^2*d^2
+2*B*a^2*b^3*c*d+B*a*b^4*c^2-25*C*a^4*b*d^2+6*C*a^3*b^2*c*d+C*a^2*b^3*c^2+
125*D*a^5*d^2-50*D*a^4*b*c*d+3*D*a^3*b^2*c^2)/b^6-(B*b^2*d^2-4*C*a*b*d^2+2
*C*b^2*c*d+20*D*a^2*d^2-8*D*a*b*c*d+D*b^2*c^2)/b^3*x^3-1/2*(A*b^3*d^2+3*B*
a*b^2*d^2+2*B*b^3*c*d-18*C*a^2*b*d^2+6*C*a*b^2*c*d+C*b^3*c^2+90*D*a^3*d^2-
36*D*a^2*b*c*d+3*D*a*b^2*c^2)/b^4*x^2-1/3*(A*a*b^3*d^2+2*A*b^4*c*d+3*B*a^2
*b^2*d^2+2*B*a*b^3*c*d+B*b^4*c^2-22*C*a^3*b*d^2+6*C*a^2*b^2*c*d+C*a*b^3*c^
2+110*D*a^4*d^2-44*D*a^3*b*c*d+3*D*a^2*b^2*c^2)/b^5*x)/(b*x+a)^4+d*(C*b*d-
5*D*a*d+2*D*b*c)*ln(b*x+a)/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(288) = 576.

Time = 0.08 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^5}\,dx$$
$$=\frac{12\,Db^5d^2x^5+48\,Dab^4d^2x^4-12\,(Db^5c^2-2\,(4\,Dab^4-Cb^5)cd+(4\,Da^2b^3-4\,Cab^4+Bb^5)d^2)x^3-(3\,D}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/12*(12*D*b^5*d^2*x^5 + 48*D*a*b^4*d^2*x^4 - 12*(D*b^5*c^2 - 2*(4*D*a*b^4
 - C*b^5)*c*d + (4*D*a^2*b^3 - 4*C*a*b^4 + B*b^5)*d^2)*x^3 - (3*D*a^3*b^2
+ C*a^2*b^3 + B*a*b^4 + 3*A*b^5)*c^2 + 2*(25*D*a^4*b - 3*C*a^3*b^2 - B*a^2
*b^3 - A*a*b^4)*c*d - (77*D*a^5 - 25*C*a^4*b + 3*B*a^3*b^2 + A*a^2*b^3)*d^
2 - 6*((3*D*a*b^4 + C*b^5)*c^2 - 2*(18*D*a^2*b^3 - 3*C*a*b^4 - B*b^5)*c*d
+ (42*D*a^3*b^2 - 18*C*a^2*b^3 + 3*B*a*b^4 + A*b^5)*d^2)*x^2 - 4*((3*D*a^2
*b^3 + C*a*b^4 + B*b^5)*c^2 - 2*(22*D*a^3*b^2 - 3*C*a^2*b^3 - B*a*b^4 - A*
b^5)*c*d + (62*D*a^4*b - 22*C*a^3*b^2 + 3*B*a^2*b^3 + A*a*b^4)*d^2)*x + 12
*(2*D*a^4*b*c*d + (2*D*b^5*c*d - (5*D*a*b^4 - C*b^5)*d^2)*x^4 + 4*(2*D*a*b
^4*c*d - (5*D*a^2*b^3 - C*a*b^4)*d^2)*x^3 - (5*D*a^5 - C*a^4*b)*d^2 + 6*(2
*D*a^2*b^3*c*d - (5*D*a^3*b^2 - C*a^2*b^3)*d^2)*x^2 + 4*(2*D*a^3*b^2*c*d -
 (5*D*a^4*b - C*a^3*b^2)*d^2)*x)*log(b*x + a))/(b^10*x^4 + 4*a*b^9*x^3 + 6
*a^2*b^8*x^2 + 4*a^3*b^7*x + a^4*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a)**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^5}\,dx =$$
$$-\frac{12\,(Db^5c^2 - 2\,(4\,Dab^4 - Cb^5)cd + (10\,Da^2b^3 - 4\,Cab^4 + Bb^5)d^2)x^3 + (3\,Da^3b^2 + Ca^2b^3 + Bab^4 + 3\,A}{}$$
$$+ \frac{Dd^2x}{b^5} + \frac{(2\,Dbcd - (5\,Da - Cb)d^2)\log(bx+a)}{b^6}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x, algorithm="maxima")
```

output
```
-1/12*(12*(D*b^5*c^2 - 2*(4*D*a*b^4 - C*b^5)*c*d + (10*D*a^2*b^3 - 4*C*a*b
^4 + B*b^5)*d^2)*x^3 + (3*D*a^3*b^2 + C*a^2*b^3 + B*a*b^4 + 3*A*b^5)*c^2 -
 2*(25*D*a^4*b - 3*C*a^3*b^2 - B*a^2*b^3 - A*a*b^4)*c*d + (77*D*a^5 - 25*C
*a^4*b + 3*B*a^3*b^2 + A*a^2*b^3)*d^2 + 6*((3*D*a*b^4 + C*b^5)*c^2 - 2*(18
*D*a^2*b^3 - 3*C*a*b^4 - B*b^5)*c*d + (50*D*a^3*b^2 - 18*C*a^2*b^3 + 3*B*a
*b^4 + A*b^5)*d^2)*x^2 + 4*((3*D*a^2*b^3 + C*a*b^4 + B*b^5)*c^2 - 2*(22*D*
a^3*b^2 - 3*C*a^2*b^3 - B*a*b^4 - A*b^5)*c*d + (65*D*a^4*b - 22*C*a^3*b^2
+ 3*B*a^2*b^3 + A*a*b^4)*d^2)*x)/(b^10*x^4 + 4*a*b^9*x^3 + 6*a^2*b^8*x^2 +
 4*a^3*b^7*x + a^4*b^6) + D*d^2*x/b^5 + (2*D*b*c*d - (5*D*a - C*b)*d^2)*lo
g(b*x + a)/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(288) = 576$.

Time = 0.13 (sec) , antiderivative size = 726, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^5} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x, algorithm="giac")
```

output
```
(b*x + a)*D*d^2/b^6 - (2*D*b*c*d - 5*D*a*d^2 + C*b*d^2)*log(abs(b*x + a)/(
(b*x + a)^2*abs(b)))/b^6 - 1/12*(12*D*b^24*c^2/(b*x + a) - 18*D*a*b^24*c^2
/(b*x + a)^2 + 12*D*a^2*b^24*c^2/(b*x + a)^3 - 3*D*a^3*b^24*c^2/(b*x + a)^
4 + 6*C*b^25*c^2/(b*x + a)^2 - 8*C*a*b^25*c^2/(b*x + a)^3 + 3*C*a^2*b^25*c
^2/(b*x + a)^4 + 4*B*b^26*c^2/(b*x + a)^3 - 3*B*a*b^26*c^2/(b*x + a)^4 + 3
*A*b^27*c^2/(b*x + a)^4 - 96*D*a*b^23*c*d/(b*x + a) + 72*D*a^2*b^23*c*d/(b
*x + a)^2 - 32*D*a^3*b^23*c*d/(b*x + a)^3 + 6*D*a^4*b^23*c*d/(b*x + a)^4 +
 24*C*b^24*c*d/(b*x + a) - 36*C*a*b^24*c*d/(b*x + a)^2 + 24*C*a^2*b^24*c*d
/(b*x + a)^3 - 6*C*a^3*b^24*c*d/(b*x + a)^4 + 12*B*b^25*c*d/(b*x + a)^2 -
16*B*a*b^25*c*d/(b*x + a)^3 + 6*B*a^2*b^25*c*d/(b*x + a)^4 + 8*A*b^26*c*d/
(b*x + a)^3 - 6*A*a*b^26*c*d/(b*x + a)^4 + 120*D*a^2*b^22*d^2/(b*x + a) -
60*D*a^3*b^22*d^2/(b*x + a)^2 + 20*D*a^4*b^22*d^2/(b*x + a)^3 - 3*D*a^5*b^
22*d^2/(b*x + a)^4 - 48*C*a*b^23*d^2/(b*x + a) + 36*C*a^2*b^23*d^2/(b*x +
a)^2 - 16*C*a^3*b^23*d^2/(b*x + a)^3 + 3*C*a^4*b^23*d^2/(b*x + a)^4 + 12*B
*b^24*d^2/(b*x + a) - 18*B*a*b^24*d^2/(b*x + a)^2 + 12*B*a^2*b^24*d^2/(b*x
 + a)^3 - 3*B*a^3*b^24*d^2/(b*x + a)^4 + 6*A*b^25*d^2/(b*x + a)^2 - 8*A*a*
b^25*d^2/(b*x + a)^3 + 3*A*a^2*b^25*d^2/(b*x + a)^4)/b^28
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^5}\,dx=\int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^5}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^5,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.68

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^5} \, dx$$
$$= \frac{-60\log(bx+a)\, a^2b^4d^3x^4 + 144\log(bx+a)\, a^4b^2c\, d^2x + 216\log(bx+a)\, a^3b^3c\, d^2x^2 + 36\log(bx+a)\, a^5bc}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^5,x)
```

output
```
( - 60*log(a + b*x)*a**6*d**3 + 36*log(a + b*x)*a**5*b*c*d**2 - 240*log(a
+ b*x)*a**5*b*d**3*x + 144*log(a + b*x)*a**4*b**2*c*d**2*x - 360*log(a + b
*x)*a**4*b**2*d**3*x**2 + 216*log(a + b*x)*a**3*b**3*c*d**2*x**2 - 240*log
(a + b*x)*a**3*b**3*d**3*x**3 + 144*log(a + b*x)*a**2*b**4*c*d**2*x**3 - 6
0*log(a + b*x)*a**2*b**4*d**3*x**4 + 36*log(a + b*x)*a*b**5*c*d**2*x**4 -
65*a**6*d**3 + 39*a**5*b*c*d**2 - 200*a**5*b*d**3*x - a**4*b**3*d**2 + 120
*a**4*b**2*c*d**2*x - 180*a**4*b**2*d**3*x**2 - 4*a**3*b**4*c*d - 4*a**3*b
**4*d**2*x - a**3*b**3*c**3 + 108*a**3*b**3*c*d**2*x**2 - 4*a**2*b**5*c**2
 - 16*a**2*b**5*c*d*x - 6*a**2*b**5*d**2*x**2 - 4*a**2*b**4*c**3*x + 60*a*
*2*b**4*d**3*x**4 - 4*a*b**6*c**2*x - 12*a*b**6*c*d*x**2 - 6*a*b**5*c**3*x
**2 - 36*a*b**5*c*d**2*x**4 + 12*a*b**5*d**3*x**5 + 3*b**7*d**2*x**4 + 9*b
**6*c**2*d*x**4)/(12*a*b**6*(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b*
*3*x**3 + b**4*x**4))
```

3.22 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^6}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 301

$$
\begin{aligned}
\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^6}\,dx &= -\frac{(bc-ad)^2(Ab^3-a(b^2B-abC+a^2D))}{5b^6(a+bx)^5} \\
&\quad -\frac{(bc-ad)(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))}{4b^6(a+bx)^4} \\
&\quad -\frac{b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D)}{3b^6(a+bx)^3} \\
&\quad -\frac{10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D)}{2b^6(a+bx)^2} \\
&\quad -\frac{d(bCd+2bcD-5adD)}{b^6(a+bx)}+\frac{d^2D\log(a+bx)}{b^6}
\end{aligned}
$$

output
```
-1/5*(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^6/(b*x+a)^5-1/4*(-a*d+b*
c)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))/b^6
/(b*x+a)^4-1/3*(b^3*(A*d^2+2*B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*
c)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^3-1/2*(10*a^2*d^2*D-4*a*b*d*
(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^2-d*(C*b*d-5*D*a*d+2*D*
b*c)/b^6/(b*x+a)+d^2*D*ln(b*x+a)/b^6
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^6} \, dx =$$
$$-\frac{\frac{12(bc-ad)^2(Ab^3-a(b^2B-abC+a^2D))}{(a+bx)^5}+\frac{15(bc-ad)(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))}{(a+bx)^4}+\frac{20(b^3(c^2C+2Bcd+Ad}{}}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^6,x]
```

output
```
-1/60*((12*(b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(a + b*x)^5
+ (15*(b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D +
 a^2*b*(4*C*d + 3*c*D)))/(a + b*x)^4 + (20*(b^3*(c^2*C + 2*B*c*d + A*d^2)
- 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^2*
D)))/(a + b*x)^3 + (30*(10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2*c*C*
d + B*d^2 + c^2*D)))/(a + b*x)^2 + (60*d*(b*C*d + 2*b*c*D - 5*a*d*D))/(a +
 b*x) - 60*d^2*D*Log[a + b*x])/b^6
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^6} \, dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{(bc-ad)^2 (Ab^3-a(a^2D-abC+b^2B))}{b^5(a+bx)^6}+\frac{10a^2d^2D-4abd(2cD+Cd)+b^2(Bd^2+c^2D+2cCd)}{b^5(a+bx)^3}+\frac{-10a}{}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^6(a+bx)^5}-\frac{10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)}{2b^6(a+bx)^2}-\frac{-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)}{3b^6(a+bx)^3}-\frac{(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{4b^6(a+bx)^4}-\frac{d(-5adD+2bcD+bCd)}{b^6(a+bx)}+\frac{d^2D\log(a+bx)}{b^6}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^6,x]
```

output
```
-1/5*((b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^6*(a + b*x)^5)
 - ((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a
^2*b*(4*C*d + 3*c*D)))/(4*b^6*(a + b*x)^4) - (b^3*(c^2*C + 2*B*c*d + A*d^2
) - 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^
2*D))/(3*b^6*(a + b*x)^3) - (10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2
*c*C*d + B*d^2 + c^2*D))/(2*b^6*(a + b*x)^2) - (d*(b*C*d + 2*b*c*D - 5*a*d
*D))/(b^6*(a + b*x)) + (d^2*D*Log[a + b*x])/b^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.50

method	result
norman	$-\frac{2A\,a^{2}b^{3}d^{2}+6Aa\,b^{4}cd+12A\,b^{5}c^{2}+3B\,a^{3}b^{2}d^{2}+4B\,a^{2}b^{3}cd+3Ba\,b^{4}c^{2}+12C\,a^{4}b\,d^{2}+6C\,a^{3}b^{2}cd+2C\,a^{2}b^{3}c^{2}-137Da^{5}d^{2}+24Da^{4}bcd+3Da^{3}}{60b^{6}}$
default	$-\frac{A\,a^{2}b^{3}d^{2}-2Aa\,b^{4}cd+A\,b^{5}c^{2}-B\,a^{3}b^{2}d^{2}+2B\,a^{2}b^{3}cd-Ba\,b^{4}c^{2}+C\,a^{4}b\,d^{2}-2C\,a^{3}b^{2}cd+C\,a^{2}b^{3}c^{2}-Da^{5}d^{2}+2Da^{4}bcd-Da^{3}b^{2}}{5b^{6}(bx+a)^{5}}$
parallelrisch	$-\frac{60C\,x^{3}b^{5}cd-900Dx^{3}a^{2}b^{3}d^{2}+30B\,x^{2}a\,b^{4}d^{2}+240Dx^{2}a^{2}b^{3}cd+20Bxa\,b^{4}cd+240Dx^{3}a\,b^{4}cd-300D\ln(bx+a)x\,a^{4}b\,d^{2}-300D}{}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
(-1/60*(2*A*a^2*b^3*d^2+6*A*a*b^4*c*d+12*A*b^5*c^2+3*B*a^3*b^2*d^2+4*B*a^2
*b^3*c*d+3*B*a*b^4*c^2+12*C*a^4*b*d^2+6*C*a^3*b^2*c*d+2*C*a^2*b^3*c^2-137*
D*a^5*d^2+24*D*a^4*b*c*d+3*D*a^3*b^2*c^2)/b^6-(C*b*d^2-5*D*a*d^2+2*D*b*c*d
)/b^2*x^4-1/2*(B*b^2*d^2+4*C*a*b*d^2+2*C*b^2*c*d-30*D*a^2*d^2+8*D*a*b*c*d+
D*b^2*c^2)/b^3*x^3-1/6*(2*A*b^3*d^2+3*B*a*b^2*d^2+4*B*b^3*c*d+12*C*a^2*b*d
^2+6*C*a*b^2*c*d+2*C*b^3*c^2-110*D*a^3*d^2+24*D*a^2*b*c*d+3*D*a*b^2*c^2)/b
^4*x^2-1/12*(2*A*a*b^3*d^2+6*A*b^4*c*d+3*B*a^2*b^2*d^2+4*B*a*b^3*c*d+3*B*b
^4*c^2+12*C*a^3*b*d^2+6*C*a^2*b^2*c*d+2*C*a*b^3*c^2-125*D*a^4*d^2+24*D*a^3
*b*c*d+3*D*a^2*b^2*c^2)/b^5*x)/(b*x+a)^5+d^2*D*ln(b*x+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx =$$
$$-\frac{60\left(2\,Db^5cd-(5\,Dab^4-Cb^5)d^2\right)x^4+30\left(Db^5c^2+2\,(4\,Dab^4+Cb^5)cd-(30\,Da^2b^3-4\,Cab^4-Bb^5)d\right.}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x, algorithm="fricas")
```

output
```
-1/60*(60*(2*D*b^5*c*d - (5*D*a*b^4 - C*b^5)*d^2)*x^4 + 30*(D*b^5*c^2 + 2*
(4*D*a*b^4 + C*b^5)*c*d - (30*D*a^2*b^3 - 4*C*a*b^4 - B*b^5)*d^2)*x^3 + (3
*D*a^3*b^2 + 2*C*a^2*b^3 + 3*B*a*b^4 + 12*A*b^5)*c^2 + 2*(12*D*a^4*b + 3*C
*a^3*b^2 + 2*B*a^2*b^3 + 3*A*a*b^4)*c*d - (137*D*a^5 - 12*C*a^4*b - 3*B*a^
3*b^2 - 2*A*a^2*b^3)*d^2 + 10*((3*D*a*b^4 + 2*C*b^5)*c^2 + 2*(12*D*a^2*b^3
 + 3*C*a*b^4 + 2*B*b^5)*c*d - (110*D*a^3*b^2 - 12*C*a^2*b^3 - 3*B*a*b^4 -
2*A*b^5)*d^2)*x^2 + 5*((3*D*a^2*b^3 + 2*C*a*b^4 + 3*B*b^5)*c^2 + 2*(12*D*a
^3*b^2 + 3*C*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c*d - (125*D*a^4*b - 12*C*a^3*
b^2 - 3*B*a^2*b^3 - 2*A*a*b^4)*d^2)*x - 60*(D*b^5*d^2*x^5 + 5*D*a*b^4*d^2*
x^4 + 10*D*a^2*b^3*d^2*x^3 + 10*D*a^3*b^2*d^2*x^2 + 5*D*a^4*b*d^2*x + D*a^
5*d^2)*log(b*x + a))/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b^9*x^3 + 10*a^3*b^
8*x^2 + 5*a^4*b^7*x + a^5*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a)**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^6}\,dx = -\frac{60\,(2\,Db^5cd-(5\,Dab^4-Cb^5)d^2)x^4+30\,(Db^5c^2+2\,(4\,Dab^4+Cb^5)cd-(30\,Da^2b^3-4\,Cab^4-Bb^5)d}{} + \frac{Dd^2\log(bx+a)}{b^6}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x, algorithm="maxima")
```

output
```
-1/60*(60*(2*D*b^5*c*d - (5*D*a*b^4 - C*b^5)*d^2)*x^4 + 30*(D*b^5*c^2 + 2*
(4*D*a*b^4 + C*b^5)*c*d - (30*D*a^2*b^3 - 4*C*a*b^4 - B*b^5)*d^2)*x^3 + (3
*D*a^3*b^2 + 2*C*a^2*b^3 + 3*B*a*b^4 + 12*A*b^5)*c^2 + 2*(12*D*a^4*b + 3*C
*a^3*b^2 + 2*B*a^2*b^3 + 3*A*a*b^4)*c*d - (137*D*a^5 - 12*C*a^4*b - 3*B*a^
3*b^2 - 2*A*a^2*b^3)*d^2 + 10*((3*D*a*b^4 + 2*C*b^5)*c^2 + 2*(12*D*a^2*b^3
 + 3*C*a*b^4 + 2*B*b^5)*c*d - (110*D*a^3*b^2 - 12*C*a^2*b^3 - 3*B*a*b^4 -
2*A*b^5)*d^2)*x^2 + 5*((3*D*a^2*b^3 + 2*C*a*b^4 + 3*B*b^5)*c^2 + 2*(12*D*a
^3*b^2 + 3*C*a^2*b^3 + 2*B*a*b^4 + 3*A*b^5)*c*d - (125*D*a^4*b - 12*C*a^3*
b^2 - 3*B*a^2*b^3 - 2*A*a*b^4)*d^2)*x)/(b^11*x^5 + 5*a*b^10*x^4 + 10*a^2*b
^9*x^3 + 10*a^3*b^8*x^2 + 5*a^4*b^7*x + a^5*b^6) + D*d^2*log(b*x + a)/b^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^6}\,dx = \frac{Dd^2 \log(|bx+a|)}{b^6} - \frac{60\,(2\,Db^4cd - 5\,Dab^3d^2 + Cb^4d^2)x^4 + 30\,(Db^4c^2 + 8\,Dab^3cd + 2\,Cb^4cd - 30\,Da^2b^2d^2 + 4\,Cab^3d^2 + B}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x, algorithm="giac")
```

output
```
D*d^2*log(abs(b*x + a))/b^6 - 1/60*(60*(2*D*b^4*c*d - 5*D*a*b^3*d^2 + C*b^
4*d^2)*x^4 + 30*(D*b^4*c^2 + 8*D*a*b^3*c*d + 2*C*b^4*c*d - 30*D*a^2*b^2*d^
2 + 4*C*a*b^3*d^2 + B*b^4*d^2)*x^3 + 10*(3*D*a*b^3*c^2 + 2*C*b^4*c^2 + 24*
D*a^2*b^2*c*d + 6*C*a*b^3*c*d + 4*B*b^4*c*d - 110*D*a^3*b*d^2 + 12*C*a^2*b
^2*d^2 + 3*B*a*b^3*d^2 + 2*A*b^4*d^2)*x^2 + 5*(3*D*a^2*b^2*c^2 + 2*C*a*b^3
*c^2 + 3*B*b^4*c^2 + 24*D*a^3*b*c*d + 6*C*a^2*b^2*c*d + 4*B*a*b^3*c*d + 6*
A*b^4*c*d - 125*D*a^4*d^2 + 12*C*a^3*b*d^2 + 3*B*a^2*b^2*d^2 + 2*A*a*b^3*d
^2)*x + (3*D*a^3*b^2*c^2 + 2*C*a^2*b^3*c^2 + 3*B*a*b^4*c^2 + 12*A*b^5*c^2
+ 24*D*a^4*b*c*d + 6*C*a^3*b^2*c*d + 4*B*a^2*b^3*c*d + 6*A*a*b^4*c*d - 137
*D*a^5*d^2 + 12*C*a^4*b*d^2 + 3*B*a^3*b^2*d^2 + 2*A*a^2*b^3*d^2)/b)/((b*x
+ a)^5*b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^6} \, dx = \int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^6} \, dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^6,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^6} \, dx$$
$$= \frac{300 \log(bx+a)\, a^2 b^4 d^3 x^4 + 300 \log(bx+a)\, a^5 b\, d^3 x + 60 \log(bx+a)\, a\, b^5 d^3 x^5 + 36 b^6 c\, d^2 x^5 + 600 \log(bx+}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^6,x)
```

output
```
(60*log(a + b*x)*a**6*d**3 + 300*log(a + b*x)*a**5*b*d**3*x + 600*log(a +
b*x)*a**4*b**2*d**3*x**2 + 600*log(a + b*x)*a**3*b**3*d**3*x**3 + 300*log(
a + b*x)*a**2*b**4*d**3*x**4 + 60*log(a + b*x)*a*b**5*d**3*x**5 + 77*a**6*
d**3 + 325*a**5*b*d**3*x - 5*a**4*b**3*d**2 - 9*a**4*b**2*c**2*d + 500*a**
4*b**2*d**3*x**2 - 10*a**3*b**4*c*d - 25*a**3*b**4*d**2*x - 2*a**3*b**3*c*
*3 - 45*a**3*b**3*c**2*d*x + 300*a**3*b**3*d**3*x**3 - 15*a**2*b**5*c**2 -
 50*a**2*b**5*c*d*x - 50*a**2*b**5*d**2*x**2 - 10*a**2*b**4*c**3*x - 90*a*
*2*b**4*c**2*d*x**2 - 15*a*b**6*c**2*x - 40*a*b**6*c*d*x**2 - 30*a*b**6*d*
*2*x**3 - 20*a*b**5*c**3*x**2 - 90*a*b**5*c**2*d*x**3 - 60*a*b**5*d**3*x**
5 + 36*b**6*c*d**2*x**5)/(60*a*b**6*(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2
 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5))
```

3.23 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^7}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 305

$$
\begin{aligned}
\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^7}\,dx &= -\frac{(bc-ad)^2(Ab^3-a(b^2B-abC+a^2D))}{6b^6(a+bx)^6} \\
&\quad -\frac{(bc-ad)(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))}{5b^6(a+bx)^5} \\
&\quad -\frac{b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D)}{4b^6(a+bx)^4} \\
&\quad -\frac{10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D)}{3b^6(a+bx)^3} \\
&\quad -\frac{d(bCd+2bcD-5adD)}{2b^6(a+bx)^2}-\frac{d^2D}{b^6(a+bx)}
\end{aligned}
$$

output

```
-1/6*(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^6/(b*x+a)^6-1/5*(-a*d+b*
c)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))/b^6
/(b*x+a)^5-1/4*(b^3*(A*d^2+2*B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*
c)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^4-1/3*(10*a^2*d^2*D-4*a*b*d*
(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^3-1/2*d*(C*b*d-5*D*a*d+
2*D*b*c)/b^6/(b*x+a)^2-d^2*D/b^6/(b*x+a)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^7} \, dx = -\frac{10a^5d^2D + 2a^4bd(Cd+2D(c+15dx)) + Ab^3(a^2d^2+2abd(2c+3dx)+b^2(10c^2+24cdx+15d^2x^2))}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^7,x]
```

output
```
-1/60*(10*a^5*d^2*D + 2*a^4*b*d*(C*d + 2*D*(c + 15*d*x)) + A*b^3*(a^2*d^2
+ 2*a*b*d*(2*c + 3*d*x) + b^2*(10*c^2 + 24*c*d*x + 15*d^2*x^2)) + b^5*x*(2
*B*(6*c^2 + 15*c*d*x + 10*d^2*x^2) + 5*x*(6*d^2*x^2*(C + 2*D*x) + 4*c*d*x*
(2*C + 3*D*x) + c^2*(3*C + 4*D*x))) + a*b^4*(B*(2*c^2 + 12*c*d*x + 15*d^2*
x^2) + x*(3*c^2*(2*C + 5*D*x) + 10*c*d*x*(3*C + 8*D*x) + 10*d^2*x^2*(4*C +
 15*D*x))) + a^2*b^3*(c^2*(C + 6*D*x) + 2*c*d*(B + 6*x*(C + 5*D*x)) + 2*d^
2*x*(3*B + 5*x*(3*C + 20*D*x))) + a^3*b^2*(c^2*D + 2*c*d*(C + 12*D*x) + d^
2*(B + 6*x*(2*C + 25*D*x))))/(b^6*(a + b*x)^6)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^7} \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)^2 (Ab^3-a(a^2D-abC+b^2B))}{b^5(a+bx)^7} + \frac{10a^2d^2D-4abd(2cD+Cd)+b^2(Bd^2+c^2D+2cCd)}{b^5(a+bx)^4} + \frac{-10a}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{6b^6(a+bx)^6}-\frac{10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)}{3b^6(a+bx)^3}-\frac{-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)}{4b^6(a+bx)^4}-\frac{(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{5b^6(a+bx)^5}-\frac{d(-5adD+2bcD+bCd)}{2b^6(a+bx)^2}-\frac{d^2D}{b^6(a+bx)}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^7,x]
```

output
```
-1/6*((b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^6*(a + b*x)^6)
 - ((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a
^2*b*(4*C*d + 3*c*D)))/(5*b^6*(a + b*x)^5) - (b^3*(c^2*C + 2*B*c*d + A*d^2
) - 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^
2*D))/(4*b^6*(a + b*x)^4) - (10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2
*c*C*d + B*d^2 + c^2*D))/(3*b^6*(a + b*x)^3) - (d*(b*C*d + 2*b*c*D - 5*a*d
*D))/(2*b^6*(a + b*x)^2) - (d^2*D)/(b^6*(a + b*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.43

method	result
norman	$-\frac{Dd^2x^5}{b}-\frac{\left(b\,d^2C+5Da\,d^2+2Dbcd\right)x^4}{2b^2}-\frac{\left(b^2B\,d^2+2Cab\,d^2+2C\,b^2cd+10a^2d^2D+4Dabcd+Db^2c^2\right)x^3}{3b^3}-\frac{\left(b^3d^2A+Ba\,b^2d^2+2B\,b^3cd+2C\right.}{}$
default	$-\frac{-2Aa\,b^3d^2+2A\,b^4cd+3B\,a^2b^2d^2-4Ba\,b^3cd+B\,b^4c^2-4C\,a^3b\,d^2+6C\,a^2b^2cd-2Ca\,b^3c^2+5Da^4d^2-8Da^3bcd+3Da^2b^2c^2}{5b^6(bx+a)^5}-$
gosper	$-\frac{60Dx^5d^2b^5+30C\,x^4b^5d^2+150Dx^4a\,b^4d^2+60Dx^4b^5cd+20B\,x^3b^5d^2+40C\,x^3a\,b^4d^2+40C\,x^3b^5cd+200Dx^3a^2b^3d^2+80Dx^3}{}$
parallelrisch	$-\frac{60Dx^5d^2b^5+30C\,x^4b^5d^2+150Dx^4a\,b^4d^2+60Dx^4b^5cd+20B\,x^3b^5d^2+40C\,x^3a\,b^4d^2+40C\,x^3b^5cd+200Dx^3a^2b^3d^2+80Dx^3}{}$
orering	$-\frac{60Dx^5d^2b^5+30C\,x^4b^5d^2+150Dx^4a\,b^4d^2+60Dx^4b^5cd+20B\,x^3b^5d^2+40C\,x^3a\,b^4d^2+40C\,x^3b^5cd+200Dx^3a^2b^3d^2+80Dx^3}{}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x,method=_RETURNVERBOSE)
```

output
```
1/(b*x+a)^6*(-D/b*d^2*x^5-1/2*(C*b*d^2+5*D*a*d^2+2*D*b*c*d)/b^2*x^4-1/3*(B
*b^2*d^2+2*C*a*b*d^2+2*C*b^2*c*d+10*D*a^2*d^2+4*D*a*b*c*d+D*b^2*c^2)/b^3*x
^3-1/4*(A*b^3*d^2+B*a*b^2*d^2+2*B*b^3*c*d+2*C*a^2*b*d^2+2*C*a*b^2*c*d+C*b^
3*c^2+10*D*a^3*d^2+4*D*a^2*b*c*d+D*a*b^2*c^2)/b^4*x^2-1/10*(A*a*b^3*d^2+4*
A*b^4*c*d+B*a^2*b^2*d^2+2*B*a*b^3*c*d+2*B*b^4*c^2+2*C*a^3*b*d^2+2*C*a^2*b^
2*c*d+C*a*b^3*c^2+10*D*a^4*d^2+4*D*a^3*b*c*d+D*a^2*b^2*c^2)/b^5*x-1/60*(A*
a^2*b^3*d^2+4*A*a*b^4*c*d+10*A*b^5*c^2+B*a^3*b^2*d^2+2*B*a^2*b^3*c*d+2*B*a
*b^4*c^2+2*C*a^4*b*d^2+2*C*a^3*b^2*c*d+C*a^2*b^3*c^2+10*D*a^5*d^2+4*D*a^4*
b*c*d+D*a^3*b^2*c^2)/b^6)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^7}\,dx =$$
$$-\frac{60\,Db^5d^2x^5+30\,(2\,Db^5cd+(5\,Dab^4+Cb^5)d^2)x^4+20\,(Db^5c^2+2\,(2\,Dab^4+Cb^5)cd+(10\,Da^2b^3+2}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x, algorithm="fricas")
```

output
```
-1/60*(60*D*b^5*d^2*x^5 + 30*(2*D*b^5*c*d + (5*D*a*b^4 + C*b^5)*d^2)*x^4 +
 20*(D*b^5*c^2 + 2*(2*D*a*b^4 + C*b^5)*c*d + (10*D*a^2*b^3 + 2*C*a*b^4 + B
*b^5)*d^2)*x^3 + (D*a^3*b^2 + C*a^2*b^3 + 2*B*a*b^4 + 10*A*b^5)*c^2 + 2*(2
*D*a^4*b + C*a^3*b^2 + B*a^2*b^3 + 2*A*a*b^4)*c*d + (10*D*a^5 + 2*C*a^4*b
+ B*a^3*b^2 + A*a^2*b^3)*d^2 + 15*((D*a*b^4 + C*b^5)*c^2 + 2*(2*D*a^2*b^3
+ C*a*b^4 + B*b^5)*c*d + (10*D*a^3*b^2 + 2*C*a^2*b^3 + B*a*b^4 + A*b^5)*d^
2)*x^2 + 6*((D*a^2*b^3 + C*a*b^4 + 2*B*b^5)*c^2 + 2*(2*D*a^3*b^2 + C*a^2*b
^3 + B*a*b^4 + 2*A*b^5)*c*d + (10*D*a^4*b + 2*C*a^3*b^2 + B*a^2*b^3 + A*a*
b^4)*d^2)*x)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 +
 15*a^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^7} \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a)**7,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^7} \, dx = -\frac{60\, Db^5d^2x^5 + 30\,(2\, Db^5cd + (5\, Dab^4 + Cb^5)d^2)x^4 + 20\,(Db^5c^2 + 2\,(2\, Dab^4 + Cb^5)cd + (10\, Da^2b^3 + 2}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x, algorithm="maxima")
```

output
```
-1/60*(60*D*b^5*d^2*x^5 + 30*(2*D*b^5*c*d + (5*D*a*b^4 + C*b^5)*d^2)*x^4 +
 20*(D*b^5*c^2 + 2*(2*D*a*b^4 + C*b^5)*c*d + (10*D*a^2*b^3 + 2*C*a*b^4 + B
*b^5)*d^2)*x^3 + (D*a^3*b^2 + C*a^2*b^3 + 2*B*a*b^4 + 10*A*b^5)*c^2 + 2*(2
*D*a^4*b + C*a^3*b^2 + B*a^2*b^3 + 2*A*a*b^4)*c*d + (10*D*a^5 + 2*C*a^4*b
+ B*a^3*b^2 + A*a^2*b^3)*d^2 + 15*((D*a*b^4 + C*b^5)*c^2 + 2*(2*D*a^2*b^3
+ C*a*b^4 + B*b^5)*c*d + (10*D*a^3*b^2 + 2*C*a^2*b^3 + B*a*b^4 + A*b^5)*d^
2)*x^2 + 6*((D*a^2*b^3 + C*a*b^4 + 2*B*b^5)*c^2 + 2*(2*D*a^3*b^2 + C*a^2*b
^3 + B*a*b^4 + 2*A*b^5)*c*d + (10*D*a^4*b + 2*C*a^3*b^2 + B*a^2*b^3 + A*a*
b^4)*d^2)*x)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 +
 15*a^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.64

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^7}\,dx =$$
$$-\frac{60\,Db^5d^2x^5 + 60\,Db^5cdx^4 + 150\,Dab^4d^2x^4 + 30\,Cb^5d^2x^4 + 20\,Db^5c^2x^3 + 80\,Dab^4cdx^3 + 40\,Cb^5cdx^3 +}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x, algorithm="giac")
```

output
```
-1/60*(60*D*b^5*d^2*x^5 + 60*D*b^5*c*d*x^4 + 150*D*a*b^4*d^2*x^4 + 30*C*b^
5*d^2*x^4 + 20*D*b^5*c^2*x^3 + 80*D*a*b^4*c*d*x^3 + 40*C*b^5*c*d*x^3 + 200
*D*a^2*b^3*d^2*x^3 + 40*C*a*b^4*d^2*x^3 + 20*B*b^5*d^2*x^3 + 15*D*a*b^4*c^
2*x^2 + 15*C*b^5*c^2*x^2 + 60*D*a^2*b^3*c*d*x^2 + 30*C*a*b^4*c*d*x^2 + 30*
B*b^5*c*d*x^2 + 150*D*a^3*b^2*d^2*x^2 + 30*C*a^2*b^3*d^2*x^2 + 15*B*a*b^4*
d^2*x^2 + 15*A*b^5*d^2*x^2 + 6*D*a^2*b^3*c^2*x + 6*C*a*b^4*c^2*x + 12*B*b^
5*c^2*x + 24*D*a^3*b^2*c*d*x + 12*C*a^2*b^3*c*d*x + 12*B*a*b^4*c*d*x + 24*
A*b^5*c*d*x + 60*D*a^4*b*d^2*x + 12*C*a^3*b^2*d^2*x + 6*B*a^2*b^3*d^2*x +
6*A*a*b^4*d^2*x + D*a^3*b^2*c^2 + C*a^2*b^3*c^2 + 2*B*a*b^4*c^2 + 10*A*b^5
*c^2 + 4*D*a^4*b*c*d + 2*C*a^3*b^2*c*d + 2*B*a^2*b^3*c*d + 4*A*a*b^4*c*d +
 10*D*a^5*d^2 + 2*C*a^4*b*d^2 + B*a^3*b^2*d^2 + A*a^2*b^3*d^2)/((b*x + a)^
6*b^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^7}\,dx = \int \frac{(c+d\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^7}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^7,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^7}\,dx$$
$$= \frac{10b^5d^3x^6 - 90a\,b^4c\,d^2x^4 - 120a^2b^3c\,d^2x^3 - 20a\,b^5d^2x^3 - 60a\,b^4c^2d\,x^3 - 90a^3b^2c\,d^2x^2 - 30a^2b^4d^2x^2 - 45c}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^7,x)
```

output
```
( - 6*a**5*c*d**2 - 2*a**4*b**2*d**2 - 3*a**4*b*c**2*d - 36*a**4*b*c*d**2*
x - 6*a**3*b**3*c*d - 12*a**3*b**3*d**2*x - a**3*b**2*c**3 - 18*a**3*b**2*
c**2*d*x - 90*a**3*b**2*c*d**2*x**2 - 12*a**2*b**4*c**2 - 36*a**2*b**4*c*d
*x - 30*a**2*b**4*d**2*x**2 - 6*a**2*b**3*c**3*x - 45*a**2*b**3*c**2*d*x**
2 - 120*a**2*b**3*c*d**2*x**3 - 12*a*b**5*c**2*x - 30*a*b**5*c*d*x**2 - 20
*a*b**5*d**2*x**3 - 15*a*b**4*c**3*x**2 - 60*a*b**4*c**2*d*x**3 - 90*a*b**
4*c*d**2*x**4 + 10*b**5*d**3*x**6)/(60*a*b**5*(a**6 + 6*a**5*b*x + 15*a**4
*b**2*x**2 + 20*a**3*b**3*x**3 + 15*a**2*b**4*x**4 + 6*a*b**5*x**5 + b**6*
x**6))
```

3.24 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 307

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx &= -\frac{(bc-ad)^2(Ab^3-a(b^2B-abC+a^2D))}{7b^6(a+bx)^7}\\ &-\frac{(bc-ad)(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))}{6b^6(a+bx)^6}\\ &-\frac{b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D)}{5b^6(a+bx)^5}\\ &-\frac{10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D)}{4b^6(a+bx)^4}\\ &-\frac{d(bCd+2bcD-5adD)}{3b^6(a+bx)^3}-\frac{d^2D}{2b^6(a+bx)^2}\end{aligned}$$

output
```
-1/7*(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^6/(b*x+a)^7-1/6*(-a*d+b*
c)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))/b^6
/(b*x+a)^6-1/5*(b^3*(A*d^2+2*B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*
c)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^5-1/4*(10*a^2*d^2*D-4*a*b*d*
(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^4-1/3*d*(C*b*d-5*D*a*d+
2*D*b*c)/b^6/(b*x+a)^3-1/2*d^2*D/b^6/(b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^8} dx = -\frac{10a^5d^2D+2a^4bd(2Cd+4cD+35dDx)+4Ab^3(a^2d^2+abd(5c+7dx)+b^2(15c^2+35cdx+21d^2x^2))}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^8,x]
```

output
```
-1/420*(10*a^5*d^2*D + 2*a^4*b*d*(2*C*d + 4*c*D + 35*d*D*x) + 4*A*b^3*(a^2
*d^2 + a*b*d*(5*c + 7*d*x) + b^2*(15*c^2 + 35*c*d*x + 21*d^2*x^2)) + a^3*b
^2*(3*c^2*D + c*(6*C*d + 56*d*D*x) + d^2*(3*B + 28*C*x + 210*D*x^2)) + 7*b
^5*x*(B*(10*c^2 + 24*c*d*x + 15*d^2*x^2) + x*(10*d^2*x^2*(2*C + 3*D*x) + 1
0*c*d*x*(3*C + 4*D*x) + 3*c^2*(4*C + 5*D*x))) + a*b^4*(B*(10*c^2 + 56*c*d*
x + 63*d^2*x^2) + 7*x*(10*d^2*x^2*(2*C + 5*D*x) + c^2*(4*C + 9*D*x) + 2*c*
d*x*(9*C + 20*D*x))) + a^2*b^3*(c^2*(4*C + 21*D*x) + 2*c*d*(4*B + 21*x*(C
+ 4*D*x)) + 7*d^2*x*(3*B + 2*x*(6*C + 25*D*x))))/(b^6*(a + b*x)^7)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^8} dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)^2 (Ab^3-a(a^2D-abC+b^2B))}{b^5(a+bx)^8}+\frac{10a^2d^2D-4abd(2cD+Cd)+b^2(Bd^2+c^2D+2cCd)}{b^5(a+bx)^5}+\frac{-10a}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^6(a+bx)^7}-\frac{10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)}{4b^6(a+bx)^4}-\frac{-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)}{5b^6(a+bx)^5}-\frac{(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{6b^6(a+bx)^6}-\frac{d(-5adD+2bcD+bCd)}{3b^6(a+bx)^3}-\frac{d^2D}{2b^6(a+bx)^2}$$

input

```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^8,x]
```

output

```
-1/7*((b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^6*(a + b*x)^7)
 - ((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a
^2*b*(4*C*d + 3*c*D)))/(6*b^6*(a + b*x)^6) - (b^3*(c^2*C + 2*B*c*d + A*d^2
) - 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^
2*D))/(5*b^6*(a + b*x)^5) - (10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2
*c*C*d + B*d^2 + c^2*D))/(4*b^6*(a + b*x)^4) - (d*(b*C*d + 2*b*c*D - 5*a*d
*D))/(3*b^6*(a + b*x)^3) - (d^2*D)/(2*b^6*(a + b*x)^2)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.49

method	result
default	$-\frac{b^3d^2A-3Bab^2d^2+2Bb^3cd+6Ca^2bd^2-6Cab^2cd+Cb^3c^2-10a^3d^2D+12a^2bcdD-3ab^2c^2D}{5b^6(bx+a)^5}-\frac{b^2Bd^2-4Cabd^2+2Cb^2c}{4b^6}$
norman	$\frac{-\frac{Dd^2x^5}{2b}-\frac{\left(2b^2d^2C+5Dabd^2+4Db^2cd\right)x^4}{6b^3}-\frac{\left(3b^3Bd^2+4Cab^2d^2+6Cb^3cd+10Da^2bd^2+8Dab^2cd+3Db^3c^2\right)x^3}{12b^4}-\frac{\left(4d^2Ab^4+3Bab^3d^2+\right.}{}}{}$
gosper	$-\frac{210Dx^5d^2b^5+140Cx^4b^5d^2+350Dx^4ab^4d^2+280Dx^4b^5cd+105Bx^3b^5d^2+140Cx^3ab^4d^2+210Cx^3b^5cd+350Dx^3a^2b^3d^2+}{}$
orering	$-\frac{210Dx^5d^2b^5+140Cx^4b^5d^2+350Dx^4ab^4d^2+280Dx^4b^5cd+105Bx^3b^5d^2+140Cx^3ab^4d^2+210Cx^3b^5cd+350Dx^3a^2b^3d^2+}{}$
parallelrisch	$-\frac{210d^2Dx^5b^6+140Cb^6d^2x^4+350Dab^5d^2x^4+280Db^6cdx^4+105Bb^6d^2x^3+140Cab^5d^2x^3+210Cb^6cdx^3+350Da^2b^4d^2x^3+}{}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b^6*(A*b^3*d^2-3*B*a*b^2*d^2+2*B*b^3*c*d+6*C*a^2*b*d^2-6*C*a*b^2*c*d+
C*b^3*c^2-10*D*a^3*d^2+12*D*a^2*b*c*d-3*D*a*b^2*c^2)/(b*x+a)^5-1/4/b^6*(B*
b^2*d^2-4*C*a*b*d^2+2*C*b^2*c*d+10*D*a^2*d^2-8*D*a*b*c*d+D*b^2*c^2)/(b*x+a
)^4-1/7*(A*a^2*b^3*d^2-2*A*a*b^4*c*d+A*b^5*c^2-B*a^3*b^2*d^2+2*B*a^2*b^3*c
*d-B*a*b^4*c^2+C*a^4*b*d^2-2*C*a^3*b^2*c*d+C*a^2*b^3*c^2-D*a^5*d^2+2*D*a^4
*b*c*d-D*a^3*b^2*c^2)/b^6/(b*x+a)^7-1/2*d^2*D/b^6/(b*x+a)^2-1/3*d*(C*b*d-5
*D*a*d+2*D*b*c)/b^6/(b*x+a)^3-1/6*(-2*A*a*b^3*d^2+2*A*b^4*c*d+3*B*a^2*b^2*
d^2-4*B*a*b^3*c*d+B*b^4*c^2-4*C*a^3*b*d^2+6*C*a^2*b^2*c*d-2*C*a*b^3*c^2+5*
D*a^4*d^2-8*D*a^3*b*c*d+3*D*a^2*b^2*c^2)/b^6/(b*x+a)^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^8}\,dx = -\frac{210\,Db^5d^2x^5+70\,(4\,Db^5cd+(5\,Dab^4+2\,Cb^5)d^2)x^4+35\,(3\,Db^5c^2+2\,(4\,Dab^4+3\,Cb^5)cd+(10\,Da^2b}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x, algorithm="fricas")
```

output
```
-1/420*(210*D*b^5*d^2*x^5 + 70*(4*D*b^5*c*d + (5*D*a*b^4 + 2*C*b^5)*d^2)*x
^4 + 35*(3*D*b^5*c^2 + 2*(4*D*a*b^4 + 3*C*b^5)*c*d + (10*D*a^2*b^3 + 4*C*a
*b^4 + 3*B*b^5)*d^2)*x^3 + (3*D*a^3*b^2 + 4*C*a^2*b^3 + 10*B*a*b^4 + 60*A*
b^5)*c^2 + 2*(4*D*a^4*b + 3*C*a^3*b^2 + 4*B*a^2*b^3 + 10*A*a*b^4)*c*d + (1
0*D*a^5 + 4*C*a^4*b + 3*B*a^3*b^2 + 4*A*a^2*b^3)*d^2 + 21*((3*D*a*b^4 + 4*
C*b^5)*c^2 + 2*(4*D*a^2*b^3 + 3*C*a*b^4 + 4*B*b^5)*c*d + (10*D*a^3*b^2 + 4
*C*a^2*b^3 + 3*B*a*b^4 + 4*A*b^5)*d^2)*x^2 + 7*((3*D*a^2*b^3 + 4*C*a*b^4 +
 10*B*b^5)*c^2 + 2*(4*D*a^3*b^2 + 3*C*a^2*b^3 + 4*B*a*b^4 + 10*A*b^5)*c*d
+ (10*D*a^4*b + 4*C*a^3*b^2 + 3*B*a^2*b^3 + 4*A*a*b^4)*d^2)*x)/(b^13*x^7 +
 7*a*b^12*x^6 + 21*a^2*b^11*x^5 + 35*a^3*b^10*x^4 + 35*a^4*b^9*x^3 + 21*a^
5*b^8*x^2 + 7*a^6*b^7*x + a^7*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a)**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx = -\frac{210\,Db^5d^2x^5 + 70\,(4\,Db^5cd + (5\,Dab^4 + 2\,Cb^5)d^2)x^4 + 35\,(3\,Db^5c^2 + 2\,(4\,Dab^4 + 3\,Cb^5)cd + (10\,Da^2b}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x, algorithm="maxima")
```

output
```
-1/420*(210*D*b^5*d^2*x^5 + 70*(4*D*b^5*c*d + (5*D*a*b^4 + 2*C*b^5)*d^2)*x
^4 + 35*(3*D*b^5*c^2 + 2*(4*D*a*b^4 + 3*C*b^5)*c*d + (10*D*a^2*b^3 + 4*C*a
*b^4 + 3*B*b^5)*d^2)*x^3 + (3*D*a^3*b^2 + 4*C*a^2*b^3 + 10*B*a*b^4 + 60*A*
b^5)*c^2 + 2*(4*D*a^4*b + 3*C*a^3*b^2 + 4*B*a^2*b^3 + 10*A*a*b^4)*c*d + (1
0*D*a^5 + 4*C*a^4*b + 3*B*a^3*b^2 + 4*A*a^2*b^3)*d^2 + 21*((3*D*a*b^4 + 4*
C*b^5)*c^2 + 2*(4*D*a^2*b^3 + 3*C*a*b^4 + 4*B*b^5)*c*d + (10*D*a^3*b^2 + 4
*C*a^2*b^3 + 3*B*a*b^4 + 4*A*b^5)*d^2)*x^2 + 7*((3*D*a^2*b^3 + 4*C*a*b^4 +
 10*B*b^5)*c^2 + 2*(4*D*a^3*b^2 + 3*C*a^2*b^3 + 4*B*a*b^4 + 10*A*b^5)*c*d
+ (10*D*a^4*b + 4*C*a^3*b^2 + 3*B*a^2*b^3 + 4*A*a*b^4)*d^2)*x)/(b^13*x^7 +
 7*a*b^12*x^6 + 21*a^2*b^11*x^5 + 35*a^3*b^10*x^4 + 35*a^4*b^9*x^3 + 21*a^
5*b^8*x^2 + 7*a^6*b^7*x + a^7*b^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.64

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^8}\,dx = -\frac{210\,Db^5d^2x^5+280\,Db^5cdx^4+350\,Dab^4d^2x^4+140\,Cb^5d^2x^4+105\,Db^5c^2x^3+280\,Dab^4cdx^3+210\,Cb^5}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x, algorithm="giac")
```

output
```
-1/420*(210*D*b^5*d^2*x^5 + 280*D*b^5*c*d*x^4 + 350*D*a*b^4*d^2*x^4 + 140*
C*b^5*d^2*x^4 + 105*D*b^5*c^2*x^3 + 280*D*a*b^4*c*d*x^3 + 210*C*b^5*c*d*x^
3 + 350*D*a^2*b^3*d^2*x^3 + 140*C*a*b^4*d^2*x^3 + 105*B*b^5*d^2*x^3 + 63*D
*a*b^4*c^2*x^2 + 84*C*b^5*c^2*x^2 + 168*D*a^2*b^3*c*d*x^2 + 126*C*a*b^4*c*
d*x^2 + 168*B*b^5*c*d*x^2 + 210*D*a^3*b^2*d^2*x^2 + 84*C*a^2*b^3*d^2*x^2 +
 63*B*a*b^4*d^2*x^2 + 84*A*b^5*d^2*x^2 + 21*D*a^2*b^3*c^2*x + 28*C*a*b^4*c
^2*x + 70*B*b^5*c^2*x + 56*D*a^3*b^2*c*d*x + 42*C*a^2*b^3*c*d*x + 56*B*a*b
^4*c*d*x + 140*A*b^5*c*d*x + 70*D*a^4*b*d^2*x + 28*C*a^3*b^2*d^2*x + 21*B*
a^2*b^3*d^2*x + 28*A*a*b^4*d^2*x + 3*D*a^3*b^2*c^2 + 4*C*a^2*b^3*c^2 + 10*
B*a*b^4*c^2 + 60*A*b^5*c^2 + 8*D*a^4*b*c*d + 6*C*a^3*b^2*c*d + 8*B*a^2*b^3
*c*d + 20*A*a*b^4*c*d + 10*D*a^5*d^2 + 4*C*a^4*b*d^2 + 3*B*a^3*b^2*d^2 + 4
*A*a^2*b^3*d^2)/((b*x + a)^7*b^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^8} dx = \int \frac{(c+d\,x)^2 \,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^8} dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^8,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^8} dx = \frac{-210b^5d^3x^5 - 350a\,b^4d^3x^4 - 420b^5c\,d^2x^4 - 350a^2b^3d^3x^3 - 420a\,b^4c\,d^2x^3 - 105b^6d^2x^3 - 315b^5c^2d\,x^3 - 2}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^8,x)
```

output
```
( - 10*a**5*d**3 - 12*a**4*b*c*d**2 - 70*a**4*b*d**3*x - 7*a**3*b**3*d**2
- 9*a**3*b**2*c**2*d - 84*a**3*b**2*c*d**2*x - 210*a**3*b**2*d**3*x**2 - 2
8*a**2*b**4*c*d - 49*a**2*b**4*d**2*x - 4*a**2*b**3*c**3 - 63*a**2*b**3*c*
*2*d*x - 252*a**2*b**3*c*d**2*x**2 - 350*a**2*b**3*d**3*x**3 - 70*a*b**5*c
**2 - 196*a*b**5*c*d*x - 147*a*b**5*d**2*x**2 - 28*a*b**4*c**3*x - 189*a*b
**4*c**2*d*x**2 - 420*a*b**4*c*d**2*x**3 - 350*a*b**4*d**3*x**4 - 70*b**6*
c**2*x - 168*b**6*c*d*x**2 - 105*b**6*d**2*x**3 - 84*b**5*c**3*x**2 - 315*
b**5*c**2*d*x**3 - 420*b**5*c*d**2*x**4 - 210*b**5*d**3*x**5)/(420*b**6*(a
**7 + 7*a**6*b*x + 21*a**5*b**2*x**2 + 35*a**4*b**3*x**3 + 35*a**3*b**4*x*
*4 + 21*a**2*b**5*x**5 + 7*a*b**6*x**6 + b**7*x**7))
```

3.25 $\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^9}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 307

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2+Dx^3)}{(a+bx)^9}\,dx &= -\frac{(bc-ad)^2(Ab^3-a(b^2B-abC+a^2D))}{8b^6(a+bx)^8}\\ &-\frac{(bc-ad)(b^3(Bc+2Ad)-ab^2(2cC+3Bd)-5a^3dD+a^2b(4Cd+3cD))}{7b^6(a+bx)^7}\\ &-\frac{b^3(c^2C+2Bcd+Ad^2)-10a^3d^2D+6a^2bd(Cd+2cD)-3ab^2(2cCd+Bd^2+c^2D)}{6b^6(a+bx)^6}\\ &-\frac{10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D)}{5b^6(a+bx)^5}\\ &-\frac{d(bCd+2bcD-5adD)}{4b^6(a+bx)^4}-\frac{d^2D}{3b^6(a+bx)^3}\end{aligned}$$

output
```
-1/8*(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^6/(b*x+a)^8-1/7*(-a*d+b*
c)*(b^3*(2*A*d+B*c)-a*b^2*(3*B*d+2*C*c)-5*a^3*d*D+a^2*b*(4*C*d+3*D*c))/b^6
/(b*x+a)^7-1/6*(b^3*(A*d^2+2*B*c*d+C*c^2)-10*a^3*d^2*D+6*a^2*b*d*(C*d+2*D*
c)-3*a*b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^6-1/5*(10*a^2*d^2*D-4*a*b*d*
(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))/b^6/(b*x+a)^5-1/4*d*(C*b*d-5*D*a*d+
2*D*b*c)/b^6/(b*x+a)^4-1/3*d^2*D/b^6/(b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^9} dx = -\frac{5a^5d^2D+a^4bd(3Cd+6cD+40dDx)+5Ab^3(a^2d^2+2abd(3c+4dx)+b^2(21c^2+48cdx+28d^2x^2))}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^9,x]
```

output
```
-1/840*(5*a^5*d^2*D + a^4*b*d*(3*C*d + 6*c*D + 40*d*D*x) + 5*A*b^3*(a^2*d^
2 + 2*a*b*d*(3*c + 4*d*x) + b^2*(21*c^2 + 48*c*d*x + 28*d^2*x^2)) + a^2*b^
3*(c^2*(5*C + 24*D*x) + 4*d^2*x*(6*B + 21*C*x + 70*D*x^2) + 2*c*d*(5*B + 2
4*C*x + 84*D*x^2)) + 2*b^5*x*(4*B*(15*c^2 + 35*c*d*x + 21*d^2*x^2) + 7*x*(
5*d^2*x^2*(3*C + 4*D*x) + 6*c*d*x*(4*C + 5*D*x) + 2*c^2*(5*C + 6*D*x))) +
a^3*b^2*(3*c^2*D + 6*c*d*(C + 8*D*x) + d^2*(3*B + 4*x*(6*C + 35*D*x))) + a
*b^4*(B*(15*c^2 + 80*c*d*x + 84*d^2*x^2) + 2*x*(84*c*d*x*(C + 2*D*x) + 7*d
^2*x^2*(12*C + 25*D*x) + c^2*(20*C + 42*D*x))))/(b^6*(a + b*x)^8)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^9} dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad)^2 (Ab^3-a(a^2D-abC+b^2B))}{b^5(a+bx)^9}+\frac{10a^2d^2D-4abd(2cD+Cd)+b^2(Bd^2+c^2D+2cCd)}{b^5(a+bx)^6}+\frac{-10a\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{8b^6(a+bx)^8}-\frac{10a^2d^2D-4abd(2cD+Cd)+b^2\left(Bd^2+c^2D+2cCd\right)}{5b^6(a+bx)^5}-\frac{-10a^3d^2D+6a^2bd(2cD+Cd)-3ab^2\left(Bd^2+c^2D+2cCd\right)+b^3\left(Ad^2+2Bcd+c^2C\right)}{6b^6(a+bx)^6}-\frac{(bc-ad)\left(-5a^3dD+a^2b(3cD+4Cd)-ab^2(3Bd+2cC)+b^3(2Ad+Bc)\right)}{7b^6(a+bx)^7}-\frac{d(-5adD+2bcD+bCd)}{4b^6(a+bx)^4}-\frac{d^2D}{3b^6(a+bx)^3}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^9,x]
```

output
```
-1/8*((b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^6*(a + b*x)^8)
 - ((b*c - a*d)*(b^3*(B*c + 2*A*d) - a*b^2*(2*c*C + 3*B*d) - 5*a^3*d*D + a
^2*b*(4*C*d + 3*c*D)))/(7*b^6*(a + b*x)^7) - (b^3*(c^2*C + 2*B*c*d + A*d^2
) - 10*a^3*d^2*D + 6*a^2*b*d*(C*d + 2*c*D) - 3*a*b^2*(2*c*C*d + B*d^2 + c^
2*D))/(6*b^6*(a + b*x)^6) - (10*a^2*d^2*D - 4*a*b*d*(C*d + 2*c*D) + b^2*(2
*c*C*d + B*d^2 + c^2*D))/(5*b^6*(a + b*x)^5) - (d*(b*C*d + 2*b*c*D - 5*a*d
*D))/(4*b^6*(a + b*x)^4) - (d^2*D)/(3*b^6*(a + b*x)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.49

method	result
default	$-\frac{b^2Bd^2-4Cabd^2+2Cb^2cd+10a^2d^2D-8Dabcd+Db^2c^2}{5b^6(bx+a)^5}-\frac{d(Cbd-5Dad+2Dbc)}{4b^6(bx+a)^4}-\frac{-2Aab^3d^2+2Ab^4cd+3Ba^2b^2d^2-\cdots}{\cdots}$
norman	$\frac{-\frac{Dd^2x^5}{3b}-\frac{\left(3b^3d^2C+5Dab^2d^2+6Db^3cd\right)x^4}{12b^4}-\frac{\left(3Bd^2b^4+3Cab^3d^2+6Ccdb^4+5Da^2b^2d^2+6Dab^3cd+3Dc^2b^4\right)x^3}{15b^5}-\frac{\left(5d^2Ab^5+3Bab^4d^2\cdots\right.}{\cdots}}{\cdots}$
gosper	$-\frac{280Dx^5d^2b^5+210Cx^4b^5d^2+350Dx^4ab^4d^2+420Dx^4b^5cd+168Bx^3b^5d^2+168Cx^3ab^4d^2+336Cx^3b^5cd+280Dx^3a^2b^3d^2+\cdots}{\cdots}$
orering	$-\frac{280Dx^5d^2b^5+210Cx^4b^5d^2+350Dx^4ab^4d^2+420Dx^4b^5cd+168Bx^3b^5d^2+168Cx^3ab^4d^2+336Cx^3b^5cd+280Dx^3a^2b^3d^2+\cdots}{\cdots}$
parallelrisch	$-\frac{280d^2Dx^5b^7+210Cb^7d^2x^4+350Dab^6d^2x^4+420Db^7cdx^4+168Bb^7d^2x^3+168Cab^6d^2x^3+336Cb^7cdx^3+280Da^2b^5d^2x^3+\cdots}{\cdots}$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^9,x,method=_RETURNVERBOSE)
```

output
```
-1/5/b^6*(B*b^2*d^2-4*C*a*b*d^2+2*C*b^2*c*d+10*D*a^2*d^2-8*D*a*b*c*d+D*b^2
*c^2)/(b*x+a)^5-1/4*d*(C*b*d-5*D*a*d+2*D*b*c)/b^6/(b*x+a)^4-1/7*(-2*A*a*b^
3*d^2+2*A*b^4*c*d+3*B*a^2*b^2*d^2-4*B*a*b^3*c*d+B*b^4*c^2-4*C*a^3*b*d^2+6*
C*a^2*b^2*c*d-2*C*a*b^3*c^2+5*D*a^4*d^2-8*D*a^3*b*c*d+3*D*a^2*b^2*c^2)/b^6
/(b*x+a)^7-1/8*(A*a^2*b^3*d^2-2*A*a*b^4*c*d+A*b^5*c^2-B*a^3*b^2*d^2+2*B*a^
2*b^3*c*d-B*a*b^4*c^2+C*a^4*b*d^2-2*C*a^3*b^2*c*d+C*a^2*b^3*c^2-D*a^5*d^2+
2*D*a^4*b*c*d-D*a^3*b^2*c^2)/b^6/(b*x+a)^8-1/3*d^2*D/b^6/(b*x+a)^3-1/6/b^6
*(A*b^3*d^2-3*B*a*b^2*d^2+2*B*b^3*c*d+6*C*a^2*b*d^2-6*C*a*b^2*c*d+C*b^3*c^
2-10*D*a^3*d^2+12*D*a^2*b*c*d-3*D*a*b^2*c^2)/(b*x+a)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^9}\,dx =$$
$$-\frac{280\,Db^5d^2x^5+70\,(6\,Db^5cd+(5\,Dab^4+3\,Cb^5)d^2)x^4+56\,(3\,Db^5c^2+6\,(Dab^4+Cb^5)cd+(5\,Da^2b^3+\cdots}{\cdots}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^9,x, algorithm="fricas")
```

output
```
-1/840*(280*D*b^5*d^2*x^5 + 70*(6*D*b^5*c*d + (5*D*a*b^4 + 3*C*b^5)*d^2)*x
^4 + 56*(3*D*b^5*c^2 + 6*(D*a*b^4 + C*b^5)*c*d + (5*D*a^2*b^3 + 3*C*a*b^4
+ 3*B*b^5)*d^2)*x^3 + (3*D*a^3*b^2 + 5*C*a^2*b^3 + 15*B*a*b^4 + 105*A*b^5)
*c^2 + 2*(3*D*a^4*b + 3*C*a^3*b^2 + 5*B*a^2*b^3 + 15*A*a*b^4)*c*d + (5*D*a
^5 + 3*C*a^4*b + 3*B*a^3*b^2 + 5*A*a^2*b^3)*d^2 + 28*((3*D*a*b^4 + 5*C*b^5
)*c^2 + 2*(3*D*a^2*b^3 + 3*C*a*b^4 + 5*B*b^5)*c*d + (5*D*a^3*b^2 + 3*C*a^2
*b^3 + 3*B*a*b^4 + 5*A*b^5)*d^2)*x^2 + 8*((3*D*a^2*b^3 + 5*C*a*b^4 + 15*B*
b^5)*c^2 + 2*(3*D*a^3*b^2 + 3*C*a^2*b^3 + 5*B*a*b^4 + 15*A*b^5)*c*d + (5*D
*a^4*b + 3*C*a^3*b^2 + 3*B*a^2*b^3 + 5*A*a*b^4)*d^2)*x)/(b^14*x^8 + 8*a*b^
13*x^7 + 28*a^2*b^12*x^6 + 56*a^3*b^11*x^5 + 70*a^4*b^10*x^4 + 56*a^5*b^9*
x^3 + 28*a^6*b^8*x^2 + 8*a^7*b^7*x + a^8*b^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^9}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(D*x**3+C*x**2+B*x+A)/(b*x+a)**9,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^9}\,dx = -\frac{280\,Db^5d^2x^5 + 70\,(6\,Db^5cd + (5\,Dab^4 + 3\,Cb^5)d^2)x^4 + 56\,(3\,Db^5c^2 + 6\,(Dab^4 + Cb^5)cd + (5\,Da^2b^3 + \cdots}{\cdots}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^9,x, algorithm="maxima")
```

output
```
-1/840*(280*D*b^5*d^2*x^5 + 70*(6*D*b^5*c*d + (5*D*a*b^4 + 3*C*b^5)*d^2)*x
^4 + 56*(3*D*b^5*c^2 + 6*(D*a*b^4 + C*b^5)*c*d + (5*D*a^2*b^3 + 3*C*a*b^4
+ 3*B*b^5)*d^2)*x^3 + (3*D*a^3*b^2 + 5*C*a^2*b^3 + 15*B*a*b^4 + 105*A*b^5)
*c^2 + 2*(3*D*a^4*b + 3*C*a^3*b^2 + 5*B*a^2*b^3 + 15*A*a*b^4)*c*d + (5*D*a
^5 + 3*C*a^4*b + 3*B*a^3*b^2 + 5*A*a^2*b^3)*d^2 + 28*((3*D*a*b^4 + 5*C*b^5
)*c^2 + 2*(3*D*a^2*b^3 + 3*C*a*b^4 + 5*B*b^5)*c*d + (5*D*a^3*b^2 + 3*C*a^2
*b^3 + 3*B*a*b^4 + 5*A*b^5)*d^2)*x^2 + 8*((3*D*a^2*b^3 + 5*C*a*b^4 + 15*B*
b^5)*c^2 + 2*(3*D*a^3*b^2 + 3*C*a^2*b^3 + 5*B*a*b^4 + 15*A*b^5)*c*d + (5*D
*a^4*b + 3*C*a^3*b^2 + 3*B*a^2*b^3 + 5*A*a*b^4)*d^2)*x)/(b^14*x^8 + 8*a*b^
13*x^7 + 28*a^2*b^12*x^6 + 56*a^3*b^11*x^5 + 70*a^4*b^10*x^4 + 56*a^5*b^9*
x^3 + 28*a^6*b^8*x^2 + 8*a^7*b^7*x + a^8*b^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.64

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2+Dx^3)}{(a+bx)^9}\,dx = -\frac{280\,Db^5d^2x^5+420\,Db^5cdx^4+350\,Dab^4d^2x^4+210\,Cb^5d^2x^4+168\,Db^5c^2x^3+336\,Dab^4cdx^3+336\,Cb}{}$$

input
```
integrate((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^9,x, algorithm="giac")
```

output
```
-1/840*(280*D*b^5*d^2*x^5 + 420*D*b^5*c*d*x^4 + 350*D*a*b^4*d^2*x^4 + 210*
C*b^5*d^2*x^4 + 168*D*b^5*c^2*x^3 + 336*D*a*b^4*c*d*x^3 + 336*C*b^5*c*d*x^
3 + 280*D*a^2*b^3*d^2*x^3 + 168*C*a*b^4*d^2*x^3 + 168*B*b^5*d^2*x^3 + 84*D
*a*b^4*c^2*x^2 + 140*C*b^5*c^2*x^2 + 168*D*a^2*b^3*c*d*x^2 + 168*C*a*b^4*c
*d*x^2 + 280*B*b^5*c*d*x^2 + 140*D*a^3*b^2*d^2*x^2 + 84*C*a^2*b^3*d^2*x^2
+ 84*B*a*b^4*d^2*x^2 + 140*A*b^5*d^2*x^2 + 24*D*a^2*b^3*c^2*x + 40*C*a*b^4
*c^2*x + 120*B*b^5*c^2*x + 48*D*a^3*b^2*c*d*x + 48*C*a^2*b^3*c*d*x + 80*B*
a*b^4*c*d*x + 240*A*b^5*c*d*x + 40*D*a^4*b*d^2*x + 24*C*a^3*b^2*d^2*x + 24
*B*a^2*b^3*d^2*x + 40*A*a*b^4*d^2*x + 3*D*a^3*b^2*c^2 + 5*C*a^2*b^3*c^2 +
15*B*a*b^4*c^2 + 105*A*b^5*c^2 + 6*D*a^4*b*c*d + 6*C*a^3*b^2*c*d + 10*B*a^
2*b^3*c*d + 30*A*a*b^4*c*d + 5*D*a^5*d^2 + 3*C*a^4*b*d^2 + 3*B*a^3*b^2*d^2
 + 5*A*a^2*b^3*d^2)/((b*x + a)^8*b^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^9}\,dx=\int \frac{{\left(c+d\,x\right)}^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^9}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^9,x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^9}\,dx = \frac{-280b^5d^3x^5-350a\,b^4d^3x^4-630b^5c\,d^2x^4-280a^2b^3d^3x^3-504a\,b^4c\,d^2x^3-168b^6d^2x^3-504b^5c^2d\,x^3-1}{}$$

input
```
int((d*x+c)^2*(D*x^3+C*x^2+B*x+A)/(b*x+a)^9,x)
```

output
```
( - 5*a**5*d**3 - 9*a**4*b*c*d**2 - 40*a**4*b*d**3*x - 8*a**3*b**3*d**2 -
9*a**3*b**2*c**2*d - 72*a**3*b**2*c*d**2*x - 140*a**3*b**2*d**3*x**2 - 40*
a**2*b**4*c*d - 64*a**2*b**4*d**2*x - 5*a**2*b**3*c**3 - 72*a**2*b**3*c**2
*d*x - 252*a**2*b**3*c*d**2*x**2 - 280*a**2*b**3*d**3*x**3 - 120*a*b**5*c*
*2 - 320*a*b**5*c*d*x - 224*a*b**5*d**2*x**2 - 40*a*b**4*c**3*x - 252*a*b*
*4*c**2*d*x**2 - 504*a*b**4*c*d**2*x**3 - 350*a*b**4*d**3*x**4 - 120*b**6*
c**2*x - 280*b**6*c*d*x**2 - 168*b**6*d**2*x**3 - 140*b**5*c**3*x**2 - 504
*b**5*c**2*d*x**3 - 630*b**5*c*d**2*x**4 - 280*b**5*d**3*x**5)/(840*b**6*(
a**8 + 8*a**7*b*x + 28*a**6*b**2*x**2 + 56*a**5*b**3*x**3 + 70*a**4*b**4*x
**4 + 56*a**3*b**5*x**5 + 28*a**2*b**6*x**6 + 8*a*b**7*x**7 + b**8*x**8))
```

3.26 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 413

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx$$
$$= -\frac{(bc-ad)^2\left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right)x}{d^6}$$
$$-\frac{(bc-ad)\left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)(}{2d^7}$$
$$+\frac{\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D)\right)}{3d^7}$$
$$+\frac{b(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D))(c+dx)^4}{4d^7}$$
$$+\frac{b^2(bCd-6bcD+3adD)(c+dx)^5}{5d^7}+\frac{b^3D(c+dx)^6}{6d^7}$$
$$-\frac{(bc-ad)^3\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)\log(c+dx)}{d^7}$$

output
```
-(-a*d+b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d
-6*D*c^3))*x/d^6-1/2*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^2+8*C*c
*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^2/d^7+1/
3*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d-10*D*c^2)+b
^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^3/d^7+1/4*b*(3*a^2*d^2*D
+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*x+c)^4/d^7+1/5*b^2*
(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^5/d^7+1/6*b^3*D*(d*x+c)^6/d^7-(-a*d+b*c)^3
*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^7
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{dx(10a^3d^3(6c^2D-3cd(2C+Dx)+d^2(6B+x(3C+2Dx)))+15a^2bd^2(-12c^3D+6c^2d(2C+Dx)-2}{}$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(d*x*(10*a^3*d^3*(6*c^2*D - 3*c*d*(2*C + D*x) + d^2*(6*B + x*(3*C + 2*D*x)
)) + 15*a^2*b*d^2*(-12*c^3*D + 6*c^2*d*(2*C + D*x) - 2*c*d^2*(6*B + x*(3*C
 + 2*D*x)) + d^3*(12*A + x*(6*B + 4*C*x + 3*D*x^2))) + 3*a*b^2*d*(60*c^4*D
 - 30*c^3*d*(2*C + D*x) + 10*c^2*d^2*(6*B + x*(3*C + 2*D*x)) - 5*c*d^3*(12
*A + x*(6*B + x*(4*C + 3*D*x))) + d^4*x*(30*A + x*(20*B + 3*x*(5*C + 4*D*x
)))) + b^3*(-60*c^5*D + 30*c^4*d*(2*C + D*x) - 10*c^3*d^2*(6*B + x*(3*C +
2*D*x)) + 5*c^2*d^3*(12*A + x*(6*B + x*(4*C + 3*D*x))) - c*d^4*x*(30*A + x
*(20*B + 3*x*(5*C + 4*D*x))) + d^5*x^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)
)))) + 60*(b*c - a*d)^3*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*Log[c + d*x
])/(60*d^7)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{(c+dx)(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+10c\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(c+dx)^2(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right.}{2d^7}$$
$$\frac{b(c+dx)^4\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{4d^7}+$$
$$\frac{(c+dx)^3\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2Cd\right.\right.}{3d^7}$$
$$\frac{(bc-ad)^3\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^7}-$$
$$\frac{x(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{d^6}+$$
$$\frac{b^2(c+dx)^5(3adD-6bcD+bCd)}{5d^7}+\frac{b^3D(c+dx)^6}{6d^7}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
-(((b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d - 4*B*c*d
^2 + 3*A*d^3 - 6*c^3*D))*x)/d^6) - ((b*c - a*d)*(a^2*d^2*(C*d - 3*c*D) - a
*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^2*C*d - 6*B*c*d^2 + 3*A*d^
3 - 15*c^3*D))*(c + d*x)^2)/(2*d^7) + ((a^3*d^3*D + 3*a^2*b*d^2*(C*d - 4*c
*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*D) + b^3*(10*c^2*C*d - 4*B*c*d^2
 + A*d^3 - 20*c^3*D))*(c + d*x)^3)/(3*d^7) + (b*(3*a^2*d^2*D + 3*a*b*d*(C*
d - 5*c*D) - b^2*(5*c*C*d - B*d^2 - 15*c^2*D))*(c + d*x)^4)/(4*d^7) + (b^2
*(b*C*d - 6*b*c*D + 3*a*d*D)*(c + d*x)^5)/(5*d^7) + (b^3*D*(c + d*x)^6)/(6
*d^7) - ((b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/d
^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.73

method	result
norman	$\frac{(3A\,a^2b\,d^5-3Aa\,b^2c\,d^4+A\,b^3c^2d^3+B\,a^3d^5-3B\,a^2bc\,d^4+3Ba\,b^2c^2d^3-B\,b^3c^3d^2-C\,a^3c\,d^4+3C\,a^2b\,c^2d^3-3Ca\,b^2c^3d^2+C\,b^3c}{d^6}$
default	$\frac{3}{2}Da^2b\,c^2d^3x^2-\frac{1}{3}B\,b^3c\,d^4x^3+\frac{1}{3}C\,b^3c^2d^3x^3-\frac{1}{3}Db^3c^3d^2x^3-\frac{1}{5}Db^3c\,d^4x^5-Ca\,b^2c\,d^4x^3-3Da^2b\,c^3d^2x+3Da\,b^2c^4dx+\frac{1}{4}Db^3c$
parallelrisch	$180D\ln(xd+c)a^2b\,c^4d^2-180D\ln(xd+c)a\,b^2c^5d-45Dx^4a\,b^2c\,d^5-60C\,x^3a\,b^2c\,d^5-60Dx^3a^2bc\,d^5+60Dx^3a\,b^2c^2d^4-180A\ln($

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(3*A*a^2*b*d^5-3*A*a*b^2*c*d^4+A*b^3*c^2*d^3+B*a^3*d^5-3*B*a^2*b*c*d^4+3*B
*a*b^2*c^2*d^3-B*b^3*c^3*d^2-C*a^3*c*d^4+3*C*a^2*b*c^2*d^3-3*C*a*b^2*c^3*d
^2+C*b^3*c^4*d+D*a^3*c^2*d^3-3*D*a^2*b*c^3*d^2+3*D*a*b^2*c^4*d-D*b^3*c^5)/
d^6*x+1/2/d^5*(3*A*a*b^2*d^4-A*b^3*c*d^3+3*B*a^2*b*d^4-3*B*a*b^2*c*d^3+B*b
^3*c^2*d^2+C*a^3*d^4-3*C*a^2*b*c*d^3+3*C*a*b^2*c^2*d^2-C*b^3*c^3*d-D*a^3*c
*d^3+3*D*a^2*b*c^2*d^2-3*D*a*b^2*c^3*d+D*b^3*c^4)*x^2+1/3/d^4*(A*b^3*d^3+3
*B*a*b^2*d^3-B*b^3*c*d^2+3*C*a^2*b*d^3-3*C*a*b^2*c*d^2+C*b^3*c^2*d+D*a^3*d
^3-3*D*a^2*b*c*d^2+3*D*a*b^2*c^2*d-D*b^3*c^3)*x^3+1/6*D*b^3/d*x^6+1/4*b/d^
3*(B*b^2*d^2+3*C*a*b*d^2-C*b^2*c*d+3*D*a^2*d^2-3*D*a*b*c*d+D*b^2*c^2)*x^4+
1/5*b^2/d^2*(C*b*d+3*D*a*d-D*b*c)*x^5+(A*a^3*d^6-3*A*a^2*b*c*d^5+3*A*a*b^2
*c^2*d^4-A*b^3*c^3*d^3-B*a^3*c*d^5+3*B*a^2*b*c^2*d^4-3*B*a*b^2*c^3*d^3+B*b
^3*c^4*d^2+C*a^3*c^2*d^4-3*C*a^2*b*c^3*d^3+3*C*a*b^2*c^4*d^2-C*b^3*c^5*d-D
*a^3*c^3*d^3+3*D*a^2*b*c^4*d^2-3*D*a*b^2*c^5*d+D*b^3*c^6)/d^7*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{10\,Db^3d^6x^6 - 12\,(Db^3cd^5 - (3\,Dab^2 + Cb^3)d^6)x^5 + 15\,(Db^3c^2d^4 - (3\,Dab^2 + Cb^3)cd^5 + (3\,Da^2b + 3\,Cab}{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
1/60*(10*D*b^3*d^6*x^6 - 12*(D*b^3*c*d^5 - (3*D*a*b^2 + C*b^3)*d^6)*x^5 +
15*(D*b^3*c^2*d^4 - (3*D*a*b^2 + C*b^3)*c*d^5 + (3*D*a^2*b + 3*C*a*b^2 + B
*b^3)*d^6)*x^4 - 20*(D*b^3*c^3*d^3 - (3*D*a*b^2 + C*b^3)*c^2*d^4 + (3*D*a^
2*b + 3*C*a*b^2 + B*b^3)*c*d^5 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d
^6)*x^3 + 30*(D*b^3*c^4*d^2 - (3*D*a*b^2 + C*b^3)*c^3*d^3 + (3*D*a^2*b + 3
*C*a*b^2 + B*b^3)*c^2*d^4 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^5
+ (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^6)*x^2 - 60*(D*b^3*c^5*d - (3*D*a*b^2
+ C*b^3)*c^4*d^2 + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^3 - (D*a^3 + 3*C*
a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^4 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^5
 - (B*a^3 + 3*A*a^2*b)*d^6)*x + 60*(D*b^3*c^6 + A*a^3*d^6 - (3*D*a*b^2 + C
*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b
 + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 -
(B*a^3 + 3*A*a^2*b)*c*d^5)*log(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx = \frac{Db^3x^6}{6d}+x^5\left(\frac{Cb^3}{5d}+\frac{3Dab^2}{5d}-\frac{Db^3c}{5d^2}\right)+x^4\left(\frac{Bb^3}{4d}+\frac{3Cab^2}{4d}-\frac{Cb^3c}{4d^2}+\frac{3Da^2b}{4d}-\frac{3Dab^2c}{4d^2}+\frac{Db^3c^2}{4d^3}\right)+x^3\left(\frac{Ab^3}{3d}+\frac{Bab^2}{d}-\frac{Bb^3c}{3d^2}+\frac{Ca^2b}{d}-\frac{Cab^2c}{d^2}+\frac{Cb^3c^2}{3d^3}+\frac{Da^3}{3d}-\frac{Da^2bc}{d^2}+\frac{Dab^2c^2}{d^3}-\frac{Db^3c^3}{3d^4}\right)+x^2\cdot\left(\frac{3Aab^2}{2d}-\frac{Ab^3c}{2d^2}+\frac{3Ba^2b}{2d}-\frac{3Bab^2c}{2d^2}+\frac{Bb^3c^2}{2d^3}+\frac{Ca^3}{2d}-\frac{3Ca^2bc}{2d^2}+\frac{3Cab^2c^2}{2d^3}-\frac{Cb^3c^3}{2d^4}-\frac{Da^3c}{2d^2}+\frac{3Da^2bc^2}{2d^3}-\frac{3Dab^2c^3}{2d^4}+\frac{Db^3c^4}{2d^5}\right)+x\left(\frac{3Aa^2b}{d}-\frac{3Aab^2c}{d^2}+\frac{Ab^3c^2}{d^3}+\frac{Ba^3}{d}-\frac{3Ba^2bc}{d^2}+\frac{3Bab^2c^2}{d^3}-\frac{Bb^3c^3}{d^4}-\frac{Ca^3c}{d^2}+\frac{3Ca^2bc^2}{d^3}-\frac{3Cab^2c^3}{d^4}+\frac{Cb^3c^4}{d^5}+\frac{Da^3c^2}{d^3}-\frac{3Da^2bc^3}{d^4}+\frac{3Dab^2c^4}{d^5}-\frac{Db^3c^5}{d^6}\right)-\frac{(ad-bc)^3\left(-Ad^3+Bcd^2-Cc^2d+Dc^3\right)\log\left(c+dx\right)}{d^7}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
D*b**3*x**6/(6*d) + x**5*(C*b**3/(5*d) + 3*D*a*b**2/(5*d) - D*b**3*c/(5*d*
*2)) + x**4*(B*b**3/(4*d) + 3*C*a*b**2/(4*d) - C*b**3*c/(4*d**2) + 3*D*a**
2*b/(4*d) - 3*D*a*b**2*c/(4*d**2) + D*b**3*c**2/(4*d**3)) + x**3*(A*b**3/(
3*d) + B*a*b**2/d - B*b**3*c/(3*d**2) + C*a**2*b/d - C*a*b**2*c/d**2 + C*b
**3*c**2/(3*d**3) + D*a**3/(3*d) - D*a**2*b*c/d**2 + D*a*b**2*c**2/d**3 -
D*b**3*c**3/(3*d**4)) + x**2*(3*A*a*b**2/(2*d) - A*b**3*c/(2*d**2) + 3*B*a
**2*b/(2*d) - 3*B*a*b**2*c/(2*d**2) + B*b**3*c**2/(2*d**3) + C*a**3/(2*d)
- 3*C*a**2*b*c/(2*d**2) + 3*C*a*b**2*c**2/(2*d**3) - C*b**3*c**3/(2*d**4)
- D*a**3*c/(2*d**2) + 3*D*a**2*b*c**2/(2*d**3) - 3*D*a*b**2*c**3/(2*d**4)
+ D*b**3*c**4/(2*d**5)) + x*(3*A*a**2*b/d - 3*A*a*b**2*c/d**2 + A*b**3*c**
2/d**3 + B*a**3/d - 3*B*a**2*b*c/d**2 + 3*B*a*b**2*c**2/d**3 - B*b**3*c**3
/d**4 - C*a**3*c/d**2 + 3*C*a**2*b*c**2/d**3 - 3*C*a*b**2*c**3/d**4 + C*b*
*3*c**4/d**5 + D*a**3*c**2/d**3 - 3*D*a**2*b*c**3/d**4 + 3*D*a*b**2*c**4/d
**5 - D*b**3*c**5/d**6) - (a*d - b*c)**3*(-A*d**3 + B*c*d**2 - C*c**2*d +
D*c**3)*log(c + d*x)/d**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{10\,Db^3d^5x^6 - 12\,(Db^3cd^4 - (3\,Dab^2 + Cb^3)d^5)x^5 + 15\,(Db^3c^2d^3 - (3\,Dab^2 + Cb^3)cd^4 + (3\,Da^2b + 3\,Ca\cdots}{\cdots}$$
$$+ \frac{(Db^3c^6 + Aa^3d^6 - (3\,Dab^2 + Cb^3)c^5d + (3\,Da^2b + 3\,Cab^2 + Bb^3)c^4d^2 - (Da^3 + 3\,Ca^2b + 3\,Bab^2 + A\cdots}{d^7}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/60*(10*D*b^3*d^5*x^6 - 12*(D*b^3*c*d^4 - (3*D*a*b^2 + C*b^3)*d^5)*x^5 +
15*(D*b^3*c^2*d^3 - (3*D*a*b^2 + C*b^3)*c*d^4 + (3*D*a^2*b + 3*C*a*b^2 + B
*b^3)*d^5)*x^4 - 20*(D*b^3*c^3*d^2 - (3*D*a*b^2 + C*b^3)*c^2*d^3 + (3*D*a^
2*b + 3*C*a*b^2 + B*b^3)*c*d^4 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d
^5)*x^3 + 30*(D*b^3*c^4*d - (3*D*a*b^2 + C*b^3)*c^3*d^2 + (3*D*a^2*b + 3*C
*a*b^2 + B*b^3)*c^2*d^3 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^4 +
(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^5)*x^2 - 60*(D*b^3*c^5 - (3*D*a*b^2 + C*
b^3)*c^4*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2 - (D*a^3 + 3*C*a^2*b
+ 3*B*a*b^2 + A*b^3)*c^2*d^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^4 - (B*
a^3 + 3*A*a^2*b)*d^5)*x)/d^6 + (D*b^3*c^6 + A*a^3*d^6 - (3*D*a*b^2 + C*b^3
)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b + 3
*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 - (B*a
^3 + 3*A*a^2*b)*c*d^5)*log(d*x + c)/d^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(403) = 806$.

Time = 0.12 (sec) , antiderivative size = 835, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{10\,Db^3d^5x^6 - 12\,Db^3cd^4x^5 + 36\,Dab^2d^5x^5 + 12\,Cb^3d^5x^5 + 15\,Db^3c^2d^3x^4 - 45\,Dab^2cd^4x^4 - 15\,Cb^3cd^4x^4}{}$$
$$+ \frac{(Db^3c^6 - 3\,Dab^2c^5d - Cb^3c^5d + 3\,Da^2bc^4d^2 + 3\,Cab^2c^4d^2 + Bb^3c^4d^2 - Da^3c^3d^3 - 3\,Ca^2bc^3d^3 - 3\,Ba}{d}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
1/60*(10*D*b^3*d^5*x^6 - 12*D*b^3*c*d^4*x^5 + 36*D*a*b^2*d^5*x^5 + 12*C*b^
3*d^5*x^5 + 15*D*b^3*c^2*d^3*x^4 - 45*D*a*b^2*c*d^4*x^4 - 15*C*b^3*c*d^4*x
^4 + 45*D*a^2*b*d^5*x^4 + 45*C*a*b^2*d^5*x^4 + 15*B*b^3*d^5*x^4 - 20*D*b^3
*c^3*d^2*x^3 + 60*D*a*b^2*c^2*d^3*x^3 + 20*C*b^3*c^2*d^3*x^3 - 60*D*a^2*b*
c*d^4*x^3 - 60*C*a*b^2*c*d^4*x^3 - 20*B*b^3*c*d^4*x^3 + 20*D*a^3*d^5*x^3 +
 60*C*a^2*b*d^5*x^3 + 60*B*a*b^2*d^5*x^3 + 20*A*b^3*d^5*x^3 + 30*D*b^3*c^4
*d*x^2 - 90*D*a*b^2*c^3*d^2*x^2 - 30*C*b^3*c^3*d^2*x^2 + 90*D*a^2*b*c^2*d^
3*x^2 + 90*C*a*b^2*c^2*d^3*x^2 + 30*B*b^3*c^2*d^3*x^2 - 30*D*a^3*c*d^4*x^2
 - 90*C*a^2*b*c*d^4*x^2 - 90*B*a*b^2*c*d^4*x^2 - 30*A*b^3*c*d^4*x^2 + 30*C
*a^3*d^5*x^2 + 90*B*a^2*b*d^5*x^2 + 90*A*a*b^2*d^5*x^2 - 60*D*b^3*c^5*x +
180*D*a*b^2*c^4*d*x + 60*C*b^3*c^4*d*x - 180*D*a^2*b*c^3*d^2*x - 180*C*a*b
^2*c^3*d^2*x - 60*B*b^3*c^3*d^2*x + 60*D*a^3*c^2*d^3*x + 180*C*a^2*b*c^2*d
^3*x + 180*B*a*b^2*c^2*d^3*x + 60*A*b^3*c^2*d^3*x - 60*C*a^3*c*d^4*x - 180
*B*a^2*b*c*d^4*x - 180*A*a*b^2*c*d^4*x + 60*B*a^3*d^5*x + 180*A*a^2*b*d^5*
x)/d^6 + (D*b^3*c^6 - 3*D*a*b^2*c^5*d - C*b^3*c^5*d + 3*D*a^2*b*c^4*d^2 +
3*C*a*b^2*c^4*d^2 + B*b^3*c^4*d^2 - D*a^3*c^3*d^3 - 3*C*a^2*b*c^3*d^3 - 3*
B*a*b^2*c^3*d^3 - A*b^3*c^3*d^3 + C*a^3*c^2*d^4 + 3*B*a^2*b*c^2*d^4 + 3*A*
a*b^2*c^2*d^4 - B*a^3*c*d^5 - 3*A*a^2*b*c*d^5 + A*a^3*d^6)*log(abs(d*x + c
))/d^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx=\int \frac{{\left(a+b\,x\right)}^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{c+d\,x}\,dx$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{60\log(dx+c)\,a^4d^4 - 240\log(dx+c)\,a^3bc\,d^3 + 360\log(dx+c)\,a^2b^2c^2d^2 - 240\log(dx+c)\,a\,b^3c^3d + 60\,\mathrm{lo}}{}$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
(60*log(c + d*x)*a**4*d**4 - 240*log(c + d*x)*a**3*b*c*d**3 + 360*log(c +
d*x)*a**2*b**2*c**2*d**2 - 240*log(c + d*x)*a*b**3*c**3*d + 60*log(c + d*x
)*b**4*c**4 + 240*a**3*b*d**4*x + 20*a**3*d**5*x**3 - 360*a**2*b**2*c*d**3
*x + 180*a**2*b**2*d**4*x**2 + 45*a**2*b*d**5*x**4 + 240*a*b**3*c**2*d**2*
x - 120*a*b**3*c*d**3*x**2 + 80*a*b**3*d**4*x**3 + 36*a*b**2*d**5*x**5 - 6
0*b**4*c**3*d*x + 30*b**4*c**2*d**2*x**2 - 20*b**4*c*d**3*x**3 + 15*b**4*d
**4*x**4 + 10*b**3*d**5*x**6)/(60*d**5)
```

3.27 $\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$

Optimal result 296
Mathematica [A] (verified) 297
Rubi [A] (verified) 297
Maple [A] (verified) 299
Fricas [A] (verification not implemented) 299
Sympy [A] (verification not implemented) 300
Maxima [A] (verification not implemented) 301
Giac [A] (verification not implemented) 301
Mupad [F(-1)] 302
Reduce [B] (verification not implemented) 302

Optimal result

Integrand size = 30, antiderivative size = 301

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \frac{(bc-ad)\left(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D)\right)x}{d^5} + \frac{\left(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D)\right)(c+dx)^2}{2d^6} + \frac{\left(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D)\right)(c+dx)^3}{3d^6} + \frac{b(bCd-5bcD+2adD)(c+dx)^4}{4d^6} + \frac{b^2D(c+dx)^5}{5d^6} + \frac{(bc-ad)^2\,(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^6}$$

output
```
(-a*d+b*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*
D*c^3))*x/d^5+1/2*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*c^2)+b^
2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^2/d^6+1/3*(a^2*d^2*D+2*a*b
*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^3/d^6+1/4*b*(C*b*d+2
*D*a*d-5*D*b*c)*(d*x+c)^4/d^6+1/5*b^2*D*(d*x+c)^5/d^6+(-a*d+b*c)^2*(A*d^3-
B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^6
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{c+dx} \, dx$$
$$= \frac{dx(10a^2d^2(6c^2D-3cd(2C+Dx)+d^2(6B+x(3C+2Dx)))+10abd(-12c^3D+6c^2d(2C+Dx)-2cc}{}$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(d*x*(10*a^2*d^2*(6*c^2*D - 3*c*d*(2*C + D*x) + d^2*(6*B + x*(3*C + 2*D*x)
)) + 10*a*b*d*(-12*c^3*D + 6*c^2*d*(2*C + D*x) - 2*c*d^2*(6*B + x*(3*C + 2
*D*x)) + d^3*(12*A + x*(6*B + 4*C*x + 3*D*x^2))) + b^2*(60*c^4*D - 30*c^3*
d*(2*C + D*x) + 10*c^2*d^2*(6*B + x*(3*C + 2*D*x)) - 5*c*d^3*(12*A + x*(6*
B + x*(4*C + 3*D*x))) + d^4*x*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))) - 60
*(b*c - a*d)^2*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*Log[c + d*x])/(60*d^
6)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{c+dx} \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c+d}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^2\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{2d^6}+\frac{(c+dx)^3\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{3d^6}+\frac{(bc-ad)^2\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^6}+\frac{x(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{d^5}+\frac{b(c+dx)^4(2adD-5bcD+bCd)}{4d^6}+\frac{b^2D(c+dx)^5}{5d^6}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
((b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B*c*d^2 +
 2*A*d^3 - 5*c^3*D))*x)/d^5 + ((a^2*d^2*(C*d - 3*c*D) - 2*a*b*d*(3*c*C*d -
 B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*(c + d
*x)^2)/(2*d^6) + ((a^2*d^2*D + 2*a*b*d*(C*d - 4*c*D) - b^2*(4*c*C*d - B*d^
2 - 10*c^2*D))*(c + d*x)^3)/(3*d^6) + (b*(b*C*d - 5*b*c*D + 2*a*d*D)*(c +
d*x)^4)/(4*d^6) + (b^2*D*(c + d*x)^5)/(5*d^6) + ((b*c - a*d)^2*(c^2*C*d -
B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/d^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.42

method	result
norman	$\frac{(2Aab\,d^4-A\,b^2c\,d^3+B\,a^2d^4-2Babc\,d^3+B\,b^2c^2d^2-C\,a^2c\,d^3+2Cab\,c^2d^2-C\,b^2c^3d+Da^2c^2d^2-2Dab\,c^3d+Db^2c^4)x}{d^5}+\frac{(A\,b^2}{}$
default	$\frac{\frac{2}{3}Cab\,d^4x^3+\frac{1}{2}C\,a^2d^4x^2+\frac{1}{2}Dab\,d^4x^4-\frac{1}{4}Db^2c\,d^3x^4+\frac{1}{3}Db^2c^2d^2x^3-A\,b^2c\,d^3x+B\,b^2c^2d^2x-C\,a^2c\,d^3x-C\,b^2c^3dx+Da^2c^2d^2x}{}$
parallelrisch	$\frac{-120A\ln(xd+c)abc\,d^4-120Dxab\,c^3d^2+40C\,x^3ab\,d^5-20C\,x^3b^2c\,d^4+20Dx^3b^2c^2d^3+60A\ln(xd+c)b^2c^2d^3-60B\ln(xd+c)a^2}{}$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(2*A*a*b*d^4-A*b^2*c*d^3+B*a^2*d^4-2*B*a*b*c*d^3+B*b^2*c^2*d^2-C*a^2*c*d^3
+2*C*a*b*c^2*d^2-C*b^2*c^3*d+D*a^2*c^2*d^2-2*D*a*b*c^3*d+D*b^2*c^4)/d^5*x+
1/2/d^4*(A*b^2*d^3+2*B*a*b*d^3-B*b^2*c*d^2+C*a^2*d^3-2*C*a*b*c*d^2+C*b^2*c
^2*d-D*a^2*c*d^2+2*D*a*b*c^2*d-D*b^2*c^3)*x^2+1/3/d^3*(B*b^2*d^2+2*C*a*b*d
^2-C*b^2*c*d+D*a^2*d^2-2*D*a*b*c*d+D*b^2*c^2)*x^3+1/5*D*b^2/d*x^5+1/4*b/d^
2*(C*b*d+2*D*a*d-D*b*c)*x^4+(A*a^2*d^5-2*A*a*b*c*d^4+A*b^2*c^2*d^3-B*a^2*c
*d^4+2*B*a*b*c^2*d^3-B*b^2*c^3*d^2+C*a^2*c^2*d^3-2*C*a*b*c^3*d^2+C*b^2*c^4
*d-D*a^2*c^3*d^2+2*D*a*b*c^4*d-D*b^2*c^5)/d^6*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$=\frac{12\,Db^2d^5x^5-15\,(Db^2cd^4-(2\,Dab+Cb^2)d^5)x^4+20\,(Db^2c^2d^3-(2\,Dab+Cb^2)cd^4+(Da^2+2\,Cab+B}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
1/60*(12*D*b^2*d^5*x^5 - 15*(D*b^2*c*d^4 - (2*D*a*b + C*b^2)*d^5)*x^4 + 20
*(D*b^2*c^2*d^3 - (2*D*a*b + C*b^2)*c*d^4 + (D*a^2 + 2*C*a*b + B*b^2)*d^5)
*x^3 - 30*(D*b^2*c^3*d^2 - (2*D*a*b + C*b^2)*c^2*d^3 + (D*a^2 + 2*C*a*b +
B*b^2)*c*d^4 - (C*a^2 + 2*B*a*b + A*b^2)*d^5)*x^2 + 60*(D*b^2*c^4*d - (2*D
*a*b + C*b^2)*c^3*d^2 + (D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 - (C*a^2 + 2*B*a
*b + A*b^2)*c*d^4 + (B*a^2 + 2*A*a*b)*d^5)*x - 60*(D*b^2*c^5 - A*a^2*d^5 -
 (2*D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 + 2*
B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)*log(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \frac{Db^2x^5}{5d} + x^4\left(\frac{Cb^2}{4d} + \frac{Dab}{2d} - \frac{Db^2c}{4d^2}\right) + x^3\left(\frac{Bb^2}{3d} + \frac{2Cab}{3d} - \frac{Cb^2c}{3d^2} + \frac{Da^2}{3d} - \frac{2Dabc}{3d^2} + \frac{Db^2c^2}{3d^3}\right) + x^2\left(\frac{Ab^2}{2d} + \frac{Bab}{d} - \frac{Bb^2c}{2d^2} + \frac{Ca^2}{2d} - \frac{Cabc}{d^2} + \frac{Cb^2c^2}{2d^3} - \frac{Da^2c}{2d^2} + \frac{Dabc^2}{d^3} - \frac{Db^2c^3}{2d^4}\right) + x\left(\frac{2Aab}{d} - \frac{Ab^2c}{d^2} + \frac{Ba^2}{d} - \frac{2Babc}{d^2} + \frac{Bb^2c^2}{d^3} - \frac{Ca^2c}{d^2} + \frac{2Cabc^2}{d^3} - \frac{Cb^2c^3}{d^4} + \frac{Da^2c^2}{d^3} - \frac{2Dabc^3}{d^4} + \frac{Db^2c^4}{d^5}\right) - \frac{(ad-bc)^2(-Ad^3+Bcd^2-Cc^2d+Dc^3)\log(c+dx)}{d^6}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
D*b**2*x**5/(5*d) + x**4*(C*b**2/(4*d) + D*a*b/(2*d) - D*b**2*c/(4*d**2))
+ x**3*(B*b**2/(3*d) + 2*C*a*b/(3*d) - C*b**2*c/(3*d**2) + D*a**2/(3*d) -
2*D*a*b*c/(3*d**2) + D*b**2*c**2/(3*d**3)) + x**2*(A*b**2/(2*d) + B*a*b/d
- B*b**2*c/(2*d**2) + C*a**2/(2*d) - C*a*b*c/d**2 + C*b**2*c**2/(2*d**3) -
 D*a**2*c/(2*d**2) + D*a*b*c**2/d**3 - D*b**2*c**3/(2*d**4)) + x*(2*A*a*b/
d - A*b**2*c/d**2 + B*a**2/d - 2*B*a*b*c/d**2 + B*b**2*c**2/d**3 - C*a**2*
c/d**2 + 2*C*a*b*c**2/d**3 - C*b**2*c**3/d**4 + D*a**2*c**2/d**3 - 2*D*a*b
*c**3/d**4 + D*b**2*c**4/d**5) - (a*d - b*c)**2*(-A*d**3 + B*c*d**2 - C*c*
*2*d + D*c**3)*log(c + d*x)/d**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{12\,Db^2d^4x^5 - 15\,(Db^2cd^3 - (2\,Dab+Cb^2)d^4)x^4 + 20\,(Db^2c^2d^2 - (2\,Dab+Cb^2)cd^3 + (Da^2+2\,Cab+B}{}$$
$$- \frac{(Db^2c^5 - Aa^2d^5 - (2\,Dab+Cb^2)c^4d + (Da^2+2\,Cab+Bb^2)c^3d^2 - (Ca^2+2\,Bab+Ab^2)c^2d^3 + (Ba^2}{d^6}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/60*(12*D*b^2*d^4*x^5 - 15*(D*b^2*c*d^3 - (2*D*a*b + C*b^2)*d^4)*x^4 + 20
*(D*b^2*c^2*d^2 - (2*D*a*b + C*b^2)*c*d^3 + (D*a^2 + 2*C*a*b + B*b^2)*d^4)
*x^3 - 30*(D*b^2*c^3*d - (2*D*a*b + C*b^2)*c^2*d^2 + (D*a^2 + 2*C*a*b + B*
b^2)*c*d^3 - (C*a^2 + 2*B*a*b + A*b^2)*d^4)*x^2 + 60*(D*b^2*c^4 - (2*D*a*b
 + C*b^2)*c^3*d + (D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - (C*a^2 + 2*B*a*b + A
*b^2)*c*d^3 + (B*a^2 + 2*A*a*b)*d^4)*x)/d^5 - (D*b^2*c^5 - A*a^2*d^5 - (2*
D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 + 2*B*a*
b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)*log(d*x + c)/d^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$= \frac{12\,Db^2d^4x^5 - 15\,Db^2cd^3x^4 + 30\,Dabd^4x^4 + 15\,Cb^2d^4x^4 + 20\,Db^2c^2d^2x^3 - 40\,Dabcd^3x^3 - 20\,Cb^2cd^3x^3 +}{}$$
$$- \frac{(Db^2c^5 - 2\,Dabc^4d - Cb^2c^4d + Da^2c^3d^2 + 2\,Cabc^3d^2 + Bb^2c^3d^2 - Ca^2c^2d^3 - 2\,Babc^2d^3 - Ab^2c^2d^3 +}{d^6}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
1/60*(12*D*b^2*d^4*x^5 - 15*D*b^2*c*d^3*x^4 + 30*D*a*b*d^4*x^4 + 15*C*b^2*
d^4*x^4 + 20*D*b^2*c^2*d^2*x^3 - 40*D*a*b*c*d^3*x^3 - 20*C*b^2*c*d^3*x^3 +
 20*D*a^2*d^4*x^3 + 40*C*a*b*d^4*x^3 + 20*B*b^2*d^4*x^3 - 30*D*b^2*c^3*d*x
^2 + 60*D*a*b*c^2*d^2*x^2 + 30*C*b^2*c^2*d^2*x^2 - 30*D*a^2*c*d^3*x^2 - 60
*C*a*b*c*d^3*x^2 - 30*B*b^2*c*d^3*x^2 + 30*C*a^2*d^4*x^2 + 60*B*a*b*d^4*x^
2 + 30*A*b^2*d^4*x^2 + 60*D*b^2*c^4*x - 120*D*a*b*c^3*d*x - 60*C*b^2*c^3*d
*x + 60*D*a^2*c^2*d^2*x + 120*C*a*b*c^2*d^2*x + 60*B*b^2*c^2*d^2*x - 60*C*
a^2*c*d^3*x - 120*B*a*b*c*d^3*x - 60*A*b^2*c*d^3*x + 60*B*a^2*d^4*x + 120*
A*a*b*d^4*x)/d^5 - (D*b^2*c^5 - 2*D*a*b*c^4*d - C*b^2*c^4*d + D*a^2*c^3*d^
2 + 2*C*a*b*c^3*d^2 + B*b^2*c^3*d^2 - C*a^2*c^2*d^3 - 2*B*a*b*c^2*d^3 - A*
b^2*c^2*d^3 + B*a^2*c*d^4 + 2*A*a*b*c*d^4 - A*a^2*d^5)*log(abs(d*x + c))/d
^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx=\int \frac{(a+b\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{c+d\,x}\,dx$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$$
$$=\frac{30\log(dx+c)\,a^3d^3-90\log(dx+c)\,a^2bc\,d^2+90\log(dx+c)\,a\,b^2c^2d-30\log(dx+c)\,b^3c^3+90a^2b\,d^3x+}{30d^4}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
(30*log(c + d*x)*a**3*d**3 - 90*log(c + d*x)*a**2*b*c*d**2 + 90*log(c + d*
x)*a*b**2*c**2*d - 30*log(c + d*x)*b**3*c**3 + 90*a**2*b*d**3*x + 10*a**2*
d**4*x**3 - 90*a*b**2*c*d**2*x + 45*a*b**2*d**3*x**2 + 15*a*b*d**4*x**4 +
30*b**3*c**2*d*x - 15*b**3*c*d**2*x**2 + 10*b**3*d**3*x**3 + 6*b**2*d**4*x
**5)/(30*d**4)
```

3.28 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx$

Optimal result 304
Mathematica [A] (verified) 305
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Optimal result

Integrand size = 28, antiderivative size = 178

$$\begin{aligned}\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx &= -\frac{(ad(cCd-Bd^2-c^2D)-b(c^2Cd-Bcd^2+Ad^3-c^3D))\,x}{d^4} \\ &\quad+\frac{(ad(Cd-cD)-b(cCd-Bd^2-c^2D))\,x^2}{2d^3}+\frac{(bCd-bcD+adD)x^3}{3d^2} \\ &\quad+\frac{bDx^4}{4d}-\frac{(bc-ad)\,(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^5}\end{aligned}$$

output
```
-(a*d*(-B*d^2+C*c*d-D*c^2)-b*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*x/d^4+1/2*(a*d
*(C*d-D*c)-b*(-B*d^2+C*c*d-D*c^2))*x^2/d^3+1/3*(C*b*d+D*a*d-D*b*c)*x^3/d^2
+1/4*b*D*x^4/d-(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx$$
$$= \frac{dx(2ad(6c^2D-3cd(2C+Dx)+d^2(6B+3Cx+2Dx^2))+b(-12c^3D+6c^2d(2C+Dx)-2cd^2(6B+3\ldots}{\ldots}$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
(d*x*(2*a*d*(6*c^2*D - 3*c*d*(2*C + D*x) + d^2*(6*B + 3*C*x + 2*D*x^2)) +
b*(-12*c^3*D + 6*c^2*d*(2*C + D*x) - 2*c*d^2*(6*B + 3*C*x + 2*D*x^2) + d^3
*(12*A + 6*B*x + 4*C*x^2 + 3*D*x^3))) + 12*(b*c - a*d)*(-(c^2*C*d) + B*c*d
^2 - A*d^3 + c^3*D)*Log[c + d*x])/(12*d^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(ad-bc)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^4(c+dx)}+\frac{b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)-ad\left(-Bd^2+c^2(-D)+cC\ldots\right.}{d^4}\ldots\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^5}-\frac{x\left(ad\left(-Bd^2+c^2(-D)+cCd\right)-b\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)\right)}{d^4}+\frac{x^2\left(ad(Cd-cD)-b\left(-Bd^2+c^2(-D)+cCd\right)\right)}{2d^3}+\frac{x^3(adD-bcD+bCd)}{3d^2}+\frac{bDx^4}{4d}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x),x]
```

output
```
-(((a*d*(c*C*d - B*d^2 - c^2*D) - b*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))*x
)/d^4) + ((a*d*(C*d - c*D) - b*(c*C*d - B*d^2 - c^2*D))*x^2)/(2*d^3) + ((b
*C*d - b*c*D + a*d*D)*x^3)/(3*d^2) + (b*D*x^4)/(4*d) - ((b*c - a*d)*(c^2*C
*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.13

method	result
norman	$\frac{\left(Ab\,d^3+Ba\,d^3-Bbc\,d^2-Cac\,d^2+Cb\,c^2d+Da\,c^2d-Db\,c^3\right)x}{d^4}+\frac{\left(bB\,d^2+Ca\,d^2-Cbcd-Dacd+Db\,c^2\right)x^2}{2d^3}+\frac{(Cbd+Dad-Db}{3d^2}$
default	$\frac{\frac{1}{4}Db\,x^4d^3+\frac{1}{3}Cb\,d^3x^3+\frac{1}{3}Da\,d^3x^3-\frac{1}{3}Dbc\,d^2x^3+\frac{1}{2}Bb\,d^3x^2+\frac{1}{2}Ca\,d^3x^2-\frac{1}{2}Cbc\,d^2x^2-\frac{1}{2}Dac\,d^2x^2+\frac{1}{2}Db\,c^2d\,x^2+Ab\,d^3x+Ba\,d}{d^4}$
parallelrisch	$-12D\ln(xd+c)a\,c^3d+3Db\,x^4d^4+12Bxa\,d^4+12A\ln(xd+c)a\,d^4+12D\ln(xd+c)b\,c^4+4C\,x^3b\,d^4+4Dx^3a\,d^4+6B\,x^2b\,d^4+6C\,x$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(A*b*d^3+B*a*d^3-B*b*c*d^2-C*a*c*d^2+C*b*c^2*d+D*a*c^2*d-D*b*c^3)/d^4*x+1/
2/d^3*(B*b*d^2+C*a*d^2-C*b*c*d-D*a*c*d+D*b*c^2)*x^2+1/3*(C*b*d+D*a*d-D*b*c
)*x^3/d^2+1/4*b*D*x^4/d+(A*a*d^4-A*b*c*d^3-B*a*c*d^3+B*b*c^2*d^2+C*a*c^2*d
^2-C*b*c^3*d-D*a*c^3*d+D*b*c^4)/d^5*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx$$
$$= \frac{3\,Dbd^4x^4 - 4\,(Dbcd^3 - (Da+Cb)d^4)x^3 + 6\,(Dbc^2d^2 - (Da+Cb)cd^3 + (Ca+Bb)d^4)x^2 - 12\,(Dbc^3d -}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
1/12*(3*D*b*d^4*x^4 - 4*(D*b*c*d^3 - (D*a + C*b)*d^4)*x^3 + 6*(D*b*c^2*d^2
 - (D*a + C*b)*c*d^3 + (C*a + B*b)*d^4)*x^2 - 12*(D*b*c^3*d - (D*a + C*b)*
c^2*d^2 + (C*a + B*b)*c*d^3 - (B*a + A*b)*d^4)*x + 12*(D*b*c^4 + A*a*d^4 -
 (D*a + C*b)*c^3*d + (C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)*log(d*x + c)
)/d^5
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx$$
$$= \frac{Dbx^4}{4d} + x^3\left(\frac{Cb}{3d} + \frac{Da}{3d} - \frac{Dbc}{3d^2}\right) + x^2\left(\frac{Bb}{2d} + \frac{Ca}{2d} - \frac{Cbc}{2d^2} - \frac{Dac}{2d^2} + \frac{Dbc^2}{2d^3}\right)$$
$$+ x\left(\frac{Ab}{d} + \frac{Ba}{d} - \frac{Bbc}{d^2} - \frac{Cac}{d^2} + \frac{Cbc^2}{d^3} + \frac{Dac^2}{d^3} - \frac{Dbc^3}{d^4}\right)$$
$$- \frac{(ad-bc)\left(-Ad^3 + Bcd^2 - Cc^2d + Dc^3\right)\log(c+dx)}{d^5}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
D*b*x**4/(4*d) + x**3*(C*b/(3*d) + D*a/(3*d) - D*b*c/(3*d**2)) + x**2*(B*b
/(2*d) + C*a/(2*d) - C*b*c/(2*d**2) - D*a*c/(2*d**2) + D*b*c**2/(2*d**3))
+ x*(A*b/d + B*a/d - B*b*c/d**2 - C*a*c/d**2 + C*b*c**2/d**3 + D*a*c**2/d*
*3 - D*b*c**3/d**4) - (a*d - b*c)*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)
*log(c + d*x)/d**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx = \frac{3\,Dbd^3x^4 - 4\,(Dbcd^2 - (Da+Cb)d^3)x^3 + 6\,(Dbc^2d - (Da+Cb)cd^2 + (Ca+Bb)d^3)x^2 - 12\,(Dbc^3 - (}{12\,d^4} + \frac{(Dbc^4 + Aad^4 - (Da+Cb)c^3d + (Ca+Bb)c^2d^2 - (Ba+Ab)cd^3)\log\left(dx+c\right)}{d^5}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/12*(3*D*b*d^3*x^4 - 4*(D*b*c*d^2 - (D*a + C*b)*d^3)*x^3 + 6*(D*b*c^2*d -
 (D*a + C*b)*c*d^2 + (C*a + B*b)*d^3)*x^2 - 12*(D*b*c^3 - (D*a + C*b)*c^2*
d + (C*a + B*b)*c*d^2 - (B*a + A*b)*d^3)*x)/d^4 + (D*b*c^4 + A*a*d^4 - (D*
a + C*b)*c^3*d + (C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{c+dx}\,dx = \frac{3\,Dbd^3x^4 - 4\,Dbcd^2x^3 + 4\,Dad^3x^3 + 4\,Cbd^3x^3 + 6\,Dbc^2dx^2 - 6\,Dacd^2x^2 - 6\,Cbcd^2x^2 + 6\,Cad^3x^2 + 6\,B}{12\,d^4} + \frac{(Dbc^4 - Dac^3d - Cbc^3d + Cac^2d^2 + Bbc^2d^2 - Bacd^3 - Abcd^3 + Aad^4)\log\left(|dx+c|\right)}{d^5}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
1/12*(3*D*b*d^3*x^4 - 4*D*b*c*d^2*x^3 + 4*D*a*d^3*x^3 + 4*C*b*d^3*x^3 + 6*
D*b*c^2*d*x^2 - 6*D*a*c*d^2*x^2 - 6*C*b*c*d^2*x^2 + 6*C*a*d^3*x^2 + 6*B*b*
d^3*x^2 - 12*D*b*c^3*x + 12*D*a*c^2*d*x + 12*C*b*c^2*d*x - 12*C*a*c*d^2*x
- 12*B*b*c*d^2*x + 12*B*a*d^3*x + 12*A*b*d^3*x)/d^4 + (D*b*c^4 - D*a*c^3*d
 - C*b*c^3*d + C*a*c^2*d^2 + B*b*c^2*d^2 - B*a*c*d^3 - A*b*c*d^3 + A*a*d^4
)*log(abs(d*x + c))/d^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \int \frac{(a+b\,x)\,(A+B\,x+C\,x^2+x^3\,D)}{c+d\,x}\,dx$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x),x)
```

output
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{c+dx}\,dx = \frac{12\log(dx+c)\,a^2d^2-24\log(dx+c)\,abcd+12\log(dx+c)\,b^2c^2+24ab\,d^2x+4a\,d^3x^3-12b^2cdx+6b^2d^2x}{12d^3}$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
(12*log(c + d*x)*a**2*d**2 - 24*log(c + d*x)*a*b*c*d + 12*log(c + d*x)*b**
2*c**2 + 24*a*b*d**2*x + 4*a*d**3*x**3 - 12*b**2*c*d*x + 6*b**2*d**2*x**2
+ 3*b*d**3*x**4)/(12*d**3)
```

3.29 $\int \frac{A+Bx+Cx^2+Dx^3}{c+dx} dx$

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Mathematica [A] (verified) 310
Rubi [A] (verified) 311
Maple [A] (verified) 312
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Sympy [A] (verification not implemented) 313
Maxima [A] (verification not implemented) 313
Giac [A] (verification not implemented) 314
Mupad [F(-1)] 314
Reduce [B] (verification not implemented) 314

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx = -\frac{(cCd-Bd^2-c^2D)\,x}{d^3}+\frac{(Cd-cD)x^2}{2d^2}+\frac{Dx^3}{3d}+\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^4}$$

output
```
-(-B*d^2+C*c*d-D*c^2)*x/d^3+1/2*(C*d-D*c)*x^2/d^2+1/3*D*x^3/d+(A*d^3-B*c*d
^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx = \frac{dx(6c^2D-3cd(2C+Dx)+d^2(6B+3Cx+2Dx^2))-6(-c^2Cd+Bcd^2-Ad^3+c^3D)\log(c+dx)}{6d^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(c + d*x),x]
```

output
```
(d*x*(6*c^2*D - 3*c*d*(2*C + D*x) + d^2*(6*B + 3*C*x + 2*D*x^2)) - 6*(-(c^
2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*Log[c + d*x])/(6*d^4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx$$

$$\Big\downarrow \ 2389$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)}+\frac{Bd^2+c^2D-cCd}{d^3}+\frac{x(Cd-cD)}{d^2}+\frac{Dx^2}{d}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^4}-\frac{x\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}+\frac{x^2(Cd-cD)}{2d^2}+\frac{Dx^3}{3d}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(c + d*x),x]
```

output
```
-(((c*C*d - B*d^2 - c^2*D)*x)/d^3) + ((C*d - c*D)*x^2)/(2*d^2) + (D*x^3)/(
3*d) + ((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

method	result
norman	$\frac{(B\,d^2-Ccd+Dc^2)x}{d^3}+\frac{Dx^3}{3d}+\frac{(Cd-Dc)x^2}{2d^2}+\frac{(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3)\ln(xd+c)}{d^4}$
default	$\frac{\frac{1}{3}Dx^3d^2+\frac{1}{2}C\,d^2x^2-\frac{1}{2}Dcd\,x^2+B\,d^2x-Ccdx+Dc^2x}{d^3}+\frac{(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3)\ln(xd+c)}{d^4}$
parallelrisch	$\frac{2Dx^3d^3+3C\,x^2d^3-3Dx^2c\,d^2+6A\ln(xd+c)d^3-6B\ln(xd+c)c\,d^2+6Bx\,d^3+6C\ln(xd+c)c^2d-6Cxc\,d^2-6D\ln(xd+c)c^3+6Dx}{6d^4}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(B*d^2-C*c*d+D*c^2)/d^3*x+1/3*D*x^3/d+1/2*(C*d-D*c)*x^2/d^2+(A*d^3-B*c*d^2
+C*c^2*d-D*c^3)*ln(d*x+c)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx = \frac{2\,Dd^3x^3-3\,(Dcd^2-Cd^3)x^2+6\,(Dc^2d-Ccd^2+Bd^3)x-6\,(Dc^3-Cc^2d+Bcd^2-Ad^3)\log\left(dx+c\right)}{6\,d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
1/6*(2*D*d^3*x^3 - 3*(D*c*d^2 - C*d^3)*x^2 + 6*(D*c^2*d - C*c*d^2 + B*d^3)
*x - 6*(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx = \frac{Dx^3}{3d} + x^2\left(\frac{C}{2d} - \frac{Dc}{2d^2}\right) + x\left(\frac{B}{d} - \frac{Cc}{d^2} + \frac{Dc^2}{d^3}\right) - \frac{(-Ad^3+Bcd^2-Cc^2d+Dc^3)\log(c+dx)}{d^4}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c),x)
```

output
```
D*x**3/(3*d) + x**2*(C/(2*d) - D*c/(2*d**2)) + x*(B/d - C*c/d**2 + D*c**2/
d**3) - (-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)*log(c + d*x)/d**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx = \frac{2\,Dd^2x^3 - 3\,(Dcd-Cd^2)x^2 + 6\,(Dc^2-Ccd+Bd^2)x}{6\,d^3} - \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(dx+c)}{d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/6*(2*D*d^2*x^3 - 3*(D*c*d - C*d^2)*x^2 + 6*(D*c^2 - C*c*d + B*d^2)*x)/d^
3 - (D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(d*x + c)/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx = \frac{2\,Dd^2x^3-3\,Dcdx^2+3\,Cd^2x^2+6\,Dc^2x-6\,Ccdx+6\,Bd^2x}{6\,d^3} - \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(|dx+c|)}{d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
1/6*(2*D*d^2*x^3 - 3*D*c*d*x^2 + 3*C*d^2*x^2 + 6*D*c^2*x - 6*C*c*d*x + 6*B
*d^2*x)/d^3 - (D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(abs(d*x + c))/d^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{c+d\,x}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.43

$$\int \frac{A+Bx+Cx^2+Dx^3}{c+dx}\,dx = \frac{3\log(dx+c)\,ad-3\log(dx+c)\,bc+3bdx+d^2x^3}{3d^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c),x)
```

output
```
(3*log(c + d*x)*a*d - 3*log(c + d*x)*b*c + 3*b*d*x + d**2*x**3)/(3*d**2)
```

3.30 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx$

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Rubi [A] (verified) . . . 317
Maple [A] (verified) . . . 318
Fricas [A] (verification not implemented) . . . 318
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Giac [A] (verification not implemented) . . . 320
Mupad [F(-1)] . . . 320
Reduce [B] (verification not implemented) . . . 321

Optimal result

Integrand size = 30, antiderivative size = 129

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx=\frac{(bCd-bcD-adD)x}{b^2d^2}+\frac{Dx^2}{2bd}+\frac{(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{b^3(bc-ad)}-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^3(bc-ad)}$$

output
```
(C*b*d-D*a*d-D*b*c)*x/b^2/d^2+1/2*D*x^2/b/d+(A*b^3-a*(B*b^2-C*a*b+D*a^2))*
ln(b*x+a)/b^3/(-a*d+b*c)-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^3/(-a*d
+b*c)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx=\frac{bd(bc-ad)x(2bCd-2bcD-2adD+bdDx)+2d^3(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)+2b^3(-c^2}{2b^3d^3(bc-ad)}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)),x]
```

output
```
(b*d*(b*c - a*d)*x*(2*b*C*d - 2*b*c*D - 2*a*d*D + b*d*D*x) + 2*d^3*(A*b^3
- a*(b^2*B - a*b*C + a^2*D))*Log[a + b*x] + 2*b^3*(-(c^2*C*d) + B*c*d^2 -
A*d^3 + c^3*D)*Log[c + d*x])/(2*b^3*d^3*(b*c - a*d))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{b^2(a+bx)(bc-ad)}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^2(c+dx)(ad-bc)}+\frac{-adD-bcD+bCd}{b^2d^2}+\frac{Dx}{bd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(a+bx)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^3(bc-ad)}-\frac{\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3(bc-ad)}+\frac{x(-adD-bcD+bCd)}{b^2d^2}+\frac{Dx^2}{2bd}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)),x]
```

output
```
((b*C*d - b*c*D - a*d*D)*x)/(b^2*d^2) + (D*x^2)/(2*b*d) + ((A*b^3 - a*(b^2
*B - a*b*C + a^2*D))*Log[a + b*x])/(b^3*(b*c - a*d)) - ((c^2*C*d - B*c*d^2
 + A*d^3 - c^3*D)*Log[c + d*x])/(d^3*(b*c - a*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.97

method	result
default	$\frac{\frac{1}{2}dDx^2b+Cbdx-Dadx-Dbcx}{b^2d^2}+\frac{(Ad^3-Bcd^2+Cc^2d-Dc^3)\ln(xd+c)}{d^3(ad-bc)}+\frac{(-b^3A+ab^2B-a^2bC+a^3D)\ln(bx+a)}{b^3(ad-bc)}$
norman	$\frac{(Cbd-Dad-Dbc)x}{b^2d^2}+\frac{Dx^2}{2bd}+\frac{(Ad^3-Bcd^2+Cc^2d-Dc^3)\ln(xd+c)}{d^3(ad-bc)}-\frac{(b^3A-ab^2B+a^2bC-a^3D)\ln(bx+a)}{(ad-bc)b^3}$
parallelrisch	$-\frac{-Dx^2ab^2d^3+Dx^2b^3cd^2+2A\ln(bx+a)b^3d^3-2A\ln(xd+c)b^3d^3-2B\ln(bx+a)ab^2d^3+2B\ln(xd+c)b^3cd^2+2C\ln(bx+a)a^2b}{2d^3b^3(ad-b}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/d^2*(1/2*d*D*x^2*b+C*b*d*x-D*a*d*x-D*b*c*x)+1/d^3*(A*d^3-B*c*d^2+C*c
^2*d-D*c^3)/(a*d-b*c)*ln(d*x+c)+(-A*b^3+B*a*b^2-C*a^2*b+D*a^3)/b^3/(a*d-b*
c)*ln(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx = -\frac{2\,(Da^3-Ca^2b+Bab^2-Ab^3)d^3\log{(bx+a)}-(Db^3cd^2-Dab^2d^3)x^2+2\,(Db^3c^2d-Cb^3cd^2-(Da^2b}{2\,(b^4cd^3-ab^3d^4)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^3*log(b*x + a) - (D*b^3*c*d^
2 - D*a*b^2*d^3)*x^2 + 2*(D*b^3*c^2*d - C*b^3*c*d^2 - (D*a^2*b - C*a*b^2)*
d^3)*x - 2*(D*b^3*c^3 - C*b^3*c^2*d + B*b^3*c*d^2 - A*b^3*d^3)*log(d*x + c
))/(b^4*c*d^3 - a*b^3*d^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\log(bx+a)}{b^4c-ab^3d} + \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(dx+c)}{bcd^3-ad^4} + \frac{Dbdx^2-2(Dbc+(Da-Cb)d)x}{2b^2d^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*log(b*x + a)/(b^4*c - a*b^3*d) + (D*c
^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(d*x + c)/(b*c*d^3 - a*d^4) + 1/2*(D*b*
d*x^2 - 2*(D*b*c + (D*a - C*b)*d)*x)/(b^2*d^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\log(|bx+a|)}{b^4c-ab^3d} + \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(|dx+c|)}{bcd^3-ad^4} + \frac{Dbdx^2-2\,Dbcx-2\,Dadx+2\,Cbdx}{2\,b^2d^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*log(abs(b*x + a))/(b^4*c - a*b^3*d) +
 (D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(abs(d*x + c))/(b*c*d^3 - a*d^4) +
 1/2*(D*b*d*x^2 - 2*D*b*c*x - 2*D*a*d*x + 2*C*b*d*x)/(b^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.36

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)}\,dx = \frac{2\log(bx+a)\,a^2d+2\log(dx+c)\,b^3-2abdx+b^2d\,x^2}{2b^3d}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c),x)
```

output
```
(2*log(a + b*x)*a**2*d + 2*log(c + d*x)*b**3 - 2*a*b*d*x + b**2*d*x**2)/(2
*b**3*d)
```

3.31 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx$

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Rubi [A] (verified) 323
Maple [A] (verified) 324
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Sympy [F(-1)] 325
Maxima [A] (verification not implemented) 326
Giac [A] (verification not implemented) 326
Mupad [F(-1)] 327
Reduce [B] (verification not implemented) 327

Optimal result

Integrand size = 30, antiderivative size = 166

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx &= \frac{Dx}{b^2d}-\frac{Ab^3-a(b^2B-abC+a^2D)}{b^3(bc-ad)(a+bx)}\\ &\quad-\frac{(2ab^2cC-b^3(Bc-Ad)+2a^3dD-a^2b(Cd+3cD))\log(a+bx)}{b^3(bc-ad)^2}\\ &\quad+\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^2(bc-ad)^2}\end{aligned}$$

output

```
D*x/b^2/d-(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^3/(-a*d+b*c)/(b*x+a)-(2*a*b^2*c*
C-b^3*(-A*d+B*c)+2*a^3*d*D-a^2*b*(C*d+3*D*c))*ln(b*x+a)/b^3/(-a*d+b*c)^2+(
A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d^2/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx = \frac{Dx}{b^2d}+\frac{-Ab^3+a(b^2B-abC+a^2D)}{b^3(bc-ad)(a+bx)} + \frac{(-2ab^2cC+b^3(Bc-Ad)-2a^3dD+a^2b(Cd+3cD))\log(a+bx)}{b^3(bc-ad)^2} + \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d^2(bc-ad)^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)),x]
```

output
```
(D*x)/(b^2*d) + (-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))/(b^3*(b*c - a*d)*(a
 + b*x)) + ((-2*a*b^2*c*C + b^3*(B*c - A*d) - 2*a^3*d*D + a^2*b*(C*d + 3*c
*D))*Log[a + b*x])/(b^3*(b*c - a*d)^2) + ((c^2*C*d - B*c*d^2 + A*d^3 - c^3
*D)*Log[c + d*x])/(d^2*(b*c - a*d)^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{Ab^3-a(a^2D-abC+b^2B)}{b^2(a+bx)^2(bc-ad)}+\frac{-2a^3dD+a^2b(3cD+Cd)-2ab^2cC+b^3(Bc-Ad)}{b^2(a+bx)(bc-ad)^2}+\frac{Ad^3-Bcd^2+c^3(-\cdots}{d(c+dx)(ad-\cdots}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{b^3(a+bx)(bc-ad)}-\frac{\log(a+bx)\left(2a^3dD-a^2b(3cD+Cd)+2ab^2cC-b^3(Bc-Ad)\right)}{b^3(bc-ad)^2}+\frac{\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^2(bc-ad)^2}+\frac{Dx}{b^2d}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)),x]
```

output
```
(D*x)/(b^2*d) - (A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b^3*(b*c - a*d)*(a + 
b*x)) - ((2*a*b^2*c*C - b^3*(B*c - A*d) + 2*a^3*d*D - a^2*b*(C*d + 3*c*D))
*Log[a + b*x])/(b^3*(b*c - a*d)^2) + ((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*
Log[c + d*x])/(d^2*(b*c - a*d)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

method	result
default	$\frac{Dx}{b^2d}+\frac{\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)\ln(xd+c)}{d^2(ad-bc)^2}-\frac{-b^3A+a\,b^2B-a^2bC+a^3D}{b^3(ad-bc)(bx+a)}+\frac{\left(-b^3dA+B\,b^3c+C\,a^2bd-2a\,b^2cC-2a^3dD\right.}{b^3(ad-bc)^2}$
norman	$\frac{\frac{Dx^2}{bd}+\frac{b^3dA-Ba\,b^2d+C\,a^2bd-2a^3dD+a^2bcD}{d\,b^3(ad-bc)}}{bx+a}+\frac{\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)\ln(xd+c)}{d^2(a^2d^2-2abcd+b^2c^2)}-\frac{\left(b^3dA-B\,b^3c-C\,a^2bd+2a\,b^2cC+2\right.}{\left(a^2d^2-2abcd+b\right.}$
parallelrisch	$-\frac{-Ba\,b^3c\,d^2+C\,a^2b^2c\,d^2-3Da^3bc\,d^2+Da^2b^2c^2d-Aa\,b^3d^3+A\,b^4c\,d^2+B\,a^2b^2d^3-C\,a^3b\,d^3+2D\ln(bx+a)a^4d^3+D\ln(xd+c)}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
D*x/b^2/d+1/d^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^2*ln(d*x+c)-(-A*b^
3+B*a*b^2-C*a^2*b+D*a^3)/b^3/(a*d-b*c)/(b*x+a)+(-A*b^3*d+B*b^3*c+C*a^2*b*d
-2*C*a*b^2*c-2*D*a^3*d+3*D*a^2*b*c)/b^3/(a*d-b*c)^2*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(167) = 334$.

Time = 0.11 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.49

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)}\,dx$$
$$= \frac{(Da^3b - Ca^2b^2 + Bab^3 - Ab^4)cd^2 - (Da^4 - Ca^3b + Ba^2b^2 - Aab^3)d^3 + (Db^4c^2d - 2\,Dab^3cd^2 + Da^2b^2c}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
((D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d^2 - (D*a^4 - C*a^3*b + B*a^2*
b^2 - A*a*b^3)*d^3 + (D*b^4*c^2*d - 2*D*a*b^3*c*d^2 + D*a^2*b^2*d^3)*x^2 +
 (D*a*b^3*c^2*d - 2*D*a^2*b^2*c*d^2 + D*a^3*b*d^3)*x + ((3*D*a^3*b - 2*C*a
^2*b^2 + B*a*b^3)*c*d^2 - (2*D*a^4 - C*a^3*b + A*a*b^3)*d^3 + ((3*D*a^2*b^
2 - 2*C*a*b^3 + B*b^4)*c*d^2 - (2*D*a^3*b - C*a^2*b^2 + A*b^4)*d^3)*x)*log
(b*x + a) - (D*a*b^3*c^3 - C*a*b^3*c^2*d + B*a*b^3*c*d^2 - A*a*b^3*d^3 + (
D*b^4*c^3 - C*b^4*c^2*d + B*b^4*c*d^2 - A*b^4*d^3)*x)*log(d*x + c))/(a*b^5
*c^2*d^2 - 2*a^2*b^4*c*d^3 + a^3*b^3*d^4 + (b^6*c^2*d^2 - 2*a*b^5*c*d^3 +
a^2*b^4*d^4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx = \frac{((3\,Da^2b-2\,Cab^2+Bb^3)c-(2\,Da^3-Ca^2b+Ab^3)d)\log(bx+a)}{b^5c^2-2\,ab^4cd+a^2b^3d^2} - \frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(dx+c)}{b^2c^2d^2-2\,abcd^3+a^2d^4} + \frac{Da^3-Ca^2b+Bab^2-Ab^3}{ab^4c-a^2b^3d+(b^5c-ab^4d)x} + \frac{Dx}{b^2d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
((3*D*a^2*b - 2*C*a*b^2 + B*b^3)*c - (2*D*a^3 - C*a^2*b + A*b^3)*d)*log(b*
x + a)/(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2) - (D*c^3 - C*c^2*d + B*c*d^2
- A*d^3)*log(d*x + c)/(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + (D*a^3 - C*a
^2*b + B*a*b^2 - A*b^3)/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x) + D*x/
(b^2*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx = -\frac{(Dbc^3-Cbc^2d+Bbcd^2-Abd^3)\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^3c^2d^2-2\,ab^2cd^3+a^2bd^4} + \frac{\frac{Da^3b^2}{bx+a}-\frac{Ca^2b^3}{bx+a}+\frac{Bab^4}{bx+a}-\frac{Ab^5}{bx+a}}{b^6c-ab^5d} + \frac{(bx+a)D}{b^3d} + \frac{(Dbc+2\,Dad-Cbd)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3d^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
-(D*b*c^3 - C*b*c^2*d + B*b*c*d^2 - A*b*d^3)*log(abs(b*c/(b*x + a) - a*d/(
b*x + a) + d))/(b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4) + (D*a^3*b^2/(b*x
 + a) - C*a^2*b^3/(b*x + a) + B*a*b^4/(b*x + a) - A*b^5/(b*x + a))/(b^6*c
- a*b^5*d) + (b*x + a)*D/(b^3*d) + (D*b*c + 2*D*a*d - C*b*d)*log(abs(b*x +
 a)/((b*x + a)^2*abs(b)))/(b^3*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)}\,dx$$
$$= \frac{-2\log(bx+a)\,a^3d+2\log(bx+a)\,a^2bc-2\log(bx+a)\,a^2bdx-\log(bx+a)\,a\,b^3+2\log(bx+a)\,a\,b^2cx-}{b^3\,(abdx-b^2cx+a^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c),x)
```

output
```
( - 2*log(a + b*x)*a**3*d + 2*log(a + b*x)*a**2*b*c - 2*log(a + b*x)*a**2*
b*d*x - log(a + b*x)*a*b**3 + 2*log(a + b*x)*a*b**2*c*x - log(a + b*x)*b**
4*x + log(c + d*x)*a*b**3 + log(c + d*x)*b**4*x + 2*a**2*b*d*x - 2*a*b**2*
c*x + a*b**2*d*x**2 - b**3*c*x**2)/(b**3*(a**2*d - a*b*c + a*b*d*x - b**2*
c*x))
```

3.32 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx$

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Maxima [A] (verification not implemented) . . . 333
Giac [A] (verification not implemented) . . . 333
Mupad [F(-1)] . . . 334
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Optimal result

Integrand size = 30, antiderivative size = 229

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx &= \frac{-Ab^3+a(b^2B-abC+a^2D)}{2b^3(bc-ad)(a+bx)^2}+\frac{2ab^2cC-b^3(Bc-Ad)+2a^3dD-a^2b(Cd+3cD)}{b^3(bc-ad)^2(a+bx)}\\ &+\frac{(b^3(c^2C-Bcd+Ad^2)-3ab^2c^2D+3a^2bcdD-a^3d^2D)\log(a+bx)}{b^3(bc-ad)^3}\\ &-\frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{d(bc-ad)^3}\end{aligned}$$

output
```
1/2*(-A*b^3+a*(B*b^2-C*a*b+D*a^2))/b^3/(-a*d+b*c)/(b*x+a)^2+(2*a*b^2*c*C-b
^3*(-A*d+B*c)+2*a^3*d*D-a^2*b*(C*d+3*D*c))/b^3/(-a*d+b*c)^2/(b*x+a)+(b^3*(
A*d^2-B*c*d+C*c^2)-3*a*b^2*c^2*D+3*a^2*b*c*d*D-a^3*d^2*D)*ln(b*x+a)/b^3/(-
a*d+b*c)^3-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/d/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx = \frac{\frac{(bc-ad)^2(-Ab^3+a(b^2B-abC+a^2D))}{b^3(a+bx)^2} - \frac{2(bc-ad)(-2ab^2cC+b^3(Bc-Ad)-2a^3dD+a^2b(Cd+3cD))}{b^3(a+bx)} + \frac{2(b^3(c^2C-Bcd+Ad^2)-3ab^2c^2D+}{b^3}}{2(bc-ad)^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)),x]
```

output
```
(((b*c - a*d)^2*(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D)))/(b^3*(a + b*x)^2)
- (2*(b*c - a*d)*(-2*a*b^2*c*C + b^3*(B*c - A*d) - 2*a^3*d*D + a^2*b*(C*d
+ 3*c*D)))/(b^3*(a + b*x)) + (2*(b^3*(c^2*C - B*c*d + A*d^2) - 3*a*b^2*c^2
*D + 3*a^2*b*c*d*D - a^3*d^2*D)*Log[a + b*x])/b^3 + (2*(-(c^2*C*d) + B*c*d
^2 - A*d^3 + c^3*D)*Log[c + d*x])/d)/(2*(b*c - a*d)^3)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{Ab^3-a(a^2D-abC+b^2B)}{b^2(a+bx)^3(bc-ad)} + \frac{a^3(-d^2)D+3a^2bcdD-3ab^2c^2D+b^3(Ad^2-Bcd+c^2C)}{b^2(a+bx)(bc-ad)^3} + \frac{-2a^3dD+a^2}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{Ab^3 - a(a^2D - abC + b^2B)}{2b^3(a+bx)^2(bc-ad)} + \frac{\log(a+bx)\left(a^3(-d^2)D + 3a^2bcdD - 3ab^2c^2D + b^3(Ad^2 - Bcd + c^2C)\right)}{b^3(bc-ad)^3} + \frac{2a^3dD - a^2b(3cD + Cd) + 2ab^2cC - b^3(Bc - Ad)}{b^3(a+bx)(bc-ad)^2} - \frac{\log(c+dx)\left(Ad^3 - Bcd^2 + c^3(-D) + c^2Cd\right)}{d(bc-ad)^3}
$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)),x]
```

output
```
-1/2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b^3*(b*c - a*d)*(a + b*x)^2) + (
2*a*b^2*c*C - b^3*(B*c - A*d) + 2*a^3*d*D - a^2*b*(C*d + 3*c*D))/(b^3*(b*c
 - a*d)^2*(a + b*x)) + ((b^3*(c^2*C - B*c*d + A*d^2) - 3*a*b^2*c^2*D + 3*a
^2*b*c*d*D - a^3*d^2*D)*Log[a + b*x])/(b^3*(b*c - a*d)^3) - ((c^2*C*d - B*
c*d^2 + A*d^3 - c^3*D)*Log[c + d*x])/(d*(b*c - a*d)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.02

method	result
default	$\frac{(A d^3 - Bc d^2 + C c^2 d - D c^3)\ln(xd+c)}{(ad-bc)^3 d} - \frac{-b^3 A + a b^2 B - a^2 bC + a^3 D}{2b^3(ad-bc)(bx+a)^2} - \frac{-b^3 dA + B b^3 c + C a^2 bd - 2a b^2 cC - 2a^3 dD + 3a^2 bcD}{b^3(ad-bc)^2(bx+a)}$
norman	$\frac{\frac{(b^3 dA - B b^3 c - C a^2 bd + 2a b^2 cC + 2a^3 dD - 3a^2 bcD)x}{b^2(a^2d^2 - 2abcd + b^2c^2)} + \frac{3Aa b^3 d - A b^4 c - B a^2 b^2 d - Ba b^3 c - C a^3 bd + 3C a^2 b^2 c + 3Da^4 d - 5Da^3 bc}{2b^3(a^2d^2 - 2abcd + b^2c^2)}}{(bx+a)^2} + \frac{(A d}{d(a}$
parallelrisch	$-\frac{4C\ln(bx+a)xa b^4 c^2 d - 4C\ln(xd+c)xa b^4 c^2 d + 12D\ln(bx+a)x a^3 b^2 c d^2 - 2D\ln(bx+a)a^5 d^3 - 12D\ln(bx+a)x a^2 b^3 c^2 d - 3A a}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^3/d*ln(d*x+c)-1/2*(-A*b^3+B*a*b^2-
C*a^2*b+D*a^3)/b^3/(a*d-b*c)/(b*x+a)^2-(-A*b^3*d+B*b^3*c+C*a^2*b*d-2*C*a*b
^2*c-2*D*a^3*d+3*D*a^2*b*c)/b^3/(a*d-b*c)^2/(b*x+a)+1/(a*d-b*c)^3*(-A*b^3*
d^2+B*b^3*c*d-C*b^3*c^2+D*a^3*d^2-3*D*a^2*b*c*d+3*D*a*b^2*c^2)/b^3*ln(b*x+
a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(227) = 454$.

Time = 0.12 (sec) , antiderivative size = 695, normalized size of antiderivative = 3.03

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)}\,dx = -\frac{(5\,Da^3b^2 - 3\,Ca^2b^3 + Bab^4 + Ab^5)c^2d - 4\,(2\,Da^4b - Ca^3b^2 + Aab^4)cd^2 + (3\,Da^5 - Ca^4b - Ba^3b^2 + 3}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*((5*D*a^3*b^2 - 3*C*a^2*b^3 + B*a*b^4 + A*b^5)*c^2*d - 4*(2*D*a^4*b -
 C*a^3*b^2 + A*a*b^4)*c*d^2 + (3*D*a^5 - C*a^4*b - B*a^3*b^2 + 3*A*a^2*b^3
)*d^3 + 2*((3*D*a^2*b^3 - 2*C*a*b^4 + B*b^5)*c^2*d - (5*D*a^3*b^2 - 3*C*a^
2*b^3 + B*a*b^4 + A*b^5)*c*d^2 + (2*D*a^4*b - C*a^3*b^2 + A*a*b^4)*d^3)*x
+ 2*((3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - (3*D*a^4*b - B*a^2*b^3)*c*d^2 + (D*
a^5 - A*a^2*b^3)*d^3 + ((3*D*a*b^4 - C*b^5)*c^2*d - (3*D*a^2*b^3 - B*b^5)*
c*d^2 + (D*a^3*b^2 - A*b^5)*d^3)*x^2 + 2*((3*D*a^2*b^3 - C*a*b^4)*c^2*d -
(3*D*a^3*b^2 - B*a*b^4)*c*d^2 + (D*a^4*b - A*a*b^4)*d^3)*x)*log(b*x + a) -
 2*(D*a^2*b^3*c^3 - C*a^2*b^3*c^2*d + B*a^2*b^3*c*d^2 - A*a^2*b^3*d^3 + (D
*b^5*c^3 - C*b^5*c^2*d + B*b^5*c*d^2 - A*b^5*d^3)*x^2 + 2*(D*a*b^4*c^3 - C
*a*b^4*c^2*d + B*a*b^4*c*d^2 - A*a*b^4*d^3)*x)*log(d*x + c))/(a^2*b^6*c^3*
d - 3*a^3*b^5*c^2*d^2 + 3*a^4*b^4*c*d^3 - a^5*b^3*d^4 + (b^8*c^3*d - 3*a*b
^7*c^2*d^2 + 3*a^2*b^6*c*d^3 - a^3*b^5*d^4)*x^2 + 2*(a*b^7*c^3*d - 3*a^2*b
^6*c^2*d^2 + 3*a^3*b^5*c*d^3 - a^4*b^4*d^4)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1306 vs. 2(211) = 422.

Time = 146.57 (sec) , antiderivative size = 1306, normalized size of antiderivative = 5.70

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c),x)
```

output
```
(3*A*a*b**3*d - A*b**4*c - B*a**2*b**2*d - B*a*b**3*c - C*a**3*b*d + 3*C*a
**2*b**2*c + 3*D*a**4*d - 5*D*a**3*b*c + x*(2*A*b**4*d - 2*B*b**4*c - 2*C*
a**2*b**2*d + 4*C*a*b**3*c + 4*D*a**3*b*d - 6*D*a**2*b**2*c))/(2*a**4*b**3
*d**2 - 4*a**3*b**4*c*d + 2*a**2*b**5*c**2 + x**2*(2*a**2*b**5*d**2 - 4*a*
b**6*c*d + 2*b**7*c**2) + x*(4*a**3*b**4*d**2 - 8*a**2*b**5*c*d + 4*a*b**6
*c**2)) - (-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)*log(x + (-A*a*b**2*d**3
 - A*b**3*c*d**2 + B*a*b**2*c*d**2 + B*b**3*c**2*d - C*a*b**2*c**2*d - C*b
**3*c**3 + D*a**3*c*d**2 - 3*D*a**2*b*c**2*d + 4*D*a*b**2*c**3 - a**4*b**2
*d**3*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**3 + 4*a**3*b**
3*c*d**2*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**3 - 6*a**2*
b**4*c**2*d*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**3 + 4*a*
b**5*c**3*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**3 - b**6*c
**4*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(d*(a*d - b*c)**3))/(-2*A*b**
3*d**3 + 2*B*b**3*c*d**2 - 2*C*b**3*c**2*d + D*a**3*d**3 - 3*D*a**2*b*c*d*
*2 + 3*D*a*b**2*c**2*d + D*b**3*c**3))/(d*(a*d - b*c)**3) + (-A*b**3*d**2
+ B*b**3*c*d - C*b**3*c**2 + D*a**3*d**2 - 3*D*a**2*b*c*d + 3*D*a*b**2*c**
2)*log(x + (-A*a*b**2*d**3 - A*b**3*c*d**2 + B*a*b**2*c*d**2 + B*b**3*c**2
*d - C*a*b**2*c**2*d - C*b**3*c**3 + D*a**3*c*d**2 - 3*D*a**2*b*c**2*d + 4
*D*a*b**2*c**3 + a**4*d**4*(-A*b**3*d**2 + B*b**3*c*d - C*b**3*c**2 + D*a*
*3*d**2 - 3*D*a**2*b*c*d + 3*D*a*b**2*c**2)/(b*(a*d - b*c)**3) - 4*a**3...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx$$
$$= -\frac{((3\,Dab^2-Cb^3)c^2-(3\,Da^2b-Bb^3)cd+(Da^3-Ab^3)d^2)\log(bx+a)}{b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3}$$
$$+\frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(dx+c)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4}$$
$$-\frac{(5\,Da^3b-3\,Ca^2b^2+Bab^3+Ab^4)c-(3\,Da^4-Ca^3b-Ba^2b^2+3\,Aab^3)d+2\,((3\,Da^2b^2-2\,Cab^3+B}{2\,(a^2b^5c^2-2\,a^3b^4cd+a^4b^3d^2+(b^7c^2-2\,ab^6cd+a^2b^5d^2)x^2+2\,(ab^6c^2-2\,a^2b^5c}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-((3*D*a*b^2 - C*b^3)*c^2 - (3*D*a^2*b - B*b^3)*c*d + (D*a^3 - A*b^3)*d^2)
*log(b*x + a)/(b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3) +
(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(d*x + c)/(b^3*c^3*d - 3*a*b^2*c^2*
d^2 + 3*a^2*b*c*d^3 - a^3*d^4) - 1/2*((5*D*a^3*b - 3*C*a^2*b^2 + B*a*b^3 +
 A*b^4)*c - (3*D*a^4 - C*a^3*b - B*a^2*b^2 + 3*A*a*b^3)*d + 2*((3*D*a^2*b^
2 - 2*C*a*b^3 + B*b^4)*c - (2*D*a^3*b - C*a^2*b^2 + A*b^4)*d)*x)/(a^2*b^5*
c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2 + (b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*
x^2 + 2*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx$$
$$= -\frac{(3\,Dab^2c^2-Cb^3c^2-3\,Da^2bcd+Bb^3cd+Da^3d^2-Ab^3d^2)\log(|bx+a|)}{b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3}$$
$$+\frac{(Dc^3-Cc^2d+Bcd^2-Ad^3)\log(|dx+c|)}{b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4}$$
$$-\frac{2\,(3\,Da^2b^2c^2-2\,Cab^3c^2+Bb^4c^2-5\,Da^3bcd+3\,Ca^2b^2cd-Bab^3cd-Ab^4cd+2\,Da^4d^2-Ca^3bd^2+A}{2\,(bc-ad)^3(b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
-(3*D*a*b^2*c^2 - C*b^3*c^2 - 3*D*a^2*b*c*d + B*b^3*c*d + D*a^3*d^2 - A*b^
3*d^2)*log(abs(b*x + a))/(b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*
b^3*d^3) + (D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(abs(d*x + c))/(b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4) - 1/2*(2*(3*D*a^2*b^2*c^2 -
 2*C*a*b^3*c^2 + B*b^4*c^2 - 5*D*a^3*b*c*d + 3*C*a^2*b^2*c*d - B*a*b^3*c*d
 - A*b^4*c*d + 2*D*a^4*d^2 - C*a^3*b*d^2 + A*a*b^3*d^2)*x + (5*D*a^3*b^2*c
^2 - 3*C*a^2*b^3*c^2 + B*a*b^4*c^2 + A*b^5*c^2 - 8*D*a^4*b*c*d + 4*C*a^3*b
^2*c*d - 4*A*a*b^4*c*d + 3*D*a^5*d^2 - C*a^4*b*d^2 - B*a^3*b^2*d^2 + 3*A*a
^2*b^3*d^2)/b)/((b*c - a*d)^3*(b*x + a)^2*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{(a+b\,x)}^3\,(c+d\,x)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)}\,dx$$
$$=\frac{2\log(bx+a)\,a^5d^2+a^5d^2-4\log(bx+a)\,a^4bcd+4\log(bx+a)\,a^4b\,d^2x+2\log(bx+a)\,a^3b^2d^2x^2-4\log(}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c),x)
```

output
```
(2*log(a + b*x)*a**5*d**2 - 4*log(a + b*x)*a**4*b*c*d + 4*log(a + b*x)*a**
4*b*d**2*x - 2*log(a + b*x)*a**3*b**3*d + 2*log(a + b*x)*a**3*b**2*c**2 -
8*log(a + b*x)*a**3*b**2*c*d*x + 2*log(a + b*x)*a**3*b**2*d**2*x**2 - 4*lo
g(a + b*x)*a**2*b**4*d*x + 4*log(a + b*x)*a**2*b**3*c**2*x - 4*log(a + b*x
)*a**2*b**3*c*d*x**2 - 2*log(a + b*x)*a*b**5*d*x**2 + 2*log(a + b*x)*a*b**
4*c**2*x**2 + 2*log(c + d*x)*a**3*b**3*d + 4*log(c + d*x)*a**2*b**4*d*x +
2*log(c + d*x)*a*b**5*d*x**2 + a**5*d**2 - 2*a**4*b*c*d + a**3*b**3*d + a*
*3*b**2*c**2 - 2*a**3*b**2*d**2*x**2 - a**2*b**4*c + 4*a**2*b**3*c*d*x**2
- a*b**5*d*x**2 - 2*a*b**4*c**2*x**2 + b**6*c*x**2)/(2*a*b**3*(a**4*d**2 -
 2*a**3*b*c*d + 2*a**3*b*d**2*x + a**2*b**2*c**2 - 4*a**2*b**2*c*d*x + a**
2*b**2*d**2*x**2 + 2*a*b**3*c**2*x - 2*a*b**3*c*d*x**2 + b**4*c**2*x**2))
```

3.33 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4(c+dx)}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 273

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4(c+dx)}\,dx = \frac{-Ab^3+a(b^2B-abC+a^2D)}{3b^3(bc-ad)(a+bx)^3} + \frac{2ab^2cC-b^3(Bc-Ad)+2a^3dD-a^2b(Cd+3cD)}{2b^3(bc-ad)^2(a+bx)^2} - \frac{b^3(c^2C-Bcd+Ad^2)-3ab^2c^2D+3a^2bcdD-a^3d^2D}{b^3(bc-ad)^3(a+bx)} - \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(a+bx)}{(bc-ad)^4} + \frac{(c^2Cd-Bcd^2+Ad^3-c^3D)\log(c+dx)}{(bc-ad)^4}$$

output
```
1/3*(-A*b^3+a*(B*b^2-C*a*b+D*a^2))/b^3/(-a*d+b*c)/(b*x+a)^3+1/2*(2*a*b^2*c
*C-b^3*(-A*d+B*c)+2*a^3*d*D-a^2*b*(C*d+3*D*c))/b^3/(-a*d+b*c)^2/(b*x+a)^2-
(b^3*(A*d^2-B*c*d+C*c^2)-3*a*b^2*c^2*D+3*a^2*b*c*d*D-a^3*d^2*D)/b^3/(-a*d+
b*c)^3/(b*x+a)-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*ln(b*x+a)/(-a*d+b*c)^4+(A*d^3
-B*c*d^2+C*c^2*d-D*c^3)*ln(d*x+c)/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4(c+dx)}\,dx = \frac{\frac{2(bc-ad)^3(-Ab^3+a(b^2B-abC+a^2D))}{b^3(a+bx)^3} - \frac{3(bc-ad)^2(-2ab^2cC+b^3(Bc-Ad)-2a^3dD+a^2b(Cd+3cD))}{b^3(a+bx)^2} - \frac{6(bc-ad)(b^3(c^2C-Bcd+Ad^2)-\cdots}{b^3(a+\cdots}}{6\cdots}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^4*(c + d*x)),x]
```

output
```
((2*(b*c - a*d)^3*(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D)))/(b^3*(a + b*x)^3
) - (3*(b*c - a*d)^2*(-2*a*b^2*c*C + b^3*(B*c - A*d) - 2*a^3*d*D + a^2*b*(
C*d + 3*c*D)))/(b^3*(a + b*x)^2) - (6*(b*c - a*d)*(b^3*(c^2*C - B*c*d + A*
d^2) - 3*a*b^2*c^2*D + 3*a^2*b*c*d*D - a^3*d^2*D))/(b^3*(a + b*x)) + 6*(-(
c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*Log[a + b*x] + 6*(c^2*C*d - B*c*d^2 +
A*d^3 - c^3*D)*Log[c + d*x])/(6*(b*c - a*d)^4)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4(c+dx)}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{Ab^3-a(a^2D-abC+b^2B)}{b^2(a+bx)^4(bc-ad)} + \frac{a^3(-d^2)D+3a^2bcdD-3ab^2c^2D+b^3(Ad^2-Bcd+c^2C)}{b^2(a+bx)^2(bc-ad)^3} + \frac{-2a^3dD+a^2\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{3b^3(a+bx)^3(bc-ad)}-\frac{a^3\left(-d^2\right)D+3a^2bcdD-3ab^2c^2D+b^3\left(Ad^2-Bcd+c^2C\right)}{b^3(a+bx)(bc-ad)^3}+\frac{2a^3dD-a^2b(3cD+Cd)+2ab^2cC-b^3(Bc-Ad)}{2b^3(a+bx)^2(bc-ad)^2}-\frac{\log(a+bx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{(bc-ad)^4}+\frac{\log(c+dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{(bc-ad)^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^4*(c + d*x)),x]
```

output
```
-1/3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b^3*(b*c - a*d)*(a + b*x)^3) + (
2*a*b^2*c*C - b^3*(B*c - A*d) + 2*a^3*d*D - a^2*b*(C*d + 3*c*D))/(2*b^3*(b
*c - a*d)^2*(a + b*x)^2) - (b^3*(c^2*C - B*c*d + A*d^2) - 3*a*b^2*c^2*D +
3*a^2*b*c*d*D - a^3*d^2*D)/(b^3*(b*c - a*d)^3*(a + b*x)) - ((c^2*C*d - B*c
*d^2 + A*d^3 - c^3*D)*Log[a + b*x])/(b*c - a*d)^4 + ((c^2*C*d - B*c*d^2 +
A*d^3 - c^3*D)*Log[c + d*x])/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.01

method	result
default	$\frac{\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)\ln(xd+c)}{(ad-bc)^4}-\frac{-b^3A+a\,b^2B-a^2bC+a^3D}{3b^3(ad-bc)(bx+a)^3}-\frac{-b^3dA+B\,b^3c+C\,a^2bd-2a\,b^2cC-2a^3dD+3a^2bcD}{2b^3(ad-bc)^2(bx+a)^2}$
norman	$\frac{\frac{\left(b^3d^2A-B\,b^3cd+C\,b^3c^2-a^3d^2D+3a^2bcdD-3a\,b^2c^2D\right)x^2}{b\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{11A\,a^2b^3d^2-7Aa\,b^4cd+2A\,b^5c^2-2B\,a^3b^2d^2-5B\,a^2b^3cd+Ba\,b^4c^2-C\,a^4b\,d}{6b^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2\right.}}{}$
parallelrisch	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^4*ln(d*x+c)-1/3*(-A*b^3+B*a*b^2-C*
a^2*b+D*a^3)/b^3/(a*d-b*c)/(b*x+a)^3-1/2*(-A*b^3*d+B*b^3*c+C*a^2*b*d-2*C*a
*b^2*c-2*D*a^3*d+3*D*a^2*b*c)/b^3/(a*d-b*c)^2/(b*x+a)^2-(-A*b^3*d^2+B*b^3*
c*d-C*b^3*c^2+D*a^3*d^2-3*D*a^2*b*c*d+3*D*a*b^2*c^2)/(a*d-b*c)^3/b^3/(b*x+
a)-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^4*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. $2(268) = 536$.

Time = 0.10 (sec) , antiderivative size = 1031, normalized size of antiderivative = 3.78

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^4(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c),x, algorithm="fricas")
```

output
```
1/6*((11*D*a^3*b^3 - 2*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c^3 - 3*(6*D*a^4*b^2
 + C*a^3*b^3 - 2*B*a^2*b^4 - 3*A*a*b^5)*c^2*d + 3*(3*D*a^5*b + 2*C*a^4*b^2
 - B*a^3*b^3 - 6*A*a^2*b^4)*c*d^2 - (2*D*a^6 + C*a^5*b + 2*B*a^4*b^2 - 11*
A*a^3*b^3)*d^3 + 6*((3*D*a*b^5 - C*b^6)*c^3 - (6*D*a^2*b^4 - C*a*b^5 - B*b
^6)*c^2*d + (4*D*a^3*b^3 - B*a*b^5 - A*b^6)*c*d^2 - (D*a^4*b^2 - A*a*b^5)*
d^3)*x^2 + 3*((9*D*a^2*b^4 - 2*C*a*b^5 - B*b^6)*c^3 - (16*D*a^3*b^3 + C*a^
2*b^4 - 6*B*a*b^5 - A*b^6)*c^2*d + (9*D*a^4*b^2 + 4*C*a^3*b^3 - 5*B*a^2*b^
4 - 6*A*a*b^5)*c*d^2 - (2*D*a^5*b + C*a^4*b^2 - 5*A*a^2*b^4)*d^3)*x + 6*(D
*a^3*b^3*c^3 - C*a^3*b^3*c^2*d + B*a^3*b^3*c*d^2 - A*a^3*b^3*d^3 + (D*b^6*
c^3 - C*b^6*c^2*d + B*b^6*c*d^2 - A*b^6*d^3)*x^3 + 3*(D*a*b^5*c^3 - C*a*b^
5*c^2*d + B*a*b^5*c*d^2 - A*a*b^5*d^3)*x^2 + 3*(D*a^2*b^4*c^3 - C*a^2*b^4*
c^2*d + B*a^2*b^4*c*d^2 - A*a^2*b^4*d^3)*x)*log(b*x + a) - 6*(D*a^3*b^3*c^
3 - C*a^3*b^3*c^2*d + B*a^3*b^3*c*d^2 - A*a^3*b^3*d^3 + (D*b^6*c^3 - C*b^6
*c^2*d + B*b^6*c*d^2 - A*b^6*d^3)*x^3 + 3*(D*a*b^5*c^3 - C*a*b^5*c^2*d + B
*a*b^5*c*d^2 - A*a*b^5*d^3)*x^2 + 3*(D*a^2*b^4*c^3 - C*a^2*b^4*c^2*d + B*a
^2*b^4*c*d^2 - A*a^2*b^4*d^3)*x)*log(d*x + c))/(a^3*b^7*c^4 - 4*a^4*b^6*c^
3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4 + (b^10*c^4 - 4*a*
b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*x^3 + 3*(a*
b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*
d^4)*x^2 + 3*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(248) = 496$.

Time = 28.97 (sec) , antiderivative size = 1360, normalized size of antiderivative = 4.98

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^4(c + dx)}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**4/(d*x+c),x)
```

output
```
(11*A*a**2*b**3*d**2 - 7*A*a*b**4*c*d + 2*A*b**5*c**2 - 2*B*a**3*b**2*d**2
 - 5*B*a**2*b**3*c*d + B*a*b**4*c**2 - C*a**4*b*d**2 + 5*C*a**3*b**2*c*d +
 2*C*a**2*b**3*c**2 - 2*D*a**5*d**2 + 7*D*a**4*b*c*d - 11*D*a**3*b**2*c**2
 + x**2*(6*A*b**5*d**2 - 6*B*b**5*c*d + 6*C*b**5*c**2 - 6*D*a**3*b**2*d**2
 + 18*D*a**2*b**3*c*d - 18*D*a*b**4*c**2) + x*(15*A*a*b**4*d**2 - 3*A*b**5
*c*d - 15*B*a*b**4*c*d + 3*B*b**5*c**2 - 3*C*a**3*b**2*d**2 + 9*C*a**2*b**
3*c*d + 6*C*a*b**4*c**2 - 6*D*a**4*b*d**2 + 21*D*a**3*b**2*c*d - 27*D*a**2
*b**3*c**2))/(6*a**6*b**3*d**3 - 18*a**5*b**4*c*d**2 + 18*a**4*b**5*c**2*d
 - 6*a**3*b**6*c**3 + x**3*(6*a**3*b**6*d**3 - 18*a**2*b**7*c*d**2 + 18*a*
b**8*c**2*d - 6*b**9*c**3) + x**2*(18*a**4*b**5*d**3 - 54*a**3*b**6*c*d**2
 + 54*a**2*b**7*c**2*d - 18*a*b**8*c**3) + x*(18*a**5*b**4*d**3 - 54*a**4*
b**5*c*d**2 + 54*a**3*b**6*c**2*d - 18*a**2*b**7*c**3)) - (-A*d**3 + B*c*d
**2 - C*c**2*d + D*c**3)*log(x + (-A*a*d**4 - A*b*c*d**3 + B*a*c*d**3 + B*
b*c**2*d**2 - C*a*c**2*d**2 - C*b*c**3*d + D*a*c**3*d + D*b*c**4 - a**5*d*
*5*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**4 + 5*a**4*b*c*d*
*4*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**4 - 10*a**3*b**2*
c**2*d**3*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**4 + 10*a**
2*b**3*c**3*d**2*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**4 -
 5*a*b**4*c**4*d*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**4 +
 b**5*c**5*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(a*d - b*c)**4)/(-2...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(268) = 536$.

Time = 0.06 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.30

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^4(c + dx)}\,dx = \frac{(Dc^3 - Cc^2d + Bcd^2 - Ad^3)\log(bx + a)}{b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4} - \frac{(Dc^3 - Cc^2d + Bcd^2 - Ad^3)\log(dx + c)}{b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4} + \frac{(11\,Da^3b^2 - 2\,Ca^2b^3 - Bab^4 - 2\,Ab^5)c^2 - (7\,Da^4b + 5\,Ca^3b^2 - 5\,Ba^2b^3 - 7\,Aab^4)cd + (2\,Da^5 + Ca^4b}{6\,(a^3b^6c^3 - 3\,a^4b^5c^2d + 3\,a^5b^4cd^2 - a^6b^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c),x, algorithm="maxima")
```

output
```
(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - (D*c^3 - C*c^2*d + B*c*d^
2 - A*d^3)*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4) + 1/6*((11*D*a^3*b^2 - 2*C*a^2*b^3 - B*a*b^4 - 2*A*b
^5)*c^2 - (7*D*a^4*b + 5*C*a^3*b^2 - 5*B*a^2*b^3 - 7*A*a*b^4)*c*d + (2*D*a
^5 + C*a^4*b + 2*B*a^3*b^2 - 11*A*a^2*b^3)*d^2 + 6*((3*D*a*b^4 - C*b^5)*c^
2 - (3*D*a^2*b^3 - B*b^5)*c*d + (D*a^3*b^2 - A*b^5)*d^2)*x^2 + 3*((9*D*a^2
*b^3 - 2*C*a*b^4 - B*b^5)*c^2 - (7*D*a^3*b^2 + 3*C*a^2*b^3 - 5*B*a*b^4 - A
*b^5)*c*d + (2*D*a^4*b + C*a^3*b^2 - 5*A*a*b^4)*d^2)*x)/(a^3*b^6*c^3 - 3*a
^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3 + (b^9*c^3 - 3*a*b^8*c^2*d +
3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*x^3 + 3*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^
3*b^6*c*d^2 - a^4*b^5*d^3)*x^2 + 3*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*
b^5*c*d^2 - a^5*b^4*d^3)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(268) = 536$.

Time = 0.13 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.45

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^4(c + dx)}\,dx = \frac{(Dbc^3 - Cbc^2d + Bbcd^2 - Abd^3)\log(|bx + a|)}{b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4} - \frac{(Dc^3d - Cc^2d^2 + Bcd^3 - Ad^4)\log(|dx + c|)}{b^4c^4d - 4\,ab^3c^3d^2 + 6\,a^2b^2c^2d^3 - 4\,a^3bcd^4 + a^4d^5} + \frac{11\,Da^3b^3c^3 - 2\,Ca^2b^4c^3 - Bab^5c^3 - 2\,Ab^6c^3 - 18\,Da^4b^2c^2d - 3\,Ca^3b^3c^2d + 6\,Ba^2b^4c^2d + 9\,Aab^5c^2d +}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c),x, algorithm="giac")
```

output
```
(D*b*c^3 - C*b*c^2*d + B*b*c*d^2 - A*b*d^3)*log(abs(b*x + a))/(b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - (D*c^3*d
 - C*c^2*d^2 + B*c*d^3 - A*d^4)*log(abs(d*x + c))/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) + 1/6*(11*D*a^3*b^3*c^
3 - 2*C*a^2*b^4*c^3 - B*a*b^5*c^3 - 2*A*b^6*c^3 - 18*D*a^4*b^2*c^2*d - 3*C
*a^3*b^3*c^2*d + 6*B*a^2*b^4*c^2*d + 9*A*a*b^5*c^2*d + 9*D*a^5*b*c*d^2 + 6
*C*a^4*b^2*c*d^2 - 3*B*a^3*b^3*c*d^2 - 18*A*a^2*b^4*c*d^2 - 2*D*a^6*d^3 -
C*a^5*b*d^3 - 2*B*a^4*b^2*d^3 + 11*A*a^3*b^3*d^3 + 6*(3*D*a*b^5*c^3 - C*b^
6*c^3 - 6*D*a^2*b^4*c^2*d + C*a*b^5*c^2*d + B*b^6*c^2*d + 4*D*a^3*b^3*c*d^
2 - B*a*b^5*c*d^2 - A*b^6*c*d^2 - D*a^4*b^2*d^3 + A*a*b^5*d^3)*x^2 + 3*(9*
D*a^2*b^4*c^3 - 2*C*a*b^5*c^3 - B*b^6*c^3 - 16*D*a^3*b^3*c^2*d - C*a^2*b^4
*c^2*d + 6*B*a*b^5*c^2*d + A*b^6*c^2*d + 9*D*a^4*b^2*c*d^2 + 4*C*a^3*b^3*c
*d^2 - 5*B*a^2*b^4*c*d^2 - 6*A*a*b^5*c*d^2 - 2*D*a^5*b*d^3 - C*a^4*b^2*d^3
 + 5*A*a^2*b^4*d^3)*x)/((b*c - a*d)^4*(b*x + a)^3*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4(c+dx)}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^4\,(c+d\,x)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^4*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^4*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.72

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4(c+dx)}\,dx$$
$$=\frac{-6\log(bx+a)\,a^4d^2-18\log(bx+a)\,a^3b\,d^2x-18\log(bx+a)\,a^2b^2d^2x^2-6\log(bx+a)\,a\,b^3d^2x^3+6\log(}{6a\,(a^3b^3d^3x^3-3a^2b^4c\,d^2x^3+3a\,b^5c^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c),x)
```

output
```
( - 6*log(a + b*x)*a**4*d**2 - 18*log(a + b*x)*a**3*b*d**2*x - 18*log(a +
b*x)*a**2*b**2*d**2*x**2 - 6*log(a + b*x)*a*b**3*d**2*x**3 + 6*log(c + d*x
)*a**4*d**2 + 18*log(c + d*x)*a**3*b*d**2*x + 18*log(c + d*x)*a**2*b**2*d*
*2*x**2 + 6*log(c + d*x)*a*b**3*d**2*x**3 + 7*a**4*d**2 - 10*a**3*b*c*d +
9*a**3*b*d**2*x + 2*a**3*d**3*x**3 + 3*a**2*b**2*c**2 - 12*a**2*b**2*c*d*x
 - 6*a**2*b*c*d**2*x**3 + 3*a*b**3*c**2*x - 2*a*b**3*d**2*x**3 + 6*a*b**2*
c**2*d*x**3 + 2*b**4*c*d*x**3 - 2*b**3*c**3*x**3)/(6*a*(a**6*d**3 - 3*a**5
*b*c*d**2 + 3*a**5*b*d**3*x + 3*a**4*b**2*c**2*d - 9*a**4*b**2*c*d**2*x +
3*a**4*b**2*d**3*x**2 - a**3*b**3*c**3 + 9*a**3*b**3*c**2*d*x - 9*a**3*b**
3*c*d**2*x**2 + a**3*b**3*d**3*x**3 - 3*a**2*b**4*c**3*x + 9*a**2*b**4*c**
2*d*x**2 - 3*a**2*b**4*c*d**2*x**3 - 3*a*b**5*c**3*x**2 + 3*a*b**5*c**2*d*
x**3 - b**6*c**3*x**3))
```

3.34 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 410

$$\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx =$$
$$-\frac{(bc-ad)\left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)x}{d^6}$$
$$+\frac{(bc-ad)^3\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)}{d^7(c+dx)}$$
$$+\frac{\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D)\right)}{2d^7}$$
$$+\frac{b(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D))(c+dx)^3}{3d^7}$$
$$+\frac{b^2(bCd-6bcD+3adD)(c+dx)^4}{4d^7}+\frac{b^3D(c+dx)^5}{5d^7}$$
$$-\frac{(bc-ad)^2\left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right)\log(c+dx)}{d^7}$$

output
```
-(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^2+8*C*c*d-15*D*c^2)+b^2*(3*
A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*x/d^6+(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*
c^2*d-D*c^3)/d^7/(d*x+c)+1/2*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*
(-B*d^2+4*C*c*d-10*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+
c)^2/d^7+1/3*b*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c
^2))*(d*x+c)^3/d^7+1/4*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^4/d^7+1/5*b^3*D
*(d*x+c)^5/d^7-(-a*d+b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c
*d^2+5*C*c^2*d-6*D*c^3))*ln(d*x+c)/d^7
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} \, dx$$

$$= \frac{60d(a^3d^3(Cd-2cD)+3a^2bd^2(-2cCd+Bd^2+3c^2D)+3ab^2d(3c^2Cd-2Bcd^2+Ad^3-4c^3D)+b^3c(-$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
(60*d*(a^3*d^3*(C*d - 2*c*D) + 3*a^2*b*d^2*(-2*c*C*d + B*d^2 + 3*c^2*D) +
3*a*b^2*d*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D) + b^3*c*(-4*c^2*C*d +
3*B*c*d^2 - 2*A*d^3 + 5*c^3*D))*x + 30*d^2*(a^3*d^3*D + 3*a^2*b*d^2*(C*d -
 2*c*D) + 3*a*b^2*d*(-2*c*C*d + B*d^2 + 3*c^2*D) + b^3*(3*c^2*C*d - 2*B*c*
d^2 + A*d^3 - 4*c^3*D))*x^2 + 20*b*d^3*(3*a^2*d^2*D + 3*a*b*d*(C*d - 2*c*D
) + b^2*(-2*c*C*d + B*d^2 + 3*c^2*D))*x^3 + 15*b^2*d^4*(b*C*d - 2*b*c*D +
3*a*d*D)*x^4 + 12*b^3*d^5*D*x^5 - (60*(b*c - a*d)^3*(-(c^2*C*d) + B*c*d^2
- A*d^3 + c^3*D))/(c + d*x) - 60*(b*c - a*d)^2*(-(a*d*(-2*c*C*d + B*d^2 +
3*c^2*D)) + b*(-5*c^2*C*d + 4*B*c*d^2 - 3*A*d^3 + 6*c^3*D))*Log[c + d*x])/
(60*d^7)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad) \left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)\right)}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{x(bc-ad) \left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)}{d^6}+\frac{b(c+dx)^3 \left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{3d^7}+\frac{(c+dx)^2 \left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2Cd\right.\right.}{2d^7}$$

$$\frac{(bc-ad)^3 \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^7(c+dx)}-\frac{(bc-ad)^2 \log(c+dx) \left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{d^7}+\frac{b^2(c+dx)^4(3adD-6bcD+bCd)}{4d^7}+\frac{b^3D(c+dx)^5}{5d^7}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
-(((b*c - a*d)*(a^2*d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*
D) + b^2*(10*c^2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*x)/d^6) + ((b*c -
a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^7*(c + d*x)) + ((a^3*d^3*D
+ 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*D) + b^3
*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^2)/(2*d^7) + (b*(3
*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^2 - 15*c^2*D))*(c
+ d*x)^3)/(3*d^7) + (b^2*(b*C*d - 6*b*c*D + 3*a*d*D)*(c + d*x)^4)/(4*d^7)
+ (b^3*D*(c + d*x)^5)/(5*d^7) - ((b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c
^2*D) - b*(5*c^2*C*d - 4*B*c*d^2 + 3*A*d^3 - 6*c^3*D))*Log[c + d*x])/d^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.80

method	result
norman	$\frac{\frac{\left(A a^3 d^6-3A a^2 bc d^5+6Aa b^2 c^2 d^4-3A b^3 c^3 d^3-B a^3 c d^5+6B a^2 b c^2 d^4-9Ba b^2 c^3 d^3+4B b^3 c^4 d^2+2C a^3 c^2 d^4-9C a^2 b c^3 d^3+12Ca b^2 c^4 d^2}{d^6 c}}{}$
default	$\frac{-2Da b^2 c d^3 x^3-3Ca b^2 c d^3 x^2-3Da^2 bc d^3 x^2+\frac{9}{2}Da b^2 c^2 d^2 x^2+3a b^2 A d^4 x-2A b^3 c d^3 x+3a^2 bB d^4 x+3B b^3 c^2 d^2 x-4C b^3 c^3 dx}{}$
parallelrisch	Expression too large to display

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
((A*a^3*d^6-3*A*a^2*b*c*d^5+6*A*a*b^2*c^2*d^4-3*A*b^3*c^3*d^3-B*a^3*c*d^5+
6*B*a^2*b*c^2*d^4-9*B*a*b^2*c^3*d^3+4*B*b^3*c^4*d^2+2*C*a^3*c^2*d^4-9*C*a^
2*b*c^3*d^3+12*C*a*b^2*c^4*d^2-5*C*b^3*c^5*d-3*D*a^3*c^3*d^3+12*D*a^2*b*c^
4*d^2-15*D*a*b^2*c^5*d+6*D*b^3*c^6)/d^6/c*x+1/6*(3*A*b^3*d^3+9*B*a*b^2*d^3
-4*B*b^3*c*d^2+9*C*a^2*b*d^3-12*C*a*b^2*c*d^2+5*C*b^3*c^2*d+3*D*a^3*d^3-12
*D*a^2*b*c*d^2+15*D*a*b^2*c^2*d-6*D*b^3*c^3)/d^4*x^3+1/2*(6*A*a*b^2*d^4-3*
A*b^3*c*d^3+6*B*a^2*b*d^4-9*B*a*b^2*c*d^3+4*B*b^3*c^2*d^2+2*C*a^3*d^4-9*C*
a^2*b*c*d^3+12*C*a*b^2*c^2*d^2-5*C*b^3*c^3*d-3*D*a^3*c*d^3+12*D*a^2*b*c^2*
d^2-15*D*a*b^2*c^3*d+6*D*b^3*c^4)/d^5*x^2+1/5*D*b^3/d*x^6+1/12*b*(4*B*b^2*
d^2+12*C*a*b*d^2-5*C*b^2*c*d+12*D*a^2*d^2-15*D*a*b*c*d+6*D*b^2*c^2)/d^3*x^
4+1/20*b^2*(5*C*b*d+15*D*a*d-6*D*b*c)/d^2*x^5)/(d*x+c)+1/d^7*(3*A*a^2*b*d^
5-6*A*a*b^2*c*d^4+3*A*b^3*c^2*d^3+B*a^3*d^5-6*B*a^2*b*c*d^4+9*B*a*b^2*c^2*
d^3-4*B*b^3*c^3*d^2-2*C*a^3*c*d^4+9*C*a^2*b*c^2*d^3-12*C*a*b^2*c^3*d^2+5*C
*b^3*c^4*d+3*D*a^3*c^2*d^3-12*D*a^2*b*c^3*d^2+15*D*a*b^2*c^4*d-6*D*b^3*c^5
)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(403) = 806$.

Time = 0.08 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.18

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output

```
1/60*(12*D*b^3*d^6*x^6 - 60*D*b^3*c^6 - 60*A*a^3*d^6 + 60*(3*D*a*b^2 + C*b
^3)*c^5*d - 60*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 + 60*(D*a^3 + 3*C*a
^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 - 60*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2
*d^4 + 60*(B*a^3 + 3*A*a^2*b)*c*d^5 - 3*(6*D*b^3*c*d^5 - 5*(3*D*a*b^2 + C*
b^3)*d^6)*x^5 + 5*(6*D*b^3*c^2*d^4 - 5*(3*D*a*b^2 + C*b^3)*c*d^5 + 4*(3*D*
a^2*b + 3*C*a*b^2 + B*b^3)*d^6)*x^4 - 10*(6*D*b^3*c^3*d^3 - 5*(3*D*a*b^2 +
 C*b^3)*c^2*d^4 + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^5 - 3*(D*a^3 + 3*C
*a^2*b + 3*B*a*b^2 + A*b^3)*d^6)*x^3 + 30*(6*D*b^3*c^4*d^2 - 5*(3*D*a*b^2
+ C*b^3)*c^3*d^3 + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^4 - 3*(D*a^3 +
3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^5 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d
^6)*x^2 + 60*(5*D*b^3*c^5*d - 4*(3*D*a*b^2 + C*b^3)*c^4*d^2 + 3*(3*D*a^2*b
 + 3*C*a*b^2 + B*b^3)*c^3*d^3 - 2*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*
c^2*d^4 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^5)*x - 60*(6*D*b^3*c^6 - 5*(
3*D*a*b^2 + C*b^3)*c^5*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - 3*(
D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + 2*(C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5 + (6*D*b^3*c^5*d - 5*(3*D*a*b
^2 + C*b^3)*c^4*d^2 + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^3 - 3*(D*a^3
 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^4 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b
^2)*c*d^5 - (B*a^3 + 3*A*a^2*b)*d^6)*x)*log(d*x + c))/(d^8*x + c*d^7)
```

Sympy [A] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \frac{Db^3x^5}{5d^2} + x^4\left(\frac{Cb^3}{4d^2} + \frac{3Dab^2}{4d^2} - \frac{Db^3c}{2d^3}\right) + x^3\left(\frac{Bb^3}{3d^2} + \frac{Cab^2}{d^2} - \frac{2Cb^3c}{3d^3} + \frac{Da^2b}{d^2} - \frac{2Dab^2c}{d^3} + \frac{Db^3c^2}{d^4}\right) + x^2\left(\frac{Ab^3}{2d^2} + \frac{3Bab^2}{2d^2} - \frac{Bb^3c}{d^3} + \frac{3Ca^2b}{2d^2} - \frac{3Cab^2c}{d^3} + \frac{3Cb^3c^2}{2d^4} + \frac{Da^3}{2d^2} - \frac{3Da^2bc}{d^3} + \frac{9Dab^2c^2}{2d^4} - \frac{2Db^3c^3}{d^5}\right) + x\left(\frac{3Aab^2}{d^2} - \frac{2Ab^3c}{d^3} + \frac{3Ba^2b}{d^2} - \frac{6Bab^2c}{d^3} + \frac{3Bb^3c^2}{d^4} + \frac{Ca^3}{d^2} - \frac{6Ca^2bc}{d^3} + \frac{9Cab^2c^2}{d^4} - \frac{4Cb^3c^3}{d^5} - \frac{2Da^3c}{d^3} + \frac{9Da^2bc^2}{d^4} - \frac{12Dab^2c^3}{d^5} + \frac{5Db^3c^4}{d^6}\right) + \frac{-Aa^3d^6 + 3Aa^2bcd^5 - 3Aab^2c^2d^4 + Ab^3c^3d^3 + Ba^3cd^5 - 3Ba^2bc^2d^4 + 3Bab^2c^3d^3 - Bb^3c^4d^2 - Ca^3c^2}{cd^7 + d^8x} + \frac{(ad-bc)^2\cdot(3Abd^3 + Bad^3 - 4Bbcd^2 - 2Cacd^2 + 5Cbc^2d + 3Dac^2d - 6Dbc^3)\log(c+dx)}{d^7}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
D*b**3*x**5/(5*d**2) + x**4*(C*b**3/(4*d**2) + 3*D*a*b**2/(4*d**2) - D*b**
3*c/(2*d**3)) + x**3*(B*b**3/(3*d**2) + C*a*b**2/d**2 - 2*C*b**3*c/(3*d**3
) + D*a**2*b/d**2 - 2*D*a*b**2*c/d**3 + D*b**3*c**2/d**4) + x**2*(A*b**3/(
2*d**2) + 3*B*a*b**2/(2*d**2) - B*b**3*c/d**3 + 3*C*a**2*b/(2*d**2) - 3*C*
a*b**2*c/d**3 + 3*C*b**3*c**2/(2*d**4) + D*a**3/(2*d**2) - 3*D*a**2*b*c/d*
*3 + 9*D*a*b**2*c**2/(2*d**4) - 2*D*b**3*c**3/d**5) + x*(3*A*a*b**2/d**2 -
 2*A*b**3*c/d**3 + 3*B*a**2*b/d**2 - 6*B*a*b**2*c/d**3 + 3*B*b**3*c**2/d**
4 + C*a**3/d**2 - 6*C*a**2*b*c/d**3 + 9*C*a*b**2*c**2/d**4 - 4*C*b**3*c**3
/d**5 - 2*D*a**3*c/d**3 + 9*D*a**2*b*c**2/d**4 - 12*D*a*b**2*c**3/d**5 + 5
*D*b**3*c**4/d**6) + (-A*a**3*d**6 + 3*A*a**2*b*c*d**5 - 3*A*a*b**2*c**2*d
**4 + A*b**3*c**3*d**3 + B*a**3*c*d**5 - 3*B*a**2*b*c**2*d**4 + 3*B*a*b**2
*c**3*d**3 - B*b**3*c**4*d**2 - C*a**3*c**2*d**4 + 3*C*a**2*b*c**3*d**3 -
3*C*a*b**2*c**4*d**2 + C*b**3*c**5*d + D*a**3*c**3*d**3 - 3*D*a**2*b*c**4*
d**2 + 3*D*a*b**2*c**5*d - D*b**3*c**6)/(c*d**7 + d**8*x) + (a*d - b*c)**2
*(3*A*b*d**3 + B*a*d**3 - 4*B*b*c*d**2 - 2*C*a*c*d**2 + 5*C*b*c**2*d + 3*D
*a*c**2*d - 6*D*b*c**3)*log(c + d*x)/d**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2}\,dx =$$
$$-\frac{Db^3c^6 + Aa^3d^6 - (3\,Dab^2 + Cb^3)c^5d + (3\,Da^2b + 3\,Cab^2 + Bb^3)c^4d^2 - (Da^3 + 3\,Ca^2b + 3\,Bab^2 + Ab}{d^8x + cd^7}$$
$$+\frac{12\,Db^3d^4x^5 - 15\,(2\,Db^3cd^3 - (3\,Dab^2 + Cb^3)d^4)x^4 + 20\,(3\,Db^3c^2d^2 - 2\,(3\,Dab^2 + Cb^3)cd^3 + (3\,Da^2b}{}$$
$$-\frac{(6\,Db^3c^5 - 5\,(3\,Dab^2 + Cb^3)c^4d + 4\,(3\,Da^2b + 3\,Cab^2 + Bb^3)c^3d^2 - 3\,(Da^3 + 3\,Ca^2b + 3\,Bab^2 + Ab^3)}{d^7}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(D*b^3*c^6 + A*a^3*d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b
^2 + B*b^3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C
*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)/(d^8*x
+ c*d^7) + 1/60*(12*D*b^3*d^4*x^5 - 15*(2*D*b^3*c*d^3 - (3*D*a*b^2 + C*b^3
)*d^4)*x^4 + 20*(3*D*b^3*c^2*d^2 - 2*(3*D*a*b^2 + C*b^3)*c*d^3 + (3*D*a^2*
b + 3*C*a*b^2 + B*b^3)*d^4)*x^3 - 30*(4*D*b^3*c^3*d - 3*(3*D*a*b^2 + C*b^3
)*c^2*d^2 + 2*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^3 - (D*a^3 + 3*C*a^2*b +
 3*B*a*b^2 + A*b^3)*d^4)*x^2 + 60*(5*D*b^3*c^4 - 4*(3*D*a*b^2 + C*b^3)*c^3
*d + 3*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^2 - 2*(D*a^3 + 3*C*a^2*b + 3*
B*a*b^2 + A*b^3)*c*d^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^4)*x)/d^6 - (6*
D*b^3*c^5 - 5*(3*D*a*b^2 + C*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3
)*c^3*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 +
 3*B*a^2*b + 3*A*a*b^2)*c*d^4 - (B*a^3 + 3*A*a^2*b)*d^5)*log(d*x + c)/d^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(403) = 806$.

Time = 0.13 (sec) , antiderivative size = 912, normalized size of antiderivative = 2.22

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/60*(12*D*b^3 - 15*(6*D*b^3*c*d - 3*D*a*b^2*d^2 - C*b^3*d^2)/((d*x + c)*d
) + 20*(15*D*b^3*c^2*d^2 - 15*D*a*b^2*c*d^3 - 5*C*b^3*c*d^3 + 3*D*a^2*b*d^
4 + 3*C*a*b^2*d^4 + B*b^3*d^4)/((d*x + c)^2*d^2) - 30*(20*D*b^3*c^3*d^3 -
30*D*a*b^2*c^2*d^4 - 10*C*b^3*c^2*d^4 + 12*D*a^2*b*c*d^5 + 12*C*a*b^2*c*d^
5 + 4*B*b^3*c*d^5 - D*a^3*d^6 - 3*C*a^2*b*d^6 - 3*B*a*b^2*d^6 - A*b^3*d^6)
/((d*x + c)^3*d^3) + 60*(15*D*b^3*c^4*d^4 - 30*D*a*b^2*c^3*d^5 - 10*C*b^3*
c^3*d^5 + 18*D*a^2*b*c^2*d^6 + 18*C*a*b^2*c^2*d^6 + 6*B*b^3*c^2*d^6 - 3*D*
a^3*c*d^7 - 9*C*a^2*b*c*d^7 - 9*B*a*b^2*c*d^7 - 3*A*b^3*c*d^7 + C*a^3*d^8
+ 3*B*a^2*b*d^8 + 3*A*a*b^2*d^8)/((d*x + c)^4*d^4))*(d*x + c)^5/d^7 + (6*D
*b^3*c^5 - 15*D*a*b^2*c^4*d - 5*C*b^3*c^4*d + 12*D*a^2*b*c^3*d^2 + 12*C*a*
b^2*c^3*d^2 + 4*B*b^3*c^3*d^2 - 3*D*a^3*c^2*d^3 - 9*C*a^2*b*c^2*d^3 - 9*B*
a*b^2*c^2*d^3 - 3*A*b^3*c^2*d^3 + 2*C*a^3*c*d^4 + 6*B*a^2*b*c*d^4 + 6*A*a*
b^2*c*d^4 - B*a^3*d^5 - 3*A*a^2*b*d^5)*log(abs(d*x + c)/((d*x + c)^2*abs(d
)))/d^7 - (D*b^3*c^6*d^5/(d*x + c) - 3*D*a*b^2*c^5*d^6/(d*x + c) - C*b^3*c
^5*d^6/(d*x + c) + 3*D*a^2*b*c^4*d^7/(d*x + c) + 3*C*a*b^2*c^4*d^7/(d*x +
c) + B*b^3*c^4*d^7/(d*x + c) - D*a^3*c^3*d^8/(d*x + c) - 3*C*a^2*b*c^3*d^8
/(d*x + c) - 3*B*a*b^2*c^3*d^8/(d*x + c) - A*b^3*c^3*d^8/(d*x + c) + C*a^3
*c^2*d^9/(d*x + c) + 3*B*a^2*b*c^2*d^9/(d*x + c) + 3*A*a*b^2*c^2*d^9/(d*x
+ c) - B*a^3*c*d^10/(d*x + c) - 3*A*a^2*b*c*d^10/(d*x + c) + A*a^3*d^11/(d
*x + c))/d^12
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \int \frac{{(a+b\,x)}^3\,(A+B\,x+C\,x^2+x^3\,D)}{{(c+d\,x)}^2}\,dx$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} \, dx = \frac{240 \log(dx+c) a^3 bc d^5 x - 720 \log(dx+c) a^2 b^2 c^2 d^4 x - 180 \log(dx+c) a^2 b c^4 d^3 x + 720 \log(dx+c) a b^3 c^3}{}$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
(240*log(c + d*x)*a**3*b*c**2*d**4 + 240*log(c + d*x)*a**3*b*c*d**5*x + 60
*log(c + d*x)*a**3*c**4*d**3 + 60*log(c + d*x)*a**3*c**3*d**4*x - 720*log(
c + d*x)*a**2*b**2*c**3*d**3 - 720*log(c + d*x)*a**2*b**2*c**2*d**4*x - 18
0*log(c + d*x)*a**2*b*c**5*d**2 - 180*log(c + d*x)*a**2*b*c**4*d**3*x + 72
0*log(c + d*x)*a*b**3*c**4*d**2 + 720*log(c + d*x)*a*b**3*c**3*d**3*x + 18
0*log(c + d*x)*a*b**2*c**6*d + 180*log(c + d*x)*a*b**2*c**5*d**2*x - 240*l
og(c + d*x)*b**4*c**5*d - 240*log(c + d*x)*b**4*c**4*d**2*x - 60*log(c + d
*x)*b**3*c**7 - 60*log(c + d*x)*b**3*c**6*d*x + 60*a**4*d**6*x - 240*a**3*
b*c*d**5*x - 60*a**3*c**3*d**4*x - 30*a**3*c**2*d**5*x**2 + 30*a**3*c*d**6
*x**3 + 720*a**2*b**2*c**2*d**4*x + 360*a**2*b**2*c*d**5*x**2 + 180*a**2*b
*c**4*d**3*x + 90*a**2*b*c**3*d**4*x**2 - 30*a**2*b*c**2*d**5*x**3 + 60*a*
*2*b*c*d**6*x**4 - 720*a*b**3*c**3*d**3*x - 360*a*b**3*c**2*d**4*x**2 + 12
0*a*b**3*c*d**5*x**3 - 180*a*b**2*c**5*d**2*x - 90*a*b**2*c**4*d**3*x**2 +
 30*a*b**2*c**3*d**4*x**3 - 15*a*b**2*c**2*d**5*x**4 + 45*a*b**2*c*d**6*x*
*5 + 240*b**4*c**4*d**2*x + 120*b**4*c**3*d**3*x**2 - 40*b**4*c**2*d**4*x*
*3 + 20*b**4*c*d**5*x**4 + 60*b**3*c**6*d*x + 30*b**3*c**5*d**2*x**2 - 10*
b**3*c**4*d**3*x**3 + 5*b**3*c**3*d**4*x**4 - 3*b**3*c**2*d**5*x**5 + 12*b
**3*c*d**6*x**6)/(60*c*d**6*(c + d*x))
```

3.35 $\int \frac{(a+bx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2} \, dx$

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Optimal result

Integrand size = 30, antiderivative size = 287

$$\int \frac{(a+bx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2} \, dx = \frac{\left(a^2d^2(Cd-2cD)-2abd(2cCd-Bd^2-3c^2D)+b^2(3c^2Cd-2Bcd^2+Ad^3-4c^3D)\right)x}{d^5} + \frac{\left(a^2d^2D+2abd(Cd-2cD)-b^2(2cCd-Bd^2-3c^2D)\right)x^2}{2d^4} + \frac{b(bCd-2bcD+2adD)x^3}{3d^3} + \frac{b^2Dx^4}{4d^2} - \frac{(bc-ad)^2\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)}{d^6(c+dx)} + \frac{(bc-ad)\left(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D)\right)\log(c+dx)}{d^6}$$

output
```
(a^2*d^2*(C*d-2*D*c)-2*a*b*d*(-B*d^2+2*C*c*d-3*D*c^2)+b^2*(A*d^3-2*B*c*d^2
+3*C*c^2*d-4*D*c^3))*x/d^5+1/2*(a^2*d^2*D+2*a*b*d*(C*d-2*D*c)-b^2*(-B*d^2+
2*C*c*d-3*D*c^2))*x^2/d^4+1/3*b*(C*b*d+2*D*a*d-2*D*b*c)*x^3/d^3+1/4*b^2*D*
x^4/d^2-(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^6/(d*x+c)+(-a*d+b*c)*
(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))*ln(
d*x+c)/d^6
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx$$
$$= \frac{12d(a^2d^2(Cd-2cD)+2abd(-2cCd+Bd^2+3c^2D)+b^2(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,x+6d^2(a^2d^2}{}$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
(12*d*(a^2*d^2*(C*d - 2*c*D) + 2*a*b*d*(-2*c*C*d + B*d^2 + 3*c^2*D) + b^2*
(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*x + 6*d^2*(a^2*d^2*D + 2*a*b*d*
(C*d - 2*c*D) + b^2*(-2*c*C*d + B*d^2 + 3*c^2*D))*x^2 + 4*b*d^3*(b*C*d - 2
*b*c*D + 2*a*d*D)*x^3 + 3*b^2*d^4*D*x^4 + (12*(b*c - a*d)^2*(-(c^2*C*d) +
B*c*d^2 - A*d^3 + c^3*D))/(c + d*x) + 12*(b*c - a*d)*(-(a*d*(-2*c*C*d + B*
d^2 + 3*c^2*D)) + b*(-4*c^2*C*d + 3*B*c*d^2 - 2*A*d^3 + 5*c^3*D))*Log[c +
d*x])/(12*d^6)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{a^2d^2(Cd-2cD)-2abd\left(-Bd^2-3c^2D+2cCd\right)+b^2\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)}{d^5}+\frac{x\left(a^2d^2D+2abd(\right.}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a^2d^2(Cd-2cD)-2abd\left(-Bd^2-3c^2D+2cCd\right)+b^2\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{d^5}+\frac{x^2\left(a^2d^2D+2abd(Cd-2cD)-\left(b^2\left(-Bd^2-3c^2D+2cCd\right)\right)\right)}{2d^4}-\frac{(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^6(c+dx)}+\frac{(bc-ad)\log(c+dx)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{d^6}+\frac{bx^3(2adD-2bcD+bCd)}{3d^3}+\frac{b^2Dx^4}{4d^2}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
((a^2*d^2*(C*d - 2*c*D) - 2*a*b*d*(2*c*C*d - B*d^2 - 3*c^2*D) + b^2*(3*c^2
*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*x)/d^5 + ((a^2*d^2*D + 2*a*b*d*(C*d -
 2*c*D) - b^2*(2*c*C*d - B*d^2 - 3*c^2*D))*x^2)/(2*d^4) + (b*(b*C*d - 2*b*
c*D + 2*a*d*D)*x^3)/(3*d^3) + (b^2*D*x^4)/(4*d^2) - ((b*c - a*d)^2*(c^2*C*
d - B*c*d^2 + A*d^3 - c^3*D))/(d^6*(c + d*x)) + ((b*c - a*d)*(a*d*(2*c*C*d
 - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B*c*d^2 + 2*A*d^3 - 5*c^3*D))*Log[c
 + d*x])/d^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.57

method	result
norman	$\frac{\left(A a^2 d^5-2Aabc d^4+2A b^2 c^2 d^3-B a^2 c d^4+4Bab c^2 d^3-3B b^2 c^3 d^2+2C a^2 c^2 d^3-6Cab c^3 d^2+4C b^2 c^4 d-3Da^2 c^3 d^2+8Dab c^4 d-5Db^2 c^5\right)x}{d^5 c}$
default	$\frac{\frac{1}{4}b^2 Dx^4 d^3+\frac{1}{3}C b^2 d^3 x^3+\frac{2}{3}Dab d^3 x^3-\frac{2}{3}Db^2 c d^2 x^3+\frac{1}{2}B b^2 d^3 x^2+Cab d^3 x^2-C b^2 c d^2 x^2+\frac{1}{2}Da^2 d^3 x^2-2Dabc d^2 x^2+\frac{3}{2}Db^2 c^2}{d^5}$
parallelrisch	$12B a^2 c d^4-24A b^2 c^2 d^3+36B b^2 c^3 d^2-24C a^2 c^2 d^3+36Da^2 c^3 d^2+24Aabc d^4-48Bab c^2 d^3-96Dab c^4 d+72Cab c^3 d^2+12C x^3$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
((A*a^2*d^5-2*A*a*b*c*d^4+2*A*b^2*c^2*d^3-B*a^2*c*d^4+4*B*a*b*c^2*d^3-3*B*
b^2*c^3*d^2+2*C*a^2*c^2*d^3-6*C*a*b*c^3*d^2+4*C*b^2*c^4*d-3*D*a^2*c^3*d^2+
8*D*a*b*c^4*d-5*D*b^2*c^5)/d^5/c*x+1/6*(3*B*b^2*d^2+6*C*a*b*d^2-4*C*b^2*c*
d+3*D*a^2*d^2-8*D*a*b*c*d+5*D*b^2*c^2)/d^3*x^3+1/2*(2*A*b^2*d^3+4*B*a*b*d^
3-3*B*b^2*c*d^2+2*C*a^2*d^3-6*C*a*b*c*d^2+4*C*b^2*c^2*d-3*D*a^2*c*d^2+8*D*
a*b*c^2*d-5*D*b^2*c^3)/d^4*x^2+1/4*D*b^2/d*x^5+1/12*b*(4*C*b*d+8*D*a*d-5*D
*b*c)/d^2*x^4)/(d*x+c)+1/d^6*(2*A*a*b*d^4-2*A*b^2*c*d^3+B*a^2*d^4-4*B*a*b*
c*d^3+3*B*b^2*c^2*d^2-2*C*a^2*c*d^3+6*C*a*b*c^2*d^2-4*C*b^2*c^3*d+3*D*a^2*
c^2*d^2-8*D*a*b*c^3*d+5*D*b^2*c^4)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(282) = 564.

Time = 0.08 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.98

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{3\,Db^2d^5x^5+12\,Db^2c^5-12\,Aa^2d^5-12\,(2\,Dab+Cb^2)c^4d+12\,(Da^2+2\,Cab+Bb^2)c^3d^2-12\,(Ca^2+2}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/12*(3*D*b^2*d^5*x^5 + 12*D*b^2*c^5 - 12*A*a^2*d^5 - 12*(2*D*a*b + C*b^2)
*c^4*d + 12*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - 12*(C*a^2 + 2*B*a*b + A*b^
2)*c^2*d^3 + 12*(B*a^2 + 2*A*a*b)*c*d^4 - (5*D*b^2*c*d^4 - 4*(2*D*a*b + C*
b^2)*d^5)*x^4 + 2*(5*D*b^2*c^2*d^3 - 4*(2*D*a*b + C*b^2)*c*d^4 + 3*(D*a^2
+ 2*C*a*b + B*b^2)*d^5)*x^3 - 6*(5*D*b^2*c^3*d^2 - 4*(2*D*a*b + C*b^2)*c^2
*d^3 + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^4 - 2*(C*a^2 + 2*B*a*b + A*b^2)*d^5
)*x^2 - 12*(4*D*b^2*c^4*d - 3*(2*D*a*b + C*b^2)*c^3*d^2 + 2*(D*a^2 + 2*C*a
*b + B*b^2)*c^2*d^3 - (C*a^2 + 2*B*a*b + A*b^2)*c*d^4)*x + 12*(5*D*b^2*c^5
 - 4*(2*D*a*b + C*b^2)*c^4*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - 2*(C*
a^2 + 2*B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4 + (5*D*b^2*c^4*d
- 4*(2*D*a*b + C*b^2)*c^3*d^2 + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 - 2*(C
*a^2 + 2*B*a*b + A*b^2)*c*d^4 + (B*a^2 + 2*A*a*b)*d^5)*x)*log(d*x + c))/(d
^7*x + c*d^6)
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2}\,dx = \frac{Db^2x^4}{4d^2} + x^3\left(\frac{Cb^2}{3d^2} + \frac{2Dab}{3d^2} - \frac{2Db^2c}{3d^3}\right)$$
$$+ x^2\left(\frac{Bb^2}{2d^2} + \frac{Cab}{d^2} - \frac{Cb^2c}{d^3} + \frac{Da^2}{2d^2} - \frac{2Dabc}{d^3} + \frac{3Db^2c^2}{2d^4}\right)$$
$$+ x\left(\frac{Ab^2}{d^2} + \frac{2Bab}{d^2} - \frac{2Bb^2c}{d^3} + \frac{Ca^2}{d^2} - \frac{4Cabc}{d^3} + \frac{3Cb^2c^2}{d^4} - \frac{2Da^2c}{d^3} + \frac{6Dabc^2}{d^4} - \frac{4Db^2c^3}{d^5}\right)$$
$$+ \frac{-Aa^2d^5 + 2Aabcd^4 - Ab^2c^2d^3 + Ba^2cd^4 - 2Babc^2d^3 + Bb^2c^3d^2 - Ca^2c^2d^3 + 2Cabc^3d^2 - Cb^2c^4d + L}{cd^6 + d^7x}$$
$$+ \frac{(ad-bc)\left(2Abd^3 + Bad^3 - 3Bbcd^2 - 2Cacd^2 + 4Cbc^2d + 3Dac^2d - 5Dbc^3\right)\log{(c+dx)}}{d^6}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
D*b**2*x**4/(4*d**2) + x**3*(C*b**2/(3*d**2) + 2*D*a*b/(3*d**2) - 2*D*b**2
*c/(3*d**3)) + x**2*(B*b**2/(2*d**2) + C*a*b/d**2 - C*b**2*c/d**3 + D*a**2
/(2*d**2) - 2*D*a*b*c/d**3 + 3*D*b**2*c**2/(2*d**4)) + x*(A*b**2/d**2 + 2*
B*a*b/d**2 - 2*B*b**2*c/d**3 + C*a**2/d**2 - 4*C*a*b*c/d**3 + 3*C*b**2*c**
2/d**4 - 2*D*a**2*c/d**3 + 6*D*a*b*c**2/d**4 - 4*D*b**2*c**3/d**5) + (-A*a
**2*d**5 + 2*A*a*b*c*d**4 - A*b**2*c**2*d**3 + B*a**2*c*d**4 - 2*B*a*b*c**
2*d**3 + B*b**2*c**3*d**2 - C*a**2*c**2*d**3 + 2*C*a*b*c**3*d**2 - C*b**2*
c**4*d + D*a**2*c**3*d**2 - 2*D*a*b*c**4*d + D*b**2*c**5)/(c*d**6 + d**7*x
) + (a*d - b*c)*(2*A*b*d**3 + B*a*d**3 - 3*B*b*c*d**2 - 2*C*a*c*d**2 + 4*C
*b*c**2*d + 3*D*a*c**2*d - 5*D*b*c**3)*log(c + d*x)/d**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{Db^2c^5 - Aa^2d^5 - (2\,Dab + Cb^2)c^4d + (Da^2 + 2\,Cab + Bb^2)c^3d^2 - (Ca^2 + 2\,Bab + Ab^2)c^2d^3 + (Ba^2 + 2}{d^7x + cd^6}$$
$$+ \frac{3\,Db^2d^3x^4 - 4\,(2\,Db^2cd^2 - (2\,Dab + Cb^2)d^3)x^3 + 6\,(3\,Db^2c^2d - 2\,(2\,Dab + Cb^2)cd^2 + (Da^2 + 2\,Cab}{}$$
$$+ \frac{(5\,Db^2c^4 - 4\,(2\,Dab + Cb^2)c^3d + 3\,(Da^2 + 2\,Cab + Bb^2)c^2d^2 - 2\,(Ca^2 + 2\,Bab + Ab^2)cd^3 + (Ba^2 + 2}{d^6}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(D*b^2*c^5 - A*a^2*d^5 - (2*D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^
2)*c^3*d^2 - (C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)/
(d^7*x + c*d^6) + 1/12*(3*D*b^2*d^3*x^4 - 4*(2*D*b^2*c*d^2 - (2*D*a*b + C*
b^2)*d^3)*x^3 + 6*(3*D*b^2*c^2*d - 2*(2*D*a*b + C*b^2)*c*d^2 + (D*a^2 + 2*
C*a*b + B*b^2)*d^3)*x^2 - 12*(4*D*b^2*c^3 - 3*(2*D*a*b + C*b^2)*c^2*d + 2*
(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*b + A*b^2)*d^3)*x)/d^5 +
(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^2)*c^3*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2
*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 + (B*a^2 + 2*A*a*b)*d^4)*log(d*x
+ c)/d^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(282) = 564$.

Time = 0.13 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{\left(3\,Db^2 - \frac{4\,(5\,Db^2cd-2\,Dabd^2-Cb^2d^2)}{(dx+c)d} + \frac{6\,(10\,Db^2c^2d^2-8\,Dabcd^3-4\,Cb^2cd^3+Da^2d^4+2\,Cabd^4+Bb^2d^4)}{(dx+c)^2d^2} - \frac{12\,(10\,Db^2c^3d^3-12\,Dabc^2}{}\right.}{12\,d^6}$$
$$- \frac{(5\,Db^2c^4 - 8\,Dabc^3d - 4\,Cb^2c^3d + 3\,Da^2c^2d^2 + 6\,Cabc^2d^2 + 3\,Bb^2c^2d^2 - 2\,Ca^2cd^3 - 4\,Babcd^3 - 2\,Ab}{d^6}$$
$$+ \frac{\frac{Db^2c^5d^4}{dx+c} - \frac{2\,Dabc^4d^5}{dx+c} - \frac{Cb^2c^4d^5}{dx+c} + \frac{Da^2c^3d^6}{dx+c} + \frac{2\,Cabc^3d^6}{dx+c} + \frac{Bb^2c^3d^6}{dx+c} - \frac{Ca^2c^2d^7}{dx+c} - \frac{2\,Babc^2d^7}{dx+c} - \frac{Ab^2c^2d^7}{dx+c} + \frac{Ba^2cd^8}{dx+c} + 2}{d^{10}}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/12*(3*D*b^2 - 4*(5*D*b^2*c*d - 2*D*a*b*d^2 - C*b^2*d^2)/((d*x + c)*d) +
6*(10*D*b^2*c^2*d^2 - 8*D*a*b*c*d^3 - 4*C*b^2*c*d^3 + D*a^2*d^4 + 2*C*a*b*
d^4 + B*b^2*d^4)/((d*x + c)^2*d^2) - 12*(10*D*b^2*c^3*d^3 - 12*D*a*b*c^2*d
^4 - 6*C*b^2*c^2*d^4 + 3*D*a^2*c*d^5 + 6*C*a*b*c*d^5 + 3*B*b^2*c*d^5 - C*a
^2*d^6 - 2*B*a*b*d^6 - A*b^2*d^6)/((d*x + c)^3*d^3))*(d*x + c)^4/d^6 - (5*
D*b^2*c^4 - 8*D*a*b*c^3*d - 4*C*b^2*c^3*d + 3*D*a^2*c^2*d^2 + 6*C*a*b*c^2*
d^2 + 3*B*b^2*c^2*d^2 - 2*C*a^2*c*d^3 - 4*B*a*b*c*d^3 - 2*A*b^2*c*d^3 + B*
a^2*d^4 + 2*A*a*b*d^4)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 + (D*b^2
*c^5*d^4/(d*x + c) - 2*D*a*b*c^4*d^5/(d*x + c) - C*b^2*c^4*d^5/(d*x + c) +
 D*a^2*c^3*d^6/(d*x + c) + 2*C*a*b*c^3*d^6/(d*x + c) + B*b^2*c^3*d^6/(d*x
+ c) - C*a^2*c^2*d^7/(d*x + c) - 2*B*a*b*c^2*d^7/(d*x + c) - A*b^2*c^2*d^7
/(d*x + c) + B*a^2*c*d^8/(d*x + c) + 2*A*a*b*c*d^8/(d*x + c) - A*a^2*d^9/(
d*x + c))/d^10
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx=\int \frac{(a+b\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$=\frac{36\log(dx+c)\,a^2bc\,d^4x-72\log(dx+c)\,a\,b^2c^2d^3x-24\log(dx+c)\,ab\,c^4d^2x+12\log(dx+c)\,b^2c^6+12a^3}{}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
(36*log(c + d*x)*a**2*b*c**2*d**3 + 36*log(c + d*x)*a**2*b*c*d**4*x + 12*l
og(c + d*x)*a**2*c**4*d**2 + 12*log(c + d*x)*a**2*c**3*d**3*x - 72*log(c +
 d*x)*a*b**2*c**3*d**2 - 72*log(c + d*x)*a*b**2*c**2*d**3*x - 24*log(c + d
*x)*a*b*c**5*d - 24*log(c + d*x)*a*b*c**4*d**2*x + 36*log(c + d*x)*b**3*c*
*4*d + 36*log(c + d*x)*b**3*c**3*d**2*x + 12*log(c + d*x)*b**2*c**6 + 12*l
og(c + d*x)*b**2*c**5*d*x + 12*a**3*d**5*x - 36*a**2*b*c*d**4*x - 12*a**2*
c**3*d**3*x - 6*a**2*c**2*d**4*x**2 + 6*a**2*c*d**5*x**3 + 72*a*b**2*c**2*
d**3*x + 36*a*b**2*c*d**4*x**2 + 24*a*b*c**4*d**2*x + 12*a*b*c**3*d**3*x**
2 - 4*a*b*c**2*d**4*x**3 + 8*a*b*c*d**5*x**4 - 36*b**3*c**3*d**2*x - 18*b*
*3*c**2*d**3*x**2 + 6*b**3*c*d**4*x**3 - 12*b**2*c**5*d*x - 6*b**2*c**4*d*
*2*x**2 + 2*b**2*c**3*d**3*x**3 - b**2*c**2*d**4*x**4 + 3*b**2*c*d**5*x**5
)/(12*c*d**5*(c + d*x))
```

3.36 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2} dx$

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Optimal result

Integrand size = 28, antiderivative size = 181

$$\begin{aligned}\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx &= \frac{(ad(Cd-2cD)-b(2cCd-Bd^2-3c^2D))\,x}{d^4}+\frac{(bCd-2bcD+adD)x^2}{2d^3}\\&\quad+\frac{bDx^3}{3d^2}+\frac{(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)}{d^5(c+dx)}\\&\quad-\frac{(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\log(c+dx)}{d^5}\end{aligned}$$

output
```
(a*d*(C*d-2*D*c)-b*(-B*d^2+2*C*c*d-3*D*c^2))*x/d^4+1/2*(C*b*d+D*a*d-2*D*b*
c)*x^2/d^3+1/3*b*D*x^3/d^2+(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^5/(d
*x+c)-(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))
*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{6d(ad(Cd-2cD)+b(-2cCd+Bd^2+3c^2D))\,x+3d^2(bCd-2bcD+adD)x^2+2bd^3Dx^3-\frac{6(bc-ad)(-c^2\cdots}{}}{6d^5}$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
(6*d*(a*d*(C*d - 2*c*D) + b*(-2*c*C*d + B*d^2 + 3*c^2*D))*x + 3*d^2*(b*C*d
 - 2*b*c*D + a*d*D)*x^2 + 2*b*d^3*D*x^3 - (6*(b*c - a*d)*(-(c^2*C*d) + B*c
*d^2 - A*d^3 + c^3*D))/(c + d*x) + 6*(a*d*(-2*c*C*d + B*d^2 + 3*c^2*D) + b
*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*Log[c + d*x])/(6*d^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{b(Ad^3-2Bcd^2-4c^3D+3c^2Cd)-ad(-Bd^2-3c^2D+2cCd)}{d^4(c+dx)}+\frac{(ad-bc)\,(Ad^3-Bcd^2+c^3(-D)+c^2C\cdots}{d^4(c+dx)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^5(c+dx)}-\frac{\log(c+dx)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{d^5}+\frac{x\left(ad(Cd-2cD)-b\left(-Bd^2-3c^2D+2cCd\right)\right)}{d^4}+\frac{x^2(adD-2bcD+bCd)}{2d^3}+\frac{bDx^3}{3d^2}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^2,x]
```

output
```
((a*d*(C*d - 2*c*D) - b*(2*c*C*d - B*d^2 - 3*c^2*D))*x)/d^4 + ((b*C*d - 2*
b*c*D + a*d*D)*x^2)/(2*d^3) + (b*D*x^3)/(3*d^2) + ((b*c - a*d)*(c^2*C*d -
B*c*d^2 + A*d^3 - c^3*D))/(d^5*(c + d*x)) - ((a*d*(2*c*C*d - B*d^2 - 3*c^2
*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))*Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.21

method	result
default	$\frac{\frac{1}{3}bDx^3d^2+\frac{1}{2}Cb\,d^2x^2+\frac{1}{2}Da\,d^2x^2-Dbcd\,x^2+bB\,d^2x+Ca\,d^2x-2Cbcdx-2Dacdx+3Db\,c^2x}{d^4}-$ $Aa\,d^4-Ac\,d^3b-Bc\,d^3a+Bb\,c$ …
norman	$\frac{\left(Aa\,d^4-Ac\,d^3b-Bc\,d^3a+2Bb\,c^2d^2+2Ca\,c^2d^2-3Cb\,c^3d-3Da\,c^3d+4Db\,c^4\right)x}{d^4c}+\frac{(3Cbd+3Dad-4Dbc)x^3}{6d^2}+$ $\left(2bB\,d^2+2Ca\,d^2-3Cbcd-3Da\right.$ … $/\,2d^3$ … (all over $xd+c$)
parallelrisch	$-12C\ln(xd+c)xac\,d^3+18C\ln(xd+c)xb\,c^2d^2+18D\ln(xd+c)xa\,c^2d^2-24D\ln(xd+c)xb\,c^3d-12Bb\,c^2d^2-12B\ln(xd+c)xbc\,d^3$ …

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(1/3*b*D*x^3*d^2+1/2*C*b*d^2*x^2+1/2*D*a*d^2*x^2-D*b*c*d*x^2+b*B*d^2
*x+C*a*d^2*x-2*C*b*c*d*x-2*D*a*c*d*x+3*D*b*c^2*x)-(A*a*d^4-A*b*c*d^3-B*a*c
*d^3+B*b*c^2*d^2+C*a*c^2*d^2-C*b*c^3*d-D*a*c^3*d+D*b*c^4)/d^5/(d*x+c)+1/d^
5*(A*b*d^3+B*a*d^3-2*B*b*c*d^2-2*C*a*c*d^2+3*C*b*c^2*d+3*D*a*c^2*d-4*D*b*c
^3)*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2}\,dx$$
$$= \frac{2\,Dbd^4x^4 - 6\,Dbc^4 - 6\,Aad^4 + 6\,(Da+Cb)c^3d - 6\,(Ca+Bb)c^2d^2 + 6\,(Ba+Ab)cd^3 - (4\,Dbcd^3 - 3\,(D$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/6*(2*D*b*d^4*x^4 - 6*D*b*c^4 - 6*A*a*d^4 + 6*(D*a + C*b)*c^3*d - 6*(C*a
+ B*b)*c^2*d^2 + 6*(B*a + A*b)*c*d^3 - (4*D*b*c*d^3 - 3*(D*a + C*b)*d^4)*x
^3 + 3*(4*D*b*c^2*d^2 - 3*(D*a + C*b)*c*d^3 + 2*(C*a + B*b)*d^4)*x^2 + 6*(
3*D*b*c^3*d - 2*(D*a + C*b)*c^2*d^2 + (C*a + B*b)*c*d^3)*x - 6*(4*D*b*c^4
- 3*(D*a + C*b)*c^3*d + 2*(C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3 + (4*D*b
*c^3*d - 3*(D*a + C*b)*c^2*d^2 + 2*(C*a + B*b)*c*d^3 - (B*a + A*b)*d^4)*x)
*log(d*x + c))/(d^6*x + c*d^5)
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2}\,dx$$
$$= \frac{Dbx^3}{3d^2} + x^2\left(\frac{Cb}{2d^2} + \frac{Da}{2d^2} - \frac{Dbc}{d^3}\right) + x\left(\frac{Bb}{d^2} + \frac{Ca}{d^2} - \frac{2Cbc}{d^3} - \frac{2Dac}{d^3} + \frac{3Dbc^2}{d^4}\right)$$
$$+ \frac{-Aad^4 + Abcd^3 + Bacd^3 - Bbc^2d^2 - Cac^2d^2 + Cbc^3d + Dac^3d - Dbc^4}{cd^5 + d^6x}$$
$$+ \frac{(Abd^3 + Bad^3 - 2Bbcd^2 - 2Cacd^2 + 3Cbc^2d + 3Dac^2d - 4Dbc^3)\log(c+dx)}{d^5}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
D*b*x**3/(3*d**2) + x**2*(C*b/(2*d**2) + D*a/(2*d**2) - D*b*c/d**3) + x*(B
*b/d**2 + C*a/d**2 - 2*C*b*c/d**3 - 2*D*a*c/d**3 + 3*D*b*c**2/d**4) + (-A*
a*d**4 + A*b*c*d**3 + B*a*c*d**3 - B*b*c**2*d**2 - C*a*c**2*d**2 + C*b*c**
3*d + D*a*c**3*d - D*b*c**4)/(c*d**5 + d**6*x) + (A*b*d**3 + B*a*d**3 - 2*
B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D*b*c**3)*log(
c + d*x)/d**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= -\frac{Dbc^4+Aad^4-(Da+Cb)c^3d+(Ca+Bb)c^2d^2-(Ba+Ab)cd^3}{d^6x+cd^5} + \frac{2\,Dbd^2x^3-3\,(2\,Dbcd-(Da+Cb)d^2)x^2+6\,(3\,Dbc^2-2\,(Da+Cb)cd+(Ca+Bb)d^2)x}{6\,d^4} - \frac{(4\,Dbc^3-3\,(Da+Cb)c^2d+2\,(Ca+Bb)cd^2-(Ba+Ab)d^3)\log(dx+c)}{d^5}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c^3*d + (C*a + B*b)*c^2*d^2 - (B*a + A*b
)*c*d^3)/(d^6*x + c*d^5) + 1/6*(2*D*b*d^2*x^3 - 3*(2*D*b*c*d - (D*a + C*b)
*d^2)*x^2 + 6*(3*D*b*c^2 - 2*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*x)/d^4 - (
4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 - (B*a + A*b)*d^3)*l
og(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx$$
$$= \frac{\left(2\,Db - \frac{3\,(4\,Dbcd-Dad^2-Cbd^2)}{(dx+c)d} + \frac{6\,(6\,Dbc^2d^2-3\,Dacd^3-3\,Cbcd^3+Cad^4+Bbd^4)}{(dx+c)^2d^2}\right)(dx+c)^3}{6\,d^5}$$
$$+ \frac{(4\,Dbc^3 - 3\,Dac^2d - 3\,Cbc^2d + 2\,Cacd^2 + 2\,Bbcd^2 - Bad^3 - Abd^3)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5}$$
$$- \frac{\frac{Dbc^4d^3}{dx+c} - \frac{Dac^3d^4}{dx+c} - \frac{Cbc^3d^4}{dx+c} + \frac{Cac^2d^5}{dx+c} + \frac{Bbc^2d^5}{dx+c} - \frac{Bacd^6}{dx+c} - \frac{Abcd^6}{dx+c} + \frac{Aad^7}{dx+c}}{d^8}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/6*(2*D*b - 3*(4*D*b*c*d - D*a*d^2 - C*b*d^2)/((d*x + c)*d) + 6*(6*D*b*c^
2*d^2 - 3*D*a*c*d^3 - 3*C*b*c*d^3 + C*a*d^4 + B*b*d^4)/((d*x + c)^2*d^2))*
(d*x + c)^3/d^5 + (4*D*b*c^3 - 3*D*a*c^2*d - 3*C*b*c^2*d + 2*C*a*c*d^2 + 2
*B*b*c*d^2 - B*a*d^3 - A*b*d^3)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^5
 - (D*b*c^4*d^3/(d*x + c) - D*a*c^3*d^4/(d*x + c) - C*b*c^3*d^4/(d*x + c)
+ C*a*c^2*d^5/(d*x + c) + B*b*c^2*d^5/(d*x + c) - B*a*c*d^6/(d*x + c) - A*
b*c*d^6/(d*x + c) + A*a*d^7/(d*x + c))/d^8
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^2}\,dx = \int \frac{(a+b\,x)\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2,x)
```

output
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^2}\,dx$$
$$= \frac{12\log(dx+c)\,ab\,c^2d^2+12\log(dx+c)\,abc\,d^3x+6\log(dx+c)\,a\,c^4d+6\log(dx+c)\,a\,c^3d^2x-12\log(dx+}{}$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
(12*log(c + d*x)*a*b*c**2*d**2 + 12*log(c + d*x)*a*b*c*d**3*x + 6*log(c +
d*x)*a*c**4*d + 6*log(c + d*x)*a*c**3*d**2*x - 12*log(c + d*x)*b**2*c**3*d
 - 12*log(c + d*x)*b**2*c**2*d**2*x - 6*log(c + d*x)*b*c**5 - 6*log(c + d*
x)*b*c**4*d*x + 6*a**2*d**4*x - 12*a*b*c*d**3*x - 6*a*c**3*d**2*x - 3*a*c*
*2*d**3*x**2 + 3*a*c*d**4*x**3 + 12*b**2*c**2*d**2*x + 6*b**2*c*d**3*x**2
+ 6*b*c**4*d*x + 3*b*c**3*d**2*x**2 - b*c**2*d**3*x**3 + 2*b*c*d**4*x**4)/
(6*c*d**4*(c + d*x))
```

3.37 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2} dx$

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Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2} dx = \frac{(Cd-2cD)x}{d^3}+\frac{Dx^2}{2d^2}-\frac{c^2Cd-Bcd^2+Ad^3-c^3D}{d^4(c+dx)} - \frac{(2cCd-Bd^2-3c^2D)\log(c+dx)}{d^4}$$

output
```
(C*d-2*D*c)*x/d^3+1/2*D*x^2/d^2-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4/(d*x+c)-
(-B*d^2+2*C*c*d-3*D*c^2)*ln(d*x+c)/d^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2} dx = \frac{(Cd-2cD)x}{d^3}+\frac{Dx^2}{2d^2}+\frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{d^4(c+dx)} + \frac{(-2cCd+Bd^2+3c^2D)\log(c+dx)}{d^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^2,x]
```

output
```
((C*d - 2*c*D)*x)/d^3 + (D*x^2)/(2*d^2) + (-(c^2*C*d) + B*c*d^2 - A*d^3 +
c^3*D)/(d^4*(c + d*x)) + ((-2*c*C*d + B*d^2 + 3*c^2*D)*Log[c + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)^2}+\frac{Bd^2+3c^2D-2cCd}{d^3(c+dx)}+\frac{Cd-2cD}{d^3}+\frac{Dx}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^4(c+dx)}-\frac{\log(c+dx)\left(-Bd^2-3c^2D+2cCd\right)}{d^4}+\frac{x(Cd-2cD)}{d^3}+\frac{Dx^2}{2d^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^2,x]
```

output
```
((C*d - 2*c*D)*x)/d^3 + (D*x^2)/(2*d^2) - (c^2*C*d - B*c*d^2 + A*d^3 - c^3
*D)/(d^4*(c + d*x)) - ((2*c*C*d - B*d^2 - 3*c^2*D)*Log[c + d*x])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

method	result
default	$\frac{\frac{1}{2}Dx^2d+Cdx-2Dcx}{d^3}-\frac{A\,d^3-Bc\,d^2+C\,c^2d-Dc^3}{d^4(xd+c)}+\frac{(B\,d^2-2Ccd+3Dc^2)\ln(xd+c)}{d^4}$
norman	$\frac{-\frac{A\,d^3-Bc\,d^2+2C\,c^2d-3Dc^3}{d^4}+\frac{Dx^3}{2d}+\frac{(2Cd-3Dc)x^2}{2d^2}}{xd+c}+\frac{(B\,d^2-2Ccd+3Dc^2)\ln(xd+c)}{d^4}$
parallelrisch	$-\frac{-Dx^3d^3-2B\ln(xd+c)x\,d^3+4C\ln(xd+c)xc\,d^2-2C\,x^2d^3-6D\ln(xd+c)x\,c^2d+3Dx^2c\,d^2-2B\ln(xd+c)c\,d^2+4C\ln(xd+c)c}{2d^4(xd+c)}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(1/2*D*x^2*d+C*d*x-2*D*c*x)-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4/(d*x+c
)+1/d^4*(B*d^2-2*C*c*d+3*D*c^2)*ln(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2}\,dx$$
$$=\frac{Dd^3x^3+2\,Dc^3-2\,Cc^2d+2\,Bcd^2-2\,Ad^3-(3\,Dcd^2-2\,Cd^3)x^2-2\,(2\,Dc^2d-Ccd^2)x+2\,(3\,Dc^3-2\,C}{2\,(d^5x+cd^4)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(D*d^3*x^3 + 2*D*c^3 - 2*C*c^2*d + 2*B*c*d^2 - 2*A*d^3 - (3*D*c*d^2 -
2*C*d^3)*x^2 - 2*(2*D*c^2*d - C*c*d^2)*x + 2*(3*D*c^3 - 2*C*c^2*d + B*c*d^
2 + (3*D*c^2*d - 2*C*c*d^2 + B*d^3)*x)*log(d*x + c))/(d^5*x + c*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2}\,dx = \frac{Dx^2}{2d^2} + x\left(\frac{C}{d^2} - \frac{2Dc}{d^3}\right) + \frac{-Ad^3+Bcd^2-Cc^2d+Dc^3}{cd^4+d^5x} + \frac{(Bd^2-2Ccd+3Dc^2)\log(c+dx)}{d^4}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
D*x**2/(2*d**2) + x*(C/d**2 - 2*D*c/d**3) + (-A*d**3 + B*c*d**2 - C*c**2*d
 + D*c**3)/(c*d**4 + d**5*x) + (B*d**2 - 2*C*c*d + 3*D*c**2)*log(c + d*x)/
d**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2}\,dx = \frac{Dc^3-Cc^2d+Bcd^2-Ad^3}{d^5x+cd^4} + \frac{Ddx^2-2\,(2\,Dc-Cd)x}{2\,d^3} + \frac{(3\,Dc^2-2\,Ccd+Bd^2)\log(dx+c)}{d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)/(d^5*x + c*d^4) + 1/2*(D*d*x^2 - 2*(2*
D*c - C*d)*x)/d^3 + (3*D*c^2 - 2*C*c*d + B*d^2)*log(d*x + c)/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(88) = 176$.

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2}\,dx = -\frac{1}{2}D\left(\frac{(dx+c)^2\left(\frac{6c}{dx+c}-1\right)}{d^4}+\frac{6c^2\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4}-\frac{2c^3}{(dx+c)d^4}\right) + C\left(\frac{2c\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3}+\frac{dx+c}{d^3}-\frac{c^2}{(dx+c)d^3}\right) - \frac{B\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d}-\frac{c}{(dx+c)d}\right)}{d} - \frac{A}{(dx+c)d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*D*((d*x + c)^2*(6*c/(d*x + c) - 1)/d^4 + 6*c^2*log(abs(d*x + c)/((d*x
 + c)^2*abs(d)))/d^4 - 2*c^3/((d*x + c)*d^4)) + C*(2*c*log(abs(d*x + c)/((
d*x + c)^2*abs(d)))/d^3 + (d*x + c)/d^3 - c^2/((d*x + c)*d^3)) - B*(log(ab
s(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d))/d - A/((d*x + c)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^2}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{{(c+d\,x)}^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^2,x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^2} \, dx$$
$$= \frac{2\log(dx+c)\, b\, c^2 d + 2\log(dx+c)\, bc\, d^2 x + 2\log(dx+c)\, c^4 + 2\log(dx+c)\, c^3 dx + 2a\, d^3 x - 2bc\, d^2 x - 2c}{2c\, d^3\, (dx+c)}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
(2*log(c + d*x)*b*c**2*d + 2*log(c + d*x)*b*c*d**2*x + 2*log(c + d*x)*c**4
 + 2*log(c + d*x)*c**3*d*x + 2*a*d**3*x - 2*b*c*d**2*x - 2*c**3*d*x - c**2
*d**2*x**2 + c*d**3*x**3)/(2*c*d**3*(c + d*x))
```

3.38 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2} \, dx$

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Optimal result

Integrand size = 30, antiderivative size = 166

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2} \, dx = \frac{Dx}{bd^2}+\frac{c^2Cd-Bcd^2+Ad^3-c^3D}{d^3(bc-ad)(c+dx)}+\frac{(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{b^2(bc-ad)^2} - \frac{(ad(2cCd-Bd^2-3c^2D)-b(c^2Cd-Ad^3-2c^3D))\log(c+dx)}{d^3(bc-ad)^2}$$

output
```
D*x/b/d^2+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^3/(-a*d+b*c)/(d*x+c)+(A*b^3-a*(B
*b^2-C*a*b+D*a^2))*ln(b*x+a)/b^2/(-a*d+b*c)^2-(a*d*(-B*d^2+2*C*c*d-3*D*c^2
)-b*(-A*d^3+C*c^2*d-2*D*c^3))*ln(d*x+c)/d^3/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2} \, dx = \frac{Dx}{bd^2}+\frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{d^3(-bc+ad)(c+dx)}+\frac{(Ab^3-ab^2B+a^2bC-a^3D)\log(a+bx)}{b^2(bc-ad)^2} + \frac{(bc^2Cd-2acCd^2-Abd^3+aBd^3-2bc^3D+3ac^2dD)\log(c+dx)}{d^3(bc-ad)^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^2),x]
```

output
```
(D*x)/(b*d^2) + (-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)/(d^3*(-(b*c) + a*d)
*(c + d*x)) + ((A*b^3 - a*b^2*B + a^2*b*C - a^3*D)*Log[a + b*x])/(b^2*(b*c
 - a*d)^2) + ((b*c^2*C*d - 2*a*c*C*d^2 - A*b*d^3 + a*B*d^3 - 2*b*c^3*D + 3
*a*c^2*d*D)*Log[c + d*x])/(d^3*(b*c - a*d)^2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2}dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{b(a+bx)(bc-ad)^2}+\frac{b\left(-Ad^3-2c^3D+c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)}{d^2(c+dx)(bc-ad)^2}+\frac{Ad^3-Bcd^2+c^3(}{d^2(c+dx)^2(}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(a+bx)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^2(bc-ad)^2}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)(bc-ad)}-\frac{\log(c+dx)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-Ad^3-2c^3D+c^2Cd\right)\right)}{d^3(bc-ad)^2}+\frac{Dx}{bd^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^2),x]
```

output
```
(D*x)/(b*d^2) + (c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(d^3*(b*c - a*d)*(c +
d*x)) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Log[a + b*x])/(b^2*(b*c - a*d
)^2) - ((a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(c^2*C*d - A*d^3 - 2*c^3*D))*
Log[c + d*x])/(d^3*(b*c - a*d)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01

method	result
default	$\frac{Dx}{b\,d^2}-\frac{A\,d^3-Bc\,d^2+C\,c^2d-Dc^3}{d^3(ad-bc)(xd+c)}+\frac{\left(-Ab\,d^3+Ba\,d^3-2Cac\,d^2+Cb\,c^2d+3Da\,c^2d-2Db\,c^3\right)\ln(xd+c)}{d^3(ad-bc)^2}+\frac{(b^3A-a\,b^2B+a^2 \ldots}{b^2(a \ldots}$
norman	$\frac{\frac{Dx^2}{bd}-\frac{Ab\,d^3-Bbc\,d^2+Cb\,c^2d+Da\,c^2d-2Db\,c^3}{d^3b(ad-bc)}}{xd+c}+\frac{\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\ln(bx+a)}{(a^2d^2-2abcd+b^2c^2)b^2}-\frac{(Ab\,d^3-Ba\,d^3+2Cac\,d^2-Cb\,c^2d- \ldots}{d^3(a^2d^2-2abc \ldots}$
parallelrisch	$\frac{-Ca\,b^2c^2d^2-Da^2b\,c^2d^2+3Da\,b^2c^3d+A\ln(bx+a)x\,b^3d^4-A\ln(xd+c)x\,b^3d^4-D\ln(bx+a)x\,a^3d^4+A\ln(bx+a)b^3c\,d^3-A\ln(xd \ldots}{\ldots}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
D*x/b/d^2-1/d^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)/(d*x+c)+(-A*b*d^3+
B*a*d^3-2*C*a*c*d^2+C*b*c^2*d+3*D*a*c^2*d-2*D*b*c^3)/d^3/(a*d-b*c)^2*ln(d*
x+c)+1/b^2*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(a*d-b*c)^2*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(165) = 330.

Time = 0.12 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2}\,dx = -\frac{Db^3c^4+Aab^2d^4-(Dab^2+Cb^3)c^3d+(Cab^2+Bb^3)c^2d^2-(Bab^2+Ab^3)cd^3-(Db^3c^2d^2-2\,Dab^2cd^3 \ldots}{\ldots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(D*b^3*c^4 + A*a*b^2*d^4 - (D*a*b^2 + C*b^3)*c^3*d + (C*a*b^2 + B*b^3)*c^
2*d^2 - (B*a*b^2 + A*b^3)*c*d^3 - (D*b^3*c^2*d^2 - 2*D*a*b^2*c*d^3 + D*a^2
*b*d^4)*x^2 - (D*b^3*c^3*d - 2*D*a*b^2*c^2*d^2 + D*a^2*b*c*d^3)*x + ((D*a^
3 - C*a^2*b + B*a*b^2 - A*b^3)*d^4*x + (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)
*c*d^3)*log(b*x + a) + (2*D*b^3*c^4 + 2*C*a*b^2*c^2*d^2 - (3*D*a*b^2 + C*b
^3)*c^3*d - (B*a*b^2 - A*b^3)*c*d^3 + (2*D*b^3*c^3*d + 2*C*a*b^2*c*d^3 - (
3*D*a*b^2 + C*b^3)*c^2*d^2 - (B*a*b^2 - A*b^3)*d^4)*x)*log(d*x + c))/(b^4*
c^3*d^3 - 2*a*b^3*c^2*d^4 + a^2*b^2*c*d^5 + (b^4*c^2*d^4 - 2*a*b^3*c*d^5 +
 a^2*b^2*d^6)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2}\,dx \\
&= -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\log(bx+a)}{b^4c^2-2\,ab^3cd+a^2b^2d^2} \\
&\quad -\frac{(2\,Dbc^3+2\,Cacd^2-(3\,Da+Cb)c^2d-(Ba-Ab)d^3)\log(dx+c)}{b^2c^2d^3-2\,abcd^4+a^2d^5} \\
&\quad -\frac{Dc^3-Cc^2d+Bcd^2-Ad^3}{bc^2d^3-acd^4+(bcd^4-ad^5)x}+\frac{Dx}{bd^2}
\end{aligned}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*log(b*x + a)/(b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2) - (2*D*b*c^3 + 2*C*a*c*d^2 - (3*D*a + C*b)*c^2*d - (B*a - A*
b)*d^3)*log(d*x + c)/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) - (D*c^3 - C*c^
2*d + B*c*d^2 - A*d^3)/(b*c^2*d^3 - a*c*d^4 + (b*c*d^4 - a*d^5)*x) + D*x/(
b*d^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2}\,dx = -\frac{(Da^3d-Ca^2bd+Bab^2d-Ab^3d)\log\left(\left|b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right|\right)}{b^4c^2d-2\,ab^3cd^2+a^2b^2d^3} - \frac{\frac{Dc^3d^2}{dx+c}-\frac{Cc^2d^3}{dx+c}+\frac{Bcd^4}{dx+c}-\frac{Ad^5}{dx+c}}{bcd^5-ad^6} + \frac{(dx+c)D}{bd^3} + \frac{(2\,Dbc+Dad-Cbd)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{b^2d^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-(D*a^3*d - C*a^2*b*d + B*a*b^2*d - A*b^3*d)*log(abs(b - b*c/(d*x + c) + a
*d/(d*x + c)))/(b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3) - (D*c^3*d^2/(d*x
 + c) - C*c^2*d^3/(d*x + c) + B*c*d^4/(d*x + c) - A*d^5/(d*x + c))/(b*c*d^
5 - a*d^6) + (d*x + c)*D/(b*d^3) + (2*D*b*c + D*a*d - C*b*d)*log(abs(d*x +
 c)/((d*x + c)^2*abs(d)))/(b^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^2}\,dx = \frac{-\log(bx+a)\,a^2c^2d^2 - \log(bx+a)\,a^2c\,d^3x + \log(dx+c)\,b^2c^4 + \log(dx+c)\,b^2c^3dx + a\,b^2d^3x + ab\,c^2d^2x +}{b^2c\,d^2\left(a\,d^2x - bcdx + acd - b\,c^2\right)}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^2,x)
```

output
```
( - log(a + b*x)*a**2*c**2*d**2 - log(a + b*x)*a**2*c*d**3*x + log(c + d*x
)*b**2*c**4 + log(c + d*x)*b**2*c**3*d*x + a*b**2*d**3*x + a*b*c**2*d**2*x
 + a*b*c*d**3*x**2 - b**3*c*d**2*x - b**2*c**3*d*x - b**2*c**2*d**2*x**2)/
(b**2*c*d**2*(a*c*d + a*d**2*x - b*c**2 - b*c*d*x))
```

3.39 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 222

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx
&= -\frac{Ab^3-a(b^2B-abC+a^2D)}{b^2(bc-ad)^2(a+bx)} - \frac{c^2Cd-Bcd^2+Ad^3-c^3D}{d^2(bc-ad)^2(c+dx)} \\
&\quad + \frac{(b^3(Bc-2Ad)-ab^2(2cC-Bd)+3a^2bcD-a^3dD)\log(a+bx)}{b^2(bc-ad)^3} \\
&\quad + \frac{(ad(2cCd-Bd^2-3c^2D)-b(Bcd^2-2Ad^3-c^3D))\log(c+dx)}{d^2(bc-ad)^3}
\end{aligned}$$

output
```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^2/(-a*d+b*c)^2/(b*x+a)-(A*d^3-B*c*d^2+C*c
^2*d-D*c^3)/d^2/(-a*d+b*c)^2/(d*x+c)+(b^3*(-2*A*d+B*c)-a*b^2*(-B*d+2*C*c)+
3*a^2*b*c*D-a^3*d*D)*ln(b*x+a)/b^2/(-a*d+b*c)^3+(a*d*(-B*d^2+2*C*c*d-3*D*c
^2)-b*(-2*A*d^3+B*c*d^2-D*c^3))*ln(d*x+c)/d^2/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx = \frac{-Ab^3+a(b^2B-abC+a^2D)}{b^2(bc-ad)^2(a+bx)}+\frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{d^2(bc-ad)^2(c+dx)} + \frac{(b^3(Bc-2Ad)+ab^2(-2cC+Bd)+3a^2bcD-a^3dD)\log(a+bx)}{b^2(bc-ad)^3} + \frac{(ad(-2cCd+Bd^2+3c^2D)+b(Bcd^2-2Ad^3-c^3D))\log(c+dx)}{d^2(-bc+ad)^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^2),x]
```

output
```
(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))/(b^2*(b*c - a*d)^2*(a + b*x)) + (-(
c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)/(d^2*(b*c - a*d)^2*(c + d*x)) + ((b^3*
(B*c - 2*A*d) + a*b^2*(-2*c*C + B*d) + 3*a^2*b*c*D - a^3*d*D)*Log[a + b*x]
)/(b^2*(b*c - a*d)^3) + ((a*d*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*(B*c*d^2 -
2*A*d^3 - c^3*D))*Log[c + d*x])/(d^2*(-(b*c) + a*d)^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{Ab^3-a(a^2D-abC+b^2B)}{b(a+bx)^2(bc-ad)^2}+\frac{a^3(-d)D+3a^2bcD-ab^2(2cC-Bd)+b^3(Bc-2Ad)}{b(a+bx)(bc-ad)^3}+\frac{Ad^3-Bcd^2+c^3(}{d(c+dx)^2(a}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^3-a(a^2D-abC+b^2B)}{b^2(a+bx)(bc-ad)^2}+\frac{\log(a+bx)\left(a^3(-d)D+3a^2bcD-ab^2(2cC-Bd)+b^3(Bc-2Ad)\right)}{b^2(bc-ad)^3}-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^2(c+dx)(bc-ad)^2}+\frac{\log(c+dx)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3+Bcd^2+c^3(-D)\right)\right)}{d^2(bc-ad)^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^2),x]
```

output
```
-((A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b^2*(b*c - a*d)^2*(a + b*x))) - (c^
2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(d^2*(b*c - a*d)^2*(c + d*x)) + ((b^3*(B*
c - 2*A*d) - a*b^2*(2*c*C - B*d) + 3*a^2*b*c*D - a^3*d*D)*Log[a + b*x])/(b
^2*(b*c - a*d)^3) + ((a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(B*c*d^2 - 2*A*d
^3 - c^3*D))*Log[c + d*x])/(d^2*(b*c - a*d)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.01

method	result
default	$-\frac{A\,d^{3}-Bc\,d^{2}+C\,c^{2}d-Dc^{3}}{d^{2}\left(ad-bc\right)^{2}\left(xd+c\right)}+\frac{\left(-2Ab\,d^{3}+Ba\,d^{3}+Bbc\,d^{2}-2Cac\,d^{2}+3Da\,c^{2}d-Db\,c^{3}\right)\ln\left(xd+c\right)}{\left(ad-bc\right)^{3}d^{2}}-\frac{b^{3}A-a\,b^{2}B+a^{2}bC-a^{3}}{b^{2}\left(ad-bc\right)^{2}\left(bx+a\right)}$
norman	$\frac{-\frac{Aa\,b^{2}d^{3}+A\,b^{3}c\,d^{2}-2Ba\,b^{2}c\,d^{2}+C\,a^{2}bc\,d^{2}+Ca\,b^{2}c^{2}d-Da^{3}c\,d^{2}-Da\,b^{2}c^{3}}{d^{2}b^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}-\frac{\left(2A\,b^{3}d^{3}-Ba\,b^{2}d^{3}-B\,b^{3}c\,d^{2}+a^{2}bC\,d^{3}+C\,b^{3}c^{2}d-a^{3}d^{3}D-\right.}{d^{2}b^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}}{\left(bx+a\right)\left(xd+c\right)}$
parallelrisch	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^2/(a*d-b*c)^2/(d*x+c)+1/(a*d-b*c)^3*(-2*A
*b*d^3+B*a*d^3+B*b*c*d^2-2*C*a*c*d^2+3*D*a*c^2*d-D*b*c^3)/d^2*ln(d*x+c)-(A
*b^3-B*a*b^2+C*a^2*b-D*a^3)/b^2/(a*d-b*c)^2/(b*x+a)+(2*A*b^3*d-B*a*b^2*d-B
*b^3*c+2*C*a*b^2*c+D*a^3*d-3*D*a^2*b*c)/(a*d-b*c)^3/b^2*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(221) = 442.

Time = 0.13 (sec) , antiderivative size = 814, normalized size of antiderivative = 3.67

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
(D*a*b^3*c^4 + A*a^2*b^2*d^4 - (D*a^2*b^2 + C*a*b^3)*c^3*d + (D*a^3*b + 2*
B*a*b^3 - A*b^4)*c^2*d^2 - (D*a^4 - C*a^3*b + 2*B*a^2*b^2)*c*d^3 + (D*b^4*
c^4 - (D*a*b^3 + C*b^4)*c^3*d + (C*a*b^3 + B*b^4)*c^2*d^2 + (D*a^3*b - C*a
^2*b^2 - 2*A*b^4)*c*d^3 - (D*a^4 - C*a^3*b + B*a^2*b^2 - 2*A*a*b^3)*d^4)*x
 + ((3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c^2*d^2 - (D*a^4 - B*a^2*b^2 + 2*A
*a*b^3)*c*d^3 + ((3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c*d^3 - (D*a^3*b - B*a*
b^3 + 2*A*b^4)*d^4)*x^2 + ((3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c^2*d^2 + 2*(
D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d^3 - (D*a^4 - B*a^2*b^2 + 2*A*a*
b^3)*d^4)*x)*log(b*x + a) + (D*a*b^3*c^4 - 3*D*a^2*b^2*c^3*d + (2*C*a^2*b^
2 - B*a*b^3)*c^2*d^2 - (B*a^2*b^2 - 2*A*a*b^3)*c*d^3 + (D*b^4*c^3*d - 3*D*
a*b^3*c^2*d^2 + (2*C*a*b^3 - B*b^4)*c*d^3 - (B*a*b^3 - 2*A*b^4)*d^4)*x^2 +
 (D*b^4*c^4 - 2*D*a*b^3*c^3*d - (3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c^2*d^2
+ 2*(C*a^2*b^2 - B*a*b^3 + A*b^4)*c*d^3 - (B*a^2*b^2 - 2*A*a*b^3)*d^4)*x)*
log(d*x + c))/(a*b^5*c^4*d^2 - 3*a^2*b^4*c^3*d^3 + 3*a^3*b^3*c^2*d^4 - a^4
*b^2*c*d^5 + (b^6*c^3*d^3 - 3*a*b^5*c^2*d^4 + 3*a^2*b^4*c*d^5 - a^3*b^3*d^
6)*x^2 + (b^6*c^4*d^2 - 2*a*b^5*c^3*d^3 + 2*a^3*b^3*c*d^5 - a^4*b^2*d^6)*x
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1525 vs. $2(201) = 402$.

Time = 165.30 (sec) , antiderivative size = 1525, normalized size of antiderivative = 6.87

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2 (c + dx)^2} \, dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**2,x)
```

output
```
(-A*a*b**2*d**3 - A*b**3*c*d**2 + 2*B*a*b**2*c*d**2 - C*a**2*b*c*d**2 - C*
a*b**2*c**2*d + D*a**3*c*d**2 + D*a*b**2*c**3 + x*(-2*A*b**3*d**3 + B*a*b*
*2*d**3 + B*b**3*c*d**2 - C*a**2*b*d**3 - C*b**3*c**2*d + D*a**3*d**3 + D*
b**3*c**3))/(a**3*b**2*c*d**4 - 2*a**2*b**3*c**2*d**3 + a*b**4*c**3*d**2 +
 x**2*(a**2*b**3*d**5 - 2*a*b**4*c*d**4 + b**5*c**2*d**3) + x*(a**3*b**2*d
**5 - a**2*b**3*c*d**4 - a*b**4*c**2*d**3 + b**5*c**3*d**2)) + (-2*A*b*d**
3 + B*a*d**3 + B*b*c*d**2 - 2*C*a*c*d**2 + 3*D*a*c**2*d - D*b*c**3)*log(x
+ (2*A*a*b**2*d**3 + 2*A*b**3*c*d**2 - B*a**2*b*d**3 - 2*B*a*b**2*c*d**2 -
 B*b**3*c**2*d + 2*C*a**2*b*c*d**2 + 2*C*a*b**2*c**2*d + D*a**3*c*d**2 - 6
*D*a**2*b*c**2*d + D*a*b**2*c**3 + a**4*b*d**3*(-2*A*b*d**3 + B*a*d**3 + B
*b*c*d**2 - 2*C*a*c*d**2 + 3*D*a*c**2*d - D*b*c**3)/(a*d - b*c)**3 - 4*a**
3*b**2*c*d**2*(-2*A*b*d**3 + B*a*d**3 + B*b*c*d**2 - 2*C*a*c*d**2 + 3*D*a*
c**2*d - D*b*c**3)/(a*d - b*c)**3 + 6*a**2*b**3*c**2*d*(-2*A*b*d**3 + B*a*
d**3 + B*b*c*d**2 - 2*C*a*c*d**2 + 3*D*a*c**2*d - D*b*c**3)/(a*d - b*c)**3
 - 4*a*b**4*c**3*(-2*A*b*d**3 + B*a*d**3 + B*b*c*d**2 - 2*C*a*c*d**2 + 3*D
*a*c**2*d - D*b*c**3)/(a*d - b*c)**3 + b**5*c**4*(-2*A*b*d**3 + B*a*d**3 +
 B*b*c*d**2 - 2*C*a*c*d**2 + 3*D*a*c**2*d - D*b*c**3)/(d*(a*d - b*c)**3))/
(4*A*b**3*d**3 - 2*B*a*b**2*d**3 - 2*B*b**3*c*d**2 + 4*C*a*b**2*c*d**2 + D
*a**3*d**3 - 3*D*a**2*b*c*d**2 - 3*D*a*b**2*c**2*d + D*b**3*c**3))/(d**2*(
a*d - b*c)**3) + (2*A*b**3*d - B*a*b**2*d - B*b**3*c + 2*C*a*b**2*c + D...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.94

$$
\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx \\
&= \frac{((3\,Da^2b-2\,Cab^2+Bb^3)c-(Da^3-Bab^2+2\,Ab^3)d)\log(bx+a)}{b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3} \\
&\quad+\frac{(Dbc^3-3\,Dac^2d+(2\,Ca-Bb)cd^2-(Ba-2\,Ab)d^3)\log(dx+c)}{b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3d^5} \\
&\quad+\frac{Dab^2c^3-Cab^2c^2d-Aab^2d^3+(Da^3-Ca^2b+2\,Bab^2-Ab^3)cd^2+(Db^3c^3-Cb^3c^2d+Bb^3cd^2+(Da}{ab^4c^3d^2-2\,a^2b^3c^2d^3+a^3b^2cd^4+(b^5c^2d^3-2\,ab^4cd^4+a^2b^3d^5)x^2+(b^5c^3d^2-ab^4c^2d^3-a}
\end{aligned}
$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
((3*D*a^2*b - 2*C*a*b^2 + B*b^3)*c - (D*a^3 - B*a*b^2 + 2*A*b^3)*d)*log(b*
x + a)/(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3) + (D*b*c^
3 - 3*D*a*c^2*d + (2*C*a - B*b)*c*d^2 - (B*a - 2*A*b)*d^3)*log(d*x + c)/(b
^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5) + (D*a*b^2*c^3 - C
*a*b^2*c^2*d - A*a*b^2*d^3 + (D*a^3 - C*a^2*b + 2*B*a*b^2 - A*b^3)*c*d^2 +
 (D*b^3*c^3 - C*b^3*c^2*d + B*b^3*c*d^2 + (D*a^3 - C*a^2*b + B*a*b^2 - 2*A
*b^3)*d^3)*x)/(a*b^4*c^3*d^2 - 2*a^2*b^3*c^2*d^3 + a^3*b^2*c*d^4 + (b^5*c^
2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^2 + (b^5*c^3*d^2 - a*b^4*c^2*d^3 -
a^2*b^3*c*d^4 + a^3*b^2*d^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx = \frac{\left(Db^2c^3-3\,Dabc^2d+2\,Cabcd^2-Bb^2cd^2-Babd^3+2\,Ab^2d^3\right)\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5} + \frac{\frac{Da^3b^2}{bx+a}-\frac{Ca^2b^3}{bx+a}+\frac{Bab^4}{bx+a}-\frac{Ab^5}{bx+a}}{b^6c^2-2\,ab^5cd+a^2b^4d^2} - \frac{D\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^2d^2} - \frac{Dbc^3d-Cbc^2d^2+Bbcd^3-Abd^4}{(bc-ad)^3\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
(D*b^2*c^3 - 3*D*a*b*c^2*d + 2*C*a*b*c*d^2 - B*b^2*c*d^2 - B*a*b*d^3 + 2*A
*b^2*d^3)*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^4*c^3*d^2 - 3*a*b
^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5) + (D*a^3*b^2/(b*x + a) - C*a^2*b
^3/(b*x + a) + B*a*b^4/(b*x + a) - A*b^5/(b*x + a))/(b^6*c^2 - 2*a*b^5*c*d
 + a^2*b^4*d^2) - D*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^2*d^2) - (D*
b*c^3*d - C*b*c^2*d^2 + B*b*c*d^3 - A*b*d^4)/((b*c - a*d)^3*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 744, normalized size of antiderivative = 3.35

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^2,x)
```

output
```
(log(a + b*x)*a**4*c*d**3 + log(a + b*x)*a**4*d**4*x - log(a + b*x)*a**3*b
*c**2*d**2 + log(a + b*x)*a**3*b*d**4*x**2 + log(a + b*x)*a**2*b**3*c*d**2
 + log(a + b*x)*a**2*b**3*d**3*x - 2*log(a + b*x)*a**2*b**2*c**3*d - 3*log
(a + b*x)*a**2*b**2*c**2*d**2*x - log(a + b*x)*a**2*b**2*c*d**3*x**2 + log
(a + b*x)*a*b**4*c**2*d + 2*log(a + b*x)*a*b**4*c*d**2*x + log(a + b*x)*a*
b**4*d**3*x**2 - 2*log(a + b*x)*a*b**3*c**3*d*x - 2*log(a + b*x)*a*b**3*c*
*2*d**2*x**2 + log(a + b*x)*b**5*c**2*d*x + log(a + b*x)*b**5*c*d**2*x**2
- log(c + d*x)*a**2*b**3*c*d**2 - log(c + d*x)*a**2*b**3*d**3*x + log(c +
d*x)*a**2*b**2*c**3*d + log(c + d*x)*a**2*b**2*c**2*d**2*x - log(c + d*x)*
a*b**4*c**2*d - 2*log(c + d*x)*a*b**4*c*d**2*x - log(c + d*x)*a*b**4*d**3*
x**2 + log(c + d*x)*a*b**3*c**4 + 2*log(c + d*x)*a*b**3*c**3*d*x + log(c +
 d*x)*a*b**3*c**2*d**2*x**2 - log(c + d*x)*b**5*c**2*d*x - log(c + d*x)*b*
*5*c*d**2*x**2 + log(c + d*x)*b**4*c**4*x + log(c + d*x)*b**4*c**3*d*x**2
- a**3*b**2*d**3 + a**3*b*c**2*d**2 - a**3*b*d**4*x**2 + a**2*b**3*c*d**2
- a**2*b**2*c**3*d + a**2*b**2*c*d**3*x**2 + a*b**4*d**3*x**2 - b**5*c*d**
2*x**2)/(b**2*d*(a**4*c*d**3 + a**4*d**4*x - a**3*b*c**2*d**2 + a**3*b*d**
4*x**2 - a**2*b**2*c**3*d - 2*a**2*b**2*c**2*d**2*x - a**2*b**2*c*d**3*x**
2 + a*b**3*c**4 - a*b**3*c**2*d**2*x**2 + b**4*c**4*x + b**4*c**3*d*x**2))
```

3.40 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 276

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx = \frac{-Ab^3+a(b^2B-abC+a^2D)}{2b^2(bc-ad)^2(a+bx)^2} - \frac{b^3(Bc-2Ad)-ab^2(2cC-Bd)+3a^2bcD-a^3dD}{b^2(bc-ad)^3(a+bx)} + \frac{c^2Cd-Bcd^2+Ad^3-c^3D}{d(bc-ad)^3(c+dx)} + \frac{(b(c^2C-2Bcd+3Ad^2)+a(2cCd-Bd^2-3c^2D))\log(a+bx)}{(bc-ad)^4} - \frac{(b(c^2C-2Bcd+3Ad^2)+a(2cCd-Bd^2-3c^2D))\log(c+dx)}{(bc-ad)^4}$$

output
```
1/2*(-A*b^3+a*(B*b^2-C*a*b+D*a^2))/b^2/(-a*d+b*c)^2/(b*x+a)^2-(b^3*(-2*A*d
+B*c)-a*b^2*(-B*d+2*C*c)+3*a^2*b*c*D-a^3*d*D)/b^2/(-a*d+b*c)^3/(b*x+a)+(A*
d^3-B*c*d^2+C*c^2*d-D*c^3)/d/(-a*d+b*c)^3/(d*x+c)+(b*(3*A*d^2-2*B*c*d+C*c^
2)+a*(-B*d^2+2*C*c*d-3*D*c^2))*ln(b*x+a)/(-a*d+b*c)^4-(b*(3*A*d^2-2*B*c*d+
C*c^2)+a*(-B*d^2+2*C*c*d-3*D*c^2))*ln(d*x+c)/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx = \frac{-Ab^3+a(b^2B-abC+a^2D)}{2b^2(bc-ad)^2(a+bx)^2} + \frac{b^3(-Bc+2Ad)+ab^2(2cC-Bd)-3a^2bcD+a^3dD}{b^2(bc-ad)^3(a+bx)} + \frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{d(-bc+ad)^3(c+dx)} + \frac{(b(c^2C-2Bcd+3Ad^2)-a(-2cCd+Bd^2+3c^2D))\log(a+bx)}{(bc-ad)^4} + \frac{(-b(c^2C-2Bcd+3Ad^2)+a(-2cCd+Bd^2+3c^2D))\log(c+dx)}{(bc-ad)^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^2),x]
```

output
```
(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))/(2*b^2*(b*c - a*d)^2*(a + b*x)^2) +
 (b^3*(-(B*c) + 2*A*d) + a*b^2*(2*c*C - B*d) - 3*a^2*b*c*D + a^3*d*D)/(b^2
*(b*c - a*d)^3*(a + b*x)) + (-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)/(d*(-(b
*c) + a*d)^3*(c + d*x)) + ((b*(c^2*C - 2*B*c*d + 3*A*d^2) - a*(-2*c*C*d +
B*d^2 + 3*c^2*D))*Log[a + b*x])/(b*c - a*d)^4 + ((-(b*(c^2*C - 2*B*c*d + 3
*A*d^2)) + a*(-2*c*C*d + B*d^2 + 3*c^2*D))*Log[c + d*x])/(b*c - a*d)^4
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{Ab^3 - a(a^2D - abC + b^2B)}{b(a+bx)^3(bc-ad)^2} + \frac{a^3(-d)D + 3a^2bcD - ab^2(2cC - Bd) + b^3(Bc - 2Ad)}{b(a+bx)^2(bc-ad)^3} + \frac{b(a(-Bd^2 - 3c^2D\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^3 - a(a^2D - abC + b^2B)}{2b^2(a+bx)^2(bc-ad)^2} - \frac{a^3(-d)D + 3a^2bcD - ab^2(2cC - Bd) + b^3(Bc - 2Ad)}{b^2(a+bx)(bc-ad)^3} + \frac{\log(a+bx)\,(a(-Bd^2 - 3c^2D + 2cCd) + b(3Ad^2 - 2Bcd + c^2C))}{(bc-ad)^4} - \frac{\log(c+dx)\,(a(-Bd^2 - 3c^2D + 2cCd) + b(3Ad^2 - 2Bcd + c^2C))}{(bc-ad)^4} + \frac{Ad^3 - Bcd^2 + c^3(-D) + c^2Cd}{d(c+dx)(bc-ad)^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^2),x]
```

output
```
-1/2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b^2*(b*c - a*d)^2*(a + b*x)^2) -
 (b^3*(B*c - 2*A*d) - a*b^2*(2*c*C - B*d) + 3*a^2*b*c*D - a^3*d*D)/(b^2*(b
*c - a*d)^3*(a + b*x)) + (c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(d*(b*c - a*d
)^3*(c + d*x)) + ((b*(c^2*C - 2*B*c*d + 3*A*d^2) + a*(2*c*C*d - B*d^2 - 3*
c^2*D))*Log[a + b*x])/(b*c - a*d)^4 - ((b*(c^2*C - 2*B*c*d + 3*A*d^2) + a*
(2*c*C*d - B*d^2 - 3*c^2*D))*Log[c + d*x])/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.01

method	result
default	$-\frac{A d^3-B c d^2+C c^2 d-D c^3}{(ad-bc)^3 d(xd+c)}-\frac{(3b d^2 A-B a d^2-2Bbcd+2Cacd+Cb c^2-3a c^2 D)\ln(xd+c)}{(ad-bc)^4}+\frac{(3b d^2 A-B a d^2-2Bbcd+2C}{(ac}$
norman	$\frac{-\frac{2A a^2 b^2 d^3+5Aa b^3 c d^2-A b^4 c^2 d-5B a^2 b^2 c d^2-B a b^3 c^2 d+C a^3 bc d^2+5C a^2 b^2 c^2 d+D a^4 c d^2-5D a^3 b c^2 d-2D a^2 b^2 c^3}{2b^2 d\left(a^3 d^3-3a^2 bc d^2+3a b^2 c^2 d-b^3 c^3\right)}-\frac{(3A b^3 d^3-B a b}{}}{}$
parallelrisch	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^3/d/(d*x+c)-(3*A*b*d^2-B*a*d^2-2*
B*b*c*d+2*C*a*c*d+C*b*c^2-3*D*a*c^2)/(a*d-b*c)^4*ln(d*x+c)+(3*A*b*d^2-B*a*
d^2-2*B*b*c*d+2*C*a*c*d+C*b*c^2-3*D*a*c^2)/(a*d-b*c)^4*ln(b*x+a)-1/2*(A*b^
3-B*a*b^2+C*a^2*b-D*a^3)/b^2/(a*d-b*c)^2/(b*x+a)^2-(2*A*b^3*d-B*a*b^2*d-B*
b^3*c+2*C*a*b^2*c+D*a^3*d-3*D*a^2*b*c)/(a*d-b*c)^3/b^2/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. 2(275) = 550.

Time = 0.12 (sec) , antiderivative size = 1420, normalized size of antiderivative = 5.14

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*D*a^2*b^3*c^4 + 2*A*a^3*b^2*d^4 + (3*D*a^3*b^2 - 5*C*a^2*b^3 + B*a
*b^4 + A*b^5)*c^3*d - 2*(3*D*a^4*b - 2*C*a^3*b^2 - 2*B*a^2*b^3 + 3*A*a*b^4
)*c^2*d^2 + (D*a^5 + C*a^4*b - 5*B*a^3*b^2 + 3*A*a^2*b^3)*c*d^3 + 2*(D*b^5
*c^4 - (D*a*b^4 + C*b^5)*c^3*d + (3*D*a^2*b^3 - C*a*b^4 + 2*B*b^5)*c^2*d^2
 - (4*D*a^3*b^2 - 2*C*a^2*b^3 + B*a*b^4 + 3*A*b^5)*c*d^3 + (D*a^4*b - B*a^
2*b^3 + 3*A*a*b^4)*d^4)*x^2 + (4*D*a*b^4*c^4 + 2*(D*a^2*b^3 - 4*C*a*b^4 +
B*b^5)*c^3*d - (3*D*a^3*b^2 - 5*C*a^2*b^3 - 5*B*a*b^4 + 3*A*b^5)*c^2*d^2 -
 2*(2*D*a^4*b - C*a^3*b^2 + 2*B*a^2*b^3 + 3*A*a*b^4)*c*d^3 + (D*a^5 + C*a^
4*b - 3*B*a^3*b^2 + 9*A*a^2*b^3)*d^4)*x + 2*((3*D*a^3*b^2 - C*a^2*b^3)*c^3
*d - 2*(C*a^3*b^2 - B*a^2*b^3)*c^2*d^2 + (B*a^3*b^2 - 3*A*a^2*b^3)*c*d^3 +
 ((3*D*a*b^4 - C*b^5)*c^2*d^2 - 2*(C*a*b^4 - B*b^5)*c*d^3 + (B*a*b^4 - 3*A
*b^5)*d^4)*x^3 + ((3*D*a*b^4 - C*b^5)*c^3*d + 2*(3*D*a^2*b^3 - 2*C*a*b^4 +
 B*b^5)*c^2*d^2 - (4*C*a^2*b^3 - 5*B*a*b^4 + 3*A*b^5)*c*d^3 + 2*(B*a^2*b^3
 - 3*A*a*b^4)*d^4)*x^2 + (2*(3*D*a^2*b^3 - C*a*b^4)*c^3*d + (3*D*a^3*b^2 -
 5*C*a^2*b^3 + 4*B*a*b^4)*c^2*d^2 - 2*(C*a^3*b^2 - 2*B*a^2*b^3 + 3*A*a*b^4
)*c*d^3 + (B*a^3*b^2 - 3*A*a^2*b^3)*d^4)*x)*log(b*x + a) - 2*((3*D*a^3*b^2
 - C*a^2*b^3)*c^3*d - 2*(C*a^3*b^2 - B*a^2*b^3)*c^2*d^2 + (B*a^3*b^2 - 3*A
*a^2*b^3)*c*d^3 + ((3*D*a*b^4 - C*b^5)*c^2*d^2 - 2*(C*a*b^4 - B*b^5)*c*d^3
 + (B*a*b^4 - 3*A*b^5)*d^4)*x^3 + ((3*D*a*b^4 - C*b^5)*c^3*d + 2*(3*D*a^2*
b^3 - 2*C*a*b^4 + B*b^5)*c^2*d^2 - (4*C*a^2*b^3 - 5*B*a*b^4 + 3*A*b^5)*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. $2(253) = 506$.

Time = 42.77 (sec) , antiderivative size = 1912, normalized size of antiderivative = 6.93

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**2,x)
```

output
```
(-2*A*a**2*b**2*d**3 - 5*A*a*b**3*c*d**2 + A*b**4*c**2*d + 5*B*a**2*b**2*c
*d**2 + B*a*b**3*c**2*d - C*a**3*b*c*d**2 - 5*C*a**2*b**2*c**2*d - D*a**4*
c*d**2 + 5*D*a**3*b*c**2*d + 2*D*a**2*b**2*c**3 + x**2*(-6*A*b**4*d**3 + 2
*B*a*b**3*d**3 + 4*B*b**4*c*d**2 - 4*C*a*b**3*c*d**2 - 2*C*b**4*c**2*d - 2
*D*a**3*b*d**3 + 6*D*a**2*b**2*c*d**2 + 2*D*b**4*c**3) + x*(-9*A*a*b**3*d*
*3 - 3*A*b**4*c*d**2 + 3*B*a**2*b**2*d**3 + 7*B*a*b**3*c*d**2 + 2*B*b**4*c
**2*d - C*a**3*b*d**3 - 3*C*a**2*b**2*c*d**2 - 8*C*a*b**3*c**2*d - D*a**4*
d**3 + 3*D*a**3*b*c*d**2 + 6*D*a**2*b**2*c**2*d + 4*D*a*b**3*c**3))/(2*a**
5*b**2*c*d**4 - 6*a**4*b**3*c**2*d**3 + 6*a**3*b**4*c**3*d**2 - 2*a**2*b**
5*c**4*d + x**3*(2*a**3*b**4*d**5 - 6*a**2*b**5*c*d**4 + 6*a*b**6*c**2*d**
3 - 2*b**7*c**3*d**2) + x**2*(4*a**4*b**3*d**5 - 10*a**3*b**4*c*d**4 + 6*a
**2*b**5*c**2*d**3 + 2*a*b**6*c**3*d**2 - 2*b**7*c**4*d) + x*(2*a**5*b**2*
d**5 - 2*a**4*b**3*c*d**4 - 6*a**3*b**4*c**2*d**3 + 10*a**2*b**5*c**3*d**2
 - 4*a*b**6*c**4*d)) + (-3*A*b*d**2 + B*a*d**2 + 2*B*b*c*d - 2*C*a*c*d - C
*b*c**2 + 3*D*a*c**2)*log(x + (-3*A*a*b*d**3 - 3*A*b**2*c*d**2 + B*a**2*d*
*3 + 3*B*a*b*c*d**2 + 2*B*b**2*c**2*d - 2*C*a**2*c*d**2 - 3*C*a*b*c**2*d -
 C*b**2*c**3 + 3*D*a**2*c**2*d + 3*D*a*b*c**3 - a**5*d**5*(-3*A*b*d**2 + B
*a*d**2 + 2*B*b*c*d - 2*C*a*c*d - C*b*c**2 + 3*D*a*c**2)/(a*d - b*c)**4 +
5*a**4*b*c*d**4*(-3*A*b*d**2 + B*a*d**2 + 2*B*b*c*d - 2*C*a*c*d - C*b*c**2
 + 3*D*a*c**2)/(a*d - b*c)**4 - 10*a**3*b**2*c**2*d**3*(-3*A*b*d**2 + B...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(275) = 550$.

Time = 0.07 (sec) , antiderivative size = 703, normalized size of antiderivative = 2.55

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^2}\,dx$$
$$= -\frac{\left((3\,Da - Cb)c^2 - 2\,(Ca - Bb)cd + (Ba - 3\,Ab)d^2\right)\log\left(bx + a\right)}{b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4}$$
$$+ \frac{\left((3\,Da - Cb)c^2 - 2\,(Ca - Bb)cd + (Ba - 3\,Ab)d^2\right)\log\left(dx + c\right)}{b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4}$$
$$- \frac{2\,Da^2b^2c^3 - 2\,Aa^2b^2d^3 + (5\,Da^3b - 5\,Ca^2b^2 + Bab^3 + Ab^4)c^2d - (Da^4 + Ca^3b - 5\,Ba^2b^2 + 5\,Aab^3)cd \cdots}{2\,(a^2b^5c^4d - 3\,a^3b^4c^3d^2 + 3\,a^4b^3c^2d^3 - a^5b^2cd^4 \cdots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((3*D*a - C*b)*c^2 - 2*(C*a - B*b)*c*d + (B*a - 3*A*b)*d^2)*log(b*x + a)/
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) +
((3*D*a - C*b)*c^2 - 2*(C*a - B*b)*c*d + (B*a - 3*A*b)*d^2)*log(d*x + c)/(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1
/2*(2*D*a^2*b^2*c^3 - 2*A*a^2*b^2*d^3 + (5*D*a^3*b - 5*C*a^2*b^2 + B*a*b^3
 + A*b^4)*c^2*d - (D*a^4 + C*a^3*b - 5*B*a^2*b^2 + 5*A*a*b^3)*c*d^2 + 2*(D
*b^4*c^3 - C*b^4*c^2*d + (3*D*a^2*b^2 - 2*C*a*b^3 + 2*B*b^4)*c*d^2 - (D*a^
3*b - B*a*b^3 + 3*A*b^4)*d^3)*x^2 + (4*D*a*b^3*c^3 + 2*(3*D*a^2*b^2 - 4*C*
a*b^3 + B*b^4)*c^2*d + (3*D*a^3*b - 3*C*a^2*b^2 + 7*B*a*b^3 - 3*A*b^4)*c*d
^2 - (D*a^4 + C*a^3*b - 3*B*a^2*b^2 + 9*A*a*b^3)*d^3)*x)/(a^2*b^5*c^4*d -
3*a^3*b^4*c^3*d^2 + 3*a^4*b^3*c^2*d^3 - a^5*b^2*c*d^4 + (b^7*c^3*d^2 - 3*a
*b^6*c^2*d^3 + 3*a^2*b^5*c*d^4 - a^3*b^4*d^5)*x^3 + (b^7*c^4*d - a*b^6*c^3
*d^2 - 3*a^2*b^5*c^2*d^3 + 5*a^3*b^4*c*d^4 - 2*a^4*b^3*d^5)*x^2 + (2*a*b^6
*c^4*d - 5*a^2*b^5*c^3*d^2 + 3*a^3*b^4*c^2*d^3 + a^4*b^3*c*d^4 - a^5*b^2*d
^5)*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.63

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx$$
$$= -\frac{\left(3\,Dac^2d-Cbc^2d-2\,Cacd^2+2\,Bbcd^2+Bad^3-3\,Abd^3\right)\log\left(\left|b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right|\right)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5}$$
$$-\frac{\frac{Dc^3d^2}{dx+c}-\frac{Cc^2d^3}{dx+c}+\frac{Bcd^4}{dx+c}-\frac{Ad^5}{dx+c}}{b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6}$$
$$-\frac{6\,Da^2bcd-4\,Cab^2cd+2\,Bb^3cd-Da^3d^2-Ca^2bd^2+3\,Bab^2d^2-5\,Ab^3d^2-\frac{2\left(3\,Da^2bc^2d^2-2\,Cab^2c^2d^2+Bb^3c\right.}{}}{2\,(bc-ad)^4\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-(3*D*a*c^2*d - C*b*c^2*d - 2*C*a*c*d^2 + 2*B*b*c*d^2 + B*a*d^3 - 3*A*b*d^
3)*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^4*c^4*d - 4*a*b^3*c^3*d^
2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - (D*c^3*d^2/(d*x + c) -
C*c^2*d^3/(d*x + c) + B*c*d^4/(d*x + c) - A*d^5/(d*x + c))/(b^3*c^3*d^3 -
3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6) - 1/2*(6*D*a^2*b*c*d - 4*C*a*b^
2*c*d + 2*B*b^3*c*d - D*a^3*d^2 - C*a^2*b*d^2 + 3*B*a*b^2*d^2 - 5*A*b^3*d^
2 - 2*(3*D*a^2*b*c^2*d^2 - 2*C*a*b^2*c^2*d^2 + B*b^3*c^2*d^2 - 3*D*a^3*c*d
^3 + C*a^2*b*c*d^3 + B*a*b^2*c*d^3 - 3*A*b^3*c*d^3 + C*a^3*d^4 - 2*B*a^2*b
*d^4 + 3*A*a*b^2*d^4)/((d*x + c)*d))/((b*c - a*d)^4*(b - b*c/(d*x + c) + a
*d/(d*x + c))^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1223, normalized size of antiderivative = 4.43

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^2,x)
```

output
```
(8*log(a + b*x)*a**3*b**2*c*d**2 + 8*log(a + b*x)*a**3*b**2*d**3*x - 4*log
(a + b*x)*a**3*b*c**3*d - 4*log(a + b*x)*a**3*b*c**2*d**2*x + 4*log(a + b*
x)*a**2*b**3*c**2*d + 20*log(a + b*x)*a**2*b**3*c*d**2*x + 16*log(a + b*x)
*a**2*b**3*d**3*x**2 - 2*log(a + b*x)*a**2*b**2*c**4 - 10*log(a + b*x)*a**
2*b**2*c**3*d*x - 8*log(a + b*x)*a**2*b**2*c**2*d**2*x**2 + 8*log(a + b*x)
*a*b**4*c**2*d*x + 16*log(a + b*x)*a*b**4*c*d**2*x**2 + 8*log(a + b*x)*a*b
**4*d**3*x**3 - 4*log(a + b*x)*a*b**3*c**4*x - 8*log(a + b*x)*a*b**3*c**3*
d*x**2 - 4*log(a + b*x)*a*b**3*c**2*d**2*x**3 + 4*log(a + b*x)*b**5*c**2*d
*x**2 + 4*log(a + b*x)*b**5*c*d**2*x**3 - 2*log(a + b*x)*b**4*c**4*x**2 -
2*log(a + b*x)*b**4*c**3*d*x**3 - 8*log(c + d*x)*a**3*b**2*c*d**2 - 8*log(
c + d*x)*a**3*b**2*d**3*x + 4*log(c + d*x)*a**3*b*c**3*d + 4*log(c + d*x)*
a**3*b*c**2*d**2*x - 4*log(c + d*x)*a**2*b**3*c**2*d - 20*log(c + d*x)*a**
2*b**3*c*d**2*x - 16*log(c + d*x)*a**2*b**3*d**3*x**2 + 2*log(c + d*x)*a**
2*b**2*c**4 + 10*log(c + d*x)*a**2*b**2*c**3*d*x + 8*log(c + d*x)*a**2*b**
2*c**2*d**2*x**2 - 8*log(c + d*x)*a*b**4*c**2*d*x - 16*log(c + d*x)*a*b**4
*c*d**2*x**2 - 8*log(c + d*x)*a*b**4*d**3*x**3 + 4*log(c + d*x)*a*b**3*c**
4*x + 8*log(c + d*x)*a*b**3*c**3*d*x**2 + 4*log(c + d*x)*a*b**3*c**2*d**2*
x**3 - 4*log(c + d*x)*b**5*c**2*d*x**2 - 4*log(c + d*x)*b**5*c*d**2*x**3 +
 2*log(c + d*x)*b**4*c**4*x**2 + 2*log(c + d*x)*b**4*c**3*d*x**3 - 4*a**4*
b*d**3 + a**4*c**2*d**2 + a**4*c*d**3*x + 2*a**3*b**2*c*d**2 - 8*a**3*b...
```

3.41 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 397

$$\begin{aligned}
\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx
&= \frac{(a^3d^3D+3a^2bd^2(Cd-3cD)-3ab^2d(3cCd-Bd^2-6c^2D)+b^3(6c^2Cd-3Bcd^2+Ad^3-10c^3D))\,x}{d^6} \\
&+ \frac{b(3a^2d^2D+3abd(Cd-3cD)-b^2(3cCd-Bd^2-6c^2D))\,x^2}{2d^5} \\
&+ \frac{b^2(bCd-3bcD+3adD)x^3}{3d^4} + \frac{b^3Dx^4}{4d^3} + \frac{(bc-ad)^3\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)}{2d^7(c+dx)^2} \\
&+ \frac{(bc-ad)^2\left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right)}{d^7(c+dx)} \\
&- \frac{(bc-ad)\left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)\mathrm{lo}}{d^7}
\end{aligned}$$

output
```
(a^3*d^3*D+3*a^2*b*d^2*(C*d-3*D*c)-3*a*b^2*d*(-B*d^2+3*C*c*d-6*D*c^2)+b^3*
(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*x/d^6+1/2*b*(3*a^2*d^2*D+3*a*b*d*(C*
d-3*D*c)-b^2*(-B*d^2+3*C*c*d-6*D*c^2))*x^2/d^5+1/3*b^2*(C*b*d+3*D*a*d-3*D*
b*c)*x^3/d^4+1/4*b^3*D*x^4/d^3+1/2*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c
^3)/d^7/(d*x+c)^2+(-a*d+b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*
B*c*d^2+5*C*c^2*d-6*D*c^3))/d^7/(d*x+c)-(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*
b*d*(-3*B*d^2+8*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3
))*ln(d*x+c)/d^7
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3} dx$$
$$= \frac{12d(a^3d^3D+3a^2bd^2(Cd-3cD)+3ab^2d(-3cCd+Bd^2+6c^2D)+b^3(6c^2Cd-3Bcd^2+Ad^3-10c^3D)}{}$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
(12*d*(a^3*d^3*D + 3*a^2*b*d^2*(C*d - 3*c*D) + 3*a*b^2*d*(-3*c*C*d + B*d^2
 + 6*c^2*D) + b^3*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*x + 6*b*d^2*
(3*a^2*d^2*D + 3*a*b*d*(C*d - 3*c*D) + b^2*(-3*c*C*d + B*d^2 + 6*c^2*D))*x
^2 + 4*b^2*d^3*(b*C*d - 3*b*c*D + 3*a*d*D)*x^3 + 3*b^3*d^4*D*x^4 - (6*(b*c
 - a*d)^3*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(c + d*x)^2 + (12*(b*c -
 a*d)^2*(-(a*d*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-5*c^2*C*d + 4*B*c*d^2 -
 3*A*d^3 + 6*c^3*D)))/(c + d*x) + 12*(b*c - a*d)*(a^2*d^2*(-(C*d) + 3*c*D)
 + a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(-10*c^2*C*d + 6*B*c*d^2 - 3
*A*d^3 + 15*c^3*D))*Log[c + d*x])/(12*d^7)
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3} dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{(bc-ad)\left(-a^2d^2(Cd-3cD)+abd(-3Bd^2-15c^2D+8cCd)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)\right)}{d^6(c+dx)}\right.$$

$\downarrow$ 2009

$$-\frac{(bc-ad)\log(c+dx)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2C\right.\right.}{d^7}$$

$$\frac{bx^2\left(3a^2d^2D+3abd(Cd-3cD)-\left(b^2\left(-Bd^2-6c^2D+3cCd\right)\right)\right)}{2d^5}+\frac{x\left(a^3d^3D+3a^2bd^2(Cd-3cD)-3ab^2d\left(-Bd^2-6c^2D+3cCd\right)+b^3\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^6}+\frac{(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{d^7(c+dx)}+\frac{(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^7(c+dx)^2}+\frac{b^2x^3(3adD-3bcD+bCd)}{3d^4}+\frac{b^3Dx^4}{4d^3}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((a^3*d^3*D + 3*a^2*b*d^2*(C*d - 3*c*D) - 3*a*b^2*d*(3*c*C*d - B*d^2 - 6*c
^2*D) + b^3*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*x)/d^6 + (b*(3*a^2
*d^2*D + 3*a*b*d*(C*d - 3*c*D) - b^2*(3*c*C*d - B*d^2 - 6*c^2*D))*x^2)/(2*
d^5) + (b^2*(b*C*d - 3*b*c*D + 3*a*d*D)*x^3)/(3*d^4) + (b^3*D*x^4)/(4*d^3)
 + ((b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(2*d^7*(c + d*x)^2)
 + ((b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d - 4*B*c*
d^2 + 3*A*d^3 - 6*c^3*D)))/(d^7*(c + d*x)) - ((b*c - a*d)*(a^2*d^2*(C*d -
3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^2*C*d - 6*B*c*d^
2 + 3*A*d^3 - 15*c^3*D))*Log[c + d*x])/d^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.85

method	result
norman	$-\frac{A a^3 d^6+3A a^2 bc d^5-9Aa b^2 c^2 d^4+9A b^3 c^3 d^3+B a^3 c d^5-9B a^2 b c^2 d^4+27Ba b^2 c^3 d^3-18B b^3 c^4 d^2-3C a^3 c^2 d^4+27C a^2 b c^3 d^3-54Ca b^2 c}{2d^7}$
default	$\frac{1}{4}Dx^4 b^3 d^3+\frac{1}{3}C b^3 d^3 x^3+Dx^3 a b^2 d^3-Dx^3 b^3 c d^2+\frac{1}{2}b^3 B d^3 x^2+\frac{3}{2}C x^2 a b^2 d^3-\frac{3}{2}C x^2 b^3 c d^2+\frac{3}{2}Dx^2 a^2 b d^3-\frac{9}{2}Da b^2 c d^2 x^2+3D$
parallelrisch	Expression too large to display

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(A*a^3*d^6+3*A*a^2*b*c*d^5-9*A*a*b^2*c^2*d^4+9*A*b^3*c^3*d^3+B*a^3*c
*d^5-9*B*a^2*b*c^2*d^4+27*B*a*b^2*c^3*d^3-18*B*b^3*c^4*d^2-3*C*a^3*c^2*d^4
+27*C*a^2*b*c^3*d^3-54*C*a*b^2*c^4*d^2+30*C*b^3*c^5*d+9*D*a^3*c^3*d^3-54*D
*a^2*b*c^4*d^2+90*D*a*b^2*c^5*d-45*D*b^3*c^6)/d^7+1/3*(3*A*b^3*d^3+9*B*a*b
^2*d^3-6*B*b^3*c*d^2+9*C*a^2*b*d^3-18*C*a*b^2*c*d^2+10*C*b^3*c^2*d+3*D*a^3
*d^3-18*D*a^2*b*c*d^2+30*D*a*b^2*c^2*d-15*D*b^3*c^3)/d^4*x^3-(3*A*a^2*b*d^
5-6*A*a*b^2*c*d^4+6*A*b^3*c^2*d^3+B*a^3*d^5-6*B*a^2*b*c*d^4+18*B*a*b^2*c^2
*d^3-12*B*b^3*c^3*d^2-2*C*a^3*c*d^4+18*C*a^2*b*c^2*d^3-36*C*a*b^2*c^3*d^2+
20*C*b^3*c^4*d+6*D*a^3*c^2*d^3-36*D*a^2*b*c^3*d^2+60*D*a*b^2*c^4*d-30*D*b^
3*c^5)/d^6*x+1/4*D*b^3/d*x^6+1/12*b*(6*B*b^2*d^2+18*C*a*b*d^2-10*C*b^2*c*d
+18*D*a^2*d^2-30*D*a*b*c*d+15*D*b^2*c^2)/d^3*x^4+1/6*b^2*(2*C*b*d+6*D*a*d-
3*D*b*c)/d^2*x^5)/(d*x+c)^2+1/d^7*(3*A*a*b^2*d^4-3*A*b^3*c*d^3+3*B*a^2*b*d
^4-9*B*a*b^2*c*d^3+6*B*b^3*c^2*d^2+C*a^3*d^4-9*C*a^2*b*c*d^3+18*C*a*b^2*c^
2*d^2-10*C*b^3*c^3*d-3*D*a^3*c*d^3+18*D*a^2*b*c^2*d^2-30*D*a*b^2*c^3*d+15*
D*b^3*c^4)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. 2(391) = 782.

Time = 0.09 (sec) , antiderivative size = 978, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/12*(3*D*b^3*d^6*x^6 + 66*D*b^3*c^6 - 6*A*a^3*d^6 - 54*(3*D*a*b^2 + C*b^3
)*c^5*d + 42*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - 30*(D*a^3 + 3*C*a^2
*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + 18*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d
^4 - 6*(B*a^3 + 3*A*a^2*b)*c*d^5 - 2*(3*D*b^3*c*d^5 - 2*(3*D*a*b^2 + C*b^3
)*d^6)*x^5 + (15*D*b^3*c^2*d^4 - 10*(3*D*a*b^2 + C*b^3)*c*d^5 + 6*(3*D*a^2
*b + 3*C*a*b^2 + B*b^3)*d^6)*x^4 - 4*(15*D*b^3*c^3*d^3 - 10*(3*D*a*b^2 + C
*b^3)*c^2*d^4 + 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^5 - 3*(D*a^3 + 3*C*a
^2*b + 3*B*a*b^2 + A*b^3)*d^6)*x^3 - 6*(34*D*b^3*c^4*d^2 - 21*(3*D*a*b^2 +
 C*b^3)*c^3*d^3 + 11*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^4 - 4*(D*a^3 +
3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^5)*x^2 - 12*(4*D*b^3*c^5*d - (3*D*a*b^2
 + C*b^3)*c^4*d^2 - (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^3 + 2*(D*a^3 + 3
*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^4 - 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*
c*d^5 + (B*a^3 + 3*A*a^2*b)*d^6)*x + 12*(15*D*b^3*c^6 - 10*(3*D*a*b^2 + C*
b^3)*c^5*d + 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - 3*(D*a^3 + 3*C*a^
2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4
 + (15*D*b^3*c^4*d^2 - 10*(3*D*a*b^2 + C*b^3)*c^3*d^3 + 6*(3*D*a^2*b + 3*C
*a*b^2 + B*b^3)*c^2*d^4 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^5
+ (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^6)*x^2 + 2*(15*D*b^3*c^5*d - 10*(3*D*a
*b^2 + C*b^3)*c^4*d^2 + 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^3 - 3*(D*a
^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^4 + (C*a^3 + 3*B*a^2*b + 3*A*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(400) = 800$.

Time = 30.36 (sec) , antiderivative size = 814, normalized size of antiderivative = 2.05

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx = \frac{Db^3x^4}{4d^3} + x^3\left(\frac{Cb^3}{3d^3} + \frac{Dab^2}{d^3} - \frac{Db^3c}{d^4}\right) + x^2\left(\frac{Bb^3}{2d^3} + \frac{3Cab^2}{2d^3} - \frac{3Cb^3c}{2d^4} + \frac{3Da^2b}{2d^3} - \frac{9Dab^2c}{2d^4} + \frac{3Db^3c^2}{d^5}\right) + x\left(\frac{Ab^3}{d^3} + \frac{3Bab^2}{d^3} - \frac{3Bb^3c}{d^4} + \frac{3Ca^2b}{d^3} - \frac{9Cab^2c}{d^4} + \frac{6Cb^3c^2}{d^5} + \frac{Da^3}{d^3} - \frac{9Da^2bc}{d^4} + \frac{18Dab^2c^2}{d^5} - \frac{10Db^3c^3}{d^6}\right) + \frac{-Aa^3d^6 - 3Aa^2bcd^5 + 9Aab^2c^2d^4 - 5Ab^3c^3d^3 - Ba^3cd^5 + 9Ba^2bc^2d^4 - 15Bab^2c^3d^3 + 7Bb^3c^4d^2 + 3C}{} - \frac{(ad-bc)\left(-3Ab^2d^3 - 3Babd^3 + 6Bb^2cd^2 - Ca^2d^3 + 8Cabcd^2 - 10Cb^2c^2d + 3Da^2cd^2 - 15Dabc^2d + \right.}{d^7}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
D*b**3*x**4/(4*d**3) + x**3*(C*b**3/(3*d**3) + D*a*b**2/d**3 - D*b**3*c/d*
*4) + x**2*(B*b**3/(2*d**3) + 3*C*a*b**2/(2*d**3) - 3*C*b**3*c/(2*d**4) +
3*D*a**2*b/(2*d**3) - 9*D*a*b**2*c/(2*d**4) + 3*D*b**3*c**2/d**5) + x*(A*b
**3/d**3 + 3*B*a*b**2/d**3 - 3*B*b**3*c/d**4 + 3*C*a**2*b/d**3 - 9*C*a*b**
2*c/d**4 + 6*C*b**3*c**2/d**5 + D*a**3/d**3 - 9*D*a**2*b*c/d**4 + 18*D*a*b
**2*c**2/d**5 - 10*D*b**3*c**3/d**6) + (-A*a**3*d**6 - 3*A*a**2*b*c*d**5 +
 9*A*a*b**2*c**2*d**4 - 5*A*b**3*c**3*d**3 - B*a**3*c*d**5 + 9*B*a**2*b*c*
*2*d**4 - 15*B*a*b**2*c**3*d**3 + 7*B*b**3*c**4*d**2 + 3*C*a**3*c**2*d**4
- 15*C*a**2*b*c**3*d**3 + 21*C*a*b**2*c**4*d**2 - 9*C*b**3*c**5*d - 5*D*a*
*3*c**3*d**3 + 21*D*a**2*b*c**4*d**2 - 27*D*a*b**2*c**5*d + 11*D*b**3*c**6
 + x*(-6*A*a**2*b*d**6 + 12*A*a*b**2*c*d**5 - 6*A*b**3*c**2*d**4 - 2*B*a**
3*d**6 + 12*B*a**2*b*c*d**5 - 18*B*a*b**2*c**2*d**4 + 8*B*b**3*c**3*d**3 +
 4*C*a**3*c*d**5 - 18*C*a**2*b*c**2*d**4 + 24*C*a*b**2*c**3*d**3 - 10*C*b*
*3*c**4*d**2 - 6*D*a**3*c**2*d**4 + 24*D*a**2*b*c**3*d**3 - 30*D*a*b**2*c*
*4*d**2 + 12*D*b**3*c**5*d))/(2*c**2*d**7 + 4*c*d**8*x + 2*d**9*x**2) - (a
*d - b*c)*(-3*A*b**2*d**3 - 3*B*a*b*d**3 + 6*B*b**2*c*d**2 - C*a**2*d**3 +
 8*C*a*b*c*d**2 - 10*C*b**2*c**2*d + 3*D*a**2*c*d**2 - 15*D*a*b*c**2*d + 1
5*D*b**2*c**3)*log(c + d*x)/d**7
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$
$$= \frac{11\,Db^3c^6 - Aa^3d^6 - 9\,(3\,Dab^2 + Cb^3)c^5d + 7\,(3\,Da^2b + 3\,Cab^2 + Bb^3)c^4d^2 - 5\,(Da^3 + 3\,Ca^2b + 3\,Bab^2}{}$$
$$+ \frac{3\,Db^3d^3x^4 - 4\,(3\,Db^3cd^2 - (3\,Dab^2 + Cb^3)d^3)x^3 + 6\,(6\,Db^3c^2d - 3\,(3\,Dab^2 + Cb^3)cd^2 + (3\,Da^2b + 3\,C}{}$$
$$+ \frac{(15\,Db^3c^4 - 10\,(3\,Dab^2 + Cb^3)c^3d + 6\,(3\,Da^2b + 3\,Cab^2 + Bb^3)c^2d^2 - 3\,(Da^3 + 3\,Ca^2b + 3\,Bab^2 + A}{d^7}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*(11*D*b^3*c^6 - A*a^3*d^6 - 9*(3*D*a*b^2 + C*b^3)*c^5*d + 7*(3*D*a^2*b
 + 3*C*a*b^2 + B*b^3)*c^4*d^2 - 5*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*
c^3*d^3 + 3*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*
c*d^5 + 2*(6*D*b^3*c^5*d - 5*(3*D*a*b^2 + C*b^3)*c^4*d^2 + 4*(3*D*a^2*b +
3*C*a*b^2 + B*b^3)*c^3*d^3 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2
*d^4 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^5 - (B*a^3 + 3*A*a^2*b)*d^6)*
x)/(d^9*x^2 + 2*c*d^8*x + c^2*d^7) + 1/12*(3*D*b^3*d^3*x^4 - 4*(3*D*b^3*c*
d^2 - (3*D*a*b^2 + C*b^3)*d^3)*x^3 + 6*(6*D*b^3*c^2*d - 3*(3*D*a*b^2 + C*b
^3)*c*d^2 + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^3)*x^2 - 12*(10*D*b^3*c^3 -
6*(3*D*a*b^2 + C*b^3)*c^2*d + 3*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^2 - (D
*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*x)/d^6 + (15*D*b^3*c^4 - 10*(3*
D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^2 - 3*(D*
a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^
2)*d^4)*log(d*x + c)/d^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.97

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$
$$= \frac{(15\,Db^3c^4 - 30\,Dab^2c^3d - 10\,Cb^3c^3d + 18\,Da^2bc^2d^2 + 18\,Cab^2c^2d^2 + 6\,Bb^3c^2d^2 - 3\,Da^3cd^3 - 9\,Ca^2bcd^3}{d^7}$$
$$+ \frac{11\,Db^3c^6 - 27\,Dab^2c^5d - 9\,Cb^3c^5d + 21\,Da^2bc^4d^2 + 21\,Cab^2c^4d^2 + 7\,Bb^3c^4d^2 - 5\,Da^3c^3d^3 - 15\,Ca^2bc}{}$$
$$+ \frac{3\,Db^3d^9x^4 - 12\,Db^3cd^8x^3 + 12\,Dab^2d^9x^3 + 4\,Cb^3d^9x^3 + 36\,Db^3c^2d^7x^2 - 54\,Dab^2cd^8x^2 - 18\,Cb^3cd^8x^2}{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac")
```

output
```
(15*D*b^3*c^4 - 30*D*a*b^2*c^3*d - 10*C*b^3*c^3*d + 18*D*a^2*b*c^2*d^2 + 1
8*C*a*b^2*c^2*d^2 + 6*B*b^3*c^2*d^2 - 3*D*a^3*c*d^3 - 9*C*a^2*b*c*d^3 - 9*
B*a*b^2*c*d^3 - 3*A*b^3*c*d^3 + C*a^3*d^4 + 3*B*a^2*b*d^4 + 3*A*a*b^2*d^4)
*log(abs(d*x + c))/d^7 + 1/2*(11*D*b^3*c^6 - 27*D*a*b^2*c^5*d - 9*C*b^3*c^
5*d + 21*D*a^2*b*c^4*d^2 + 21*C*a*b^2*c^4*d^2 + 7*B*b^3*c^4*d^2 - 5*D*a^3*
c^3*d^3 - 15*C*a^2*b*c^3*d^3 - 15*B*a*b^2*c^3*d^3 - 5*A*b^3*c^3*d^3 + 3*C*
a^3*c^2*d^4 + 9*B*a^2*b*c^2*d^4 + 9*A*a*b^2*c^2*d^4 - B*a^3*c*d^5 - 3*A*a^
2*b*c*d^5 - A*a^3*d^6 + 2*(6*D*b^3*c^5*d - 15*D*a*b^2*c^4*d^2 - 5*C*b^3*c^
4*d^2 + 12*D*a^2*b*c^3*d^3 + 12*C*a*b^2*c^3*d^3 + 4*B*b^3*c^3*d^3 - 3*D*a^
3*c^2*d^4 - 9*C*a^2*b*c^2*d^4 - 9*B*a*b^2*c^2*d^4 - 3*A*b^3*c^2*d^4 + 2*C*
a^3*c*d^5 + 6*B*a^2*b*c*d^5 + 6*A*a*b^2*c*d^5 - B*a^3*d^6 - 3*A*a^2*b*d^6)
*x)/((d*x + c)^2*d^7) + 1/12*(3*D*b^3*d^9*x^4 - 12*D*b^3*c*d^8*x^3 + 12*D*
a*b^2*d^9*x^3 + 4*C*b^3*d^9*x^3 + 36*D*b^3*c^2*d^7*x^2 - 54*D*a*b^2*c*d^8*
x^2 - 18*C*b^3*c*d^8*x^2 + 18*D*a^2*b*d^9*x^2 + 18*C*a*b^2*d^9*x^2 + 6*B*b
^3*d^9*x^2 - 120*D*b^3*c^3*d^6*x + 216*D*a*b^2*c^2*d^7*x + 72*C*b^3*c^2*d^
7*x - 108*D*a^2*b*c*d^8*x - 108*C*a*b^2*c*d^8*x - 36*B*b^3*c*d^8*x + 12*D*
a^3*d^9*x + 36*C*a^2*b*d^9*x + 36*B*a*b^2*d^9*x + 12*A*b^3*d^9*x)/d^12
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \int \frac{(a+b\,x)^3\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^3}\,dx$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 794, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\, dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
( - 24*log(c + d*x)*a**3*c**4*d**3 - 48*log(c + d*x)*a**3*c**3*d**4*x - 24
*log(c + d*x)*a**3*c**2*d**5*x**2 + 72*log(c + d*x)*a**2*b**2*c**3*d**3 +
144*log(c + d*x)*a**2*b**2*c**2*d**4*x + 72*log(c + d*x)*a**2*b**2*c*d**5*
x**2 + 108*log(c + d*x)*a**2*b*c**5*d**2 + 216*log(c + d*x)*a**2*b*c**4*d*
*3*x + 108*log(c + d*x)*a**2*b*c**3*d**4*x**2 - 144*log(c + d*x)*a*b**3*c*
*4*d**2 - 288*log(c + d*x)*a*b**3*c**3*d**3*x - 144*log(c + d*x)*a*b**3*c*
*2*d**4*x**2 - 144*log(c + d*x)*a*b**2*c**6*d - 288*log(c + d*x)*a*b**2*c*
*5*d**2*x - 144*log(c + d*x)*a*b**2*c**4*d**3*x**2 + 72*log(c + d*x)*b**4*
c**5*d + 144*log(c + d*x)*b**4*c**4*d**2*x + 72*log(c + d*x)*b**4*c**3*d**
3*x**2 + 60*log(c + d*x)*b**3*c**7 + 120*log(c + d*x)*b**3*c**6*d*x + 60*l
og(c + d*x)*b**3*c**5*d**2*x**2 - 6*a**4*c*d**5 + 24*a**3*b*d**6*x**2 - 12
*a**3*c**4*d**3 + 24*a**3*c**2*d**5*x**2 + 12*a**3*c*d**6*x**3 + 36*a**2*b
**2*c**3*d**3 - 72*a**2*b**2*c*d**5*x**2 + 54*a**2*b*c**5*d**2 - 108*a**2*
b*c**3*d**4*x**2 - 36*a**2*b*c**2*d**5*x**3 + 18*a**2*b*c*d**6*x**4 - 72*a
*b**3*c**4*d**2 + 144*a*b**3*c**2*d**4*x**2 + 48*a*b**3*c*d**5*x**3 - 72*a
*b**2*c**6*d + 144*a*b**2*c**4*d**3*x**2 + 48*a*b**2*c**3*d**4*x**3 - 12*a
*b**2*c**2*d**5*x**4 + 12*a*b**2*c*d**6*x**5 + 36*b**4*c**5*d - 72*b**4*c*
*3*d**3*x**2 - 24*b**4*c**2*d**4*x**3 + 6*b**4*c*d**5*x**4 + 30*b**3*c**7
- 60*b**3*c**5*d**2*x**2 - 20*b**3*c**4*d**3*x**3 + 5*b**3*c**3*d**4*x**4
- 2*b**3*c**2*d**5*x**5 + 3*b**3*c*d**6*x**6)/(12*c*d**6*(c**2 + 2*c*d*...
```

3.42 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 291

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx = \frac{\left(a^2d^2D+2abd(Cd-3cD)-b^2\left(3cCd-Bd^2-6c^2D\right)\right)x}{d^5} + \frac{b(bCd-3bcD+2adD)x^2}{2d^4} + \frac{b^2Dx^3}{3d^3} - \frac{(bc-ad)^2\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)}{2d^6(c+dx)^2} - \frac{(bc-ad)\left(ad\left(2cCd-Bd^2-3c^2D\right)-b\left(4c^2Cd-3Bcd^2+2Ad^3-5c^3D\right)\right)}{d^6(c+dx)} + \frac{\left(a^2d^2(Cd-3cD)-2abd\left(3cCd-Bd^2-6c^2D\right)+b^2\left(6c^2Cd-3Bcd^2+Ad^3-10c^3D\right)\right)\log(c+dx)}{d^6}$$

output
```
(a^2*d^2*D+2*a*b*d*(C*d-3*D*c)-b^2*(-B*d^2+3*C*c*d-6*D*c^2))*x/d^5+1/2*b*(
C*b*d+2*D*a*d-3*D*b*c)*x^2/d^4+1/3*b^2*D*x^3/d^3-1/2*(-a*d+b*c)^2*(A*d^3-B
*c*d^2+C*c^2*d-D*c^3)/d^6/(d*x+c)^2-(-a*d+b*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^
2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))/d^6/(d*x+c)+(a^2*d^2*(C*d-3*D*
c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*c^2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^
3))*ln(d*x+c)/d^6
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3} \, dx$$
$$= \frac{6d(a^2d^2D+2abd(Cd-3cD)+b^2(-3cCd+Bd^2+6c^2D))\,x+3bd^2(bCd-3bcD+2adD)x^2+2b^2d^3D}{}$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
(6*d*(a^2*d^2*D + 2*a*b*d*(C*d - 3*c*D) + b^2*(-3*c*C*d + B*d^2 + 6*c^2*D)
)*x + 3*b*d^2*(b*C*d - 3*b*c*D + 2*a*d*D)*x^2 + 2*b^2*d^3*D*x^3 + (3*(b*c
- a*d)^2*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(c + d*x)^2 - (6*(b*c - a
*d)*(-(a*d*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-4*c^2*C*d + 3*B*c*d^2 - 2*A
*d^3 + 5*c^3*D)))/(c + d*x) + 6*(a^2*d^2*(C*d - 3*c*D) + 2*a*b*d*(-3*c*C*d
 + B*d^2 + 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))*Log[
c + d*x])/(6*d^6)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3} \, dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)}{d^5(c+dx)}+\frac{a^2d^2D+2abd(C}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\log(c+dx)\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^6}+\frac{x\left(a^2d^2D+2abd(Cd-3cD)-\left(b^2\left(-Bd^2-6c^2D+3cCd\right)\right)\right)}{d^5}-\frac{(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{d^6(c+dx)}-\frac{(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^6(c+dx)^2}+\frac{bx^2(2adD-3bcD+bCd)}{2d^4}+\frac{b^2Dx^3}{3d^3}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((a^2*d^2*D + 2*a*b*d*(C*d - 3*c*D) - b^2*(3*c*C*d - B*d^2 - 6*c^2*D))*x)/
d^5 + (b*(b*C*d - 3*b*c*D + 2*a*d*D)*x^2)/(2*d^4) + (b^2*D*x^3)/(3*d^3) -
((b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(2*d^6*(c + d*x)^2) -
((b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B*c*d^2 +
 2*A*d^3 - 5*c^3*D)))/(d^6*(c + d*x)) + ((a^2*d^2*(C*d - 3*c*D) - 2*a*b*d*
(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*
D))*Log[c + d*x])/d^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.53

method	result
norman	$-\frac{A a^2 d^5+2Aabc\, d^4-3A\, b^2 c^2 d^3+B\, a^2 c\, d^4-6Bab\, c^2 d^3+9B\, b^2 c^3 d^2-3C\, a^2 c^2 d^3+18Cab\, c^3 d^2-18C\, b^2 c^4 d+9Da^2 c^3 d^2-36Dab\, c^4 d+30Db^2 c}{2d^6}$
default	$\frac{\frac{1}{3}d^2 Dx^3 b^2+\frac{1}{2}C\, b^2 d^2 x^2+Dab\, d^2 x^2-\frac{3}{2}Db^2 cd\, x^2+b^2 B\, d^2 x+2Cab\, d^2 x-3C\, b^2 cdx+a^2 d^2 Dx-6Dabcdx+6Db^2 c^2 x}{d^5}-\frac{2Aab\, d^4}{}$
parallelrisch	$\frac{-3B\, a^2 c\, d^4+9A\, b^2 c^2 d^3-27B\, b^2 c^3 d^2+9C\, a^2 c^2 d^3-27Da^2 c^3 d^2-36B\, b^2 c^2 d^3 x+72C\, b^2 c^3 d^2 x-36Da^2 c^2 d^3 x-120Db^2 c^4 dx-6A}{}$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*(A*a^2*d^5+2*A*a*b*c*d^4-3*A*b^2*c^2*d^3+B*a^2*c*d^4-6*B*a*b*c^2*d^3
+9*B*b^2*c^3*d^2-3*C*a^2*c^2*d^3+18*C*a*b*c^3*d^2-18*C*b^2*c^4*d+9*D*a^2*c
^3*d^2-36*D*a*b*c^4*d+30*D*b^2*c^5)/d^6+1/3*(3*B*b^2*d^2+6*C*a*b*d^2-6*C*b
^2*c*d+3*D*a^2*d^2-12*D*a*b*c*d+10*D*b^2*c^2)/d^3*x^3-(2*A*a*b*d^4-2*A*b^2
*c*d^3+B*a^2*d^4-4*B*a*b*c*d^3+6*B*b^2*c^2*d^2-2*C*a^2*c*d^3+12*C*a*b*c^2*
d^2-12*C*b^2*c^3*d+6*D*a^2*c^2*d^2-24*D*a*b*c^3*d+20*D*b^2*c^4)/d^5*x+1/3*
D*b^2/d*x^5+1/6*b*(3*C*b*d+6*D*a*d-5*D*b*c)/d^2*x^4)/(d*x+c)^2+1/d^6*(A*b^
2*d^3+2*B*a*b*d^3-3*B*b^2*c*d^2+C*a^2*d^3-6*C*a*b*c*d^2+6*C*b^2*c^2*d-3*D*
a^2*c*d^2+12*D*a*b*c^2*d-10*D*b^2*c^3)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(285) = 570$.

Time = 0.08 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.13

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\, dx$$
$$= \frac{2\, Db^2 d^5 x^5 - 27\, Db^2 c^5 - 3\, Aa^2 d^5 + 21\, (2\, Dab + Cb^2)c^4 d - 15\, (Da^2 + 2\, Cab + Bb^2)c^3 d^2 + 9\, (Ca^2 + 2\, Ba}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/6*(2*D*b^2*d^5*x^5 - 27*D*b^2*c^5 - 3*A*a^2*d^5 + 21*(2*D*a*b + C*b^2)*c
^4*d - 15*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 + 9*(C*a^2 + 2*B*a*b + A*b^2)*
c^2*d^3 - 3*(B*a^2 + 2*A*a*b)*c*d^4 - (5*D*b^2*c*d^4 - 3*(2*D*a*b + C*b^2)
*d^5)*x^4 + 2*(10*D*b^2*c^2*d^3 - 6*(2*D*a*b + C*b^2)*c*d^4 + 3*(D*a^2 + 2
*C*a*b + B*b^2)*d^5)*x^3 + 3*(21*D*b^2*c^3*d^2 - 11*(2*D*a*b + C*b^2)*c^2*
d^3 + 4*(D*a^2 + 2*C*a*b + B*b^2)*c*d^4)*x^2 + 6*(D*b^2*c^4*d + (2*D*a*b +
 C*b^2)*c^3*d^2 - 2*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 + 2*(C*a^2 + 2*B*a*b
 + A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*x - 6*(10*D*b^2*c^5 - 6*(2*D*a*b
+ C*b^2)*c^4*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 + 2*B*a*b +
A*b^2)*c^2*d^3 + (10*D*b^2*c^3*d^2 - 6*(2*D*a*b + C*b^2)*c^2*d^3 + 3*(D*a^
2 + 2*C*a*b + B*b^2)*c*d^4 - (C*a^2 + 2*B*a*b + A*b^2)*d^5)*x^2 + 2*(10*D*
b^2*c^4*d - 6*(2*D*a*b + C*b^2)*c^3*d^2 + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2*
d^3 - (C*a^2 + 2*B*a*b + A*b^2)*c*d^4)*x)*log(d*x + c))/(d^8*x^2 + 2*c*d^7
*x + c^2*d^6)
```

Sympy [A] (verification not implemented)

Time = 17.05 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx = \frac{Db^2x^3}{3d^3} + x^2\left(\frac{Cb^2}{2d^3} + \frac{Dab}{d^3} - \frac{3Db^2c}{2d^4}\right) + x\left(\frac{Bb^2}{d^3} + \frac{2Cab}{d^3} - \frac{3Cb^2c}{d^4} + \frac{Da^2}{d^3} - \frac{6Dabc}{d^4} + \frac{6Db^2c^2}{d^5}\right) + \frac{-Aa^2d^5 - 2Aabcd^4 + 3Ab^2c^2d^3 - Ba^2cd^4 + 6Babc^2d^3 - 5Bb^2c^3d^2 + 3Ca^2c^2d^3 - 10Cabc^3d^2 + 7Cb^2c^{\cdots}}{\cdots} - \frac{(-Ab^2d^3 - 2Babd^3 + 3Bb^2cd^2 - Ca^2d^3 + 6Cabcd^2 - 6Cb^2c^2d + 3Da^2cd^2 - 12Dabc^2d + 10Db^2c^3)\log\cdots}{d^6}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
D*b**2*x**3/(3*d**3) + x**2*(C*b**2/(2*d**3) + D*a*b/d**3 - 3*D*b**2*c/(2*
d**4)) + x*(B*b**2/d**3 + 2*C*a*b/d**3 - 3*C*b**2*c/d**4 + D*a**2/d**3 - 6
*D*a*b*c/d**4 + 6*D*b**2*c**2/d**5) + (-A*a**2*d**5 - 2*A*a*b*c*d**4 + 3*A
*b**2*c**2*d**3 - B*a**2*c*d**4 + 6*B*a*b*c**2*d**3 - 5*B*b**2*c**3*d**2 +
 3*C*a**2*c**2*d**3 - 10*C*a*b*c**3*d**2 + 7*C*b**2*c**4*d - 5*D*a**2*c**3
*d**2 + 14*D*a*b*c**4*d - 9*D*b**2*c**5 + x*(-4*A*a*b*d**5 + 4*A*b**2*c*d*
*4 - 2*B*a**2*d**5 + 8*B*a*b*c*d**4 - 6*B*b**2*c**2*d**3 + 4*C*a**2*c*d**4
 - 12*C*a*b*c**2*d**3 + 8*C*b**2*c**3*d**2 - 6*D*a**2*c**2*d**3 + 16*D*a*b
*c**3*d**2 - 10*D*b**2*c**4*d))/(2*c**2*d**6 + 4*c*d**7*x + 2*d**8*x**2) -
 (-A*b**2*d**3 - 2*B*a*b*d**3 + 3*B*b**2*c*d**2 - C*a**2*d**3 + 6*C*a*b*c*
d**2 - 6*C*b**2*c**2*d + 3*D*a**2*c*d**2 - 12*D*a*b*c**2*d + 10*D*b**2*c**
3)*log(c + d*x)/d**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx =$$

$$-\frac{9\,Db^2c^5+Aa^2d^5-7\,(2\,Dab+Cb^2)c^4d+5\,(Da^2+2\,Cab+Bb^2)c^3d^2-3\,(Ca^2+2\,Bab+Ab^2)c^2d^3+}{}$$

$$+\frac{2\,Db^2d^2x^3-3\,(3\,Db^2cd-(2\,Dab+Cb^2)d^2)x^2+6\,(6\,Db^2c^2-3\,(2\,Dab+Cb^2)cd+(Da^2+2\,Cab+B}{6\,d^5}$$

$$-\frac{(10\,Db^2c^3-6\,(2\,Dab+Cb^2)c^2d+3\,(Da^2+2\,Cab+Bb^2)cd^2-(Ca^2+2\,Bab+Ab^2)d^3)\log(dx+c)}{d^6}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(9*D*b^2*c^5 + A*a^2*d^5 - 7*(2*D*a*b + C*b^2)*c^4*d + 5*(D*a^2 + 2*C
*a*b + B*b^2)*c^3*d^2 - 3*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A
*a*b)*c*d^4 + 2*(5*D*b^2*c^4*d - 4*(2*D*a*b + C*b^2)*c^3*d^2 + 3*(D*a^2 +
2*C*a*b + B*b^2)*c^2*d^3 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + (B*a^2 + 2*
A*a*b)*d^5)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6) + 1/6*(2*D*b^2*d^2*x^3 - 3*
(3*D*b^2*c*d - (2*D*a*b + C*b^2)*d^2)*x^2 + 6*(6*D*b^2*c^2 - 3*(2*D*a*b +
C*b^2)*c*d + (D*a^2 + 2*C*a*b + B*b^2)*d^2)*x)/d^5 - (10*D*b^2*c^3 - 6*(2*
D*a*b + C*b^2)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*
b + A*b^2)*d^3)*log(d*x + c)/d^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx =$$
$$-\frac{(10\,Db^2c^3-12\,Dabc^2d-6\,Cb^2c^2d+3\,Da^2cd^2+6\,Cabcd^2+3\,Bb^2cd^2-Ca^2d^3-2\,Babd^3-Ab^2d^3)\,\mathrm{lo}}{d^6}$$
$$-\frac{9\,Db^2c^5-14\,Dabc^4d-7\,Cb^2c^4d+5\,Da^2c^3d^2+10\,Cabc^3d^2+5\,Bb^2c^3d^2-3\,Ca^2c^2d^3-6\,Babc^2d^3-3}{}$$
$$+\frac{2\,Db^2d^6x^3-9\,Db^2cd^5x^2+6\,Dabd^6x^2+3\,Cb^2d^6x^2+36\,Db^2c^2d^4x-36\,Dabcd^5x-18\,Cb^2cd^5x+6\,Da}{6\,d^9}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(10*D*b^2*c^3 - 12*D*a*b*c^2*d - 6*C*b^2*c^2*d + 3*D*a^2*c*d^2 + 6*C*a*b*
c*d^2 + 3*B*b^2*c*d^2 - C*a^2*d^3 - 2*B*a*b*d^3 - A*b^2*d^3)*log(abs(d*x +
 c))/d^6 - 1/2*(9*D*b^2*c^5 - 14*D*a*b*c^4*d - 7*C*b^2*c^4*d + 5*D*a^2*c^3
*d^2 + 10*C*a*b*c^3*d^2 + 5*B*b^2*c^3*d^2 - 3*C*a^2*c^2*d^3 - 6*B*a*b*c^2*
d^3 - 3*A*b^2*c^2*d^3 + B*a^2*c*d^4 + 2*A*a*b*c*d^4 + A*a^2*d^5 + 2*(5*D*b
^2*c^4*d - 8*D*a*b*c^3*d^2 - 4*C*b^2*c^3*d^2 + 3*D*a^2*c^2*d^3 + 6*C*a*b*c
^2*d^3 + 3*B*b^2*c^2*d^3 - 2*C*a^2*c*d^4 - 4*B*a*b*c*d^4 - 2*A*b^2*c*d^4 +
 B*a^2*d^5 + 2*A*a*b*d^5)*x)/((d*x + c)^2*d^6) + 1/6*(2*D*b^2*d^6*x^3 - 9*
D*b^2*c*d^5*x^2 + 6*D*a*b*d^6*x^2 + 3*C*b^2*d^6*x^2 + 36*D*b^2*c^2*d^4*x -
 36*D*a*b*c*d^5*x - 18*C*b^2*c*d^5*x + 6*D*a^2*d^6*x + 12*C*a*b*d^6*x + 6*
B*b^2*d^6*x)/d^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx = \int \frac{{\left(a+b\,x\right)}^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$$
$$= \frac{36\log(dx+c)\,a\,b^2c^2d^3x+72\log(dx+c)\,ab\,c^4d^2x+36\log(dx+c)\,ab\,c^3d^3x^2-24\log(dx+c)\,b^2c^6+9a^2}{}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
( - 12*log(c + d*x)*a**2*c**4*d**2 - 24*log(c + d*x)*a**2*c**3*d**3*x - 12
*log(c + d*x)*a**2*c**2*d**4*x**2 + 18*log(c + d*x)*a*b**2*c**3*d**2 + 36*
log(c + d*x)*a*b**2*c**2*d**3*x + 18*log(c + d*x)*a*b**2*c*d**4*x**2 + 36*
log(c + d*x)*a*b*c**5*d + 72*log(c + d*x)*a*b*c**4*d**2*x + 36*log(c + d*x
)*a*b*c**3*d**3*x**2 - 18*log(c + d*x)*b**3*c**4*d - 36*log(c + d*x)*b**3*
c**3*d**2*x - 18*log(c + d*x)*b**3*c**2*d**3*x**2 - 24*log(c + d*x)*b**2*c
**6 - 48*log(c + d*x)*b**2*c**5*d*x - 24*log(c + d*x)*b**2*c**4*d**2*x**2
- 3*a**3*c*d**4 + 9*a**2*b*d**5*x**2 - 6*a**2*c**4*d**2 + 12*a**2*c**2*d**
4*x**2 + 6*a**2*c*d**5*x**3 + 9*a*b**2*c**3*d**2 - 18*a*b**2*c*d**4*x**2 +
 18*a*b*c**5*d - 36*a*b*c**3*d**3*x**2 - 12*a*b*c**2*d**4*x**3 + 6*a*b*c*d
**5*x**4 - 9*b**3*c**4*d + 18*b**3*c**2*d**3*x**2 + 6*b**3*c*d**4*x**3 - 1
2*b**2*c**6 + 24*b**2*c**4*d**2*x**2 + 8*b**2*c**3*d**3*x**3 - 2*b**2*c**2
*d**4*x**4 + 2*b**2*c*d**5*x**5)/(6*c*d**5*(c**2 + 2*c*d*x + d**2*x**2))
```

3.43 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 184

$$\begin{aligned}\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^3}\,dx &= \frac{(bCd-3bcD+adD)x}{d^4}+\frac{bDx^2}{2d^3}+\frac{(bc-ad)(c^2Cd-Bcd^2+Ad^3-c^3D)}{2d^5(c+dx)^2}\\ &\quad+\frac{ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D)}{d^5(c+dx)}\\ &\quad+\frac{(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))\log(c+dx)}{d^5}\end{aligned}$$

output
```
(C*b*d+D*a*d-3*D*b*c)*x/d^4+1/2*b*D*x^2/d^3+1/2*(-a*d+b*c)*(A*d^3-B*c*d^2+
C*c^2*d-D*c^3)/d^5/(d*x+c)^2+(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*
d^2+3*C*c^2*d-4*D*c^3))/d^5/(d*x+c)+(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D
*c^2))*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx = \frac{2d(bCd-3bcD+adD)x+bd^2Dx^2-\frac{(bc-ad)\left(-c^2Cd+Bcd^2-Ad^3+c^3D\right)}{(c+dx)^2}-\frac{2\left(ad\left(-2cCd+Bd^2+3c^2D\right)+b\left(3c^2Cd-2Bcd^2+\right.\right.}{c+dx}}{2d^5}$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
(2*d*(b*C*d - 3*b*c*D + a*d*D)*x + b*d^2*D*x^2 - ((b*c - a*d)*(-(c^2*C*d)
+ B*c*d^2 - A*d^3 + c^3*D))/(c + d*x)^2 - (2*(a*d*(-2*c*C*d + B*d^2 + 3*c^
2*D) + b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D)))/(c + d*x) + 2*(a*d*(C
*d - 3*c*D) + b*(-3*c*C*d + B*d^2 + 6*c^2*D))*Log[c + d*x])/(2*d^5)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(ad-bc)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^4(c+dx)^3}+\frac{b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cC\right.}{d^4(c+dx)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{ad(-Bd^2-3c^2D+2cCd)-b(Ad^3-2Bcd^2-4c^3D+3c^2Cd)}{d^5(c+dx)}+\frac{(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{2d^5(c+dx)^2}+\frac{\log(c+dx)\left(ad(Cd-3cD)-b(-Bd^2-6c^2D+3cCd)\right)}{d^5}+\frac{x(adD-3bcD+bCd)}{d^4}+\frac{bDx^2}{2d^3}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^3,x]
```

output
```
((b*C*d - 3*b*c*D + a*d*D)*x)/d^4 + (b*D*x^2)/(2*d^3) + ((b*c - a*d)*(c^2*
C*d - B*c*d^2 + A*d^3 - c^3*D))/(2*d^5*(c + d*x)^2) + (a*d*(2*c*C*d - B*d^
2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*c^3*D))/(d^5*(c + d*x)
) + ((a*d*(C*d - 3*c*D) - b*(3*c*C*d - B*d^2 - 6*c^2*D))*Log[c + d*x])/d^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.15

method	result
norman	$\frac{\frac{(Cbd+Dad-2Dbc)x^3}{d^2}-\frac{Aa\,d^4+Ac\,d^3b+Bc\,d^3a-3Bb\,c^2d^2-3Ca\,c^2d^2+9Cb\,c^3d+9Da\,c^3d-18Db\,c^4}{2d^5}-\frac{(Ab\,d^3+Ba\,d^3-2Bbc\,d^2-2Cac\,d^2+6}{d^4}}{(xd+c)^2}$
default	$\frac{\frac{1}{2}dDx^2b+Cbdx+Dadx-3Dbcx}{d^4}-\frac{Ab\,d^3+Ba\,d^3-2Bbc\,d^2-2Cac\,d^2+3Cb\,c^2d+3Da\,c^2d-4Db\,c^3}{d^5(xd+c)}+\frac{(bB\,d^2+Ca\,d^2-3Cbc}{}$
parallelrisch	$-\frac{-4C\ln(xd+c)xac\,d^3+12C\ln(xd+c)xb\,c^2d^2+12D\ln(xd+c)xa\,c^2d^2-24D\ln(xd+c)xb\,c^3d-3Bb\,c^2d^2-4B\ln(xd+c)xbc\,d^3-}{}$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
((C*b*d+D*a*d-2*D*b*c)/d^2*x^3-1/2*(A*a*d^4+A*b*c*d^3+B*a*c*d^3-3*B*b*c^2*
d^2-3*C*a*c^2*d^2+9*C*b*c^3*d+9*D*a*c^3*d-18*D*b*c^4)/d^5-(A*b*d^3+B*a*d^3
-2*B*b*c*d^2-2*C*a*c*d^2+6*C*b*c^2*d+6*D*a*c^2*d-12*D*b*c^3)/d^4*x+1/2*b*D
*x^4/d)/(d*x+c)^2+1/d^5*(B*b*d^2+C*a*d^2-3*C*b*c*d-3*D*a*c*d+6*D*b*c^2)*ln
(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$
$$= \frac{Dbd^4x^4+7\,Dbc^4-Aad^4-5\,(Da+Cb)c^3d+3\,(Ca+Bb)c^2d^2-(Ba+Ab)cd^3-2\,(2\,Dbcd^3-(Da+ \dots}{\dots}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(D*b*d^4*x^4 + 7*D*b*c^4 - A*a*d^4 - 5*(D*a + C*b)*c^3*d + 3*(C*a + B*
b)*c^2*d^2 - (B*a + A*b)*c*d^3 - 2*(2*D*b*c*d^3 - (D*a + C*b)*d^4)*x^3 - (
11*D*b*c^2*d^2 - 4*(D*a + C*b)*c*d^3)*x^2 + 2*(D*b*c^3*d - 2*(D*a + C*b)*c
^2*d^2 + 2*(C*a + B*b)*c*d^3 - (B*a + A*b)*d^4)*x + 2*(6*D*b*c^4 - 3*(D*a
+ C*b)*c^3*d + (C*a + B*b)*c^2*d^2 + (6*D*b*c^2*d^2 - 3*(D*a + C*b)*c*d^3
+ (C*a + B*b)*d^4)*x^2 + 2*(6*D*b*c^3*d - 3*(D*a + C*b)*c^2*d^2 + (C*a + B
*b)*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [A] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.41

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx = \frac{Dbx^2}{2d^3}+x\left(\frac{Cb}{d^3}+\frac{Da}{d^3}-\frac{3Dbc}{d^4}\right)$$
$$+\frac{-Aad^4-Abcd^3-Bacd^3+3Bbc^2d^2+3Cac^2d^2-5Cbc^3d-5Dac^3d+7Dbc^4+x(-2Abd^4-2Bad^4 \dots}{2c^2d^5+4cd^6x+2d^7x^2}$$
$$-\frac{\left(-Bbd^2-Cad^2+3Cbcd+3Dacd-6Dbc^2\right)\log{(c+dx)}}{d^5}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
D*b*x**2/(2*d**3) + x*(C*b/d**3 + D*a/d**3 - 3*D*b*c/d**4) + (-A*a*d**4 -
A*b*c*d**3 - B*a*c*d**3 + 3*B*b*c**2*d**2 + 3*C*a*c**2*d**2 - 5*C*b*c**3*d
 - 5*D*a*c**3*d + 7*D*b*c**4 + x*(-2*A*b*d**4 - 2*B*a*d**4 + 4*B*b*c*d**3
+ 4*C*a*c*d**3 - 6*C*b*c**2*d**2 - 6*D*a*c**2*d**2 + 8*D*b*c**3*d))/(2*c**
2*d**5 + 4*c*d**6*x + 2*d**7*x**2) - (-B*b*d**2 - C*a*d**2 + 3*C*b*c*d + 3
*D*a*c*d - 6*D*b*c**2)*log(c + d*x)/d**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$
$$= \frac{7\,Dbc^4 - Aad^4 - 5\,(Da+Cb)c^3d + 3\,(Ca+Bb)c^2d^2 - (Ba+Ab)cd^3 + 2\,(4\,Dbc^3d - 3\,(Da+Cb)c^2d^2 - \cdots}{2\,(d^7x^2 + 2\,cd^6x + c^2d^5)}$$
$$+ \frac{Dbdx^2 - 2\,(3\,Dbc - (Da+Cb)d)x}{2\,d^4}$$
$$+ \frac{\left(6\,Dbc^2 - 3\,(Da+Cb)cd + (Ca+Bb)d^2\right)\log\left(dx+c\right)}{d^5}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*(7*D*b*c^4 - A*a*d^4 - 5*(D*a + C*b)*c^3*d + 3*(C*a + B*b)*c^2*d^2 - (
B*a + A*b)*c*d^3 + 2*(4*D*b*c^3*d - 3*(D*a + C*b)*c^2*d^2 + 2*(C*a + B*b)*
c*d^3 - (B*a + A*b)*d^4)*x)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(D*b*d*x
^2 - 2*(3*D*b*c - (D*a + C*b)*d)*x)/d^4 + (6*D*b*c^2 - 3*(D*a + C*b)*c*d +
 (C*a + B*b)*d^2)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$
$$= \frac{\left(6\,Dbc^2-3\,Dacd-3\,Cbcd+Cad^2+Bbd^2\right)\log\left(|dx+c|\right)}{d^5} + \frac{Dbd^3x^2-6\,Dbcd^2x+2\,Dad^3x+2\,Cbd^3x}{2\,d^6} + \frac{7\,Dbc^4-5\,Dac^3d-5\,Cbc^3d+3\,Cac^2d^2+3\,Bbc^2d^2-Bacd^3-Abcd^3-Aad^4+2\,(4\,Dbc^3d-3\,Dac^2d}{2\,(dx+c)^2d^5}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac")
```

output
```
(6*D*b*c^2 - 3*D*a*c*d - 3*C*b*c*d + C*a*d^2 + B*b*d^2)*log(abs(d*x + c))/
d^5 + 1/2*(D*b*d^3*x^2 - 6*D*b*c*d^2*x + 2*D*a*d^3*x + 2*C*b*d^3*x)/d^6 +
1/2*(7*D*b*c^4 - 5*D*a*c^3*d - 5*C*b*c^3*d + 3*C*a*c^2*d^2 + 3*B*b*c^2*d^2
 - B*a*c*d^3 - A*b*c*d^3 - A*a*d^4 + 2*(4*D*b*c^3*d - 3*D*a*c^2*d^2 - 3*C*
b*c^2*d^2 + 2*C*a*c*d^3 + 2*B*b*c*d^3 - B*a*d^4 - A*b*d^4)*x)/((d*x + c)^2
*d^5)
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$
$$= \frac{\frac{3\,B\,b\,c^2}{2\,d^3}+\frac{2\,B\,b\,c\,x}{d^2}}{c^2+2\,c\,d\,x+d^2\,x^2} + \frac{\frac{3\,C\,a\,c^2}{2\,d^3}+\frac{2\,C\,a\,c\,x}{d^2}}{c^2+2\,c\,d\,x+d^2\,x^2} - \frac{\frac{A\,b\,c}{2\,d^2}+\frac{A\,b\,x}{d}}{c^2+2\,c\,d\,x+d^2\,x^2} - \frac{\frac{B\,a\,c}{2\,d^2}+\frac{B\,a\,x}{d}}{c^2+2\,c\,d\,x+d^2\,x^2} + \frac{B\,b\,\ln\left(c+d\,x\right)}{d^3} + \frac{C\,a\,\ln\left(c+d\,x\right)}{d^3} + \frac{b\,D\left(\frac{(c+d\,x)^2}{2}+\frac{4\,c^3}{c+d\,x}-\frac{c^4}{2\,(c+d\,x)^2}+6\,c^2\,\ln\left(c+d\,x\right)-4\,c\,d\,x\right)}{d^5} - \frac{C\,b\left(3\,c\,\ln\left(c+d\,x\right)-d\,x+\frac{3\,c^2}{c+d\,x}-\frac{c^3}{2\,(c+d\,x)^2}\right)}{d^4} - \frac{A\,a}{2\,d\,\left(c^2+2\,c\,d\,x+d^2\,x^2\right)} - \frac{a\,D\left(3\,c\,\ln\left(c+d\,x\right)-d\,x+\frac{3\,c^2}{c+d\,x}-\frac{c^3}{2\,(c+d\,x)^2}\right)}{d^4}$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^3,x)
```

output
```
((3*B*b*c^2)/(2*d^3) + (2*B*b*c*x)/d^2)/(c^2 + d^2*x^2 + 2*c*d*x) + ((3*C*
a*c^2)/(2*d^3) + (2*C*a*c*x)/d^2)/(c^2 + d^2*x^2 + 2*c*d*x) - ((A*b*c)/(2*
d^2) + (A*b*x)/d)/(c^2 + d^2*x^2 + 2*c*d*x) - ((B*a*c)/(2*d^2) + (B*a*x)/d
)/(c^2 + d^2*x^2 + 2*c*d*x) + (B*b*log(c + d*x))/d^3 + (C*a*log(c + d*x))/
d^3 + (b*D*((c + d*x)^2/2 + (4*c^3)/(c + d*x) - c^4/(2*(c + d*x)^2) + 6*c^
2*log(c + d*x) - 4*c*d*x))/d^5 - (C*b*(3*c*log(c + d*x) - d*x + (3*c^2)/(c
 + d*x) - c^3/(2*(c + d*x)^2)))/d^4 - (A*a)/(2*d*(c^2 + d^2*x^2 + 2*c*d*x)
) - (a*D*(3*c*log(c + d*x) - d*x + (3*c^2)/(c + d*x) - c^3/(2*(c + d*x)^2)
))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^3}\,dx$$
$$= \frac{-4\log(dx+c)\,a\,c^4d-8\log(dx+c)\,a\,c^3d^2x-4\log(dx+c)\,a\,c^2d^3x^2+2\log(dx+c)\,b^2c^3d+4\log(dx+c}{}$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
( - 4*log(c + d*x)*a*c**4*d - 8*log(c + d*x)*a*c**3*d**2*x - 4*log(c + d*x
)*a*c**2*d**3*x**2 + 2*log(c + d*x)*b**2*c**3*d + 4*log(c + d*x)*b**2*c**2
*d**2*x + 2*log(c + d*x)*b**2*c*d**3*x**2 + 6*log(c + d*x)*b*c**5 + 12*log
(c + d*x)*b*c**4*d*x + 6*log(c + d*x)*b*c**3*d**2*x**2 - a**2*c*d**3 + 2*a
*b*d**4*x**2 - 2*a*c**4*d + 4*a*c**2*d**3*x**2 + 2*a*c*d**4*x**3 + b**2*c*
*3*d - 2*b**2*c*d**3*x**2 + 3*b*c**5 - 6*b*c**3*d**2*x**2 - 2*b*c**2*d**3*
x**3 + b*c*d**4*x**4)/(2*c*d**4*(c**2 + 2*c*d*x + d**2*x**2))
```

3.44 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3} dx$

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Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3} dx = \frac{Dx}{d^3} - \frac{c^2Cd - Bcd^2 + Ad^3 - c^3D}{2d^4(c+dx)^2} + \frac{2cCd - Bd^2 - 3c^2D}{d^4(c+dx)} + \frac{(Cd-3cD)\log(c+dx)}{d^4}$$

output
```
D*x/d^3-1/2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4/(d*x+c)^2+(-B*d^2+2*C*c*d-3*
D*c^2)/d^4/(d*x+c)+(C*d-3*D*c)*ln(d*x+c)/d^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3} dx = \frac{Dx}{d^3} + \frac{-c^2Cd + Bcd^2 - Ad^3 + c^3D}{2d^4(c+dx)^2} + \frac{2cCd - Bd^2 - 3c^2D}{d^4(c+dx)} + \frac{(Cd-3cD)\log(c+dx)}{d^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^3,x]
```

output
```
(D*x)/d^3 + (-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)/(2*d^4*(c + d*x)^2) + (
2*c*C*d - B*d^2 - 3*c^2*D)/(d^4*(c + d*x)) + ((C*d - 3*c*D)*Log[c + d*x])/
d^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)^3}+\frac{Bd^2+3c^2D-2cCd}{d^3(c+dx)^2}+\frac{Cd-3cD}{d^3(c+dx)}+\frac{D}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{2d^4(c+dx)^2}+\frac{-Bd^2-3c^2D+2cCd}{d^4(c+dx)}+\frac{(Cd-3cD)\log(c+dx)}{d^4}+\frac{Dx}{d^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^3,x]
```

output
```
(D*x)/d^3 - (c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(2*d^4*(c + d*x)^2) + (2*c
*C*d - B*d^2 - 3*c^2*D)/(d^4*(c + d*x)) + ((C*d - 3*c*D)*Log[c + d*x])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{D x^{3}}{d}-\frac{A\,d^{3}+B c\,d^{2}-3 C\,c^{2} d+9 D c^{3}}{2 d^{4}}-\frac{\left(B\,d^{2}-2 C c d+6 D c^{2}\right) x}{d^{3}}}{\left(x d+c\right)^{2}}+\frac{\left(C d-3 D c\right) \ln \left(x d+c\right)}{d^{4}}$
default	$\frac{D x}{d^{3}}-\frac{B\,d^{2}-2 C c d+3 D c^{2}}{d^{4} \left(x d+c\right)}+\frac{\left(C d-3 D c\right) \ln \left(x d+c\right)}{d^{4}}-\frac{A\,d^{3}-B c\,d^{2}+C\,c^{2} d-D c^{3}}{2 d^{4} \left(x d+c\right)^{2}}$
parallelrisch	$-\frac{-2 C \ln \left(x d+c\right) x^{2} d^{3}+6 D \ln \left(x d+c\right) x^{2} c\,d^{2}-2 D x^{3} d^{3}-4 C \ln \left(x d+c\right) x c\,d^{2}+12 D \ln \left(x d+c\right) x\,c^{2} d+2 B x\,d^{3}-2 C \ln \left(x d+c\right) c^{2} d-4C}{2 d^{4} \left(x d+c\right)^{2}}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(D*x^3/d-1/2*(A*d^3+B*c*d^2-3*C*c^2*d+9*D*c^3)/d^4-(B*d^2-2*C*c*d+6*D*c^2)
/d^3*x)/(d*x+c)^2+(C*d-3*D*c)*ln(d*x+c)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3}\,dx = \frac{2\,Dd^3x^3+4\,Dcd^2x^2-5\,Dc^3+3\,Cc^2d-Bcd^2-Ad^3-2\,(2\,Dc^2d-2\,Ccd^2+Bd^3)x-2\,(3\,Dc^3-Cc^2d}{2\,(d^6x^2+2\,cd^5x+c^2d^4)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(2*D*d^3*x^3 + 4*D*c*d^2*x^2 - 5*D*c^3 + 3*C*c^2*d - B*c*d^2 - A*d^3 -
 2*(2*D*c^2*d - 2*C*c*d^2 + B*d^3)*x - 2*(3*D*c^3 - C*c^2*d + (3*D*c*d^2 -
 C*d^3)*x^2 + 2*(3*D*c^2*d - C*c*d^2)*x)*log(d*x + c))/(d^6*x^2 + 2*c*d^5*
x + c^2*d^4)
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3}\,dx = \frac{Dx}{d^3}+\frac{-Ad^3-Bcd^2+3Cc^2d-5Dc^3+x(-2Bd^3+4Ccd^2-6Dc^2d)}{2c^2d^4+4cd^5x+2d^6x^2} - \frac{(-Cd+3Dc)\log{(c+dx)}}{d^4}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**3,x)
```

output
```
D*x/d**3 + (-A*d**3 - B*c*d**2 + 3*C*c**2*d - 5*D*c**3 + x*(-2*B*d**3 + 4*
C*c*d**2 - 6*D*c**2*d))/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - (-C*d +
 3*D*c)*log(c + d*x)/d**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3}\,dx = -\frac{5\,Dc^3-3\,Cc^2d+Bcd^2+Ad^3+2\,(3\,Dc^2d-2\,Ccd^2+Bd^3)x}{2\,(d^6x^2+2\,cd^5x+c^2d^4)} + \frac{Dx}{d^3}-\frac{(3\,Dc-Cd)\log{(dx+c)}}{d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(5*D*c^3 - 3*C*c^2*d + B*c*d^2 + A*d^3 + 2*(3*D*c^2*d - 2*C*c*d^2 + B
*d^3)*x)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4) + D*x/d^3 - (3*D*c - C*d)*log(d*x
 + c)/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3}\,dx = \frac{Dx}{d^3} - \frac{(3\,Dc-Cd)\log(|dx+c|)}{d^4} - \frac{5\,Dc^3-3\,Cc^2d+Bcd^2+Ad^3+2\,(3\,Dc^2d-2\,Ccd^2+Bd^3)x}{2\,(dx+c)^2d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x, algorithm="giac")
```

output
```
D*x/d^3 - (3*D*c - C*d)*log(abs(d*x + c))/d^4 - 1/2*(5*D*c^3 - 3*C*c^2*d +
 B*c*d^2 + A*d^3 + 2*(3*D*c^2*d - 2*C*c*d^2 + B*d^3)*x)/((d*x + c)^2*d^4)
```

Mupad [B] (verification not implemented)

Time = 3.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.62

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3}\,dx = \frac{\frac{3\,C\,c^2}{2\,d^3}+\frac{2\,C\,c\,x}{d^2}}{c^2+2\,c\,d\,x+d^2\,x^2} - \frac{\frac{B\,c}{2\,d^2}+\frac{B\,x}{d}}{c^2+2\,c\,d\,x+d^2\,x^2} - \frac{A}{2\,d\,\left(c^2+2\,c\,d\,x+d^2\,x^2\right)} - \frac{D\,\left(3\,c\,\ln\left(c+d\,x\right)-d\,x+\frac{3\,c^2}{c+d\,x}-\frac{c^3}{2\,{\left(c+d\,x\right)}^2}\right)}{d^4} + \frac{C\,\ln\left(c+d\,x\right)}{d^3}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^3,x)
```

output
```
((3*C*c^2)/(2*d^3) + (2*C*c*x)/d^2)/(c^2 + d^2*x^2 + 2*c*d*x) - ((B*c)/(2*
d^2) + (B*x)/d)/(c^2 + d^2*x^2 + 2*c*d*x) - A/(2*d*(c^2 + d^2*x^2 + 2*c*d*
x)) - (D*(3*c*log(c + d*x) - d*x + (3*c^2)/(c + d*x) - c^3/(2*(c + d*x)^2)
))/d^4 + (C*log(c + d*x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^3}\,dx = \frac{-4\log(dx+c)\,c^4-8\log(dx+c)\,c^3dx-4\log(dx+c)\,c^2d^2x^2-ac\,d^2+b\,d^3x^2-2c^4+4c^2d^2x^2+2c\,d^3x^3}{2c\,d^3\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^3,x)
```

output
```
( - 4*log(c + d*x)*c**4 - 8*log(c + d*x)*c**3*d*x - 4*log(c + d*x)*c**2*d*
*2*x**2 - a*c*d**2 + b*d**3*x**2 - 2*c**4 + 4*c**2*d**2*x**2 + 2*c*d**3*x*
*3)/(2*c*d**3*(c**2 + 2*c*d*x + d**2*x**2))
```

3.45 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 231

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx &= \frac{c^2Cd-Bcd^2+Ad^3-c^3D}{2d^3(bc-ad)(c+dx)^2}+\frac{ad(2cCd-Bd^2-3c^2D)-b(c^2Cd-Ad^3-2c^3D)}{d^3(bc-ad)^2(c+dx)}\\ &\quad+\frac{(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{b(bc-ad)^3}\\ &\quad-\frac{(Ab^2d^3-b^2c^3D+a^2d^2(Cd-3cD)-ab(Bd^3-3c^2dD))\log(c+dx)}{d^3(bc-ad)^3}\end{aligned}$$

output

```
1/2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^3/(-a*d+b*c)/(d*x+c)^2+(a*d*(-B*d^2+2*
C*c*d-3*D*c^2)-b*(-A*d^3+C*c^2*d-2*D*c^3))/d^3/(-a*d+b*c)^2/(d*x+c)+(A*b^3
-a*(B*b^2-C*a*b+D*a^2))*ln(b*x+a)/b/(-a*d+b*c)^3-(A*b^2*d^3-b^2*c^3*D+a^2*
d^2*(C*d-3*D*c)-a*b*(B*d^3-3*D*c^2*d))*ln(d*x+c)/d^3/(-a*d+b*c)^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx$$
$$= \frac{b(bc-ad)^2\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)+2b(bc-ad)\left(-ad(-2cCd+Bd^2+3c^2D)+b(-c^2Cd+Ad^3\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^3),x]
```

output
```
(b*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D) + 2*b*(b*c - a*d)*(-(
a*d*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-(c^2*C*d) + A*d^3 + 2*c^3*D))*(c +
 d*x) + 2*d^3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^2*Log[a + b*x]
 - 2*b*(A*b^2*d^3 - b^2*c^3*D + a^2*d^2*(C*d - 3*c*D) + a*b*(-(B*d^3) + 3*
c^2*d*D))*(c + d*x)^2*Log[c + d*x])/(2*b*d^3*(b*c - a*d)^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{-a^2d^2(Cd-3cD)+ab\left(Bd^3-3c^2dD\right)-Ab^2d^3+b^2c^3D}{d^2(c+dx)(bc-ad)^3}+\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{(a+bx)(bc-ad)^3}+\frac{Ad^3-Bcd^2+}{d^2(c+dx)}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\log(c+dx)\left(a^2d^2(Cd-3cD)-ab\left(Bd^3-3c^2dD\right)+Ab^2d^3-b^2c^3D\right)}{d^3(bc-ad)^3}+\frac{\log(a+bx)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b(bc-ad)^3}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{2d^3(c+dx)^2(bc-ad)}+\frac{ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-Ad^3-2c^3D+c^2Cd\right)}{d^3(c+dx)(bc-ad)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^3),x]
```

output
```
(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(2*d^3*(b*c - a*d)*(c + d*x)^2) + (a*d
*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(c^2*C*d - A*d^3 - 2*c^3*D))/(d^3*(b*c -
a*d)^2*(c + d*x)) + ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Log[a + b*x])/(b*
(b*c - a*d)^3) - ((A*b^2*d^3 - b^2*c^3*D + a^2*d^2*(C*d - 3*c*D) - a*b*(B*
d^3 - 3*c^2*d*D))*Log[c + d*x])/(d^3*(b*c - a*d)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.01

method	result
default	$-\frac{A\,d^3-Bc\,d^2+C\,c^2d-Dc^3}{2d^3(ad-bc)(xd+c)^2}-\frac{-Ab\,d^3+Ba\,d^3-2Cac\,d^2+Cb\,c^2d+3Da\,c^2d-2Db\,c^3}{d^3(ad-bc)^2(xd+c)}+\frac{(A\,b^2d^3-abB\,d^3+C\,a^2d^3-3Da^2c}{(ad-bc}$
norman	$\frac{\frac{\left(Ab\,d^3-Ba\,d^3+2Cac\,d^2-Cb\,c^2d-3Da\,c^2d+2Db\,c^3\right)x}{d^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{Aa\,d^4-3Ac\,d^3b+Bc\,d^3a+Bb\,c^2d^2-3Ca\,c^2d^2+Cb\,c^3d+5Da\,c^3d-3Db\,c^4}{2d^3\left(a^2d^2-2abcd+b^2c^2\right)}}{(xd+c)^2}+(A$
parallelrisch	$-\frac{4Dx\,b^3c^4d+2A\ln(bx+a)x^2b^3d^5-2A\ln(xd+c)x^2b^3d^5-2D\ln(bx+a)x^2a^3d^5+2A\ln(bx+a)b^3c^2d^3-8Da\,b^2c^4d-2C\ln(xd+}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)/(d*x+c)^2-(-A*b*d^3+B*a*d
^3-2*C*a*c*d^2+C*b*c^2*d+3*D*a*c^2*d-2*D*b*c^3)/d^3/(a*d-b*c)^2/(d*x+c)+(A
*b^2*d^3-B*a*b*d^3+C*a^2*d^3-3*D*a^2*c*d^2+3*D*a*b*c^2*d-D*b^2*c^3)/(a*d-b
*c)^3/d^3*ln(d*x+c)+(-A*b^3+B*a*b^2-C*a^2*b+D*a^3)/(a*d-b*c)^3/b*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(230) = 460$.

Time = 0.13 (sec) , antiderivative size = 658, normalized size of antiderivative = 2.85

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)(c + dx)^3} \, dx = \frac{3\,Db^3c^5 + Aa^2bd^5 - (8\,Dab^2 + Cb^3)c^4d + (5\,Da^2b + 4\,Cab^2 - Bb^3)c^3d^2 - 3\,(Ca^2b - Ab^3)c^2d^3 + (Ba^2b - \cdots}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(3*D*b^3*c^5 + A*a^2*b*d^5 - (8*D*a*b^2 + C*b^3)*c^4*d + (5*D*a^2*b +
4*C*a*b^2 - B*b^3)*c^3*d^2 - 3*(C*a^2*b - A*b^3)*c^2*d^3 + (B*a^2*b - 4*A*
a*b^2)*c*d^4 + 2*(2*D*b^3*c^4*d - (5*D*a*b^2 + C*b^3)*c^3*d^2 + 3*(D*a^2*b
 + C*a*b^2)*c^2*d^3 - (2*C*a^2*b + B*a*b^2 - A*b^3)*c*d^4 + (B*a^2*b - A*a
*b^2)*d^5)*x - 2*((D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^5*x^2 + 2*(D*a^3 -
 C*a^2*b + B*a*b^2 - A*b^3)*c*d^4*x + (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*
c^2*d^3)*log(b*x + a) + 2*(D*b^3*c^5 - 3*D*a*b^2*c^4*d + 3*D*a^2*b*c^3*d^2
 - (C*a^2*b - B*a*b^2 + A*b^3)*c^2*d^3 + (D*b^3*c^3*d^2 - 3*D*a*b^2*c^2*d^
3 + 3*D*a^2*b*c*d^4 - (C*a^2*b - B*a*b^2 + A*b^3)*d^5)*x^2 + 2*(D*b^3*c^4*
d - 3*D*a*b^2*c^3*d^2 + 3*D*a^2*b*c^2*d^3 - (C*a^2*b - B*a*b^2 + A*b^3)*c*
d^4)*x)*log(d*x + c))/(b^4*c^5*d^3 - 3*a*b^3*c^4*d^4 + 3*a^2*b^2*c^3*d^5 -
 a^3*b*c^2*d^6 + (b^4*c^3*d^5 - 3*a*b^3*c^2*d^6 + 3*a^2*b^2*c*d^7 - a^3*b*
d^8)*x^2 + 2*(b^4*c^4*d^4 - 3*a*b^3*c^3*d^5 + 3*a^2*b^2*c^2*d^6 - a^3*b*c*
d^7)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\log(bx+a)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} + \frac{(Db^2c^3-3\,Dabc^2d+3\,Da^2cd^2-(Ca^2-Bab+Ab^2)d^3)\log(dx+c)}{b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6} + \frac{3\,Dbc^4-Aad^4-(5\,Da+Cb)c^3d+(3\,Ca-Bb)c^2d^2-(Ba-3\,Ab)cd^3+2\,(2\,Dbc^3d+2\,Cacd^3-(3\,I}{2\,(b^2c^4d^3-2\,abc^3d^4+a^2c^2d^5+(b^2c^2d^5-2\,abcd^6+a^2d^7)x^2+2\,(b^2c^3d^4-2\,abc^2d^5}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d
 + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (D*b^2*c^3 - 3*D*a*b*c^2*d + 3*D*a^2*c*d
^2 - (C*a^2 - B*a*b + A*b^2)*d^3)*log(d*x + c)/(b^3*c^3*d^3 - 3*a*b^2*c^2*
d^4 + 3*a^2*b*c*d^5 - a^3*d^6) + 1/2*(3*D*b*c^4 - A*a*d^4 - (5*D*a + C*b)*
c^3*d + (3*C*a - B*b)*c^2*d^2 - (B*a - 3*A*b)*c*d^3 + 2*(2*D*b*c^3*d + 2*C
*a*c*d^3 - (3*D*a + C*b)*c^2*d^2 - (B*a - A*b)*d^4)*x)/(b^2*c^4*d^3 - 2*a*
b*c^3*d^4 + a^2*c^2*d^5 + (b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7)*x^2 + 2*(b
^2*c^3*d^4 - 2*a*b*c^2*d^5 + a^2*c*d^6)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.82

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\log(|bx+a|)}{b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3} + \frac{(Db^2c^3-3\,Dabc^2d+3\,Da^2cd^2-Ca^2d^3+Babd^3-Ab^2d^3)\log(|dx+c|)}{b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6} + \frac{2\,(2\,Db^2c^4-5\,Dabc^3d-Cb^2c^3d+3\,Da^2c^2d^2+3\,Cabc^2d^2-2\,Ca^2cd^3-Babcd^3+Ab^2cd^3+Ba^2d^4-}{2\,(bc-ad)^3(d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (D*b^2*c^3 - 3*D*a*b*c^2*d + 3*D*a^
2*c*d^2 - C*a^2*d^3 + B*a*b*d^3 - A*b^2*d^3)*log(abs(d*x + c))/(b^3*c^3*d^
3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6) + 1/2*(2*(2*D*b^2*c^4 - 5*D
*a*b*c^3*d - C*b^2*c^3*d + 3*D*a^2*c^2*d^2 + 3*C*a*b*c^2*d^2 - 2*C*a^2*c*d
^3 - B*a*b*c*d^3 + A*b^2*c*d^3 + B*a^2*d^4 - A*a*b*d^4)*x + (3*D*b^2*c^5 -
 8*D*a*b*c^4*d - C*b^2*c^4*d + 5*D*a^2*c^3*d^2 + 4*C*a*b*c^3*d^2 - B*b^2*c
^3*d^2 - 3*C*a^2*c^2*d^3 + 3*A*b^2*c^2*d^3 + B*a^2*c*d^4 - 4*A*a*b*c*d^4 +
 A*a^2*d^5)/d)/((b*c - a*d)^3*(d*x + c)^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,(c+d\,x)^3}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^3}\,dx$$
$$= \frac{2\log(bx+a)\,a^2c^2d^2+4\log(bx+a)\,a^2c\,d^3x+2\log(bx+a)\,a^2d^4x^2-4\log(dx+c)\,ab\,c^3d-8\log(dx+c}{2b\,d^2\,(a^2d^4x^2-2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^3,x)
```

output
```
(2*log(a + b*x)*a**2*c**2*d**2 + 4*log(a + b*x)*a**2*c*d**3*x + 2*log(a +
b*x)*a**2*d**4*x**2 - 4*log(c + d*x)*a*b*c**3*d - 8*log(c + d*x)*a*b*c**2*
d**2*x - 4*log(c + d*x)*a*b*c*d**3*x**2 + 2*log(c + d*x)*b**2*c**4 + 4*log
(c + d*x)*b**2*c**3*d*x + 2*log(c + d*x)*b**2*c**2*d**2*x**2 - a**2*b*d**3
 + 2*a*b**2*c*d**2 - a*b*c**3*d + a*b*c*d**3*x**2 - b**3*c**2*d + b**2*c**
4 - b**2*c**2*d**2*x**2)/(2*b*d**2*(a**2*c**2*d**2 + 2*a**2*c*d**3*x + a**
2*d**4*x**2 - 2*a*b*c**3*d - 4*a*b*c**2*d**2*x - 2*a*b*c*d**3*x**2 + b**2*
c**4 + 2*b**2*c**3*d*x + b**2*c**2*d**2*x**2))
```

3.46 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 276

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx
&= -\frac{Ab^3-a(b^2B-abC+a^2D)}{b(bc-ad)^3(a+bx)} - \frac{c^2Cd-Bcd^2+Ad^3-c^3D}{2d^2(bc-ad)^2(c+dx)^2} \\
&\quad - \frac{ad(2cCd-Bd^2-3c^2D)-b(Bcd^2-2Ad^3-c^3D)}{d^2(bc-ad)^3(c+dx)} \\
&\quad + \frac{(b^2(Bc-3Ad)-ab(2cC-2Bd)-a^2(Cd-3cD))\log(a+bx)}{(bc-ad)^4} \\
&\quad - \frac{(b^2(Bc-3Ad)-ab(2cC-2Bd)-a^2(Cd-3cD))\log(c+dx)}{(bc-ad)^4}
\end{aligned}$$

output
```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b/(-a*d+b*c)^3/(b*x+a)-1/2*(A*d^3-B*c*d^2+C
*c^2*d-D*c^3)/d^2/(-a*d+b*c)^2/(d*x+c)^2-(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(
-2*A*d^3+B*c*d^2-D*c^3))/d^2/(-a*d+b*c)^3/(d*x+c)+(b^2*(-3*A*d+B*c)-a*b*(-
2*B*d+2*C*c)-a^2*(C*d-3*D*c))*ln(b*x+a)/(-a*d+b*c)^4-(b^2*(-3*A*d+B*c)-a*b
*(-2*B*d+2*C*c)-a^2*(C*d-3*D*c))*ln(d*x+c)/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx = \frac{-Ab^3+a(b^2B-abC+a^2D)}{b(bc-ad)^3(a+bx)}+\frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{2d^2(bc-ad)^2(c+dx)^2} + \frac{-ad(-2cCd+Bd^2+3c^2D)+b(-Bcd^2+2Ad^3+c^3D)}{d^2(-bc+ad)^3(c+dx)} + \frac{(b^2(Bc-3Ad)+ab(-2cC+2Bd)+a^2(-Cd+3cD))\log(a+bx)}{(bc-ad)^4} + \frac{(b^2(-Bc+3Ad)+2ab(cC-Bd)+a^2(Cd-3cD))\log(c+dx)}{(bc-ad)^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^3),x]
```

output
```
(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))/(b*(b*c - a*d)^3*(a + b*x)) + (-(c^
2*C*d) + B*c*d^2 - A*d^3 + c^3*D)/(2*d^2*(b*c - a*d)^2*(c + d*x)^2) + (-(a
*d*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-(B*c*d^2) + 2*A*d^3 + c^3*D))/(d^2*
(-(b*c) + a*d)^3*(c + d*x)) + ((b^2*(B*c - 3*A*d) + a*b*(-2*c*C + 2*B*d) +
 a^2*(-(C*d) + 3*c*D))*Log[a + b*x])/(b*c - a*d)^4 + ((b^2*(-(B*c) + 3*A*d
) + 2*a*b*(c*C - B*d) + a^2*(C*d - 3*c*D))*Log[c + d*x])/(b*c - a*d)^4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{b(-(a^2(Cd-3cD))-ab(2cC-2Bd)+b^2(Bc-3Ad))}{(a+bx)(bc-ad)^4} + \frac{d(a^2(Cd-3cD)+2ab(cC-Bd)-b^2(Bc-3A}{(c+dx)(bc-ad)^4}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\log(a+bx)\left(-(a^2(Cd-3cD))-ab(2cC-2Bd)+b^2(Bc-3Ad)\right)}{(bc-ad)^4} - \frac{\log(c+dx)\left(-(a^2(Cd-3cD))-ab(2cC-2Bd)+b^2(Bc-3Ad)\right)}{(bc-ad)^4} - \frac{Ab^3-a(a^2D-abC+b^2B)}{b(a+bx)(bc-ad)^3} - \frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{2d^2(c+dx)^2(bc-ad)^2} - \frac{ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3+Bcd^2+c^3(-D))}{d^2(c+dx)(bc-ad)^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^3),x]
```

output
```
-((A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b*(b*c - a*d)^3*(a + b*x))) - (c^2*
C*d - B*c*d^2 + A*d^3 - c^3*D)/(2*d^2*(b*c - a*d)^2*(c + d*x)^2) - (a*d*(2
*c*C*d - B*d^2 - 3*c^2*D) - b*(B*c*d^2 - 2*A*d^3 - c^3*D))/(d^2*(b*c - a*d
)^3*(c + d*x)) + ((b^2*(B*c - 3*A*d) - a*b*(2*c*C - 2*B*d) - a^2*(C*d - 3*
c*D))*Log[a + b*x])/(b*c - a*d)^4 - ((b^2*(B*c - 3*A*d) - a*b*(2*c*C - 2*B
*d) - a^2*(C*d - 3*c*D))*Log[c + d*x])/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00

method	result
default	$\frac{(3d\,b^2A-2Babd-B\,b^2c+C\,a^2d+2Cabc-3a^2cD)\ln(xd+c)}{(ad-bc)^4}-\frac{A\,d^3-Bc\,d^2+C\,c^2d-Dc^3}{2d^2(ad-bc)^2(xd+c)^2}-\frac{-2Ab\,d^3+Ba\,d^3+Bbc\,d^2-2C\ldots}{(ad-bc)^3d^2(x\ldots}$
norman	$\frac{\frac{(3A\,b^3d^3-2Ba\,b^2d^3-B\,b^3c\,d^2+a^2bC\,d^3+2Ca\,b^2c\,d^2-a^3d^3D-3Da\,b^2c^2d+Db^3c^3)x^2}{bd(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)}-\frac{A\,a^2b\,d^4-5Aa\,b^2c\,d^3-2A\,b^3c^2d^2+B\,a^2bc\,d^3+5\ldots}{2d^2b(a^3d\ldots}}{\ldots}$
parallelrisch	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
(3*A*b^2*d-2*B*a*b*d-B*b^2*c+C*a^2*d+2*C*a*b*c-3*D*a^2*c)/(a*d-b*c)^4*ln(d
*x+c)-1/2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^2/(a*d-b*c)^2/(d*x+c)^2-(-2*A*b*
d^3+B*a*d^3+B*b*c*d^2-2*C*a*c*d^2+3*D*a*c^2*d-D*b*c^3)/(a*d-b*c)^3/d^2/(d*
x+c)-(-A*b^3+B*a*b^2-C*a^2*b+D*a^3)/(a*d-b*c)^3/b/(b*x+a)-(3*A*b^2*d-2*B*a
*b*d-B*b^2*c+C*a^2*d+2*C*a*b*c-3*D*a^2*c)/(a*d-b*c)^4*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. 2(270) = 540.

Time = 0.11 (sec) , antiderivative size = 1351, normalized size of antiderivative = 4.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(D*a*b^3*c^5 + A*a^3*b*d^5 - (6*D*a^2*b^2 - C*a*b^3)*c^4*d + (3*D*a^3
*b + 4*C*a^2*b^2 - 5*B*a*b^3 + 2*A*b^4)*c^3*d^2 + (2*D*a^4 - 5*C*a^3*b + 4
*B*a^2*b^2 + 3*A*a*b^3)*c^2*d^3 + (B*a^3*b - 6*A*a^2*b^2)*c*d^4 + 2*(D*b^4
*c^4*d - 4*D*a*b^3*c^3*d^2 + (3*D*a^2*b^2 + 2*C*a*b^3 - B*b^4)*c^2*d^3 - (
D*a^3*b + C*a^2*b^2 + B*a*b^3 - 3*A*b^4)*c*d^4 + (D*a^4 - C*a^3*b + 2*B*a^
2*b^2 - 3*A*a*b^3)*d^5)*x^2 + (D*b^4*c^5 - (4*D*a*b^3 - C*b^4)*c^4*d - (3*
D*a^2*b^2 - 2*C*a*b^3 + 3*B*b^4)*c^3*d^2 + (2*D*a^3*b + 5*C*a^2*b^2 - 4*B*
a*b^3 + 9*A*b^4)*c^2*d^3 + (4*D*a^4 - 8*C*a^3*b + 5*B*a^2*b^2 - 6*A*a*b^3)
*c*d^4 + (2*B*a^3*b - 3*A*a^2*b^2)*d^5)*x - 2*((3*D*a^3*b - 2*C*a^2*b^2 +
B*a*b^3)*c^3*d^2 - (C*a^3*b - 2*B*a^2*b^2 + 3*A*a*b^3)*c^2*d^3 + ((3*D*a^2
*b^2 - 2*C*a*b^3 + B*b^4)*c*d^4 - (C*a^2*b^2 - 2*B*a*b^3 + 3*A*b^4)*d^5)*x
^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c^2*d^3 + (3*D*a^3*b - 4*C*a^2*b
^2 + 5*B*a*b^3 - 6*A*b^4)*c*d^4 - (C*a^3*b - 2*B*a^2*b^2 + 3*A*a*b^3)*d^5)
*x^2 + ((3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c^3*d^2 + (6*D*a^3*b - 5*C*a^2*b
^2 + 4*B*a*b^3 - 3*A*b^4)*c^2*d^3 - 2*(C*a^3*b - 2*B*a^2*b^2 + 3*A*a*b^3)*
c*d^4)*x)*log(b*x + a) + 2*((3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c^3*d^2 -
(C*a^3*b - 2*B*a^2*b^2 + 3*A*a*b^3)*c^2*d^3 + ((3*D*a^2*b^2 - 2*C*a*b^3 +
B*b^4)*c*d^4 - (C*a^2*b^2 - 2*B*a*b^3 + 3*A*b^4)*d^5)*x^3 + (2*(3*D*a^2*b^
2 - 2*C*a*b^3 + B*b^4)*c^2*d^3 + (3*D*a^3*b - 4*C*a^2*b^2 + 5*B*a*b^3 - 6*
A*b^4)*c*d^4 - (C*a^3*b - 2*B*a^2*b^2 + 3*A*a*b^3)*d^5)*x^2 + ((3*D*a^2...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1912 vs. $2(250) = 500$.

Time = 42.63 (sec) , antiderivative size = 1912, normalized size of antiderivative = 6.93

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**3,x)
```

output
```
(-A*a**2*b*d**4 + 5*A*a*b**2*c*d**3 + 2*A*b**3*c**2*d**2 - B*a**2*b*c*d**3
 - 5*B*a*b**2*c**2*d**2 + 5*C*a**2*b*c**2*d**2 + C*a*b**2*c**3*d - 2*D*a**
3*c**2*d**2 - 5*D*a**2*b*c**3*d + D*a*b**2*c**4 + x**2*(6*A*b**3*d**4 - 4*
B*a*b**2*d**4 - 2*B*b**3*c*d**3 + 2*C*a**2*b*d**4 + 4*C*a*b**2*c*d**3 - 2*
D*a**3*d**4 - 6*D*a*b**2*c**2*d**2 + 2*D*b**3*c**3*d) + x*(3*A*a*b**2*d**4
 + 9*A*b**3*c*d**3 - 2*B*a**2*b*d**4 - 7*B*a*b**2*c*d**3 - 3*B*b**3*c**2*d
**2 + 8*C*a**2*b*c*d**3 + 3*C*a*b**2*c**2*d**2 + C*b**3*c**3*d - 4*D*a**3*
c*d**3 - 6*D*a**2*b*c**2*d**2 - 3*D*a*b**2*c**3*d + D*b**3*c**4))/(2*a**4*
b*c**2*d**5 - 6*a**3*b**2*c**3*d**4 + 6*a**2*b**3*c**4*d**3 - 2*a*b**4*c**
5*d**2 + x**3*(2*a**3*b**2*d**7 - 6*a**2*b**3*c*d**6 + 6*a*b**4*c**2*d**5
- 2*b**5*c**3*d**4) + x**2*(2*a**4*b*d**7 - 2*a**3*b**2*c*d**6 - 6*a**2*b*
*3*c**2*d**5 + 10*a*b**4*c**3*d**4 - 4*b**5*c**4*d**3) + x*(4*a**4*b*c*d**
6 - 10*a**3*b**2*c**2*d**5 + 6*a**2*b**3*c**3*d**4 + 2*a*b**4*c**4*d**3 -
2*b**5*c**5*d**2)) - (-3*A*b**2*d + 2*B*a*b*d + B*b**2*c - C*a**2*d - 2*C*
a*b*c + 3*D*a**2*c)*log(x + (-3*A*a*b**2*d**2 - 3*A*b**3*c*d + 2*B*a**2*b*
d**2 + 3*B*a*b**2*c*d + B*b**3*c**2 - C*a**3*d**2 - 3*C*a**2*b*c*d - 2*C*a
*b**2*c**2 + 3*D*a**3*c*d + 3*D*a**2*b*c**2 - a**5*d**5*(-3*A*b**2*d + 2*B
*a*b*d + B*b**2*c - C*a**2*d - 2*C*a*b*c + 3*D*a**2*c)/(a*d - b*c)**4 + 5*
a**4*b*c*d**4*(-3*A*b**2*d + 2*B*a*b*d + B*b**2*c - C*a**2*d - 2*C*a*b*c +
 3*D*a**2*c)/(a*d - b*c)**4 - 10*a**3*b**2*c**2*d**3*(-3*A*b**2*d + 2*B...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(270) = 540$.

Time = 0.07 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.59

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx = \frac{((3\,Da^2-2\,Cab+Bb^2)c-(Ca^2-2\,Bab+3\,Ab^2)d)\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} - \frac{((3\,Da^2-2\,Cab+Bb^2)c-(Ca^2-2\,Bab+3\,Ab^2)d)\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} - \frac{Dab^2c^4-Aa^2bd^4-(5\,Da^2b-Cab^2)c^3d-(2\,Da^3-5\,Ca^2b+5\,Bab^2-2\,Ab^3)c^2d^2-(Ba^2b-5\,Aab^2)}{2\,(ab^4c^5d^2-3\,a^2b^3c^4d^3+3\,a^3b^2c^3d^4-a^4bc^2d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
((3*D*a^2 - 2*C*a*b + B*b^2)*c - (C*a^2 - 2*B*a*b + 3*A*b^2)*d)*log(b*x +
a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
 - ((3*D*a^2 - 2*C*a*b + B*b^2)*c - (C*a^2 - 2*B*a*b + 3*A*b^2)*d)*log(d*x
 + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d
^4) - 1/2*(D*a*b^2*c^4 - A*a^2*b*d^4 - (5*D*a^2*b - C*a*b^2)*c^3*d - (2*D*
a^3 - 5*C*a^2*b + 5*B*a*b^2 - 2*A*b^3)*c^2*d^2 - (B*a^2*b - 5*A*a*b^2)*c*d
^3 + 2*(D*b^3*c^3*d - 3*D*a*b^2*c^2*d^2 + (2*C*a*b^2 - B*b^3)*c*d^3 - (D*a
^3 - C*a^2*b + 2*B*a*b^2 - 3*A*b^3)*d^4)*x^2 + (D*b^3*c^4 - (3*D*a*b^2 - C
*b^3)*c^3*d - 3*(2*D*a^2*b - C*a*b^2 + B*b^3)*c^2*d^2 - (4*D*a^3 - 8*C*a^2
*b + 7*B*a*b^2 - 9*A*b^3)*c*d^3 - (2*B*a^2*b - 3*A*a*b^2)*d^4)*x)/(a*b^4*c
^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c^2*d^5 + (b^5*c^3*
d^4 - 3*a*b^4*c^2*d^5 + 3*a^2*b^3*c*d^6 - a^3*b^2*d^7)*x^3 + (2*b^5*c^4*d^
3 - 5*a*b^4*c^3*d^4 + 3*a^2*b^3*c^2*d^5 + a^3*b^2*c*d^6 - a^4*b*d^7)*x^2 +
 (b^5*c^5*d^2 - a*b^4*c^4*d^3 - 3*a^2*b^3*c^3*d^4 + 5*a^3*b^2*c^2*d^5 - 2*
a^4*b*c*d^6)*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx$$
$$= -\frac{(3\,Da^2bc-2\,Cab^2c+Bb^3c-Ca^2bd+2\,Bab^2d-3\,Ab^3d)\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4}$$
$$+\frac{\frac{Da^3b^2}{bx+a}-\frac{Ca^2b^3}{bx+a}+\frac{Bab^4}{bx+a}-\frac{Ab^5}{bx+a}}{b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3}$$
$$+\frac{Db^2c^3-6\,Dabc^2d+Cb^2c^2d+4\,Cabcd^2-3\,Bb^2cd^2-2\,Babd^3+5\,Ab^2d^3-\frac{2\,(3\,Dab^3c^3-Cb^4c^3-3\,Da^2b^2c^2d-}{}}{2\,(bc-ad)^4\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(3*D*a^2*b*c - 2*C*a*b^2*c + B*b^3*c - C*a^2*b*d + 2*B*a*b^2*d - 3*A*b^3*
d)*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^5*c^4 - 4*a*b^4*c^3*d +
6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (D*a^3*b^2/(b*x + a) -
C*a^2*b^3/(b*x + a) + B*a*b^4/(b*x + a) - A*b^5/(b*x + a))/(b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3) + 1/2*(D*b^2*c^3 - 6*D*a*b*c^2*
d + C*b^2*c^2*d + 4*C*a*b*c*d^2 - 3*B*b^2*c*d^2 - 2*B*a*b*d^3 + 5*A*b^2*d^
3 - 2*(3*D*a*b^3*c^3 - C*b^4*c^3 - 3*D*a^2*b^2*c^2*d - C*a*b^3*c^2*d + 2*B
*b^4*c^2*d + 2*C*a^2*b^2*c*d^2 - B*a*b^3*c*d^2 - 3*A*b^4*c*d^2 - B*a^2*b^2
*d^3 + 3*A*a*b^3*d^3)/((b*x + a)*b))/((b*c - a*d)^4*(b*c/(b*x + a) - a*d/(
b*x + a) + d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1252, normalized size of antiderivative = 4.54

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^3}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^3,x)
```

output
```
(4*log(a + b*x)*a**3*c**3*d**2 + 8*log(a + b*x)*a**3*c**2*d**3*x + 4*log(a
 + b*x)*a**3*c*d**4*x**2 - 2*log(a + b*x)*a**2*b**2*c**2*d**2 - 4*log(a +
b*x)*a**2*b**2*c*d**3*x - 2*log(a + b*x)*a**2*b**2*d**4*x**2 + 8*log(a + b
*x)*a**2*b*c**4*d + 20*log(a + b*x)*a**2*b*c**3*d**2*x + 16*log(a + b*x)*a
**2*b*c**2*d**3*x**2 + 4*log(a + b*x)*a**2*b*c*d**4*x**3 - 4*log(a + b*x)*
a*b**3*c**3*d - 10*log(a + b*x)*a*b**3*c**2*d**2*x - 8*log(a + b*x)*a*b**3
*c*d**3*x**2 - 2*log(a + b*x)*a*b**3*d**4*x**3 + 8*log(a + b*x)*a*b**2*c**
4*d*x + 16*log(a + b*x)*a*b**2*c**3*d**2*x**2 + 8*log(a + b*x)*a*b**2*c**2
*d**3*x**3 - 4*log(a + b*x)*b**4*c**3*d*x - 8*log(a + b*x)*b**4*c**2*d**2*
x**2 - 4*log(a + b*x)*b**4*c*d**3*x**3 - 4*log(c + d*x)*a**3*c**3*d**2 - 8
*log(c + d*x)*a**3*c**2*d**3*x - 4*log(c + d*x)*a**3*c*d**4*x**2 + 2*log(c
 + d*x)*a**2*b**2*c**2*d**2 + 4*log(c + d*x)*a**2*b**2*c*d**3*x + 2*log(c
+ d*x)*a**2*b**2*d**4*x**2 - 8*log(c + d*x)*a**2*b*c**4*d - 20*log(c + d*x
)*a**2*b*c**3*d**2*x - 16*log(c + d*x)*a**2*b*c**2*d**3*x**2 - 4*log(c + d
*x)*a**2*b*c*d**4*x**3 + 4*log(c + d*x)*a*b**3*c**3*d + 10*log(c + d*x)*a*
b**3*c**2*d**2*x + 8*log(c + d*x)*a*b**3*c*d**3*x**2 + 2*log(c + d*x)*a*b*
*3*d**4*x**3 - 8*log(c + d*x)*a*b**2*c**4*d*x - 16*log(c + d*x)*a*b**2*c**
3*d**2*x**2 - 8*log(c + d*x)*a*b**2*c**2*d**3*x**3 + 4*log(c + d*x)*b**4*c
**3*d*x + 8*log(c + d*x)*b**4*c**2*d**2*x**2 + 4*log(c + d*x)*b**4*c*d**3*
x**3 - a**4*d**4 + 2*a**3*b*c*d**3 + a**3*b*d**4*x - 6*a**3*c**3*d**2 -...
```

3.47 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^3}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 362

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^3}\,dx &= \frac{-Ab^3+a(b^2B-abC+a^2D)}{2b(bc-ad)^3(a+bx)^2}-\frac{b^2(Bc-3Ad)-ab(2cC-2Bd)-a^2(Cd-3cD)}{(bc-ad)^4(a+bx)}\\ &+\frac{c^2Cd-Bcd^2+Ad^3-c^3D}{2d(bc-ad)^3(c+dx)^2}+\frac{b(c^2C-2Bcd+3Ad^2)+a(2cCd-Bd^2-3c^2D)}{(bc-ad)^4(c+dx)}\\ &+\frac{(b^2(c^2C-3Bcd+6Ad^2)+a^2d(Cd-3cD)+ab(4cCd-3Bd^2-3c^2D))\log(a+bx)}{(bc-ad)^5}\\ &-\frac{(b^2(c^2C-3Bcd+6Ad^2)+a^2d(Cd-3cD)+ab(4cCd-3Bd^2-3c^2D))\log(c+dx)}{(bc-ad)^5}\end{aligned}$$

output
```
1/2*(-A*b^3+a*(B*b^2-C*a*b+D*a^2))/b/(-a*d+b*c)^3/(b*x+a)^2-(b^2*(-3*A*d+B
*c)-a*b*(-2*B*d+2*C*c)-a^2*(C*d-3*D*c))/(-a*d+b*c)^4/(b*x+a)+1/2*(A*d^3-B*
c*d^2+C*c^2*d-D*c^3)/d/(-a*d+b*c)^3/(d*x+c)^2+(b*(3*A*d^2-2*B*c*d+C*c^2)+a
*(-B*d^2+2*C*c*d-3*D*c^2))/(-a*d+b*c)^4/(d*x+c)+(b^2*(6*A*d^2-3*B*c*d+C*c^
2)+a^2*d*(C*d-3*D*c)+a*b*(-3*B*d^2+4*C*c*d-3*D*c^2))*ln(b*x+a)/(-a*d+b*c)^
5-(b^2*(6*A*d^2-3*B*c*d+C*c^2)+a^2*d*(C*d-3*D*c)+a*b*(-3*B*d^2+4*C*c*d-3*D
*c^2))*ln(d*x+c)/(-a*d+b*c)^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^3}\,dx$$
$$= \frac{-Ab^3+a(b^2B-abC+a^2D)}{2b(bc-ad)^3(a+bx)^2}+\frac{b^2(-Bc+3Ad)+2ab(cC-Bd)+a^2(Cd-3cD)}{(bc-ad)^4(a+bx)}$$
$$+\frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{2d(-bc+ad)^3(c+dx)^2}+\frac{b(c^2C-2Bcd+3Ad^2)-a(-2cCd+Bd^2+3c^2D)}{(bc-ad)^4(c+dx)}$$
$$+\frac{(b^2(c^2C-3Bcd+6Ad^2)+a^2d(Cd-3cD)+ab(4cCd-3Bd^2-3c^2D))\log(a+bx)}{(bc-ad)^5}$$
$$-\frac{(b^2(c^2C-3Bcd+6Ad^2)+a^2d(Cd-3cD)+ab(4cCd-3Bd^2-3c^2D))\log(c+dx)}{(bc-ad)^5}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^3),x]
```

output
```
(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))/(2*b*(b*c - a*d)^3*(a + b*x)^2) + (
b^2*(-(B*c) + 3*A*d) + 2*a*b*(c*C - B*d) + a^2*(C*d - 3*c*D))/((b*c - a*d)
^4*(a + b*x)) + (-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)/(2*d*(-(b*c) + a*d)
^3*(c + d*x)^2) + (b*(c^2*C - 2*B*c*d + 3*A*d^2) - a*(-2*c*C*d + B*d^2 + 3
*c^2*D))/((b*c - a*d)^4*(c + d*x)) + ((b^2*(c^2*C - 3*B*c*d + 6*A*d^2) + a
^2*d*(C*d - 3*c*D) + a*b*(4*c*C*d - 3*B*d^2 - 3*c^2*D))*Log[a + b*x])/(b*c
 - a*d)^5 - ((b^2*(c^2*C - 3*B*c*d + 6*A*d^2) + a^2*d*(C*d - 3*c*D) + a*b*
(4*c*C*d - 3*B*d^2 - 3*c^2*D))*Log[c + d*x])/(b*c - a*d)^5
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^3}\,dx$$

$$\downarrow \text{2123}$$

$$\int \left(\frac{b\left(a^2d(Cd-3cD)+ab\left(-3Bd^2-3c^2D+4cCd\right)+b^2\left(6Ad^2-3Bcd+c^2C\right)\right)}{(a+bx)(bc-ad)^5}+\frac{d\left(a^2(-d)(Cd-3cD)-ab\right.}{}\right.$$

$$\downarrow \text{2009}$$

$$\frac{\log(a+bx)\left(a^2d(Cd-3cD)+ab\left(-3Bd^2-3c^2D+4cCd\right)+b^2\left(6Ad^2-3Bcd+c^2C\right)\right)}{(bc-ad)^5}-\frac{\log(c+dx)\left(a^2d(Cd-3cD)+ab\left(-3Bd^2-3c^2D+4cCd\right)+b^2\left(6Ad^2-3Bcd+c^2C\right)\right)}{(bc-ad)^5}-\frac{-\left(a^2(Cd-3cD)\right)-ab(2cC-2Bd)+b^2(Bc-3Ad)}{(a+bx)(bc-ad)^4}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b(a+bx)^2(bc-ad)^3}+\frac{a\left(-Bd^2-3c^2D+2cCd\right)+b\left(3Ad^2-2Bcd+c^2C\right)}{(c+dx)(bc-ad)^4}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{2d(c+dx)^2(bc-ad)^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^3),x]
```

output
```
-1/2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b*(b*c - a*d)^3*(a + b*x)^2) - (
b^2*(B*c - 3*A*d) - a*b*(2*c*C - 2*B*d) - a^2*(C*d - 3*c*D))/((b*c - a*d)^
4*(a + b*x)) + (c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(2*d*(b*c - a*d)^3*(c +
 d*x)^2) + (b*(c^2*C - 2*B*c*d + 3*A*d^2) + a*(2*c*C*d - B*d^2 - 3*c^2*D))
/((b*c - a*d)^4*(c + d*x)) + ((b^2*(c^2*C - 3*B*c*d + 6*A*d^2) + a^2*d*(C*
d - 3*c*D) + a*b*(4*c*C*d - 3*B*d^2 - 3*c^2*D))*Log[a + b*x])/(b*c - a*d)^
5 - ((b^2*(c^2*C - 3*B*c*d + 6*A*d^2) + a^2*d*(C*d - 3*c*D) + a*b*(4*c*C*d
 - 3*B*d^2 - 3*c^2*D))*Log[c + d*x])/(b*c - a*d)^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.04

method	result
default	$-\frac{A\,d^{3}-B c\,d^{2}+C\,c^{2} d-D c^{3}}{2\left(a d-b c\right)^{3} d\left(x d+c\right)^{2}}+\frac{\left(6 A\,b^{2} d^{2}-3 B a b\,d^{2}-3 B\,b^{2} c d+C\,a^{2} d^{2}+4 C a b c d+C\,b^{2} c^{2}-3 D a^{2} c d-3 D a b\,c^{2}\right) \ln \left(x d+c\right)}{\left(a d-b c\right)^{5}}+$
norman	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^3/d/(d*x+c)^2+(6*A*b^2*d^2-3*
B*a*b*d^2-3*B*b^2*c*d+C*a^2*d^2+4*C*a*b*c*d+C*b^2*c^2-3*D*a^2*c*d-3*D*a*b*
c^2)/(a*d-b*c)^5*ln(d*x+c)+(3*A*b*d^2-B*a*d^2-2*B*b*c*d+2*C*a*c*d+C*b*c^2-
3*D*a*c^2)/(a*d-b*c)^4/(d*x+c)-1/2*(-A*b^3+B*a*b^2-C*a^2*b+D*a^3)/(a*d-b*c
)^3/b/(b*x+a)^2+(3*A*b^2*d-2*B*a*b*d-B*b^2*c+C*a^2*d+2*C*a*b*c-3*D*a^2*c)/
(a*d-b*c)^4/(b*x+a)-(6*A*b^2*d^2-3*B*a*b*d^2-3*B*b^2*c*d+C*a^2*d^2+4*C*a*b
*c*d+C*b^2*c^2-3*D*a^2*c*d-3*D*a*b*c^2)/(a*d-b*c)^5*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2183 vs. 2(362) = 724.

Time = 0.15 (sec) , antiderivative size = 2183, normalized size of antiderivative = 6.03

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(D*a^2*b^3*c^5 - A*a^4*b*d^5 + (9*D*a^3*b^2 - 6*C*a^2*b^3 + B*a*b^4 +
 A*b^5)*c^4*d - (9*D*a^4*b - 9*B*a^2*b^3 + 8*A*a*b^4)*c^3*d^2 - (D*a^5 - 6
*C*a^4*b + 9*B*a^3*b^2)*c^2*d^3 - (B*a^4*b - 8*A*a^3*b^2)*c*d^4 + 2*((3*D*
a*b^4 - C*b^5)*c^3*d^2 - 3*(C*a*b^4 - B*b^5)*c^2*d^3 - 3*(D*a^3*b^2 - C*a^
2*b^3 + 2*A*b^5)*c*d^4 + (C*a^3*b^2 - 3*B*a^2*b^3 + 6*A*a*b^4)*d^5)*x^3 +
(D*b^5*c^5 + (4*D*a*b^4 - 3*C*b^5)*c^4*d + (19*D*a^2*b^3 - 12*C*a*b^4 + 9*
B*b^5)*c^3*d^2 - (19*D*a^3*b^2 - 9*B*a*b^4 + 18*A*b^5)*c^2*d^3 - (4*D*a^4*
b - 12*C*a^3*b^2 + 9*B*a^2*b^3)*c*d^4 - (D*a^5 - 3*C*a^4*b + 9*B*a^3*b^2 -
 18*A*a^2*b^3)*d^5)*x^2 + 2*(D*a*b^4*c^5 + (7*D*a^2*b^3 - 5*C*a*b^4 + B*b^
5)*c^4*d - (3*C*a^2*b^3 - 7*B*a*b^4 + 2*A*b^5)*c^3*d^2 - (7*D*a^4*b - 3*C*
a^3*b^2 + 12*A*a*b^4)*c^2*d^3 - (D*a^5 - 5*C*a^4*b + 7*B*a^3*b^2 - 12*A*a^
2*b^3)*c*d^4 - (B*a^4*b - 2*A*a^3*b^2)*d^5)*x + 2*((3*D*a^3*b^2 - C*a^2*b^
3)*c^4*d + (3*D*a^4*b - 4*C*a^3*b^2 + 3*B*a^2*b^3)*c^3*d^2 - (C*a^4*b - 3*
B*a^3*b^2 + 6*A*a^2*b^3)*c^2*d^3 + ((3*D*a*b^4 - C*b^5)*c^2*d^3 + (3*D*a^2
*b^3 - 4*C*a*b^4 + 3*B*b^5)*c*d^4 - (C*a^2*b^3 - 3*B*a*b^4 + 6*A*b^5)*d^5)
*x^4 + 2*((3*D*a*b^4 - C*b^5)*c^3*d^2 + (6*D*a^2*b^3 - 5*C*a*b^4 + 3*B*b^5
)*c^2*d^3 + (3*D*a^3*b^2 - 5*C*a^2*b^3 + 6*B*a*b^4 - 6*A*b^5)*c*d^4 - (C*a
^3*b^2 - 3*B*a^2*b^3 + 6*A*a*b^4)*d^5)*x^3 + ((3*D*a*b^4 - C*b^5)*c^4*d +
(15*D*a^2*b^3 - 8*C*a*b^4 + 3*B*b^5)*c^3*d^2 + 3*(5*D*a^3*b^2 - 6*C*a^2*b^
3 + 5*B*a*b^4 - 2*A*b^5)*c^2*d^3 + (3*D*a^4*b - 8*C*a^3*b^2 + 15*B*a^2*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3074 vs. $2(343) = 686$.

Time = 149.07 (sec) , antiderivative size = 3074, normalized size of antiderivative = 8.49

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**3,x)
```

output
```
(-A*a**3*b*d**4 + 7*A*a**2*b**2*c*d**3 + 7*A*a*b**3*c**2*d**2 - A*b**4*c**
3*d - B*a**3*b*c*d**3 - 10*B*a**2*b**2*c**2*d**2 - B*a*b**3*c**3*d + 6*C*a
**3*b*c**2*d**2 + 6*C*a**2*b**2*c**3*d - D*a**4*c**2*d**2 - 10*D*a**3*b*c*
*3*d - D*a**2*b**2*c**4 + x**3*(12*A*b**4*d**4 - 6*B*a*b**3*d**4 - 6*B*b**
4*c*d**3 + 2*C*a**2*b**2*d**4 + 8*C*a*b**3*c*d**3 + 2*C*b**4*c**2*d**2 - 6
*D*a**2*b**2*c*d**3 - 6*D*a*b**3*c**2*d**2) + x**2*(18*A*a*b**3*d**4 + 18*
A*b**4*c*d**3 - 9*B*a**2*b**2*d**4 - 18*B*a*b**3*c*d**3 - 9*B*b**4*c**2*d*
*2 + 3*C*a**3*b*d**4 + 15*C*a**2*b**2*c*d**3 + 15*C*a*b**3*c**2*d**2 + 3*C
*b**4*c**3*d - D*a**4*d**4 - 5*D*a**3*b*c*d**3 - 24*D*a**2*b**2*c**2*d**2
- 5*D*a*b**3*c**3*d - D*b**4*c**4) + x*(4*A*a**2*b**2*d**4 + 28*A*a*b**3*c
*d**3 + 4*A*b**4*c**2*d**2 - 2*B*a**3*b*d**4 - 16*B*a**2*b**2*c*d**3 - 16*
B*a*b**3*c**2*d**2 - 2*B*b**4*c**3*d + 10*C*a**3*b*c*d**3 + 16*C*a**2*b**2
*c**2*d**2 + 10*C*a*b**3*c**3*d - 2*D*a**4*c*d**3 - 16*D*a**3*b*c**2*d**2
- 16*D*a**2*b**2*c**3*d - 2*D*a*b**3*c**4))/(2*a**6*b*c**2*d**5 - 8*a**5*b
**2*c**3*d**4 + 12*a**4*b**3*c**4*d**3 - 8*a**3*b**4*c**5*d**2 + 2*a**2*b*
*5*c**6*d + x**4*(2*a**4*b**3*d**7 - 8*a**3*b**4*c*d**6 + 12*a**2*b**5*c**
2*d**5 - 8*a*b**6*c**3*d**4 + 2*b**7*c**4*d**3) + x**3*(4*a**5*b**2*d**7 -
 12*a**4*b**3*c*d**6 + 8*a**3*b**4*c**2*d**5 + 8*a**2*b**5*c**3*d**4 - 12*
a*b**6*c**4*d**3 + 4*b**7*c**5*d**2) + x**2*(2*a**6*b*d**7 - 18*a**4*b**3*
c**2*d**5 + 32*a**3*b**4*c**3*d**4 - 18*a**2*b**5*c**4*d**3 + 2*b**7*c*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(362) = 724$.

Time = 0.07 (sec) , antiderivative size = 1075, normalized size of antiderivative = 2.97

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-((3*D*a*b - C*b^2)*c^2 + (3*D*a^2 - 4*C*a*b + 3*B*b^2)*c*d - (C*a^2 - 3*B
*a*b + 6*A*b^2)*d^2)*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) + ((3*D*a*b - C*b^2)
*c^2 + (3*D*a^2 - 4*C*a*b + 3*B*b^2)*c*d - (C*a^2 - 3*B*a*b + 6*A*b^2)*d^2
)*log(d*x + c)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*
c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 1/2*(D*a^2*b^2*c^4 + A*a^3*b*d^4 + (1
0*D*a^3*b - 6*C*a^2*b^2 + B*a*b^3 + A*b^4)*c^3*d + (D*a^4 - 6*C*a^3*b + 10
*B*a^2*b^2 - 7*A*a*b^3)*c^2*d^2 + (B*a^3*b - 7*A*a^2*b^2)*c*d^3 + 2*((3*D*
a*b^3 - C*b^4)*c^2*d^2 + (3*D*a^2*b^2 - 4*C*a*b^3 + 3*B*b^4)*c*d^3 - (C*a^
2*b^2 - 3*B*a*b^3 + 6*A*b^4)*d^4)*x^3 + (D*b^4*c^4 + (5*D*a*b^3 - 3*C*b^4)
*c^3*d + 3*(8*D*a^2*b^2 - 5*C*a*b^3 + 3*B*b^4)*c^2*d^2 + (5*D*a^3*b - 15*C
*a^2*b^2 + 18*B*a*b^3 - 18*A*b^4)*c*d^3 + (D*a^4 - 3*C*a^3*b + 9*B*a^2*b^2
 - 18*A*a*b^3)*d^4)*x^2 + 2*(D*a*b^3*c^4 + (8*D*a^2*b^2 - 5*C*a*b^3 + B*b^
4)*c^3*d + 2*(4*D*a^3*b - 4*C*a^2*b^2 + 4*B*a*b^3 - A*b^4)*c^2*d^2 + (D*a^
4 - 5*C*a^3*b + 8*B*a^2*b^2 - 14*A*a*b^3)*c*d^3 + (B*a^3*b - 2*A*a^2*b^2)*
d^4)*x)/(a^2*b^5*c^6*d - 4*a^3*b^4*c^5*d^2 + 6*a^4*b^3*c^4*d^3 - 4*a^5*b^2
*c^3*d^4 + a^6*b*c^2*d^5 + (b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*
d^5 - 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^4 + 2*(b^7*c^5*d^2 - 3*a*b^6*c^4*d^
3 + 2*a^2*b^5*c^3*d^4 + 2*a^3*b^4*c^2*d^5 - 3*a^4*b^3*c*d^6 + a^5*b^2*d^7)
*x^3 + (b^7*c^6*d - 9*a^2*b^5*c^4*d^3 + 16*a^3*b^4*c^3*d^4 - 9*a^4*b^3*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(362) = 724$.

Time = 0.13 (sec) , antiderivative size = 1012, normalized size of antiderivative = 2.80

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^3} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
-(3*D*a*b^2*c^2 - C*b^3*c^2 + 3*D*a^2*b*c*d - 4*C*a*b^2*c*d + 3*B*b^3*c*d
- C*a^2*b*d^2 + 3*B*a*b^2*d^2 - 6*A*b^3*d^2)*log(abs(b*x + a))/(b^6*c^5 -
5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4
- a^5*b*d^5) + (3*D*a*b*c^2*d - C*b^2*c^2*d + 3*D*a^2*c*d^2 - 4*C*a*b*c*d^
2 + 3*B*b^2*c*d^2 - C*a^2*d^3 + 3*B*a*b*d^3 - 6*A*b^2*d^3)*log(abs(d*x + c
))/(b^5*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2*d^4
+ 5*a^4*b*c*d^5 - a^5*d^6) - 1/2*(6*D*a*b^3*c^2*d^2*x^3 - 2*C*b^4*c^2*d^2*
x^3 + 6*D*a^2*b^2*c*d^3*x^3 - 8*C*a*b^3*c*d^3*x^3 + 6*B*b^4*c*d^3*x^3 - 2*
C*a^2*b^2*d^4*x^3 + 6*B*a*b^3*d^4*x^3 - 12*A*b^4*d^4*x^3 + D*b^4*c^4*x^2 +
 5*D*a*b^3*c^3*d*x^2 - 3*C*b^4*c^3*d*x^2 + 24*D*a^2*b^2*c^2*d^2*x^2 - 15*C
*a*b^3*c^2*d^2*x^2 + 9*B*b^4*c^2*d^2*x^2 + 5*D*a^3*b*c*d^3*x^2 - 15*C*a^2*
b^2*c*d^3*x^2 + 18*B*a*b^3*c*d^3*x^2 - 18*A*b^4*c*d^3*x^2 + D*a^4*d^4*x^2
- 3*C*a^3*b*d^4*x^2 + 9*B*a^2*b^2*d^4*x^2 - 18*A*a*b^3*d^4*x^2 + 2*D*a*b^3
*c^4*x + 16*D*a^2*b^2*c^3*d*x - 10*C*a*b^3*c^3*d*x + 2*B*b^4*c^3*d*x + 16*
D*a^3*b*c^2*d^2*x - 16*C*a^2*b^2*c^2*d^2*x + 16*B*a*b^3*c^2*d^2*x - 4*A*b^
4*c^2*d^2*x + 2*D*a^4*c*d^3*x - 10*C*a^3*b*c*d^3*x + 16*B*a^2*b^2*c*d^3*x
- 28*A*a*b^3*c*d^3*x + 2*B*a^3*b*d^4*x - 4*A*a^2*b^2*d^4*x + D*a^2*b^2*c^4
 + 10*D*a^3*b*c^3*d - 6*C*a^2*b^2*c^3*d + B*a*b^3*c^3*d + A*b^4*c^3*d + D*
a^4*c^2*d^2 - 6*C*a^3*b*c^2*d^2 + 10*B*a^2*b^2*c^2*d^2 - 7*A*a*b^3*c^2*d^2
 + B*a^3*b*c*d^3 - 7*A*a^2*b^2*c*d^3 + A*a^3*b*d^4)/((b^5*c^4*d - 4*a*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^3}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2124, normalized size of antiderivative = 5.87

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^3} \, dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^3,x)
```

output
```
(4*log(a + b*x)*a**4*b*c**3*d**2 + 8*log(a + b*x)*a**4*b*c**2*d**3*x + 4*l
og(a + b*x)*a**4*b*c*d**4*x**2 - 6*log(a + b*x)*a**3*b**3*c**2*d**2 - 12*l
og(a + b*x)*a**3*b**3*c*d**3*x - 6*log(a + b*x)*a**3*b**3*d**4*x**2 + 6*lo
g(a + b*x)*a**3*b**2*c**4*d + 20*log(a + b*x)*a**3*b**2*c**3*d**2*x + 22*l
og(a + b*x)*a**3*b**2*c**2*d**3*x**2 + 8*log(a + b*x)*a**3*b**2*c*d**4*x**
3 - 6*log(a + b*x)*a**2*b**4*c**3*d - 24*log(a + b*x)*a**2*b**4*c**2*d**2*
x - 30*log(a + b*x)*a**2*b**4*c*d**3*x**2 - 12*log(a + b*x)*a**2*b**4*d**4
*x**3 + 2*log(a + b*x)*a**2*b**3*c**5 + 16*log(a + b*x)*a**2*b**3*c**4*d*x
 + 30*log(a + b*x)*a**2*b**3*c**3*d**2*x**2 + 20*log(a + b*x)*a**2*b**3*c*
*2*d**3*x**3 + 4*log(a + b*x)*a**2*b**3*c*d**4*x**4 - 12*log(a + b*x)*a*b*
*5*c**3*d*x - 30*log(a + b*x)*a*b**5*c**2*d**2*x**2 - 24*log(a + b*x)*a*b*
*5*c*d**3*x**3 - 6*log(a + b*x)*a*b**5*d**4*x**4 + 4*log(a + b*x)*a*b**4*c
**5*x + 14*log(a + b*x)*a*b**4*c**4*d*x**2 + 16*log(a + b*x)*a*b**4*c**3*d
**2*x**3 + 6*log(a + b*x)*a*b**4*c**2*d**3*x**4 - 6*log(a + b*x)*b**6*c**3
*d*x**2 - 12*log(a + b*x)*b**6*c**2*d**2*x**3 - 6*log(a + b*x)*b**6*c*d**3
*x**4 + 2*log(a + b*x)*b**5*c**5*x**2 + 4*log(a + b*x)*b**5*c**4*d*x**3 +
2*log(a + b*x)*b**5*c**3*d**2*x**4 - 4*log(c + d*x)*a**4*b*c**3*d**2 - 8*l
og(c + d*x)*a**4*b*c**2*d**3*x - 4*log(c + d*x)*a**4*b*c*d**4*x**2 + 6*log
(c + d*x)*a**3*b**3*c**2*d**2 + 12*log(c + d*x)*a**3*b**3*c*d**3*x + 6*log
(c + d*x)*a**3*b**3*d**4*x**2 - 6*log(c + d*x)*a**3*b**2*c**4*d - 20*lo...
```

3.48 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 400

$$\begin{aligned}
&\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx \\
&= \frac{b(3a^2d^2D+3abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))\,x}{d^6} \\
&+ \frac{b^2(bCd-4bcD+3adD)x^2}{2d^5} + \frac{b^3Dx^3}{3d^4} + \frac{(bc-ad)^3\,(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^7(c+dx)^3} \\
&+ \frac{(bc-ad)^2\,(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D))}{2d^7(c+dx)^2} \\
&+ \frac{(bc-ad)\,(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D))}{d^7(c+dx)} \\
&+ \frac{(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D))}{d^7}
\end{aligned}$$

output

```
b*(3*a^2*d^2*D+3*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*x/d^6+1/
2*b^2*(C*b*d+3*D*a*d-4*D*b*c)*x^2/d^5+1/3*b^3*D*x^3/d^4+1/3*(-a*d+b*c)^3*(
A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^7/(d*x+c)^3+1/2*(-a*d+b*c)^2*(a*d*(-B*d^2+2
*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3))/d^7/(d*x+c)^2+(-a
*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^2+8*C*c*d-15*D*c^2)+b^2*(3*A*d^
3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))/d^7/(d*x+c)+(a^3*d^3*D+3*a^2*b*d^2*(C*d-
4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d-10*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d
-20*D*c^3))*ln(d*x+c)/d^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4} dx$$
$$= \frac{6bd(3a^2d^2D+3abd(Cd-4cD)+b^2(-4cCd+Bd^2+10c^2D))\,x+3b^2d^2(bCd-4bcD+3adD)x^2+2b^3}{}$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
(6*b*d*(3*a^2*d^2*D + 3*a*b*d*(C*d - 4*c*D) + b^2*(-4*c*C*d + B*d^2 + 10*c
^2*D))*x + 3*b^2*d^2*(b*C*d - 4*b*c*D + 3*a*d*D)*x^2 + 2*b^3*d^3*D*x^3 - (
2*(b*c - a*d)^3*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(c + d*x)^3 + (3*(
b*c - a*d)^2*(-(a*d*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-5*c^2*C*d + 4*B*c*
d^2 - 3*A*d^3 + 6*c^3*D)))/(c + d*x)^2 - (6*(b*c - a*d)*(a^2*d^2*(-(C*d) +
 3*c*D) + a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(-10*c^2*C*d + 6*B*c*
d^2 - 3*A*d^3 + 15*c^3*D)))/(c + d*x) + 6*(a^3*d^3*D + 3*a^2*b*d^2*(C*d -
4*c*D) + 3*a*b^2*d*(-4*c*C*d + B*d^2 + 10*c^2*D) + b^3*(10*c^2*C*d - 4*B*c
*d^2 + A*d^3 - 20*c^3*D))*Log[c + d*x])/(6*d^7)
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4} dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)\right)}{d^6(c+dx)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)}{d^7(c+dx)}+\frac{bx\left(3a^2d^2D+3abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{d^6}+\frac{\log(c+dx)\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2C\right.\right.}{d^7}\frac{(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^7(c+dx)^3}+\frac{(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{2d^7(c+dx)^2}+\frac{b^2x^2(3adD-4bcD+bCd)}{2d^5}+\frac{b^3Dx^3}{3d^4}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
(b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D)
)*x)/d^6 + (b^2*(b*C*d - 4*b*c*D + 3*a*d*D)*x^2)/(2*d^5) + (b^3*D*x^3)/(3*
d^4) + ((b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^7*(c + d*x
)^3) + ((b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d - 4*
B*c*d^2 + 3*A*d^3 - 6*c^3*D)))/(2*d^7*(c + d*x)^2) + ((b*c - a*d)*(a^2*d^2
*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^2*C*d -
6*B*c*d^2 + 3*A*d^3 - 15*c^3*D)))/(d^7*(c + d*x)) + ((a^3*d^3*D + 3*a^2*b*
d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*D) + b^3*(10*c^2*C
*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*Log[c + d*x])/d^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.83

method	result
norman	$-\frac{\frac{2A\,a^{3}d^{6}+3A\,a^{2}bc\,d^{5}+6Aa\,b^{2}c^{2}d^{4}-11A\,b^{3}c^{3}d^{3}+B\,a^{3}c\,d^{5}+6B\,a^{2}b\,c^{2}d^{4}-33Ba\,b^{2}c^{3}d^{3}+44B\,b^{3}c^{4}d^{2}+2C\,a^{3}c^{2}d^{4}-33C\,a^{2}b\,c^{3}d^{3}+132Ca}{6d^{7}}}{}$
default	$\frac{b\left(\frac{1}{3}d^{2}Dx^{3}b^{2}+\frac{1}{2}C\,b^{2}d^{2}x^{2}+\frac{3}{2}Dab\,d^{2}x^{2}-2Db^{2}cd\,x^{2}+b^{2}B\,d^{2}x+3Cab\,d^{2}x-4C\,b^{2}cdx+3a^{2}d^{2}Dx-12Dabcdx+10Db^{2}c^{2}x\right)}{d^{6}}-$
parallelrisch	Expression too large to display

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*(2*A*a^3*d^6+3*A*a^2*b*c*d^5+6*A*a*b^2*c^2*d^4-11*A*b^3*c^3*d^3+B*a^
3*c*d^5+6*B*a^2*b*c^2*d^4-33*B*a*b^2*c^3*d^3+44*B*b^3*c^4*d^2+2*C*a^3*c^2*
d^4-33*C*a^2*b*c^3*d^3+132*C*a*b^2*c^4*d^2-110*C*b^3*c^5*d-11*D*a^3*c^3*d^
3+132*D*a^2*b*c^4*d^2-330*D*a*b^2*c^5*d+220*D*b^3*c^6)/d^7-(3*A*a*b^2*d^4-
3*A*b^3*c*d^3+3*B*a^2*b*d^4-9*B*a*b^2*c*d^3+12*B*b^3*c^2*d^2+C*a^3*d^4-9*C
*a^2*b*c*d^3+36*C*a*b^2*c^2*d^2-30*C*b^3*c^3*d-3*D*a^3*c*d^3+36*D*a^2*b*c^
2*d^2-90*D*a*b^2*c^3*d+60*D*b^3*c^4)/d^5*x^2-1/2*(3*A*a^2*b*d^5+6*A*a*b^2*
c*d^4-9*A*b^3*c^2*d^3+B*a^3*d^5+6*B*a^2*b*c*d^4-27*B*a*b^2*c^2*d^3+36*B*b^
3*c^3*d^2+2*C*a^3*c*d^4-27*C*a^2*b*c^2*d^3+108*C*a*b^2*c^3*d^2-90*C*b^3*c^
4*d-9*D*a^3*c^2*d^3+108*D*a^2*b*c^3*d^2-270*D*a*b^2*c^4*d+180*D*b^3*c^5)/d
^6*x+1/3*D*b^3/d*x^6+1/2*b*(2*B*b^2*d^2+6*C*a*b*d^2-5*C*b^2*c*d+6*D*a^2*d^
2-15*D*a*b*c*d+10*D*b^2*c^2)/d^3*x^4+1/2*b^2*(C*b*d+3*D*a*d-2*D*b*c)/d^2*x
^5)/(d*x+c)^3+1/d^7*(A*b^3*d^3+3*B*a*b^2*d^3-4*B*b^3*c*d^2+3*C*a^2*b*d^3-1
2*C*a*b^2*c*d^2+10*C*b^3*c^2*d+D*a^3*d^3-12*D*a^2*b*c*d^2+30*D*a*b^2*c^2*d
-20*D*b^3*c^3)*ln(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(395) = 790$.

Time = 0.09 (sec) , antiderivative size = 1003, normalized size of antiderivative = 2.51

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(2*D*b^3*d^6*x^6 - 74*D*b^3*c^6 - 2*A*a^3*d^6 + 47*(3*D*a*b^2 + C*b^3)
*c^5*d - 26*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 + 11*(D*a^3 + 3*C*a^2*
b + 3*B*a*b^2 + A*b^3)*c^3*d^3 - 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4
 - (B*a^3 + 3*A*a^2*b)*c*d^5 - 3*(2*D*b^3*c*d^5 - (3*D*a*b^2 + C*b^3)*d^6)
*x^5 + 3*(10*D*b^3*c^2*d^4 - 5*(3*D*a*b^2 + C*b^3)*c*d^5 + 2*(3*D*a^2*b +
3*C*a*b^2 + B*b^3)*d^6)*x^4 + (146*D*b^3*c^3*d^3 - 63*(3*D*a*b^2 + C*b^3)*
c^2*d^4 + 18*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^5)*x^3 + 3*(26*D*b^3*c^4*
d^2 - 3*(3*D*a*b^2 + C*b^3)*c^3*d^3 - 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^
2*d^4 + 6*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^5 - 2*(C*a^3 + 3*B*a
^2*b + 3*A*a*b^2)*d^6)*x^2 - 3*(34*D*b^3*c^5*d - 27*(3*D*a*b^2 + C*b^3)*c^
4*d^2 + 18*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^3 - 9*(D*a^3 + 3*C*a^2*b
+ 3*B*a*b^2 + A*b^3)*c^2*d^4 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^5 + (
B*a^3 + 3*A*a^2*b)*d^6)*x - 6*(20*D*b^3*c^6 - 10*(3*D*a*b^2 + C*b^3)*c^5*d
 + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*
b^2 + A*b^3)*c^3*d^3 + (20*D*b^3*c^3*d^3 - 10*(3*D*a*b^2 + C*b^3)*c^2*d^4
+ 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^5 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2
 + A*b^3)*d^6)*x^3 + 3*(20*D*b^3*c^4*d^2 - 10*(3*D*a*b^2 + C*b^3)*c^3*d^3
+ 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^4 - (D*a^3 + 3*C*a^2*b + 3*B*a*b
^2 + A*b^3)*c*d^5)*x^2 + 3*(20*D*b^3*c^5*d - 10*(3*D*a*b^2 + C*b^3)*c^4*d^
2 + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^3 - (D*a^3 + 3*C*a^2*b + 3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4} \, dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx =$$

$$-\frac{74\,Db^3c^6+2\,Aa^3d^6-47\,(3\,Dab^2+Cb^3)c^5d+26\,(3\,Da^2b+3\,Cab^2+Bb^3)c^4d^2-11\,(Da^3+3\,Ca^2b+}{}$$

$$+\frac{2\,Db^3d^2x^3-3\,(4\,Db^3cd-(3\,Dab^2+Cb^3)d^2)x^2+6\,(10\,Db^3c^2-4\,(3\,Dab^2+Cb^3)cd+(3\,Da^2b+3\,Ca}{6\,d^6}$$

$$-\frac{(20\,Db^3c^3-10\,(3\,Dab^2+Cb^3)c^2d+4\,(3\,Da^2b+3\,Cab^2+Bb^3)cd^2-(Da^3+3\,Ca^2b+3\,Bab^2+Ab^3)}{d^7}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/6*(74*D*b^3*c^6 + 2*A*a^3*d^6 - 47*(3*D*a*b^2 + C*b^3)*c^5*d + 26*(3*D*
a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - 11*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A
*b^3)*c^3*d^3 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 + (B*a^3 + 3*A*a
^2*b)*c*d^5 + 6*(15*D*b^3*c^4*d^2 - 10*(3*D*a*b^2 + C*b^3)*c^3*d^3 + 6*(3*
D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^4 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 +
A*b^3)*c*d^5 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^6)*x^2 + 3*(54*D*b^3*c^5*
d - 35*(3*D*a*b^2 + C*b^3)*c^4*d^2 + 20*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^
3*d^3 - 9*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^4 + 2*(C*a^3 + 3*B
*a^2*b + 3*A*a*b^2)*c*d^5 + (B*a^3 + 3*A*a^2*b)*d^6)*x)/(d^10*x^3 + 3*c*d^
9*x^2 + 3*c^2*d^8*x + c^3*d^7) + 1/6*(2*D*b^3*d^2*x^3 - 3*(4*D*b^3*c*d - (
3*D*a*b^2 + C*b^3)*d^2)*x^2 + 6*(10*D*b^3*c^2 - 4*(3*D*a*b^2 + C*b^3)*c*d
+ (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*x)/d^6 - (20*D*b^3*c^3 - 10*(3*D*a*
b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^2 - (D*a^3 + 3*
C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*log(d*x + c)/d^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.91

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx =$$
$$-\frac{(20\,Db^3c^3 - 30\,Dab^2c^2d - 10\,Cb^3c^2d + 12\,Da^2bcd^2 + 12\,Cab^2cd^2 + 4\,Bb^3cd^2 - Da^3d^3 - 3\,Ca^2bd^3 - 3}{d^7}$$
$$-\frac{74\,Db^3c^6 - 141\,Dab^2c^5d - 47\,Cb^3c^5d + 78\,Da^2bc^4d^2 + 78\,Cab^2c^4d^2 + 26\,Bb^3c^4d^2 - 11\,Da^3c^3d^3 - 33\,C}{}$$
$$+\frac{2\,Db^3d^8x^3 - 12\,Db^3cd^7x^2 + 9\,Dab^2d^8x^2 + 3\,Cb^3d^8x^2 + 60\,Db^3c^2d^6x - 72\,Dab^2cd^7x - 24\,Cb^3cd^7x + 18}{6\,d^{12}}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="giac")
```

output
```
-(20*D*b^3*c^3 - 30*D*a*b^2*c^2*d - 10*C*b^3*c^2*d + 12*D*a^2*b*c*d^2 + 12
*C*a*b^2*c*d^2 + 4*B*b^3*c*d^2 - D*a^3*d^3 - 3*C*a^2*b*d^3 - 3*B*a*b^2*d^3
 - A*b^3*d^3)*log(abs(d*x + c))/d^7 - 1/6*(74*D*b^3*c^6 - 141*D*a*b^2*c^5*
d - 47*C*b^3*c^5*d + 78*D*a^2*b*c^4*d^2 + 78*C*a*b^2*c^4*d^2 + 26*B*b^3*c^
4*d^2 - 11*D*a^3*c^3*d^3 - 33*C*a^2*b*c^3*d^3 - 33*B*a*b^2*c^3*d^3 - 11*A*
b^3*c^3*d^3 + 2*C*a^3*c^2*d^4 + 6*B*a^2*b*c^2*d^4 + 6*A*a*b^2*c^2*d^4 + B*
a^3*c*d^5 + 3*A*a^2*b*c*d^5 + 2*A*a^3*d^6 + 6*(15*D*b^3*c^4*d^2 - 30*D*a*b
^2*c^3*d^3 - 10*C*b^3*c^3*d^3 + 18*D*a^2*b*c^2*d^4 + 18*C*a*b^2*c^2*d^4 +
6*B*b^3*c^2*d^4 - 3*D*a^3*c*d^5 - 9*C*a^2*b*c*d^5 - 9*B*a*b^2*c*d^5 - 3*A*
b^3*c*d^5 + C*a^3*d^6 + 3*B*a^2*b*d^6 + 3*A*a*b^2*d^6)*x^2 + 3*(54*D*b^3*c
^5*d - 105*D*a*b^2*c^4*d^2 - 35*C*b^3*c^4*d^2 + 60*D*a^2*b*c^3*d^3 + 60*C*
a*b^2*c^3*d^3 + 20*B*b^3*c^3*d^3 - 9*D*a^3*c^2*d^4 - 27*C*a^2*b*c^2*d^4 -
27*B*a*b^2*c^2*d^4 - 9*A*b^3*c^2*d^4 + 2*C*a^3*c*d^5 + 6*B*a^2*b*c*d^5 + 6
*A*a*b^2*c*d^5 + B*a^3*d^6 + 3*A*a^2*b*d^6)*x)/((d*x + c)^3*d^7) + 1/6*(2*
D*b^3*d^8*x^3 - 12*D*b^3*c*d^7*x^2 + 9*D*a*b^2*d^8*x^2 + 3*C*b^3*d^8*x^2 +
 60*D*b^3*c^2*d^6*x - 72*D*a*b^2*c*d^7*x - 24*C*b^3*c*d^7*x + 18*D*a^2*b*d
^8*x + 18*C*a*b^2*d^8*x + 6*B*b^3*d^8*x)/d^12
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx=\int \frac{{\left(a+b\,x\right)}^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4,x)
```

output
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.11

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx=\text{Too large to display}$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

output
```
(6*log(c + d*x)*a**3*c**4*d**3 + 18*log(c + d*x)*a**3*c**3*d**4*x + 18*log
(c + d*x)*a**3*c**2*d**5*x**2 + 6*log(c + d*x)*a**3*c*d**6*x**3 - 54*log(c
 + d*x)*a**2*b*c**5*d**2 - 162*log(c + d*x)*a**2*b*c**4*d**3*x - 162*log(c
 + d*x)*a**2*b*c**3*d**4*x**2 - 54*log(c + d*x)*a**2*b*c**2*d**5*x**3 + 24
*log(c + d*x)*a*b**3*c**4*d**2 + 72*log(c + d*x)*a*b**3*c**3*d**3*x + 72*l
og(c + d*x)*a*b**3*c**2*d**4*x**2 + 24*log(c + d*x)*a*b**3*c*d**5*x**3 + 1
08*log(c + d*x)*a*b**2*c**6*d + 324*log(c + d*x)*a*b**2*c**5*d**2*x + 324*
log(c + d*x)*a*b**2*c**4*d**3*x**2 + 108*log(c + d*x)*a*b**2*c**3*d**4*x**
3 - 24*log(c + d*x)*b**4*c**5*d - 72*log(c + d*x)*b**4*c**4*d**2*x - 72*lo
g(c + d*x)*b**4*c**3*d**3*x**2 - 24*log(c + d*x)*b**4*c**2*d**4*x**3 - 60*
log(c + d*x)*b**3*c**7 - 180*log(c + d*x)*b**3*c**6*d*x - 180*log(c + d*x)
*b**3*c**5*d**2*x**2 - 60*log(c + d*x)*b**3*c**4*d**3*x**3 - 2*a**4*c*d**5
 - 4*a**3*b*c**2*d**4 - 12*a**3*b*c*d**5*x + 5*a**3*c**4*d**3 + 9*a**3*c**
3*d**4*x - 4*a**3*c*d**6*x**3 + 12*a**2*b**2*d**6*x**3 - 45*a**2*b*c**5*d*
*2 - 81*a**2*b*c**4*d**3*x + 54*a**2*b*c**2*d**5*x**3 + 18*a**2*b*c*d**6*x
**4 + 20*a*b**3*c**4*d**2 + 36*a*b**3*c**3*d**3*x - 24*a*b**3*c*d**5*x**3
+ 90*a*b**2*c**6*d + 162*a*b**2*c**5*d**2*x - 108*a*b**2*c**3*d**4*x**3 -
27*a*b**2*c**2*d**5*x**4 + 9*a*b**2*c*d**6*x**5 - 20*b**4*c**5*d - 36*b**4
*c**4*d**2*x + 24*b**4*c**2*d**4*x**3 + 6*b**4*c*d**5*x**4 - 50*b**3*c**7
- 90*b**3*c**6*d*x + 60*b**3*c**4*d**3*x**3 + 15*b**3*c**3*d**4*x**4 - ...
```

3.49 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 295

$$\begin{aligned}
&\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx \\
&= \frac{b(bCd-4bcD+2adD)x}{d^5}+\frac{b^2Dx^2}{2d^4}-\frac{(bc-ad)^2(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^6(c+dx)^3} \\
&\quad -\frac{(bc-ad)(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))}{2d^6(c+dx)^2} \\
&\quad -\frac{a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D)}{d^6(c+dx)} \\
&\quad +\frac{(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))\log(c+dx)}{d^6}
\end{aligned}$$

output
```
b*(C*b*d+2*D*a*d-4*D*b*c)*x/d^5+1/2*b^2*D*x^2/d^4-1/3*(-a*d+b*c)^2*(A*d^3-
B*c*d^2+C*c^2*d-D*c^3)/d^6/(d*x+c)^3-1/2*(-a*d+b*c)*(a*d*(-B*d^2+2*C*c*d-3
*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))/d^6/(d*x+c)^2-(a^2*d^2*(C
*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*c^2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-
10*D*c^3))/d^6/(d*x+c)+(a^2*d^2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-
10*D*c^2))*ln(d*x+c)/d^6
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$
$$= \frac{6bd(bCd-4bcD+2adD)x+3b^2d^2Dx^2+\frac{2(bc-ad)^2(-c^2Cd+Bcd^2-Ad^3+c^3D)}{(c+dx)^3}-\frac{3(bc-ad)(-ad(-2cCd+Bd^2+3c^2D)+b}{(c+dx)}}{}$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
(6*b*d*(b*C*d - 4*b*c*D + 2*a*d*D)*x + 3*b^2*d^2*D*x^2 + (2*(b*c - a*d)^2*
(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(c + d*x)^3 - (3*(b*c - a*d)*(-(a*
d*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-4*c^2*C*d + 3*B*c*d^2 - 2*A*d^3 + 5*
c^3*D)))/(c + d*x)^2 - (6*(a^2*d^2*(C*d - 3*c*D) + 2*a*b*d*(-3*c*C*d + B*d
^2 + 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D)))/(c + d*x)
 + 6*(a^2*d^2*D + 2*a*b*d*(C*d - 4*c*D) + b^2*(-4*c*C*d + B*d^2 + 10*c^2*D
))*Log[c + d*x])/(6*d^6)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{a^2d^2(Cd-3cD)-2abd(-Bd^2-6c^2D+3cCd)+b^2(Ad^3-3Bcd^2-10c^3D+6c^2Cd)}{d^5(c+dx)^2}+\frac{a^2d^2D+2abd(C}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{a^2d^2(Cd-3cD)-2abd(-Bd^2-6c^2D+3cCd)+b^2(Ad^3-3Bcd^2-10c^3D+6c^2Cd)}{d^6(c+dx)}+\frac{\log(c+dx)\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{d^6}-\frac{(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^6(c+dx)^3}-\frac{(bc-ad)\left(ad(-Bd^2-3c^2D+2cCd)-b(2Ad^3-3Bcd^2-5c^3D+4c^2Cd)\right)}{2d^6(c+dx)^2}+\frac{bx(2adD-4bcD+bCd)}{d^5}+\frac{b^2Dx^2}{2d^4}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
(b*(b*C*d - 4*b*c*D + 2*a*d*D)*x)/d^5 + (b^2*D*x^2)/(2*d^4) - ((b*c - a*d)
^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^6*(c + d*x)^3) - ((b*c - a*d)
*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B*c*d^2 + 2*A*d^3 - 5
*c^3*D)))/(2*d^6*(c + d*x)^2) - (a^2*d^2*(C*d - 3*c*D) - 2*a*b*d*(3*c*C*d
- B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 + A*d^3 - 10*c^3*D))/(d^6*
(c + d*x)) + ((a^2*d^2*D + 2*a*b*d*(C*d - 4*c*D) - b^2*(4*c*C*d - B*d^2 -
10*c^2*D))*Log[c + d*x])/d^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.50

method	result
default	$\frac{b\left(\frac{1}{2}dDx^2b+Cbdx+2Dadx-4Dbcx\right)}{d^5}-\frac{Ab^2d^3+2abBd^3-3Bb^2cd^2+Ca^2d^3-6Cabcd^2+6Cb^2c^2d-3Da^2cd^2+12Dabc^2d-}{d^6(xd+c)}$
norman	$\frac{-\frac{2Aa^2d^5+2Aabcd^4+2Ab^2c^2d^3+Ba^2cd^4+4Babc^2d^3-11Bb^2c^3d^2+2Ca^2c^2d^3-22Cabc^3d^2+44Cb^2c^4d-11Da^2c^3d^2+88Dabc^4d-110}{6d^6}}{}$
parallelrisch	$-\frac{Ba^2cd^4+2Ab^2c^2d^3-11Bb^2c^3d^2+2Ca^2c^2d^3-11Da^2c^3d^2-27Bb^2c^2d^3x+108Cb^2c^3d^2x-27Da^2c^2d^3x-270Db^2c^4dx+2}{}$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
b/d^5*(1/2*d*D*x^2*b+C*b*d*x+2*D*a*d*x-4*D*b*c*x)-1/d^6*(A*b^2*d^3+2*B*a*b
*d^3-3*B*b^2*c*d^2+C*a^2*d^3-6*C*a*b*c*d^2+6*C*b^2*c^2*d-3*D*a^2*c*d^2+12*
D*a*b*c^2*d-10*D*b^2*c^3)/(d*x+c)+1/d^6*(B*b^2*d^2+2*C*a*b*d^2-4*C*b^2*c*d
+D*a^2*d^2-8*D*a*b*c*d+10*D*b^2*c^2)*ln(d*x+c)-1/2/d^6*(2*A*a*b*d^4-2*A*b^
2*c*d^3+B*a^2*d^4-4*B*a*b*c*d^3+3*B*b^2*c^2*d^2-2*C*a^2*c*d^3+6*C*a*b*c^2*
d^2-4*C*b^2*c^3*d+3*D*a^2*c^2*d^2-8*D*a*b*c^3*d+5*D*b^2*c^4)/(d*x+c)^2-1/3
*(A*a^2*d^5-2*A*a*b*c*d^4+A*b^2*c^2*d^3-B*a^2*c*d^4+2*B*a*b*c^2*d^3-B*b^2*
c^3*d^2+C*a^2*c^2*d^3-2*C*a*b*c^3*d^2+C*b^2*c^4*d-D*a^2*c^3*d^2+2*D*a*b*c^
4*d-D*b^2*c^5)/d^6/(d*x+c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(289) = 578$.

Time = 0.08 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.09

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}\,dx$$
$$=\frac{3\,Db^2d^5x^5+47\,Db^2c^5-2\,Aa^2d^5-26\,(2\,Dab+Cb^2)c^4d+11\,(Da^2+2\,Cab+Bb^2)c^3d^2-2\,(Ca^2+2\,Ba}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(3*D*b^2*d^5*x^5 + 47*D*b^2*c^5 - 2*A*a^2*d^5 - 26*(2*D*a*b + C*b^2)*c
^4*d + 11*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*
c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d^4 - 3*(5*D*b^2*c*d^4 - 2*(2*D*a*b + C*b^2)
*d^5)*x^4 - 9*(7*D*b^2*c^2*d^3 - 2*(2*D*a*b + C*b^2)*c*d^4)*x^3 - 3*(3*D*b
^2*c^3*d^2 + 6*(2*D*a*b + C*b^2)*c^2*d^3 - 6*(D*a^2 + 2*C*a*b + B*b^2)*c*d
^4 + 2*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*x^2 + 3*(27*D*b^2*c^4*d - 18*(2*D*a*
b + C*b^2)*c^3*d^2 + 9*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 - 2*(C*a^2 + 2*B*
a*b + A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*x + 6*(10*D*b^2*c^5 - 4*(2*D*a
*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 + (10*D*b^2*c^2*d^3
- 4*(2*D*a*b + C*b^2)*c*d^4 + (D*a^2 + 2*C*a*b + B*b^2)*d^5)*x^3 + 3*(10*D
*b^2*c^3*d^2 - 4*(2*D*a*b + C*b^2)*c^2*d^3 + (D*a^2 + 2*C*a*b + B*b^2)*c*d
^4)*x^2 + 3*(10*D*b^2*c^4*d - 4*(2*D*a*b + C*b^2)*c^3*d^2 + (D*a^2 + 2*C*a
*b + B*b^2)*c^2*d^3)*x)*log(d*x + c))/(d^9*x^3 + 3*c*d^8*x^2 + 3*c^2*d^7*x
 + c^3*d^6)
```

Sympy [A] (verification not implemented)

Time = 141.04 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \frac{Db^2x^2}{2d^4} + x\left(\frac{Cb^2}{d^4} + \frac{2Dab}{d^4} - \frac{4Db^2c}{d^5}\right)$$
$$+ \frac{-2Aa^2d^5 - 2Aabcd^4 - 2Ab^2c^2d^3 - Ba^2cd^4 - 4Babc^2d^3 + 11Bb^2c^3d^2 - 2Ca^2c^2d^3 + 22Cabc^3d^2 - 26C}{}$$
$$+ \frac{(Bb^2d^2 + 2Cabd^2 - 4Cb^2cd + Da^2d^2 - 8Dabcd + 10Db^2c^2)\log{(c+dx)}}{d^6}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**4,x)
```

output
```
D*b**2*x**2/(2*d**4) + x*(C*b**2/d**4 + 2*D*a*b/d**4 - 4*D*b**2*c/d**5) +
(-2*A*a**2*d**5 - 2*A*a*b*c*d**4 - 2*A*b**2*c**2*d**3 - B*a**2*c*d**4 - 4*
B*a*b*c**2*d**3 + 11*B*b**2*c**3*d**2 - 2*C*a**2*c**2*d**3 + 22*C*a*b*c**3
*d**2 - 26*C*b**2*c**4*d + 11*D*a**2*c**3*d**2 - 52*D*a*b*c**4*d + 47*D*b*
*2*c**5 + x**2*(-6*A*b**2*d**5 - 12*B*a*b*d**5 + 18*B*b**2*c*d**4 - 6*C*a*
*2*d**5 + 36*C*a*b*c*d**4 - 36*C*b**2*c**2*d**3 + 18*D*a**2*c*d**4 - 72*D*
a*b*c**2*d**3 + 60*D*b**2*c**3*d**2) + x*(-6*A*a*b*d**5 - 6*A*b**2*c*d**4
- 3*B*a**2*d**5 - 12*B*a*b*c*d**4 + 27*B*b**2*c**2*d**3 - 6*C*a**2*c*d**4
+ 54*C*a*b*c**2*d**3 - 60*C*b**2*c**3*d**2 + 27*D*a**2*c**2*d**3 - 120*D*a
*b*c**3*d**2 + 105*D*b**2*c**4*d))/(6*c**3*d**6 + 18*c**2*d**7*x + 18*c*d*
*8*x**2 + 6*d**9*x**3) + (B*b**2*d**2 + 2*C*a*b*d**2 - 4*C*b**2*c*d + D*a*
*2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2)*log(c + d*x)/d**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}\,dx$$
$$= \frac{47\,Db^2c^5 - 2\,Aa^2d^5 - 26\,(2\,Dab + Cb^2)c^4d + 11\,(Da^2 + 2\,Cab + Bb^2)c^3d^2 - 2\,(Ca^2 + 2\,Bab + Ab^2)c^2d^3}{}$$
$$+ \frac{Db^2dx^2 - 2\,(4\,Db^2c - (2\,Dab + Cb^2)d)x}{2\,d^5}$$
$$+ \frac{(10\,Db^2c^2 - 4\,(2\,Dab + Cb^2)cd + (Da^2 + 2\,Cab + Bb^2)d^2)\log(dx + c)}{d^6}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6*(47*D*b^2*c^5 - 2*A*a^2*d^5 - 26*(2*D*a*b + C*b^2)*c^4*d + 11*(D*a^2 +
 2*C*a*b + B*b^2)*c^3*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - (B*a^2 +
 2*A*a*b)*c*d^4 + 6*(10*D*b^2*c^3*d^2 - 6*(2*D*a*b + C*b^2)*c^2*d^3 + 3*(D
*a^2 + 2*C*a*b + B*b^2)*c*d^4 - (C*a^2 + 2*B*a*b + A*b^2)*d^5)*x^2 + 3*(35
*D*b^2*c^4*d - 20*(2*D*a*b + C*b^2)*c^3*d^2 + 9*(D*a^2 + 2*C*a*b + B*b^2)*
c^2*d^3 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*x)/(d
^9*x^3 + 3*c*d^8*x^2 + 3*c^2*d^7*x + c^3*d^6) + 1/2*(D*b^2*d*x^2 - 2*(4*D*
b^2*c - (2*D*a*b + C*b^2)*d)*x)/d^5 + (10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*
c*d + (D*a^2 + 2*C*a*b + B*b^2)*d^2)*log(d*x + c)/d^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$
$$= \frac{(10\,Db^2c^2-8\,Dabcd-4\,Cb^2cd+Da^2d^2+2\,Cabd^2+Bb^2d^2)\log(|dx+c|)}{d^6} + \frac{Db^2d^4x^2-8\,Db^2cd^3x+4\,Dabd^4x+2\,Cb^2d^4x}{2\,d^8} + \frac{47\,Db^2c^5-52\,Dabc^4d-26\,Cb^2c^4d+11\,Da^2c^3d^2+22\,Cabc^3d^2+11\,Bb^2c^3d^2-2\,Ca^2c^2d^3-4\,Babc^2d^3}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="giac")
```

output
```
(10*D*b^2*c^2 - 8*D*a*b*c*d - 4*C*b^2*c*d + D*a^2*d^2 + 2*C*a*b*d^2 + B*b^
2*d^2)*log(abs(d*x + c))/d^6 + 1/2*(D*b^2*d^4*x^2 - 8*D*b^2*c*d^3*x + 4*D*
a*b*d^4*x + 2*C*b^2*d^4*x)/d^8 + 1/6*(47*D*b^2*c^5 - 52*D*a*b*c^4*d - 26*C
*b^2*c^4*d + 11*D*a^2*c^3*d^2 + 22*C*a*b*c^3*d^2 + 11*B*b^2*c^3*d^2 - 2*C*
a^2*c^2*d^3 - 4*B*a*b*c^2*d^3 - 2*A*b^2*c^2*d^3 - B*a^2*c*d^4 - 2*A*a*b*c*
d^4 - 2*A*a^2*d^5 + 6*(10*D*b^2*c^3*d^2 - 12*D*a*b*c^2*d^3 - 6*C*b^2*c^2*d
^3 + 3*D*a^2*c*d^4 + 6*C*a*b*c*d^4 + 3*B*b^2*c*d^4 - C*a^2*d^5 - 2*B*a*b*d
^5 - A*b^2*d^5)*x^2 + 3*(35*D*b^2*c^4*d - 40*D*a*b*c^3*d^2 - 20*C*b^2*c^3*
d^2 + 9*D*a^2*c^2*d^3 + 18*C*a*b*c^2*d^3 + 9*B*b^2*c^2*d^3 - 2*C*a^2*c*d^4
 - 4*B*a*b*c*d^4 - 2*A*b^2*c*d^4 - B*a^2*d^5 - 2*A*a*b*d^5)*x)/((d*x + c)^
3*d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx = \int \frac{(a+b\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^4}\,dx$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4,x)
```

output
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$
$$= \frac{-108\log(dx+c)\,ab\,c^4d^2x - 108\log(dx+c)\,ab\,c^3d^3x^2 - 3a^2b\,c^2d^3 + 6a\,b^2d^5x^3 + 36\log(dx+c)\,b^2c^6 - 30}{}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

output
```
(6*log(c + d*x)*a**2*c**4*d**2 + 18*log(c + d*x)*a**2*c**3*d**3*x + 18*log
(c + d*x)*a**2*c**2*d**4*x**2 + 6*log(c + d*x)*a**2*c*d**5*x**3 - 36*log(c
 + d*x)*a*b*c**5*d - 108*log(c + d*x)*a*b*c**4*d**2*x - 108*log(c + d*x)*a
*b*c**3*d**3*x**2 - 36*log(c + d*x)*a*b*c**2*d**4*x**3 + 6*log(c + d*x)*b*
*3*c**4*d + 18*log(c + d*x)*b**3*c**3*d**2*x + 18*log(c + d*x)*b**3*c**2*d
**3*x**2 + 6*log(c + d*x)*b**3*c*d**4*x**3 + 36*log(c + d*x)*b**2*c**6 + 1
08*log(c + d*x)*b**2*c**5*d*x + 108*log(c + d*x)*b**2*c**4*d**2*x**2 + 36*
log(c + d*x)*b**2*c**3*d**3*x**3 - 2*a**3*c*d**4 - 3*a**2*b*c**2*d**3 - 9*
a**2*b*c*d**4*x + 5*a**2*c**4*d**2 + 9*a**2*c**3*d**3*x - 4*a**2*c*d**5*x*
*3 + 6*a*b**2*d**5*x**3 - 30*a*b*c**5*d - 54*a*b*c**4*d**2*x + 36*a*b*c**2
*d**4*x**3 + 12*a*b*c*d**5*x**4 + 5*b**3*c**4*d + 9*b**3*c**3*d**2*x - 6*b
**3*c*d**4*x**3 + 30*b**2*c**6 + 54*b**2*c**5*d*x - 36*b**2*c**3*d**3*x**3
 - 9*b**2*c**2*d**4*x**4 + 3*b**2*c*d**5*x**5)/(6*c*d**5*(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3))
```

3.50 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$

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Optimal result

Integrand size = 28, antiderivative size = 189

$$\begin{aligned}
\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx
&= \frac{bDx}{d^4} + \frac{(bc-ad)(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^5(c+dx)^3} \\
&\quad + \frac{ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D)}{2d^5(c+dx)^2} \\
&\quad - \frac{ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D)}{d^5(c+dx)} + \frac{(bCd-4bcD+adD)\log(c+dx)}{d^5}
\end{aligned}$$

output

```
b*D*x/d^4+1/3*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^5/(d*x+c)^3+1/2*(
a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))/d^5/(d
*x+c)^2-(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))/d^5/(d*x+c)+(C*b*d+D*
a*d-4*D*b*c)*ln(d*x+c)/d^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx =$$
$$-\frac{ad(-11c^3D+c^2d(2C-27Dx)+d^3(2A+3x(B+2Cx))+cd^2(B+6x(C-3Dx)))+b(26c^4D+c^3\cdots}{\cdots}$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
-1/6*(a*d*(-11*c^3*D + c^2*d*(2*C - 27*D*x) + d^3*(2*A + 3*x*(B + 2*C*x))
+ c*d^2*(B + 6*x*(C - 3*D*x))) + b*(26*c^4*D + c^3*(-11*C*d + 54*d*D*x) +
3*d^4*x*(A + 2*B*x - 2*D*x^3) + c^2*d^2*(2*B + 9*x*(-3*C + 2*D*x)) + c*d^3
*(A + 6*x*(B - 3*x*(C + D*x)))) - 6*(b*C*d - 4*b*c*D + a*d*D)*(c + d*x)^3*
Log[c + d*x])/(d^5*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{b(Ad^3-2Bcd^2-4c^3D+3c^2Cd)-ad(-Bd^2-3c^2D+2cCd)}{d^4(c+dx)^3}+\frac{(ad-bc)(Ad^3-Bcd^2+c^3(-D)+c^2C\cdots}{d^4(c+dx)^4}\cdots\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^5(c+dx)^3}+\frac{ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)}{2d^5(c+dx)^2}-\frac{ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)}{d^5(c+dx)}+\frac{\log(c+dx)(adD-4bcD+bCd)}{d^5}+\frac{bDx}{d^4}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^4,x]
```

output
```
(b*D*x)/d^4 + ((b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^5*(c
+ d*x)^3) + (a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 +
A*d^3 - 4*c^3*D))/(2*d^5*(c + d*x)^2) - (a*d*(C*d - 3*c*D) - b*(3*c*C*d -
B*d^2 - 6*c^2*D))/(d^5*(c + d*x)) + ((b*C*d - 4*b*c*D + a*d*D)*Log[c + d*x
])/d^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.12

method	result
norman	$\frac{\frac{bDx^4}{d}-\frac{2Aa\,d^4+Ac\,d^3b+Bc\,d^3a+2Bb\,c^2d^2+2Ca\,c^2d^2-11Cb\,c^3d-11Da\,c^3d+44Db\,c^4}{6d^5}-\frac{\left(bB\,d^2+Ca\,d^2-3Cbcd-3Dacd+12Db\,c^2\right)x^2}{d^3}-}{(xd+c)^3}$
default	$\frac{bDx}{d^4}-\frac{bB\,d^2+Ca\,d^2-3Cbcd-3Dacd+6Db\,c^2}{d^5(xd+c)}+\frac{(Cbd+Dad-4Dbc)\ln(xd+c)}{d^5}-\frac{Ab\,d^3+Ba\,d^3-2Bbc\,d^2-2Cac\,d^2+}{2d^5(xd+c)^2}$
parallelrisch	$-\frac{-18C\ln(xd+c)xb\,c^2d^2-18D\ln(xd+c)xa\,c^2d^2+72D\ln(xd+c)xb\,c^3d+2Bb\,c^2d^2+24D\ln(xd+c)x^3bc\,d^3+2Ca\,c^2d^2-11Da}{}$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(b*D*x^4/d-1/6*(2*A*a*d^4+A*b*c*d^3+B*a*c*d^3+2*B*b*c^2*d^2+2*C*a*c^2*d^2-
11*C*b*c^3*d-11*D*a*c^3*d+44*D*b*c^4)/d^5-(B*b*d^2+C*a*d^2-3*C*b*c*d-3*D*a
*c*d+12*D*b*c^2)/d^3*x^2-1/2*(A*b*d^3+B*a*d^3+2*B*b*c*d^2+2*C*a*c*d^2-9*C*
b*c^2*d-9*D*a*c^2*d+36*D*b*c^3)/d^4*x)/(d*x+c)^3+(C*b*d+D*a*d-4*D*b*c)*ln(
d*x+c)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}\,dx$$
$$= \frac{6\,Dbd^4x^4+18\,Dbcd^3x^3-26\,Dbc^4-2\,Aad^4+11\,(Da+Cb)c^3d-2\,(Ca+Bb)c^2d^2-(Ba+Ab)cd^3-6$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(6*D*b*d^4*x^4 + 18*D*b*c*d^3*x^3 - 26*D*b*c^4 - 2*A*a*d^4 + 11*(D*a +
 C*b)*c^3*d - 2*(C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3 - 6*(3*D*b*c^2*d^2
 - 3*(D*a + C*b)*c*d^3 + (C*a + B*b)*d^4)*x^2 - 3*(18*D*b*c^3*d - 9*(D*a +
 C*b)*c^2*d^2 + 2*(C*a + B*b)*c*d^3 + (B*a + A*b)*d^4)*x - 6*(4*D*b*c^4 -
(D*a + C*b)*c^3*d + (4*D*b*c*d^3 - (D*a + C*b)*d^4)*x^3 + 3*(4*D*b*c^2*d^2
 - (D*a + C*b)*c*d^3)*x^2 + 3*(4*D*b*c^3*d - (D*a + C*b)*c^2*d^2)*x)*log(d
*x + c))/(d^8*x^3 + 3*c*d^7*x^2 + 3*c^2*d^6*x + c^3*d^5)
```

Sympy [A] (verification not implemented)

Time = 35.80 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}\,dx = \frac{Dbx}{d^4}$$
$$+\frac{-2Aad^4-Abcd^3-Bacd^3-2Bbc^2d^2-2Cac^2d^2+11Cbc^3d+11Dac^3d-26Dbc^4+x^2(-6Bbd^4-6C}{6c^3d^5+}$$
$$+\frac{(Cbd+Dad-4Dbc)\log(c+dx)}{d^5}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**4,x)
```

output
```
D*b*x/d**4 + (-2*A*a*d**4 - A*b*c*d**3 - B*a*c*d**3 - 2*B*b*c**2*d**2 - 2*
C*a*c**2*d**2 + 11*C*b*c**3*d + 11*D*a*c**3*d - 26*D*b*c**4 + x**2*(-6*B*b
*d**4 - 6*C*a*d**4 + 18*C*b*c*d**3 + 18*D*a*c*d**3 - 36*D*b*c**2*d**2) + x
*(-3*A*b*d**4 - 3*B*a*d**4 - 6*B*b*c*d**3 - 6*C*a*c*d**3 + 27*C*b*c**2*d**
2 + 27*D*a*c**2*d**2 - 60*D*b*c**3*d))/(6*c**3*d**5 + 18*c**2*d**6*x + 18*
c*d**7*x**2 + 6*d**8*x**3) + (C*b*d + D*a*d - 4*D*b*c)*log(c + d*x)/d**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}\,dx = -\frac{26\,Dbc^4 + 2\,Aad^4 - 11\,(Da+Cb)c^3d + 2\,(Ca+Bb)c^2d^2 + (Ba+Ab)cd^3 + 6\,(6\,Dbc^2d^2 - 3\,(Da+C}{6\,(d^8x^3 + 3\,cd^7x^2 + 3} + \frac{Dbx}{d^4} - \frac{(4\,Dbc - (Da+Cb)d)\log(dx+c)}{d^5}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/6*(26*D*b*c^4 + 2*A*a*d^4 - 11*(D*a + C*b)*c^3*d + 2*(C*a + B*b)*c^2*d^
2 + (B*a + A*b)*c*d^3 + 6*(6*D*b*c^2*d^2 - 3*(D*a + C*b)*c*d^3 + (C*a + B*
b)*d^4)*x^2 + 3*(20*D*b*c^3*d - 9*(D*a + C*b)*c^2*d^2 + 2*(C*a + B*b)*c*d^
3 + (B*a + A*b)*d^4)*x)/(d^8*x^3 + 3*c*d^7*x^2 + 3*c^2*d^6*x + c^3*d^5) +
D*b*x/d^4 - (4*D*b*c - (D*a + C*b)*d)*log(d*x + c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}\,dx = \frac{Dbx}{d^4} - \frac{(4\,Dbc - Dad - Cbd)\log\left(|dx+c|\right)}{d^5} - \frac{26\,Dbc^4 - 11\,Dac^3d - 11\,Cbc^3d + 2\,Cac^2d^2 + 2\,Bbc^2d^2 + Bacd^3 + Abcd^3 + 2\,Aad^4 + 6\,(6\,Dbc^2d^2 - 3}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="giac")
```

output
```
D*b*x/d^4 - (4*D*b*c - D*a*d - C*b*d)*log(abs(d*x + c))/d^5 - 1/6*(26*D*b*
c^4 - 11*D*a*c^3*d - 11*C*b*c^3*d + 2*C*a*c^2*d^2 + 2*B*b*c^2*d^2 + B*a*c*
d^3 + A*b*c*d^3 + 2*A*a*d^4 + 6*(6*D*b*c^2*d^2 - 3*D*a*c*d^3 - 3*C*b*c*d^3
 + C*a*d^4 + B*b*d^4)*x^2 + 3*(20*D*b*c^3*d - 9*D*a*c^2*d^2 - 9*C*b*c^2*d^
2 + 2*C*a*c*d^3 + 2*B*b*c*d^3 + B*a*d^4 + A*b*d^4)*x)/((d*x + c)^3*d^5)
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.17

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}\,dx$$
$$= \frac{a\,D\left(\ln(c+d\,x) + \frac{3\,c}{c+d\,x} - \frac{3\,c^2}{2\,(c+d\,x)^2} + \frac{c^3}{3\,(c+d\,x)^3}\right)}{d^4} - \frac{C\,a\,c^2 + 3\,C\,a\,c\,d\,x + 3\,C\,a\,d^2\,x^2}{3\,c^3\,d^3 + 9\,c^2\,d^4\,x + 9\,c\,d^5\,x^2 + 3\,d^6\,x^3} - \frac{\frac{A\,b\,c}{6\,d^2} + \frac{A\,b\,x}{2\,d}}{c^3 + 3\,c^2\,d\,x + 3\,c\,d^2\,x^2 + d^3\,x^3} - \frac{\frac{B\,a\,c}{6\,d^2} + \frac{B\,a\,x}{2\,d}}{c^3 + 3\,c^2\,d\,x + 3\,c\,d^2\,x^2 + d^3\,x^3} - \frac{B\,b\,c^2 + 3\,B\,b\,c\,d\,x + 3\,B\,b\,d^2\,x^2}{3\,c^3\,d^3 + 9\,c^2\,d^4\,x + 9\,c\,d^5\,x^2 + 3\,d^6\,x^3} - \frac{b\,D\left(4\,c\,\ln(c+d\,x) - d\,x + \frac{6\,c^2}{c+d\,x} - \frac{2\,c^3}{(c+d\,x)^2} + \frac{c^4}{3\,(c+d\,x)^3}\right)}{d^5} - \frac{A\,a}{3\,d\,\left(c^3 + 3\,c^2\,d\,x + 3\,c\,d^2\,x^2 + d^3\,x^3\right)} + \frac{C\,b\left(\ln(c+d\,x) + \frac{3\,c}{c+d\,x} - \frac{3\,c^2}{2\,(c+d\,x)^2} + \frac{c^3}{3\,(c+d\,x)^3}\right)}{d^4}$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^4,x)
```

output
```
(a*D*(log(c + d*x) + (3*c)/(c + d*x) - (3*c^2)/(2*(c + d*x)^2) + c^3/(3*(c
 + d*x)^3)))/d^4 - (C*a*c^2 + 3*C*a*d^2*x^2 + 3*C*a*c*d*x)/(3*c^3*d^3 + 3*
d^6*x^3 + 9*c^2*d^4*x + 9*c*d^5*x^2) - ((A*b*c)/(6*d^2) + (A*b*x)/(2*d))/(
c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x) - ((B*a*c)/(6*d^2) + (B*a*x)/(2*d
))/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x) - (B*b*c^2 + 3*B*b*d^2*x^2 +
3*B*b*c*d*x)/(3*c^3*d^3 + 3*d^6*x^3 + 9*c^2*d^4*x + 9*c*d^5*x^2) - (b*D*(4
*c*log(c + d*x) - d*x + (6*c^2)/(c + d*x) - (2*c^3)/(c + d*x)^2 + c^4/(3*(
c + d*x)^3)))/d^5 - (A*a)/(3*d*(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x))
+ (C*b*(log(c + d*x) + (3*c)/(c + d*x) - (3*c^2)/(2*(c + d*x)^2) + c^3/(3*
(c + d*x)^3)))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^4}\,dx$$
$$= \frac{6\log(dx+c)\,a\,c^4d+18\log(dx+c)\,a\,c^3d^2x+18\log(dx+c)\,a\,c^2d^3x^2+6\log(dx+c)\,ac\,d^4x^3-18\log(dx$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

output
```
(6*log(c + d*x)*a*c**4*d + 18*log(c + d*x)*a*c**3*d**2*x + 18*log(c + d*x)
*a*c**2*d**3*x**2 + 6*log(c + d*x)*a*c*d**4*x**3 - 18*log(c + d*x)*b*c**5
- 54*log(c + d*x)*b*c**4*d*x - 54*log(c + d*x)*b*c**3*d**2*x**2 - 18*log(c
 + d*x)*b*c**2*d**3*x**3 - 2*a**2*c*d**3 - 2*a*b*c**2*d**2 - 6*a*b*c*d**3*
x + 5*a*c**4*d + 9*a*c**3*d**2*x - 4*a*c*d**4*x**3 + 2*b**2*d**4*x**3 - 15
*b*c**5 - 27*b*c**4*d*x + 18*b*c**2*d**3*x**3 + 6*b*c*d**4*x**4)/(6*c*d**4
*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))
```

3.51 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx$

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Giac [A] (verification not implemented) 482
Mupad [B] (verification not implemented) 482
Reduce [B] (verification not implemented) 483

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx = \frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{3d^4(c+dx)^3}+\frac{2cCd-Bd^2-3c^2D}{2d^4(c+dx)^2} - \frac{Cd-3cD}{d^4(c+dx)}+\frac{D\log(c+dx)}{d^4}$$

output
```
1/3*(-A*d^3+B*c*d^2-C*c^2*d+D*c^3)/d^4/(d*x+c)^3+1/2*(-B*d^2+2*C*c*d-3*D*c
^2)/d^4/(d*x+c)^2-(C*d-3*D*c)/d^4/(d*x+c)+D*ln(d*x+c)/d^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx$$
$$= \frac{11c^3D+c^2(-2Cd+27dDx)-d^3(2A+3x(B+2Cx))-cd^2(B+6x(C-3Dx))+6D(c+dx)^3\log(c}{6d^4(c+dx)^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^4,x]
```

output
```
(11*c^3*D + c^2*(-2*C*d + 27*d*D*x) - d^3*(2*A + 3*x*(B + 2*C*x)) - c*d^2*
(B + 6*x*(C - 3*D*x)) + 6*D*(c + d*x)^3*Log[c + d*x])/(6*d^4*(c + d*x)^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)^4}+\frac{Bd^2+3c^2D-2cCd}{d^3(c+dx)^3}+\frac{Cd-3cD}{d^3(c+dx)^2}+\frac{D}{d^3(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{3d^4(c+dx)^3}+\frac{-Bd^2-3c^2D+2cCd}{2d^4(c+dx)^2}-\frac{Cd-3cD}{d^4(c+dx)}+\frac{D\log(c+dx)}{d^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^4,x]
```

output
```
-1/3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(d^4*(c + d*x)^3) + (2*c*C*d - B*
d^2 - 3*c^2*D)/(2*d^4*(c + d*x)^2) - (C*d - 3*c*D)/(d^4*(c + d*x)) + (D*Lo
g[c + d*x])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.89

method	result
norman	$\frac{-\frac{2A d^3+Bc d^2+2C c^2d-11Dc^3}{6d^4}-\frac{(Cd-3Dc)x^2}{d^2}-\frac{\left(B d^2+2Ccd-9Dc^2\right)x}{2d^3}}{(xd+c)^3}+\frac{D\ln(xd+c)}{d^4}$
default	$-\frac{Cd-3Dc}{d^4(xd+c)}+\frac{D\ln(xd+c)}{d^4}-\frac{B d^2-2Ccd+3Dc^2}{2d^4(xd+c)^2}-\frac{A d^3-Bc d^2+C c^2d-Dc^3}{3d^4(xd+c)^3}$
parallelrisch	$-\frac{-6D\ln(xd+c)x^3d^3-18D\ln(xd+c)x^2c d^2+6C x^2d^3-18D\ln(xd+c)x c^2d-18Dx^2c d^2+3Bx d^3+6Cxc d^2-6D\ln(xd+c)c^3}{6d^4(xd+c)^3}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
(-1/6*(2*A*d^3+B*c*d^2+2*C*c^2*d-11*D*c^3)/d^4-(C*d-3*D*c)/d^2*x^2-1/2*(B*
d^2+2*C*c*d-9*D*c^2)/d^3*x)/(d*x+c)^3+D*ln(d*x+c)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx = \frac{11\,Dc^3-2\,Cc^2d-Bcd^2-2\,Ad^3+6\,(3\,Dcd^2-Cd^3)x^2+3\,(9\,Dc^2d-2\,Ccd^2-Bd^3)x+6\,(Dd^3x^3+3\,}{6\,(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(11*D*c^3 - 2*C*c^2*d - B*c*d^2 - 2*A*d^3 + 6*(3*D*c*d^2 - C*d^3)*x^2
+ 3*(9*D*c^2*d - 2*C*c*d^2 - B*d^3)*x + 6*(D*d^3*x^3 + 3*D*c*d^2*x^2 + 3*D
*c^2*d*x + D*c^3)*log(d*x + c))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3
*d^4)
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx = \frac{D\log{(c+dx)}}{d^4} + \frac{-2Ad^3-Bcd^2-2Cc^2d+11Dc^3+x^2(-6Cd^3+18Dcd^2)+x(-3Bd^3-6Ccd^2+27Dc^2d)}{6c^3d^4+18c^2d^5x+18cd^6x^2+6d^7x^3}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**4,x)
```

output
```
D*log(c + d*x)/d**4 + (-2*A*d**3 - B*c*d**2 - 2*C*c**2*d + 11*D*c**3 + x**
2*(-6*C*d**3 + 18*D*c*d**2) + x*(-3*B*d**3 - 6*C*c*d**2 + 27*D*c**2*d))/(6
*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx = \frac{11\,Dc^3-2\,Cc^2d-Bcd^2-2\,Ad^3+6\,(3\,Dcd^2-Cd^3)x^2+3\,(9\,Dc^2d-2\,Ccd^2-Bd^3)x}{6\,(d^7x^3+3\,cd^6x^2+3\,c^2d^5x+c^3d^4)} + \frac{D\log{(dx+c)}}{d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6*(11*D*c^3 - 2*C*c^2*d - B*c*d^2 - 2*A*d^3 + 6*(3*D*c*d^2 - C*d^3)*x^2
+ 3*(9*D*c^2*d - 2*C*c*d^2 - B*d^3)*x)/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*
x + c^3*d^4) + D*log(d*x + c)/d^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx$$
$$=\frac{D\log(|dx+c|)}{d^4}$$
$$+\frac{6\,(3\,Dcd-Cd^2)x^2+3\,(9\,Dc^2-2\,Ccd-Bd^2)x+\frac{11\,Dc^3-2\,Cc^2d-Bcd^2-2\,Ad^3}{d}}{6\,(dx+c)^3d^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x, algorithm="giac")
```

output
```
D*log(abs(d*x + c))/d^4 + 1/6*(6*(3*D*c*d - C*d^2)*x^2 + 3*(9*D*c^2 - 2*C*
c*d - B*d^2)*x + (11*D*c^3 - 2*C*c^2*d - B*c*d^2 - 2*A*d^3)/d)/((d*x + c)^
3*d^3)
```

Mupad [B] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^4}\,dx=\frac{D\left(\ln(c+d\,x)+\frac{3\,c}{c+d\,x}-\frac{3\,c^2}{2\,(c+d\,x)^2}+\frac{c^3}{3\,(c+d\,x)^3}\right)}{d^4}$$
$$-\frac{C\,c^2+3\,C\,c\,d\,x+3\,C\,d^2\,x^2}{3\,c^3\,d^3+9\,c^2\,d^4\,x+9\,c\,d^5\,x^2+3\,d^6\,x^3}$$
$$-\frac{A}{3\,d\,\left(c^3+3\,c^2\,d\,x+3\,c\,d^2\,x^2+d^3\,x^3\right)}$$
$$-\frac{\frac{B\,c}{6\,d^2}+\frac{B\,x}{2\,d}}{c^3+3\,c^2\,d\,x+3\,c\,d^2\,x^2+d^3\,x^3}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^4,x)
```

output
```
(D*(log(c + d*x) + (3*c)/(c + d*x) - (3*c^2)/(2*(c + d*x)^2) + c^3/(3*(c +
 d*x)^3)))/d^4 - (C*c^2 + 3*C*d^2*x^2 + 3*C*c*d*x)/(3*c^3*d^3 + 3*d^6*x^3
+ 9*c^2*d^4*x + 9*c*d^5*x^2) - A/(3*d*(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2
*d*x)) - ((B*c)/(6*d^2) + (B*x)/(2*d))/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^
2*d*x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.22

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)^4}\, dx$$
$$= \frac{6\log(dx+c)\,c^3 + 18\log(dx+c)\,c^2dx + 18\log(dx+c)\,c\,d^2x^2 + 6\log(dx+c)\,d^3x^3 - 2a\,d^2 - bcd - 3b\,d^2}{6d^3\left(d^3x^3 + 3c\,d^2x^2 + 3c^2dx + c^3\right)}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^4,x)
```

output
```
(6*log(c + d*x)*c**3 + 18*log(c + d*x)*c**2*d*x + 18*log(c + d*x)*c*d**2*x
**2 + 6*log(c + d*x)*d**3*x**3 - 2*a*d**2 - b*c*d - 3*b*d**2*x + 5*c**3 +
9*c**2*d*x - 4*d**3*x**3)/(6*d**3*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3))
```

3.52 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4} dx$

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Optimal result

Integrand size = 30, antiderivative size = 274

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx = \frac{c^2Cd-Bcd^2+Ad^3-c^3D}{3d^3(bc-ad)(c+dx)^3} + \frac{ad(2cCd-Bd^2-3c^2D)-b(c^2Cd-Ad^3-2c^3D)}{2d^3(bc-ad)^2(c+dx)^2} + \frac{Ab^2d^3-b^2c^3D+a^2d^2(Cd-3cD)-ab(Bd^3-3c^2dD)}{d^3(bc-ad)^3(c+dx)} + \frac{(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{(bc-ad)^4} - \frac{(Ab^3-a(b^2B-abC+a^2D))\log(c+dx)}{(bc-ad)^4}$$

output
```
1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^3/(-a*d+b*c)/(d*x+c)^3+1/2*(a*d*(-B*d^
2+2*C*c*d-3*D*c^2)-b*(-A*d^3+C*c^2*d-2*D*c^3))/d^3/(-a*d+b*c)^2/(d*x+c)^2+
(A*b^2*d^3-b^2*c^3*D+a^2*d^2*(C*d-3*D*c)-a*b*(B*d^3-3*D*c^2*d))/d^3/(-a*d+
b*c)^3/(d*x+c)+(A*b^3-a*(B*b^2-C*a*b+D*a^2))*ln(b*x+a)/(-a*d+b*c)^4-(A*b^3
-a*(B*b^2-C*a*b+D*a^2))*ln(d*x+c)/(-a*d+b*c)^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.01

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx \\
&= \frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{3d^3(-bc+ad)(c+dx)^3} \\
&\quad + \frac{-bc^2Cd+2acCd^2+Abd^3-aBd^3+2bc^3D-3ac^2dD}{2d^3(-bc+ad)^2(c+dx)^2} \\
&\quad + \frac{-Ab^2d^3+abBd^3-a^2Cd^3+b^2c^3D-3abc^2dD+3a^2cd^2D}{d^3(-bc+ad)^3(c+dx)} \\
&\quad + \frac{(Ab^3-ab^2B+a^2bC-a^3D)\log(a+bx)}{(bc-ad)^4} \\
&\quad + \frac{(-Ab^3+ab^2B-a^2bC+a^3D)\log(c+dx)}{(bc-ad)^4}
\end{aligned}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^4),x]
```

output
```
(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)/(3*d^3*(-(b*c) + a*d)*(c + d*x)^3)
+ (-(b*c^2*C*d) + 2*a*c*C*d^2 + A*b*d^3 - a*B*d^3 + 2*b*c^3*D - 3*a*c^2*d*
D)/(2*d^3*(-(b*c) + a*d)^2*(c + d*x)^2) + (-(A*b^2*d^3) + a*b*B*d^3 - a^2*
C*d^3 + b^2*c^3*D - 3*a*b*c^2*d*D + 3*a^2*c*d^2*D)/(d^3*(-(b*c) + a*d)^3*(
c + d*x)) + ((A*b^3 - a*b^2*B + a^2*b*C - a^3*D)*Log[a + b*x])/(b*c - a*d)
^4 + ((-(A*b^3) + a*b^2*B - a^2*b*C + a^3*D)*Log[c + d*x])/(b*c - a*d)^4
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{-a^2d^2(Cd-3cD)+ab\left(Bd^3-3c^2dD\right)-Ab^2d^3+b^2c^3D}{d^2(c+dx)^2(bc-ad)^3}+\frac{b\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{(a+bx)(bc-ad)^4}+\frac{d\left(a\left(a^2D-\right.\right.}{(c+a}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2d^2(Cd-3cD)-ab\left(Bd^3-3c^2dD\right)+Ab^2d^3-b^2c^3D}{d^3(c+dx)(bc-ad)^3}+\frac{\log(a+bx)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{(bc-ad)^4}-\frac{\log(c+dx)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{(bc-ad)^4}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{3d^3(c+dx)^3(bc-ad)}+\frac{ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-Ad^3-2c^3D+c^2Cd\right)}{2d^3(c+dx)^2(bc-ad)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^4),x]
```

output
```
(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(3*d^3*(b*c - a*d)*(c + d*x)^3) + (a*d
*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(c^2*C*d - A*d^3 - 2*c^3*D))/(2*d^3*(b*c
- a*d)^2*(c + d*x)^2) + (A*b^2*d^3 - b^2*c^3*D + a^2*d^2*(C*d - 3*c*D) - a
*b*(B*d^3 - 3*c^2*d*D))/(d^3*(b*c - a*d)^3*(c + d*x)) + ((A*b^3 - a*(b^2*B
 - a*b*C + a^2*D))*Log[a + b*x])/(b*c - a*d)^4 - ((A*b^3 - a*(b^2*B - a*b*
C + a^2*D))*Log[c + d*x])/(b*c - a*d)^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00

method	result
default	$-\frac{A d^3-Bc d^2+C c^2 d-D c^3}{3d^3(ad-bc)(xd+c)^3}-\frac{-Ab d^3+Ba d^3-2Cac d^2+Cb c^2 d+3Da c^2 d-2Db c^3}{2d^3(ad-bc)^2(xd+c)^2}-\frac{A b^2 d^3-abB d^3+C a^2 d^3-3Da^2 c d}{(ad-bc)^3 d^3 (xd+}$
norman	$-\frac{2A a^2 d^5-7Aabc d^4+11A b^2 c^2 d^3+B a^2 c d^4-5Bab c^2 d^3-2B b^2 c^3 d^2+2C a^2 c^2 d^3+5Cab c^3 d^2-C b^2 c^4 d-11Da^2 c^3 d^2+7Dab c^4 d-2Db^2 c^5}{6d^3\left(a^3 d^3-3a^2 bc d^2+3a b^2 c^2 d-b^3 c^3\right)}$
parallelrisch	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/d^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)/(d*x+c)^3-1/2*(-A*b*d^3+B
*a*d^3-2*C*a*c*d^2+C*b*c^2*d+3*D*a*c^2*d-2*D*b*c^3)/d^3/(a*d-b*c)^2/(d*x+c
)^2-(A*b^2*d^3-B*a*b*d^3+C*a^2*d^3-3*D*a^2*c*d^2+3*D*a*b*c^2*d-D*b^2*c^3)/
(a*d-b*c)^3/d^3/(d*x+c)-(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(a*d-b*c)^4*ln(d*x+c
)+(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(a*d-b*c)^4*ln(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 909 vs. 2(272) = 544.

Time = 0.10 (sec) , antiderivative size = 909, normalized size of antiderivative = 3.32

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(2*D*b^3*c^6 + 2*A*a^3*d^6 - (9*D*a*b^2 - C*b^3)*c^5*d + 2*(9*D*a^2*b
 - 3*C*a*b^2 + B*b^3)*c^4*d^2 - (11*D*a^3 - 3*C*a^2*b - 3*B*a*b^2 + 11*A*b
^3)*c^3*d^3 + 2*(C*a^3 - 3*B*a^2*b + 9*A*a*b^2)*c^2*d^4 + (B*a^3 - 9*A*a^2
*b)*c*d^5 + 6*(D*b^3*c^4*d^2 - 4*D*a*b^2*c^3*d^3 + 6*D*a^2*b*c^2*d^4 - (3*
D*a^3 + C*a^2*b - B*a*b^2 + A*b^3)*c*d^5 + (C*a^3 - B*a^2*b + A*a*b^2)*d^6
)*x^2 + 3*(2*D*b^3*c^5*d - (9*D*a*b^2 - C*b^3)*c^4*d^2 + 4*(4*D*a^2*b - C*
a*b^2)*c^3*d^3 - (9*D*a^3 - C*a^2*b - 5*B*a*b^2 + 5*A*b^3)*c^2*d^4 + 2*(C*
a^3 - 3*B*a^2*b + 3*A*a*b^2)*c*d^5 + (B*a^3 - A*a^2*b)*d^6)*x + 6*((D*a^3
- C*a^2*b + B*a*b^2 - A*b^3)*d^6*x^3 + 3*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^
3)*c*d^5*x^2 + 3*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*c^2*d^4*x + (D*a^3 -
C*a^2*b + B*a*b^2 - A*b^3)*c^3*d^3)*log(b*x + a) - 6*((D*a^3 - C*a^2*b + B
*a*b^2 - A*b^3)*d^6*x^3 + 3*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*c*d^5*x^2
+ 3*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*c^2*d^4*x + (D*a^3 - C*a^2*b + B*a
*b^2 - A*b^3)*c^3*d^3)*log(d*x + c))/(b^4*c^7*d^3 - 4*a*b^3*c^6*d^4 + 6*a^
2*b^2*c^5*d^5 - 4*a^3*b*c^4*d^6 + a^4*c^3*d^7 + (b^4*c^4*d^6 - 4*a*b^3*c^3
*d^7 + 6*a^2*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*x^3 + 3*(b^4*c^5*d^5
- 4*a*b^3*c^4*d^6 + 6*a^2*b^2*c^3*d^7 - 4*a^3*b*c^2*d^8 + a^4*c*d^9)*x^2 +
 3*(b^4*c^6*d^4 - 4*a*b^3*c^5*d^5 + 6*a^2*b^2*c^4*d^6 - 4*a^3*b*c^3*d^7 +
a^4*c^2*d^8)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(252) = 504$.

Time = 28.12 (sec) , antiderivative size = 1360, normalized size of antiderivative = 4.96

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)(c + dx)^4}\, dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c)**4,x)
```

output
```
(-2*A*a**2*d**5 + 7*A*a*b*c*d**4 - 11*A*b**2*c**2*d**3 - B*a**2*c*d**4 + 5
*B*a*b*c**2*d**3 + 2*B*b**2*c**3*d**2 - 2*C*a**2*c**2*d**3 - 5*C*a*b*c**3*
d**2 + C*b**2*c**4*d + 11*D*a**2*c**3*d**2 - 7*D*a*b*c**4*d + 2*D*b**2*c**
5 + x**2*(-6*A*b**2*d**5 + 6*B*a*b*d**5 - 6*C*a**2*d**5 + 18*D*a**2*c*d**4
 - 18*D*a*b*c**2*d**3 + 6*D*b**2*c**3*d**2) + x*(3*A*a*b*d**5 - 15*A*b**2*
c*d**4 - 3*B*a**2*d**5 + 15*B*a*b*c*d**4 - 6*C*a**2*c*d**4 - 9*C*a*b*c**2*
d**3 + 3*C*b**2*c**3*d**2 + 27*D*a**2*c**2*d**3 - 21*D*a*b*c**3*d**2 + 6*D
*b**2*c**4*d))/(6*a**3*c**3*d**6 - 18*a**2*b*c**4*d**5 + 18*a*b**2*c**5*d*
*4 - 6*b**3*c**6*d**3 + x**3*(6*a**3*d**9 - 18*a**2*b*c*d**8 + 18*a*b**2*c
**2*d**7 - 6*b**3*c**3*d**6) + x**2*(18*a**3*c*d**8 - 54*a**2*b*c**2*d**7
+ 54*a*b**2*c**3*d**6 - 18*b**3*c**4*d**5) + x*(18*a**3*c**2*d**7 - 54*a**
2*b*c**3*d**6 + 54*a*b**2*c**4*d**5 - 18*b**3*c**5*d**4)) + (-A*b**3 + B*a
*b**2 - C*a**2*b + D*a**3)*log(x + (-A*a*b**3*d - A*b**4*c + B*a**2*b**2*d
 + B*a*b**3*c - C*a**3*b*d - C*a**2*b**2*c + D*a**4*d + D*a**3*b*c - a**5*
d**5*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(a*d - b*c)**4 + 5*a**4*b*c*
d**4*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(a*d - b*c)**4 - 10*a**3*b**
2*c**2*d**3*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(a*d - b*c)**4 + 10*a
**2*b**3*c**3*d**2*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(a*d - b*c)**4
 - 5*a*b**4*c**4*d*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(a*d - b*c)**4
 + b**5*c**5*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)/(a*d - b*c)**4)/(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(272) = 544.

Time = 0.07 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.26

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx = -\frac{(Da^3-Ca^2b+Bab^2-Ab^3)\log(bx+a)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} + \frac{(Da^3-Ca^2b+Bab^2-Ab^3)\log(dx+c)}{b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4} - \frac{2\,Db^2c^5-2\,Aa^2d^5-(7\,Dab-Cb^2)c^4d+(11\,Da^2-5\,Cab+2\,Bb^2)c^3d^2-(2\,Ca^2-5\,Bab+11\,Ab^2)c}{6\,(b^3c^6d^3-3\,ab^2c^5d^4+3\,a^2bc^4d^5-a^3c^3d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + (D*a^3 - C*a^2*b + B*a*b
^2 - A*b^3)*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4) - 1/6*(2*D*b^2*c^5 - 2*A*a^2*d^5 - (7*D*a*b - C*b^2
)*c^4*d + (11*D*a^2 - 5*C*a*b + 2*B*b^2)*c^3*d^2 - (2*C*a^2 - 5*B*a*b + 11
*A*b^2)*c^2*d^3 - (B*a^2 - 7*A*a*b)*c*d^4 + 6*(D*b^2*c^3*d^2 - 3*D*a*b*c^2
*d^3 + 3*D*a^2*c*d^4 - (C*a^2 - B*a*b + A*b^2)*d^5)*x^2 + 3*(2*D*b^2*c^4*d
 - (7*D*a*b - C*b^2)*c^3*d^2 + 3*(3*D*a^2 - C*a*b)*c^2*d^3 - (2*C*a^2 - 5*
B*a*b + 5*A*b^2)*c*d^4 - (B*a^2 - A*a*b)*d^5)*x)/(b^3*c^6*d^3 - 3*a*b^2*c^
5*d^4 + 3*a^2*b*c^4*d^5 - a^3*c^3*d^6 + (b^3*c^3*d^6 - 3*a*b^2*c^2*d^7 + 3
*a^2*b*c*d^8 - a^3*d^9)*x^3 + 3*(b^3*c^4*d^5 - 3*a*b^2*c^3*d^6 + 3*a^2*b*c
^2*d^7 - a^3*c*d^8)*x^2 + 3*(b^3*c^5*d^4 - 3*a*b^2*c^4*d^5 + 3*a^2*b*c^3*d
^6 - a^3*c^2*d^7)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. $2(272) = 544$.

Time = 0.13 (sec) , antiderivative size = 664, normalized size of antiderivative = 2.42

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx = -\frac{(Da^3b-Ca^2b^2+Bab^3-Ab^4)\log(|bx+a|)}{b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4} + \frac{(Da^3d-Ca^2bd+Bab^2d-Ab^3d)\log(|dx+c|)}{b^4c^4d-4\,ab^3c^3d^2+6\,a^2b^2c^2d^3-4\,a^3bcd^4+a^4d^5} - \frac{2\,Db^3c^6-9\,Dab^2c^5d+Cb^3c^5d+18\,Da^2bc^4d^2-6\,Cab^2c^4d^2+2\,Bb^3c^4d^2-11\,Da^3c^3d^3+3\,Ca^2bc^3d^3+}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
-(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*log(abs(b*x + a))/(b^5*c^4 - 4*a*
b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (D*a^3*d -
C*a^2*b*d + B*a*b^2*d - A*b^3*d)*log(abs(d*x + c))/(b^4*c^4*d - 4*a*b^3*c^
3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) - 1/6*(2*D*b^3*c^6 -
9*D*a*b^2*c^5*d + C*b^3*c^5*d + 18*D*a^2*b*c^4*d^2 - 6*C*a*b^2*c^4*d^2 + 2
*B*b^3*c^4*d^2 - 11*D*a^3*c^3*d^3 + 3*C*a^2*b*c^3*d^3 + 3*B*a*b^2*c^3*d^3
- 11*A*b^3*c^3*d^3 + 2*C*a^3*c^2*d^4 - 6*B*a^2*b*c^2*d^4 + 18*A*a*b^2*c^2*
d^4 + B*a^3*c*d^5 - 9*A*a^2*b*c*d^5 + 2*A*a^3*d^6 + 6*(D*b^3*c^4*d^2 - 4*D
*a*b^2*c^3*d^3 + 6*D*a^2*b*c^2*d^4 - 3*D*a^3*c*d^5 - C*a^2*b*c*d^5 + B*a*b
^2*c*d^5 - A*b^3*c*d^5 + C*a^3*d^6 - B*a^2*b*d^6 + A*a*b^2*d^6)*x^2 + 3*(2
*D*b^3*c^5*d - 9*D*a*b^2*c^4*d^2 + C*b^3*c^4*d^2 + 16*D*a^2*b*c^3*d^3 - 4*
C*a*b^2*c^3*d^3 - 9*D*a^3*c^2*d^4 + C*a^2*b*c^2*d^4 + 5*B*a*b^2*c^2*d^4 -
5*A*b^3*c^2*d^4 + 2*C*a^3*c*d^5 - 6*B*a^2*b*c*d^5 + 6*A*a*b^2*c*d^5 + B*a^
3*d^6 - A*a^2*b*d^6)*x)/((b*c - a*d)^4*(d*x + c)^3*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,\left(c+d\,x\right)^4}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^4),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.75

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^4}\,dx$$
$$=\frac{-6\log(bx+a)\,a^2c^3d^2-18\log(bx+a)\,a^2c^2d^3x-18\log(bx+a)\,a^2c\,d^4x^2-6\log(bx+a)\,a^2d^5x^3+6\log(}{6d^2\,(a^3d^6x^3-3a^2bc\,d^5x^3+3a\,b^2c^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^4,x)
```

output
```
( - 6*log(a + b*x)*a**2*c**3*d**2 - 18*log(a + b*x)*a**2*c**2*d**3*x - 18*
log(a + b*x)*a**2*c*d**4*x**2 - 6*log(a + b*x)*a**2*d**5*x**3 + 6*log(c +
d*x)*a**2*c**3*d**2 + 18*log(c + d*x)*a**2*c**2*d**3*x + 18*log(c + d*x)*a
**2*c*d**4*x**2 + 6*log(c + d*x)*a**2*d**5*x**3 - 2*a**3*d**4 + 6*a**2*b*c
*d**3 + 5*a**2*c**3*d**2 + 9*a**2*c**2*d**3*x - 4*a**2*d**5*x**3 - 6*a*b**
2*c**2*d**2 - 6*a*b*c**4*d - 12*a*b*c**3*d**2*x + 6*a*b*c*d**4*x**3 + 2*b*
*3*c**3*d + b**2*c**5 + 3*b**2*c**4*d*x - 2*b**2*c**2*d**3*x**3)/(6*d**2*(
a**3*c**3*d**3 + 3*a**3*c**2*d**4*x + 3*a**3*c*d**5*x**2 + a**3*d**6*x**3
- 3*a**2*b*c**4*d**2 - 9*a**2*b*c**3*d**3*x - 9*a**2*b*c**2*d**4*x**2 - 3*
a**2*b*c*d**5*x**3 + 3*a*b**2*c**5*d + 9*a*b**2*c**4*d**2*x + 9*a*b**2*c**
3*d**3*x**2 + 3*a*b**2*c**2*d**4*x**3 - b**3*c**6 - 3*b**3*c**5*d*x - 3*b*
*3*c**4*d**2*x**2 - b**3*c**3*d**3*x**3))
```

3.53 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^4} dx$

Optimal result 493
Mathematica [A] (verified) 494
Rubi [A] (verified) 494
Maple [A] (verified) 496
Fricas [B] (verification not implemented) 496
Sympy [B] (verification not implemented) 497
Maxima [B] (verification not implemented) 498
Giac [B] (verification not implemented) 499
Mupad [F(-1)] 500
Reduce [B] (verification not implemented) 500

Optimal result

Integrand size = 30, antiderivative size = 352

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^4}\,dx &= -\frac{Ab^3-a(b^2B-abC+a^2D)}{(bc-ad)^4(a+bx)}-\frac{c^2Cd-Bcd^2+Ad^3-c^3D}{3d^2(bc-ad)^2(c+dx)^3}\\ &\quad-\frac{ad(2cCd-Bd^2-3c^2D)-b(Bcd^2-2Ad^3-c^3D)}{2d^2(bc-ad)^3(c+dx)^2}\\ &\quad+\frac{b^2(Bc-3Ad)-ab(2cC-2Bd)-a^2(Cd-3cD)}{(bc-ad)^4(c+dx)}\\ &\quad+\frac{(b^3(Bc-4Ad)-ab^2(2cC-3Bd)+a^3dD-a^2b(2Cd-3cD))\log(a+bx)}{(bc-ad)^5}\\ &\quad-\frac{(b^3(Bc-4Ad)-ab^2(2cC-3Bd)+a^3dD-a^2b(2Cd-3cD))\log(c+dx)}{(bc-ad)^5}\end{aligned}$$

output

```
-(A*b^3-a*(B*b^2-C*a*b+D*a^2))/(-a*d+b*c)^4/(b*x+a)-1/3*(A*d^3-B*c*d^2+C*c
^2*d-D*c^3)/d^2/(-a*d+b*c)^2/(d*x+c)^3-1/2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b
*(-2*A*d^3+B*c*d^2-D*c^3))/d^2/(-a*d+b*c)^3/(d*x+c)^2+(b^2*(-3*A*d+B*c)-a*
b*(-2*B*d+2*C*c)-a^2*(C*d-3*D*c))/(-a*d+b*c)^4/(d*x+c)+(b^3*(-4*A*d+B*c)-a
*b^2*(-3*B*d+2*C*c)+a^3*d*D-a^2*b*(2*C*d-3*D*c))*ln(b*x+a)/(-a*d+b*c)^5-(b
^3*(-4*A*d+B*c)-a*b^2*(-3*B*d+2*C*c)+a^3*d*D-a^2*b*(2*C*d-3*D*c))*ln(d*x+c
)/(-a*d+b*c)^5
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^4}\,dx$$
$$= \frac{-Ab^3+a(b^2B-abC+a^2D)}{(bc-ad)^4(a+bx)} + \frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{3d^2(bc-ad)^2(c+dx)^3}$$
$$+ \frac{-ad(-2cCd+Bd^2+3c^2D)+b(-Bcd^2+2Ad^3+c^3D)}{2d^2(-bc+ad)^3(c+dx)^2}$$
$$+ \frac{b^2(Bc-3Ad)+ab(-2cC+2Bd)+a^2(-Cd+3cD)}{(bc-ad)^4(c+dx)}$$
$$+ \frac{(b^3(Bc-4Ad)+ab^2(-2cC+3Bd)+a^3dD+a^2b(-2Cd+3cD))\log(a+bx)}{(bc-ad)^5}$$
$$- \frac{(b^3(Bc-4Ad)+ab^2(-2cC+3Bd)+a^3dD+a^2b(-2Cd+3cD))\log(c+dx)}{(bc-ad)^5}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^4),x]
```

output
```
(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))/((b*c - a*d)^4*(a + b*x)) + (-(c^2*
C*d) + B*c*d^2 - A*d^3 + c^3*D)/(3*d^2*(b*c - a*d)^2*(c + d*x)^3) + (-(a*d
*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-(B*c*d^2) + 2*A*d^3 + c^3*D))/(2*d^2*
(-(b*c) + a*d)^3*(c + d*x)^2) + (b^2*(B*c - 3*A*d) + a*b*(-2*c*C + 2*B*d)
+ a^2*(-(C*d) + 3*c*D))/((b*c - a*d)^4*(c + d*x)) + ((b^3*(B*c - 4*A*d) +
a*b^2*(-2*c*C + 3*B*d) + a^3*d*D + a^2*b*(-2*C*d + 3*c*D))*Log[a + b*x])/(
b*c - a*d)^5 - ((b^3*(B*c - 4*A*d) + a*b^2*(-2*c*C + 3*B*d) + a^3*d*D + a^
2*b*(-2*C*d + 3*c*D))*Log[c + d*x])/(b*c - a*d)^5
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^4}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{d\left(a^2(Cd-3cD)+2ab(cC-Bd)-b^2(Bc-3Ad)\right)}{(c+dx)^2(bc-ad)^4}+\frac{b\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{(a+bx)^2(bc-ad)^4}+\frac{b\left(a^3dD-a^2b(2C\right.}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{-\left(a^2(Cd-3cD)\right)-ab(2cC-2Bd)+b^2(Bc-3Ad)}{(c+dx)(bc-ad)^4}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{(a+bx)(bc-ad)^4}+\frac{\log(a+bx)\left(a^3dD-a^2b(2Cd-3cD)-ab^2(2cC-3Bd)+b^3(Bc-4Ad)\right)}{(bc-ad)^5}-\frac{\log(c+dx)\left(a^3dD-a^2b(2Cd-3cD)-ab^2(2cC-3Bd)+b^3(Bc-4Ad)\right)}{(bc-ad)^5}-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{3d^2(c+dx)^3(bc-ad)^2}-\frac{ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3+Bcd^2+c^3(-D)\right)}{2d^2(c+dx)^2(bc-ad)^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^4),x]
```

output
```
-((A*b^3 - a*(b^2*B - a*b*C + a^2*D))/((b*c - a*d)^4*(a + b*x))) - (c^2*C*
d - B*c*d^2 + A*d^3 - c^3*D)/(3*d^2*(b*c - a*d)^2*(c + d*x)^3) - (a*d*(2*c
*C*d - B*d^2 - 3*c^2*D) - b*(B*c*d^2 - 2*A*d^3 - c^3*D))/(2*d^2*(b*c - a*d
)^3*(c + d*x)^2) + (b^2*(B*c - 3*A*d) - a*b*(2*c*C - 2*B*d) - a^2*(C*d - 3
*c*D))/((b*c - a*d)^4*(c + d*x)) + ((b^3*(B*c - 4*A*d) - a*b^2*(2*c*C - 3*
B*d) + a^3*d*D - a^2*b*(2*C*d - 3*c*D))*Log[a + b*x])/(b*c - a*d)^5 - ((b^
3*(B*c - 4*A*d) - a*b^2*(2*c*C - 3*B*d) + a^3*d*D - a^2*b*(2*C*d - 3*c*D))
*Log[c + d*x])/(b*c - a*d)^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.03

method	result
default	$-\frac{3d\,b^{2}A-2Babd-B\,b^{2}c+C\,a^{2}d+2Cabc-3a^{2}cD}{(ad-bc)^{4}(xd+c)}-\frac{A\,d^{3}-Bc\,d^{2}+C\,c^{2}d-Dc^{3}}{3d^{2}(ad-bc)^{2}(xd+c)^{3}}-\frac{-2Ab\,d^{3}+Ba\,d^{3}+Bbc\,d^{2}-2Cac\,d^{2}+3D}{2d^{2}(ad-bc)^{3}(xd+c)^{2}}$
norman	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-(3*A*b^2*d-2*B*a*b*d-B*b^2*c+C*a^2*d+2*C*a*b*c-3*D*a^2*c)/(a*d-b*c)^4/(d*
x+c)-1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^2/(a*d-b*c)^2/(d*x+c)^3-1/2*(-2*A
*b*d^3+B*a*d^3+B*b*c*d^2-2*C*a*c*d^2+3*D*a*c^2*d-D*b*c^3)/d^2/(a*d-b*c)^3/
(d*x+c)^2-(4*A*b^3*d-3*B*a*b^2*d-B*b^3*c+2*C*a^2*b*d+2*C*a*b^2*c-D*a^3*d-3
*D*a^2*b*c)/(a*d-b*c)^5*ln(d*x+c)+(4*A*b^3*d-3*B*a*b^2*d-B*b^3*c+2*C*a^2*b
*d+2*C*a*b^2*c-D*a^3*d-3*D*a^2*b*c)/(a*d-b*c)^5*ln(b*x+a)-(A*b^3-B*a*b^2+C
*a^2*b-D*a^3)/(a*d-b*c)^4/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1929 vs. 2(344) = 688.

Time = 0.15 (sec) , antiderivative size = 1929, normalized size of antiderivative = 5.48

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(D*a*b^3*c^6 - 2*A*a^4*d^6 - (9*D*a^2*b^2 - 2*C*a*b^3)*c^5*d - (9*D*a
^3*b - 18*C*a^2*b^2 + 17*B*a*b^3 - 6*A*b^4)*c^4*d^2 + (17*D*a^4 - 18*C*a^3
*b + 9*B*a^2*b^2 + 20*A*a*b^3)*c^3*d^3 - (2*C*a^4 - 9*B*a^3*b + 36*A*a^2*b
^2)*c^2*d^4 - (B*a^4 - 12*A*a^3*b)*c*d^5 - 6*((3*D*a^2*b^2 - 2*C*a*b^3 + B
*b^4)*c^2*d^4 - 2*(D*a^3*b - B*a*b^3 + 2*A*b^4)*c*d^5 - (D*a^4 - 2*C*a^3*b
 + 3*B*a^2*b^2 - 4*A*a*b^3)*d^6)*x^3 + 3*(D*b^4*c^5*d - 5*D*a*b^3*c^4*d^2
- 5*(D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c^3*d^3 - (3*D*a^3*b - 2*C*a^2*b^2 + 1
1*B*a*b^3 - 20*A*b^4)*c^2*d^4 + (12*D*a^4 - 10*C*a^3*b + 13*B*a^2*b^2 - 16
*A*a*b^3)*c*d^5 - (2*C*a^4 - 3*B*a^3*b + 4*A*a^2*b^2)*d^6)*x^2 + (D*b^4*c^
6 - 2*(3*D*a*b^3 - C*b^4)*c^5*d - (18*D*a^2*b^2 - 12*C*a*b^3 + 11*B*b^4)*c
^4*d^2 - 2*(11*D*a^3*b - 18*C*a^2*b^2 + 15*B*a*b^3 - 22*A*b^4)*c^3*d^3 + (
45*D*a^4 - 44*C*a^3*b + 18*B*a^2*b^2 - 12*A*a*b^3)*c^2*d^4 - 2*(3*C*a^4 -
13*B*a^3*b + 18*A*a^2*b^2)*c*d^5 - (3*B*a^4 - 4*A*a^3*b)*d^6)*x - 6*((3*D*
a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c^4*d^2 + (D*a^4 - 2*C*a^3*b + 3*B*a^2*b^2
- 4*A*a*b^3)*c^3*d^3 + ((3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c*d^5 + (D*a^3*b
 - 2*C*a^2*b^2 + 3*B*a*b^3 - 4*A*b^4)*d^6)*x^4 + (3*(3*D*a^2*b^2 - 2*C*a*b
^3 + B*b^4)*c^2*d^4 + 2*(3*D*a^3*b - 4*C*a^2*b^2 + 5*B*a*b^3 - 6*A*b^4)*c*
d^5 + (D*a^4 - 2*C*a^3*b + 3*B*a^2*b^2 - 4*A*a*b^3)*d^6)*x^3 + 3*((3*D*a^2
*b^2 - 2*C*a*b^3 + B*b^4)*c^3*d^3 + 4*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b
^4)*c^2*d^4 + (D*a^4 - 2*C*a^3*b + 3*B*a^2*b^2 - 4*A*a*b^3)*c*d^5)*x^2 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2739 vs. $2(328) = 656$.

Time = 120.68 (sec) , antiderivative size = 2739, normalized size of antiderivative = 7.78

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^4} \, dx = \text{Too large to display}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**4,x)
```

output
```
(-2*A*a**3*d**5 + 10*A*a**2*b*c*d**4 - 26*A*a*b**2*c**2*d**3 - 6*A*b**3*c*
*3*d**2 - B*a**3*c*d**4 + 8*B*a**2*b*c**2*d**3 + 17*B*a*b**2*c**3*d**2 - 2
*C*a**3*c**2*d**3 - 20*C*a**2*b*c**3*d**2 - 2*C*a*b**2*c**4*d + 17*D*a**3*
c**3*d**2 + 8*D*a**2*b*c**4*d - D*a*b**2*c**5 + x**3*(-24*A*b**3*d**5 + 18
*B*a*b**2*d**5 + 6*B*b**3*c*d**4 - 12*C*a**2*b*d**5 - 12*C*a*b**2*c*d**4 +
 6*D*a**3*d**5 + 18*D*a**2*b*c*d**4) + x**2*(-12*A*a*b**2*d**5 - 60*A*b**3
*c*d**4 + 9*B*a**2*b*d**5 + 48*B*a*b**2*c*d**4 + 15*B*b**3*c**2*d**3 - 6*C
*a**3*d**5 - 36*C*a**2*b*c*d**4 - 30*C*a*b**2*c**2*d**3 + 36*D*a**3*c*d**4
 + 27*D*a**2*b*c**2*d**3 + 12*D*a*b**2*c**3*d**2 - 3*D*b**3*c**4*d) + x*(4
*A*a**2*b*d**5 - 32*A*a*b**2*c*d**4 - 44*A*b**3*c**2*d**3 - 3*B*a**3*d**5
+ 23*B*a**2*b*c*d**4 + 41*B*a*b**2*c**2*d**3 + 11*B*b**3*c**3*d**2 - 6*C*a
**3*c*d**4 - 50*C*a**2*b*c**2*d**3 - 14*C*a*b**2*c**3*d**2 - 2*C*b**3*c**4
*d + 45*D*a**3*c**2*d**3 + 23*D*a**2*b*c**3*d**2 + 5*D*a*b**2*c**4*d - D*b
**3*c**5))/(6*a**5*c**3*d**6 - 24*a**4*b*c**4*d**5 + 36*a**3*b**2*c**5*d**
4 - 24*a**2*b**3*c**6*d**3 + 6*a*b**4*c**7*d**2 + x**4*(6*a**4*b*d**9 - 24
*a**3*b**2*c*d**8 + 36*a**2*b**3*c**2*d**7 - 24*a*b**4*c**3*d**6 + 6*b**5*
c**4*d**5) + x**3*(6*a**5*d**9 - 6*a**4*b*c*d**8 - 36*a**3*b**2*c**2*d**7
+ 84*a**2*b**3*c**3*d**6 - 66*a*b**4*c**4*d**5 + 18*b**5*c**5*d**4) + x**2
*(18*a**5*c*d**8 - 54*a**4*b*c**2*d**7 + 36*a**3*b**2*c**3*d**6 + 36*a**2*
b**3*c**4*d**5 - 54*a*b**4*c**5*d**4 + 18*b**5*c**6*d**3) + x*(18*a**5*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(344) = 688$.

Time = 0.09 (sec) , antiderivative size = 1038, normalized size of antiderivative = 2.95

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
((3*D*a^2*b - 2*C*a*b^2 + B*b^3)*c + (D*a^3 - 2*C*a^2*b + 3*B*a*b^2 - 4*A*
b^3)*d)*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^
3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - ((3*D*a^2*b - 2*C*a*b^2 + B*b^3
)*c + (D*a^3 - 2*C*a^2*b + 3*B*a*b^2 - 4*A*b^3)*d)*log(d*x + c)/(b^5*c^5 -
 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 -
 a^5*d^5) - 1/6*(D*a*b^2*c^5 + 2*A*a^3*d^5 - 2*(4*D*a^2*b - C*a*b^2)*c^4*d
 - (17*D*a^3 - 20*C*a^2*b + 17*B*a*b^2 - 6*A*b^3)*c^3*d^2 + 2*(C*a^3 - 4*B
*a^2*b + 13*A*a*b^2)*c^2*d^3 + (B*a^3 - 10*A*a^2*b)*c*d^4 - 6*((3*D*a^2*b
- 2*C*a*b^2 + B*b^3)*c*d^4 + (D*a^3 - 2*C*a^2*b + 3*B*a*b^2 - 4*A*b^3)*d^5
)*x^3 + 3*(D*b^3*c^4*d - 4*D*a*b^2*c^3*d^2 - (9*D*a^2*b - 10*C*a*b^2 + 5*B
*b^3)*c^2*d^3 - 4*(3*D*a^3 - 3*C*a^2*b + 4*B*a*b^2 - 5*A*b^3)*c*d^4 + (2*C
*a^3 - 3*B*a^2*b + 4*A*a*b^2)*d^5)*x^2 + (D*b^3*c^5 - (5*D*a*b^2 - 2*C*b^3
)*c^4*d - (23*D*a^2*b - 14*C*a*b^2 + 11*B*b^3)*c^3*d^2 - (45*D*a^3 - 50*C*
a^2*b + 41*B*a*b^2 - 44*A*b^3)*c^2*d^3 + (6*C*a^3 - 23*B*a^2*b + 32*A*a*b^
2)*c*d^4 + (3*B*a^3 - 4*A*a^2*b)*d^5)*x)/(a*b^4*c^7*d^2 - 4*a^2*b^3*c^6*d^
3 + 6*a^3*b^2*c^5*d^4 - 4*a^4*b*c^4*d^5 + a^5*c^3*d^6 + (b^5*c^4*d^5 - 4*a
*b^4*c^3*d^6 + 6*a^2*b^3*c^2*d^7 - 4*a^3*b^2*c*d^8 + a^4*b*d^9)*x^4 + (3*b
^5*c^5*d^4 - 11*a*b^4*c^4*d^5 + 14*a^2*b^3*c^3*d^6 - 6*a^3*b^2*c^2*d^7 - a
^4*b*c*d^8 + a^5*d^9)*x^3 + 3*(b^5*c^6*d^3 - 3*a*b^4*c^5*d^4 + 2*a^2*b^3*c
^4*d^5 + 2*a^3*b^2*c^3*d^6 - 3*a^4*b*c^2*d^7 + a^5*c*d^8)*x^2 + (b^5*c^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(344) = 688$.

Time = 0.14 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.19

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^4}\,dx = -\frac{(3\,Da^2b^2c - 2\,Cab^3c + Bb^4c + Da^3bd - 2\,Ca^2b^2d + 3\,Bab^3d - 4\,Ab^4d)\log\left(\left|\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right|\right)}{b^6c^5 - 5\,ab^5c^4d + 10\,a^2b^4c^3d^2 - 10\,a^3b^3c^2d^3 + 5\,a^4b^2cd^4 - a^5bd^5} + \frac{\frac{Da^3b^4}{bx+a} - \frac{Ca^2b^5}{bx+a} + \frac{Bab^6}{bx+a} - \frac{Ab^7}{bx+a}}{b^8c^4 - 4\,ab^7c^3d + 6\,a^2b^6c^2d^2 - 4\,a^3b^5cd^3 + a^4b^4d^4} + \frac{Db^3c^3d - 9\,Dab^2c^2d^2 + 2\,Cb^3c^2d^2 - 18\,Da^2bcd^3 + 18\,Cab^2cd^3 - 11\,Bb^3cd^3 + 6\,Ca^2bd^4 - 15\,Bab^2d^4 + \cdots}{\cdots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
-(3*D*a^2*b^2*c - 2*C*a*b^3*c + B*b^4*c + D*a^3*b*d - 2*C*a^2*b^2*d + 3*B*
a*b^3*d - 4*A*b^4*d)*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^6*c^5
- 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^
4 - a^5*b*d^5) + (D*a^3*b^4/(b*x + a) - C*a^2*b^5/(b*x + a) + B*a*b^6/(b*x
 + a) - A*b^7/(b*x + a))/(b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*
a^3*b^5*c*d^3 + a^4*b^4*d^4) + 1/6*(D*b^3*c^3*d - 9*D*a*b^2*c^2*d^2 + 2*C*
b^3*c^2*d^2 - 18*D*a^2*b*c*d^3 + 18*C*a*b^2*c*d^3 - 11*B*b^3*c*d^3 + 6*C*a
^2*b*d^4 - 15*B*a*b^2*d^4 + 26*A*b^3*d^4 + 3*(D*b^5*c^4 - 10*D*a*b^4*c^3*d
 + 2*C*b^5*c^3*d - 3*D*a^2*b^3*c^2*d^2 + 12*C*a*b^4*c^2*d^2 - 9*B*b^5*c^2*
d^2 + 12*D*a^3*b^2*c*d^3 - 10*C*a^2*b^3*c*d^3 - 2*B*a*b^4*c*d^3 + 20*A*b^5
*c*d^3 - 4*C*a^3*b^2*d^4 + 11*B*a^2*b^3*d^4 - 20*A*a*b^4*d^4)/((b*x + a)*b
) - 6*(3*D*a*b^6*c^4 - C*b^7*c^4 - 3*D*a^2*b^5*c^3*d - 2*C*a*b^6*c^3*d + 3
*B*b^7*c^3*d - 3*D*a^3*b^4*c^2*d^2 + 6*C*a^2*b^5*c^2*d^2 - 3*B*a*b^6*c^2*d
^2 - 6*A*b^7*c^2*d^2 + 3*D*a^4*b^3*c*d^3 - 2*C*a^3*b^4*c*d^3 - 3*B*a^2*b^5
*c*d^3 + 12*A*a*b^6*c*d^3 - C*a^4*b^3*d^4 + 3*B*a^3*b^4*d^4 - 6*A*a^2*b^5*
d^4)/((b*x + a)^2*b^2))/((b*c - a*d)^5*(b*c/(b*x + a) - a*d/(b*x + a) + d)
^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^4}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^4}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^4),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2067, normalized size of antiderivative = 5.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^4}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^4,x)
```

output
```
( - 6*log(a + b*x)*a**4*c**3*d**3 - 18*log(a + b*x)*a**4*c**2*d**4*x - 18*
log(a + b*x)*a**4*c*d**5*x**2 - 6*log(a + b*x)*a**4*d**6*x**3 - 30*log(a +
 b*x)*a**3*b*c**4*d**2 - 96*log(a + b*x)*a**3*b*c**3*d**3*x - 108*log(a +
b*x)*a**3*b*c**2*d**4*x**2 - 48*log(a + b*x)*a**3*b*c*d**5*x**3 - 6*log(a
+ b*x)*a**3*b*d**6*x**4 + 6*log(a + b*x)*a**2*b**3*c**3*d**2 + 18*log(a +
b*x)*a**2*b**3*c**2*d**3*x + 18*log(a + b*x)*a**2*b**3*c*d**4*x**2 + 6*log
(a + b*x)*a**2*b**3*d**5*x**3 - 36*log(a + b*x)*a**2*b**2*c**5*d - 138*log
(a + b*x)*a**2*b**2*c**4*d**2*x - 198*log(a + b*x)*a**2*b**2*c**3*d**3*x**
2 - 126*log(a + b*x)*a**2*b**2*c**2*d**4*x**3 - 30*log(a + b*x)*a**2*b**2*
c*d**5*x**4 + 18*log(a + b*x)*a*b**4*c**4*d + 60*log(a + b*x)*a*b**4*c**3*
d**2*x + 72*log(a + b*x)*a*b**4*c**2*d**3*x**2 + 36*log(a + b*x)*a*b**4*c*
d**4*x**3 + 6*log(a + b*x)*a*b**4*d**5*x**4 - 36*log(a + b*x)*a*b**3*c**5*
d*x - 108*log(a + b*x)*a*b**3*c**4*d**2*x**2 - 108*log(a + b*x)*a*b**3*c**
3*d**3*x**3 - 36*log(a + b*x)*a*b**3*c**2*d**4*x**4 + 18*log(a + b*x)*b**5
*c**4*d*x + 54*log(a + b*x)*b**5*c**3*d**2*x**2 + 54*log(a + b*x)*b**5*c**
2*d**3*x**3 + 18*log(a + b*x)*b**5*c*d**4*x**4 + 6*log(c + d*x)*a**4*c**3*
d**3 + 18*log(c + d*x)*a**4*c**2*d**4*x + 18*log(c + d*x)*a**4*c*d**5*x**2
 + 6*log(c + d*x)*a**4*d**6*x**3 + 30*log(c + d*x)*a**3*b*c**4*d**2 + 96*l
og(c + d*x)*a**3*b*c**3*d**3*x + 108*log(c + d*x)*a**3*b*c**2*d**4*x**2 +
48*log(c + d*x)*a**3*b*c*d**5*x**3 + 6*log(c + d*x)*a**3*b*d**6*x**4 - ...
```

3.54 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 474

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx &= \frac{-Ab^3+a(b^2B-abC+a^2D)}{2(bc-ad)^4(a+bx)^2} \\ &- \frac{b^3(Bc-4Ad)-ab^2(2cC-3Bd)+a^3dD-a^2b(2Cd-3cD)}{(bc-ad)^5(a+bx)} \\ &+ \frac{c^2Cd-Bcd^2+Ad^3-c^3D}{3d(bc-ad)^3(c+dx)^3} + \frac{b(c^2C-2Bcd+3Ad^2)+a(2cCd-Bd^2-3c^2D)}{2(bc-ad)^4(c+dx)^2} \\ &+ \frac{b^2(c^2C-3Bcd+6Ad^2)+a^2d(Cd-3cD)+ab(4cCd-3Bd^2-3c^2D)}{(bc-ad)^5(c+dx)} \\ &+ \frac{(b^3(c^2C-4Bcd+10Ad^2)-a^3d^2D+3a^2bd(Cd-2cD)+3ab^2(2cCd-2Bd^2-c^2D))\log(a+bx)}{(bc-ad)^6} \\ &- \frac{(b^3(c^2C-4Bcd+10Ad^2)-a^3d^2D+3a^2bd(Cd-2cD)+3ab^2(2cCd-2Bd^2-c^2D))\log(c+dx)}{(bc-ad)^6}\end{aligned}$$

output
```
1/2*(-A*b^3+a*(B*b^2-C*a*b+D*a^2))/(-a*d+b*c)^4/(b*x+a)^2-(b^3*(-4*A*d+B*c
)-a*b^2*(-3*B*d+2*C*c)+a^3*d*D-a^2*b*(2*C*d-3*D*c))/(-a*d+b*c)^5/(b*x+a)+1
/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d/(-a*d+b*c)^3/(d*x+c)^3+1/2*(b*(3*A*d^2-
2*B*c*d+C*c^2)+a*(-B*d^2+2*C*c*d-3*D*c^2))/(-a*d+b*c)^4/(d*x+c)^2+(b^2*(6*
A*d^2-3*B*c*d+C*c^2)+a^2*d*(C*d-3*D*c)+a*b*(-3*B*d^2+4*C*c*d-3*D*c^2))/(-a
*d+b*c)^5/(d*x+c)+(b^3*(10*A*d^2-4*B*c*d+C*c^2)-a^3*d^2*D+3*a^2*b*d*(C*d-2
*D*c)+3*a*b^2*(-2*B*d^2+2*C*c*d-D*c^2))*ln(b*x+a)/(-a*d+b*c)^6-(b^3*(10*A*
d^2-4*B*c*d+C*c^2)-a^3*d^2*D+3*a^2*b*d*(C*d-2*D*c)+3*a*b^2*(-2*B*d^2+2*C*c
*d-D*c^2))*ln(d*x+c)/(-a*d+b*c)^6
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 471, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx = \frac{-Ab^3+a(b^2B-abC+a^2D)}{2(bc-ad)^4(a+bx)^2} + \frac{b^3(-Bc+4Ad)+ab^2(2cC-3Bd)-a^3dD+a^2b(2Cd-3cD)}{(bc-ad)^5(a+bx)} + \frac{-c^2Cd+Bcd^2-Ad^3+c^3D}{3d(-bc+ad)^3(c+dx)^3} + \frac{b(c^2C-2Bcd+3Ad^2)-a(-2cCd+Bd^2+3c^2D)}{2(bc-ad)^4(c+dx)^2} + \frac{b^2(c^2C-3Bcd+6Ad^2)+a^2d(Cd-3cD)+ab(4cCd-3Bd^2-3c^2D)}{(bc-ad)^5(c+dx)} + \frac{(b^3(c^2C-4Bcd+10Ad^2)-a^3d^2D+3a^2bd(Cd-2cD)-3ab^2(-2cCd+2Bd^2+c^2D))\log(a+bx)}{(bc-ad)^6} - \frac{(b^3(c^2C-4Bcd+10Ad^2)-a^3d^2D+3a^2bd(Cd-2cD)-3ab^2(-2cCd+2Bd^2+c^2D))\log(c+dx)}{(bc-ad)^6}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^4),x]
```

output
```
(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))/(2*(b*c - a*d)^4*(a + b*x)^2) + (b^
3*(-(B*c) + 4*A*d) + a*b^2*(2*c*C - 3*B*d) - a^3*d*D + a^2*b*(2*C*d - 3*c*
D))/((b*c - a*d)^5*(a + b*x)) + (-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)/(3*
d*(-(b*c) + a*d)^3*(c + d*x)^3) + (b*(c^2*C - 2*B*c*d + 3*A*d^2) - a*(-2*c
*C*d + B*d^2 + 3*c^2*D))/(2*(b*c - a*d)^4*(c + d*x)^2) + (b^2*(c^2*C - 3*B
*c*d + 6*A*d^2) + a^2*d*(C*d - 3*c*D) + a*b*(4*c*C*d - 3*B*d^2 - 3*c^2*D))
/((b*c - a*d)^5*(c + d*x)) + ((b^3*(c^2*C - 4*B*c*d + 10*A*d^2) - a^3*d^2*
D + 3*a^2*b*d*(C*d - 2*c*D) - 3*a*b^2*(-2*c*C*d + 2*B*d^2 + c^2*D))*Log[a
+ b*x])/(b*c - a*d)^6 - ((b^3*(c^2*C - 4*B*c*d + 10*A*d^2) - a^3*d^2*D + 3
*a^2*b*d*(C*d - 2*c*D) - 3*a*b^2*(-2*c*C*d + 2*B*d^2 + c^2*D))*Log[c + d*x
])/(b*c - a*d)^6
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{d\left(a^2(-d)(Cd-3cD)-ab\left(-3Bd^2-3c^2D+4cCd\right)-b^2\left(6Ad^2-3Bcd+c^2C\right)\right)}{(c+dx)^2(bc-ad)^5}+\frac{b\left(Ab^3-a\left(a^2D-abC\right.\right.}{(a+bx)^3(bc-ad\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2d(Cd-3cD)+ab(-3Bd^2-3c^2D+4cCd)+b^2(6Ad^2-3Bcd+c^2C)}{(c+dx)(bc-ad)^5}-\frac{Ab^3-a(a^2D-abC+b^2B)}{2(a+bx)^2(bc-ad)^4}+\frac{\log(a+bx)\left(a^3\left(-d^2\right)D+3a^2bd(Cd-2cD)+3ab^2\left(-2Bd^2+c^2(-D)+2cCd\right)+b^3\left(10Ad^2-4Bcd+c^2C\right)\right)}{(bc-ad)^6}-\frac{\log(c+dx)\left(a^3\left(-d^2\right)D+3a^2bd(Cd-2cD)+3ab^2\left(-2Bd^2+c^2(-D)+2cCd\right)+b^3\left(10Ad^2-4Bcd+c^2C\right)\right)}{(bc-ad)^6}-\frac{a^3dD-a^2b(2Cd-3cD)-ab^2(2cC-3Bd)+b^3(Bc-4Ad)}{(a+bx)(bc-ad)^5}+\frac{a(-Bd^2-3c^2D+2cCd)+b(3Ad^2-2Bcd+c^2C)}{2(c+dx)^2(bc-ad)^4}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{3d(c+dx)^3(bc-ad)^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^4),x]
```

output
```
-1/2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))/((b*c - a*d)^4*(a + b*x)^2) - (b^
3*(B*c - 4*A*d) - a*b^2*(2*c*C - 3*B*d) + a^3*d*D - a^2*b*(2*C*d - 3*c*D))
/((b*c - a*d)^5*(a + b*x)) + (c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(3*d*(b*c
 - a*d)^3*(c + d*x)^3) + (b*(c^2*C - 2*B*c*d + 3*A*d^2) + a*(2*c*C*d - B*d
^2 - 3*c^2*D))/(2*(b*c - a*d)^4*(c + d*x)^2) + (b^2*(c^2*C - 3*B*c*d + 6*A
*d^2) + a^2*d*(C*d - 3*c*D) + a*b*(4*c*C*d - 3*B*d^2 - 3*c^2*D))/((b*c - a
*d)^5*(c + d*x)) + ((b^3*(c^2*C - 4*B*c*d + 10*A*d^2) - a^3*d^2*D + 3*a^2*
b*d*(C*d - 2*c*D) + 3*a*b^2*(2*c*C*d - 2*B*d^2 - c^2*D))*Log[a + b*x])/(b*
c - a*d)^6 - ((b^3*(c^2*C - 4*B*c*d + 10*A*d^2) - a^3*d^2*D + 3*a^2*b*d*(C
*d - 2*c*D) + 3*a*b^2*(2*c*C*d - 2*B*d^2 - c^2*D))*Log[c + d*x])/(b*c - a*
d)^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.08

method	result
default	$-\frac{A\,d^{3}-B c\,d^{2}+C\,c^{2} d-D c^{3}}{3\left(a d-b c\right)^{3} d\left(x d+c\right)^{3}}-\frac{6 A\,b^{2} d^{2}-3 B a b\,d^{2}-3 B\,b^{2} c d+C\,a^{2} d^{2}+4 C a b c d+C\,b^{2} c^{2}-3 D a^{2} c d-3 D a b\,c^{2}}{\left(a d-b c\right)^{5} \left(x d+c\right)}+\frac{3 b\,d^{2} A-B a\,d}{}$
norman	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^3/d/(d*x+c)^3-(6*A*b^2*d^2-3*
B*a*b*d^2-3*B*b^2*c*d+C*a^2*d^2+4*C*a*b*c*d+C*b^2*c^2-3*D*a^2*c*d-3*D*a*b*
c^2)/(a*d-b*c)^5/(d*x+c)+1/2*(3*A*b*d^2-B*a*d^2-2*B*b*c*d+2*C*a*c*d+C*b*c^
2-3*D*a*c^2)/(a*d-b*c)^4/(d*x+c)^2-(10*A*b^3*d^2-6*B*a*b^2*d^2-4*B*b^3*c*d
+3*C*a^2*b*d^2+6*C*a*b^2*c*d+C*b^3*c^2-D*a^3*d^2-6*D*a^2*b*c*d-3*D*a*b^2*c
^2)/(a*d-b*c)^6*ln(d*x+c)-(4*A*b^3*d-3*B*a*b^2*d-B*b^3*c+2*C*a^2*b*d+2*C*a
*b^2*c-D*a^3*d-3*D*a^2*b*c)/(a*d-b*c)^5/(b*x+a)+(10*A*b^3*d^2-6*B*a*b^2*d^
2-4*B*b^3*c*d+3*C*a^2*b*d^2+6*C*a*b^2*c*d+C*b^3*c^2-D*a^3*d^2-6*D*a^2*b*c*
d-3*D*a*b^2*c^2)/(a*d-b*c)^6*ln(b*x+a)-1/2*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(
a*d-b*c)^4/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3119 vs. 2(470) = 940.

Time = 0.26 (sec) , antiderivative size = 3119, normalized size of antiderivative = 6.58

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1553 vs. $2(470) = 940$.

Time = 0.14 (sec) , antiderivative size = 1553, normalized size of antiderivative = 3.28

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-((3*D*a*b^2 - C*b^3)*c^2 + 2*(3*D*a^2*b - 3*C*a*b^2 + 2*B*b^3)*c*d + (D*a
^3 - 3*C*a^2*b + 6*B*a*b^2 - 10*A*b^3)*d^2)*log(b*x + a)/(b^6*c^6 - 6*a*b^
5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6
*a^5*b*c*d^5 + a^6*d^6) + ((3*D*a*b^2 - C*b^3)*c^2 + 2*(3*D*a^2*b - 3*C*a*
b^2 + 2*B*b^3)*c*d + (D*a^3 - 3*C*a^2*b + 6*B*a*b^2 - 10*A*b^3)*d^2)*log(d
*x + c)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6) - 1/6*(2*D*a^2*b^2*c^5 -
2*A*a^4*d^5 + (38*D*a^3*b - 20*C*a^2*b^2 + 3*B*a*b^3 + 3*A*b^4)*c^4*d + (2
0*D*a^4 - 38*C*a^3*b + 47*B*a^2*b^2 - 27*A*a*b^3)*c^3*d^2 - (2*C*a^4 - 11*
B*a^3*b + 47*A*a^2*b^2)*c^2*d^3 - (B*a^4 - 13*A*a^3*b)*c*d^4 + 6*((3*D*a*b
^3 - C*b^4)*c^2*d^3 + 2*(3*D*a^2*b^2 - 3*C*a*b^3 + 2*B*b^4)*c*d^4 + (D*a^3
*b - 3*C*a^2*b^2 + 6*B*a*b^3 - 10*A*b^4)*d^5)*x^4 + 3*(5*(3*D*a*b^3 - C*b^
4)*c^3*d^2 + (39*D*a^2*b^2 - 33*C*a*b^3 + 20*B*b^4)*c^2*d^3 + (23*D*a^3*b
- 33*C*a^2*b^2 + 42*B*a*b^3 - 50*A*b^4)*c*d^4 + 3*(D*a^4 - 3*C*a^3*b + 6*B
*a^2*b^2 - 10*A*a*b^3)*d^5)*x^3 + (2*D*b^4*c^5 + (23*D*a*b^3 - 11*C*b^4)*c
^4*d + (155*D*a^2*b^2 - 89*C*a*b^3 + 44*B*b^4)*c^3*d^2 + (135*D*a^3*b - 17
3*C*a^2*b^2 + 158*B*a*b^3 - 110*A*b^4)*c^2*d^3 + (45*D*a^4 - 81*C*a^3*b +
146*B*a^2*b^2 - 230*A*a*b^3)*c*d^4 - 2*(3*C*a^4 - 6*B*a^3*b + 10*A*a^2*b^2
)*d^5)*x^2 + (4*D*a*b^3*c^5 + 2*(32*D*a^2*b^2 - 17*C*a*b^3 + 3*B*b^4)*c^4*
d + (118*D*a^3*b - 100*C*a^2*b^2 + 79*B*a*b^3 - 15*A*b^4)*c^3*d^2 + (54...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(470) = 940$.

Time = 0.13 (sec) , antiderivative size = 1509, normalized size of antiderivative = 3.18

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)^4}\, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

output
```
-(3*D*a*b^3*c^2 - C*b^4*c^2 + 6*D*a^2*b^2*c*d - 6*C*a*b^3*c*d + 4*B*b^4*c*
d + D*a^3*b*d^2 - 3*C*a^2*b^2*d^2 + 6*B*a*b^3*d^2 - 10*A*b^4*d^2)*log(abs(
b*x + a))/(b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d
^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6) + (3*D*a*b^2*c^2*d
- C*b^3*c^2*d + 6*D*a^2*b*c*d^2 - 6*C*a*b^2*c*d^2 + 4*B*b^3*c*d^2 + D*a^3*
d^3 - 3*C*a^2*b*d^3 + 6*B*a*b^2*d^3 - 10*A*b^3*d^3)*log(abs(d*x + c))/(b^6
*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^
4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7) - 1/6*(2*D*a^2*b^3*c^6 + 36*D*a^3
*b^2*c^5*d - 20*C*a^2*b^3*c^5*d + 3*B*a*b^4*c^5*d + 3*A*b^5*c^5*d - 18*D*a
^4*b*c^4*d^2 - 18*C*a^3*b^2*c^4*d^2 + 44*B*a^2*b^3*c^4*d^2 - 30*A*a*b^4*c^
4*d^2 - 20*D*a^5*c^3*d^3 + 36*C*a^4*b*c^3*d^3 - 36*B*a^3*b^2*c^3*d^3 - 20*
A*a^2*b^3*c^3*d^3 + 2*C*a^5*c^2*d^4 - 12*B*a^4*b*c^2*d^4 + 60*A*a^3*b^2*c^
2*d^4 + B*a^5*c*d^5 - 15*A*a^4*b*c*d^5 + 2*A*a^5*d^6 + 6*(3*D*a*b^4*c^3*d^
3 - C*b^5*c^3*d^3 + 3*D*a^2*b^3*c^2*d^4 - 5*C*a*b^4*c^2*d^4 + 4*B*b^5*c^2*
d^4 - 5*D*a^3*b^2*c*d^5 + 3*C*a^2*b^3*c*d^5 + 2*B*a*b^4*c*d^5 - 10*A*b^5*c
*d^5 - D*a^4*b*d^6 + 3*C*a^3*b^2*d^6 - 6*B*a^2*b^3*d^6 + 10*A*a*b^4*d^6)*x
^4 + 3*(15*D*a*b^4*c^4*d^2 - 5*C*b^5*c^4*d^2 + 24*D*a^2*b^3*c^3*d^3 - 28*C
*a*b^4*c^3*d^3 + 20*B*b^5*c^3*d^3 - 16*D*a^3*b^2*c^2*d^4 + 22*B*a*b^4*c^2*
d^4 - 50*A*b^5*c^2*d^4 - 20*D*a^4*b*c*d^5 + 24*C*a^3*b^2*c*d^5 - 24*B*a^2*
b^3*c*d^5 + 20*A*a*b^4*c*d^5 - 3*D*a^5*d^6 + 9*C*a^4*b*d^6 - 18*B*a^3*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^4}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^4),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^4), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 3217, normalized size of antiderivative = 6.79

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^4,x)
```

output
```
( - 12*log(a + b*x)*a**5*c**3*d**3 - 36*log(a + b*x)*a**5*c**2*d**4*x - 36
*log(a + b*x)*a**5*c*d**5*x**2 - 12*log(a + b*x)*a**5*d**6*x**3 - 66*log(a
 + b*x)*a**4*b*c**4*d**2 - 222*log(a + b*x)*a**4*b*c**3*d**3*x - 270*log(a
 + b*x)*a**4*b*c**2*d**4*x**2 - 138*log(a + b*x)*a**4*b*c*d**5*x**3 - 24*l
og(a + b*x)*a**4*b*d**6*x**4 + 48*log(a + b*x)*a**3*b**3*c**3*d**2 + 144*l
og(a + b*x)*a**3*b**3*c**2*d**3*x + 144*log(a + b*x)*a**3*b**3*c*d**4*x**2
 + 48*log(a + b*x)*a**3*b**3*d**5*x**3 - 84*log(a + b*x)*a**3*b**2*c**5*d
- 384*log(a + b*x)*a**3*b**2*c**4*d**2*x - 660*log(a + b*x)*a**3*b**2*c**3
*d**3*x**2 - 516*log(a + b*x)*a**3*b**2*c**2*d**4*x**3 - 168*log(a + b*x)*
a**3*b**2*c*d**5*x**4 - 12*log(a + b*x)*a**3*b**2*d**6*x**5 + 72*log(a + b
*x)*a**2*b**4*c**4*d + 312*log(a + b*x)*a**2*b**4*c**3*d**2*x + 504*log(a
+ b*x)*a**2*b**4*c**2*d**3*x**2 + 360*log(a + b*x)*a**2*b**4*c*d**4*x**3 +
 96*log(a + b*x)*a**2*b**4*d**5*x**4 - 18*log(a + b*x)*a**2*b**3*c**6 - 22
2*log(a + b*x)*a**2*b**3*c**5*d*x - 624*log(a + b*x)*a**2*b**3*c**4*d**2*x
**2 - 720*log(a + b*x)*a**2*b**3*c**3*d**3*x**3 - 366*log(a + b*x)*a**2*b*
*3*c**2*d**4*x**4 - 66*log(a + b*x)*a**2*b**3*c*d**5*x**5 + 144*log(a + b*
x)*a*b**5*c**4*d*x + 480*log(a + b*x)*a*b**5*c**3*d**2*x**2 + 576*log(a +
b*x)*a*b**5*c**2*d**3*x**3 + 288*log(a + b*x)*a*b**5*c*d**4*x**4 + 48*log(
a + b*x)*a*b**5*d**5*x**5 - 36*log(a + b*x)*a*b**4*c**6*x - 192*log(a + b*
x)*a*b**4*c**5*d*x**2 - 360*log(a + b*x)*a*b**4*c**4*d**2*x**3 - 288*lo...
```

3.55 $\int (a+bx)^3\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 438

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= -\frac{2(bc-ad)^3\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{3/2}}{3d^7}$$
$$-\frac{2(bc-ad)^2\left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right)(c+dx)^{5/2}}{5d^7}$$
$$-\frac{2(bc-ad)\left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)}{7d^7}$$
$$+\frac{2(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D)}{9d^7}$$
$$+\frac{2b\left(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D)\right)(c+dx)^{11/2}}{11d^7}$$
$$+\frac{2b^2(bCd-6bcD+3adD)(c+dx)^{13/2}}{13d^7}+\frac{2b^3D(c+dx)^{15/2}}{15d^7}$$

output
```
-2/3*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(3/2)/d^7-2/5*(-a*
d+b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*
c^3))*(d*x+c)^(5/2)/d^7-2/7*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^
2+8*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^(
7/2)/d^7+2/9*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d-
10*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(9/2)/d^7+2/1
1*b*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*x+c
)^(11/2)/d^7+2/13*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(13/2)/d^7+2/15*b^3*
D*(d*x+c)^(15/2)/d^7
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.13

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2(c+dx)^{3/2}\left(143a^3d^3(-16c^3D+24c^2d(C+Dx)-6cd^2(7B+x(6C+5Dx))+d^3(105A+x(63B+5x\right.}{}$$

input
```
Integrate[(a + b*x)^3*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(3/2)*(143*a^3*d^3*(-16*c^3*D + 24*c^2*d*(C + D*x) - 6*c*d^2*
(7*B + x*(6*C + 5*D*x)) + d^3*(105*A + x*(63*B + 5*x*(9*C + 7*D*x)))) + b^
3*(1024*c^6*D - 256*c^5*d*(5*C + 6*D*x) + 7*d^6*x^3*(715*A + 585*B*x + 495
*C*x^2 + 429*D*x^3) + 128*c^4*d^2*(13*B + 15*x*(C + D*x)) + 8*c^2*d^4*x*(4
29*A + 5*x*(78*B + 70*C*x + 63*D*x^2)) - 16*c^3*d^3*(143*A + 2*x*(78*B + 7
5*C*x + 70*D*x^2)) - 2*c*d^5*x^2*(2145*A + 7*x*(260*B + 9*x*(25*C + 22*D*x
)))) + 39*a^2*b*d^2*(128*c^4*D - 16*c^3*d*(11*C + 12*D*x) + 24*c^2*d^2*(11
*B + x*(11*C + 10*D*x)) + d^4*x*(693*A + 5*x*(99*B + 7*x*(11*C + 9*D*x)))
- 2*c*d^3*(231*A + x*(198*B + 5*x*(33*C + 28*D*x)))) + 3*a*b^2*d*(-1280*c^
5*D + 128*c^4*d*(13*C + 15*D*x) - 16*c^3*d^2*(143*B + 6*x*(26*C + 25*D*x))
 + 5*d^5*x^2*(1287*A + 7*x*(143*B + 117*C*x + 99*D*x^2)) + 8*c^2*d^3*(429*
A + x*(429*B + 390*C*x + 350*D*x^2)) - 2*c*d^4*x*(2574*A + 5*x*(429*B + 7*
x*(52*C + 45*D*x))))))/(45045*d^7)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{5/2}(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(c+dx)^{7/2}(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2\right.\right.}{7d^7}$$
$$\frac{2b(c+dx)^{11/2}\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{11d^7}+$$
$$\frac{2(c+dx)^{9/2}\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2\right.\right.}{9d^7}$$
$$\frac{2(c+dx)^{5/2}(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{5d^7}-$$
$$\frac{2(c+dx)^{3/2}(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^7}+$$
$$\frac{2b^2(c+dx)^{13/2}(3adD-6bcD+bCd)}{13d^7}+\frac{2b^3D(c+dx)^{15/2}}{15d^7}$$

input
```
Int[(a + b*x)^3*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-2*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(3/2))/(3*
d^7) - (2*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d -
4*B*c*d^2 + 3*A*d^3 - 6*c^3*D))*(c + d*x)^(5/2))/(5*d^7) - (2*(b*c - a*d)*
(a^2*d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^
2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(7/2))/(7*d^7) + (2*(a^
3*d^3*D + 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*
D) + b^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(9/2))/(9*
d^7) + (2*b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^2 -
15*c^2*D))*(c + d*x)^(11/2))/(11*d^7) + (2*b^2*(b*C*d - 6*b*c*D + 3*a*d*D)
*(c + d*x)^(13/2))/(13*d^7) + (2*b^3*D*(c + d*x)^(15/2))/(15*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{2(xd+c)^{\frac{3}{2}}\left(\left(\frac{\left(\frac{3}{5}Dx^3+\frac{9}{13}Cx^2+\frac{9}{11}Bx+A\right)x^3b^3}{3}+\frac{9\left(\frac{7}{13}Dx^3+\frac{7}{11}Cx^2+\frac{7}{9}Bx+A\right)x^2ab^2}{7}+\frac{9x\left(\frac{5}{11}Dx^3+\frac{5}{9}Cx^2+\frac{5}{7}Bx+A\right)a^2b}{5}+\right.\right.}{}$
derivativedivides	$\frac{2b^3D(xd+c)^{\frac{15}{2}}}{15}+\frac{2\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(Bd^2-2Ccd+3Dc^2\right)\right)(}{11}$
default	$\frac{2b^3D(xd+c)^{\frac{15}{2}}}{15}+\frac{2\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(Bd^2-2Ccd+3Dc^2\right)\right)(}{11}$
gosper	$\frac{2(xd+c)^{\frac{3}{2}}(3003Dx^6b^3d^6+3465Cx^5b^3d^6+10395Dx^5ab^2d^6-2772Dx^5b^3cd^5+4095Bx^4b^3d^6+12285Cx^4ab^2d^6-3150}{}$
orering	$\frac{2(xd+c)^{\frac{3}{2}}(3003Dx^6b^3d^6+3465Cx^5b^3d^6+10395Dx^5ab^2d^6-2772Dx^5b^3cd^5+4095Bx^4b^3d^6+12285Cx^4ab^2d^6-3150}{}$
trager	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x+c)^(3/2)*((1/3*(3/5*D*x^3+9/13*C*x^2+9/11*B*x+A)*x^3*b^3+9/7*(7/1
3*D*x^3+7/11*C*x^2+7/9*B*x+A)*x^2*a*b^2+9/5*x*(5/11*D*x^3+5/9*C*x^2+5/7*B*
x+A)*a^2*b+a^3*(1/3*D*x^3+3/7*C*x^2+3/5*B*x+A))*d^6-6/5*(5/21*(42/65*D*x^3
+105/143*C*x^2+28/33*B*x+A)*x^2*b^3+6/7*(175/286*D*x^3+70/99*C*x^2+5/6*B*x
+A)*x*a*b^2+a^2*(20/33*D*x^3+5/7*C*x^2+6/7*B*x+A)*b+1/3*(5/7*D*x^2+6/7*C*x
+B)*a^3)*c*d^5+24/35*(1/3*x*(105/143*D*x^3+350/429*C*x^2+10/11*B*x+A)*b^3+
a*(350/429*D*x^3+10/11*C*x^2+B*x+A)*b^2+a^2*(10/11*D*x^2+C*x+B)*b+1/3*a^3*
(D*x+C))*c^2*d^4-16/105*((140/143*D*x^3+150/143*C*x^2+12/11*B*x+A)*b^3+3*(
150/143*D*x^2+12/11*C*x+B)*a*b^2+3*(12/11*D*x+C)*a^2*b+a^3*D)*c^3*d^3+128/
1155*c^4*b*((15/13*D*x^2+15/13*C*x+B)*b^2+3*(15/13*D*x+C)*a*b+3*D*a^2)*d^2
-256/3003*((6/5*D*x+C)*b+3*D*a)*c^5*b^2*d+1024/15015*D*b^3*c^6)/d^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.79

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
2/45045*(3003*D*b^3*d^7*x^7 + 1024*D*b^3*c^7 + 15015*A*a^3*c*d^6 - 1280*(3
*D*a*b^2 + C*b^3)*c^6*d + 1664*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^5*d^2 - 2
288*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^4*d^3 + 3432*(C*a^3 + 3*B*a^
2*b + 3*A*a*b^2)*c^3*d^4 - 6006*(B*a^3 + 3*A*a^2*b)*c^2*d^5 + 231*(D*b^3*c
*d^6 + 15*(3*D*a*b^2 + C*b^3)*d^7)*x^6 - 63*(4*D*b^3*c^2*d^5 - 5*(3*D*a*b^
2 + C*b^3)*c*d^6 - 65*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^7)*x^5 + 35*(8*D*b
^3*c^3*d^4 - 10*(3*D*a*b^2 + C*b^3)*c^2*d^5 + 13*(3*D*a^2*b + 3*C*a*b^2 +
B*b^3)*c*d^6 + 143*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^7)*x^4 - 5*(6
4*D*b^3*c^4*d^3 - 80*(3*D*a*b^2 + C*b^3)*c^3*d^4 + 104*(3*D*a^2*b + 3*C*a*
b^2 + B*b^3)*c^2*d^5 - 143*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^6 -
 1287*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^7)*x^3 + 3*(128*D*b^3*c^5*d^2 - 16
0*(3*D*a*b^2 + C*b^3)*c^4*d^3 + 208*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^
4 - 286*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^5 + 429*(C*a^3 + 3*B
*a^2*b + 3*A*a*b^2)*c*d^6 + 3003*(B*a^3 + 3*A*a^2*b)*d^7)*x^2 - (512*D*b^3
*c^6*d - 15015*A*a^3*d^7 - 640*(3*D*a*b^2 + C*b^3)*c^5*d^2 + 832*(3*D*a^2*
b + 3*C*a*b^2 + B*b^3)*c^4*d^3 - 1144*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b
^3)*c^3*d^4 + 1716*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^5 - 3003*(B*a^3 +
 3*A*a^2*b)*c*d^6)*x)*sqrt(d*x + c)/d^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(456) = 912$.

Time = 2.19 (sec) , antiderivative size = 1028, normalized size of antiderivative = 2.35

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b**3*(c + d*x)**(15/2)/(15*d**6) + (c + d*x)**(13/2)*(C*b*
*3*d + 3*D*a*b**2*d - 6*D*b**3*c)/(13*d**6) + (c + d*x)**(11/2)*(B*b**3*d*
*2 + 3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d +
15*D*b**3*c**2)/(11*d**6) + (c + d*x)**(9/2)*(A*b**3*d**3 + 3*B*a*b**2*d**
3 - 4*B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c**
2*d + D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c*
*3)/(9*d**6) + (c + d*x)**(7/2)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*B*a
**2*b*d**4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C*a*
*2*b*c*d**3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**3 +
 18*D*a**2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/(7*d**6) + (
c + d*x)**(5/2)*(3*A*a**2*b*d**5 - 6*A*a*b**2*c*d**4 + 3*A*b**3*c**2*d**3
+ B*a**3*d**5 - 6*B*a**2*b*c*d**4 + 9*B*a*b**2*c**2*d**3 - 4*B*b**3*c**3*d
**2 - 2*C*a**3*c*d**4 + 9*C*a**2*b*c**2*d**3 - 12*C*a*b**2*c**3*d**2 + 5*C
*b**3*c**4*d + 3*D*a**3*c**2*d**3 - 12*D*a**2*b*c**3*d**2 + 15*D*a*b**2*c*
*4*d - 6*D*b**3*c**5)/(5*d**6) + (c + d*x)**(3/2)*(A*a**3*d**6 - 3*A*a**2*
b*c*d**5 + 3*A*a*b**2*c**2*d**4 - A*b**3*c**3*d**3 - B*a**3*c*d**5 + 3*B*a
**2*b*c**2*d**4 - 3*B*a*b**2*c**3*d**3 + B*b**3*c**4*d**2 + C*a**3*c**2*d*
*4 - 3*C*a**2*b*c**3*d**3 + 3*C*a*b**2*c**4*d**2 - C*b**3*c**5*d - D*a**3*
c**3*d**3 + 3*D*a**2*b*c**4*d**2 - 3*D*a*b**2*c**5*d + D*b**3*c**6)/(3*d**
6))/d, Ne(d, 0)), (sqrt(c)*(A*a**3*x + D*b**3*x**7/7 + x**6*(C*b**3 + 3...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.42

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{2\left(3003\,(dx+c)^{\frac{15}{2}}Db^3 - 3465\,(6\,Db^3c-(3\,Dab^2+Cb^3)d)(dx+c)^{\frac{13}{2}} + 4095\,(15\,Db^3c^2-5\,(3\,Dab^2+C\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
2/45045*(3003*(d*x + c)^(15/2)*D*b^3 - 3465*(6*D*b^3*c - (3*D*a*b^2 + C*b^
3)*d)*(d*x + c)^(13/2) + 4095*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^3)*c*d +
(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(11/2) - 5005*(20*D*b^3*c^3
 - 10*(3*D*a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^2
- (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(9/2) + 6435*(15*
D*b^3*c^4 - 10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^
3)*c^2*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (C*a^3 + 3*
B*a^2*b + 3*A*a*b^2)*d^4)*(d*x + c)^(7/2) - 9009*(6*D*b^3*c^5 - 5*(3*D*a*b
^2 + C*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2 - 3*(D*a^3 +
 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2
)*c*d^4 - (B*a^3 + 3*A*a^2*b)*d^5)*(d*x + c)^(5/2) + 15015*(D*b^3*c^6 + A*
a^3*d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*
d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3*B*a^2*b
 + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)*(d*x + c)^(3/2))/d^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. $2(412) = 824$.

Time = 0.14 (sec) , antiderivative size = 1913, normalized size of antiderivative = 4.37

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(d*x + c)*A*a^3*c + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x
 + c)*c)*A*a^3 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*a^3*c/d + 4
5045*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*a^2*b*c/d + 3003*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a^3*c/d^2 + 9009*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*a^2*b*c
/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*A*a*b^2*c/d^2 + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sq
rt(d*x + c)*c^2)*B*a^3/d + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c
+ 15*sqrt(d*x + c)*c^2)*A*a^2*b/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)
^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*D*a^3*c/d^3 + 38
61*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35
*sqrt(d*x + c)*c^3)*C*a^2*b*c/d^3 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)
^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*a*b^2*c/d^3 +
1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
35*sqrt(d*x + c)*c^3)*A*b^3*c/d^3 + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)
^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*a^3/d^2 + 3861
*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*s
qrt(d*x + c)*c^3)*B*a^2*b/d^2 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/
2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A*a*b^2/d^2 + 429*(3
5*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 4...
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 770, normalized size of antiderivative = 1.76

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((a + b*x)^3*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(70*a^3*(c + d*x)^(9/2)*D - 270*a^3*c*(c + d*x)^(7/2)*D - 210*a^3*c^3*(c +
 d*x)^(3/2)*D + 378*a^3*c^2*(c + d*x)^(5/2)*D)/(315*d^4) + ((6*a*b^2*(c +
d*x)^(13/2)*D)/13 - 2*a*b^2*c^5*(c + d*x)^(3/2)*D + 6*a*b^2*c^4*(c + d*x)^
(5/2)*D - (60*a*b^2*c^3*(c + d*x)^(7/2)*D)/7 + (20*a*b^2*c^2*(c + d*x)^(9/
2)*D)/3 - (30*a*b^2*c*(c + d*x)^(11/2)*D)/11)/d^6 + ((2*b^3*(c + d*x)^(15/
2)*D)/15 - (12*b^3*c*(c + d*x)^(13/2)*D)/13 + (2*b^3*c^6*(c + d*x)^(3/2)*D
)/3 - (12*b^3*c^5*(c + d*x)^(5/2)*D)/5 + (30*b^3*c^4*(c + d*x)^(7/2)*D)/7
- (40*b^3*c^3*(c + d*x)^(9/2)*D)/9 + (30*b^3*c^2*(c + d*x)^(11/2)*D)/11)/d
^7 + ((6*a^2*b*(c + d*x)^(11/2)*D)/11 + 2*a^2*b*c^4*(c + d*x)^(3/2)*D - (2
4*a^2*b*c^3*(c + d*x)^(5/2)*D)/5 + (36*a^2*b*c^2*(c + d*x)^(7/2)*D)/7 - (8
*a^2*b*c*(c + d*x)^(9/2)*D)/3)/d^5 + (2*C*(c + d*x)^(7/2)*(a^3*d^3 - 10*b^
3*c^3 + 18*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(7*d^6) + (2*A*b^3*(c + d*x)^(9/2
))/(9*d^4) + (2*B*b^3*(c + d*x)^(11/2))/(11*d^5) + (2*C*b^3*(c + d*x)^(13/
2))/(13*d^6) + (2*A*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^4) + (6*A*b*(a*d -
 b*c)^2*(c + d*x)^(5/2))/(5*d^4) + (6*A*b^2*(a*d - b*c)*(c + d*x)^(7/2))/(
7*d^4) + (2*B*b^2*(3*a*d - 4*b*c)*(c + d*x)^(9/2))/(9*d^5) - (2*B*c*(a*d -
 b*c)^3*(c + d*x)^(3/2))/(3*d^5) + (2*C*b^2*(3*a*d - 5*b*c)*(c + d*x)^(11/
2))/(11*d^6) + (6*B*b*(c + d*x)^(7/2)*(a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d))/(
7*d^5) + (2*C*b*(c + d*x)^(9/2)*(3*a^2*d^2 + 10*b^2*c^2 - 12*a*b*c*d))/(9*
d^6) + (2*B*(a*d - b*c)^2*(a*d - 4*b*c)*(c + d*x)^(5/2))/(5*d^5) + (2*C...
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.35

$$\int (a+bx)^3\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2\sqrt{dx+c}\,(3003b^3d^7x^7+10395a\,b^2d^7x^6+3696b^3c\,d^6x^6+12285a^2b\,d^7x^5+13230a\,b^2c\,d^6x^5+4095b^4d^6x^5}{}$$

input
```
int((b*x+a)^3*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output

```
(2*sqrt(c + d*x)*(15015*a**4*c*d**5 + 15015*a**4*d**6*x - 24024*a**3*b*c**
2*d**4 + 12012*a**3*b*c*d**5*x + 36036*a**3*b*d**6*x**2 + 1144*a**3*c**4*d
**3 - 572*a**3*c**3*d**4*x + 429*a**3*c**2*d**5*x**2 + 7150*a**3*c*d**6*x*
*3 + 5005*a**3*d**7*x**4 + 20592*a**2*b**2*c**3*d**3 - 10296*a**2*b**2*c**
2*d**4*x + 7722*a**2*b**2*c*d**5*x**2 + 38610*a**2*b**2*d**6*x**3 - 1872*a
**2*b*c**5*d**2 + 936*a**2*b*c**4*d**3*x - 702*a**2*b*c**3*d**4*x**2 + 585
*a**2*b*c**2*d**5*x**3 + 16380*a**2*b*c*d**6*x**4 + 12285*a**2*b*d**7*x**5
 - 9152*a*b**3*c**4*d**2 + 4576*a*b**3*c**3*d**3*x - 3432*a*b**3*c**2*d**4
*x**2 + 2860*a*b**3*c*d**5*x**3 + 20020*a*b**3*d**6*x**4 + 1152*a*b**2*c**
6*d - 576*a*b**2*c**5*d**2*x + 432*a*b**2*c**4*d**3*x**2 - 360*a*b**2*c**3
*d**4*x**3 + 315*a*b**2*c**2*d**5*x**4 + 13230*a*b**2*c*d**6*x**5 + 10395*
a*b**2*d**7*x**6 + 1664*b**4*c**5*d - 832*b**4*c**4*d**2*x + 624*b**4*c**3
*d**3*x**2 - 520*b**4*c**2*d**4*x**3 + 455*b**4*c*d**5*x**4 + 4095*b**4*d*
*6*x**5 - 256*b**3*c**7 + 128*b**3*c**6*d*x - 96*b**3*c**5*d**2*x**2 + 80*
b**3*c**4*d**3*x**3 - 70*b**3*c**3*d**4*x**4 + 63*b**3*c**2*d**5*x**5 + 36
96*b**3*c*d**6*x**6 + 3003*b**3*d**7*x**7))/(45045*d**6)
```

3.56 $\int (a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result 522
Mathematica [A] (verified) 523
Rubi [A] (verified) 523
Maple [A] (verified) 525
Fricas [A] (verification not implemented) 525
Sympy [A] (verification not implemented) 526
Maxima [A] (verification not implemented) 527
Giac [B] (verification not implemented) 528
Mupad [B] (verification not implemented) 529
Reduce [B] (verification not implemented) 530

Optimal result

Integrand size = 32, antiderivative size = 326

$$\begin{aligned}\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx\\
&=\frac{2(bc-ad)^2\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{3/2}}{3d^6}\\
&+\frac{2(bc-ad)\left(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D)\right)(c+dx)^{5/2}}{5d^6}\\
&+\frac{2(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D))(c+dx)^{7/2}}{7d^6}\\
&+\frac{2(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))(c+dx)^{9/2}}{9d^6}\\
&+\frac{2b(bCd-5bcD+2adD)(c+dx)^{11/2}}{11d^6}+\frac{2b^2D(c+dx)^{13/2}}{13d^6}\end{aligned}$$

output
```
2/3*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(3/2)/d^6+2/5*(-a*d
+b*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3
))*(d*x+c)^(5/2)/d^6+2/7*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*
c^2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(7/2)/d^6+2/9*(a^2*
d^2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(9/2)/d^6
+2/11*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(11/2)/d^6+2/13*b^2*D*(d*x+c)^(13/
2)/d^6
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.99

$$\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2(c+dx)^{3/2}\left(143a^2d^2(-16c^3D+24c^2d(C+Dx)-6cd^2(7B+x(6C+5Dx))+d^3(105A+x(63B+5x\right.}{}$$

input
```
Integrate[(a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(3/2)*(143*a^2*d^2*(-16*c^3*D + 24*c^2*d*(C + D*x) - 6*c*d^2*
(7*B + x*(6*C + 5*D*x)) + d^3*(105*A + x*(63*B + 5*x*(9*C + 7*D*x)))) + 26
*a*b*d*(128*c^4*D - 16*c^3*d*(11*C + 12*D*x) + 24*c^2*d^2*(11*B + x*(11*C
+ 10*D*x)) + d^4*x*(693*A + 5*x*(99*B + 7*x*(11*C + 9*D*x))) - 2*c*d^3*(23
1*A + x*(198*B + 5*x*(33*C + 28*D*x)))) + b^2*(-1280*c^5*D + 128*c^4*d*(13
*C + 15*D*x) - 16*c^3*d^2*(143*B + 6*x*(26*C + 25*D*x)) + 5*d^5*x^2*(1287*
A + 7*x*(143*B + 117*C*x + 99*D*x^2)) + 8*c^2*d^3*(429*A + x*(429*B + 390*
C*x + 350*D*x^2)) - 2*c*d^4*x*(2574*A + 5*x*(429*B + 7*x*(52*C + 45*D*x)))
)))/(45045*d^6)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{(c+dx)^{5/2}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c}{}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{2(c+dx)^{7/2}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{7d^6}+\frac{2(c+dx)^{9/2}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{9d^6}+\frac{2(c+dx)^{5/2}(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{5d^6}+\frac{2(c+dx)^{3/2}(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^6}+\frac{2b(c+dx)^{11/2}(2adD-5bcD+bCd)}{11d^6}+\frac{2b^2D(c+dx)^{13/2}}{13d^6}$$

input
```
Int[(a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(3/2))/(3*d
^6) + (2*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B
*c*d^2 + 2*A*d^3 - 5*c^3*D))*(c + d*x)^(5/2))/(5*d^6) + (2*(a^2*d^2*(C*d -
 3*c*D) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2
 + A*d^3 - 10*c^3*D))*(c + d*x)^(7/2))/(7*d^6) + (2*(a^2*d^2*D + 2*a*b*d*(
C*d - 4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(9/2))/(9*d^6)
+ (2*b*(b*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(11/2))/(11*d^6) + (2*b^2*D*(
c + d*x)^(13/2))/(13*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{2(xd+c)^{\frac{3}{2}}\left(\left(\left(\frac{3\left(\frac{7}{13}Dx^{3}+\frac{7}{11}Cx^{2}+\frac{7}{9}Bx+A\right)x^{2}b^{2}}{7}+\frac{6x\left(\frac{5}{11}Dx^{3}+\frac{5}{9}Cx^{2}+\frac{5}{7}Bx+A\right)ab}{5}+a^{2}\left(\frac{1}{3}Dx^{3}+\frac{3}{7}Cx^{2}+\frac{3}{5}Bx+A\right)\right)d^{5}-\cdots\right.\right.}{\cdots}$
derivativedivides	$\frac{2b^{2}D(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left(2b(ad-bc)D+b^{2}(Cd-3Dc)\right)(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left((ad-bc)^{2}D+2b(ad-bc)(Cd-3Dc)+b^{2}\left(Bd^{2}-2Ccd+3Dc^{2}\right)\right)(xd+\cdots}{9}$
default	$\frac{2b^{2}D(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left(2b(ad-bc)D+b^{2}(Cd-3Dc)\right)(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left((ad-bc)^{2}D+2b(ad-bc)(Cd-3Dc)+b^{2}\left(Bd^{2}-2Ccd+3Dc^{2}\right)\right)(xd+\cdots}{9}$
gosper	$2(xd+c)^{\frac{3}{2}}\left(3465Dx^{5}b^{2}d^{5}+4095Cx^{4}b^{2}d^{5}+8190Dx^{4}abd^{5}-3150Dx^{4}b^{2}cd^{4}+5005Bx^{3}b^{2}d^{5}+10010Cx^{3}abd^{5}-3640Cx\cdots\right.$
orering	$2(xd+c)^{\frac{3}{2}}\left(3465Dx^{5}b^{2}d^{5}+4095Cx^{4}b^{2}d^{5}+8190Dx^{4}abd^{5}-3150Dx^{4}b^{2}cd^{4}+5005Bx^{3}b^{2}d^{5}+10010Cx^{3}abd^{5}-3640Cx\cdots\right.$
trager	$2\left(3465b^{2}Dd^{6}x^{6}+4095Cb^{2}d^{6}x^{5}+8190Dabd^{6}x^{5}+315Db^{2}cd^{5}x^{5}+5005Bb^{2}d^{6}x^{4}+10010Cabd^{6}x^{4}+455Cb^{2}cd^{5}x^{4}+50\cdots\right.$

input
```
int((b*x+a)^2*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x+c)^(3/2)*((3/7*(7/13*D*x^3+7/11*C*x^2+7/9*B*x+A)*x^2*b^2+6/5*x*(5
/11*D*x^3+5/9*C*x^2+5/7*B*x+A)*a*b+a^2*(1/3*D*x^3+3/7*C*x^2+3/5*B*x+A))*d^
5-4/5*c*(3/7*(175/286*D*x^3+70/99*C*x^2+5/6*B*x+A)*x*b^2+a*(20/33*D*x^3+5/
7*C*x^2+6/7*B*x+A)*b+1/2*(5/7*D*x^2+6/7*C*x+B)*a^2)*d^4+8/35*((350/429*D*x
^3+10/11*C*x^2+B*x+A)*b^2+2*(10/11*D*x^2+C*x+B)*a*b+a^2*(D*x+C))*c^2*d^3-1
6/105*c^3*((150/143*D*x^2+12/11*C*x+B)*b^2+2*(12/11*D*x+C)*a*b+D*a^2)*d^2+
128/1155*c^4*b*((15/13*D*x+C)*b+2*D*a)*d-256/3003*D*b^2*c^5)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.53

$$\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2\left(3465\,Db^2d^6x^6-1280\,Db^2c^6+15015\,Aa^2cd^5+1664\,(2\,Dab+Cb^2)c^5d-2288\,(Da^2+2\,Cab+Bb^2)c^4\cdots\right.}{\cdots}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
2/45045*(3465*D*b^2*d^6*x^6 - 1280*D*b^2*c^6 + 15015*A*a^2*c*d^5 + 1664*(2
*D*a*b + C*b^2)*c^5*d - 2288*(D*a^2 + 2*C*a*b + B*b^2)*c^4*d^2 + 3432*(C*a
^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 6006*(B*a^2 + 2*A*a*b)*c^2*d^4 + 315*(D*b^
2*c*d^5 + 13*(2*D*a*b + C*b^2)*d^6)*x^5 - 35*(10*D*b^2*c^2*d^4 - 13*(2*D*a
*b + C*b^2)*c*d^5 - 143*(D*a^2 + 2*C*a*b + B*b^2)*d^6)*x^4 + 5*(80*D*b^2*c
^3*d^3 - 104*(2*D*a*b + C*b^2)*c^2*d^4 + 143*(D*a^2 + 2*C*a*b + B*b^2)*c*d
^5 + 1287*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*x^3 - 3*(160*D*b^2*c^4*d^2 - 208*
(2*D*a*b + C*b^2)*c^3*d^3 + 286*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^4 - 429*(C
*a^2 + 2*B*a*b + A*b^2)*c*d^5 - 3003*(B*a^2 + 2*A*a*b)*d^6)*x^2 + (640*D*b
^2*c^5*d + 15015*A*a^2*d^6 - 832*(2*D*a*b + C*b^2)*c^4*d^2 + 1144*(D*a^2 +
 2*C*a*b + B*b^2)*c^3*d^3 - 1716*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 + 3003*
(B*a^2 + 2*A*a*b)*c*d^5)*x)*sqrt(d*x + c)/d^6
```

Sympy [A] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.97

$$\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$=\begin{cases}\frac{2\left(\frac{Db^2(c+dx)^{\frac{13}{2}}}{13d^5}+\frac{(c+dx)^{\frac{11}{2}}\left(Cb^2d+2Dabd-5Db^2c\right)}{11d^5}+\frac{(c+dx)^{\frac{9}{2}}\left(Bb^2d^2+2Cabd^2-4Cb^2cd+Da^2d^2-8Dabcd+10Db^2c^2\right)}{9d^5}+\frac{(c+dx)^{\frac{7}{2}}\left(Ab^2d^3+2Babd^3-3\right.}{}\right.}{}\\ \sqrt{c}\left(Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5\left(Cb^2+2Dab\right)}{5}+\frac{x^4\left(Bb^2+2Cab+Da^2\right)}{4}+\frac{x^3\left(Ab^2+2Bab+Ca^2\right)}{3}+\frac{x^2\cdot\left(2Aab+Ba^2\right)}{2}\right)\end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b**2*(c + d*x)**(13/2)/(13*d**5) + (c + d*x)**(11/2)*(C*b*
*2*d + 2*D*a*b*d - 5*D*b**2*c)/(11*d**5) + (c + d*x)**(9/2)*(B*b**2*d**2 +
 2*C*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2)
/(9*d**5) + (c + d*x)**(7/2)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d**2
 + C*a**2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 12*D
*a*b*c**2*d - 10*D*b**2*c**3)/(7*d**5) + (c + d*x)**(5/2)*(2*A*a*b*d**4 -
2*A*b**2*c*d**3 + B*a**2*d**4 - 4*B*a*b*c*d**3 + 3*B*b**2*c**2*d**2 - 2*C*
a**2*c*d**3 + 6*C*a*b*c**2*d**2 - 4*C*b**2*c**3*d + 3*D*a**2*c**2*d**2 - 8
*D*a*b*c**3*d + 5*D*b**2*c**4)/(5*d**5) + (c + d*x)**(3/2)*(A*a**2*d**5 -
2*A*a*b*c*d**4 + A*b**2*c**2*d**3 - B*a**2*c*d**4 + 2*B*a*b*c**2*d**3 - B*
b**2*c**3*d**2 + C*a**2*c**2*d**3 - 2*C*a*b*c**3*d**2 + C*b**2*c**4*d - D*
a**2*c**3*d**2 + 2*D*a*b*c**4*d - D*b**2*c**5)/(3*d**5))/d, Ne(d, 0)), (sq
rt(c)*(A*a**2*x + D*b**2*x**6/6 + x**5*(C*b**2 + 2*D*a*b)/5 + x**4*(B*b**2
 + 2*C*a*b + D*a**2)/4 + x**3*(A*b**2 + 2*B*a*b + C*a**2)/3 + x**2*(2*A*a*
b + B*a**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.19

$$\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{2\left(3465\,(dx+c)^{\frac{13}{2}}Db^2 - 4095\,(5\,Db^2c - (2\,Dab + Cb^2)d)(dx+c)^{\frac{11}{2}} + 5005\,(10\,Db^2c^2 - 4\,(2\,Dab + Cb^2\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
2/45045*(3465*(d*x + c)^(13/2)*D*b^2 - 4095*(5*D*b^2*c - (2*D*a*b + C*b^2)
*d)*(d*x + c)^(11/2) + 5005*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d + (D*a
^2 + 2*C*a*b + B*b^2)*d^2)*(d*x + c)^(9/2) - 6435*(10*D*b^2*c^3 - 6*(2*D*a
*b + C*b^2)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*b +
 A*b^2)*d^3)*(d*x + c)^(7/2) + 9009*(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^2)*c^3
*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d
^3 + (B*a^2 + 2*A*a*b)*d^4)*(d*x + c)^(5/2) - 15015*(D*b^2*c^5 - A*a^2*d^5
 - (2*D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 +
2*B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)*(d*x + c)^(3/2))/d^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1269 vs. $2(302) = 604$.

Time = 0.13 (sec) , antiderivative size = 1269, normalized size of antiderivative = 3.89

$$\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(d*x + c)*A*a^2*c + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x
 + c)*c)*A*a^2 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*a^2*c/d + 3
0030*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*a*b*c/d + 3003*(3*(d*x + c)^(
5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a^2*c/d^2 + 6006*(3*
(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*a*b*c/d^2
 + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*
A*b^2*c/d^2 + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x
 + c)*c^2)*B*a^2/d + 6006*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*s
qrt(d*x + c)*c^2)*A*a*b/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c
 + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*D*a^2*c/d^3 + 2574*(5*(d
*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)*C*a*b*c/d^3 + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*b^2*c/d^3 + 1287*(5*(d*x
 + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)*C*a^2/d^2 + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*
(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*a*b/d^2 + 1287*(5*(d*x + c)^
(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c
^3)*A*b^2/d^2 + 286*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x
 + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*D*a*b*c
/d^4 + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^...
```

Mupad [B] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.57

$$\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{2\,A\,(c+d\,x)^{3/2}\,\left(15\,b^2\,(c+d\,x)^2+35\,a^2\,d^2+35\,b^2\,c^2-42\,b^2\,c\,(c+d\,x)+42\,a\,b\,d\,(c+d\,x)-70\,a\,b\,c\,d\right)}{105\,d^3}$$
$$+\frac{2\,B\,b^2\,(c+d\,x)^{9/2}}{9\,d^4}+\frac{2\,C\,b^2\,(c+d\,x)^{11/2}}{11\,d^5}$$
$$+\frac{2\,(c+d\,x)^{3/2}\,D\,\left(3465\,b^2\,(c+d\,x)^5-15015\,b^2\,c^5+5005\,a^2\,d^2\,(c+d\,x)^3+50050\,b^2\,c^2\,(c+d\,x)^3-6435\right.}{}$$
$$+\frac{2\,B\,(c+d\,x)^{5/2}\,(a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2)}{5\,d^4}$$
$$+\frac{2\,C\,(c+d\,x)^{7/2}\,(a^2\,d^2-6\,a\,b\,c\,d+6\,b^2\,c^2)}{7\,d^5}-\frac{2\,B\,c\,(a\,d-b\,c)^2\,(c+d\,x)^{3/2}}{3\,d^4}$$
$$-\frac{4\,C\,c\,(c+d\,x)^{5/2}\,(a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2)}{5\,d^5}+\frac{2\,C\,c^2\,(a\,d-b\,c)^2\,(c+d\,x)^{3/2}}{3\,d^5}$$
$$+\frac{2\,B\,b\,(2\,a\,d-3\,b\,c)\,(c+d\,x)^{7/2}}{7\,d^4}+\frac{4\,C\,b\,(a\,d-2\,b\,c)\,(c+d\,x)^{9/2}}{9\,d^5}$$

input
```
int((a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(2*A*(c + d*x)^(3/2)*(15*b^2*(c + d*x)^2 + 35*a^2*d^2 + 35*b^2*c^2 - 42*b^
2*c*(c + d*x) + 42*a*b*d*(c + d*x) - 70*a*b*c*d))/(105*d^3) + (2*B*b^2*(c
+ d*x)^(9/2))/(9*d^4) + (2*C*b^2*(c + d*x)^(11/2))/(11*d^5) + (2*(c + d*x)
^(3/2)*D*(3465*b^2*(c + d*x)^5 - 15015*b^2*c^5 + 5005*a^2*d^2*(c + d*x)^3
+ 50050*b^2*c^2*(c + d*x)^3 - 64350*b^2*c^3*(c + d*x)^2 - 15015*a^2*c^3*d^
2 - 20475*b^2*c*(c + d*x)^4 + 45045*b^2*c^4*(c + d*x) + 30030*a*b*c^4*d -
19305*a^2*c*d^2*(c + d*x)^2 + 27027*a^2*c^2*d^2*(c + d*x) + 8190*a*b*d*(c
+ d*x)^4 - 40040*a*b*c*d*(c + d*x)^3 - 72072*a*b*c^3*d*(c + d*x) + 77220*a
*b*c^2*d*(c + d*x)^2))/(45045*d^6) + (2*B*(c + d*x)^(5/2)*(a^2*d^2 + 3*b^2
*c^2 - 4*a*b*c*d))/(5*d^4) + (2*C*(c + d*x)^(7/2)*(a^2*d^2 + 6*b^2*c^2 - 6
*a*b*c*d))/(7*d^5) - (2*B*c*(a*d - b*c)^2*(c + d*x)^(3/2))/(3*d^4) - (4*C*
c*(c + d*x)^(5/2)*(a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d))/(5*d^5) + (2*C*c^2*(a
*d - b*c)^2*(c + d*x)^(3/2))/(3*d^5) + (2*B*b*(2*a*d - 3*b*c)*(c + d*x)^(7
/2))/(7*d^4) + (4*C*b*(a*d - 2*b*c)*(c + d*x)^(9/2))/(9*d^5)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.17

$$\int (a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2\sqrt{dx+c}\,(3465b^2d^6x^6+8190ab\,d^6x^5+4410b^2c\,d^5x^5+5005a^2d^6x^4+10920abc\,d^5x^4+5005b^3d^5x^4+10\cdots}{}$$

input
```
int((b*x+a)^2*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(15015*a**3*c*d**4 + 15015*a**3*d**5*x - 18018*a**2*b*c**
2*d**3 + 9009*a**2*b*c*d**4*x + 27027*a**2*b*d**5*x**2 + 1144*a**2*c**4*d*
*2 - 572*a**2*c**3*d**3*x + 429*a**2*c**2*d**4*x**2 + 7150*a**2*c*d**5*x**
3 + 5005*a**2*d**6*x**4 + 10296*a*b**2*c**3*d**2 - 5148*a*b**2*c**2*d**3*x
 + 3861*a*b**2*c*d**4*x**2 + 19305*a*b**2*d**5*x**3 - 1248*a*b*c**5*d + 62
4*a*b*c**4*d**2*x - 468*a*b*c**3*d**3*x**2 + 390*a*b*c**2*d**4*x**3 + 1092
0*a*b*c*d**5*x**4 + 8190*a*b*d**6*x**5 - 2288*b**3*c**4*d + 1144*b**3*c**3
*d**2*x - 858*b**3*c**2*d**3*x**2 + 715*b**3*c*d**4*x**3 + 5005*b**3*d**5*
x**4 + 384*b**2*c**6 - 192*b**2*c**5*d*x + 144*b**2*c**4*d**2*x**2 - 120*b
**2*c**3*d**3*x**3 + 105*b**2*c**2*d**4*x**4 + 4410*b**2*c*d**5*x**5 + 346
5*b**2*d**6*x**6))/(45045*d**5)
```

3.57 $\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

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Optimal result

Integrand size = 30, antiderivative size = 214

$$\begin{aligned}
&\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx \\
&= -\frac{2(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{3/2}}{3d^5} \\
&\quad -\frac{2(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))(c+dx)^{5/2}}{5d^5} \\
&\quad +\frac{2(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))(c+dx)^{7/2}}{7d^5} \\
&\quad +\frac{2(bCd-4bcD+adD)(c+dx)^{9/2}}{9d^5}+\frac{2bD(c+dx)^{11/2}}{11d^5}
\end{aligned}$$

output

```
-2/3*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(3/2)/d^5-2/5*(a*d*(
-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(5/
2)/d^5+2/7*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(7/2)/d^5+
2/9*(C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(9/2)/d^5+2/11*b*D*(d*x+c)^(11/2)/d^5
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{2(c+dx)^{3/2}\left(11ad(-16c^3D+24c^2d(C+Dx)-6cd^2(7B+x(6C+5Dx))+d^3(105A+x(63B+5x(9C\right.}{}$$

input
```
Integrate[(a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(3/2)*(11*a*d*(-16*c^3*D + 24*c^2*d*(C + D*x) - 6*c*d^2*(7*B
+ x*(6*C + 5*D*x)) + d^3*(105*A + x*(63*B + 5*x*(9*C + 7*D*x)))) + b*(128*
c^4*D - 16*c^3*d*(11*C + 12*D*x) + 24*c^2*d^2*(11*B + x*(11*C + 10*D*x)) +
 d^4*x*(693*A + 5*x*(99*B + 7*x*(11*C + 9*D*x))) - 2*c*d^3*(231*A + x*(198
*B + 5*x*(33*C + 28*D*x))))))/(3465*d^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{(c+dx)^{3/2}\left(b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)}{d^4}+\frac{\sqrt{c+dx}(ad-bc)\left(Ad^3-\right.}{d^4}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2(c+dx)^{5/2}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{5d^5}-\frac{2(c+dx)^{3/2}(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^5}+\frac{2(c+dx)^{7/2}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{7d^5}+\frac{2(c+dx)^{9/2}(adD-4bcD+bCd)}{9d^5}+\frac{2bD(c+dx)^{11/2}}{11d^5}$$

input
```
Int[(a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-2*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(3/2))/(3*d^
5) - (2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^
3 - 4*c^3*D))*(c + d*x)^(5/2))/(5*d^5) + (2*(a*d*(C*d - 3*c*D) - b*(3*c*C*
d - B*d^2 - 6*c^2*D))*(c + d*x)^(7/2))/(7*d^5) + (2*(b*C*d - 4*b*c*D + a*d
*D)*(c + d*x)^(9/2))/(9*d^5) + (2*b*D*(c + d*x)^(11/2))/(11*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{2\left(\left(\frac{3Db\,x^4}{11}+\frac{(Cb+Da)x^3}{3}+\frac{3(Bb+Ca)x^2}{7}+\frac{3(Ab+Ba)x}{5}+Aa\right)d^4-\frac{2\left(\frac{20Db\,x^3}{33}+\frac{5(Cb+Da)x^2}{7}+\frac{6(Bb+Ca)x}{7}+Ab+Ba\right)c\,d^3}{5}+\cdots}{3d^5}$
derivativedivides	$\frac{\frac{2bD(xd+c)^{\frac{11}{2}}}{11}+\frac{2((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left((ad-bc)(Cd-3Dc)+b\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{7}{2}}}{7}+\frac{2\left((ad-bc)\left(B\,d^2\cdots\right.\right.}{\cdots}}{d^5}$
default	$\frac{\frac{2bD(xd+c)^{\frac{11}{2}}}{11}+\frac{2((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left((ad-bc)(Cd-3Dc)+b\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{7}{2}}}{7}+\frac{2\left((ad-bc)\left(B\,d^2\cdots\right.\right.}{\cdots}}{d^5}$
gosper	$\frac{2(xd+c)^{\frac{3}{2}}\left(315Db\,x^4d^4+385C\,x^3b\,d^4+385Dx^3a\,d^4-280Dx^3bc\,d^3+495B\,x^2b\,d^4+495C\,x^2a\,d^4-330C\,x^2bc\,d^3-330Dx\cdots\right.}{\cdots}$
orering	$\frac{2(xd+c)^{\frac{3}{2}}\left(315Db\,x^4d^4+385C\,x^3b\,d^4+385Dx^3a\,d^4-280Dx^3bc\,d^3+495B\,x^2b\,d^4+495C\,x^2a\,d^4-330C\,x^2bc\,d^3-330Dx\cdots\right.}{\cdots}$
trager	$\frac{2\left(315bDd^5x^5+385Cb\,d^5x^4+385Da\,d^5x^4+35Dbc\,d^4x^4+495Bb\,d^5x^3+495Ca\,d^5x^3+55Cbc\,d^4x^3+55Dac\,d^4x^3-40Db\,c\cdots\right.}{\cdots}$

input
```
int((b*x+a)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/3*((3/11*D*b*x^4+1/3*(C*b+D*a)*x^3+3/7*(B*b+C*a)*x^2+3/5*(A*b+B*a)*x+A*a
)*d^4-2/5*(20/33*D*b*x^3+5/7*(C*b+D*a)*x^2+6/7*(B*b+C*a)*x+A*b+B*a)*c*d^3+
8/35*c^2*(10/11*D*b*x^2+(C*b+D*a)*x+B*b+C*a)*d^2-16/105*(12/11*D*b*x+C*b+D
*a)*c^3*d+128/1155*D*b*c^4)*(d*x+c)^(3/2)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.24

$$\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2\left(315\,Dbd^5x^5+128\,Dbc^5+1155\,Aacd^4-176\,(Da+Cb)c^4d+264\,(Ca+Bb)c^3d^2-462\,(Ba+Ab)c^2d\cdots\right.}{\cdots}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
2/3465*(315*D*b*d^5*x^5 + 128*D*b*c^5 + 1155*A*a*c*d^4 - 176*(D*a + C*b)*c
^4*d + 264*(C*a + B*b)*c^3*d^2 - 462*(B*a + A*b)*c^2*d^3 + 35*(D*b*c*d^4 +
 11*(D*a + C*b)*d^5)*x^4 - 5*(8*D*b*c^2*d^3 - 11*(D*a + C*b)*c*d^4 - 99*(C
*a + B*b)*d^5)*x^3 + 3*(16*D*b*c^3*d^2 - 22*(D*a + C*b)*c^2*d^3 + 33*(C*a
+ B*b)*c*d^4 + 231*(B*a + A*b)*d^5)*x^2 - (64*D*b*c^4*d - 1155*A*a*d^5 - 8
8*(D*a + C*b)*c^3*d^2 + 132*(C*a + B*b)*c^2*d^3 - 231*(B*a + A*b)*c*d^4)*x
)*sqrt(d*x + c)/d^5
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.50

$$\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \begin{cases} \frac{2\left(\frac{Db(c+dx)^{\frac{11}{2}}}{11d^4}+\frac{(c+dx)^{\frac{9}{2}}(Cbd+Dad-4Dbc)}{9d^4}+\frac{(c+dx)^{\frac{7}{2}}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6Dbc^2\right)}{7d^4}+\frac{(c+dx)^{\frac{5}{2}}\left(Abd^3+Bad^3-2Bbcd^2-2Cacd^2+3Cbc^2d+3\cdots\right.}{5d^4}\cdots}{d} \\ \sqrt{c}\left(Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b*(c + d*x)**(11/2)/(11*d**4) + (c + d*x)**(9/2)*(C*b*d +
D*a*d - 4*D*b*c)/(9*d**4) + (c + d*x)**(7/2)*(B*b*d**2 + C*a*d**2 - 3*C*b*
c*d - 3*D*a*c*d + 6*D*b*c**2)/(7*d**4) + (c + d*x)**(5/2)*(A*b*d**3 + B*a*
d**3 - 2*B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D*b*c
**3)/(5*d**4) + (c + d*x)**(3/2)*(A*a*d**4 - A*b*c*d**3 - B*a*c*d**3 + B*b
*c**2*d**2 + C*a*c**2*d**2 - C*b*c**3*d - D*a*c**3*d + D*b*c**4)/(3*d**4))
/d, Ne(d, 0)), (sqrt(c)*(A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/4 + x**3*(B
*b + C*a)/3 + x**2*(A*b + B*a)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{2\left(315\,(dx+c)^{\frac{11}{2}}Db - 385\,(4\,Dbc-(Da+Cb)d)(dx+c)^{\frac{9}{2}} + 495\,(6\,Dbc^2-3\,(Da+Cb)cd+(Ca+Bb\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
2/3465*(315*(d*x + c)^(11/2)*D*b - 385*(4*D*b*c - (D*a + C*b)*d)*(d*x + c)
^(9/2) + 495*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*(d*x + c)^(
7/2) - 693*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 - (B*a +
 A*b)*d^3)*(d*x + c)^(5/2) + 1155*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c^3*d +
 (C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)*(d*x + c)^(3/2))/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(196) = 392.

Time = 0.13 (sec) , antiderivative size = 721, normalized size of antiderivative = 3.37

$$\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(d*x + c)*A*a*c + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)
*c)*A*a + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*a*c/d + 1155*((d*x
+ c)^(3/2) - 3*sqrt(d*x + c)*c)*A*b*c/d + 231*(3*(d*x + c)^(5/2) - 10*(d*x
 + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a*c/d^2 + 231*(3*(d*x + c)^(5/2) -
 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*b*c/d^2 + 231*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*a/d + 231*(3*(d*x
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*b/d + 99*(5*(d
*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)*D*a*c/d^3 + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*
(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*b*c/d^3 + 99*(5*(d*x + c)^(7
/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3
)*C*a/d^2 + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3
/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*b/d^2 + 11*(35*(d*x + c)^(9/2) - 180*(d*
x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*s
qrt(d*x + c)*c^4)*D*b*c/d^4 + 11*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)
*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)
*c^4)*D*a/d^3 + 11*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x
+ c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*C*b/d^3
+ 5*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2
 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x +...
```

Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.63

$$\int (a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{\frac{2\,b\,(c+d\,x)^{11/2}\,D}{11}-\frac{8\,b\,c\,(c+d\,x)^{9/2}\,D}{9}+\frac{2\,b\,c^4\,(c+d\,x)^{3/2}\,D}{3}-\frac{8\,b\,c^3\,(c+d\,x)^{5/2}\,D}{5}+\frac{12\,b\,c^2\,(c+d\,x)^{7/2}\,D}{7}}{d^5}$$
$$+\frac{70\,a\,(c+d\,x)^{9/2}\,D-270\,a\,c\,(c+d\,x)^{7/2}\,D-210\,a\,c^3\,(c+d\,x)^{3/2}\,D+378\,a\,c^2\,(c+d\,x)^{5/2}\,D}{315\,d^4}$$
$$+\frac{6\,A\,b\,(c+d\,x)^{5/2}-10\,A\,b\,c\,(c+d\,x)^{3/2}}{15\,d^2}+\frac{6\,B\,a\,(c+d\,x)^{5/2}-10\,B\,a\,c\,(c+d\,x)^{3/2}}{15\,d^2}$$
$$+\frac{30\,B\,b\,(c+d\,x)^{7/2}+70\,B\,b\,c^2\,(c+d\,x)^{3/2}-84\,B\,b\,c\,(c+d\,x)^{5/2}}{105\,d^3}$$
$$+\frac{30\,C\,a\,(c+d\,x)^{7/2}+70\,C\,a\,c^2\,(c+d\,x)^{3/2}-84\,C\,a\,c\,(c+d\,x)^{5/2}}{105\,d^3}+\frac{2\,A\,a\,(c+d\,x)^{3/2}}{3\,d}$$
$$+\frac{2\,C\,b\,(c+d\,x)^{9/2}}{9\,d^4}-\frac{6\,C\,b\,c\,(c+d\,x)^{7/2}}{7\,d^4}-\frac{2\,C\,b\,c^3\,(c+d\,x)^{3/2}}{3\,d^4}+\frac{6\,C\,b\,c^2\,(c+d\,x)^{5/2}}{5\,d^4}$$

input
```
int((a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
((2*b*(c + d*x)^(11/2)*D)/11 - (8*b*c*(c + d*x)^(9/2)*D)/9 + (2*b*c^4*(c +
 d*x)^(3/2)*D)/3 - (8*b*c^3*(c + d*x)^(5/2)*D)/5 + (12*b*c^2*(c + d*x)^(7/
2)*D)/7)/d^5 + (70*a*(c + d*x)^(9/2)*D - 270*a*c*(c + d*x)^(7/2)*D - 210*a
*c^3*(c + d*x)^(3/2)*D + 378*a*c^2*(c + d*x)^(5/2)*D)/(315*d^4) + (6*A*b*(
c + d*x)^(5/2) - 10*A*b*c*(c + d*x)^(3/2))/(15*d^2) + (6*B*a*(c + d*x)^(5/
2) - 10*B*a*c*(c + d*x)^(3/2))/(15*d^2) + (30*B*b*(c + d*x)^(7/2) + 70*B*b
*c^2*(c + d*x)^(3/2) - 84*B*b*c*(c + d*x)^(5/2))/(105*d^3) + (30*C*a*(c +
d*x)^(7/2) + 70*C*a*c^2*(c + d*x)^(3/2) - 84*C*a*c*(c + d*x)^(5/2))/(105*d
^3) + (2*A*a*(c + d*x)^(3/2))/(3*d) + (2*C*b*(c + d*x)^(9/2))/(9*d^4) - (6
*C*b*c*(c + d*x)^(7/2))/(7*d^4) - (2*C*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (6
*C*b*c^2*(c + d*x)^(5/2))/(5*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.97

$$\int (a+bx)\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$$
$$= \frac{2\sqrt{dx+c}\,(315b\,d^5x^5+385a\,d^5x^4+420bc\,d^4x^4+550ac\,d^4x^3+495b^2d^4x^3+15b\,c^2d^3x^3+1386ab\,d^4x^2+}{}$$

input
```
int((b*x+a)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(1155*a**2*c*d**3 + 1155*a**2*d**4*x - 924*a*b*c**2*d**2
+ 462*a*b*c*d**3*x + 1386*a*b*d**4*x**2 + 88*a*c**4*d - 44*a*c**3*d**2*x +
 33*a*c**2*d**3*x**2 + 550*a*c*d**4*x**3 + 385*a*d**5*x**4 + 264*b**2*c**3
*d - 132*b**2*c**2*d**2*x + 99*b**2*c*d**3*x**2 + 495*b**2*d**4*x**3 - 48*
b*c**5 + 24*b*c**4*d*x - 18*b*c**3*d**2*x**2 + 15*b*c**2*d**3*x**3 + 420*b
*c*d**4*x**4 + 315*b*d**5*x**5))/(3465*d**4)
```

3.58 $\int \sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result . . . 539
Mathematica [A] (verified) . . . 539
Rubi [A] (verified) . . . 540
Maple [A] (verified) . . . 541
Fricas [A] (verification not implemented) . . . 542
Sympy [A] (verification not implemented) . . . 542
Maxima [A] (verification not implemented) . . . 543
Giac [B] (verification not implemented) . . . 543
Mupad [B] (verification not implemented) . . . 544
Reduce [B] (verification not implemented) . . . 544

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(c+dx)^{3/2}}{3d^4} - \frac{2(2cCd-Bd^2-3c^2D)\,(c+dx)^{5/2}}{5d^4} + \frac{2(Cd-3cD)(c+dx)^{7/2}}{7d^4} + \frac{2D(c+dx)^{9/2}}{9d^4}$$

output
```
2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(3/2)/d^4-2/5*(-B*d^2+2*C*c*d-3*
D*c^2)*(d*x+c)^(5/2)/d^4+2/7*(C*d-3*D*c)*(d*x+c)^(7/2)/d^4+2/9*D*(d*x+c)^(
9/2)/d^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{2(c+dx)^{3/2}\left(-16c^3D+24c^2d(C+Dx)-6cd^2(7B+x(6C+5Dx))+d^3(105A+x(63B+5x(9C+7.\right.}{315d^4}$$

input
```
Integrate[Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(3/2)*(-16*c^3*D + 24*c^2*d*(C + D*x) - 6*c*d^2*(7*B + x*(6*C
 + 5*D*x)) + d^3*(105*A + x*(63*B + 5*x*(9*C + 7*D*x)))))/(315*d^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{\sqrt{c+dx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3}+\frac{(c+dx)^{3/2}\left(Bd^2+3c^2D-2cCd\right)}{d^3}+\frac{(c+dx)^{5/2}(Cd-3cD)}{d^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(c+dx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^4}-\frac{2(c+dx)^{5/2}\left(-Bd^2-3c^2D+2cCd\right)}{5d^4}+\frac{2(c+dx)^{7/2}(Cd-3cD)}{7d^4}+\frac{2D(c+dx)^{9/2}}{9d^4}$$

input
```
Int[Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(3/2))/(3*d^4) - (2*(2*c*
C*d - B*d^2 - 3*c^2*D)*(c + d*x)^(5/2))/(5*d^4) + (2*(C*d - 3*c*D)*(c + d*
x)^(7/2))/(7*d^4) + (2*D*(c + d*x)^(9/2))/(9*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{2(xd+c)^{\frac{3}{2}}\left(\left(\frac{1}{3}Dx^3+\frac{3}{7}Cx^2+\frac{3}{5}Bx+A\right)d^3-\frac{2\left(\frac{5}{7}Dx^2+\frac{6}{7}Cx+B\right)c d^2}{5}+\frac{8c^2(Dx+C)d}{35}-\frac{16Dc^3}{105}\right)}{3d^4}$
gosper	$\frac{2(xd+c)^{\frac{3}{2}}\left(35Dx^3d^3+45Cx^2d^3-30Dx^2c d^2+63Bx d^3-36Cxc d^2+24Dx c^2d+105A d^3-42Bc d^2+24C c^2d-16Dc^3\right)}{315d^4}$
orering	$\frac{2(xd+c)^{\frac{3}{2}}\left(35Dx^3d^3+45Cx^2d^3-30Dx^2c d^2+63Bx d^3-36Cxc d^2+24Dx c^2d+105A d^3-42Bc d^2+24C c^2d-16Dc^3\right)}{315d^4}$
derivativedivides	$\frac{\frac{2D(xd+c)^{\frac{9}{2}}}{9}+\frac{2(Cd-3Dc)(xd+c)^{\frac{7}{2}}}{7}+\frac{2\left(B d^2-2Ccd+3Dc^2\right)(xd+c)^{\frac{5}{2}}}{5}+\frac{2\left(A d^3-Bc d^2+C c^2d-Dc^3\right)(xd+c)^{\frac{3}{2}}}{3}}{d^4}$
default	$\frac{\frac{2D(xd+c)^{\frac{9}{2}}}{9}+\frac{2(Cd-3Dc)(xd+c)^{\frac{7}{2}}}{7}+\frac{2\left(B d^2-2Ccd+3Dc^2\right)(xd+c)^{\frac{5}{2}}}{5}+\frac{2\left(A d^3-Bc d^2+C c^2d-Dc^3\right)(xd+c)^{\frac{3}{2}}}{3}}{d^4}$
trager	$\frac{2(35Dd^4x^4+45C d^4x^3+5Dc d^3x^3+63B d^4x^2+9Cc d^3x^2-6Dc^2d^2x^2+105A d^4x+21Bc d^3x-12C c^2d^2x+8Dc^3dx+10}{315d^4}$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x+c)^(3/2)*((1/3*D*x^3+3/7*C*x^2+3/5*B*x+A)*d^3-2/5*(5/7*D*x^2+6/7*
C*x+B)*c*d^2+8/35*c^2*(D*x+C)*d-16/105*D*c^3)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{2\left(35\,Dd^4x^4-16\,Dc^4+24\,Cc^3d-42\,Bc^2d^2+105\,Acd^3+5\,(Dcd^3+9\,Cd^4)x^3-3\,(2\,Dc^2d^2-3\,Ccd^3-\right.}{315\,d^4}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
2/315*(35*D*d^4*x^4 - 16*D*c^4 + 24*C*c^3*d - 42*B*c^2*d^2 + 105*A*c*d^3 +
 5*(D*c*d^3 + 9*C*d^4)*x^3 - 3*(2*D*c^2*d^2 - 3*C*c*d^3 - 21*B*d^4)*x^2 +
(8*D*c^3*d - 12*C*c^2*d^2 + 21*B*c*d^3 + 105*A*d^4)*x)*sqrt(d*x + c)/d^4
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \begin{cases} \frac{2\left(\frac{D(c+dx)^{\frac{9}{2}}}{9d^3}+\frac{(c+dx)^{\frac{7}{2}}(Cd-3Dc)}{7d^3}+\frac{(c+dx)^{\frac{5}{2}}\left(Bd^2-2Ccd+3Dc^2\right)}{5d^3}+\frac{(c+dx)^{\frac{3}{2}}\left(Ad^3-Bcd^2+Cc^2d-Dc^3\right)}{3d^3}\right)}{d} & \text{for } d\neq 0 \\ \sqrt{c}\left(Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*(c + d*x)**(9/2)/(9*d**3) + (c + d*x)**(7/2)*(C*d - 3*D*c)
/(7*d**3) + (c + d*x)**(5/2)*(B*d**2 - 2*C*c*d + 3*D*c**2)/(5*d**3) + (c +
 d*x)**(3/2)*(A*d**3 - B*c*d**2 + C*c**2*d - D*c**3)/(3*d**3))/d, Ne(d, 0)
), (sqrt(c)*(A*x + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2\left(35\,(dx+c)^{\frac{9}{2}}D-45\,(3\,Dc-Cd)(dx+c)^{\frac{7}{2}}+63\,(3\,Dc^2-2\,Ccd+Bd^2)(dx+c)^{\frac{5}{2}}-105\,(Dc^3-Cc^2d\right.}{315\,d^4}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
2/315*(35*(d*x + c)^(9/2)*D - 45*(3*D*c - C*d)*(d*x + c)^(7/2) + 63*(3*D*c
^2 - 2*C*c*d + B*d^2)*(d*x + c)^(5/2) - 105*(D*c^3 - C*c^2*d + B*c*d^2 - A
*d^3)*(d*x + c)^(3/2))/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(101) = 202.

Time = 0.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.58

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{2\left(315\sqrt{dx+c}Ac+105\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)A+\frac{105\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)Bc}{d}+\frac{21\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c\right.}{d^2}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(d*x + c)*A*c + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A
 + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*c/d + 21*(3*(d*x + c)^(5/2)
 - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*c/d^2 + 21*(3*(d*x + c)^
(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B/d + 9*(5*(d*x + c)^
(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c
^3)*D*c/d^3 + 9*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(
3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C/d^2 + (35*(d*x + c)^(9/2) - 180*(d*x +
c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(
d*x + c)*c^4)*D/d^3)/d
```

Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.15

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{70\,(c+d\,x)^{9/2}\,D-210\,c^3\,(c+d\,x)^{3/2}\,D+378\,c^2\,(c+d\,x)^{5/2}\,D-270\,c\,(c+d\,x)^{7/2}\,D}{315\,d^4} + \frac{30\,C\,(c+d\,x)^{7/2}-84\,C\,c\,(c+d\,x)^{5/2}+70\,C\,c^2\,(c+d\,x)^{3/2}}{105\,d^3} + \frac{6\,B\,(c+d\,x)^{5/2}-10\,B\,c\,(c+d\,x)^{3/2}}{15\,d^2} + \frac{2\,A\,(c+d\,x)^{3/2}}{3\,d}$$

input
```
int((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(70*(c + d*x)^(9/2)*D - 210*c^3*(c + d*x)^(3/2)*D + 378*c^2*(c + d*x)^(5/2
)*D - 270*c*(c + d*x)^(7/2)*D)/(315*d^4) + (30*C*(c + d*x)^(7/2) - 84*C*c*
(c + d*x)^(5/2) + 70*C*c^2*(c + d*x)^(3/2))/(105*d^3) + (6*B*(c + d*x)^(5/
2) - 10*B*c*(c + d*x)^(3/2))/(15*d^2) + (2*A*(c + d*x)^(3/2))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.77

$$\int \sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2\sqrt{dx+c}\,(35d^4x^4+50c\,d^3x^3+63b\,d^3x^2+3c^2d^2x^2+105a\,d^3x+21bc\,d^2x-4c^3dx+105ac\,d^2-42b\,c^2d}{315d^3}$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(105*a*c*d**2 + 105*a*d**3*x - 42*b*c**2*d + 21*b*c*d**2*
x + 63*b*d**3*x**2 + 8*c**4 - 4*c**3*d*x + 3*c**2*d**2*x**2 + 50*c*d**3*x*
*3 + 35*d**4*x**4))/(315*d**3)
```

3.59 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$

Optimal result . . . 545
Mathematica [A] (verified) . . . 546
Rubi [A] (verified) . . . 546
Maple [A] (verified) . . . 548
Fricas [A] (verification not implemented) . . . 548
Sympy [A] (verification not implemented) . . . 549
Maxima [F(-2)] . . . 550
Giac [A] (verification not implemented) . . . 550
Mupad [F(-1)] . . . 551
Reduce [B] (verification not implemented) . . . 551

Optimal result

Integrand size = 32, antiderivative size = 229

$$\begin{aligned}\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx &= \frac{2(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b^4}\\ &+\frac{2(a^2d^2D-abd(Cd-cD)-b^2(cCd-Bd^2-c^2D))(c+dx)^{3/2}}{3b^3d^3}\\ &+\frac{2(bCd-2bcD-adD)(c+dx)^{5/2}}{5b^2d^3}+\frac{2D(c+dx)^{7/2}}{7bd^3}\\ &-\frac{2\sqrt{bc-ad}(Ab^3-a(b^2B-abC+a^2D))\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{9/2}}\end{aligned}$$

output
```
2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^4+2/3*(a^2*d^2*D-a*b*d*(C*
d-D*c)-b^2*(-B*d^2+C*c*d-D*c^2))*(d*x+c)^(3/2)/b^3/d^3+2/5*(C*b*d-D*a*d-2*
D*b*c)*(d*x+c)^(5/2)/b^2/d^3+2/7*D*(d*x+c)^(7/2)/b/d^3-2*(-a*d+b*c)^(1/2)*
(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/
2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$
$$= \frac{2\sqrt{c+dx}(-105a^3d^3D+35a^2bd^2(3Cd+D(c+dx))-7ab^2d(-2c^2D+cd(5C+Dx)+d^2(15B+x(5C$$
$$-\frac{2\sqrt{-bc+ad}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{9/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(2*Sqrt[c + d*x]*(-105*a^3*d^3*D + 35*a^2*b*d^2*(3*C*d + D*(c + d*x)) - 7*
a*b^2*d*(-2*c^2*D + c*d*(5*C + D*x) + d^2*(15*B + x*(5*C + 3*D*x))) + b^3*
(8*c^3*D - 2*c^2*d*(7*C + 2*D*x) + c*d^2*(35*B + x*(7*C + 3*D*x)) + d^3*(1
05*A + x*(35*B + 3*x*(7*C + 5*D*x))))))/(105*b^4*d^3) - (2*Sqrt[-(b*c) + a
*d]*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[-(b*c) + a*d]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{\sqrt{c+dx}(Ab^3-a(a^2D-abC+b^2B))}{b^3(a+bx)}+\frac{\sqrt{c+dx}(a^2d^2D-abd(Cd-cD)-(b^2(-Bd^2+c^2(-D)+cCd)}{b^3d^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{bc-ad}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{9/2}}+\frac{2\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^4}+\frac{2(c+dx)^{3/2}\left(a^2d^2D-abd(Cd-cD)-\left(b^2\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{3b^3d^3}+\frac{2(c+dx)^{5/2}(-adD-2bcD+bCd)}{5b^2d^3}+\frac{2D(c+dx)^{7/2}}{7bd^3}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[c + d*x])/b^4 + (2*(a^2*d^2*D
- a*b*d*(C*d - c*D) - b^2*(c*C*d - B*d^2 - c^2*D))*(c + d*x)^(3/2))/(3*b^3
*d^3) + (2*(b*C*d - 2*b*c*D - a*d*D)*(c + d*x)^(5/2))/(5*b^2*d^3) + (2*D*(
c + d*x)^(7/2))/(7*b*d^3) - (2*Sqrt[b*c - a*d]*(A*b^3 - a*(b^2*B - a*b*C +
 a^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(9/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-2d^3(ad-bc)(b^3A-a\,b^2B+a^2bC-a^3D)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)+2\sqrt{(ad-bc)b}\,\sqrt{xd+c}\left(\left(\left(\frac{1}{5}C\,x^2+\frac{1}{3}Bx+\frac{1}{7}Dx^3+A\right)b^3\right.\right.}{}$
derivativedivides	$\frac{2\left(\frac{D(xd+c)^{\frac{7}{2}}b^3}{7}+\frac{C\,b^3d(xd+c)^{\frac{5}{2}}}{5}-\frac{Da\,b^2d(xd+c)^{\frac{5}{2}}}{5}-\frac{2Db^3c(xd+c)^{\frac{5}{2}}}{5}+\frac{B\,b^3d^2(xd+c)^{\frac{3}{2}}}{3}-\frac{Ca\,b^2d^2(xd+c)^{\frac{3}{2}}}{3}-\frac{C\,b^3cd(xd+c)^{\frac{3}{2}}}{3}+\right.}{b^4}$
default	$\frac{2\left(\frac{D(xd+c)^{\frac{7}{2}}b^3}{7}+\frac{C\,b^3d(xd+c)^{\frac{5}{2}}}{5}-\frac{Da\,b^2d(xd+c)^{\frac{5}{2}}}{5}-\frac{2Db^3c(xd+c)^{\frac{5}{2}}}{5}+\frac{B\,b^3d^2(xd+c)^{\frac{3}{2}}}{3}-\frac{Ca\,b^2d^2(xd+c)^{\frac{3}{2}}}{3}-\frac{C\,b^3cd(xd+c)^{\frac{3}{2}}}{3}+\right.}{b^4}$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(-d^3*(a*d-b*c)*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)*arctan(b*(d*x+c)^(1/2)/((a
*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(d*x+c)^(1/2)*(((1/5*C*x^2+1/3*B*x+1
/7*D*x^3+A)*b^3-(1/5*D*x^2+1/3*C*x+B)*a*b^2+a^2*(1/3*D*x+C)*b-a^3*D)*d^3+1
/3*((3/35*D*x^2+1/5*C*x+B)*b^2-(1/5*D*x+C)*a*b+D*a^2)*c*b*d^2-2/15*((2/7*D
*x+C)*b-D*a)*c^2*b^2*d+8/105*D*b^3*c^3))/((a*d-b*c)*b)^(1/2)/d^3/b^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 579, normalized size of antiderivative = 2.53

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$
$$=\left[-\frac{105\,(Da^3-Ca^2b+Bab^2-Ab^3)d^3\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\,(15\,Db^3d^3x^3+8\,Db^3\right.$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/105*(105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^3*sqrt((b*c - a*d)/b)*l
og((b*d*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)
) - 2*(15*D*b^3*d^3*x^3 + 8*D*b^3*c^3 + 14*(D*a*b^2 - C*b^3)*c^2*d + 35*(D
*a^2*b - C*a*b^2 + B*b^3)*c*d^2 - 105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*
d^3 + 3*(D*b^3*c*d^2 - 7*(D*a*b^2 - C*b^3)*d^3)*x^2 - (4*D*b^3*c^2*d + 7*(
D*a*b^2 - C*b^3)*c*d^2 - 35*(D*a^2*b - C*a*b^2 + B*b^3)*d^3)*x)*sqrt(d*x +
 c))/(b^4*d^3), 2/105*(105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^3*sqrt(-(
b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) +
(15*D*b^3*d^3*x^3 + 8*D*b^3*c^3 + 14*(D*a*b^2 - C*b^3)*c^2*d + 35*(D*a^2*b
 - C*a*b^2 + B*b^3)*c*d^2 - 105*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^3 +
3*(D*b^3*c*d^2 - 7*(D*a*b^2 - C*b^3)*d^3)*x^2 - (4*D*b^3*c^2*d + 7*(D*a*b^
2 - C*b^3)*c*d^2 - 35*(D*a^2*b - C*a*b^2 + B*b^3)*d^3)*x)*sqrt(d*x + c))/(
b^4*d^3)]
```

Sympy [A] (verification not implemented)

Time = 8.02 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{D(c+dx)^{\frac{7}{2}}}{7bd^2}+\frac{(c+dx)^{\frac{5}{2}}(Cbd-Dad-2Dbc)}{5b^2d^2}+\frac{(c+dx)^{\frac{3}{2}}\left(Bb^2d^2-Cabd^2-Cb^2cd+Da^2d^2+Dabcd+Db^2c^2\right)}{3b^3d^2}+\frac{\sqrt{c+dx}\left(Ab^3d-Bab^2d+Ca^2bd-Da^3d\right)}{b^4}+\frac{d(a}{}\right.}{d} \\ \sqrt{c}\left(\frac{Dx^3}{3b}+\frac{x^2(Cb-Da)}{2b^2}+\frac{x(Bb^2-Cab+Da^2)}{b^3}-\frac{(-Ab^3+Bab^2-Ca^2b+Da^3)\left(\begin{cases}\frac{x}{a} & \text{for } b=0\\ \frac{\log(a+bx)}{b} & \text{otherwise}\end{cases}\right)}{b^3}\right) \end{cases}$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a),x)
```

output
```
Piecewise((2*(D*(c + d*x)**(7/2)/(7*b*d**2) + (c + d*x)**(5/2)*(C*b*d - D*
a*d - 2*D*b*c)/(5*b**2*d**2) + (c + d*x)**(3/2)*(B*b**2*d**2 - C*a*b*d**2
- C*b**2*c*d + D*a**2*d**2 + D*a*b*c*d + D*b**2*c**2)/(3*b**3*d**2) + sqrt
(c + d*x)*(A*b**3*d - B*a*b**2*d + C*a**2*b*d - D*a**3*d)/b**4 + d*(a*d -
b*c)*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*atan(sqrt(c + d*x)/sqrt((a*d
 - b*c)/b))/(b**5*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (sqrt(c)*(D*x**3/(3*
b) + x**2*(C*b - D*a)/(2*b**2) + x*(B*b**2 - C*a*b + D*a**2)/b**3 - (-A*b*
*3 + B*a*b**2 - C*a**2*b + D*a**3)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x
)/b, True))/b**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = -\frac{2\left(Da^3bc-Ca^2b^2c+Bab^3c-Ab^4c-Da^4d+Ca^3bd-Ba^2b^2d+Aab^3d\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^4} + \frac{2\left(15\left(dx+c\right)^{\frac{7}{2}}Db^6d^{18}-42\left(dx+c\right)^{\frac{5}{2}}Db^6cd^{18}+35\left(dx+c\right)^{\frac{3}{2}}Db^6c^2d^{18}-21\left(dx+c\right)^{\frac{5}{2}}Dab^5d^{19}+21\left(d\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
-2*(D*a^3*b*c - C*a^2*b^2*c + B*a*b^3*c - A*b^4*c - D*a^4*d + C*a^3*b*d -
B*a^2*b^2*d + A*a*b^3*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqr
t(-b^2*c + a*b*d)*b^4) + 2/105*(15*(d*x + c)^(7/2)*D*b^6*d^18 - 42*(d*x +
c)^(5/2)*D*b^6*c*d^18 + 35*(d*x + c)^(3/2)*D*b^6*c^2*d^18 - 21*(d*x + c)^(
5/2)*D*a*b^5*d^19 + 21*(d*x + c)^(5/2)*C*b^6*d^19 + 35*(d*x + c)^(3/2)*D*a
*b^5*c*d^19 - 35*(d*x + c)^(3/2)*C*b^6*c*d^19 + 35*(d*x + c)^(3/2)*D*a^2*b
^4*d^20 - 35*(d*x + c)^(3/2)*C*a*b^5*d^20 + 35*(d*x + c)^(3/2)*B*b^6*d^20
- 105*sqrt(d*x + c)*D*a^3*b^3*d^21 + 105*sqrt(d*x + c)*C*a^2*b^4*d^21 - 10
5*sqrt(d*x + c)*B*a*b^5*d^21 + 105*sqrt(d*x + c)*A*b^6*d^21)/(b^7*d^21)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \int \frac{\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{a+b\,x}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x),x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$
$$= \frac{2\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^3d^3 - 2\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^2bc\,d^2 - 2\sqrt{dx+c}\,a^3b\,d^3 + \cdots}{\cdots}$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

output
```
(2*(105*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
 b*c)))*a**3*d**3 - 105*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sq
rt(b)*sqrt(a*d - b*c)))*a**2*b*c*d**2 - 105*sqrt(c + d*x)*a**3*b*d**3 + 14
0*sqrt(c + d*x)*a**2*b**2*c*d**2 + 35*sqrt(c + d*x)*a**2*b**2*d**3*x - 21*
sqrt(c + d*x)*a*b**3*c**2*d - 42*sqrt(c + d*x)*a*b**3*c*d**2*x - 21*sqrt(c
 + d*x)*a*b**3*d**3*x**2 + 35*sqrt(c + d*x)*b**5*c*d + 35*sqrt(c + d*x)*b*
*5*d**2*x - 6*sqrt(c + d*x)*b**4*c**3 + 3*sqrt(c + d*x)*b**4*c**2*d*x + 24
*sqrt(c + d*x)*b**4*c*d**2*x**2 + 15*sqrt(c + d*x)*b**4*d**3*x**3))/(105*b
**5*d**2)
```

3.60 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$

Optimal result 553
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Optimal result

Integrand size = 32, antiderivative size = 228

$$\begin{aligned}\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx &= \frac{2(b^2B-2abC+3a^2D)\sqrt{c+dx}}{b^4}-\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b^4(a+bx)}\\ &\quad+\frac{2(bCd-bcD-2adD)(c+dx)^{3/2}}{3b^3d^2}+\frac{2D(c+dx)^{5/2}}{5b^2d^2}\\ &\quad-\frac{(b^3(2Bc+Ad)-ab^2(4cC+3Bd)-7a^3dD+a^2b(5Cd+6cD))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{9/2}\sqrt{bc-ad}}\end{aligned}$$

output
```
2*(B*b^2-2*C*a*b+3*D*a^2)*(d*x+c)^(1/2)/b^4-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*
(d*x+c)^(1/2)/b^4/(b*x+a)+2/3*(C*b*d-2*D*a*d-D*b*c)*(d*x+c)^(3/2)/b^3/d^2+
2/5*D*(d*x+c)^(5/2)/b^2/d^2-(b^3*(A*d+2*B*c)-a*b^2*(3*B*d+4*C*c)-7*a^3*d*D
+a^2*b*(5*C*d+6*D*c))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(9
/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$= \frac{\sqrt{c+dx}(105a^3d^2D-5a^2bd(15Cd+4cD-14dDx)-ab^2(4c^2D-2cd(5C-9Dx)+d^2(-45B+50Cx}{15b^4d^2(a+bx)}$$
$$+\frac{(b^3(2Bc+Ad)-ab^2(4cC+3Bd)-7a^3dD+a^2b(5Cd+6cD))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{9/2}\sqrt{-bc+ad}}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
(Sqrt[c + d*x]*(105*a^3*d^2*D - 5*a^2*b*d*(15*C*d + 4*c*D - 14*d*D*x) - a*
b^2*(4*c^2*D - 2*c*d*(5*C - 9*D*x) + d^2*(-45*B + 50*C*x + 14*D*x^2)) + b^
3*(-15*A*d^2 + 2*x*(-2*c^2*D + c*d*(5*C + D*x) + d^2*(15*B + 5*C*x + 3*D*x
^2)))))/(15*b^4*d^2*(a + b*x)) + ((b^3*(2*B*c + A*d) - a*b^2*(4*c*C + 3*B*
d) - 7*a^3*d*D + a^2*b*(5*C*d + 6*c*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[-(b*c) + a*d]])/(b^(9/2)*Sqrt[-(b*c) + a*d])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2124, 27, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \text{2124}$$

$$-\frac{\int -\frac{\sqrt{c+dx}\left(2\left(c-\frac{ad}{b}\right)Dx^2+\frac{2(bc-ad)(bC-aD)x}{b^2}+\frac{-3dDa^3+b(3Cd+2cD)a^2-b^2(2cC+3Bd)a+b^3(2Bc+Ad)}{b^3}\right)}{2(a+bx)}dx}{bc-ad}-\frac{(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{c+dx}\left(-\frac{3dDa^3}{b^3}+\frac{(3Cd+2cD)a^2}{b^2}-\frac{(2cC+3Bd)a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc+Ad+\frac{2(bc-ad)(bC-aD)x}{b^2}\right)}{a+bx}dx}{2(bc-ad)}-\frac{(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 1192}$$

$$\frac{\int \frac{(c+dx)\left(-2Dc^3+2Cdc^2-2Bd^2c-2\left(c-\frac{ad}{b}\right)D(c+dx)^2-d^3\left(A-\frac{3a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{2(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{bc-ad-b(c+dx)}d\sqrt{c+dx}}{d^2(bc-ad)}-\frac{(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{\int \left(\frac{\left(-7dDa^3+b(5Cd+6cD)a^2-b^2(4cC+3Bd)a+b^3(2Bc+Ad)\right)d^2}{b^4}+\frac{2(bc-ad)D(c+dx)^2}{b^2}+\frac{2(bc-ad)(bCd-2aDd-bcD)(c+dx)}{b^3}+\frac{-Acd^3b^4 \cdots}{\cdots}\right.}{d^2(bc-a\cdots}$$

$$-\frac{(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{d^2\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-7a^3dD+a^2b(6cD+5Cd)-ab^2(3Bd+4cC)+b^3(Ad+2Bc)\right)}{b^{9/2}}+\frac{d^2\sqrt{c+dx}\left(-7a^3dD+a^2b(6cD+5Cd)-ab^2(3\cdots\right.}{b^4}\cdots}{d^2(bc-ad)}$$

$$-\frac{(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
-(((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(3/2))/((b*c - a*d)*(a
+ b*x))) + ((d^2*(b^3*(2*B*c + A*d) - a*b^2*(4*c*C + 3*B*d) - 7*a^3*d*D +
a^2*b*(5*C*d + 6*c*D))*Sqrt[c + d*x])/b^4 + (2*(b*c - a*d)*(b*C*d - b*c*D
- 2*a*d*D)*(c + d*x)^(3/2))/(3*b^3) + (2*(b*c - a*d)*D*(c + d*x)^(5/2))/(5
*b^2) - (d^2*Sqrt[b*c - a*d]*(b^3*(2*B*c + A*d) - a*b^2*(4*c*C + 3*B*d) -
7*a^3*d*D + a^2*b*(5*C*d + 6*c*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*
c - a*d]])/b^(9/2))/(d^2*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\sqrt{(ad-bc)b}\left(\left(\left(-\frac{2}{5}Dx^3-\frac{2}{3}Cx^2+A-2Bx\right)b^3-3\left(-\frac{14}{45}Dx^2-\frac{10}{9}Cx+B\right)ab^2+5a^2\left(-\frac{14Dx}{15}+C\right)b-7a^3D\right)d^2-\frac{2\left(\left(\frac{Dx}{5}\right.\right.}{}\right.$
derivativedivides	$\frac{2\left(\frac{D(xd+c)^{\frac{5}{2}}b^2}{5}+\frac{Cb^2d(xd+c)^{\frac{3}{2}}}{3}-\frac{2Dabd(xd+c)^{\frac{3}{2}}}{3}-\frac{Db^2c(xd+c)^{\frac{3}{2}}}{3}+Bd^2b^2\sqrt{xd+c}-2Cad^2b\sqrt{xd+c}+3Da^2d^2\sqrt{xd+c}\right)}{b^4}+\frac{2d^2\left(\right.}{}$
default	$\frac{2\left(\frac{D(xd+c)^{\frac{5}{2}}b^2}{5}+\frac{Cb^2d(xd+c)^{\frac{3}{2}}}{3}-\frac{2Dabd(xd+c)^{\frac{3}{2}}}{3}-\frac{Db^2c(xd+c)^{\frac{3}{2}}}{3}+Bd^2b^2\sqrt{xd+c}-2Cad^2b\sqrt{xd+c}+3Da^2d^2\sqrt{xd+c}\right)}{b^4}+\frac{2d^2\left(\right.}{}$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-((a*d-b*c)*b)^(1/2)*(((-2/5*D*x^3-2/3*C*x^2+A-2*B*x)*b^3-3*(-14/45*D*x^2
-10/9*C*x+B)*a*b^2+5*a^2*(-14/15*D*x+C)*b-7*a^3*D)*d^2-2/3*((1/5*D*x+C)*b-
2*D*a)*(b*x+a)*c*b*d+4/15*D*b^2*c^2*(b*x+a))*(d*x+c)^(1/2)+((A*b^3-3*B*a*b
^2+5*C*a^2*b-7*D*a^3)*d+2*b*c*(B*b^2-2*C*a*b+3*D*a^2))*d^2*(b*x+a)*arctan(
b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))/((a*d-b*c)*b)^(1/2)/d^2/b^4/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(206) = 412.

Time = 0.11 (sec) , antiderivative size = 1045, normalized size of antiderivative = 4.58

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="fricas
")
```

output
```
[1/30*(15*(2*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c*d^2 - (7*D*a^4 - 5*C*a^
3*b + 3*B*a^2*b^2 - A*a*b^3)*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c*
d^2 - (7*D*a^3*b - 5*C*a^2*b^2 + 3*B*a*b^3 - A*b^4)*d^3)*x)*sqrt(b^2*c - a
*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x
 + a)) - 2*(4*D*a*b^4*c^3 + 2*(8*D*a^2*b^3 - 5*C*a*b^4)*c^2*d - 5*(25*D*a^
3*b^2 - 17*C*a^2*b^3 + 9*B*a*b^4 - 3*A*b^5)*c*d^2 + 15*(7*D*a^4*b - 5*C*a^
3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*d^3 - 6*(D*b^5*c*d^2 - D*a*b^4*d^3)*x^3 - 2
*(D*b^5*c^2*d - (8*D*a*b^4 - 5*C*b^5)*c*d^2 + (7*D*a^2*b^3 - 5*C*a*b^4)*d^
3)*x^2 + 2*(2*D*b^5*c^3 + (7*D*a*b^4 - 5*C*b^5)*c^2*d - (44*D*a^2*b^3 - 30
*C*a*b^4 + 15*B*b^5)*c*d^2 + 5*(7*D*a^3*b^2 - 5*C*a^2*b^3 + 3*B*a*b^4)*d^3
)*x)*sqrt(d*x + c))/(a*b^6*c*d^2 - a^2*b^5*d^3 + (b^7*c*d^2 - a*b^6*d^3)*x
), 1/15*(15*(2*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c*d^2 - (7*D*a^4 - 5*C*
a^3*b + 3*B*a^2*b^2 - A*a*b^3)*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*
c*d^2 - (7*D*a^3*b - 5*C*a^2*b^2 + 3*B*a*b^3 - A*b^4)*d^3)*x)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (4*D*a
*b^4*c^3 + 2*(8*D*a^2*b^3 - 5*C*a*b^4)*c^2*d - 5*(25*D*a^3*b^2 - 17*C*a^2*
b^3 + 9*B*a*b^4 - 3*A*b^5)*c*d^2 + 15*(7*D*a^4*b - 5*C*a^3*b^2 + 3*B*a^2*b
^3 - A*a*b^4)*d^3 - 6*(D*b^5*c*d^2 - D*a*b^4*d^3)*x^3 - 2*(D*b^5*c^2*d - (
8*D*a*b^4 - 5*C*b^5)*c*d^2 + (7*D*a^2*b^3 - 5*C*a*b^4)*d^3)*x^2 + 2*(2*D*b
^5*c^3 + (7*D*a*b^4 - 5*C*b^5)*c^2*d - (44*D*a^2*b^3 - 30*C*a*b^4 + 15*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$= \frac{(6\,Da^2bc-4\,Cab^2c+2\,Bb^3c-7\,Da^3d+5\,Ca^2bd-3\,Bab^2d+Ab^3d)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^4}$$
$$+\frac{\sqrt{dx+c}Da^3d-\sqrt{dx+c}Ca^2bd+\sqrt{dx+c}Bab^2d-\sqrt{dx+c}Ab^3d}{((dx+c)b-bc+ad)b^4}$$
$$+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}Db^8d^8-5\,(dx+c)^{\frac{3}{2}}Db^8cd^8-10\,(dx+c)^{\frac{3}{2}}Dab^7d^9+5\,(dx+c)^{\frac{3}{2}}Cb^8d^9+45\,\sqrt{dx+c}Dc\right.}{15\,b^{10}d^{10}}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(6*D*a^2*b*c - 4*C*a*b^2*c + 2*B*b^3*c - 7*D*a^3*d + 5*C*a^2*b*d - 3*B*a*b
^2*d + A*b^3*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
+ a*b*d)*b^4) + (sqrt(d*x + c)*D*a^3*d - sqrt(d*x + c)*C*a^2*b*d + sqrt(d*
x + c)*B*a*b^2*d - sqrt(d*x + c)*A*b^3*d)/(((d*x + c)*b - b*c + a*d)*b^4)
+ 2/15*(3*(d*x + c)^(5/2)*D*b^8*d^8 - 5*(d*x + c)^(3/2)*D*b^8*c*d^8 - 10*(
d*x + c)^(3/2)*D*a*b^7*d^9 + 5*(d*x + c)^(3/2)*C*b^8*d^9 + 45*sqrt(d*x + c
)*D*a^2*b^6*d^10 - 30*sqrt(d*x + c)*C*a*b^7*d^10 + 15*sqrt(d*x + c)*B*b^8*
d^10)/(b^10*d^10)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx=\int \frac{\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2,x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \frac{-105\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^3d^2+60\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^2bcd-105\sqrt{b}\,\sqrt{ad-bc}\,ata}{}$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
 b*c)))*a**3*d**2 + 60*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqr
t(b)*sqrt(a*d - b*c)))*a**2*b*c*d - 105*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt
(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b*d**2*x - 30*sqrt(b)*sqrt(a*
d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**3*d + 60*s
qrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a
*b**2*c*d*x - 30*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*s
qrt(a*d - b*c)))*b**4*d*x + 105*sqrt(c + d*x)*a**3*b*d**2 - 95*sqrt(c + d*
x)*a**2*b**2*c*d + 70*sqrt(c + d*x)*a**2*b**2*d**2*x + 30*sqrt(c + d*x)*a*
b**4*d + 6*sqrt(c + d*x)*a*b**3*c**2 - 68*sqrt(c + d*x)*a*b**3*c*d*x - 14*
sqrt(c + d*x)*a*b**3*d**2*x**2 + 30*sqrt(c + d*x)*b**5*d*x + 6*sqrt(c + d*
x)*b**4*c**2*x + 12*sqrt(c + d*x)*b**4*c*d*x**2 + 6*sqrt(c + d*x)*b**4*d**
2*x**3)/(15*b**5*d*(a + b*x))
```

3.61 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3} dx$

Optimal result 562
Mathematica [A] (verified) 563
Rubi [A] (verified) 563
Maple [A] (verified) 567
Fricas [B] (verification not implemented) 567
Sympy [F(-1)] 568
Maxima [F(-2)] 569
Giac [B] (verification not implemented) 569
Mupad [F(-1)] 570
Reduce [B] (verification not implemented) 570

Optimal result

Integrand size = 32, antiderivative size = 293

$$\begin{aligned}
&\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3} dx \\
&= \frac{2(bC-3aD)\sqrt{c+dx}}{b^4} - \frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{2b^4(a+bx)^2} \\
&\quad - \frac{(b^3(4Bc+Ad)-ab^2(8cC+5Bd)-13a^3dD+3a^2b(3Cd+4cD))\sqrt{c+dx}}{4b^4(bc-ad)(a+bx)} \\
&\quad + \frac{2D(c+dx)^{3/2}}{3b^3d} \\
&\quad - \frac{(b^3(8c^2C+4Bcd-Ad^2)-35a^3d^2D+15a^2bd(Cd+4cD)-3ab^2(8cCd+Bd^2+8c^2D))\operatorname{arctanh}\left(\frac{\sqrt{b\cdots}}{\sqrt{\cdots}}\right.}{4b^{9/2}(bc-ad)^{3/2}}
\end{aligned}$$

output
```
2*(C*b-3*D*a)*(d*x+c)^(1/2)/b^4-1/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^
(1/2)/b^4/(b*x+a)^2-1/4*(b^3*(A*d+4*B*c)-a*b^2*(5*B*d+8*C*c)-13*a^3*d*D+3*
a^2*b*(3*C*d+4*D*c))*(d*x+c)^(1/2)/b^4/(-a*d+b*c)/(b*x+a)+2/3*D*(d*x+c)^(3
/2)/b^3/d-1/4*(b^3*(-A*d^2+4*B*c*d+8*C*c^2)-35*a^3*d^2*D+15*a^2*b*d*(C*d+4
*D*c)-3*a*b^2*(B*d^2+8*C*c*d+8*D*c^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d
+b*c)^(1/2))/b^(9/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$$
$$= \frac{\sqrt{c+dx}(105a^4d^2D-3Ab^3d(2bc-ad+bdx)-5a^3bd(9Cd+22cD-35dDx)+4b^4cx(-3Bd+6Cdx-}{}$$
$$-\frac{(b^3(8c^2C+4Bcd-Ad^2)-35a^3d^2D+15a^2bd(Cd+4cD)-3ab^2(8cCd+Bd^2+8c^2D))\arctan\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{-b}}\right.}{4b^{9/2}(-bc+ad)^{3/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
(Sqrt[c + d*x]*(105*a^4*d^2*D - 3*A*b^3*d*(2*b*c - a*d + b*d*x) - 5*a^3*b*
d*(9*C*d + 22*c*D - 35*d*D*x) + 4*b^4*c*x*(-3*B*d + 6*C*d*x + 2*D*x*(c + d
*x)) + a^2*b^2*(8*c^2*D + 2*c*d*(21*C - 94*D*x) + d^2*(9*B + x*(-75*C + 56
*D*x))) + a*b^3*(3*B*d*(-2*c + 5*d*x) - 8*x*(-2*c^2*D + d^2*x*(3*C + D*x)
+ c*(-9*C*d + 8*d*D*x)))))/(12*b^4*d*(b*c - a*d)*(a + b*x)^2) - ((b^3*(8*c
^2*C + 4*B*c*d - A*d^2) - 35*a^3*d^2*D + 15*a^2*b*d*(C*d + 4*c*D) - 3*a*b^
2*(8*c*C*d + B*d^2 + 8*c^2*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c)
+ a*d]])/(4*b^(9/2)*(-(b*c) + a*d)^(3/2))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2124, 27, 1192, 25, 1580, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$
$$\Big\downarrow \text{ 2124}$$

$$-\frac{\int -\frac{\sqrt{c+dx}\left(4\left(c-\frac{ad}{b}\right)Dx^2+\frac{4(bc-ad)(bC-aD)x}{b^2}+\frac{-3dDa^3+b(3Cd+4cD)a^2-b^2(4cC+3Bd)a+b^3(4Bc-Ad)}{b^3}\right)}{2(a+bx)^2}dx}{2(bc-ad)}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\left(-\frac{3dDa^3}{b^3}+\frac{(3Cd+4cD)a^2}{b^2}-\frac{(4cC+3Bd)a}{b}+4\left(c-\frac{ad}{b}\right)Dx^2+4Bc-Ad+\frac{4(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^2}dx}{4(bc-ad)}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 1192$$

$$\frac{\int -\frac{(c+dx)\left(-4Dc^3+4Cdc^2-4Bd^2c+Ad^3-4\left(c-\frac{ad}{b}\right)D(c+dx)^2+\frac{3ad^3\left(Da^2-bCa+b^2B\right)}{b^3}-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{(c+dx)\left(-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2+d^3\left(A+\frac{3a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 1580$$

$$\frac{\frac{d^2\sqrt{c+dx}\left(-11a^3dD+a^2b(12cD+7Cd)-ab^2(3Bd+8cC)+b^3(4Bc-Ad)\right)}{2b^4(-ad-b(c+dx)+bc)}-\frac{\int \frac{\frac{\left(-11dDa^3+b(7Cd+12cD)a^2-b^2(8cC+3Bd)a+b^3(4Bc-Ad)\right)d^2}{b}+8b(bc- \cdots}{bc-ad-b(c+dx) \cdots}}{2b^3 \cdots}}{2d(bc-ad)}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 1467$$

$$\frac{\frac{d^2\sqrt{c+dx}(-11a^3dD+a^2b(12cD+7Cd)-ab^2(3Bd+8cC)+b^3(4Bc-Ad))}{2b^4(-ad-b(c+dx)+bc)}-\frac{\int\left(-\frac{8d(bc-ad)(bC-3aD)}{b}-8(bc-ad)D(c+dx)+\frac{-Ad^3b^3+4Bcd^2b^3+8c}{}\right.}{}}{2d(bc-ad)}$$

$$\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{d^2\sqrt{c+dx}(-11a^3dD+a^2b(12cD+7Cd)-ab^2(3Bd+8cC)+b^3(4Bc-Ad))}{2b^4(-ad-b(c+dx)+bc)}-\frac{\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-35a^3d^2D+15a^2bd(4cD+Cd)-3ab^2\left(Bd^2+8c^2\right.\right.}{b^{3/2}\sqrt{bc-ad}}}{}}{2d(bc-ad)}$$

$$\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
-1/2*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(3/2))/(b^3*(b*c - a*d
)*(a + b*x)^2) + ((d^2*(b^3*(4*B*c - A*d) - a*b^2*(8*c*C + 3*B*d) - 11*a^3
*d*D + a^2*b*(7*C*d + 12*c*D))*Sqrt[c + d*x])/(2*b^4*(b*c - a*d - b*(c + d
*x))) - ((-8*d*(b*c - a*d)*(b*C - 3*a*D)*Sqrt[c + d*x])/b - (8*(b*c - a*d)
*D*(c + d*x)^(3/2))/3 + (d*(b^3*(8*c^2*C + 4*B*c*d - A*d^2) - 35*a^3*d^2*D
 + 15*a^2*b*d*(C*d + 4*c*D) - 3*a*b^2*(8*c*C*d + B*d^2 + 8*c^2*D))*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/(2*b
^3))/(2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.07

method	result
pseudoelliptic	$\frac{-\sqrt{(ad-bc)b}\left(\left(-A\,b^4x+a\left(A-8C\,x^2-\frac{8}{3}Dx^3+5Bx\right)b^3+3\left(\frac{56}{9}Dx^2-\frac{25}{3}Cx+B\right)a^2b^2-15\left(-\frac{35Dx}{9}+C\right)a^3b+35Da^4\right)d^2}{}$
derivativedivides	$\frac{2\left(\frac{D(xd+c)^{\frac{3}{2}}b}{3}+Cdb\sqrt{xd+c}-3Dad\sqrt{xd+c}\right)}{b^4}+\frac{2d\left(\frac{\frac{bd\left(b^3dA-5Ba\,b^2d+4B\,b^3c+9C\,a^2bd-8a\,b^2cC-13a^3dD+12a^2bcD\right)(xd+c)^{\frac{3}{2}}}{8ad-8bc}}{((xd+c)b+}\right.}{}$
default	$\frac{2\left(\frac{D(xd+c)^{\frac{3}{2}}b}{3}+Cdb\sqrt{xd+c}-3Dad\sqrt{xd+c}\right)}{b^4}+\frac{2d\left(\frac{\frac{bd\left(b^3dA-5Ba\,b^2d+4B\,b^3c+9C\,a^2bd-8a\,b^2cC-13a^3dD+12a^2bcD\right)(xd+c)^{\frac{3}{2}}}{8ad-8bc}}{((xd+c)b+}\right.}{}$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-((a*d-b*c)*b)^(1/2)*((-A*b^4*x+a*(A-8*C*x^2-8/3*D*x^3+5*B*x)*b^3+3*(
56/9*D*x^2-25/3*C*x+B)*a^2*b^2-15*(-35/9*D*x+C)*a^3*b+35*D*a^4)*d^2-2*c*((
-4/3*D*x^3-4*C*x^2+2*B*x+A)*b^3+a*(32/3*D*x^2-12*C*x+B)*b^2-7*(-94/21*D*x+
C)*a^2*b+55/3*a^3*D)*b*d+8/3*D*b^2*c^2*(b*x+a)^2)*(d*x+c)^(1/2)+arctan(b*(
d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))*((A*b^3+3*B*a*b^2-15*C*a^2*b+35*D*a^3)*d
^2-4*b*c*(B*b^2-6*C*a*b+15*D*a^2)*d-8*b^2*c^2*(C*b-3*D*a))*d*(b*x+a)^2)/((
a*d-b*c)*b)^(1/2)/b^4/(b*x+a)^2/(a*d-b*c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. 2(269) = 538.

Time = 0.12 (sec) , antiderivative size = 1621, normalized size of antiderivative = 5.53

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="fricas
")
```

output
```
[-1/24*(3*(8*(3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - 4*(15*D*a^4*b - 6*C*a^3*b^2
 + B*a^2*b^3)*c*d^2 + (35*D*a^5 - 15*C*a^4*b + 3*B*a^3*b^2 + A*a^2*b^3)*d^
3 + (8*(3*D*a*b^4 - C*b^5)*c^2*d - 4*(15*D*a^2*b^3 - 6*C*a*b^4 + B*b^5)*c*
d^2 + (35*D*a^3*b^2 - 15*C*a^2*b^3 + 3*B*a*b^4 + A*b^5)*d^3)*x^2 + 2*(8*(3
*D*a^2*b^3 - C*a*b^4)*c^2*d - 4*(15*D*a^3*b^2 - 6*C*a^2*b^3 + B*a*b^4)*c*d
^2 + (35*D*a^4*b - 15*C*a^3*b^2 + 3*B*a^2*b^3 + A*a*b^4)*d^3)*x)*sqrt(b^2*
c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) - 2*(8*D*a^2*b^4*c^3 - 2*(59*D*a^3*b^3 - 21*C*a^2*b^4 + 3*B*a*
b^5 + 3*A*b^6)*c^2*d + (215*D*a^4*b^2 - 87*C*a^3*b^3 + 15*B*a^2*b^4 + 9*A*
a*b^5)*c*d^2 - 3*(35*D*a^5*b - 15*C*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4)*d^3
 + 8*(D*b^6*c^2*d - 2*D*a*b^5*c*d^2 + D*a^2*b^4*d^3)*x^3 + 8*(D*b^6*c^3 -
3*(3*D*a*b^5 - C*b^6)*c^2*d + 3*(5*D*a^2*b^4 - 2*C*a*b^5)*c*d^2 - (7*D*a^3
*b^3 - 3*C*a^2*b^4)*d^3)*x^2 + (16*D*a*b^5*c^3 - 12*(17*D*a^2*b^4 - 6*C*a*
b^5 + B*b^6)*c^2*d + 3*(121*D*a^3*b^3 - 49*C*a^2*b^4 + 9*B*a*b^5 - A*b^6)*
c*d^2 - (175*D*a^4*b^2 - 75*C*a^3*b^3 + 15*B*a^2*b^4 - 3*A*a*b^5)*d^3)*x)*
sqrt(d*x + c))/(a^2*b^7*c^2*d - 2*a^3*b^6*c*d^2 + a^4*b^5*d^3 + (b^9*c^2*d
 - 2*a*b^8*c*d^2 + a^2*b^7*d^3)*x^2 + 2*(a*b^8*c^2*d - 2*a^2*b^7*c*d^2 + a
^3*b^6*d^3)*x), -1/12*(3*(8*(3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - 4*(15*D*a^4*
b - 6*C*a^3*b^2 + B*a^2*b^3)*c*d^2 + (35*D*a^5 - 15*C*a^4*b + 3*B*a^3*b^2
+ A*a^2*b^3)*d^3 + (8*(3*D*a*b^4 - C*b^5)*c^2*d - 4*(15*D*a^2*b^3 - 6*C...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(269) = 538.

Time = 0.14 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx =$$
$$-\frac{(24\,Dab^2c^2 - 8\,Cb^3c^2 - 60\,Da^2bcd + 24\,Cab^2cd - 4\,Bb^3cd + 35\,Da^3d^2 - 15\,Ca^2bd^2 + 3\,Bab^2d^2 + Ab^3}{4\,(b^5c - ab^4d)\sqrt{-b^2c + abd}}$$
$$-\frac{12\,(dx+c)^{\frac{3}{2}}Da^2b^2cd - 8\,(dx+c)^{\frac{3}{2}}Cab^3cd + 4\,(dx+c)^{\frac{3}{2}}Bb^4cd - 12\,\sqrt{dx+c}Da^2b^2c^2d + 8\,\sqrt{dx+c}C}{}$$
$$+\frac{2\left((dx+c)^{\frac{3}{2}}Db^6d^2 - 9\,\sqrt{dx+c}Dab^5d^3 + 3\,\sqrt{dx+c}Cb^6d^3\right)}{3\,b^9d^3}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(24*D*a*b^2*c^2 - 8*C*b^3*c^2 - 60*D*a^2*b*c*d + 24*C*a*b^2*c*d - 4*B
*b^3*c*d + 35*D*a^3*d^2 - 15*C*a^2*b*d^2 + 3*B*a*b^2*d^2 + A*b^3*d^2)*arct
an(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c - a*b^4*d)*sqrt(-b^2*c +
a*b*d)) - 1/4*(12*(d*x + c)^(3/2)*D*a^2*b^2*c*d - 8*(d*x + c)^(3/2)*C*a*b^
3*c*d + 4*(d*x + c)^(3/2)*B*b^4*c*d - 12*sqrt(d*x + c)*D*a^2*b^2*c^2*d + 8
*sqrt(d*x + c)*C*a*b^3*c^2*d - 4*sqrt(d*x + c)*B*b^4*c^2*d - 13*(d*x + c)^
(3/2)*D*a^3*b*d^2 + 9*(d*x + c)^(3/2)*C*a^2*b^2*d^2 - 5*(d*x + c)^(3/2)*B*
a*b^3*d^2 + (d*x + c)^(3/2)*A*b^4*d^2 + 23*sqrt(d*x + c)*D*a^3*b*c*d^2 - 1
5*sqrt(d*x + c)*C*a^2*b^2*c*d^2 + 7*sqrt(d*x + c)*B*a*b^3*c*d^2 + sqrt(d*x
 + c)*A*b^4*c*d^2 - 11*sqrt(d*x + c)*D*a^4*d^3 + 7*sqrt(d*x + c)*C*a^3*b*d
^3 - 3*sqrt(d*x + c)*B*a^2*b^2*d^3 - sqrt(d*x + c)*A*a*b^3*d^3)/((b^5*c -
a*b^4*d)*((d*x + c)*b - b*c + a*d)^2) + 2/3*((d*x + c)^(3/2)*D*b^6*d^2 - 9
*sqrt(d*x + c)*D*a*b^5*d^3 + 3*sqrt(d*x + c)*C*b^6*d^3)/(b^9*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \int \frac{\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^3}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3,x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*
c)))*a**4*d**2 - 120*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(
b)*sqrt(a*d - b*c)))*a**3*b*c*d + 210*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c
 + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*d**2*x + 12*sqrt(b)*sqrt(a*d
- b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*d + 24*
sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*
a**2*b**2*c**2 - 240*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(
b)*sqrt(a*d - b*c)))*a**2*b**2*c*d*x + 105*sqrt(b)*sqrt(a*d - b*c)*atan((s
qrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*d**2*x**2 + 24*sqrt(b
)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4
*d*x + 48*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
 - b*c)))*a*b**3*c**2*x - 120*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*
b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**3*c*d*x**2 + 12*sqrt(b)*sqrt(a*d - b*c)
*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**5*d*x**2 + 24*sqrt(b
)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**4*c
**2*x**2 - 105*sqrt(c + d*x)*a**4*b*d**2 + 155*sqrt(c + d*x)*a**3*b**2*c*d
 - 175*sqrt(c + d*x)*a**3*b**2*d**2*x - 12*sqrt(c + d*x)*a**2*b**4*d - 50*
sqrt(c + d*x)*a**2*b**3*c**2 + 263*sqrt(c + d*x)*a**2*b**3*c*d*x - 56*sqrt
(c + d*x)*a**2*b**3*d**2*x**2 + 12*sqrt(c + d*x)*a*b**5*c - 12*sqrt(c + d*
x)*a*b**5*d*x - 88*sqrt(c + d*x)*a*b**4*c**2*x + 88*sqrt(c + d*x)*a*b**...
```

3.62 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4} dx$

Optimal result 572
Mathematica [A] (verified) 573
Rubi [A] (verified) 573
Maple [A] (verified) 577
Fricas [B] (verification not implemented) 577
Sympy [F(-1)] 578
Maxima [F(-2)] 579
Giac [B] (verification not implemented) 579
Mupad [F(-1)] 580
Reduce [B] (verification not implemented) 581

Optimal result

Integrand size = 32, antiderivative size = 380

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4} dx$$
$$= \frac{2D\sqrt{c+dx}}{b^4} - \frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{3b^4(a+bx)^3}$$
$$- \frac{(b^3(6Bc+Ad)-ab^2(12cC+7Bd)-19a^3dD+a^2b(13Cd+18cD))\sqrt{c+dx}}{12b^4(bc-ad)(a+bx)^2}$$
$$- \frac{(b^3(8c^2C+2Bcd-Ad^2)-29a^3d^2D+a^2bd(11Cd+54cD)-ab^2(20cCd+Bd^2+24c^2D))\sqrt{c+dx}}{8b^4(bc-ad)^2(a+bx)}$$
$$+ \frac{(35a^3d^3D-5a^2bd^2(Cd+18cD)+ab^2d(12cCd-Bd^2+72c^2D)-b^3(8c^2Cd-2Bcd^2+Ad^3+16c^3D}{8b^{9/2}(bc-ad)^{5/2}}$$

output
```
2*D*(d*x+c)^(1/2)/b^4-1/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^4/
(b*x+a)^3-1/12*(b^3*(A*d+6*B*c)-a*b^2*(7*B*d+12*C*c)-19*a^3*d*D+a^2*b*(13*
C*d+18*D*c))*(d*x+c)^(1/2)/b^4/(-a*d+b*c)/(b*x+a)^2-1/8*(b^3*(-A*d^2+2*B*c
*d+8*C*c^2)-29*a^3*d^2*D+a^2*b*d*(11*C*d+54*D*c)-a*b^2*(B*d^2+20*C*c*d+24*
D*c^2))*(d*x+c)^(1/2)/b^4/(-a*d+b*c)^2/(b*x+a)+1/8*(35*a^3*d^3*D-5*a^2*b*d
^2*(C*d+18*D*c)+a*b^2*d*(-B*d^2+12*C*c*d+72*D*c^2)-b^3*(A*d^3-2*B*c*d^2+8*
C*c^2*d+16*D*c^3))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(9/2)
/(-a*d+b*c)^(5/2)
```

Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$$
$$= \frac{\sqrt{c+dx}(105a^5d^2D - 5a^4bd(3Cd+8D(5c-7dx)) - 6b^5cx(B(2c+dx)+4cx(C-2Dx)) + Ab^3(-3a^2c}{}$$
$$+ \frac{(-35a^3d^3D + 5a^2bd^2(Cd+18cD) + ab^2d(-12cCd+Bd^2-72c^2D) + b^3(8c^2Cd - 2Bcd^2 + Ad^3 + 16}{8b^{9/2}(-bc+ad)^{5/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
(Sqrt[c + d*x]*(105*a^5*d^2*D - 5*a^4*b*d*(3*C*d + 8*D*(5*c - 7*d*x)) - 6*
b^5*c*x*(B*(2*c + d*x) + 4*c*x*(C - 2*D*x)) + A*b^3*(-3*a^2*d^2 + 2*a*b*d*
(7*c + 4*d*x) + b^2*(-8*c^2 - 2*c*d*x + 3*d^2*x^2)) + a^3*b^2*(92*c^2*D +
c*(26*C*d - 538*d*D*x) + d^2*(-3*B - 40*C*x + 231*D*x^2)) + a^2*b^3*(c^2*(
-8*C + 252*D*x) + d^2*x*(-8*B - 33*C*x + 48*D*x^2) + 2*c*d*(2*B + 5*x*(7*C
 - 45*D*x))) + a*b^4*(B*(-4*c^2 + 14*c*d*x + 3*d^2*x^2) + 12*c*x*(-2*c*(C
- 9*D*x) + d*x*(5*C - 8*D*x)))))/(24*b^4*(b*c - a*d)^2*(a + b*x)^3) + ((-3
5*a^3*d^3*D + 5*a^2*b*d^2*(C*d + 18*c*D) + a*b^2*d*(-12*c*C*d + B*d^2 - 72
*c^2*D) + b^3*(8*c^2*C*d - 2*B*c*d^2 + A*d^3 + 16*c^3*D))*ArcTan[(Sqrt[b]*
Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(8*b^(9/2)*(-(b*c) + a*d)^(5/2))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1192, 1580, 1471, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx$$
$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{3\sqrt{c+dx}\left(2\left(c-\frac{ad}{b}\right)Dx^2+\frac{2(bc-ad)(bC-aD)x}{b^2}+\frac{-dDa^3+b(Cd+2cD)a^2-b^2(2cC+Bd)a+b^3(2Bc-Ad)}{b^3}\right)}{2(a+bx)^3}dx}{3(bc-ad)}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\left(-\frac{dDa^3}{b^3}+\frac{(Cd+2cD)a^2}{b^2}-\frac{(2cC+Bd)a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc-Ad+\frac{2(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^3}dx}{2(bc-ad)}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 1192$$

$$\frac{\int \frac{(c+dx)\left(-2Dc^3+2Cdc^2-2Bd^2c-2\left(c-\frac{ad}{b}\right)D(c+dx)^2+\frac{d^3\left(Ab^3+a\left(Da^2-bCa+b^2B\right)\right)}{b^3}-\frac{2(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^3}d\sqrt{c+dx}}{bc-ad}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 1580$$

$$\frac{\frac{\int \frac{\frac{\left(-5dDa^3+3b(Cd+2cD)a^2-b^2(4cC+Bd)a+b^3(2Bc-Ad)\right)d^2}{b}+8b(bc-ad)D(c+dx)^2+8(bc-ad)(bCd-2aDd-bcD)(c+dx)}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{4b^3}-\frac{d^2\sqrt{c+dx}\left(-5a^3dD-\cdots\right.}{\cdots}}{bc-ad}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 1471$$

$$\frac{\frac{\frac{d\sqrt{c+dx}\left(-29a^3d^2D+a^2bd(54cD+11Cd)-ab^2\left(Bd^2+24c^2D+20cCd\right)+b^3\left(-Ad^2+2Bcd+8c^2C\right)\right)}{2b(bc-ad)(-ad-b(c+dx)+bc)}-\frac{\int \frac{16D(c+dx)(bc-ad)^2+d\left(-\frac{19d^2Da^3}{b}+d(5Cd+42cD)a^2-b\cdots\right.}{bc-ad-b(c-\cdots}}{2(bc-\cdots}}{4b^3}}{bc-ad}-\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{d\sqrt{c+dx}\left(-29a^3d^2D+a^2bd(54cD+11Cd)-ab^2\left(Bd^2+24c^2D+20cCd\right)+b^3\left(-Ad^2+2Bcd+8c^2C\right)\right)}{2b(bc-ad)(-ad-b(c+dx)+bc)}-\frac{-\frac{\left(35a^3d^3D-5a^2bd^2(18cD+Cd)+ab^2d\left(-Bd^2+72c^2D+12c\right.\right.}{}}{4b^3}}{}$$

$$\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{d\sqrt{c+dx}\left(-29a^3d^2D+a^2bd(54cD+11Cd)-ab^2\left(Bd^2+24c^2D+20cCd\right)+b^3\left(-Ad^2+2Bcd+8c^2C\right)\right)}{2b(bc-ad)(-ad-b(c+dx)+bc)}-\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(35a^3d^3D-5a^2bd^2(18cD+Cd)+\right.}{b}}{4b^3}}{}$$

$$\frac{(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output

```
-1/3*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(3/2))/(b^3*(b*c - a*d
)*(a + b*x)^3) + (-1/4*(d^2*(b^3*(2*B*c - A*d) - a*b^2*(4*c*C + B*d) - 5*a
^3*d*D + 3*a^2*b*(C*d + 2*c*D))*Sqrt[c + d*x])/(b^4*(b*c - a*d - b*(c + d*
x))^2) + ((d*(b^3*(8*c^2*C + 2*B*c*d - A*d^2) - 29*a^3*d^2*D + a^2*b*d*(11
*C*d + 54*c*D) - a*b^2*(20*c*C*d + B*d^2 + 24*c^2*D))*Sqrt[c + d*x])/(2*b*
(b*c - a*d)*(b*c - a*d - b*(c + d*x))) - ((-16*(b*c - a*d)^2*D*Sqrt[c + d*
x])/b - ((35*a^3*d^3*D - 5*a^2*b*d^2*(C*d + 18*c*D) + a*b^2*d*(12*c*C*d -
B*d^2 + 72*c^2*D) - b^3*(8*c^2*C*d - 2*B*c*d^2 + A*d^3 + 16*c^3*D))*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/(2*
(b*c - a*d)))/(4*b^3))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$-\frac{-\left(b^{3}\left(A d^{3}-2 B c d^{2}+8 C c^{2} d+16 D c^{3}\right)+a d\left(B d^{2}-12 C c d-72 D c^{2}\right) b^{2}+5 a^{2} b d^{2}\left(C d+18 D c\right)-35 a^{3} d^{3} D\right)\left(b x+a\right)^{3} \operatorname{arcta}}{}$
derivativedivides	$\frac{2 D \sqrt{x d+c}}{b^{4}}+\frac{2\left(\frac{b^{2} d\left(b^{3} d^{2} A+B a b^{2} d^{2}-2 B b^{3} c d-11 C a^{2} b d^{2}+20 C a b^{2} c d-8 C b^{3} c^{2}+29 a^{3} d^{2} D-54 a^{2} b c d D+24 a b^{2} c^{2} D\right)\left(x d+c\right)^{\frac{5}{2}}}{16 a^{2} d^{2}-32 a b c d+16 b^{2} c^{2}}\right.}{}$
default	$\frac{2 D \sqrt{x d+c}}{b^{4}}+\frac{2\left(\frac{b^{2} d\left(b^{3} d^{2} A+B a b^{2} d^{2}-2 B b^{3} c d-11 C a^{2} b d^{2}+20 C a b^{2} c d-8 C b^{3} c^{2}+29 a^{3} d^{2} D-54 a^{2} b c d D+24 a b^{2} c^{2} D\right)\left(x d+c\right)^{\frac{5}{2}}}{16 a^{2} d^{2}-32 a b c d+16 b^{2} c^{2}}\right.}{}$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-(b^3*(A*d^3-2*B*c*d^2+8*C*c^2*d+16*D*c^3)+a*d*(B*d^2-12*C*c*d-72*D*
c^2)*b^2+5*a^2*b*d^2*(C*d+18*D*c)-35*a^3*d^3*D)*(b*x+a)^3*arctan(b*(d*x+c)
^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((-A*d^2*x^2+2/3*c*x*(3*B*
x+A)*d+8/3*c^2*(-6*D*x^3+3*C*x^2+3/2*B*x+A))*b^5-14/3*(4/7*x*(3/8*B*x+A)*d
^2+c*(-48/7*D*x^3+30/7*C*x^2+B*x+A)*d-2/7*c^2*(-54*D*x^2+6*C*x+B))*a*b^4+(
(11*C*x^2+8/3*B*x-16*D*x^3+A)*d^2-4/3*c*(-225/2*D*x^2+35/2*C*x+B)*d+8/3*(-
63/2*D*x+C)*c^2)*a^2*b^3+((-77*D*x^2+40/3*C*x+B)*d^2-26/3*c*(-269/13*D*x+C
)*d-92/3*D*c^2)*a^3*b^2+5*((-56/3*D*x+C)*d+40/3*D*c)*d*a^4*b-35*D*a^5*d^2)
*(d*x+c)^(1/2))/((a*d-b*c)*b)^(1/2)/(a*d-b*c)^2/b^4/(b*x+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. $2(356) = 712$.

Time = 0.17 (sec) , antiderivative size = 2354, normalized size of antiderivative = 6.19

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="fricas
")
```

output
```
[1/48*(3*(16*D*a^3*b^3*c^3 - 8*(9*D*a^4*b^2 - C*a^3*b^3)*c^2*d + 2*(45*D*a
^5*b - 6*C*a^4*b^2 - B*a^3*b^3)*c*d^2 - (35*D*a^6 - 5*C*a^5*b - B*a^4*b^2
- A*a^3*b^3)*d^3 + (16*D*b^6*c^3 - 8*(9*D*a*b^5 - C*b^6)*c^2*d + 2*(45*D*a
^2*b^4 - 6*C*a*b^5 - B*b^6)*c*d^2 - (35*D*a^3*b^3 - 5*C*a^2*b^4 - B*a*b^5
- A*b^6)*d^3)*x^3 + 3*(16*D*a*b^5*c^3 - 8*(9*D*a^2*b^4 - C*a*b^5)*c^2*d +
2*(45*D*a^3*b^3 - 6*C*a^2*b^4 - B*a*b^5)*c*d^2 - (35*D*a^4*b^2 - 5*C*a^3*b
^3 - B*a^2*b^4 - A*a*b^5)*d^3)*x^2 + 3*(16*D*a^2*b^4*c^3 - 8*(9*D*a^3*b^3
- C*a^2*b^4)*c^2*d + 2*(45*D*a^4*b^2 - 6*C*a^3*b^3 - B*a^2*b^4)*c*d^2 - (3
5*D*a^5*b - 5*C*a^4*b^2 - B*a^3*b^3 - A*a^2*b^4)*d^3)*x)*sqrt(b^2*c - a*b*
d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x +
a)) + 2*(4*(23*D*a^3*b^4 - 2*C*a^2*b^5 - B*a*b^6 - 2*A*b^7)*c^3 - 2*(146*D
*a^4*b^3 - 17*C*a^3*b^4 - 4*B*a^2*b^5 - 11*A*a*b^6)*c^2*d + (305*D*a^5*b^2
 - 41*C*a^4*b^3 - 7*B*a^3*b^4 - 17*A*a^2*b^5)*c*d^2 - 3*(35*D*a^6*b - 5*C*
a^5*b^2 - B*a^4*b^3 - A*a^3*b^4)*d^3 + 48*(D*b^7*c^3 - 3*D*a*b^6*c^2*d + 3
*D*a^2*b^5*c*d^2 - D*a^3*b^4*d^3)*x^3 + 3*(8*(9*D*a*b^6 - C*b^7)*c^3 - 2*(
111*D*a^2*b^5 - 14*C*a*b^6 + B*b^7)*c^2*d + (227*D*a^3*b^4 - 31*C*a^2*b^5
+ 3*B*a*b^6 + A*b^7)*c*d^2 - (77*D*a^4*b^3 - 11*C*a^3*b^4 + B*a^2*b^5 + A*
a*b^6)*d^3)*x^2 + 2*(6*(21*D*a^2*b^5 - 2*C*a*b^6 - B*b^7)*c^3 - (395*D*a^3
*b^4 - 47*C*a^2*b^5 - 13*B*a*b^6 + A*b^7)*c^2*d + (409*D*a^4*b^3 - 55*C*a^
3*b^4 - 11*B*a^2*b^5 + 5*A*a*b^6)*c*d^2 - 4*(35*D*a^5*b^2 - 5*C*a^4*b^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(356) = 712.

Time = 0.15 (sec) , antiderivative size = 938, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="giac")
```

output
```
1/8*(16*D*b^3*c^3 - 72*D*a*b^2*c^2*d + 8*C*b^3*c^2*d + 90*D*a^2*b*c*d^2 -
12*C*a*b^2*c*d^2 - 2*B*b^3*c*d^2 - 35*D*a^3*d^3 + 5*C*a^2*b*d^3 + B*a*b^2*
d^3 + A*b^3*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^6*c^2 -
2*a*b^5*c*d + a^2*b^4*d^2)*sqrt(-b^2*c + a*b*d)) + 2*sqrt(d*x + c)*D/b^4 +
 1/24*(72*(d*x + c)^(5/2)*D*a*b^4*c^2*d - 24*(d*x + c)^(5/2)*C*b^5*c^2*d -
 144*(d*x + c)^(3/2)*D*a*b^4*c^3*d + 48*(d*x + c)^(3/2)*C*b^5*c^3*d + 72*s
qrt(d*x + c)*D*a*b^4*c^4*d - 24*sqrt(d*x + c)*C*b^5*c^4*d - 162*(d*x + c)^
(5/2)*D*a^2*b^3*c*d^2 + 60*(d*x + c)^(5/2)*C*a*b^4*c*d^2 - 6*(d*x + c)^(5/
2)*B*b^5*c*d^2 + 432*(d*x + c)^(3/2)*D*a^2*b^3*c^2*d^2 - 144*(d*x + c)^(3/
2)*C*a*b^4*c^2*d^2 - 270*sqrt(d*x + c)*D*a^2*b^3*c^3*d^2 + 84*sqrt(d*x + c
)*C*a*b^4*c^3*d^2 + 6*sqrt(d*x + c)*B*b^5*c^3*d^2 + 87*(d*x + c)^(5/2)*D*a
^3*b^2*d^3 - 33*(d*x + c)^(5/2)*C*a^2*b^3*d^3 + 3*(d*x + c)^(5/2)*B*a*b^4*
d^3 + 3*(d*x + c)^(5/2)*A*b^5*d^3 - 424*(d*x + c)^(3/2)*D*a^3*b^2*c*d^3 +
136*(d*x + c)^(3/2)*C*a^2*b^3*c*d^3 + 8*(d*x + c)^(3/2)*B*a*b^4*c*d^3 - 8*
(d*x + c)^(3/2)*A*b^5*c*d^3 + 381*sqrt(d*x + c)*D*a^3*b^2*c^2*d^3 - 111*sq
rt(d*x + c)*C*a^2*b^3*c^2*d^3 - 15*sqrt(d*x + c)*B*a*b^4*c^2*d^3 - 3*sqrt(
d*x + c)*A*b^5*c^2*d^3 + 136*(d*x + c)^(3/2)*D*a^4*b*d^4 - 40*(d*x + c)^(3
/2)*C*a^3*b^2*d^4 - 8*(d*x + c)^(3/2)*B*a^2*b^3*d^4 + 8*(d*x + c)^(3/2)*A*
a*b^4*d^4 - 240*sqrt(d*x + c)*D*a^4*b*c*d^4 + 66*sqrt(d*x + c)*C*a^3*b^2*c
*d^4 + 12*sqrt(d*x + c)*B*a^2*b^3*c*d^4 + 6*sqrt(d*x + c)*A*a*b^4*c*d^4...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx=\int \frac{\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^4}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4,x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1279, normalized size of antiderivative = 3.37

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x)
```

output
```
( - 105*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
 b*c)))*a**5*d**3 + 180*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sq
rt(b)*sqrt(a*d - b*c)))*a**4*b*c*d**2 - 315*sqrt(b)*sqrt(a*d - b*c)*atan((
sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**4*b*d**3*x + 6*sqrt(b)*sqrt
(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**3*d*
*2 - 72*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
 b*c)))*a**3*b**2*c**2*d + 540*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)
*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*c*d**2*x - 315*sqrt(b)*sqrt(a*d -
 b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*d**3*x**
2 + 18*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
b*c)))*a**2*b**4*d**2*x - 216*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*
b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c**2*d*x + 540*sqrt(b)*sqrt(a*d -
b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c*d**2*x*
*2 - 105*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
- b*c)))*a**2*b**3*d**3*x**3 + 18*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d
*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**5*d**2*x**2 - 216*sqrt(b)*sqrt(a*d
- b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c**2*d*x**
2 + 180*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
 b*c)))*a*b**4*c*d**2*x**3 + 6*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)
*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**6*d**2*x**3 - 72*sqrt(b)*sqrt(a*d - b...
```

3.63 $\int (a+bx)^3(c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result 582
Mathematica [A] (verified) 583
Rubi [A] (verified) 584
Maple [A] (verified) 585
Fricas [B] (verification not implemented) 586
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Giac [B] (verification not implemented) 589
Mupad [F(-1)] 590
Reduce [B] (verification not implemented) 591

Optimal result

Integrand size = 32, antiderivative size = 438

$$\begin{aligned}
&\int (a+bx)^3(c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \\
&-\frac{2(bc-ad)^3 \left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{5/2}}{5d^7} \\
&-\frac{2(bc-ad)^2 \left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right)(c+dx)^{7/2}}{7d^7} \\
&-\frac{2(bc-ad) \left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)(c}{9d^7} \\
&+\frac{2(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D))(c}{11d^7} \\
&+\frac{2b(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D))(c+dx)^{13/2}}{13d^7} \\
&+\frac{2b^2(bCd-6bcD+3adD)(c+dx)^{15/2}}{15d^7}+\frac{2b^3D(c+dx)^{17/2}}{17d^7}
\end{aligned}$$

output
```
-2/5*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(5/2)/d^7-2/7*(-a*
d+b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*
c^3))*(d*x+c)^(7/2)/d^7-2/9*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^
2+8*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^(
9/2)/d^7+2/11*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d
-10*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(11/2)/d^7+2
/13*b*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*x
+c)^(13/2)/d^7+2/15*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(15/2)/d^7+2/17*b^
3*D*(d*x+c)^(17/2)/d^7
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.14

$$\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2(c+dx)^{5/2}\left(221a^3d^3(-48c^3D+8c^2d(11C+15Dx)-2cd^2(99B+5x(22C+21Dx))+d^3(6\right.}{}$$

input
```
Integrate[(a + b*x)^3*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(5/2)*(221*a^3*d^3*(-48*c^3*D + 8*c^2*d*(11*C + 15*D*x) - 2*c
*d^2*(99*B + 5*x*(22*C + 21*D*x)) + d^3*(693*A + 5*x*(99*B + 77*C*x + 63*D
*x^2))) + 51*a^2*b*d^2*(384*c^4*D - 48*c^3*d*(13*C + 20*D*x) + 8*c^2*d^2*(
143*B + 15*x*(13*C + 14*D*x)) + 5*d^4*x*(1287*A + 7*x*(143*B + 117*C*x + 9
9*D*x^2)) - 2*c*d^3*(1287*A + 5*x*(286*B + 273*C*x + 252*D*x^2))) + 51*a*b
^2*d*(-256*c^5*D + 128*c^4*d*(3*C + 5*D*x) + 7*d^5*x^2*(715*A + 585*B*x +
495*C*x^2 + 429*D*x^3) - 16*c^3*d^2*(39*B + 10*x*(6*C + 7*D*x)) - 10*c*d^4
*x*(286*A + 21*x*(13*B + 12*C*x + 11*D*x^2)) + 8*c^2*d^3*(143*A + 15*x*(13
*B + 14*x*(C + D*x)))) + b^3*(3072*c^6*D - 256*c^5*d*(17*C + 30*D*x) + 128
*c^4*d^2*(51*B + 5*x*(17*C + 21*D*x)) + 120*c^2*d^4*x*(221*A + 7*x*(34*B +
 34*C*x + 33*D*x^2)) + 21*d^6*x^3*(3315*A + 11*x*(255*B + 221*C*x + 195*D*
x^2)) - 42*c*d^5*x^2*(1105*A + x*(1020*B + 935*C*x + 858*D*x^2)) - 16*c^3*
d^3*(663*A + 10*x*(102*B + 7*x*(17*C + 18*D*x))))))/(765765*d^7)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{7/2}(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(c+dx)^{9/2}(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2\right.\right.}{9d^7}$$
$$\frac{2b(c+dx)^{13/2}\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{13d^7}+$$
$$\frac{2(c+dx)^{11/2}\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c\right.\right.}{11d^7}$$
$$\frac{2(c+dx)^{7/2}(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{7d^7}-$$
$$\frac{2(c+dx)^{5/2}(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^7}+$$
$$\frac{2b^2(c+dx)^{15/2}(3adD-6bcD+bCd)}{15d^7}+\frac{2b^3D(c+dx)^{17/2}}{17d^7}$$

input

```
Int[(a + b*x)^3*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-2*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(5/2))/(5*
d^7) - (2*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d -
4*B*c*d^2 + 3*A*d^3 - 6*c^3*D))*(c + d*x)^(7/2))/(7*d^7) - (2*(b*c - a*d)*
(a^2*d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^
2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(9/2))/(9*d^7) + (2*(a^
3*d^3*D + 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*
D) + b^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(11/2))/(1
1*d^7) + (2*b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^2
- 15*c^2*D))*(c + d*x)^(13/2))/(13*d^7) + (2*b^2*(b*C*d - 6*b*c*D + 3*a*d*
D)*(c + d*x)^(15/2))/(15*d^7) + (2*b^3*D*(c + d*x)^(17/2))/(17*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$2\left(\left(\frac{5x^3\left(\frac{11}{17}Dx^3+\frac{11}{15}Cx^2+\frac{11}{13}Bx+A\right)b^3}{11}+\frac{5\left(\frac{3}{5}Dx^3+\frac{9}{13}Cx^2+\frac{9}{11}Bx+A\right)x^2ab^2}{3}+\frac{15\left(\frac{7}{13}Dx^3+\frac{7}{11}Cx^2+\frac{7}{9}Bx+A\right)xa^2b}{7}+a^3\left(\right.\right.\right.$
derivativedivides	$\frac{2b^3D(xd+c)^{\frac{17}{2}}}{17}+\frac{2\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{15}{2}}}{15}+\frac{2\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(Bd^2-2Ccd+3Dc^2\right)\right)(}{13}$
default	$\frac{2b^3D(xd+c)^{\frac{17}{2}}}{17}+\frac{2\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{15}{2}}}{15}+\frac{2\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(Bd^2-2Ccd+3Dc^2\right)\right)(}{13}$
gosper	$2(xd+c)^{\frac{5}{2}}\left(45045Dx^6b^3d^6+51051Cx^5b^3d^6+153153Dx^5ab^2d^6-36036Dx^5b^3cd^5+58905Bx^4b^3d^6+176715Cx^4ab^2d^6\right.$
orering	$2(xd+c)^{\frac{5}{2}}\left(45045Dx^6b^3d^6+51051Cx^5b^3d^6+153153Dx^5ab^2d^6-36036Dx^5b^3cd^5+58905Bx^4b^3d^6+176715Cx^4ab^2d^6\right.$
trager	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/5*((5/11*x^3*(11/17*D*x^3+11/15*C*x^2+11/13*B*x+A)*b^3+5/3*(3/5*D*x^3+9/
13*C*x^2+9/11*B*x+A)*x^2*a*b^2+15/7*(7/13*D*x^3+7/11*C*x^2+7/9*B*x+A)*x*a^
2*b+a^3*(5/11*D*x^3+5/9*C*x^2+5/7*B*x+A))*d^6-6/7*(35/99*x^2*(66/85*D*x^3+
11/13*C*x^2+12/13*B*x+A)*b^3+10/9*x*(21/26*D*x^3+126/143*C*x^2+21/22*B*x+A
)*a*b^2+a^2*(140/143*D*x^3+35/33*C*x^2+10/9*B*x+A)*b+1/3*(35/33*D*x^2+10/9
*C*x+B)*a^3)*c*d^5+8/21*(5/11*x*(231/221*D*x^3+14/13*C*x^2+14/13*B*x+A)*b^
3+a*(210/143*D*x^3+210/143*C*x^2+15/11*B*x+A)*b^2+a^2*(210/143*D*x^2+15/11
*C*x+B)*b+1/3*(15/11*D*x+C)*a^3)*c^2*d^4-16/231*((420/221*D*x^3+70/39*C*x^
2+20/13*B*x+A)*b^3+3*(70/39*D*x^2+20/13*C*x+B)*a*b^2+3*a^2*(20/13*D*x+C)*b
+a^3*D)*c^3*d^3+128/3003*((35/17*D*x^2+5/3*C*x+B)*b^2+3*(5/3*D*x+C)*a*b+3*
D*a^2)*c^4*b*d^2-256/9009*c^5*b^2*((30/17*D*x+C)*b+3*D*a)*d+1024/51051*D*b
^3*c^6)*(d*x+c)^(5/2)/d^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(412) = 824$.

Time = 0.08 (sec) , antiderivative size = 944, normalized size of antiderivative = 2.16

$$\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
2/765765*(45045*D*b^3*d^8*x^8 + 3072*D*b^3*c^8 + 153153*A*a^3*c^2*d^6 - 43
52*(3*D*a*b^2 + C*b^3)*c^7*d + 6528*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^6*d^
2 - 10608*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^5*d^3 + 19448*(C*a^3 +
 3*B*a^2*b + 3*A*a*b^2)*c^4*d^4 - 43758*(B*a^3 + 3*A*a^2*b)*c^3*d^5 + 3003
*(18*D*b^3*c*d^7 + 17*(3*D*a*b^2 + C*b^3)*d^8)*x^7 + 231*(3*D*b^3*c^2*d^6
+ 272*(3*D*a*b^2 + C*b^3)*c*d^7 + 255*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^8)
*x^6 - 63*(12*D*b^3*c^3*d^5 - 17*(3*D*a*b^2 + C*b^3)*c^2*d^6 - 1190*(3*D*a
^2*b + 3*C*a*b^2 + B*b^3)*c*d^7 - 1105*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*
b^3)*d^8)*x^5 + 35*(24*D*b^3*c^4*d^4 - 34*(3*D*a*b^2 + C*b^3)*c^3*d^5 + 51
*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^6 + 2652*(D*a^3 + 3*C*a^2*b + 3*B*a
*b^2 + A*b^3)*c*d^7 + 2431*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^8)*x^4 - 5*(1
92*D*b^3*c^5*d^3 - 272*(3*D*a*b^2 + C*b^3)*c^4*d^4 + 408*(3*D*a^2*b + 3*C*
a*b^2 + B*b^3)*c^3*d^5 - 663*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d
^6 - 24310*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^7 - 21879*(B*a^3 + 3*A*a^2*
b)*d^8)*x^3 + 3*(384*D*b^3*c^6*d^2 + 51051*A*a^3*d^8 - 544*(3*D*a*b^2 + C*
b^3)*c^5*d^3 + 816*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^4 - 1326*(D*a^3 +
 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^5 + 2431*(C*a^3 + 3*B*a^2*b + 3*A*a*
b^2)*c^2*d^6 + 58344*(B*a^3 + 3*A*a^2*b)*c*d^7)*x^2 - (1536*D*b^3*c^7*d -
306306*A*a^3*c*d^7 - 2176*(3*D*a*b^2 + C*b^3)*c^6*d^2 + 3264*(3*D*a^2*b +
3*C*a*b^2 + B*b^3)*c^5*d^3 - 5304*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(456) = 912$.

Time = 2.09 (sec) , antiderivative size = 1028, normalized size of antiderivative = 2.35

$$\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b**3*(c + d*x)**(17/2)/(17*d**6) + (c + d*x)**(15/2)*(C*b*
*3*d + 3*D*a*b**2*d - 6*D*b**3*c)/(15*d**6) + (c + d*x)**(13/2)*(B*b**3*d*
*2 + 3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d +
15*D*b**3*c**2)/(13*d**6) + (c + d*x)**(11/2)*(A*b**3*d**3 + 3*B*a*b**2*d*
*3 - 4*B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c*
*2*d + D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c
**3)/(11*d**6) + (c + d*x)**(9/2)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*B
*a**2*b*d**4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C*
a**2*b*c*d**3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**3
 + 18*D*a**2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/(9*d**6) +
 (c + d*x)**(7/2)*(3*A*a**2*b*d**5 - 6*A*a*b**2*c*d**4 + 3*A*b**3*c**2*d**
3 + B*a**3*d**5 - 6*B*a**2*b*c*d**4 + 9*B*a*b**2*c**2*d**3 - 4*B*b**3*c**3
*d**2 - 2*C*a**3*c*d**4 + 9*C*a**2*b*c**2*d**3 - 12*C*a*b**2*c**3*d**2 + 5
*C*b**3*c**4*d + 3*D*a**3*c**2*d**3 - 12*D*a**2*b*c**3*d**2 + 15*D*a*b**2*
c**4*d - 6*D*b**3*c**5)/(7*d**6) + (c + d*x)**(5/2)*(A*a**3*d**6 - 3*A*a**
2*b*c*d**5 + 3*A*a*b**2*c**2*d**4 - A*b**3*c**3*d**3 - B*a**3*c*d**5 + 3*B
*a**2*b*c**2*d**4 - 3*B*a*b**2*c**3*d**3 + B*b**3*c**4*d**2 + C*a**3*c**2*
d**4 - 3*C*a**2*b*c**3*d**3 + 3*C*a*b**2*c**4*d**2 - C*b**3*c**5*d - D*a**
3*c**3*d**3 + 3*D*a**2*b*c**4*d**2 - 3*D*a*b**2*c**5*d + D*b**3*c**6)/(5*d
**6))/d, Ne(d, 0)), (c**(3/2)*(A*a**3*x + D*b**3*x**7/7 + x**6*(C*b**3 ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.42

$$\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2\left(45045\,(dx+c)^{\frac{17}{2}}Db^3-51051\,(6\,Db^3c-(3\,Dab^2+Cb^3)d)(dx+c)^{\frac{15}{2}}+58905\,(15\,Db^3c^2-\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
2/765765*(45045*(d*x + c)^(17/2)*D*b^3 - 51051*(6*D*b^3*c - (3*D*a*b^2 + C
*b^3)*d)*(d*x + c)^(15/2) + 58905*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^3)*c*
d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(13/2) - 69615*(20*D*b^
3*c^3 - 10*(3*D*a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c
*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(11/2) + 850
85*(15*D*b^3*c^4 - 10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*b^2
 + B*b^3)*c^2*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (C*a
^3 + 3*B*a^2*b + 3*A*a*b^2)*d^4)*(d*x + c)^(9/2) - 109395*(6*D*b^3*c^5 - 5
*(3*D*a*b^2 + C*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2 - 3
*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3*B*a^2*b +
3*A*a*b^2)*c*d^4 - (B*a^3 + 3*A*a^2*b)*d^5)*(d*x + c)^(7/2) + 153153*(D*b^
3*c^6 + A*a^3*d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B
*b^3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 +
 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)*(d*x + c)^(5/
2))/d^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3198 vs. $2(412) = 824$.

Time = 0.17 (sec) , antiderivative size = 3198, normalized size of antiderivative = 7.30

$$\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/765765*(765765*sqrt(d*x + c)*A*a^3*c^2 + 510510*((d*x + c)^(3/2) - 3*sqr
t(d*x + c)*c)*A*a^3*c + 255255*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*a^3
*c^2/d + 765765*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*a^2*b*c^2/d + 5105
1*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a^3
+ 51051*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*
C*a^3*c^2/d^2 + 153153*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt
(d*x + c)*c^2)*B*a^2*b*c^2/d^2 + 153153*(3*(d*x + c)^(5/2) - 10*(d*x + c)^
(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a*b^2*c^2/d^2 + 102102*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*a^3*c/d + 306306*(3*(d*
x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a^2*b*c/d +
21879*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*D*a^3*c^2/d^3 + 65637*(5*(d*x + c)^(7/2) - 21*(d*x
+ c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*a^2*b*c^2/
d^3 + 65637*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)
*c^2 - 35*sqrt(d*x + c)*c^3)*B*a*b^2*c^2/d^3 + 21879*(5*(d*x + c)^(7/2) -
21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A*b^
3*c^2/d^3 + 43758*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)
^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*a^3*c/d^2 + 131274*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B
*a^2*b*c/d^2 + 131274*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+b\,x)^3\,(c+d\,x)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^3*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^3*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.65

$$\int (a+bx)^3(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\sqrt{dx+c}\,(45045b^3d^8x^8+153153a\,b^2d^8x^7+105105b^3c\,d^7x^7+176715a^2b\,d^8x^6+365211a\,b^2c\,d\cdots}{\cdots}$$

input
```
int((b*x+a)^3*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(153153*a**4*c**2*d**5 + 306306*a**4*c*d**6*x + 153153*a*
*4*d**7*x**2 - 175032*a**3*b*c**3*d**4 + 87516*a**3*b*c**2*d**5*x + 700128
*a**3*b*c*d**6*x**2 + 437580*a**3*b*d**7*x**3 + 8840*a**3*c**5*d**3 - 4420
*a**3*c**4*d**4*x + 3315*a**3*c**3*d**5*x**2 + 124865*a**3*c**2*d**6*x**3
+ 177905*a**3*c*d**7*x**4 + 69615*a**3*d**8*x**5 + 116688*a**2*b**2*c**4*d
**3 - 58344*a**2*b**2*c**3*d**4*x + 43758*a**2*b**2*c**2*d**5*x**2 + 72930
0*a**2*b**2*c*d**6*x**3 + 510510*a**2*b**2*d**7*x**4 - 12240*a**2*b*c**6*d
**2 + 6120*a**2*b*c**5*d**3*x - 4590*a**2*b*c**4*d**4*x**2 + 3825*a**2*b*c
**3*d**5*x**3 + 283815*a**2*b*c**2*d**6*x**4 + 433755*a**2*b*c*d**7*x**5 +
 176715*a**2*b*d**8*x**6 - 42432*a*b**3*c**5*d**2 + 21216*a*b**3*c**4*d**3
*x - 15912*a*b**3*c**3*d**4*x**2 + 13260*a*b**3*c**2*d**5*x**3 + 371280*a*
b**3*c*d**6*x**4 + 278460*a*b**3*d**7*x**5 + 6528*a*b**2*c**7*d - 3264*a*b
**2*c**6*d**2*x + 2448*a*b**2*c**5*d**3*x**2 - 2040*a*b**2*c**4*d**4*x**3
+ 1785*a*b**2*c**3*d**5*x**4 + 228123*a*b**2*c**2*d**6*x**5 + 365211*a*b**
2*c*d**7*x**6 + 153153*a*b**2*d**8*x**7 + 6528*b**4*c**6*d - 3264*b**4*c**
5*d**2*x + 2448*b**4*c**4*d**3*x**2 - 2040*b**4*c**3*d**4*x**3 + 1785*b**4
*c**2*d**5*x**4 + 74970*b**4*c*d**6*x**5 + 58905*b**4*d**7*x**6 - 1280*b**
3*c**8 + 640*b**3*c**7*d*x - 480*b**3*c**6*d**2*x**2 + 400*b**3*c**5*d**3*
x**3 - 350*b**3*c**4*d**4*x**4 + 315*b**3*c**3*d**5*x**5 + 63525*b**3*c**2
*d**6*x**6 + 105105*b**3*c*d**7*x**7 + 45045*b**3*d**8*x**8))/(765765*d...
```

3.64 $\int (a+bx)^2(c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result 592
Mathematica [A] (verified) 593
Rubi [A] (verified) 593
Maple [A] (verified) 595
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Optimal result

Integrand size = 32, antiderivative size = 326

$$\int (a+bx)^2(c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{2(bc-ad)^2\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{5/2}}{5d^6} + \frac{2(bc-ad)\left(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D)\right)(c+dx)^{7/2}}{7d^6} + \frac{2\left(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D)\right)(c+dx)^{9/2}}{9d^6} + \frac{2\left(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D)\right)(c+dx)^{11/2}}{11d^6} + \frac{2b(bCd-5bcD+2adD)(c+dx)^{13/2}}{13d^6} + \frac{2b^2D(c+dx)^{15/2}}{15d^6}$$

output
```
2/5*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(5/2)/d^6+2/7*(-a*d
+b*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3
))*(d*x+c)^(7/2)/d^6+2/9*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*
c^2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(9/2)/d^6+2/11*(a^2
*d^2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(11/2)/d
^6+2/13*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(13/2)/d^6+2/15*b^2*D*(d*x+c)^(1
5/2)/d^6
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00

$$\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2(c+dx)^{5/2}\left(13a^2d^2(-48c^3D+8c^2d(11C+15Dx)-2cd^2(99B+5x(22C+21Dx))+d^3(69\right.}{}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(5/2)*(13*a^2*d^2*(-48*c^3*D + 8*c^2*d*(11*C + 15*D*x) - 2*c*
d^2*(99*B + 5*x*(22*C + 21*D*x)) + d^3*(693*A + 5*x*(99*B + 77*C*x + 63*D*
x^2))) + 2*a*b*d*(384*c^4*D - 48*c^3*d*(13*C + 20*D*x) + 8*c^2*d^2*(143*B
+ 15*x*(13*C + 14*D*x)) + 5*d^4*x*(1287*A + 7*x*(143*B + 117*C*x + 99*D*x^
2)) - 2*c*d^3*(1287*A + 5*x*(286*B + 273*C*x + 252*D*x^2))) + b^2*(-256*c^
5*D + 128*c^4*d*(3*C + 5*D*x) + 7*d^5*x^2*(715*A + 585*B*x + 495*C*x^2 + 4
29*D*x^3) - 16*c^3*d^2*(39*B + 10*x*(6*C + 7*D*x)) - 10*c*d^4*x*(286*A + 2
1*x*(13*B + 12*C*x + 11*D*x^2)) + 8*c^2*d^3*(143*A + 15*x*(13*B + 14*x*(C
+ D*x))))))/(45045*d^6)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{7/2}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c\right.$$

$\downarrow$ 2009

$$\frac{2(c+dx)^{9/2}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{9d^6}+\frac{2(c+dx)^{11/2}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{11d^6}+\frac{2(c+dx)^{7/2}(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{7d^6}+\frac{2(c+dx)^{5/2}(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^6}+\frac{2b(c+dx)^{13/2}(2adD-5bcD+bCd)}{13d^6}+\frac{2b^2D(c+dx)^{15/2}}{15d^6}$$

input
```
Int[(a + b*x)^2*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(5/2))/(5*d
^6) + (2*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B
*c*d^2 + 2*A*d^3 - 5*c^3*D))*(c + d*x)^(7/2))/(7*d^6) + (2*(a^2*d^2*(C*d -
 3*c*D) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2
 + A*d^3 - 10*c^3*D))*(c + d*x)^(9/2))/(9*d^6) + (2*(a^2*d^2*D + 2*a*b*d*(
C*d - 4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(11/2))/(11*d^6
) + (2*b*(b*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(13/2))/(13*d^6) + (2*b^2*D
*(c + d*x)^(15/2))/(15*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$2\left(\left(\frac{5\left(\frac{3}{5}Dx^3+\frac{9}{13}Cx^2+\frac{9}{11}Bx+A\right)x^2b^2}{9}+\frac{10\left(\frac{7}{13}Dx^3+\frac{7}{11}Cx^2+\frac{7}{9}Bx+A\right)xab}{7}+a^2\left(\frac{5}{11}Dx^3+\frac{5}{9}Cx^2+\frac{5}{7}Bx+A\right)\right)d^5-\frac{4\left(\frac{5x(\cdots}{\cdots}\right.}{\cdots}\right.$
derivativedivides	$\frac{2b^2D(xd+c)^{\frac{15}{2}}}{15}+\frac{2\left(2b(ad-bc)D+b^2(Cd-3Dc)\right)(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left((ad-bc)^2D+2b(ad-bc)(Cd-3Dc)+b^2\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+\cdots}{11}$
default	$\frac{2b^2D(xd+c)^{\frac{15}{2}}}{15}+\frac{2\left(2b(ad-bc)D+b^2(Cd-3Dc)\right)(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left((ad-bc)^2D+2b(ad-bc)(Cd-3Dc)+b^2\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+\cdots}{11}$
gosper	$\frac{2(xd+c)^{\frac{5}{2}}\left(3003Dx^5b^2d^5+3465C\,x^4b^2d^5+6930Dx^4ab\,d^5-2310Dx^4b^2c\,d^4+4095B\,x^3b^2d^5+8190C\,x^3ab\,d^5-2520C\,x^3\cdots\right.}{\cdots}$
orering	$\frac{2(xd+c)^{\frac{5}{2}}\left(3003Dx^5b^2d^5+3465C\,x^4b^2d^5+6930Dx^4ab\,d^5-2310Dx^4b^2c\,d^4+4095B\,x^3b^2d^5+8190C\,x^3ab\,d^5-2520C\,x^3\cdots\right.}{\cdots}$
trager	$\frac{2\left(3003d^7b^2Dx^7+3465C\,b^2d^7x^6+6930Dab\,d^7x^6+3696Db^2c\,d^6x^6+4095B\,b^2d^7x^5+8190Cab\,d^7x^5+4410C\,b^2c\,d^6x^5+4\cdots\right.}{\cdots}$

input
```
int((b*x+a)^2*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/5*((5/9*(3/5*D*x^3+9/13*C*x^2+9/11*B*x+A)*x^2*b^2+10/7*(7/13*D*x^3+7/11*
C*x^2+7/9*B*x+A)*x*a*b+a^2*(5/11*D*x^3+5/9*C*x^2+5/7*B*x+A))*d^5-4/7*(5/9*
x*(21/26*D*x^3+126/143*C*x^2+21/22*B*x+A)*b^2+a*(140/143*D*x^3+35/33*C*x^2
+10/9*B*x+A)*b+1/2*(35/33*D*x^2+10/9*C*x+B)*a^2)*c*d^4+8/63*((210/143*D*x^
3+210/143*C*x^2+15/11*B*x+A)*b^2+2*(210/143*D*x^2+15/11*C*x+B)*a*b+a^2*(15
/11*D*x+C))*c^2*d^3-16/231*((70/39*D*x^2+20/13*C*x+B)*b^2+2*a*(20/13*D*x+C
)*b+D*a^2)*c^3*d^2+128/3003*((5/3*D*x+C)*b+2*D*a)*c^4*b*d-256/9009*D*b^2*c
^5)*(d*x+c)^(5/2)/d^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. 2(302) = 604.

Time = 0.08 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.89

$$\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\left(3003\,Db^2d^7x^7-256\,Db^2c^7+9009\,Aa^2c^2d^5+384\,(2\,Dab+Cb^2)c^6d-624\,(Da^2+2\,Cab+\cdots\right.}{\cdots}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
2/45045*(3003*D*b^2*d^7*x^7 - 256*D*b^2*c^7 + 9009*A*a^2*c^2*d^5 + 384*(2*
D*a*b + C*b^2)*c^6*d - 624*(D*a^2 + 2*C*a*b + B*b^2)*c^5*d^2 + 1144*(C*a^2
 + 2*B*a*b + A*b^2)*c^4*d^3 - 2574*(B*a^2 + 2*A*a*b)*c^3*d^4 + 231*(16*D*b
^2*c*d^6 + 15*(2*D*a*b + C*b^2)*d^7)*x^6 + 63*(D*b^2*c^2*d^5 + 70*(2*D*a*b
 + C*b^2)*c*d^6 + 65*(D*a^2 + 2*C*a*b + B*b^2)*d^7)*x^5 - 35*(2*D*b^2*c^3*
d^4 - 3*(2*D*a*b + C*b^2)*c^2*d^5 - 156*(D*a^2 + 2*C*a*b + B*b^2)*c*d^6 -
143*(C*a^2 + 2*B*a*b + A*b^2)*d^7)*x^4 + 5*(16*D*b^2*c^4*d^3 - 24*(2*D*a*b
 + C*b^2)*c^3*d^4 + 39*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^5 + 1430*(C*a^2 + 2
*B*a*b + A*b^2)*c*d^6 + 1287*(B*a^2 + 2*A*a*b)*d^7)*x^3 - 3*(32*D*b^2*c^5*
d^2 - 3003*A*a^2*d^7 - 48*(2*D*a*b + C*b^2)*c^4*d^3 + 78*(D*a^2 + 2*C*a*b
+ B*b^2)*c^3*d^4 - 143*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^5 - 3432*(B*a^2 + 2
*A*a*b)*c*d^6)*x^2 + (128*D*b^2*c^6*d + 18018*A*a^2*c*d^6 - 192*(2*D*a*b +
 C*b^2)*c^5*d^2 + 312*(D*a^2 + 2*C*a*b + B*b^2)*c^4*d^3 - 572*(C*a^2 + 2*B
*a*b + A*b^2)*c^3*d^4 + 1287*(B*a^2 + 2*A*a*b)*c^2*d^5)*x)*sqrt(d*x + c)/d
^6
```

Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.97

$$\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\begin{cases} \frac{2\left(\frac{Db^2(c+dx)^{\frac{15}{2}}}{15d^5}+\frac{(c+dx)^{\frac{13}{2}}\left(Cb^2d+2Dabd-5Db^2c\right)}{13d^5}+\frac{(c+dx)^{\frac{11}{2}}\left(Bb^2d^2+2Cabd^2-4Cb^2cd+Da^2d^2-8Dabcd+10Db^2c^2\right)}{11d^5}+\frac{(c+dx)^{\frac{9}{2}}\left(A\right.}{}\right.}{} \\ c^{\frac{3}{2}}\left(Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5\left(Cb^2+2Dab\right)}{5}+\frac{x^4\left(Bb^2+2Cab+Da^2\right)}{4}+\frac{x^3\left(Ab^2+2Bab+Ca^2\right)}{3}+\frac{x^2\cdot\left(2Aab+Ba^2\right)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b**2*(c + d*x)**(15/2)/(15*d**5) + (c + d*x)**(13/2)*(C*b*
*2*d + 2*D*a*b*d - 5*D*b**2*c)/(13*d**5) + (c + d*x)**(11/2)*(B*b**2*d**2
+ 2*C*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2
)/(11*d**5) + (c + d*x)**(9/2)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d*
*2 + C*a**2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 12
*D*a*b*c**2*d - 10*D*b**2*c**3)/(9*d**5) + (c + d*x)**(7/2)*(2*A*a*b*d**4
- 2*A*b**2*c*d**3 + B*a**2*d**4 - 4*B*a*b*c*d**3 + 3*B*b**2*c**2*d**2 - 2*
C*a**2*c*d**3 + 6*C*a*b*c**2*d**2 - 4*C*b**2*c**3*d + 3*D*a**2*c**2*d**2 -
 8*D*a*b*c**3*d + 5*D*b**2*c**4)/(7*d**5) + (c + d*x)**(5/2)*(A*a**2*d**5
- 2*A*a*b*c*d**4 + A*b**2*c**2*d**3 - B*a**2*c*d**4 + 2*B*a*b*c**2*d**3 -
B*b**2*c**3*d**2 + C*a**2*c**2*d**3 - 2*C*a*b*c**3*d**2 + C*b**2*c**4*d -
D*a**2*c**3*d**2 + 2*D*a*b*c**4*d - D*b**2*c**5)/(5*d**5))/d, Ne(d, 0)), (
c**(3/2)*(A*a**2*x + D*b**2*x**6/6 + x**5*(C*b**2 + 2*D*a*b)/5 + x**4*(B*b
**2 + 2*C*a*b + D*a**2)/4 + x**3*(A*b**2 + 2*B*a*b + C*a**2)/3 + x**2*(2*A
*a*b + B*a**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.19

$$\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2\left(3003\,(dx+c)^{\frac{15}{2}}Db^2 - 3465\,(5\,Db^2c - (2\,Dab+Cb^2)d)(dx+c)^{\frac{13}{2}} + 4095\,(10\,Db^2c^2 - 4\,(2\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
2/45045*(3003*(d*x + c)^(15/2)*D*b^2 - 3465*(5*D*b^2*c - (2*D*a*b + C*b^2)
*d)*(d*x + c)^(13/2) + 4095*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d + (D*a
^2 + 2*C*a*b + B*b^2)*d^2)*(d*x + c)^(11/2) - 5005*(10*D*b^2*c^3 - 6*(2*D*
a*b + C*b^2)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*b
+ A*b^2)*d^3)*(d*x + c)^(9/2) + 6435*(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^2)*c^
3*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*
d^3 + (B*a^2 + 2*A*a*b)*d^4)*(d*x + c)^(7/2) - 9009*(D*b^2*c^5 - A*a^2*d^5
 - (2*D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 +
2*B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)*(d*x + c)^(5/2))/d^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2150 vs. $2(302) = 604$.

Time = 0.15 (sec) , antiderivative size = 2150, normalized size of antiderivative = 6.60

$$\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(d*x + c)*A*a^2*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d
*x + c)*c)*A*a^2*c + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*a^2*c^2
/d + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*a*b*c^2/d + 3003*(3*(d*
x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a^2 + 3003*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a^2*c^2
/d^2 + 6006*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*B*a*b*c^2/d^2 + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sq
rt(d*x + c)*c^2)*A*b^2*c^2/d^2 + 6006*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3
/2)*c + 15*sqrt(d*x + c)*c^2)*B*a^2*c/d + 12012*(3*(d*x + c)^(5/2) - 10*(d
*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a*b*c/d + 1287*(5*(d*x + c)^(7/2
) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*
D*a^2*c^2/d^3 + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*a*b*c^2/d^3 + 1287*(5*(d*x + c)^(7
/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3
)*B*b^2*c^2/d^3 + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x
 + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*a^2*c/d^2 + 5148*(5*(d*x + c)^(7
/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3
)*B*a*b*c/d^2 + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A*b^2*c/d^2 + 1287*(5*(d*x + c)^(7/2
) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+b\,x)^2\,(c+d\,x)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^2*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^2*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.48

$$\int (a+bx)^2(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\sqrt{dx+c}\,(3003b^2d^7x^7+6930ab\,d^7x^6+7161b^2c\,d^6x^6+4095a^2d^7x^5+17010abc\,d^6x^5+4095b^3$$

input
```
int((b*x+a)^2*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(9009*a**3*c**2*d**4 + 18018*a**3*c*d**5*x + 9009*a**3*d*
*6*x**2 - 7722*a**2*b*c**3*d**3 + 3861*a**2*b*c**2*d**4*x + 30888*a**2*b*c
*d**5*x**2 + 19305*a**2*b*d**6*x**3 + 520*a**2*c**5*d**2 - 260*a**2*c**4*d
**3*x + 195*a**2*c**3*d**4*x**2 + 7345*a**2*c**2*d**5*x**3 + 10465*a**2*c*
d**6*x**4 + 4095*a**2*d**7*x**5 + 3432*a*b**2*c**4*d**2 - 1716*a*b**2*c**3
*d**3*x + 1287*a*b**2*c**2*d**4*x**2 + 21450*a*b**2*c*d**5*x**3 + 15015*a*
b**2*d**6*x**4 - 480*a*b*c**6*d + 240*a*b*c**5*d**2*x - 180*a*b*c**4*d**3*
x**2 + 150*a*b*c**3*d**4*x**3 + 11130*a*b*c**2*d**5*x**4 + 17010*a*b*c*d**
6*x**5 + 6930*a*b*d**7*x**6 - 624*b**3*c**5*d + 312*b**3*c**4*d**2*x - 234
*b**3*c**3*d**3*x**2 + 195*b**3*c**2*d**4*x**3 + 5460*b**3*c*d**5*x**4 + 4
095*b**3*d**6*x**5 + 128*b**2*c**7 - 64*b**2*c**6*d*x + 48*b**2*c**5*d**2*
x**2 - 40*b**2*c**4*d**3*x**3 + 35*b**2*c**3*d**4*x**4 + 4473*b**2*c**2*d*
*5*x**5 + 7161*b**2*c*d**6*x**6 + 3003*b**2*d**7*x**7))/(45045*d**5)
```

3.65 $\int (a+bx)(c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result 600
Mathematica [A] (verified) 601
Rubi [A] (verified) 601
Maple [A] (verified) 603
Fricas [A] (verification not implemented) 603
Sympy [A] (verification not implemented) 604
Maxima [A] (verification not implemented) 605
Giac [B] (verification not implemented) 605
Mupad [F(-1)] 606
Reduce [B] (verification not implemented) 607

Optimal result

Integrand size = 30, antiderivative size = 214

$$\int (a+bx)(c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx = -\frac{2(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{5/2}}{5d^5} - \frac{2(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))(c+dx)^{7/2}}{7d^5} + \frac{2(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))(c+dx)^{9/2}}{9d^5} + \frac{2(bCd-4bcD+adD)(c+dx)^{11/2}}{11d^5} + \frac{2bD(c+dx)^{13/2}}{13d^5}$$

output
```
-2/5*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(5/2)/d^5-2/7*(a*d*(
-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(7/
2)/d^5+2/9*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(9/2)/d^5+
2/11*(C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(11/2)/d^5+2/13*b*D*(d*x+c)^(13/2)/d^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.88

$$\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2(c+dx)^{5/2}\left(13ad(-48c^3D+8c^2d(11C+15Dx)-2cd^2(99B+5x(22C+21Dx))+d^3(693\ldots\right.}{}$$

input
```
Integrate[(a + b*x)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(5/2)*(13*a*d*(-48*c^3*D + 8*c^2*d*(11*C + 15*D*x) - 2*c*d^2*
(99*B + 5*x*(22*C + 21*D*x)) + d^3*(693*A + 5*x*(99*B + 77*C*x + 63*D*x^2)
)) + b*(384*c^4*D - 48*c^3*d*(13*C + 20*D*x) + 8*c^2*d^2*(143*B + 15*x*(13
*C + 14*D*x)) + 5*d^4*x*(1287*A + 7*x*(143*B + 117*C*x + 99*D*x^2)) - 2*c*
d^3*(1287*A + 5*x*(286*B + 273*C*x + 252*D*x^2)))))/(45045*d^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{(c+dx)^{5/2}\left(b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)}{d^4}+\frac{(c+dx)^{3/2}(ad-bc)\left(Ad^3\ldots\right.}{d\ldots}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{2(c+dx)^{7/2}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{7d^5}-\frac{2(c+dx)^{5/2}(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^5}+\frac{2(c+dx)^{9/2}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{9d^5}+\frac{2(c+dx)^{11/2}(adD-4bcD+bCd)}{11d^5}+\frac{2bD(c+dx)^{13/2}}{13d^5}$$

input
```
Int[(a + b*x)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-2*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(5/2))/(5*d^
5) - (2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^
3 - 4*c^3*D))*(c + d*x)^(7/2))/(7*d^5) + (2*(a*d*(C*d - 3*c*D) - b*(3*c*C*
d - B*d^2 - 6*c^2*D))*(c + d*x)^(9/2))/(9*d^5) + (2*(b*C*d - 4*b*c*D + a*d
*D)*(c + d*x)^(11/2))/(11*d^5) + (2*b*D*(c + d*x)^(13/2))/(13*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{2\left(\left(\left(\frac{5Db\,x^4}{13}+\frac{5(Cb+Da)x^3}{11}+\frac{5(Bb+Ca)x^2}{9}+\frac{5(Ab+Ba)x}{7}+Aa\right)d^4-\frac{2c\left(\frac{140Db\,x^3}{143}+\frac{35(Cb+Da)x^2}{33}+\frac{10(Bb+Ca)x}{9}+Ab+Ba\right)d}{7}\right.}{5d^5}$
derivativedivides	$\frac{\frac{2bD(xd+c)^{\frac{13}{2}}}{13}+\frac{2((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left((ad-bc)(Cd-3Dc)+b\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left((ad-bc)\left(B\,d\right.\right.}{}}{d^5}$
default	$\frac{\frac{2bD(xd+c)^{\frac{13}{2}}}{13}+\frac{2((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left((ad-bc)(Cd-3Dc)+b\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left((ad-bc)\left(B\,d\right.\right.}{}}{d^5}$
gosper	$\frac{2(xd+c)^{\frac{5}{2}}\left(3465Db\,x^4d^4+4095C\,x^3b\,d^4+4095Dx^3a\,d^4-2520Dx^3bc\,d^3+5005B\,x^2b\,d^4+5005C\,x^2a\,d^4-2730C\,x^2bc\,d^3\right.}{}$
orering	$\frac{2(xd+c)^{\frac{5}{2}}\left(3465Db\,x^4d^4+4095C\,x^3b\,d^4+4095Dx^3a\,d^4-2520Dx^3bc\,d^3+5005B\,x^2b\,d^4+5005C\,x^2a\,d^4-2730C\,x^2bc\,d^3\right.}{}$
trager	$\frac{2\left(3465d^6bDx^6+4095Cb\,d^6x^5+4095Da\,d^6x^5+4410Dbc\,d^5x^5+5005Bb\,d^6x^4+5005Ca\,d^6x^4+5460Cbc\,d^5x^4+5460Dac\right.}{}$

input
```
int((b*x+a)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/5*((5/13*D*b*x^4+5/11*(C*b+D*a)*x^3+5/9*(B*b+C*a)*x^2+5/7*(A*b+B*a)*x+A*
a)*d^4-2/7*c*(140/143*D*b*x^3+35/33*(C*b+D*a)*x^2+10/9*(B*b+C*a)*x+A*b+B*a
)*d^3+8/63*(210/143*D*b*x^2+15/11*(C*b+D*a)*x+B*b+C*a)*c^2*d^2-16/231*(20/
13*D*b*x+C*b+D*a)*c^3*d+128/3003*D*b*c^4)*(d*x+c)^(5/2)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.57

$$\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\left(3465\,Dbd^6x^6+384\,Dbc^6+9009\,Aac^2d^4-624\,(Da+Cb)c^5d+1144\,(Ca+Bb)c^4d^2-257\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
2/45045*(3465*D*b*d^6*x^6 + 384*D*b*c^6 + 9009*A*a*c^2*d^4 - 624*(D*a + C*
b)*c^5*d + 1144*(C*a + B*b)*c^4*d^2 - 2574*(B*a + A*b)*c^3*d^3 + 315*(14*D
*b*c*d^5 + 13*(D*a + C*b)*d^6)*x^5 + 35*(3*D*b*c^2*d^4 + 156*(D*a + C*b)*c
*d^5 + 143*(C*a + B*b)*d^6)*x^4 - 5*(24*D*b*c^3*d^3 - 39*(D*a + C*b)*c^2*d
^4 - 1430*(C*a + B*b)*c*d^5 - 1287*(B*a + A*b)*d^6)*x^3 + 3*(48*D*b*c^4*d^
2 + 3003*A*a*d^6 - 78*(D*a + C*b)*c^3*d^3 + 143*(C*a + B*b)*c^2*d^4 + 3432
*(B*a + A*b)*c*d^5)*x^2 - (192*D*b*c^5*d - 18018*A*a*c*d^5 - 312*(D*a + C*
b)*c^4*d^2 + 572*(C*a + B*b)*c^3*d^3 - 1287*(B*a + A*b)*c^2*d^4)*x)*sqrt(d
*x + c)/d^5
```

Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.50

$$\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\begin{cases} \frac{2\left(\frac{Db(c+dx)^{\frac{13}{2}}}{13d^4}+\frac{(c+dx)^{\frac{11}{2}}(Cbd+Dad-4Dbc)}{11d^4}+\frac{(c+dx)^{\frac{9}{2}}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6Dbc^2\right)}{9d^4}+\frac{(c+dx)^{\frac{7}{2}}\left(Abd^3+Bad^3-2Bbcd^2-2C\right.}{7d^4}\right.}{d} \\ c^{\frac{3}{2}}\left(Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b*(c + d*x)**(13/2)/(13*d**4) + (c + d*x)**(11/2)*(C*b*d +
 D*a*d - 4*D*b*c)/(11*d**4) + (c + d*x)**(9/2)*(B*b*d**2 + C*a*d**2 - 3*C*
b*c*d - 3*D*a*c*d + 6*D*b*c**2)/(9*d**4) + (c + d*x)**(7/2)*(A*b*d**3 + B*
a*d**3 - 2*B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D*b
*c**3)/(7*d**4) + (c + d*x)**(5/2)*(A*a*d**4 - A*b*c*d**3 - B*a*c*d**3 + B
*b*c**2*d**2 + C*a*c**2*d**2 - C*b*c**3*d - D*a*c**3*d + D*b*c**4)/(5*d**4
))/d, Ne(d, 0)), (c**(3/2)*(A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/4 + x**3
*(B*b + C*a)/3 + x**2*(A*b + B*a)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2\left(3465\,(dx+c)^{\frac{13}{2}}Db-4095\,(4\,Dbc-(Da+Cb)d)(dx+c)^{\frac{11}{2}}+5005\,(6\,Dbc^2-3\,(Da+Cb)\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
2/45045*(3465*(d*x + c)^(13/2)*D*b - 4095*(4*D*b*c - (D*a + C*b)*d)*(d*x +
 c)^(11/2) + 5005*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*(d*x +
 c)^(9/2) - 6435*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 -
(B*a + A*b)*d^3)*(d*x + c)^(7/2) + 9009*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c
^3*d + (C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)*(d*x + c)^(5/2))/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1246 vs. 2(196) = 392.

Time = 0.14 (sec) , antiderivative size = 1246, normalized size of antiderivative = 5.82

$$\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(d*x + c)*A*a*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x
 + c)*c)*A*a*c + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*a*c^2/d + 1
5015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*b*c^2/d + 3003*(3*(d*x + c)^(
5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a + 3003*(3*(d*x + c
)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a*c^2/d^2 + 3003*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*b*c^2/
d^2 + 6006*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*B*a*c/d + 6006*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x
+ c)*c^2)*A*b*c/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d
*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*D*a*c^2/d^3 + 1287*(5*(d*x + c)^
(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c
^3)*C*b*c^2/d^3 + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x
 + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*a*c/d^2 + 2574*(5*(d*x + c)^(7/2
) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*
B*b*c/d^2 + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*a/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*
x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A*b/d + 14
3*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 -
420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*D*b*c^2/d^4 + 286*(35*(d*
x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+b\,x)\,\left(c+d\,x\right)^{3/2}\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.28

$$\int (a+bx)(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2\sqrt{dx+c}\,(3465b\,d^6x^6+4095a\,d^6x^5+8505bc\,d^5x^5+10465ac\,d^5x^4+5005b^2d^5x^4+5565b\,c^2d^4$$

input
```
int((b*x+a)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(9009*a**2*c**2*d**3 + 18018*a**2*c*d**4*x + 9009*a**2*d*
*5*x**2 - 5148*a*b*c**3*d**2 + 2574*a*b*c**2*d**3*x + 20592*a*b*c*d**4*x**
2 + 12870*a*b*d**5*x**3 + 520*a*c**5*d - 260*a*c**4*d**2*x + 195*a*c**3*d*
*3*x**2 + 7345*a*c**2*d**4*x**3 + 10465*a*c*d**5*x**4 + 4095*a*d**6*x**5 +
 1144*b**2*c**4*d - 572*b**2*c**3*d**2*x + 429*b**2*c**2*d**3*x**2 + 7150*
b**2*c*d**4*x**3 + 5005*b**2*d**5*x**4 - 240*b*c**6 + 120*b*c**5*d*x - 90*
b*c**4*d**2*x**2 + 75*b*c**3*d**3*x**3 + 5565*b*c**2*d**4*x**4 + 8505*b*c*
d**5*x**5 + 3465*b*d**6*x**6))/(45045*d**4)
```

3.66 $\int (c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx$

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Optimal result

Integrand size = 25, antiderivative size = 117

$$\int (c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(c+dx)^{5/2}}{5d^4} - \frac{2(2cCd-Bd^2-3c^2D)\,(c+dx)^{7/2}}{7d^4} + \frac{2(Cd-3cD)(c+dx)^{9/2}}{9d^4} + \frac{2D(c+dx)^{11/2}}{11d^4}$$

output
```
2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(5/2)/d^4-2/7*(-B*d^2+2*C*c*d-3*
D*c^2)*(d*x+c)^(7/2)/d^4+2/9*(C*d-3*D*c)*(d*x+c)^(9/2)/d^4+2/11*D*(d*x+c)^
(11/2)/d^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.71

$$\int (c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{2(c+dx)^{5/2}\left(-48c^3D+8c^2d(11C+15Dx)-2cd^2(99B+5x(22C+21Dx))+d^3(693A+5\right.}{3465d^4}$$

input
```
Integrate[(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(5/2)*(-48*c^3*D + 8*c^2*d*(11*C + 15*D*x) - 2*c*d^2*(99*B +
5*x*(22*C + 21*D*x)) + d^3*(693*A + 5*x*(99*B + 77*C*x + 63*D*x^2))))/(346
5*d^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \ 2389$$

$$\int \left(\frac{(c+dx)^{3/2} \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3}+\frac{(c+dx)^{5/2} \left(Bd^2+3c^2D-2cCd\right)}{d^3}+\frac{(c+dx)^{7/2}(Cd-3cD}{d^3}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{2(c+dx)^{5/2} \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{5d^4}-\frac{2(c+dx)^{7/2} \left(-Bd^2-3c^2D+2cCd\right)}{7d^4}+\frac{2(c+dx)^{9/2}(Cd-3cD)}{9d^4}+\frac{2D(c+dx)^{11/2}}{11d^4}$$

input
```
Int[(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(5/2))/(5*d^4) - (2*(2*c*
C*d - B*d^2 - 3*c^2*D)*(c + d*x)^(7/2))/(7*d^4) + (2*(C*d - 3*c*D)*(c + d*
x)^(9/2))/(9*d^4) + (2*D*(c + d*x)^(11/2))/(11*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{2\left(\left(\frac{5}{11}Dx^{3}+\frac{5}{9}Cx^{2}+\frac{5}{7}Bx+A\right)d^{3}-\frac{2\left(\frac{35}{33}Dx^{2}+\frac{10}{9}Cx+B\right)c\,d^{2}}{7}+\frac{8\left(\frac{15Dx}{11}+C\right)c^{2}d}{63}-\frac{16Dc^{3}}{231}\right)(xd+c)^{\frac{5}{2}}}{5d^{4}}$
gosper	$\frac{2(xd+c)^{\frac{5}{2}}\left(315Dx^{3}d^{3}+385Cx^{2}d^{3}-210Dx^{2}c\,d^{2}+495Bx\,d^{3}-220Cxc\,d^{2}+120Dx\,c^{2}d+693A\,d^{3}-198Bc\,d^{2}+88C\,c^{2}d-4\right.}{3465d^{4}}$
orering	$\frac{2(xd+c)^{\frac{5}{2}}\left(315Dx^{3}d^{3}+385Cx^{2}d^{3}-210Dx^{2}c\,d^{2}+495Bx\,d^{3}-220Cxc\,d^{2}+120Dx\,c^{2}d+693A\,d^{3}-198Bc\,d^{2}+88C\,c^{2}d-4\right.}{3465d^{4}}$
derivativedivides	$\frac{\frac{2D(xd+c)^{\frac{11}{2}}}{11}+\frac{2(Cd-3Dc)(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left(B\,d^{2}-2Ccd+3Dc^{2}\right)(xd+c)^{\frac{7}{2}}}{7}+\frac{2\left(A\,d^{3}-Bc\,d^{2}+C\,c^{2}d-Dc^{3}\right)(xd+c)^{\frac{5}{2}}}{5}}{d^{4}}$
default	$\frac{\frac{2D(xd+c)^{\frac{11}{2}}}{11}+\frac{2(Cd-3Dc)(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left(B\,d^{2}-2Ccd+3Dc^{2}\right)(xd+c)^{\frac{7}{2}}}{7}+\frac{2\left(A\,d^{3}-Bc\,d^{2}+C\,c^{2}d-Dc^{3}\right)(xd+c)^{\frac{5}{2}}}{5}}{d^{4}}$
trager	$\frac{2\left(315d^{5}Dx^{5}+385C\,d^{5}x^{4}+420Dc\,d^{4}x^{4}+495B\,d^{5}x^{3}+550Cc\,d^{4}x^{3}+15Dc^{2}d^{3}x^{3}+693A\,d^{5}x^{2}+792Bc\,d^{4}x^{2}+33C\,c^{2}d^{3}x^{2}\right.}{3465d}$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/5*((5/11*D*x^3+5/9*C*x^2+5/7*B*x+A)*d^3-2/7*(35/33*D*x^2+10/9*C*x+B)*c*d
^2+8/63*(15/11*D*x+C)*c^2*d-16/231*D*c^3)*(d*x+c)^(5/2)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.47

$$\int (c+dx)^{3/2} \left(A+Bx+Cx^2 +Dx^3\right) \, dx = \frac{2\left(315\, Dd^5x^5 - 48\, Dc^5 + 88\, Cc^4d - 198\, Bc^3d^2 + 693\, Ac^2d^3 + 35\,(12\, Dcd^4 + 11\, Cd^5)x^4 + 5\,(3\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
2/3465*(315*D*d^5*x^5 - 48*D*c^5 + 88*C*c^4*d - 198*B*c^3*d^2 + 693*A*c^2*
d^3 + 35*(12*D*c*d^4 + 11*C*d^5)*x^4 + 5*(3*D*c^2*d^3 + 110*C*c*d^4 + 99*B
*d^5)*x^3 - 3*(6*D*c^3*d^2 - 11*C*c^2*d^3 - 264*B*c*d^4 - 231*A*d^5)*x^2 +
 (24*D*c^4*d - 44*C*c^3*d^2 + 99*B*c^2*d^3 + 1386*A*c*d^4)*x)*sqrt(d*x + c
)/d^4
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int (c+dx)^{3/2} \left(A+Bx+Cx^2 +Dx^3\right) \, dx = \begin{cases} \frac{2\left(\frac{D(c+dx)^{\frac{11}{2}}}{11d^3} + \frac{(c+dx)^{\frac{9}{2}}(Cd-3Dc)}{9d^3} + \frac{(c+dx)^{\frac{7}{2}}\left(Bd^2-2Ccd+3Dc^2\right)}{7d^3} + \frac{(c+dx)^{\frac{5}{2}}\left(Ad^3-Bcd^2+Cc^2d-Dc^3\right)}{5d^3}\right)}{d} & \text{for } d \neq 0 \\ c^{\frac{3}{2}}\left(Ax + \frac{Bx^2}{2} + \frac{Cx^3}{3} + \frac{Dx^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*(c + d*x)**(11/2)/(11*d**3) + (c + d*x)**(9/2)*(C*d - 3*D*
c)/(9*d**3) + (c + d*x)**(7/2)*(B*d**2 - 2*C*c*d + 3*D*c**2)/(7*d**3) + (c
 + d*x)**(5/2)*(A*d**3 - B*c*d**2 + C*c**2*d - D*c**3)/(5*d**3))/d, Ne(d,
0)), (c**(3/2)*(A*x + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int (c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2\left(315\,(dx+c)^{\frac{11}{2}}D-385\,(3\,Dc-Cd)(dx+c)^{\frac{9}{2}}+495\,(3\,Dc^2-2\,Ccd+Bd^2)(dx+c)^{\frac{7}{2}}-69\right.}{3465\,d^4}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
2/3465*(315*(d*x + c)^(11/2)*D - 385*(3*D*c - C*d)*(d*x + c)^(9/2) + 495*(
3*D*c^2 - 2*C*c*d + B*d^2)*(d*x + c)^(7/2) - 693*(D*c^3 - C*c^2*d + B*c*d^
2 - A*d^3)*(d*x + c)^(5/2))/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(101) = 202.

Time = 0.13 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.58

$$\int (c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(d*x + c)*A*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c)
*c)*A*c + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*c^2/d + 231*(3*(d*x
 + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A + 231*(3*(d*x
 + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*c^2/d^2 + 462
*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*c/d +
 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 3
5*sqrt(d*x + c)*c^3)*D*c^2/d^3 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/
2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*c/d^2 + 99*(5*(d*x
 + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)*B/d + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x
+ c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*D*c/d^3
+ 11*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2
 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*C/d^2 + 5*(63*(d*x + c
)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c
)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*D/d^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int (c+d\,x)^{3/2} \left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07

$$\int (c+dx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{2\sqrt{dx+c}\,(315d^5x^5+805c\,d^4x^4+495b\,d^4x^3+565c^2d^3x^3+693a\,d^4x^2+792bc\,d^3x^2+15c^3d^2x}{3465d^3}$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(693*a*c**2*d**2 + 1386*a*c*d**3*x + 693*a*d**4*x**2 - 19
8*b*c**3*d + 99*b*c**2*d**2*x + 792*b*c*d**3*x**2 + 495*b*d**4*x**3 + 40*c
**5 - 20*c**4*d*x + 15*c**3*d**2*x**2 + 565*c**2*d**3*x**3 + 805*c*d**4*x*
*4 + 315*d**5*x**5))/(3465*d**3)
```

3.67 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$

Optimal result . . . 615
Mathematica [A] (verified) . . . 616
Rubi [A] (verified) . . . 616
Maple [A] (verified) . . . 618
Fricas [A] (verification not implemented) . . . 618
Sympy [A] (verification not implemented) . . . 619
Maxima [F(-2)] . . . 620
Giac [B] (verification not implemented) . . . 620
Mupad [F(-1)] . . . 621
Reduce [B] (verification not implemented) . . . 622

Optimal result

Integrand size = 32, antiderivative size = 278

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \frac{2(bc-ad)\left(Ab^3-a(b^2B-abC+a^2D)\right)\sqrt{c+dx}}{b^5} + \frac{2(Ab^3-a(b^2B-abC+a^2D))\,(c+dx)^{3/2}}{3b^4} + \frac{2(a^2d^2D-abd(Cd-cD)-b^2(cCd-Bd^2-c^2D))\,(c+dx)^{5/2}}{5b^3d^3} + \frac{2(bCd-2bcD-adD)(c+dx)^{7/2}}{7b^2d^3} + \frac{2D(c+dx)^{9/2}}{9bd^3} - \frac{2(bc-ad)^{3/2}\left(Ab^3-a(b^2B-abC+a^2D)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{11/2}}$$

output
```
2*(-a*d+b*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^5+2/3*(A*b^3-a*
(B*b^2-C*a*b+D*a^2))*(d*x+c)^(3/2)/b^4+2/5*(a^2*d^2*D-a*b*d*(C*d-D*c)-b^2*
(-B*d^2+C*c*d-D*c^2))*(d*x+c)^(5/2)/b^3/d^3+2/7*(C*b*d-D*a*d-2*D*b*c)*(d*x
+c)^(7/2)/b^2/d^3+2/9*D*(d*x+c)^(9/2)/b/d^3-2*(-a*d+b*c)^(3/2)*(A*b^3-a*(B
*b^2-C*a*b+D*a^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(11/2
)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx = \frac{2\sqrt{c+dx}(315a^4d^4D-105a^3bd^3(3Cd+4cD+dDx)+21a^2b}{}$$

$$+\frac{2(-bc+ad)^{3/2}\left(Ab^3-a(b^2B-abC+a^2D)\right)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{11/2}}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(2*Sqrt[c + d*x]*(315*a^4*d^4*D - 105*a^3*b*d^3*(3*C*d + 4*c*D + d*D*x) +
21*a^2*b^2*d^2*(3*c^2*D + c*(20*C*d + 6*d*D*x) + d^2*(15*B + x*(5*C + 3*D*
x))) - 3*a*b^3*d*(-6*c^3*D + 3*c^2*d*(7*C + D*x) + 2*c*d^2*(70*B + 3*x*(7*
C + 4*D*x)) + d^3*(105*A + x*(35*B + 3*x*(7*C + 5*D*x)))) + b^4*(8*c^4*D -
 2*c^3*d*(9*C + 2*D*x) + 3*c^2*d^2*(21*B + x*(3*C + D*x)) + d^4*x*(105*A +
 x*(63*B + 5*x*(9*C + 7*D*x))) + 2*c*d^3*(210*A + x*(63*B + x*(36*C + 25*D
*x))))))/(315*b^5*d^3) + (2*(-(b*c) + a*d)^(3/2)*(A*b^3 - a*(b^2*B - a*b*C
 + a^2*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/b^(11/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{3/2}\left(Ab^3-a(a^2D-abC+b^2B)\right)}{b^3(a+bx)}+\frac{(c+dx)^{3/2}\left(a^2d^2D-abd(Cd-cD)-(b^2(-Bd^2+c^2(-D)+\right.}{b^3d^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(bc-ad)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{11/2}}+\frac{2\sqrt{c+dx}(bc-ad)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{b^5}+\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^4}+\frac{2(c+dx)^{5/2}\left(a^2d^2D-abd(Cd-cD)-\left(b^2\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{5b^3d^3}+\frac{2(c+dx)^{7/2}(-adD-2bcD+bCd)}{7b^2d^3}+\frac{2D(c+dx)^{9/2}}{9bd^3}$$

input
```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(2*(b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[c + d*x])/b^5 + (2
*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(3/2))/(3*b^4) + (2*(a^2*d^
2*D - a*b*d*(C*d - c*D) - b^2*(c*C*d - B*d^2 - c^2*D))*(c + d*x)^(5/2))/(5
*b^3*d^3) + (2*(b*C*d - 2*b*c*D - a*d*D)*(c + d*x)^(7/2))/(7*b^2*d^3) + (2
*D*(c + d*x)^(9/2))/(9*b*d^3) - (2*(b*c - a*d)^(3/2)*(A*b^3 - a*(b^2*B - a
*b*C + a^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(11/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.20

method	result
pseudoelliptic	$-\frac{2\left(-d^3(ad-bc)^2\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)+\sqrt{(ad-bc)b}\left(\left(\left(-\frac{x\left(\frac{1}{3}Dx^3+\frac{3}{7}C\,x^2+\frac{3}{5}Bx+A\right)b^4}{3}+\cdots\right.\right.\right.\right.}{\cdots}$
derivativedivides	$-\frac{2\left(-\frac{D(xd+c)^{\frac{9}{2}}b^4}{9}-\frac{C\,b^4d(xd+c)^{\frac{7}{2}}}{7}+\frac{Da\,b^3d(xd+c)^{\frac{7}{2}}}{7}+\frac{2Db^4c(xd+c)^{\frac{7}{2}}}{7}-\frac{B\,b^4d^2(xd+c)^{\frac{5}{2}}}{5}+\frac{Ca\,b^3d^2(xd+c)^{\frac{5}{2}}}{5}+\frac{C\,b^4cd(xd+c)^{\frac{5}{2}}}{5}\cdots\right.}{\cdots}$
default	$-\frac{2\left(-\frac{D(xd+c)^{\frac{9}{2}}b^4}{9}-\frac{C\,b^4d(xd+c)^{\frac{7}{2}}}{7}+\frac{Da\,b^3d(xd+c)^{\frac{7}{2}}}{7}+\frac{2Db^4c(xd+c)^{\frac{7}{2}}}{7}-\frac{B\,b^4d^2(xd+c)^{\frac{5}{2}}}{5}+\frac{Ca\,b^3d^2(xd+c)^{\frac{5}{2}}}{5}+\frac{C\,b^4cd(xd+c)^{\frac{5}{2}}}{5}\cdots\right.}{\cdots}$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/((a*d-b*c)*b)^(1/2)*(-d^3*(a*d-b*c)^2*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)*arc
tan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((-1/3*x*(1/3
*D*x^3+3/7*C*x^2+3/5*B*x+A)*b^4+a*(1/5*C*x^2+1/3*B*x+1/7*D*x^3+A)*b^3-(1/5
*D*x^2+1/3*C*x+B)*a^2*b^2+a^3*(1/3*D*x+C)*b-D*a^4)*d^4-4/3*c*b*((5/42*D*x^
3+6/35*C*x^2+3/10*B*x+A)*b^3-(6/35*D*x^2+3/10*C*x+B)*a*b^2+a^2*(3/10*D*x+C
)*b-a^3*D)*d^3-1/5*((1/21*D*x^2+1/7*C*x+B)*b^2-(1/7*D*x+C)*a*b+D*a^2)*c^2*
b^2*d^2+2/35*c^3*b^3*((2/9*D*x+C)*b-D*a)*d-8/315*D*b^4*c^4)*(d*x+c)^(1/2))
/d^3/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.38

$$\int\frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
[1/315*(315*((D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d^3 - (D*a^4 - C*a^
3*b + B*a^2*b^2 - A*a*b^3)*d^4)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a
*d + 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(35*D*b^4*d^4*x
^4 + 8*D*b^4*c^4 + 18*(D*a*b^3 - C*b^4)*c^3*d + 63*(D*a^2*b^2 - C*a*b^3 +
B*b^4)*c^2*d^2 - 420*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d^3 + 315*(
D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^3)*d^4 + 5*(10*D*b^4*c*d^3 - 9*(D*a*b^
3 - C*b^4)*d^4)*x^3 + 3*(D*b^4*c^2*d^2 - 24*(D*a*b^3 - C*b^4)*c*d^3 + 21*(
D*a^2*b^2 - C*a*b^3 + B*b^4)*d^4)*x^2 - (4*D*b^4*c^3*d + 9*(D*a*b^3 - C*b^
4)*c^2*d^2 - 126*(D*a^2*b^2 - C*a*b^3 + B*b^4)*c*d^3 + 105*(D*a^3*b - C*a^
2*b^2 + B*a*b^3 - A*b^4)*d^4)*x)*sqrt(d*x + c))/(b^5*d^3), 2/315*(315*((D*
a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d^3 - (D*a^4 - C*a^3*b + B*a^2*b^2
- A*a*b^3)*d^4)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c -
a*d)/b)/(b*c - a*d)) + (35*D*b^4*d^4*x^4 + 8*D*b^4*c^4 + 18*(D*a*b^3 - C*b
^4)*c^3*d + 63*(D*a^2*b^2 - C*a*b^3 + B*b^4)*c^2*d^2 - 420*(D*a^3*b - C*a^
2*b^2 + B*a*b^3 - A*b^4)*c*d^3 + 315*(D*a^4 - C*a^3*b + B*a^2*b^2 - A*a*b^
3)*d^4 + 5*(10*D*b^4*c*d^3 - 9*(D*a*b^3 - C*b^4)*d^4)*x^3 + 3*(D*b^4*c^2*d
^2 - 24*(D*a*b^3 - C*b^4)*c*d^3 + 21*(D*a^2*b^2 - C*a*b^3 + B*b^4)*d^4)*x^
2 - (4*D*b^4*c^3*d + 9*(D*a*b^3 - C*b^4)*c^2*d^2 - 126*(D*a^2*b^2 - C*a*b^
3 + B*b^4)*c*d^3 + 105*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)*d^4)*x)*sqr
t(d*x + c))/(b^5*d^3)]
```

Sympy [A] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx = \begin{cases} \frac{2\left(\frac{D(c+dx)^{\frac{9}{2}}}{9bd^2} + \frac{(c+dx)^{\frac{7}{2}}(Cbd-Dad-2Dbc)}{7b^2d^2} + \frac{(c+dx)^{\frac{5}{2}}\left(Bb^2d^2-Cabd^2-Cb^2cd+Da^2d^2 \cdots\right.}{5b^3d^2} \cdots\right.}{\cdots} \\ c^{\frac{3}{2}}\left(\frac{Dx^3}{3b} + \frac{x^2(Cb-Da)}{2b^2} + \frac{x\left(Bb^2-Cab+Da^2\right)}{b^3} - \frac{\left(-Ab^3+Bab^2-Ca^2b+D \cdots\right.}{\cdots} \cdots\right. \end{cases}$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a),x)
```

output
```
Piecewise((2*(D*(c + d*x)**(9/2)/(9*b*d**2) + (c + d*x)**(7/2)*(C*b*d - D*
a*d - 2*D*b*c)/(7*b**2*d**2) + (c + d*x)**(5/2)*(B*b**2*d**2 - C*a*b*d**2
- C*b**2*c*d + D*a**2*d**2 + D*a*b*c*d + D*b**2*c**2)/(5*b**3*d**2) + (c +
 d*x)**(3/2)*(A*b**3*d - B*a*b**2*d + C*a**2*b*d - D*a**3*d)/(3*b**4) + sq
rt(c + d*x)*(-A*a*b**3*d**2 + A*b**4*c*d + B*a**2*b**2*d**2 - B*a*b**3*c*d
 - C*a**3*b*d**2 + C*a**2*b**2*c*d + D*a**4*d**2 - D*a**3*b*c*d)/b**5 - d*
(a*d - b*c)**2*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*atan(sqrt(c + d*x)
/sqrt((a*d - b*c)/b))/(b**6*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), (c**(3/2)*
(D*x**3/(3*b) + x**2*(C*b - D*a)/(2*b**2) + x*(B*b**2 - C*a*b + D*a**2)/b*
*3 - (-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*Piecewise((x/a, Eq(b, 0)), (
log(a + b*x)/b, True))/b**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. 2(247) = 494.

Time = 0.14 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.04

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx =$$

$$-\frac{2\left(Da^3b^2c^2 - Ca^2b^3c^2 + Bab^4c^2 - Ab^5c^2 - 2\,Da^4bcd + 2\,Ca^3b^2cd - 2\,Ba^2b^3cd + 2\,Aab^4cd + Da^5d^2 - C\right.}{\sqrt{-b^2c+abd}\,b^5}$$

$$+\frac{2\left(35\,(dx+c)^{\frac{9}{2}}Db^8d^{24} - 90\,(dx+c)^{\frac{7}{2}}Db^8cd^{24} + 63\,(dx+c)^{\frac{5}{2}}Db^8c^2d^{24} - 45\,(dx+c)^{\frac{7}{2}}Dab^7d^{25} + 45\,(dx\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
-2*(D*a^3*b^2*c^2 - C*a^2*b^3*c^2 + B*a*b^4*c^2 - A*b^5*c^2 - 2*D*a^4*b*c*
d + 2*C*a^3*b^2*c*d - 2*B*a^2*b^3*c*d + 2*A*a*b^4*c*d + D*a^5*d^2 - C*a^4*
b*d^2 + B*a^3*b^2*d^2 - A*a^2*b^3*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*b^5) + 2/315*(35*(d*x + c)^(9/2)*D*b^8*d^2
4 - 90*(d*x + c)^(7/2)*D*b^8*c*d^24 + 63*(d*x + c)^(5/2)*D*b^8*c^2*d^24 -
45*(d*x + c)^(7/2)*D*a*b^7*d^25 + 45*(d*x + c)^(7/2)*C*b^8*d^25 + 63*(d*x
+ c)^(5/2)*D*a*b^7*c*d^25 - 63*(d*x + c)^(5/2)*C*b^8*c*d^25 + 63*(d*x + c)
^(5/2)*D*a^2*b^6*d^26 - 63*(d*x + c)^(5/2)*C*a*b^7*d^26 + 63*(d*x + c)^(5/
2)*B*b^8*d^26 - 105*(d*x + c)^(3/2)*D*a^3*b^5*d^27 + 105*(d*x + c)^(3/2)*C
*a^2*b^6*d^27 - 105*(d*x + c)^(3/2)*B*a*b^7*d^27 + 105*(d*x + c)^(3/2)*A*b
^8*d^27 - 315*sqrt(d*x + c)*D*a^3*b^5*c*d^27 + 315*sqrt(d*x + c)*C*a^2*b^6
*c*d^27 - 315*sqrt(d*x + c)*B*a*b^7*c*d^27 + 315*sqrt(d*x + c)*A*b^8*c*d^2
7 + 315*sqrt(d*x + c)*D*a^4*b^4*d^28 - 315*sqrt(d*x + c)*C*a^3*b^5*d^28 +
315*sqrt(d*x + c)*B*a^2*b^6*d^28 - 315*sqrt(d*x + c)*A*a*b^7*d^28)/(b^9*d^
27)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx=\int \frac{(c+d\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{a+b\,x}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x),x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.66

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx = \frac{-2\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^4d^4+4\sqrt{b}\,\sqrt{ad-bc}\,atan\Big(}{}$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

output
```
(2*( - 315*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*
d - b*c)))*a**4*d**4 + 630*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/
(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*c*d**3 - 315*sqrt(b)*sqrt(a*d - b*c)*ata
n((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*c**2*d**2 + 315*s
qrt(c + d*x)*a**4*b*d**4 - 735*sqrt(c + d*x)*a**3*b**2*c*d**3 - 105*sqrt(c
 + d*x)*a**3*b**2*d**4*x + 483*sqrt(c + d*x)*a**2*b**3*c**2*d**2 + 231*sqr
t(c + d*x)*a**2*b**3*c*d**3*x + 63*sqrt(c + d*x)*a**2*b**3*d**4*x**2 - 45*
sqrt(c + d*x)*a*b**4*c**3*d - 135*sqrt(c + d*x)*a*b**4*c**2*d**2*x - 135*s
qrt(c + d*x)*a*b**4*c*d**3*x**2 - 45*sqrt(c + d*x)*a*b**4*d**4*x**3 + 63*s
qrt(c + d*x)*b**6*c**2*d + 126*sqrt(c + d*x)*b**6*c*d**2*x + 63*sqrt(c + d
*x)*b**6*d**3*x**2 - 10*sqrt(c + d*x)*b**5*c**4 + 5*sqrt(c + d*x)*b**5*c**
3*d*x + 75*sqrt(c + d*x)*b**5*c**2*d**2*x**2 + 95*sqrt(c + d*x)*b**5*c*d**
3*x**3 + 35*sqrt(c + d*x)*b**5*d**4*x**4))/(315*b**6*d**2)
```

3.68 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$

Optimal result 623
Mathematica [A] (verified) 624
Rubi [A] (verified) 624
Maple [A] (verified) 627
Fricas [A] (verification not implemented) 628
Sympy [F(-1)] 629
Maxima [F(-2)] 629
Giac [B] (verification not implemented) 629
Mupad [F(-1)] 630
Reduce [B] (verification not implemented) 631

Optimal result

Integrand size = 32, antiderivative size = 294

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx = \frac{(b^3(2Bc+3Ad)-ab^2(4cC+5Bd)-9a^3dD+a^2b(7Cd+6c}{b^5}$$
$$+\frac{2(b^2B-2abC+3a^2D)\,(c+dx)^{3/2}}{3b^4}-\frac{(Ab^3-a(b^2B-abC+a^2D))\,(c+dx)^{3/2}}{b^4(a+bx)}$$
$$+\frac{2(bCd-bcD-2adD)(c+dx)^{5/2}}{5b^3d^2}+\frac{2D(c+dx)^{7/2}}{7b^2d^2}$$
$$-\frac{\sqrt{bc-ad}(b^3(2Bc+3Ad)-ab^2(4cC+5Bd)-9a^3dD+a^2b(7Cd+6cD))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{11/2}}$$

output
```
(b^3*(3*A*d+2*B*c)-a*b^2*(5*B*d+4*C*c)-9*a^3*d*D+a^2*b*(7*C*d+6*D*c))*(d*x
+c)^(1/2)/b^5+2/3*(B*b^2-2*C*a*b+3*D*a^2)*(d*x+c)^(3/2)/b^4-(A*b^3-a*(B*b^
2-C*a*b+D*a^2))*(d*x+c)^(3/2)/b^4/(b*x+a)+2/5*(C*b*d-2*D*a*d-D*b*c)*(d*x+c
)^(5/2)/b^3/d^2+2/7*D*(d*x+c)^(7/2)/b^2/d^2-(-a*d+b*c)^(1/2)*(b^3*(3*A*d+2
*B*c)-a*b^2*(5*B*d+4*C*c)-9*a^3*d*D+a^2*b*(7*C*d+6*D*c))*arctanh(b^(1/2)*(
d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx = \frac{\sqrt{c+dx}(-945a^4d^3D+105a^3bd^2(7Cd+9cD-6dDx)-ab^3}{}$$

$$-\frac{\sqrt{-bc+ad}(b^3(2Bc+3Ad)-ab^2(4cC+5Bd)-9a^3dD+a^2b(7Cd+6cD))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{11/2}}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
(Sqrt[c + d*x]*(-945*a^4*d^3*D + 105*a^3*b*d^2*(7*C*d + 9*c*D - 6*d*D*x) -
 a*b^3*(12*c^3*D + c^2*(-42*C*d + 78*d*D*x) + c*d^2*(-385*B + 476*C*x + 12
0*D*x^2) + d^3*(-315*A + 350*B*x + 98*C*x^2 + 54*D*x^3)) - 7*a^2*b^2*d*(12
*c^2*D + c*d*(95*C - 96*D*x) + d^2*(75*B - 2*x*(35*C + 9*D*x))) + b^4*(-10
5*A*d^2*(c - 2*d*x) + 2*x*(-6*c^3*D + 3*c^2*d*(7*C + D*x) + 2*c*d^2*(70*B
+ 3*x*(7*C + 4*D*x)) + d^3*x*(35*B + 3*x*(7*C + 5*D*x))))))/(105*b^5*d^2*(
a + b*x)) - (Sqrt[-(b*c) + a*d]*(b^3*(2*B*c + 3*A*d) - a*b^2*(4*c*C + 5*B*
d) - 9*a^3*d*D + a^2*b*(7*C*d + 6*c*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[-(b*c) + a*d]])/b^(11/2)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2124, 27, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{\int -\frac{(c+dx)^{3/2}\left(2\left(c-\frac{ad}{b}\right)Dx^2+\frac{2(bc-ad)(bC-aD)x}{b^2}+\frac{-5dDa^3+b(5Cd+2cD)a^2-b^2(2cC+5Bd)a+b^3(2Bc+3Ad)}{b^3}\right)}{2(a+bx)}dx}{bc-ad}-\frac{(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)^{3/2}\left(-\frac{5dDa^3}{b^3}+\frac{(5Cd+2cD)a^2}{b^2}-\frac{(2cC+5Bd)a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc+3Ad+\frac{2(bc-ad)(bC-aD)x}{b^2}\right)}{a+bx}dx}{2(bc-ad)}-\frac{(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 1192$$

$$\frac{\int \frac{(c+dx)^2\left(-2Dc^3+2Cdc^2-2Bd^2c-2\left(c-\frac{ad}{b}\right)D(c+dx)^2-d^3\left(3A-\frac{5a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{2(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{bc-ad-b(c+dx)}d\sqrt{c+dx}}{d^2(bc-ad)}-\frac{(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 1584$$

$$\frac{\int \left(\frac{2(bc-ad)D(c+dx)^3}{b^2}+\frac{2(bc-ad)(bCd-2aDd-bcD)(c+dx)^2}{b^3}+\frac{d^2\left(-9dDa^3+b(7Cd+6cD)a^2-b^2(4cC+5Bd)a+b^3(2Bc+3Ad)\right)(c+dx)}{b^4}+\right.}{}$$

$$\frac{(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{d^2(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-9a^3dD+a^2b(6cD+7Cd)-ab^2(5Bd+4cC)+b^3(3Ad+2Bc)\right)}{b^{11/2}}+\frac{d^2\sqrt{c+dx}(bc-ad)\left(-9a^3dD+a^2b(6cD+\right.}{b}}{}$$

$$\frac{(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

input
```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
-(((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(5/2))/((b*c - a*d)*(a
+ b*x))) + ((d^2*(b*c - a*d)*(b^3*(2*B*c + 3*A*d) - a*b^2*(4*c*C + 5*B*d)
- 9*a^3*d*D + a^2*b*(7*C*d + 6*c*D))*Sqrt[c + d*x])/b^5 + (d^2*(b^3*(2*B*c
 + 3*A*d) - a*b^2*(4*c*C + 5*B*d) - 9*a^3*d*D + a^2*b*(7*C*d + 6*c*D))*(c
+ d*x)^(3/2))/(3*b^4) + (2*(b*c - a*d)*(b*C*d - b*c*D - 2*a*d*D)*(c + d*x)
^(5/2))/(5*b^3) + (2*(b*c - a*d)*D*(c + d*x)^(7/2))/(7*b^2) - (d^2*(b*c -
a*d)^(3/2)*(b^3*(2*B*c + 3*A*d) - a*b^2*(4*c*C + 5*B*d) - 9*a^3*d*D + a^2*
b*(7*C*d + 6*c*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(11
/2))/(d^2*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.19

method	result
pseudoelliptic	$\frac{-3\left(\left(Ad+\frac{2Bc}{3}\right)b^3-\frac{5\left(Bd+\frac{4Cc}{5}\right)a\,b^2}{3}+a^2\left(\frac{7Cd}{3}+2Dc\right)b-3a^3dD\right)(ad-bc)d^2(bx+a)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)+3\sqrt{(ad-bc}}{}$
derivativedivides	$\frac{2\left(\frac{D(xd+c)^{\frac{7}{2}}b^3}{7}+\frac{C\,b^3d(xd+c)^{\frac{5}{2}}}{5}-\frac{2Da\,b^2d(xd+c)^{\frac{5}{2}}}{5}-\frac{Db^3c(xd+c)^{\frac{5}{2}}}{5}+\frac{B\,b^3d^2(xd+c)^{\frac{3}{2}}}{3}-\frac{2Ca\,b^2d^2(xd+c)^{\frac{3}{2}}}{3}+Da^2b\,d^2(xd+c)^{\frac{3}{2}}\right.}{b^5}$
default	$\frac{2\left(\frac{D(xd+c)^{\frac{7}{2}}b^3}{7}+\frac{C\,b^3d(xd+c)^{\frac{5}{2}}}{5}-\frac{2Da\,b^2d(xd+c)^{\frac{5}{2}}}{5}-\frac{Db^3c(xd+c)^{\frac{5}{2}}}{5}+\frac{B\,b^3d^2(xd+c)^{\frac{3}{2}}}{3}-\frac{2Ca\,b^2d^2(xd+c)^{\frac{3}{2}}}{3}+Da^2b\,d^2(xd+c)^{\frac{3}{2}}\right.}{b^5}$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/((a*d-b*c)*b)^(1/2)*(-((A*d+2/3*B*c)*b^3-5/3*(B*d+4/5*C*c)*a*b^2+a^2*(7/
3*C*d+2*D*c)*b-3*a^3*d*D)*(a*d-b*c)*d^2*(b*x+a)*arctan(b*(d*x+c)^(1/2)/((a
*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(1/3*(2*x*(1/5*C*x^2+1/3*B*x+1/7*D*x
^3+A)*d^3-(-16/35*D*x^3-4/5*C*x^2-8/3*B*x+A)*c*d^2+2/5*x*(1/7*D*x+C)*c^2*d
-4/35*D*c^3*x)*b^4+a*((-6/35*D*x^3-14/45*C*x^2-10/9*B*x+A)*d^3+11/9*(-24/7
7*D*x^2-68/55*C*x+B)*c*d^2+2/15*(-13/7*D*x+C)*c^2*d-4/105*D*c^3)*b^3-5/3*(
(-6/25*D*x^2-14/15*C*x+B)*d^2+19/15*(-96/95*D*x+C)*c*d+4/25*D*c^2)*d*a^2*b
^2+7/3*((-6/7*D*x+C)*d+9/7*D*c)*d^2*a^3*b-3*D*a^4*d^3)*(d*x+c)^(1/2))/d^2/
b^5/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 1066, normalized size of antiderivative = 3.63

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="fricas
")
```

output
```
[1/210*(105*(2*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c*d^2 - (9*D*a^4 - 7*C*
a^3*b + 5*B*a^2*b^2 - 3*A*a*b^3)*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4
)*c*d^2 - (9*D*a^3*b - 7*C*a^2*b^2 + 5*B*a*b^3 - 3*A*b^4)*d^3)*x)*sqrt((b*
c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/
b))/(b*x + a)) + 2*(30*D*b^4*d^3*x^4 - 12*D*a*b^3*c^3 - 42*(2*D*a^2*b^2 -
C*a*b^3)*c^2*d + 35*(27*D*a^3*b - 19*C*a^2*b^2 + 11*B*a*b^3 - 3*A*b^4)*c*d
^2 - 105*(9*D*a^4 - 7*C*a^3*b + 5*B*a^2*b^2 - 3*A*a*b^3)*d^3 + 6*(8*D*b^4*
c*d^2 - (9*D*a*b^3 - 7*C*b^4)*d^3)*x^3 + 2*(3*D*b^4*c^2*d - 6*(10*D*a*b^3
- 7*C*b^4)*c*d^2 + 7*(9*D*a^2*b^2 - 7*C*a*b^3 + 5*B*b^4)*d^3)*x^2 - 2*(6*D
*b^4*c^3 + 3*(13*D*a*b^3 - 7*C*b^4)*c^2*d - 14*(24*D*a^2*b^2 - 17*C*a*b^3
+ 10*B*b^4)*c*d^2 + 35*(9*D*a^3*b - 7*C*a^2*b^2 + 5*B*a*b^3 - 3*A*b^4)*d^3
)*x)*sqrt(d*x + c))/(b^6*d^2*x + a*b^5*d^2), -1/105*(105*(2*(3*D*a^3*b - 2
*C*a^2*b^2 + B*a*b^3)*c*d^2 - (9*D*a^4 - 7*C*a^3*b + 5*B*a^2*b^2 - 3*A*a*b
^3)*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c*d^2 - (9*D*a^3*b - 7*C*a^
2*b^2 + 5*B*a*b^3 - 3*A*b^4)*d^3)*x)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x
 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (30*D*b^4*d^3*x^4 - 12*D*a*b^3
*c^3 - 42*(2*D*a^2*b^2 - C*a*b^3)*c^2*d + 35*(27*D*a^3*b - 19*C*a^2*b^2 +
11*B*a*b^3 - 3*A*b^4)*c*d^2 - 105*(9*D*a^4 - 7*C*a^3*b + 5*B*a^2*b^2 - 3*A
*a*b^3)*d^3 + 6*(8*D*b^4*c*d^2 - (9*D*a*b^3 - 7*C*b^4)*d^3)*x^3 + 2*(3*D*b
^4*c^2*d - 6*(10*D*a*b^3 - 7*C*b^4)*c*d^2 + 7*(9*D*a^2*b^2 - 7*C*a*b^3 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. 2(270) = 540.

Time = 0.14 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx=\frac{\left(6\,Da^2b^2c^2-4\,Cab^3c^2+2\,Bb^4c^2-15\,Da^3bcd+11\,Ca^2b^2cd-\right.}{}$$
$$+\frac{\sqrt{dx+c}Da^3bcd-\sqrt{dx+c}Ca^2b^2cd+\sqrt{dx+c}Bab^3cd-\sqrt{dx+c}Ab^4cd-\sqrt{dx+c}Da^4d^2+\sqrt{dx+c}Ca}{((dx+c)b-bc+ad)b^5}$$
$$+\frac{2\left(15\,(dx+c)^{\frac{7}{2}}Db^{12}d^{12}-21\,(dx+c)^{\frac{5}{2}}Db^{12}cd^{12}-42\,(dx+c)^{\frac{5}{2}}Dab^{11}d^{13}+21\,(dx+c)^{\frac{5}{2}}Cb^{12}d^{13}+105\,(d\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(6*D*a^2*b^2*c^2 - 4*C*a*b^3*c^2 + 2*B*b^4*c^2 - 15*D*a^3*b*c*d + 11*C*a^2
*b^2*c*d - 7*B*a*b^3*c*d + 3*A*b^4*c*d + 9*D*a^4*d^2 - 7*C*a^3*b*d^2 + 5*B
*a^2*b^2*d^2 - 3*A*a*b^3*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))
/(sqrt(-b^2*c + a*b*d)*b^5) + (sqrt(d*x + c)*D*a^3*b*c*d - sqrt(d*x + c)*C
*a^2*b^2*c*d + sqrt(d*x + c)*B*a*b^3*c*d - sqrt(d*x + c)*A*b^4*c*d - sqrt(
d*x + c)*D*a^4*d^2 + sqrt(d*x + c)*C*a^3*b*d^2 - sqrt(d*x + c)*B*a^2*b^2*d
^2 + sqrt(d*x + c)*A*a*b^3*d^2)/(((d*x + c)*b - b*c + a*d)*b^5) + 2/105*(1
5*(d*x + c)^(7/2)*D*b^12*d^12 - 21*(d*x + c)^(5/2)*D*b^12*c*d^12 - 42*(d*x
 + c)^(5/2)*D*a*b^11*d^13 + 21*(d*x + c)^(5/2)*C*b^12*d^13 + 105*(d*x + c)
^(3/2)*D*a^2*b^10*d^14 - 70*(d*x + c)^(3/2)*C*a*b^11*d^14 + 35*(d*x + c)^(
3/2)*B*b^12*d^14 + 315*sqrt(d*x + c)*D*a^2*b^10*c*d^14 - 210*sqrt(d*x + c)
*C*a*b^11*c*d^14 + 105*sqrt(d*x + c)*B*b^12*c*d^14 - 420*sqrt(d*x + c)*D*a
^3*b^9*d^15 + 315*sqrt(d*x + c)*C*a^2*b^10*d^15 - 210*sqrt(d*x + c)*B*a*b^
11*d^15 + 105*sqrt(d*x + c)*A*b^12*d^15)/(b^14*d^14)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2} (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} \, dx = \int \frac{(c+d\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^2} \, dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2,x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.69

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

output
```
(945*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*
c)))*a**4*d**3 - 1365*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt
(b)*sqrt(a*d - b*c)))*a**3*b*c*d**2 + 945*sqrt(b)*sqrt(a*d - b*c)*atan((sq
rt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*d**3*x + 210*sqrt(b)*sqrt
(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*d*
*2 + 420*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
- b*c)))*a**2*b**2*c**2*d - 1365*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*
x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*c*d**2*x - 210*sqrt(b)*sqrt(a*d
 - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c*d + 210
*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))
*a*b**4*d**2*x + 420*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(
b)*sqrt(a*d - b*c)))*a*b**3*c**2*d*x - 210*sqrt(b)*sqrt(a*d - b*c)*atan((s
qrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**5*c*d*x - 945*sqrt(c + d*x)*
a**4*b*d**3 + 1680*sqrt(c + d*x)*a**3*b**2*c*d**2 - 630*sqrt(c + d*x)*a**3
*b**2*d**3*x - 210*sqrt(c + d*x)*a**2*b**4*d**2 - 749*sqrt(c + d*x)*a**2*b
**3*c**2*d + 1162*sqrt(c + d*x)*a**2*b**3*c*d**2*x + 126*sqrt(c + d*x)*a**
2*b**3*d**3*x**2 + 280*sqrt(c + d*x)*a*b**5*c*d - 140*sqrt(c + d*x)*a*b**5
*d**2*x + 30*sqrt(c + d*x)*a*b**4*c**3 - 554*sqrt(c + d*x)*a*b**4*c**2*d*x
 - 218*sqrt(c + d*x)*a*b**4*c*d**2*x**2 - 54*sqrt(c + d*x)*a*b**4*d**3*x**
3 + 280*sqrt(c + d*x)*b**6*c*d*x + 70*sqrt(c + d*x)*b**6*d**2*x**2 + 30...
```

3.69 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$

Optimal result 632
Mathematica [A] (verified) 633
Rubi [A] (verified) 633
Maple [A] (verified) 637
Fricas [B] (verification not implemented) 637
Sympy [F(-1)] 638
Maxima [F(-2)] 639
Giac [B] (verification not implemented) 639
Mupad [F(-1)] 640
Reduce [B] (verification not implemented) 640

Optimal result

Integrand size = 32, antiderivative size = 331

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \frac{2(b^2(cC+Bd)+6a^2dD-3ab(Cd+cD))\sqrt{c+dx}}{b^5}$$
$$-\frac{(b^3(4Bc+3Ad)-ab^2(8cC+7Bd)-15a^3dD+a^2b(11Cd+12cD))\sqrt{c+dx}}{4b^5(a+bx)}$$
$$+\frac{2(bC-3aD)(c+dx)^{3/2}}{3b^4}-\frac{(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{3/2}}{2b^4(a+bx)^2}+\frac{2D(c+dx)^{5/2}}{5b^3d}$$
$$-\frac{(b^3(8c^2C+12Bcd+3Ad^2)-63a^3d^2D+7a^2bd(5Cd+12cD)-ab^2(40cCd+15Bd^2+24c^2D))\operatorname{arctanh}}{4b^{11/2}\sqrt{bc-ad}}$$

output
```
2*(b^2*(B*d+C*c)+6*a^2*d*D-3*a*b*(C*d+D*c))*(d*x+c)^(1/2)/b^5-1/4*(b^3*(3*
A*d+4*B*c)-a*b^2*(7*B*d+8*C*c)-15*a^3*d*D+a^2*b*(11*C*d+12*D*c))*(d*x+c)^(
1/2)/b^5/(b*x+a)+2/3*(C*b-3*D*a)*(d*x+c)^(3/2)/b^4-1/2*(A*b^3-a*(B*b^2-C*a
*b+D*a^2))*(d*x+c)^(3/2)/b^4/(b*x+a)^2+2/5*D*(d*x+c)^(5/2)/b^3/d-1/4*(b^3*
(3*A*d^2+12*B*c*d+8*C*c^2)-63*a^3*d^2*D+7*a^2*b*d*(5*C*d+12*D*c)-a*b^2*(15
*B*d^2+40*C*c*d+24*D*c^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))
/b^(11/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx = \frac{\sqrt{c+dx}(945a^4d^2D-15Ab^3d(2bc+3ad+5bdx)-105a^3bd(5}{}$$

$$+\frac{(b^3(8c^2C+12Bcd+3Ad^2)-63a^3d^2D+7a^2bd(5Cd+12cD)-ab^2(40cCd+15Bd^2+24c^2D))\arctan\Big(}{4b^{11/2}\sqrt{-bc+ad}}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
(Sqrt[c + d*x]*(945*a^4*d^2*D - 15*A*b^3*d*(2*b*c + 3*a*d + 5*b*d*x) - 105
*a^3*b*d*(5*C*d + 6*c*D - 15*d*D*x) + a*b^3*(15*B*d*(-2*c + 25*d*x) + 8*x*
(6*c^2*D + c*d*(55*C - 48*D*x) - d^2*x*(35*C + 9*D*x))) + a^2*b^2*(24*c^2*
D + 2*c*d*(125*C - 546*D*x) + d^2*(225*B + 7*x*(-125*C + 72*D*x))) + 4*b^4
*x*(-15*B*d*(c - 2*d*x) + 2*x*(3*c^2*D + d^2*x*(5*C + 3*D*x) + c*(20*C*d +
 6*d*D*x)))))/(60*b^5*d*(a + b*x)^2) + ((b^3*(8*c^2*C + 12*B*c*d + 3*A*d^2
) - 63*a^3*d^2*D + 7*a^2*b*d*(5*C*d + 12*c*D) - a*b^2*(40*c*C*d + 15*B*d^2
 + 24*c^2*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*b^(11
/2)*Sqrt[-(b*c) + a*d])
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2124, 27, 1192, 25, 1580, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{\int -\frac{(c+dx)^{3/2}\left(4\left(c-\frac{ad}{b}\right)Dx^2+\frac{4(bc-ad)(bC-aD)x}{b^2}+\frac{-5dDa^3+b(5Cd+4cD)a^2-b^2(4cC+5Bd)a+b^3(4Bc+Ad)}{b^3}\right)}{2(a+bx)^2}dx}{2(bc-ad)}-\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(c+dx)^{3/2}\left(-\frac{5dDa^3}{b^3}+\frac{(5Cd+4cD)a^2}{b^2}-\frac{(4cC+5Bd)a}{b}+4\left(c-\frac{ad}{b}\right)Dx^2+4Bc+Ad+\frac{4(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^2}dx}{4(bc-ad)}-\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\Big\downarrow$ 1192

$$\frac{\int -\frac{(c+dx)^2\left(-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2-d^3\left(A-\frac{5a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{(c+dx)^2\left(-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2-d^3\left(A-\frac{5a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\Big\downarrow$ 1580

$$\frac{\int -\frac{\frac{8b^2(bc-ad)D(c+dx)^3+8b(bc-ad)(bCd-2aDd-bcD)(c+dx)^2+2d^2\left(-13dDa^3+3b(3Cd+4cD)a^2-b^2(8cC+5Bd)a+b^3(4Bc+Ad)\right)(c+dx)+\frac{d^2(bc-ad)\left(-13dD\cdots\right.}{\cdots}}{bc-ad-b(c+dx)}}{2b^4}}{2d(bc-\cdots}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\frac{d^2\sqrt{c+dx}(bc-ad)\left(-13a^3dD+3a^2b(4cD+3Cd)-ab^2(5Bd+8cC)+b^3(Ad+4Bc)\right)}{2b^5(-ad-b(c+dx)+bc)}-\frac{\int\frac{8b^2(bc-ad)D(c+dx)^3+8b(bc-ad)(bCd-2aDd-bcD)(c+dx)^2+2}{}}{}}{2d(bc-}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\Big\downarrow$ 2341

$$\frac{d^2\sqrt{c+dx}(bc-ad)\left(-13a^3dD+3a^2b(4cD+3Cd)-ab^2(5Bd+8cC)+b^3(Ad+4Bc)\right)}{2b^5(-ad-b(c+dx)+bc)}-\int\left(-8b(bc-ad)D(c+dx)^2-8d(bc-ad)(bC-3aD)(c+dx)-2\right.$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\Big\downarrow$ 2009

$$\frac{d^2\sqrt{c+dx}(bc-ad)\left(-13a^3dD+3a^2b(4cD+3Cd)-ab^2(5Bd+8cC)+b^3(Ad+4Bc)\right)}{2b^5(-ad-b(c+dx)+bc)}-\frac{d\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-63a^3d^2D+7a^2bd(12cD+5Cd\right.}{b^{3/2}}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

input
```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
-1/2*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(5/2))/(b^3*(b*c - a*d
)*(a + b*x)^2) + ((d^2*(b*c - a*d)*(b^3*(4*B*c + A*d) - a*b^2*(8*c*C + 5*B
*d) - 13*a^3*d*D + 3*a^2*b*(3*C*d + 4*c*D))*Sqrt[c + d*x])/(2*b^5*(b*c - a
*d - b*(c + d*x))) - (-2*d*(b^2*(4*c^2*C + 4*B*c*d + A*d^2) - (25*a^3*d^2*
D)/b + a^2*d*(13*C*d + 36*c*D) - a*b*(16*c*C*d + 5*B*d^2 + 12*c^2*D))*Sqrt
[c + d*x] - (8*d*(b*c - a*d)*(b*C - 3*a*D)*(c + d*x)^(3/2))/3 - (8*b*(b*c
- a*d)*D*(c + d*x)^(5/2))/5 + (d*Sqrt[b*c - a*d]*(b^3*(8*c^2*C + 12*B*c*d
+ 3*A*d^2) - 63*a^3*d^2*D + 7*a^2*b*d*(5*C*d + 12*c*D) - a*b^2*(40*c*C*d +
 15*B*d^2 + 24*c^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b
^(3/2))/(2*b^4))/(2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$-\frac{3\left(-\left(\left(d^2A+4cdB+\frac{8}{3}C\,c^2\right)b^3-5\left(B\,d^2+\frac{8}{3}Ccd+\frac{8}{5}Dc^2\right)a\,b^2+7a^2\left(\frac{5}{3}d^2C+4cdD\right)b-21a^3d^2D\right)d(bx+a)^2\arctan\left(\frac{b\sqrt{}}{\sqrt{(a}}\right.\right.}{}$
derivativedivides	$\frac{2\left(\frac{D(xd+c)^{\frac{5}{2}}b^2}{5}+\frac{C\,b^2d(xd+c)^{\frac{3}{2}}}{3}-Dabd(xd+c)^{\frac{3}{2}}+B\,d^2b^2\sqrt{xd+c}-3Ca\,d^2b\sqrt{xd+c}+C\,b^2cd\sqrt{xd+c}+6Da^2d^2\sqrt{xd+c}-3Dabcd\sqrt{xd-}\right.}{b^5}$
default	$\frac{2\left(\frac{D(xd+c)^{\frac{5}{2}}b^2}{5}+\frac{C\,b^2d(xd+c)^{\frac{3}{2}}}{3}-Dabd(xd+c)^{\frac{3}{2}}+B\,d^2b^2\sqrt{xd+c}-3Ca\,d^2b\sqrt{xd+c}+C\,b^2cd\sqrt{xd+c}+6Da^2d^2\sqrt{xd+c}-3Dabcd\sqrt{xd-}\right.}{b^5}$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4*(-((d^2*A+4*c*d*B+8/3*C*c^2)*b^3-5*(B*d^2+8/3*C*c*d+8/5*D*c^2)*a*b^2+
7*a^2*(5/3*d^2*C+4*c*d*D)*b-21*a^3*d^2*D)*d*(b*x+a)^2*arctan(b*(d*x+c)^(1/
2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(1/3*(5*x*(-8/25*D*x^3-8/15*C*
x^2-8/5*B*x+A)*d^2+2*(-8/5*D*x^3-16/3*C*x^2+2*B*x+A)*c*d-8/5*D*c^2*x^2)*b^
4+((56/9*C*x^2+8/5*D*x^3-25/3*B*x+A)*d^2+2/3*c*(64/5*D*x^2-44/3*C*x+B)*d-1
6/15*D*c^2*x)*a*b^3-5*((56/25*D*x^2-35/9*C*x+B)*d^2+10/9*(-546/125*D*x+C)*
c*d+8/75*D*c^2)*a^2*b^2+35/3*((-3*D*x+C)*d+6/5*D*c)*d*a^3*b-21*D*a^4*d^2)*
(d*x+c)^(1/2))/((a*d-b*c)*b)^(1/2)/(b*x+a)^2/b^5/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(303) = 606.

Time = 0.12 (sec) , antiderivative size = 1675, normalized size of antiderivative = 5.06

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="fricas
")
```

output
```
[-1/120*(15*(8*(3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - 4*(21*D*a^4*b - 10*C*a^3*
b^2 + 3*B*a^2*b^3)*c*d^2 + (63*D*a^5 - 35*C*a^4*b + 15*B*a^3*b^2 - 3*A*a^2
*b^3)*d^3 + (8*(3*D*a*b^4 - C*b^5)*c^2*d - 4*(21*D*a^2*b^3 - 10*C*a*b^4 +
3*B*b^5)*c*d^2 + (63*D*a^3*b^2 - 35*C*a^2*b^3 + 15*B*a*b^4 - 3*A*b^5)*d^3)
*x^2 + 2*(8*(3*D*a^2*b^3 - C*a*b^4)*c^2*d - 4*(21*D*a^3*b^2 - 10*C*a^2*b^3
 + 3*B*a*b^4)*c*d^2 + (63*D*a^4*b - 35*C*a^3*b^2 + 15*B*a^2*b^3 - 3*A*a*b^
4)*d^3)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a
*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(24*D*a^2*b^4*c^3 + 24*(D*b^6*c*d^2 -
D*a*b^5*d^3)*x^4 - 2*(327*D*a^3*b^3 - 125*C*a^2*b^4 + 15*B*a*b^5 + 15*A*b^
6)*c^2*d + 5*(315*D*a^4*b^2 - 155*C*a^3*b^3 + 51*B*a^2*b^4 - 3*A*a*b^5)*c*
d^2 - 15*(63*D*a^5*b - 35*C*a^4*b^2 + 15*B*a^3*b^3 - 3*A*a^2*b^4)*d^3 + 8*
(6*D*b^6*c^2*d - 5*(3*D*a*b^5 - C*b^6)*c*d^2 + (9*D*a^2*b^4 - 5*C*a*b^5)*d
^3)*x^3 + 8*(3*D*b^6*c^3 - (51*D*a*b^5 - 20*C*b^6)*c^2*d + (111*D*a^2*b^4
- 55*C*a*b^5 + 15*B*b^6)*c*d^2 - (63*D*a^3*b^3 - 35*C*a^2*b^4 + 15*B*a*b^5
)*d^3)*x^2 + (48*D*a*b^5*c^3 - 20*(57*D*a^2*b^4 - 22*C*a*b^5 + 3*B*b^6)*c^
2*d + (2667*D*a^3*b^3 - 1315*C*a^2*b^4 + 435*B*a*b^5 - 75*A*b^6)*c*d^2 - 2
5*(63*D*a^4*b^2 - 35*C*a^3*b^3 + 15*B*a^2*b^4 - 3*A*a*b^5)*d^3)*x)*sqrt(d*
x + c))/(a^2*b^7*c*d - a^3*b^6*d^2 + (b^9*c*d - a*b^8*d^2)*x^2 + 2*(a*b^8*
c*d - a^2*b^7*d^2)*x), -1/60*(15*(8*(3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - 4*(2
1*D*a^4*b - 10*C*a^3*b^2 + 3*B*a^2*b^3)*c*d^2 + (63*D*a^5 - 35*C*a^4*b ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. 2(303) = 606.

Time = 0.15 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx =$$

$$-\frac{(24\,Dab^2c^2 - 8\,Cb^3c^2 - 84\,Da^2bcd + 40\,Cab^2cd - 12\,Bb^3cd + 63\,Da^3d^2 - 35\,Ca^2bd^2 + 15\,Bab^2d^2 - 3\,Ab}{4\sqrt{-b^2c+abd}\,b^5}$$

$$-\frac{12\,(dx+c)^{\frac{3}{2}}Da^2b^2cd - 8\,(dx+c)^{\frac{3}{2}}Cab^3cd + 4\,(dx+c)^{\frac{3}{2}}Bb^4cd - 12\,\sqrt{dx+c}Da^2b^2c^2d + 8\,\sqrt{dx+c}Cab^3}{}$$

$$+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}Db^{12}d^4 - 15\,(dx+c)^{\frac{3}{2}}Dab^{11}d^5 + 5\,(dx+c)^{\frac{3}{2}}Cb^{12}d^5 - 45\,\sqrt{dx+c}Dab^{11}cd^5 + 15\,\sqrt{dx+c}C\right.}{15\,b^{15}d^5}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(24*D*a*b^2*c^2 - 8*C*b^3*c^2 - 84*D*a^2*b*c*d + 40*C*a*b^2*c*d - 12*
B*b^3*c*d + 63*D*a^3*d^2 - 35*C*a^2*b*d^2 + 15*B*a*b^2*d^2 - 3*A*b^3*d^2)*
arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^5) -
1/4*(12*(d*x + c)^(3/2)*D*a^2*b^2*c*d - 8*(d*x + c)^(3/2)*C*a*b^3*c*d + 4*
(d*x + c)^(3/2)*B*b^4*c*d - 12*sqrt(d*x + c)*D*a^2*b^2*c^2*d + 8*sqrt(d*x
+ c)*C*a*b^3*c^2*d - 4*sqrt(d*x + c)*B*b^4*c^2*d - 17*(d*x + c)^(3/2)*D*a^
3*b*d^2 + 13*(d*x + c)^(3/2)*C*a^2*b^2*d^2 - 9*(d*x + c)^(3/2)*B*a*b^3*d^2
 + 5*(d*x + c)^(3/2)*A*b^4*d^2 + 27*sqrt(d*x + c)*D*a^3*b*c*d^2 - 19*sqrt(
d*x + c)*C*a^2*b^2*c*d^2 + 11*sqrt(d*x + c)*B*a*b^3*c*d^2 - 3*sqrt(d*x + c
)*A*b^4*c*d^2 - 15*sqrt(d*x + c)*D*a^4*d^3 + 11*sqrt(d*x + c)*C*a^3*b*d^3
- 7*sqrt(d*x + c)*B*a^2*b^2*d^3 + 3*sqrt(d*x + c)*A*a*b^3*d^3)/(((d*x + c)
*b - b*c + a*d)^2*b^5) + 2/15*(3*(d*x + c)^(5/2)*D*b^12*d^4 - 15*(d*x + c)
^(3/2)*D*a*b^11*d^5 + 5*(d*x + c)^(3/2)*C*b^12*d^5 - 45*sqrt(d*x + c)*D*a*
b^11*c*d^5 + 15*sqrt(d*x + c)*C*b^12*c*d^5 + 90*sqrt(d*x + c)*D*a^2*b^10*d
^6 - 45*sqrt(d*x + c)*C*a*b^11*d^6 + 15*sqrt(d*x + c)*B*b^12*d^6)/(b^15*d^
5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx=\int \frac{\left(c+d\,x\right)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\left(a+b\,x\right)^3}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3,x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 838, normalized size of antiderivative = 2.53

$$\int \frac{(c+dx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
( - 945*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
 b*c)))*a**4*d**2 + 840*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sq
rt(b)*sqrt(a*d - b*c)))*a**3*b*c*d - 1890*sqrt(b)*sqrt(a*d - b*c)*atan((sq
rt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*d**2*x - 180*sqrt(b)*sqrt
(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*d
- 120*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b
*c)))*a**2*b**2*c**2 + 1680*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)
/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*c*d*x - 945*sqrt(b)*sqrt(a*d - b*c)*
atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*d**2*x**2 - 36
0*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c))
)*a*b**4*d*x - 240*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)
*sqrt(a*d - b*c)))*a*b**3*c**2*x + 840*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(
c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**3*c*d*x**2 - 180*sqrt(b)*sqrt(
a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**5*d*x**2 -
 120*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*
c)))*b**4*c**2*x**2 + 945*sqrt(c + d*x)*a**4*b*d**2 - 1155*sqrt(c + d*x)*a
**3*b**2*c*d + 1575*sqrt(c + d*x)*a**3*b**2*d**2*x + 180*sqrt(c + d*x)*a**
2*b**4*d + 274*sqrt(c + d*x)*a**2*b**3*c**2 - 1967*sqrt(c + d*x)*a**2*b**3
*c*d*x + 504*sqrt(c + d*x)*a**2*b**3*d**2*x**2 - 60*sqrt(c + d*x)*a*b**5*c
 + 300*sqrt(c + d*x)*a*b**5*d*x + 488*sqrt(c + d*x)*a*b**4*c**2*x - 664...
```

3.70 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$

Optimal result 642
Mathematica [A] (verified) 643
Rubi [A] (verified) 643
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Sympy [F(-1)] 649
Maxima [F(-2)] 650
Giac [B] (verification not implemented) 650
Mupad [F(-1)] 651
Reduce [B] (verification not implemented) 652

Optimal result

Integrand size = 32, antiderivative size = 401

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx = \frac{2(bCd+bcD-4adD)\sqrt{c+dx}}{b^5}$$
$$-\frac{\left(b^3(2Bc+Ad)-ab^2(4cC+3Bd)-7a^3dD+a^2b(5Cd+6cD)\right)\sqrt{c+dx}}{4b^5(a+bx)^2}$$
$$-\frac{\left(b^3(8c^2C+10Bcd+Ad^2)-55a^3d^2D+a^2bd(29Cd+78cD)-ab^2(36cCd+11Bd^2+24c^2D)\right)\sqrt{c+dx}}{8b^5(bc-ad)(a+bx)}$$
$$+\frac{2D(c+dx)^{3/2}}{3b^4}-\frac{\left(Ab^3-a(b^2B-abC+a^2D)\right)(c+dx)^{3/2}}{3b^4(a+bx)^3}$$
$$+\frac{(105a^3d^3D-35a^2bd^2(Cd+6cD)+5ab^2d(12cCd+Bd^2+24c^2D)-b^3(24c^2Cd+6Bcd^2-Ad^3+16c^3D}{8b^{11/2}(bc-ad)^{3/2}}$$

output
```
2*(C*b*d-4*D*a*d+D*b*c)*(d*x+c)^(1/2)/b^5-1/4*(b^3*(A*d+2*B*c)-a*b^2*(3*B*
d+4*C*c)-7*a^3*d*D+a^2*b*(5*C*d+6*D*c))*(d*x+c)^(1/2)/b^5/(b*x+a)^2-1/8*(b
^3*(A*d^2+10*B*c*d+8*C*c^2)-55*a^3*d^2*D+a^2*b*d*(29*C*d+78*D*c)-a*b^2*(11
*B*d^2+36*C*c*d+24*D*c^2))*(d*x+c)^(1/2)/b^5/(-a*d+b*c)/(b*x+a)+2/3*D*(d*x
+c)^(3/2)/b^4-1/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(3/2)/b^4/(b*x+a)^
3+1/8*(105*a^3*d^3*D-35*a^2*b*d^2*(C*d+6*D*c)+5*a*b^2*d*(B*d^2+12*C*c*d+24
*D*c^2)-b^3*(-A*d^3+6*B*c*d^2+24*C*c^2*d+16*D*c^3))*arctanh(b^(1/2)*(d*x+c
)^(1/2)/(-a*d+b*c)^(1/2))/b^(11/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \frac{\sqrt{c+dx}(315a^5d^2D-105a^4bd(Cd+4D(c-2dx))+Ab^3(3a^2}{}$$

$$-\frac{(-105a^3d^3D+35a^2bd^2(Cd+6cD)-5ab^2d(12cCd+Bd^2+24c^2D)+b^3(24c^2Cd+6Bcd^2-Ad^3+16c}{8b^{11/2}(-bc+ad)^{3/2}}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
(Sqrt[c + d*x]*(315*a^5*d^2*D - 105*a^4*b*d*(C*d + 4*D*(c - 2*d*x)) + A*b^
3*(3*a^2*d^2 + 2*a*b*d*(c + 4*d*x) - b^2*(8*c^2 + 14*c*d*x + 3*d^2*x^2)) +
 2*b^5*c*x*(-3*B*(2*c + 5*d*x) + 4*x*(-3*c*C + 6*C*d*x + 8*c*D*x + 2*d*D*x
^2)) - a*b^4*(B*(4*c^2 + 22*c*d*x - 33*d^2*x^2) + 4*x*(6*c^2*(C - 11*D*x)
+ 4*d^2*x^2*(3*C + D*x) + c*d*x*(-63*C + 52*D*x))) + a^3*b^2*(108*c^2*D +
2*c*d*(55*C - 567*D*x) + d^2*(15*B + 7*x*(-40*C + 99*D*x))) + a^2*b^3*(c^2
*(-8*C + 300*D*x) + d^2*x*(40*B + 3*x*(-77*C + 48*D*x)) - 2*c*d*(4*B + x*(
-149*C + 477*D*x)))))/(24*b^5*(b*c - a*d)*(a + b*x)^3) - ((-105*a^3*d^3*D
+ 35*a^2*b*d^2*(C*d + 6*c*D) - 5*a*b^2*d*(12*c*C*d + B*d^2 + 24*c^2*D) + b
^3*(24*c^2*C*d + 6*B*c*d^2 - A*d^3 + 16*c^3*D))*ArcTan[(Sqrt[b]*Sqrt[c + d
*x])/Sqrt[-(b*c) + a*d]])/(8*b^(11/2)*(-(b*c) + a*d)^(3/2))
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2124, 27, 1192, 1580, 25, 2345, 27, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{(c+dx)^{3/2}\left(6\left(c-\frac{ad}{b}\right)Dx^2+\frac{6(bc-ad)(bC-aD)x}{b^2}+\frac{-5dDa^3+b(5Cd+6cD)a^2-b^2(6cC+5Bd)a+b^3(6Bc-Ad)}{b^3}\right)}{2(a+bx)^3}dx}{3(bc-ad)}-\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(c+dx)^{3/2}\left(-\frac{5dDa^3}{b^3}+\frac{(5Cd+6cD)a^2}{b^2}-\frac{(6cC+5Bd)a}{b}+6\left(c-\frac{ad}{b}\right)Dx^2+6Bc-Ad+\frac{6(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^3}dx}{6(bc-ad)}-\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 1192

$$\frac{\int \frac{(c+dx)^2\left(-6Dc^3+6Cdc^2-6Bd^2c+Ad^3-6\left(c-\frac{ad}{b}\right)D(c+dx)^2+\frac{5ad^3\left(Da^2-bCa+b^2B\right)}{b^3}-\frac{6(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^3}d\sqrt{c+dx}}{3(bc-ad)}-\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 1580

$$-\frac{\int -\frac{\frac{24b^2(bc-ad)D(c+dx)^3+24b(bc-ad)(bCd-2aDd-bcD)(c+dx)^2+4d^2\left(-17dDa^3+b(11Cd+18cD)a^2-b^2(12cC+5Bd)a+b^3(6Bc-Ad)\right)(c+dx)+\frac{d^2(bc-ad)\left(\cdots\right.}{\cdots}}{(bc-ad-b(c+dx))^2}}{4b^4}}{3(\cdots}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\frac{24b^2(bc-ad)D(c+dx)^3+24b(bc-ad)(bCd-2aDd-bcD)(c+dx)^2+4d^2\left(-17dDa^3+b(11Cd+18cD)a^2-b^2(12cC+5Bd)a+b^3(6Bc-Ad)\right)(c+dx)+\frac{d^2(bc-ad)\left(-17\cdots\right.}{\cdots}}{(bc-ad-b(c+dx))^2}}{4b^4}}{3(b\cdots}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 2345

$$\frac{\frac{d\sqrt{c+dx}\left(-157a^3d^2D+a^2bd(234cD+79Cd)-ab^2\left(25Bd^2+72c^2D+108cCd\right)+b^3\left(-5Ad^2+30Bcd+24c^2C\right)\right)}{2b(-ad-b(c+dx)+bc)}-\frac{\int\frac{3\left(16bD(c+dx)^2(bc-ad)^2+16d(bC-3aD)(c+dx)\cdots\right.}{\cdots}}{\cdots}}{4b^4}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{d\sqrt{c+dx}\left(-157a^3d^2D+a^2bd(234cD+79Cd)-ab^2\left(25Bd^2+72c^2D+108cCd\right)+b^3\left(-5Ad^2+30Bcd+24c^2C\right)\right)}{2b(-ad-b(c+dx)+bc)}-\frac{3\int\frac{16bD(c+dx)^2(bc-ad)^2+16d(bC-3aD)(c+dx)(b\cdots}{\cdots}}{\cdots}}{4b^4}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\downarrow$ 1467

$$\frac{d\sqrt{c+dx}\left(-157a^3d^2D+a^2bd(234cD+79Cd)-ab^2\left(25Bd^2+72c^2D+108cCd\right)+b^3\left(-5Ad^2+30Bcd+24c^2C\right)\right)}{2b(-ad-b(c+dx)+bc)}-\frac{3\int\left(-\frac{16(bCd-4aDd+bcD)(bc-ad)^2}{b}-16D(c+dx)(\cdots\right.}{\cdots}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\downarrow$ 2009

$$\frac{d\sqrt{c+dx}\left(-157a^3d^2D+a^2bd(234cD+79Cd)-ab^2\left(25Bd^2+72c^2D+108cCd\right)+b^3\left(-5Ad^2+30Bcd+24c^2C\right)\right)}{2b(-ad-b(c+dx)+bc)}-\frac{3\left(-\frac{\sqrt{bc-ad}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(105a^3d^3D-\cdots\right.}{\cdots}\right.}{\cdots}$$

$$\frac{(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

input

```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
-1/3*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(5/2))/(b^3*(b*c - a*d
)*(a + b*x)^3) + (-1/4*(d^2*(b*c - a*d)*(b^3*(6*B*c - A*d) - a*b^2*(12*c*C
 + 5*B*d) - 17*a^3*d*D + a^2*b*(11*C*d + 18*c*D))*Sqrt[c + d*x])/(b^5*(b*c
 - a*d - b*(c + d*x))^2) + ((d*(b^3*(24*c^2*C + 30*B*c*d - 5*A*d^2) - 157*
a^3*d^2*D + a^2*b*d*(79*C*d + 234*c*D) - a*b^2*(108*c*C*d + 25*B*d^2 + 72*
c^2*D))*Sqrt[c + d*x])/(2*b*(b*c - a*d - b*(c + d*x))) - (3*((-16*(b*c - a
*d)^2*(b*C*d + b*c*D - 4*a*d*D)*Sqrt[c + d*x])/b - (16*(b*c - a*d)^2*D*(c
+ d*x)^(3/2))/3 - (Sqrt[b*c - a*d]*(105*a^3*d^3*D - 35*a^2*b*d^2*(C*d + 6*
c*D) + 5*a*b^2*d*(12*c*C*d + B*d^2 + 24*c^2*D) - b^3*(24*c^2*C*d + 6*B*c*d
^2 - A*d^3 + 16*c^3*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/
b^(3/2)))/(2*(b*c - a*d)))/(4*b^4))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.06

method	result
pseudoelliptic	$-\frac{-\left(\left(A\,d^{3}-6Bc\,d^{2}-24C\,c^{2}d-16Dc^{3}\right)b^{3}+5a\,b^{2}d\left(B\,d^{2}+12Ccd+24Dc^{2}\right)-35a^{2}b\,d^{2}\left(Cd+6Dc\right)+105a^{3}d^{3}D\right)\left(bx+a\right)^{3}\text{a}}{}$
derivativedivides	$\frac{\frac{2D(xd+c)^{\frac{3}{2}}b}{3}+2Cdb\sqrt{xd+c}-8Dad\sqrt{xd+c}+2Dbc\sqrt{xd+c}}{b^{5}}+\frac{2\left(\frac{b^{2}d\left(b^{3}d^{2}A-11Ba\,b^{2}d^{2}+10B\,b^{3}cd+29C\,a^{2}b\,d^{2}-36Ca\,b^{2}cd\right.}{16ad-1}\right.}{}$
default	$\frac{\frac{2D(xd+c)^{\frac{3}{2}}b}{3}+2Cdb\sqrt{xd+c}-8Dad\sqrt{xd+c}+2Dbc\sqrt{xd+c}}{b^{5}}+\frac{2\left(\frac{b^{2}d\left(b^{3}d^{2}A-11Ba\,b^{2}d^{2}+10B\,b^{3}cd+29C\,a^{2}b\,d^{2}-36Ca\,b^{2}cd\right.}{16ad-1}\right.}{}$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-((A*d^3-6*B*c*d^2-24*C*c^2*d-16*D*c^3)*b^3+5*a*b^2*d*(B*d^2+12*C*c*
d+24*D*c^2)-35*a^2*b*d^2*(C*d+6*D*c)+105*a^3*d^3*D)*(b*x+a)^3*arctan(b*(d*
x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((-A*d^2*x^2-14/3*x*(-
8/7*D*x^3-24/7*C*x^2+15/7*B*x+A)*c*d-8/3*(-8*D*x^3+3*C*x^2+3/2*B*x+A)*c^2)
*b^5+2/3*(4*x*(-2*D*x^3-6*C*x^2+33/8*B*x+A)*d^2+c*(-104*D*x^3+126*C*x^2-11
*B*x+A)*d-2*c^2*(-66*D*x^2+6*C*x+B))*a*b^4+((40/3*B*x+48*D*x^3-77*C*x^2+A)
*d^2-8/3*c*(477/4*D*x^2-149/4*C*x+B)*d-8/3*(-75/2*D*x+C)*c^2)*a^2*b^3+5*((
231/5*D*x^2-56/3*C*x+B)*d^2+22/3*(-567/55*D*x+C)*c*d+36/5*D*c^2)*a^3*b^2-3
5*((-8*D*x+C)*d+4*D*c)*d*a^4*b+105*D*a^5*d^2)*(d*x+c)^(1/2))/((a*d-b*c)*b)
^(1/2)/(b*x+a)^3/(a*d-b*c)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(373) = 746$.

Time = 0.16 (sec) , antiderivative size = 2362, normalized size of antiderivative = 5.89

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="fricas
")
```

output
```
[1/48*(3*(16*D*a^3*b^3*c^3 - 24*(5*D*a^4*b^2 - C*a^3*b^3)*c^2*d + 6*(35*D*
a^5*b - 10*C*a^4*b^2 + B*a^3*b^3)*c*d^2 - (105*D*a^6 - 35*C*a^5*b + 5*B*a^
4*b^2 + A*a^3*b^3)*d^3 + (16*D*b^6*c^3 - 24*(5*D*a*b^5 - C*b^6)*c^2*d + 6*
(35*D*a^2*b^4 - 10*C*a*b^5 + B*b^6)*c*d^2 - (105*D*a^3*b^3 - 35*C*a^2*b^4
+ 5*B*a*b^5 + A*b^6)*d^3)*x^3 + 3*(16*D*a*b^5*c^3 - 24*(5*D*a^2*b^4 - C*a*
b^5)*c^2*d + 6*(35*D*a^3*b^3 - 10*C*a^2*b^4 + B*a*b^5)*c*d^2 - (105*D*a^4*
b^2 - 35*C*a^3*b^3 + 5*B*a^2*b^4 + A*a*b^5)*d^3)*x^2 + 3*(16*D*a^2*b^4*c^3
 - 24*(5*D*a^3*b^3 - C*a^2*b^4)*c^2*d + 6*(35*D*a^4*b^2 - 10*C*a^3*b^3 + B
*a^2*b^4)*c*d^2 - (105*D*a^5*b - 35*C*a^4*b^2 + 5*B*a^3*b^3 + A*a^2*b^4)*d
^3)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d
)*sqrt(d*x + c))/(b*x + a)) + 2*(16*(D*b^7*c^2*d - 2*D*a*b^6*c*d^2 + D*a^2
*b^5*d^3)*x^4 + 4*(27*D*a^3*b^4 - 2*C*a^2*b^5 - B*a*b^6 - 2*A*b^7)*c^3 - 2
*(264*D*a^4*b^3 - 59*C*a^3*b^4 + 2*B*a^2*b^5 - 5*A*a*b^6)*c^2*d + (735*D*a
^5*b^2 - 215*C*a^4*b^3 + 23*B*a^3*b^4 + A*a^2*b^5)*c*d^2 - 3*(105*D*a^6*b
- 35*C*a^5*b^2 + 5*B*a^4*b^3 + A*a^3*b^4)*d^3 + 16*(4*D*b^7*c^3 - (17*D*a*
b^6 - 3*C*b^7)*c^2*d + 2*(11*D*a^2*b^5 - 3*C*a*b^6)*c*d^2 - 3*(3*D*a^3*b^4
 - C*a^2*b^5)*d^3)*x^3 + 3*(8*(11*D*a*b^6 - C*b^7)*c^3 - 2*(203*D*a^2*b^5
- 46*C*a*b^6 + 5*B*b^7)*c^2*d + (549*D*a^3*b^4 - 161*C*a^2*b^5 + 21*B*a*b^
6 - A*b^7)*c*d^2 - (231*D*a^4*b^3 - 77*C*a^3*b^4 + 11*B*a^2*b^5 - A*a*b^6)
*d^3)*x^2 + 2*(6*(25*D*a^2*b^5 - 2*C*a*b^6 - B*b^7)*c^3 - (717*D*a^3*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. 2(373) = 746.

Time = 0.15 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.45

$$\int \frac{(c+dx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="giac")
```

output
```
1/8*(16*D*b^3*c^3 - 120*D*a*b^2*c^2*d + 24*C*b^3*c^2*d + 210*D*a^2*b*c*d^2
 - 60*C*a*b^2*c*d^2 + 6*B*b^3*c*d^2 - 105*D*a^3*d^3 + 35*C*a^2*b*d^3 - 5*B
*a*b^2*d^3 - A*b^3*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^6
*c - a*b^5*d)*sqrt(-b^2*c + a*b*d)) + 1/24*(72*(d*x + c)^(5/2)*D*a*b^4*c^2
*d - 24*(d*x + c)^(5/2)*C*b^5*c^2*d - 144*(d*x + c)^(3/2)*D*a*b^4*c^3*d +
48*(d*x + c)^(3/2)*C*b^5*c^3*d + 72*sqrt(d*x + c)*D*a*b^4*c^4*d - 24*sqrt(
d*x + c)*C*b^5*c^4*d - 234*(d*x + c)^(5/2)*D*a^2*b^3*c*d^2 + 108*(d*x + c)
^(5/2)*C*a*b^4*c*d^2 - 30*(d*x + c)^(5/2)*B*b^5*c*d^2 + 576*(d*x + c)^(3/2
)*D*a^2*b^3*c^2*d^2 - 240*(d*x + c)^(3/2)*C*a*b^4*c^2*d^2 + 48*(d*x + c)^(
3/2)*B*b^5*c^2*d^2 - 342*sqrt(d*x + c)*D*a^2*b^3*c^3*d^2 + 132*sqrt(d*x +
c)*C*a*b^4*c^3*d^2 - 18*sqrt(d*x + c)*B*b^5*c^3*d^2 + 165*(d*x + c)^(5/2)*
D*a^3*b^2*d^3 - 87*(d*x + c)^(5/2)*C*a^2*b^3*d^3 + 33*(d*x + c)^(5/2)*B*a*
b^4*d^3 - 3*(d*x + c)^(5/2)*A*b^5*d^3 - 712*(d*x + c)^(3/2)*D*a^3*b^2*c*d^
3 + 328*(d*x + c)^(3/2)*C*a^2*b^3*c*d^3 - 88*(d*x + c)^(3/2)*B*a*b^4*c*d^3
 - 8*(d*x + c)^(3/2)*A*b^5*c*d^3 + 591*sqrt(d*x + c)*D*a^3*b^2*c^2*d^3 - 2
49*sqrt(d*x + c)*C*a^2*b^3*c^2*d^3 + 51*sqrt(d*x + c)*B*a*b^4*c^2*d^3 + 3*
sqrt(d*x + c)*A*b^5*c^2*d^3 + 280*(d*x + c)^(3/2)*D*a^4*b*d^4 - 136*(d*x +
 c)^(3/2)*C*a^3*b^2*d^4 + 40*(d*x + c)^(3/2)*B*a^2*b^3*d^4 + 8*(d*x + c)^(
3/2)*A*a*b^4*d^4 - 444*sqrt(d*x + c)*D*a^4*b*c*d^4 + 198*sqrt(d*x + c)*C*a
^3*b^2*c*d^4 - 48*sqrt(d*x + c)*B*a^2*b^3*c*d^4 - 6*sqrt(d*x + c)*A*a*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx=\int \frac{(c+d\,x)^{3/2}\left(A+B\,x+C\,x^2+x^3\,D\right)}{(a+b\,x)^4}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4,x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1272, normalized size of antiderivative = 3.17

$$\int \frac{(c+dx)^{3/2} (A+Bx+Cx^2+Dx^3)}{(a+bx)^4} dx = \text{Too large to display}$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x)
```

output
```
(315*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*
c)))*a**5*d**3 - 420*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(
b)*sqrt(a*d - b*c)))*a**4*b*c*d**2 + 945*sqrt(b)*sqrt(a*d - b*c)*atan((sqr
t(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**4*b*d**3*x + 18*sqrt(b)*sqrt(a
*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**3*d**2
 + 120*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
b*c)))*a**3*b**2*c**2*d - 1260*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)
*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*c*d**2*x + 945*sqrt(b)*sqrt(a*d -
 b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*d**3*x**
2 + 54*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
b*c)))*a**2*b**4*d**2*x + 360*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*
b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c**2*d*x - 1260*sqrt(b)*sqrt(a*d -
 b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c*d**2*x
**2 + 315*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
 - b*c)))*a**2*b**3*d**3*x**3 + 54*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c +
d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**5*d**2*x**2 + 360*sqrt(b)*sqrt(a*d
 - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c**2*d*x*
*2 - 420*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
- b*c)))*a*b**4*c*d**2*x**3 + 18*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*
x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**6*d**2*x**3 + 120*sqrt(b)*sqrt(a*d ...
```

3.71 $\int (a+bx)^3(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result 653
Mathematica [A] (verified) 654
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Optimal result

Integrand size = 32, antiderivative size = 438

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx =$$
$$-\frac{2(bc-ad)^3\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{7/2}}{7d^7}$$
$$-\frac{2(bc-ad)^2\left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right)(c+dx)^{9/2}}{9d^7}$$
$$-\frac{2(bc-ad)\left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)(c}{11d^7}$$
$$+\frac{2(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D))(c}{13d^7}$$
$$+\frac{2b\left(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D)\right)(c+dx)^{15/2}}{15d^7}$$
$$+\frac{2b^2(bCd-6bcD+3adD)(c+dx)^{17/2}}{17d^7}+\frac{2b^3D(c+dx)^{19/2}}{19d^7}$$

output
```
-2/7*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(7/2)/d^7-2/9*(-a*
d+b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*
c^3))*(d*x+c)^(9/2)/d^7-2/11*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d
^2+8*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^
(11/2)/d^7+2/13*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c
*d-10*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(13/2)/d^7
+2/15*b*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d
*x+c)^(15/2)/d^7+2/17*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(17/2)/d^7+2/19*
b^3*D*(d*x+c)^(19/2)/d^7
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.15

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2(c+dx)^{7/2}\left(969a^2bd^2(128c^4D-16c^3d(15C+28Dx)+7d^4x(715A+585Bx+495Cx^2+42\right.}{}$$

input
```
Integrate[(a + b*x)^3*(c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(7/2)*(969*a^2*b*d^2*(128*c^4*D - 16*c^3*d*(15*C + 28*D*x) +
7*d^4*x*(715*A + 585*B*x + 495*C*x^2 + 429*D*x^3) - 2*c*d^3*(715*A + 910*B
*x + 945*C*x^2 + 924*D*x^3) + 8*c^2*d^2*(65*B + 21*x*(5*C + 6*D*x))) + 161
5*a^3*d^3*(-48*c^3*D + 8*c^2*d*(13*C + 21*D*x) - 2*c*d^2*(143*B + 7*x*(26*
C + 27*D*x)) + d^3*(1287*A + 7*x*(143*B + 117*C*x + 99*D*x^2))) + b^3*(153
60*c^6*D - 1280*c^5*d*(19*C + 42*D*x) + 128*c^4*d^2*(323*B + 35*x*(19*C +
27*D*x)) + 231*d^6*x^3*(4845*A + 13*x*(323*B + 15*x*(19*C + 17*D*x))) - 42
*c*d^5*x^2*(14535*A + 11*x*(1292*B + 65*x*(19*C + 18*D*x))) - 16*c^3*d^3*(
4845*A + 14*x*(646*B + 45*x*(19*C + 22*D*x))) + 168*c^2*d^4*x*(1615*A + x*
(1938*B + 55*x*(38*C + 39*D*x)))) + 57*a*b^2*d*(-1280*c^5*D + 128*c^4*d*(1
7*C + 35*D*x) - 16*c^3*d^2*(255*B + 14*x*(34*C + 45*D*x)) + 21*d^5*x^2*(33
15*A + 11*x*(255*B + 221*C*x + 195*D*x^2)) - 14*c*d^4*x*(2210*A + 3*x*(765
*B + 748*C*x + 715*D*x^2)) + 8*c^2*d^3*(1105*A + 21*x*(85*B + 2*x*(51*C +
55*D*x))))))/(14549535*d^7)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{9/2}(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(c+dx)^{11/2}(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2\right.\right.}{11d^7}$$
$$\frac{2b(c+dx)^{15/2}\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{15d^7}+$$
$$\frac{2(c+dx)^{13/2}\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c\right.\right.}{13d^7}$$
$$\frac{2(c+dx)^{9/2}(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{9d^7}-$$
$$\frac{2(c+dx)^{7/2}(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^7}+$$
$$\frac{2b^2(c+dx)^{17/2}(3adD-6bcD+bCd)}{17d^7}+\frac{2b^3D(c+dx)^{19/2}}{19d^7}$$

input
```
Int[(a + b*x)^3*(c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-2*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(7/2))/(7*
d^7) - (2*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d -
4*B*c*d^2 + 3*A*d^3 - 6*c^3*D))*(c + d*x)^(9/2))/(9*d^7) - (2*(b*c - a*d)*
(a^2*d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^
2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(11/2))/(11*d^7) + (2*(
a^3*d^3*D + 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^
2*D) + b^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(13/2))/
(13*d^7) + (2*b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^
2 - 15*c^2*D))*(c + d*x)^(15/2))/(15*d^7) + (2*b^2*(b*C*d - 6*b*c*D + 3*a*
d*D)*(c + d*x)^(17/2))/(17*d^7) + (2*b^3*D*(c + d*x)^(19/2))/(19*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$2(xd+c)^{\frac{7}{2}}\left(\left(\frac{7x^3\left(\frac{13}{19}Dx^3+\frac{13}{17}Cx^2+\frac{13}{15}Bx+A\right)b^3}{13}+\frac{21x^2\left(\frac{11}{17}Dx^3+\frac{11}{15}Cx^2+\frac{11}{13}Bx+A\right)ab^2}{11}+\frac{7\left(\frac{3}{5}Dx^3+\frac{9}{13}Cx^2+\frac{9}{11}Bx+A\right)x}{3}\right.\right.$
derivativedivides	$\frac{2b^3D(xd+c)^{\frac{19}{2}}}{19}+\frac{2\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{17}{2}}}{17}+\frac{2\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(Bd^2-2Ccd+3Dc^2\right)\right)}{15}$
default	$\frac{2b^3D(xd+c)^{\frac{19}{2}}}{19}+\frac{2\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{17}{2}}}{17}+\frac{2\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(Bd^2-2Ccd+3Dc^2\right)\right)}{15}$
gosper	$2(xd+c)^{\frac{7}{2}}(765765Dx^6b^3d^6+855855Cx^5b^3d^6+2567565Dx^5ab^2d^6-540540Dx^5b^3cd^5+969969Bx^4b^3d^6+2909907C$
orering	$2(xd+c)^{\frac{7}{2}}(765765Dx^6b^3d^6+855855Cx^5b^3d^6+2567565Dx^5ab^2d^6-540540Dx^5b^3cd^5+969969Bx^4b^3d^6+2909907C$
trager	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/7*(d*x+c)^(7/2)*((7/13*x^3*(13/19*D*x^3+13/17*C*x^2+13/15*B*x+A)*b^3+21/
11*x^2*(11/17*D*x^3+11/15*C*x^2+11/13*B*x+A)*a*b^2+7/3*(3/5*D*x^3+9/13*C*x
^2+9/11*B*x+A)*x*a^2*b+a^3*(7/13*D*x^3+7/11*C*x^2+7/9*B*x+A))*d^6-2/3*(63/
143*x^2*(286/323*D*x^3+143/153*C*x^2+44/45*B*x+A)*b^3+14/11*x*(33/34*D*x^3
+66/65*C*x^2+27/26*B*x+A)*a*b^2+a^2*(84/65*D*x^3+189/143*C*x^2+14/11*B*x+A
)*b+1/3*(189/143*D*x^2+14/11*C*x+B)*a^3)*c*d^5+8/33*(7/13*x*(429/323*D*x^3
+22/17*C*x^2+6/5*B*x+A)*b^3+a*(462/221*D*x^3+126/65*C*x^2+21/13*B*x+A)*b^2
+a^2*(126/65*D*x^2+21/13*C*x+B)*b+1/3*a^3*(21/13*D*x+C))*c^2*d^4-16/429*((
924/323*D*x^3+42/17*C*x^2+28/15*B*x+A)*b^3+3*(42/17*D*x^2+28/15*C*x+B)*a*b
^2+3*(28/15*D*x+C)*a^2*b+a^3*D)*c^3*d^3+128/6435*((945/323*D*x^2+35/17*C*x
+B)*b^2+3*(35/17*D*x+C)*a*b+3*D*a^2)*c^4*b*d^2-256/21879*((42/19*D*x+C)*b+
3*D*a)*c^5*b^2*d+1024/138567*D*b^3*c^6)/d^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(412) = 824$.

Time = 0.09 (sec) , antiderivative size = 1105, normalized size of antiderivative = 2.52

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
2/14549535*(765765*D*b^3*d^9*x^9 + 15360*D*b^3*c^9 + 2078505*A*a^3*c^3*d^6
 - 24320*(3*D*a*b^2 + C*b^3)*c^8*d + 41344*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)
*c^7*d^2 - 77520*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^6*d^3 + 167960*
(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^5*d^4 - 461890*(B*a^3 + 3*A*a^2*b)*c^4*d
^5 + 45045*(39*D*b^3*c*d^8 + 19*(3*D*a*b^2 + C*b^3)*d^9)*x^8 + 3003*(345*D
*b^3*c^2*d^7 + 665*(3*D*a*b^2 + C*b^3)*c*d^8 + 323*(3*D*a^2*b + 3*C*a*b^2
+ B*b^3)*d^9)*x^7 + 231*(15*D*b^3*c^3*d^6 + 5225*(3*D*a*b^2 + C*b^3)*c^2*d
^7 + 10013*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^8 + 4845*(D*a^3 + 3*C*a^2*b
 + 3*B*a*b^2 + A*b^3)*d^9)*x^6 - 63*(60*D*b^3*c^4*d^5 - 95*(3*D*a*b^2 + C*
b^3)*c^3*d^6 - 22933*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^7 - 43605*(D*a^
3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^8 - 20995*(C*a^3 + 3*B*a^2*b + 3*A*
a*b^2)*d^9)*x^5 + 35*(120*D*b^3*c^5*d^4 - 190*(3*D*a*b^2 + C*b^3)*c^4*d^5
+ 323*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^6 + 51357*(D*a^3 + 3*C*a^2*b +
 3*B*a*b^2 + A*b^3)*c^2*d^7 + 96577*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^8
+ 46189*(B*a^3 + 3*A*a^2*b)*d^9)*x^4 - 5*(960*D*b^3*c^6*d^3 - 415701*A*a^3
*d^9 - 1520*(3*D*a*b^2 + C*b^3)*c^5*d^4 + 2584*(3*D*a^2*b + 3*C*a*b^2 + B*
b^3)*c^4*d^5 - 4845*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^6 - 4744
87*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^7 - 877591*(B*a^3 + 3*A*a^2*b)*c*
d^8)*x^3 + 3*(1920*D*b^3*c^7*d^2 + 2078505*A*a^3*c*d^8 - 3040*(3*D*a*b^2 +
 C*b^3)*c^6*d^3 + 5168*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^5*d^4 - 9690*(...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(456) = 912$.

Time = 2.41 (sec) , antiderivative size = 1028, normalized size of antiderivative = 2.35

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b**3*(c + d*x)**(19/2)/(19*d**6) + (c + d*x)**(17/2)*(C*b*
*3*d + 3*D*a*b**2*d - 6*D*b**3*c)/(17*d**6) + (c + d*x)**(15/2)*(B*b**3*d*
*2 + 3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d +
15*D*b**3*c**2)/(15*d**6) + (c + d*x)**(13/2)*(A*b**3*d**3 + 3*B*a*b**2*d*
*3 - 4*B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c*
*2*d + D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c
**3)/(13*d**6) + (c + d*x)**(11/2)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*
B*a**2*b*d**4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C
*a**2*b*c*d**3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**
3 + 18*D*a**2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/(11*d**6)
 + (c + d*x)**(9/2)*(3*A*a**2*b*d**5 - 6*A*a*b**2*c*d**4 + 3*A*b**3*c**2*d
**3 + B*a**3*d**5 - 6*B*a**2*b*c*d**4 + 9*B*a*b**2*c**2*d**3 - 4*B*b**3*c*
*3*d**2 - 2*C*a**3*c*d**4 + 9*C*a**2*b*c**2*d**3 - 12*C*a*b**2*c**3*d**2 +
 5*C*b**3*c**4*d + 3*D*a**3*c**2*d**3 - 12*D*a**2*b*c**3*d**2 + 15*D*a*b**
2*c**4*d - 6*D*b**3*c**5)/(9*d**6) + (c + d*x)**(7/2)*(A*a**3*d**6 - 3*A*a
**2*b*c*d**5 + 3*A*a*b**2*c**2*d**4 - A*b**3*c**3*d**3 - B*a**3*c*d**5 + 3
*B*a**2*b*c**2*d**4 - 3*B*a*b**2*c**3*d**3 + B*b**3*c**4*d**2 + C*a**3*c**
2*d**4 - 3*C*a**2*b*c**3*d**3 + 3*C*a*b**2*c**4*d**2 - C*b**3*c**5*d - D*a
**3*c**3*d**3 + 3*D*a**2*b*c**4*d**2 - 3*D*a*b**2*c**5*d + D*b**3*c**6)/(7
*d**6))/d, Ne(d, 0)), (c**(5/2)*(A*a**3*x + D*b**3*x**7/7 + x**6*(C*b**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.42

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2\left(765765\,(dx+c)^{\frac{19}{2}}Db^3 - 855855\,(6\,Db^3c-(3\,Dab^2+Cb^3)d)(dx+c)^{\frac{17}{2}} + 969969\,(15\,Db^3c\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
2/14549535*(765765*(d*x + c)^(19/2)*D*b^3 - 855855*(6*D*b^3*c - (3*D*a*b^2
 + C*b^3)*d)*(d*x + c)^(17/2) + 969969*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^
3)*c*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(15/2) - 1119195*(
20*D*b^3*c^3 - 10*(3*D*a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B
*b^3)*c*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(13/2
) + 1322685*(15*D*b^3*c^4 - 10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b +
3*C*a*b^2 + B*b^3)*c^2*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d
^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^4)*(d*x + c)^(11/2) - 1616615*(6*D*
b^3*c^5 - 5*(3*D*a*b^2 + C*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*
c^3*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3
*B*a^2*b + 3*A*a*b^2)*c*d^4 - (B*a^3 + 3*A*a^2*b)*d^5)*(d*x + c)^(9/2) + 2
078505*(D*b^3*c^6 + A*a^3*d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3
*C*a*b^2 + B*b^3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^
3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)*(
d*x + c)^(7/2))/d^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4675 vs. $2(412) = 824$.

Time = 0.19 (sec) , antiderivative size = 4675, normalized size of antiderivative = 10.67

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/14549535*(14549535*sqrt(d*x + c)*A*a^3*c^3 + 14549535*((d*x + c)^(3/2) -
 3*sqrt(d*x + c)*c)*A*a^3*c^2 + 4849845*((d*x + c)^(3/2) - 3*sqrt(d*x + c)
*c)*B*a^3*c^3/d + 14549535*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*a^2*b*c
^3/d + 2909907*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c
)*c^2)*A*a^3*c + 969969*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqr
t(d*x + c)*c^2)*C*a^3*c^3/d^2 + 2909907*(3*(d*x + c)^(5/2) - 10*(d*x + c)^
(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*a^2*b*c^3/d^2 + 2909907*(3*(d*x + c)^(5/
2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a*b^2*c^3/d^2 + 290990
7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*a^3*
c^2/d + 8729721*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x +
c)*c^2)*A*a^2*b*c^2/d + 415701*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A*a^3 + 415701*(5*(d*x + c
)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)
*c^3)*D*a^3*c^3/d^3 + 1247103*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*a^2*b*c^3/d^3 + 1247103*(
5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqr
t(d*x + c)*c^3)*B*a*b^2*c^3/d^3 + 415701*(5*(d*x + c)^(7/2) - 21*(d*x + c)
^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A*b^3*c^3/d^3 +
1247103*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2
 - 35*sqrt(d*x + c)*c^3)*C*a^3*c^2/d^2 + 3741309*(5*(d*x + c)^(7/2) - 2...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+b\,x)^3\,(c+d\,x)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^3*(c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^3*(c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.95

$$\int (a+bx)^3(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(2078505*a**4*c**3*d**5 + 6235515*a**4*c**2*d**6*x + 6235
515*a**4*c*d**7*x**2 + 2078505*a**4*d**8*x**3 - 1847560*a**3*b*c**4*d**4 +
 923780*a**3*b*c**3*d**5*x + 13856700*a**3*b*c**2*d**6*x**2 + 17551820*a**
3*b*c*d**7*x**3 + 6466460*a**3*b*d**8*x**4 + 90440*a**3*c**6*d**3 - 45220*
a**3*c**5*d**4*x + 33915*a**3*c**4*d**5*x**2 + 2396660*a**3*c**3*d**6*x**3
 + 5177690*a**3*c**2*d**7*x**4 + 4069800*a**3*c*d**8*x**5 + 1119195*a**3*d
**9*x**6 + 1007760*a**2*b**2*c**5*d**3 - 503880*a**2*b**2*c**4*d**4*x + 37
7910*a**2*b**2*c**3*d**5*x**2 + 14234610*a**2*b**2*c**2*d**6*x**3 + 202811
70*a**2*b**2*c*d**7*x**4 + 7936110*a**2*b**2*d**8*x**5 - 108528*a**2*b*c**
7*d**2 + 54264*a**2*b*c**6*d**3*x - 40698*a**2*b*c**5*d**4*x**2 + 33915*a*
*2*b*c**4*d**5*x**3 + 5426400*a**2*b*c**3*d**6*x**4 + 12575682*a**2*b*c**2
*d**7*x**5 + 10296594*a**2*b*c*d**8*x**6 + 2909907*a**2*b*d**9*x**7 - 3100
80*a*b**3*c**6*d**2 + 155040*a*b**3*c**5*d**3*x - 116280*a*b**3*c**4*d**4*
x**2 + 96900*a*b**3*c**3*d**5*x**3 + 7189980*a*b**3*c**2*d**6*x**4 + 10988
460*a*b**3*c*d**7*x**5 + 4476780*a*b**3*d**8*x**6 + 51072*a*b**2*c**8*d -
25536*a*b**2*c**7*d**2*x + 19152*a*b**2*c**6*d**3*x**2 - 15960*a*b**2*c**5
*d**4*x**3 + 13965*a*b**2*c**4*d**5*x**4 + 4352292*a*b**2*c**3*d**6*x**5 +
 10559934*a*b**2*c**2*d**7*x**6 + 8900892*a*b**2*c*d**8*x**7 + 2567565*a*b
**2*d**9*x**8 + 41344*b**4*c**7*d - 20672*b**4*c**6*d**2*x + 15504*b**4*c*
*5*d**3*x**2 - 12920*b**4*c**4*d**4*x**3 + 11305*b**4*c**3*d**5*x**4 + ...
```

3.72 $\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result 663
Mathematica [A] (verified) 664
Rubi [A] (verified) 664
Maple [A] (verified) 666
Fricas [B] (verification not implemented) 666
Sympy [A] (verification not implemented) 667
Maxima [A] (verification not implemented) 668
Giac [B] (verification not implemented) 669
Mupad [F(-1)] 670
Reduce [B] (verification not implemented) 670

Optimal result

Integrand size = 32, antiderivative size = 326

$$\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2(bc-ad)^2\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{7/2}}{7d^6}+\frac{2(bc-ad)\left(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D)\right)(c+dx)^{9/2}}{9d^6}+\frac{2\left(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D)\right)(c+dx)^{11/2}}{11d^6}+\frac{2\left(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D)\right)(c+dx)^{13/2}}{13d^6}+\frac{2b(bCd-5bcD+2adD)(c+dx)^{15/2}}{15d^6}+\frac{2b^2D(c+dx)^{17/2}}{17d^6}$$

output
```
2/7*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(7/2)/d^6+2/9*(-a*d
+b*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3
))*(d*x+c)^(9/2)/d^6+2/11*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D
*c^2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(11/2)/d^6+2/13*(a
^2*d^2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(13/2)
/d^6+2/15*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(15/2)/d^6+2/17*b^2*D*(d*x+c)^
(17/2)/d^6
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.01

$$\int (a+bx)^2(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{2(c+dx)^{7/2} (34abd(128c^4D-16c^3d(15C+28Dx)+7d^4x(715A+585Bx+495Cx^2+429D$$

input
```
Integrate[(a + b*x)^2*(c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(7/2)*(34*a*b*d*(128*c^4*D - 16*c^3*d*(15*C + 28*D*x) + 7*d^4
*x*(715*A + 585*B*x + 495*C*x^2 + 429*D*x^3) - 2*c*d^3*(715*A + 910*B*x +
945*C*x^2 + 924*D*x^3) + 8*c^2*d^2*(65*B + 21*x*(5*C + 6*D*x))) + 85*a^2*d
^2*(-48*c^3*D + 8*c^2*d*(13*C + 21*D*x) - 2*c*d^2*(143*B + 7*x*(26*C + 27*
D*x)) + d^3*(1287*A + 7*x*(143*B + 117*C*x + 99*D*x^2))) + b^2*(-1280*c^5*
D + 128*c^4*d*(17*C + 35*D*x) - 16*c^3*d^2*(255*B + 14*x*(34*C + 45*D*x))
+ 21*d^5*x^2*(3315*A + 11*x*(255*B + 221*C*x + 195*D*x^2)) - 14*c*d^4*x*(2
210*A + 3*x*(765*B + 748*C*x + 715*D*x^2)) + 8*c^2*d^3*(1105*A + 21*x*(85*
B + 2*x*(51*C + 55*D*x))))))/(765765*d^6)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{(c+dx)^{9/2} \left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c-$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(c+dx)^{11/2}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{11d^6}+\frac{2(c+dx)^{13/2}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{13d^6}+\frac{2(c+dx)^{9/2}(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{9d^6}+\frac{2(c+dx)^{7/2}(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^6}+\frac{2b(c+dx)^{15/2}(2adD-5bcD+bCd)}{15d^6}+\frac{2b^2D(c+dx)^{17/2}}{17d^6}$$

input
```
Int[(a + b*x)^2*(c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(7/2))/(7*d
^6) + (2*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B
*c*d^2 + 2*A*d^3 - 5*c^3*D))*(c + d*x)^(9/2))/(9*d^6) + (2*(a^2*d^2*(C*d -
 3*c*D) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2
 + A*d^3 - 10*c^3*D))*(c + d*x)^(11/2))/(11*d^6) + (2*(a^2*d^2*D + 2*a*b*d
*(C*d - 4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(13/2))/(13*d
^6) + (2*b*(b*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(15/2))/(15*d^6) + (2*b^2
*D*(c + d*x)^(17/2))/(17*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$2\left(\left(\frac{7x^2\left(\frac{11}{17}Dx^3+\frac{11}{15}Cx^2+\frac{11}{13}Bx+A\right)b^2}{11}+\frac{14\left(\frac{3}{5}Dx^3+\frac{9}{13}Cx^2+\frac{9}{11}Bx+A\right)xab}{9}+a^2\left(\frac{7}{13}Dx^3+\frac{7}{11}Cx^2+\frac{7}{9}Bx+A\right)\right)d^5-\frac{4\left(\frac{7}{}\right.}{}\right.$
derivativedivides	$\frac{2b^2D(xd+c)^{\frac{17}{2}}}{17}+\frac{2\left(2b(ad-bc)D+b^2(Cd-3Dc)\right)(xd+c)^{\frac{15}{2}}}{15}+\frac{2\left((ad-bc)^2D+2b(ad-bc)(Cd-3Dc)+b^2\left(Bd^2-2Ccd+3Dc^2\right)\right)(xd+c}{13}$
default	$\frac{2b^2D(xd+c)^{\frac{17}{2}}}{17}+\frac{2\left(2b(ad-bc)D+b^2(Cd-3Dc)\right)(xd+c)^{\frac{15}{2}}}{15}+\frac{2\left((ad-bc)^2D+2b(ad-bc)(Cd-3Dc)+b^2\left(Bd^2-2Ccd+3Dc^2\right)\right)(xd+c}{13}$
gosper	$\frac{2(xd+c)^{\frac{7}{2}}\left(45045Dx^5b^2d^5+51051Cx^4b^2d^5+102102Dx^4abd^5-30030Dx^4b^2cd^4+58905Bx^3b^2d^5+117810Cx^3abd^5-\right.}{}$
orering	$\frac{2(xd+c)^{\frac{7}{2}}\left(45045Dx^5b^2d^5+51051Cx^4b^2d^5+102102Dx^4abd^5-30030Dx^4b^2cd^4+58905Bx^3b^2d^5+117810Cx^3abd^5-\right.}{}$
trager	Expression too large to display

input
```
int((b*x+a)^2*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/7*((7/11*x^2*(11/17*D*x^3+11/15*C*x^2+11/13*B*x+A)*b^2+14/9*(3/5*D*x^3+9
/13*C*x^2+9/11*B*x+A)*x*a*b+a^2*(7/13*D*x^3+7/11*C*x^2+7/9*B*x+A))*d^5-4/9
*(7/11*x*(33/34*D*x^3+66/65*C*x^2+27/26*B*x+A)*b^2+a*(84/65*D*x^3+189/143*
C*x^2+14/11*B*x+A)*b+1/2*(189/143*D*x^2+14/11*C*x+B)*a^2)*c*d^4+8/99*((462
/221*D*x^3+126/65*C*x^2+21/13*B*x+A)*b^2+2*(126/65*D*x^2+21/13*C*x+B)*a*b+
a^2*(21/13*D*x+C))*c^2*d^3-16/429*((42/17*D*x^2+28/15*C*x+B)*b^2+2*(28/15*
D*x+C)*a*b+D*a^2)*c^3*d^2+128/6435*((35/17*D*x+C)*b+2*D*a)*c^4*b*d-256/218
79*D*b^2*c^5)*(d*x+c)^(7/2)/d^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(302) = 604$.

Time = 0.08 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.25

$$\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
2/765765*(45045*D*b^2*d^8*x^8 - 1280*D*b^2*c^8 + 109395*A*a^2*c^3*d^5 + 21
76*(2*D*a*b + C*b^2)*c^7*d - 4080*(D*a^2 + 2*C*a*b + B*b^2)*c^6*d^2 + 8840
*(C*a^2 + 2*B*a*b + A*b^2)*c^5*d^3 - 24310*(B*a^2 + 2*A*a*b)*c^4*d^4 + 300
3*(35*D*b^2*c*d^7 + 17*(2*D*a*b + C*b^2)*d^8)*x^7 + 231*(275*D*b^2*c^2*d^6
 + 527*(2*D*a*b + C*b^2)*c*d^7 + 255*(D*a^2 + 2*C*a*b + B*b^2)*d^8)*x^6 +
63*(5*D*b^2*c^3*d^5 + 1207*(2*D*a*b + C*b^2)*c^2*d^6 + 2295*(D*a^2 + 2*C*a
*b + B*b^2)*c*d^7 + 1105*(C*a^2 + 2*B*a*b + A*b^2)*d^8)*x^5 - 35*(10*D*b^2
*c^4*d^4 - 17*(2*D*a*b + C*b^2)*c^3*d^5 - 2703*(D*a^2 + 2*C*a*b + B*b^2)*c
^2*d^6 - 5083*(C*a^2 + 2*B*a*b + A*b^2)*c*d^7 - 2431*(B*a^2 + 2*A*a*b)*d^8
)*x^4 + 5*(80*D*b^2*c^5*d^3 + 21879*A*a^2*d^8 - 136*(2*D*a*b + C*b^2)*c^4*
d^4 + 255*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^5 + 24973*(C*a^2 + 2*B*a*b + A*b
^2)*c^2*d^6 + 46189*(B*a^2 + 2*A*a*b)*c*d^7)*x^3 - 3*(160*D*b^2*c^6*d^2 -
109395*A*a^2*c*d^7 - 272*(2*D*a*b + C*b^2)*c^5*d^3 + 510*(D*a^2 + 2*C*a*b
+ B*b^2)*c^4*d^4 - 1105*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^5 - 60775*(B*a^2 +
 2*A*a*b)*c^2*d^6)*x^2 + (640*D*b^2*c^7*d + 328185*A*a^2*c^2*d^6 - 1088*(2
*D*a*b + C*b^2)*c^6*d^2 + 2040*(D*a^2 + 2*C*a*b + B*b^2)*c^5*d^3 - 4420*(C
*a^2 + 2*B*a*b + A*b^2)*c^4*d^4 + 12155*(B*a^2 + 2*A*a*b)*c^3*d^5)*x)*sqrt
(d*x + c)/d^6
```

Sympy [A] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.97

$$\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \begin{cases} \frac{2\left(\frac{Db^2(c+dx)^{\frac{17}{2}}}{17d^5}+\frac{(c+dx)^{\frac{15}{2}}\left(Cb^2d+2Dabd-5Db^2c\right)}{15d^5}+\frac{(c+dx)^{\frac{13}{2}}\left(Bb^2d^2+2Cabd^2-4Cb^2cd+Da^2d^2-8Dabcd+10Db^2c^2\right)}{13d^5}+\frac{(c+dx)^{\frac{11}{2}}\left(\ldots\right.}{\ldots}\right.}{\ldots} \\ c^{\frac{5}{2}}\left(Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5\left(Cb^2+2Dab\right)}{5}+\frac{x^4\left(Bb^2+2Cab+Da^2\right)}{4}+\frac{x^3\left(Ab^2+2Bab+Ca^2\right)}{3}+\frac{x^2\cdot\left(2Aab+Ba^2\right)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b**2*(c + d*x)**(17/2)/(17*d**5) + (c + d*x)**(15/2)*(C*b*
*2*d + 2*D*a*b*d - 5*D*b**2*c)/(15*d**5) + (c + d*x)**(13/2)*(B*b**2*d**2
+ 2*C*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2
)/(13*d**5) + (c + d*x)**(11/2)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d
**2 + C*a**2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 1
2*D*a*b*c**2*d - 10*D*b**2*c**3)/(11*d**5) + (c + d*x)**(9/2)*(2*A*a*b*d**
4 - 2*A*b**2*c*d**3 + B*a**2*d**4 - 4*B*a*b*c*d**3 + 3*B*b**2*c**2*d**2 -
2*C*a**2*c*d**3 + 6*C*a*b*c**2*d**2 - 4*C*b**2*c**3*d + 3*D*a**2*c**2*d**2
 - 8*D*a*b*c**3*d + 5*D*b**2*c**4)/(9*d**5) + (c + d*x)**(7/2)*(A*a**2*d**
5 - 2*A*a*b*c*d**4 + A*b**2*c**2*d**3 - B*a**2*c*d**4 + 2*B*a*b*c**2*d**3
- B*b**2*c**3*d**2 + C*a**2*c**2*d**3 - 2*C*a*b*c**3*d**2 + C*b**2*c**4*d
- D*a**2*c**3*d**2 + 2*D*a*b*c**4*d - D*b**2*c**5)/(7*d**5))/d, Ne(d, 0)),
 (c**(5/2)*(A*a**2*x + D*b**2*x**6/6 + x**5*(C*b**2 + 2*D*a*b)/5 + x**4*(B
*b**2 + 2*C*a*b + D*a**2)/4 + x**3*(A*b**2 + 2*B*a*b + C*a**2)/3 + x**2*(2
*A*a*b + B*a**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.19

$$\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\left(45045\,(dx+c)^{\frac{17}{2}}Db^2-51051\,(5\,Db^2c-(2\,Dab+Cb^2)d)(dx+c)^{\frac{15}{2}}+58905\,(10\,Db^2c^2-4\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
2/765765*(45045*(d*x + c)^(17/2)*D*b^2 - 51051*(5*D*b^2*c - (2*D*a*b + C*b
^2)*d)*(d*x + c)^(15/2) + 58905*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d +
(D*a^2 + 2*C*a*b + B*b^2)*d^2)*(d*x + c)^(13/2) - 69615*(10*D*b^2*c^3 - 6*
(2*D*a*b + C*b^2)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B
*a*b + A*b^2)*d^3)*(d*x + c)^(11/2) + 85085*(5*D*b^2*c^4 - 4*(2*D*a*b + C*
b^2)*c^3*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*
b^2)*c*d^3 + (B*a^2 + 2*A*a*b)*d^4)*(d*x + c)^(9/2) - 109395*(D*b^2*c^5 -
A*a^2*d^5 - (2*D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 -
(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)*(d*x + c)^(7/
2))/d^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3175 vs. $2(302) = 604$.

Time = 0.16 (sec) , antiderivative size = 3175, normalized size of antiderivative = 9.74

$$\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/765765*(765765*sqrt(d*x + c)*A*a^2*c^3 + 765765*((d*x + c)^(3/2) - 3*sqr
t(d*x + c)*c)*A*a^2*c^2 + 255255*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*a
^2*c^3/d + 510510*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*a*b*c^3/d + 1531
53*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a^2
*c + 51051*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*C*a^2*c^3/d^2 + 102102*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*s
qrt(d*x + c)*c^2)*B*a*b*c^3/d^2 + 51051*(3*(d*x + c)^(5/2) - 10*(d*x + c)^
(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*b^2*c^3/d^2 + 153153*(3*(d*x + c)^(5/2)
- 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*a^2*c^2/d + 306306*(3*(d*
x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a*b*c^2/d +
21879*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*A*a^2 + 21879*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/
2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*D*a^2*c^3/d^3 + 4375
8*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*
sqrt(d*x + c)*c^3)*C*a*b*c^3/d^3 + 21879*(5*(d*x + c)^(7/2) - 21*(d*x + c)
^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*b^2*c^3/d^3 +
65637*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*C*a^2*c^2/d^2 + 131274*(5*(d*x + c)^(7/2) - 21*(d*x
 + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*a*b*c^2/d
^2 + 65637*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+b\,x)^2\,(c+d\,x)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^2*(c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^2*(c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.78

$$\int (a+bx)^2(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\sqrt{dx+c}\,(45045b^2d^8x^8+102102ab\,d^8x^7+156156b^2c\,d^7x^7+58905a^2d^8x^6+361284abc\,d^7x^6$$

input
```
int((b*x+a)^2*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(109395*a**3*c**3*d**4 + 328185*a**3*c**2*d**5*x + 328185
*a**3*c*d**6*x**2 + 109395*a**3*d**7*x**3 - 72930*a**2*b*c**4*d**3 + 36465
*a**2*b*c**3*d**4*x + 546975*a**2*b*c**2*d**5*x**2 + 692835*a**2*b*c*d**6*
x**3 + 255255*a**2*b*d**7*x**4 + 4760*a**2*c**6*d**2 - 2380*a**2*c**5*d**3
*x + 1785*a**2*c**4*d**4*x**2 + 126140*a**2*c**3*d**5*x**3 + 272510*a**2*c
**2*d**6*x**4 + 214200*a**2*c*d**7*x**5 + 58905*a**2*d**8*x**6 + 26520*a*b
**2*c**5*d**2 - 13260*a*b**2*c**4*d**3*x + 9945*a*b**2*c**3*d**4*x**2 + 37
4595*a*b**2*c**2*d**5*x**3 + 533715*a*b**2*c*d**6*x**4 + 208845*a*b**2*d**
7*x**5 - 3808*a*b*c**7*d + 1904*a*b*c**6*d**2*x - 1428*a*b*c**5*d**3*x**2
+ 1190*a*b*c**4*d**4*x**3 + 190400*a*b*c**3*d**5*x**4 + 441252*a*b*c**2*d*
*6*x**5 + 361284*a*b*c*d**7*x**6 + 102102*a*b*d**8*x**7 - 4080*b**3*c**6*d
 + 2040*b**3*c**5*d**2*x - 1530*b**3*c**4*d**3*x**2 + 1275*b**3*c**3*d**4*
x**3 + 94605*b**3*c**2*d**5*x**4 + 144585*b**3*c*d**6*x**5 + 58905*b**3*d*
*7*x**6 + 896*b**2*c**8 - 448*b**2*c**7*d*x + 336*b**2*c**6*d**2*x**2 - 28
0*b**2*c**5*d**3*x**3 + 245*b**2*c**4*d**4*x**4 + 76356*b**2*c**3*d**5*x**
5 + 185262*b**2*c**2*d**6*x**6 + 156156*b**2*c*d**7*x**7 + 45045*b**2*d**8
*x**8))/(765765*d**5)
```

3.73 $\int (a+bx)(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx$

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Optimal result

Integrand size = 30, antiderivative size = 214

$$\begin{aligned}\int (a+bx)(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx =&-\frac{2(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{7/2}}{7d^5}\\ &-\frac{2(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))(c+dx)^{9/2}}{9d^5}\\ &+\frac{2(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))(c+dx)^{11/2}}{11d^5}\\ &+\frac{2(bCd-4bcD+adD)(c+dx)^{13/2}}{13d^5}+\frac{2bD(c+dx)^{15/2}}{15d^5}\end{aligned}$$

output
```
-2/7*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(7/2)/d^5-2/9*(a*d*(
-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(9/
2)/d^5+2/11*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(11/2)/d^
5+2/13*(C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(13/2)/d^5+2/15*b*D*(d*x+c)^(15/2)/d^
5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int (a+bx)(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2(c+dx)^{7/2}\left(b(128c^4D-16c^3d(15C+28Dx)+7d^4x(715A+585Bx+495Cx^2+429Dx^3)\right.}{}$$

input
```
Integrate[(a + b*x)*(c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(7/2)*(b*(128*c^4*D - 16*c^3*d*(15*C + 28*D*x) + 7*d^4*x*(715
*A + 585*B*x + 495*C*x^2 + 429*D*x^3) - 2*c*d^3*(715*A + 910*B*x + 945*C*x
^2 + 924*D*x^3) + 8*c^2*d^2*(65*B + 21*x*(5*C + 6*D*x))) + 5*a*d*(-48*c^3*
D + 8*c^2*d*(13*C + 21*D*x) - 2*c*d^2*(143*B + 7*x*(26*C + 27*D*x)) + d^3*
(1287*A + 7*x*(143*B + 117*C*x + 99*D*x^2)))))/(45045*d^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{7/2}\left(b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)}{d^4}+\frac{(c+dx)^{5/2}(ad-bc)\left(Ad^3\right.}{d}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(c+dx)^{9/2}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{9d^5}-\frac{2(c+dx)^{7/2}(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^5}+\frac{2(c+dx)^{11/2}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{11d^5}+\frac{2(c+dx)^{13/2}(adD-4bcD+bCd)}{13d^5}+\frac{2bD(c+dx)^{15/2}}{15d^5}$$

input
```
Int[(a + b*x)*(c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-2*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(7/2))/(7*d^
5) - (2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^
3 - 4*c^3*D))*(c + d*x)^(9/2))/(9*d^5) + (2*(a*d*(C*d - 3*c*D) - b*(3*c*C*
d - B*d^2 - 6*c^2*D))*(c + d*x)^(11/2))/(11*d^5) + (2*(b*C*d - 4*b*c*D + a
*d*D)*(c + d*x)^(13/2))/(13*d^5) + (2*b*D*(c + d*x)^(15/2))/(15*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{2\left(\left(\left(\frac{7Dbx^4}{15}+\frac{7(Cb+Da)x^3}{13}+\frac{7(Bb+Ca)x^2}{11}+\frac{7(Ab+Ba)x}{9}+Aa\right)d^4-\frac{2\left(\frac{84Dbx^3}{65}+\frac{189(Cb+Da)x^2}{143}+\frac{14(Bb+Ca)x}{11}+Ab+Ba\right)cd}{9}\right.\right.}{7d^5}$
derivativedivides	$\frac{\frac{2bD(xd+c)^{\frac{15}{2}}}{15}+\frac{2((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left((ad-bc)(Cd-3Dc)+b\left(Bd^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left((ad-bc)\left(B\right.\right.}{}}{d^5}$
default	$\frac{\frac{2bD(xd+c)^{\frac{15}{2}}}{15}+\frac{2((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left((ad-bc)(Cd-3Dc)+b\left(Bd^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left((ad-bc)\left(B\right.\right.}{}}{d^5}$
gosper	$2(xd+c)^{\frac{7}{2}}(3003Dbx^4d^4+3465Cx^3bd^4+3465Dx^3ad^4-1848Dx^3bcd^3+4095Bx^2bd^4+4095Cx^2ad^4-1890Cx^2bcd^3$
orering	$2(xd+c)^{\frac{7}{2}}(3003Dbx^4d^4+3465Cx^3bd^4+3465Dx^3ad^4-1848Dx^3bcd^3+4095Bx^2bd^4+4095Cx^2ad^4-1890Cx^2bcd^3$
trager	$2(3003bd^7Dx^7+3465Cbd^7x^6+3465Dad^7x^6+7161Dbcd^6x^6+4095Bbd^7x^5+4095Cad^7x^5+8505Cbcd^6x^5+8505Dac$

input
```
int((b*x+a)*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/7*((7/15*D*b*x^4+7/13*(C*b+D*a)*x^3+7/11*(B*b+C*a)*x^2+7/9*(A*b+B*a)*x+A
*a)*d^4-2/9*(84/65*D*b*x^3+189/143*(C*b+D*a)*x^2+14/11*(B*b+C*a)*x+A*b+B*a
)*c*d^3+8/99*(126/65*D*b*x^2+21/13*(C*b+D*a)*x+B*b+C*a)*c^2*d^2-16/429*(28
/15*D*b*x+C*b+D*a)*c^3*d+128/6435*D*b*c^4)*(d*x+c)^(7/2)/d^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(196) = 392.

Time = 0.08 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.90

$$\int (a+bx)(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\left(3003\,Dbd^7x^7+128\,Dbc^7+6435\,Aac^3d^4-240\,(Da+Cb)c^6d+520\,(Ca+Bb)c^5d^2-1430\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
2/45045*(3003*D*b*d^7*x^7 + 128*D*b*c^7 + 6435*A*a*c^3*d^4 - 240*(D*a + C*
b)*c^6*d + 520*(C*a + B*b)*c^5*d^2 - 1430*(B*a + A*b)*c^4*d^3 + 231*(31*D*
b*c*d^6 + 15*(D*a + C*b)*d^7)*x^6 + 63*(71*D*b*c^2*d^5 + 135*(D*a + C*b)*c
*d^6 + 65*(C*a + B*b)*d^7)*x^5 + 35*(D*b*c^3*d^4 + 159*(D*a + C*b)*c^2*d^5
 + 299*(C*a + B*b)*c*d^6 + 143*(B*a + A*b)*d^7)*x^4 - 5*(8*D*b*c^4*d^3 - 1
287*A*a*d^7 - 15*(D*a + C*b)*c^3*d^4 - 1469*(C*a + B*b)*c^2*d^5 - 2717*(B*
a + A*b)*c*d^6)*x^3 + 3*(16*D*b*c^5*d^2 + 6435*A*a*c*d^6 - 30*(D*a + C*b)*
c^4*d^3 + 65*(C*a + B*b)*c^3*d^4 + 3575*(B*a + A*b)*c^2*d^5)*x^2 - (64*D*b
*c^6*d - 19305*A*a*c^2*d^5 - 120*(D*a + C*b)*c^5*d^2 + 260*(C*a + B*b)*c^4
*d^3 - 715*(B*a + A*b)*c^3*d^4)*x)*sqrt(d*x + c)/d^5
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.50

$$\int (a+bx)(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\begin{cases}\frac{2\left(\frac{Db(c+dx)^{\frac{15}{2}}}{15d^4}+\frac{(c+dx)^{\frac{13}{2}}(Cbd+Dad-4Dbc)}{13d^4}+\frac{(c+dx)^{\frac{11}{2}}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6Dbc^2\right)}{11d^4}+\frac{(c+dx)^{\frac{9}{2}}\left(Abd^3+Bad^3-2Bbcd^2-2C\cdots\right)}{9d\cdots}\right)}{d} \\ c^{\frac{5}{2}}\left(Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}\right)\end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*(D*b*(c + d*x)**(15/2)/(15*d**4) + (c + d*x)**(13/2)*(C*b*d +
 D*a*d - 4*D*b*c)/(13*d**4) + (c + d*x)**(11/2)*(B*b*d**2 + C*a*d**2 - 3*C
*b*c*d - 3*D*a*c*d + 6*D*b*c**2)/(11*d**4) + (c + d*x)**(9/2)*(A*b*d**3 +
B*a*d**3 - 2*B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D
*b*c**3)/(9*d**4) + (c + d*x)**(7/2)*(A*a*d**4 - A*b*c*d**3 - B*a*c*d**3 +
 B*b*c**2*d**2 + C*a*c**2*d**2 - C*b*c**3*d - D*a*c**3*d + D*b*c**4)/(7*d*
*4))/d, Ne(d, 0)), (c**(5/2)*(A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/4 + x*
*3*(B*b + C*a)/3 + x**2*(A*b + B*a)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int (a+bx)(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{2\left(3003\,(dx+c)^{\frac{15}{2}}Db-3465\,(4\,Dbc-(Da+Cb)d)(dx+c)^{\frac{13}{2}}+4095\,(6\,Dbc^2-3\,(Da+Cb)\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
2/45045*(3003*(d*x + c)^(15/2)*D*b - 3465*(4*D*b*c - (D*a + C*b)*d)*(d*x +
 c)^(13/2) + 4095*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*(d*x +
 c)^(11/2) - 5005*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 -
 (B*a + A*b)*d^3)*(d*x + c)^(9/2) + 6435*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*
c^3*d + (C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)*(d*x + c)^(7/2))/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. 2(196) = 392.

Time = 0.14 (sec) , antiderivative size = 1867, normalized size of antiderivative = 8.72

$$\int (a+bx)(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(d*x + c)*A*a*c^3 + 45045*((d*x + c)^(3/2) - 3*sqrt(d*x
 + c)*c)*A*a*c^2 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*a*c^3/d +
 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*b*c^3/d + 9009*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*a*c + 3003*(3*(d*x
 + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a*c^3/d^2 + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*b*
c^3/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c
)*c^2)*B*a*c^2/d + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqr
t(d*x + c)*c^2)*A*b*c^2/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c
 + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A*a + 1287*(5*(d*x + c)^
(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c
^3)*D*a*c^3/d^3 + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x
 + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*b*c^3/d^3 + 3861*(5*(d*x + c)^(7
/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3
)*C*a*c^2/d^2 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*b*c^2/d^2 + 3861*(5*(d*x + c)^(7/2
) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*
B*a*c/d + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3
/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A*b*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d
*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 3...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+b\,x)\,\left(c+d\,x\right)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)*(c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)*(c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.58

$$\int (a+bx)(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{2\sqrt{dx+c}\,(3003b\,d^7x^7+3465a\,d^7x^6+10626bc\,d^6x^6+12600ac\,d^6x^5+4095b^2d^6x^5+12978b\,c^2}{}$$

input
```
int((b*x+a)*(d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(6435*a**2*c**3*d**3 + 19305*a**2*c**2*d**4*x + 19305*a**
2*c*d**5*x**2 + 6435*a**2*d**6*x**3 - 2860*a*b*c**4*d**2 + 1430*a*b*c**3*d
**3*x + 21450*a*b*c**2*d**4*x**2 + 27170*a*b*c*d**5*x**3 + 10010*a*b*d**6*
x**4 + 280*a*c**6*d - 140*a*c**5*d**2*x + 105*a*c**4*d**3*x**2 + 7420*a*c*
*3*d**4*x**3 + 16030*a*c**2*d**5*x**4 + 12600*a*c*d**6*x**5 + 3465*a*d**7*
x**6 + 520*b**2*c**5*d - 260*b**2*c**4*d**2*x + 195*b**2*c**3*d**3*x**2 +
7345*b**2*c**2*d**4*x**3 + 10465*b**2*c*d**5*x**4 + 4095*b**2*d**6*x**5 -
112*b*c**7 + 56*b*c**6*d*x - 42*b*c**5*d**2*x**2 + 35*b*c**4*d**3*x**3 + 5
600*b*c**3*d**4*x**4 + 12978*b*c**2*d**5*x**5 + 10626*b*c*d**6*x**6 + 3003
*b*d**7*x**7))/(45045*d**4)
```

3.74 $\int (c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result 680
Mathematica [A] (verified) 680
Rubi [A] (verified) 681
Maple [A] (verified) 682
Fricas [B] (verification not implemented) 683
Sympy [B] (verification not implemented) 683
Maxima [A] (verification not implemented) 684
Giac [B] (verification not implemented) 684
Mupad [F(-1)] 685
Reduce [B] (verification not implemented) 686

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int (c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx=\frac{2\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{7/2}}{7d^4}-\frac{2\left(2cCd-Bd^2-3c^2D\right)(c+dx)^{9/2}}{9d^4}+\frac{2(Cd-3cD)(c+dx)^{11/2}}{11d^4}+\frac{2D(c+dx)^{13/2}}{13d^4}$$

output
```
2/7*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(7/2)/d^4-2/9*(-B*d^2+2*C*c*d-3*
D*c^2)*(d*x+c)^(9/2)/d^4+2/11*(C*d-3*D*c)*(d*x+c)^(11/2)/d^4+2/13*D*(d*x+c
)^(13/2)/d^4
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.71

$$\int (c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx=\frac{2(c+dx)^{7/2}\left(-48c^3D+8c^2d(13C+21Dx)-2cd^2(143B+7x(26C+27Dx))+d^3(1287A+\right.}{9009d^4}$$

input
```
Integrate[(c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c + d*x)^(7/2)*(-48*c^3*D + 8*c^2*d*(13*C + 21*D*x) - 2*c*d^2*(143*B +
 7*x*(26*C + 27*D*x)) + d^3*(1287*A + 7*x*(143*B + 117*C*x + 99*D*x^2))))/
(9009*d^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{(c+dx)^{5/2} \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3}+\frac{(c+dx)^{7/2} \left(Bd^2+3c^2D-2cCd\right)}{d^3}+\frac{(c+dx)^{9/2}(Cd-3cD}{d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(c+dx)^{7/2} \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{7d^4}-\frac{2(c+dx)^{9/2} \left(-Bd^2-3c^2D+2cCd\right)}{9d^4}+\frac{2(c+dx)^{11/2}(Cd-3cD)}{11d^4}+\frac{2D(c+dx)^{13/2}}{13d^4}$$

input
```
Int[(c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(7/2))/(7*d^4) - (2*(2*c*
C*d - B*d^2 - 3*c^2*D)*(c + d*x)^(9/2))/(9*d^4) + (2*(C*d - 3*c*D)*(c + d*
x)^(11/2))/(11*d^4) + (2*D*(c + d*x)^(13/2))/(13*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{2\left(\left(\frac{7}{13}D x^{3}+\frac{7}{11}C x^{2}+\frac{7}{9}B x+A\right)d^{3}-\frac{2\left(\frac{189}{143}D x^{2}+\frac{14}{11}C x+B\right)c d^{2}}{9}+\frac{8c^{2}\left(\frac{21Dx}{13}+C\right)d}{99}-\frac{16Dc^{3}}{429}\right)(xd+c)^{\frac{7}{2}}}{7d^{4}}$
gosper	$\frac{2(xd+c)^{\frac{7}{2}}(693Dx^{3}d^{3}+819C x^{2}d^{3}-378Dx^{2}c d^{2}+1001Bx d^{3}-364Cxc d^{2}+168Dx c^{2}d+1287A d^{3}-286Bc d^{2}+104C c^{2}}{9009d^{4}}$
orering	$\frac{2(xd+c)^{\frac{7}{2}}(693Dx^{3}d^{3}+819C x^{2}d^{3}-378Dx^{2}c d^{2}+1001Bx d^{3}-364Cxc d^{2}+168Dx c^{2}d+1287A d^{3}-286Bc d^{2}+104C c^{2}}{9009d^{4}}$
derivativedivides	$\frac{\frac{2D(xd+c)^{\frac{13}{2}}}{13}+\frac{2(Cd-3Dc)(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left(B d^{2}-2Ccd+3Dc^{2}\right)(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left(A d^{3}-Bc d^{2}+C c^{2}d-Dc^{3}\right)(xd+c)^{\frac{7}{2}}}{7}}{d^{4}}$
default	$\frac{\frac{2D(xd+c)^{\frac{13}{2}}}{13}+\frac{2(Cd-3Dc)(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left(B d^{2}-2Ccd+3Dc^{2}\right)(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left(A d^{3}-Bc d^{2}+C c^{2}d-Dc^{3}\right)(xd+c)^{\frac{7}{2}}}{7}}{d^{4}}$
trager	$2(693d^{6}Dx^{6}+819C d^{6}x^{5}+1701Dc d^{5}x^{5}+1001B d^{6}x^{4}+2093Cc d^{5}x^{4}+1113Dc^{2}d^{4}x^{4}+1287A d^{6}x^{3}+2717Bc d^{5}x^{3}+146$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
2/7*((7/13*D*x^3+7/11*C*x^2+7/9*B*x+A)*d^3-2/9*(189/143*D*x^2+14/11*C*x+B)
*c*d^2+8/99*c^2*(21/13*D*x+C)*d-16/429*D*c^3)*(d*x+c)^(7/2)/d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(101) = 202$.

Time = 0.07 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.82

$$\int (c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{2\left(693\,Dd^6x^6 - 48\,Dc^6 + 104\,Cc^5d - 286\,Bc^4d^2 + 1287\,Ac^3d^3 + 63\,(27\,Dcd^5 + 13\,Cd^6)x^5 + 7\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
2/9009*(693*D*d^6*x^6 - 48*D*c^6 + 104*C*c^5*d - 286*B*c^4*d^2 + 1287*A*c^
3*d^3 + 63*(27*D*c*d^5 + 13*C*d^6)*x^5 + 7*(159*D*c^2*d^4 + 299*C*c*d^5 +
143*B*d^6)*x^4 + (15*D*c^3*d^3 + 1469*C*c^2*d^4 + 2717*B*c*d^5 + 1287*A*d^
6)*x^3 - 3*(6*D*c^4*d^2 - 13*C*c^3*d^3 - 715*B*c^2*d^4 - 1287*A*c*d^5)*x^2
 + (24*D*c^5*d - 52*C*c^4*d^2 + 143*B*c^3*d^3 + 3861*A*c^2*d^4)*x)*sqrt(d*
x + c)/d^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(128) = 256$.

Time = 0.45 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.13

$$\int (c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) \, dx = \begin{cases} \frac{2Ac^3\sqrt{c+dx}}{7d} + \frac{6Ac^2x\sqrt{c+dx}}{7} + \frac{6Acdx^2\sqrt{c+dx}}{7} + \frac{2Ad^2x^3\sqrt{c+dx}}{7} - \frac{4Bc^4\sqrt{c+dx}}{63d^2} + \frac{2Bc^3x\sqrt{c+dx}}{63d} + \frac{10Bc^2x^2\sqrt{c+dx}}{21} \\ c^{\frac{5}{2}}\left(Ax + \frac{Bx^2}{2} + \frac{Cx^3}{3} + \frac{Dx^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((2*A*c**3*sqrt(c + d*x)/(7*d) + 6*A*c**2*x*sqrt(c + d*x)/7 + 6*A
*c*d*x**2*sqrt(c + d*x)/7 + 2*A*d**2*x**3*sqrt(c + d*x)/7 - 4*B*c**4*sqrt(
c + d*x)/(63*d**2) + 2*B*c**3*x*sqrt(c + d*x)/(63*d) + 10*B*c**2*x**2*sqrt
(c + d*x)/21 + 38*B*c*d*x**3*sqrt(c + d*x)/63 + 2*B*d**2*x**4*sqrt(c + d*x
)/9 + 16*C*c**5*sqrt(c + d*x)/(693*d**3) - 8*C*c**4*x*sqrt(c + d*x)/(693*d
**2) + 2*C*c**3*x**2*sqrt(c + d*x)/(231*d) + 226*C*c**2*x**3*sqrt(c + d*x)
/693 + 46*C*c*d*x**4*sqrt(c + d*x)/99 + 2*C*d**2*x**5*sqrt(c + d*x)/11 - 3
2*D*c**6*sqrt(c + d*x)/(3003*d**4) + 16*D*c**5*x*sqrt(c + d*x)/(3003*d**3)
 - 4*D*c**4*x**2*sqrt(c + d*x)/(1001*d**2) + 10*D*c**3*x**3*sqrt(c + d*x)/
(3003*d) + 106*D*c**2*x**4*sqrt(c + d*x)/429 + 54*D*c*d*x**5*sqrt(c + d*x)
/143 + 2*D*d**2*x**6*sqrt(c + d*x)/13, Ne(d, 0)), (c**(5/2)*(A*x + B*x**2/
2 + C*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int (c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{2\left(693\,(dx+c)^{\frac{13}{2}}D-819\,(3\,Dc-Cd)(dx+c)^{\frac{11}{2}}+1001\,(3\,Dc^2-2\,Ccd+Bd^2)(dx+c)^{\frac{9}{2}}-\right.}{9009\,d^4}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
2/9009*(693*(d*x + c)^(13/2)*D - 819*(3*D*c - C*d)*(d*x + c)^(11/2) + 1001
*(3*D*c^2 - 2*C*c*d + B*d^2)*(d*x + c)^(9/2) - 1287*(D*c^3 - C*c^2*d + B*c
*d^2 - A*d^3)*(d*x + c)^(7/2))/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. 2(101) = 202.

Time = 0.13 (sec) , antiderivative size = 817, normalized size of antiderivative = 6.98

$$\int (c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right) dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(d*x + c)*A*c^3 + 45045*((d*x + c)^(3/2) - 3*sqrt(d*x +
 c)*c)*A*c^2 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B*c^3/d + 9009*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*A*c + 30
03*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*c^3
/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*B*c^2/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*A + 1287*(5*(d*x + c)^(7/2) - 21*(d*x
+ c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*D*c^3/d^3 +
3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
35*sqrt(d*x + c)*c^3)*C*c^2/d^2 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(
5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*c/d + 429*(35*(d
*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x
 + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*D*c^2/d^3 + 429*(35*(d*x + c)^(9/
2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)
*c^3 + 315*sqrt(d*x + c)*c^4)*C*c/d^2 + 143*(35*(d*x + c)^(9/2) - 180*(d*x
 + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sq
rt(d*x + c)*c^4)*B/d + 195*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c +
990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*
c^4 - 693*sqrt(d*x + c)*c^5)*D*c/d^3 + 65*(63*(d*x + c)^(11/2) - 385*(d*x
+ c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 115...
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (c+d\,x)^{5/2}\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.37

$$\int (c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{2\sqrt{dx+c}\,(693d^6x^6+2520c\,d^5x^5+1001b\,d^5x^4+3206c^2d^4x^4+1287a\,d^5x^3+2717bc\,d^4x^3+14}{}$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*sqrt(c + d*x)*(1287*a*c**3*d**2 + 3861*a*c**2*d**3*x + 3861*a*c*d**4*x*
*2 + 1287*a*d**5*x**3 - 286*b*c**4*d + 143*b*c**3*d**2*x + 2145*b*c**2*d**
3*x**2 + 2717*b*c*d**4*x**3 + 1001*b*d**5*x**4 + 56*c**6 - 28*c**5*d*x + 2
1*c**4*d**2*x**2 + 1484*c**3*d**3*x**3 + 3206*c**2*d**4*x**4 + 2520*c*d**5
*x**5 + 693*d**6*x**6))/(9009*d**3)
```

3.75 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$

Optimal result 687
Mathematica [A] (verified) 688
Rubi [A] (verified) 688
Maple [A] (verified) 690
Fricas [B] (verification not implemented) 690
Sympy [A] (verification not implemented) 691
Maxima [F(-2)] 692
Giac [B] (verification not implemented) 693
Mupad [F(-1)] 694
Reduce [B] (verification not implemented) 694

Optimal result

Integrand size = 32, antiderivative size = 329

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \frac{2(bc-ad)^2(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b^6} + \frac{2(bc-ad)(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{3/2}}{3b^5} + \frac{2(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{5/2}}{5b^4} + \frac{2(a^2d^2D-abd(Cd-cD)-b^2(cCd-Bd^2-c^2D))(c+dx)^{7/2}}{7b^3d^3} + \frac{2(bCd-2bcD-adD)(c+dx)^{9/2}}{9b^2d^3} + \frac{2D(c+dx)^{11/2}}{11bd^3} - \frac{2(bc-ad)^{5/2}(Ab^3-a(b^2B-abC+a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{13/2}}$$

output
```
2*(-a*d+b*c)^2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^6+2/3*(-a*d+b
*c)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(3/2)/b^5+2/5*(A*b^3-a*(B*b^2-C*
a*b+D*a^2))*(d*x+c)^(5/2)/b^4+2/7*(a^2*d^2*D-a*b*d*(C*d-D*c)-b^2*(-B*d^2+C
*c*d-D*c^2))*(d*x+c)^(7/2)/b^3/d^3+2/9*(C*b*d-D*a*d-2*D*b*c)*(d*x+c)^(9/2)
/b^2/d^3+2/11*D*(d*x+c)^(11/2)/b/d^3-2*(-a*d+b*c)^(5/2)*(A*b^3-a*(B*b^2-C*
a*b+D*a^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx = \frac{2\sqrt{c+dx}(-3465a^5d^5D+1155a^4bd^4(3Cd+7cD+dDx)-2}{} \\ -\frac{2(-bc+ad)^{5/2}\left(Ab^3-a(b^2B-abC+a^2D)\right)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{13/2}}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(2*Sqrt[c + d*x]*(-3465*a^5*d^5*D + 1155*a^4*b*d^4*(3*C*d + 7*c*D + d*D*x)
 - 231*a^3*b^2*d^3*(23*c^2*D + c*d*(35*C + 11*D*x) + d^2*(15*B + x*(5*C +
3*D*x))) + b^5*(40*c^5*D - 10*c^4*d*(11*C + 2*D*x) + 5*c^3*d^2*(99*B + x*(
11*C + 3*D*x)) + d^5*x^2*(693*A + 5*x*(99*B + 77*C*x + 63*D*x^2)) + c^2*d^
3*(5313*A + 5*x*(297*B + 165*C*x + 113*D*x^2)) + c*d^4*x*(2541*A + 5*x*(29
7*B + 209*C*x + 161*D*x^2))) + 33*a^2*b^3*d^2*(15*c^3*D + c^2*d*(161*C + 4
5*D*x) + c*d^2*(245*B + x*(77*C + 45*D*x)) + d^3*(105*A + x*(35*B + 3*x*(7
*C + 5*D*x)))) - 11*a*b^4*d*(-10*c^4*D + 5*c^3*d*(9*C + D*x) + 3*c^2*d^2*(
161*B + 5*x*(9*C + 5*D*x)) + d^4*x*(105*A + x*(63*B + 5*x*(9*C + 7*D*x)))
+ c*d^3*(735*A + x*(231*B + 5*x*(27*C + 19*D*x))))))/(3465*b^6*d^3) - (2*(
-(b*c) + a*d)^(5/2)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[-(b*c) + a*d]])/b^(13/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{(c+dx)^{5/2} \left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right)}{b^3(a+bx)} + \frac{(c+dx)^{5/2} \left(a^2d^2D - abd(Cd - cD) - \left(b^2\left(-Bd^2 + c^2(-D) + \right.\right.\right.}{b^3d^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(bc-ad)^{5/2} \left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right) \operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{13/2}} + \frac{2\sqrt{c+dx}(bc-ad)^2 \left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right)}{b^6} + \frac{2(c+dx)^{3/2}(bc-ad) \left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right)}{3b^5} + \frac{2(c+dx)^{5/2} \left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right)}{5b^4} + \frac{2(c+dx)^{7/2} \left(a^2d^2D - abd(Cd - cD) - \left(b^2\left(-Bd^2 + c^2(-D) + cCd\right)\right)\right)}{7b^3d^3} + \frac{2(c+dx)^{9/2}(-adD - 2bcD + bCd)}{9b^2d^3} + \frac{2D(c+dx)^{11/2}}{11bd^3}$$

input
```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(2*(b*c - a*d)^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[c + d*x])/b^6 +
(2*(b*c - a*d)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(3/2))/(3*b^5
) + (2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(5/2))/(5*b^4) + (2*(
a^2*d^2*D - a*b*d*(C*d - c*D) - b^2*(c*C*d - B*d^2 - c^2*D))*(c + d*x)^(7/
2))/(7*b^3*d^3) + (2*(b*C*d - 2*b*c*D - a*d*D)*(c + d*x)^(9/2))/(9*b^2*d^3
) + (2*D*(c + d*x)^(11/2))/(11*b*d^3) - (2*(b*c - a*d)^(5/2)*(A*b^3 - a*(b
^2*B - a*b*C + a^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b
^(13/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.39

method	result
pseudoelliptic	$\frac{-2d^3(ad-bc)^3\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)+2\sqrt{(ad-bc)b}\left(\left(\frac{x^2\left(\frac{5}{11}Dx^3+\frac{5}{9}C\,x^2+\frac{5}{7}Bx+A\right)b^5}{5}-\frac{x(}{}\right.\right.}{}$
derivativedivides	$\frac{2\left(-B\,a^3b^2d^5\sqrt{xd+c}+\frac{C\,a^2b^3c\,d^3(xd+c)^{\frac{3}{2}}}{3}-\frac{Da^3b^2c\,d^3(xd+c)^{\frac{3}{2}}}{3}-\frac{Ba\,b^4d^3(xd+c)^{\frac{5}{2}}}{5}+A\,a^2b^3d^5\sqrt{xd+c}+A\,b^5c^2d^3\sqrt{xd+c}+\frac{Da^2b}{}\right.}{}$
default	$\frac{2\left(-B\,a^3b^2d^5\sqrt{xd+c}+\frac{C\,a^2b^3c\,d^3(xd+c)^{\frac{3}{2}}}{3}-\frac{Da^3b^2c\,d^3(xd+c)^{\frac{3}{2}}}{3}-\frac{Ba\,b^4d^3(xd+c)^{\frac{5}{2}}}{5}+A\,a^2b^3d^5\sqrt{xd+c}+A\,b^5c^2d^3\sqrt{xd+c}+\frac{Da^2b}{}\right.}{}$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/((a*d-b*c)*b)^(1/2)*(-d^3*(a*d-b*c)^3*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)*arct
an(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((1/5*x^2*(5/1
1*D*x^3+5/9*C*x^2+5/7*B*x+A)*b^5-1/3*x*(1/3*D*x^3+3/7*C*x^2+3/5*B*x+A)*a*b
^4+a^2*(1/5*C*x^2+1/3*B*x+1/7*D*x^3+A)*b^3-(1/5*D*x^2+1/3*C*x+B)*a^3*b^2+a
^4*(1/3*D*x+C)*b-D*a^5)*d^5-7/3*(-11/35*x*(115/363*D*x^3+95/231*C*x^2+45/7
7*B*x+A)*b^4+a*(19/147*D*x^3+9/49*C*x^2+11/35*B*x+A)*b^3-(9/49*D*x^2+11/35
*C*x+B)*a^2*b^2+a^3*(11/35*D*x+C)*b-D*a^4)*c*b*d^4+23/15*((565/5313*D*x^3+
25/161*C*x^2+45/161*B*x+A)*b^3-(25/161*D*x^2+45/161*C*x+B)*a*b^2+a^2*(45/1
61*D*x+C)*b-a^3*D)*c^2*b^2*d^3+1/7*((1/33*D*x^2+1/9*C*x+B)*b^2-(1/9*D*x+C)
*a*b+D*a^2)*c^3*b^3*d^2-2/63*((2/11*D*x+C)*b-D*a)*c^4*b^4*d+8/693*D*b^5*c^
5)*(d*x+c)^(1/2))/d^3/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(294) = 588$.

Time = 0.10 (sec) , antiderivative size = 1397, normalized size of antiderivative = 4.25

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
[-1/3465*(3465*((D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*c^2*d^3 - 2*(D*a
^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*c*d^4 + (D*a^5 - C*a^4*b + B*a^3*b
^2 - A*a^2*b^3)*d^5)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt
(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) - 2*(315*D*b^5*d^5*x^5 + 40*D*
b^5*c^5 + 110*(D*a*b^4 - C*b^5)*c^4*d + 495*(D*a^2*b^3 - C*a*b^4 + B*b^5)*
c^3*d^2 - 5313*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*c^2*d^3 + 8085*(D
*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*c*d^4 - 3465*(D*a^5 - C*a^4*b +
B*a^3*b^2 - A*a^2*b^3)*d^5 + 35*(23*D*b^5*c*d^4 - 11*(D*a*b^4 - C*b^5)*d^5
)*x^4 + 5*(113*D*b^5*c^2*d^3 - 209*(D*a*b^4 - C*b^5)*c*d^4 + 99*(D*a^2*b^3
 - C*a*b^4 + B*b^5)*d^5)*x^3 + 3*(5*D*b^5*c^3*d^2 - 275*(D*a*b^4 - C*b^5)*
c^2*d^3 + 495*(D*a^2*b^3 - C*a*b^4 + B*b^5)*c*d^4 - 231*(D*a^3*b^2 - C*a^2
*b^3 + B*a*b^4 - A*b^5)*d^5)*x^2 - (20*D*b^5*c^4*d + 55*(D*a*b^4 - C*b^5)*
c^3*d^2 - 1485*(D*a^2*b^3 - C*a*b^4 + B*b^5)*c^2*d^3 + 2541*(D*a^3*b^2 - C
*a^2*b^3 + B*a*b^4 - A*b^5)*c*d^4 - 1155*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3
- A*a*b^4)*d^5)*x)*sqrt(d*x + c))/(b^6*d^3), 2/3465*(3465*((D*a^3*b^2 - C*
a^2*b^3 + B*a*b^4 - A*b^5)*c^2*d^3 - 2*(D*a^4*b - C*a^3*b^2 + B*a^2*b^3 -
A*a*b^4)*c*d^4 + (D*a^5 - C*a^4*b + B*a^3*b^2 - A*a^2*b^3)*d^5)*sqrt(-(b*c
 - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (31
5*D*b^5*d^5*x^5 + 40*D*b^5*c^5 + 110*(D*a*b^4 - C*b^5)*c^4*d + 495*(D*a^2*
b^3 - C*a*b^4 + B*b^5)*c^3*d^2 - 5313*(D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 ...
```

Sympy [A] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx = \begin{cases} \frac{2\left(\frac{D(c+dx)^{\frac{11}{2}}}{11bd^2} + \frac{(c+dx)^{\frac{9}{2}}(Cbd-Dad-2Dbc)}{9b^2d^2} + \frac{(c+dx)^{\frac{7}{2}}\left(Bb^2d^2-Cabd^2-Cb^2cd+Da^2d^2 \right.}{7b^3d^2} \right.}{} \\ c^{\frac{5}{2}}\left(\frac{Dx^3}{3b} + \frac{x^2(Cb-Da)}{2b^2} + \frac{x\left(Bb^2-Cab+Da^2\right)}{b^3} - \frac{\left(-Ab^3+Bab^2-Ca^2b+D\right.}{} \right. \end{cases}$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a),x)
```

output
```
Piecewise((2*(D*(c + d*x)**(11/2)/(11*b*d**2) + (c + d*x)**(9/2)*(C*b*d -
D*a*d - 2*D*b*c)/(9*b**2*d**2) + (c + d*x)**(7/2)*(B*b**2*d**2 - C*a*b*d**
2 - C*b**2*c*d + D*a**2*d**2 + D*a*b*c*d + D*b**2*c**2)/(7*b**3*d**2) + (c
 + d*x)**(5/2)*(A*b**3*d - B*a*b**2*d + C*a**2*b*d - D*a**3*d)/(5*b**4) +
(c + d*x)**(3/2)*(-A*a*b**3*d**2 + A*b**4*c*d + B*a**2*b**2*d**2 - B*a*b**
3*c*d - C*a**3*b*d**2 + C*a**2*b**2*c*d + D*a**4*d**2 - D*a**3*b*c*d)/(3*b
**5) + sqrt(c + d*x)*(A*a**2*b**3*d**3 - 2*A*a*b**4*c*d**2 + A*b**5*c**2*d
 - B*a**3*b**2*d**3 + 2*B*a**2*b**3*c*d**2 - B*a*b**4*c**2*d + C*a**4*b*d*
*3 - 2*C*a**3*b**2*c*d**2 + C*a**2*b**3*c**2*d - D*a**5*d**3 + 2*D*a**4*b*
c*d**2 - D*a**3*b**2*c**2*d)/b**6 + d*(a*d - b*c)**3*(-A*b**3 + B*a*b**2 -
 C*a**2*b + D*a**3)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**7*sqrt((a*
d - b*c)/b)))/d, Ne(d, 0)), (c**(5/2)*(D*x**3/(3*b) + x**2*(C*b - D*a)/(2*
b**2) + x*(B*b**2 - C*a*b + D*a**2)/b**3 - (-A*b**3 + B*a*b**2 - C*a**2*b
+ D*a**3)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/b**3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(294) = 588$.

Time = 0.15 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.62

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
-2*(D*a^3*b^3*c^3 - C*a^2*b^4*c^3 + B*a*b^5*c^3 - A*b^6*c^3 - 3*D*a^4*b^2*
c^2*d + 3*C*a^3*b^3*c^2*d - 3*B*a^2*b^4*c^2*d + 3*A*a*b^5*c^2*d + 3*D*a^5*
b*c*d^2 - 3*C*a^4*b^2*c*d^2 + 3*B*a^3*b^3*c*d^2 - 3*A*a^2*b^4*c*d^2 - D*a^
6*d^3 + C*a^5*b*d^3 - B*a^4*b^2*d^3 + A*a^3*b^3*d^3)*arctan(sqrt(d*x + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^6) + 2/3465*(315*(d*x + c)
^(11/2)*D*b^10*d^30 - 770*(d*x + c)^(9/2)*D*b^10*c*d^30 + 495*(d*x + c)^(7
/2)*D*b^10*c^2*d^30 - 385*(d*x + c)^(9/2)*D*a*b^9*d^31 + 385*(d*x + c)^(9/
2)*C*b^10*d^31 + 495*(d*x + c)^(7/2)*D*a*b^9*c*d^31 - 495*(d*x + c)^(7/2)*
C*b^10*c*d^31 + 495*(d*x + c)^(7/2)*D*a^2*b^8*d^32 - 495*(d*x + c)^(7/2)*C
*a*b^9*d^32 + 495*(d*x + c)^(7/2)*B*b^10*d^32 - 693*(d*x + c)^(5/2)*D*a^3*
b^7*d^33 + 693*(d*x + c)^(5/2)*C*a^2*b^8*d^33 - 693*(d*x + c)^(5/2)*B*a*b^
9*d^33 + 693*(d*x + c)^(5/2)*A*b^10*d^33 - 1155*(d*x + c)^(3/2)*D*a^3*b^7*
c*d^33 + 1155*(d*x + c)^(3/2)*C*a^2*b^8*c*d^33 - 1155*(d*x + c)^(3/2)*B*a*
b^9*c*d^33 + 1155*(d*x + c)^(3/2)*A*b^10*c*d^33 - 3465*sqrt(d*x + c)*D*a^3
*b^7*c^2*d^33 + 3465*sqrt(d*x + c)*C*a^2*b^8*c^2*d^33 - 3465*sqrt(d*x + c)
*B*a*b^9*c^2*d^33 + 3465*sqrt(d*x + c)*A*b^10*c^2*d^33 + 1155*(d*x + c)^(3
/2)*D*a^4*b^6*d^34 - 1155*(d*x + c)^(3/2)*C*a^3*b^7*d^34 + 1155*(d*x + c)^
(3/2)*B*a^2*b^8*d^34 - 1155*(d*x + c)^(3/2)*A*a*b^9*d^34 + 6930*sqrt(d*x +
 c)*D*a^4*b^6*c*d^34 - 6930*sqrt(d*x + c)*C*a^3*b^7*c*d^34 + 6930*sqrt(d*x
 + c)*B*a^2*b^8*c*d^34 - 6930*sqrt(d*x + c)*A*a*b^9*c*d^34 - 3465*sqrt(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx=\int \frac{{\left(c+d\,x\right)}^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{a+b\,x} \,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x),x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.98

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{a+bx}\,dx=\frac{-\frac{4\sqrt{dx+c}\,a\,b^5c^2d^3x^2}{3}-\frac{8\sqrt{dx+c}\,a\,b^5c\,d^4x^3}{9}-2\sqrt{dx+c}\,a^5b\,d^5+\frac{2\sqrt{dx+c}}{7}}{}$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

output

```
(2*(3465*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
- b*c)))*a**5*d**5 - 10395*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/
(sqrt(b)*sqrt(a*d - b*c)))*a**4*b*c*d**4 + 10395*sqrt(b)*sqrt(a*d - b*c)*a
tan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*c**2*d**3 - 346
5*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c))
)*a**2*b**3*c**3*d**2 - 3465*sqrt(c + d*x)*a**5*b*d**5 + 11550*sqrt(c + d*
x)*a**4*b**2*c*d**4 + 1155*sqrt(c + d*x)*a**4*b**2*d**5*x - 13398*sqrt(c +
 d*x)*a**3*b**3*c**2*d**3 - 3696*sqrt(c + d*x)*a**3*b**3*c*d**4*x - 693*sq
rt(c + d*x)*a**3*b**3*d**5*x**2 + 5808*sqrt(c + d*x)*a**2*b**4*c**3*d**2 +
 4026*sqrt(c + d*x)*a**2*b**4*c**2*d**3*x + 2178*sqrt(c + d*x)*a**2*b**4*c
*d**4*x**2 + 495*sqrt(c + d*x)*a**2*b**4*d**5*x**3 - 385*sqrt(c + d*x)*a*b
**5*c**4*d - 1540*sqrt(c + d*x)*a*b**5*c**3*d**2*x - 2310*sqrt(c + d*x)*a*
b**5*c**2*d**3*x**2 - 1540*sqrt(c + d*x)*a*b**5*c*d**4*x**3 - 385*sqrt(c +
 d*x)*a*b**5*d**5*x**4 + 495*sqrt(c + d*x)*b**7*c**3*d + 1485*sqrt(c + d*x
)*b**7*c**2*d**2*x + 1485*sqrt(c + d*x)*b**7*c*d**3*x**2 + 495*sqrt(c + d*
x)*b**7*d**4*x**3 - 70*sqrt(c + d*x)*b**6*c**5 + 35*sqrt(c + d*x)*b**6*c**
4*d*x + 840*sqrt(c + d*x)*b**6*c**3*d**2*x**2 + 1610*sqrt(c + d*x)*b**6*c*
*2*d**3*x**3 + 1190*sqrt(c + d*x)*b**6*c*d**4*x**4 + 315*sqrt(c + d*x)*b**
6*d**5*x**5))/(3465*b**7*d**2)
```

3.76 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$

Optimal result 696
Mathematica [A] (verified) 697
Rubi [A] (verified) 697
Maple [A] (verified) 700
Fricas [B] (verification not implemented) 701
Sympy [F(-1)] 702
Maxima [F(-2)] 703
Giac [B] (verification not implemented) 703
Mupad [F(-1)] 704
Reduce [B] (verification not implemented) 705

Optimal result

Integrand size = 32, antiderivative size = 371

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \frac{(bc-ad)\left(b^3(2Bc+5Ad)-ab^2(4cC+7Bd)-11a^3dD+3a^2\right.}{b^6}$$
$$+\frac{(b^3(2Bc+5Ad)-ab^2(4cC+7Bd)-11a^3dD+3a^2b(3Cd+2cD))\,(c+dx)^{3/2}}{3b^5}$$
$$+\frac{2(b^2B-2abC+3a^2D)\,(c+dx)^{5/2}}{5b^4}-\frac{(Ab^3-a(b^2B-abC+a^2D))\,(c+dx)^{5/2}}{b^4(a+bx)}$$
$$+\frac{2(bCd-bcD-2adD)(c+dx)^{7/2}}{7b^3d^2}+\frac{2D(c+dx)^{9/2}}{9b^2d^2}$$
$$-\frac{(bc-ad)^{3/2}\,(b^3(2Bc+5Ad)-ab^2(4cC+7Bd)-11a^3dD+3a^2b(3Cd+2cD))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{13/2}}$$

output
```
(-a*d+b*c)*(b^3*(5*A*d+2*B*c)-a*b^2*(7*B*d+4*C*c)-11*a^3*d*D+3*a^2*b*(3*C*
d+2*D*c))*(d*x+c)^(1/2)/b^6+1/3*(b^3*(5*A*d+2*B*c)-a*b^2*(7*B*d+4*C*c)-11*
a^3*d*D+3*a^2*b*(3*C*d+2*D*c))*(d*x+c)^(3/2)/b^5+2/5*(B*b^2-2*C*a*b+3*D*a^
2)*(d*x+c)^(5/2)/b^4-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(5/2)/b^4/(b*x+
a)+2/7*(C*b*d-2*D*a*d-D*b*c)*(d*x+c)^(7/2)/b^3/d^2+2/9*D*(d*x+c)^(9/2)/b^2
/d^2-(-a*d+b*c)^(3/2)*(b^3*(5*A*d+2*B*c)-a*b^2*(7*B*d+4*C*c)-11*a^3*d*D+3*
a^2*b*(3*C*d+2*D*c))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(13
/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx = \sqrt{c+dx}(3465a^5d^4D-105a^4bd^3(27Cd+62cD-22dDx)+2$$

$$+\frac{(-bc+ad)^{3/2}\left(b^3(2Bc+5Ad)-ab^2(4cC+7Bd)-11a^3dD+3a^2b(3Cd+2cD)\right)\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{13/2}}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
(Sqrt[c + d*x]*(3465*a^5*d^4*D - 105*a^4*b*d^3*(27*C*d + 62*c*D - 22*d*D*x
) + 21*a^3*b^2*d^2*(153*c^2*D + c*(240*C*d - 214*d*D*x) + d^2*(105*B - 90*
C*x - 22*D*x^2)) + 3*a^2*b^3*d*(-60*c^3*D + c^2*(-749*C*d + 786*d*D*x) + 2
*c*d^2*(-595*B + 581*C*x + 141*D*x^2) + d^3*(-525*A + 490*B*x + 126*C*x^2
+ 66*D*x^3)) - a*b^4*(20*c^4*D + c^3*(-90*C*d + 170*d*D*x) + 3*c^2*d^2*(-4
27*B + 554*C*x + 130*D*x^2) + 2*d^4*x*(525*A + 147*B*x + 81*C*x^2 + 55*D*x
^3) + 2*c*d^3*(-1050*A + 1239*B*x + 327*C*x^2 + 175*D*x^3)) + b^5*(105*A*d
^2*(-3*c^2 + 14*c*d*x + 2*d^2*x^2) + 2*x*(-10*c^4*D + 5*c^3*d*(9*C + D*x)
+ 3*c^2*d^2*(161*B + 5*x*(9*C + 5*D*x)) + d^4*x^2*(63*B + 5*x*(9*C + 7*D*x
)) + c*d^3*x*(231*B + 5*x*(27*C + 19*D*x))))))/(315*b^6*d^2*(a + b*x)) + (
(-(b*c) + a*d)^(3/2)*(b^3*(2*B*c + 5*A*d) - a*b^2*(4*c*C + 7*B*d) - 11*a^3
*d*D + 3*a^2*b*(3*C*d + 2*c*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c)
 + a*d]])/b^(13/2)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2124, 27, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx$$

$$\Big\downarrow \ 2124$$

$$-\frac{\int -\frac{(c+dx)^{5/2}\left(2\left(c-\frac{ad}{b}\right)Dx^2+\frac{2(bc-ad)(bC-aD)x}{b^2}+\frac{-7dDa^3+b(7Cd+2cD)a^2-b^2(2cC+7Bd)a+b^3(2Bc+5Ad)}{b^3}\right)}{2(a+bx)}dx}{bc-ad}-\frac{(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(c+dx)^{5/2}\left(-\frac{7dDa^3}{b^3}+\frac{(7Cd+2cD)a^2}{b^2}-\frac{(2cC+7Bd)a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc+5Ad+\frac{2(bc-ad)(bC-aD)x}{b^2}\right)}{a+bx}dx}{2(bc-ad)}-\frac{(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \ 1192$$

$$\frac{\int \frac{(c+dx)^3\left(-2Dc^3+2Cdc^2-2Bd^2c-2\left(c-\frac{ad}{b}\right)D(c+dx)^2-d^3\left(5A-\frac{7a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{2(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{bc-ad-b(c+dx)}d\sqrt{c+dx}}{d^2(bc-ad)}-\frac{(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \ 1584$$

$$\int \Big(\frac{2(bc-ad)D(c+dx)^4}{b^2}+\frac{2(bc-ad)(bCd-2aDd-bcD)(c+dx)^3}{b^3}+\frac{d^2(-11dDa^3+3b(3Cd+2cD)a^2-b^2(4cC+7Bd)a+b^3(2Bc+5Ad))(c+dx)^2}{b^4}\cdots$$

$$\frac{(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \ 2009$$

$$-\frac{d^2(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-11a^3dD+3a^2b(2cD+3Cd)-ab^2(7Bd+4cC)+b^3(5Ad+2Bc)\right)}{b^{13/2}}+d^2\sqrt{c+dx}(bc-ad)^2(-11a^3dD+3a^2b(\cdots$$

$$\frac{(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

input
```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
-(((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(7/2))/((b*c - a*d)*(a
+ b*x))) + ((d^2*(b*c - a*d)^2*(b^3*(2*B*c + 5*A*d) - a*b^2*(4*c*C + 7*B*d
) - 11*a^3*d*D + 3*a^2*b*(3*C*d + 2*c*D))*Sqrt[c + d*x])/b^6 + (d^2*(b*c -
 a*d)*(b^3*(2*B*c + 5*A*d) - a*b^2*(4*c*C + 7*B*d) - 11*a^3*d*D + 3*a^2*b*
(3*C*d + 2*c*D))*(c + d*x)^(3/2))/(3*b^5) + (d^2*(b^3*(2*B*c + 5*A*d) - a*
b^2*(4*c*C + 7*B*d) - 11*a^3*d*D + 3*a^2*b*(3*C*d + 2*c*D))*(c + d*x)^(5/2
))/(5*b^4) + (2*(b*c - a*d)*(b*C*d - b*c*D - 2*a*d*D)*(c + d*x)^(7/2))/(7*
b^3) + (2*(b*c - a*d)*D*(c + d*x)^(9/2))/(9*b^2) - (d^2*(b*c - a*d)^(5/2)*
(b^3*(2*B*c + 5*A*d) - a*b^2*(4*c*C + 7*B*d) - 11*a^3*d*D + 3*a^2*b*(3*C*d
 + 2*c*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(13/2))/(d^
2*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.28

method	result
pseudoelliptic	$-\frac{5 (- ((Ad+\frac{2Bc}{5}) b^{3}-\frac{7 (Bd+\frac{4Cc}{7}) a\,b^{2}}{5}+\frac{9 (Cd+\frac{2Dc}{3}) a^{2}b}{5}-\frac{11a^{3}dD}{5}) (ad-bc)^{2}d^{2}(bx+a)\arctan (\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}) + ((\cdots}{\cdots}$
derivativedivides	$-\frac{\frac{2 (\frac{2Ba\,b^{3}d^{3}(xd+c)^{\frac{3}{2}}}{3}-C\,a^{2}b^{2}d^{3}(xd+c)^{\frac{3}{2}}+\frac{4Da^{3}b\,d^{3}(xd+c)^{\frac{3}{2}}}{3}+2Aa\,b^{3}d^{4}\sqrt{xd+c}-2A\,b^{4}c\,d^{3}\sqrt{xd+c}-3B\,a^{2}b^{2}d^{4}\sqrt{xd+c}+4C\,a^{3}b \cdots}{\cdots}}{\cdots}$
default	$-\frac{\frac{2 (\frac{2Ba\,b^{3}d^{3}(xd+c)^{\frac{3}{2}}}{3}-C\,a^{2}b^{2}d^{3}(xd+c)^{\frac{3}{2}}+\frac{4Da^{3}b\,d^{3}(xd+c)^{\frac{3}{2}}}{3}+2Aa\,b^{3}d^{4}\sqrt{xd+c}-2A\,b^{4}c\,d^{3}\sqrt{xd+c}-3B\,a^{2}b^{2}d^{4}\sqrt{xd+c}+4C\,a^{3}b \cdots}{\cdots}}{\cdots}$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-5*(-((A*d+2/5*B*c)*b^3-7/5*(B*d+4/7*C*c)*a*b^2+9/5*(C*d+2/3*D*c)*a^2*b-11
/5*a^3*d*D)*(a*d-b*c)^2*d^2*(b*x+a)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(
1/2))+(1/5*(-2/3*x^2*(1/3*D*x^3+3/7*C*x^2+3/5*B*x+A)*d^4-14/3*x*(19/147*D*
x^3+9/49*C*x^2+11/35*B*x+A)*c*d^3+(-10/21*D*x^3-6/7*C*x^2-46/15*B*x+A)*c^2
*d^2-2/7*x*(1/9*D*x+C)*c^3*d+4/63*D*c^4*x)*b^5-4/3*(-1/2*x*(11/105*D*x^3+2
7/175*C*x^2+7/25*B*x+A)*d^4+c*(-1/6*D*x^3-109/350*C*x^2-59/50*B*x+A)*d^3+6
1/100*(-130/427*D*x^2-554/427*C*x+B)*c^2*d^2+3/70*(-17/9*D*x+C)*c^3*d-1/10
5*D*c^4)*a*b^4+((-6/25*C*x^2-14/15*B*x-22/175*D*x^3+A)*d^3+34/15*(-141/595
*D*x^2-83/85*C*x+B)*c*d^2+107/75*(-786/749*D*x+C)*c^2*d+4/35*D*c^3)*d*a^2*
b^3-7/5*d^2*a^3*((-22/105*D*x^2-6/7*C*x+B)*d^2+16/7*(-107/120*D*x+C)*c*d+5
1/35*D*c^2)*b^2+9/5*((-22/27*D*x+C)*d+62/27*D*c)*d^3*a^4*b-11/5*D*a^5*d^4)
*((a*d-b*c)*b)^(1/2)*(d*x+c)^(1/2))/((a*d-b*c)*b)^(1/2)/d^2/(b*x+a)/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(341) = 682$.

Time = 0.11 (sec) , antiderivative size = 1626, normalized size of antiderivative = 4.38

$$\int \frac{(c+dx)^{5/2} (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="fricas
")
```

output
```
[-1/630*(315*(2*(3*D*a^3*b^2 - 2*C*a^2*b^3 + B*a*b^4)*c^2*d^2 - (17*D*a^4*
b - 13*C*a^3*b^2 + 9*B*a^2*b^3 - 5*A*a*b^4)*c*d^3 + (11*D*a^5 - 9*C*a^4*b
+ 7*B*a^3*b^2 - 5*A*a^2*b^3)*d^4 + (2*(3*D*a^2*b^3 - 2*C*a*b^4 + B*b^5)*c^
2*d^2 - (17*D*a^3*b^2 - 13*C*a^2*b^3 + 9*B*a*b^4 - 5*A*b^5)*c*d^3 + (11*D*
a^4*b - 9*C*a^3*b^2 + 7*B*a^2*b^3 - 5*A*a*b^4)*d^4)*x)*sqrt((b*c - a*d)/b)
*log((b*d*x + 2*b*c - a*d + 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x +
a)) - 2*(70*D*b^5*d^4*x^5 - 20*D*a*b^4*c^4 - 90*(2*D*a^2*b^3 - C*a*b^4)*c^
3*d + 21*(153*D*a^3*b^2 - 107*C*a^2*b^3 + 61*B*a*b^4 - 15*A*b^5)*c^2*d^2 -
 210*(31*D*a^4*b - 24*C*a^3*b^2 + 17*B*a^2*b^3 - 10*A*a*b^4)*c*d^3 + 315*(
11*D*a^5 - 9*C*a^4*b + 7*B*a^3*b^2 - 5*A*a^2*b^3)*d^4 + 10*(19*D*b^5*c*d^3
 - (11*D*a*b^4 - 9*C*b^5)*d^4)*x^4 + 2*(75*D*b^5*c^2*d^2 - 5*(35*D*a*b^4 -
 27*C*b^5)*c*d^3 + 9*(11*D*a^2*b^3 - 9*C*a*b^4 + 7*B*b^5)*d^4)*x^3 + 2*(5*
D*b^5*c^3*d - 15*(13*D*a*b^4 - 9*C*b^5)*c^2*d^2 + 3*(141*D*a^2*b^3 - 109*C
*a*b^4 + 77*B*b^5)*c*d^3 - 21*(11*D*a^3*b^2 - 9*C*a^2*b^3 + 7*B*a*b^4 - 5*
A*b^5)*d^4)*x^2 - 2*(10*D*b^5*c^4 + 5*(17*D*a*b^4 - 9*C*b^5)*c^3*d - 3*(39
3*D*a^2*b^3 - 277*C*a*b^4 + 161*B*b^5)*c^2*d^2 + 21*(107*D*a^3*b^2 - 83*C*
a^2*b^3 + 59*B*a*b^4 - 35*A*b^5)*c*d^3 - 105*(11*D*a^4*b - 9*C*a^3*b^2 + 7
*B*a^2*b^3 - 5*A*a*b^4)*d^4)*x)*sqrt(d*x + c))/(b^7*d^2*x + a*b^6*d^2), -1
/315*(315*(2*(3*D*a^3*b^2 - 2*C*a^2*b^3 + B*a*b^4)*c^2*d^2 - (17*D*a^4*b -
 13*C*a^3*b^2 + 9*B*a^2*b^3 - 5*A*a*b^4)*c*d^3 + (11*D*a^5 - 9*C*a^4*b ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 914 vs. $2(341) = 682$.

Time = 0.14 (sec) , antiderivative size = 914, normalized size of antiderivative = 2.46

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
(6*D*a^2*b^3*c^3 - 4*C*a*b^4*c^3 + 2*B*b^5*c^3 - 23*D*a^3*b^2*c^2*d + 17*C
*a^2*b^3*c^2*d - 11*B*a*b^4*c^2*d + 5*A*b^5*c^2*d + 28*D*a^4*b*c*d^2 - 22*
C*a^3*b^2*c*d^2 + 16*B*a^2*b^3*c*d^2 - 10*A*a*b^4*c*d^2 - 11*D*a^5*d^3 + 9
*C*a^4*b*d^3 - 7*B*a^3*b^2*d^3 + 5*A*a^2*b^3*d^3)*arctan(sqrt(d*x + c)*b/s
qrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^6) + (sqrt(d*x + c)*D*a^3*b^2
*c^2*d - sqrt(d*x + c)*C*a^2*b^3*c^2*d + sqrt(d*x + c)*B*a*b^4*c^2*d - sqr
t(d*x + c)*A*b^5*c^2*d - 2*sqrt(d*x + c)*D*a^4*b*c*d^2 + 2*sqrt(d*x + c)*C
*a^3*b^2*c*d^2 - 2*sqrt(d*x + c)*B*a^2*b^3*c*d^2 + 2*sqrt(d*x + c)*A*a*b^4
*c*d^2 + sqrt(d*x + c)*D*a^5*d^3 - sqrt(d*x + c)*C*a^4*b*d^3 + sqrt(d*x +
c)*B*a^3*b^2*d^3 - sqrt(d*x + c)*A*a^2*b^3*d^3)/(((d*x + c)*b - b*c + a*d)
*b^6) + 2/315*(35*(d*x + c)^(9/2)*D*b^16*d^16 - 45*(d*x + c)^(7/2)*D*b^16*
c*d^16 - 90*(d*x + c)^(7/2)*D*a*b^15*d^17 + 45*(d*x + c)^(7/2)*C*b^16*d^17
 + 189*(d*x + c)^(5/2)*D*a^2*b^14*d^18 - 126*(d*x + c)^(5/2)*C*a*b^15*d^18
 + 63*(d*x + c)^(5/2)*B*b^16*d^18 + 315*(d*x + c)^(3/2)*D*a^2*b^14*c*d^18
- 210*(d*x + c)^(3/2)*C*a*b^15*c*d^18 + 105*(d*x + c)^(3/2)*B*b^16*c*d^18
+ 945*sqrt(d*x + c)*D*a^2*b^14*c^2*d^18 - 630*sqrt(d*x + c)*C*a*b^15*c^2*d
^18 + 315*sqrt(d*x + c)*B*b^16*c^2*d^18 - 420*(d*x + c)^(3/2)*D*a^3*b^13*d
^19 + 315*(d*x + c)^(3/2)*C*a^2*b^14*d^19 - 210*(d*x + c)^(3/2)*B*a*b^15*d
^19 + 105*(d*x + c)^(3/2)*A*b^16*d^19 - 2520*sqrt(d*x + c)*D*a^3*b^13*c*d^
19 + 1890*sqrt(d*x + c)*C*a^2*b^14*c*d^19 - 1260*sqrt(d*x + c)*B*a*b^15...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx=\int \frac{{\left(c+d\,x\right)}^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2,x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1188, normalized size of antiderivative = 3.20

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

output
```
( - 3465*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
 - b*c)))*a**5*d**4 + 8190*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(
sqrt(b)*sqrt(a*d - b*c)))*a**4*b*c*d**3 - 3465*sqrt(b)*sqrt(a*d - b*c)*ata
n((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**4*b*d**4*x - 630*sqrt(b)
*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*
*3*d**3 - 5985*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqr
t(a*d - b*c)))*a**3*b**2*c**2*d**2 + 8190*sqrt(b)*sqrt(a*d - b*c)*atan((sq
rt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*c*d**3*x + 1260*sqrt(b
)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b
**4*c*d**2 - 630*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*s
qrt(a*d - b*c)))*a**2*b**4*d**3*x + 1260*sqrt(b)*sqrt(a*d - b*c)*atan((sqr
t(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c**3*d - 5985*sqrt(b)*s
qrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3
*c**2*d**2*x - 630*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)
*sqrt(a*d - b*c)))*a*b**5*c**2*d + 1260*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt
(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**5*c*d**2*x + 1260*sqrt(b)*sqr
t(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c**3
*d*x - 630*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*
d - b*c)))*b**6*c**2*d*x + 3465*sqrt(c + d*x)*a**5*b*d**4 - 9345*sqrt(c +
d*x)*a**4*b**2*c*d**3 + 2310*sqrt(c + d*x)*a**4*b**2*d**4*x + 630*sqrt(...
```

3.77 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$

Optimal result . . . 706
Mathematica [A] (verified) . . . 707
Rubi [A] (verified) . . . 707
Maple [A] (verified) . . . 711
Fricas [B] (verification not implemented) . . . 712
Sympy [F(-1)] . . . 713
Maxima [F(-2)] . . . 713
Giac [B] (verification not implemented) . . . 713
Mupad [F(-1)] . . . 714
Reduce [B] (verification not implemented) . . . 715

Optimal result

Integrand size = 32, antiderivative size = 421

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \frac{(b^3(8c^2C+20Bcd+15Ad^2)-99a^3d^2D+9a^2bd(7Cd+12cD)}{4b^6}$$
$$+\frac{2(b^2(cC+Bd)+6a^2dD-3ab(Cd+cD))(c+dx)^{3/2}}{3b^5}$$
$$-\frac{(b^3(4Bc+5Ad)-ab^2(8cC+9Bd)-17a^3dD+a^2b(13Cd+12cD))(c+dx)^{3/2}}{4b^5(a+bx)}$$
$$+\frac{2(bC-3aD)(c+dx)^{5/2}}{5b^4}-\frac{(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{5/2}}{2b^4(a+bx)^2}+\frac{2D(c+dx)^{7/2}}{7b^3d}$$
$$-\frac{\sqrt{bc-ad}(b^3(8c^2C+20Bcd+15Ad^2)-99a^3d^2D+9a^2bd(7Cd+12cD)-ab^2(56cCd+35Bd^2+24c^2D}{4b^{13/2}}$$

output
```
1/4*(b^3*(15*A*d^2+20*B*c*d+8*C*c^2)-99*a^3*d^2*D+9*a^2*b*d*(7*C*d+12*D*c)
-a*b^2*(35*B*d^2+56*C*c*d+24*D*c^2))*(d*x+c)^(1/2)/b^6+2/3*(b^2*(B*d+C*c)+
6*a^2*d*D-3*a*b*(C*d+D*c))*(d*x+c)^(3/2)/b^5-1/4*(b^3*(5*A*d+4*B*c)-a*b^2*
(9*B*d+8*C*c)-17*a^3*d*D+a^2*b*(13*C*d+12*D*c))*(d*x+c)^(3/2)/b^5/(b*x+a)+
2/5*(C*b-3*D*a)*(d*x+c)^(5/2)/b^4-1/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c
)^(5/2)/b^4/(b*x+a)^2+2/7*D*(d*x+c)^(7/2)/b^3/d-1/4*(-a*d+b*c)^(1/2)*(b^3*
(15*A*d^2+20*B*c*d+8*C*c^2)-99*a^3*d^2*D+9*a^2*b*d*(7*C*d+12*D*c)-a*b^2*(3
5*B*d^2+56*C*c*d+24*D*c^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)
)/b^(13/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx = \frac{\sqrt{c+dx}(-10395a^5d^3D+315a^4bd^2(21Cd+47cD-55dDx)}{}$$

$$-\frac{\sqrt{-bc+ad}(b^3(8c^2C+20Bcd+15Ad^2)-99a^3d^2D+9a^2bd(7Cd+12cD)-ab^2(56cCd+35Bd^2+24c}{4b^{13/2}}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
(Sqrt[c + d*x]*(-10395*a^5*d^3*D + 315*a^4*b*d^2*(21*C*d + 47*c*D - 55*d*D
*x) - 105*A*b^3*d*(-15*a^2*d^2 + 5*a*b*d*(c - 5*d*x) + b^2*(2*c^2 + 9*c*d*
x - 8*d^2*x^2)) - 21*a^3*b^2*d*(234*c^2*D + 7*c*d*(55*C - 171*D*x) + d^2*(
175*B - 525*C*x + 264*D*x^2)) + 4*b^5*x*(35*B*d*(-3*c^2 + 14*c*d*x + 2*d^2
*x^2) + 2*x*(15*c^3*D + 3*d^3*x^2*(7*C + 5*D*x) + c*d^2*x*(77*C + 45*D*x)
+ c^2*d*(161*C + 45*D*x))) + a^2*b^3*(120*c^3*D + 2*c^2*d*(959*C - 4314*D*
x) + d^3*x*(-6125*B + 72*x*(49*C + 11*D*x)) + c*d^2*(3325*B + x*(-13769*C
+ 8424*D*x))) - a*b^4*(35*B*d*(6*c^2 - 163*c*d*x + 56*d^2*x^2) + 8*x*(-30*
c^3*D + 3*d^3*x^2*(21*C + 11*D*x) + c*d^2*x*(581*C + 141*D*x) + c^2*(-427*
C*d + 393*d*D*x)))))/(420*b^6*d*(a + b*x)^2) - (Sqrt[-(b*c) + a*d]*(b^3*(8
*c^2*C + 20*B*c*d + 15*A*d^2) - 99*a^3*d^2*D + 9*a^2*b*d*(7*C*d + 12*c*D)
- a*b^2*(56*c*C*d + 35*B*d^2 + 24*c^2*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[-(b*c) + a*d]])/(4*b^(13/2))
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1192, 25, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{(c+dx)^{5/2}\left(4\left(c-\frac{ad}{b}\right)Dx^2+\frac{4(bc-ad)(bC-aD)x}{b^2}+\frac{-7dDa^3+b(7Cd+4cD)a^2-b^2(4cC+7Bd)a+b^3(4Bc+3Ad)}{b^3}\right)}{2(a+bx)^2}dx}{2(bc-ad)}-\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)^{5/2}\left(-\frac{7dDa^3}{b^3}+\frac{(7Cd+4cD)a^2}{b^2}-\frac{(4cC+7Bd)a}{b}+4\left(c-\frac{ad}{b}\right)Dx^2+4Bc+3Ad+\frac{4(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^2}dx}{4(bc-ad)}-\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 1192$$

$$\frac{\int -\frac{(c+dx)^3\left(-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2-d^3\left(3A-\frac{7a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{(c+dx)^3\left(-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2-d^3\left(3A-\frac{7a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 1580$$

$$\frac{\frac{d^2\sqrt{c+dx}(bc-ad)^2\left(-15a^3dD+a^2b(12cD+11Cd)-ab^2(7Bd+8cC)+b^3(3Ad+4Bc)\right)}{2b^6(-ad-b(c+dx)+bc)}-\frac{\int \frac{8b^3(bc-ad)D(c+dx)^4+8b^2(bc-ad)(bCd-2aDd-bcD)(c+dx}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 2341$$

$$\frac{\frac{d^2\sqrt{c+dx}(bc-ad)^2\left(-15a^3dD+a^2b(12cD+11Cd)-ab^2(7Bd+8cC)+b^3(3Ad+4Bc)\right)}{2b^6(-ad-b(c+dx)+bc)}-\frac{\int\left(-8b^2(bc-ad)D(c+dx)^3-8bd(bc-ad)(bC-3aD)(c+\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{d^2\sqrt{c+dx}(bc-ad)^2\left(-15a^3dD+a^2b(12cD+11Cd)-ab^2(7Bd+8cC)+b^3(3Ad+4Bc)\right)}{2b^6(-ad-b(c+dx)+bc)}-\frac{d(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-99a^3d^2D+9a^2bd(12c\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

input
```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
-1/2*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(7/2))/(b^3*(b*c - a*d
)*(a + b*x)^2) + ((d^2*(b*c - a*d)^2*(b^3*(4*B*c + 3*A*d) - a*b^2*(8*c*C +
 7*B*d) - 15*a^3*d*D + a^2*b*(11*C*d + 12*c*D))*Sqrt[c + d*x])/(2*b^6*(b*c
 - a*d - b*(c + d*x))) - ((-4*d*(b*c - a*d)*(b^3*(2*c^2*C + 4*B*c*d + 3*A*
d^2) - 21*a^3*d^2*D + a^2*b*d*(13*C*d + 24*c*D) - a*b^2*(12*c*C*d + 7*B*d^
2 + 6*c^2*D))*Sqrt[c + d*x])/b - (2*d*(b^3*(4*c^2*C + 4*B*c*d + 3*A*d^2) -
 27*a^3*d^2*D + 3*a^2*b*d*(5*C*d + 12*c*D) - a*b^2*(16*c*C*d + 7*B*d^2 + 1
2*c^2*D))*(c + d*x)^(3/2))/3 - (8*b*d*(b*c - a*d)*(b*C - 3*a*D)*(c + d*x)^
(5/2))/5 - (8*b^2*(b*c - a*d)*D*(c + d*x)^(7/2))/7 + (d*(b*c - a*d)^(3/2)*
(b^3*(8*c^2*C + 20*B*c*d + 15*A*d^2) - 99*a^3*d^2*D + 9*a^2*b*d*(7*C*d + 1
2*c*D) - a*b^2*(56*c*C*d + 35*B*d^2 + 24*c^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[b*c - a*d]])/b^(3/2))/(2*b^5))/(2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.11

method	result
pseudoelliptic	$-\frac{15\left(\left(d^{2}A+\frac{4}{3}cdB+\frac{8}{15}C\,c^{2}\right)b^{3}-\frac{7\left(B\,d^{2}+\frac{8}{5}Ccd+\frac{24}{35}Dc^{2}\right)a\,b^{2}}{3}+\frac{21\left(Cd+\frac{12Dc}{7}\right)d\,a^{2}b}{5}-\frac{33a^{3}d^{2}D}{5}\right)(ad-bc)d(bx+a)^{2}\arctan\left(\frac{b\sqrt{}}{\sqrt{(c}}\right.}{4}$
derivativedivides	$\frac{2\left(\frac{D(xd+c)^{\frac{7}{2}}b^{3}}{7}+\frac{C\,b^{3}d(xd+c)^{\frac{5}{2}}}{5}-\frac{3Da\,b^{2}d(xd+c)^{\frac{5}{2}}}{5}+\frac{B\,b^{3}d^{2}(xd+c)^{\frac{3}{2}}}{3}-Ca\,b^{2}d^{2}(xd+c)^{\frac{3}{2}}+\frac{C\,b^{3}cd(xd+c)^{\frac{3}{2}}}{3}+2Da^{2}b\,d^{2}(xd+c)^{\frac{3}{2}}\right.}{}$
default	$\frac{2\left(\frac{D(xd+c)^{\frac{7}{2}}b^{3}}{7}+\frac{C\,b^{3}d(xd+c)^{\frac{5}{2}}}{5}-\frac{3Da\,b^{2}d(xd+c)^{\frac{5}{2}}}{5}+\frac{B\,b^{3}d^{2}(xd+c)^{\frac{3}{2}}}{3}-Ca\,b^{2}d^{2}(xd+c)^{\frac{3}{2}}+\frac{C\,b^{3}cd(xd+c)^{\frac{3}{2}}}{3}+2Da^{2}b\,d^{2}(xd+c)^{\frac{3}{2}}\right.}{}$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
15/4*(-((d^2*A+4/3*c*d*B+8/15*C*c^2)*b^3-7/3*(B*d^2+8/5*C*c*d+24/35*D*c^2)
*a*b^2+21/5*(C*d+12/7*D*c)*d*a^2*b-33/5*a^3*d^2*D)*(a*d-b*c)*d*(b*x+a)^2*a
rctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(d*x+c)^(1/
2)*((8/15*x^2*(1/5*C*x^2+1/3*B*x+1/7*D*x^3+A)*d^3-3/5*(-8/21*D*x^3-88/135*
C*x^2-56/27*B*x+A)*x*c*d^2-2/15*(-12/7*D*x^3-92/15*C*x^2+2*B*x+A)*c^2*d+8/
105*D*c^3*x^2)*b^5-1/3*(-5*x*(-88/875*D*x^3-24/125*C*x^2-56/75*B*x+A)*d^3+
c*(376/175*D*x^3+664/75*C*x^2-163/15*B*x+A)*d^2+2/5*(524/35*D*x^2-244/15*C
*x+B)*c^2*d-16/35*D*c^3*x)*a*b^4+((A+88/175*D*x^3-35/9*B*x+56/25*C*x^2)*d^
3+19/9*(8424/3325*D*x^2-1967/475*C*x+B)*c*d^2+274/225*(-4314/959*D*x+C)*c^
2*d+8/105*D*c^3)*a^2*b^3-7/3*((264/175*D*x^2-3*C*x+B)*d^2+11/5*(-171/55*D*
x+C)*c*d+234/175*D*c^2)*d*a^3*b^2+21/5*((-55/21*D*x+C)*d+47/21*D*c)*d^2*a^
4*b-33/5*D*a^5*d^3))/((a*d-b*c)*b)^(1/2)/(b*x+a)^2/b^6/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(387) = 774$.

Time = 0.12 (sec) , antiderivative size = 1663, normalized size of antiderivative = 3.95

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="fricas
")
```

output
```
[-1/840*(105*(8*(3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - 4*(27*D*a^4*b - 14*C*a^3
*b^2 + 5*B*a^2*b^3)*c*d^2 + (99*D*a^5 - 63*C*a^4*b + 35*B*a^3*b^2 - 15*A*a
^2*b^3)*d^3 + (8*(3*D*a*b^4 - C*b^5)*c^2*d - 4*(27*D*a^2*b^3 - 14*C*a*b^4
+ 5*B*b^5)*c*d^2 + (99*D*a^3*b^2 - 63*C*a^2*b^3 + 35*B*a*b^4 - 15*A*b^5)*d
^3)*x^2 + 2*(8*(3*D*a^2*b^3 - C*a*b^4)*c^2*d - 4*(27*D*a^3*b^2 - 14*C*a^2*
b^3 + 5*B*a*b^4)*c*d^2 + (99*D*a^4*b - 63*C*a^3*b^2 + 35*B*a^2*b^3 - 15*A*
a*b^4)*d^3)*x)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x +
 c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) - 2*(120*D*b^5*d^3*x^5 + 120*D*a^2*b
^3*c^3 + 24*(15*D*b^5*c*d^2 - (11*D*a*b^4 - 7*C*b^5)*d^3)*x^4 - 14*(351*D*
a^3*b^2 - 137*C*a^2*b^3 + 15*B*a*b^4 + 15*A*b^5)*c^2*d + 35*(423*D*a^4*b -
 231*C*a^3*b^2 + 95*B*a^2*b^3 - 15*A*a*b^4)*c*d^2 - 105*(99*D*a^5 - 63*C*a
^4*b + 35*B*a^3*b^2 - 15*A*a^2*b^3)*d^3 + 8*(45*D*b^5*c^2*d - (141*D*a*b^4
 - 77*C*b^5)*c*d^2 + (99*D*a^2*b^3 - 63*C*a*b^4 + 35*B*b^5)*d^3)*x^3 + 8*(
15*D*b^5*c^3 - (393*D*a*b^4 - 161*C*b^5)*c^2*d + (1053*D*a^2*b^3 - 581*C*a
*b^4 + 245*B*b^5)*c*d^2 - 7*(99*D*a^3*b^2 - 63*C*a^2*b^3 + 35*B*a*b^4 - 15
*A*b^5)*d^3)*x^2 + (240*D*a*b^4*c^3 - 4*(2157*D*a^2*b^3 - 854*C*a*b^4 + 10
5*B*b^5)*c^2*d + 7*(3591*D*a^3*b^2 - 1967*C*a^2*b^3 + 815*B*a*b^4 - 135*A*
b^5)*c*d^2 - 175*(99*D*a^4*b - 63*C*a^3*b^2 + 35*B*a^2*b^3 - 15*A*a*b^4)*d
^3)*x)*sqrt(d*x + c))/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d), 1/420*(105*(8
*(3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - 4*(27*D*a^4*b - 14*C*a^3*b^2 + 5*B*a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(387) = 774$.

Time = 0.15 (sec) , antiderivative size = 1002, normalized size of antiderivative = 2.38

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(24*D*a*b^3*c^3 - 8*C*b^4*c^3 - 132*D*a^2*b^2*c^2*d + 64*C*a*b^3*c^2*
d - 20*B*b^4*c^2*d + 207*D*a^3*b*c*d^2 - 119*C*a^2*b^2*c*d^2 + 55*B*a*b^3*
c*d^2 - 15*A*b^4*c*d^2 - 99*D*a^4*d^3 + 63*C*a^3*b*d^3 - 35*B*a^2*b^2*d^3
+ 15*A*a*b^3*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*b^6) - 1/4*(12*(d*x + c)^(3/2)*D*a^2*b^3*c^2*d - 8*(d*x + c)^(3
/2)*C*a*b^4*c^2*d + 4*(d*x + c)^(3/2)*B*b^5*c^2*d - 12*sqrt(d*x + c)*D*a^2
*b^3*c^3*d + 8*sqrt(d*x + c)*C*a*b^4*c^3*d - 4*sqrt(d*x + c)*B*b^5*c^3*d -
 33*(d*x + c)^(3/2)*D*a^3*b^2*c*d^2 + 25*(d*x + c)^(3/2)*C*a^2*b^3*c*d^2 -
 17*(d*x + c)^(3/2)*B*a*b^4*c*d^2 + 9*(d*x + c)^(3/2)*A*b^5*c*d^2 + 43*sqr
t(d*x + c)*D*a^3*b^2*c^2*d^2 - 31*sqrt(d*x + c)*C*a^2*b^3*c^2*d^2 + 19*sqr
t(d*x + c)*B*a*b^4*c^2*d^2 - 7*sqrt(d*x + c)*A*b^5*c^2*d^2 + 21*(d*x + c)^
(3/2)*D*a^4*b*d^3 - 17*(d*x + c)^(3/2)*C*a^3*b^2*d^3 + 13*(d*x + c)^(3/2)*
B*a^2*b^3*d^3 - 9*(d*x + c)^(3/2)*A*a*b^4*d^3 - 50*sqrt(d*x + c)*D*a^4*b*c
*d^3 + 38*sqrt(d*x + c)*C*a^3*b^2*c*d^3 - 26*sqrt(d*x + c)*B*a^2*b^3*c*d^3
 + 14*sqrt(d*x + c)*A*a*b^4*c*d^3 + 19*sqrt(d*x + c)*D*a^5*d^4 - 15*sqrt(d
*x + c)*C*a^4*b*d^4 + 11*sqrt(d*x + c)*B*a^3*b^2*d^4 - 7*sqrt(d*x + c)*A*a
^2*b^3*d^4)/(((d*x + c)*b - b*c + a*d)^2*b^6) + 2/105*(15*(d*x + c)^(7/2)*
D*b^18*d^6 - 63*(d*x + c)^(5/2)*D*a*b^17*d^7 + 21*(d*x + c)^(5/2)*C*b^18*d
^7 - 105*(d*x + c)^(3/2)*D*a*b^17*c*d^7 + 35*(d*x + c)^(3/2)*C*b^18*c*d^7
- 315*sqrt(d*x + c)*D*a*b^17*c^2*d^7 + 105*sqrt(d*x + c)*C*b^18*c^2*d^7...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx=\int \frac{(c+d\,x)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^3}\,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3,x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.16

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
(10395*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
b*c)))*a**5*d**3 - 17955*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(s
qrt(b)*sqrt(a*d - b*c)))*a**4*b*c*d**2 + 20790*sqrt(b)*sqrt(a*d - b*c)*ata
n((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**4*b*d**3*x + 2100*sqrt(b
)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b
**3*d**2 + 8400*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sq
rt(a*d - b*c)))*a**3*b**2*c**2*d - 35910*sqrt(b)*sqrt(a*d - b*c)*atan((sqr
t(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*c*d**2*x + 10395*sqrt(b
)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b
**2*d**3*x**2 - 2100*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(
b)*sqrt(a*d - b*c)))*a**2*b**4*c*d + 4200*sqrt(b)*sqrt(a*d - b*c)*atan((sq
rt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**4*d**2*x - 840*sqrt(b)*s
qrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3
*c**3 + 16800*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt
(a*d - b*c)))*a**2*b**3*c**2*d*x - 17955*sqrt(b)*sqrt(a*d - b*c)*atan((sqr
t(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c*d**2*x**2 - 4200*sqrt
(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b*
*5*c*d*x + 2100*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sq
rt(a*d - b*c)))*a*b**5*d**2*x**2 - 1680*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt
(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c**3*x + 8400*sqrt(b)*sq...
```

3.78 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$

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Mathematica [A] (verified) . . . 717
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Reduce [B] (verification not implemented) . . . 726

Optimal result

Integrand size = 32, antiderivative size = 454

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \frac{2(10a^2d^2D-4abd(Cd+2cD)+b^2(2cCd+Bd^2+c^2D))\sqrt{c}}{b^6}$$

$$-\frac{(b^3(8c^2C+14Bcd+5Ad^2)-71a^3d^2D+a^2bd(41Cd+90cD)-ab^2(44cCd+19Bd^2+24c^2D))\sqrt{c+d}}{8b^6(a+bx)}$$

$$+\frac{2(bCd+bcD-4adD)(c+dx)^{3/2}}{3b^5}$$

$$-\frac{(b^3(6Bc+5Ad)-ab^2(12cC+11Bd)-23a^3dD+a^2b(17Cd+18cD))(c+dx)^{3/2}}{12b^5(a+bx)^2}$$

$$+\frac{2D(c+dx)^{5/2}}{5b^4}-\frac{(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{5/2}}{3b^4(a+bx)^3}$$

$$+\frac{(231a^3d^3D-21a^2bd^2(5Cd+18cD)+7ab^2d(20cCd+5Bd^2+24c^2D)-b^3(40c^2Cd+30Bcd^2+5Ad^3+}{8b^{13/2}\sqrt{bc-ad}}$$

output
```
2*(10*a^2*d^2*D-4*a*b*d*(C*d+2*D*c)+b^2*(B*d^2+2*C*c*d+D*c^2))*(d*x+c)^(1/
2)/b^6-1/8*(b^3*(5*A*d^2+14*B*c*d+8*C*c^2)-71*a^3*d^2*D+a^2*b*d*(41*C*d+90
*D*c)-a*b^2*(19*B*d^2+44*C*c*d+24*D*c^2))*(d*x+c)^(1/2)/b^6/(b*x+a)+2/3*(C
*b*d-4*D*a*d+D*b*c)*(d*x+c)^(3/2)/b^5-1/12*(b^3*(5*A*d+6*B*c)-a*b^2*(11*B*
d+12*C*c)-23*a^3*d*D+a^2*b*(17*C*d+18*D*c))*(d*x+c)^(3/2)/b^5/(b*x+a)^2+2/
5*D*(d*x+c)^(5/2)/b^4-1/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(5/2)/b^4/
(b*x+a)^3+1/8*(231*a^3*d^3*D-21*a^2*b*d^2*(5*C*d+18*D*c)+7*a*b^2*d*(5*B*d^
2+20*C*c*d+24*D*c^2)-b^3*(5*A*d^3+30*B*c*d^2+40*C*c^2*d+16*D*c^3))*arctanh
(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(13/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \frac{\sqrt{c+dx}(3465a^5d^2D-105a^4bd(15Cd+32cD-88dDx)-5\ldots}{}$$
$$+\frac{(-231a^3d^3D+21a^2bd^2(5Cd+18cD)-7ab^2d(20cCd+5Bd^2+24c^2D)+b^3(40c^2Cd+30Bcd^2+5Ad^3\ldots}{8b^{13/2}\sqrt{-bc+ad}}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
(Sqrt[c + d*x]*(3465*a^5*d^2*D - 105*a^4*b*d*(15*C*d + 32*c*D - 88*d*D*x)
- 5*A*b^3*(15*a^2*d^2 + 10*a*b*d*(c + 4*d*x) + b^2*(8*c^2 + 26*c*d*x + 33*
d^2*x^2)) + 21*a^3*b^2*(28*c^2*D + c*(50*C*d - 434*d*D*x) + d^2*(25*B - 20
0*C*x + 363*D*x^2)) + 2*b^5*x*(-15*B*(2*c^2 + 9*c*d*x - 8*d^2*x^2) + 4*x*(
2*d^2*x^2*(5*C + 3*D*x) + 2*c*d*x*(35*C + 11*D*x) + c^2*(-15*C + 46*D*x)))
 - a*b^4*(5*B*(4*c^2 + 58*c*d*x - 231*d^2*x^2) + 4*x*(6*c^2*(5*C - 61*D*x)
 + 4*d^2*x^2*(45*C + 11*D*x) + c*d*x*(-615*C + 428*D*x))) + a^2*b^3*(c^2*(
-40*C + 1644*D*x) + d^2*x*(1400*B + 99*x*(-35*C + 16*D*x)) - 2*c*d*(50*B +
 x*(-1435*C + 3861*D*x)))))/(120*b^6*(a + b*x)^3) + ((-231*a^3*d^3*D + 21*
a^2*b*d^2*(5*C*d + 18*c*D) - 7*a*b^2*d*(20*c*C*d + 5*B*d^2 + 24*c^2*D) + b
^3*(40*c^2*C*d + 30*B*c*d^2 + 5*A*d^3 + 16*c^3*D))*ArcTan[(Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[-(b*c) + a*d]])/(8*b^(13/2)*Sqrt[-(b*c) + a*d])
```

Rubi [A] (verified)

Time = 2.34 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1192, 1580, 2345, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{\int -\frac{(c+dx)^{5/2}\left(6\left(c-\frac{ad}{b}\right)Dx^2+\frac{6(bc-ad)(bC-aD)x}{b^2}+\frac{-7dDa^3+b(7Cd+6cD)a^2-b^2(6cC+7Bd)a+b^3(6Bc+Ad)}{b^3}\right)}{2(a+bx)^3}dx}{3(bc-ad)}-\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^{5/2}\left(-\frac{7dDa^3}{b^3}+\frac{(7Cd+6cD)a^2}{b^2}-\frac{(6cC+7Bd)a}{b}+6\left(c-\frac{ad}{b}\right)Dx^2+6Bc+Ad+\frac{6(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^3}dx}{6(bc-ad)}-\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \text{1192}$$

$$\frac{\int \frac{(c+dx)^3\left(-6Dc^3+6Cdc^2-6Bd^2c-6\left(c-\frac{ad}{b}\right)D(c+dx)^2-d^3\left(A-\frac{7a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{6(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}\right)}{(bc-ad-b(c+dx))^3}d\sqrt{c+dx}}{3(bc-ad)}-\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \text{1580}$$

$$\frac{\int \frac{24b^3(bc-ad)D(c+dx)^4+24b^2(bc-ad)(bCd-2aDd-bcD)(c+dx)^3+4bd^2\left(-19dDa^3+b(13Cd+18cD)a^2-b^2(12cC+7Bd)a+b^3(6Bc+Ad)\right)(c+dx)^2+4d^2(bc-ad)\left(\cdots\right.}{(bc-ad-b(c+d\cdots}}{4b^5\cdots}$$

$$\frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 2345$$

$$\frac{\frac{3d\sqrt{c+dx}(bc-ad)\left(-81a^3d^2D+a^2bd(102cD+47Cd)-ab^2\left(21Bd^2+24c^2D+52cCd\right)+b^3\left(3Ad^2+18Bcd+8c^2C\right)\right)}{2b(-ad-b(c+dx)+bc)}-\frac{\int \frac{48b^2(bc-ad)^2D(c+dx)^3+48bd(bc-ad)^2(bC-}{}}{}}{} \\ \frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 2341$$

$$\frac{\frac{3d\sqrt{c+dx}(bc-ad)\left(-81a^3d^2D+a^2bd(102cD+47Cd)-ab^2\left(21Bd^2+24c^2D+52cCd\right)+b^3\left(3Ad^2+18Bcd+8c^2C\right)\right)}{2b(-ad-b(c+dx)+bc)}-\frac{\int \left(-48bD(c+dx)^2(bc-ad)^2-48(bCd-4aDd+b\right.}{}}{} \\ \frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3d\sqrt{c+dx}(bc-ad)\left(-81a^3d^2D+a^2bd(102cD+47Cd)-ab^2\left(21Bd^2+24c^2D+52cCd\right)+b^3\left(3Ad^2+18Bcd+8c^2C\right)\right)}{2b(-ad-b(c+dx)+bc)}-\frac{-\frac{3(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(231a^3\right.}{}}{}}{} \\ \frac{(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

input

```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
-1/3*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(7/2))/(b^3*(b*c - a*d
)*(a + b*x)^3) + (-1/4*(d^2*(b*c - a*d)^2*(b^3*(6*B*c + A*d) - a*b^2*(12*c
*C + 7*B*d) - 19*a^3*d*D + a^2*b*(13*C*d + 18*c*D))*Sqrt[c + d*x])/(b^6*(b
*c - a*d - b*(c + d*x))^2) + ((3*d*(b*c - a*d)*(b^3*(8*c^2*C + 18*B*c*d +
3*A*d^2) - 81*a^3*d^2*D + a^2*b*d*(47*C*d + 102*c*D) - a*b^2*(52*c*C*d + 2
1*B*d^2 + 24*c^2*D))*Sqrt[c + d*x])/(2*b*(b*c - a*d - b*(c + d*x))) - ((8*
(b*c - a*d)*(61*a^3*d^3*D - a^2*b*d^2*(25*C*d + 108*c*D) + a*b^2*d*(36*c*C
*d + 7*B*d^2 + 54*c^2*D) - b^3*(12*c^2*C*d + 6*B*c*d^2 + A*d^3 + 6*c^3*D))
*Sqrt[c + d*x])/b - 16*(b*c - a*d)^2*(b*C*d + b*c*D - 4*a*d*D)*(c + d*x)^(
3/2) - (48*b*(b*c - a*d)^2*D*(c + d*x)^(5/2))/5 - (3*(b*c - a*d)^(3/2)*(23
1*a^3*d^3*D - 21*a^2*b*d^2*(5*C*d + 18*c*D) + 7*a*b^2*d*(20*c*C*d + 5*B*d^
2 + 24*c^2*D) - b^3*(40*c^2*C*d + 30*B*c*d^2 + 5*A*d^3 + 16*c^3*D))*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(3/2))/(2*(b*c - a*d)))/(4*b
^5))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{5\left(-(bx+a)^3\left(\left(A d^3+6Bc d^2+8C c^2 d+\frac{16}{5}Dc^3\right)b^3-7\left(B d^2+4Ccd+\frac{24}{5}Dc^2\right)da b^2+21\left(Cd+\frac{18Dc}{5}\right)d^2a^2b-\frac{231a^3d^3D}{5}\right.\right.}{}$
derivativedivides	$\frac{\frac{2D(xd+c)^{\frac{5}{2}}b^2}{5}+\frac{2C b^2 d(xd+c)^{\frac{3}{2}}}{3}-\frac{8Dabd(xd+c)^{\frac{3}{2}}}{3}+\frac{2Db^2c(xd+c)^{\frac{3}{2}}}{3}+2B d^2b^2\sqrt{xd+c}-8Ca d^2b\sqrt{xd+c}+4C b^2cd\sqrt{xd+c}+2}{b^6}$
default	$\frac{\frac{2D(xd+c)^{\frac{5}{2}}b^2}{5}+\frac{2C b^2 d(xd+c)^{\frac{3}{2}}}{3}-\frac{8Dabd(xd+c)^{\frac{3}{2}}}{3}+\frac{2Db^2c(xd+c)^{\frac{3}{2}}}{3}+2B d^2b^2\sqrt{xd+c}-8Ca d^2b\sqrt{xd+c}+4C b^2cd\sqrt{xd+c}+2}{b^6}$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/8*(-(b*x+a)^3*((A*d^3+6*B*c*d^2+8*C*c^2*d+16/5*D*c^3)*b^3-7*(B*d^2+4*C*
c*d+24/5*D*c^2)*d*a*b^2+21*(C*d+18/5*D*c)*d^2*a^2*b-231/5*a^3*d^3*D)*arcta
n(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((11/5*x^2*(-16
/55*D*x^3-16/33*C*x^2-16/11*B*x+A)*d^2+26/15*x*c*(-88/65*D*x^3-56/13*C*x^2
+27/13*B*x+A)*d+8/15*c^2*(-46/5*D*x^3+3*C*x^2+3/2*B*x+A))*b^5+2/3*(4*(22/2
5*D*x^3+18/5*C*x^2-231/40*B*x+A)*x*d^2+c*(856/25*D*x^3-246/5*C*x^2+29/5*B*
x+A)*d+2/5*c^2*(-366/5*D*x^2+6*C*x+B))*a*b^4+a^2*((-528/25*D*x^3+231/5*C*x
^2-56/3*B*x+A)*d^2+4/3*(3861/50*D*x^2-287/10*C*x+B)*c*d+8/15*(-411/10*D*x+
C)*c^2)*b^3-7*((363/25*D*x^2-8*C*x+B)*d^2+2*(-217/25*D*x+C)*c*d+28/25*D*c^
2)*a^3*b^2+21*d*a^4*((-88/15*D*x+C)*d+32/15*D*c)*b-231/5*D*a^5*d^2)*(d*x+c
)^(1/2))/((a*d-b*c)*b)^(1/2)/(b*x+a)^3/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(422) = 844$.

Time = 0.14 (sec) , antiderivative size = 2398, normalized size of antiderivative = 5.28

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="fricas
")
```

output
```
[1/240*(15*(16*D*a^3*b^3*c^3 - 8*(21*D*a^4*b^2 - 5*C*a^3*b^3)*c^2*d + 2*(1
89*D*a^5*b - 70*C*a^4*b^2 + 15*B*a^3*b^3)*c*d^2 - (231*D*a^6 - 105*C*a^5*b
 + 35*B*a^4*b^2 - 5*A*a^3*b^3)*d^3 + (16*D*b^6*c^3 - 8*(21*D*a*b^5 - 5*C*b
^6)*c^2*d + 2*(189*D*a^2*b^4 - 70*C*a*b^5 + 15*B*b^6)*c*d^2 - (231*D*a^3*b
^3 - 105*C*a^2*b^4 + 35*B*a*b^5 - 5*A*b^6)*d^3)*x^3 + 3*(16*D*a*b^5*c^3 -
8*(21*D*a^2*b^4 - 5*C*a*b^5)*c^2*d + 2*(189*D*a^3*b^3 - 70*C*a^2*b^4 + 15*
B*a*b^5)*c*d^2 - (231*D*a^4*b^2 - 105*C*a^3*b^3 + 35*B*a^2*b^4 - 5*A*a*b^5
)*d^3)*x^2 + 3*(16*D*a^2*b^4*c^3 - 8*(21*D*a^3*b^3 - 5*C*a^2*b^4)*c^2*d +
2*(189*D*a^4*b^2 - 70*C*a^3*b^3 + 15*B*a^2*b^4)*c*d^2 - (231*D*a^5*b - 105
*C*a^4*b^2 + 35*B*a^3*b^3 - 5*A*a^2*b^4)*d^3)*x)*sqrt(b^2*c - a*b*d)*log((
b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*
(48*(D*b^7*c*d^2 - D*a*b^6*d^3)*x^5 + 16*(11*D*b^7*c^2*d - (22*D*a*b^6 - 5
*C*b^7)*c*d^2 + (11*D*a^2*b^5 - 5*C*a*b^6)*d^3)*x^4 + 4*(147*D*a^3*b^4 - 1
0*C*a^2*b^5 - 5*B*a*b^6 - 10*A*b^7)*c^3 - 2*(1974*D*a^4*b^3 - 545*C*a^3*b^
4 + 40*B*a^2*b^5 + 5*A*a*b^6)*c^2*d + 25*(273*D*a^5*b^2 - 105*C*a^4*b^3 +
25*B*a^3*b^4 - A*a^2*b^5)*c*d^2 - 15*(231*D*a^6*b - 105*C*a^5*b^2 + 35*B*a
^4*b^3 - 5*A*a^3*b^4)*d^3 + 16*(23*D*b^7*c^3 - 5*(26*D*a*b^6 - 7*C*b^7)*c^
2*d + (206*D*a^2*b^5 - 80*C*a*b^6 + 15*B*b^7)*c*d^2 - 3*(33*D*a^3*b^4 - 15
*C*a^2*b^5 + 5*B*a*b^6)*d^3)*x^3 + 3*(8*(61*D*a*b^6 - 5*C*b^7)*c^3 - 2*(15
31*D*a^2*b^5 - 430*C*a*b^6 + 45*B*b^7)*c^2*d + 5*(1023*D*a^3*b^4 - 395*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(422) = 844$.

Time = 0.17 (sec) , antiderivative size = 1057, normalized size of antiderivative = 2.33

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="giac")
```

output
```
1/8*(16*D*b^3*c^3 - 168*D*a*b^2*c^2*d + 40*C*b^3*c^2*d + 378*D*a^2*b*c*d^2
 - 140*C*a*b^2*c*d^2 + 30*B*b^3*c*d^2 - 231*D*a^3*d^3 + 105*C*a^2*b*d^3 -
35*B*a*b^2*d^3 + 5*A*b^3*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))
/(sqrt(-b^2*c + a*b*d)*b^6) + 1/24*(72*(d*x + c)^(5/2)*D*a*b^4*c^2*d - 24*
(d*x + c)^(5/2)*C*b^5*c^2*d - 144*(d*x + c)^(3/2)*D*a*b^4*c^3*d + 48*(d*x
+ c)^(3/2)*C*b^5*c^3*d + 72*sqrt(d*x + c)*D*a*b^4*c^4*d - 24*sqrt(d*x + c)
*C*b^5*c^4*d - 306*(d*x + c)^(5/2)*D*a^2*b^3*c*d^2 + 156*(d*x + c)^(5/2)*C
*a*b^4*c*d^2 - 54*(d*x + c)^(5/2)*B*b^5*c*d^2 + 720*(d*x + c)^(3/2)*D*a^2*
b^3*c^2*d^2 - 336*(d*x + c)^(3/2)*C*a*b^4*c^2*d^2 + 96*(d*x + c)^(3/2)*B*b
^5*c^2*d^2 - 414*sqrt(d*x + c)*D*a^2*b^3*c^3*d^2 + 180*sqrt(d*x + c)*C*a*b
^4*c^3*d^2 - 42*sqrt(d*x + c)*B*b^5*c^3*d^2 + 267*(d*x + c)^(5/2)*D*a^3*b^
2*d^3 - 165*(d*x + c)^(5/2)*C*a^2*b^3*d^3 + 87*(d*x + c)^(5/2)*B*a*b^4*d^3
 - 33*(d*x + c)^(5/2)*A*b^5*d^3 - 1048*(d*x + c)^(3/2)*D*a^3*b^2*c*d^3 + 5
68*(d*x + c)^(3/2)*C*a^2*b^3*c*d^3 - 232*(d*x + c)^(3/2)*B*a*b^4*c*d^3 + 4
0*(d*x + c)^(3/2)*A*b^5*c*d^3 + 825*sqrt(d*x + c)*D*a^3*b^2*c^2*d^3 - 411*
sqrt(d*x + c)*C*a^2*b^3*c^2*d^3 + 141*sqrt(d*x + c)*B*a*b^4*c^2*d^3 - 15*s
qrt(d*x + c)*A*b^5*c^2*d^3 + 472*(d*x + c)^(3/2)*D*a^4*b*d^4 - 280*(d*x +
c)^(3/2)*C*a^3*b^2*d^4 + 136*(d*x + c)^(3/2)*B*a^2*b^3*d^4 - 40*(d*x + c)^
(3/2)*A*a*b^4*d^4 - 696*sqrt(d*x + c)*D*a^4*b*c*d^4 + 378*sqrt(d*x + c)*C*
a^3*b^2*c*d^4 - 156*sqrt(d*x + c)*B*a^2*b^3*c*d^4 + 30*sqrt(d*x + c)*A*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx=\int \frac{(c+d\,x)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^4}\,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4,x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1262, normalized size of antiderivative = 2.78

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x)
```

output
```
( - 3465*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
- b*c)))*a**5*d**3 + 3780*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(
sqrt(b)*sqrt(a*d - b*c)))*a**4*b*c*d**2 - 10395*sqrt(b)*sqrt(a*d - b*c)*at
an((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**4*b*d**3*x - 450*sqrt(b
)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b
**3*d**2 - 840*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqr
t(a*d - b*c)))*a**3*b**2*c**2*d + 11340*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt
(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*c*d**2*x - 10395*sqrt(b)
*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*
*2*d**3*x**2 - 1350*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b
)*sqrt(a*d - b*c)))*a**2*b**4*d**2*x - 2520*sqrt(b)*sqrt(a*d - b*c)*atan((
sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c**2*d*x + 11340*sqr
t(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**
2*b**3*c*d**2*x**2 - 3465*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(
sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*d**3*x**3 - 1350*sqrt(b)*sqrt(a*d - b*
c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**5*d**2*x**2 - 25
20*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)
))*a*b**4*c**2*d*x**2 + 3780*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b
)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c*d**2*x**3 - 450*sqrt(b)*sqrt(a*d - b
*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**6*d**2*x**3 - ...
```

3.79 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 436

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$

$$= -\frac{2(bc-ad)^3\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)\sqrt{c+dx}}{d^7}$$

$$-\frac{2(bc-ad)^2\left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right)(c+dx)^{3/2}}{3d^7}$$

$$-\frac{2(bc-ad)\left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)}{5d^7}$$

$$+\frac{2(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D)}{7d^7}$$

$$+\frac{2b\left(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D)\right)(c+dx)^{9/2}}{9d^7}$$

$$+\frac{2b^2(bCd-6bcD+3adD)(c+dx)^{11/2}}{11d^7}+\frac{2b^3D(c+dx)^{13/2}}{13d^7}$$

output
```
-2*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1/2)/d^7-2/3*(-a*d+
b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^
3))*(d*x+c)^(3/2)/d^7-2/5*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^2+
8*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^(5/
2)/d^7+2/7*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d-10
*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(7/2)/d^7+2/9*b
*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*x+c)^(
9/2)/d^7+2/11*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(11/2)/d^7+2/13*b^3*D*(d
*x+c)^(13/2)/d^7
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\sqrt{c+dx}(429a^3d^3(-48c^3D+8c^2d(7C+3Dx)-2cd^2(35B+x(14C+9Dx))+d^3(105A+x(35B+3$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(429*a^3*d^3*(-48*c^3*D + 8*c^2*d*(7*C + 3*D*x) - 2*c*d^2
*(35*B + x*(14*C + 9*D*x)) + d^3*(105*A + x*(35*B + 3*x*(7*C + 5*D*x)))) +
 429*a^2*b*d^2*(128*c^4*D - 16*c^3*d*(9*C + 4*D*x) + 24*c^2*d^2*(7*B + x*(
3*C + 2*D*x)) + d^4*x*(105*A + x*(63*B + 5*x*(9*C + 7*D*x))) - 2*c*d^3*(10
5*A + x*(42*B + x*(27*C + 20*D*x)))) + 39*a*b^2*d*(-1280*c^5*D + 128*c^4*d
*(11*C + 5*D*x) - 16*c^3*d^2*(99*B + 44*C*x + 30*D*x^2) + 8*c^2*d^3*(231*A
 + x*(99*B + 66*C*x + 50*D*x^2)) + d^5*x^2*(693*A + 5*x*(99*B + 7*x*(11*C
+ 9*D*x))) - 2*c*d^4*x*(462*A + x*(297*B + 5*x*(44*C + 35*D*x)))) + b^3*(1
5360*c^6*D - 1280*c^5*d*(13*C + 6*D*x) - 16*c^3*d^3*(1287*A + 572*B*x + 39
0*C*x^2 + 300*D*x^3) + 128*c^4*d^2*(143*B + 5*x*(13*C + 9*D*x)) + 5*d^6*x^
3*(1287*A + 7*x*(143*B + 117*C*x + 99*D*x^2)) + 8*c^2*d^4*x*(1287*A + x*(8
58*B + 650*C*x + 525*D*x^2)) - 2*c*d^5*x^2*(3861*A + 5*x*(572*B + 7*x*(65*
C + 54*D*x))))))/(45045*d^7)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}} \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{3/2}(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(c+dx)^{5/2}(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2C\right.\right.}{5d^7}$$
$$\frac{2b(c+dx)^{9/2}\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{9d^7}+$$
$$\frac{2(c+dx)^{7/2}\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2\right.\right.}{7d^7}$$
$$\frac{2(c+dx)^{3/2}(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{3d^7}-$$
$$\frac{2\sqrt{c+dx}(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^7}+$$
$$\frac{2b^2(c+dx)^{11/2}(3adD-6bcD+bCd)}{11d^7}+\frac{2b^3D(c+dx)^{13/2}}{13d^7}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(-2*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[c + d*x])/d^7 -
 (2*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d - 4*B*c*
d^2 + 3*A*d^3 - 6*c^3*D))*(c + d*x)^(3/2))/(3*d^7) - (2*(b*c - a*d)*(a^2*d
^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^2*C*d
- 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(5/2))/(5*d^7) + (2*(a^3*d^3*
D + 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*D) + b
^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(7/2))/(7*d^7) +
 (2*b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^2 - 15*c^2
*D))*(c + d*x)^(9/2))/(9*d^7) + (2*b^2*(b*C*d - 6*b*c*D + 3*a*d*D)*(c + d*
x)^(11/2))/(11*d^7) + (2*b^3*D*(c + d*x)^(13/2))/(13*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{2b^3D(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(B\,d^2-2Ccd+3Dc^2\right)\right)(}{9}$
default	$\frac{\frac{2b^3D(xd+c)^{\frac{13}{2}}}{13}+\frac{2\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(B\,d^2-2Ccd+3Dc^2\right)\right)(}{9}$
pseudoelliptic	$\frac{2\left(\left(\frac{\left(\frac{7}{13}Dx^3+\frac{7}{11}C\,x^2+\frac{7}{9}Bx+A\right)x^3b^3}{7}+\frac{3x^2\left(\frac{5}{11}Dx^3+\frac{5}{9}C\,x^2+\frac{5}{7}Bx+A\right)a\,b^2}{5}+a^2x\left(\frac{1}{3}Dx^3+\frac{3}{7}C\,x^2+\frac{3}{5}Bx+A\right)b+a^3\left(\frac{1}{5}C\,x^2\right.\right.\right.}{}$
gosper	$\frac{2\sqrt{xd+c}\,(3465Dx^6b^3d^6+4095C\,x^5b^3d^6+12285Dx^5a\,b^2d^6-3780Dx^5b^3c\,d^5+5005B\,x^4b^3d^6+15015C\,x^4a\,b^2d^6-4550C}{}$
trager	$\frac{2\sqrt{xd+c}\,(3465Dx^6b^3d^6+4095C\,x^5b^3d^6+12285Dx^5a\,b^2d^6-3780Dx^5b^3c\,d^5+5005B\,x^4b^3d^6+15015C\,x^4a\,b^2d^6-4550C}{}$
orering	$\frac{2\sqrt{xd+c}\,(3465Dx^6b^3d^6+4095C\,x^5b^3d^6+12285Dx^5a\,b^2d^6-3780Dx^5b^3c\,d^5+5005B\,x^4b^3d^6+15015C\,x^4a\,b^2d^6-4550C}{}$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d^7*(1/13*b^3*D*(d*x+c)^(13/2)+1/11*(3*(a*d-b*c)*b^2*D+b^3*(C*d-3*D*c))*
(d*x+c)^(11/2)+1/9*(3*(a*d-b*c)^2*b*D+3*(a*d-b*c)*b^2*(C*d-3*D*c)+b^3*(B*d
^2-2*C*c*d+3*D*c^2))*(d*x+c)^(9/2)+1/7*((a*d-b*c)^3*D+3*(a*d-b*c)^2*b*(C*d
-3*D*c)+3*(a*d-b*c)*b^2*(B*d^2-2*C*c*d+3*D*c^2)+b^3*(A*d^3-B*c*d^2+C*c^2*d
-D*c^3))*(d*x+c)^(7/2)+1/5*((a*d-b*c)^3*(C*d-3*D*c)+3*(a*d-b*c)^2*b*(B*d^2
-2*C*c*d+3*D*c^2)+3*(a*d-b*c)*b^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*(d*x+c)^(
5/2)+1/3*((a*d-b*c)^3*(B*d^2-2*C*c*d+3*D*c^2)+3*(a*d-b*c)^2*b*(A*d^3-B*c*d
^2+C*c^2*d-D*c^3))*(d*x+c)^(3/2)+(a*d-b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)
*(d*x+c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \frac{2\,(3465\,Db^3d^6x^6 + 15360\,Db^3c^6 + 45045\,Aa^3d^6 - 16640\,(3\,Dab^2 + Cb^3)c^5d + 18304\,(3\,Da^2b + 3\,Cab^2 + }{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fricas
")
```

output
```
2/45045*(3465*D*b^3*d^6*x^6 + 15360*D*b^3*c^6 + 45045*A*a^3*d^6 - 16640*(3
*D*a*b^2 + C*b^3)*c^5*d + 18304*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 -
20592*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + 24024*(C*a^3 + 3*B
*a^2*b + 3*A*a*b^2)*c^2*d^4 - 30030*(B*a^3 + 3*A*a^2*b)*c*d^5 - 315*(12*D*
b^3*c*d^5 - 13*(3*D*a*b^2 + C*b^3)*d^6)*x^5 + 35*(120*D*b^3*c^2*d^4 - 130*
(3*D*a*b^2 + C*b^3)*c*d^5 + 143*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^6)*x^4 -
 5*(960*D*b^3*c^3*d^3 - 1040*(3*D*a*b^2 + C*b^3)*c^2*d^4 + 1144*(3*D*a^2*b
 + 3*C*a*b^2 + B*b^3)*c*d^5 - 1287*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)
*d^6)*x^3 + 3*(1920*D*b^3*c^4*d^2 - 2080*(3*D*a*b^2 + C*b^3)*c^3*d^3 + 228
8*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^4 - 2574*(D*a^3 + 3*C*a^2*b + 3*B*
a*b^2 + A*b^3)*c*d^5 + 3003*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^6)*x^2 - (76
80*D*b^3*c^5*d - 8320*(3*D*a*b^2 + C*b^3)*c^4*d^2 + 9152*(3*D*a^2*b + 3*C*
a*b^2 + B*b^3)*c^3*d^3 - 10296*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2
*d^4 + 12012*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^5 - 15015*(B*a^3 + 3*A*a^
2*b)*d^6)*x)*sqrt(d*x + c)/d^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(454) = 908$.

Time = 1.78 (sec) , antiderivative size = 1027, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Piecewise((2*(D*b**3*(c + d*x)**(13/2)/(13*d**6) + (c + d*x)**(11/2)*(C*b*
*3*d + 3*D*a*b**2*d - 6*D*b**3*c)/(11*d**6) + (c + d*x)**(9/2)*(B*b**3*d**
2 + 3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d + 1
5*D*b**3*c**2)/(9*d**6) + (c + d*x)**(7/2)*(A*b**3*d**3 + 3*B*a*b**2*d**3
- 4*B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c**2*
d + D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c**3
)/(7*d**6) + (c + d*x)**(5/2)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*B*a**
2*b*d**4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C*a**2
*b*c*d**3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**3 + 1
8*D*a**2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/(5*d**6) + (c
+ d*x)**(3/2)*(3*A*a**2*b*d**5 - 6*A*a*b**2*c*d**4 + 3*A*b**3*c**2*d**3 +
B*a**3*d**5 - 6*B*a**2*b*c*d**4 + 9*B*a*b**2*c**2*d**3 - 4*B*b**3*c**3*d**
2 - 2*C*a**3*c*d**4 + 9*C*a**2*b*c**2*d**3 - 12*C*a*b**2*c**3*d**2 + 5*C*b
**3*c**4*d + 3*D*a**3*c**2*d**3 - 12*D*a**2*b*c**3*d**2 + 15*D*a*b**2*c**4
*d - 6*D*b**3*c**5)/(3*d**6) + sqrt(c + d*x)*(A*a**3*d**6 - 3*A*a**2*b*c*d
**5 + 3*A*a*b**2*c**2*d**4 - A*b**3*c**3*d**3 - B*a**3*c*d**5 + 3*B*a**2*b
*c**2*d**4 - 3*B*a*b**2*c**3*d**3 + B*b**3*c**4*d**2 + C*a**3*c**2*d**4 -
3*C*a**2*b*c**3*d**3 + 3*C*a*b**2*c**4*d**2 - C*b**3*c**5*d - D*a**3*c**3*
d**3 + 3*D*a**2*b*c**4*d**2 - 3*D*a*b**2*c**5*d + D*b**3*c**6)/d**6)/d, Ne
(d, 0)), ((A*a**3*x + D*b**3*x**7/7 + x**6*(C*b**3 + 3*D*a*b**2)/6 + x*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\left(3465\,(dx+c)^{\frac{13}{2}}Db^3 - 4095\,(6\,Db^3c - (3\,Dab^2 + Cb^3)d)(dx+c)^{\frac{11}{2}} + 5005\,(15\,Db^3c^2 - 5\,(3\,Dab^2 + C\right.}{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="maxima
")
```

output
```
2/45045*(3465*(d*x + c)^(13/2)*D*b^3 - 4095*(6*D*b^3*c - (3*D*a*b^2 + C*b^
3)*d)*(d*x + c)^(11/2) + 5005*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^3)*c*d +
(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(9/2) - 6435*(20*D*b^3*c^3
- 10*(3*D*a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^2 -
 (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(7/2) + 9009*(15*D
*b^3*c^4 - 10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^3
)*c^2*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (C*a^3 + 3*B
*a^2*b + 3*A*a*b^2)*d^4)*(d*x + c)^(5/2) - 15015*(6*D*b^3*c^5 - 5*(3*D*a*b
^2 + C*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2 - 3*(D*a^3 +
 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2
)*c*d^4 - (B*a^3 + 3*A*a^2*b)*d^5)*(d*x + c)^(3/2) + 45045*(D*b^3*c^6 + A*
a^3*d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*
d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3*B*a^2*b
 + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)*sqrt(d*x + c))/d^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(412) = 824$.

Time = 0.13 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/45045*(45045*sqrt(d*x + c)*A*a^3 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x +
 c)*c)*B*a^3/d + 45045*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*a^2*b/d + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a^
3/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*
c^2)*B*a^2*b/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqr
t(d*x + c)*c^2)*A*a*b^2/d^2 + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)
*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*D*a^3/d^3 + 3861*(5*(d
*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)*C*a^2*b/d^3 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*B*a*b^2/d^3 + 1287*(5*(d*x
 + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)*A*b^3/d^3 + 429*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 37
8*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*D
*a^2*b/d^4 + 429*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x +
c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*C*a*b^2/d^
4 + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*
c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*B*b^3/d^4 + 195*(63
*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386
*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*D
*a*b^2/d^5 + 65*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x...
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}} \, dx = \text{Too large to display}$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2),x)
```

output
```
(5544*b^3*c^6*(c + d*x)^(1/2)*D - 504*b^3*c*(c + d*x)^(11/2)*D - 9240*b^3*
c^5*(c + d*x)^(3/2)*D + 11088*b^3*c^4*(c + d*x)^(5/2)*D - 7920*b^3*c^3*(c
+ d*x)^(7/2)*D + 3080*b^3*c^2*(c + d*x)^(9/2)*D + 462*b^3*d^6*x^6*(c + d*x
)^(1/2)*D)/(3003*d^7) + (2*C*(c + d*x)^(5/2)*(a^3*d^3 - 10*b^3*c^3 + 18*a*
b^2*c^2*d - 9*a^2*b*c*d^2))/(5*d^6) + (2*A*b^3*(c + d*x)^(7/2))/(7*d^4) +
(2*B*b^3*(c + d*x)^(9/2))/(9*d^5) + (2*C*b^3*(c + d*x)^(11/2))/(11*d^6) +
(2*A*(a*d - b*c)^3*(c + d*x)^(1/2))/d^4 + (2*A*b*(a*d - b*c)^2*(c + d*x)^(
3/2))/d^4 + (6*A*b^2*(a*d - b*c)*(c + d*x)^(5/2))/(5*d^4) + (2*B*b^2*(3*a*
d - 4*b*c)*(c + d*x)^(7/2))/(7*d^5) - (2*B*c*(a*d - b*c)^3*(c + d*x)^(1/2)
)/d^5 + (2*C*b^2*(3*a*d - 5*b*c)*(c + d*x)^(9/2))/(9*d^6) + (6*B*b*(c + d*
x)^(5/2)*(a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d))/(5*d^5) + (2*C*b*(c + d*x)^(7/
2)*(3*a^2*d^2 + 10*b^2*c^2 - 12*a*b*c*d))/(7*d^6) + (2*B*(a*d - b*c)^2*(a*
d - 4*b*c)*(c + d*x)^(3/2))/(3*d^5) + (2*C*c^2*(a*d - b*c)^3*(c + d*x)^(1/
2))/d^6 - (2*a^3*(c + d*x)^(1/2)*D*(6*c*(c + d*x)^2 - 20*c^2*(c + d*x) + 3
0*c^3 - 5*d^3*x^3))/(35*d^4) - (2*a*b^2*(c + d*x)^(1/2)*D*(70*c*(c + d*x)^
4 - 840*c^4*(c + d*x) - 360*c^2*(c + d*x)^3 + 756*c^3*(c + d*x)^2 + 630*c^
5 - 63*d^5*x^5))/(231*d^6) + (2*a^2*b*(c + d*x)^(1/2)*D*(168*c^2*(c + d*x)
^2 - 280*c^3*(c + d*x) - 40*c*(c + d*x)^3 + 280*c^4 + 35*d^4*x^4))/(105*d^
5) - (2*C*c*(a*d - b*c)^2*(2*a*d - 5*b*c)*(c + d*x)^(3/2))/(3*d^6)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\sqrt{dx+c}\,(3465b^3d^6x^6+12285a\,b^2d^6x^5+315b^3c\,d^5x^5+15015a^2b\,d^6x^4+1365a\,b^2c\,d^5x^4+5005b^4d^5x^4-$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(c + d*x)*(45045*a**4*d**5 - 120120*a**3*b*c*d**4 + 60060*a**3*b*d*
*5*x + 3432*a**3*c**3*d**3 - 1716*a**3*c**2*d**4*x + 1287*a**3*c*d**5*x**2
 + 6435*a**3*d**6*x**3 + 144144*a**2*b**2*c**2*d**3 - 72072*a**2*b**2*c*d*
*4*x + 54054*a**2*b**2*d**5*x**2 - 6864*a**2*b*c**4*d**2 + 3432*a**2*b*c**
3*d**3*x - 2574*a**2*b*c**2*d**4*x**2 + 2145*a**2*b*c*d**5*x**3 + 15015*a*
*2*b*d**6*x**4 - 82368*a*b**3*c**3*d**2 + 41184*a*b**3*c**2*d**3*x - 30888
*a*b**3*c*d**4*x**2 + 25740*a*b**3*d**5*x**3 + 4992*a*b**2*c**5*d - 2496*a
*b**2*c**4*d**2*x + 1872*a*b**2*c**3*d**3*x**2 - 1560*a*b**2*c**2*d**4*x**
3 + 1365*a*b**2*c*d**5*x**4 + 12285*a*b**2*d**6*x**5 + 18304*b**4*c**4*d -
 9152*b**4*c**3*d**2*x + 6864*b**4*c**2*d**3*x**2 - 5720*b**4*c*d**4*x**3
+ 5005*b**4*d**5*x**4 - 1280*b**3*c**6 + 640*b**3*c**5*d*x - 480*b**3*c**4
*d**2*x**2 + 400*b**3*c**3*d**3*x**3 - 350*b**3*c**2*d**4*x**4 + 315*b**3*
c*d**5*x**5 + 3465*b**3*d**6*x**6))/(45045*d**6)
```

3.80 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 324

$$\begin{aligned}
&\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx \\
&= \frac{2(bc-ad)^2 (c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{c+dx}}{d^6} \\
&+ \frac{2(bc-ad)(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))(c+dx)^{3/2}}{3d^6} \\
&+ \frac{2(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D))(c+dx)^{5/2}}{5d^6} \\
&+ \frac{2(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))(c+dx)^{7/2}}{7d^6} \\
&+ \frac{2b(bCd-5bcD+2adD)(c+dx)^{9/2}}{9d^6} + \frac{2b^2D(c+dx)^{11/2}}{11d^6}
\end{aligned}$$

output
```
2*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1/2)/d^6+2/3*(-a*d+b
*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))
*(d*x+c)^(3/2)/d^6+2/5*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*c^
2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(5/2)/d^6+2/7*(a^2*d^
2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(7/2)/d^6+2
/9*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(9/2)/d^6+2/11*b^2*D*(d*x+c)^(11/2)/d
^6
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\sqrt{c+dx}(33a^2d^2(-48c^3D+8c^2d(7C+3Dx)-2cd^2(35B+x(14C+9Dx))+d^3(105A+x(35B+3x$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(33*a^2*d^2*(-48*c^3*D + 8*c^2*d*(7*C + 3*D*x) - 2*c*d^2*
(35*B + x*(14*C + 9*D*x)) + d^3*(105*A + x*(35*B + 3*x*(7*C + 5*D*x)))) +
22*a*b*d*(128*c^4*D - 16*c^3*d*(9*C + 4*D*x) + 24*c^2*d^2*(7*B + x*(3*C +
2*D*x)) + d^4*x*(105*A + x*(63*B + 5*x*(9*C + 7*D*x))) - 2*c*d^3*(105*A +
x*(42*B + x*(27*C + 20*D*x)))) + b^2*(-1280*c^5*D + 128*c^4*d*(11*C + 5*D*
x) - 16*c^3*d^2*(99*B + 44*C*x + 30*D*x^2) + 8*c^2*d^3*(231*A + x*(99*B +
66*C*x + 50*D*x^2)) + d^5*x^2*(693*A + 5*x*(99*B + 7*x*(11*C + 9*D*x))) -
2*c*d^4*x*(462*A + x*(297*B + 5*x*(44*C + 35*D*x))))))/(3465*d^6)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{(c+dx)^{3/2} \left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c\cdot}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2(c+dx)^{5/2}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{5d^6}+\frac{2(c+dx)^{7/2}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{7d^6}+\frac{2(c+dx)^{3/2}(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{3d^6}+\frac{2\sqrt{c+dx}(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^6}+\frac{2b(c+dx)^{9/2}(2adD-5bcD+bCd)}{9d^6}+\frac{2b^2D(c+dx)^{11/2}}{11d^6}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(2*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[c + d*x])/d^6 +
(2*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B*c*d^2
 + 2*A*d^3 - 5*c^3*D))*(c + d*x)^(3/2))/(3*d^6) + (2*(a^2*d^2*(C*d - 3*c*D
) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 + A*d
^3 - 10*c^3*D))*(c + d*x)^(5/2))/(5*d^6) + (2*(a^2*d^2*D + 2*a*b*d*(C*d -
4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(7/2))/(7*d^6) + (2*b
*(b*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(9/2))/(9*d^6) + (2*b^2*D*(c + d*x)
^(11/2))/(11*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$2\left(\left(\frac{x^2\left(\frac{5}{11}Dx^3+\frac{5}{9}Cx^2+\frac{5}{7}Bx+A\right)b^2}{5}+\frac{2x\left(\frac{1}{3}Dx^3+\frac{3}{7}Cx^2+\frac{3}{5}Bx+A\right)ab}{3}+a^2\left(\frac{1}{5}Cx^2+\frac{1}{3}Bx+\frac{1}{7}Dx^3+A\right)\right)d^5-\frac{4\left(\left(\frac{25}{66}Dx^3+\right.\right.}{}\right.$
derivativedivides	$\frac{\frac{2b^2D(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left(2b(ad-bc)D+b^2(Cd-3Dc)\right)(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left((ad-bc)^2D+2b(ad-bc)(Cd-3Dc)+b^2\left(Bd^2-2Ccd+3Dc^2\right)\right)(xd+c)}{7}}{}$
default	$\frac{\frac{2b^2D(xd+c)^{\frac{11}{2}}}{11}+\frac{2\left(2b(ad-bc)D+b^2(Cd-3Dc)\right)(xd+c)^{\frac{9}{2}}}{9}+\frac{2\left((ad-bc)^2D+2b(ad-bc)(Cd-3Dc)+b^2\left(Bd^2-2Ccd+3Dc^2\right)\right)(xd+c)}{7}}{}$
gosper	$\frac{2\sqrt{xd+c}\,(315Dx^5b^2d^5+385Cx^4b^2d^5+770Dx^4abd^5-350Dx^4b^2cd^4+495Bx^3b^2d^5+990Cx^3abd^5-440Cx^3b^2cd^4+49}{}$
trager	$\frac{2\sqrt{xd+c}\,(315Dx^5b^2d^5+385Cx^4b^2d^5+770Dx^4abd^5-350Dx^4b^2cd^4+495Bx^3b^2d^5+990Cx^3abd^5-440Cx^3b^2cd^4+49}{}$
orering	$\frac{2\sqrt{xd+c}\,(315Dx^5b^2d^5+385Cx^4b^2d^5+770Dx^4abd^5-350Dx^4b^2cd^4+495Bx^3b^2d^5+990Cx^3abd^5-440Cx^3b^2cd^4+49}{}$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((1/5*x^2*(5/11*D*x^3+5/9*C*x^2+5/7*B*x+A)*b^2+2/3*x*(1/3*D*x^3+3/7*C*x^
2+3/5*B*x+A)*a*b+a^2*(1/5*C*x^2+1/3*B*x+1/7*D*x^3+A))*d^5-4/3*(1/5*(25/66*
D*x^3+10/21*C*x^2+9/14*B*x+A)*x*b^2+a*(4/21*D*x^3+9/35*C*x^2+2/5*B*x+A)*b+
1/2*(9/35*D*x^2+2/5*C*x+B)*a^2)*c*d^4+8/15*((50/231*D*x^3+2/7*C*x^2+3/7*B*
x+A)*b^2+2*(2/7*D*x^2+3/7*C*x+B)*a*b+a^2*(3/7*D*x+C))*c^2*d^3-16/35*((10/3
3*D*x^2+4/9*C*x+B)*b^2+2*(4/9*D*x+C)*a*b+D*a^2)*c^3*d^2+128/315*c^4*b*((5/
11*D*x+C)*b+2*D*a)*d-256/693*D*b^2*c^5)*(d*x+c)^(1/2)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$=\frac{2\,(315\,Db^2d^5x^5-1280\,Db^2c^5+3465\,Aa^2d^5+1408\,(2\,Dab+Cb^2)c^4d-1584\,(Da^2+2\,Cab+Bb^2)c^3d^2}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fricas
")
```

output
```
2/3465*(315*D*b^2*d^5*x^5 - 1280*D*b^2*c^5 + 3465*A*a^2*d^5 + 1408*(2*D*a*
b + C*b^2)*c^4*d - 1584*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 + 1848*(C*a^2 +
2*B*a*b + A*b^2)*c^2*d^3 - 2310*(B*a^2 + 2*A*a*b)*c*d^4 - 35*(10*D*b^2*c*d
^4 - 11*(2*D*a*b + C*b^2)*d^5)*x^4 + 5*(80*D*b^2*c^2*d^3 - 88*(2*D*a*b + C
*b^2)*c*d^4 + 99*(D*a^2 + 2*C*a*b + B*b^2)*d^5)*x^3 - 3*(160*D*b^2*c^3*d^2
 - 176*(2*D*a*b + C*b^2)*c^2*d^3 + 198*(D*a^2 + 2*C*a*b + B*b^2)*c*d^4 - 2
31*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*x^2 + (640*D*b^2*c^4*d - 704*(2*D*a*b +
C*b^2)*c^3*d^2 + 792*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 - 924*(C*a^2 + 2*B*
a*b + A*b^2)*c*d^4 + 1155*(B*a^2 + 2*A*a*b)*d^5)*x)*sqrt(d*x + c)/d^6
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.97

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{Db^2(c+dx)^{\frac{11}{2}}}{11d^5} + \frac{(c+dx)^{\frac{9}{2}}\left(Cb^2d+2Dabd-5Db^2c\right)}{9d^5} + \frac{(c+dx)^{\frac{7}{2}}\left(Bb^2d^2+2Cabd^2-4Cb^2cd+Da^2d^2-8Dabcd+10Db^2c^2\right)}{7d^5} + \frac{(c+dx)^{\frac{5}{2}}\left(Ab^2d^3+2Babd^3-3B\right.}{} \right.}{} \\ \frac{Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5\left(Cb^2+2Dab\right)}{5}+\frac{x^4\left(Bb^2+2Cab+Da^2\right)}{4}+\frac{x^3\left(Ab^2+2Bab+Ca^2\right)}{3}+\frac{x^2\cdot\left(2Aab+Ba^2\right)}{2}}{\sqrt{c}} \end{cases}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Piecewise((2*(D*b**2*(c + d*x)**(11/2)/(11*d**5) + (c + d*x)**(9/2)*(C*b**
2*d + 2*D*a*b*d - 5*D*b**2*c)/(9*d**5) + (c + d*x)**(7/2)*(B*b**2*d**2 + 2
*C*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2)/(
7*d**5) + (c + d*x)**(5/2)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d**2 +
 C*a**2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 12*D*a
*b*c**2*d - 10*D*b**2*c**3)/(5*d**5) + (c + d*x)**(3/2)*(2*A*a*b*d**4 - 2*
A*b**2*c*d**3 + B*a**2*d**4 - 4*B*a*b*c*d**3 + 3*B*b**2*c**2*d**2 - 2*C*a*
*2*c*d**3 + 6*C*a*b*c**2*d**2 - 4*C*b**2*c**3*d + 3*D*a**2*c**2*d**2 - 8*D
*a*b*c**3*d + 5*D*b**2*c**4)/(3*d**5) + sqrt(c + d*x)*(A*a**2*d**5 - 2*A*a
*b*c*d**4 + A*b**2*c**2*d**3 - B*a**2*c*d**4 + 2*B*a*b*c**2*d**3 - B*b**2*
c**3*d**2 + C*a**2*c**2*d**3 - 2*C*a*b*c**3*d**2 + C*b**2*c**4*d - D*a**2*
c**3*d**2 + 2*D*a*b*c**4*d - D*b**2*c**5)/d**5)/d, Ne(d, 0)), ((A*a**2*x +
 D*b**2*x**6/6 + x**5*(C*b**2 + 2*D*a*b)/5 + x**4*(B*b**2 + 2*C*a*b + D*a*
*2)/4 + x**3*(A*b**2 + 2*B*a*b + C*a**2)/3 + x**2*(2*A*a*b + B*a**2)/2)/sq
rt(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx = \frac{2\left(315\,(dx+c)^{\frac{11}{2}}Db^2 - 385\,(5\,Db^2c-(2\,Dab+Cb^2)d)(dx+c)^{\frac{9}{2}} + 495\,(10\,Db^2c^2-4\,(2\,Dab+Cb^2)cd \cdots\right.}{\cdots}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="maxima
")
```

output
```
2/3465*(315*(d*x + c)^(11/2)*D*b^2 - 385*(5*D*b^2*c - (2*D*a*b + C*b^2)*d)
*(d*x + c)^(9/2) + 495*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d + (D*a^2 +
2*C*a*b + B*b^2)*d^2)*(d*x + c)^(7/2) - 693*(10*D*b^2*c^3 - 6*(2*D*a*b + C
*b^2)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*b + A*b^2
)*d^3)*(d*x + c)^(5/2) + 1155*(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^2)*c^3*d + 3
*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 + (
B*a^2 + 2*A*a*b)*d^4)*(d*x + c)^(3/2) - 3465*(D*b^2*c^5 - A*a^2*d^5 - (2*D
*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 + 2*B*a*b
 + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)*sqrt(d*x + c))/d^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(3465*sqrt(d*x + c)*A*a^2 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)
*c)*B*a^2/d + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*a*b/d + 231*(3*
(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a^2/d^2 +
 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*a
*b/d^2 + 231*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*
c^2)*A*b^2/d^2 + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*D*a^2/d^3 + 198*(5*(d*x + c)^(7/2) -
21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*a*
b/d^3 + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*
c^2 - 35*sqrt(d*x + c)*c^3)*B*b^2/d^3 + 22*(35*(d*x + c)^(9/2) - 180*(d*x
+ c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqr
t(d*x + c)*c^4)*D*a*b/d^4 + 11*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c
 + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c
^4)*C*b^2/d^4 + 5*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x
+ c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693
*sqrt(d*x + c)*c^5)*D*b^2/d^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2A\sqrt{c+d\,x}\,\left(3\,b^2\,(c+d\,x)^2+15\,a^2\,d^2+15\,b^2\,c^2-10\,b^2\,c\,(c+d\,x)+10\,a\,b\,d\,(c+d\,x)-30\,a\,b\,c\,d\right)}{15\,d^3}$$
$$+\frac{2\,B\,b^2\,(c+d\,x)^{7/2}}{7\,d^4}+\frac{2\,C\,b^2\,(c+d\,x)^{9/2}}{9\,d^5}+\frac{2\,B\,(c+d\,x)^{3/2}\,(a^2\,d^2-4\,a\,b\,c\,d+3\,b^2\,c^2)}{3\,d^4}$$
$$+\frac{2\,C\,(c+d\,x)^{5/2}\,(a^2\,d^2-6\,a\,b\,c\,d+6\,b^2\,c^2)}{5\,d^5}-\frac{2\,B\,c\,(a\,d-b\,c)^2\,\sqrt{c+d\,x}}{d^4}$$
$$-\frac{4\,C\,c\,(c+d\,x)^{3/2}\,(a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2)}{3\,d^5}$$
$$-\frac{10\,b^2\,c\,D\left(\frac{2\,(c+d\,x)^{9/2}}{9\,d^5}+\frac{2\,c^4\,\sqrt{c+d\,x}}{d^5}-\frac{8\,c^3\,(c+d\,x)^{3/2}}{3\,d^5}+\frac{12\,c^2\,(c+d\,x)^{5/2}}{5\,d^5}-\frac{8\,c\,(c+d\,x)^{7/2}}{7\,d^5}\right)}{11\,d}$$
$$+\frac{2\,C\,c^2\,(a\,d-b\,c)^2\,\sqrt{c+d\,x}}{d^5}$$
$$-\frac{2\,a^2\,\sqrt{c+d\,x}\,D\,\left(6\,c\,(c+d\,x)^2-20\,c^2\,(c+d\,x)+30\,c^3-5\,d^3\,x^3\right)}{35\,d^4}$$
$$+\frac{2\,b^2\,x^5\,\sqrt{c+d\,x}\,D}{11\,d}+\frac{2\,B\,b\,(2\,a\,d-3\,b\,c)\,\left(c+d\,x\right)^{5/2}}{5\,d^4}+\frac{4\,C\,b\,(a\,d-2\,b\,c)\,\left(c+d\,x\right)^{7/2}}{7\,d^5}$$
$$+\frac{4\,a\,b\,\sqrt{c+d\,x}\,D\,\left(168\,c^2\,(c+d\,x)^2-280\,c^3\,(c+d\,x)-40\,c\,(c+d\,x)^3+280\,c^4+35\,d^4\,x^4\right)}{315\,d^5}$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2),x)
```

output
```
(2*A*(c + d*x)^(1/2)*(3*b^2*(c + d*x)^2 + 15*a^2*d^2 + 15*b^2*c^2 - 10*b^2
*c*(c + d*x) + 10*a*b*d*(c + d*x) - 30*a*b*c*d))/(15*d^3) + (2*B*b^2*(c +
d*x)^(7/2))/(7*d^4) + (2*C*b^2*(c + d*x)^(9/2))/(9*d^5) + (2*B*(c + d*x)^(
3/2)*(a^2*d^2 + 3*b^2*c^2 - 4*a*b*c*d))/(3*d^4) + (2*C*(c + d*x)^(5/2)*(a^
2*d^2 + 6*b^2*c^2 - 6*a*b*c*d))/(5*d^5) - (2*B*c*(a*d - b*c)^2*(c + d*x)^(
1/2))/d^4 - (4*C*c*(c + d*x)^(3/2)*(a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d))/(3*d
^5) - (10*b^2*c*D*((2*(c + d*x)^(9/2))/(9*d^5) + (2*c^4*(c + d*x)^(1/2))/d
^5 - (8*c^3*(c + d*x)^(3/2))/(3*d^5) + (12*c^2*(c + d*x)^(5/2))/(5*d^5) -
(8*c*(c + d*x)^(7/2))/(7*d^5)))/(11*d) + (2*C*c^2*(a*d - b*c)^2*(c + d*x)^
(1/2))/d^5 - (2*a^2*(c + d*x)^(1/2)*D*(6*c*(c + d*x)^2 - 20*c^2*(c + d*x)
+ 30*c^3 - 5*d^3*x^3))/(35*d^4) + (2*b^2*x^5*(c + d*x)^(1/2)*D)/(11*d) + (
2*B*b*(2*a*d - 3*b*c)*(c + d*x)^(5/2))/(5*d^4) + (4*C*b*(a*d - 2*b*c)*(c +
 d*x)^(7/2))/(7*d^5) + (4*a*b*(c + d*x)^(1/2)*D*(168*c^2*(c + d*x)^2 - 280
*c^3*(c + d*x) - 40*c*(c + d*x)^3 + 280*c^4 + 35*d^4*x^4))/(315*d^5)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\sqrt{dx+c}\,(315b^2d^5x^5 + 770ab\,d^5x^4 + 35b^2c\,d^4x^4 + 495a^2d^5x^3 + 110abc\,d^4x^3 + 495b^3d^4x^3 - 40b^2c^2d^3x^3 + \cdots}{}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(c + d*x)*(3465*a**3*d**4 - 6930*a**2*b*c*d**3 + 3465*a**2*b*d**4*x
 + 264*a**2*c**3*d**2 - 132*a**2*c**2*d**3*x + 99*a**2*c*d**4*x**2 + 495*a
**2*d**5*x**3 + 5544*a*b**2*c**2*d**2 - 2772*a*b**2*c*d**3*x + 2079*a*b**2
*d**4*x**2 - 352*a*b*c**4*d + 176*a*b*c**3*d**2*x - 132*a*b*c**2*d**3*x**2
 + 110*a*b*c*d**4*x**3 + 770*a*b*d**5*x**4 - 1584*b**3*c**3*d + 792*b**3*c
**2*d**2*x - 594*b**3*c*d**3*x**2 + 495*b**3*d**4*x**3 + 128*b**2*c**5 - 6
4*b**2*c**4*d*x + 48*b**2*c**3*d**2*x**2 - 40*b**2*c**2*d**3*x**3 + 35*b**
2*c*d**4*x**4 + 315*b**2*d**5*x**5))/(3465*d**5)
```

3.81 $\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$

Optimal result . . . 747
Mathematica [A] (verified) . . . 748
Rubi [A] (verified) . . . 748
Maple [A] (verified) . . . 750
Fricas [A] (verification not implemented) . . . 750
Sympy [A] (verification not implemented) . . . 751
Maxima [A] (verification not implemented) . . . 752
Giac [A] (verification not implemented) . . . 752
Mupad [B] (verification not implemented) . . . 753
Reduce [B] (verification not implemented) . . . 754

Optimal result

Integrand size = 30, antiderivative size = 212

$$\begin{aligned}\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx &= -\frac{2(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)\sqrt{c+dx}}{d^5} \\ &\quad -\frac{2(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,(c+dx)^{3/2}}{3d^5} \\ &\quad +\frac{2(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))\,(c+dx)^{5/2}}{5d^5} \\ &\quad +\frac{2(bCd-4bcD+adD)(c+dx)^{7/2}}{7d^5}+\frac{2bD(c+dx)^{9/2}}{9d^5}\end{aligned}$$

output
```
-2*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1/2)/d^5-2/3*(a*d*(-B
*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(3/2)
/d^5+2/5*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(5/2)/d^5+2/
7*(C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(7/2)/d^5+2/9*b*D*(d*x+c)^(9/2)/d^5
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\sqrt{c+dx}(3ad(-48c^3D+8c^2d(7C+3Dx)-2cd^2(35B+x(14C+9Dx))+d^3(105A+x(35B+3x(7C$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(3*a*d*(-48*c^3*D + 8*c^2*d*(7*C + 3*D*x) - 2*c*d^2*(35*B
 + x*(14*C + 9*D*x)) + d^3*(105*A + x*(35*B + 3*x*(7*C + 5*D*x)))) + b*(12
8*c^4*D - 16*c^3*d*(9*C + 4*D*x) + 24*c^2*d^2*(7*B + x*(3*C + 2*D*x)) + d^
4*x*(105*A + x*(63*B + 5*x*(9*C + 7*D*x))) - 2*c*d^3*(105*A + x*(42*B + x*
(27*C + 20*D*x))))))/(315*d^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{\sqrt{c+dx}\left(b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)}{d^4}+\frac{(ad-bc)\left(Ad^3-Bcd^2+c^3(-\right.}{d^4\sqrt{c+dx}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(c+dx)^{3/2}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{3d^5}-\frac{2\sqrt{c+dx}(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^5}+\frac{2(c+dx)^{5/2}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{5d^5}+\frac{2(c+dx)^{7/2}(adD-4bcD+bCd)}{7d^5}+\frac{2bD(c+dx)^{9/2}}{9d^5}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(-2*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[c + d*x])/d^5 - (
2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4*
c^3*D))*(c + d*x)^(3/2))/(3*d^5) + (2*(a*d*(C*d - 3*c*D) - b*(3*c*C*d - B*
d^2 - 6*c^2*D))*(c + d*x)^(5/2))/(5*d^5) + (2*(b*C*d - 4*b*c*D + a*d*D)*(c
 + d*x)^(7/2))/(7*d^5) + (2*b*D*(c + d*x)^(9/2))/(9*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{2\left(\left(\frac{Db\,x^4}{9}+\frac{(Cb+Da)x^3}{7}+\frac{(Bb+Ca)x^2}{5}+\frac{(Ab+Ba)x}{3}+Aa\right)d^4-\frac{2\left(\frac{4Db\,x^3}{21}+\frac{9(Cb+Da)x^2}{35}+\frac{2(Bb+Ca)x}{5}+Ab+Ba\right)c\,d^3}{3}+\frac{8(2\cdots}{}\right.}{d^5}$
derivativedivides	$\frac{\frac{2bD(xd+c)^{\frac{9}{2}}}{9}+\frac{2((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{7}{2}}}{7}+\frac{2\left((ad-bc)(Cd-3Dc)+b\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{5}{2}}}{5}+\frac{2\left((ad-bc)\left(B\,d^2-\cdots\right.\right.}{}}{d^5}$
default	$\frac{\frac{2bD(xd+c)^{\frac{9}{2}}}{9}+\frac{2((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{7}{2}}}{7}+\frac{2\left((ad-bc)(Cd-3Dc)+b\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{5}{2}}}{5}+\frac{2\left((ad-bc)\left(B\,d^2-\cdots\right.\right.}{}}{d^5}$
gosper	$2\sqrt{xd+c}\,(35Db\,x^4d^4+45C\,x^3b\,d^4+45Dx^3a\,d^4-40Dx^3bc\,d^3+63B\,x^2b\,d^4+63C\,x^2a\,d^4-54C\,x^2bc\,d^3-54Dx^2ac\,d^3+48\cdots$
trager	$2\sqrt{xd+c}\,(35Db\,x^4d^4+45C\,x^3b\,d^4+45Dx^3a\,d^4-40Dx^3bc\,d^3+63B\,x^2b\,d^4+63C\,x^2a\,d^4-54C\,x^2bc\,d^3-54Dx^2ac\,d^3+48\cdots$
orering	$2\sqrt{xd+c}\,(35Db\,x^4d^4+45C\,x^3b\,d^4+45Dx^3a\,d^4-40Dx^3bc\,d^3+63B\,x^2b\,d^4+63C\,x^2a\,d^4-54C\,x^2bc\,d^3-54Dx^2ac\,d^3+48\cdots$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((1/9*D*b*x^4+1/7*(C*b+D*a)*x^3+1/5*(B*b+C*a)*x^2+1/3*(A*b+B*a)*x+A*a)*d
^4-2/3*(4/21*D*b*x^3+9/35*(C*b+D*a)*x^2+2/5*(B*b+C*a)*x+A*b+B*a)*c*d^3+8/1
5*(2/7*D*b*x^2+3/7*(C*b+D*a)*x+B*b+C*a)*c^2*d^2-16/35*(4/9*D*b*x+C*b+D*a)*
c^3*d+128/315*D*b*c^4)*(d*x+c)^(1/2)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$
$$=\frac{2\left(35\,Dbd^4x^4+128\,Dbc^4+315\,Aad^4-144\,(Da+Cb)c^3d+168\,(Ca+Bb)c^2d^2-210\,(Ba+Ab)cd^3-\cdots\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*D*b*d^4*x^4 + 128*D*b*c^4 + 315*A*a*d^4 - 144*(D*a + C*b)*c^3*d
+ 168*(C*a + B*b)*c^2*d^2 - 210*(B*a + A*b)*c*d^3 - 5*(8*D*b*c*d^3 - 9*(D*
a + C*b)*d^4)*x^3 + 3*(16*D*b*c^2*d^2 - 18*(D*a + C*b)*c*d^3 + 21*(C*a + B
*b)*d^4)*x^2 - (64*D*b*c^3*d - 72*(D*a + C*b)*c^2*d^2 + 84*(C*a + B*b)*c*d
^3 - 105*(B*a + A*b)*d^4)*x)*sqrt(d*x + c)/d^5
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{Db(c+dx)^{\frac{9}{2}}}{9d^4}+\frac{(c+dx)^{\frac{7}{2}}(Cbd+Dad-4Dbc)}{7d^4}+\frac{(c+dx)^{\frac{5}{2}}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6Dbc^2\right)}{5d^4}+\frac{(c+dx)^{\frac{3}{2}}\left(Abd^3+Bad^3-2Bbcd^2-2Cacd^2+3Cbc^2d+3D\right.}{3d^4}}{d} \\ \frac{Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}}{\sqrt{c}} \end{cases}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Piecewise((2*(D*b*(c + d*x)**(9/2)/(9*d**4) + (c + d*x)**(7/2)*(C*b*d + D*
a*d - 4*D*b*c)/(7*d**4) + (c + d*x)**(5/2)*(B*b*d**2 + C*a*d**2 - 3*C*b*c*
d - 3*D*a*c*d + 6*D*b*c**2)/(5*d**4) + (c + d*x)**(3/2)*(A*b*d**3 + B*a*d*
*3 - 2*B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D*b*c**
3)/(3*d**4) + sqrt(c + d*x)*(A*a*d**4 - A*b*c*d**3 - B*a*c*d**3 + B*b*c**2
*d**2 + C*a*c**2*d**2 - C*b*c**3*d - D*a*c**3*d + D*b*c**4)/d**4)/d, Ne(d,
 0)), ((A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/4 + x**3*(B*b + C*a)/3 + x**
2*(A*b + B*a)/2)/sqrt(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\left(35\,(dx+c)^{\frac{9}{2}}Db - 45\,(4\,Dbc-(Da+Cb)d)(dx+c)^{\frac{7}{2}} + 63\,(6\,Dbc^2-3\,(Da+Cb)cd+(Ca+Bb)d^2)\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*(d*x + c)^(9/2)*D*b - 45*(4*D*b*c - (D*a + C*b)*d)*(d*x + c)^(7/
2) + 63*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*(d*x + c)^(5/2)
- 105*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 - (B*a + A*b)
*d^3)*(d*x + c)^(3/2) + 315*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c^3*d + (C*a
+ B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)*sqrt(d*x + c))/d^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\left(315\,\sqrt{dx+c}Aa + \frac{105\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}c\right)Ba}{d} + \frac{105\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}c\right)Ab}{d} + \frac{21\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}c\right.}{d^2}\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(d*x + c)*A*a + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B
*a/d + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*A*b/d + 21*(3*(d*x + c)^(
5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C*a/d^2 + 21*(3*(d*x +
 c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*B*b/d^2 + 9*(5*(d
*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)*D*a/d^3 + 9*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*
x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*C*b/d^3 + (35*(d*x + c)^(9/2) - 1
80*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 +
 315*sqrt(d*x + c)*c^4)*D*b/d^4)/d
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx = \frac{2\,A\,b\,{\left(c+d\,x\right)}^{3/2}-6\,A\,b\,c\,\sqrt{c+d\,x}}{3\,d^2}+\frac{2\,B\,a\,{\left(c+d\,x\right)}^{3/2}-6\,B\,a\,c\,\sqrt{c+d\,x}}{3\,d^2} + \frac{6\,B\,b\,{\left(c+d\,x\right)}^{5/2}+30\,B\,b\,c^2\,\sqrt{c+d\,x}-20\,B\,b\,c\,{\left(c+d\,x\right)}^{3/2}}{15\,d^3} + \frac{6\,C\,a\,{\left(c+d\,x\right)}^{5/2}+30\,C\,a\,c^2\,\sqrt{c+d\,x}-20\,C\,a\,c\,{\left(c+d\,x\right)}^{3/2}}{15\,d^3} + \frac{2\,A\,a\,\sqrt{c+d\,x}}{d}+\frac{2\,C\,b\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4} - \frac{2\,a\,\sqrt{c+d\,x}\,D\,\left(6\,c\,{\left(c+d\,x\right)}^2-20\,c^2\,\left(c+d\,x\right)+30\,c^3-5\,d^3\,x^3\right)}{35\,d^4} + \frac{2\,b\,x^4\,\sqrt{c+d\,x}\,D}{9\,d}-\frac{6\,C\,b\,c\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4} - \frac{8\,b\,c\,D\,\left(\frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}-\frac{2\,c^3\,\sqrt{c+d\,x}}{d^4}+\frac{2\,c^2\,{\left(c+d\,x\right)}^{3/2}}{d^4}-\frac{6\,c\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}\right)}{9\,d} - \frac{2\,C\,b\,c^3\,\sqrt{c+d\,x}}{d^4}+\frac{2\,C\,b\,c^2\,{\left(c+d\,x\right)}^{3/2}}{d^4}$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2),x)
```

output
```
(2*A*b*(c + d*x)^(3/2) - 6*A*b*c*(c + d*x)^(1/2))/(3*d^2) + (2*B*a*(c + d*
x)^(3/2) - 6*B*a*c*(c + d*x)^(1/2))/(3*d^2) + (6*B*b*(c + d*x)^(5/2) + 30*
B*b*c^2*(c + d*x)^(1/2) - 20*B*b*c*(c + d*x)^(3/2))/(15*d^3) + (6*C*a*(c +
 d*x)^(5/2) + 30*C*a*c^2*(c + d*x)^(1/2) - 20*C*a*c*(c + d*x)^(3/2))/(15*d
^3) + (2*A*a*(c + d*x)^(1/2))/d + (2*C*b*(c + d*x)^(7/2))/(7*d^4) - (2*a*(
c + d*x)^(1/2)*D*(6*c*(c + d*x)^2 - 20*c^2*(c + d*x) + 30*c^3 - 5*d^3*x^3)
)/(35*d^4) + (2*b*x^4*(c + d*x)^(1/2)*D)/(9*d) - (6*C*b*c*(c + d*x)^(5/2))
/(5*d^4) - (8*b*c*D*((2*(c + d*x)^(7/2))/(7*d^4) - (2*c^3*(c + d*x)^(1/2))
/d^4 + (2*c^2*(c + d*x)^(3/2))/d^4 - (6*c*(c + d*x)^(5/2))/(5*d^4)))/(9*d)
 - (2*C*b*c^3*(c + d*x)^(1/2))/d^4 + (2*C*b*c^2*(c + d*x)^(3/2))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\sqrt{dx+c}\,(35b\,d^4x^4+45a\,d^4x^3+5bc\,d^3x^3+9ac\,d^3x^2+63b^2d^3x^2-6b\,c^2d^2x^2+210ab\,d^3x-12a\,c^2d^2x-}{315d^4}$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(c + d*x)*(315*a**2*d**3 - 420*a*b*c*d**2 + 210*a*b*d**3*x + 24*a*c
**3*d - 12*a*c**2*d**2*x + 9*a*c*d**3*x**2 + 45*a*d**4*x**3 + 168*b**2*c**
2*d - 84*b**2*c*d**2*x + 63*b**2*d**3*x**2 - 16*b*c**4 + 8*b*c**3*d*x - 6*
b*c**2*d**2*x**2 + 5*b*c*d**3*x**3 + 35*b*d**4*x**4))/(315*d**4)
```

3.82 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$

Optimal result 755
Mathematica [A] (verified) 755
Rubi [A] (verified) 756
Maple [A] (verified) 757
Fricas [A] (verification not implemented) 758
Sympy [A] (verification not implemented) 758
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Reduce [B] (verification not implemented) 760

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{c+dx}}{d^4} - \frac{2(2cCd-Bd^2-3c^2D)(c+dx)^{3/2}}{3d^4} + \frac{2(Cd-3cD)(c+dx)^{5/2}}{5d^4} + \frac{2D(c+dx)^{7/2}}{7d^4}$$

output
```
2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1/2)/d^4-2/3*(-B*d^2+2*C*c*d-3*D*
c^2)*(d*x+c)^(3/2)/d^4+2/5*(C*d-3*D*c)*(d*x+c)^(5/2)/d^4+2/7*D*(d*x+c)^(7/
2)/d^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}(-48c^3D+8c^2d(7C+3Dx)-2cd^2(35B+x(14C+9Dx))+d^3(105A+x(35B+3x(7C+5}{105d^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[c + d*x]*(-48*c^3*D + 8*c^2*d*(7*C + 3*D*x) - 2*c*d^2*(35*B + x*(1
4*C + 9*D*x)) + d^3*(105*A + x*(35*B + 3*x*(7*C + 5*D*x)))))/(105*d^4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3\sqrt{c+dx}}+\frac{\sqrt{c+dx}\,(Bd^2+3c^2D-2cCd)}{d^3}+\frac{(c+dx)^{3/2}(Cd-3cD)}{d^3}+\frac{D(c+dx)^{5/2}}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{c+dx}\,(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{d^4}-\frac{2(c+dx)^{3/2}\left(-Bd^2-3c^2D+2cCd\right)}{3d^4}+\frac{2(c+dx)^{5/2}(Cd-3cD)}{5d^4}+\frac{2D(c+dx)^{7/2}}{7d^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/Sqrt[c + d*x],x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[c + d*x])/d^4 - (2*(2*c*C*d -
B*d^2 - 3*c^2*D)*(c + d*x)^(3/2))/(3*d^4) + (2*(C*d - 3*c*D)*(c + d*x)^(5/
2))/(5*d^4) + (2*D*(c + d*x)^(7/2))/(7*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{2\sqrt{xd+c}\left(\left(\frac{1}{5}C\,x^{2}+\frac{1}{3}Bx+\frac{1}{7}Dx^{3}+A\right)d^{3}-\frac{2\left(\frac{9}{35}Dx^{2}+\frac{2}{5}Cx+B\right)c\,d^{2}}{3}+\frac{8\left(\frac{3Dx}{7}+C\right)c^{2}d}{15}-\frac{16Dc^{3}}{35}\right)}{d^{4}}$
gosper	$\frac{2\sqrt{xd+c}\,(15Dx^{3}d^{3}+21C\,x^{2}d^{3}-18Dx^{2}c\,d^{2}+35Bx\,d^{3}-28Cxc\,d^{2}+24Dx\,c^{2}d+105A\,d^{3}-70Bc\,d^{2}+56C\,c^{2}d-48Dc^{3})}{105d^{4}}$
trager	$\frac{2\sqrt{xd+c}\,(15Dx^{3}d^{3}+21C\,x^{2}d^{3}-18Dx^{2}c\,d^{2}+35Bx\,d^{3}-28Cxc\,d^{2}+24Dx\,c^{2}d+105A\,d^{3}-70Bc\,d^{2}+56C\,c^{2}d-48Dc^{3})}{105d^{4}}$
orering	$\frac{2\sqrt{xd+c}\,(15Dx^{3}d^{3}+21C\,x^{2}d^{3}-18Dx^{2}c\,d^{2}+35Bx\,d^{3}-28Cxc\,d^{2}+24Dx\,c^{2}d+105A\,d^{3}-70Bc\,d^{2}+56C\,c^{2}d-48Dc^{3})}{105d^{4}}$
derivativedivides	$\frac{\frac{2D(xd+c)^{\frac{7}{2}}}{7}+\frac{2Cd(xd+c)^{\frac{5}{2}}}{5}-\frac{6Dc(xd+c)^{\frac{5}{2}}}{5}+\frac{2B\,d^{2}(xd+c)^{\frac{3}{2}}}{3}-\frac{4Ccd(xd+c)^{\frac{3}{2}}}{3}+2Dc^{2}(xd+c)^{\frac{3}{2}}+2A\,d^{3}\sqrt{xd+c}-2Bc\,d^{2}\sqrt{xd+}}{d^{4}}$
default	$\frac{\frac{2D(xd+c)^{\frac{7}{2}}}{7}+\frac{2Cd(xd+c)^{\frac{5}{2}}}{5}-\frac{6Dc(xd+c)^{\frac{5}{2}}}{5}+\frac{2B\,d^{2}(xd+c)^{\frac{3}{2}}}{3}-\frac{4Ccd(xd+c)^{\frac{3}{2}}}{3}+2Dc^{2}(xd+c)^{\frac{3}{2}}+2A\,d^{3}\sqrt{xd+c}-2Bc\,d^{2}\sqrt{xd+}}{d^{4}}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(d*x+c)^(1/2)*((1/5*C*x^2+1/3*B*x+1/7*D*x^3+A)*d^3-2/3*(9/35*D*x^2+2/5*C
*x+B)*c*d^2+8/15*(3/7*D*x+C)*c^2*d-16/35*D*c^3)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\left(15\,Dd^3x^3-48\,Dc^3+56\,Cc^2d-70\,Bcd^2+105\,Ad^3-3\,(6\,Dcd^2-7\,Cd^3)x^2+(24\,Dc^2d-28\,Ccd^2+\cdots\right.}{105\,d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*D*d^3*x^3 - 48*D*c^3 + 56*C*c^2*d - 70*B*c*d^2 + 105*A*d^3 - 3*(
6*D*c*d^2 - 7*C*d^3)*x^2 + (24*D*c^2*d - 28*C*c*d^2 + 35*B*d^3)*x)*sqrt(d*
x + c)/d^4
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$$
$$= \begin{cases} \frac{2A\sqrt{c+dx}+\frac{2B\left(-c\sqrt{c+dx}+\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}+\frac{2C\left(c^2\sqrt{c+dx}-\frac{2c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{2D\left(-c^3\sqrt{c+dx}+c^2(c+dx)^{\frac{3}{2}}-\frac{3c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{for } d \ldots \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}}{\sqrt{c}} & \text{othe}\ldots \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Piecewise(((2*A*sqrt(c + d*x) + 2*B*(-c*sqrt(c + d*x) + (c + d*x)**(3/2)/3
)/d + 2*C*(c**2*sqrt(c + d*x) - 2*c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/
5)/d**2 + 2*D*(-c**3*sqrt(c + d*x) + c**2*(c + d*x)**(3/2) - 3*c*(c + d*x)
**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3)/d, Ne(d, 0)), ((A*x + B*x**2/2 + C*x
**3/3 + D*x**4/4)/sqrt(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\left(105\sqrt{dx+c}A + \frac{35\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)B}{d} + \frac{7\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}c^2\right)C}{d^2} + \frac{3\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+3\right.}{d}}{105\,d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(105*sqrt(d*x + c)*A + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B/d
+ 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C/d^
2 + 3*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*D/d^3)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\left(105\sqrt{dx+c}A + \frac{35\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)B}{d} + \frac{7\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}c^2\right)C}{d^2} + \frac{3\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+3\right.}{d}}{105\,d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*sqrt(d*x + c)*A + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*B/d
+ 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*C/d^
2 + 3*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*D/d^3)/d
```

Mupad [B] (verification not implemented)

Time = 3.66 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$$
$$=\frac{6\,C\,(c+d\,x)^{5/2}-20\,C\,c\,(c+d\,x)^{3/2}+30\,C\,c^2\,\sqrt{c+d\,x}}{15\,d^3}$$
$$+\frac{2\,B\,(c+d\,x)^{3/2}-6\,B\,c\,\sqrt{c+d\,x}}{3\,d^2}+\frac{2\,A\,\sqrt{c+d\,x}}{d}$$
$$-\frac{2\,\sqrt{c+d\,x}\,D\,\left(6\,c\,(c+d\,x)^2-20\,c^2\,(c+d\,x)+30\,c^3-5\,d^3\,x^3\right)}{35\,d^4}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(1/2),x)
```

output
```
(6*C*(c + d*x)^(5/2) - 20*C*c*(c + d*x)^(3/2) + 30*C*c^2*(c + d*x)^(1/2))/
(15*d^3) + (2*B*(c + d*x)^(3/2) - 6*B*c*(c + d*x)^(1/2))/(3*d^2) + (2*A*(c
 + d*x)^(1/2))/d - (2*(c + d*x)^(1/2)*D*(6*c*(c + d*x)^2 - 20*c^2*(c + d*x
) + 30*c^3 - 5*d^3*x^3))/(35*d^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.51

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}}\,dx$$
$$=\frac{2\sqrt{dx+c}\,(15d^3x^3+3c\,d^2x^2+35b\,d^2x-4c^2dx+105a\,d^2-70bcd+8c^3)}{105d^3}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
(2*sqrt(c + d*x)*(105*a*d**2 - 70*b*c*d + 35*b*d**2*x + 8*c**3 - 4*c**2*d*
x + 3*c*d**2*x**2 + 15*d**3*x**3))/(105*d**3)
```

3.83 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx$

Optimal result . . . 761
Mathematica [A] (verified) . . . 762
Rubi [A] (verified) . . . 762
Maple [A] (verified) . . . 764
Fricas [A] (verification not implemented) . . . 764
Sympy [A] (verification not implemented) . . . 765
Maxima [F(-2)] . . . 766
Giac [A] (verification not implemented) . . . 766
Mupad [F(-1)] . . . 767
Reduce [B] (verification not implemented) . . . 767

Optimal result

Integrand size = 32, antiderivative size = 188

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx = \frac{2(a^2d^2D-abd(Cd-cD)-b^2(cCd-Bd^2-c^2D))\sqrt{c+dx}}{b^3d^3} + \frac{2(bCd-2bcD-adD)(c+dx)^{3/2}}{3b^2d^3} + \frac{2D(c+dx)^{5/2}}{5bd^3} - \frac{2(Ab^3-a(b^2B-abC+a^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}\sqrt{bc-ad}}$$

output
```
2*(a^2*d^2*D-a*b*d*(C*d-D*c)-b^2*(-B*d^2+C*c*d-D*c^2))*(d*x+c)^(1/2)/b^3/d
^3+2/3*(C*b*d-D*a*d-2*D*b*c)*(d*x+c)^(3/2)/b^2/d^3+2/5*D*(d*x+c)^(5/2)/b/d
^3-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(7/2)/(-a*d+b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx$$
$$= \frac{2\sqrt{c+dx}(15a^2d^2D-5abd(3Cd-2cD+dDx)+b^2(8c^2D-2cd(5C+2Dx)+d^2(15B+5Cx+3Dx^2}{15b^3d^3}$$
$$+\frac{2(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{7/2}\sqrt{-bc+ad}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*Sqrt[c + d*x]),x]
```

output
```
(2*Sqrt[c + d*x]*(15*a^2*d^2*D - 5*a*b*d*(3*C*d - 2*c*D + d*D*x) + b^2*(8*
c^2*D - 2*c*d*(5*C + 2*D*x) + d^2*(15*B + 5*C*x + 3*D*x^2))))/(15*b^3*d^3)
 + (2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[-(b*c) + a*d]])/(b^(7/2)*Sqrt[-(b*c) + a*d])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{b^3(a+bx)\sqrt{c+dx}}+\frac{a^2d^2D-abd(Cd-cD)-\left(b^2\left(-Bd^2+c^2(-D)+cCd\right)\right)}{b^3d^2\sqrt{c+dx}}+\frac{\sqrt{c+dx}(-adD}{b^2}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx}\left(a^2d^2D-abd(Cd-cD)-\left(b^2\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{b^3d^3}+\frac{2(c+dx)^{3/2}(-adD-2bcD+bCd)}{3b^2d^3}+\frac{2D(c+dx)^{5/2}}{5bd^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*Sqrt[c + d*x]),x]
```

output
```
(2*(a^2*d^2*D - a*b*d*(C*d - c*D) - b^2*(c*C*d - B*d^2 - c^2*D))*Sqrt[c +
d*x])/(b^3*d^3) + (2*(b*C*d - 2*b*c*D - a*d*D)*(c + d*x)^(3/2))/(3*b^2*d^3
) + (2*D*(c + d*x)^(5/2))/(5*b*d^3) - (2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D
))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b^(7/2)*Sqrt[b*c - a
*d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{2d^3(b^3A-a\,b^2B+a^2bC-a^3D)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)+2\sqrt{(ad-bc)b}\left(\left(\left(\frac{1}{5}Dx^2+\frac{1}{3}Cx+B\right)b^2-\left(\frac{Dx}{3}+C\right)ab+Da^2\right)d^2-}{d^3b^3\sqrt{(ad-bc)b}}$
derivativedivides	$\frac{\frac{2\left(\frac{D(xd+c)^{\frac{5}{2}}b^2}{5}+\frac{C\,b^2d(xd+c)^{\frac{3}{2}}}{3}-\frac{Dabd(xd+c)^{\frac{3}{2}}}{3}-\frac{2Db^2c(xd+c)^{\frac{3}{2}}}{3}+B\,d^2b^2\sqrt{xd+c}-Ca\,d^2b\sqrt{xd+c}-C\,b^2cd\sqrt{xd+c}+Da^2d^2\sqrt{xd-}\right.}{b^3}}{d^3}$
default	$\frac{\frac{2\left(\frac{D(xd+c)^{\frac{5}{2}}b^2}{5}+\frac{C\,b^2d(xd+c)^{\frac{3}{2}}}{3}-\frac{Dabd(xd+c)^{\frac{3}{2}}}{3}-\frac{2Db^2c(xd+c)^{\frac{3}{2}}}{3}+B\,d^2b^2\sqrt{xd+c}-Ca\,d^2b\sqrt{xd+c}-C\,b^2cd\sqrt{xd+c}+Da^2d^2\sqrt{xd-}\right.}{b^3}}{d^3}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/((a*d-b*c)*b)^(1/2)*(d^3*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)*arctan(b*(d*x+c)^
(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*(((1/5*D*x^2+1/3*C*x+B)*b^2
-(1/3*D*x+C)*a*b+D*a^2)*d^2-2/3*((2/5*D*x+C)*b-D*a)*c*b*d+8/15*D*b^2*c^2)*
(d*x+c)^(1/2))/d^3/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx$$

$$= \left[-\frac{15\left(Da^3-Ca^2b+Bab^2-Ab^3\right)\sqrt{b^2c-abd}\,d^3\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)-2\left(8\,Db^4c^3+2\left(Dab\right.\right.}{}\right.$$

$$-\frac{2\left(15\left(Da^3-Ca^2b+Bab^2-Ab^3\right)\sqrt{-b^2c+abd}\,d^3\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)-\left(8\,Db^4c^3+2\left(Dab^3-5\right.\right.\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(b^2*c - a*b*d)*d^3*log
((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) -
2*(8*D*b^4*c^3 + 2*(D*a*b^3 - 5*C*b^4)*c^2*d + 5*(D*a^2*b^2 - C*a*b^3 + 3*
B*b^4)*c*d^2 - 15*(D*a^3*b - C*a^2*b^2 + B*a*b^3)*d^3 + 3*(D*b^4*c*d^2 - D
*a*b^3*d^3)*x^2 - (4*D*b^4*c^2*d + (D*a*b^3 - 5*C*b^4)*c*d^2 - 5*(D*a^2*b^
2 - C*a*b^3)*d^3)*x)*sqrt(d*x + c))/(b^5*c*d^3 - a*b^4*d^4), -2/15*(15*(D*
a^3 - C*a^2*b + B*a*b^2 - A*b^3)*sqrt(-b^2*c + a*b*d)*d^3*arctan(sqrt(-b^2
*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*D*b^4*c^3 + 2*(D*a*b^3 - 5*C
*b^4)*c^2*d + 5*(D*a^2*b^2 - C*a*b^3 + 3*B*b^4)*c*d^2 - 15*(D*a^3*b - C*a^
2*b^2 + B*a*b^3)*d^3 + 3*(D*b^4*c*d^2 - D*a*b^3*d^3)*x^2 - (4*D*b^4*c^2*d
+ (D*a*b^3 - 5*C*b^4)*c*d^2 - 5*(D*a^2*b^2 - C*a*b^3)*d^3)*x)*sqrt(d*x + c
))/(b^5*c*d^3 - a*b^4*d^4)]
```

Sympy [A] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(\frac{D(c+dx)^{\frac{5}{2}}}{5bd^2}+\frac{(c+dx)^{\frac{3}{2}}(Cbd-Dad-2Dbc)}{3b^2d^2}+\frac{\sqrt{c+dx}\left(Bb^2d^2-Cabd^2-Cb^2cd+Da^2d^2+Dabcd+Db^2c^2\right)}{b^3d^2}-\frac{d\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right.\cdots}{b^4\sqrt{\frac{ad-bc}{b}}}\right.}{d} \\ \frac{\frac{Dx^3}{3b}+\frac{x^2(Cb-Da)}{2b^2}+\frac{x\left(Bb^2-Cab+Da^2\right)}{b^3}-\frac{\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\left(\begin{cases}\frac{x}{a} & \text{for } b=0\\ \frac{\log(a+bx)}{b} & \text{otherwise}\end{cases}\right)}{b^3}}{\sqrt{c}} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Piecewise((2*(D*(c + d*x)**(5/2)/(5*b*d**2) + (c + d*x)**(3/2)*(C*b*d - D*
a*d - 2*D*b*c)/(3*b**2*d**2) + sqrt(c + d*x)*(B*b**2*d**2 - C*a*b*d**2 - C
*b**2*c*d + D*a**2*d**2 + D*a*b*c*d + D*b**2*c**2)/(b**3*d**2) - d*(-A*b**
3 + B*a*b**2 - C*a**2*b + D*a**3)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/
(b**4*sqrt((a*d - b*c)/b)))/d, Ne(d, 0)), ((D*x**3/(3*b) + x**2*(C*b - D*a
)/(2*b**2) + x*(B*b**2 - C*a*b + D*a**2)/b**3 - (-A*b**3 + B*a*b**2 - C*a*
*2*b + D*a**3)*Piecewise((x/a, Eq(b, 0)), (log(a + b*x)/b, True))/b**3)/sq
rt(c), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx = -\frac{2\left(Da^3-Ca^2b+Bab^2-Ab^3\right)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}b^3} + \frac{2\left(3\,(dx+c)^{\frac{5}{2}}Db^4d^{12}-10\,(dx+c)^{\frac{3}{2}}Db^4cd^{12}+15\sqrt{dx+c}Db^4c^2d^{12}-5\,(dx+c)^{\frac{3}{2}}Dab^3d^{13}+5\,(dx+\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-2*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/15*(3*(d*x + c)^(5/2)*D*b^4*d^12
- 10*(d*x + c)^(3/2)*D*b^4*c*d^12 + 15*sqrt(d*x + c)*D*b^4*c^2*d^12 - 5*(d
*x + c)^(3/2)*D*a*b^3*d^13 + 5*(d*x + c)^(3/2)*C*b^4*d^13 + 15*sqrt(d*x +
c)*D*a*b^3*c*d^13 - 15*sqrt(d*x + c)*C*b^4*c*d^13 + 15*sqrt(d*x + c)*D*a^2
*b^2*d^14 - 15*sqrt(d*x + c)*C*a*b^3*d^14 + 15*sqrt(d*x + c)*B*b^4*d^14)/(
b^5*d^15)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\sqrt{c+dx}}\,dx = \frac{-2\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^2d^2+2\sqrt{dx+c}\,a^2b\,d^2-\frac{2\sqrt{dx+c}\,a\,b^2cd}{3}-\frac{2\sqrt{dx+c}\,a\,b^2d^2x}{3}+2\sqrt{dx+c}\,b^4d-}{b^4d^2}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
(2*( - 15*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
 - b*c)))*a**2*d**2 + 15*sqrt(c + d*x)*a**2*b*d**2 - 5*sqrt(c + d*x)*a*b**
2*c*d - 5*sqrt(c + d*x)*a*b**2*d**2*x + 15*sqrt(c + d*x)*b**4*d - 2*sqrt(c
 + d*x)*b**3*c**2 + sqrt(c + d*x)*b**3*c*d*x + 3*sqrt(c + d*x)*b**3*d**2*x
**2))/(15*b**4*d**2)
```

3.84 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx$

Optimal result 768
Mathematica [A] (verified) 769
Rubi [A] (verified) 769
Maple [A] (verified) 772
Fricas [B] (verification not implemented) 772
Sympy [F(-1)] 773
Maxima [F(-2)] 774
Giac [A] (verification not implemented) 774
Mupad [F(-1)] 775
Reduce [B] (verification not implemented) 775

Optimal result

Integrand size = 32, antiderivative size = 205

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx &= \frac{2(bCd-bcD-2adD)\sqrt{c+dx}}{b^3d^2}\\ &\quad -\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b^3(bc-ad)(a+bx)}+\frac{2D(c+dx)^{3/2}}{3b^2d^2}\\ &\quad -\frac{(b^3(2Bc-Ad)-ab^2(4cC+Bd)-5a^3dD+3a^2b(Cd+2cD))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}(bc-ad)^{3/2}}\end{aligned}$$

output

```
2*(C*b*d-2*D*a*d-D*b*c)*(d*x+c)^(1/2)/b^3/d^2-(A*b^3-a*(B*b^2-C*a*b+D*a^2)
)*(d*x+c)^(1/2)/b^3/(-a*d+b*c)/(b*x+a)+2/3*D*(d*x+c)^(3/2)/b^2/d^2-(b^3*(-
A*d+2*B*c)-a*b^2*(B*d+4*C*c)-5*a^3*d*D+3*a^2*b*(C*d+2*D*c))*arctanh(b^(1/2
)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx =$$
$$-\frac{\sqrt{c+dx}(-15a^3d^2D+a^2bd(9Cd+8cD-10dDx)+b^3(3Ad^2-2cx(3Cd-2cD+dDx))+ab^2(4c^2D}{3b^3d^2(bc-ad)(a+bx)}$$
$$-\frac{(b^3(2Bc-Ad)-ab^2(4cC+Bd)-5a^3dD+3a^2b(Cd+2cD))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{7/2}(-bc+ad)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*Sqrt[c + d*x]),x]
```

output
```
-1/3*(Sqrt[c + d*x]*(-15*a^3*d^2*D + a^2*b*d*(9*C*d + 8*c*D - 10*d*D*x) +
b^3*(3*A*d^2 - 2*c*x*(3*C*d - 2*c*D + d*D*x)) + a*b^2*(4*c^2*D - 6*c*d*(C
- D*x) + d^2*(-3*B + 6*C*x + 2*D*x^2))))/(b^3*d^2*(b*c - a*d)*(a + b*x)) -
 ((b^3*(2*B*c - A*d) - a*b^2*(4*c*C + B*d) - 5*a^3*d*D + 3*a^2*b*(C*d + 2*
c*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(b^(7/2)*(-(b*c)
 + a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2124, 27, 1192, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{ 2124}$$

$$-\frac{\int -\frac{2\left(c-\frac{ad}{b}\right)Dx^2+\frac{2(bc-ad)(bC-aD)x}{b^2}+\frac{-dDa^3+b(Cd+2cD)a^2-b^2(2cC+Bd)a+b^3(2Bc-Ad)}{b^3}}{2(a+bx)\sqrt{c+dx}}dx}{bc-ad}-\frac{\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$\downarrow$ 27

$$\frac{\int \frac{-\frac{dDa^3}{b^3}+\frac{(Cd+2cD)a^2}{b^2}-\frac{(2cC+Bd)a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc-Ad+\frac{2(bc-ad)(bC-aD)x}{b^2}}{(a+bx)\sqrt{c+dx}}dx}{2(bc-ad)}-\frac{\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$\downarrow$ 1192

$$\frac{\int \frac{-2Dc^3+2Cdc^2-2Bd^2c-2\left(c-\frac{ad}{b}\right)D(c+dx)^2+\frac{d^3\left(Ab^3+a\left(Da^2-bCa+b^2B\right)\right)}{b^3}-\frac{2(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{bc-ad-b(c+dx)}d\sqrt{c+dx}}{d^2(bc-ad)}-\frac{\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$\downarrow$ 1467

$$\frac{\int \left(\frac{2(bc-ad)(bCd-2aDd-bcD)}{b^3}+\frac{2(bc-ad)D(c+dx)}{b^2}+\frac{Ad^3b^3-2Bcd^2b^3+aBd^3b^2+4acCd^2b^2-3a^2Cd^3b-6a^2cd^2Db+5a^3d^3D}{b^3(bc-ad-b(c+dx))}\right)d\sqrt{c+d}}{d^2(bc-ad)}$$
$$\frac{\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$\downarrow$ 2009

$$\frac{-\frac{d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-5a^3dD+3a^2b(2cD+Cd)-ab^2(Bd+4cC)+b^3(2Bc-Ad)\right)}{b^{7/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx}(bc-ad)(-2adD-bcD+bCd)}{b^3}+\frac{2D(c+dx)^3}{3b}}{d^2(bc-ad)}$$
$$\frac{\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*Sqrt[c + d*x]),x]
```

output
```
-(((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*Sqrt[c + d*x])/((b*c - a*d)*(a +
b*x))) + ((2*(b*c - a*d)*(b*C*d - b*c*D - 2*a*d*D)*Sqrt[c + d*x])/b^3 + (2
*(b*c - a*d)*D*(c + d*x)^(3/2))/(3*b^2) - (d^2*(b^3*(2*B*c - A*d) - a*b^2*
(4*c*C + B*d) - 5*a^3*d*D + 3*a^2*b*(C*d + 2*c*D))*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[b*c - a*d]])/(b^(7/2)*Sqrt[b*c - a*d]))/(d^2*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{2\left(\frac{D(xd+c)^{\frac{3}{2}}b}{3}+Cdb\sqrt{xd+c}-2Dad\sqrt{xd+c}-Dbc\sqrt{xd+c}\right)}{b^3}+\frac{2d^2\left(\frac{d\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\sqrt{xd+c}}{2(ad-bc)((xd+c)b+ad-bc)}+\frac{\left(b^3dA+Ba\,b^2d-2B\,b^3c-\right.}{}\right.}{b^3}}{d^2}$
default	$\frac{\frac{2\left(\frac{D(xd+c)^{\frac{3}{2}}b}{3}+Cdb\sqrt{xd+c}-2Dad\sqrt{xd+c}-Dbc\sqrt{xd+c}\right)}{b^3}+\frac{2d^2\left(\frac{d\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\sqrt{xd+c}}{2(ad-bc)((xd+c)b+ad-bc)}+\frac{\left(b^3dA+Ba\,b^2d-2B\,b^3c-\right.}{}\right.}{b^3}}{d^2}$
pseudoelliptic	$\frac{((Ad-2Bc)b^3+a\,b^2(Bd+4Cc)-3a^2b(Cd+2Dc)+5a^3dD)d^2(bx+a)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)+\left(\left(d^2A-2\left(\frac{Dx}{3}+C\right)xcd+\right.\right.}{\sqrt{(ad-bc)b}}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(1/b^3*(1/3*D*(d*x+c)^(3/2)*b+C*d*b*(d*x+c)^(1/2)-2*D*a*d*(d*x+c)^(1
/2)-D*b*c*(d*x+c)^(1/2))+d^2/b^3*(1/2*d*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(a*d
-b*c)*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+1/2*(A*b^3*d+B*a*b^2*d-2*B*b^3*c-3
*C*a^2*b*d+4*C*a*b^2*c+5*D*a^3*d-6*D*a^2*b*c)/(a*d-b*c)/((a*d-b*c)*b)^(1/2
)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(186) = 372$.

Time = 0.11 (sec) , antiderivative size = 969, normalized size of antiderivative = 4.73

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas
")
```

output
```
[1/6*(3*(2*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c*d^2 - (5*D*a^4 - 3*C*a^3*
b + B*a^2*b^2 + A*a*b^3)*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c*d^2
- (5*D*a^3*b - 3*C*a^2*b^2 + B*a*b^3 + A*b^4)*d^3)*x)*sqrt(b^2*c - a*b*d)*
log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a))
 - 2*(4*D*a*b^4*c^3 + 2*(2*D*a^2*b^3 - 3*C*a*b^4)*c^2*d - (23*D*a^3*b^2 -
15*C*a^2*b^3 + 3*B*a*b^4 - 3*A*b^5)*c*d^2 + 3*(5*D*a^4*b - 3*C*a^3*b^2 + B
*a^2*b^3 - A*a*b^4)*d^3 - 2*(D*b^5*c^2*d - 2*D*a*b^4*c*d^2 + D*a^2*b^3*d^3
)*x^2 + 2*(2*D*b^5*c^3 + (D*a*b^4 - 3*C*b^5)*c^2*d - 2*(4*D*a^2*b^3 - 3*C*
a*b^4)*c*d^2 + (5*D*a^3*b^2 - 3*C*a^2*b^3)*d^3)*x)*sqrt(d*x + c))/(a*b^6*c
^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 - 2*a*b^6*c*d^3 + a^
2*b^5*d^4)*x), 1/3*(3*(2*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c*d^2 - (5*D*
a^4 - 3*C*a^3*b + B*a^2*b^2 + A*a*b^3)*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 +
 B*b^4)*c*d^2 - (5*D*a^3*b - 3*C*a^2*b^2 + B*a*b^3 + A*b^4)*d^3)*x)*sqrt(-
b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) -
(4*D*a*b^4*c^3 + 2*(2*D*a^2*b^3 - 3*C*a*b^4)*c^2*d - (23*D*a^3*b^2 - 15*C*
a^2*b^3 + 3*B*a*b^4 - 3*A*b^5)*c*d^2 + 3*(5*D*a^4*b - 3*C*a^3*b^2 + B*a^2*
b^3 - A*a*b^4)*d^3 - 2*(D*b^5*c^2*d - 2*D*a*b^4*c*d^2 + D*a^2*b^3*d^3)*x^2
 + 2*(2*D*b^5*c^3 + (D*a*b^4 - 3*C*b^5)*c^2*d - 2*(4*D*a^2*b^3 - 3*C*a*b^4
)*c*d^2 + (5*D*a^3*b^2 - 3*C*a^2*b^3)*d^3)*x)*sqrt(d*x + c))/(a*b^6*c^2*d^
2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 - 2*a*b^6*c*d^3 + a^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx = \frac{(6\,Da^2bc-4\,Cab^2c+2\,Bb^3c-5\,Da^3d+3\,Ca^2bd-Bab^2d-Ab^3d)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^4c-ab^3d)\sqrt{-b^2c+abd}} + \frac{\sqrt{dx+c}Da^3d-\sqrt{dx+c}Ca^2bd+\sqrt{dx+c}Bab^2d-\sqrt{dx+c}Ab^3d}{(b^4c-ab^3d)((dx+c)b-bc+ad)} + \frac{2\left((dx+c)^{\frac{3}{2}}Db^4d^4-3\,\sqrt{dx+c}Db^4cd^4-6\,\sqrt{dx+c}Dab^3d^5+3\,\sqrt{dx+c}Cb^4d^5\right)}{3\,b^6d^6}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
(6*D*a^2*b*c - 4*C*a*b^2*c + 2*B*b^3*c - 5*D*a^3*d + 3*C*a^2*b*d - B*a*b^2
*d - A*b^3*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c - a*b^3
*d)*sqrt(-b^2*c + a*b*d)) + (sqrt(d*x + c)*D*a^3*d - sqrt(d*x + c)*C*a^2*b
*d + sqrt(d*x + c)*B*a*b^2*d - sqrt(d*x + c)*A*b^3*d)/((b^4*c - a*b^3*d)*(
(d*x + c)*b - b*c + a*d)) + 2/3*((d*x + c)^(3/2)*D*b^4*d^4 - 3*sqrt(d*x +
c)*D*b^4*c*d^4 - 6*sqrt(d*x + c)*D*a*b^3*d^5 + 3*sqrt(d*x + c)*C*b^4*d^5)/
(b^6*d^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(a+b\,x\right)^2\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2\sqrt{c+dx}}\,dx = \frac{15\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^3d^2-12\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^2bcd+15\sqrt{b}\,\sqrt{ad-bc}\,atan\left(\frac{}{\sqrt{}}\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2),x)
```

output
```
(15*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c
)))*a**3*d**2 - 12*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)
*sqrt(a*d - b*c)))*a**2*b*c*d + 15*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c +
d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b*d**2*x + 6*sqrt(b)*sqrt(a*d - b*
c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**3*d - 12*sqrt(b)
*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**2*
c*d*x + 6*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
 - b*c)))*b**4*d*x - 15*sqrt(c + d*x)*a**3*b*d**2 + 17*sqrt(c + d*x)*a**2*
b**2*c*d - 10*sqrt(c + d*x)*a**2*b**2*d**2*x - 2*sqrt(c + d*x)*a*b**3*c**2
 + 12*sqrt(c + d*x)*a*b**3*c*d*x + 2*sqrt(c + d*x)*a*b**3*d**2*x**2 - 2*sq
rt(c + d*x)*b**4*c**2*x - 2*sqrt(c + d*x)*b**4*c*d*x**2)/(3*b**4*d*(a**2*d
 - a*b*c + a*b*d*x - b**2*c*x))
```

3.85 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx$

Optimal result 777
Mathematica [A] (verified) 778
Rubi [A] (verified) 778
Maple [A] (verified) 782
Fricas [B] (verification not implemented) 783
Sympy [F(-1)] 784
Maxima [F(-2)] 784
Giac [B] (verification not implemented) 784
Mupad [F(-1)] 785
Reduce [B] (verification not implemented) 786

Optimal result

Integrand size = 32, antiderivative size = 279

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx=\frac{2D\sqrt{c+dx}}{b^3d}-\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{2b^3(bc-ad)(a+bx)^2}$$
$$-\frac{(b^3(4Bc-3Ad)-ab^2(8cC+Bd)-9a^3dD+a^2b(5Cd+12cD))\sqrt{c+dx}}{4b^3(bc-ad)^2(a+bx)}$$
$$-\frac{(b^3(8c^2C-4Bcd+3Ad^2)-15a^3d^2D+3a^2bd(Cd+12cD)-ab^2(8cCd-Bd^2+24c^2D))\operatorname{arctanh}\Big(\sqrt{\ }}{4b^{7/2}(bc-ad)^{5/2}}$$

output
```
2*D*(d*x+c)^(1/2)/b^3/d-1/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^
3/(-a*d+b*c)/(b*x+a)^2-1/4*(b^3*(-3*A*d+4*B*c)-a*b^2*(B*d+8*C*c)-9*a^3*d*D
+a^2*b*(5*C*d+12*D*c))*(d*x+c)^(1/2)/b^3/(-a*d+b*c)^2/(b*x+a)-1/4*(b^3*(3*
A*d^2-4*B*c*d+8*C*c^2)-15*a^3*d^2*D+3*a^2*b*d*(C*d+12*D*c)-a*b^2*(-B*d^2+8
*C*c*d+24*D*c^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/
(-a*d+b*c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx$$
$$= \frac{\frac{\sqrt{b}\sqrt{c+dx}(15a^4d^2D+Ab^3d(-2bc+5ad+3bdx)+4b^4cx(-Bd+2cDx)+a^3bd(-3Cd-26cD+25dDx)+ab^3(Bd(-2c+dx)+8cx(Cd+2cD-2dDx))}{d(bc-ad)^2(a+bx)^2}}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*Sqrt[c + d*x]),x]
```

output
```
((Sqrt[b]*Sqrt[c + d*x]*(15*a^4*d^2*D + A*b^3*d*(-2*b*c + 5*a*d + 3*b*d*x)
 + 4*b^4*c*x*(-(B*d) + 2*c*D*x) + a^3*b*d*(-3*C*d - 26*c*D + 25*d*D*x) + a
*b^3*(B*d*(-2*c + d*x) + 8*c*x*(C*d + 2*c*D - 2*d*D*x)) + a^2*b^2*(8*c^2*D
 + c*(6*C*d - 44*d*D*x) - d^2*(B + 5*C*x - 8*D*x^2))))/(d*(b*c - a*d)^2*(a
 + b*x)^2) + ((b^3*(8*c^2*C - 4*B*c*d + 3*A*d^2) - 15*a^3*d^2*D + 3*a^2*b*
d*(C*d + 12*c*D) + a*b^2*(-8*c*C*d + B*d^2 - 24*c^2*D))*ArcTan[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(5/2))/(4*b^(7/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2124, 27, 1192, 25, 1471, 25, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \; 2124$$
$$-\frac{\int -\frac{4\left(c-\frac{ad}{b}\right)Dx^2+\frac{4(bc-ad)(bC-aD)x}{b^2}+\frac{-dDa^3+b(Cd+4cD)a^2-b^2(4cC+Bd)a+b^3(4Bc-3Ad)}{b^3}}{2(a+bx)^2\sqrt{c+dx}}dx}{2(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 27

$$\frac{\int \frac{-\frac{dDa^3}{b^3}+\frac{(Cd+4cD)a^2}{b^2}-\frac{(4cC+Bd)a}{b}+4\left(c-\frac{ad}{b}\right)Dx^2+4Bc-3Ad+\frac{4(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^2\sqrt{c+dx}}dx}{4(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 1192

$$\frac{\int -\frac{-4Dc^3+4Cdc^2-4Bd^2c+3Ad^3-4\left(c-\frac{ad}{b}\right)D(c+dx)^2+\frac{ad^3\left(Da^2-bCa+b^2B\right)}{b^3}-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 25

$$-\frac{\int \frac{-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2+d^3\left(3A+\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 1471

$$\frac{\frac{\int -\frac{\left(-8Dc^3+8Cdc^2-\frac{4d^2\left(-3Da^2+2bCa+b^2B\right)c}{b^2}+3Ad^3+\frac{ad^3\left(-7Da^2+3bCa+b^2B\right)}{b^3}\right)b^2+8(bc-ad)^2D(c+dx)}{b^2(bc-ad-b(c+dx))}d\sqrt{c+dx}}{2(bc-ad)}+\frac{d^2\sqrt{c+dx}(-9a^3dD+a^2b(12c\ldots}{2b^3(bc-a\ldots}}{2d(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 25

$$\frac{\frac{d^2\sqrt{c+dx}(-9a^3dD+a^2b(12cD+5Cd)-ab^2(Bd+8cC)+b^3(4Bc-3Ad))}{2b^3(bc-ad)(-ad-b(c+dx)+bc)}-\frac{\int \frac{\left(-8Dc^3+8Cdc^2-\frac{4d^2\left(-3Da^2+2bCa+b^2B\right)c}{b^2}+d^3\left(3A+\frac{a\left(-7Da^2+3bC\ldots\right.}{b^3}\ldots\right.}{b^2(bc-ad-b(c+dx))}\ldots}{2(bc-ad)}}{2d(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{d^2\sqrt{c+dx}\left(-9a^3dD+a^2b(12cD+5Cd)-ab^2(Bd+8cC)+b^3(4Bc-3Ad)\right)}{2b^3(bc-ad)(-ad-b(c+dx)+bc)}-\frac{\int\frac{\left(-8Dc^3+8Cdc^2-\frac{4d^2\left(-3Da^2+2bCa+b^2B\right)c}{b^2}+d^3\left(3A+\frac{a\left(-7Da^2+3bC\cdots\right.}{b^3}\right.\right.}{bc-ad-b(c+dx)}\cdots}{2b^2(bc-ad)}}{2d(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{d^2\sqrt{c+dx}\left(-9a^3dD+a^2b(12cD+5Cd)-ab^2(Bd+8cC)+b^3(4Bc-3Ad)\right)}{2b^3(bc-ad)(-ad-b(c+dx)+bc)}-\frac{\frac{d\left(-15a^3d^2D+3a^2bd(12cD+Cd)-ab^2\left(-Bd^2+24c^2D+8cCd\right)+b^3\left(3Ad^2-4\cdots\right.\right.}{b}\cdots}{2b^2(bc-ad)}}{2d(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{d^2\sqrt{c+dx}\left(-9a^3dD+a^2b(12cD+5Cd)-ab^2(Bd+8cC)+b^3(4Bc-3Ad)\right)}{2b^3(bc-ad)(-ad-b(c+dx)+bc)}-\frac{\frac{d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-15a^3d^2D+3a^2bd(12cD+Cd)-ab^2\left(-Bd^2+24c^2\cdots\right.\right.}{b^{3/2}\sqrt{bc-ad}}\cdots}{2b^2(bc-ad)}}{2d(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*Sqrt[c + d*x]),x]
```

output

```
-1/2*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[c + d*x])/(b^3*(b*c - a*d)*
(a + b*x)^2) + ((d^2*(b^3*(4*B*c - 3*A*d) - a*b^2*(8*c*C + B*d) - 9*a^3*d*
D + a^2*b*(5*C*d + 12*c*D))*Sqrt[c + d*x])/(2*b^3*(b*c - a*d)*(b*c - a*d -
 b*(c + d*x))) - ((-8*(b*c - a*d)^2*D*Sqrt[c + d*x])/b + (d*(b^3*(8*c^2*C
- 4*B*c*d + 3*A*d^2) - 15*a^3*d^2*D + 3*a^2*b*d*(C*d + 12*c*D) - a*b^2*(8*
c*C*d - B*d^2 + 24*c^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]
])/(b^(3/2)*Sqrt[b*c - a*d]))/(2*b^2*(b*c - a*d)))/(2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{3\left(\left(d^{2}A-\frac{4}{3}cdB+\frac{8}{3}Cc^{2}\right)b^{3}+\frac{a\left(Bd^{2}-8Ccd-24Dc^{2}\right)b^{2}}{3}+a^{2}bd(Cd+12Dc)-5a^{3}d^{2}D\right)(bx+a)^{2}d\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{4}+\frac{5\left(\frac{\left(3A d^{2} \cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{2D\sqrt{xd+c}}{b^{3}}+\frac{2d\left(\frac{\frac{bd\left(3b^{3}dA+Bab^{2}d-4Bb^{3}c-5Ca^{2}bd+8ab^{2}cC+9a^{3}dD-12a^{2}bcD\right)(xd+c)^{\frac{3}{2}}}{8a^{2}d^{2}-16abcd+8b^{2}c^{2}}+\frac{\left(5b^{3}dA-Bab^{2}d-4Bb^{3}c-3Ca^{2}b\cdots\right.}{8a\cdots}}{((xd+c)b+ad-bc)^{2}}\cdots\right.}{\cdots}$
default	$\frac{2D\sqrt{xd+c}}{b^{3}}+\frac{2d\left(\frac{\frac{bd\left(3b^{3}dA+Bab^{2}d-4Bb^{3}c-5Ca^{2}bd+8ab^{2}cC+9a^{3}dD-12a^{2}bcD\right)(xd+c)^{\frac{3}{2}}}{8a^{2}d^{2}-16abcd+8b^{2}c^{2}}+\frac{\left(5b^{3}dA-Bab^{2}d-4Bb^{3}c-3Ca^{2}b\cdots\right.}{8a\cdots}}{((xd+c)b+ad-bc)^{2}}\cdots\right.}{\cdots}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/4*(3/5*((d^2*A-4/3*c*d*B+8/3*C*c^2)*b^3+1/3*a*(B*d^2-8*C*c*d-24*D*c^2)*b
^2+a^2*b*d*(C*d+12*D*c)-5*a^3*d^2*D)*(b*x+a)^2*d*arctan(b*(d*x+c)^(1/2)/((
a*d-b*c)*b)^(1/2))+(1/5*(3*A*d^2*x-2*c*(2*B*x+A)*d+8*D*c^2*x^2)*b^4+((1/5*
B*x+A)*d^2-2/5*c*(8*D*x^2-4*C*x+B)*d+16/5*D*c^2*x)*a*b^3-1/5*((-8*D*x^2+5*
C*x+B)*d^2-6*(-22/3*D*x+C)*c*d-8*D*c^2)*a^2*b^2-3/5*d*((-25/3*D*x+C)*d+26/
3*D*c)*a^3*b+3*D*a^4*d^2)*((a*d-b*c)*b)^(1/2)*(d*x+c)^(1/2))/(a*d-b*c)^2/(
(a*d-b*c)*b)^(1/2)/b^3/(b*x+a)^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(258) = 516$.

Time = 0.13 (sec) , antiderivative size = 1577, normalized size of antiderivative = 5.65

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 \sqrt{c + dx}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/8*((8*(3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - 4*(9*D*a^4*b - 2*C*a^3*b^2 - B
*a^2*b^3)*c*d^2 + (15*D*a^5 - 3*C*a^4*b - B*a^3*b^2 - 3*A*a^2*b^3)*d^3 + (
8*(3*D*a*b^4 - C*b^5)*c^2*d - 4*(9*D*a^2*b^3 - 2*C*a*b^4 - B*b^5)*c*d^2 +
(15*D*a^3*b^2 - 3*C*a^2*b^3 - B*a*b^4 - 3*A*b^5)*d^3)*x^2 + 2*(8*(3*D*a^2*
b^3 - C*a*b^4)*c^2*d - 4*(9*D*a^3*b^2 - 2*C*a^2*b^3 - B*a*b^4)*c*d^2 + (15
*D*a^4*b - 3*C*a^3*b^2 - B*a^2*b^3 - 3*A*a*b^4)*d^3)*x)*sqrt(b^2*c - a*b*d
)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a
)) - 2*(8*D*a^2*b^4*c^3 - 2*(17*D*a^3*b^3 - 3*C*a^2*b^4 + B*a*b^5 + A*b^6)
*c^2*d + (41*D*a^4*b^2 - 9*C*a^3*b^3 + B*a^2*b^4 + 7*A*a*b^5)*c*d^2 - (15*
D*a^5*b - 3*C*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4)*d^3 + 8*(D*b^6*c^3 - 3*D*
a*b^5*c^2*d + 3*D*a^2*b^4*c*d^2 - D*a^3*b^3*d^3)*x^2 + (16*D*a*b^5*c^3 - 4
*(15*D*a^2*b^4 - 2*C*a*b^5 + B*b^6)*c^2*d + (69*D*a^3*b^3 - 13*C*a^2*b^4 +
 5*B*a*b^5 + 3*A*b^6)*c*d^2 - (25*D*a^4*b^2 - 5*C*a^3*b^3 + B*a^2*b^4 + 3*
A*a*b^5)*d^3)*x)*sqrt(d*x + c))/(a^2*b^7*c^3*d - 3*a^3*b^6*c^2*d^2 + 3*a^4
*b^5*c*d^3 - a^5*b^4*d^4 + (b^9*c^3*d - 3*a*b^8*c^2*d^2 + 3*a^2*b^7*c*d^3
- a^3*b^6*d^4)*x^2 + 2*(a*b^8*c^3*d - 3*a^2*b^7*c^2*d^2 + 3*a^3*b^6*c*d^3
- a^4*b^5*d^4)*x), -1/4*((8*(3*D*a^3*b^2 - C*a^2*b^3)*c^2*d - 4*(9*D*a^4*b
 - 2*C*a^3*b^2 - B*a^2*b^3)*c*d^2 + (15*D*a^5 - 3*C*a^4*b - B*a^3*b^2 - 3*
A*a^2*b^3)*d^3 + (8*(3*D*a*b^4 - C*b^5)*c^2*d - 4*(9*D*a^2*b^3 - 2*C*a*b^4
 - B*b^5)*c*d^2 + (15*D*a^3*b^2 - 3*C*a^2*b^3 - B*a*b^4 - 3*A*b^5)*d^3)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(258) = 516$.

Time = 0.14 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx =$$
$$-\frac{(24\,Dab^2c^2-8\,Cb^3c^2-36\,Da^2bcd+8\,Cab^2cd+4\,Bb^3cd+15\,Da^3d^2-3\,Ca^2bd^2-Bab^2d^2-3\,Ab^3d^2}{4\,(b^5c^2-2\,ab^4cd+a^2b^3d^2)\sqrt{-b^2c+abd}}$$
$$-\frac{12\,(dx+c)^{\frac{3}{2}}Da^2b^2cd-8\,(dx+c)^{\frac{3}{2}}Cab^3cd+4\,(dx+c)^{\frac{3}{2}}Bb^4cd-12\,\sqrt{dx+c}Da^2b^2c^2d+8\,\sqrt{dx+c}C}{}$$
$$+\frac{2\,\sqrt{dx+c}D}{b^3d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(24*D*a*b^2*c^2 - 8*C*b^3*c^2 - 36*D*a^2*b*c*d + 8*C*a*b^2*c*d + 4*B*
b^3*c*d + 15*D*a^3*d^2 - 3*C*a^2*b*d^2 - B*a*b^2*d^2 - 3*A*b^3*d^2)*arctan
(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d
^2)*sqrt(-b^2*c + a*b*d)) - 1/4*(12*(d*x + c)^(3/2)*D*a^2*b^2*c*d - 8*(d*x
 + c)^(3/2)*C*a*b^3*c*d + 4*(d*x + c)^(3/2)*B*b^4*c*d - 12*sqrt(d*x + c)*D
*a^2*b^2*c^2*d + 8*sqrt(d*x + c)*C*a*b^3*c^2*d - 4*sqrt(d*x + c)*B*b^4*c^2
*d - 9*(d*x + c)^(3/2)*D*a^3*b*d^2 + 5*(d*x + c)^(3/2)*C*a^2*b^2*d^2 - (d*
x + c)^(3/2)*B*a*b^3*d^2 - 3*(d*x + c)^(3/2)*A*b^4*d^2 + 19*sqrt(d*x + c)*
D*a^3*b*c*d^2 - 11*sqrt(d*x + c)*C*a^2*b^2*c*d^2 + 3*sqrt(d*x + c)*B*a*b^3
*c*d^2 + 5*sqrt(d*x + c)*A*b^4*c*d^2 - 7*sqrt(d*x + c)*D*a^4*d^3 + 3*sqrt(
d*x + c)*C*a^3*b*d^3 + sqrt(d*x + c)*B*a^2*b^2*d^3 - 5*sqrt(d*x + c)*A*a*b
^3*d^3)/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*((d*x + c)*b - b*c + a*d)^2
) + 2*sqrt(d*x + c)*D/(b^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^3\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 847, normalized size of antiderivative = 3.04

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d -
b*c)))*a**4*d**2 + 24*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt
(b)*sqrt(a*d - b*c)))*a**3*b*c*d - 30*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c
 + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*d**2*x + 4*sqrt(b)*sqrt(a*d -
 b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*d - 8*sq
rt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*
*2*b**2*c**2 + 48*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*
sqrt(a*d - b*c)))*a**2*b**2*c*d*x - 15*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(
c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*d**2*x**2 + 8*sqrt(b)*sqr
t(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*d*x
- 16*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*
c)))*a*b**3*c**2*x + 24*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sq
rt(b)*sqrt(a*d - b*c)))*a*b**3*c*d*x**2 + 4*sqrt(b)*sqrt(a*d - b*c)*atan((
sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**5*d*x**2 - 8*sqrt(b)*sqrt(a
*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**4*c**2*x**2
 + 15*sqrt(c + d*x)*a**4*b*d**2 - 29*sqrt(c + d*x)*a**3*b**2*c*d + 25*sqrt
(c + d*x)*a**3*b**2*d**2*x + 4*sqrt(c + d*x)*a**2*b**4*d + 14*sqrt(c + d*x
)*a**2*b**3*c**2 - 49*sqrt(c + d*x)*a**2*b**3*c*d*x + 8*sqrt(c + d*x)*a**2
*b**3*d**2*x**2 - 4*sqrt(c + d*x)*a*b**5*c + 4*sqrt(c + d*x)*a*b**5*d*x +
24*sqrt(c + d*x)*a*b**4*c**2*x - 16*sqrt(c + d*x)*a*b**4*c*d*x**2 - 4*s...
```

3.86 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4\sqrt{c+dx}}\,dx$

Optimal result 787
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Optimal result

Integrand size = 32, antiderivative size = 375

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4\sqrt{c+dx}}\,dx &= -\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{3b^3(bc-ad)(a+bx)^3}\\
&-\frac{(b^3(6Bc-5Ad)-ab^2(12cC+Bd)-13a^3dD+a^2b(7Cd+18cD))\sqrt{c+dx}}{12b^3(bc-ad)^2(a+bx)^2}\\
&-\frac{(b^3(8c^2C-6Bcd+5Ad^2)-11a^3d^2D+a^2bd(Cd+30cD)-ab^2(4cCd-Bd^2+24c^2D))\sqrt{c+dx}}{8b^3(bc-ad)^3(a+bx)}\\
&+\frac{(5a^3d^3D+a^2bd^2(Cd-18cD)-ab^2d(4cCd-Bd^2-24c^2D)+b^3(8c^2Cd-6Bcd^2+5Ad^3-16c^3D))}{8b^{7/2}(bc-ad)^{7/2}}\end{aligned}$$

output
```
-1/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^3/(-a*d+b*c)/(b*x+a)^3-
1/12*(b^3*(-5*A*d+6*B*c)-a*b^2*(B*d+12*C*c)-13*a^3*d*D+a^2*b*(7*C*d+18*D*c
))*(d*x+c)^(1/2)/b^3/(-a*d+b*c)^2/(b*x+a)^2-1/8*(b^3*(5*A*d^2-6*B*c*d+8*C*
c^2)-11*a^3*d^2*D+a^2*b*d*(C*d+30*D*c)-a*b^2*(-B*d^2+4*C*c*d+24*D*c^2))*(d
*x+c)^(1/2)/b^3/(-a*d+b*c)^3/(b*x+a)+1/8*(5*a^3*d^3*D+a^2*b*d^2*(C*d-18*D*
c)-a*b^2*d*(-B*d^2+4*C*c*d-24*D*c^2)+b^3*(5*A*d^3-6*B*c*d^2+8*C*c^2*d-16*D
*c^3))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)/(-a*d+b*c)^
(7/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4\sqrt{c+dx}}\,dx =$$
$$-\frac{\sqrt{c+dx}(-15a^5d^2D+6b^5cx(2Bc+4cCx-3Bdx)+a^4bd(-3Cd+44cD-40dDx)+ab^4(-12cx(-$$

$$-\frac{(-5a^3d^3D+a^2bd^2(-Cd+18cD)-ab^2d(-4cCd+Bd^2+24c^2D)+b^3(-8c^2Cd+6Bcd^2-5Ad^3+1}{8b^{7/2}(-bc+ad)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^4*Sqrt[c + d*x]),x]
```

output
```
-1/24*(Sqrt[c + d*x]*(-15*a^5*d^2*D + 6*b^5*c*x*(2*B*c + 4*c*C*x - 3*B*d*x
) + a^4*b*d*(-3*C*d + 44*c*D - 40*d*D*x) + a*b^4*(-12*c*x*(-2*c*C + C*d*x
+ 6*c*D*x) + B*(4*c^2 - 50*c*d*x + 3*d^2*x^2)) + A*b^3*(33*a^2*d^2 + 2*a*b
*d*(-13*c + 20*d*x) + b^2*(8*c^2 - 10*c*d*x + 15*d^2*x^2)) - a^3*b^2*(44*c
^2*D - 2*c*d*(5*C + 59*D*x) + d^2*(3*B + 8*C*x + 33*D*x^2)) + a^2*b^3*(d^2
*x*(8*B + 3*C*x) + 4*c^2*(2*C - 27*D*x) + 2*c*d*(-8*B + 7*C*x + 45*D*x^2))
))/(b^3*(b*c - a*d)^3*(a + b*x)^3) - ((-5*a^3*d^3*D + a^2*b*d^2*(-(C*d) +
18*c*D) - a*b^2*d*(-4*c*C*d + B*d^2 + 24*c^2*D) + b^3*(-8*c^2*C*d + 6*B*c*
d^2 - 5*A*d^3 + 16*c^3*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*
d]])/(8*b^(7/2)*(-(b*c) + a*d)^(7/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1192, 1471, 27, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{6\left(c-\frac{ad}{b}\right)Dx^2+\frac{6(bc-ad)(bC-aD)x}{b^2}+\frac{-dDa^3+b(Cd+6cD)a^2-b^2(6cC+Bd)a+b^3(6Bc-5Ad)}{b^3}}{2(a+bx)^3\sqrt{c+dx}}dx}{3(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-\frac{dDa^3}{b^3}+\frac{(Cd+6cD)a^2}{b^2}-\frac{(6cC+Bd)a}{b}+6\left(c-\frac{ad}{b}\right)Dx^2+6Bc-5Ad+\frac{6(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^3\sqrt{c+dx}}dx}{6(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 1192$$

$$\frac{\int \frac{-6Dc^3+6Cdc^2-6Bd^2c+5Ad^3-6\left(c-\frac{ad}{b}\right)D(c+dx)^2+\frac{ad^3\left(Da^2-bCa+b^2B\right)}{b^3}-\frac{6(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(bc-ad-b(c+dx))^3}d\sqrt{c+dx}}{3(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 1471$$

$$\frac{-\frac{\int \frac{3\left(-\left(\left(-8Dc^3+8Cdc^2-6Bd^2c+5Ad^3\right)b^3\right)+ad^2(4cC-Bd)b^2-a^2d^2(Cd+6cD)b-8(bc-ad)^2D(c+dx)b+3a^3d^3D\right)}{b^3(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{4(bc-ad)}-\frac{d^2\sqrt{c+dx}\left(-13a^3dD+a^2b\cdots\right.}{4b^3(b\cdots}}{3(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{3\int \frac{-\left(\left(-8Dc^3+8Cdc^2-6Bd^2c+5Ad^3\right)b^3\right)+ad^2(4cC-Bd)b^2-a^2d^2(Cd+6cD)b-8(bc-ad)^2D(c+dx)b+3a^3d^3D}{(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{4b^3(bc-ad)}-\frac{d^2\sqrt{c+dx}\left(-13a^3dD+a^2b(\cdots\right.}{4b^3(bc\cdots}}{3(bc-ad)}-\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \; 298$$

$$-\frac{3\left(-\frac{\left(5a^3d^3D+a^2bd^2(Cd-18cD)-ab^2d\left(-Bd^2-24c^2D+4cCd\right)+b^3\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{1}{bc-ad-b(c+dx)}d\sqrt{c+dx}}{2(bc-ad)}-\frac{d\sqrt{c+dx}\left(-11a^3d^2D+a^2\cdots\right.}{\cdots}\cdots\right)}{4b^3(bc-ad)} \over 3(bc-\cdots$$

$$\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 221

$$-\frac{3\left(-\frac{\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(5a^3d^3D+a^2bd^2(Cd-18cD)-ab^2d\left(-Bd^2-24c^2D+4cCd\right)+b^3\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)}{2\sqrt{b}(bc-ad)^{3/2}}-\frac{d\sqrt{c+dx}\left(-11a^3d^2D+a^2bd\cdots\right.}{\cdots}\cdots\right)}{4b^3(bc-ad)} \over 3(bc-\cdots$$

$$\frac{\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^4*Sqrt[c + d*x]),x]
```

output
```
-1/3*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[c + d*x])/(b^3*(b*c - a*d)*
(a + b*x)^3) + (-1/4*(d^2*(b^3*(6*B*c - 5*A*d) - a*b^2*(12*c*C + B*d) - 13
*a^3*d*D + a^2*b*(7*C*d + 18*c*D))*Sqrt[c + d*x])/(b^3*(b*c - a*d)*(b*c -
a*d - b*(c + d*x))^2) - (3*(-1/2*(d*(b^3*(8*c^2*C - 6*B*c*d + 5*A*d^2) - 1
1*a^3*d^2*D + a^2*b*d*(C*d + 30*c*D) - a*b^2*(4*c*C*d - B*d^2 + 24*c^2*D))
*Sqrt[c + d*x])/((b*c - a*d)*(b*c - a*d - b*(c + d*x))) - ((5*a^3*d^3*D +
a^2*b*d^2*(C*d - 18*c*D) - a*b^2*d*(4*c*C*d - B*d^2 - 24*c^2*D) + b^3*(8*c
^2*C*d - 6*B*c*d^2 + 5*A*d^3 - 16*c^3*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[b*c - a*d]])/(2*Sqrt[b]*(b*c - a*d)^(3/2))))/(4*b^3*(b*c - a*d)))/(3*
(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{5\left(\left(A\,d^{3}-\frac{6}{5}B c\,d^{2}+\frac{8}{5}C\,c^{2}d-\frac{16}{5}D c^{3}\right)b^{3}+\frac{a d\left(B\,d^{2}-4C c d+24D c^{2}\right)b^{2}}{5}+\frac{a^{2}b\,d^{2}\left(C d-18D c\right)}{5}+a^{3}d^{3}D\right)\left(b x+a\right)^{3}\arctan\left(\frac{b\sqrt{x d+c}}{\sqrt{\left(a d-b c\right)b}}\right.}{8}$
derivativedivides	$\frac{d\left(5b^{3}d^{2}A+B a\,b^{2}d^{2}-6B\,b^{3}c d+C\,a^{2}b\,d^{2}-4C a\,b^{2}c d+8C\,b^{3}c^{2}-11a^{3}d^{2}D+30a^{2}b c d D-24a\,b^{2}c^{2}D\right)\left(x d+c\right)^{\frac{5}{2}}}{8b\left(a^{3}d^{3}-3a^{2}b c\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{\left(5b^{3}d^{2}A+B a\,b^{2}d^{2}-6\right.}{}$
default	$\frac{d\left(5b^{3}d^{2}A+B a\,b^{2}d^{2}-6B\,b^{3}c d+C\,a^{2}b\,d^{2}-4C a\,b^{2}c d+8C\,b^{3}c^{2}-11a^{3}d^{2}D+30a^{2}b c d D-24a\,b^{2}c^{2}D\right)\left(x d+c\right)^{\frac{5}{2}}}{8b\left(a^{3}d^{3}-3a^{2}b c\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}+\frac{\left(5b^{3}d^{2}A+B a\,b^{2}d^{2}-6\right.}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
11/8*(5/11*((A*d^3-6/5*B*c*d^2+8/5*C*c^2*d-16/5*D*c^3)*b^3+1/5*a*d*(B*d^2-
4*C*c*d+24*D*c^2)*b^2+1/5*a^2*b*d^2*(C*d-18*D*c)+a^3*d^3*D)*(b*x+a)^3*arct
an(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+((a*d-b*c)*b)^(1/2)*((5/11*A*d^2*x
^2-10/33*x*(9/5*B*x+A)*c*d+8/33*c^2*(3*C*x^2+3/2*B*x+A))*b^5-26/33*(-20/13
*x*(3/40*B*x+A)*d^2+c*(6/13*C*x^2+25/13*B*x+A)*d-2/13*c^2*(-18*D*x^2+6*C*x
+B))*a*b^4+((1/11*C*x^2+8/33*B*x+A)*d^2-16/33*(-45/8*D*x^2-7/8*C*x+B)*c*d+
8/33*(-27/2*D*x+C)*c^2)*a^2*b^3-1/11*((11*D*x^2+8/3*C*x+B)*d^2-10/3*c*(59/
5*D*x+C)*d+44/3*D*c^2)*a^3*b^2-1/11*d*((40/3*D*x+C)*d-44/3*D*c)*a^4*b-5/11
*D*a^5*d^2)*(d*x+c)^(1/2))/((a*d-b*c)*b)^(1/2)/(b*x+a)^3/(a*d-b*c)^3/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. 2(354) = 708.

Time = 0.17 (sec) , antiderivative size = 2338, normalized size of antiderivative = 6.23

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas
")
```

output
```
[1/48*(3*(16*D*a^3*b^3*c^3 - 8*(3*D*a^4*b^2 + C*a^3*b^3)*c^2*d + 2*(9*D*a^
5*b + 2*C*a^4*b^2 + 3*B*a^3*b^3)*c*d^2 - (5*D*a^6 + C*a^5*b + B*a^4*b^2 +
5*A*a^3*b^3)*d^3 + (16*D*b^6*c^3 - 8*(3*D*a*b^5 + C*b^6)*c^2*d + 2*(9*D*a^
2*b^4 + 2*C*a*b^5 + 3*B*b^6)*c*d^2 - (5*D*a^3*b^3 + C*a^2*b^4 + B*a*b^5 +
5*A*b^6)*d^3)*x^3 + 3*(16*D*a*b^5*c^3 - 8*(3*D*a^2*b^4 + C*a*b^5)*c^2*d +
2*(9*D*a^3*b^3 + 2*C*a^2*b^4 + 3*B*a*b^5)*c*d^2 - (5*D*a^4*b^2 + C*a^3*b^3
 + B*a^2*b^4 + 5*A*a*b^5)*d^3)*x^2 + 3*(16*D*a^2*b^4*c^3 - 8*(3*D*a^3*b^3
+ C*a^2*b^4)*c^2*d + 2*(9*D*a^4*b^2 + 2*C*a^3*b^3 + 3*B*a^2*b^4)*c*d^2 - (
5*D*a^5*b + C*a^4*b^2 + B*a^3*b^3 + 5*A*a^2*b^4)*d^3)*x)*sqrt(b^2*c - a*b*
d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x +
a)) + 2*(4*(11*D*a^3*b^4 - 2*C*a^2*b^5 - B*a*b^6 - 2*A*b^7)*c^3 - 2*(44*D*
a^4*b^3 + C*a^3*b^4 - 10*B*a^2*b^5 - 17*A*a*b^6)*c^2*d + (59*D*a^5*b^2 + 1
3*C*a^4*b^3 - 13*B*a^3*b^4 - 59*A*a^2*b^5)*c*d^2 - 3*(5*D*a^6*b + C*a^5*b^
2 + B*a^4*b^3 - 11*A*a^3*b^4)*d^3 + 3*(8*(3*D*a*b^6 - C*b^7)*c^3 - 6*(9*D*
a^2*b^5 - 2*C*a*b^6 - B*b^7)*c^2*d + (41*D*a^3*b^4 - 5*C*a^2*b^5 - 7*B*a*b
^6 - 5*A*b^7)*c*d^2 - (11*D*a^4*b^3 - C*a^3*b^4 - B*a^2*b^5 - 5*A*a*b^6)*d
^3)*x^2 + 2*(6*(9*D*a^2*b^5 - 2*C*a*b^6 - B*b^7)*c^3 - (113*D*a^3*b^4 - 5*
C*a^2*b^5 - 31*B*a*b^6 - 5*A*b^7)*c^2*d + (79*D*a^4*b^3 + 11*C*a^3*b^4 - 2
9*B*a^2*b^5 - 25*A*a*b^6)*c*d^2 - 4*(5*D*a^5*b^2 + C*a^4*b^3 - B*a^3*b^4 -
 5*A*a^2*b^5)*d^3)*x)*sqrt(d*x + c))/(a^3*b^8*c^4 - 4*a^4*b^7*c^3*d + 6...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^4 \sqrt{c + dx}}\, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**4/(d*x+c)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^4\sqrt{c + dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. 2(354) = 708.

Time = 0.14 (sec) , antiderivative size = 976, normalized size of antiderivative = 2.60

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^4\sqrt{c + dx}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*(16*D*b^3*c^3 - 24*D*a*b^2*c^2*d - 8*C*b^3*c^2*d + 18*D*a^2*b*c*d^2 +
4*C*a*b^2*c*d^2 + 6*B*b^3*c*d^2 - 5*D*a^3*d^3 - C*a^2*b*d^3 - B*a*b^2*d^3
- 5*A*b^3*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^6*c^3 - 3*
a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*sqrt(-b^2*c + a*b*d)) + 1/24*
(72*(d*x + c)^(5/2)*D*a*b^4*c^2*d - 24*(d*x + c)^(5/2)*C*b^5*c^2*d - 144*(
d*x + c)^(3/2)*D*a*b^4*c^3*d + 48*(d*x + c)^(3/2)*C*b^5*c^3*d + 72*sqrt(d*
x + c)*D*a*b^4*c^4*d - 24*sqrt(d*x + c)*C*b^5*c^4*d - 90*(d*x + c)^(5/2)*D
*a^2*b^3*c*d^2 + 12*(d*x + c)^(5/2)*C*a*b^4*c*d^2 + 18*(d*x + c)^(5/2)*B*b
^5*c*d^2 + 288*(d*x + c)^(3/2)*D*a^2*b^3*c^2*d^2 - 48*(d*x + c)^(3/2)*C*a*
b^4*c^2*d^2 - 48*(d*x + c)^(3/2)*B*b^5*c^2*d^2 - 198*sqrt(d*x + c)*D*a^2*b
^3*c^3*d^2 + 36*sqrt(d*x + c)*C*a*b^4*c^3*d^2 + 30*sqrt(d*x + c)*B*b^5*c^3
*d^2 + 33*(d*x + c)^(5/2)*D*a^3*b^2*d^3 - 3*(d*x + c)^(5/2)*C*a^2*b^3*d^3
- 3*(d*x + c)^(5/2)*B*a*b^4*d^3 - 15*(d*x + c)^(5/2)*A*b^5*d^3 - 184*(d*x
+ c)^(3/2)*D*a^3*b^2*c*d^3 - 8*(d*x + c)^(3/2)*C*a^2*b^3*c*d^3 + 56*(d*x +
 c)^(3/2)*B*a*b^4*c*d^3 + 40*(d*x + c)^(3/2)*A*b^5*c*d^3 + 195*sqrt(d*x +
c)*D*a^3*b^2*c^2*d^3 + 3*sqrt(d*x + c)*C*a^2*b^3*c^2*d^3 - 57*sqrt(d*x + c
)*B*a*b^4*c^2*d^3 - 33*sqrt(d*x + c)*A*b^5*c^2*d^3 + 40*(d*x + c)^(3/2)*D*
a^4*b*d^4 + 8*(d*x + c)^(3/2)*C*a^3*b^2*d^4 - 8*(d*x + c)^(3/2)*B*a^2*b^3*
d^4 - 40*(d*x + c)^(3/2)*A*a*b^4*d^4 - 84*sqrt(d*x + c)*D*a^4*b*c*d^4 - 18
*sqrt(d*x + c)*C*a^3*b^2*c*d^4 + 24*sqrt(d*x + c)*B*a^2*b^3*c*d^4 + 66*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4\sqrt{c+dx}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^4\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^4*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^4*(c + d*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1305, normalized size of antiderivative = 3.48

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^4\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2),x)
```

output
```
(15*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c
)))*a**5*d**3 - 36*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)
*sqrt(a*d - b*c)))*a**4*b*c*d**2 + 45*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c
 + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**4*b*d**3*x + 18*sqrt(b)*sqrt(a*d
- b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**3*d**2 +
24*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)
))*a**3*b**2*c**2*d - 108*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(
sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*c*d**2*x + 45*sqrt(b)*sqrt(a*d - b*c)*
atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b**2*d**3*x**2 + 54
*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))
*a**2*b**4*d**2*x + 72*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqr
t(b)*sqrt(a*d - b*c)))*a**2*b**3*c**2*d*x - 108*sqrt(b)*sqrt(a*d - b*c)*at
an((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c*d**2*x**2 + 15
*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))
*a**2*b**3*d**3*x**3 + 54*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(
sqrt(b)*sqrt(a*d - b*c)))*a*b**5*d**2*x**2 + 72*sqrt(b)*sqrt(a*d - b*c)*at
an((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c**2*d*x**2 - 36*sq
rt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*
b**4*c*d**2*x**3 + 18*sqrt(b)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt
(b)*sqrt(a*d - b*c)))*b**6*d**2*x**3 + 24*sqrt(b)*sqrt(a*d - b*c)*atan(...
```

3.87 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$

Optimal result 797
Mathematica [A] (verified) 798
Rubi [A] (verified) 799
Maple [A] (verified) 800
Fricas [A] (verification not implemented) 801
Sympy [A] (verification not implemented) 802
Maxima [A] (verification not implemented) 803
Giac [B] (verification not implemented) 803
Mupad [F(-1)] 804
Reduce [B] (verification not implemented) 805

Optimal result

Integrand size = 32, antiderivative size = 434

$$\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2(bc-ad)^3(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^7\sqrt{c+dx}}$$
$$-\frac{2(bc-ad)^2(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D))\sqrt{c+dx}}{d^7}$$
$$-\frac{2(bc-ad)(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D))(c \cdots}{3d^7}$$
$$+\frac{2(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D))(\cdots}{5d^7}$$
$$+\frac{2b(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D))(c+dx)^{7/2}}{7d^7}$$
$$+\frac{2b^2(bCd-6bcD+3adD)(c+dx)^{9/2}}{9d^7}+\frac{2b^3D(c+dx)^{11/2}}{11d^7}$$

output
```
2*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^7/(d*x+c)^(1/2)-2*(-a*d+b*c
)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3))
*(d*x+c)^(1/2)/d^7-2/3*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^2+8*C
*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^(3/2)/
d^7+2/5*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d-10*D*
c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(5/2)/d^7+2/7*b*(3
*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*x+c)^(7/2
)/d^7+2/9*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(9/2)/d^7+2/11*b^3*D*(d*x+c)
^(11/2)/d^7
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2(231a^3d^3(48c^3D-8c^2d(5C-3Dx)+2cd^2(15B-x(10C+3$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*(231*a^3*d^3*(48*c^3*D - 8*c^2*d*(5*C - 3*D*x) + 2*c*d^2*(15*B - x*(10*
C + 3*D*x)) + d^3*(-15*A + x*(15*B + 5*C*x + 3*D*x^2))) + 99*a^2*b*d^2*(-3
84*c^4*D + 48*c^3*d*(7*C - 4*D*x) - 8*c^2*d^2*(35*B - 3*x*(7*C + 2*D*x)) +
 2*c*d^3*(105*A - x*(70*B + 3*x*(7*C + 4*D*x))) + d^4*x*(105*A + x*(35*B +
 3*x*(7*C + 5*D*x)))) + 33*a*b^2*d*(1280*c^5*D - 128*c^4*d*(9*C - 5*D*x) +
 16*c^3*d^2*(63*B - 2*x*(18*C + 5*D*x)) + 8*c^2*d^3*(-105*A + x*(63*B + 2*
x*(9*C + 5*D*x))) + d^5*x^2*(105*A + x*(63*B + 5*x*(9*C + 7*D*x))) - 2*c*d
^4*x*(210*A + x*(63*B + x*(36*C + 25*D*x)))) + b^3*(-15360*c^6*D + 1280*c^
5*d*(11*C - 6*D*x) - 128*c^4*d^2*(99*B - 5*x*(11*C + 3*D*x)) + 16*c^3*d^3*
(693*A - 2*x*(198*B + 5*x*(11*C + 6*D*x))) + d^6*x^3*(693*A + 5*x*(99*B +
7*x*(11*C + 9*D*x))) + 8*c^2*d^4*x*(693*A + x*(198*B + 5*x*(22*C + 15*D*x)
)) - 2*c*d^5*x^2*(693*A + x*(396*B + 5*x*(55*C + 42*D*x))))))/(3465*d^7*Sq
rt[c + d*x])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{\sqrt{c+dx}(bc-ad) \left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+10c\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(c+dx)^{3/2}(bc-ad) \left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2C\right.\right.}{3d^7}$$
$$\frac{2b(c+dx)^{7/2} \left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{7d^7}+$$
$$\frac{2(c+dx)^{5/2} \left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2\right.\right.}{5d^7}$$
$$\frac{2\sqrt{c+dx}(bc-ad)^2 \left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{d^7}+$$
$$\frac{2(bc-ad)^3 \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^7\sqrt{c+dx}}+\frac{2b^2(c+dx)^{9/2}(3adD-6bcD+bCd)}{9d^7}+$$
$$\frac{2b^3D(c+dx)^{11/2}}{11d^7}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^7*Sqrt[c + d*x])
- (2*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d - 4*B*c
*d^2 + 3*A*d^3 - 6*c^3*D))*Sqrt[c + d*x])/d^7 - (2*(b*c - a*d)*(a^2*d^2*(C
*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^2*C*d - 6*B
*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(3/2))/(3*d^7) + (2*(a^3*d^3*D + 3
*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*D) + b^3*(1
0*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(5/2))/(5*d^7) + (2*b
*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^2 - 15*c^2*D))*
(c + d*x)^(7/2))/(7*d^7) + (2*b^2*(b*C*d - 6*b*c*D + 3*a*d*D)*(c + d*x)^(9
/2))/(9*d^7) + (2*b^3*D*(c + d*x)^(11/2))/(11*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{x^{3}\left(\frac{5}{11}Dx^{3}+\frac{5}{9}Cx^{2}+\frac{5}{7}Bx+A\right)b^{3}}{5}-x^{2}\left(\frac{1}{3}Dx^{3}+\frac{3}{7}Cx^{2}+\frac{3}{5}Bx+A\right)ab^{2}-3x\left(\frac{1}{5}Cx^{2}+\frac{1}{3}Bx+\frac{1}{7}Dx^{3}+A\right)a^{2}b+a^{3}(-\right.\right.}{}$
gosper	$-\frac{2(-315Dx^{6}b^{3}d^{6}-385Cx^{5}b^{3}d^{6}-1155Dx^{5}ab^{2}d^{6}+420Dx^{5}b^{3}cd^{5}-495Bx^{4}b^{3}d^{6}-1485Cx^{4}ab^{2}d^{6}+550Cx^{4}b^{3}cd^{5}-}{}$
trager	$-\frac{2(-315Dx^{6}b^{3}d^{6}-385Cx^{5}b^{3}d^{6}-1155Dx^{5}ab^{2}d^{6}+420Dx^{5}b^{3}cd^{5}-495Bx^{4}b^{3}d^{6}-1485Cx^{4}ab^{2}d^{6}+550Cx^{4}b^{3}cd^{5}-}{}$
orering	$-\frac{2(-315Dx^{6}b^{3}d^{6}-385Cx^{5}b^{3}d^{6}-1155Dx^{5}ab^{2}d^{6}+420Dx^{5}b^{3}cd^{5}-495Bx^{4}b^{3}d^{6}-1485Cx^{4}ab^{2}d^{6}+550Cx^{4}b^{3}cd^{5}-}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((-1/5*x^3*(5/11*D*x^3+5/9*C*x^2+5/7*B*x+A)*b^3-x^2*(1/3*D*x^3+3/7*C*x^
2+3/5*B*x+A)*a*b^2-3*x*(1/5*C*x^2+1/3*B*x+1/7*D*x^3+A)*a^2*b+a^3*(-1/5*D*x
^3-1/3*C*x^2-B*x+A))*d^6-6*(-1/15*x^2*(10/33*D*x^3+25/63*C*x^2+4/7*B*x+A)*
b^3-2/3*x*(5/42*D*x^3+6/35*C*x^2+3/10*B*x+A)*a*b^2+a^2*(-4/35*D*x^3-1/5*C*
x^2-2/3*B*x+A)*b+1/3*(-1/5*D*x^2-2/3*C*x+B)*a^3)*c*d^5+8*(-1/5*x*(25/231*D
*x^3+10/63*C*x^2+2/7*B*x+A)*b^3+a*(-2/21*D*x^3-6/35*C*x^2-3/5*B*x+A)*b^2+a
^2*(-6/35*D*x^2-3/5*C*x+B)*b+1/3*(-3/5*D*x+C)*a^3)*c^2*d^4-16/5*((-20/231*
D*x^3-10/63*C*x^2-4/7*B*x+A)*b^3+3*(-10/63*D*x^2-4/7*C*x+B)*a*b^2+3*(-4/7*
D*x+C)*a^2*b+a^3*D)*c^3*d^3+128/35*((-5/33*D*x^2-5/9*C*x+B)*b^2+3*(-5/9*D*
x+C)*a*b+3*D*a^2)*c^4*b*d^2-256/63*c^5*b^2*((-6/11*D*x+C)*b+3*D*a)*d+1024/
231*D*b^3*c^6)/(d*x+c)^(1/2)/d^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2\,(315\,Db^3d^6x^6 - 15360\,Db^3c^6 - 3465\,Aa^3d^6 + 14080\,(3\,Dab^2 +$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="fricas
")
```

output
```
2/3465*(315*D*b^3*d^6*x^6 - 15360*D*b^3*c^6 - 3465*A*a^3*d^6 + 14080*(3*D*
a*b^2 + C*b^3)*c^5*d - 12672*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 + 110
88*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 - 9240*(C*a^3 + 3*B*a^2
*b + 3*A*a*b^2)*c^2*d^4 + 6930*(B*a^3 + 3*A*a^2*b)*c*d^5 - 35*(12*D*b^3*c*
d^5 - 11*(3*D*a*b^2 + C*b^3)*d^6)*x^5 + 5*(120*D*b^3*c^2*d^4 - 110*(3*D*a*
b^2 + C*b^3)*c*d^5 + 99*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^6)*x^4 - (960*D*
b^3*c^3*d^3 - 880*(3*D*a*b^2 + C*b^3)*c^2*d^4 + 792*(3*D*a^2*b + 3*C*a*b^2
 + B*b^3)*c*d^5 - 693*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^6)*x^3 + (
1920*D*b^3*c^4*d^2 - 1760*(3*D*a*b^2 + C*b^3)*c^3*d^3 + 1584*(3*D*a^2*b +
3*C*a*b^2 + B*b^3)*c^2*d^4 - 1386*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*
c*d^5 + 1155*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^6)*x^2 - (7680*D*b^3*c^5*d
- 7040*(3*D*a*b^2 + C*b^3)*c^4*d^2 + 6336*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*
c^3*d^3 - 5544*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^4 + 4620*(C*a
^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^5 - 3465*(B*a^3 + 3*A*a^2*b)*d^6)*x)*sqrt(
d*x + c)/(d^8*x + c*d^7)
```

Sympy [A] (verification not implemented)

Time = 84.52 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.97

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Db^3(c+dx)^{\frac{11}{2}}}{11d^6}+\frac{(c+dx)^{\frac{9}{2}}\left(Cb^3d+3Dab^2d-6Db^3c\right)}{9d^6}+\frac{(c+dx)^{\frac{7}{2}}\left(Bb^3d^2+3Cab^2d^2-5Cb^3c\ldots\right.}{7\ldots}\right.}{\ldots} \\ \frac{Aa^3x+\frac{Db^3x^7}{7}+\frac{x^6\left(Cb^3+3Dab^2\right)}{6}+\frac{x^5\left(Bb^3+3Cab^2+3Da^2b\right)}{5}+\frac{x^4\left(Ab^3+3Bab^2+3Ca^2b+\ldots\right.}{4}}{c^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Piecewise((2*(D*b**3*(c + d*x)**(11/2)/(11*d**6) + (c + d*x)**(9/2)*(C*b**
3*d + 3*D*a*b**2*d - 6*D*b**3*c)/(9*d**6) + (c + d*x)**(7/2)*(B*b**3*d**2
+ 3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d + 15*
D*b**3*c**2)/(7*d**6) + (c + d*x)**(5/2)*(A*b**3*d**3 + 3*B*a*b**2*d**3 -
4*B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c**2*d
+ D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c**3)/
(5*d**6) + (c + d*x)**(3/2)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*B*a**2*
b*d**4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C*a**2*b
*c*d**3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**3 + 18*
D*a**2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/(3*d**6) + sqrt(
c + d*x)*(3*A*a**2*b*d**5 - 6*A*a*b**2*c*d**4 + 3*A*b**3*c**2*d**3 + B*a**
3*d**5 - 6*B*a**2*b*c*d**4 + 9*B*a*b**2*c**2*d**3 - 4*B*b**3*c**3*d**2 - 2
*C*a**3*c*d**4 + 9*C*a**2*b*c**2*d**3 - 12*C*a*b**2*c**3*d**2 + 5*C*b**3*c
**4*d + 3*D*a**3*c**2*d**3 - 12*D*a**2*b*c**3*d**2 + 15*D*a*b**2*c**4*d -
6*D*b**3*c**5)/d**6 + (a*d - b*c)**3*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c*
*3)/(d**6*sqrt(c + d*x)))/d, Ne(d, 0)), ((A*a**3*x + D*b**3*x**7/7 + x**6*
(C*b**3 + 3*D*a*b**2)/6 + x**5*(B*b**3 + 3*C*a*b**2 + 3*D*a**2*b)/5 + x**4
*(A*b**3 + 3*B*a*b**2 + 3*C*a**2*b + D*a**3)/4 + x**3*(3*A*a*b**2 + 3*B*a*
*2*b + C*a**3)/3 + x**2*(3*A*a**2*b + B*a**3)/2)/c**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2\left(\frac{315\,(dx+c)^{\frac{11}{2}}Db^3-385\,(6\,Db^3c-(3\,Dab^2+Cb^3)\,d)\,(dx+c)^{\frac{9}{2}}+495\,(15\,Db^3c^2-5\,(3\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="maxima
")
```

output
```
2/3465*((315*(d*x + c)^(11/2)*D*b^3 - 385*(6*D*b^3*c - (3*D*a*b^2 + C*b^3)
*d)*(d*x + c)^(9/2) + 495*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^3)*c*d + (3*D
*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(7/2) - 693*(20*D*b^3*c^3 - 10*
(3*D*a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^2 - (D*a
^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(5/2) + 1155*(15*D*b^3*
c^4 - 10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2
*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (C*a^3 + 3*B*a^2*
b + 3*A*a*b^2)*d^4)*(d*x + c)^(3/2) - 3465*(6*D*b^3*c^5 - 5*(3*D*a*b^2 + C
*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2 - 3*(D*a^3 + 3*C*a
^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^
4 - (B*a^3 + 3*A*a^2*b)*d^5)*sqrt(d*x + c))/d^6 - 3465*(D*b^3*c^6 + A*a^3*
d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2
- (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3*B*a^2*b + 3
*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)/(sqrt(d*x + c)*d^6))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. 2(412) = 824.

Time = 0.14 (sec) , antiderivative size = 1067, normalized size of antiderivative = 2.46

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-2*(D*b^3*c^6 - 3*D*a*b^2*c^5*d - C*b^3*c^5*d + 3*D*a^2*b*c^4*d^2 + 3*C*a*
b^2*c^4*d^2 + B*b^3*c^4*d^2 - D*a^3*c^3*d^3 - 3*C*a^2*b*c^3*d^3 - 3*B*a*b^
2*c^3*d^3 - A*b^3*c^3*d^3 + C*a^3*c^2*d^4 + 3*B*a^2*b*c^2*d^4 + 3*A*a*b^2*
c^2*d^4 - B*a^3*c*d^5 - 3*A*a^2*b*c*d^5 + A*a^3*d^6)/(sqrt(d*x + c)*d^7) +
 2/3465*(315*(d*x + c)^(11/2)*D*b^3*d^70 - 2310*(d*x + c)^(9/2)*D*b^3*c*d^
70 + 7425*(d*x + c)^(7/2)*D*b^3*c^2*d^70 - 13860*(d*x + c)^(5/2)*D*b^3*c^3
*d^70 + 17325*(d*x + c)^(3/2)*D*b^3*c^4*d^70 - 20790*sqrt(d*x + c)*D*b^3*c
^5*d^70 + 1155*(d*x + c)^(9/2)*D*a*b^2*d^71 + 385*(d*x + c)^(9/2)*C*b^3*d^
71 - 7425*(d*x + c)^(7/2)*D*a*b^2*c*d^71 - 2475*(d*x + c)^(7/2)*C*b^3*c*d^
71 + 20790*(d*x + c)^(5/2)*D*a*b^2*c^2*d^71 + 6930*(d*x + c)^(5/2)*C*b^3*c
^2*d^71 - 34650*(d*x + c)^(3/2)*D*a*b^2*c^3*d^71 - 11550*(d*x + c)^(3/2)*C
*b^3*c^3*d^71 + 51975*sqrt(d*x + c)*D*a*b^2*c^4*d^71 + 17325*sqrt(d*x + c)
*C*b^3*c^4*d^71 + 1485*(d*x + c)^(7/2)*D*a^2*b*d^72 + 1485*(d*x + c)^(7/2)
*C*a*b^2*d^72 + 495*(d*x + c)^(7/2)*B*b^3*d^72 - 8316*(d*x + c)^(5/2)*D*a^
2*b*c*d^72 - 8316*(d*x + c)^(5/2)*C*a*b^2*c*d^72 - 2772*(d*x + c)^(5/2)*B*
b^3*c*d^72 + 20790*(d*x + c)^(3/2)*D*a^2*b*c^2*d^72 + 20790*(d*x + c)^(3/2
)*C*a*b^2*c^2*d^72 + 6930*(d*x + c)^(3/2)*B*b^3*c^2*d^72 - 41580*sqrt(d*x
+ c)*D*a^2*b*c^3*d^72 - 41580*sqrt(d*x + c)*C*a*b^2*c^3*d^72 - 13860*sqrt(
d*x + c)*B*b^3*c^3*d^72 + 693*(d*x + c)^(5/2)*D*a^3*d^73 + 2079*(d*x + c)^
(5/2)*C*a^2*b*d^73 + 2079*(d*x + c)^(5/2)*B*a*b^2*d^73 + 693*(d*x + c)^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{(a+b\,x)^3\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2),x)
```

output
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{\frac{64}{5}a\,b^3c^2d^3x - \frac{96}{35}a^2b\,c^4d^2 - 2a^4d^5 - \frac{512}{693}b^3c^6 + 16a^3bc\,d^4 + 8a^3b\,c}{}$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output
```
(2*( - 3465*a**4*d**5 + 27720*a**3*b*c*d**4 + 13860*a**3*b*d**5*x + 1848*a
**3*c**3*d**3 + 924*a**3*c**2*d**4*x - 231*a**3*c*d**5*x**2 + 693*a**3*d**
6*x**3 - 55440*a**2*b**2*c**2*d**3 - 27720*a**2*b**2*c*d**4*x + 6930*a**2*
b**2*d**5*x**2 - 4752*a**2*b*c**4*d**2 - 2376*a**2*b*c**3*d**3*x + 594*a**
2*b*c**2*d**4*x**2 - 297*a**2*b*c*d**5*x**3 + 1485*a**2*b*d**6*x**4 + 4435
2*a*b**3*c**3*d**2 + 22176*a*b**3*c**2*d**3*x - 5544*a*b**3*c*d**4*x**2 +
2772*a*b**3*d**5*x**3 + 4224*a*b**2*c**5*d + 2112*a*b**2*c**4*d**2*x - 528
*a*b**2*c**3*d**3*x**2 + 264*a*b**2*c**2*d**4*x**3 - 165*a*b**2*c*d**5*x**
4 + 1155*a*b**2*d**6*x**5 - 12672*b**4*c**4*d - 6336*b**4*c**3*d**2*x + 15
84*b**4*c**2*d**3*x**2 - 792*b**4*c*d**4*x**3 + 495*b**4*d**5*x**4 - 1280*
b**3*c**6 - 640*b**3*c**5*d*x + 160*b**3*c**4*d**2*x**2 - 80*b**3*c**3*d**
3*x**3 + 50*b**3*c**2*d**4*x**4 - 35*b**3*c*d**5*x**5 + 315*b**3*d**6*x**6
))/(3465*sqrt(c + d*x)*d**6)
```

3.88 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$

Optimal result 806
Mathematica [A] (verified) 807
Rubi [A] (verified) 807
Maple [A] (verified) 809
Fricas [A] (verification not implemented) 809
Sympy [A] (verification not implemented) 810
Maxima [A] (verification not implemented) 811
Giac [B] (verification not implemented) 811
Mupad [F(-1)] 812
Reduce [B] (verification not implemented) 813

Optimal result

Integrand size = 32, antiderivative size = 322

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = -\frac{2(bc-ad)^2(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^6\sqrt{c+dx}}$$
$$+\frac{2(bc-ad)(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))\sqrt{c+dx}}{d^6}$$
$$+\frac{2(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D))(c+dx)^{3/2}}{3d^6}$$
$$+\frac{2(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))(c+dx)^{5/2}}{5d^6}$$
$$+\frac{2b(bCd-5bcD+2adD)(c+dx)^{7/2}}{7d^6}+\frac{2b^2D(c+dx)^{9/2}}{9d^6}$$

output
```
-2*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^6/(d*x+c)^(1/2)+2*(-a*d+b*
c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))*
(d*x+c)^(1/2)/d^6+2/3*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*c^2
)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(3/2)/d^6+2/5*(a^2*d^2
*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(5/2)/d^6+2/
7*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(7/2)/d^6+2/9*b^2*D*(d*x+c)^(9/2)/d^6
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2(21a^2d^2(48c^3D-8c^2d(5C-3Dx)+2cd^2(15B-x(10C+3L$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*(21*a^2*d^2*(48*c^3*D - 8*c^2*d*(5*C - 3*D*x) + 2*c*d^2*(15*B - x*(10*C
 + 3*D*x)) + d^3*(-15*A + x*(15*B + 5*C*x + 3*D*x^2))) + 6*a*b*d*(-384*c^4
*D + 48*c^3*d*(7*C - 4*D*x) - 8*c^2*d^2*(35*B - 3*x*(7*C + 2*D*x)) + 2*c*d
^3*(105*A - x*(70*B + 3*x*(7*C + 4*D*x))) + d^4*x*(105*A + x*(35*B + 3*x*(
7*C + 5*D*x)))) + b^2*(1280*c^5*D - 128*c^4*d*(9*C - 5*D*x) + 16*c^3*d^2*(
63*B - 2*x*(18*C + 5*D*x)) + 8*c^2*d^3*(-105*A + x*(63*B + 2*x*(9*C + 5*D*
x))) + d^5*x^2*(105*A + x*(63*B + 5*x*(9*C + 7*D*x))) - 2*c*d^4*x*(210*A +
 x*(63*B + x*(36*C + 25*D*x))))))/(315*d^6*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{\sqrt{c+dx}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c+d}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(c+dx)^{3/2}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{3d^6}+\frac{2(c+dx)^{5/2}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{5d^6}+\frac{2\sqrt{c+dx}(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{d^6}-\frac{2(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^6\sqrt{c+dx}}+\frac{2b(c+dx)^{7/2}(2adD-5bcD+bCd)}{7d^6}+\frac{2b^2D(c+dx)^{9/2}}{9d^6}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(-2*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^6*Sqrt[c + d*x])
 + (2*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B*c*
d^2 + 2*A*d^3 - 5*c^3*D))*Sqrt[c + d*x])/d^6 + (2*(a^2*d^2*(C*d - 3*c*D) -
 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 + A*d^3
 - 10*c^3*D))*(c + d*x)^(3/2))/(3*d^6) + (2*(a^2*d^2*D + 2*a*b*d*(C*d - 4*c
*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(5/2))/(5*d^6) + (2*b*(b
*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(7/2))/(7*d^6) + (2*b^2*D*(c + d*x)^(9
/2))/(9*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{((70Dx^5+90Cx^4+126Bx^3+210Ax^2)b^2+1260x(\frac{1}{5}Cx^2+\frac{1}{3}Bx+\frac{1}{7}Dx^3+A)ab-630a^2(-\frac{1}{5}Dx^3-\frac{1}{3}Cx^2-Bx+A))d^5}{}$
gosper	$-\frac{2(-35Dx^5b^2d^5-45Cx^4b^2d^5-90Dx^4abd^5+50Dx^4b^2cd^4-63Bx^3b^2d^5-126Cx^3abd^5+72Cx^3b^2cd^4-63Dx^3a^2d^5}{}$
trager	$-\frac{2(-35Dx^5b^2d^5-45Cx^4b^2d^5-90Dx^4abd^5+50Dx^4b^2cd^4-63Bx^3b^2d^5-126Cx^3abd^5+72Cx^3b^2cd^4-63Dx^3a^2d^5}{}$
orering	$-\frac{2(-35Dx^5b^2d^5-45Cx^4b^2d^5-90Dx^4abd^5+50Dx^4b^2cd^4-63Bx^3b^2d^5-126Cx^3abd^5+72Cx^3b^2cd^4-63Dx^3a^2d^5}{}$
derivativedivides	$-\frac{\frac{2(Aa^2d^5-2Aabcd^4+Ab^2c^2d^3-Ba^2cd^4+2Babc^2d^3-Bb^2c^3d^2+Ca^2c^2d^3-2Cabc^3d^2+Cb^2c^4d-Da^2c^3d^2+2Dabc^4d-Db^2c^5}{\sqrt{xd+c}}}{}$
default	$-\frac{\frac{2(Aa^2d^5-2Aabcd^4+Ab^2c^2d^3-Ba^2cd^4+2Babc^2d^3-Bb^2c^3d^2+Ca^2c^2d^3-2Cabc^3d^2+Cb^2c^4d-Da^2c^3d^2+2Dabc^4d-Db^2c^5}{\sqrt{xd+c}}}{}$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(((70*D*x^5+90*C*x^4+126*B*x^3+210*A*x^2)*b^2+1260*x*(1/5*C*x^2+1/3*
B*x+1/7*D*x^3+A)*a*b-630*a^2*(-1/5*D*x^3-1/3*C*x^2-B*x+A))*d^5+2520*c*(-1/
3*x*(5/42*D*x^3+6/35*C*x^2+3/10*B*x+A)*b^2+a*(-4/35*D*x^3-1/5*C*x^2-2/3*B*
x+A)*b+1/2*(-1/5*D*x^2-2/3*C*x+B)*a^2)*d^4-1680*((-2/21*D*x^3-6/35*C*x^2-3
/5*B*x+A)*b^2+2*(-6/35*D*x^2-3/5*C*x+B)*a*b+a^2*(-3/5*D*x+C))*c^2*d^3+2016
*c^3*((-10/63*D*x^2-4/7*C*x+B)*b^2+2*(-4/7*D*x+C)*a*b+D*a^2)*d^2-2304*((-5
/9*D*x+C)*b+2*D*a)*c^4*b*d+2560*D*b^2*c^5)/(d*x+c)^(1/2)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2(35\,Db^2d^5x^5+1280\,Db^2c^5-315\,Aa^2d^5-1152\,(2\,Dab+Cb^2)}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="fricas
")
```

output
```
2/315*(35*D*b^2*d^5*x^5 + 1280*D*b^2*c^5 - 315*A*a^2*d^5 - 1152*(2*D*a*b +
 C*b^2)*c^4*d + 1008*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - 840*(C*a^2 + 2*B*
a*b + A*b^2)*c^2*d^3 + 630*(B*a^2 + 2*A*a*b)*c*d^4 - 5*(10*D*b^2*c*d^4 - 9
*(2*D*a*b + C*b^2)*d^5)*x^4 + (80*D*b^2*c^2*d^3 - 72*(2*D*a*b + C*b^2)*c*d
^4 + 63*(D*a^2 + 2*C*a*b + B*b^2)*d^5)*x^3 - (160*D*b^2*c^3*d^2 - 144*(2*D
*a*b + C*b^2)*c^2*d^3 + 126*(D*a^2 + 2*C*a*b + B*b^2)*c*d^4 - 105*(C*a^2 +
 2*B*a*b + A*b^2)*d^5)*x^2 + (640*D*b^2*c^4*d - 576*(2*D*a*b + C*b^2)*c^3*
d^2 + 504*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 - 420*(C*a^2 + 2*B*a*b + A*b^2
)*c*d^4 + 315*(B*a^2 + 2*A*a*b)*d^5)*x)*sqrt(d*x + c)/(d^7*x + c*d^6)
```

Sympy [A] (verification not implemented)

Time = 30.44 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \begin{cases} \dfrac{2\left(\frac{Db^2(c+dx)^{\frac{9}{2}}}{9d^5}+\frac{(c+dx)^{\frac{7}{2}}\left(Cb^2d+2Dabd-5Db^2c\right)}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\left(Bb^2d^2+2Cabd^2-4Cb^2cd+\cdots\right)}{5d^5}\cdots\right.}{\cdots} \\ \dfrac{Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5\left(Cb^2+2Dab\right)}{5}+\frac{x^4\left(Bb^2+2Cab+Da^2\right)}{4}+\frac{x^3\left(Ab^2+2Bab+Ca^2\right)}{3}+\frac{x^2\cdot\left(2A\cdots\right)}{\cdots}}{c^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Piecewise((2*(D*b**2*(c + d*x)**(9/2)/(9*d**5) + (c + d*x)**(7/2)*(C*b**2*
d + 2*D*a*b*d - 5*D*b**2*c)/(7*d**5) + (c + d*x)**(5/2)*(B*b**2*d**2 + 2*C
*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2)/(5*
d**5) + (c + d*x)**(3/2)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d**2 + C
*a**2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 12*D*a*b
*c**2*d - 10*D*b**2*c**3)/(3*d**5) + sqrt(c + d*x)*(2*A*a*b*d**4 - 2*A*b**
2*c*d**3 + B*a**2*d**4 - 4*B*a*b*c*d**3 + 3*B*b**2*c**2*d**2 - 2*C*a**2*c*
d**3 + 6*C*a*b*c**2*d**2 - 4*C*b**2*c**3*d + 3*D*a**2*c**2*d**2 - 8*D*a*b*
c**3*d + 5*D*b**2*c**4)/d**5 + (a*d - b*c)**2*(-A*d**3 + B*c*d**2 - C*c**2
*d + D*c**3)/(d**5*sqrt(c + d*x)))/d, Ne(d, 0)), ((A*a**2*x + D*b**2*x**6/
6 + x**5*(C*b**2 + 2*D*a*b)/5 + x**4*(B*b**2 + 2*C*a*b + D*a**2)/4 + x**3*
(A*b**2 + 2*B*a*b + C*a**2)/3 + x**2*(2*A*a*b + B*a**2)/2)/c**(3/2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2\left(\frac{35\,(dx+c)^{\frac{9}{2}}Db^2-45\,(5\,Db^2c-(2\,Dab+Cb^2)d)(dx+c)^{\frac{7}{2}}+63\,(10\,Db^2c^2-4\,(2\,Dab+\right.}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="maxima
")
```

output
```
2/315*((35*(d*x + c)^(9/2)*D*b^2 - 45*(5*D*b^2*c - (2*D*a*b + C*b^2)*d)*(d
*x + c)^(7/2) + 63*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d + (D*a^2 + 2*C*
a*b + B*b^2)*d^2)*(d*x + c)^(5/2) - 105*(10*D*b^2*c^3 - 6*(2*D*a*b + C*b^2
)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*b + A*b^2)*d^
3)*(d*x + c)^(3/2) + 315*(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^2)*c^3*d + 3*(D*a
^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 + (B*a^2
 + 2*A*a*b)*d^4)*sqrt(d*x + c))/d^5 + 315*(D*b^2*c^5 - A*a^2*d^5 - (2*D*a*
b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 + 2*B*a*b +
A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)/(sqrt(d*x + c)*d^5))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. 2(302) = 604.

Time = 0.14 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{2\,(Db^2c^5-2\,Dabc^4d-Cb^2c^4d+Da^2c^3d^2+2\,Cabc^3d^2+Bb^2c^3}{\sqrt{c}} + \frac{2\left(35\,(dx+c)^{\frac{9}{2}}Db^2d^{48}-225\,(dx+c)^{\frac{7}{2}}Db^2cd^{48}+630\,(dx+c)^{\frac{5}{2}}Db^2c^2d^{48}-1050\,(dx+c)^{\frac{3}{2}}Db^2c^3d^{48}+15\right.}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
2*(D*b^2*c^5 - 2*D*a*b*c^4*d - C*b^2*c^4*d + D*a^2*c^3*d^2 + 2*C*a*b*c^3*d
^2 + B*b^2*c^3*d^2 - C*a^2*c^2*d^3 - 2*B*a*b*c^2*d^3 - A*b^2*c^2*d^3 + B*a
^2*c*d^4 + 2*A*a*b*c*d^4 - A*a^2*d^5)/(sqrt(d*x + c)*d^6) + 2/315*(35*(d*x
 + c)^(9/2)*D*b^2*d^48 - 225*(d*x + c)^(7/2)*D*b^2*c*d^48 + 630*(d*x + c)^
(5/2)*D*b^2*c^2*d^48 - 1050*(d*x + c)^(3/2)*D*b^2*c^3*d^48 + 1575*sqrt(d*x
 + c)*D*b^2*c^4*d^48 + 90*(d*x + c)^(7/2)*D*a*b*d^49 + 45*(d*x + c)^(7/2)*
C*b^2*d^49 - 504*(d*x + c)^(5/2)*D*a*b*c*d^49 - 252*(d*x + c)^(5/2)*C*b^2*
c*d^49 + 1260*(d*x + c)^(3/2)*D*a*b*c^2*d^49 + 630*(d*x + c)^(3/2)*C*b^2*c
^2*d^49 - 2520*sqrt(d*x + c)*D*a*b*c^3*d^49 - 1260*sqrt(d*x + c)*C*b^2*c^3
*d^49 + 63*(d*x + c)^(5/2)*D*a^2*d^50 + 126*(d*x + c)^(5/2)*C*a*b*d^50 + 6
3*(d*x + c)^(5/2)*B*b^2*d^50 - 315*(d*x + c)^(3/2)*D*a^2*c*d^50 - 630*(d*x
 + c)^(3/2)*C*a*b*c*d^50 - 315*(d*x + c)^(3/2)*B*b^2*c*d^50 + 945*sqrt(d*x
 + c)*D*a^2*c^2*d^50 + 1890*sqrt(d*x + c)*C*a*b*c^2*d^50 + 945*sqrt(d*x +
c)*B*b^2*c^2*d^50 + 105*(d*x + c)^(3/2)*C*a^2*d^51 + 210*(d*x + c)^(3/2)*B
*a*b*d^51 + 105*(d*x + c)^(3/2)*A*b^2*d^51 - 630*sqrt(d*x + c)*C*a^2*c*d^5
1 - 1260*sqrt(d*x + c)*B*a*b*c*d^51 - 630*sqrt(d*x + c)*A*b^2*c*d^51 + 315
*sqrt(d*x + c)*B*a^2*d^52 + 630*sqrt(d*x + c)*A*a*b*d^52)/d^54
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx=\int \frac{(a+b\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2),x)
```

output
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{12a^2bc\,d^3+\frac{8}{15}a^2c^2d^3x-16a\,b^2c^2d^2-\frac{64}{35}ab\,c^4d+\frac{128}{315}b^2c^4dx+\frac{16}{31}}{}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output
```
(2*( - 315*a**3*d**4 + 1890*a**2*b*c*d**3 + 945*a**2*b*d**4*x + 168*a**2*c
**3*d**2 + 84*a**2*c**2*d**3*x - 21*a**2*c*d**4*x**2 + 63*a**2*d**5*x**3 -
 2520*a*b**2*c**2*d**2 - 1260*a*b**2*c*d**3*x + 315*a*b**2*d**4*x**2 - 288
*a*b*c**4*d - 144*a*b*c**3*d**2*x + 36*a*b*c**2*d**3*x**2 - 18*a*b*c*d**4*
x**3 + 90*a*b*d**5*x**4 + 1008*b**3*c**3*d + 504*b**3*c**2*d**2*x - 126*b*
*3*c*d**3*x**2 + 63*b**3*d**4*x**3 + 128*b**2*c**5 + 64*b**2*c**4*d*x - 16
*b**2*c**3*d**2*x**2 + 8*b**2*c**2*d**3*x**3 - 5*b**2*c*d**4*x**4 + 35*b**
2*d**5*x**5))/(315*sqrt(c + d*x)*d**5)
```

3.89 $\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx$

Optimal result 814
Mathematica [A] (verified) 815
Rubi [A] (verified) 815
Maple [A] (verified) 817
Fricas [A] (verification not implemented) 817
Sympy [A] (verification not implemented) 818
Maxima [A] (verification not implemented) 818
Giac [A] (verification not implemented) 819
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Reduce [B] (verification not implemented) 820

Optimal result

Integrand size = 30, antiderivative size = 210

$$\begin{aligned}\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx &= \frac{2(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)}{d^5\sqrt{c+dx}}\\ &- \frac{2(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\sqrt{c+dx}}{d^5}\\ &+ \frac{2(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))(c+dx)^{3/2}}{3d^5}\\ &+ \frac{2(bCd-4bcD+adD)(c+dx)^{5/2}}{5d^5}+\frac{2bD(c+dx)^{7/2}}{7d^5}\end{aligned}$$

output
```
2*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^5/(d*x+c)^(1/2)-2*(a*d*(-B*d^
2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(1/2)/d^
5+2/3*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(3/2)/d^5+2/5*(
C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(5/2)/d^5+2/7*b*D*(d*x+c)^(7/2)/d^5
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx = \frac{14ad(48c^3D-8c^2d(5C-3Dx)+2cd^2(15B-x(10C+3Dx))+}{}$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(14*a*d*(48*c^3*D - 8*c^2*d*(5*C - 3*D*x) + 2*c*d^2*(15*B - x*(10*C + 3*D*
x)) + d^3*(-15*A + x*(15*B + 5*C*x + 3*D*x^2))) + b*(-768*c^4*D + 96*c^3*d
*(7*C - 4*D*x) + 16*c^2*d^2*(-35*B + 3*x*(7*C + 2*D*x)) + 4*c*d^3*(105*A -
 x*(70*B + 3*x*(7*C + 4*D*x))) + 2*d^4*x*(105*A + x*(35*B + 3*x*(7*C + 5*D
*x)))))/(105*d^5*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)}{d^4\sqrt{c+dx}}+\frac{(ad-bc)\left(Ad^3-Bcd^2+c^3(-D)+c^2C\right.}{d^4(c+dx)^{3/2}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2\sqrt{c+dx}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{d^5}+\frac{2(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^5\sqrt{c+dx}}+\frac{2(c+dx)^{3/2}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{3d^5}+\frac{2(c+dx)^{5/2}(adD-4bcD+bCd)}{5d^5}+\frac{2bD(c+dx)^{7/2}}{7d^5}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^5*Sqrt[c + d*x]) -
(2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 - 4
*c^3*D))*Sqrt[c + d*x])/d^5 + (2*(a*d*(C*d - 3*c*D) - b*(3*c*C*d - B*d^2 -
 6*c^2*D))*(c + d*x)^(3/2))/(3*d^5) + (2*(b*C*d - 4*b*c*D + a*d*D)*(c + d*
x)^(5/2))/(5*d^5) + (2*b*D*(c + d*x)^(7/2))/(7*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{Db\,x^4}{7}+\frac{(-Cb-Da)x^3}{5}+\frac{(-Bb-Ca)x^2}{3}+(-Ab-Ba)x+Aa\right)d^4-2\left(-\frac{4Db\,x^3}{35}+\frac{(-Cb-Da)x^2}{5}+\frac{2(-Bb-Ca)x}{3}+Ab\cdots\right.\right.}{\sqrt{xd+c}\,d^5}$
gosper	$-\frac{2(-15Db\,x^4d^4-21C\,x^3b\,d^4-21Dx^3a\,d^4+24Dx^3bc\,d^3-35B\,x^2b\,d^4-35C\,x^2a\,d^4+42C\,x^2bc\,d^3+42Dx^2ac\,d^3-48Dx\cdots}{}$
trager	$-\frac{2(-15Db\,x^4d^4-21C\,x^3b\,d^4-21Dx^3a\,d^4+24Dx^3bc\,d^3-35B\,x^2b\,d^4-35C\,x^2a\,d^4+42C\,x^2bc\,d^3+42Dx^2ac\,d^3-48Dx\cdots}{}$
orering	$-\frac{2(-15Db\,x^4d^4-21C\,x^3b\,d^4-21Dx^3a\,d^4+24Dx^3bc\,d^3-35B\,x^2b\,d^4-35C\,x^2a\,d^4+42C\,x^2bc\,d^3+42Dx^2ac\,d^3-48Dx\cdots}{}$
derivativedivides	$\frac{\frac{2bD(xd+c)^{\frac{7}{2}}}{7}+\frac{2Cbd(xd+c)^{\frac{5}{2}}}{5}+\frac{2Dad(xd+c)^{\frac{5}{2}}}{5}-\frac{8Dbc(xd+c)^{\frac{5}{2}}}{5}+\frac{2Bb\,d^2(xd+c)^{\frac{3}{2}}}{3}+\frac{2Ca\,d^2(xd+c)^{\frac{3}{2}}}{3}-2Cbcd(xd+c)^{\frac{3}{2}}-2Dac\cdots}{}$
default	$\frac{\frac{2bD(xd+c)^{\frac{7}{2}}}{7}+\frac{2Cbd(xd+c)^{\frac{5}{2}}}{5}+\frac{2Dad(xd+c)^{\frac{5}{2}}}{5}-\frac{8Dbc(xd+c)^{\frac{5}{2}}}{5}+\frac{2Bb\,d^2(xd+c)^{\frac{3}{2}}}{3}+\frac{2Ca\,d^2(xd+c)^{\frac{3}{2}}}{3}-2Cbcd(xd+c)^{\frac{3}{2}}-2Dac\cdots}{}$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((-1/7*D*b*x^4+1/5*(-C*b-D*a)*x^3+1/3*(-B*b-C*a)*x^2+(-A*b-B*a)*x+A*a)*
d^4-2*(-4/35*D*b*x^3+1/5*(-C*b-D*a)*x^2+2/3*(-B*b-C*a)*x+A*b+B*a)*c*d^3+8/
3*(-6/35*D*b*x^2+3/5*(-C*b-D*a)*x+B*b+C*a)*c^2*d^2-16/5*c^3*(-4/7*D*b*x+C*
b+D*a)*d+128/35*D*b*c^4)/(d*x+c)^(1/2)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx = \frac{2\left(15\,Dbd^4x^4-384\,Dbc^4-105\,Aad^4+336\,(Da+Cb)c^3d-280\cdots\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*D*b*d^4*x^4 - 384*D*b*c^4 - 105*A*a*d^4 + 336*(D*a + C*b)*c^3*d
- 280*(C*a + B*b)*c^2*d^2 + 210*(B*a + A*b)*c*d^3 - 3*(8*D*b*c*d^3 - 7*(D*
a + C*b)*d^4)*x^3 + (48*D*b*c^2*d^2 - 42*(D*a + C*b)*c*d^3 + 35*(C*a + B*b
)*d^4)*x^2 - (192*D*b*c^3*d - 168*(D*a + C*b)*c^2*d^2 + 140*(C*a + B*b)*c*
d^3 - 105*(B*a + A*b)*d^4)*x)*sqrt(d*x + c)/(d^6*x + c*d^5)
```

Sympy [A] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{Db(c+dx)^{\frac{7}{2}}}{7d^4}+\frac{(c+dx)^{\frac{5}{2}}(Cbd+Dad-4Dbc)}{5d^4}+\frac{(c+dx)^{\frac{3}{2}}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6Dbc\right.}{3d^4}\right.}{} \\ \frac{Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}}{c^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Piecewise((2*(D*b*(c + d*x)**(7/2)/(7*d**4) + (c + d*x)**(5/2)*(C*b*d + D*
a*d - 4*D*b*c)/(5*d**4) + (c + d*x)**(3/2)*(B*b*d**2 + C*a*d**2 - 3*C*b*c*
d - 3*D*a*c*d + 6*D*b*c**2)/(3*d**4) + sqrt(c + d*x)*(A*b*d**3 + B*a*d**3
- 2*B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D*b*c**3)/
d**4 + (a*d - b*c)*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(d**4*sqrt(c +
 d*x)))/d, Ne(d, 0)), ((A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/4 + x**3*(B*
b + C*a)/3 + x**2*(A*b + B*a)/2)/c**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx = \frac{2\left(\frac{15\,(dx+c)^{\frac{7}{2}}Db-21\,(4\,Dbc-(Da+Cb)d)(dx+c)^{\frac{5}{2}}+35\,(6\,Dbc^2-3\,(Da+Cb)cd+(Ca+\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(d*x + c)^(7/2)*D*b - 21*(4*D*b*c - (D*a + C*b)*d)*(d*x + c)^(5
/2) + 35*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*(d*x + c)^(3/2)
 - 105*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 - (B*a + A*b
)*d^3)*sqrt(d*x + c))/d^4 - 105*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c^3*d + (
C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)/(sqrt(d*x + c)*d^4))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx =$$
$$-\frac{2\left(Dbc^4-Dac^3d-Cbc^3d+Cac^2d^2+Bbc^2d^2-Bacd^3-Abcd^3+Aad^4\right)}{\sqrt{dx+c}d^5}$$
$$+\frac{2\left(15\,(dx+c)^{\frac{7}{2}}Dbd^{30}-84\,(dx+c)^{\frac{5}{2}}Dbcd^{30}+210\,(dx+c)^{\frac{3}{2}}Dbc^2d^{30}-420\,\sqrt{dx+c}Dbc^3d^{30}+21\,(dx+c\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-2*(D*b*c^4 - D*a*c^3*d - C*b*c^3*d + C*a*c^2*d^2 + B*b*c^2*d^2 - B*a*c*d^
3 - A*b*c*d^3 + A*a*d^4)/(sqrt(d*x + c)*d^5) + 2/105*(15*(d*x + c)^(7/2)*D
*b*d^30 - 84*(d*x + c)^(5/2)*D*b*c*d^30 + 210*(d*x + c)^(3/2)*D*b*c^2*d^30
 - 420*sqrt(d*x + c)*D*b*c^3*d^30 + 21*(d*x + c)^(5/2)*D*a*d^31 + 21*(d*x
+ c)^(5/2)*C*b*d^31 - 105*(d*x + c)^(3/2)*D*a*c*d^31 - 105*(d*x + c)^(3/2)
*C*b*c*d^31 + 315*sqrt(d*x + c)*D*a*c^2*d^31 + 315*sqrt(d*x + c)*C*b*c^2*d
^31 + 35*(d*x + c)^(3/2)*C*a*d^32 + 35*(d*x + c)^(3/2)*B*b*d^32 - 210*sqrt
(d*x + c)*C*a*c*d^32 - 210*sqrt(d*x + c)*B*b*c*d^32 + 105*sqrt(d*x + c)*B*
a*d^33 + 105*sqrt(d*x + c)*A*b*d^33)/d^35
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx=\int \frac{\left(a+b\,x\right)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2),x)
```

output
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx=\frac{\frac{2}{7}b\,d^4x^4+\frac{2}{5}a\,d^4x^3-\frac{2}{35}bc\,d^3x^3-\frac{2}{15}ac\,d^3x^2+\frac{2}{3}b^2d^3x^2+\frac{4}{35}b\,c^2d^2x}{}$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output
```
(2*( - 105*a**2*d**3 + 420*a*b*c*d**2 + 210*a*b*d**3*x + 56*a*c**3*d + 28*
a*c**2*d**2*x - 7*a*c*d**3*x**2 + 21*a*d**4*x**3 - 280*b**2*c**2*d - 140*b
**2*c*d**2*x + 35*b**2*d**3*x**2 - 48*b*c**4 - 24*b*c**3*d*x + 6*b*c**2*d*
*2*x**2 - 3*b*c*d**3*x**3 + 15*b*d**4*x**4))/(105*sqrt(c + d*x)*d**4)
```

3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx$

Optimal result 821
Mathematica [A] (verified) 821
Rubi [A] (verified) 822
Maple [A] (verified) 823
Fricas [A] (verification not implemented) 824
Sympy [A] (verification not implemented) 824
Maxima [A] (verification not implemented) 825
Giac [A] (verification not implemented) 825
Mupad [F(-1)] 826
Reduce [B] (verification not implemented) 826

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx=-\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^4\sqrt{c+dx}}-\frac{2(2cCd-Bd^2-3c^2D)\sqrt{c+dx}}{d^4}+\frac{2(Cd-3cD)(c+dx)^{3/2}}{3d^4}+\frac{2D(c+dx)^{5/2}}{5d^4}$$

output
```
(-2*A*d^3+2*B*c*d^2-2*C*c^2*d+2*D*c^3)/d^4/(d*x+c)^(1/2)-2*(-B*d^2+2*C*c*d
-3*D*c^2)*(d*x+c)^(1/2)/d^4+2/3*(C*d-3*D*c)*(d*x+c)^(3/2)/d^4+2/5*D*(d*x+c
)^(5/2)/d^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx=\frac{2(48c^3D-8c^2d(5C-3Dx)+2cd^2(15B-x(10C+3Dx))+d^3(-15A+\cdots}{15d^4\sqrt{c+dx}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^(3/2),x]
```

output
```
(2*(48*c^3*D - 8*c^2*d*(5*C - 3*D*x) + 2*c*d^2*(15*B - x*(10*C + 3*D*x)) +
 d^3*(-15*A + x*(15*B + 5*C*x + 3*D*x^2))))/(15*d^4*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)^{3/2}}+\frac{Bd^2+3c^2D-2cCd}{d^3\sqrt{c+dx}}+\frac{\sqrt{c+dx}(Cd-3cD)}{d^3}+\frac{D(c+dx)^{3/2}}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^4\sqrt{c+dx}}-\frac{2\sqrt{c+dx}\left(-Bd^2-3c^2D+2cCd\right)}{d^4}+\frac{2(c+dx)^{3/2}(Cd-3cD)}{3d^4}+\frac{2D(c+dx)^{5/2}}{5d^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^(3/2),x]
```

output
```
(-2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^4*Sqrt[c + d*x]) - (2*(2*c*C*d
 - B*d^2 - 3*c^2*D)*Sqrt[c + d*x])/d^4 + (2*(C*d - 3*c*D)*(c + d*x)^(3/2))
/(3*d^4) + (2*D*(c + d*x)^(5/2))/(5*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{(6Dx^3+10Cx^2+30Bx-30A)d^3+60(-\frac{1}{5}Dx^2-\frac{2}{3}Cx+B)c\,d^2-80\left(-\frac{3Dx}{5}+C\right)c^2d+96Dc^3}{15\sqrt{xd+c}\,d^4}$
gosper	$-\frac{2(-3Dx^3d^3-5Cx^2d^3+6Dx^2c\,d^2-15Bx\,d^3+20Cxc\,d^2-24Dx\,c^2d+15A\,d^3-30Bc\,d^2+40C\,c^2d-48Dc^3)}{15\sqrt{xd+c}\,d^4}$
trager	$-\frac{2(-3Dx^3d^3-5Cx^2d^3+6Dx^2c\,d^2-15Bx\,d^3+20Cxc\,d^2-24Dx\,c^2d+15A\,d^3-30Bc\,d^2+40C\,c^2d-48Dc^3)}{15\sqrt{xd+c}\,d^4}$
orering	$-\frac{2(-3Dx^3d^3-5Cx^2d^3+6Dx^2c\,d^2-15Bx\,d^3+20Cxc\,d^2-24Dx\,c^2d+15A\,d^3-30Bc\,d^2+40C\,c^2d-48Dc^3)}{15\sqrt{xd+c}\,d^4}$
derivativedivides	$\frac{\frac{2D(xd+c)^{\frac{5}{2}}}{5}+\frac{2Cd(xd+c)^{\frac{3}{2}}}{3}-2Dc(xd+c)^{\frac{3}{2}}+2B\,d^2\sqrt{xd+c}-4Ccd\sqrt{xd+c}+6Dc^2\sqrt{xd+c}-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{\sqrt{xd+c}}}{d^4}$
default	$\frac{\frac{2D(xd+c)^{\frac{5}{2}}}{5}+\frac{2Cd(xd+c)^{\frac{3}{2}}}{3}-2Dc(xd+c)^{\frac{3}{2}}+2B\,d^2\sqrt{xd+c}-4Ccd\sqrt{xd+c}+6Dc^2\sqrt{xd+c}-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{\sqrt{xd+c}}}{d^4}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((6*D*x^3+10*C*x^2+30*B*x-30*A)*d^3+60*(-1/5*D*x^2-2/3*C*x+B)*c*d^2-8
0*(-3/5*D*x+C)*c^2*d+96*D*c^3)/(d*x+c)^(1/2)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx = \frac{2\left(3\,Dd^3x^3+48\,Dc^3-40\,Cc^2d+30\,Bcd^2-15\,Ad^3-(6\,Dcd^2-5\,Cd^3)x^2\right.}{15\,(d^5x+cd^4)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*D*d^3*x^3 + 48*D*c^3 - 40*C*c^2*d + 30*B*c*d^2 - 15*A*d^3 - (6*D*c
*d^2 - 5*C*d^3)*x^2 + (24*D*c^2*d - 20*C*c*d^2 + 15*B*d^3)*x)*sqrt(d*x + c
)/(d^5*x + c*d^4)
```

Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{D(c+dx)^{\frac{5}{2}}}{5d^3}+\frac{(c+dx)^{\frac{3}{2}}(Cd-3Dc)}{3d^3}+\frac{\sqrt{c+dx}\left(Bd^2-2Ccd+3Dc^2\right)}{d^3}+\frac{-Ad^3+Bcd^2-Cc^2d+Dc^3}{d^3\sqrt{c+dx}}\right)}{d} & \text{for } d \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}}{c^{\frac{3}{2}}} & \text{other} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Piecewise((2*(D*(c + d*x)**(5/2)/(5*d**3) + (c + d*x)**(3/2)*(C*d - 3*D*c)
/(3*d**3) + sqrt(c + d*x)*(B*d**2 - 2*C*c*d + 3*D*c**2)/d**3 + (-A*d**3 +
B*c*d**2 - C*c**2*d + D*c**3)/(d**3*sqrt(c + d*x)))/d, Ne(d, 0)), ((A*x +
B*x**2/2 + C*x**3/3 + D*x**4/4)/c**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx = \frac{2\left(\frac{3\,(dx+c)^{\frac{5}{2}}D-5\,(3\,Dc-Cd)(dx+c)^{\frac{3}{2}}+15\,(3\,Dc^2-2\,Ccd+Bd^2)\sqrt{dx+c}}{d^3}+\frac{15\,(Dc^3-Cc^2d+Bcd^2-\cdots}{\sqrt{dx+c}d^3}\cdots\right.}{15\,d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*(d*x + c)^(5/2)*D - 5*(3*D*c - C*d)*(d*x + c)^(3/2) + 15*(3*D*c^2
 - 2*C*c*d + B*d^2)*sqrt(d*x + c))/d^3 + 15*(D*c^3 - C*c^2*d + B*c*d^2 - A
*d^3)/(sqrt(d*x + c)*d^3))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx = \frac{2\,(Dc^3-Cc^2d+Bcd^2-Ad^3)}{\sqrt{dx+c}d^4} + \frac{2\left(3\,(dx+c)^{\frac{5}{2}}Dd^{16}-15\,(dx+c)^{\frac{3}{2}}Dcd^{16}+45\,\sqrt{dx+c}Dc^2d^{16}+5\,(dx+c)^{\frac{3}{2}}Cd^{17}-30\,\sqrt{dx+c}Ccd^{17}+\cdots\right.}{15\,d^{20}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
2*(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)/(sqrt(d*x + c)*d^4) + 2/15*(3*(d*x +
 c)^(5/2)*D*d^16 - 15*(d*x + c)^(3/2)*D*c*d^16 + 45*sqrt(d*x + c)*D*c^2*d^
16 + 5*(d*x + c)^(3/2)*C*d^17 - 30*sqrt(d*x + c)*C*c*d^17 + 15*sqrt(d*x +
c)*B*d^18)/d^20
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(c+d\,x)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(3/2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{3/2}}\,dx=\frac{\frac{2}{5}d^3x^3-\frac{2}{15}c\,d^2x^2+2b\,d^2x+\frac{8}{15}c^2dx-2a\,d^2+4bcd+\frac{16}{15}c^3}{\sqrt{dx+c}\,d^3}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output
```
(2*( - 15*a*d**2 + 30*b*c*d + 15*b*d**2*x + 8*c**3 + 4*c**2*d*x - c*d**2*x
**2 + 3*d**3*x**3))/(15*sqrt(c + d*x)*d**3)
```

3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx$

Optimal result 827
Mathematica [A] (verified) 828
Rubi [A] (verified) 828
Maple [A] (verified) 829
Fricas [B] (verification not implemented) 830
Sympy [A] (verification not implemented) 831
Maxima [F(-2)] 831
Giac [A] (verification not implemented) 832
Mupad [F(-1)] 832
Reduce [B] (verification not implemented) 833

Optimal result

Integrand size = 32, antiderivative size = 174

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^3(bc-ad)\sqrt{c+dx}} + \frac{2(bCd-2bcD-adD)\sqrt{c+dx}}{b^2d^3} + \frac{2D(c+dx)^{3/2}}{3bd^3} - \frac{2(Ab^3-a(b^2B-abC+a^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}$$

output
```
2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^3/(-a*d+b*c)/(d*x+c)^(1/2)+2*(C*b*d-D*a*
d-2*D*b*c)*(d*x+c)^(1/2)/b^2/d^3+2/3*D*(d*x+c)^(3/2)/b/d^3-2*(A*b^3-a*(B*b
^2-C*a*b+D*a^2))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(
-a*d+b*c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx = \frac{-6a^2d^2D(c+dx)+2abd(c+dx)(3Cd-2cD+dDx)+2b^2(-3Ad^3+8c^3 \cdots}{3b^2d^3(-bc+ad)\sqrt{c+dx}}$$
$$-\frac{2(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{5/2}(-bc+ad)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^(3/2)),x]
```

output
```
(-6*a^2*d^2*D*(c + d*x) + 2*a*b*d*(c + d*x)*(3*C*d - 2*c*D + d*D*x) + 2*b^
2*(-3*A*d^3 + 8*c^3*D + c^2*(-6*C*d + 4*d*D*x) + c*d^2*(3*B - x*(3*C + D*x
))))/(3*b^2*d^3*(-(b*c) + a*d)*Sqrt[c + d*x]) - (2*(A*b^3 - a*(b^2*B - a*b
*C + a^2*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(b^(5/2)*
(-(b*c) + a*d)^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2122, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx$$
$$\Big\downarrow \ 2122$$
$$\int \left(\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{b^2(a+bx)\sqrt{c+dx}(bc-ad)}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^2(c+dx)^{3/2}(ad-bc)}+\frac{-adD-bcD+bCd}{b^2d^2\sqrt{c+dx}}+\frac{Dx}{bd\sqrt{c+dx}}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}+\frac{2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}+\frac{2\sqrt{c+dx}(-adD-bcD+bCd)}{b^2d^3}+\frac{2D(c+dx)^{3/2}}{3bd^3}-\frac{2cD\sqrt{c+dx}}{bd^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^(3/2)),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^3*(b*c - a*d)*Sqrt[c + d*x]) -
(2*c*D*Sqrt[c + d*x])/(b*d^3) + (2*(b*C*d - b*c*D - a*d*D)*Sqrt[c + d*x])/
(b^2*d^3) + (2*D*(c + d*x)^(3/2))/(3*b*d^3) - (2*(A*b^3 - a*(b^2*B - a*b*C
 + a^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2122
```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_))/((a_.) + (b_.)*(x_)), x_Symbol] :> In
t[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{\frac{2\sqrt{xd+c}\,(Dbdx+3Cbd-3Dad-5Dbc)}{3b^2}-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{(ad-bc)\sqrt{xd+c}}-\frac{2d^3\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)b^2\sqrt{(ad-bc)b}}}{d^3}$
derivativedivides	$\frac{\frac{2\left(\frac{D(xd+c)^{\frac{3}{2}}b}{3}+Cdb\sqrt{xd+c}-Dad\sqrt{xd+c}-2Dbc\sqrt{xd+c}\right)}{b^2}-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{(ad-bc)\sqrt{xd+c}}-\frac{2d^3\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\arctan}{(ad-bc)b^2\sqrt{(ad-bc)b}}}{d^3}$
default	$\frac{\frac{2\left(\frac{D(xd+c)^{\frac{3}{2}}b}{3}+Cdb\sqrt{xd+c}-Dad\sqrt{xd+c}-2Dbc\sqrt{xd+c}\right)}{b^2}-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{(ad-bc)\sqrt{xd+c}}-\frac{2d^3\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\arctan}{(ad-bc)b^2\sqrt{(ad-bc)b}}}{d^3}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/3*(d*x+c)^(1/2)*(D*b*d*x+3*C*b*d-3*D*a*d-5*D*b*c)/b^2-(A*d^3-B*c*d^2+
C*c^2*d-D*c^3)/(a*d-b*c)/(d*x+c)^(1/2)-d^3*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(
a*d-b*c)/b^2/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2
)))/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(156) = 312$.

Time = 0.10 (sec) , antiderivative size = 834, normalized size of antiderivative = 4.79

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)(c + dx)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*((D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^4*x + (D*a^3 - C*a^2*b + B*
a*b^2 - A*b^3)*c*d^3)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d + 2*sqr
t(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(8*D*b^4*c^4 + 3*A*a*b^3*d^
4 - 2*(5*D*a*b^3 + 3*C*b^4)*c^3*d - (D*a^2*b^2 - 9*C*a*b^3 - 3*B*b^4)*c^2*
d^2 + 3*(D*a^3*b - C*a^2*b^2 - B*a*b^3 - A*b^4)*c*d^3 - (D*b^4*c^2*d^2 - 2
*D*a*b^3*c*d^3 + D*a^2*b^2*d^4)*x^2 + (4*D*b^4*c^3*d - (5*D*a*b^3 + 3*C*b^
4)*c^2*d^2 - 2*(D*a^2*b^2 - 3*C*a*b^3)*c*d^3 + 3*(D*a^3*b - C*a^2*b^2)*d^4
)*x)*sqrt(d*x + c))/(b^5*c^3*d^3 - 2*a*b^4*c^2*d^4 + a^2*b^3*c*d^5 + (b^5*
c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x), -2/3*(3*((D*a^3 - C*a^2*b + B*a
*b^2 - A*b^3)*d^4*x + (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*c*d^3)*sqrt(-b^2
*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (8*
D*b^4*c^4 + 3*A*a*b^3*d^4 - 2*(5*D*a*b^3 + 3*C*b^4)*c^3*d - (D*a^2*b^2 - 9
*C*a*b^3 - 3*B*b^4)*c^2*d^2 + 3*(D*a^3*b - C*a^2*b^2 - B*a*b^3 - A*b^4)*c*
d^3 - (D*b^4*c^2*d^2 - 2*D*a*b^3*c*d^3 + D*a^2*b^2*d^4)*x^2 + (4*D*b^4*c^3
*d - (5*D*a*b^3 + 3*C*b^4)*c^2*d^2 - 2*(D*a^2*b^2 - 3*C*a*b^3)*c*d^3 + 3*(
D*a^3*b - C*a^2*b^2)*d^4)*x)*sqrt(d*x + c))/(b^5*c^3*d^3 - 2*a*b^4*c^2*d^4
 + a^2*b^3*c*d^5 + (b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x)]
```

Sympy [A] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.51

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{D(c+dx)^{\frac{3}{2}}}{3bd^2}+\frac{-Ad^3+Bcd^2-Cc^2d+Dc^3}{d^2\sqrt{c+dx}(ad-bc)}+\frac{\sqrt{c+dx}(Cbd-Dad-2Dbc)}{b^2d^2}+\frac{d\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\operatorname{atan}\left(\right.}{b^3\sqrt{\frac{ad-bc}{b}}(ad-bc)}\right.}{d} \\ \frac{\frac{Dx^3}{3b}+\frac{x^2(Cb-Da)}{2b^2}+\frac{x\left(Bb^2-Cab+Da^2\right)}{b^3}-\frac{\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\left(\begin{cases}\frac{x}{a} & \text{for } b=0\\ \frac{\log(a+bx)}{b} & \text{otherwise}\end{cases}\right)}{b^3}}{c^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(3/2),x)
```

output
```
Piecewise((2*(D*(c + d*x)**(3/2)/(3*b*d**2) + (-A*d**3 + B*c*d**2 - C*c**2
*d + D*c**3)/(d**2*sqrt(c + d*x)*(a*d - b*c)) + sqrt(c + d*x)*(C*b*d - D*a
*d - 2*D*b*c)/(b**2*d**2) + d*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*ata
n(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b)*(a*d - b*c)
))/d, Ne(d, 0)), ((D*x**3/(3*b) + x**2*(C*b - D*a)/(2*b**2) + x*(B*b**2 -
C*a*b + D*a**2)/b**3 - (-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*Piecewise(
(x/a, Eq(b, 0)), (log(a + b*x)/b, True))/b**3)/c**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx = -\frac{2\,(Da^3-Ca^2b+Bab^2-Ab^3)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c-ab^2d)\sqrt{-b^2c+abd}} - \frac{2\,(Dc^3-Cc^2d+Bcd^2-Ad^3)}{(bcd^3-ad^4)\sqrt{dx+c}} + \frac{2\left((dx+c)^{\frac{3}{2}}Db^2d^6-6\sqrt{dx+c}Db^2cd^6-3\sqrt{dx+c}Dabd^7+3\sqrt{dx+c}Cb^2d^7\right)}{3\,b^3d^9}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-2*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
+ a*b*d))/((b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) - 2*(D*c^3 - C*c^2*d +
B*c*d^2 - A*d^3)/((b*c*d^3 - a*d^4)*sqrt(d*x + c)) + 2/3*((d*x + c)^(3/2)*
D*b^2*d^6 - 6*sqrt(d*x + c)*D*b^2*c*d^6 - 3*sqrt(d*x + c)*D*a*b*d^7 + 3*sq
rt(d*x + c)*C*b^2*d^7)/(b^3*d^9)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{3/2}}\,dx = \frac{2\sqrt{b}\,\sqrt{dx+c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^2d^2-2a^2bc\,d^2-2a^2b\,d^3x-2a\,b^3}{\sqrt{dx+c}\,b^3d^2\,(}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(3/2),x)
```

output
```
(2*(3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b
)*sqrt(a*d - b*c)))*a**2*d**2 - 3*a**2*b*c*d**2 - 3*a**2*b*d**3*x - 3*a*b*
*3*d**2 + a*b**2*c**2*d + 2*a*b**2*c*d**2*x + a*b**2*d**3*x**2 + 3*b**4*c*
d + 2*b**3*c**3 + b**3*c**2*d*x - b**3*c*d**2*x**2))/(3*sqrt(c + d*x)*b**3
*d**2*(a*d - b*c))
```

3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx$

Optimal result 834
Mathematica [A] (verified) 835
Rubi [A] (verified) 835
Maple [A] (verified) 838
Fricas [B] (verification not implemented) 838
Sympy [F(-1)] 839
Maxima [F(-2)] 840
Giac [A] (verification not implemented) 840
Mupad [F(-1)] 841
Reduce [B] (verification not implemented) 841

Optimal result

Integrand size = 32, antiderivative size = 220

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx=-\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^2(bc-ad)^2\sqrt{c+dx}}+\frac{2D\sqrt{c+dx}}{b^2d^2}-\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b^2(bc-ad)^2(a+bx)}-\frac{(b^3(2Bc-3Ad)-ab^2(4cC-Bd)-3a^3dD+a^2b(Cd+6cD))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{5/2}}$$

output

```
(-2*A*d^3+2*B*c*d^2-2*C*c^2*d+2*D*c^3)/d^2/(-a*d+b*c)^2/(d*x+c)^(1/2)+2*D*
(d*x+c)^(1/2)/b^2/d^2-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^2/(-a*
d+b*c)^2/(b*x+a)-(b^3*(-3*A*d+2*B*c)-a*b^2*(-B*d+4*C*c)-3*a^3*d*D+a^2*b*(C
*d+6*D*c))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b
*c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx = \frac{3a^3d^2D(c+dx)+a^2bd(c+dx)(-Cd-4cD+2dDx)+b^3(-Ad^2(c+3dx)}{}$$
$$+\frac{(b^3(2Bc-3Ad)+ab^2(-4cC+Bd)-3a^3dD+a^2b(Cd+6cD))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{5/2}(-bc+ad)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

output
```
(3*a^3*d^2*D*(c + d*x) + a^2*b*d*(c + d*x)*(-(C*d) - 4*c*D + 2*d*D*x) + b^
3*(-(A*d^2*(c + 3*d*x)) + 2*c*x*(-(c*C*d) + B*d^2 + 2*c^2*D + c*d*D*x)) +
a*b^2*(4*c^3*D + d^3*(-2*A + B*x) - 2*c^2*d*(C + D*x) + c*d^2*(3*B - 4*D*x
^2)))/(b^2*d^2*(b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x]) + ((b^3*(2*B*c - 3*A
*d) + a*b^2*(-4*c*C + B*d) - 3*a^3*d*D + a^2*b*(C*d + 6*c*D))*ArcTan[(Sqrt
[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(b^(5/2)*(-(b*c) + a*d)^(5/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2124, 27, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx$$
$$\Big\downarrow \text{2124}$$
$$-\frac{\int -\frac{2\left(c-\frac{ad}{b}\right)Dx^2+\frac{2(bc-ad)(bC-aD)x}{b^2}+\frac{dDa^3-b(Cd-2cD)a^2-b^2(2cC-Bd)a+b^3(2Bc-3Ad)}{b^3}}{2(a+bx)(c+dx)^{3/2}}dx}{bc-ad}-\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt{c+dx}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\frac{dDa^3}{b^3}-\frac{(Cd-2cD)a^2}{b^2}-\frac{(2cC-Bd)a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc-3Ad+\frac{2(bc-ad)(bC-aD)x}{b^2}}{(a+bx)(c+dx)^{3/2}}dx}{2(bc-ad)}-\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt{c+dx}(bc-ad)}$$

$\Big\downarrow$ 1192

$$\frac{\int \frac{-2Dc^3+2Cdc^2-2Bd^2c-2\left(c-\frac{ad}{b}\right)D(c+dx)^2+d^3\left(3A-\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{2(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(c+dx)(bc-ad-b(c+dx))}d\sqrt{c+dx}}{d^2(bc-ad)}-\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt{c+dx}(bc-ad)}$$

$\Big\downarrow$ 1584

$$\frac{\int \left(\frac{\left(3dDa^3-b(Cd+6cD)a^2+b^2(4cC-Bd)a-b^3(2Bc-3Ad)\right)d^2}{b^2(bc-ad)(bc-ad-b(c+dx))}+\frac{2(bc-ad)D}{b^2}+\frac{\left(-2Dc^3+2Cdc^2-2Bd^2c+3Ad^3\right)b^3-aBd^3b^2+a^2Cd^3b-a^3d^3 \cdots}{b^3(bc-ad)(c+dx)}\right.}{d^2(bc-ad)}$$

$$\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt{c+dx}(bc-ad)}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-3a^3dD+a^2b(6cD+Cd)-ab^2(4cC-Bd)+b^3(2Bc-3Ad)\right)}{b^{5/2}(bc-ad)^{3/2}}+\frac{a^3d^3D-a^2bCd^3+ab^2Bd^3-\left(b^3\left(3Ad^3-2Bcd^2-2c^3D \cdots\right.\right.}{b^3\sqrt{c+dx}(bc-ad)}}{d^2(bc-ad)}$$

$$\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt{c+dx}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

output
```
-((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)/((b*c - a*d)*(a + b*x)*Sqrt[c + d*
x])) + ((a*b^2*B*d^3 - a^2*b*C*d^3 + a^3*d^3*D - b^3*(2*c^2*C*d - 2*B*c*d^
2 + 3*A*d^3 - 2*c^3*D))/(b^3*(b*c - a*d)*Sqrt[c + d*x]) + (2*(b*c - a*d)*D
*Sqrt[c + d*x])/b^2 - (d^2*(b^3*(2*B*c - 3*A*d) - a*b^2*(4*c*C - B*d) - 3*
a^3*d*D + a^2*b*(C*d + 6*c*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c -
a*d]])/(b^(5/2)*(b*c - a*d)^(3/2)))/(d^2*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{2D\sqrt{xd+c}}{b^2}-\frac{2d^2\left(\frac{\left(\frac{1}{2}b^3dA-\frac{1}{2}Ba\,b^2d+\frac{1}{2}C\,a^2bd-\frac{1}{2}a^3dD\right)\sqrt{xd+c}}{(xd+c)b+ad-bc}+\frac{\left(3b^3dA-Ba\,b^2d-2B\,b^3c-C\,a^2bd+4a\,b^2cC+3a^3dD-6a^2bcD\right)}{2\sqrt{(ad-bc)b}}\right.}{(ad-bc)^2b^2}}{d^2}$
default	$\frac{\frac{2D\sqrt{xd+c}}{b^2}-\frac{2d^2\left(\frac{\left(\frac{1}{2}b^3dA-\frac{1}{2}Ba\,b^2d+\frac{1}{2}C\,a^2bd-\frac{1}{2}a^3dD\right)\sqrt{xd+c}}{(xd+c)b+ad-bc}+\frac{\left(3b^3dA-Ba\,b^2d-2B\,b^3c-C\,a^2bd+4a\,b^2cC+3a^3dD-6a^2bcD\right)}{2\sqrt{(ad-bc)b}}\right.}{(ad-bc)^2b^2}}{d^2}$
pseudoelliptic	$-\frac{2\left(\frac{3\left(\left(b^3A-\frac{1}{3}a\,b^2B-\frac{1}{3}a^2bC+a^3D\right)d-\frac{2bc\left(B\,b^2-2Cab+3Da^2\right)}{3}\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)d^2(bx+a)\sqrt{xd+c}}{2}+\left(\left(\frac{3A\,b^3x}{2}+a\left(-\right.\right.\right.\right.}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(D/b^2*(d*x+c)^(1/2)-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^2/(d*x+
c)^(1/2)-d^2/(a*d-b*c)^2/b^2*((1/2*b^3*d*A-1/2*B*a*b^2*d+1/2*C*a^2*b*d-1/2
*a^3*d*D)*(d*x+c)^(1/2)/((d*x+c)*b+a*d-b*c)+1/2*(3*A*b^3*d-B*a*b^2*d-2*B*b
^3*c-C*a^2*b*d+4*C*a*b^2*c+3*D*a^3*d-6*D*a^2*b*c)/((a*d-b*c)*b)^(1/2)*arct
an(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(204) = 408$.

Time = 0.12 (sec) , antiderivative size = 1505, normalized size of antiderivative = 6.84

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/2*((2*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c^2*d^2 - (3*D*a^4 - C*a^3*b
 - B*a^2*b^2 + 3*A*a*b^3)*c*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c*d
^3 - (3*D*a^3*b - C*a^2*b^2 - B*a*b^3 + 3*A*b^4)*d^4)*x^2 + (2*(3*D*a^2*b^
2 - 2*C*a*b^3 + B*b^4)*c^2*d^2 + 3*(D*a^3*b - C*a^2*b^2 + B*a*b^3 - A*b^4)
*c*d^3 - (3*D*a^4 - C*a^3*b - B*a^2*b^2 + 3*A*a*b^3)*d^4)*x)*sqrt(b^2*c -
a*b*d)*log((b*d*x + 2*b*c - a*d + 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*
x + a)) - 2*(4*D*a*b^4*c^4 + 2*A*a^2*b^3*d^4 - 2*(4*D*a^2*b^3 + C*a*b^4)*c
^3*d + (7*D*a^3*b^2 + C*a^2*b^3 + 3*B*a*b^4 - A*b^5)*c^2*d^2 - (3*D*a^4*b
- C*a^3*b^2 + 3*B*a^2*b^3 + A*a*b^4)*c*d^3 + 2*(D*b^5*c^3*d - 3*D*a*b^4*c^
2*d^2 + 3*D*a^2*b^3*c*d^3 - D*a^3*b^2*d^4)*x^2 + (4*D*b^5*c^4 - 2*(3*D*a*b
^4 + C*b^5)*c^3*d + 2*(C*a*b^4 + B*b^5)*c^2*d^2 + (5*D*a^3*b^2 - C*a^2*b^3
 - B*a*b^4 - 3*A*b^5)*c*d^3 - (3*D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - 3*A*a*b
^4)*d^4)*x)*sqrt(d*x + c))/(a*b^6*c^4*d^2 - 3*a^2*b^5*c^3*d^3 + 3*a^3*b^4*
c^2*d^4 - a^4*b^3*c*d^5 + (b^7*c^3*d^3 - 3*a*b^6*c^2*d^4 + 3*a^2*b^5*c*d^5
 - a^3*b^4*d^6)*x^2 + (b^7*c^4*d^2 - 2*a*b^6*c^3*d^3 + 2*a^3*b^4*c*d^5 - a
^4*b^3*d^6)*x), ((2*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c^2*d^2 - (3*D*a^4
 - C*a^3*b - B*a^2*b^2 + 3*A*a*b^3)*c*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 +
B*b^4)*c*d^3 - (3*D*a^3*b - C*a^2*b^2 - B*a*b^3 + 3*A*b^4)*d^4)*x^2 + (2*(
3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c^2*d^2 + 3*(D*a^3*b - C*a^2*b^2 + B*a*b^
3 - A*b^4)*c*d^3 - (3*D*a^4 - C*a^3*b - B*a^2*b^2 + 3*A*a*b^3)*d^4)*x)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx = \frac{(6\,Da^2bc-4\,Cab^2c+2\,Bb^3c-3\,Da^3d+Ca^2bd+Bab^2d-3\,Ab^3d)\arctan}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)\sqrt{-b^2c+abd}}$$
$$+\frac{2\,(dx+c)Db^3c^3-2\,Db^3c^4-2\,(dx+c)Cb^3c^2d+2\,Dab^2c^3d+2\,Cb^3c^3d+2\,(dx+c)Bb^3cd^2-2\,Cab^2c^2d^2}{(b^4c^2d^2-2\,ab^3cd^3+a^2b^2c}$$
$$+\frac{2\sqrt{dx+c}D}{b^2d^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
(6*D*a^2*b*c - 4*C*a*b^2*c + 2*B*b^3*c - 3*D*a^3*d + C*a^2*b*d + B*a*b^2*d
 - 3*A*b^3*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^2 - 2*a
*b^3*c*d + a^2*b^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x + c)*D*b^3*c^3 - 2
*D*b^3*c^4 - 2*(d*x + c)*C*b^3*c^2*d + 2*D*a*b^2*c^3*d + 2*C*b^3*c^3*d + 2
*(d*x + c)*B*b^3*c*d^2 - 2*C*a*b^2*c^2*d^2 - 2*B*b^3*c^2*d^2 + (d*x + c)*D
*a^3*d^3 - (d*x + c)*C*a^2*b*d^3 + (d*x + c)*B*a*b^2*d^3 - 3*(d*x + c)*A*b
^3*d^3 + 2*B*a*b^2*c*d^3 + 2*A*b^3*c*d^3 - 2*A*a*b^2*d^4)/((b^4*c^2*d^2 -
2*a*b^3*c*d^3 + a^2*b^2*d^4)*((d*x + c)^(3/2)*b - sqrt(d*x + c)*b*c + sqrt
(d*x + c)*a*d)) + 2*sqrt(d*x + c)*D/(b^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^2\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.33

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{3/2}}\,dx=\frac{-3\sqrt{b}\,\sqrt{dx+c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^3d^2+4\sqrt{b}\,\sqrt{dx+c}\,\sqrt{ad-bc}\,at}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(3/2),x)
```

output

```
( - 3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b
)*sqrt(a*d - b*c)))*a**3*d**2 + 4*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*at
an((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b*c*d - 3*sqrt(b)*sqr
t(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)
))*a**2*b*d**2*x - 2*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c +
d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**3*d + 4*sqrt(b)*sqrt(c + d*x)*sqrt
(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**2*c*d*x
 - 2*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)
*sqrt(a*d - b*c)))*b**4*d*x + 3*a**3*b*c*d**2 + 3*a**3*b*d**3*x - 2*a**2*b
**3*d**2 - 5*a**2*b**2*c**2*d - 3*a**2*b**2*c*d**2*x + 2*a**2*b**2*d**3*x*
*2 + 2*a*b**4*c*d - 2*a*b**4*d**2*x + 2*a*b**3*c**3 - 2*a*b**3*c**2*d*x -
4*a*b**3*c*d**2*x**2 + 2*b**5*c*d*x + 2*b**4*c**3*x + 2*b**4*c**2*d*x**2)/
(sqrt(c + d*x)*b**3*d*(a**3*d**2 - 2*a**2*b*c*d + a**2*b*d**2*x + a*b**2*c
**2 - 2*a*b**2*c*d*x + b**3*c**2*x))
```

3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{3/2}}\,dx$

Optimal result 843
Mathematica [A] (verified) 844
Rubi [A] (verified) 844
Maple [A] (verified) 848
Fricas [B] (verification not implemented) 849
Sympy [F(-1)] 850
Maxima [F(-2)] 850
Giac [B] (verification not implemented) 850
Mupad [F(-1)] 852
Reduce [B] (verification not implemented) 852

Optimal result

Integrand size = 32, antiderivative size = 309

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{3/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)}{d(bc-ad)^3\sqrt{c+dx}}$$
$$-\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{2b^2(bc-ad)^2(a+bx)^2}$$
$$-\frac{(b^3(4Bc-7Ad)-ab^2(8cC-3Bd)-5a^3dD+a^2b(Cd+12cD))\sqrt{c+dx}}{4b^2(bc-ad)^3(a+bx)}$$
$$-\frac{(b^3(8c^2C-12Bcd+15Ad^2)-3a^3d^2D-a^2bd(Cd-12cD)+ab^2(8cCd-3Bd^2-24c^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}\cdots}{\sqrt{\cdots}}\right.}{4b^{5/2}(bc-ad)^{7/2}}$$

output
```
2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d/(-a*d+b*c)^3/(d*x+c)^(1/2)-1/2*(A*b^3-a*
(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)^2-1/4*(b^3*(-7
*A*d+4*B*c)-a*b^2*(-3*B*d+8*C*c)-5*a^3*d*D+a^2*b*(C*d+12*D*c))*(d*x+c)^(1/
2)/b^2/(-a*d+b*c)^3/(b*x+a)-1/4*(b^3*(15*A*d^2-12*B*c*d+8*C*c^2)-3*a^3*d^2
*D-a^2*b*d*(C*d-12*D*c)+a*b^2*(-3*B*d^2+8*C*c*d-24*D*c^2))*arctanh(b^(1/2)
*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(7/2)
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{3/2}}\,dx = \frac{\sqrt{b}(-3a^4d^2D(c+dx)-a^3bd(c+dx)(Cd+5D(-2c+dx))+4b^4cx(2c(-Cd+cD)x+Bd(c+3dx))+ab^3(-8c}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

output
```
((Sqrt[b]*(-3*a^4*d^2*D*(c + d*x) - a^3*b*d*(c + d*x)*(C*d + 5*D*(-2*c + d
*x)) + 4*b^4*c*x*(2*c*(-(C*d) + c*D)*x + B*d*(c + 3*d*x)) + a*b^3*(-8*c*x*
(3*c*C*d - 2*c^2*D + C*d^2*x) + B*d*(2*c^2 + 21*c*d*x + 3*d^2*x^2)) - A*b^
2*d*(8*a^2*d^2 + a*b*d*(9*c + 25*d*x) + b^2*(-2*c^2 + 5*c*d*x + 15*d^2*x^2
)) + a^2*b^2*(8*c^3*D + d^3*x*(5*B + C*x) - 2*c^2*d*(7*C - 6*D*x) + c*d^2*
(13*B - 5*C*x + 12*D*x^2))))/(d*(-(b*c) + a*d)^3*(a + b*x)^2*Sqrt[c + d*x]
) - ((b^3*(8*c^2*C - 12*B*c*d + 15*A*d^2) - 3*a^3*d^2*D + a^2*b*d*(-(C*d)
+ 12*c*D) + a*b^2*(8*c*C*d - 3*B*d^2 - 24*c^2*D))*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[-(b*c) + a*d]])/(-(b*c) + a*d)^(7/2))/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2124, 27, 1192, 25, 1582, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{\int -\frac{4\left(c-\frac{ad}{b}\right)Dx^2+\frac{4(bc-ad)(bC-aD)x}{b^2}+\frac{dDa^3-b(Cd-4cD)a^2-b^2(4cC-Bd)a+b^3(4Bc-5Ad)}{b^3}}{2(a+bx)^2(c+dx)^{3/2}}dx}{2(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\frac{dDa^3}{b^3}-\frac{(Cd-4cD)a^2}{b^2}-\frac{(4cC-Bd)a}{b}+4\left(c-\frac{ad}{b}\right)Dx^2+4Bc-5Ad+\frac{4(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^2(c+dx)^{3/2}}dx}{4(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow 1192$$

$$\frac{\int -\frac{-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2+d^3\left(5A-\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(c+dx)(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow 25$$

$$-\frac{\int \frac{-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2+d^3\left(5A-\frac{a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(c+dx)(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow 1582$$

$$\frac{\int \frac{2(bc-ad)\left(-\left(\left(-4Dc^3+4Cdc^2-4Bd^2c+5Ad^3\right)b^3\right)+aBd^3b^2-a^2Cd^3b+a^3d^3D\right)+b\left(\left(-8Dc^3+4Bd^2c-5Ad^3\right)b^3-ad\left(-24Dc^2+8Cdc-Bd^2\right)b^2+3a^2d^2(Cd-4c\cdots}{b(c+dx)(bc-ad-b(c+dx))}}{2b^2(bc-ad)^2} \cdots}{2d(bc-ad)}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{2(bc-ad)\left(-\left(\left(-4Dc^3+4Cdc^2-4Bd^2c+5Ad^3\right)b^3\right)+aBd^3b^2-a^2Cd^3b+a^3d^3D\right)+b\left(\left(-8Dc^3+4Bd^2c-5Ad^3\right)b^3-ad\left(-24Dc^2+8Cdc-Bd^2\right)b^2+3a^2d^2(Cd-4c\cdots}{(c+dx)(bc-ad-b(c+dx))}}{2b^3(bc-ad)^2} \cdots}{2d(bc-ad)}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow 359$$

$$\frac{\frac{-bd\left(-3a^3d^2D-a^2bd(Cd-12cD)+ab^2\left(-3Bd^2-24c^2D+8cCd\right)+b^3\left(15Ad^2-12Bcd+8c^2C\right)\right)\int \frac{1}{bc-ad-b(c+dx)}d\sqrt{c+dx}-\frac{2\left(a^3d^3D-a^2bCd^3+ab^2B\right.}{}}{2b^3(bc-ad)^2}}{2d(bc-ad)}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{-\frac{\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(-3a^3d^2D-a^2bd(Cd-12cD)+ab^2\left(-3Bd^2-24c^2D+8cCd\right)+b^3\left(15Ad^2-12Bcd+8c^2C\right)\right)}{\sqrt{bc-ad}}-\frac{2\left(a^3d^3D-a^2bCd^3+ab^2Bd^3-\left(b^3\left(5A\right.\right.\right.}{\sqrt{c+dx}}}{2b^3(bc-ad)^2}}{2d(bc-ad)}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt{c+dx}(bc-ad)}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

output

```
-1/2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b^3*(b*c - a*d)*(a + b*x)^2*Sqrt
[c + d*x]) + ((d^2*(b^3*(4*B*c - 5*A*d) - a*b^2*(8*c*C - B*d) - 7*a^3*d*D
+ 3*a^2*b*(C*d + 4*c*D))*Sqrt[c + d*x])/(2*b^2*(b*c - a*d)^2*(b*c - a*d -
b*(c + d*x))) + ((-2*(a*b^2*B*d^3 - a^2*b*C*d^3 + a^3*d^3*D - b^3*(4*c^2*C
*d - 4*B*c*d^2 + 5*A*d^3 - 4*c^3*D)))/Sqrt[c + d*x] - (Sqrt[b]*d*(b^3*(8*c
^2*C - 12*B*c*d + 15*A*d^2) - 3*a^3*d^2*D - a^2*b*d*(C*d - 12*c*D) + a*b^2
*(8*c*C*d - 3*B*d^2 - 24*c^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c
- a*d]])/Sqrt[b*c - a*d])/(2*b^3*(b*c - a*d)^2))/(2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.23

method	result
pseudoelliptic	$-\frac{15\left(\left(\left(d^{2}A-\frac{4}{5}cdB+\frac{8}{15}Cc^{2}\right)b^{3}-\frac{\left(Bd^{2}-\frac{8}{3}Ccd+8Dc^{2}\right)ab^{2}}{5}-\frac{a^{2}bd(Cd-12Dc)}{15}-\frac{a^{3}d^{2}D}{5}\right)\sqrt{xd+c}\,d(bx+a)^{2}\arctan\left(\frac{b\sqrt{}}{\sqrt{(}}\right.\right.}{}$
derivativedivides	$-\frac{2d\left(\frac{d\left(7b^{3}dA-3Bab^{2}d-4Bb^{3}c-Ca^{2}bd+8ab^{2}cC+5a^{3}dD-12a^{2}bcD\right)(xd+c)^{\frac{3}{2}}}{8b}+\frac{d\left(9Aab^{3}d^{2}-9Ab^{4}cd-5Ba^{2}b^{2}d^{2}+Bab^{3}cd+\right.}{((xd+c)b+ad-bc)^{2}}\right.}{}$
default	$-\frac{2d\left(\frac{d\left(7b^{3}dA-3Bab^{2}d-4Bb^{3}c-Ca^{2}bd+8ab^{2}cC+5a^{3}dD-12a^{2}bcD\right)(xd+c)^{\frac{3}{2}}}{8b}+\frac{d\left(9Aab^{3}d^{2}-9Ab^{4}cd-5Ba^{2}b^{2}d^{2}+Bab^{3}cd+\right.}{((xd+c)b+ad-bc)^{2}}\right.}{}$

input

```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-15/4/((a*d-b*c)*b)^(1/2)*(((d^2*A-4/5*c*d*B+8/15*C*c^2)*b^3-1/5*(B*d^2-8/
3*C*c*d+8*D*c^2)*a*b^2-1/15*a^2*b*d*(C*d-12*D*c)-1/5*a^3*d^2*D)*(d*x+c)^(1
/2)*d*(b*x+a)^2*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2))+8/15*((a*d-b*c
)*b)^(1/2)*((15/8*A*d^3*x^2+5/8*x*(-12/5*B*x+A)*c*d^2-1/4*c^2*(-4*C*x^2+2*
B*x+A)*d-D*c^3*x^2)*b^4+9/8*((-1/3*x^2*B+25/9*x*A)*d^3+c*(8/9*C*x^2-7/3*B*
x+A)*d^2-2/9*c^2*(-12*C*x+B)*d-16/9*D*c^3*x)*a*b^3+((A-1/8*C*x^2-5/8*B*x)*
d^3-13/8*(12/13*D*x^2-5/13*C*x+B)*c*d^2+7/4*(-6/7*D*x+C)*c^2*d-D*c^3)*a^2*
b^2+1/8*(d*x+c)*d*((5*D*x+C)*d-10*D*c)*a^3*b+3/8*D*a^4*d^2*(d*x+c)))/(d*x+
c)^(1/2)/(a*d-b*c)^3/(b*x+a)^2/b^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. $2(290) = 580$.

Time = 0.18 (sec) , antiderivative size = 2436, normalized size of antiderivative = 7.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas
")
```

output
```
[1/8*((8*(3*D*a^3*b^2 - C*a^2*b^3)*c^3*d - 4*(3*D*a^4*b + 2*C*a^3*b^2 - 3*
B*a^2*b^3)*c^2*d^2 + (3*D*a^5 + C*a^4*b + 3*B*a^3*b^2 - 15*A*a^2*b^3)*c*d^
3 + (8*(3*D*a*b^4 - C*b^5)*c^2*d^2 - 4*(3*D*a^2*b^3 + 2*C*a*b^4 - 3*B*b^5)
*c*d^3 + (3*D*a^3*b^2 + C*a^2*b^3 + 3*B*a*b^4 - 15*A*b^5)*d^4)*x^3 + (8*(3
*D*a*b^4 - C*b^5)*c^3*d + 12*(3*D*a^2*b^3 - 2*C*a*b^4 + B*b^5)*c^2*d^2 - 3
*(7*D*a^3*b^2 + 5*C*a^2*b^3 - 9*B*a*b^4 + 5*A*b^5)*c*d^3 + 2*(3*D*a^4*b +
C*a^3*b^2 + 3*B*a^2*b^3 - 15*A*a*b^4)*d^4)*x^2 + (16*(3*D*a^2*b^3 - C*a*b^
4)*c^3*d - 24*(C*a^2*b^3 - B*a*b^4)*c^2*d^2 - 6*(D*a^4*b + C*a^3*b^2 - 3*B
*a^2*b^3 + 5*A*a*b^4)*c*d^3 + (3*D*a^5 + C*a^4*b + 3*B*a^3*b^2 - 15*A*a^2*
b^3)*d^4)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d + 2*sqrt(b^2*c -
 a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(8*D*a^2*b^4*c^4 + 8*A*a^3*b^3*d^4 +
 2*(D*a^3*b^3 - 7*C*a^2*b^4 + B*a*b^5 + A*b^6)*c^3*d - (13*D*a^4*b^2 - 13*
C*a^3*b^3 - 11*B*a^2*b^4 + 11*A*a*b^5)*c^2*d^2 + (3*D*a^5*b + C*a^4*b^2 -
13*B*a^3*b^3 + A*a^2*b^4)*c*d^3 + (8*D*b^6*c^4 - 8*(D*a*b^5 + C*b^6)*c^3*d
 + 12*(D*a^2*b^4 + B*b^6)*c^2*d^2 - (17*D*a^3*b^3 - 9*C*a^2*b^4 + 9*B*a*b^
5 + 15*A*b^6)*c*d^3 + (5*D*a^4*b^2 - C*a^3*b^3 - 3*B*a^2*b^4 + 15*A*a*b^5)
*d^4)*x^2 + (16*D*a*b^5*c^4 - 4*(D*a^2*b^4 + 6*C*a*b^5 - B*b^6)*c^3*d - (7
*D*a^3*b^3 - 19*C*a^2*b^4 - 17*B*a*b^5 + 5*A*b^6)*c^2*d^2 - 4*(2*D*a^4*b^2
 - C*a^3*b^3 + 4*B*a^2*b^4 + 5*A*a*b^5)*c*d^3 + (3*D*a^5*b + C*a^4*b^2 - 5
*B*a^3*b^3 + 25*A*a^2*b^4)*d^4)*x)*sqrt(d*x + c))/(a^2*b^7*c^5*d - 4*a^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(290) = 580$.

Time = 0.15 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.00

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{3/2}} \, dx =$$

$$-\frac{(24 Dab^2c^2 - 8 Cb^3c^2 - 12 Da^2bcd - 8 Cab^2cd + 12 Bb^3cd + 3 Da^3d^2 + Ca^2bd^2 + 3 Bab^2d^2 - 15 Ab^3d^2)}{4 (b^5c^3 - 3 ab^4c^2d + 3 a^2b^3cd^2 - a^3b^2d^3)\sqrt{-b^2c + abd}}$$

$$-\frac{2 (Dc^3 - Cc^2d + Bcd^2 - Ad^3)}{(b^3c^3d - 3 ab^2c^2d^2 + 3 a^2bcd^3 - a^3d^4)\sqrt{dx + c}}$$

$$-\frac{12 (dx + c)^{\frac{3}{2}} Da^2b^2cd - 8 (dx + c)^{\frac{3}{2}} Cab^3cd + 4 (dx + c)^{\frac{3}{2}} Bb^4cd - 12 \sqrt{dx + c} Da^2b^2c^2d + 8 \sqrt{dx + c} Cab^3}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
-1/4*(24*D*a*b^2*c^2 - 8*C*b^3*c^2 - 12*D*a^2*b*c*d - 8*C*a*b^2*c*d + 12*B
*b^3*c*d + 3*D*a^3*d^2 + C*a^2*b*d^2 + 3*B*a*b^2*d^2 - 15*A*b^3*d^2)*arcta
n(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*
b^3*c*d^2 - a^3*b^2*d^3)*sqrt(-b^2*c + a*b*d)) - 2*(D*c^3 - C*c^2*d + B*c*
d^2 - A*d^3)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt
(d*x + c)) - 1/4*(12*(d*x + c)^(3/2)*D*a^2*b^2*c*d - 8*(d*x + c)^(3/2)*C*a
*b^3*c*d + 4*(d*x + c)^(3/2)*B*b^4*c*d - 12*sqrt(d*x + c)*D*a^2*b^2*c^2*d
+ 8*sqrt(d*x + c)*C*a*b^3*c^2*d - 4*sqrt(d*x + c)*B*b^4*c^2*d - 5*(d*x + c
)^(3/2)*D*a^3*b*d^2 + (d*x + c)^(3/2)*C*a^2*b^2*d^2 + 3*(d*x + c)^(3/2)*B*
a*b^3*d^2 - 7*(d*x + c)^(3/2)*A*b^4*d^2 + 15*sqrt(d*x + c)*D*a^3*b*c*d^2 -
 7*sqrt(d*x + c)*C*a^2*b^2*c*d^2 - sqrt(d*x + c)*B*a*b^3*c*d^2 + 9*sqrt(d*
x + c)*A*b^4*c*d^2 - 3*sqrt(d*x + c)*D*a^4*d^3 - sqrt(d*x + c)*C*a^3*b*d^3
 + 5*sqrt(d*x + c)*B*a^2*b^2*d^3 - 9*sqrt(d*x + c)*A*a*b^3*d^3)/((b^5*c^3
- 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*((d*x + c)*b - b*c + a*d)
^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^{3/2}} \,d x$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 970, normalized size of antiderivative = 3.14

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(3/2),x)
```

output

```
(3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*s
qrt(a*d - b*c)))*a**4*d**2 - 8*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan(
(sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*c*d + 6*sqrt(b)*sqrt(c
 + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*
a**3*b*d**2*x - 12*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*
x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*d + 8*sqrt(b)*sqrt(c + d*x)*sqr
t(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*c
**2 - 16*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqr
t(b)*sqrt(a*d - b*c)))*a**2*b**2*c*d*x + 3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d
- b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*d**2*x*
*2 - 24*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt
(b)*sqrt(a*d - b*c)))*a*b**4*d*x + 16*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c
)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**3*c**2*x - 8*sqrt
(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d
 - b*c)))*a*b**3*c*d*x**2 - 12*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan(
(sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**5*d*x**2 + 8*sqrt(b)*sqrt(
c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))
*b**4*c**2*x**2 - 3*a**4*b*c*d**2 - 3*a**4*b*d**3*x - 8*a**3*b**3*d**2 + 9
*a**3*b**2*c**2*d + 4*a**3*b**2*c*d**2*x - 5*a**3*b**2*d**3*x**2 + 4*a**2*
b**4*c*d - 20*a**2*b**4*d**2*x - 6*a**2*b**3*c**3 + 7*a**2*b**3*c**2*d*...
```

3.94 $\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$

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Optimal result

Integrand size = 32, antiderivative size = 434

$$\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{2(bc-ad)^3(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^7(c+dx)^{3/2}}$$
$$+\frac{2(bc-ad)^2(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D))}{d^7\sqrt{c+dx}}$$
$$-\frac{2(bc-ad)(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D))\sqrt{c\cdots}}{d^7}$$
$$+\frac{2(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D))(c\cdots}{3d^7}$$
$$+\frac{2b(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D))(c+dx)^{5/2}}{5d^7}$$
$$+\frac{2b^2(bCd-6bcD+3adD)(c+dx)^{7/2}}{7d^7}+\frac{2b^3D(c+dx)^{9/2}}{9d^7}$$

output
```
2/3*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^7/(d*x+c)^(3/2)+2*(-a*d+b
*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3
))/d^7/(d*x+c)^(1/2)-2*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^2+8*C
*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^(1/2)/
d^7+2/3*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d-10*D*
c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(3/2)/d^7+2/5*b*(3
*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*x+c)^(5/2
)/d^7+2/7*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(7/2)/d^7+2/9*b^3*D*(d*x+c)^
(9/2)/d^7
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}} dx = \frac{2(-105a^3d^3(16c^3D-8c^2d(C-3Dx)+2cd^2(B+3x(-2C+D$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(-105*a^3*d^3*(16*c^3*D - 8*c^2*d*(C - 3*D*x) + 2*c*d^2*(B + 3*x*(-2*C
+ D*x)) + d^3*(A + 3*B*x - x^2*(3*C + D*x))) + 63*a^2*b*d^2*(128*c^4*D + c
^3*(-80*C*d + 192*d*D*x) + 8*c^2*d^2*(5*B + 3*x*(-5*C + 2*D*x)) + d^4*x*(-
15*A + x*(15*B + 5*C*x + 3*D*x^2)) - 2*c*d^3*(5*A + x*(-30*B + 15*C*x + 4*
D*x^2))) + b^3*(5120*c^6*D - 3840*c^5*d*(C - 2*D*x) + 384*c^4*d^2*(7*B + 5
*x*(-3*C + D*x)) + 24*c^2*d^4*x*(-105*A + x*(42*B + 5*x*(2*C + D*x))) - 6*
c*d^5*x^2*(105*A + x*(28*B + 5*x*(3*C + 2*D*x))) - 16*c^3*d^3*(105*A + 2*x
*(-126*B + 5*x*(9*C + 2*D*x))) + d^6*x^3*(105*A + x*(63*B + 5*x*(9*C + 7*D
*x)))) + 9*a*b^2*d*(-1280*c^5*D + 128*c^4*d*(7*C - 15*D*x) - 16*c^3*d^2*(3
5*B + 6*x*(-14*C + 5*D*x)) + d^5*x^2*(105*A + x*(35*B + 3*x*(7*C + 5*D*x))
) + 8*c^2*d^3*(35*A + x*(-105*B + 2*x*(21*C + 5*D*x))) - 2*c*d^4*x*(-210*A
 + x*(105*B + x*(28*C + 15*D*x))))))/(315*d^7*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}} \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(bc-ad) \left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right)\right)\right)}{d^6\sqrt{c+dx}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{c+dx}(bc-ad) \left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2Cd\right.\right.}{d^7}$$
$$\frac{2b(c+dx)^{5/2} \left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{5d^7}+$$
$$\frac{2(c+dx)^{3/2} \left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2\right.\right.}{3d^7}$$
$$\frac{2(bc-ad)^2 \left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{d^7\sqrt{c+dx}}+$$
$$\frac{2(bc-ad)^3 \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^7(c+dx)^{3/2}}+\frac{2b^2(c+dx)^{7/2}(3adD-6bcD+bCd)}{7d^7}+$$
$$\frac{2b^3D(c+dx)^{9/2}}{9d^7}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^7*(c + d*x)^(3/
2)) + (2*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d - 4
*B*c*d^2 + 3*A*d^3 - 6*c^3*D)))/(d^7*Sqrt[c + d*x]) - (2*(b*c - a*d)*(a^2*
d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^2*C*d
 - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*Sqrt[c + d*x])/d^7 + (2*(a^3*d^3*D + 3
*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*D) + b^3*(1
0*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(3/2))/(3*d^7) + (2*b
*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^2 - 15*c^2*D))*
(c + d*x)^(5/2))/(5*d^7) + (2*b^2*(b*C*d - 6*b*c*D + 3*a*d*D)*(c + d*x)^(7
/2))/(7*d^7) + (2*b^3*D*(c + d*x)^(9/2))/(9*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{2\left(\left(-x^3\left(\frac{1}{3}Dx^3+\frac{3}{7}Cx^2+\frac{3}{5}Bx+A\right)b^3-9x^2\left(\frac{1}{5}Cx^2+\frac{1}{3}Bx+\frac{1}{7}Dx^3+A\right)ab^2+9x\left(-\frac{1}{5}Dx^3-\frac{1}{3}Cx^2-Bx+A\right)a^2b+a^3(\right.\right.}{}$
gosper	$-\frac{2(-35Dx^6b^3d^6-45Cx^5b^3d^6-135Dx^5ab^2d^6+60Dx^5b^3cd^5-63Bx^4b^3d^6-189Cx^4ab^2d^6+90Cx^4b^3cd^5-189Dx^4}{}$
trager	$-\frac{2(-35Dx^6b^3d^6-45Cx^5b^3d^6-135Dx^5ab^2d^6+60Dx^5b^3cd^5-63Bx^4b^3d^6-189Cx^4ab^2d^6+90Cx^4b^3cd^5-189Dx^4}{}$
orering	$-\frac{2(-35Dx^6b^3d^6-45Cx^5b^3d^6-135Dx^5ab^2d^6+60Dx^5b^3cd^5-63Bx^4b^3d^6-189Cx^4ab^2d^6+90Cx^4b^3cd^5-189Dx^4}{}$
derivativedivides	$\frac{-18Ca^2bcd^3\sqrt{xd+c}+36Cab^2c^2d^2\sqrt{xd+c}+30Db^3c^4\sqrt{xd+c}+2Ca^3d^4\sqrt{xd+c}+\frac{2Cb^3d(xd+c)^{\frac{7}{2}}}{7}-\frac{12Db^3c(xd+c)^{\frac{7}{2}}}{7}+\frac{2B}{}}{}$
default	$\frac{-18Ca^2bcd^3\sqrt{xd+c}+36Cab^2c^2d^2\sqrt{xd+c}+30Db^3c^4\sqrt{xd+c}+2Ca^3d^4\sqrt{xd+c}+\frac{2Cb^3d(xd+c)^{\frac{7}{2}}}{7}-\frac{12Db^3c(xd+c)^{\frac{7}{2}}}{7}+\frac{2B}{}}{}$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((-x^3*(1/3*D*x^3+3/7*C*x^2+3/5*B*x+A)*b^3-9*x^2*(1/5*C*x^2+1/3*B*x+1
/7*D*x^3+A)*a*b^2+9*x*(-1/5*D*x^3-1/3*C*x^2-B*x+A)*a^2*b+a^3*(-D*x^3-3*C*x
^2+3*B*x+A))*d^6+6*(x^2*(2/21*D*x^3+1/7*C*x^2+4/15*B*x+A)*b^3-6*x*(-1/14*D
*x^3-2/15*C*x^2-1/2*B*x+A)*a*b^2+a^2*(4/5*D*x^3+3*C*x^2-6*B*x+A)*b+1/3*a^3
*(3*D*x^2-6*C*x+B))*c*d^5-24*(-x*(-1/21*D*x^3-2/21*C*x^2-2/5*B*x+A)*b^3+a*
(2/7*D*x^3+6/5*C*x^2-3*B*x+A)*b^2+a^2*(6/5*D*x^2-3*C*x+B)*b+1/3*a^3*(-3*D*
x+C))*c^2*d^4+16*c^3*((4/21*D*x^3+6/7*C*x^2-12/5*B*x+A)*b^3+3*a*(6/7*D*x^2
-12/5*C*x+B)*b^2+3*(-12/5*D*x+C)*a^2*b+a^3*D)*d^3-128/5*((5/7*D*x^2-15/7*C
*x+B)*b^2+3*(-15/7*D*x+C)*a*b+3*D*a^2)*c^4*b*d^2+256/7*((-2*D*x+C)*b+3*D*a
)*c^5*b^2*d-1024/21*D*b^3*c^6)/(d*x+c)^(3/2)/d^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{2\,(35\,Db^3d^6x^6+5120\,Db^3c^6-105\,Aa^3d^6-3840\,(3\,Dab^2+Cb^3}{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="fricas
")
```

output
```
2/315*(35*D*b^3*d^6*x^6 + 5120*D*b^3*c^6 - 105*A*a^3*d^6 - 3840*(3*D*a*b^2
 + C*b^3)*c^5*d + 2688*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - 1680*(D*a
^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + 840*(C*a^3 + 3*B*a^2*b + 3*A
*a*b^2)*c^2*d^4 - 210*(B*a^3 + 3*A*a^2*b)*c*d^5 - 15*(4*D*b^3*c*d^5 - 3*(3
*D*a*b^2 + C*b^3)*d^6)*x^5 + 3*(40*D*b^3*c^2*d^4 - 30*(3*D*a*b^2 + C*b^3)*
c*d^5 + 21*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^6)*x^4 - (320*D*b^3*c^3*d^3 -
 240*(3*D*a*b^2 + C*b^3)*c^2*d^4 + 168*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d
^5 - 105*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^6)*x^3 + 3*(640*D*b^3*c
^4*d^2 - 480*(3*D*a*b^2 + C*b^3)*c^3*d^3 + 336*(3*D*a^2*b + 3*C*a*b^2 + B*
b^3)*c^2*d^4 - 210*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^5 + 105*(C*
a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^6)*x^2 + 3*(2560*D*b^3*c^5*d - 1920*(3*D*a*
b^2 + C*b^3)*c^4*d^2 + 1344*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^3 - 840*
(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^4 + 420*(C*a^3 + 3*B*a^2*b +
 3*A*a*b^2)*c*d^5 - 105*(B*a^3 + 3*A*a^2*b)*d^6)*x)*sqrt(d*x + c)/(d^9*x^2
 + 2*c*d^8*x + c^2*d^7)
```

Sympy [A] (verification not implemented)

Time = 80.90 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Db^3(c+dx)^{\frac{9}{2}}}{9d^6}+\frac{(c+dx)^{\frac{7}{2}}\left(Cb^3d+3Dab^2d-6Db^3c\right)}{7d^6}+\frac{(c+dx)^{\frac{5}{2}}\left(Bb^3d^2+3Cab^2d^2-5Cb^3cd\right.}{5d}\right.}{} \\ \frac{Aa^3x+\frac{Db^3x^7}{7}+\frac{x^6\left(Cb^3+3Dab^2\right)}{6}+\frac{x^5\left(Bb^3+3Cab^2+3Da^2b\right)}{5}+\frac{x^4\left(Ab^3+3Bab^2+3Ca^2b+\right.}{4}}{c^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Piecewise((2*(D*b**3*(c + d*x)**(9/2)/(9*d**6) + (c + d*x)**(7/2)*(C*b**3*
d + 3*D*a*b**2*d - 6*D*b**3*c)/(7*d**6) + (c + d*x)**(5/2)*(B*b**3*d**2 +
3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d + 15*D*
b**3*c**2)/(5*d**6) + (c + d*x)**(3/2)*(A*b**3*d**3 + 3*B*a*b**2*d**3 - 4*
B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c**2*d +
D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c**3)/(3
*d**6) + sqrt(c + d*x)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*B*a**2*b*d**
4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C*a**2*b*c*d*
*3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**3 + 18*D*a**
2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/d**6 - (a*d - b*c)**2
*(3*A*b*d**3 + B*a*d**3 - 4*B*b*c*d**2 - 2*C*a*c*d**2 + 5*C*b*c**2*d + 3*D
*a*c**2*d - 6*D*b*c**3)/(d**6*sqrt(c + d*x)) + (a*d - b*c)**3*(-A*d**3 + B
*c*d**2 - C*c**2*d + D*c**3)/(3*d**6*(c + d*x)**(3/2)))/d, Ne(d, 0)), ((A*
a**3*x + D*b**3*x**7/7 + x**6*(C*b**3 + 3*D*a*b**2)/6 + x**5*(B*b**3 + 3*C
*a*b**2 + 3*D*a**2*b)/5 + x**4*(A*b**3 + 3*B*a*b**2 + 3*C*a**2*b + D*a**3)
/4 + x**3*(3*A*a*b**2 + 3*B*a**2*b + C*a**3)/3 + x**2*(3*A*a**2*b + B*a**3
)/2)/c**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \frac{2\left(\frac{35\,(dx+c)^{\frac{9}{2}}\,Db^3-45\,\left(6\,Db^3c-\left(3\,Dab^2+Cb^3\right)d\right)(dx+c)^{\frac{7}{2}}+63\,\left(15\,Db^3c^2-5\,\left(3\,Dab^2\right.\right.}{}\right.}{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="maxima
")
```

output
```
2/315*((35*(d*x + c)^(9/2)*D*b^3 - 45*(6*D*b^3*c - (3*D*a*b^2 + C*b^3)*d)*
(d*x + c)^(7/2) + 63*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^3)*c*d + (3*D*a^2*
b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(5/2) - 105*(20*D*b^3*c^3 - 10*(3*D*
a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^2 - (D*a^3 +
3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(3/2) + 315*(15*D*b^3*c^4 -
10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^2 -
 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (C*a^3 + 3*B*a^2*b + 3*
A*a*b^2)*d^4)*sqrt(d*x + c))/d^6 - 105*(D*b^3*c^6 + A*a^3*d^6 - (3*D*a*b^2
 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2 - (D*a^3 + 3*C*a
^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^
4 - (B*a^3 + 3*A*a^2*b)*c*d^5 - 3*(6*D*b^3*c^5 - 5*(3*D*a*b^2 + C*b^3)*c^4
*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*
B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^4 - (B*a^
3 + 3*A*a^2*b)*d^5)*(d*x + c))/((d*x + c)^(3/2)*d^6))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(412) = 824$.

Time = 0.16 (sec) , antiderivative size = 1030, normalized size of antiderivative = 2.37

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
2/3*(18*(d*x + c)*D*b^3*c^5 - D*b^3*c^6 - 45*(d*x + c)*D*a*b^2*c^4*d - 15*
(d*x + c)*C*b^3*c^4*d + 3*D*a*b^2*c^5*d + C*b^3*c^5*d + 36*(d*x + c)*D*a^2
*b*c^3*d^2 + 36*(d*x + c)*C*a*b^2*c^3*d^2 + 12*(d*x + c)*B*b^3*c^3*d^2 - 3
*D*a^2*b*c^4*d^2 - 3*C*a*b^2*c^4*d^2 - B*b^3*c^4*d^2 - 9*(d*x + c)*D*a^3*c
^2*d^3 - 27*(d*x + c)*C*a^2*b*c^2*d^3 - 27*(d*x + c)*B*a*b^2*c^2*d^3 - 9*(
d*x + c)*A*b^3*c^2*d^3 + D*a^3*c^3*d^3 + 3*C*a^2*b*c^3*d^3 + 3*B*a*b^2*c^3
*d^3 + A*b^3*c^3*d^3 + 6*(d*x + c)*C*a^3*c*d^4 + 18*(d*x + c)*B*a^2*b*c*d^
4 + 18*(d*x + c)*A*a*b^2*c*d^4 - C*a^3*c^2*d^4 - 3*B*a^2*b*c^2*d^4 - 3*A*a
*b^2*c^2*d^4 - 3*(d*x + c)*B*a^3*d^5 - 9*(d*x + c)*A*a^2*b*d^5 + B*a^3*c*d
^5 + 3*A*a^2*b*c*d^5 - A*a^3*d^6)/((d*x + c)^(3/2)*d^7) + 2/315*(35*(d*x +
 c)^(9/2)*D*b^3*d^56 - 270*(d*x + c)^(7/2)*D*b^3*c*d^56 + 945*(d*x + c)^(5
/2)*D*b^3*c^2*d^56 - 2100*(d*x + c)^(3/2)*D*b^3*c^3*d^56 + 4725*sqrt(d*x +
 c)*D*b^3*c^4*d^56 + 135*(d*x + c)^(7/2)*D*a*b^2*d^57 + 45*(d*x + c)^(7/2)
*C*b^3*d^57 - 945*(d*x + c)^(5/2)*D*a*b^2*c*d^57 - 315*(d*x + c)^(5/2)*C*b
^3*c*d^57 + 3150*(d*x + c)^(3/2)*D*a*b^2*c^2*d^57 + 1050*(d*x + c)^(3/2)*C
*b^3*c^2*d^57 - 9450*sqrt(d*x + c)*D*a*b^2*c^3*d^57 - 3150*sqrt(d*x + c)*C
*b^3*c^3*d^57 + 189*(d*x + c)^(5/2)*D*a^2*b*d^58 + 189*(d*x + c)^(5/2)*C*a
*b^2*d^58 + 63*(d*x + c)^(5/2)*B*b^3*d^58 - 1260*(d*x + c)^(3/2)*D*a^2*b*c
*d^58 - 1260*(d*x + c)^(3/2)*C*a*b^2*c*d^58 - 420*(d*x + c)^(3/2)*B*b^3*c*
d^58 + 5670*sqrt(d*x + c)*D*a^2*b*c^2*d^58 + 5670*sqrt(d*x + c)*C*a*b^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{(a+b\,x)^3\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2),x)
```

output
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}} \, dx = \frac{-64a\,b^3c^2d^3x + \frac{96}{5}a^2b\,c^4d^2 - \frac{2}{3}a^4d^5 + \frac{512}{63}b^3c^6 - \frac{16}{3}a^3bc\,d^4 - 8a^3}{}$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
(2*( - 105*a**4*d**5 - 840*a**3*b*c*d**4 - 1260*a**3*b*d**5*x - 840*a**3*c
**3*d**3 - 1260*a**3*c**2*d**4*x - 315*a**3*c*d**5*x**2 + 105*a**3*d**6*x*
*3 + 5040*a**2*b**2*c**2*d**3 + 7560*a**2*b**2*c*d**4*x + 1890*a**2*b**2*d
**5*x**2 + 3024*a**2*b*c**4*d**2 + 4536*a**2*b*c**3*d**3*x + 1134*a**2*b*c
**2*d**4*x**2 - 189*a**2*b*c*d**5*x**3 + 189*a**2*b*d**6*x**4 - 6720*a*b**
3*c**3*d**2 - 10080*a*b**3*c**2*d**3*x - 2520*a*b**3*c*d**4*x**2 + 420*a*b
**3*d**5*x**3 - 3456*a*b**2*c**5*d - 5184*a*b**2*c**4*d**2*x - 1296*a*b**2
*c**3*d**3*x**2 + 216*a*b**2*c**2*d**4*x**3 - 81*a*b**2*c*d**5*x**4 + 135*
a*b**2*d**6*x**5 + 2688*b**4*c**4*d + 4032*b**4*c**3*d**2*x + 1008*b**4*c*
*2*d**3*x**2 - 168*b**4*c*d**4*x**3 + 63*b**4*d**5*x**4 + 1280*b**3*c**6 +
 1920*b**3*c**5*d*x + 480*b**3*c**4*d**2*x**2 - 80*b**3*c**3*d**3*x**3 + 3
0*b**3*c**2*d**4*x**4 - 15*b**3*c*d**5*x**5 + 35*b**3*d**6*x**6))/(315*sqr
t(c + d*x)*d**6*(c + d*x))
```

3.95 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$

Optimal result 864
Mathematica [A] (verified) 865
Rubi [A] (verified) 865
Maple [A] (verified) 867
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Optimal result

Integrand size = 32, antiderivative size = 322

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = -\frac{2(bc-ad)^2(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^6(c+dx)^{3/2}} - \frac{2(bc-ad)(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))}{d^6\sqrt{c+dx}} + \frac{2(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D))\sqrt{c+dx}}{d^6} + \frac{2(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))(c+dx)^{3/2}}{3d^6} + \frac{2b(bCd-5bcD+2adD)(c+dx)^{5/2}}{5d^6} + \frac{2b^2D(c+dx)^{7/2}}{7d^6}$$

output
```
-2/3*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^6/(d*x+c)^(3/2)-2*(-a*d+
b*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3)
)/d^6/(d*x+c)^(1/2)+2*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*c^2
)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(1/2)/d^6+2/3*(a^2*d^2
*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(3/2)/d^6+2/
5*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(5/2)/d^6+2/7*b^2*D*(d*x+c)^(7/2)/d^6
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}} dx = \frac{-70a^2d^2(16c^3D-8c^2d(C-3Dx)+2cd^2(B+3x(-2C+Dx)) \cdots}{}$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(-70*a^2*d^2*(16*c^3*D - 8*c^2*d*(C - 3*D*x) + 2*c*d^2*(B + 3*x*(-2*C + D*
x)) + d^3*(A + 3*B*x - x^2*(3*C + D*x))) + 28*a*b*d*(128*c^4*D + c^3*(-80*
C*d + 192*d*D*x) + 8*c^2*d^2*(5*B + 3*x*(-5*C + 2*D*x)) + d^4*x*(-15*A + x
*(15*B + 5*C*x + 3*D*x^2)) - 2*c*d^3*(5*A + x*(-30*B + 15*C*x + 4*D*x^2)))
 + 2*b^2*(-1280*c^5*D + 128*c^4*d*(7*C - 15*D*x) - 16*c^3*d^2*(35*B + 6*x*
(-14*C + 5*D*x)) + d^5*x^2*(105*A + x*(35*B + 3*x*(7*C + 5*D*x))) + 8*c^2*
d^3*(35*A + x*(-105*B + 2*x*(21*C + 5*D*x))) - 2*c*d^4*x*(-210*A + x*(105*
B + x*(28*C + 15*D*x)))))/(105*d^6*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}} dx$$

$$\Big\downarrow \ 2123$$

$$\int \left(\frac{a^2d^2(Cd-3cD)-2abd(-Bd^2-6c^2D+3cCd)+b^2(Ad^3-3Bcd^2-10c^3D+6c^2Cd)}{d^5\sqrt{c+dx}} + \frac{\sqrt{c+dx}(a^2d^2D \cdots}{} \right.$$

$$\Big\downarrow \ 2009$$

$$\frac{2\sqrt{c+dx}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^6}+\frac{2(c+dx)^{3/2}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{3d^6}-\frac{2(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{d^6\sqrt{c+dx}}-\frac{2(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^6(c+dx)^{3/2}}+\frac{2b(c+dx)^{5/2}(2adD-5bcD+bCd)}{5d^6}+\frac{2b^2D(c+dx)^{7/2}}{7d^6}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(-2*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^6*(c + d*x)^(3
/2)) - (2*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*
B*c*d^2 + 2*A*d^3 - 5*c^3*D)))/(d^6*Sqrt[c + d*x]) + (2*(a^2*d^2*(C*d - 3*
c*D) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 +
A*d^3 - 10*c^3*D))*Sqrt[c + d*x])/d^6 + (2*(a^2*d^2*D + 2*a*b*d*(C*d - 4*c
*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(3/2))/(3*d^6) + (2*b*(b
*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(5/2))/(5*d^6) + (2*b^2*D*(c + d*x)^(7
/2))/(7*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{2\left(\left(\left(-\frac{3}{7} D x^{5}-3 A x^{2}-\frac{3}{5} C x^{4}-B x^{3}\right) b^{2}+6 x\left(-\frac{1}{5} D x^{3}-\frac{1}{3} C x^{2}-B x+A\right) a b+a^{2}\left(-D x^{3}-3 C x^{2}+3 B x+A\right)\right) d^{5}+4\left(-3\right.\right.}{}$
gosper	$-\frac{2\left(-15 D x^{5} b^{2} d^{5}-21 C x^{4} b^{2} d^{5}-42 D x^{4} a b d^{5}+30 D x^{4} b^{2} c d^{4}-35 B x^{3} b^{2} d^{5}-70 C x^{3} a b d^{5}+56 C x^{3} b^{2} c d^{4}-35 D x^{3} a^{2} d^{5}+\right.}{}$
trager	$-\frac{2\left(-15 D x^{5} b^{2} d^{5}-21 C x^{4} b^{2} d^{5}-42 D x^{4} a b d^{5}+30 D x^{4} b^{2} c d^{4}-35 B x^{3} b^{2} d^{5}-70 C x^{3} a b d^{5}+56 C x^{3} b^{2} c d^{4}-35 D x^{3} a^{2} d^{5}+\right.}{}$
orering	$-\frac{2\left(-15 D x^{5} b^{2} d^{5}-21 C x^{4} b^{2} d^{5}-42 D x^{4} a b d^{5}+30 D x^{4} b^{2} c d^{4}-35 B x^{3} b^{2} d^{5}-70 C x^{3} a b d^{5}+56 C x^{3} b^{2} c d^{4}-35 D x^{3} a^{2} d^{5}+\right.}{}$
derivativedivides	$\frac{\frac{2 b^{2} D (x d+c)^{\frac{7}{2}}}{7}+\frac{2 C b^{2} d (x d+c)^{\frac{5}{2}}}{5}+\frac{4 D a b d (x d+c)^{\frac{5}{2}}}{5}-2 D b^{2} c (x d+c)^{\frac{5}{2}}+\frac{2 B b^{2} d^{2} (x d+c)^{\frac{3}{2}}}{3}+\frac{4 C a b d^{2} (x d+c)^{\frac{3}{2}}}{3}-\frac{8 C b^{2} c d (x d+c)^{\frac{3}{2}}}{3}}{}$
default	$\frac{\frac{2 b^{2} D (x d+c)^{\frac{7}{2}}}{7}+\frac{2 C b^{2} d (x d+c)^{\frac{5}{2}}}{5}+\frac{4 D a b d (x d+c)^{\frac{5}{2}}}{5}-2 D b^{2} c (x d+c)^{\frac{5}{2}}+\frac{2 B b^{2} d^{2} (x d+c)^{\frac{3}{2}}}{3}+\frac{4 C a b d^{2} (x d+c)^{\frac{3}{2}}}{3}-\frac{8 C b^{2} c d (x d+c)^{\frac{3}{2}}}{3}}{}$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(((-3/7*D*x^5-3*A*x^2-3/5*C*x^4-B*x^3)*b^2+6*x*(-1/5*D*x^3-1/3*C*x^2-
B*x+A)*a*b+a^2*(-D*x^3-3*C*x^2+3*B*x+A))*d^5+4*(-3*x*(-1/14*D*x^3-2/15*C*x
^2-1/2*B*x+A)*b^2+a*(4/5*D*x^3+3*C*x^2-6*B*x+A)*b+1/2*a^2*(3*D*x^2-6*C*x+B
))*c*d^4-8*((2/7*D*x^3+6/5*C*x^2-3*B*x+A)*b^2+2*(6/5*D*x^2-3*C*x+B)*a*b+a^
2*(-3*D*x+C))*c^2*d^3+16*((6/7*D*x^2-12/5*C*x+B)*b^2+2*(-12/5*D*x+C)*a*b+D
*a^2)*c^3*d^2-128/5*((-15/7*D*x+C)*b+2*D*a)*c^4*b*d+256/7*D*b^2*c^5)/(d*x+
c)^(3/2)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{2\,(15\,Db^2d^5x^5-1280\,Db^2c^5-35\,Aa^2d^5+896\,(2\,Dab+Cb^2)c^4}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="fricas
")
```

output
```
2/105*(15*D*b^2*d^5*x^5 - 1280*D*b^2*c^5 - 35*A*a^2*d^5 + 896*(2*D*a*b + C
*b^2)*c^4*d - 560*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 + 280*(C*a^2 + 2*B*a*b
 + A*b^2)*c^2*d^3 - 70*(B*a^2 + 2*A*a*b)*c*d^4 - 3*(10*D*b^2*c*d^4 - 7*(2*
D*a*b + C*b^2)*d^5)*x^4 + (80*D*b^2*c^2*d^3 - 56*(2*D*a*b + C*b^2)*c*d^4 +
 35*(D*a^2 + 2*C*a*b + B*b^2)*d^5)*x^3 - 3*(160*D*b^2*c^3*d^2 - 112*(2*D*a
*b + C*b^2)*c^2*d^3 + 70*(D*a^2 + 2*C*a*b + B*b^2)*c*d^4 - 35*(C*a^2 + 2*B
*a*b + A*b^2)*d^5)*x^2 - 3*(640*D*b^2*c^4*d - 448*(2*D*a*b + C*b^2)*c^3*d^
2 + 280*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 - 140*(C*a^2 + 2*B*a*b + A*b^2)*
c*d^4 + 35*(B*a^2 + 2*A*a*b)*d^5)*x)*sqrt(d*x + c)/(d^8*x^2 + 2*c*d^7*x +
c^2*d^6)
```

Sympy [A] (verification not implemented)

Time = 30.30 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Db^2(c+dx)^{\frac{7}{2}}}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\left(Cb^2d+2Dabd-5Db^2c\right)}{5d^5}+\frac{(c+dx)^{\frac{3}{2}}\left(Bb^2d^2+2Cabd^2-4Cb^2cd+\cdots\right)}{3d^5}+\cdots\right.}{\cdots} \\ \frac{Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5\left(Cb^2+2Dab\right)}{5}+\frac{x^4\left(Bb^2+2Cab+Da^2\right)}{4}+\frac{x^3\left(Ab^2+2Bab+Ca^2\right)}{3}+\frac{x^2\cdot\left(2A\cdots\right.}{\cdots}}{c^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Piecewise((2*(D*b**2*(c + d*x)**(7/2)/(7*d**5) + (c + d*x)**(5/2)*(C*b**2*
d + 2*D*a*b*d - 5*D*b**2*c)/(5*d**5) + (c + d*x)**(3/2)*(B*b**2*d**2 + 2*C
*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2)/(3*
d**5) + sqrt(c + d*x)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d**2 + C*a*
*2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 12*D*a*b*c*
*2*d - 10*D*b**2*c**3)/d**5 - (a*d - b*c)*(2*A*b*d**3 + B*a*d**3 - 3*B*b*c
*d**2 - 2*C*a*c*d**2 + 4*C*b*c**2*d + 3*D*a*c**2*d - 5*D*b*c**3)/(d**5*sqr
t(c + d*x)) + (a*d - b*c)**2*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(3*d
**5*(c + d*x)**(3/2)))/d, Ne(d, 0)), ((A*a**2*x + D*b**2*x**6/6 + x**5*(C*
b**2 + 2*D*a*b)/5 + x**4*(B*b**2 + 2*C*a*b + D*a**2)/4 + x**3*(A*b**2 + 2*
B*a*b + C*a**2)/3 + x**2*(2*A*a*b + B*a**2)/2)/c**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{2\left(\frac{15\,(dx+c)^{\frac{7}{2}}Db^2-21\,(5\,Db^2c-(2\,Dab+Cb^2)d)(dx+c)^{\frac{5}{2}}+35\,(10\,Db^2c^2-4\,(2\,Dab+\right.}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="maxima
")
```

output
```
2/105*((15*(d*x + c)^(7/2)*D*b^2 - 21*(5*D*b^2*c - (2*D*a*b + C*b^2)*d)*(d
*x + c)^(5/2) + 35*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d + (D*a^2 + 2*C*
a*b + B*b^2)*d^2)*(d*x + c)^(3/2) - 105*(10*D*b^2*c^3 - 6*(2*D*a*b + C*b^2
)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*b + A*b^2)*d^
3)*sqrt(d*x + c))/d^5 + 35*(D*b^2*c^5 - A*a^2*d^5 - (2*D*a*b + C*b^2)*c^4*
d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3
+ (B*a^2 + 2*A*a*b)*c*d^4 - 3*(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^2)*c^3*d + 3
*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 + (
B*a^2 + 2*A*a*b)*d^4)*(d*x + c))/((d*x + c)^(3/2)*d^5))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(302) = 604.

Time = 0.14 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{2\,(15\,(dx+c)Db^2c^4-Db^2c^5-24\,(dx+c)Dabc^3d-12\,(dx+c)Cb^2c^3d+2\,Dabc^4d+Cb^2c^4d+9\,(dx+}{}$$

$$+\frac{2\left(15\,(dx+c)^{\frac{7}{2}}Db^2d^{36}-105\,(dx+c)^{\frac{5}{2}}Db^2cd^{36}+350\,(dx+c)^{\frac{3}{2}}Db^2c^2d^{36}-1050\,\sqrt{dx+c}Db^2c^3d^{36}+42\,(\right.}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(15*(d*x + c)*D*b^2*c^4 - D*b^2*c^5 - 24*(d*x + c)*D*a*b*c^3*d - 12*(
d*x + c)*C*b^2*c^3*d + 2*D*a*b*c^4*d + C*b^2*c^4*d + 9*(d*x + c)*D*a^2*c^2
*d^2 + 18*(d*x + c)*C*a*b*c^2*d^2 + 9*(d*x + c)*B*b^2*c^2*d^2 - D*a^2*c^3*
d^2 - 2*C*a*b*c^3*d^2 - B*b^2*c^3*d^2 - 6*(d*x + c)*C*a^2*c*d^3 - 12*(d*x
+ c)*B*a*b*c*d^3 - 6*(d*x + c)*A*b^2*c*d^3 + C*a^2*c^2*d^3 + 2*B*a*b*c^2*d
^3 + A*b^2*c^2*d^3 + 3*(d*x + c)*B*a^2*d^4 + 6*(d*x + c)*A*a*b*d^4 - B*a^2
*c*d^4 - 2*A*a*b*c*d^4 + A*a^2*d^5)/((d*x + c)^(3/2)*d^6) + 2/105*(15*(d*x
 + c)^(7/2)*D*b^2*d^36 - 105*(d*x + c)^(5/2)*D*b^2*c*d^36 + 350*(d*x + c)^
(3/2)*D*b^2*c^2*d^36 - 1050*sqrt(d*x + c)*D*b^2*c^3*d^36 + 42*(d*x + c)^(5
/2)*D*a*b*d^37 + 21*(d*x + c)^(5/2)*C*b^2*d^37 - 280*(d*x + c)^(3/2)*D*a*b
*c*d^37 - 140*(d*x + c)^(3/2)*C*b^2*c*d^37 + 1260*sqrt(d*x + c)*D*a*b*c^2*
d^37 + 630*sqrt(d*x + c)*C*b^2*c^2*d^37 + 35*(d*x + c)^(3/2)*D*a^2*d^38 +
70*(d*x + c)^(3/2)*C*a*b*d^38 + 35*(d*x + c)^(3/2)*B*b^2*d^38 - 315*sqrt(d
*x + c)*D*a^2*c*d^38 - 630*sqrt(d*x + c)*C*a*b*c*d^38 - 315*sqrt(d*x + c)*
B*b^2*c*d^38 + 105*sqrt(d*x + c)*C*a^2*d^39 + 210*sqrt(d*x + c)*B*a*b*d^39
 + 105*sqrt(d*x + c)*A*b^2*d^39)/d^42
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{(a+b\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2),x)
```

output
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{-4a^2bcd^3 - 8a^2c^2d^3x + 16ab^2c^2d^2 + \frac{64}{5}abc^4d - \frac{384}{35}b^2c^4dx + \frac{16}{35}}{}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
(2*( - 35*a**3*d**4 - 210*a**2*b*c*d**3 - 315*a**2*b*d**4*x - 280*a**2*c**
3*d**2 - 420*a**2*c**2*d**3*x - 105*a**2*c*d**4*x**2 + 35*a**2*d**5*x**3 +
 840*a*b**2*c**2*d**2 + 1260*a*b**2*c*d**3*x + 315*a*b**2*d**4*x**2 + 672*
a*b*c**4*d + 1008*a*b*c**3*d**2*x + 252*a*b*c**2*d**3*x**2 - 42*a*b*c*d**4
*x**3 + 42*a*b*d**5*x**4 - 560*b**3*c**3*d - 840*b**3*c**2*d**2*x - 210*b*
*3*c*d**3*x**2 + 35*b**3*d**4*x**3 - 384*b**2*c**5 - 576*b**2*c**4*d*x - 1
44*b**2*c**3*d**2*x**2 + 24*b**2*c**2*d**3*x**3 - 9*b**2*c*d**4*x**4 + 15*
b**2*d**5*x**5))/(105*sqrt(c + d*x)*d**5*(c + d*x))
```

3.96 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}} \, dx$

Optimal result . . . 872
Mathematica [A] (verified) . . . 873
Rubi [A] (verified) . . . 873
Maple [A] (verified) . . . 875
Fricas [A] (verification not implemented) . . . 875
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Mupad [F(-1)] . . . 877
Reduce [B] (verification not implemented) . . . 878

Optimal result

Integrand size = 30, antiderivative size = 210

$$\begin{aligned}\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}} \, dx &= \frac{2(bc-ad)(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^5(c+dx)^{3/2}} \\ &+ \frac{2(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))}{d^5\sqrt{c+dx}} \\ &+ \frac{2(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))\sqrt{c+dx}}{d^5} \\ &+ \frac{2(bCd-4bcD+adD)(c+dx)^{3/2}}{3d^5} + \frac{2bD(c+dx)^{5/2}}{5d^5}\end{aligned}$$

output

```
2/3*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^5/(d*x+c)^(3/2)+2*(a*d*(-B*
d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))/d^5/(d*x+c)^(1
/2)+2*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(1/2)/d^5+2/3*(
C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(3/2)/d^5+2/5*b*D*(d*x+c)^(5/2)/d^5
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \frac{2(-5ad(16c^3D-8c^2d(C-3Dx)+2cd^2(B+3x(-2C+Dx))\ldots}{}$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(-5*a*d*(16*c^3*D - 8*c^2*d*(C - 3*D*x) + 2*c*d^2*(B + 3*x*(-2*C + D*x)
) + d^3*(A + 3*B*x - x^2*(3*C + D*x))) + b*(128*c^4*D + c^3*(-80*C*d + 192
*d*D*x) + 8*c^2*d^2*(5*B + 3*x*(-5*C + 2*D*x)) + d^4*x*(-15*A + x*(15*B +
5*C*x + 3*D*x^2)) - 2*c*d^3*(5*A + x*(-30*B + 15*C*x + 4*D*x^2)))))/(15*d^
5*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)}{d^4(c+dx)^{3/2}}+\frac{(ad-bc)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\ldots\right.}{d^4(c+dx)^{5/2}}\ldots\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{d^5\sqrt{c+dx}}+\frac{2(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^5(c+dx)^{3/2}}+\frac{2\sqrt{c+dx}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{d^5}+\frac{2(c+dx)^{3/2}(adD-4bcD+bCd)}{3d^5}+\frac{2bD(c+dx)^{5/2}}{5d^5}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^5*(c + d*x)^(3/2)
) + (2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3
 - 4*c^3*D)))/(d^5*Sqrt[c + d*x]) + (2*(a*d*(C*d - 3*c*D) - b*(3*c*C*d - B
*d^2 - 6*c^2*D))*Sqrt[c + d*x])/d^5 + (2*(b*C*d - 4*b*c*D + a*d*D)*(c + d*
x)^(3/2))/(3*d^5) + (2*b*D*(c + d*x)^(5/2))/(5*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{2\left(\left(-\frac{3Db\,x^4}{5}+(-Cb-Da)x^3+(-3Bb-3Ca)x^2+(3Ab+3Ba)x+Aa\right)d^4+2c\left(\frac{4Db\,x^3}{5}+(3Cb+3Da)x^2+(-6Bb-6Ca)\cdots\right.\right.}{3(xd+c)^{\frac{3}{2}}d^5}$
gosper	$-\frac{2(-3Db\,x^4d^4-5C\,x^3b\,d^4-5Dx^3a\,d^4+8Dx^3bc\,d^3-15B\,x^2b\,d^4-15C\,x^2a\,d^4+30C\,x^2bc\,d^3+30Dx^2ac\,d^3-48Dx^2b\,c^2\cdots}{\cdots}$
trager	$-\frac{2(-3Db\,x^4d^4-5C\,x^3b\,d^4-5Dx^3a\,d^4+8Dx^3bc\,d^3-15B\,x^2b\,d^4-15C\,x^2a\,d^4+30C\,x^2bc\,d^3+30Dx^2ac\,d^3-48Dx^2b\,c^2\cdots}{\cdots}$
orering	$-\frac{2(-3Db\,x^4d^4-5C\,x^3b\,d^4-5Dx^3a\,d^4+8Dx^3bc\,d^3-15B\,x^2b\,d^4-15C\,x^2a\,d^4+30C\,x^2bc\,d^3+30Dx^2ac\,d^3-48Dx^2b\,c^2\cdots}{\cdots}$
derivativedivides	$\frac{\frac{2bD(xd+c)^{\frac{5}{2}}}{5}+\frac{2Cbd(xd+c)^{\frac{3}{2}}}{3}+\frac{2Dad(xd+c)^{\frac{3}{2}}}{3}-\frac{8Dbc(xd+c)^{\frac{3}{2}}}{3}+2Bb\,d^2\sqrt{xd+c}+2Ca\,d^2\sqrt{xd+c}-6Cbcd\sqrt{xd+c}-6Dacd\sqrt{\cdots}}{\cdots}$
default	$\frac{\frac{2bD(xd+c)^{\frac{5}{2}}}{5}+\frac{2Cbd(xd+c)^{\frac{3}{2}}}{3}+\frac{2Dad(xd+c)^{\frac{3}{2}}}{3}-\frac{8Dbc(xd+c)^{\frac{3}{2}}}{3}+2Bb\,d^2\sqrt{xd+c}+2Ca\,d^2\sqrt{xd+c}-6Cbcd\sqrt{xd+c}-6Dacd\sqrt{\cdots}}{\cdots}$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((-3/5*D*b*x^4+(-C*b-D*a)*x^3+(-3*B*b-3*C*a)*x^2+(3*A*b+3*B*a)*x+A*a)
*d^4+2*c*(4/5*D*b*x^3+(3*C*b+3*D*a)*x^2+(-6*B*b-6*C*a)*x+A*b+B*a)*d^3-8*(6
/5*D*b*x^2+(-3*C*b-3*D*a)*x+B*b+C*a)*c^2*d^2+16*(-12/5*D*b*x+C*b+D*a)*c^3*
d-128/5*D*b*c^4)/(d*x+c)^(3/2)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \frac{2\left(3\,Dbd^4x^4+128\,Dbc^4-5\,Aad^4-80\,(Da+Cb)c^3d+40\,(Ca+\cdots\right.}{\cdots}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*D*b*d^4*x^4 + 128*D*b*c^4 - 5*A*a*d^4 - 80*(D*a + C*b)*c^3*d + 40*
(C*a + B*b)*c^2*d^2 - 10*(B*a + A*b)*c*d^3 - (8*D*b*c*d^3 - 5*(D*a + C*b)*
d^4)*x^3 + 3*(16*D*b*c^2*d^2 - 10*(D*a + C*b)*c*d^3 + 5*(C*a + B*b)*d^4)*x
^2 + 3*(64*D*b*c^3*d - 40*(D*a + C*b)*c^2*d^2 + 20*(C*a + B*b)*c*d^3 - 5*(
B*a + A*b)*d^4)*x)*sqrt(d*x + c)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

Sympy [A] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{Db(c+dx)^{\frac{5}{2}}}{5d^4}+\frac{(c+dx)^{\frac{3}{2}}(Cbd+Dad-4Dbc)}{3d^4}+\frac{\sqrt{c+dx}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6Dbc^2\right)}{d^4}\right.}{} \\ \frac{Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}}{c^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Piecewise((2*(D*b*(c + d*x)**(5/2)/(5*d**4) + (c + d*x)**(3/2)*(C*b*d + D*
a*d - 4*D*b*c)/(3*d**4) + sqrt(c + d*x)*(B*b*d**2 + C*a*d**2 - 3*C*b*c*d -
 3*D*a*c*d + 6*D*b*c**2)/d**4 - (A*b*d**3 + B*a*d**3 - 2*B*b*c*d**2 - 2*C*
a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D*b*c**3)/(d**4*sqrt(c + d*x))
+ (a*d - b*c)*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(3*d**4*(c + d*x)**
(3/2)))/d, Ne(d, 0)), ((A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/4 + x**3*(B*
b + C*a)/3 + x**2*(A*b + B*a)/2)/c**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \frac{2\left(\frac{3(dx+c)^{\frac{5}{2}}Db-5(4Dbc-(Da+Cb)d)(dx+c)^{\frac{3}{2}}+15\left(6Dbc^2-3(Da+Cb)cd+(Ca+B\right.}{d^4}\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
2/15*((3*(d*x + c)^(5/2)*D*b - 5*(4*D*b*c - (D*a + C*b)*d)*(d*x + c)^(3/2)
 + 15*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*sqrt(d*x + c))/d^4
 - 5*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c^3*d + (C*a + B*b)*c^2*d^2 - (B*a +
 A*b)*c*d^3 - 3*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 - (
B*a + A*b)*d^3)*(d*x + c))/((d*x + c)^(3/2)*d^4))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \frac{2\left(12\,(dx+c)Dbc^3 - Dbc^4 - 9\,(dx+c)Dac^2d - 9\,(dx+c)Cbc^2d\right.}{}$$

$$+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}Dbd^{20} - 20\,(dx+c)^{\frac{3}{2}}Dbcd^{20} + 90\,\sqrt{dx+c}Dbc^2d^{20} + 5\,(dx+c)^{\frac{3}{2}}Dad^{21} + 5\,(dx+c)^{\frac{3}{2}}Cbd^2\right.}{15\,d^{25}}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
2/3*(12*(d*x + c)*D*b*c^3 - D*b*c^4 - 9*(d*x + c)*D*a*c^2*d - 9*(d*x + c)*
C*b*c^2*d + D*a*c^3*d + C*b*c^3*d + 6*(d*x + c)*C*a*c*d^2 + 6*(d*x + c)*B*
b*c*d^2 - C*a*c^2*d^2 - B*b*c^2*d^2 - 3*(d*x + c)*B*a*d^3 - 3*(d*x + c)*A*
b*d^3 + B*a*c*d^3 + A*b*c*d^3 - A*a*d^4)/((d*x + c)^(3/2)*d^5) + 2/15*(3*(
d*x + c)^(5/2)*D*b*d^20 - 20*(d*x + c)^(3/2)*D*b*c*d^20 + 90*sqrt(d*x + c)
*D*b*c^2*d^20 + 5*(d*x + c)^(3/2)*D*a*d^21 + 5*(d*x + c)^(3/2)*C*b*d^21 -
45*sqrt(d*x + c)*D*a*c*d^21 - 45*sqrt(d*x + c)*C*b*c*d^21 + 15*sqrt(d*x +
c)*C*a*d^22 + 15*sqrt(d*x + c)*B*b*d^22)/d^25
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \int \frac{(a+b\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{(c+d\,x)}^{5/2}}\,dx$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2),x)
```

output
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \frac{\frac{2}{5}b\,d^4x^4+\frac{2}{3}a\,d^4x^3-\frac{2}{5}bc\,d^3x^3-2ac\,d^3x^2+2b^2d^3x^2+\frac{12}{5}b\,c^2d^2x^2}{}$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
(2*( - 5*a**2*d**3 - 20*a*b*c*d**2 - 30*a*b*d**3*x - 40*a*c**3*d - 60*a*c*
*2*d**2*x - 15*a*c*d**3*x**2 + 5*a*d**4*x**3 + 40*b**2*c**2*d + 60*b**2*c*
d**2*x + 15*b**2*d**3*x**2 + 48*b*c**4 + 72*b*c**3*d*x + 18*b*c**2*d**2*x*
*2 - 3*b*c*d**3*x**3 + 3*b*d**4*x**4))/(15*sqrt(c + d*x)*d**4*(c + d*x))
```

3.97 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx$

Optimal result 879
Mathematica [A] (verified) 879
Rubi [A] (verified) 880
Maple [A] (verified) 881
Fricas [A] (verification not implemented) 882
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Maxima [A] (verification not implemented) 883
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Mupad [F(-1)] 884
Reduce [B] (verification not implemented) 884

Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^4(c+dx)^{3/2}} + \frac{2(2cCd-Bd^2-3c^2D)}{d^4\sqrt{c+dx}} + \frac{2(Cd-3cD)\sqrt{c+dx}}{d^4} + \frac{2D(c+dx)^{3/2}}{3d^4}$$

output
```
1/3*(-2*A*d^3+2*B*c*d^2-2*C*c^2*d+2*D*c^3)/d^4/(d*x+c)^(3/2)+2*(-B*d^2+2*C
*c*d-3*D*c^2)/d^4/(d*x+c)^(1/2)+2*(C*d-3*D*c)*(d*x+c)^(1/2)/d^4+2/3*D*(d*x
+c)^(3/2)/d^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx = -\frac{2(16c^3D-8c^2d(C-3Dx)+2cd^2(B+3x(-2C+Dx))+d^3(A+3Bx-x^2(3C+Dx)))}{3d^4(c+dx)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^(5/2),x]
```

output
```
(-2*(16*c^3*D - 8*c^2*d*(C - 3*D*x) + 2*c*d^2*(B + 3*x*(-2*C + D*x)) + d^3
*(A + 3*B*x - x^2*(3*C + D*x))))/(3*d^4*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)^{5/2}}+\frac{Bd^2+3c^2D-2cCd}{d^3(c+dx)^{3/2}}+\frac{Cd-3cD}{d^3\sqrt{c+dx}}+\frac{D\sqrt{c+dx}}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^4(c+dx)^{3/2}}+\frac{2\left(-Bd^2-3c^2D+2cCd\right)}{d^4\sqrt{c+dx}}+\frac{2\sqrt{c+dx}(Cd-3cD)}{d^4}+\frac{2D(c+dx)^{3/2}}{3d^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^(5/2),x]
```

output
```
(-2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^4*(c + d*x)^(3/2)) + (2*(2*c
*C*d - B*d^2 - 3*c^2*D))/(d^4*Sqrt[c + d*x]) + (2*(C*d - 3*c*D)*Sqrt[c + d
*x])/d^4 + (2*D*(c + d*x)^(3/2))/(3*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{2\left(\left(-D x^{3}-3C\,x^{2}+3Bx+A\right)d^{3}+2c\left(3D x^{2}-6Cx+B\right)d^{2}-8c^{2}\left(-3Dx+C\right)d+16D c^{3}\right)}{3\left(xd+c\right)^{\frac{3}{2}}d^{4}}$	72
gosper	$-\frac{2\left(-D x^{3} d^{3}-3C\,x^{2} d^{3}+6D x^{2} c\,d^{2}+3Bx\,d^{3}-12Cxc\,d^{2}+24Dx\,c^{2} d+A\,d^{3}+2Bc\,d^{2}-8C\,c^{2} d+16D c^{3}\right)}{3\left(xd+c\right)^{\frac{3}{2}}d^{4}}$	90
trager	$-\frac{2\left(-D x^{3} d^{3}-3C\,x^{2} d^{3}+6D x^{2} c\,d^{2}+3Bx\,d^{3}-12Cxc\,d^{2}+24Dx\,c^{2} d+A\,d^{3}+2Bc\,d^{2}-8C\,c^{2} d+16D c^{3}\right)}{3\left(xd+c\right)^{\frac{3}{2}}d^{4}}$	90
orering	$-\frac{2\left(-D x^{3} d^{3}-3C\,x^{2} d^{3}+6D x^{2} c\,d^{2}+3Bx\,d^{3}-12Cxc\,d^{2}+24Dx\,c^{2} d+A\,d^{3}+2Bc\,d^{2}-8C\,c^{2} d+16D c^{3}\right)}{3\left(xd+c\right)^{\frac{3}{2}}d^{4}}$	90
derivativedivides	$\frac{\frac{2D\left(xd+c\right)^{\frac{3}{2}}}{3}+2Cd\sqrt{xd+c}-6Dc\sqrt{xd+c}-\frac{2\left(B\,d^{2}-2Ccd+3D c^{2}\right)}{\sqrt{xd+c}}-\frac{2\left(A\,d^{3}-Bc\,d^{2}+C\,c^{2} d-D c^{3}\right)}{3\left(xd+c\right)^{\frac{3}{2}}}}{d^{4}}$	98
default	$\frac{\frac{2D\left(xd+c\right)^{\frac{3}{2}}}{3}+2Cd\sqrt{xd+c}-6Dc\sqrt{xd+c}-\frac{2\left(B\,d^{2}-2Ccd+3D c^{2}\right)}{\sqrt{xd+c}}-\frac{2\left(A\,d^{3}-Bc\,d^{2}+C\,c^{2} d-D c^{3}\right)}{3\left(xd+c\right)^{\frac{3}{2}}}}{d^{4}}$	98

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((-D*x^3-3*C*x^2+3*B*x+A)*d^3+2*c*(3*D*x^2-6*C*x+B)*d^2-8*c^2*(-3*D*x
+C)*d+16*D*c^3)/(d*x+c)^(3/2)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx = \frac{2\left(Dd^3x^3-16\,Dc^3+8\,Cc^2d-2\,Bcd^2-Ad^3-3\,(2\,Dcd^2-Cd^3)x^2-3\,(8\ldots\right.}{3\,(d^6x^2+2\,cd^5x+c^2d^4)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(D*d^3*x^3 - 16*D*c^3 + 8*C*c^2*d - 2*B*c*d^2 - A*d^3 - 3*(2*D*c*d^2 -
 C*d^3)*x^2 - 3*(8*D*c^2*d - 4*C*c*d^2 + B*d^3)*x)*sqrt(d*x + c)/(d^6*x^2
+ 2*c*d^5*x + c^2*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(119) = 238.

Time = 0.41 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx = \begin{cases} -\frac{2Ad^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{4Bcd^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{6Bd^3x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{\ldots}{3cd^4\ldots} \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}}{c^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Piecewise((-2*A*d**3/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 4
*B*c*d**2/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 6*B*d**3*x/(
3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 16*C*c**2*d/(3*c*d**4*s
qrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 24*C*c*d**2*x/(3*c*d**4*sqrt(c +
d*x) + 3*d**5*x*sqrt(c + d*x)) + 6*C*d**3*x**2/(3*c*d**4*sqrt(c + d*x) + 3
*d**5*x*sqrt(c + d*x)) - 32*D*c**3/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt
(c + d*x)) - 48*D*c**2*d*x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x
)) - 12*D*c*d**2*x**2/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) +
2*D*d**3*x**3/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)), Ne(d, 0))
, ((A*x + B*x**2/2 + C*x**3/3 + D*x**4/4)/c**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx = \frac{2\left(\frac{(dx+c)^{\frac{3}{2}}D-3\,(3\,Dc-Cd)\sqrt{dx+c}}{d^3}+\frac{Dc^3-Cc^2d+Bcd^2-Ad^3-3\,(3\,Dc^2-2\,Ccd+Bd^2)(dx+c)}{(dx+c)^{\frac{3}{2}}d^3}\right)}{3\,d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(((d*x + c)^(3/2)*D - 3*(3*D*c - C*d)*sqrt(d*x + c))/d^3 + (D*c^3 - C*
c^2*d + B*c*d^2 - A*d^3 - 3*(3*D*c^2 - 2*C*c*d + B*d^2)*(d*x + c))/((d*x +
 c)^(3/2)*d^3))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx = -\frac{2\left(9\,(dx+c)Dc^2-Dc^3-6\,(dx+c)Ccd+Cc^2d+3\,(dx+c)Bd^2-Bcd^2+Ad^3\right)}{3\,(dx+c)^{\frac{3}{2}}d^4} + \frac{2\left((dx+c)^{\frac{3}{2}}Dd^8-9\,\sqrt{dx+c}Dcd^8+3\,\sqrt{dx+c}Cd^9\right)}{3\,d^{12}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(9*(d*x + c)*D*c^2 - D*c^3 - 6*(d*x + c)*C*c*d + C*c^2*d + 3*(d*x + c
)*B*d^2 - B*c*d^2 + A*d^3)/((d*x + c)^(3/2)*d^4) + 2/3*((d*x + c)^(3/2)*D*
d^8 - 9*sqrt(d*x + c)*D*c*d^8 + 3*sqrt(d*x + c)*C*d^9)/d^12
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(c+d\,x)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(5/2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{5/2}}\,dx=\frac{\frac{2}{3}d^3x^3-2c\,d^2x^2-2b\,d^2x-8c^2dx-\frac{2}{3}a\,d^2-\frac{4}{3}bcd-\frac{16}{3}c^3}{\sqrt{dx+c}\,d^3\,(dx+c)}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
(2*( - a*d**2 - 2*b*c*d - 3*b*d**2*x - 8*c**3 - 12*c**2*d*x - 3*c*d**2*x**
2 + d**3*x**3))/(3*sqrt(c + d*x)*d**3*(c + d*x))
```

3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{5/2}}\,dx$

Optimal result 885
Mathematica [A] (verified) 886
Rubi [A] (verified) 886
Maple [A] (verified) 888
Fricas [B] (verification not implemented) 888
Sympy [A] (verification not implemented) 889
Maxima [F(-2)] 890
Giac [A] (verification not implemented) 890
Mupad [F(-1)] 891
Reduce [B] (verification not implemented) 891

Optimal result

Integrand size = 32, antiderivative size = 210

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{5/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^3(bc-ad)(c+dx)^{3/2}} + \frac{2(ad(2cCd-Bd^2-3c^2D)-b(c^2Cd-Ad^3-2c^3D))}{d^3(bc-ad)^2\sqrt{c+dx}} + \frac{2D\sqrt{c+dx}}{bd^3} - \frac{2(Ab^3-a(b^2B-abC+a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{5/2}}$$

output
```
2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^3/(-a*d+b*c)/(d*x+c)^(3/2)+2*(a*d*(-B*
d^2+2*C*c*d-3*D*c^2)-b*(-A*d^3+C*c^2*d-2*D*c^3))/d^3/(-a*d+b*c)^2/(d*x+c)^
(1/2)+2*D*(d*x+c)^(1/2)/b/d^3-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctanh(b^(1
/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{5/2}}\,dx = \frac{6a^2d^2D(c+dx)^2-2abd(14c^3D+d^3(A+3Bx)+c^2(-5Cd+21dDx)+2}{} + \frac{2(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}(-bc+ad)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^(5/2)),x]
```

output
```
(6*a^2*d^2*D*(c + d*x)^2 - 2*a*b*d*(14*c^3*D + d^3*(A + 3*B*x) + c^2*(-5*C
*d + 21*d*D*x) + 2*c*d^2*(B - 3*C*x + 3*D*x^2)) + 2*b^2*(4*A*c*d^3 + 8*c^4
*D + 3*A*d^4*x - 2*c^3*d*(C - 6*D*x) - c^2*d^2*(B + 3*x*(C - D*x))))/(3*b*
d^3*(b*c - a*d)^2*(c + d*x)^(3/2)) + (2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)
)*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(b^(3/2)*(-(b*c) + a
*d)^(5/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2122, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2122}$$

$$\int \left(\frac{Ab^3-a(a^2D-abC+b^2B)}{b(a+bx)\sqrt{c+dx}(bc-ad)^2}+\frac{b(-Ad^3-2c^3D+c^2Cd)-ad(-Bd^2-3c^2D+2cCd)}{d^2(c+dx)^{3/2}(bc-ad)^2}+\frac{Ad^3-Bcd^2+c^3(}{d^2(c+dx)^{5/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{5/2}}+\frac{2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-Ad^3-2c^3D+c^2Cd\right)\right)}{d^3\sqrt{c+dx}(bc-ad)^2}+\frac{2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3d^3(c+dx)^{3/2}(bc-ad)}+\frac{2D\sqrt{c+dx}}{bd^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^(5/2)),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(3*d^3*(b*c - a*d)*(c + d*x)^(3/2)
) + (2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(c^2*C*d - A*d^3 - 2*c^3*D)))/
(d^3*(b*c - a*d)^2*Sqrt[c + d*x]) + (2*D*Sqrt[c + d*x])/(b*d^3) - (2*(A*b^
3 - a*(b^2*B - a*b*C + a^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c -
a*d]])/(b^(3/2)*(b*c - a*d)^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2122
```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_))/((a_.) + (b_.)*(x_)), x_Symbol] :> In
t[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{2D\sqrt{xd+c}}{b}+\frac{2\left(b^3A-a\,b^2B+a^2bC-a^3D\right)d^3\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{b(ad-bc)^2\sqrt{(ad-bc)b}}-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{3(ad-bc)(xd+c)^{\frac{3}{2}}}-\frac{2\left(-Ab\,d^3+Ba\,d^3-2Cac\,d^2+\cdots\right.}{(ad-bc)^2\cdots}}{d^3}$
default	$\frac{\frac{2D\sqrt{xd+c}}{b}+\frac{2\left(b^3A-a\,b^2B+a^2bC-a^3D\right)d^3\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{b(ad-bc)^2\sqrt{(ad-bc)b}}-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{3(ad-bc)(xd+c)^{\frac{3}{2}}}-\frac{2\left(-Ab\,d^3+Ba\,d^3-2Cac\,d^2+\cdots\right.}{(ad-bc)^2\cdots}}{d^3}$
pseudoelliptic	$\frac{\frac{2D\sqrt{xd+c}}{b}+\frac{2\left(b^3A-a\,b^2B+a^2bC-a^3D\right)d^3\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{b(ad-bc)^2\sqrt{(ad-bc)b}}-\frac{2\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)}{3(ad-bc)(xd+c)^{\frac{3}{2}}}+\frac{2\left(Ab\,d^3-Ba\,d^3+2Cac\,d^2-C\cdots\right.}{(ad-bc)^2\cdots}}{d^3}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d^3*(1/b*D*(d*x+c)^(1/2)-1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)/(d*
x+c)^(3/2)-1/(a*d-b*c)^2*(-A*b*d^3+B*a*d^3-2*C*a*c*d^2+C*b*c^2*d+3*D*a*c^2
*d-2*D*b*c^3)/(d*x+c)^(1/2)+1/b*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(a*d-b*c)^2*
d^3/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 607 vs. $2(191) = 382$.

Time = 0.12 (sec) , antiderivative size = 1227, normalized size of antiderivative = 5.84

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)(c + dx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*((D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^5*x^2 + 2*(D*a^3 - C*a^2*b
 + B*a*b^2 - A*b^3)*c*d^4*x + (D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*c^2*d^3)
*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt
(d*x + c))/(b*x + a)) - 2*(8*D*b^4*c^5 + A*a^2*b^2*d^5 - 2*(11*D*a*b^3 + C
*b^4)*c^4*d + (17*D*a^2*b^2 + 7*C*a*b^3 - B*b^4)*c^3*d^2 - (3*D*a^3*b + 5*
C*a^2*b^2 + B*a*b^3 - 4*A*b^4)*c^2*d^3 + (2*B*a^2*b^2 - 5*A*a*b^3)*c*d^4 +
 3*(D*b^4*c^3*d^2 - 3*D*a*b^3*c^2*d^3 + 3*D*a^2*b^2*c*d^4 - D*a^3*b*d^5)*x
^2 + 3*(4*D*b^4*c^4*d - (11*D*a*b^3 + C*b^4)*c^3*d^2 + 3*(3*D*a^2*b^2 + C*
a*b^3)*c^2*d^3 - (2*D*a^3*b + 2*C*a^2*b^2 + B*a*b^3 - A*b^4)*c*d^4 + (B*a^
2*b^2 - A*a*b^3)*d^5)*x)*sqrt(d*x + c))/(b^5*c^5*d^3 - 3*a*b^4*c^4*d^4 + 3
*a^2*b^3*c^3*d^5 - a^3*b^2*c^2*d^6 + (b^5*c^3*d^5 - 3*a*b^4*c^2*d^6 + 3*a^
2*b^3*c*d^7 - a^3*b^2*d^8)*x^2 + 2*(b^5*c^4*d^4 - 3*a*b^4*c^3*d^5 + 3*a^2*
b^3*c^2*d^6 - a^3*b^2*c*d^7)*x), -2/3*(3*((D*a^3 - C*a^2*b + B*a*b^2 - A*b
^3)*d^5*x^2 + 2*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*c*d^4*x + (D*a^3 - C*a
^2*b + B*a*b^2 - A*b^3)*c^2*d^3)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c +
 a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*D*b^4*c^5 + A*a^2*b^2*d^5 - 2*(1
1*D*a*b^3 + C*b^4)*c^4*d + (17*D*a^2*b^2 + 7*C*a*b^3 - B*b^4)*c^3*d^2 - (3
*D*a^3*b + 5*C*a^2*b^2 + B*a*b^3 - 4*A*b^4)*c^2*d^3 + (2*B*a^2*b^2 - 5*A*a
*b^3)*c*d^4 + 3*(D*b^4*c^3*d^2 - 3*D*a*b^3*c^2*d^3 + 3*D*a^2*b^2*c*d^4 - D
*a^3*b*d^5)*x^2 + 3*(4*D*b^4*c^4*d - (11*D*a*b^3 + C*b^4)*c^3*d^2 + 3*(...
```

Sympy [A] (verification not implemented)

Time = 13.08 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{D\sqrt{c+dx}}{bd^2} - \frac{-Abd^3+Bad^3-2Cacd^2+Cbc^2d+3Dac^2d-2Dbc^3}{d^2\sqrt{c+dx}(ad-bc)^2} + \frac{-Ad^3+Bcd^2-Cc^2d+Dc^3}{3d^2(c+dx)^{\frac{3}{2}}(ad-bc)} - \frac{d\left(-Ab^3+Bab^2 \cdots\right.}{b \cdots}\right.}{d} \\ \frac{\frac{Dx^3}{3b} + \frac{x^2(Cb-Da)}{2b^2} + \frac{x\left(Bb^2-Cab+Da^2\right)}{b^3} - \frac{\left(-Ab^3+Bab^2-Ca^2b+Da^3\right)\left(\begin{cases}\frac{x}{a} & \text{for } b=0\\ \frac{\log(a+bx)}{b} & \text{otherwise}\end{cases}\right)}{b^3}}{c^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(5/2),x)
```

output
```
Piecewise((2*(D*sqrt(c + d*x)/(b*d**2) - (-A*b*d**3 + B*a*d**3 - 2*C*a*c*d
**2 + C*b*c**2*d + 3*D*a*c**2*d - 2*D*b*c**3)/(d**2*sqrt(c + d*x)*(a*d - b
*c)**2) + (-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(3*d**2*(c + d*x)**(3/2
)*(a*d - b*c)) - d*(-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*atan(sqrt(c +
d*x)/sqrt((a*d - b*c)/b))/(b**2*sqrt((a*d - b*c)/b)*(a*d - b*c)**2))/d, Ne
(d, 0)), ((D*x**3/(3*b) + x**2*(C*b - D*a)/(2*b**2) + x*(B*b**2 - C*a*b +
D*a**2)/b**3 - (-A*b**3 + B*a*b**2 - C*a**2*b + D*a**3)*Piecewise((x/a, Eq
(b, 0)), (log(a + b*x)/b, True))/b**3)/c**(5/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)(c + dx)^{5/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.34

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)(c + dx)^{5/2}}\, dx = -\frac{2\,(Da^3 - Ca^2b + Bab^2 - Ab^3)\arctan\left(\frac{\sqrt{dx+c}b}{\sqrt{-b^2c+abd}}\right)}{(b^3c^2 - 2\,ab^2cd + a^2bd^2)\sqrt{-b^2c + abd}} + \frac{2\,(6\,(dx + c)Dbc^3 - Dbc^4 - 9\,(dx + c)Dac^2d - 3\,(dx + c)Cbc^2d + Dac^3d + Cbc^3d + 6\,(dx + c)Cacd^2 - \cdots}{3\,(b^2c^2d^3 - 2\,abcd^4 + a^2d^5)(dx \cdots} + \frac{2\sqrt{dx + c}D}{bd^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
-2*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
+ a*b*d))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) + 2/3
*(6*(d*x + c)*D*b*c^3 - D*b*c^4 - 9*(d*x + c)*D*a*c^2*d - 3*(d*x + c)*C*b*
c^2*d + D*a*c^3*d + C*b*c^3*d + 6*(d*x + c)*C*a*c*d^2 - C*a*c^2*d^2 - B*b*
c^2*d^2 - 3*(d*x + c)*B*a*d^3 + 3*(d*x + c)*A*b*d^3 + B*a*c*d^3 + A*b*c*d^
3 - A*a*d^4)/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*(d*x + c)^(3/2)) + 2*s
qrt(d*x + c)*D/(b*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{5/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{5/2}}\,dx=\frac{-2\sqrt{b}\,\sqrt{dx+c}\,\sqrt{ad-bc}\,atan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{b}\,\sqrt{ad-bc}}\right)a^2c\,d^2-2\sqrt{b}\,\sqrt{dx+c}\,\sqrt{ad-bc}\,}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(5/2),x)
```

output
```
(2*( - 3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqr
t(b)*sqrt(a*d - b*c)))*a**2*c*d**2 - 3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*
c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*d**3*x - a**2*b*
*2*d**3 + 3*a**2*b*c**2*d**2 + 6*a**2*b*c*d**3*x + 3*a**2*b*d**4*x**2 + 2*
a*b**3*c*d**2 - 9*a*b**2*c**3*d - 15*a*b**2*c**2*d**2*x - 6*a*b**2*c*d**3*
x**2 - b**4*c**2*d + 6*b**3*c**4 + 9*b**3*c**3*d*x + 3*b**3*c**2*d**2*x**2
))/(3*sqrt(c + d*x)*b**2*d**2*(a**2*c*d**2 + a**2*d**3*x - 2*a*b*c**2*d -
2*a*b*c*d**2*x + b**2*c**3 + b**2*c**2*d*x))
```

3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx$

Optimal result 893
Mathematica [A] (verified) 894
Rubi [A] (verified) 894
Maple [A] (verified) 897
Fricas [B] (verification not implemented) 897
Sympy [F(-1)] 898
Maxima [F(-2)] 899
Giac [A] (verification not implemented) 899
Mupad [F(-1)] 900
Reduce [B] (verification not implemented) 900

Optimal result

Integrand size = 32, antiderivative size = 273

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx &= -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d^2(bc-ad)^2(c+dx)^{3/2}} \\ &-\frac{2(ad(2cCd-Bd^2-3c^2D)-b(Bcd^2-2Ad^3-c^3D))}{d^2(bc-ad)^3\sqrt{c+dx}} \\ &-\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b(bc-ad)^3(a+bx)} \\ &-\frac{(b^3(2Bc-5Ad)-ab^2(4cC-3Bd)-a^3dD-a^2b(Cd-6cD))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}(bc-ad)^{7/2}}\end{aligned}$$

output

```
1/3*(-2*A*d^3+2*B*c*d^2-2*C*c^2*d+2*D*c^3)/d^2/(-a*d+b*c)^2/(d*x+c)^(3/2)-
2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-2*A*d^3+B*c*d^2-D*c^3))/d^2/(-a*d+b*c)
^3/(d*x+c)^(1/2)-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b/(-a*d+b*c)^
3/(b*x+a)-(b^3*(-5*A*d+2*B*c)-a*b^2*(-3*B*d+4*C*c)-a^3*d*D-a^2*b*(C*d-6*D*
c))*arctanh(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(7/
2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.30

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx = \frac{-3a^3d^2D(c+dx)^2 - a^2bd(16c^3D + 2cd^2(2B-9Cx) + c^2(-13Cd+18dD}{}$$

$$- \frac{(b^3(2Bc-5Ad)+ab^2(-4cC+3Bd)-a^3dD+a^2b(-Cd+6cD))\arctan\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{-bc+ad}}\right)}{b^{3/2}(-bc+ad)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

output
```
(-3*a^3*d^2*D*(c + d*x)^2 - a^2*b*d*(16*c^3*D + 2*c*d^2*(2*B - 9*C*x) + c^
2*(-13*C*d + 18*d*D*x) + d^3*(2*A + 6*B*x - 3*C*x^2)) + a*b^2*(4*c^4*D + d
^4*x*(10*A - 9*B*x) + 2*c^3*d*(C - 5*D*x) + 2*c*d^3*(7*A - 8*B*x + 6*C*x^2
) + c^2*d^2*(-11*B + 2*x*(5*C - 9*D*x))) + b^3*(A*d^2*(3*c^2 + 20*c*d*x +
15*d^2*x^2) + 2*c*x*(-4*B*c*d^2 + 2*c^3*D - 3*B*d^3*x + c^2*d*(C + 3*D*x))
))/(3*b*d^2*(-(b*c) + a*d)^3*(a + b*x)*(c + d*x)^(3/2)) - ((b^3*(2*B*c - 5
*A*d) + a*b^2*(-4*c*C + 3*B*d) - a^3*d*D + a^2*b*(-(C*d) + 6*c*D))*ArcTan[
(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(b^(3/2)*(-(b*c) + a*d)^(7/2)
)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2124, 27, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{2\left(c-\frac{ad}{b}\right)Dx^2+\frac{2(bc-ad)(bC-aD)x}{b^2}+\frac{3dDa^3-b(3Cd-2cD)a^2-b^2(2cC-3Bd)a+b^3(2Bc-5Ad)}{b^3}}{2(a+bx)(c+dx)^{5/2}}dx}{bc-ad}-\frac{A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}}{(a+bx)(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{3dDa^3}{b^3}-\frac{(3Cd-2cD)a^2}{b^2}-\frac{(2cC-3Bd)a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc-5Ad+\frac{2(bc-ad)(bC-aD)x}{b^2}}{(a+bx)(c+dx)^{5/2}}dx}{2(bc-ad)}-\frac{A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}}{(a+bx)(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1192$$

$$\frac{\int \frac{-2Dc^3+2Cdc^2-2Bd^2c-2\left(c-\frac{ad}{b}\right)D(c+dx)^2+d^3\left(5A-\frac{3a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{2(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(c+dx)^2(bc-ad-b(c+dx))}d\sqrt{c+dx}}{d^2(bc-ad)}-\frac{A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}}{(a+bx)(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1584$$

$$\frac{\int \left(\frac{\left(dDa^3+b(Cd-6cD)a^2+b^2(4cC-3Bd)a-b^3(2Bc-5Ad)\right)d^2}{b(bc-ad)^2(bc-ad-b(c+dx))}+\frac{-\left(\left(-2Dc^3+2Bd^2c-5Ad^3\right)b^3\right)+ad\left(-6Dc^2+4Cdc-3Bd^2\right)b^2+a^2Cd^3b-a^3d\cdots}{b^2(bc-ad)^2(c+dx)}\cdots\right)}{d^2(bc-ad)}$$

$$\frac{A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}}{(a+bx)(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{d^2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(a^3(-d)D-a^2b(Cd-6cD)-ab^2(4cC-3Bd)+b^3(2Bc-5Ad)\right)}{b^{3/2}(bc-ad)^{5/2}}-\frac{-a^3d^3D+a^2bCd^3+ab^2d\left(-3Bd^2-6c^2D+4cCd\right)-(\cdots}{b^2\sqrt{c+dx}(bc-ad)^2}\cdots}{d^2(bc-ad)}$$

$$\frac{A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}}{(a+bx)(c+dx)^{3/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

output
```
-((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)/((b*c - a*d)*(a + b*x)*(c + d*x)^(
3/2))) + ((3*a*b^2*B*d^3 - 3*a^2*b*C*d^3 + 3*a^3*d^3*D - b^3*(2*c^2*C*d -
2*B*c*d^2 + 5*A*d^3 - 2*c^3*D))/(3*b^3*(b*c - a*d)*(c + d*x)^(3/2)) - (a^2
*b*C*d^3 - a^3*d^3*D + a*b^2*d*(4*c*C*d - 3*B*d^2 - 6*c^2*D) - b^3*(2*B*c*
d^2 - 5*A*d^3 - 2*c^3*D))/(b^2*(b*c - a*d)^2*Sqrt[c + d*x]) - (d^2*(b^3*(2
*B*c - 5*A*d) - a*b^2*(4*c*C - 3*B*d) - a^3*d*D - a^2*b*(C*d - 6*c*D))*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b^(3/2)*(b*c - a*d)^(5/2))
)/(d^2*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{2d^2\left(\frac{d\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\sqrt{xd+c}}{2b((xd+c)b+ad-bc)}+\frac{\left(5b^3dA-3Ba\,b^2d-2B\,b^3c+C\,a^2bd+4a\,b^2cC+a^3dD-6a^2bcD\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{2b\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3}\cdots}{d^2}$
default	$\frac{\frac{2d^2\left(\frac{d\left(b^3A-a\,b^2B+a^2bC-a^3D\right)\sqrt{xd+c}}{2b((xd+c)b+ad-bc)}+\frac{\left(5b^3dA-3Ba\,b^2d-2B\,b^3c+C\,a^2bd+4a\,b^2cC+a^3dD-6a^2bcD\right)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)}{2b\sqrt{(ad-bc)b}}\right)}{(ad-bc)^3}\cdots}{d^2}$
pseudoelliptic	$5\left(\left(b^3A-\frac{3}{5}a\,b^2B+\frac{1}{5}a^2bC+\frac{1}{5}a^3D\right)d-\frac{2bc\left(B\,b^2-2Cab+3Da^2\right)}{5}\right)(xd+c)^{\frac{3}{2}}d^2(bx+a)\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)b}}\right)-\frac{2\sqrt{(ad-bc)b}\cdots}{\cdots}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(d^2/(a*d-b*c)^3*(1/2*d*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/b*(d*x+c)^(1/2
)/((d*x+c)*b+a*d-b*c)+1/2*(5*A*b^3*d-3*B*a*b^2*d-2*B*b^3*c+C*a^2*b*d+4*C*a
*b^2*c+D*a^3*d-6*D*a^2*b*c)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/(
(a*d-b*c)*b)^(1/2)))-1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^2/(d*x+c)
^(3/2)-1/(a*d-b*c)^3*(-2*A*b*d^3+B*a*d^3+B*b*c*d^2-2*C*a*c*d^2+3*D*a*c^2*d
-D*b*c^3)/(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(253) = 506$.

Time = 0.16 (sec) , antiderivative size = 2281, normalized size of antiderivative = 8.36

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas
")
```

output
```
[1/6*(3*(2*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c^3*d^2 - (D*a^4 + C*a^3*b
- 3*B*a^2*b^2 + 5*A*a*b^3)*c^2*d^3 + (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*
c*d^4 - (D*a^3*b + C*a^2*b^2 - 3*B*a*b^3 + 5*A*b^4)*d^5)*x^3 + (4*(3*D*a^2
*b^2 - 2*C*a*b^3 + B*b^4)*c^2*d^3 + 2*(2*D*a^3*b - 3*C*a^2*b^2 + 4*B*a*b^3
 - 5*A*b^4)*c*d^4 - (D*a^4 + C*a^3*b - 3*B*a^2*b^2 + 5*A*a*b^3)*d^5)*x^2 +
 (2*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c^3*d^2 + (11*D*a^3*b - 9*C*a^2*b^2
+ 7*B*a*b^3 - 5*A*b^4)*c^2*d^3 - 2*(D*a^4 + C*a^3*b - 3*B*a^2*b^2 + 5*A*a*
b^3)*c*d^4)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
 - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(4*D*a*b^4*c^5 + 2*A*a^3*b^2*d^5 -
 2*(10*D*a^2*b^3 - C*a*b^4)*c^4*d + (13*D*a^3*b^2 + 11*C*a^2*b^3 - 11*B*a*
b^4 + 3*A*b^5)*c^3*d^2 + (3*D*a^4*b - 13*C*a^3*b^2 + 7*B*a^2*b^3 + 11*A*a*
b^4)*c^2*d^3 + 4*(B*a^3*b^2 - 4*A*a^2*b^3)*c*d^4 + 3*(2*D*b^5*c^4*d - 8*D*
a*b^4*c^3*d^2 + 2*(3*D*a^2*b^3 + 2*C*a*b^4 - B*b^5)*c^2*d^3 - (D*a^3*b^2 +
 3*C*a^2*b^3 + B*a*b^4 - 5*A*b^5)*c*d^4 + (D*a^4*b - C*a^3*b^2 + 3*B*a^2*b
^3 - 5*A*a*b^4)*d^5)*x^2 + 2*(2*D*b^5*c^5 - (7*D*a*b^4 - C*b^5)*c^4*d - 4*
(D*a^2*b^3 - C*a*b^4 + B*b^5)*c^3*d^2 + 2*(3*D*a^3*b^2 + 2*C*a^2*b^3 - 2*B
*a*b^4 + 5*A*b^5)*c^2*d^3 + (3*D*a^4*b - 9*C*a^3*b^2 + 5*B*a^2*b^3 - 5*A*a
*b^4)*c*d^4 + (3*B*a^3*b^2 - 5*A*a^2*b^3)*d^5)*x)*sqrt(d*x + c))/(a*b^6*c^
6*d^2 - 4*a^2*b^5*c^5*d^3 + 6*a^3*b^4*c^4*d^4 - 4*a^4*b^3*c^3*d^5 + a^5*b^
2*c^2*d^6 + (b^7*c^4*d^4 - 4*a*b^6*c^3*d^5 + 6*a^2*b^5*c^2*d^6 - 4*a^3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx = \frac{(6\,Da^2bc-4\,Cab^2c+2\,Bb^3c-Da^3d-Ca^2bd+3\,Bab^2d-5\,Ab^3d)\arctan}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)\sqrt{-b^2c+abd}}$$
$$+\frac{\sqrt{dx+c}Da^3d-\sqrt{dx+c}Ca^2bd+\sqrt{dx+c}Bab^2d-\sqrt{dx+c}Ab^3d}{(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3)((dx+c)b-bc+ad)}$$
$$-\frac{2\,(3\,(dx+c)Dbc^3-Dbc^4-9\,(dx+c)Dac^2d+Dac^3d+Cbc^3d+6\,(dx+c)Cacd^2-3\,(dx+c)Bbcd^2-}{3\,(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
(6*D*a^2*b*c - 4*C*a*b^2*c + 2*B*b^3*c - D*a^3*d - C*a^2*b*d + 3*B*a*b^2*d
 - 5*A*b^3*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^3 - 3*a
*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(-b^2*c + a*b*d)) + (sqrt(d*
x + c)*D*a^3*d - sqrt(d*x + c)*C*a^2*b*d + sqrt(d*x + c)*B*a*b^2*d - sqrt(
d*x + c)*A*b^3*d)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)
*((d*x + c)*b - b*c + a*d)) - 2/3*(3*(d*x + c)*D*b*c^3 - D*b*c^4 - 9*(d*x
+ c)*D*a*c^2*d + D*a*c^3*d + C*b*c^3*d + 6*(d*x + c)*C*a*c*d^2 - 3*(d*x +
c)*B*b*c*d^2 - C*a*c^2*d^2 - B*b*c^2*d^2 - 3*(d*x + c)*B*a*d^3 + 6*(d*x +
c)*A*b*d^3 + B*a*c*d^3 + A*b*c*d^3 - A*a*d^4)/((b^3*c^3*d^2 - 3*a*b^2*c^2*
d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(d*x + c)^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 913, normalized size of antiderivative = 3.34

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(5/2),x)
```

output
```
(3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*s
qrt(a*d - b*c)))*a**3*c*d**2 + 3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*ata
n((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*d**3*x - 12*sqrt(b)*sq
rt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c
)))*a**2*b*c**2*d - 9*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c +
 d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b*c*d**2*x + 3*sqrt(b)*sqrt(c + d
*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2
*b*d**3*x**2 + 6*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)
*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**3*c*d + 6*sqrt(b)*sqrt(c + d*x)*sqrt(a
*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**3*d**2*x
- 12*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)
*sqrt(a*d - b*c)))*a*b**2*c**2*d*x - 12*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b
*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**2*c*d**2*x**2 +
 6*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*s
qrt(a*d - b*c)))*b**4*c*d*x + 6*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan
((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*b**4*d**2*x**2 - 2*a**3*b**2
*d**3 - 3*a**3*b*c**2*d**2 - 6*a**3*b*c*d**3*x - 3*a**3*b*d**4*x**2 + 10*a
**2*b**3*c*d**2 + 4*a**2*b**3*d**3*x - 3*a**2*b**2*c**3*d + 3*a**2*b**2*c*
d**3*x**2 - 8*a*b**4*c**2*d + 4*a*b**4*c*d**2*x + 6*a*b**4*d**3*x**2 + 6*a
*b**3*c**4 - 6*a*b**3*c**2*d**2*x**2 - 8*b**5*c**2*d*x - 6*b**5*c*d**2*...
```

3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{5/2}}\,dx$

Optimal result 902
Mathematica [A] (verified) 903
Rubi [A] (verified) 903
Maple [A] (verified) 907
Fricas [B] (verification not implemented) 908
Sympy [F(-1)] 908
Maxima [F(-2)] 909
Giac [B] (verification not implemented) 909
Mupad [F(-1)] 910
Reduce [B] (verification not implemented) 911

Optimal result

Integrand size = 32, antiderivative size = 373

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{5/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)}{3d(bc-ad)^3(c+dx)^{3/2}}$$
$$+\frac{2(b(c^2C-2Bcd+3Ad^2)+a(2cCd-Bd^2-3c^2D))}{(bc-ad)^4\sqrt{c+dx}}$$
$$-\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{2b(bc-ad)^3(a+bx)^2}$$
$$-\frac{(b^3(4Bc-11Ad)-ab^2(8cC-7Bd)-a^3dD-3a^2b(Cd-4cD))\sqrt{c+dx}}{4b(bc-ad)^4(a+bx)}$$
$$-\frac{(b^3(8c^2C-20Bcd+35Ad^2)+a^3d^2D+3a^2bd(Cd-4cD)+3ab^2(8cCd-5Bd^2-8c^2D))\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{bc}}\right)}{4b^{3/2}(bc-ad)^{9/2}}$$

output
```
2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d/(-a*d+b*c)^3/(d*x+c)^(3/2)+2*(b*(3*A*d
^2-2*B*c*d+C*c^2)+a*(-B*d^2+2*C*c*d-3*D*c^2))/(-a*d+b*c)^4/(d*x+c)^(1/2)-1
/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b/(-a*d+b*c)^3/(b*x+a)^2-1/
4*(b^3*(-11*A*d+4*B*c)-a*b^2*(-7*B*d+8*C*c)-a^3*d*D-3*a^2*b*(C*d-4*D*c))*(
d*x+c)^(1/2)/b/(-a*d+b*c)^4/(b*x+a)-1/4*(b^3*(35*A*d^2-20*B*c*d+8*C*c^2)+a
^3*d^2*D+3*a^2*b*d*(C*d-4*D*c)+3*a*b^2*(-5*B*d^2+8*C*c*d-8*D*c^2))*arctanh
(b^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^(9/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{5/2}}\,dx = \frac{-3a^4d^2D(c+dx)^2-4b^4cx(2cx(-4cCd+c^2D-3Cd^2x)+Bd(3c^2+20cd}{}$$

$$+\frac{(b^3(8c^2C-20Bcd+35Ad^2)+a^3d^2D+3a^2bd(Cd-4cD)-3ab^2(-8cCd+5Bd^2+8c^2D))\arctan\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{-b}}\right.}{4b^{3/2}(-bc+ad)^{9/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

output
```
(-3*a^4*d^2*D*(c + d*x)^2 - 4*b^4*c*x*(2*c*x*(-4*c*C*d + c^2*D - 3*C*d^2*x
) + B*d*(3*c^2 + 20*c*d*x + 15*d^2*x^2)) + a^3*b*d*(-94*c^3*D + c^2*d*(55*
C - 129*D*x) + 3*d^3*x*(-8*B + 5*C*x + D*x^2) - 2*c*d^2*(8*B - 39*C*x + 12
*D*x^2)) - a^2*b^2*(8*c^4*D + 3*d^4*x^2*(25*B - 3*C*x) + c^3*(-50*C*d + 16
4*d*D*x) + 2*c*d^3*x*(67*B - 66*C*x + 18*D*x^2) + c^2*d^2*(83*B - 149*C*x
+ 216*D*x^2)) + A*b*d*(-8*a^3*d^3 + 8*a^2*b*d^2*(10*c + 7*d*x) + a*b^2*d*(
39*c^2 + 238*c*d*x + 175*d^2*x^2) + b^3*(-6*c^3 + 21*c^2*d*x + 140*c*d^2*x
^2 + 105*d^3*x^3)) - a*b^3*(B*d*(6*c^3 + 145*c^2*d*x + 160*c*d^2*x^2 + 45*
d^3*x^3) + 8*c*x*(2*c^3*D - 9*C*d^3*x^2 + c*d^2*x*(-17*C + 9*D*x) + c^2*(-
11*C*d + 8*d*D*x))))/(12*b*d*(b*c - a*d)^4*(a + b*x)^2*(c + d*x)^(3/2)) +
((b^3*(8*c^2*C - 20*B*c*d + 35*A*d^2) + a^3*d^2*D + 3*a^2*b*d*(C*d - 4*c*D
) - 3*a*b^2*(-8*c*C*d + 5*B*d^2 + 8*c^2*D))*ArcTan[(Sqrt[b]*Sqrt[c + d*x])
/Sqrt[-(b*c) + a*d]])/(4*b^(3/2)*(-(b*c) + a*d)^(9/2))
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2124, 27, 1192, 25, 1582, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{\int -\frac{4\left(c-\frac{ad}{b}\right)Dx^2+\frac{4(bc-ad)(bC-aD)x}{b^2}+\frac{3dDa^3-b(3Cd-4cD)a^2-b^2(4cC-3Bd)a+b^3(4Bc-7Ad)}{b^3}}{2(a+bx)^2(c+dx)^{5/2}}dx}{2(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{3dDa^3}{b^3}-\frac{(3Cd-4cD)a^2}{b^2}-\frac{(4cC-3Bd)a}{b}+4\left(c-\frac{ad}{b}\right)Dx^2+4Bc-7Ad+\frac{4(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^2(c+dx)^{5/2}}dx}{4(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{1192}$$

$$\frac{\int -\frac{-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2+d^3\left(7A-\frac{3a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(c+dx)^2(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{-4Dc^3+4Cdc^2-4Bd^2c-4\left(c-\frac{ad}{b}\right)D(c+dx)^2+d^3\left(7A-\frac{3a\left(Da^2-bCa+b^2B\right)}{b^3}\right)-\frac{4(bc-ad)(bCd-aDd-2bcD)(c+dx)}{b^2}}{(c+dx)^2(bc-ad-b(c+dx))^2}d\sqrt{c+dx}}{2d(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{d^2\sqrt{c+dx}\left(-5a^3dD+a^2b(12cD+Cd)-ab^2(8cC-3Bd)+b^3(4Bc-7Ad)\right)}{2b(bc-ad)^3(-ad-b(c+dx)+bc)}-\int -\frac{\frac{2\left(-\left(\left(-4Dc^3+4Cdc^2-4Bd^2c+7Ad^3\right)b^3\right)+3aBd^3b^2-3a^2Cd^3b+3a^3d^3 \cdots}{b}}{\cdots}}{\cdots}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\frac{\frac{2\left(-\left(\left(-4Dc^3+4Cdc^2-4Bd^2c+7Ad^3\right)b^3\right)+3aBd^3b^2-3a^2Cd^3b+3a^3d^3D\right)(bc-ad)^2}{b}+2\left(\left(-4Dc^3+4Bd^2c-7Ad^3\right)b^3-ad\left(-12Dc^2+8Cdc-3Bd^2\right)b^2+a^2Cd^3b \cdots}{(c+dx)^2(bc-ad-b(c+dx))}}{2b^2(bc-ad)^3}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1584$$

$$\int \frac{\left(\frac{bd\left(-d^2Da^3-3bd(Cd-4cD)a^2-3b^2\left(-8Dc^2+8Cdc-5Bd^2\right)a-b^3\left(8Cc^2-20Bdc+35Ad^2\right)\right)}{bc-ad-b(c+dx)}+\frac{4d\left(d^2Da^3-bCd^2a^2-b^2\left(-6Dc^2+4Cdc-3Bd^2\right)a-b^3\left(2Cc^2-4 \cdots\right.\right.}{c+dx}\right.}{2b^2(bc-ad)^3}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\left(a^3d^2D+3a^2bd(Cd-4cD)+3ab^2\left(-5Bd^2-8c^2D+8cCd\right)+b^3\left(35Ad^2-20Bcd+8c^2C\right)\right)}{\sqrt{bc-ad}}+\frac{4d\left(a^3\left(-d^2\right)D+a^2bCd^2+ab^2\left(-3Bd^2- \cdots\right.\right.}{\sqrt{c \cdots}}}{2b^2(bc-ad)^3}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

output
```
-1/2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b^3*(b*c - a*d)*(a + b*x)^2*(c +
 d*x)^(3/2)) + ((d^2*(b^3*(4*B*c - 7*A*d) - a*b^2*(8*c*C - 3*B*d) - 5*a^3*
d*D + a^2*b*(C*d + 12*c*D))*Sqrt[c + d*x])/(2*b*(b*c - a*d)^3*(b*c - a*d -
 b*(c + d*x))) + ((-2*(b*c - a*d)*(3*a*b^2*B*d^3 - 3*a^2*b*C*d^3 + 3*a^3*d
^3*D - b^3*(4*c^2*C*d - 4*B*c*d^2 + 7*A*d^3 - 4*c^3*D)))/(3*b*(c + d*x)^(3
/2)) + (4*d*(a^2*b*C*d^2 + b^3*(2*c^2*C - 4*B*c*d + 7*A*d^2) - a^3*d^2*D +
 a*b^2*(4*c*C*d - 3*B*d^2 - 6*c^2*D)))/Sqrt[c + d*x] - (Sqrt[b]*d*(b^3*(8*
c^2*C - 20*B*c*d + 35*A*d^2) + a^3*d^2*D + 3*a^2*b*d*(C*d - 4*c*D) + 3*a*b
^2*(8*c*C*d - 5*B*d^2 - 8*c^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c
 - a*d]])/Sqrt[b*c - a*d])/(2*b^2*(b*c - a*d)^3))/(2*d*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.23

method	result
derivativedivides	$2d\left(\left(\frac{11}{8}b^3d^2A-\frac{7}{8}Bab^2d^2-\frac{1}{2}Bb^3cd+\frac{3}{8}Ca^2bd^2+Cab^2cd+\frac{1}{8}a^3d^2D-\frac{3}{2}a^2bcdD\right)(xd+c)^{\frac{3}{2}}+\frac{d\left(13Aab^3d^2-13Ab^4cd-9Ba^2b^2d^2\cdots\right.}{((xd+c)b+ad-bc)^2}\cdots\right.$
default	$2d\left(\left(\frac{11}{8}b^3d^2A-\frac{7}{8}Bab^2d^2-\frac{1}{2}Bb^3cd+\frac{3}{8}Ca^2bd^2+Cab^2cd+\frac{1}{8}a^3d^2D-\frac{3}{2}a^2bcdD\right)(xd+c)^{\frac{3}{2}}+\frac{d\left(13Aab^3d^2-13Ab^4cd-9Ba^2b^2d^2\cdots\right.}{((xd+c)b+ad-bc)^2}\cdots\right.$
pseudoelliptic	$\frac{35\left(\left(b^3A-\frac{3}{7}ab^2B+\frac{3}{35}a^2bC+\frac{1}{35}a^3D\right)d^2-\frac{4cb\left(Bb^2-\frac{6}{5}Cab+\frac{3}{5}Da^2\right)d}{7}+\frac{8b^2c^2(Cb-3Da)}{35}\right)(xd+c)^{\frac{3}{2}}d(bx+a)^2\arctan\left(\frac{b\sqrt{xd+c}}{\sqrt{(ad-bc)\cdots}}\cdots\right.}{4}$

input

```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(d/(a*d-b*c)^4*(((11/8*b^3*d^2*A-7/8*B*a*b^2*d^2-1/2*B*b^3*c*d+3/8*C*a
^2*b*d^2+C*a*b^2*c*d+1/8*a^3*d^2*D-3/2*a^2*b*c*d*D)*(d*x+c)^(3/2)+1/8*d*(1
3*A*a*b^3*d^2-13*A*b^4*c*d-9*B*a^2*b^2*d^2+5*B*a*b^3*c*d+4*B*b^4*c^2+5*C*a
^3*b*d^2+3*C*a^2*b^2*c*d-8*C*a*b^3*c^2-D*a^4*d^2-11*D*a^3*b*c*d+12*D*a^2*b
^2*c^2)/b*(d*x+c)^(1/2))/((d*x+c)*b+a*d-b*c)^2+1/8*(35*A*b^3*d^2-15*B*a*b^
2*d^2-20*B*b^3*c*d+3*C*a^2*b*d^2+24*C*a*b^2*c*d+8*C*b^3*c^2+D*a^3*d^2-12*D
*a^2*b*c*d-24*D*a*b^2*c^2)/b/((a*d-b*c)*b)^(1/2)*arctan(b*(d*x+c)^(1/2)/((
a*d-b*c)*b)^(1/2)))-1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^3/(d*x+c)^
(3/2)+d*(3*A*b*d^2-B*a*d^2-2*B*b*c*d+2*C*a*c*d+C*b*c^2-3*D*a*c^2)/(a*d-b*c
)^4/(d*x+c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1800 vs. 2(348) = 696.

Time = 0.28 (sec) , antiderivative size = 3614, normalized size of antiderivative = 9.69

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(5/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(348) = 696$.

Time = 0.16 (sec) , antiderivative size = 767, normalized size of antiderivative = 2.06

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
-1/4*(24*D*a*b^2*c^2 - 8*C*b^3*c^2 + 12*D*a^2*b*c*d - 24*C*a*b^2*c*d + 20*
B*b^3*c*d - D*a^3*d^2 - 3*C*a^2*b*d^2 + 15*B*a*b^2*d^2 - 35*A*b^3*d^2)*arc
tan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^
2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(-b^2*c + a*b*d)) - 2/3*(
D*b*c^4 + 9*(d*x + c)*D*a*c^2*d - 3*(d*x + c)*C*b*c^2*d - D*a*c^3*d - C*b*
c^3*d - 6*(d*x + c)*C*a*c*d^2 + 6*(d*x + c)*B*b*c*d^2 + C*a*c^2*d^2 + B*b*
c^2*d^2 + 3*(d*x + c)*B*a*d^3 - 9*(d*x + c)*A*b*d^3 - B*a*c*d^3 - A*b*c*d^
3 + A*a*d^4)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c
*d^4 + a^4*d^5)*(d*x + c)^(3/2)) - 1/4*(12*(d*x + c)^(3/2)*D*a^2*b^2*c*d -
 8*(d*x + c)^(3/2)*C*a*b^3*c*d + 4*(d*x + c)^(3/2)*B*b^4*c*d - 12*sqrt(d*x
 + c)*D*a^2*b^2*c^2*d + 8*sqrt(d*x + c)*C*a*b^3*c^2*d - 4*sqrt(d*x + c)*B*
b^4*c^2*d - (d*x + c)^(3/2)*D*a^3*b*d^2 - 3*(d*x + c)^(3/2)*C*a^2*b^2*d^2
+ 7*(d*x + c)^(3/2)*B*a*b^3*d^2 - 11*(d*x + c)^(3/2)*A*b^4*d^2 + 11*sqrt(d
*x + c)*D*a^3*b*c*d^2 - 3*sqrt(d*x + c)*C*a^2*b^2*c*d^2 - 5*sqrt(d*x + c)*
B*a*b^3*c*d^2 + 13*sqrt(d*x + c)*A*b^4*c*d^2 + sqrt(d*x + c)*D*a^4*d^3 - 5
*sqrt(d*x + c)*C*a^3*b*d^3 + 9*sqrt(d*x + c)*B*a^2*b^2*d^3 - 13*sqrt(d*x +
 c)*A*a*b^3*d^3)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2
*c*d^3 + a^4*b*d^4)*((d*x + c)*b - b*c + a*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{5/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^3\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1692, normalized size of antiderivative = 4.54

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(5/2),x)
```

output
```
(3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*s
qrt(a*d - b*c)))*a**4*c*d**2 + 3*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*ata
n((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**4*d**3*x - 24*sqrt(b)*sq
rt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c
)))*a**3*b*c**2*d - 18*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c
+ d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**3*b*c*d**2*x + 6*sqrt(b)*sqrt(c +
d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**
3*b*d**3*x**2 + 60*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*
x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**3*c*d + 60*sqrt(b)*sqrt(c + d*x)*
sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**
3*d**2*x - 24*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)
/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*c**3 - 72*sqrt(b)*sqrt(c + d*x)*sqrt
(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*c*
*2*d*x - 45*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(
sqrt(b)*sqrt(a*d - b*c)))*a**2*b**2*c*d**2*x**2 + 3*sqrt(b)*sqrt(c + d*x)*
sqrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a**2*b**
2*d**3*x**3 + 120*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x
)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*c*d*x + 120*sqrt(b)*sqrt(c + d*x)*s
qrt(a*d - b*c)*atan((sqrt(c + d*x)*b)/(sqrt(b)*sqrt(a*d - b*c)))*a*b**4*d*
*2*x**2 - 48*sqrt(b)*sqrt(c + d*x)*sqrt(a*d - b*c)*atan((sqrt(c + d*x)*...
```

3.101 $\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result 912
Mathematica [A] (verified) 913
Rubi [A] (verified) 914
Maple [B] (verified) 918
Fricas [A] (verification not implemented) 919
Sympy [F] 920
Maxima [F(-2)] 921
Giac [B] (verification not implemented) 921
Mupad [F(-1)] 922
Reduce [B] (verification not implemented) 923

Optimal result

Integrand size = 34, antiderivative size = 639

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx =$$

$$-\frac{(bc-ad)^2\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d(24cCd-40Bd^2-15c^2D)-b^3(12c^2Cd-24Bcd^2+\right.}{512b^5d^4}$$

$$-\frac{(bc-ad)\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d(24cCd-40Bd^2-15c^2D)-b^3(12c^2Cd-24Bcd^2+6\right.}{256b^5d^3}$$

$$-\frac{\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d(24cCd-40Bd^2-15c^2D)-b^3(12c^2Cd-24Bcd^2+64Ad^3-7\right.}{192b^4d^3}$$

$$-\frac{\left(12bcC-24bBd+36aCd-22acD-\frac{7bc^2D}{d}-\frac{43a^2dD}{b}\right)(a+bx)^{3/2}(c+dx)^{5/2}}{96b^2d^2}$$

$$+\frac{(12bCd-7bcD-29adD)(a+bx)^{5/2}(c+dx)^{5/2}}{60b^3d^2}+\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$$+\frac{(bc-ad)^3\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d(24cCd-40Bd^2-15c^2D)-b^3(12c^2Cd-24Bcd^2+\right.}{512b^{11/2}d^{9/2}}$$

output
```
-1/512*(-a*d+b*c)^2*(21*a^3*d^3*D-7*a^2*b*d^2*(4*C*d-3*D*c)-a*b^2*d*(-40*B
*d^2+24*C*c*d-15*D*c^2)-b^3*(64*A*d^3-24*B*c*d^2+12*C*c^2*d-7*D*c^3))*(b*x
+a)^(1/2)*(d*x+c)^(1/2)/b^5/d^4-1/256*(-a*d+b*c)*(21*a^3*d^3*D-7*a^2*b*d^2
*(4*C*d-3*D*c)-a*b^2*d*(-40*B*d^2+24*C*c*d-15*D*c^2)-b^3*(64*A*d^3-24*B*c*
d^2+12*C*c^2*d-7*D*c^3))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^5/d^3-1/192*(21*a^3
*d^3*D-7*a^2*b*d^2*(4*C*d-3*D*c)-a*b^2*d*(-40*B*d^2+24*C*c*d-15*D*c^2)-b^3
*(64*A*d^3-24*B*c*d^2+12*C*c^2*d-7*D*c^3))*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b^4
/d^3-1/96*(12*C*b*c-24*B*b*d+36*C*a*d-22*D*a*c-7*b*c^2*D/d-43*a^2*d*D/b)*(
b*x+a)^(3/2)*(d*x+c)^(5/2)/b^2/d^2+1/60*(12*C*b*d-29*D*a*d-7*D*b*c)*(b*x+a
)^(5/2)*(d*x+c)^(5/2)/b^3/d^2+1/6*D*(b*x+a)^(7/2)*(d*x+c)^(5/2)/b^3/d+1/51
2*(-a*d+b*c)^3*(21*a^3*d^3*D-7*a^2*b*d^2*(4*C*d-3*D*c)-a*b^2*d*(-40*B*d^2+
24*C*c*d-15*D*c^2)-b^3*(64*A*d^3-24*B*c*d^2+12*C*c^2*d-7*D*c^3))*arctanh(d
^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(11/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx}(315a^5d^5D-105a^4bd^4(4Cd+5cD+2dDx)+2a^3b^2d^3(39c^2D+4cd(}{}$$

input
```
Integrate[Sqrt[a + b*x]*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]*(315*a^5*d^5*D - 105*a^4*b*d^
4*(4*C*d + 5*c*D + 2*d*D*x) + 2*a^3*b^2*d^3*(39*c^2*D + 4*c*d*(95*C + 42*D
*x) + 4*d^2*(75*B + 7*x*(5*C + 3*D*x))) + a*b^4*d*(55*c^4*D - 8*c^3*d*(15*
C + 4*D*x) + 24*c^2*d^2*(15*B + x*(3*C + D*x)) + 64*d^4*x*(10*A + x*(5*B +
 3*C*x + 2*D*x^2)) + 32*c*d^3*(80*A + x*(25*B + 12*C*x + 7*D*x^2))) - 2*a^
2*b^3*d^2*(-27*c^3*D + 18*c^2*d*(4*C + D*x) + 4*c*d^2*(155*B + x*(61*C + 3
3*D*x)) + 8*d^3*(60*A + x*(25*B + 14*C*x + 9*D*x^2))) + b^5*(-105*c^5*D +
10*c^4*d*(18*C + 7*D*x) - 8*c^3*d^2*(45*B + x*(15*C + 7*D*x)) + 48*c^2*d^3
*(20*A + x*(5*B + x*(2*C + D*x))) + 128*d^5*x^2*(20*A + x*(15*B + 2*x*(6*C
 + 5*D*x))) + 64*c*d^4*x*(70*A + x*(45*B + x*(33*C + 26*D*x))))) + 15*(b*c
 - a*d)^3*(21*a^3*d^3*D - 7*a^2*b*d^2*(4*C*d - 3*c*D) + a*b^2*d*(-24*c*C*d
 + 40*B*d^2 + 15*c^2*D) + b^3*(-12*c^2*C*d + 24*B*c*d^2 - 64*A*d^3 + 7*c^3
*D))*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(7680*b^(11
/2)*d^(9/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2125, 27, 1194, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2125}$$

$$\frac{\int \frac{1}{2}\sqrt{a+bx}(c+dx)^{3/2}\left(12Adb^3+(12bCd-29aDd-7bcD)x^2b^2+2\left(-11dDa^2-7bcDa+6b^2Bd\right)xb-a^2(7bc}{6b^3d}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+bx}(c+dx)^{3/2}\left(12Adb^3+(12bCd-29aDd-7bcD)x^2b^2+2\left(-11dDa^2-7bcDa+6b^2Bd\right)xb-a^2(7bc+}{12b^3d}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$\Big\downarrow$ 1194

$$\frac{\frac{\int \frac{5}{2}b^2\sqrt{a+bx}(c+dx)^{3/2}\left(19d^2Da^3-2bd(6Cd-11cD)a^2-b^2c(12Cd-7cD)a+24Ab^3d^2+b\left(-\left(\left(-7Dc^2+12Cdc-24Bd^2\right)b^2\right)-2ad(18Cd-11cD)b+\cdots\right.\right.}{5b^2d}}{12b^3d}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \sqrt{a+bx}(c+dx)^{3/2}\left(19d^2Da^3-2bd(6Cd-11cD)a^2-b^2c(12Cd-7cD)a+24Ab^3d^2+b\left(-\left(\left(-7Dc^2+12Cdc-24Bd^2\right)b^2\right)-2ad(18Cd-11cD)b+43a^2\cdots\right.\right.}{2d}}{12b^3d}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$\Big\downarrow$ 90

$$\frac{\frac{\frac{(a+bx)^{3/2}(c+dx)^{5/2}\left(43a^2d^2D-2abd(18Cd-11cD)-\left(b^2\left(-24Bd^2-7c^2D+12cCd\right)\right)\right)}{4d}-\frac{3\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d\left(-40Bd^2-15c^2D+24cCd\right)-\left(b^3\cdots\right.\right.}{8d}}{2d}}{12b^3d}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$\Big\downarrow$ 60

$$\frac{\frac{\frac{(a+bx)^{3/2}(c+dx)^{5/2}\left(43a^2d^2D-2abd(18Cd-11cD)-\left(b^2\left(-24Bd^2-7c^2D+12cCd\right)\right)\right)}{4d}-\frac{3\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d\left(-40Bd^2-15c^2D+24cCd\right)-\left(b^3\cdots\right.\right.}{\cdots}}{2d}}{12b^3d}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$\Big\downarrow$ 60

$$\frac{\frac{(a+bx)^{3/2}(c+dx)^{5/2}\left(43a^2d^2D-2abd(18Cd-11cD)-\left(b^2\left(-24Bd^2-7c^2D+12cCd\right)\right)\right)}{4d}-\frac{3\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d\left(-40Bd^2-15c^2D+24cCd\right)-\left(b^3\cdots\right.\right.}{\cdots}}{2d}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$\Big\downarrow$ 60

$$\frac{\frac{(a+bx)^{3/2}(c+dx)^{5/2}\left(43a^2d^2D-2abd(18Cd-11cD)-\left(b^2\left(-24Bd^2-7c^2D+12cCd\right)\right)\right)}{4d}-\frac{3\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d\left(-40Bd^2-15c^2D+24cCd\right)-\left(b^3\right.\right.}{}}{2d}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$\Big\downarrow$ 66

$$\frac{\frac{(a+bx)^{3/2}(c+dx)^{5/2}\left(43a^2d^2D-2abd(18Cd-11cD)-\left(b^2\left(-24Bd^2-7c^2D+12cCd\right)\right)\right)}{4d}-\frac{3\left(21a^3d^3D-7a^2bd^2(4Cd-3cD)-ab^2d\left(-40Bd^2-15c^2D+24cCd\right)-\left(b^3\right.\right.}{}}{2}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

$\Big\downarrow$ 221

$$\frac{\frac{(a+bx)^{3/2}(c+dx)^{5/2}\left(43a^2d^2D-2abd(18Cd-11cD)-\left(b^2\left(-24Bd^2-7c^2D+12cCd\right)\right)\right)}{4d}-\frac{3\left(\frac{(bc-ad)\left(\frac{(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+b}}{\sqrt{b}\sqrt{c+d}}\right.}{\sqrt{b}d^{3/2}}\right.}{4b}\right.}{2b}\right.}{}}{2}$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{5/2}}{6b^3d}$$

input
```
Int[Sqrt[a + b*x]*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(a + b*x)^(7/2)*(c + d*x)^(5/2))/(6*b^3*d) + (((12*b*C*d - 7*b*c*D - 29
*a*d*D)*(a + b*x)^(5/2)*(c + d*x)^(5/2))/(5*d) + (((43*a^2*d^2*D - 2*a*b*d
*(18*C*d - 11*c*D) - b^2*(12*c*C*d - 24*B*d^2 - 7*c^2*D))*(a + b*x)^(3/2)*
(c + d*x)^(5/2))/(4*d) - (3*(21*a^3*d^3*D - 7*a^2*b*d^2*(4*C*d - 3*c*D) -
a*b^2*d*(24*c*C*d - 40*B*d^2 - 15*c^2*D) - b^3*(12*c^2*C*d - 24*B*c*d^2 +
64*A*d^3 - 7*c^3*D))*(((a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*b) + ((b*c - a*
d)*(((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*b) + ((b*c - a*d)*((Sqrt[a + b*x]*S
qrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*b)))/(2*b)))/(8*d))/(2*d))/(12*b^3*d
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2805 vs. $2(589) = 1178$.

Time = 0.54 (sec) , antiderivative size = 2806, normalized size of antiderivative = 4.39

method	result	size
default	Expression too large to display	2806

input
```
int((b*x+a)^(1/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOS
E)
```

output
```
1/15360*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(3840*B*b^5*d^5*x^3*((b*x+a)*(d*x+c))^
(1/2)*(d*b)^(1/2)+5120*A*b^5*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-6
4*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c^3*d^2*x+1600*B*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c*d^4*x-976*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)*a^2*b^3*c*d^4*x+144*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c^2
*d^3*x-72*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*c^2*d^3*x+960*A*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*
a^3*b^3*d^6+1280*A*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*d^5*x-1050*D*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^4*b*c*d^4+156*D*((b*x+a)*(d*x+c))^(1
/2)*(d*b)^(1/2)*a^3*b^2*c^2*d^3+108*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*
a^2*b^3*c^3*d^2+110*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c^4*d-240*
C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^3*d^2*x+480*B*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)*b^5*c^2*d^3*x+672*D*a^3*b^2*c*d^4*x*((b*x+a)*(d*x+c))^
(1/2)*(d*b)^(1/2)+2560*D*b^5*d^5*x^5*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+3
072*C*b^5*d^5*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+140*D*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)*b^5*c^4*d*x+720*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)*a*b^4*c^2*d^3+1520*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^3*b^2*c*d^4-1
920*A*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*d^5+1920*A*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)*b^5*c^2*d^3+1200*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)*a^3*b^2*d^5-720*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^3*d^2+9...
```

Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1750, normalized size of antiderivative = 2.74

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fr
icas")
```

output
```
[1/30720*(15*(7*D*b^6*c^6 - 6*(D*a*b^5 + 2*C*b^6)*c^5*d - 3*(D*a^2*b^4 - 4
*C*a*b^5 - 8*B*b^6)*c^4*d^2 - 4*(D*a^3*b^3 - 2*C*a^2*b^4 + 8*B*a*b^5 + 16*
A*b^6)*c^3*d^3 - 3*(5*D*a^4*b^2 - 8*C*a^3*b^3 + 16*B*a^2*b^4 - 64*A*a*b^5)
*c^2*d^4 + 6*(7*D*a^5*b - 10*C*a^4*b^2 + 16*B*a^3*b^3 - 32*A*a^2*b^4)*c*d^
5 - (21*D*a^6 - 28*C*a^5*b + 40*B*a^4*b^2 - 64*A*a^3*b^3)*d^6)*sqrt(b*d)*l
og(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)
*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(128
0*D*b^6*d^6*x^5 - 105*D*b^6*c^5*d + 5*(11*D*a*b^5 + 36*C*b^6)*c^4*d^2 + 6*
(9*D*a^2*b^4 - 20*C*a*b^5 - 60*B*b^6)*c^3*d^3 + 6*(13*D*a^3*b^3 - 24*C*a^2
*b^4 + 60*B*a*b^5 + 160*A*b^6)*c^2*d^4 - 5*(105*D*a^4*b^2 - 152*C*a^3*b^3
+ 248*B*a^2*b^4 - 512*A*a*b^5)*c*d^5 + 15*(21*D*a^5*b - 28*C*a^4*b^2 + 40*
B*a^3*b^3 - 64*A*a^2*b^4)*d^6 + 128*(13*D*b^6*c*d^5 + (D*a*b^5 + 12*C*b^6)
*d^6)*x^4 + 16*(3*D*b^6*c^2*d^4 + 2*(7*D*a*b^5 + 66*C*b^6)*c*d^5 - 3*(3*D*
a^2*b^4 - 4*C*a*b^5 - 40*B*b^6)*d^6)*x^3 - 8*(7*D*b^6*c^3*d^3 - 3*(D*a*b^5
 + 4*C*b^6)*c^2*d^4 + 3*(11*D*a^2*b^4 - 16*C*a*b^5 - 120*B*b^6)*c*d^5 - (2
1*D*a^3*b^3 - 28*C*a^2*b^4 + 40*B*a*b^5 + 320*A*b^6)*d^6)*x^2 + 2*(35*D*b^
6*c^4*d^2 - 4*(4*D*a*b^5 + 15*C*b^6)*c^3*d^3 - 6*(3*D*a^2*b^4 - 6*C*a*b^5
- 20*B*b^6)*c^2*d^4 + 4*(42*D*a^3*b^3 - 61*C*a^2*b^4 + 100*B*a*b^5 + 560*A
*b^6)*c*d^5 - 5*(21*D*a^4*b^2 - 28*C*a^3*b^3 + 40*B*a^2*b^4 - 64*A*a*b^5)*
d^6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*d^5), -1/15360*(15*(7*D*b^6*c...
```

Sympy [F]

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \sqrt{a+bx}(c+dx)^{\frac{3}{2}}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(a + b*x)*(c + d*x)**(3/2)*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3946 vs. $2(581) = 1162$.

Time = 0.62 (sec) , antiderivative size = 3946, normalized size of antiderivative = 6.18

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="gi
ac")
```

output
```
-1/7680*(7680*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2
*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*sqrt(b*x + a))*A*a*c*abs(b)/b^2 - 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d
^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b
^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*
a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*
x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*D*a*c*
abs(b)/b^2 - 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x
+ a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13
*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3
*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))
*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3
*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*C*c*abs(b)/b - 40*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12
*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 -
163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*
a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \sqrt{a+b\,x}\,(c+d\,x)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^(1/2)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^(1/2)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1334, normalized size of antiderivative = 2.09

$$\int \sqrt{a+bx}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(315*sqrt(c + d*x)*sqrt(a + b*x)*a**5*b*d**6 - 945*sqrt(c + d*x)*sqrt(a +
b*x)*a**4*b**2*c*d**5 - 210*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b**2*d**6*x -
 360*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**4*d**5 + 838*sqrt(c + d*x)*sqrt(a
 + b*x)*a**3*b**3*c**2*d**4 + 616*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**3*c*
d**5*x + 168*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**3*d**6*x**2 + 1320*sqrt(c
 + d*x)*sqrt(a + b*x)*a**2*b**5*c*d**4 + 240*sqrt(c + d*x)*sqrt(a + b*x)*a
**2*b**5*d**5*x - 90*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*c**3*d**3 - 524
*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*c**2*d**4*x - 488*sqrt(c + d*x)*sqr
t(a + b*x)*a**2*b**4*c*d**5*x**2 - 144*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b*
*4*d**6*x**3 + 1320*sqrt(c + d*x)*sqrt(a + b*x)*a*b**6*c**2*d**3 + 5280*sq
rt(c + d*x)*sqrt(a + b*x)*a*b**6*c*d**4*x + 2880*sqrt(c + d*x)*sqrt(a + b*
x)*a*b**6*d**5*x**2 - 65*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*c**4*d**2 + 40
*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*c**3*d**3*x + 408*sqrt(c + d*x)*sqrt(a
 + b*x)*a*b**5*c**2*d**4*x**2 + 416*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*c*d
**5*x**3 + 128*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*d**6*x**4 - 360*sqrt(c +
 d*x)*sqrt(a + b*x)*b**7*c**3*d**2 + 240*sqrt(c + d*x)*sqrt(a + b*x)*b**7*
c**2*d**3*x + 2880*sqrt(c + d*x)*sqrt(a + b*x)*b**7*c*d**4*x**2 + 1920*sqr
t(c + d*x)*sqrt(a + b*x)*b**7*d**5*x**3 + 75*sqrt(c + d*x)*sqrt(a + b*x)*b
**6*c**5*d - 50*sqrt(c + d*x)*sqrt(a + b*x)*b**6*c**4*d**2*x + 40*sqrt(c +
 d*x)*sqrt(a + b*x)*b**6*c**3*d**3*x**2 + 2160*sqrt(c + d*x)*sqrt(a + b...
```

3.102 $\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result 924
Mathematica [A] (verified) 925
Rubi [A] (verified) 926
Maple [B] (verified) 930
Fricas [A] (verification not implemented) 931
Sympy [F] 932
Maxima [F(-2)] 933
Giac [B] (verification not implemented) 933
Mupad [F(-1)] 934
Reduce [B] (verification not implemented) 935

Optimal result

Integrand size = 34, antiderivative size = 516

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx =$$

$$-\frac{(bc-ad)\left(7a^3d^3D-a^2bd^2(10Cd-9cD)-ab^2d(12cCd-16Bd^2-9c^2D)-b^3(10c^2Cd-16Bcd^2+3\right.}{128b^4d^4}$$

$$-\frac{(7a^3d^3D-a^2bd^2(10Cd-9cD)-ab^2d(12cCd-16Bd^2-9c^2D)-b^3(10c^2Cd-16Bcd^2+32Ad^3-7}{64b^4d^3}$$

$$-\frac{\left(10bcC-16bBd+22aCd-16acD-\frac{7bc^2D}{d}-\frac{25a^2dD}{b}\right)(a+bx)^{3/2}(c+dx)^{3/2}}{48b^2d^2}$$

$$+\frac{(10bCd-7bcD-23adD)(a+bx)^{5/2}(c+dx)^{3/2}}{40b^3d^2}+\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$$+\frac{(bc-ad)^2\left(7a^3d^3D-a^2bd^2(10Cd-9cD)-ab^2d(12cCd-16Bd^2-9c^2D)-b^3(10c^2Cd-16Bcd^2+\right.}{128b^{9/2}d^{9/2}}$$

output
```
-1/128*(-a*d+b*c)*(7*a^3*d^3*D-a^2*b*d^2*(10*C*d-9*D*c)-a*b^2*d*(-16*B*d^2
+12*C*c*d-9*D*c^2)-b^3*(32*A*d^3-16*B*c*d^2+10*C*c^2*d-7*D*c^3))*(b*x+a)^(
1/2)*(d*x+c)^(1/2)/b^4/d^4-1/64*(7*a^3*d^3*D-a^2*b*d^2*(10*C*d-9*D*c)-a*b^
2*d*(-16*B*d^2+12*C*c*d-9*D*c^2)-b^3*(32*A*d^3-16*B*c*d^2+10*C*c^2*d-7*D*c
^3))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^4/d^3-1/48*(10*C*b*c-16*B*b*d+22*C*a*d-
16*D*a*c-7*b*c^2*D/d-25*a^2*d*D/b)*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b^2/d^2+1/4
0*(10*C*b*d-23*D*a*d-7*D*b*c)*(b*x+a)^(5/2)*(d*x+c)^(3/2)/b^3/d^2+1/5*D*(b
*x+a)^(7/2)*(d*x+c)^(3/2)/b^3/d+1/128*(-a*d+b*c)^2*(7*a^3*d^3*D-a^2*b*d^2*
(10*C*d-9*D*c)-a*b^2*d*(-16*B*d^2+12*C*c*d-9*D*c^2)-b^3*(32*A*d^3-16*B*c*d
^2+10*C*c^2*d-7*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2
))/b^(9/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.81

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{\sqrt{a+bx}\sqrt{c+dx}\left(-105a^4d^4D+10a^3bd^3(15Cd+4cD+7dDx)-2a^2b^2d^2(-17c^2D+cd(35C+11Dx)\right.}{}$$

$$+\frac{(bc-ad)^2\left(7a^3d^3D+a^2bd^2(-10Cd+9cD)+ab^2d(-12cCd+16Bd^2+9c^2D)+b^3(-10c^2Cd+16B\right.}{128b^{9/2}d^{9/2}}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(-105*a^4*d^4*D + 10*a^3*b*d^3*(15*C*d + 4*c*
D + 7*d*D*x) - 2*a^2*b^2*d^2*(-17*c^2*D + c*d*(35*C + 11*D*x) + 2*d^2*(60*
B + x*(25*C + 14*D*x))) + 2*a*b^3*d*(20*c^3*D - c^2*d*(35*C + 11*D*x) + 4*
c*d^2*(20*B + x*(5*C + 2*D*x)) + 8*d^3*(30*A + x*(10*B + 5*C*x + 3*D*x^2))
) + b^4*(-105*c^4*D + 10*c^3*d*(15*C + 7*D*x) - 4*c^2*d^2*(60*B + x*(25*C
+ 14*D*x)) + 16*c*d^3*(30*A + x*(10*B + x*(5*C + 3*D*x))) + 32*d^4*x*(30*A
 + x*(20*B + 3*x*(5*C + 4*D*x))))))/(1920*b^4*d^4) + ((b*c - a*d)^2*(7*a^3
*d^3*D + a^2*b*d^2*(-10*C*d + 9*c*D) + a*b^2*d*(-12*c*C*d + 16*B*d^2 + 9*c
^2*D) + b^3*(-10*c^2*C*d + 16*B*c*d^2 - 32*A*d^3 + 7*c^3*D))*ArcTanh[(Sqrt
[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(9/2)*d^(9/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {2125, 27, 1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \; 2125$$

$$\frac{\int \frac{1}{2}\sqrt{a+bx}\sqrt{c+dx}\left(10Adb^3+(10bCd-23aDd-7bcD)x^2b^2+2\left(-8dDa^2-7bcDa+5b^2Bd\right)xb-a^2(7bc+3a\cdots}{5b^3d}$$
$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sqrt{a+bx}\sqrt{c+dx}\left(10Adb^3+(10bCd-23aDd-7bcD)x^2b^2+2\left(-8dDa^2-7bcDa+5b^2Bd\right)xb-a^2(7bc+3ad\cdots}{10b^3d}$$
$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$$\Big\downarrow \; 1194$$

$$\frac{\frac{\int \frac{5}{2}b^2\sqrt{a+bx}\sqrt{c+dx}\left(9d^2Da^3-2bd(3Cd-8cD)a^2-b^2c(10Cd-7cD)a+16Ab^3d^2+b\left(-\left(\left(-7Dc^2+10Cdc-16Bd^2\right)b^2\right)-2ad(11Cd-8cD)b+25a^2d^2\cdots}{4b^2d}\cdots}{10b^3d}$$
$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{5\int \sqrt{a+bx}\sqrt{c+dx}\left(9d^2Da^3-2bd(3Cd-8cD)a^2-b^2c(10Cd-7cD)a+16Ab^3d^2+b\left(-\left(\left(-7Dc^2+10Cdc-16Bd^2\right)b^2\right)-2ad(11Cd-8cD)b+25a^2d^2D\cdots}{8d}\cdots}{10b^3d}$$
$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$$\Big\downarrow \; 90$$

$$\frac{5\left(\frac{(a+bx)^{3/2}(c+dx)^{3/2}\left(25a^2d^2D-2abd(11Cd-8cD)-\left(b^2\left(-16Bd^2-7c^2D+10cCd\right)\right)\right)}{3d}-\frac{\left(7a^3d^3D-a^2bd^2(10Cd-9cD)-ab^2d\left(-16Bd^2-9c^2D+12cCd\right)-\left(b^3\right.\cdots}{2d}\cdots}{8d}\cdots$$

$$10b^3d$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$\downarrow$ 60

$$\frac{5\left(\frac{(a+bx)^{3/2}(c+dx)^{3/2}\left(25a^2d^2D-2abd(11Cd-8cD)-\left(b^2\left(-16Bd^2-7c^2D+10cCd\right)\right)\right)}{3d}-\frac{\left(7a^3d^3D-a^2bd^2(10Cd-9cD)-ab^2d\left(-16Bd^2-9c^2D+12cCd\right)-\left(b^3\right.\cdots}{\cdots}\cdots}{8d}\cdots$$

$$10b^3d$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$\downarrow$ 60

$$\frac{5\left(\frac{(a+bx)^{3/2}(c+dx)^{3/2}\left(25a^2d^2D-2abd(11Cd-8cD)-\left(b^2\left(-16Bd^2-7c^2D+10cCd\right)\right)\right)}{3d}-\frac{\left(7a^3d^3D-a^2bd^2(10Cd-9cD)-ab^2d\left(-16Bd^2-9c^2D+12cCd\right)-\left(b^3\right.\cdots}{\cdots}\cdots}{8d}\cdots$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$\downarrow$ 66

$$5\left(\frac{(a+bx)^{3/2}(c+dx)^{3/2}\left(25a^2d^2D-2abd(11Cd-8cD)-\left(b^2\left(-16Bd^2-7c^2D+10cCd\right)\right)\right)}{3d}-\frac{\left(7a^3d^3D-a^2bd^2(10Cd-9cD)-ab^2d\left(-16Bd^2-9c^2D+12cCd\right)-\left(b^3\left(\right.\right.\right.}{}\right.$$

$$8d$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

$\Big\downarrow$ 221

$$5\left(\frac{(a+bx)^{3/2}(c+dx)^{3/2}\left(25a^2d^2D-2abd(11Cd-8cD)-\left(b^2\left(-16Bd^2-7c^2D+10cCd\right)\right)\right)}{3d}-\frac{\left(\frac{(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4b}+\frac{(}{}\right.}{}\right.$$

$$8d$$

$$\frac{D(a+bx)^{7/2}(c+dx)^{3/2}}{5b^3d}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(a + b*x)^(7/2)*(c + d*x)^(3/2))/(5*b^3*d) + (((10*b*C*d - 7*b*c*D - 23
*a*d*D)*(a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*d) + (5*(((25*a^2*d^2*D - 2*a*
b*d*(11*C*d - 8*c*D) - b^2*(10*c*C*d - 16*B*d^2 - 7*c^2*D))*(a + b*x)^(3/2
)*(c + d*x)^(3/2))/(3*d) - ((7*a^3*d^3*D - a^2*b*d^2*(10*C*d - 9*c*D) - a*
b^2*d*(12*c*C*d - 16*B*d^2 - 9*c^2*D) - b^3*(10*c^2*C*d - 16*B*c*d^2 + 32*
A*d^3 - 7*c^3*D))*(((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*b) + ((b*c - a*d)*((
Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*b)))/(2*d)))/(8*d))/(
10*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(472) = 944$.

Time = 0.53 (sec) , antiderivative size = 1998, normalized size of antiderivative = 3.87

method	result	size
default	Expression too large to display	1998

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOS
E)
```

output
```
-1/3840*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(480*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*d^5+480*A*ln(1/2*(2*b
*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^2*d
^3+140*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c*d^3+140*C*(d*b)^(1/
2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^2*d^2-80*D*(d*b)^(1/2)*((b*x+a)*(d*x+c)
)^(1/2)*a^3*b*c*d^3-68*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c^2*d
^2-320*B*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^4*c*d^3*x+200*C*(d*b)^(1/2)
*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*d^4*x+200*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))
^(1/2)*b^4*c^2*d^2*x-768*D*b^4*d^4*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
-960*C*b^4*d^4*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-1280*B*b^4*d^4*x^2*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-80*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/
2)*a*b^3*c*d^3*x+44*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c*d^3*x+
44*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^2*d^2*x-80*D*(d*b)^(1/2)*
((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^3*d-32*D*a*b^3*c*d^3*x^2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)-320*B*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c*d^3+2
40*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^
(1/2))*a^2*b^3*c*d^4+240*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d^3-120*C*ln(1/2*(2*b*d*x+2*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2*c*d^4-60*C*ln(1
/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))...
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1266, normalized size of antiderivative = 2.45

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fr
icas")
```

output
```
[-1/7680*(15*(7*D*b^5*c^5 - 5*(D*a*b^4 + 2*C*b^5)*c^4*d - 2*(D*a^2*b^3 - 4
*C*a*b^4 - 8*B*b^5)*c^3*d^2 - 2*(D*a^3*b^2 - 2*C*a^2*b^3 + 8*B*a*b^4 + 16*
A*b^5)*c^2*d^3 - (5*D*a^4*b - 8*C*a^3*b^2 + 16*B*a^2*b^3 - 64*A*a*b^4)*c*d
^4 + (7*D*a^5 - 10*C*a^4*b + 16*B*a^3*b^2 - 32*A*a^2*b^3)*d^5)*sqrt(b*d)*l
og(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)
*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(384
*D*b^5*d^5*x^4 - 105*D*b^5*c^4*d + 10*(4*D*a*b^4 + 15*C*b^5)*c^3*d^2 + 2*(
17*D*a^2*b^3 - 35*C*a*b^4 - 120*B*b^5)*c^2*d^3 + 10*(4*D*a^3*b^2 - 7*C*a^2
*b^3 + 16*B*a*b^4 + 48*A*b^5)*c*d^4 - 15*(7*D*a^4*b - 10*C*a^3*b^2 + 16*B*
a^2*b^3 - 32*A*a*b^4)*d^5 + 48*(D*b^5*c*d^4 + (D*a*b^4 + 10*C*b^5)*d^5)*x^
3 - 8*(7*D*b^5*c^2*d^3 - 2*(D*a*b^4 + 5*C*b^5)*c*d^4 + (7*D*a^2*b^3 - 10*C
*a*b^4 - 80*B*b^5)*d^5)*x^2 + 2*(35*D*b^5*c^3*d^2 - (11*D*a*b^4 + 50*C*b^5
)*c^2*d^3 - (11*D*a^2*b^3 - 20*C*a*b^4 - 80*B*b^5)*c*d^4 + 5*(7*D*a^3*b^2
- 10*C*a^2*b^3 + 16*B*a*b^4 + 96*A*b^5)*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b^5*d^5), -1/3840*(15*(7*D*b^5*c^5 - 5*(D*a*b^4 + 2*C*b^5)*c^4*d - 2*(
D*a^2*b^3 - 4*C*a*b^4 - 8*B*b^5)*c^3*d^2 - 2*(D*a^3*b^2 - 2*C*a^2*b^3 + 8*
B*a*b^4 + 16*A*b^5)*c^2*d^3 - (5*D*a^4*b - 8*C*a^3*b^2 + 16*B*a^2*b^3 - 64
*A*a*b^4)*c*d^4 + (7*D*a^5 - 10*C*a^4*b + 16*B*a^3*b^2 - 32*A*a^2*b^3)*d^5
)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqr
t(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(384*D*...
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1668 vs. $2(463) = 926$.

Time = 0.35 (sec) , antiderivative size = 1668, normalized size of antiderivative = 3.23

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="gi
ac")
```

output
```
-1/1920*(1920*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2
*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*sqrt(b*x + a))*A*a*abs(b)/b^2 - 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6
)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^1
4*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^
3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x
+ a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*D*a*abs(
b)/b^2 - 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)
*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2
*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3
 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqr
t(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c
*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*C*abs(b)/b - 480*(sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (
b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sq
rt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*B*a*abs(b)/b^3 - 480*(s
qrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int \sqrt{a+b\,x}\,\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 974, normalized size of antiderivative = 1.89

$$\int \sqrt{a+bx}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 105*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b*d**5 + 190*sqrt(c + d*x)*sqrt(a
 + b*x)*a**3*b**2*c*d**4 + 70*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**2*d**5*x
 + 240*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*d**4 - 36*sqrt(c + d*x)*sqrt(
a + b*x)*a**2*b**3*c**2*d**3 - 122*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**3*c
*d**4*x - 56*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**3*d**5*x**2 + 640*sqrt(c
+ d*x)*sqrt(a + b*x)*a*b**5*c*d**3 + 1120*sqrt(c + d*x)*sqrt(a + b*x)*a*b*
*5*d**4*x - 30*sqrt(c + d*x)*sqrt(a + b*x)*a*b**4*c**3*d**2 + 18*sqrt(c +
d*x)*sqrt(a + b*x)*a*b**4*c**2*d**3*x + 96*sqrt(c + d*x)*sqrt(a + b*x)*a*b
**4*c*d**4*x**2 + 48*sqrt(c + d*x)*sqrt(a + b*x)*a*b**4*d**5*x**3 - 240*sq
rt(c + d*x)*sqrt(a + b*x)*b**6*c**2*d**2 + 160*sqrt(c + d*x)*sqrt(a + b*x)
*b**6*c*d**3*x + 640*sqrt(c + d*x)*sqrt(a + b*x)*b**6*d**4*x**2 + 45*sqrt(
c + d*x)*sqrt(a + b*x)*b**5*c**4*d - 30*sqrt(c + d*x)*sqrt(a + b*x)*b**5*c
**3*d**2*x + 24*sqrt(c + d*x)*sqrt(a + b*x)*b**5*c**2*d**3*x**2 + 528*sqrt
(c + d*x)*sqrt(a + b*x)*b**5*c*d**4*x**3 + 384*sqrt(c + d*x)*sqrt(a + b*x)
*b**5*d**5*x**4 + 105*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)
*sqrt(c + d*x))/sqrt(a*d - b*c))*a**5*d**5 - 225*sqrt(d)*sqrt(b)*log((sqrt
(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**4*b*c*d**4
- 240*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/
sqrt(a*d - b*c))*a**3*b**3*d**4 + 90*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a +
 b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**3*b**2*c**2*d**3 + 7...
```

3.103 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$

Optimal result . . . 936
Mathematica [A] (verified) . . . 937
Rubi [A] (verified) . . . 937
Maple [B] (verified) . . . 941
Fricas [A] (verification not implemented) . . . 942
Sympy [F] . . . 943
Maxima [F(-2)] . . . 944
Giac [A] (verification not implemented) . . . 944
Mupad [F(-1)] . . . 945
Reduce [B] (verification not implemented) . . . 945

Optimal result

Integrand size = 34, antiderivative size = 393

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = -\frac{(5a^3d^3D-a^2bd^2(8Cd-9cD)-ab^2d(16cCd-16Bd^2-15c^2D)-b^3(40c^2Cd-48Bcd^2+64Ad^3-3}{64b^3d^4}$$
$$-\frac{\left(40bcC-48bBd+56aCd-50acD-\frac{35bc^2D}{d}-\frac{59a^2dD}{b}\right)(a+bx)^{3/2}\sqrt{c+dx}}{96b^2d^2}$$
$$+\frac{(8bCd-7bcD-17adD)(a+bx)^{5/2}\sqrt{c+dx}}{24b^3d^2}+\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$
$$+\frac{(bc-ad)\left(5a^3d^3D-a^2bd^2(8Cd-9cD)-ab^2d(16cCd-16Bd^2-15c^2D)-b^3(40c^2Cd-48Bcd^2+6\right.}{64b^{7/2}d^{9/2}}$$

output
```
-1/64*(5*a^3*d^3*D-a^2*b*d^2*(8*C*d-9*D*c)-a*b^2*d*(-16*B*d^2+16*C*c*d-15*
D*c^2)-b^3*(64*A*d^3-48*B*c*d^2+40*C*c^2*d-35*D*c^3))*(b*x+a)^(1/2)*(d*x+c
)^(1/2)/b^3/d^4-1/96*(40*C*b*c-48*B*b*d+56*C*a*d-50*D*a*c-35*b*c^2*D/d-59*
a^2*d*D/b)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^2/d^2+1/24*(8*C*b*d-17*D*a*d-7*D*
b*c)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^3/d^2+1/4*D*(b*x+a)^(7/2)*(d*x+c)^(1/2)
/b^3/d+1/64*(-a*d+b*c)*(5*a^3*d^3*D-a^2*b*d^2*(8*C*d-9*D*c)-a*b^2*d*(-16*B
*d^2+16*C*c*d-15*D*c^2)-b^3*(64*A*d^3-48*B*c*d^2+40*C*c^2*d-35*D*c^3))*arc
tanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 5.35 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$

$$= \frac{\sqrt{b}\sqrt{d}\sqrt{a+bx}\sqrt{c+dx}(15a^3d^3D+a^2bd^2(-24Cd+17cD-10dDx)+ab^2d(25c^2D-4cd(8C+3Dx)+}{}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*Sqrt[c + d*x]*(15*a^3*d^3*D + a^2*b*d^2*(-2
4*C*d + 17*c*D - 10*d*D*x) + a*b^2*d*(25*c^2*D - 4*c*d*(8*C + 3*D*x) + 8*d
^2*(6*B + 2*C*x + D*x^2)) + b^3*(-105*c^3*D + 10*c^2*d*(12*C + 7*D*x) - 8*
c*d^2*(18*B + 10*C*x + 7*D*x^2) + 16*d^3*(12*A + 6*B*x + 4*C*x^2 + 3*D*x^3
))) - 6*(b*c - a*d)*(5*a^3*d^3*D + a^2*b*d^2*(-8*C*d + 9*c*D) + a*b^2*d*(-
16*c*C*d + 16*B*d^2 + 15*c^2*D) + b^3*(-40*c^2*C*d + 48*B*c*d^2 - 64*A*d^3
 + 35*c^3*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*(Sqrt[a - (b*c)/d]
- Sqrt[a + b*x]))])/(192*b^(7/2)*d^(9/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2125, 27, 1194, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2125}$$

$$\frac{\int \frac{\sqrt{a+bx}\left(8Adb^3+(8bCd-17aDd-7bcD)x^2b^2+2\left(-5dDa^2-7bcDa+4b^2Bd\right)xb-a^2(7bc+ad)D\right)}{2\sqrt{c+dx}}dx}{4b^3d}+\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{a+bx}\left(8Adb^3+(8bCd-17aDd-7bcD)x^2b^2+2\left(-5dDa^2-7bcDa+4b^2Bd\right)xb-a^2(7bc+ad)D\right)}{\sqrt{c+dx}}dx}{8b^3d}+\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$

$\downarrow$ 1194

$$\frac{\frac{\int \frac{b^2\sqrt{a+bx}\left(11d^2Da^3-2bd(4Cd-25cD)a^2-5b^2c(8Cd-7cD)a+48Ab^3d^2+b\left(-\left(\left(-35Dc^2+40Cdc-48Bd^2\right)b^2\right)-2ad(28Cd-25cD)b+59a^2d^2D\right)x\right)}{2\sqrt{c+dx}}dx}{3b^2d}+\frac{(a+bx}{}}{8b^3d}$$

$$\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{a+bx}\left(11d^2Da^3-2bd(4Cd-25cD)a^2-5b^2c(8Cd-7cD)a+48Ab^3d^2+b\left(-\left(\left(-35Dc^2+40Cdc-48Bd^2\right)b^2\right)-2ad(28Cd-25cD)b+59a^2d^2D\right)x\right)}{\sqrt{c+dx}}dx}{6d}+\frac{(a+bx)^5}{}}{8b^3d}$$

$$\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$

$\downarrow$ 90

$$\frac{\frac{\frac{(a+bx)^{3/2}\sqrt{c+dx}\left(59a^2d^2D-2abd(28Cd-25cD)-\left(b^2\left(-48Bd^2-35c^2D+40cCd\right)\right)\right)}{2d}-\frac{3\left(5a^3d^3D-a^2bd^2(8Cd-9cD)-ab^2d\left(-16Bd^2-15c^2D+16cCd\right)-\left(b^3\left(64A\right.\right.}{4d}}{6d}}{8b^3d}$$

$$\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$

$\downarrow$ 60

$$\frac{\frac{\frac{(a+bx)^{3/2}\sqrt{c+dx}\left(59a^2d^2D-2abd(28Cd-25cD)-\left(b^2\left(-48Bd^2-35c^2D+40cCd\right)\right)\right)}{2d}-\frac{3\left(5a^3d^3D-a^2bd^2(8Cd-9cD)-ab^2d\left(-16Bd^2-15c^2D+16cCd\right)-\left(b^3\left(64A\right.\right.}{}}{6d}}{8b^3d}$$

$$\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$

$\downarrow$ 66

$$\frac{\frac{(a+bx)^{3/2}\sqrt{c+dx}\left(59a^2d^2D-2abd(28Cd-25cD)-\left(b^2\left(-48Bd^2-35c^2D+40cCd\right)\right)\right)}{2d}-\frac{3\left(5a^3d^3D-a^2bd^2(8Cd-9cD)-ab^2d\left(-16Bd^2-15c^2D+16cCd\right)-\left(b^3\left(64A\ldots\right.\right.\right.}{6d}}{8b^3d}$$

$$\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(a+bx)^{3/2}\sqrt{c+dx}\left(59a^2d^2D-2abd(28Cd-25cD)-\left(b^2\left(-48Bd^2-35c^2D+40cCd\right)\right)\right)}{2d}-\frac{3\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)\left(5a^3d^3D-a^2bd^2(\ldots\right.}{6d}}{8b^3d}$$

$$\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4b^3d}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(D*(a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b^3*d) + (((8*b*C*d - 7*b*c*D - 17*a*
d*D)*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d) + (((59*a^2*d^2*D - 2*a*b*d*(28*
C*d - 25*c*D) - b^2*(40*c*C*d - 48*B*d^2 - 35*c^2*D))*(a + b*x)^(3/2)*Sqrt
[c + d*x])/(2*d) - (3*(5*a^3*d^3*D - a^2*b*d^2*(8*C*d - 9*c*D) - a*b^2*d*(
16*c*C*d - 16*B*d^2 - 15*c^2*D) - b^3*(40*c^2*C*d - 48*B*c*d^2 + 64*A*d^3
- 35*c^3*D))*((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt
[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*d))/(6
*d))/(8*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. $2(355) = 710$.

Time = 0.53 (sec) , antiderivative size = 1334, normalized size of antiderivative = 3.39

method	result	size
default	Expression too large to display	1334

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/384*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(144*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^2*d^2-120*C*ln(1/2*(2*b*d
*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^3*d+2
40*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^3*c^2*d+384*A*(d*b)^(1/2)*((b*x
+a)*(d*x+c))^(1/2)*b^3*d^3-24*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^2*
c*d^2*x+24*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c
)/(d*b)^(1/2))*a^3*b*d^4-60*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^3*d+30*D*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)*a^3*d^3-48*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*d^4-210*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^
(1/2)*b^3*c^3-48*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b*d^3-12*D*ln(1
/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^
3*b*c*d^3-18*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b
*c)/(d*b)^(1/2))*a^2*b^2*c^2*d^2+96*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*
a*b^2*d^3+24*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b
*c)/(d*b)^(1/2))*a^2*b^2*c*d^3+192*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b
^3*d^3*x+192*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b
*c)/(d*b)^(1/2))*a*b^3*d^4-192*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c*d^3+16*D*a*b^2*d^3*x^2*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)-112*D*b^3*c*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(...
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 886, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fr
icas")
```

output
```
[1/768*(3*(35*D*b^4*c^4 - 20*(D*a*b^3 + 2*C*b^4)*c^3*d - 6*(D*a^2*b^2 - 4*
C*a*b^3 - 8*B*b^4)*c^2*d^2 - 4*(D*a^3*b - 2*C*a^2*b^2 + 8*B*a*b^3 + 16*A*b
^4)*c*d^3 - (5*D*a^4 - 8*C*a^3*b + 16*B*a^2*b^2 - 64*A*a*b^3)*d^4)*sqrt(b*
d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c +
a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*
(48*D*b^4*d^4*x^3 - 105*D*b^4*c^3*d + 5*(5*D*a*b^3 + 24*C*b^4)*c^2*d^2 + (
17*D*a^2*b^2 - 32*C*a*b^3 - 144*B*b^4)*c*d^3 + 3*(5*D*a^3*b - 8*C*a^2*b^2
+ 16*B*a*b^3 + 64*A*b^4)*d^4 - 8*(7*D*b^4*c*d^3 - (D*a*b^3 + 8*C*b^4)*d^4)
*x^2 + 2*(35*D*b^4*c^2*d^2 - 2*(3*D*a*b^3 + 20*C*b^4)*c*d^3 - (5*D*a^2*b^2
 - 8*C*a*b^3 - 48*B*b^4)*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^5), -
1/384*(3*(35*D*b^4*c^4 - 20*(D*a*b^3 + 2*C*b^4)*c^3*d - 6*(D*a^2*b^2 - 4*C
*a*b^3 - 8*B*b^4)*c^2*d^2 - 4*(D*a^3*b - 2*C*a^2*b^2 + 8*B*a*b^3 + 16*A*b^
4)*c*d^3 - (5*D*a^4 - 8*C*a^3*b + 16*B*a^2*b^2 - 64*A*a*b^3)*d^4)*sqrt(-b*
d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)
/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(48*D*b^4*d^4*x^3 -
105*D*b^4*c^3*d + 5*(5*D*a*b^3 + 24*C*b^4)*c^2*d^2 + (17*D*a^2*b^2 - 32*C*
a*b^3 - 144*B*b^4)*c*d^3 + 3*(5*D*a^3*b - 8*C*a^2*b^2 + 16*B*a*b^3 + 64*A*
b^4)*d^4 - 8*(7*D*b^4*c*d^3 - (D*a*b^3 + 8*C*b^4)*d^4)*x^2 + 2*(35*D*b^4*c
^2*d^2 - 2*(3*D*a*b^3 + 20*C*b^4)*c*d^3 - (5*D*a^2*b^2 - 8*C*a*b^3 - 48*B*
b^4)*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^5)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2 + D*x**3)/sqrt(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)D}{b^4d}-\frac{7\,Db^{13}cd^5+17\,Dab^{12}d^6-8\,Cb^{13}d^6}{b^{16}d^7}\right)+\frac{35\,Db^{14}c^2d^4+5}{}\right.\right.\right.}{}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="gi
ac")
```

output
```
1/192*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b
*x + a)*D/(b^4*d) - (7*D*b^13*c*d^5 + 17*D*a*b^12*d^6 - 8*C*b^13*d^6)/(b^1
6*d^7)) + (35*D*b^14*c^2*d^4 + 50*D*a*b^13*c*d^5 - 40*C*b^14*c*d^5 + 59*D*
a^2*b^12*d^6 - 56*C*a*b^13*d^6 + 48*B*b^14*d^6)/(b^16*d^7)) - 3*(35*D*b^15
*c^3*d^3 + 15*D*a*b^14*c^2*d^4 - 40*C*b^15*c^2*d^4 + 9*D*a^2*b^13*c*d^5 -
16*C*a*b^14*c*d^5 + 48*B*b^15*c*d^5 + 5*D*a^3*b^12*d^6 - 8*C*a^2*b^13*d^6
+ 16*B*a*b^14*d^6 - 64*A*b^15*d^6)/(b^16*d^7))*sqrt(b*x + a) - 3*(35*D*b^4
*c^4 - 20*D*a*b^3*c^3*d - 40*C*b^4*c^3*d - 6*D*a^2*b^2*c^2*d^2 + 24*C*a*b^
3*c^2*d^2 + 48*B*b^4*c^2*d^2 - 4*D*a^3*b*c*d^3 + 8*C*a^2*b^2*c*d^3 - 32*B*
a*b^3*c*d^3 - 64*A*b^4*c*d^3 - 5*D*a^4*d^4 + 8*C*a^3*b*d^4 - 16*B*a^2*b^2*
d^4 + 64*A*a*b^3*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Hanged}$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \frac{15\sqrt{dx+c}\,\sqrt{bx+a}\,a^3b\,d^4-7\sqrt{dx+c}\,\sqrt{bx+a}\,a^2b^2c\,d^3-10\sqrt{dx+c}\,\sqrt{bx+a}\,a^2b^2d^4x+240\sqrt{dx+c}\,\sqrt{}}{}$$

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output

```
(15*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b*d**4 - 7*sqrt(c + d*x)*sqrt(a + b*x
)*a**2*b**2*c*d**3 - 10*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**2*d**4*x + 240
*sqrt(c + d*x)*sqrt(a + b*x)*a*b**4*d**3 - 7*sqrt(c + d*x)*sqrt(a + b*x)*a
*b**3*c**2*d**2 + 4*sqrt(c + d*x)*sqrt(a + b*x)*a*b**3*c*d**3*x + 8*sqrt(c
 + d*x)*sqrt(a + b*x)*a*b**3*d**4*x**2 - 144*sqrt(c + d*x)*sqrt(a + b*x)*b
**5*c*d**2 + 96*sqrt(c + d*x)*sqrt(a + b*x)*b**5*d**3*x + 15*sqrt(c + d*x)
*sqrt(a + b*x)*b**4*c**3*d - 10*sqrt(c + d*x)*sqrt(a + b*x)*b**4*c**2*d**2
*x + 8*sqrt(c + d*x)*sqrt(a + b*x)*b**4*c*d**3*x**2 + 48*sqrt(c + d*x)*sqr
t(a + b*x)*b**4*d**4*x**3 - 15*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x)
+ sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**4*d**4 + 12*sqrt(d)*sqrt(b)*l
og((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**3*b
*c*d**3 + 144*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c
+ d*x))/sqrt(a*d - b*c))*a**2*b**3*d**3 + 6*sqrt(d)*sqrt(b)*log((sqrt(d)*s
qrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b**2*c**2*d**2
 - 288*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))
/sqrt(a*d - b*c))*a*b**4*c*d**2 + 12*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a +
 b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**3*c**3*d + 144*sqrt(d
)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b
*c))*b**5*c**2*d - 15*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)
*sqrt(c + d*x))/sqrt(a*d - b*c))*b**4*c**4)/(192*b**4*d**4)
```

3.104 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$

Optimal result 947
Mathematica [A] (verified) 948
Rubi [A] (verified) 948
Maple [B] (verified) 952
Fricas [A] (verification not implemented) 953
Sympy [F] 954
Maxima [F(-2)] 955
Giac [A] (verification not implemented) 955
Mupad [F(-1)] 956
Reduce [F] 956

Optimal result

Integrand size = 34, antiderivative size = 318

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{d^4\sqrt{c+dx}}$$
$$-\frac{\left(14bcC-8bBd+2aCd-4acD-\frac{19bc^2D}{d}-\frac{a^2dD}{b}\right)\sqrt{a+bx}\sqrt{c+dx}}{8bd^3}$$
$$+\frac{(2bCd-5bcD-adD)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2d^3}+\frac{D(a+bx)^{3/2}(c+dx)^{3/2}}{3bd^3}$$
$$+\frac{(a^3d^3D-a^2bd^2(2Cd-3cD)-ab^2d(12cCd-8Bd^2-15c^2D)+b^3(30c^2Cd-24Bcd^2+16Ad^3-35c^3D}{8b^{5/2}d^{9/2}}$$

output
```
-2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d^4/(d*x+c)^(1/2)-1/8*(14*C
*b*c-8*B*b*d+2*C*a*d-4*D*a*c-19*b*c^2*D/d-a^2*d*D/b)*(b*x+a)^(1/2)*(d*x+c)
^(1/2)/b/d^3+1/4*(2*C*b*d-D*a*d-5*D*b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^2/d
^3+1/3*D*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b/d^3+1/8*(a^3*d^3*D-a^2*b*d^2*(2*C*d
-3*D*c)-a*b^2*d*(-8*B*d^2+12*C*c*d-15*D*c^2)+b^3*(16*A*d^3-24*B*c*d^2+30*C
*c^2*d-35*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(
5/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{a+bx}(-3a^2d^2D(c+dx)+2abd(c+dx)(3Cd-5cD+dDx)}{}$$
$$+\frac{(a^3d^3D+a^2bd^2(-2Cd+3cD)+ab^2d(-12cCd+8Bd^2+15c^2D)+b^3(30c^2Cd-24Bcd^2+16Ad^3-35c}{8b^{5/2}d^{9/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[a + b*x]*(-3*a^2*d^2*D*(c + d*x) + 2*a*b*d*(c + d*x)*(3*C*d - 5*c*D
+ d*D*x) + b^2*(105*c^3*D + c^2*(-90*C*d + 35*d*D*x) + 2*c*d^2*(36*B - 15*
C*x - 7*D*x^2) + 4*d^3*(-12*A + 6*B*x + 3*C*x^2 + 2*D*x^3))))/(24*b^2*d^4*
Sqrt[c + d*x]) + ((a^3*d^3*D + a^2*b*d^2*(-2*C*d + 3*c*D) + a*b^2*d*(-12*c
*C*d + 8*B*d^2 + 15*c^2*D) + b^3*(30*c^2*C*d - 24*B*c*d^2 + 16*A*d^3 - 35*
c^3*D))*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^(5/
2)*d^(9/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2124, 27, 1194, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \ 2124$$

$$\frac{2\int \frac{\sqrt{a+bx}\left(-\left(\left(a-\frac{bc}{d}\right)Dx^2\right)+\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-3Dc^3+3Cdc^2-3Bd^2c+2Ad^3\right)}{d^3}\right)}{2\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a+bx}\left(-\left(\left(a-\frac{bc}{d}\right)Dx^2\right)+\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-3Dc^3+3Cdc^2-3Bd^2c+2Ad^3\right)}{d^3}\right)}{\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{1194}$$

$$\frac{\frac{\int \frac{\sqrt{a+bx}\left(-6\left(-3Dc^3+3Cdc^2-3Bd^2c+2Ad^3\right)b^3+ad\left(-11Dc^2+6Cdc-6Bd^2\right)b^2+4a^2cd^2Db+d(bc-ad)(6bCd-7aDd-11bcD)xb+a^3d^3D\right)}{2d^2\sqrt{c+dx}}dx}{3b^2d}+\frac{D(a+bx)^{5/2}\cdots}{3b\cdots}}{bc-ad}+\frac{2(a+bx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{a+bx}\left(-6\left(-3Dc^3+3Cdc^2-3Bd^2c+2Ad^3\right)b^3+ad\left(-11Dc^2+6Cdc-6Bd^2\right)b^2+4a^2cd^2Db+d(bc-ad)(6bCd-7aDd-11bcD)xb+a^3d^3D\right)}{\sqrt{c+dx}}dx}{6b^2d^3}+\frac{D(a+bx)^{5/2}\cdots}{3b\cdots}}{bc-ad}+\frac{2(a+bx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{\frac{1}{2}(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)(-7adD-11bcD+6bCd)-\frac{3}{4}\left(a^3d^3D-a^2bd^2(2Cd-3cD)-ab^2d\left(-8Bd^2-15c^2D+12cCd\right)+b^3\left(16Ad^3-24Bcd^2-35c^3\cdots\right.\right.}{6b^2d^3}\cdots}{bc-ad}+\frac{2(a+bx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{60}$$

$$\frac{\frac{\frac{1}{2}(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)(-7adD-11bcD+6bCd)-\frac{3}{4}\left(a^3d^3D-a^2bd^2(2Cd-3cD)-ab^2d\left(-8Bd^2-15c^2D+12cCd\right)+b^3\left(16Ad^3-24Bcd^2-35c^3\cdots\right.\right.}{6b^2d^3}\cdots}{bc-ad}+\frac{2(a+bx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{66}$$

$$\frac{\frac{\frac{1}{2}(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)(-7adD-11bcD+6bCd)-\frac{3}{4}\left(a^3d^3D-a^2bd^2(2Cd-3cD)-ab^2d\left(-8Bd^2-15c^2D+12cCd\right)+b^3\left(16Ad^3-24Bcd^2-35c^3\right.\right.}{6b^2d^3}}{bc-ad}$$

$$\frac{2(a+bx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{1}{2}(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)(-7adD-11bcD+6bCd)-\frac{3}{4}\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)\left(a^3d^3D-a^2bd^2(2Cd-3cD)-ab^2d(-8\right.}{6b^2d^3}}{bc-ad}$$

$$\frac{2(a+bx)^{3/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

input

```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output

```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x)^(3/2))/(d^3*(b*c - a*d)*S
qrt[c + d*x]) + (((b*c - a*d)*D*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b^2*d^2)
 + ((((b*c - a*d)*(6*b*C*d - 11*b*c*D - 7*a*d*D)*(a + b*x)^(3/2)*Sqrt[c + d
*x])/2 - (3*(a^3*d^3*D - a^2*b*d^2*(2*C*d - 3*c*D) - a*b^2*d*(12*c*C*d - 8
*B*d^2 - 15*c^2*D) + b^3*(30*c^2*C*d - 24*B*c*d^2 + 16*A*d^3 - 35*c^3*D))*
((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/4)/(6*b^2*d^3))/(b*c -
 a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1547 vs. $2(282) = 564$.

Time = 0.57 (sec) , antiderivative size = 1548, normalized size of antiderivative = 4.87

method	result	size
default	Expression too large to display	1548

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/48*(b*x+a)^(1/2)*(-6*D*a^2*d^3*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+144
*B*b^2*c*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-6*D*a^2*c*d^2*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)+24*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*d^4*x-72*B*ln(1/2*(2*b*d*x+2*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c*d^3*x+24*C*b^2*d^
3*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-180*C*b^2*c^2*d*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)+12*C*a*b*d^3*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+210
*D*b^2*c^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+48*A*ln(1/2*(2*b*d*x+2*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*d^4*x+3*D*ln(1/2
*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*
d^4*x+48*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*b^3*c*d^3-72*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b
)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^2*d^2+90*C*ln(1/2*(2*b*d*x+2*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^3*d+3*D*ln(1/2*(2*b
*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*c*d^3
-96*A*b^2*d^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-16*D*a*b*c*d^2*x*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+16*D*b^2*d^3*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b
)^(1/2)+45*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c
)/(d*b)^(1/2))*a*b^2*c^2*d^2*x+4*D*a*b*d^3*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)-28*D*b^2*c*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-60*C*b^...
```

Fricas [A] (verification not implemented)

Time = 4.77 (sec) , antiderivative size = 948, normalized size of antiderivative = 2.98

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(35*D*b^3*c^4 - 15*(D*a*b^2 + 2*C*b^3)*c^3*d - 3*(D*a^2*b - 4*C*
a*b^2 - 8*B*b^3)*c^2*d^2 - (D*a^3 - 2*C*a^2*b + 8*B*a*b^2 + 16*A*b^3)*c*d^
3 + (35*D*b^3*c^3*d - 15*(D*a*b^2 + 2*C*b^3)*c^2*d^2 - 3*(D*a^2*b - 4*C*a*
b^2 - 8*B*b^3)*c*d^3 - (D*a^3 - 2*C*a^2*b + 8*B*a*b^2 + 16*A*b^3)*d^4)*x)*
sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x +
 b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*
x) - 4*(8*D*b^3*d^4*x^3 + 105*D*b^3*c^3*d - 48*A*b^3*d^4 - 10*(D*a*b^2 + 9
*C*b^3)*c^2*d^2 - 3*(D*a^2*b - 2*C*a*b^2 - 24*B*b^3)*c*d^3 - 2*(7*D*b^3*c*
d^3 - (D*a*b^2 + 6*C*b^3)*d^4)*x^2 + (35*D*b^3*c^2*d^2 - 2*(4*D*a*b^2 + 15
*C*b^3)*c*d^3 - 3*(D*a^2*b - 2*C*a*b^2 - 8*B*b^3)*d^4)*x)*sqrt(b*x + a)*sq
rt(d*x + c))/(b^3*d^6*x + b^3*c*d^5), 1/48*(3*(35*D*b^3*c^4 - 15*(D*a*b^2
+ 2*C*b^3)*c^3*d - 3*(D*a^2*b - 4*C*a*b^2 - 8*B*b^3)*c^2*d^2 - (D*a^3 - 2*
C*a^2*b + 8*B*a*b^2 + 16*A*b^3)*c*d^3 + (35*D*b^3*c^3*d - 15*(D*a*b^2 + 2*
C*b^3)*c^2*d^2 - 3*(D*a^2*b - 4*C*a*b^2 - 8*B*b^3)*c*d^3 - (D*a^3 - 2*C*a^
2*b + 8*B*a*b^2 + 16*A*b^3)*d^4)*x)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c +
 a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2
*c*d + a*b*d^2)*x)) + 2*(8*D*b^3*d^4*x^3 + 105*D*b^3*c^3*d - 48*A*b^3*d^4
- 10*(D*a*b^2 + 9*C*b^3)*c^2*d^2 - 3*(D*a^2*b - 2*C*a*b^2 - 24*B*b^3)*c*d^
3 - 2*(7*D*b^3*c*d^3 - (D*a*b^2 + 6*C*b^3)*d^4)*x^2 + (35*D*b^3*c^2*d^2 -
2*(4*D*a*b^2 + 15*C*b^3)*c*d^3 - 3*(D*a^2*b - 2*C*a*b^2 - 8*B*b^3)*d^4)...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx=\int \frac{\sqrt{a+bx}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{\left(\left(bx+a\right)\left(2\left(bx+a\right)\left(\frac{4\left(bx+a\right)D|b|}{b^3d} - \frac{7Db^6cd^5|b|+11Dab^5d^6|b|-6Cb^6d^6|b|}{b^8d^7}\right.\right.\right.}{}$$
$$+ \frac{(35Db^3c^3|b| - 15Dab^2c^2d|b| - 30Cb^3c^2d|b| - 3Da^2bcd^2|b| + 12Cab^2cd^2|b| + 24Bb^3cd^2|b| - Da^3d^3|b| +}{8\sqrt{bd}b^3d^4}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="gi
ac")
```

output
```
1/24*((b*x + a)*(2*(b*x + a)*(4*(b*x + a)*D*abs(b)/(b^3*d) - (7*D*b^6*c*d^
5*abs(b) + 11*D*a*b^5*d^6*abs(b) - 6*C*b^6*d^6*abs(b))/(b^8*d^7)) + (35*D*
b^7*c^2*d^4*abs(b) + 20*D*a*b^6*c*d^5*abs(b) - 30*C*b^7*c*d^5*abs(b) + 17*
D*a^2*b^5*d^6*abs(b) - 18*C*a*b^6*d^6*abs(b) + 24*B*b^7*d^6*abs(b))/(b^8*d
^7)) + 3*(35*D*b^8*c^3*d^3*abs(b) - 15*D*a*b^7*c^2*d^4*abs(b) - 30*C*b^8*c
^2*d^4*abs(b) - 3*D*a^2*b^6*c*d^5*abs(b) + 12*C*a*b^7*c*d^5*abs(b) + 24*B*
b^8*c*d^5*abs(b) - D*a^3*b^5*d^6*abs(b) + 2*C*a^2*b^6*d^6*abs(b) - 8*B*a*b
^7*d^6*abs(b) - 16*A*b^8*d^6*abs(b))/(b^8*d^7))*sqrt(b*x + a)/sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d) + 1/8*(35*D*b^3*c^3*abs(b) - 15*D*a*b^2*c^2*d*abs(
b) - 30*C*b^3*c^2*d*abs(b) - 3*D*a^2*b*c*d^2*abs(b) + 12*C*a*b^2*c*d^2*abs
(b) + 24*B*b^3*c*d^2*abs(b) - D*a^3*d^3*abs(b) + 2*C*a^2*b*d^3*abs(b) - 8*
B*a*b^2*d^3*abs(b) - 16*A*b^3*d^3*abs(b))*log(abs(-sqrt(b*d)*sqrt(b*x + a)
 + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{a+b\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2),x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{\sqrt{bx+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

3.105 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$

Optimal result 957
Mathematica [A] (verified) 958
Rubi [A] (verified) 958
Maple [B] (verified) 962
Fricas [B] (verification not implemented) 963
Sympy [F] 964
Maxima [F(-2)] 965
Giac [B] (verification not implemented) 965
Mupad [F(-1)] 966
Reduce [F] 966

Optimal result

Integrand size = 34, antiderivative size = 264

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{3/2}}{3d^3(bc-ad)(c+dx)^{3/2}} + \frac{2(2cCd-Bd^2-3c^2D)\sqrt{a+bx}}{d^4\sqrt{c+dx}} + \frac{(4bCd-11bcD-adD)\sqrt{a+bx}\sqrt{c+dx}}{4bd^4} + \frac{D(a+bx)^{3/2}\sqrt{c+dx}}{2bd^3} - \frac{(a^2d^2D-2abd(2Cd-5cD)+b^2(20cCd-8Bd^2-35c^2D))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{9/2}}$$

output

```
2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(3/2)/d^3/(-a*d+b*c)/(d*x+c)^(3/
2)+2*(-B*d^2+2*C*c*d-3*D*c^2)*(b*x+a)^(1/2)/d^4/(d*x+c)^(1/2)+1/4*(4*C*b*d
-D*a*d-11*D*b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/d^4+1/2*D*(b*x+a)^(3/2)*(d*
x+c)^(1/2)/b/d^3-1/4*(a^2*d^2*D-2*a*b*d*(2*C*d-5*D*c)+b^2*(-8*B*d^2+20*C*c
*d-35*D*c^2))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/2)
/d^(9/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \frac{\sqrt{b}\sqrt{d}\sqrt{a+bx}(3a^2d^2D(c+dx)^2+b^2(105c^4D-8Ad^4x-20c^3d(}{}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*(3*a^2*d^2*D*(c + d*x)^2 + b^2*(105*c^4*D -
 8*A*d^4*x - 20*c^3*d*(3*C - 7*D*x) + 2*c*d^3*x*(16*B - 3*x*(2*C + D*x)) +
 c^2*d^2*(24*B + x*(-80*C + 21*D*x))) - 2*a*b*d*(50*c^3*D + c^2*(-26*C*d +
 69*d*D*x) + 4*c*d^2*(2*B + 3*x*(-3*C + D*x)) + d^3*(4*A - 3*x*(-4*B + 2*C
*x + D*x^2)))) - 3*(b*c - a*d)*(-(a^2*d^2*D) + 2*a*b*d*(2*C*d - 5*c*D) + b
^2*(-20*c*C*d + 8*B*d^2 + 35*c^2*D))*(c + d*x)^(3/2)*ArcTanh[(Sqrt[d]*Sqrt
[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(12*b^(3/2)*d^(9/2)*(-(b*c) + a*d)*(c
 + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2124, 27, 1193, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{2124}$$

$$\frac{2\int -\frac{3\sqrt{a+bx}\left(-\frac{(bc-ad)Dx^2}{d}-\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{(bc-ad)\left(-Dc^2+Cdc-Bd^2\right)}{d^3}\right)}{2(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ \color{blue}{27}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\int \frac{\sqrt{a+bx}\left(-\frac{(bc-ad)Dx^2}{d}-\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{(bc-ad)\left(-Dc^2+Cdc-Bd^2\right)}{d^3}\right)}{(c+dx)^{3/2}}dx}{bc-ad}$$

$$\Big\downarrow \ \color{blue}{1193}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\frac{2\int \frac{(bc-ad)\sqrt{a+bx}\left(ad(Cd-2cD)-b\left(-8Dc^2+5Cdc-2Bd^2\right)-d(bc-ad)Dx\right)}{2d^3\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{3/2}\left(-Bd^2-3c^2D+2cCd\right)}{d^3\sqrt{c+dx}}}{bc-ad}$$

$$\Big\downarrow \ \color{blue}{27}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\frac{\int \frac{\sqrt{a+bx}\left(ad(Cd-2cD)-b\left(-8Dc^2+5Cdc-2Bd^2\right)-d(bc-ad)Dx\right)}{\sqrt{c+dx}}dx}{d^3}+\frac{2(a+bx)^{3/2}\left(-Bd^2-3c^2D+2cCd\right)}{d^3\sqrt{c+dx}}}{bc-ad}$$

$$\Big\downarrow \ \color{blue}{90}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\frac{-\frac{\left(a^2d^2D-2abd(2Cd-5cD)+b^2\left(-8Bd^2-35c^2D+20cCd\right)\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{4b}-\frac{D(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{2b}}{d^3}+\frac{2(a+bx)^{3/2}\left(-Bd^2-3c^2D+2cCd\right)}{d^3\sqrt{c+dx}}}{bc-ad}$$

$$\Big\downarrow \ \color{blue}{60}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\frac{-\frac{\left(a^2d^2D-2abd(2Cd-5cD)+b^2\left(-8Bd^2-35c^2D+20cCd\right)\right)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2d}\right)}{4b}-\frac{D(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{2b}}{d^3}+\frac{2(a+bx)^{3/2}(-}{d^3}}{bc-ad}$$

$$\Big\downarrow \ \color{blue}{66}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{-\frac{-\frac{\left(a^2d^2D-2abd(2Cd-5cD)+b^2\left(-8Bd^2-35c^2D+20cCd\right)\right)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}\right)}{4b}-\frac{D(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{2b}}{d^3}+\frac{2(a+bx)^{3/}}{}}{bc-ad}$$

$$\Big\downarrow \; 221$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{-\frac{-\frac{\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)\left(a^2d^2D-2abd(2Cd-5cD)+b^2\left(-8Bd^2-35c^2D+20cCd\right)\right)}{4b}-\frac{D(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{2b}}{d^3}+\frac{2(a+bx)^{3}}{}}{bc-ad}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(3/2))/(3*(b*c - a*d)*(
c + d*x)^(3/2)) - ((2*(2*c*C*d - B*d^2 - 3*c^2*D)*(a + b*x)^(3/2))/(d^3*Sq
rt[c + d*x]) + (-1/2*((b*c - a*d)*D*(a + b*x)^(3/2)*Sqrt[c + d*x])/b - ((a
^2*d^2*D - 2*a*b*d*(2*C*d - 5*c*D) + b^2*(20*c*C*d - 8*B*d^2 - 35*c^2*D))*
((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*b))/d^3)/(b*c - a*d
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. $2(228) = 456$.

Time = 0.53 (sec) , antiderivative size = 2145, normalized size of antiderivative = 8.12

method	result	size
default	Expression too large to display	2145

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/24*(6*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*a^3*c*d^4*x-12*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c^2*d^3+72*C*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^3*d^2+27*D*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*
a^2*b*c^3*d^2-12*D*a*b*d^4*x^3*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+12*D*b^
2*c*d^3*x^3*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+200*D*a*b*c^3*d*(d*b)^(1/2
)*((b*x+a)*(d*x+c))^(1/2)+72*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d^4*x^2+27*D*ln(1/2*(2*b*d*x+2*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c*d^4*x^2-13
5*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(
1/2))*a*b^2*c^2*d^3*x^2-48*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d^4*x-48*B*b^2*c^2*d^2*(d*b)^(1/2)*
((b*x+a)*(d*x+c))^(1/2)+120*C*b^2*c^3*d*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2
)-6*D*a^2*c^2*d^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-24*B*ln(1/2*(2*b*d*x
+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*d^5*x^2
+24*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)
^(1/2))*b^3*c*d^4*x^2-12*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*d^5*x^2-120*C*ln(1/2*(2*b*d*x+2*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^3*d^2*x-24*C*a...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(227) = 454$.

Time = 12.26 (sec) , antiderivative size = 1360, normalized size of antiderivative = 5.15

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="fr
icas")
```

output
```
[1/48*(3*(35*D*b^3*c^5 - 5*(9*D*a*b^2 + 4*C*b^3)*c^4*d + (9*D*a^2*b + 24*C
*a*b^2 + 8*B*b^3)*c^3*d^2 + (D*a^3 - 4*C*a^2*b - 8*B*a*b^2)*c^2*d^3 + (35*
D*b^3*c^3*d^2 - 5*(9*D*a*b^2 + 4*C*b^3)*c^2*d^3 + (9*D*a^2*b + 24*C*a*b^2
+ 8*B*b^3)*c*d^4 + (D*a^3 - 4*C*a^2*b - 8*B*a*b^2)*d^5)*x^2 + 2*(35*D*b^3*
c^4*d - 5*(9*D*a*b^2 + 4*C*b^3)*c^3*d^2 + (9*D*a^2*b + 24*C*a*b^2 + 8*B*b^
3)*c^2*d^3 + (D*a^3 - 4*C*a^2*b - 8*B*a*b^2)*c*d^4)*x)*sqrt(b*d)*log(8*b^2
*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*
d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(105*D*b^3*c
^4*d - 16*B*a*b^2*c*d^4 - 8*A*a*b^2*d^5 - 20*(5*D*a*b^2 + 3*C*b^3)*c^3*d^2
 + (3*D*a^2*b + 52*C*a*b^2 + 24*B*b^3)*c^2*d^3 - 6*(D*b^3*c*d^4 - D*a*b^2*
d^5)*x^3 + 3*(7*D*b^3*c^2*d^3 - 4*(2*D*a*b^2 + C*b^3)*c*d^4 + (D*a^2*b + 4
*C*a*b^2)*d^5)*x^2 + 2*(70*D*b^3*c^3*d^2 - (69*D*a*b^2 + 40*C*b^3)*c^2*d^3
 + (3*D*a^2*b + 36*C*a*b^2 + 16*B*b^3)*c*d^4 - 4*(3*B*a*b^2 + A*b^3)*d^5)*
x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*c^3*d^5 - a*b^2*c^2*d^6 + (b^3*c*d^7
- a*b^2*d^8)*x^2 + 2*(b^3*c^2*d^6 - a*b^2*c*d^7)*x), -1/24*(3*(35*D*b^3*c^
5 - 5*(9*D*a*b^2 + 4*C*b^3)*c^4*d + (9*D*a^2*b + 24*C*a*b^2 + 8*B*b^3)*c^3
*d^2 + (D*a^3 - 4*C*a^2*b - 8*B*a*b^2)*c^2*d^3 + (35*D*b^3*c^3*d^2 - 5*(9*
D*a*b^2 + 4*C*b^3)*c^2*d^3 + (9*D*a^2*b + 24*C*a*b^2 + 8*B*b^3)*c*d^4 + (D
*a^3 - 4*C*a^2*b - 8*B*a*b^2)*d^5)*x^2 + 2*(35*D*b^3*c^4*d - 5*(9*D*a*b^2
+ 4*C*b^3)*c^3*d^2 + (9*D*a^2*b + 24*C*a*b^2 + 8*B*b^3)*c^2*d^3 + (D*a^...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx=\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(227) = 454$.

Time = 0.24 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="gi
ac")
```

output
```
1/12*((3*(b*x + a)*(2*(D*b^5*c*d^6*abs(b) - D*a*b^4*d^7*abs(b))*(b*x + a)/
(b^6*c*d^7 - a*b^5*d^8) - (7*D*b^6*c^2*d^5*abs(b) - 2*D*a*b^5*c*d^6*abs(b)
 - 4*C*b^6*c*d^6*abs(b) - 5*D*a^2*b^4*d^7*abs(b) + 4*C*a*b^5*d^7*abs(b))/(
b^6*c*d^7 - a*b^5*d^8)) - 4*(35*D*b^7*c^3*d^4*abs(b) - 45*D*a*b^6*c^2*d^5*
abs(b) - 20*C*b^7*c^2*d^5*abs(b) + 9*D*a^2*b^5*c*d^6*abs(b) + 24*C*a*b^6*c
*d^6*abs(b) + 8*B*b^7*c*d^6*abs(b) + 3*D*a^3*b^4*d^7*abs(b) - 6*C*a^2*b^5*
d^7*abs(b) - 6*B*a*b^6*d^7*abs(b) - 2*A*b^7*d^7*abs(b))/(b^6*c*d^7 - a*b^5
*d^8))*(b*x + a) - 3*(35*D*b^8*c^4*d^3*abs(b) - 80*D*a*b^7*c^3*d^4*abs(b)
- 20*C*b^8*c^3*d^4*abs(b) + 54*D*a^2*b^6*c^2*d^5*abs(b) + 44*C*a*b^7*c^2*d
^5*abs(b) + 8*B*b^8*c^2*d^5*abs(b) - 8*D*a^3*b^5*c*d^6*abs(b) - 28*C*a^2*b
^6*c*d^6*abs(b) - 16*B*a*b^7*c*d^6*abs(b) - D*a^4*b^4*d^7*abs(b) + 4*C*a^3
*b^5*d^7*abs(b) + 8*B*a^2*b^6*d^7*abs(b))/(b^6*c*d^7 - a*b^5*d^8))*sqrt(b*
x + a)/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) - 1/4*(35*D*b^2*c^2*abs(b) -
10*D*a*b*c*d*abs(b) - 20*C*b^2*c*d*abs(b) - D*a^2*d^2*abs(b) + 4*C*a*b*d^2
*abs(b) + 8*B*b^2*d^2*abs(b))*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*
c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{\sqrt{a+b\,x}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2),x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{\sqrt{bx+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

3.106 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx$

Optimal result 968
Mathematica [A] (verified) 969
Rubi [A] (verified) 969
Maple [B] (verified) 973
Fricas [B] (verification not implemented) 974
Sympy [F] 975
Maxima [F(-2)] 976
Giac [B] (verification not implemented) 976
Mupad [F(-1)] 977
Reduce [F] 978

Optimal result

Integrand size = 34, antiderivative size = 259

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{3/2}}{5d^3(bc-ad)(c+dx)^{5/2}} + \frac{2(5ad(2cCd-Bd^2-3c^2D)-b(8c^2Cd-3Bcd^2-2Ad^3-13c^3D))\,(a+bx)^{3/2}}{15d^3(bc-ad)^2(c+dx)^{3/2}} - \frac{2(Cd-3cD)\sqrt{a+bx}}{d^4\sqrt{c+dx}} + \frac{D\sqrt{a+bx}\sqrt{c+dx}}{d^4} + \frac{(2bCd-7bcD+adD)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{9/2}}$$

output

```
2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(3/2)/d^3/(-a*d+b*c)/(d*x+c)^(5/
2)+2/15*(5*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-2*A*d^3-3*B*c*d^2+8*C*c^2*d-13
*D*c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(3/2)-2*(C*d-3*D*c)*(b*x+a
)^(1/2)/d^4/(d*x+c)^(1/2)+D*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^4+(2*C*b*d+D*a*d
-7*D*b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/d^(
9/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \frac{\sqrt{a+bx}(b^2(105c^5D+4Ad^5x^2+2cd^4x(5A+3Bx)-5c^4d(6C-\right.}{}$$

$$+\frac{(2bCd-7bcD+adD)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{9/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
(Sqrt[a + b*x]*(b^2*(105*c^5*D + 4*A*d^5*x^2 + 2*c*d^4*x*(5*A + 3*B*x) - 5
*c^4*d*(6*C - 49*D*x) + c^2*d^3*x^2*(-46*C + 15*D*x) + 7*c^3*d^2*x*(-10*C
+ 23*D*x)) - a^2*d^2*(-81*c^3*D + c^2*d*(16*C - 195*D*x) + c*d^2*(4*B + 5*
x*(8*C - 27*D*x)) + d^3*(6*A + 5*x*(2*B + 6*C*x - 3*D*x^2))) - 2*a*b*d*(95
*c^4*D + d^4*x*(A + 5*B*x) + c^2*d^2*x*(-59*C + 150*D*x) + c^3*(-25*C*d +
224*d*D*x) - c*d^3*(5*A + x*(B + 5*x*(8*C - 3*D*x))))))/(15*d^4*(b*c - a*d
)^2*(c + d*x)^(5/2)) + ((2*b*C*d - 7*b*c*D + a*d*D)*ArcTanh[(Sqrt[d]*Sqrt[
a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(9/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2124, 27, 1193, 27, 87, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{2124}$$

$$\frac{2\int \frac{\sqrt{a+bx}\left(-5\left(a-\frac{bc}{d}\right)Dx^2+\frac{5(bc-ad)(Cd-cD)x}{d^2}+\frac{5ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-3Dc^3+3Cdc^2-3Bd^2c-2Ad^3\right)}{d^3}\right)}{2(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{a+bx}\left(-5\left(a-\frac{bc}{d}\right)Dx^2+\frac{5(bc-ad)(Cd-cD)x}{d^2}+\frac{5ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-3Dc^3+3Cdc^2-3Bd^2c-2Ad^3\right)}{d^3}\right)}{(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{\frac{2\int \frac{15(bc-ad)^2\sqrt{a+bx}(Cd+Dxd-2cD)}{2d^3(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2(a+bx)^{3/2}(5ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-3Bcd^2-13c^3D+8c^2Cd))}{3d^3(c+dx)^{3/2}(bc-ad)}}{5(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{5(bc-ad)\int \frac{\sqrt{a+bx}(Cd+Dxd-2cD)}{(c+dx)^{3/2}}dx}{d^3}+\frac{2(a+bx)^{3/2}(5ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-3Bcd^2-13c^3D+8c^2Cd))}{3d^3(c+dx)^{3/2}(bc-ad)}}{5(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{5(bc-ad)\left(\frac{2(a+bx)^{3/2}(Cd-3cD)}{\sqrt{c+dx}(bc-ad)}-\frac{(adD-7bcD+2bCd)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{bc-ad}\right)}{d^3}+\frac{2(a+bx)^{3/2}(5ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-3Bcd^2-13c^3D+8c^2C}{3d^3(c+dx)^{3/2}(bc-ad)}}{5(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{5(bc-ad)\left(\frac{2(a+bx)^{3/2}(Cd-3cD)}{\sqrt{c+dx}(bc-ad)}-\frac{(adD-7bcD+2bCd)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2d}\right)}{bc-ad}\right)}{d^3}+\frac{2(a+bx)^{3/2}(5ad(-Bd^2-3c^2D+2cCd)-\cdots}{3d^3(c+dx)^{3/2}\cdots}}{5(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{5(bc-ad)\left(\frac{2(a+bx)^{3/2}(Cd-3cD)}{\sqrt{c+dx}(bc-ad)}-\frac{(adD-7bcD+2bCd)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}\right)}{bc-ad}\right)}{d^3}+\frac{2(a+bx)^{3/2}(5ad(-Bd^2-3c^2D+2cC\cdots}{3d^3(c+dx\cdots}}{5(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2(a+bx)^{3/2}(5ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-3Bcd^2-13c^3D+8c^2Cd))}{3d^3(c+dx)^{3/2}(bc-ad)}+\frac{5(bc-ad)\left(\frac{2(a+bx)^{3/2}(Cd-3cD)}{\sqrt{c+dx}(bc-ad)}-\frac{\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arc}\cdots}{\cdots}\right.}{\cdots}\right.}{d^3}}{5(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(3/2))/(5*(b*c - a*d)*(
c + d*x)^(5/2)) + ((2*(5*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(8*c^2*C*d -
3*B*c*d^2 - 2*A*d^3 - 13*c^3*D))*(a + b*x)^(3/2))/(3*d^3*(b*c - a*d)*(c +
d*x)^(3/2)) + (5*(b*c - a*d)*((2*(C*d - 3*c*D)*(a + b*x)^(3/2))/((b*c - a*
d)*Sqrt[c + d*x]) - ((2*b*C*d - 7*b*c*D + a*d*D)*((Sqrt[a + b*x]*Sqrt[c +
d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d
*x])])/(Sqrt[b]*d^(3/2))))/(b*c - a*d)))/d^3)/(5*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2463 vs. $2(227) = 454$.

Time = 0.54 (sec) , antiderivative size = 2464, normalized size of antiderivative = 9.51

method	result	size
default	Expression too large to display	2464

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/30*(-490*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^4*d*x-100*C*(d*b)^
(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^3*d^2+380*D*(d*b)^(1/2)*((b*x+a)*(d*x+
c))^(1/2)*a*b*c^4*d+135*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c*d^5*x^3+4*A*a*b*d^5*x*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)-20*A*b^2*c*d^4*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-2
10*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^5-30*D*(d*b)^(1/2)*((b*x+a)
*(d*x+c))^(1/2)*b^2*c^2*d^3*x^3+405*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^
(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c^2*d^4*x^2-675*D*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^
3*d^3*x^2+8*B*a^2*c*d^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-8*A*b^2*d^5*x^
2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+60*C*a^2*d^5*x^2*((b*x+a)*(d*x+c))^(
1/2)*(d*b)^(1/2)-15*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)+a*d+b*c)/(d*b)^(1/2))*a^3*d^6*x^3+80*C*a^2*c*d^4*x*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)-30*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+
a*d+b*c)/(d*b)^(1/2))*a^2*b*c^3*d^3+315*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^5*d*x+60*D*(d*b)^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)*a*b*c*d^4*x^3-160*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(
1/2)*a*b*c*d^4*x^2+600*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^2*d^3*x
^2-236*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^2*d^3*x+896*D*(d*b)^(1/
2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^3*d^2*x-12*B*(d*b)^(1/2)*((b*x+a)*(d*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(229) = 458$.

Time = 38.84 (sec) , antiderivative size = 1774, normalized size of antiderivative = 6.85

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="fr
icas")
```

output
```
[-1/60*(15*(7*D*b^3*c^6 - (15*D*a*b^2 + 2*C*b^3)*c^5*d + (9*D*a^2*b + 4*C*
a*b^2)*c^4*d^2 - (D*a^3 + 2*C*a^2*b)*c^3*d^3 + (7*D*b^3*c^3*d^3 - (15*D*a*
b^2 + 2*C*b^3)*c^2*d^4 + (9*D*a^2*b + 4*C*a*b^2)*c*d^5 - (D*a^3 + 2*C*a^2*
b)*d^6)*x^3 + 3*(7*D*b^3*c^4*d^2 - (15*D*a*b^2 + 2*C*b^3)*c^3*d^3 + (9*D*a
^2*b + 4*C*a*b^2)*c^2*d^4 - (D*a^3 + 2*C*a^2*b)*c*d^5)*x^2 + 3*(7*D*b^3*c^
5*d - (15*D*a*b^2 + 2*C*b^3)*c^4*d^2 + (9*D*a^2*b + 4*C*a*b^2)*c^3*d^3 - (
D*a^3 + 2*C*a^2*b)*c^2*d^4)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*
x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(105*D*b^3*c^5*d - 16*C*a^2*b*c^2*d^
4 - 6*A*a^2*b*d^6 - 10*(19*D*a*b^2 + 3*C*b^3)*c^4*d^2 + (81*D*a^2*b + 50*C
*a*b^2)*c^3*d^3 - 2*(2*B*a^2*b - 5*A*a*b^2)*c*d^5 + 15*(D*b^3*c^2*d^4 - 2*
D*a*b^2*c*d^5 + D*a^2*b*d^6)*x^3 + (161*D*b^3*c^3*d^3 - 2*(150*D*a*b^2 + 2
3*C*b^3)*c^2*d^4 + (135*D*a^2*b + 80*C*a*b^2 + 6*B*b^3)*c*d^5 - 2*(15*C*a^
2*b + 5*B*a*b^2 - 2*A*b^3)*d^6)*x^2 + (245*D*b^3*c^4*d^2 - 14*(32*D*a*b^2
+ 5*C*b^3)*c^3*d^3 + (195*D*a^2*b + 118*C*a*b^2)*c^2*d^4 - 2*(20*C*a^2*b -
 B*a*b^2 - 5*A*b^3)*c*d^5 - 2*(5*B*a^2*b + A*a*b^2)*d^6)*x)*sqrt(b*x + a)*
sqrt(d*x + c))/(b^3*c^5*d^5 - 2*a*b^2*c^4*d^6 + a^2*b*c^3*d^7 + (b^3*c^2*d
^8 - 2*a*b^2*c*d^9 + a^2*b*d^10)*x^3 + 3*(b^3*c^3*d^7 - 2*a*b^2*c^2*d^8 +
a^2*b*c*d^9)*x^2 + 3*(b^3*c^4*d^6 - 2*a*b^2*c^3*d^7 + a^2*b*c^2*d^8)*x), 1
/30*(15*(7*D*b^3*c^6 - (15*D*a*b^2 + 2*C*b^3)*c^5*d + (9*D*a^2*b + 4*C*...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx=\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(7/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. $2(229) = 458$.

Time = 0.30 (sec) , antiderivative size = 790, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="gi
ac")
```

output
```
1/15*(((b*x + a)*(15*(D*b^9*c^2*d^6*abs(b) - 2*D*a*b^8*c*d^7*abs(b) + D*a^
2*b^7*d^8*abs(b))*(b*x + a)/(b^8*c^2*d^7 - 2*a*b^7*c*d^8 + a^2*b^6*d^9) +
(161*D*b^10*c^3*d^5*abs(b) - 345*D*a*b^9*c^2*d^6*abs(b) - 46*C*b^10*c^2*d^
6*abs(b) + 225*D*a^2*b^8*c*d^7*abs(b) + 80*C*a*b^9*c*d^7*abs(b) + 6*B*b^10
*c*d^7*abs(b) - 45*D*a^3*b^7*d^8*abs(b) - 30*C*a^2*b^8*d^8*abs(b) - 10*B*a
*b^9*d^8*abs(b) + 4*A*b^10*d^8*abs(b))/(b^8*c^2*d^7 - 2*a*b^7*c*d^8 + a^2*
b^6*d^9)) + 5*(49*D*b^11*c^4*d^4*abs(b) - 154*D*a*b^10*c^3*d^5*abs(b) - 14
*C*b^11*c^3*d^5*abs(b) + 168*D*a^2*b^9*c^2*d^6*abs(b) + 42*C*a*b^10*c^2*d^
6*abs(b) - 72*D*a^3*b^8*c*d^7*abs(b) - 40*C*a^2*b^9*c*d^7*abs(b) - 2*B*a*b
^10*c*d^7*abs(b) + 2*A*b^11*c*d^7*abs(b) + 9*D*a^4*b^7*d^8*abs(b) + 12*C*a
^3*b^8*d^8*abs(b) + 2*B*a^2*b^9*d^8*abs(b) - 2*A*a*b^10*d^8*abs(b))/(b^8*c
^2*d^7 - 2*a*b^7*c*d^8 + a^2*b^6*d^9))*(b*x + a) + 15*(7*D*b^12*c^5*d^3*ab
s(b) - 29*D*a*b^11*c^4*d^4*abs(b) - 2*C*b^12*c^4*d^4*abs(b) + 46*D*a^2*b^1
0*c^3*d^5*abs(b) + 8*C*a*b^11*c^3*d^5*abs(b) - 34*D*a^3*b^9*c^2*d^6*abs(b)
 - 12*C*a^2*b^10*c^2*d^6*abs(b) + 11*D*a^4*b^8*c*d^7*abs(b) + 8*C*a^3*b^9*
c*d^7*abs(b) - D*a^5*b^7*d^8*abs(b) - 2*C*a^4*b^8*d^8*abs(b))/(b^8*c^2*d^7
 - 2*a*b^7*c*d^8 + a^2*b^6*d^9))*sqrt(b*x + a)/(b^2*c + (b*x + a)*b*d - a*
b*d)^(5/2) + (7*D*b*c*abs(b) - D*a*d*abs(b) - 2*C*b*d*abs(b))*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*
d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{a+b\,x}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2),x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{\sqrt{bx+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

output
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

3.107 $\int \frac{\sqrt{a+bx}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx$

Optimal result 979
Mathematica [A] (verified) 980
Rubi [A] (verified) 980
Maple [B] (verified) 984
Fricas [B] (verification not implemented) 985
Sympy [F] 986
Maxima [F(-2)] 987
Giac [B] (verification not implemented) 987
Mupad [F(-1)] 988
Reduce [F] 989

Optimal result

Integrand size = 34, antiderivative size = 321

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{3/2}}{7d^3(bc-ad)(c+dx)^{7/2}}$$
$$+\frac{2(7ad(2cCd-Bd^2-3c^2D)-b(10c^2Cd-3Bcd^2-4Ad^3-17c^3D))\,(a+bx)^{3/2}}{35d^3(bc-ad)^2(c+dx)^{5/2}}$$
$$+\frac{2(35a^2d^2(Cd-3cD)-14abd(3cCd+Bd^2-12c^2D)+b^2(15c^2Cd+6Bcd^2+8Ad^3-71c^3D))\,(a+bx)^3}{105d^3(bc-ad)^3(c+dx)^{3/2}}$$
$$-\frac{2D\sqrt{a+bx}}{d^4\sqrt{c+dx}}+\frac{2\sqrt{b}D\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{9/2}}$$

output
```
2/7*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(3/2)/d^3/(-a*d+b*c)/(d*x+c)^(7/
2)+2/35*(7*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-4*A*d^3-3*B*c*d^2+10*C*c^2*d-1
7*D*c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(5/2)+2/105*(35*a^2*d^2*(
C*d-3*D*c)-14*a*b*d*(B*d^2+3*C*c*d-12*D*c^2)+b^2*(8*A*d^3+6*B*c*d^2+15*C*c
^2*d-71*D*c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(3/2)-2*D*(b*x+a)^(
1/2)/d^4/(d*x+c)^(1/2)+2*b^(1/2)*D*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(
d*x+c)^(1/2))/d^(9/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \frac{2\sqrt{a+bx}(15c^2Cd^4(a+bx)^3-15Bcd^5(a+bx)^3+15Ad^6(a+bx$$
$$+\frac{2\sqrt{b}D\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{9/2}}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(9/2),x]
```

output
```
(2*Sqrt[a + b*x]*(15*c^2*C*d^4*(a + b*x)^3 - 15*B*c*d^5*(a + b*x)^3 + 15*A
*d^6*(a + b*x)^3 - 15*c^3*d^3*D*(a + b*x)^3 + 21*b*B*c*d^4*(a + b*x)^2*(c
+ d*x) - 42*a*c*C*d^4*(a + b*x)^2*(c + d*x) - 42*A*b*d^5*(a + b*x)^2*(c +
d*x) + 21*a*B*d^5*(a + b*x)^2*(c + d*x) - 21*b*c^3*d^2*D*(a + b*x)^2*(c +
d*x) + 63*a*c^2*d^3*D*(a + b*x)^2*(c + d*x) + 35*A*b^2*d^4*(a + b*x)*(c +
d*x)^2 - 35*a*b*B*d^4*(a + b*x)*(c + d*x)^2 + 35*a^2*C*d^4*(a + b*x)*(c +
d*x)^2 - 35*b^2*c^3*d*D*(a + b*x)*(c + d*x)^2 + 105*a*b*c^2*d^2*D*(a + b*x
)*(c + d*x)^2 - 105*a^2*c*d^3*D*(a + b*x)*(c + d*x)^2 - 105*b^3*c^3*D*(c +
 d*x)^3 + 315*a*b^2*c^2*d*D*(c + d*x)^3 - 315*a^2*b*c*d^2*D*(c + d*x)^3 +
105*a^3*d^3*D*(c + d*x)^3))/(105*d^4*(b*c - a*d)^3*(c + d*x)^(7/2)) + (2*S
qrt[b]*D*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(9/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2124, 27, 1193, 27, 87, 57, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 2124}$$

$$\frac{2\int \frac{\sqrt{a+bx}\left(-7\left(a-\frac{bc}{d}\right)Dx^2+\frac{7(bc-ad)(Cd-cD)x}{d^2}+\frac{7ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-3Dc^3+3Cdc^2-3Bd^2c-4Ad^3\right)}{d^3}\right)}{2(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{a+bx}\left(-7\left(a-\frac{bc}{d}\right)Dx^2+\frac{7(bc-ad)(Cd-cD)x}{d^2}+\frac{7ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-3Dc^3+3Cdc^2-3Bd^2c-4Ad^3\right)}{d^3}\right)}{(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$\frac{\frac{2\int \frac{\sqrt{a+bx}\left(\left(-36Dc^3+15Cdc^2+6Bd^2c+8Ad^3\right)b^2-14ad\left(-7Dc^2+3Cdc+Bd^2\right)b+35a^2d^2(Cd-2cD)+35d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2(a+bx)^{3/2}(7ad(-Bd^2-3c^2 D\ldots}{5d\ldots}}{7(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{a+bx}\left(\left(-36Dc^3+15Cdc^2+6Bd^2c+8Ad^3\right)b^2-14ad\left(-7Dc^2+3Cdc+Bd^2\right)b+35a^2d^2(Cd-2cD)+35d(bc-ad)^2Dx\right)}{(c+dx)^{5/2}}dx}{5d^3(bc-ad)}+\frac{2(a+bx)^{3/2}(7ad(-Bd^2-3c^2D-\ldots}{5d^3\ldots}}{7(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$\frac{\frac{35D(bc-ad)^2\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}dx+\frac{2(a+bx)^{3/2}\left(35a^2d^2(Cd-3cD)-14abd\left(Bd^2-12c^2D+3cCd\right)+b^2\left(8Ad^3+6Bcd^2-71c^3D+15c^2Cd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}}{5d^3(bc-ad)}+\frac{2(a+bx)^{3/2}(7ad(-\ldots}{\ldots}}{7(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 57

$$\frac{\frac{35D(bc-ad)^2\left(\frac{b\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}\right)+\frac{2(a+bx)^{3/2}\left(35a^2d^2(Cd-3cD)-14abd\left(Bd^2-12c^2D+3cCd\right)+b^2\left(8Ad^3+6Bcd^2-71c^3D+15c^2Cd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}}{5d^3(bc-ad)}+2}{7(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \ 66$$

$$\frac{\frac{35D(bc-ad)^2\left(\frac{2b\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}\right)+\frac{2(a+bx)^{3/2}\left(35a^2d^2(Cd-3cD)-14abd\left(Bd^2-12c^2D+3cCd\right)+b^2\left(8Ad^3+6Bcd^2-71c^3D+15c^2Cd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}}{5d^3(bc-ad)}}{7(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{2(a+bx)^{3/2}\left(35a^2d^2(Cd-3cD)-14abd\left(Bd^2-12c^2D+3cCd\right)+b^2\left(8Ad^3+6Bcd^2-71c^3D+15c^2Cd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}+35D(bc-ad)^2\left(\frac{2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}\right.}{5d^3(bc-ad)}}{7(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(9/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(3/2))/(7*(b*c - a*d)*(
c + d*x)^(7/2)) + ((2*(7*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(10*c^2*C*d -
 3*B*c*d^2 - 4*A*d^3 - 17*c^3*D))*(a + b*x)^(3/2))/(5*d^3*(b*c - a*d)*(c +
 d*x)^(5/2)) + ((2*(35*a^2*d^2*(C*d - 3*c*D) - 14*a*b*d*(3*c*C*d + B*d^2 -
 12*c^2*D) + b^2*(15*c^2*C*d + 6*B*c*d^2 + 8*A*d^3 - 71*c^3*D))*(a + b*x)^
(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2)) + 35*(b*c - a*d)^2*D*((-2*Sqrt[a +
b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt
[b]*Sqrt[c + d*x])])/d^(3/2)))/(5*d^3*(b*c - a*d)))/(7*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2500 vs. $2(287) = 574$.

Time = 0.53 (sec) , antiderivative size = 2501, normalized size of antiderivative = 7.79

method	result	size
default	Expression too large to display	2501

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/105*(336*D*a^3*c^2*d^4*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+315*D*ln(1
/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^
2*b^2*c^5*d^2-315*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+
a*d+b*c)/(d*b)^(1/2))*a*b^3*c^6*d+16*A*b^3*d^6*x^3*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)+210*D*a^3*d^6*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+70*C*a^
3*d^6*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+42*B*a^3*d^6*x*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)-28*B*a*b^2*d^6*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)+12*B*b^3*c*d^5*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+70*C*a^2*b*d^6*
x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-28*C*a^2*b*c*d^5*x^2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)-1260*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^5*d^2*x-352*D*b^3*c^3*d^3*x^3*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-8*A*a*b^2*d^6*x^2*((b*x+a)*(d*x+c))^(1/2)*
(d*b)^(1/2)+56*A*b^3*c*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+14*B*a^
2*b*d^6*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+42*B*b^3*c^2*d^4*x^2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+30*C*b^3*c^2*d^4*x^3*((b*x+a)*(d*x+c))^(1/2
)*(d*b)^(1/2)+420*D*a^3*c*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-812*
D*b^3*c^4*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+6*A*a^2*b*d^6*x*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+70*A*b^3*c^2*d^4*x*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)+56*C*a^3*c*d^5*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-1560*D*a
^2*b*c^3*d^3*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1876*D*a*b^2*c^4*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(288) = 576$.

Time = 95.65 (sec) , antiderivative size = 2077, normalized size of antiderivative = 6.47

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="fr
icas")
```

output
```
[1/210*(105*(D*b^3*c^7 - 3*D*a*b^2*c^6*d + 3*D*a^2*b*c^5*d^2 - D*a^3*c^4*d
^3 + (D*b^3*c^3*d^4 - 3*D*a*b^2*c^2*d^5 + 3*D*a^2*b*c*d^6 - D*a^3*d^7)*x^4
 + 4*(D*b^3*c^4*d^3 - 3*D*a*b^2*c^3*d^4 + 3*D*a^2*b*c^2*d^5 - D*a^3*c*d^6)
*x^3 + 6*(D*b^3*c^5*d^2 - 3*D*a*b^2*c^4*d^3 + 3*D*a^2*b*c^3*d^4 - D*a^3*c^
2*d^5)*x^2 + 4*(D*b^3*c^6*d - 3*D*a*b^2*c^5*d^2 + 3*D*a^2*b*c^4*d^3 - D*a^
3*c^3*d^4)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
+ 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(105*D*b^3*c^6 - 280*D*a*b^2*c^5*d + 231*D*a^2*
b*c^4*d^2 - 48*D*a^3*c^3*d^3 - 15*A*a^3*d^6 - (8*C*a^3 - 14*B*a^2*b + 35*A
*a*b^2)*c^2*d^4 - 6*(B*a^3 - 7*A*a^2*b)*c*d^5 + (176*D*b^3*c^3*d^3 - 3*(16
1*D*a*b^2 + 5*C*b^3)*c^2*d^4 + 6*(70*D*a^2*b + 7*C*a*b^2 - B*b^3)*c*d^5 -
(105*D*a^3 + 35*C*a^2*b - 14*B*a*b^2 + 8*A*b^3)*d^6)*x^3 + (406*D*b^3*c^4*
d^2 - 1096*D*a*b^2*c^3*d^3 + 3*(308*D*a^2*b - C*a*b^2 - 7*B*b^3)*c^2*d^4 -
 2*(105*D*a^3 - 7*C*a^2*b - 26*B*a*b^2 + 14*A*b^3)*c*d^5 - (35*C*a^3 + 7*B
*a^2*b - 4*A*a*b^2)*d^6)*x^2 + (350*D*b^3*c^5*d - 938*D*a*b^2*c^4*d^2 + 78
0*D*a^2*b*c^3*d^3 - (168*D*a^3 - 4*C*a^2*b + 7*B*a*b^2 + 35*A*b^3)*c^2*d^4
 - 2*(14*C*a^3 - 26*B*a^2*b - 7*A*a*b^2)*c*d^5 - 3*(7*B*a^3 + A*a^2*b)*d^6
)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*c^7*d^4 - 3*a*b^2*c^6*d^5 + 3*a^2*b
*c^5*d^6 - a^3*c^4*d^7 + (b^3*c^3*d^8 - 3*a*b^2*c^2*d^9 + 3*a^2*b*c*d^10 -
 a^3*d^11)*x^4 + 4*(b^3*c^4*d^7 - 3*a*b^2*c^3*d^8 + 3*a^2*b*c^2*d^9 - a...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx=\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(9/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(288) = 576$.

Time = 0.43 (sec) , antiderivative size = 951, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="gi
ac")
```

output
```
-2*D*abs(b)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
- a*b*d)))/(sqrt(b*d)*d^4) - 2/105*(((b*x + a)*((176*D*b^10*c^3*d^6*abs(b)
 - 483*D*a*b^9*c^2*d^7*abs(b) - 15*C*b^10*c^2*d^7*abs(b) + 420*D*a^2*b^8*c
*d^8*abs(b) + 42*C*a*b^9*c*d^8*abs(b) - 6*B*b^10*c*d^8*abs(b) - 105*D*a^3*
b^7*d^9*abs(b) - 35*C*a^2*b^8*d^9*abs(b) + 14*B*a*b^9*d^9*abs(b) - 8*A*b^1
0*d^9*abs(b))*(b*x + a)/(b^7*c^3*d^7 - 3*a*b^6*c^2*d^8 + 3*a^2*b^5*c*d^9 -
 a^3*b^4*d^10) + 7*(58*D*b^11*c^4*d^5*abs(b) - 232*D*a*b^10*c^3*d^6*abs(b)
 + 339*D*a^2*b^9*c^2*d^7*abs(b) + 6*C*a*b^10*c^2*d^7*abs(b) - 3*B*b^11*c^2
*d^7*abs(b) - 210*D*a^3*b^8*c*d^8*abs(b) - 16*C*a^2*b^9*c*d^8*abs(b) + 10*
B*a*b^10*c*d^8*abs(b) - 4*A*b^11*c*d^8*abs(b) + 45*D*a^4*b^7*d^9*abs(b) +
10*C*a^3*b^8*d^9*abs(b) - 7*B*a^2*b^9*d^9*abs(b) + 4*A*a*b^10*d^9*abs(b))/
(b^7*c^3*d^7 - 3*a*b^6*c^2*d^8 + 3*a^2*b^5*c*d^9 - a^3*b^4*d^10)) + 35*(10
*D*b^12*c^5*d^4*abs(b) - 50*D*a*b^11*c^4*d^5*abs(b) + 100*D*a^2*b^10*c^3*d
^6*abs(b) - 99*D*a^3*b^9*c^2*d^7*abs(b) - C*a^2*b^10*c^2*d^7*abs(b) + B*a*
b^11*c^2*d^7*abs(b) - A*b^12*c^2*d^7*abs(b) + 48*D*a^4*b^8*c*d^8*abs(b) +
2*C*a^3*b^9*c*d^8*abs(b) - 2*B*a^2*b^10*c*d^8*abs(b) + 2*A*a*b^11*c*d^8*ab
s(b) - 9*D*a^5*b^7*d^9*abs(b) - C*a^4*b^8*d^9*abs(b) + B*a^3*b^9*d^9*abs(b
) - A*a^2*b^10*d^9*abs(b))/(b^7*c^3*d^7 - 3*a*b^6*c^2*d^8 + 3*a^2*b^5*c*d^
9 - a^3*b^4*d^10))*(b*x + a) + 105*(D*b^13*c^6*d^3*abs(b) - 6*D*a*b^12*c^5
*d^4*abs(b) + 15*D*a^2*b^11*c^4*d^5*abs(b) - 20*D*a^3*b^10*c^3*d^6*abs(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \int \frac{\sqrt{a+b\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(9/2),x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \int \frac{\sqrt{bx+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x)
```

output
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x)
```

3.108 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx$

Optimal result 990
Mathematica [A] (verified) 991
Rubi [A] (verified) 991
Maple [A] (verified) 994
Fricas [F(-1)] 995
Sympy [F] 995
Maxima [F(-2)] 996
Giac [B] (verification not implemented) 996
Mupad [F(-1)] 997
Reduce [F] 998

Optimal result

Integrand size = 34, antiderivative size = 375

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{3/2}}{9d^3(bc-ad)(c+dx)^{9/2}}$$
$$+\frac{2(3ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-Bcd^2-2Ad^3-7c^3D))\,(a+bx)^{3/2}}{21d^3(bc-ad)^2(c+dx)^{7/2}}$$
$$+\frac{2(21a^2d^2(Cd-3cD)-6abd(3cCd+2Bd^2-15c^2D)+b^2(5c^2Cd+4Bcd^2+8Ad^3-35c^3D))\,(a+bx)^{3/2}}{105d^3(bc-ad)^3(c+dx)^{5/2}}$$
$$-\frac{2(105a^3d^3D-21a^2bd^2(2Cd+9cD)+3ab^2d(12cCd+8Bd^2+45c^2D)-b^3(10c^2Cd+8Bcd^2+16Ad^3+}{315d^3(bc-ad)^4(c+dx)^{3/2}}$$

output
```
2/9*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(3/2)/d^3/(-a*d+b*c)/(d*x+c)^(9/
2)+2/21*(3*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-2*A*d^3-B*c*d^2+4*C*c^2*d-7*D*
c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(7/2)+2/105*(21*a^2*d^2*(C*d-
3*D*c)-6*a*b*d*(2*B*d^2+3*C*c*d-15*D*c^2)+b^2*(8*A*d^3+4*B*c*d^2+5*C*c^2*d
-35*D*c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(5/2)-2/315*(105*a^3*d^
3*D-21*a^2*b*d^2*(2*C*d+9*D*c)+3*a*b^2*d*(8*B*d^2+12*C*c*d+45*D*c^2)-b^3*(
16*A*d^3+8*B*c*d^2+10*C*c^2*d+35*D*c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^4/(d
*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \frac{2(a+bx)^{3/2}\left(105Ab^3-105ab^2B+105a^2bC-105a^3D-\frac{35c^2Cd(}{(c+d}\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(11/2),x]
```

output
```
(2*(a + b*x)^(3/2)*(105*A*b^3 - 105*a*b^2*B + 105*a^2*b*C - 105*a^3*D - (3
5*c^2*C*d*(a + b*x)^3)/(c + d*x)^3 + (35*B*c*d^2*(a + b*x)^3)/(c + d*x)^3
- (35*A*d^3*(a + b*x)^3)/(c + d*x)^3 + (35*c^3*D*(a + b*x)^3)/(c + d*x)^3
+ (45*b*c^2*C*(a + b*x)^2)/(c + d*x)^2 - (90*b*B*c*d*(a + b*x)^2)/(c + d*x
)^2 + (90*a*c*C*d*(a + b*x)^2)/(c + d*x)^2 + (135*A*b*d^2*(a + b*x)^2)/(c
+ d*x)^2 - (45*a*B*d^2*(a + b*x)^2)/(c + d*x)^2 - (135*a*c^2*D*(a + b*x)^2
)/(c + d*x)^2 + (63*b^2*B*c*(a + b*x))/(c + d*x) - (126*a*b*c*C*(a + b*x))
/(c + d*x) - (189*A*b^2*d*(a + b*x))/(c + d*x) + (126*a*b*B*d*(a + b*x))/(
c + d*x) - (63*a^2*C*d*(a + b*x))/(c + d*x) + (189*a^2*c*D*(a + b*x))/(c +
 d*x)))/(315*(b*c - a*d)^4*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2124, 27, 1193, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$\frac{2\int \frac{3\sqrt{a+bx}\left(-3\left(a-\frac{bc}{d}\right)Dx^2+\frac{3(bc-ad)(Cd-cD)x}{d^2}+\frac{3ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-Dc^3+Cdc^2-Bd^2c-2Ad^3\right)}{d^3}\right)}{2(c+dx)^{9/2}}dx}{9(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{a+bx}\left(-3\left(a-\frac{bc}{d}\right)Dx^2+\frac{3(bc-ad)(Cd-cD)x}{d^2}+\frac{3ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-Dc^3+Cdc^2-Bd^2c-2Ad^3\right)}{d^3}\right)}{(c+dx)^{9/2}}dx}{3(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{\frac{2\int \frac{\sqrt{a+bx}\left(\left(-14Dc^3+5Cdc^2+4Bd^2c+8Ad^3\right)b^2-6ad\left(-8Dc^2+3Cdc+2Bd^2\right)b+21a^2d^2(Cd-2cD)+21d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2(a+bx)^{3/2}(3ad(-Bd^2-3c^2D}{7d^3(}}{3(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{a+bx}\left(\left(-14Dc^3+5Cdc^2+4Bd^2c+8Ad^3\right)b^2-6ad\left(-8Dc^2+3Cdc+2Bd^2\right)b+21a^2d^2(Cd-2cD)+21d(bc-ad)^2Dx\right)}{(c+dx)^{7/2}}dx}{7d^3(bc-ad)}+\frac{2(a+bx)^{3/2}(3ad(-Bd^2-3c^2D+}{7d^3(c-}}{3(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{\frac{2(a+bx)^{3/2}\left(21a^2d^2(Cd-3cD)-6abd\left(2Bd^2-15c^2D+3cCd\right)+b^2\left(8Ad^3+4Bcd^2-35c^3D+5c^2Cd\right)\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\left(105a^3d^3D-21a^2bd^2(9cD+2Cd)+3ab^2d\left(8Bd^2+45c^2D+1\right.\right.}{5(}}{7d^3(bc-ad)}}{3(bc-a}+\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 48$$

$$\frac{\frac{\frac{2(a+bx)^{3/2}\left(21a^2d^2(Cd-3cD)-6abd\left(2Bd^2-15c^2D+3cCd\right)+b^2\left(8Ad^3+4Bcd^2-35c^3D+5c^2Cd\right)\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{2(a+bx)^{3/2}\left(105a^3d^3D-21a^2bd^2(9cD+2Cd)+3ab^2d\left(8Bd^2 \cdots\right.\right.}{15(c+dx)^3 \cdots}}{7d^3(bc-ad)}}{3(bc-ad \cdots}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(11/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(3/2))/(9*(b*c - a*d)*(
c + d*x)^(9/2)) + ((2*(3*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d -
B*c*d^2 - 2*A*d^3 - 7*c^3*D))*(a + b*x)^(3/2))/(7*d^3*(b*c - a*d)*(c + d*x
)^(7/2)) + ((2*(21*a^2*d^2*(C*d - 3*c*D) - 6*a*b*d*(3*c*C*d + 2*B*d^2 - 15
*c^2*D) + b^2*(5*c^2*C*d + 4*B*c*d^2 + 8*A*d^3 - 35*c^3*D))*(a + b*x)^(3/2
))/(5*(b*c - a*d)*(c + d*x)^(5/2)) - (2*(105*a^3*d^3*D - 21*a^2*b*d^2*(2*C
*d + 9*c*D) + 3*a*b^2*d*(12*c*C*d + 8*B*d^2 + 45*c^2*D) - b^3*(10*c^2*C*d
+ 8*B*c*d^2 + 16*A*d^3 + 35*c^3*D))*(a + b*x)^(3/2))/(15*(b*c - a*d)^2*(c
+ d*x)^(3/2)))/(7*d^3*(b*c - a*d)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.47

method	result
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-16A\,b^{3}d^{3}x^{3}+24Ba\,b^{2}d^{3}x^{3}-8B\,b^{3}c\,d^{2}x^{3}-42C\,a^{2}b\,d^{3}x^{3}+36Ca\,b^{2}c\,d^{2}x^{3}-10C\,b^{3}c^{2}d\,x^{3}+105Da^{3}d^{3}x^{3}-189Da^{2}bc\,d\right.}{}$
gosper	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-16A\,b^{3}d^{3}x^{3}+24Ba\,b^{2}d^{3}x^{3}-8B\,b^{3}c\,d^{2}x^{3}-42C\,a^{2}b\,d^{3}x^{3}+36Ca\,b^{2}c\,d^{2}x^{3}-10C\,b^{3}c^{2}d\,x^{3}+105Da^{3}d^{3}x^{3}-189Da^{2}bc\,d\right.}{}$
orering	$-\frac{2(bx+a)^{\frac{3}{2}}\left(-16A\,b^{3}d^{3}x^{3}+24Ba\,b^{2}d^{3}x^{3}-8B\,b^{3}c\,d^{2}x^{3}-42C\,a^{2}b\,d^{3}x^{3}+36Ca\,b^{2}c\,d^{2}x^{3}-10C\,b^{3}c^{2}d\,x^{3}+105Da^{3}d^{3}x^{3}-189Da^{2}bc\,d\right.}{}$

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/315*(b*x+a)^(3/2)/(d*x+c)^(9/2)*(-16*A*b^3*d^3*x^3+24*B*a*b^2*d^3*x^3-8
*B*b^3*c*d^2*x^3-42*C*a^2*b*d^3*x^3+36*C*a*b^2*c*d^2*x^3-10*C*b^3*c^2*d*x^
3+105*D*a^3*d^3*x^3-189*D*a^2*b*c*d^2*x^3+135*D*a*b^2*c^2*d*x^3-35*D*b^3*c
^3*x^3+24*A*a*b^2*d^3*x^2-72*A*b^3*c*d^2*x^2-36*B*a^2*b*d^3*x^2+120*B*a*b^
2*c*d^2*x^2-36*B*b^3*c^2*d*x^2+63*C*a^3*d^3*x^2-243*C*a^2*b*c*d^2*x^2+177*
C*a*b^2*c^2*d*x^2-45*C*b^3*c^3*x^2+126*D*a^3*c*d^2*x^2-108*D*a^2*b*c^2*d*x
^2+30*D*a*b^2*c^3*x^2-30*A*a^2*b*d^3*x+108*A*a*b^2*c*d^2*x-126*A*b^3*c^2*d
*x+45*B*a^3*d^3*x-177*B*a^2*b*c*d^2*x+243*B*a*b^2*c^2*d*x-63*B*b^3*c^3*x+3
6*C*a^3*c*d^2*x-120*C*a^2*b*c^2*d*x+36*C*a*b^2*c^3*x+72*D*a^3*c^2*d*x-24*D
*a^2*b*c^3*x+35*A*a^3*d^3-135*A*a^2*b*c*d^2+189*A*a*b^2*c^2*d-105*A*b^3*c^
3+10*B*a^3*c*d^2-36*B*a^2*b*c^2*d+42*B*a*b^2*c^3+8*C*a^3*c^2*d-24*C*a^2*b*
c^3+16*D*a^3*c^3)/(a*d-b*c)^4
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \int \frac{\sqrt{a+bx}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(11/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(11/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1086 vs. $2(350) = 700$.

Time = 0.57 (sec) , antiderivative size = 1086, normalized size of antiderivative = 2.90

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="g
iac")
```

output
```
2/315*(((b*x + a)*((35*D*b^12*c^3*d^4*abs(b) - 135*D*a*b^11*c^2*d^5*abs(b)
 + 10*C*b^12*c^2*d^5*abs(b) + 189*D*a^2*b^10*c*d^6*abs(b) - 36*C*a*b^11*c*
d^6*abs(b) + 8*B*b^12*c*d^6*abs(b) - 105*D*a^3*b^9*d^7*abs(b) + 42*C*a^2*b
^10*d^7*abs(b) - 24*B*a*b^11*d^7*abs(b) + 16*A*b^12*d^7*abs(b))*(b*x + a)/
(b^8*c^4*d^4 - 4*a*b^7*c^3*d^5 + 6*a^2*b^6*c^2*d^6 - 4*a^3*b^5*c*d^7 + a^4
*b^4*d^8) - 9*(15*D*a*b^12*c^3*d^4*abs(b) - 5*C*b^13*c^3*d^4*abs(b) - 57*D
*a^2*b^11*c^2*d^5*abs(b) + 23*C*a*b^12*c^2*d^5*abs(b) - 4*B*b^13*c^2*d^5*a
bs(b) + 77*D*a^3*b^10*c*d^6*abs(b) - 39*C*a^2*b^11*c*d^6*abs(b) + 16*B*a*b
^12*c*d^6*abs(b) - 8*A*b^13*c*d^6*abs(b) - 35*D*a^4*b^9*d^7*abs(b) + 21*C*
a^3*b^10*d^7*abs(b) - 12*B*a^2*b^11*d^7*abs(b) + 8*A*a*b^12*d^7*abs(b))/(b
^8*c^4*d^4 - 4*a*b^7*c^3*d^5 + 6*a^2*b^6*c^2*d^6 - 4*a^3*b^5*c*d^7 + a^4*b
^4*d^8)) + 63*(3*D*a^2*b^12*c^3*d^4*abs(b) - 2*C*a*b^13*c^3*d^4*abs(b) + B
*b^14*c^3*d^4*abs(b) - 11*D*a^3*b^11*c^2*d^5*abs(b) + 8*C*a^2*b^12*c^2*d^5
*abs(b) - 5*B*a*b^13*c^2*d^5*abs(b) + 2*A*b^14*c^2*d^5*abs(b) + 13*D*a^4*b
^10*c*d^6*abs(b) - 10*C*a^3*b^11*c*d^6*abs(b) + 7*B*a^2*b^12*c*d^6*abs(b)
- 4*A*a*b^13*c*d^6*abs(b) - 5*D*a^5*b^9*d^7*abs(b) + 4*C*a^4*b^10*d^7*abs(
b) - 3*B*a^3*b^11*d^7*abs(b) + 2*A*a^2*b^12*d^7*abs(b))/(b^8*c^4*d^4 - 4*a
*b^7*c^3*d^5 + 6*a^2*b^6*c^2*d^6 - 4*a^3*b^5*c*d^7 + a^4*b^4*d^8))*(b*x +
a) - 105*(D*a^3*b^12*c^3*d^4*abs(b) - C*a^2*b^13*c^3*d^4*abs(b) + B*a*b^14
*c^3*d^4*abs(b) - A*b^15*c^3*d^4*abs(b) - 3*D*a^4*b^11*c^2*d^5*abs(b) +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \int \frac{\sqrt{a+b\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{11/2}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(11/2),x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \int \frac{\sqrt{bx+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x)
```

output
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x)
```

3.109 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx$

Optimal result 999
Mathematica [A] (verified) 1000
Rubi [A] (verified) 1001
Maple [B] (verified) 1004
Fricas [F(-1)] 1005
Sympy [F] 1006
Maxima [F(-2)] 1006
Giac [B] (verification not implemented) 1006
Mupad [F(-1)] 1007
Reduce [F] 1008

Optimal result

Integrand size = 34, antiderivative size = 494

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{3/2}}{11d^3(bc-ad)(c+dx)^{11/2}}$$
$$+\frac{2(11ad(2cCd-Bd^2-3c^2D)-b(14c^2Cd-3Bcd^2-8Ad^3-25c^3D))\,(a+bx)^{3/2}}{99d^3(bc-ad)^2(c+dx)^{9/2}}$$
$$+\frac{2(33a^2d^2(Cd-3cD)-22abd(cCd+Bd^2-6c^2D)+b^2(5c^2Cd+6Bcd^2+16Ad^3-49c^3D))\,(a+bx)^{3/2}}{231d^3(bc-ad)^3(c+dx)^{7/2}}$$
$$-\frac{2(231a^3d^3D-33a^2bd^2(4Cd+9cD)+11ab^2d(8cCd+8Bd^2+15c^2D)-b^3(20c^2Cd+24Bcd^2+64Ad^3}{1155d^3(bc-ad)^4(c+dx)^{5/2}}$$
$$-\frac{4b(231a^3d^3D-33a^2bd^2(4Cd+9cD)+11ab^2d(8cCd+8Bd^2+15c^2D)-b^3(20c^2Cd+24Bcd^2+64Ad^3}{3465d^3(bc-ad)^5(c+dx)^{3/2}}$$

output
```
2/11*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(3/2)/d^3/(-a*d+b*c)/(d*x+c)^(1
1/2)+2/99*(11*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-8*A*d^3-3*B*c*d^2+14*C*c^2*
d-25*D*c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(9/2)+2/231*(33*a^2*d^
2*(C*d-3*D*c)-22*a*b*d*(B*d^2+C*c*d-6*D*c^2)+b^2*(16*A*d^3+6*B*c*d^2+5*C*c
^2*d-49*D*c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(7/2)-2/1155*(231*a
^3*d^3*D-33*a^2*b*d^2*(4*C*d+9*D*c)+11*a*b^2*d*(8*B*d^2+8*C*c*d+15*D*c^2)-
b^3*(64*A*d^3+24*B*c*d^2+20*C*c^2*d+35*D*c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c
)^4/(d*x+c)^(5/2)-4/3465*b*(231*a^3*d^3*D-33*a^2*b*d^2*(4*C*d+9*D*c)+11*a*
b^2*d*(8*B*d^2+8*C*c*d+15*D*c^2)-b^3*(64*A*d^3+24*B*c*d^2+20*C*c^2*d+35*D*
c^3))*(b*x+a)^(3/2)/d^3/(-a*d+b*c)^5/(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 613, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx = \frac{2(a+bx)^{3/2}\left(1155Ab^4-1155ab^3B+1155a^2b^2C-1155a^3bD+\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(13/2),x]
```

output
```
(2*(a + b*x)^(3/2)*(1155*A*b^4 - 1155*a*b^3*B + 1155*a^2*b^2*C - 1155*a^3*
b*D + (315*c^2*C*d^2*(a + b*x)^4)/(c + d*x)^4 - (315*B*c*d^3*(a + b*x)^4)/
(c + d*x)^4 + (315*A*d^4*(a + b*x)^4)/(c + d*x)^4 - (315*c^3*d*D*(a + b*x)
^4)/(c + d*x)^4 - (770*b*c^2*C*d*(a + b*x)^3)/(c + d*x)^3 + (1155*b*B*c*d^
2*(a + b*x)^3)/(c + d*x)^3 - (770*a*c*C*d^2*(a + b*x)^3)/(c + d*x)^3 - (15
40*A*b*d^3*(a + b*x)^3)/(c + d*x)^3 + (385*a*B*d^3*(a + b*x)^3)/(c + d*x)^
3 + (385*b*c^3*D*(a + b*x)^3)/(c + d*x)^3 + (1155*a*c^2*d*D*(a + b*x)^3)/(
c + d*x)^3 + (495*b^2*c^2*C*(a + b*x)^2)/(c + d*x)^2 - (1485*b^2*B*c*d*(a
+ b*x)^2)/(c + d*x)^2 + (1980*a*b*c*C*d*(a + b*x)^2)/(c + d*x)^2 + (2970*A
*b^2*d^2*(a + b*x)^2)/(c + d*x)^2 - (1485*a*b*B*d^2*(a + b*x)^2)/(c + d*x)
^2 + (495*a^2*C*d^2*(a + b*x)^2)/(c + d*x)^2 - (1485*a*b*c^2*D*(a + b*x)^2
)/(c + d*x)^2 - (1485*a^2*c*d*D*(a + b*x)^2)/(c + d*x)^2 + (693*b^3*B*c*(a
 + b*x))/(c + d*x) - (1386*a*b^2*c*C*(a + b*x))/(c + d*x) - (2772*A*b^3*d*
(a + b*x))/(c + d*x) + (2079*a*b^2*B*d*(a + b*x))/(c + d*x) - (1386*a^2*b*
C*d*(a + b*x))/(c + d*x) + (2079*a^2*b*c*D*(a + b*x))/(c + d*x) + (693*a^3
*d*D*(a + b*x))/(c + d*x)))/(3465*(b*c - a*d)^5*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1193, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}dx$$

$$\Big\downarrow \text{2124}$$

$$\frac{2\int \frac{\sqrt{a+bx}\left(-11\left(a-\frac{bc}{d}\right)Dx^2+\frac{11(bc-ad)(Cd-cD)x}{d^2}+\frac{11ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-3Dc^3+3Cdc^2-3Bd^2c-8Ad^3\right)}{d^3}\right)}{2(c+dx)^{11/2}}dx}{11(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a+bx}\left(-11\left(a-\frac{bc}{d}\right)Dx^2+\frac{11(bc-ad)(Cd-cD)x}{d^2}+\frac{11ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-3Dc^3+3Cdc^2-3Bd^2c-8Ad^3\right)}{d^3}\right)}{(c+dx)^{11/2}}dx}{11(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$$\Big\downarrow \text{1193}$$

$$\frac{\frac{2\int \frac{3\sqrt{a+bx}\left(\left(-16Dc^3+5Cdc^2+6Bd^2c+16Ad^3\right)b^2-22ad\left(-3Dc^2+Cdc+Bd^2\right)b+33a^2d^2(Cd-2cD)+33d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{9/2}}dx}{9(bc-ad)}+\frac{2(a+bx)^{3/2}(11ad(-Bd^2-3c^2 \cdots}{9 \cdots}}{11(bc-ad)}+\frac{2(a+bx)^{3/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{a+bx}\left(\left(-16Dc^3+5Cdc^2+6Bd^2c+16Ad^3\right)b^2-22ad\left(-3Dc^2+Cdc+Bd^2\right)b+33a^2d^2(Cd-2cD)+33d(bc-ad)^2Dx\right)}{(c+dx)^{9/2}}dx}{3d^3(bc-ad)}+\frac{2(a+bx)^{3/2}(11ad(-Bd^2-3c^2D\cdots}{9d^3\cdots}}{11(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$\frac{\frac{\frac{2(a+bx)^{3/2}\left(33a^2d^2(Cd-3cD)-22abd\left(Bd^2-6c^2D+cCd\right)+b^2\left(16Ad^3+6Bcd^2-49c^3D+5c^2Cd\right)\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\left(231a^3d^3D-33a^2bd^2(9cD+4Cd)+11ab^2d\left(8Bd^2+15c^2D+8\cdots\right.\right.}{7(\cdots}}{3d^3(bc-ad)}}{11(bc-\cdots}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\Big\downarrow$ 55

$$\frac{\frac{2(a+bx)^{3/2}\left(33a^2d^2(Cd-3cD)-22abd\left(Bd^2-6c^2D+cCd\right)+b^2\left(16Ad^3+6Bcd^2-49c^3D+5c^2Cd\right)\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\left(231a^3d^3D-33a^2bd^2(9cD+4Cd)+11ab^2d\left(8Bd^2+15c^2D+8\cdots\right.\right.}{\cdots}}{3d^3(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\Big\downarrow$ 48

$$\frac{\frac{2(a+bx)^{3/2}\left(33a^2d^2(Cd-3cD)-22abd\left(Bd^2-6c^2D+cCd\right)+b^2\left(16Ad^3+6Bcd^2-49c^3D+5c^2Cd\right)\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\left(\frac{4b(a+bx)^{3/2}}{15(c+dx)^{3/2}(bc-ad)^2}+\frac{2(a+bx)^{3/2}}{5(c+dx)^{5/2}(bc-ad)}\right)\left(231a^3d^3D\cdots\right.}{\cdots}}{3d^3(bc-ad)}$$

$$\frac{2(a+bx)^{3/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

input

```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(13/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(3/2))/(11*(b*c - a*d)*
(c + d*x)^(11/2)) + ((2*(11*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(14*c^2*C*
d - 3*B*c*d^2 - 8*A*d^3 - 25*c^3*D))*(a + b*x)^(3/2))/(9*d^3*(b*c - a*d)*(
c + d*x)^(9/2)) + ((2*(33*a^2*d^2*(C*d - 3*c*D) - 22*a*b*d*(c*C*d + B*d^2
- 6*c^2*D) + b^2*(5*c^2*C*d + 6*B*c*d^2 + 16*A*d^3 - 49*c^3*D))*(a + b*x)^
(3/2))/(7*(b*c - a*d)*(c + d*x)^(7/2)) - ((231*a^3*d^3*D - 33*a^2*b*d^2*(4
*C*d + 9*c*D) + 11*a*b^2*d*(8*c*C*d + 8*B*d^2 + 15*c^2*D) - b^3*(20*c^2*C*
d + 24*B*c*d^2 + 64*A*d^3 + 35*c^3*D))*((2*(a + b*x)^(3/2))/(5*(b*c - a*d)
*(c + d*x)^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*(b*c - a*d)^2*(c + d*x)^(3/2
))))/(7*(b*c - a*d)))/(3*d^3*(b*c - a*d)))/(11*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(464) = 928$.

Time = 0.52 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.92

method	result
default	$-\frac{2(bx+a)^{\frac{3}{2}}\left(128A\,b^4d^4x^4-176Ba\,b^3d^4x^4+48B\,b^4c\,d^3x^4+264C\,a^2b^2d^4x^4-176Ca\,b^3c\,d^3x^4+40C\,b^4c^2d^2x^4-462Da^3b\,d^4x^4+594D\right.}{}$
gosper	Expression too large to display
orering	Expression too large to display

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/3465*(b*x+a)^(3/2)/(d*x+c)^(11/2)*(128*A*b^4*d^4*x^4-176*B*a*b^3*d^4*x^
4+48*B*b^4*c*d^3*x^4+264*C*a^2*b^2*d^4*x^4-176*C*a*b^3*c*d^3*x^4+40*C*b^4*
c^2*d^2*x^4-462*D*a^3*b*d^4*x^4+594*D*a^2*b^2*c*d^3*x^4-330*D*a*b^3*c^2*d^
2*x^4+70*D*b^4*c^3*d*x^4-192*A*a*b^3*d^4*x^3+704*A*b^4*c*d^3*x^3+264*B*a^2
*b^2*d^4*x^3-1040*B*a*b^3*c*d^3*x^3+264*B*b^4*c^2*d^2*x^3-396*C*a^3*b*d^4*
x^3+1716*C*a^2*b^2*c*d^3*x^3-1028*C*a*b^3*c^2*d^2*x^3+220*C*b^4*c^3*d*x^3+
693*D*a^4*d^4*x^3-3432*D*a^3*b*c*d^3*x^3+3762*D*a^2*b^2*c^2*d^2*x^3-1920*D
*a*b^3*c^3*d*x^3+385*D*b^4*c^4*x^3+240*A*a^2*b^2*d^4*x^2-1056*A*a*b^3*c*d^
3*x^2+1584*A*b^4*c^2*d^2*x^2-330*B*a^3*b*d^4*x^2+1542*B*a^2*b^2*c*d^3*x^2-
2574*B*a*b^3*c^2*d^2*x^2+594*B*b^4*c^3*d*x^2+495*C*a^4*d^4*x^2-2508*C*a^3*
b*c*d^3*x^2+4794*C*a^2*b^2*c^2*d^2*x^2-2508*C*a*b^3*c^3*d*x^2+495*C*b^4*c^
4*x^2+594*D*a^4*c*d^3*x^2-2574*D*a^3*b*c^2*d^2*x^2+1542*D*a^2*b^2*c^3*d*x^
2-330*D*a*b^3*c^4*x^2-280*A*a^3*b*d^4*x+1320*A*a^2*b^2*c*d^3*x-2376*A*a*b^
3*c^2*d^2*x+1848*A*b^4*c^3*d*x+385*B*a^4*d^4*x-1920*B*a^3*b*c*d^3*x+3762*B
*a^2*b^2*c^2*d^2*x-3432*B*a*b^3*c^3*d*x+693*B*b^4*c^4*x+220*C*a^4*c*d^3*x-
1028*C*a^3*b*c^2*d^2*x+1716*C*a^2*b^2*c^3*d*x-396*C*a*b^3*c^4*x+264*D*a^4*
c^2*d^2*x-1040*D*a^3*b*c^3*d*x+264*D*a^2*b^2*c^4*x+315*A*a^4*d^4-1540*A*a^
3*b*c*d^3+2970*A*a^2*b^2*c^2*d^2-2772*A*a*b^3*c^3*d+1155*A*b^4*c^4+70*B*a^
4*c*d^3-330*B*a^3*b*c^2*d^2+594*B*a^2*b^2*c^3*d-462*B*a*b^3*c^4+40*C*a^4*c
^2*d^2-176*C*a^3*b*c^3*d+264*C*a^2*b^2*c^4+48*D*a^4*c^3*d-176*D*a^3*b*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx = \int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(13/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(13/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1636 vs. $2(466) = 932$.

Time = 0.68 (sec) , antiderivative size = 1636, normalized size of antiderivative = 3.31

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="g
iac")
```

output
```
2/3465*((((b*x + a)*(2*(35*D*b^14*c^3*d^6*abs(b) - 165*D*a*b^13*c^2*d^7*ab
s(b) + 20*C*b^14*c^2*d^7*abs(b) + 297*D*a^2*b^12*c*d^8*abs(b) - 88*C*a*b^1
3*c*d^8*abs(b) + 24*B*b^14*c*d^8*abs(b) - 231*D*a^3*b^11*d^9*abs(b) + 132*
C*a^2*b^12*d^9*abs(b) - 88*B*a*b^13*d^9*abs(b) + 64*A*b^14*d^9*abs(b))*(b*
x + a)/(b^9*c^5*d^5 - 5*a*b^8*c^4*d^6 + 10*a^2*b^7*c^3*d^7 - 10*a^3*b^6*c^
2*d^8 + 5*a^4*b^5*c*d^9 - a^5*b^4*d^10) + 11*(35*D*b^15*c^4*d^5*abs(b) - 2
00*D*a*b^14*c^3*d^6*abs(b) + 20*C*b^15*c^3*d^6*abs(b) + 462*D*a^2*b^13*c^2
*d^7*abs(b) - 108*C*a*b^14*c^2*d^7*abs(b) + 24*B*b^15*c^2*d^7*abs(b) - 528
*D*a^3*b^12*c*d^8*abs(b) + 220*C*a^2*b^13*c*d^8*abs(b) - 112*B*a*b^14*c*d^
8*abs(b) + 64*A*b^15*c*d^8*abs(b) + 231*D*a^4*b^11*d^9*abs(b) - 132*C*a^3*
b^12*d^9*abs(b) + 88*B*a^2*b^13*d^9*abs(b) - 64*A*a*b^14*d^9*abs(b))/(b^9*
c^5*d^5 - 5*a*b^8*c^4*d^6 + 10*a^2*b^7*c^3*d^7 - 10*a^3*b^6*c^2*d^8 + 5*a^
4*b^5*c*d^9 - a^5*b^4*d^10)) - 99*(15*D*a*b^15*c^4*d^5*abs(b) - 5*C*b^16*c
^4*d^5*abs(b) - 78*D*a^2*b^14*c^3*d^6*abs(b) + 32*C*a*b^15*c^3*d^6*abs(b)
- 6*B*b^16*c^3*d^6*abs(b) + 160*D*a^3*b^13*c^2*d^7*abs(b) - 82*C*a^2*b^14*
c^2*d^7*abs(b) + 34*B*a*b^15*c^2*d^7*abs(b) - 16*A*b^16*c^2*d^7*abs(b) - 1
46*D*a^4*b^12*c*d^8*abs(b) + 88*C*a^3*b^13*c*d^8*abs(b) - 50*B*a^2*b^14*c*
d^8*abs(b) + 32*A*a*b^15*c*d^8*abs(b) + 49*D*a^5*b^11*d^9*abs(b) - 33*C*a^
4*b^12*d^9*abs(b) + 22*B*a^3*b^13*d^9*abs(b) - 16*A*a^2*b^14*d^9*abs(b))/(
b^9*c^5*d^5 - 5*a*b^8*c^4*d^6 + 10*a^2*b^7*c^3*d^7 - 10*a^3*b^6*c^2*d^8...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx = \int \frac{\sqrt{a+b\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{13/2}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(13/2),x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(13/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx=\int \frac{\sqrt{bx+a}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{13}{2}}}dx$$

input
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x)
```

output
```
int((b*x+a)^(1/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x)
```

3.110 $\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result 1009
Mathematica [A] (verified) 1010
Rubi [A] (verified) 1011
Maple [B] (verified) 1018
Fricas [A] (verification not implemented) 1019
Sympy [F] 1020
Maxima [F(-2)] 1021
Giac [B] (verification not implemented) 1021
Mupad [F(-1)] 1022
Reduce [B] (verification not implemented) 1023

Optimal result

Integrand size = 34, antiderivative size = 762

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{(bc-ad)^3\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d(20cCd-24Bd^2-\right.}{1024b}$$

$$-\frac{(bc-ad)^2\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d(20cCd-24Bd^2-15c^2D)-b^3(14c^2Cd-24Bcd^2+\right.}{1536b^5d^4}$$

$$-\frac{(bc-ad)\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d(20cCd-24Bd^2-15c^2D)-b^3(14c^2Cd-24Bcd^2+4\right.}{384b^5d^3}$$

$$-\frac{\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d(20cCd-24Bd^2-15c^2D)-b^3(14c^2Cd-24Bcd^2+48Ad^3-9\right.}{192b^4d^3}$$

$$-\frac{\left(14bcC-24bBd+34aCd-24acD-\frac{9bc^2D}{d}-\frac{39a^2dD}{b}\right)(a+bx)^{5/2}(c+dx)^{5/2}}{120b^2d^2}$$

$$+\frac{(14bCd-9bcD-33adD)(a+bx)^{7/2}(c+dx)^{5/2}}{84b^3d^2}+\frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$$-\frac{(bc-ad)^4\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d(20cCd-24Bd^2-15c^2D)-b^3(14c^2Cd-24Bcd^2+\right.}{1024b^{11/2}d^{11/2}}$$

output
```
1/1024*(-a*d+b*c)^3*(9*a^3*d^3*D-a^2*b*d^2*(14*C*d-15*D*c)-a*b^2*d*(-24*B*
d^2+20*C*c*d-15*D*c^2)-b^3*(48*A*d^3-24*B*c*d^2+14*C*c^2*d-9*D*c^3))*(b*x+
a)^(1/2)*(d*x+c)^(1/2)/b^5/d^5-1/1536*(-a*d+b*c)^2*(9*a^3*d^3*D-a^2*b*d^2*
(14*C*d-15*D*c)-a*b^2*d*(-24*B*d^2+20*C*c*d-15*D*c^2)-b^3*(48*A*d^3-24*B*c
*d^2+14*C*c^2*d-9*D*c^3))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^5/d^4-1/384*(-a*d+
b*c)*(9*a^3*d^3*D-a^2*b*d^2*(14*C*d-15*D*c)-a*b^2*d*(-24*B*d^2+20*C*c*d-15
*D*c^2)-b^3*(48*A*d^3-24*B*c*d^2+14*C*c^2*d-9*D*c^3))*(b*x+a)^(5/2)*(d*x+c
)^(1/2)/b^5/d^3-1/192*(9*a^3*d^3*D-a^2*b*d^2*(14*C*d-15*D*c)-a*b^2*d*(-24*
B*d^2+20*C*c*d-15*D*c^2)-b^3*(48*A*d^3-24*B*c*d^2+14*C*c^2*d-9*D*c^3))*(b*
x+a)^(5/2)*(d*x+c)^(3/2)/b^4/d^3-1/120*(14*C*b*c-24*B*b*d+34*C*a*d-24*D*a*
c-9*b*c^2*D/d-39*a^2*d*D/b)*(b*x+a)^(5/2)*(d*x+c)^(5/2)/b^2/d^2+1/84*(14*C
*b*d-33*D*a*d-9*D*b*c)*(b*x+a)^(7/2)*(d*x+c)^(5/2)/b^3/d^2+1/7*D*(b*x+a)^(
9/2)*(d*x+c)^(5/2)/b^3/d-1/1024*(-a*d+b*c)^4*(9*a^3*d^3*D-a^2*b*d^2*(14*C*
d-15*D*c)-a*b^2*d*(-24*B*d^2+20*C*c*d-15*D*c^2)-b^3*(48*A*d^3-24*B*c*d^2+1
4*C*c^2*d-9*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b
^(11/2)/d^(11/2)
```

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 728, normalized size of antiderivative = 0.96

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{\sqrt{a+bx}\sqrt{c+dx}(945a^6d^6D-210a^5bd^5(7Cd+9cD+3dDx)+7a^4b^2}{}$$

$$-\frac{(bc-ad)^4\left(9a^3d^3D+a^2bd^2(-14Cd+15cD)+ab^2d(-20cCd+24Bd^2+15c^2D)+b^3(-14c^2Cd+24Bc}{1024b^{11/2}d^{11/2}}$$

input
```
Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(945*a^6*d^6*D - 210*a^5*b*d^5*(7*C*d + 9*c*D
 + 3*d*D*x) + 7*a^4*b^2*d^4*(57*c^2*D + 2*c*d*(235*C + 87*D*x) + 4*d^2*(90
*B + x*(35*C + 18*D*x))) + 3*a^2*b^4*d^2*(133*c^4*D - 4*c^3*d*(77*C + 17*D
*x) + 24*c^2*d^2*(42*B + x*(7*C + 2*D*x)) + 32*d^4*x*(35*A + x*(14*B + 7*C
*x + 4*D*x^2)) + 16*c*d^3*(385*A + x*(91*B + 35*C*x + 17*D*x^2))) - 4*a^3*
b^3*d^3*(-81*c^3*D + 3*c^2*d*(77*C + 17*D*x) + 4*c*d^2*(420*B + x*(133*C +
 60*D*x)) + 4*d^3*(315*A + x*(105*B + 49*C*x + 27*D*x^2))) + b^6*(945*c^6*
D - 210*c^5*d*(7*C + 3*D*x) + 28*c^4*d^2*(90*B + x*(35*C + 18*D*x)) + 96*c
^2*d^4*x*(35*A + x*(14*B + x*(7*C + 4*D*x))) + 256*d^6*x^3*(105*A + 2*x*(4
2*B + 5*x*(7*C + 6*D*x))) - 16*c^3*d^3*(315*A + x*(105*B + x*(49*C + 27*D*
x))) + 128*c*d^5*x^2*(315*A + x*(231*B + 2*x*(91*C + 75*D*x)))) + 2*a*b^5*
d*(-945*c^5*D + 7*c^4*d*(235*C + 87*D*x) - 8*c^3*d^2*(420*B + x*(133*C + 6
0*D*x)) + 24*c^2*d^3*(385*A + x*(91*B + x*(35*C + 17*D*x))) + 64*d^5*x^2*(
315*A + x*(231*B + 2*x*(91*C + 75*D*x))) + 32*c*d^4*x*(1155*A + x*(714*B +
 x*(511*C + 396*D*x))))))/(107520*b^5*d^5) - ((b*c - a*d)^4*(9*a^3*d^3*D +
 a^2*b*d^2*(-14*C*d + 15*c*D) + a*b^2*d*(-20*c*C*d + 24*B*d^2 + 15*c^2*D)
+ b^3*(-14*c^2*C*d + 24*B*c*d^2 - 48*A*d^3 + 9*c^3*D))*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(1024*b^(11/2)*d^(11/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$, Rules used = {2125, 27, 1194, 27, 90, 60, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2125}$$

$$\frac{\int \frac{1}{2}(a+bx)^{3/2}(c+dx)^{3/2}\left(14Adb^3+(14bCd-33aDd-9bcD)x^2b^2+2\left(-12dDa^2-9bcDa+7b^2Bd\right)xb-a^2(9\right.}{7b^3d}$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int (a+bx)^{3/2}(c+dx)^{3/2}\left(14Adb^3+(14bCd-33aDd-9bcD)x^2b^2+2\left(-12dDa^2-9bcDa+7b^2Bd\right)xb-a^2(9b}{14b^3d}}{} \quad \frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\Big\downarrow$ 1194

$$\frac{\frac{\int \frac{7}{2}b^2(a+bx)^{3/2}(c+dx)^{3/2}\left(15d^2Da^3-2bd(5Cd-12cD)a^2-b^2c(14Cd-9cD)a+24Ab^3d^2+b\left(-\left(\left(-9Dc^2+14Cdc-24Bd^2\right)b^2\right)-2ad(17Cd-12cD)\right.\right.}{6b^2d}}{14b^3d} \quad \frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\Big\downarrow$ 27

$$\frac{\frac{7\int (a+bx)^{3/2}(c+dx)^{3/2}\left(15d^2Da^3-2bd(5Cd-12cD)a^2-b^2c(14Cd-9cD)a+24Ab^3d^2+b\left(-\left(\left(-9Dc^2+14Cdc-24Bd^2\right)b^2\right)-2ad(17Cd-12cD)b+\right.\right.}{12d}}{14b^3d} \quad \frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\Big\downarrow$ 90

$$\frac{\frac{7\left(\frac{(a+bx)^{5/2}(c+dx)^{5/2}\left(39a^2d^2D-2abd(17Cd-12cD)-\left(b^2\left(-24Bd^2-9c^2D+14cCd\right)\right)\right)}{5d}-\frac{\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d\left(-24Bd^2-15c^2D+20cCd\right)-\right.}{2d}\right.}{12d}}{14b^3d} \quad \frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\Big\downarrow$ 60

$$\frac{\frac{7\left(\frac{(a+bx)^{5/2}(c+dx)^{5/2}\left(39a^2d^2D-2abd(17Cd-12cD)-\left(b^2\left(-24Bd^2-9c^2D+14cCd\right)\right)\right)}{5d}-\frac{\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d\left(-24Bd^2-15c^2D+20cCd\right)-\right.}{}\right.}{12d}}{14b} \quad \frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\Big\downarrow$ 60

$$7\left(\frac{(a+bx)^{5/2}(c+dx)^{5/2}\left(39a^2d^2D-2abd(17Cd-12cD)-\left(b^2\left(-24Bd^2-9c^2D+14cCd\right)\right)\right)}{5d}-\frac{\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d\left(-24Bd^2-15c^2D+20cCd\right)-\right(}{}\right.$$

$$\frac{}{12d}$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\downarrow$ 60

$$7\left(\frac{(a+bx)^{5/2}(c+dx)^{5/2}\left(39a^2d^2D-2abd(17Cd-12cD)-\left(b^2\left(-24Bd^2-9c^2D+14cCd\right)\right)\right)}{5d}-\frac{\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d\left(-24Bd^2-15c^2D+20cCd\right)-\right(}{}\right.$$

$$\frac{}{12}$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\downarrow$ 60

$$7\left(\frac{(a+bx)^{5/2}(c+dx)^{5/2}\left(39a^2d^2D-2abd(17Cd-12cD)-\left(b^2\left(-24Bd^2-9c^2D+14cCd\right)\right)\right)}{5d}-\frac{\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d\left(-24Bd^2-15c^2D+20cCd\right)-\left(\right.\right.}{}\right.$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\downarrow$ 66

$$\left(9a^3d^3D-a^2bd^2(14Cd-15cD)-ab^2d\left(-24Bd^2-15c^2D+20cCd\right)-\left(\right.\right.$$

$$7\left(\frac{(a+bx)^{5/2}(c+dx)^{5/2}\left(39a^2d^2D-2abd(17Cd-12cD)-\left(b^2\left(-24Bd^2-9c^2D+14cCd\right)\right)\right)}{5d}-\frac{\quad}{\quad}\right.$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

$\downarrow$ 221

$$7\left(\frac{(a+bx)^{5/2}(c+dx)^{5/2}\left(39a^2d^2D-2abd(17Cd-12cD)-\left(b^2\left(-24Bd^2-9c^2D+14cCd\right)\right)\right)}{5d}-\frac{3(bc-ad)\left((bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+}}{d}\right.}{}\right)\right.}{6b}\ \cdots\ 8$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{5/2}}{7b^3d}$$

input
```
Int[(a + b*x)^(3/2)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(a + b*x)^(9/2)*(c + d*x)^(5/2))/(7*b^3*d) + (((14*b*C*d - 9*b*c*D - 33
*a*d*D)*(a + b*x)^(7/2)*(c + d*x)^(5/2))/(6*d) + (7*(((39*a^2*d^2*D - 2*a*
b*d*(17*C*d - 12*c*D) - b^2*(14*c*C*d - 24*B*d^2 - 9*c^2*D))*(a + b*x)^(5/
2)*(c + d*x)^(5/2))/(5*d) - ((9*a^3*d^3*D - a^2*b*d^2*(14*C*d - 15*c*D) -
a*b^2*d*(20*c*C*d - 24*B*d^2 - 15*c^2*D) - b^3*(14*c^2*C*d - 24*B*c*d^2 +
48*A*d^3 - 9*c^3*D))*(((a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*b) + (3*(b*c -
a*d)*(((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b) + ((b*c - a*d)*(((a + b*x)^(3/
2)*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d
- ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(
Sqrt[b]*d^(3/2))))/(4*d)))/(6*b)))/(8*b)))/(2*d)))/(12*d))/(14*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3757 vs. $2(706) = 1412$.

Time = 0.53 (sec) , antiderivative size = 3758, normalized size of antiderivative = 4.93

method	result	size
default	Expression too large to display	3758

input
```
int((b*x+a)^(3/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOS
E)
```

output
```
1/215040*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(-13440*B*a*b^5*c^3*d^3*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)-3360*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^3*b^3*
d^6*x-3360*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^6*c^3*d^3*x+5040*B*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^4*b^2*d^6+5040*B*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)*b^6*c^4*d^2-2940*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^5*b*
d^6-2940*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^6*c^5*d-20160*A*ln(1/2*(2
*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^4
*c*d^6+30240*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b
*c)/(d*b)^(1/2))*a^2*b^5*c^2*d^5-20160*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^6*c^3*d^4+7560*B*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b^3*
c*d^6-5040*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c
)/(d*b)^(1/2))*a^3*b^4*c^2*d^5-5040*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^
(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^5*c^3*d^4+7560*B*ln(1/2*(2*b
*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^6*c^4
*d^3-3780*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)
/(d*b)^(1/2))*a^5*b^2*c*d^6+1890*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b^3*c^2*d^5+840*C*ln(1/2*(2*b*d*x
+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^4*c^3*d
^4+1890*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c...
```

Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 2320, normalized size of antiderivative = 3.04

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fr
icas")
```

output
```
[-1/430080*(105*(9*D*b^7*c^7 - 7*(3*D*a*b^6 + 2*C*b^7)*c^6*d + 3*(3*D*a^2*
b^5 + 12*C*a*b^6 + 8*B*b^7)*c^5*d^2 + 3*(D*a^3*b^4 - 6*C*a^2*b^5 - 24*B*a*
b^6 - 16*A*b^7)*c^4*d^3 + (3*D*a^4*b^3 - 8*C*a^3*b^4 + 48*B*a^2*b^5 + 192*
A*a*b^6)*c^3*d^4 + 3*(3*D*a^5*b^2 - 6*C*a^4*b^3 + 16*B*a^3*b^4 - 96*A*a^2*
b^5)*c^2*d^5 - 3*(7*D*a^6*b - 12*C*a^5*b^2 + 24*B*a^4*b^3 - 64*A*a^3*b^4)*
c*d^6 + (9*D*a^7 - 14*C*a^6*b + 24*B*a^5*b^2 - 48*A*a^4*b^3)*d^7)*sqrt(b*d
)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a
*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(
15360*D*b^7*d^7*x^6 + 945*D*b^7*c^6*d - 210*(9*D*a*b^6 + 7*C*b^7)*c^5*d^2
+ 7*(57*D*a^2*b^5 + 470*C*a*b^6 + 360*B*b^7)*c^4*d^3 + 12*(27*D*a^3*b^4 -
77*C*a^2*b^5 - 560*B*a*b^6 - 420*A*b^7)*c^3*d^4 + 21*(19*D*a^4*b^3 - 44*C*
a^3*b^4 + 144*B*a^2*b^5 + 880*A*a*b^6)*c^2*d^5 - 70*(27*D*a^5*b^2 - 47*C*a
^4*b^3 + 96*B*a^3*b^4 - 264*A*a^2*b^5)*c*d^6 + 105*(9*D*a^6*b - 14*C*a^5*b
^2 + 24*B*a^4*b^3 - 48*A*a^3*b^4)*d^7 + 1280*(15*D*b^7*c*d^6 + (15*D*a*b^6
 + 14*C*b^7)*d^7)*x^5 + 128*(3*D*b^7*c^2*d^5 + 2*(99*D*a*b^6 + 91*C*b^7)*c
*d^6 + (3*D*a^2*b^5 + 182*C*a*b^6 + 168*B*b^7)*d^7)*x^4 - 16*(27*D*b^7*c^3
*d^4 - 3*(17*D*a*b^6 + 14*C*b^7)*c^2*d^5 - (51*D*a^2*b^5 + 2044*C*a*b^6 +
1848*B*b^7)*c*d^6 + 3*(9*D*a^3*b^4 - 14*C*a^2*b^5 - 616*B*a*b^6 - 560*A*b^
7)*d^7)*x^3 + 8*(63*D*b^7*c^4*d^3 - 2*(60*D*a*b^6 + 49*C*b^7)*c^3*d^4 + 6*
(3*D*a^2*b^5 + 35*C*a*b^6 + 28*B*b^7)*c^2*d^5 - 6*(20*D*a^3*b^4 - 35*C*...
```

Sympy [F]

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+bx)^{\frac{3}{2}}(c+dx)^{\frac{3}{2}}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
integrate((b*x+a)**(3/2)*(d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral((a + b*x)**(3/2)*(c + d*x)**(3/2)*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6936 vs. $2(691) = 1382$.

Time = 0.95 (sec) , antiderivative size = 6936, normalized size of antiderivative = 9.10

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="gi
ac")
```

output
```
1/107520*(560*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x +
a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c
^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d
^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*s
qrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b
*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*B*c*abs(b) + 14*(sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)*(2*(4*(2*(b*x + a)*(8*(b*x + a)*(10*(b*x + a)/b^5 + (b
^30*c*d^9 - 61*a*b^29*d^10)/(b^34*d^10)) - 3*(3*b^31*c^2*d^8 + 14*a*b^30*c
*d^9 - 417*a^2*b^29*d^10)/(b^34*d^10)) + (21*b^32*c^3*d^7 + 77*a*b^31*c^2*
d^8 + 183*a^2*b^30*c*d^9 - 3481*a^3*b^29*d^10)/(b^34*d^10))*(b*x + a) - 5*
(21*b^33*c^4*d^6 + 56*a*b^32*c^3*d^7 + 106*a^2*b^31*c^2*d^8 + 176*a^3*b^30
*c*d^9 - 2279*a^4*b^29*d^10)/(b^34*d^10))*(b*x + a) + 15*(21*b^34*c^5*d^5
+ 35*a*b^33*c^4*d^6 + 50*a^2*b^32*c^3*d^7 + 70*a^3*b^31*c^2*d^8 + 105*a^4*
b^30*c*d^9 - 793*a^5*b^29*d^10)/(b^34*d^10))*sqrt(b*x + a) + 15*(21*b^6*c^
6 + 14*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 + 20*a^3*b^3*c^3*d^3 + 35*a^4*b^2*
c^2*d^4 + 126*a^5*b*c*d^5 - 231*a^6*d^6)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
+ sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^4*d^5))*D*c*abs(b) -
107520*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+b\,x)^{3/2}\,(c+d\,x)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^(3/2)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^(3/2)*(c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 1752, normalized size of antiderivative = 2.30

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(3/2)*(d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(945*sqrt(c + d*x)*sqrt(a + b*x)*a**6*b*d**7 - 3360*sqrt(c + d*x)*sqrt(a +
 b*x)*a**5*b**2*c*d**6 - 630*sqrt(c + d*x)*sqrt(a + b*x)*a**5*b**2*d**7*x
- 2520*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b**4*d**6 + 3689*sqrt(c + d*x)*sqr
t(a + b*x)*a**4*b**3*c**2*d**5 + 2198*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b**
3*c*d**6*x + 504*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b**3*d**7*x**2 + 11760*s
qrt(c + d*x)*sqrt(a + b*x)*a**3*b**5*c*d**5 + 1680*sqrt(c + d*x)*sqrt(a +
b*x)*a**3*b**5*d**6*x - 600*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**4*c**3*d**
4 - 2332*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**4*c**2*d**5*x - 1744*sqrt(c +
 d*x)*sqrt(a + b*x)*a**3*b**4*c*d**6*x**2 - 432*sqrt(c + d*x)*sqrt(a + b*x
)*a**3*b**4*d**7*x**3 + 21504*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**6*c**2*d
**4 + 78288*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**6*c*d**5*x + 41664*sqrt(c
+ d*x)*sqrt(a + b*x)*a**2*b**6*d**6*x**2 - 525*sqrt(c + d*x)*sqrt(a + b*x)
*a**2*b**5*c**4*d**3 + 300*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**5*c**3*d**4
*x + 1824*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**5*c**2*d**5*x**2 + 1488*sqrt
(c + d*x)*sqrt(a + b*x)*a**2*b**5*c*d**6*x**3 + 384*sqrt(c + d*x)*sqrt(a +
 b*x)*a**2*b**5*d**7*x**4 - 11760*sqrt(c + d*x)*sqrt(a + b*x)*a*b**7*c**3*
d**3 + 7728*sqrt(c + d*x)*sqrt(a + b*x)*a*b**7*c**2*d**4*x + 86016*sqrt(c
+ d*x)*sqrt(a + b*x)*a*b**7*c*d**5*x**2 + 56448*sqrt(c + d*x)*sqrt(a + b*x
)*a*b**7*d**6*x**3 + 1400*sqrt(c + d*x)*sqrt(a + b*x)*a*b**6*c**5*d**2 - 9
10*sqrt(c + d*x)*sqrt(a + b*x)*a*b**6*c**4*d**3*x + 720*sqrt(c + d*x)*s...
```

3.111 $\int (a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result 1024
Mathematica [A] (verified) 1025
Rubi [A] (verified) 1026
Maple [B] (verified) 1030
Fricas [A] (verification not implemented) 1031
Sympy [F] 1032
Maxima [F(-2)] 1033
Giac [B] (verification not implemented) 1033
Mupad [F(-1)] 1034
Reduce [B] (verification not implemented) 1035

Optimal result

Integrand size = 34, antiderivative size = 639

$$\int (a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx = \frac{(bc-ad)^2\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d(8cCd-8Bd^2-7c^2D)-b^3(28c^2Cd-40\right.}{512b^4d^5}$$

$$-\frac{(bc-ad)\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d(8cCd-8Bd^2-7c^2D)-b^3(28c^2Cd-40Bcd^2+64A\right.}{768b^4d^4}$$

$$-\frac{(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d(8cCd-8Bd^2-7c^2D)-b^3(28c^2Cd-40Bcd^2+64Ad^3-21c^3}{192b^4d^3}$$

$$-\frac{\left(28bcC-40bBd+52aCd-42acD-\frac{21bc^2D}{d}-\frac{57a^2dD}{b}\right)(a+bx)^{5/2}(c+dx)^{3/2}}{160b^2d^2}$$

$$+\frac{(4bCd-3bcD-9adD)(a+bx)^{7/2}(c+dx)^{3/2}}{20b^3d^2}+\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$$-\frac{(bc-ad)^3\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d(8cCd-8Bd^2-7c^2D)-b^3(28c^2Cd-40Bcd^2+64A\right.}{512b^{9/2}d^{11/2}}$$

output
```
1/512*(-a*d+b*c)^2*(7*a^3*d^3*D-3*a^2*b*d^2*(4*C*d-5*D*c)-3*a*b^2*d*(-8*B*
d^2+8*C*c*d-7*D*c^2)-b^3*(64*A*d^3-40*B*c*d^2+28*C*c^2*d-21*D*c^3))*(b*x+a
)^(1/2)*(d*x+c)^(1/2)/b^4/d^5-1/768*(-a*d+b*c)*(7*a^3*d^3*D-3*a^2*b*d^2*(4
*C*d-5*D*c)-3*a*b^2*d*(-8*B*d^2+8*C*c*d-7*D*c^2)-b^3*(64*A*d^3-40*B*c*d^2+
28*C*c^2*d-21*D*c^3))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^4/d^4-1/192*(7*a^3*d^3
*D-3*a^2*b*d^2*(4*C*d-5*D*c)-3*a*b^2*d*(-8*B*d^2+8*C*c*d-7*D*c^2)-b^3*(64*
A*d^3-40*B*c*d^2+28*C*c^2*d-21*D*c^3))*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^4/d^3
-1/160*(28*C*b*c-40*B*b*d+52*C*a*d-42*D*a*c-21*b*c^2*D/d-57*a^2*d*D/b)*(b*
x+a)^(5/2)*(d*x+c)^(3/2)/b^2/d^2+1/20*(4*C*b*d-9*D*a*d-3*D*b*c)*(b*x+a)^(7
/2)*(d*x+c)^(3/2)/b^3/d^2+1/6*D*(b*x+a)^(9/2)*(d*x+c)^(3/2)/b^3/d-1/512*(-
a*d+b*c)^3*(7*a^3*d^3*D-3*a^2*b*d^2*(4*C*d-5*D*c)-3*a*b^2*d*(-8*B*d^2+8*C*
c*d-7*D*c^2)-b^3*(64*A*d^3-40*B*c*d^2+28*C*c^2*d-21*D*c^3))*arctanh(d^(1/2
)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(9/2)/d^(11/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.87

$$\int (a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}(-105a^5d^5D+5a^4bd^4(36Cd+11cD+14dDx)-2a^3b^2d^3(-27c^2D+4cd(1\cdots}{\cdots} - \frac{(bc-ad)^3\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)+3ab^2d(-8cCd+8Bd^2+7c^2D)+b^3(-28c^2Cd+40Bcd^2-\cdots\right.}{512b^{9/2}d^{11/2}}$$

input
```
Integrate[(a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(-105*a^5*d^5*D + 5*a^4*b*d^4*(36*C*d + 11*c*
D + 14*d*D*x) - 2*a^3*b^2*d^3*(-27*c^2*D + 4*c*d*(15*C + 4*D*x) + 4*d^2*(4
5*B + x*(15*C + 7*D*x))) + 6*a^2*b^3*d^2*(13*c^3*D - 6*c^2*d*(4*C + D*x) +
 4*c*d^2*(15*B + x*(3*C + D*x)) + 8*d^3*(20*A + x*(5*B + 2*C*x + D*x^2)))
+ a*b^4*d*(-525*c^4*D + 8*c^3*d*(95*C + 42*D*x) - 8*c^2*d^2*(155*B + x*(61
*C + 33*D*x)) + 32*c*d^3*(80*A + x*(25*B + 12*C*x + 7*D*x^2)) + 64*d^4*x*(
70*A + x*(45*B + 33*C*x + 26*D*x^2))) + b^5*(315*c^5*D - 210*c^4*d*(2*C +
D*x) + 8*c^3*d^2*(75*B + 7*x*(5*C + 3*D*x)) + 64*c*d^4*x*(10*A + x*(5*B +
x*(3*C + 2*D*x))) + 128*d^5*x^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) - 16
*c^2*d^3*(60*A + x*(25*B + x*(14*C + 9*D*x))))))/(7680*b^4*d^5) - ((b*c -
a*d)^3*(7*a^3*d^3*D - 3*a^2*b*d^2*(4*C*d - 5*c*D) + 3*a*b^2*d*(-8*c*C*d +
8*B*d^2 + 7*c^2*D) + b^3*(-28*c^2*C*d + 40*B*c*d^2 - 64*A*d^3 + 21*c^3*D))
*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(512*b^(9/2)*d^
(11/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2125, 27, 1194, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2125}$$

$$\frac{\int \frac{3}{2}(a+bx)^{3/2}\sqrt{c+dx}\left(4Adb^3+(4bCd-9aDd-3bcD)x^2b^2+2\left(-3dDa^2-3bcDa+2b^2Bd\right)xb-a^2(3bc+ad\ldots}{6b^3d}$$
$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (a+bx)^{3/2}\sqrt{c+dx}\left(4Adb^3+(4bCd-9aDd-3bcD)x^2b^2+2\left(-3dDa^2-3bcDa+2b^2Bd\right)xb-a^2(3bc+ad)\ldots}{4b^3d}$$
$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$$\Big\downarrow \; 1194$$

$$\frac{\int \frac{1}{2}b^2(a+bx)^{3/2}\sqrt{c+dx}\left(17d^2Da^3-6bd(2Cd-7cD)a^2-7b^2c(4Cd-3cD)a+40Ab^3d^2+b\left(-\left(\left(-21Dc^2+28Cdc-40Bd^2\right)b^2\right)-2ad(26Cd-21cD)b+\right.\right.}{5b^2d}$$

$$4b^3d$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (a+bx)^{3/2}\sqrt{c+dx}\left(17d^2Da^3-6bd(2Cd-7cD)a^2-7b^2c(4Cd-3cD)a+40Ab^3d^2+b\left(-\left(\left(-21Dc^2+28Cdc-40Bd^2\right)b^2\right)-2ad(26Cd-21cD)b+57a^2\right.\right.}{10d}$$

$$4b^3d$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{(a+bx)^{5/2}(c+dx)^{3/2}\left(57a^2d^2D-2abd(26Cd-21cD)-\left(b^2\left(-40Bd^2-21c^2D+28cCd\right)\right)\right)}{4d}-\frac{5\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d\left(-8Bd^2-7c^2D+8cCd\right)-\left(b^3\left(\right.\right.\right.}{8d}}{10d}$$

$$4b^3d$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(a+bx)^{5/2}(c+dx)^{3/2}\left(57a^2d^2D-2abd(26Cd-21cD)-\left(b^2\left(-40Bd^2-21c^2D+28cCd\right)\right)\right)}{4d}-\frac{5\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d\left(-8Bd^2-7c^2D+8cCd\right)-\left(b^3\left(\right.\right.\right.}{}}{10d}$$

$$4b^3d$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(a+bx)^{5/2}(c+dx)^{3/2}\left(57a^2d^2D-2abd(26Cd-21cD)-\left(b^2\left(-40Bd^2-21c^2D+28cCd\right)\right)\right)}{4d}-\frac{5\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d\left(-8Bd^2-7c^2D+8cCd\right)-\left(b^3\left(\right.\right.\right.}{}}{10d}$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$$\Big\downarrow \; 60$$

$$5\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d\left(-8Bd^2-7c^2D+8cCd\right)-\left(b^3\left(6\right.\right.\right.$$

$$\frac{\frac{(a+bx)^{5/2}(c+dx)^{3/2}\left(57a^2d^2D-2abd(26Cd-21cD)-\left(b^2\left(-40Bd^2-21c^2D+28cCd\right)\right)\right)}{4d}-\frac{\quad}{\quad}}{10d}$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$\big\downarrow$ 66

$$5\left(7a^3d^3D-3a^2bd^2(4Cd-5cD)-3ab^2d\left(-8Bd^2-7c^2D+8cCd\right)-\left(b^3\left(6\right.\right.\right.$$

$$\frac{\frac{(a+bx)^{5/2}(c+dx)^{3/2}\left(57a^2d^2D-2abd(26Cd-21cD)-\left(b^2\left(-40Bd^2-21c^2D+28cCd\right)\right)\right)}{4d}-\frac{\quad}{\quad}}{10d}$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

$\big\downarrow$ 221

$$\frac{\frac{(a+bx)^{5/2}(c+dx)^{3/2}\left(57a^2d^2D-2abd(26Cd-21cD)-\left(b^2\left(-40Bd^2-21c^2D+28cCd\right)\right)\right)}{4d}-\frac{5\left(\frac{(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)}{\quad}\right.}{4d}\right.}{6b}\right.}{\quad}}{10d}$$

$$\frac{D(a+bx)^{9/2}(c+dx)^{3/2}}{6b^3d}$$

input
```
Int[(a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(a + b*x)^(9/2)*(c + d*x)^(3/2))/(6*b^3*d) + (((4*b*C*d - 3*b*c*D - 9*a
*d*D)*(a + b*x)^(7/2)*(c + d*x)^(3/2))/(5*d) + (((57*a^2*d^2*D - 2*a*b*d*(
26*C*d - 21*c*D) - b^2*(28*c*C*d - 40*B*d^2 - 21*c^2*D))*(a + b*x)^(5/2)*(
c + d*x)^(3/2))/(4*d) - (5*(7*a^3*d^3*D - 3*a^2*b*d^2*(4*C*d - 5*c*D) - 3*
a*b^2*d*(8*c*C*d - 8*B*d^2 - 7*c^2*D) - b^3*(28*c^2*C*d - 40*B*c*d^2 + 64*
A*d^3 - 21*c^3*D))*(((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b) + ((b*c - a*d)*(
((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqr
t[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt
[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*b)))/(8*d))/(10*d))/(4*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2805 vs. $2(589) = 1178$.

Time = 0.53 (sec) , antiderivative size = 2806, normalized size of antiderivative = 4.39

method	result	size
default	Expression too large to display	2806

input
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOS
E)
```

output
```
-1/15360*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(-3840*B*b^5*d^5*x^3*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)-5120*A*b^5*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
-672*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c^3*d^2*x-1600*B*((b*x+a)
*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c*d^4*x-144*C*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)*a^2*b^3*c*d^4*x+976*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*
c^2*d^3*x+72*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*c^2*d^3*x+960*A
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2
))*a^3*b^3*d^6-8960*A*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*d^5*x-110*
D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^4*b*c*d^4-108*D*((b*x+a)*(d*x+c))^
(1/2)*(d*b)^(1/2)*a^3*b^2*c^2*d^3-156*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)*a^2*b^3*c^3*d^2+1050*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c^4*d-5
60*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^3*d^2*x+800*B*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)*b^5*c^2*d^3*x+64*D*a^3*b^2*c*d^4*x*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)-2560*D*b^5*d^5*x^5*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
-3072*C*b^5*d^5*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+420*D*((b*x+a)*(d*
x+c))^(1/2)*(d*b)^(1/2)*b^5*c^4*d*x+2480*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)*a*b^4*c^2*d^3+240*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^3*b^2*c*d^4
-1920*A*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*d^5+1920*A*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^2*d^3+720*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)*a^3*b^2*d^5-1200*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^3*d^2...
```

Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1750, normalized size of antiderivative = 2.74

$$\int (a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fr
icas")
```

output
```
[1/30720*(15*(21*D*b^6*c^6 - 14*(3*D*a*b^5 + 2*C*b^6)*c^5*d + 5*(3*D*a^2*b
^4 + 12*C*a*b^5 + 8*B*b^6)*c^4*d^2 + 4*(D*a^3*b^3 - 6*C*a^2*b^4 - 24*B*a*b
^5 - 16*A*b^6)*c^3*d^3 + (3*D*a^4*b^2 - 8*C*a^3*b^3 + 48*B*a^2*b^4 + 192*A
*a*b^5)*c^2*d^4 + 2*(3*D*a^5*b - 6*C*a^4*b^2 + 16*B*a^3*b^3 - 96*A*a^2*b^4
)*c*d^5 - (7*D*a^6 - 12*C*a^5*b + 24*B*a^4*b^2 - 64*A*a^3*b^3)*d^6)*sqrt(b
*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c +
 a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4
*(1280*D*b^6*d^6*x^5 + 315*D*b^6*c^5*d - 105*(5*D*a*b^5 + 4*C*b^6)*c^4*d^2
 + 2*(39*D*a^2*b^4 + 380*C*a*b^5 + 300*B*b^6)*c^3*d^3 + 2*(27*D*a^3*b^3 -
72*C*a^2*b^4 - 620*B*a*b^5 - 480*A*b^6)*c^2*d^4 + 5*(11*D*a^4*b^2 - 24*C*a
^3*b^3 + 72*B*a^2*b^4 + 512*A*a*b^5)*c*d^5 - 15*(7*D*a^5*b - 12*C*a^4*b^2
+ 24*B*a^3*b^3 - 64*A*a^2*b^4)*d^6 + 128*(D*b^6*c*d^5 + (13*D*a*b^5 + 12*C
*b^6)*d^6)*x^4 - 16*(9*D*b^6*c^2*d^4 - 2*(7*D*a*b^5 + 6*C*b^6)*c*d^5 - 3*(
D*a^2*b^4 + 44*C*a*b^5 + 40*B*b^6)*d^6)*x^3 + 8*(21*D*b^6*c^3*d^3 - (33*D*
a*b^5 + 28*C*b^6)*c^2*d^4 + (3*D*a^2*b^4 + 48*C*a*b^5 + 40*B*b^6)*c*d^5 -
(7*D*a^3*b^3 - 12*C*a^2*b^4 - 360*B*a*b^5 - 320*A*b^6)*d^6)*x^2 - 2*(105*D
*b^6*c^4*d^2 - 28*(6*D*a*b^5 + 5*C*b^6)*c^3*d^3 + 2*(9*D*a^2*b^4 + 122*C*a
*b^5 + 100*B*b^6)*c^2*d^4 + 4*(4*D*a^3*b^3 - 9*C*a^2*b^4 - 100*B*a*b^5 - 8
0*A*b^6)*c*d^5 - 5*(7*D*a^4*b^2 - 12*C*a^3*b^3 + 24*B*a^2*b^4 + 448*A*a*b^
5)*d^6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*d^6), 1/15360*(15*(21*D*b^...
```

Sympy [F]

$$\int (a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+bx)^{\frac{3}{2}}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
integrate((b*x+a)**(3/2)*(d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral((a + b*x)**(3/2)*sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2977 vs. $2(585) = 1170$.

Time = 0.50 (sec) , antiderivative size = 2977, normalized size of antiderivative = 4.66

$$\int (a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="gi
ac")
```

output
```
1/7680*(40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*
(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*
d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3
+ 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt
(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*
d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*B*abs(b) + (sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)*(2*(4*(2*(b*x + a)*(8*(b*x + a)*(10*(b*x + a)/b^5 + (b^30*c*d^
9 - 61*a*b^29*d^10)/(b^34*d^10)) - 3*(3*b^31*c^2*d^8 + 14*a*b^30*c*d^9 - 4
17*a^2*b^29*d^10)/(b^34*d^10)) + (21*b^32*c^3*d^7 + 77*a*b^31*c^2*d^8 + 18
3*a^2*b^30*c*d^9 - 3481*a^3*b^29*d^10)/(b^34*d^10))*(b*x + a) - 5*(21*b^33
*c^4*d^6 + 56*a*b^32*c^3*d^7 + 106*a^2*b^31*c^2*d^8 + 176*a^3*b^30*c*d^9 -
 2279*a^4*b^29*d^10)/(b^34*d^10))*(b*x + a) + 15*(21*b^34*c^5*d^5 + 35*a*b
^33*c^4*d^6 + 50*a^2*b^32*c^3*d^7 + 70*a^3*b^31*c^2*d^8 + 105*a^4*b^30*c*d
^9 - 793*a^5*b^29*d^10)/(b^34*d^10))*sqrt(b*x + a) + 15*(21*b^6*c^6 + 14*a
*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 + 20*a^3*b^3*c^3*d^3 + 35*a^4*b^2*c^2*d^4
+ 126*a^5*b*c*d^5 - 231*a^6*d^6)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b
^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^4*d^5))*D*abs(b) - 7680*((b^2
*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
- a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\ dx = \int (a+b\,x)^{3/2}\,\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1334, normalized size of antiderivative = 2.09

$$\int (a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 105*sqrt(c + d*x)*sqrt(a + b*x)*a**5*b*d**6 + 235*sqrt(c + d*x)*sqrt(a
 + b*x)*a**4*b**2*c*d**5 + 70*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b**2*d**6*x
 + 600*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**4*d**5 - 66*sqrt(c + d*x)*sqrt(
a + b*x)*a**3*b**3*c**2*d**4 - 152*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**3*c
*d**5*x - 56*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**3*d**6*x**2 + 2920*sqrt(c
 + d*x)*sqrt(a + b*x)*a**2*b**5*c*d**4 + 4720*sqrt(c + d*x)*sqrt(a + b*x)*
a**2*b**5*d**5*x - 66*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*c**3*d**3 + 36
*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*c**2*d**4*x + 120*sqrt(c + d*x)*sqr
t(a + b*x)*a**2*b**4*c*d**5*x**2 + 48*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**
4*d**6*x**3 - 2200*sqrt(c + d*x)*sqrt(a + b*x)*a*b**6*c**2*d**3 + 1440*sqr
t(c + d*x)*sqrt(a + b*x)*a*b**6*c*d**4*x + 5440*sqrt(c + d*x)*sqrt(a + b*x
)*a*b**6*d**5*x**2 + 235*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*c**4*d**2 - 15
2*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*c**3*d**3*x + 120*sqrt(c + d*x)*sqrt(
a + b*x)*a*b**5*c**2*d**4*x**2 + 2336*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*c
*d**5*x**3 + 1664*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*d**6*x**4 + 600*sqrt(
c + d*x)*sqrt(a + b*x)*b**7*c**3*d**2 - 400*sqrt(c + d*x)*sqrt(a + b*x)*b*
*7*c**2*d**3*x + 320*sqrt(c + d*x)*sqrt(a + b*x)*b**7*c*d**4*x**2 + 1920*s
qrt(c + d*x)*sqrt(a + b*x)*b**7*d**5*x**3 - 105*sqrt(c + d*x)*sqrt(a + b*x
)*b**6*c**5*d + 70*sqrt(c + d*x)*sqrt(a + b*x)*b**6*c**4*d**2*x - 56*sqrt(
c + d*x)*sqrt(a + b*x)*b**6*c**3*d**3*x**2 + 48*sqrt(c + d*x)*sqrt(a + ...
```

3.112 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$

Optimal result . . . 1036
Mathematica [A] (verified) . . . 1037
Rubi [A] (verified) . . . 1038
Maple [B] (verified) . . . 1042
Fricas [A] (verification not implemented) . . . 1043
Sympy [F] . . . 1044
Maxima [F(-2)] . . . 1044
Giac [A] (verification not implemented) . . . 1044
Mupad [F(-1)] . . . 1045
Reduce [B] (verification not implemented) . . . 1046

Optimal result

Integrand size = 34, antiderivative size = 516

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx = \frac{(bc-ad)\left(3a^3d^3D-3a^2bd^2(2Cd-3cD)-ab^2d(20cCd-16\right.}{}$$

$$-\frac{(3a^3d^3D-3a^2bd^2(2Cd-3cD)-ab^2d(20cCd-16Bd^2-21c^2D)-b^3(70c^2Cd-80Bcd^2+96Ad^3-63}{192b^3d^4}$$

$$-\frac{\left(70bcC-80bBd+90aCd-84acD-\frac{63bc^2D}{d}-\frac{93a^2dD}{b}\right)(a+bx)^{5/2}\sqrt{c+dx}}{240b^2d^2}$$

$$+\frac{(10bCd-9bcD-21adD)(a+bx)^{7/2}\sqrt{c+dx}}{40b^3d^2}+\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$-\frac{(bc-ad)^2\left(3a^3d^3D-3a^2bd^2(2Cd-3cD)-ab^2d(20cCd-16Bd^2-21c^2D)-b^3(70c^2Cd-80Bcd^2+9\right.}{128b^{7/2}d^{11/2}}$$

output
```
1/128*(-a*d+b*c)*(3*a^3*d^3*D-3*a^2*b*d^2*(2*C*d-3*D*c)-a*b^2*d*(-16*B*d^2
+20*C*c*d-21*D*c^2)-b^3*(96*A*d^3-80*B*c*d^2+70*C*c^2*d-63*D*c^3))*(b*x+a)
^(1/2)*(d*x+c)^(1/2)/b^3/d^5-1/192*(3*a^3*d^3*D-3*a^2*b*d^2*(2*C*d-3*D*c)-
a*b^2*d*(-16*B*d^2+20*C*c*d-21*D*c^2)-b^3*(96*A*d^3-80*B*c*d^2+70*C*c^2*d-
63*D*c^3))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3/d^4-1/240*(70*C*b*c-80*B*b*d+90
*C*a*d-84*D*a*c-63*b*c^2*D/d-93*a^2*d*D/b)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^2
/d^2+1/40*(10*C*b*d-21*D*a*d-9*D*b*c)*(b*x+a)^(7/2)*(d*x+c)^(1/2)/b^3/d^2+
1/5*D*(b*x+a)^(9/2)*(d*x+c)^(1/2)/b^3/d-1/128*(-a*d+b*c)^2*(3*a^3*d^3*D-3*
a^2*b*d^2*(2*C*d-3*D*c)-a*b^2*d*(-16*B*d^2+20*C*c*d-21*D*c^2)-b^3*(96*A*d^
3-80*B*c*d^2+70*C*c^2*d-63*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(
d*x+c)^(1/2))/b^(7/2)/d^(11/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}(45a^4d^4D-30a^3bd^3(3Cd-2cD+dDx)+\cdots}{\cdots}$$

$$-\frac{(bc-ad)^2\left(3a^3d^3D+3a^2bd^2(-2Cd+3cD)+ab^2d(-20cCd+16Bd^2+21c^2D)+b^3(-70c^2Cd+80Bc\cdots\right)}{128b^{7/2}d^{11/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(45*a^4*d^4*D - 30*a^3*b*d^3*(3*C*d - 2*c*D +
 d*D*x) + 6*a^2*b^2*d^2*(19*c^2*D - c*d*(25*C + 7*D*x) + 2*d^2*(20*B + x*(
5*C + 2*D*x))) + 2*a*b^3*d*(-630*c^3*D + c^2*d*(725*C + 399*D*x) - 4*c*d^2
*(220*B + x*(115*C + 78*D*x)) + 8*d^3*(150*A + x*(70*B + 45*C*x + 33*D*x^2
))) + b^4*(945*c^4*D - 210*c^3*d*(5*C + 3*D*x) + 4*c^2*d^2*(300*B + 7*x*(2
5*C + 18*D*x)) + 32*d^4*x*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) - 16*c*d^3
*(90*A + x*(50*B + x*(35*C + 27*D*x))))))/(1920*b^3*d^5) - ((b*c - a*d)^2*
(3*a^3*d^3*D + 3*a^2*b*d^2*(-2*C*d + 3*c*D) + a*b^2*d*(-20*c*C*d + 16*B*d^
2 + 21*c^2*D) + b^3*(-70*c^2*C*d + 80*B*c*d^2 - 96*A*d^3 + 63*c^3*D))*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(7/2)*d^(11/2
))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {2125, 27, 1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 2125}$$

$$\frac{\int \frac{(a+bx)^{3/2}\left(10Adb^3+(10bCd-21aDd-9bcD)x^2b^2+2\left(-6dDa^2-9bcDa+5b^2Bd\right)xb-a^2(9bc+ad)D\right)}{2\sqrt{c+dx}}dx}{5b^3d}+$$
$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+bx)^{3/2}\left(10Adb^3+(10bCd-21aDd-9bcD)x^2b^2+2\left(-6dDa^2-9bcDa+5b^2Bd\right)xb-a^2(9bc+ad)D\right)}{\sqrt{c+dx}}dx}{10b^3d}+$$
$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$\Big\downarrow \text{ 1194}$$

$$\frac{\frac{\int \frac{b^2(a+bx)^{3/2}\left(13d^2Da^3-2bd(5Cd-42cD)a^2-7b^2c(10Cd-9cD)a+80Ab^3d^2+b\left(-\left(\left(-63Dc^2+70Cdc-80Bd^2\right)b^2\right)-6ad(15Cd-14cD)b+93a^2d^2D\right)x\right)}{2\sqrt{c+dx}}dx}{4b^2d}+\frac{(a}{}}{10b^3d}$$
$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{(a+bx)^{3/2}\left(13d^2Da^3-2bd(5Cd-42cD)a^2-7b^2c(10Cd-9cD)a+80Ab^3d^2+b\left(-\left(\left(-63Dc^2+70Cdc-80Bd^2\right)b^2\right)-6ad(15Cd-14cD)b+93a^2d^2D\right)x\right)}{\sqrt{c+dx}}dx}{8d}+\frac{(a+}{}}{10b^3d}$$
$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\frac{(a+bx)^{5/2}\sqrt{c+dx}\left(93a^2d^2D-6abd(15Cd-14cD)-\left(b^2\left(-80Bd^2-63c^2D+70cCd\right)\right)\right)}{3d}-\frac{5\left(3a^3d^3D-3a^2bd^2(2Cd-3cD)-ab^2d\left(-16Bd^2-21c^2D+20cCd\right)-\left(b^3\left(96\right.\right.\right.}{6d}}{8d}}{10b^3d}$$

$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(a+bx)^{5/2}\sqrt{c+dx}\left(93a^2d^2D-6abd(15Cd-14cD)-\left(b^2\left(-80Bd^2-63c^2D+70cCd\right)\right)\right)}{3d}-\frac{5\left(3a^3d^3D-3a^2bd^2(2Cd-3cD)-ab^2d\left(-16Bd^2-21c^2D+20cCd\right)-\left(b^3\left(96\right.\right.\right.}{8d}}{10b^3d}$$

$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{(a+bx)^{5/2}\sqrt{c+dx}\left(93a^2d^2D-6abd(15Cd-14cD)-\left(b^2\left(-80Bd^2-63c^2D+70cCd\right)\right)\right)}{3d}-\frac{5\left(3a^3d^3D-3a^2bd^2(2Cd-3cD)-ab^2d\left(-16Bd^2-21c^2D+20cCd\right)-\left(b^3\left(96\right.\right.\right.}{8d}}{}$$

$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{(a+bx)^{5/2}\sqrt{c+dx}\left(93a^2d^2D-6abd(15Cd-14cD)-\left(b^2\left(-80Bd^2-63c^2D+70cCd\right)\right)\right)}{3d}-\frac{5\left(3a^3d^3D-3a^2bd^2(2Cd-3cD)-ab^2d\left(-16Bd^2-21c^2D+20cCd\right)-\left(b^3\left(96\right.\right.\right.}{8d}}{}$$

$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{(a+bx)^{5/2}\sqrt{c+dx}\left(93a^2d^2D-6abd(15Cd-14cD)-\left(b^2\left(-80Bd^2-63c^2D+70cCd\right)\right)\right)}{3d}-\frac{5\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\cdots\right.}{\sqrt{b}d^{3/2}}\right.}{4d}\right.}{\cdots}}{8d}}{\cdots}$$

$$\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^3d}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(D*(a + b*x)^(9/2)*Sqrt[c + d*x])/(5*b^3*d) + (((10*b*C*d - 9*b*c*D - 21*a
*d*D)*(a + b*x)^(7/2)*Sqrt[c + d*x])/(4*d) + (((93*a^2*d^2*D - 6*a*b*d*(15
*C*d - 14*c*D) - b^2*(70*c*C*d - 80*B*d^2 - 63*c^2*D))*(a + b*x)^(5/2)*Sqr
t[c + d*x])/(3*d) - (5*(3*a^3*d^3*D - 3*a^2*b*d^2*(2*C*d - 3*c*D) - a*b^2*
d*(20*c*C*d - 16*B*d^2 - 21*c^2*D) - b^3*(70*c^2*C*d - 80*B*c*d^2 + 96*A*d
^3 - 63*c^3*D))*(((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((
Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d))/(8*d))/(1
0*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. 2(472) = 944.

Time = 0.54 (sec) , antiderivative size = 1998, normalized size of antiderivative = 3.87

method	result	size
default	Expression too large to display	1998

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/3840*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(1440*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*d^5+1440*A*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^2*
d^3-300*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c*d^3+2900*C*(d*b)^(
1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^2*d^2+120*D*(d*b)^(1/2)*((b*x+a)*(d*x
+c))^(1/2)*a^3*b*c*d^3+228*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c
^2*d^2-1600*B*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^4*c*d^3*x+120*C*(d*b)^
(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*d^4*x+1400*C*(d*b)^(1/2)*((b*x+a)*(d
*x+c))^(1/2)*b^4*c^2*d^2*x+768*D*b^4*d^4*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+960*C*b^4*d^4*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1280*B*b^4*d^
4*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-1840*C*(d*b)^(1/2)*((b*x+a)*(d*x
+c))^(1/2)*a*b^3*c*d^3*x-84*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*
c*d^3*x+1596*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^2*d^2*x-2520*D*
(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^3*d-1248*D*a*b^3*c*d^3*x^2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-3520*B*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2
)*a*b^3*c*d^3-720*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+
a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c*d^4+2160*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d^3+120*C*ln(1/2*(2
*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2
*c*d^4+540*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+...
```

Fricas [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 1274, normalized size of antiderivative = 2.47

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fr
icas")
```

output
```
[-1/7680*(15*(63*D*b^5*c^5 - 35*(3*D*a*b^4 + 2*C*b^5)*c^4*d + 10*(3*D*a^2*
b^3 + 12*C*a*b^4 + 8*B*b^5)*c^3*d^2 + 6*(D*a^3*b^2 - 6*C*a^2*b^3 - 24*B*a*
b^4 - 16*A*b^5)*c^2*d^3 + (3*D*a^4*b - 8*C*a^3*b^2 + 48*B*a^2*b^3 + 192*A*
a*b^4)*c*d^4 + (3*D*a^5 - 6*C*a^4*b + 16*B*a^3*b^2 - 96*A*a^2*b^3)*d^5)*sq
rt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b
*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x)
 - 4*(384*D*b^5*d^5*x^4 + 945*D*b^5*c^4*d - 210*(6*D*a*b^4 + 5*C*b^5)*c^3*
d^2 + 2*(57*D*a^2*b^3 + 725*C*a*b^4 + 600*B*b^5)*c^2*d^3 + 10*(6*D*a^3*b^2
 - 15*C*a^2*b^3 - 176*B*a*b^4 - 144*A*b^5)*c*d^4 + 15*(3*D*a^4*b - 6*C*a^3
*b^2 + 16*B*a^2*b^3 + 160*A*a*b^4)*d^5 - 48*(9*D*b^5*c*d^4 - (11*D*a*b^4 +
 10*C*b^5)*d^5)*x^3 + 8*(63*D*b^5*c^2*d^3 - 2*(39*D*a*b^4 + 35*C*b^5)*c*d^
4 + (3*D*a^2*b^3 + 90*C*a*b^4 + 80*B*b^5)*d^5)*x^2 - 2*(315*D*b^5*c^3*d^2
- 7*(57*D*a*b^4 + 50*C*b^5)*c^2*d^3 + (21*D*a^2*b^3 + 460*C*a*b^4 + 400*B*
b^5)*c*d^4 + 5*(3*D*a^3*b^2 - 6*C*a^2*b^3 - 112*B*a*b^4 - 96*A*b^5)*d^5)*x
)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^6), 1/3840*(15*(63*D*b^5*c^5 - 35*(3
*D*a*b^4 + 2*C*b^5)*c^4*d + 10*(3*D*a^2*b^3 + 12*C*a*b^4 + 8*B*b^5)*c^3*d^
2 + 6*(D*a^3*b^2 - 6*C*a^2*b^3 - 24*B*a*b^4 - 16*A*b^5)*c^2*d^3 + (3*D*a^4
*b - 8*C*a^3*b^2 + 48*B*a^2*b^3 + 192*A*a*b^4)*c*d^4 + (3*D*a^5 - 6*C*a^4*
b + 16*B*a^3*b^2 - 96*A*a^2*b^3)*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c
 + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +...
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/sqrt(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="gi
ac")
```

output
```
1/1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(
8*(b*x + a)*D/(b^4*d) - (9*D*b^13*c*d^7 + 21*D*a*b^12*d^8 - 10*C*b^13*d^8)
/(b^16*d^9)) + (63*D*b^14*c^2*d^6 + 84*D*a*b^13*c*d^7 - 70*C*b^14*c*d^7 +
93*D*a^2*b^12*d^8 - 90*C*a*b^13*d^8 + 80*B*b^14*d^8)/(b^16*d^9)) - 5*(63*D
*b^15*c^3*d^5 + 21*D*a*b^14*c^2*d^6 - 70*C*b^15*c^2*d^6 + 9*D*a^2*b^13*c*d
^7 - 20*C*a*b^14*c*d^7 + 80*B*b^15*c*d^7 + 3*D*a^3*b^12*d^8 - 6*C*a^2*b^13
*d^8 + 16*B*a*b^14*d^8 - 96*A*b^15*d^8)/(b^16*d^9))*(b*x + a) + 15*(63*D*b
^16*c^4*d^4 - 42*D*a*b^15*c^3*d^5 - 70*C*b^16*c^3*d^5 - 12*D*a^2*b^14*c^2*
d^6 + 50*C*a*b^15*c^2*d^6 + 80*B*b^16*c^2*d^6 - 6*D*a^3*b^13*c*d^7 + 14*C*
a^2*b^14*c*d^7 - 64*B*a*b^15*c*d^7 - 96*A*b^16*c*d^7 - 3*D*a^4*b^12*d^8 +
6*C*a^3*b^13*d^8 - 16*B*a^2*b^14*d^8 + 96*A*a*b^15*d^8)/(b^16*d^9))*sqrt(b
*x + a) + 15*(63*D*b^5*c^5 - 105*D*a*b^4*c^4*d - 70*C*b^5*c^4*d + 30*D*a^2
*b^3*c^3*d^2 + 120*C*a*b^4*c^3*d^2 + 80*B*b^5*c^3*d^2 + 6*D*a^3*b^2*c^2*d^
3 - 36*C*a^2*b^3*c^2*d^3 - 144*B*a*b^4*c^2*d^3 - 96*A*b^5*c^2*d^3 + 3*D*a^
4*b*c*d^4 - 8*C*a^3*b^2*c*d^4 + 48*B*a^2*b^3*c*d^4 + 192*A*a*b^4*c*d^4 + 3
*D*a^5*d^5 - 6*C*a^4*b*d^5 + 16*B*a^3*b^2*d^5 - 96*A*a^2*b^3*d^5)*log(abs(
-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d
)*b^3*d^5))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 974, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
(45*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b*d**5 - 30*sqrt(c + d*x)*sqrt(a + b*
x)*a**3*b**2*c*d**4 - 30*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**2*d**5*x + 26
40*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*d**4 - 36*sqrt(c + d*x)*sqrt(a +
b*x)*a**2*b**3*c**2*d**3 + 18*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**3*c*d**4
*x + 24*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**3*d**5*x**2 - 3200*sqrt(c + d*
x)*sqrt(a + b*x)*a*b**5*c*d**3 + 2080*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*d
**4*x + 190*sqrt(c + d*x)*sqrt(a + b*x)*a*b**4*c**3*d**2 - 122*sqrt(c + d*
x)*sqrt(a + b*x)*a*b**4*c**2*d**3*x + 96*sqrt(c + d*x)*sqrt(a + b*x)*a*b**
4*c*d**4*x**2 + 528*sqrt(c + d*x)*sqrt(a + b*x)*a*b**4*d**5*x**3 + 1200*sq
rt(c + d*x)*sqrt(a + b*x)*b**6*c**2*d**2 - 800*sqrt(c + d*x)*sqrt(a + b*x)
*b**6*c*d**3*x + 640*sqrt(c + d*x)*sqrt(a + b*x)*b**6*d**4*x**2 - 105*sqrt
(c + d*x)*sqrt(a + b*x)*b**5*c**4*d + 70*sqrt(c + d*x)*sqrt(a + b*x)*b**5*
c**3*d**2*x - 56*sqrt(c + d*x)*sqrt(a + b*x)*b**5*c**2*d**3*x**2 + 48*sqrt
(c + d*x)*sqrt(a + b*x)*b**5*c*d**4*x**3 + 384*sqrt(c + d*x)*sqrt(a + b*x)
*b**5*d**5*x**4 - 45*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*
sqrt(c + d*x))/sqrt(a*d - b*c))*a**5*d**5 + 45*sqrt(d)*sqrt(b)*log((sqrt(d
)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**4*b*c*d**4 +
1200*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/s
qrt(a*d - b*c))*a**3*b**3*d**4 + 30*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a +
b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**3*b**2*c**2*d**3 - 36...
```

3.113 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$

Optimal result 1047
Mathematica [C] (verified) 1048
Rubi [A] (verified) 1048
Maple [B] (verified) 1052
Fricas [A] (verification not implemented) 1053
Sympy [F] 1054
Maxima [F(-2)] 1055
Giac [A] (verification not implemented) 1055
Mupad [F(-1)] 1056
Reduce [F] 1057

Optimal result

Integrand size = 34, antiderivative size = 439

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{3/2}}{d^4\sqrt{c+dx}}$$
$$+\frac{(3a^3d^3D-a^2bd^2(8Cd-15cD)-ab^2d(80cCd-48Bd^2-105c^2D)+b^3(280c^2Cd-240Bcd^2+192Ad^3-}{64b^2d^5}$$
$$-\frac{\left(88bcC-48bBd+8aCd-18acD-\frac{123bc^2D}{d}-\frac{3a^2dD}{b}\right)(a+bx)^{3/2}\sqrt{c+dx}}{96bd^3}$$
$$+\frac{(8bCd-21bcD-3adD)(a+bx)^{5/2}\sqrt{c+dx}}{24b^2d^3}+\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4bd^3}$$
$$-\frac{(bc-ad)\,(3a^3d^3D-a^2bd^2(8Cd-15cD)-ab^2d(80cCd-48Bd^2-105c^2D)+b^3(280c^2Cd-240Bcd^2-}{64b^{5/2}d^{11/2}}$$

output
```
-2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(3/2)/d^4/(d*x+c)^(1/2)+1/64*(3*a
^3*d^3*D-a^2*b*d^2*(8*C*d-15*D*c)-a*b^2*d*(-48*B*d^2+80*C*c*d-105*D*c^2)+b
^3*(192*A*d^3-240*B*c*d^2+280*C*c^2*d-315*D*c^3))*(b*x+a)^(1/2)*(d*x+c)^(1
/2)/b^2/d^5-1/96*(88*C*b*c-48*B*b*d+8*C*a*d-18*D*a*c-123*b*c^2*D/d-3*a^2*d
*D/b)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b/d^3+1/24*(8*C*b*d-3*D*a*d-21*D*b*c)*(b
*x+a)^(5/2)*(d*x+c)^(1/2)/b^2/d^3+1/4*D*(b*x+a)^(5/2)*(d*x+c)^(3/2)/b/d^3-
1/64*(-a*d+b*c)*(3*a^3*d^3*D-a^2*b*d^2*(8*C*d-15*D*c)-a*b^2*d*(-48*B*d^2+8
0*C*c*d-105*D*c^2)+b^3*(192*A*d^3-240*B*c*d^2+280*C*c^2*d-315*D*c^3))*arct
anh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(11/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 15.09 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \frac{\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}\left(240b^5(bc-ad)^{3/2}\,(2cCd-Bd^2-3c^2D)\left(3b^2d(\right.\right.}{}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(((b*(c + d*x))/(b*c - a*d))^(3/2)*(240*b^5*(b*c - a*d)^(3/2)*(2*c*C*d - B
*d^2 - 3*c^2*D)*(3*b^2*d*(b*c - a*d)^(3/2)*(a + b*x)*Sqrt[(b*(c + d*x))/(b
*c - a*d)] - 2*b^2*d^2*Sqrt[b*c - a*d]*(a + b*x)^2*Sqrt[(b*(c + d*x))/(b*c
 - a*d)] - 3*b^2*Sqrt[d]*(b*c - a*d)^2*Sqrt[a + b*x]*ArcSinh[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]]) + 15*b^5*Sqrt[d]*D*(-3*Sqrt[d]*(b*c - a*d)^5*
(a + b*x)*Sqrt[(b*(c + d*x))/(b*c - a*d)] + 2*d^(3/2)*(b*c - a*d)^4*(a + b
*x)^2*Sqrt[(b*(c + d*x))/(b*c - a*d)] + 8*d^(5/2)*(b*c - a*d)^2*(a + b*x)^
3*Sqrt[(b*(c + d*x))/(b*c - a*d)]*(3*b*c - a*d + 2*b*d*x) + 3*(b*c - a*d)^
(11/2)*Sqrt[a + b*x]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]) + 4
0*b^3*(b*c - a*d)^(3/2)*(C*d - 3*c*D)*(8*b^3*d^3*Sqrt[b*c - a*d]*(a + b*x)
^3*Sqrt[(b*(c + d*x))/(b*c - a*d)] - b*(b*c - a*d)*(3*b^2*d*(b*c - a*d)^(3
/2)*(a + b*x)*Sqrt[(b*(c + d*x))/(b*c - a*d)] - 2*b^2*d^2*Sqrt[b*c - a*d]*
(a + b*x)^2*Sqrt[(b*(c + d*x))/(b*c - a*d)] - 3*b^2*Sqrt[d]*(b*c - a*d)^2*
Sqrt[a + b*x]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])) + 384*b^8
*d^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x)^3*Hypergeometric2F1[3/2
, 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)]))/(960*b^9*d^6*Sqrt[a + b*x]*(c
+ d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2124, 27, 1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$\frac{2\int \frac{(a+bx)^{3/2}\left(-\left(\left(a-\frac{bc}{d}\right)Dx^2\right)+\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c+4Ad^3\right)}{d^3}\right)}{2\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)^{3/2}\left(-\left(\left(a-\frac{bc}{d}\right)Dx^2\right)+\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c+4Ad^3\right)}{d^3}\right)}{\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 1194$$

$$\frac{\frac{\int \frac{(a+bx)^{3/2}\left(-8\left(-5Dc^3+5Cdc^2-5Bd^2c+4Ad^3\right)b^3+ad\left(-15Dc^2+8Cdc-8Bd^2\right)b^2+6a^2cd^2Db+d(bc-ad)(8bCd-9aDd-15bcD)xb+a^3d^3D\right)}{2d^2\sqrt{c+dx}}dx}{4b^2d}+\frac{D(a+bx)^{7}\cdots}{\cdots}}{bc-ad}+\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(a+bx)^{3/2}\left(-8\left(-5Dc^3+5Cdc^2-5Bd^2c+4Ad^3\right)b^3+ad\left(-15Dc^2+8Cdc-8Bd^2\right)b^2+6a^2cd^2Db+d(bc-ad)(8bCd-9aDd-15bcD)xb+a^3d^3D\right)}{\sqrt{c+dx}}dx}{8b^2d^3}+\frac{D(a+bx)^{7}\cdots}{\cdots}}{bc-ad}+\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{\frac{1}{3}(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)(-9adD-15bcD+8bCd)-\frac{1}{6}\left(3a^3d^3D-a^2bd^2(8Cd-15cD)-ab^2d\left(-48Bd^2-105c^2D+80cCd\right)+b^3\left(192Ad^3-240Bcd^2\cdots\right.\right.}{8b^2d^3}\cdots}{bc-ad}+\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\frac{1}{3}(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)(-9adD-15bcD+8bCd)-\frac{1}{6}\left(3a^3d^3D-a^2bd^2(8Cd-15cD)-ab^2d\left(-48Bd^2-105c^2D+80cCd\right)+b^3\left(192Ad^3-240Bcd^2\right.\right.}{8b^2d^3}}{bc-ad}$$

$$\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\frac{1}{3}(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)(-9adD-15bcD+8bCd)-\frac{1}{6}\left(3a^3d^3D-a^2bd^2(8Cd-15cD)-ab^2d\left(-48Bd^2-105c^2D+80cCd\right)+b^3\left(192Ad^3-240Bcd^2\right.\right.}{8b^2d^3}}{bc-ad}$$

$$\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{\frac{1}{3}(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)(-9adD-15bcD+8bCd)-\frac{1}{6}\left(3a^3d^3D-a^2bd^2(8Cd-15cD)-ab^2d\left(-48Bd^2-105c^2D+80cCd\right)+b^3\left(192Ad^3-240Bcd^2\right.\right.}{8b^2d^3}}{bc-ad}$$

$$\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{1}{3}(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)(-9adD-15bcD+8bCd)-\frac{1}{6}\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)\left(3a^3d^3\right.}{8b^2d^3}}{bc-ad}$$

$$\frac{2(a+bx)^{5/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x)^(5/2))/(d^3*(b*c - a*d)*S
qrt[c + d*x]) + (((b*c - a*d)*D*(a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b^2*d^2)
 + (((b*c - a*d)*(8*b*C*d - 15*b*c*D - 9*a*d*D)*(a + b*x)^(5/2)*Sqrt[c + d
*x])/3 - ((3*a^3*d^3*D - a^2*b*d^2*(8*C*d - 15*c*D) - a*b^2*d*(80*c*C*d -
48*B*d^2 - 105*c^2*D) + b^3*(280*c^2*C*d - 240*B*c*d^2 + 192*A*d^3 - 315*c
^3*D))*(((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a +
b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt
[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*d)))/6)/(8*b^2*d^3))/(b*c - a*
d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2427 vs. $2(397) = 794$.

Time = 0.50 (sec) , antiderivative size = 2428, normalized size of antiderivative = 5.53

method	result	size
default	Expression too large to display	2428

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/384*(b*x+a)^(1/2)*(144*D*a*b^2*d^4*x^3*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/
2)-144*D*b^3*c*d^3*x^3*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+224*C*a*b^2*d^4
*x^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-224*C*b^3*c*d^3*x^2*(d*b)^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)+270*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c^2*d^3*x-1260*D*ln(1/2*(2*b*d*x+2*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^3*d^2*x+
12*D*a^2*b*d^4*x^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+252*D*b^3*c^2*d^2*x
^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+480*B*a*b^2*d^4*x*(d*b)^(1/2)*((b*x
+a)*(d*x+c))^(1/2)-480*B*b^3*c*d^3*x*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-1
890*D*b^3*c^4*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+9*D*ln(1/2*(2*b*d*x+2*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*d^5*x-576*A*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*
b^4*c^2*d^3+720*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*
d+b*c)/(d*b)^(1/2))*b^4*c^3*d^2-840*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^
(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^4*d+9*D*ln(1/2*(2*b*d*x+2*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*c*d^4+1890*D*a
*b^2*c^3*d*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-864*B*ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c*d^4*x-216*C
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2
))*a^2*b^2*c*d^4*x+1080*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*...
```

Fricas [A] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 1424, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="fr
icas")
```

output
```
[1/768*(3*(315*D*b^4*c^5 - 140*(3*D*a*b^3 + 2*C*b^4)*c^4*d + 30*(3*D*a^2*b
^2 + 12*C*a*b^3 + 8*B*b^4)*c^3*d^2 + 12*(D*a^3*b - 6*C*a^2*b^2 - 24*B*a*b^
3 - 16*A*b^4)*c^2*d^3 + (3*D*a^4 - 8*C*a^3*b + 48*B*a^2*b^2 + 192*A*a*b^3)
*c*d^4 + (315*D*b^4*c^4*d - 140*(3*D*a*b^3 + 2*C*b^4)*c^3*d^2 + 30*(3*D*a^
2*b^2 + 12*C*a*b^3 + 8*B*b^4)*c^2*d^3 + 12*(D*a^3*b - 6*C*a^2*b^2 - 24*B*a
*b^3 - 16*A*b^4)*c*d^4 + (3*D*a^4 - 8*C*a^3*b + 48*B*a^2*b^2 + 192*A*a*b^3
)*d^5)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d +
 a*b*d^2)*x) + 4*(48*D*b^4*d^5*x^4 - 945*D*b^4*c^4*d - 384*A*a*b^3*d^5 + 1
05*(9*D*a*b^3 + 8*C*b^4)*c^3*d^2 - (39*D*a^2*b^2 + 800*C*a*b^3 + 720*B*b^4
)*c^2*d^3 - 3*(3*D*a^3*b - 8*C*a^2*b^2 - 208*B*a*b^3 - 192*A*b^4)*c*d^4 -
8*(9*D*b^4*c*d^4 - (9*D*a*b^3 + 8*C*b^4)*d^5)*x^3 + 2*(63*D*b^4*c^2*d^3 -
2*(33*D*a*b^3 + 28*C*b^4)*c*d^4 + (3*D*a^2*b^2 + 56*C*a*b^3 + 48*B*b^4)*d^
5)*x^2 - (315*D*b^4*c^3*d^2 - 7*(51*D*a*b^3 + 40*C*b^4)*c^2*d^3 + (33*D*a^
2*b^2 + 304*C*a*b^3 + 240*B*b^4)*c*d^4 + 3*(3*D*a^3*b - 8*C*a^2*b^2 - 80*B
*a*b^3 - 64*A*b^4)*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^7*x + b^3*c
*d^6), -1/384*(3*(315*D*b^4*c^5 - 140*(3*D*a*b^3 + 2*C*b^4)*c^4*d + 30*(3*
D*a^2*b^2 + 12*C*a*b^3 + 8*B*b^4)*c^3*d^2 + 12*(D*a^3*b - 6*C*a^2*b^2 - 24
*B*a*b^3 - 16*A*b^4)*c^2*d^3 + (3*D*a^4 - 8*C*a^3*b + 48*B*a^2*b^2 + 192*A
*a*b^3)*c*d^4 + (315*D*b^4*c^4*d - 140*(3*D*a*b^3 + 2*C*b^4)*c^3*d^2 + ...
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 785, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="gi
ac")
```

output
```
1/192*((2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)*D*abs(b)/(b^3*d) - (9*D*b^6*
c*d^7*abs(b) + 15*D*a*b^5*d^8*abs(b) - 8*C*b^6*d^8*abs(b))/(b^8*d^9)) + (6
3*D*b^7*c^2*d^6*abs(b) + 42*D*a*b^6*c*d^7*abs(b) - 56*C*b^7*c*d^7*abs(b) +
 39*D*a^2*b^5*d^8*abs(b) - 40*C*a*b^6*d^8*abs(b) + 48*B*b^7*d^8*abs(b))/(b
^8*d^9)) - (315*D*b^8*c^3*d^5*abs(b) - 105*D*a*b^7*c^2*d^6*abs(b) - 280*C*
b^8*c^2*d^6*abs(b) - 15*D*a^2*b^6*c*d^7*abs(b) + 80*C*a*b^7*c*d^7*abs(b) +
 240*B*b^8*c*d^7*abs(b) - 3*D*a^3*b^5*d^8*abs(b) + 8*C*a^2*b^6*d^8*abs(b)
- 48*B*a*b^7*d^8*abs(b) - 192*A*b^8*d^8*abs(b))/(b^8*d^9))*(b*x + a) - 3*(
315*D*b^9*c^4*d^4*abs(b) - 420*D*a*b^8*c^3*d^5*abs(b) - 280*C*b^9*c^3*d^5*
abs(b) + 90*D*a^2*b^7*c^2*d^6*abs(b) + 360*C*a*b^8*c^2*d^6*abs(b) + 240*B*
b^9*c^2*d^6*abs(b) + 12*D*a^3*b^6*c*d^7*abs(b) - 72*C*a^2*b^7*c*d^7*abs(b)
 - 288*B*a*b^8*c*d^7*abs(b) - 192*A*b^9*c*d^7*abs(b) + 3*D*a^4*b^5*d^8*abs
(b) - 8*C*a^3*b^6*d^8*abs(b) + 48*B*a^2*b^7*d^8*abs(b) + 192*A*a*b^8*d^8*a
bs(b))/(b^8*d^9))*sqrt(b*x + a)/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) - 1/64
*(315*D*b^4*c^4*abs(b) - 420*D*a*b^3*c^3*d*abs(b) - 280*C*b^4*c^3*d*abs(b)
 + 90*D*a^2*b^2*c^2*d^2*abs(b) + 360*C*a*b^3*c^2*d^2*abs(b) + 240*B*b^4*c^
2*d^2*abs(b) + 12*D*a^3*b*c*d^3*abs(b) - 72*C*a^2*b^2*c*d^3*abs(b) - 288*B
*a*b^3*c*d^3*abs(b) - 192*A*b^4*c*d^3*abs(b) + 3*D*a^4*d^4*abs(b) - 8*C*a^
3*b*d^4*abs(b) + 48*B*a^2*b^2*d^4*abs(b) + 192*A*a*b^3*d^4*abs(b))*log(abs
(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx=\int \frac{(a+b\,x)^{3/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

3.114 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$

Optimal result . . . 1058
Mathematica [A] (verified) . . . 1059
Rubi [A] (verified) . . . 1059
Maple [B] (verified) . . . 1063
Fricas [A] (verification not implemented) . . . 1064
Sympy [F] . . . 1065
Maxima [F(-2)] . . . 1066
Giac [B] (verification not implemented) . . . 1066
Mupad [F(-1)] . . . 1067
Reduce [F] . . . 1068

Optimal result

Integrand size = 34, antiderivative size = 399

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx)^{3/2}}{3d^4(c+dx)^{3/2}}$$
$$+\frac{2(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\sqrt{a+bx}}{d^5\sqrt{c+dx}}$$
$$-\frac{(a^2d^2D-2abd(3Cd-8cD)+b^2(22cCd-8Bd^2-41c^2D))\sqrt{a+bx}\sqrt{c+dx}}{8bd^5}$$
$$+\frac{(6bCd-17bcD-adD)(a+bx)^{3/2}\sqrt{c+dx}}{12bd^4}+\frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3bd^3}$$
$$-\frac{(a^3d^3D-3a^2bd^2(2Cd-5cD)+3ab^2d(20cCd-8Bd^2-35c^2D)-b^3(70c^2Cd-40Bcd^2+16Ad^3-105}{8b^{3/2}d^{11/2}}$$

output
```
-2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(3/2)/d^4/(d*x+c)^(3/2)+2*(a*d*
(-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(b*x+a)^(1
/2)/d^5/(d*x+c)^(1/2)-1/8*(a^2*d^2*D-2*a*b*d*(3*C*d-8*D*c)+b^2*(-8*B*d^2+2
2*C*c*d-41*D*c^2))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/d^5+1/12*(6*C*b*d-D*a*d-1
7*D*b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b/d^4+1/3*D*(b*x+a)^(5/2)*(d*x+c)^(1/
2)/b/d^3-1/8*(a^3*d^3*D-3*a^2*b*d^2*(2*C*d-5*D*c)+3*a*b^2*d*(-8*B*d^2+20*C
*c*d-35*D*c^2)-b^3*(16*A*d^3-40*B*c*d^2+70*C*c^2*d-105*D*c^3))*arctanh(d^(
1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(3/2)/d^(11/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \frac{\sqrt{a+bx}(3a^2d^2D(c+dx)^2-2abd(105c^3D+c^2(-55Cd+147\ldots}{} - \frac{(a^3d^3D+3a^2bd^2(-2Cd+5cD)-3ab^2d(-20cCd+8Bd^2+35c^2D)+b^3(-70c^2Cd+40Bcd^2-16Ad^3\ldots}{8b^{3/2}d^{11/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(Sqrt[a + b*x]*(3*a^2*d^2*D*(c + d*x)^2 - 2*a*b*d*(105*c^3*D + c^2*(-55*C*
d + 147*d*D*x) + c*d^2*(16*B - 78*C*x + 27*D*x^2) + d^3*(8*A + 24*B*x - 15
*C*x^2 - 7*D*x^3)) + b^2*(315*c^4*D - 210*c^3*d*(C - 2*D*x) + c^2*d^2*(120
*B + 7*x*(-40*C + 9*D*x)) + 4*d^4*x*(-16*A + x*(6*B + 3*C*x + 2*D*x^2)) -
2*c*d^3*(24*A + x*(-80*B + 21*C*x + 9*D*x^2)))))/(24*b*d^5*(c + d*x)^(3/2)
) - ((a^3*d^3*D + 3*a^2*b*d^2*(-2*C*d + 5*c*D) - 3*a*b^2*d*(-20*c*C*d + 8*
B*d^2 + 35*c^2*D) + b^3*(-70*c^2*C*d + 40*B*c*d^2 - 16*A*d^3 + 105*c^3*D))
*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*b^(3/2)*d^(1
1/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2124, 27, 1193, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$\frac{2\int \frac{(a+bx)^{3/2}\left(-3\left(a-\frac{bc}{d}\right)Dx^2+\frac{3(bc-ad)(Cd-cD)x}{d^2}+\frac{3ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c+2Ad^3\right)}{d^3}\right)}{2(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+bx)^{3/2}\left(-3\left(a-\frac{bc}{d}\right)Dx^2+\frac{3(bc-ad)(Cd-cD)x}{d^2}+\frac{3ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c+2Ad^3\right)}{d^3}\right)}{(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$\frac{\frac{2\int \frac{(a+bx)^{3/2}\left(\left(-50Dc^3+35Cdc^2-20Bd^2c+8Ad^3\right)b^2-6ad\left(-8Dc^2+5Cdc-2Bd^2\right)b+3a^2d^2(Cd-2cD)+3d(bc-ad)^2Dx\right)}{2d^3\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{5/2}(3ad(-Bd^2-3c}{}}{3(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(a+bx)^{3/2}\left(\left(-50Dc^3+35Cdc^2-20Bd^2c+8Ad^3\right)b^2-6ad\left(-8Dc^2+5Cdc-2Bd^2\right)b+3a^2d^2(Cd-2cD)+3d(bc-ad)^2Dx\right)}{\sqrt{c+dx}}dx}{d^3(bc-ad)}+\frac{2(a+bx)^{5/2}(3ad(-Bd^2-3c^2}{}}{3(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 90

$$\frac{\frac{\frac{D(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)^2}{b}-\frac{\left(a^3d^3D-3a^2bd^2(2Cd-5cD)+3ab^2d\left(-8Bd^2-35c^2D+20cCd\right)-\left(b^3\left(16Ad^3-40Bcd^2-105c^3D+70c^2Cd\right)\right)\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}dx}{2b}}{d^3(bc-ad)}+}{3(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 60

$$\frac{\frac{\frac{D(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)^2}{b}-\frac{\left(a^3d^3D-3a^2bd^2(2Cd-5cD)+3ab^2d\left(-8Bd^2-35c^2D+20cCd\right)-\left(b^3\left(16Ad^3-40Bcd^2-105c^3D+70c^2Cd\right)\right)\right)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}\cdots\right.}{2b}}{d^3(bc-ad)}}{3(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{D(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)^2}{b}-\frac{\left(a^3d^3D-3a^2bd^2(2Cd-5cD)+3ab^2d\left(-8Bd^2-35c^2D+20cCd\right)-\left(b^3\left(16Ad^3-40Bcd^2-105c^3D+70c^2Cd\right)\right)\right)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}\cdots\right.}{2b}}{d^3(bc-ad)}}{3(bc-a\cdots}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\downarrow$ 66

$$\frac{\frac{\frac{D(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)^2}{b}-\frac{\left(a^3d^3D-3a^2bd^2(2Cd-5cD)+3ab^2d\left(-8Bd^2-35c^2D+20cCd\right)-\left(b^3\left(16Ad^3-40Bcd^2-105c^3D+70c^2Cd\right)\right)\right)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}\cdots\right.}{2b}}{d^3(bc-ad)}}{3(bc-\cdots}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{\frac{D(a+bx)^{5/2}\sqrt{c+dx}(bc-ad)^2}{b}-\frac{\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)\left(a^3d^3D-3a^2bd^2(2Cd-5cD)+3ab^2d\left(-\cdots\right.\right.}{2b}}{d^3(bc-ad)}}{3(bc-\cdots}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(5/2))/(3*(b*c - a*d)*(
c + d*x)^(3/2)) + ((2*(3*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(8*c^2*C*d -
5*B*c*d^2 + 2*A*d^3 - 11*c^3*D))*(a + b*x)^(5/2))/(d^3*(b*c - a*d)*Sqrt[c
+ d*x]) + (((b*c - a*d)^2*D*(a + b*x)^(5/2)*Sqrt[c + d*x])/b - ((a^3*d^3*D
 - 3*a^2*b*d^2*(2*C*d - 5*c*D) + 3*a*b^2*d*(20*c*C*d - 8*B*d^2 - 35*c^2*D)
 - b^3*(70*c^2*C*d - 40*B*c*d^2 + 16*A*d^3 - 105*c^3*D))*(((a + b*x)^(3/2)
*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d -
((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sq
rt[b]*d^(3/2))))/(4*d)))/(2*b))/(d^3*(b*c - a*d)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2407 vs. $2(357) = 714$.

Time = 0.53 (sec) , antiderivative size = 2408, normalized size of antiderivative = 6.04

method	result	size
default	Expression too large to display	2408

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
1/48*(b*x+a)^(1/2)*(-6*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*c*d^4*x+18*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*
x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c^2*d^3-180*C*ln(1/2*(
2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*
c^3*d^2-45*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c
)/(d*b)^(1/2))*a^2*b*c^3*d^2+28*D*a*b*d^4*x^3*(d*b)^(1/2)*((b*x+a)*(d*x+c)
)^(1/2)-36*D*b^2*c*d^3*x^3*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-420*D*a*b*c
^3*d*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-180*C*ln(1/2*(2*b*d*x+2*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d^4*x^2-45*D*ln(1
/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^
2*b*c*d^4*x^2+315*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+
a*d+b*c)/(d*b)^(1/2))*a*b^2*c^2*d^3*x^2+144*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d^4*x+48*A*ln(1/2*
(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*d
^5*x^2+48*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)
/(d*b)^(1/2))*b^3*c^2*d^3+240*B*b^2*c^2*d^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))^
(1/2)-420*C*b^2*c^3*d*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+48*B*b^2*d^4*x^2
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+96*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c*d^4*x+6*D*a^2*c^2*d^2*(d
*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+72*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+...
```

Fricas [A] (verification not implemented)

Time = 16.85 (sec) , antiderivative size = 1400, normalized size of antiderivative = 3.51

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(105*D*b^3*c^5 - 35*(3*D*a*b^2 + 2*C*b^3)*c^4*d + 5*(3*D*a^2*b +
 12*C*a*b^2 + 8*B*b^3)*c^3*d^2 + (D*a^3 - 6*C*a^2*b - 24*B*a*b^2 - 16*A*b^
3)*c^2*d^3 + (105*D*b^3*c^3*d^2 - 35*(3*D*a*b^2 + 2*C*b^3)*c^2*d^3 + 5*(3*
D*a^2*b + 12*C*a*b^2 + 8*B*b^3)*c*d^4 + (D*a^3 - 6*C*a^2*b - 24*B*a*b^2 -
16*A*b^3)*d^5)*x^2 + 2*(105*D*b^3*c^4*d - 35*(3*D*a*b^2 + 2*C*b^3)*c^3*d^2
 + 5*(3*D*a^2*b + 12*C*a*b^2 + 8*B*b^3)*c^2*d^3 + (D*a^3 - 6*C*a^2*b - 24*
B*a*b^2 - 16*A*b^3)*c*d^4)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*
b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*D*b^3*d^5*x^4 + 315*D*b^3*c^4*d -
16*A*a*b^2*d^5 - 210*(D*a*b^2 + C*b^3)*c^3*d^2 + (3*D*a^2*b + 110*C*a*b^2
+ 120*B*b^3)*c^2*d^3 - 16*(2*B*a*b^2 + 3*A*b^3)*c*d^4 - 2*(9*D*b^3*c*d^4 -
 (7*D*a*b^2 + 6*C*b^3)*d^5)*x^3 + 3*(21*D*b^3*c^2*d^3 - 2*(9*D*a*b^2 + 7*C
*b^3)*c*d^4 + (D*a^2*b + 10*C*a*b^2 + 8*B*b^3)*d^5)*x^2 + 2*(210*D*b^3*c^3
*d^2 - 7*(21*D*a*b^2 + 20*C*b^3)*c^2*d^3 + (3*D*a^2*b + 78*C*a*b^2 + 80*B*
b^3)*c*d^4 - 8*(3*B*a*b^2 + 4*A*b^3)*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/
(b^2*d^8*x^2 + 2*b^2*c*d^7*x + b^2*c^2*d^6), 1/48*(3*(105*D*b^3*c^5 - 35*(
3*D*a*b^2 + 2*C*b^3)*c^4*d + 5*(3*D*a^2*b + 12*C*a*b^2 + 8*B*b^3)*c^3*d^2
+ (D*a^3 - 6*C*a^2*b - 24*B*a*b^2 - 16*A*b^3)*c^2*d^3 + (105*D*b^3*c^3*d^2
 - 35*(3*D*a*b^2 + 2*C*b^3)*c^2*d^3 + 5*(3*D*a^2*b + 12*C*a*b^2 + 8*B*b^3)
*c*d^4 + (D*a^3 - 6*C*a^2*b - 24*B*a*b^2 - 16*A*b^3)*d^5)*x^2 + 2*(105*...
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(356) = 712$.

Time = 0.31 (sec) , antiderivative size = 1006, normalized size of antiderivative = 2.52

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="gi
ac")
```

output
```
1/24*(((2*(b*x + a)*(4*(D*b^5*c*d^8*abs(b) - D*a*b^4*d^9*abs(b))*(b*x + a)
/(b^6*c*d^9 - a*b^5*d^10) - 3*(3*D*b^6*c^2*d^7*abs(b) - 2*C*b^6*c*d^8*abs(
b) - 3*D*a^2*b^4*d^9*abs(b) + 2*C*a*b^5*d^9*abs(b))/(b^6*c*d^9 - a*b^5*d^1
0)) + 3*(21*D*b^7*c^3*d^6*abs(b) - 21*D*a*b^6*c^2*d^7*abs(b) - 14*C*b^7*c^
2*d^7*abs(b) + 3*D*a^2*b^5*c*d^8*abs(b) + 12*C*a*b^6*c*d^8*abs(b) + 8*B*b^
7*c*d^8*abs(b) - 3*D*a^3*b^4*d^9*abs(b) + 2*C*a^2*b^5*d^9*abs(b) - 8*B*a*b
^6*d^9*abs(b))/(b^6*c*d^9 - a*b^5*d^10))*(b*x + a) + 4*(105*D*b^8*c^4*d^5*
abs(b) - 210*D*a*b^7*c^3*d^6*abs(b) - 70*C*b^8*c^3*d^6*abs(b) + 120*D*a^2*
b^6*c^2*d^7*abs(b) + 130*C*a*b^7*c^2*d^7*abs(b) + 40*B*b^8*c^2*d^7*abs(b)
- 14*D*a^3*b^5*c*d^8*abs(b) - 66*C*a^2*b^6*c*d^8*abs(b) - 64*B*a*b^7*c*d^8
*abs(b) - 16*A*b^8*c*d^8*abs(b) - D*a^4*b^4*d^9*abs(b) + 6*C*a^3*b^5*d^9*a
bs(b) + 24*B*a^2*b^6*d^9*abs(b) + 16*A*a*b^7*d^9*abs(b))/(b^6*c*d^9 - a*b^
5*d^10))*(b*x + a) + 3*(105*D*b^9*c^5*d^4*abs(b) - 315*D*a*b^8*c^4*d^5*abs
(b) - 70*C*b^9*c^4*d^5*abs(b) + 330*D*a^2*b^7*c^3*d^6*abs(b) + 200*C*a*b^8
*c^3*d^6*abs(b) + 40*B*b^9*c^3*d^6*abs(b) - 134*D*a^3*b^6*c^2*d^7*abs(b) -
 196*C*a^2*b^7*c^2*d^7*abs(b) - 104*B*a*b^8*c^2*d^7*abs(b) - 16*A*b^9*c^2*
d^7*abs(b) + 13*D*a^4*b^5*c*d^8*abs(b) + 72*C*a^3*b^6*c*d^8*abs(b) + 88*B*
a^2*b^7*c*d^8*abs(b) + 32*A*a*b^8*c*d^8*abs(b) + D*a^5*b^4*d^9*abs(b) - 6*
C*a^4*b^5*d^9*abs(b) - 24*B*a^3*b^6*d^9*abs(b) - 16*A*a^2*b^7*d^9*abs(b))/
(b^6*c*d^9 - a*b^5*d^10))*sqrt(b*x + a)/(b^2*c + (b*x + a)*b*d - a*b*d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{(a+b\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

3.115 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx$

Optimal result 1069
Mathematica [A] (verified) 1070
Rubi [A] (verified) 1070
Maple [B] (verified) 1074
Fricas [B] (verification not implemented) 1075
Sympy [F] 1076
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Giac [B] (verification not implemented) 1077
Mupad [F(-1)] 1078
Reduce [F] 1079

Optimal result

Integrand size = 34, antiderivative size = 338

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx)^{5/2}}{5d^3(bc-ad)(c+dx)^{5/2}} + \frac{2(2cCd-Bd^2-3c^2D)(a+bx)^{3/2}}{3d^4(c+dx)^{3/2}} - \frac{(a^2d^2D+2abd(2Cd-7cD)-b^2(12cCd-4Bd^2-25c^2D))\sqrt{a+bx}}{2bd^5\sqrt{c+dx}} + \frac{D(a+bx)^{5/2}}{2bd^3\sqrt{c+dx}} + \frac{(4bCd-13bcD+adD)\sqrt{a+bx}\sqrt{c+dx}}{4d^5} + \frac{(3a^2d^2D+6abd(2Cd-7cD)-b^2(28cCd-8Bd^2-63c^2D))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4\sqrt{b}d^{11/2}}$$

output
```
2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(5/2)/d^3/(-a*d+b*c)/(d*x+c)^(5/
2)+2/3*(-B*d^2+2*C*c*d-3*D*c^2)*(b*x+a)^(3/2)/d^4/(d*x+c)^(3/2)-1/2*(a^2*d
^2*D+2*a*b*d*(2*C*d-7*D*c)-b^2*(-4*B*d^2+12*C*c*d-25*D*c^2))*(b*x+a)^(1/2)
/b/d^5/(d*x+c)^(1/2)+1/2*D*(b*x+a)^(5/2)/b/d^3/(d*x+c)^(1/2)+1/4*(4*C*b*d+
D*a*d-13*D*b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^5+1/4*(3*a^2*d^2*D+6*a*b*d*(
2*C*d-7*D*c)-b^2*(-8*B*d^2+28*C*c*d-63*D*c^2))*arctanh(d^(1/2)*(b*x+a)^(1/
2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/d^(11/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^{3/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}} \, dx =$$
$$-\frac{\sqrt{a+bx}(b^2(-945c^5D+24Ad^5x^2+105c^4d(4C-21Dx)+c^3d^2(-120B+7x(140C-207Dx))+c^2d^3x(}{}$$
$$+\frac{(3a^2d^2D+6abd(2Cd-7cD)+b^2(-28cCd+8Bd^2+63c^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4\sqrt{b}d^{11/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
-1/60*(Sqrt[a + b*x]*(b^2*(-945*c^5*D + 24*A*d^5*x^2 + 105*c^4*d*(4*C - 21
*D*x) + c^3*d^2*(-120*B + 7*x*(140*C - 207*D*x)) + c^2*d^3*x*(-280*B + x*(
644*C - 135*D*x)) + 2*c*d^4*x^2*(-92*B + 15*x*(2*C + D*x))) + 2*a*b*d*(630
*c^4*D + c^3*(-230*C*d + 1491*d*D*x) + d^4*x*(24*A + 80*B*x - 15*x^2*(2*C
+ D*x)) + c*d^3*x*(96*B + 5*x*(-74*C + 21*D*x)) + c^2*d^2*(40*B + 3*x*(-18
2*C + 335*D*x))) + a^2*d^2*(-339*c^3*D + c^2*d*(64*C - 825*D*x) + c*d^2*(1
6*B + 5*x*(32*C - 117*D*x)) + d^3*(24*A + 5*x*(8*B + 3*x*(8*C - 5*D*x)))))
)/(d^5*(-(b*c) + a*d)*(c + d*x)^(5/2)) + ((3*a^2*d^2*D + 6*a*b*d*(2*C*d -
7*c*D) + b^2*(-28*c*C*d + 8*B*d^2 + 63*c^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*Sqrt[b]*d^(11/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2124, 27, 1193, 27, 87, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}} \, dx$$
$$\Big\downarrow \text{2124}$$

$$\frac{2\int -\frac{5(a+bx)^{3/2}\left(-\frac{(bc-ad)Dx^2}{d}-\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{(bc-ad)\left(-Dc^2+Cdc-Bd^2\right)}{d^3}\right)}{2(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\int \frac{(a+bx)^{3/2}\left(-\frac{(bc-ad)Dx^2}{d}-\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{(bc-ad)\left(-Dc^2+Cdc-Bd^2\right)}{d^3}\right)}{(c+dx)^{5/2}}dx}{bc-ad}$$

$$\Big\downarrow \ 1193$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\frac{2\int \frac{(bc-ad)(a+bx)^{3/2}\left(3ad(Cd-2cD)-b\left(-12Dc^2+7Cdc-2Bd^2\right)-3d(bc-ad)Dx\right)}{2d^3(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2(a+bx)^{5/2}(-Bd^2-3c^2D+2cCd)}{3d^3(c+dx)^{3/2}}}{bc-ad}$$

$$\Big\downarrow \ 27$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\frac{\int \frac{(a+bx)^{3/2}\left(3ad(Cd-2cD)-b\left(-12Dc^2+7Cdc-2Bd^2\right)-3d(bc-ad)Dx\right)}{(c+dx)^{3/2}}dx}{3d^3}+\frac{2(a+bx)^{5/2}(-Bd^2-3c^2D+2cCd)}{3d^3(c+dx)^{3/2}}}{bc-ad}$$

$$\Big\downarrow \ 87$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{5/2}\left(3ad(Cd-3cD)-b\left(-2Bd^2-15c^2D+7cCd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{\left(3a^2d^2D+6abd(2Cd-7cD)-\left(b^2\left(-8Bd^2-63c^2D+28cCd\right)\right)\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}dx}{bc-ad}}{3d^3}+\frac{2(a+bx)^{5/2}(-B}{3d^3(c-}}{bc-ad}$$

$$\Big\downarrow \ 60$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{5/2}\left(3ad(Cd-3cD)-b\left(-2Bd^2-15c^2D+7cCd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{\left(3a^2d^2D+6abd(2Cd-7cD)-\left(b^2\left(-8Bd^2-63c^2D+28cCd\right)\right)\right)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\int\frac{\sqrt{a+b}}{\sqrt{c+d}}}{4d}\right.}{bc-ad}}{3d^3}}{bc-ad}$$

$\downarrow$ 60

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{5/2}\left(3ad(Cd-3cD)-b\left(-2Bd^2-15c^2D+7cCd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{\left(3a^2d^2D+6abd(2Cd-7cD)-\left(b^2\left(-8Bd^2-63c^2D+28cCd\right)\right)\right)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\sqrt{a+b}\right.}{}\right.}{bc-ad}}{3d^3}}{bc-ad}$$

$\downarrow$ 66

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{5/2}\left(3ad(Cd-3cD)-b\left(-2Bd^2-15c^2D+7cCd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{\left(3a^2d^2D+6abd(2Cd-7cD)-\left(b^2\left(-8Bd^2-63c^2D+28cCd\right)\right)\right)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\sqrt{a+b}\right.}{}\right.}{bc-ad}}{3d^3}}{bc-ad}$$

$\downarrow$ 221

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{5/2}\left(3ad(Cd-3cD)-b\left(-2Bd^2-15c^2D+7cCd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)\left(3a^2d^2D+6\right.}{bc-ad}}{3d^3}}{bc-ad}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(5/2))/(5*(b*c - a*d)*(
c + d*x)^(5/2)) - ((2*(2*c*C*d - B*d^2 - 3*c^2*D)*(a + b*x)^(5/2))/(3*d^3*
(c + d*x)^(3/2)) + ((2*(3*a*d*(C*d - 3*c*D) - b*(7*c*C*d - 2*B*d^2 - 15*c^
2*D))*(a + b*x)^(5/2))/((b*c - a*d)*Sqrt[c + d*x]) - ((3*a^2*d^2*D + 6*a*b
*d*(2*C*d - 7*c*D) - b^2*(28*c*C*d - 8*B*d^2 - 63*c^2*D))*(((a + b*x)^(3/2
)*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d -
 ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(S
qrt[b]*d^(3/2))))/(4*d)))/(b*c - a*d))/(3*d^3))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3044 vs. $2(294) = 588$.

Time = 0.52 (sec) , antiderivative size = 3045, normalized size of antiderivative = 9.01

method	result	size
default	Expression too large to display	3045

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/120*(b*x+a)^(1/2)*(-4410*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^4*
d*x-920*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^3*d^2+2520*D*(d*b)^(1/
2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^4*d+675*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c*d^5*x^3+96*A*a*b*d^5*
x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-1890*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))
^(1/2)*b^2*c^5-270*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^2*d^3*x^3+2
025*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)
^(1/2))*a^2*b*c^2*d^4*x^2-4725*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^3*d^3*x^2+32*B*a^2*c*d^4*((b*x+
a)*(d*x+c))^(1/2)*(d*b)^(1/2)+48*A*b^2*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+240*C*a^2*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-45*D*ln(1/2
*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*
d^6*x^3+320*C*a^2*c*d^4*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-120*B*ln(1/2
*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^
2*c^3*d^3-180*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a^2*b*c^3*d^3+2835*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^5*d*x+420*D*(d*b)^(1/2)*((b
*x+a)*(d*x+c))^(1/2)*a*b*c*d^4*x^3-1480*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1
/2)*a*b*c*d^4*x^2+4020*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^2*d^3*x
^2-2184*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^2*d^3*x+5964*D*(d*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(292) = 584$.

Time = 49.95 (sec) , antiderivative size = 1884, normalized size of antiderivative = 5.57

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="fr
icas")
```

output
```
[1/240*(15*(63*D*b^3*c^6 - 7*(15*D*a*b^2 + 4*C*b^3)*c^5*d + (45*D*a^2*b +
40*C*a*b^2 + 8*B*b^3)*c^4*d^2 - (3*D*a^3 + 12*C*a^2*b + 8*B*a*b^2)*c^3*d^3
 + (63*D*b^3*c^3*d^3 - 7*(15*D*a*b^2 + 4*C*b^3)*c^2*d^4 + (45*D*a^2*b + 40
*C*a*b^2 + 8*B*b^3)*c*d^5 - (3*D*a^3 + 12*C*a^2*b + 8*B*a*b^2)*d^6)*x^3 +
3*(63*D*b^3*c^4*d^2 - 7*(15*D*a*b^2 + 4*C*b^3)*c^3*d^3 + (45*D*a^2*b + 40*
C*a*b^2 + 8*B*b^3)*c^2*d^4 - (3*D*a^3 + 12*C*a^2*b + 8*B*a*b^2)*c*d^5)*x^2
 + 3*(63*D*b^3*c^5*d - 7*(15*D*a*b^2 + 4*C*b^3)*c^4*d^2 + (45*D*a^2*b + 40
*C*a*b^2 + 8*B*b^3)*c^3*d^3 - (3*D*a^3 + 12*C*a^2*b + 8*B*a*b^2)*c^2*d^4)*
x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*
x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^
2)*x) - 4*(945*D*b^3*c^5*d - 16*B*a^2*b*c*d^5 - 24*A*a^2*b*d^6 - 420*(3*D*
a*b^2 + C*b^3)*c^4*d^2 + (339*D*a^2*b + 460*C*a*b^2 + 120*B*b^3)*c^3*d^3 -
 16*(4*C*a^2*b + 5*B*a*b^2)*c^2*d^4 - 30*(D*b^3*c*d^5 - D*a*b^2*d^6)*x^4 +
 15*(9*D*b^3*c^2*d^4 - 2*(7*D*a*b^2 + 2*C*b^3)*c*d^5 + (5*D*a^2*b + 4*C*a*
b^2)*d^6)*x^3 + (1449*D*b^3*c^3*d^3 - 2*(1005*D*a*b^2 + 322*C*b^3)*c^2*d^4
 + (585*D*a^2*b + 740*C*a*b^2 + 184*B*b^3)*c*d^5 - 8*(15*C*a^2*b + 20*B*a*
b^2 + 3*A*b^3)*d^6)*x^2 + (2205*D*b^3*c^4*d^2 - 14*(213*D*a*b^2 + 70*C*b^3
)*c^3*d^3 + (825*D*a^2*b + 1092*C*a*b^2 + 280*B*b^3)*c^2*d^4 - 32*(5*C*a^2
*b + 6*B*a*b^2)*c*d^5 - 8*(5*B*a^2*b + 6*A*a*b^2)*d^6)*x)*sqrt(b*x + a)*sq
rt(d*x + c))/(b^2*c^4*d^6 - a*b*c^3*d^7 + (b^2*c*d^9 - a*b*d^10)*x^3 + ...
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(a+bx)^{\frac{3}{2}}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(292) = 584$.

Time = 0.41 (sec) , antiderivative size = 1190, normalized size of antiderivative = 3.52

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="gi
ac")
```

output
```
1/60*(((15*(b*x + a)*(2*(D*b^9*c^2*d^8*abs(b) - 2*D*a*b^8*c*d^9*abs(b) + D
*a^2*b^7*d^10*abs(b))*(b*x + a)/(b^8*c^2*d^9 - 2*a*b^7*c*d^10 + a^2*b^6*d^
11) - (9*D*b^10*c^3*d^7*abs(b) - 15*D*a*b^9*c^2*d^8*abs(b) - 4*C*b^10*c^2*
d^8*abs(b) + 3*D*a^2*b^8*c*d^9*abs(b) + 8*C*a*b^9*c*d^9*abs(b) + 3*D*a^3*b
^7*d^10*abs(b) - 4*C*a^2*b^8*d^10*abs(b))/(b^8*c^2*d^9 - 2*a*b^7*c*d^10 +
a^2*b^6*d^11)) - (1449*D*b^11*c^4*d^6*abs(b) - 3864*D*a*b^10*c^3*d^7*abs(b
) - 644*C*b^11*c^3*d^7*abs(b) + 3450*D*a^2*b^9*c^2*d^8*abs(b) + 1564*C*a*b
^10*c^2*d^8*abs(b) + 184*B*b^11*c^2*d^8*abs(b) - 1080*D*a^3*b^8*c*d^9*abs(
b) - 1220*C*a^2*b^9*c*d^9*abs(b) - 344*B*a*b^10*c*d^9*abs(b) - 24*A*b^11*c
*d^9*abs(b) + 45*D*a^4*b^7*d^10*abs(b) + 300*C*a^3*b^8*d^10*abs(b) + 160*B
*a^2*b^9*d^10*abs(b) + 24*A*a*b^10*d^10*abs(b))/(b^8*c^2*d^9 - 2*a*b^7*c*d
^10 + a^2*b^6*d^11))*(b*x + a) - 35*(63*D*b^12*c^5*d^5*abs(b) - 231*D*a*b^
11*c^4*d^6*abs(b) - 28*C*b^12*c^4*d^6*abs(b) + 318*D*a^2*b^10*c^3*d^7*abs(
b) + 96*C*a*b^11*c^3*d^7*abs(b) + 8*B*b^12*c^3*d^7*abs(b) - 198*D*a^3*b^9*
c^2*d^8*abs(b) - 120*C*a^2*b^10*c^2*d^8*abs(b) - 24*B*a*b^11*c^2*d^8*abs(b
) + 51*D*a^4*b^8*c*d^9*abs(b) + 64*C*a^3*b^9*c*d^9*abs(b) + 24*B*a^2*b^10*
c*d^9*abs(b) - 3*D*a^5*b^7*d^10*abs(b) - 12*C*a^4*b^8*d^10*abs(b) - 8*B*a^
3*b^9*d^10*abs(b))/(b^8*c^2*d^9 - 2*a*b^7*c*d^10 + a^2*b^6*d^11))*(b*x + a
) - 15*(63*D*b^13*c^6*d^4*abs(b) - 294*D*a*b^12*c^5*d^5*abs(b) - 28*C*b^13
*c^5*d^5*abs(b) + 549*D*a^2*b^11*c^4*d^6*abs(b) + 124*C*a*b^12*c^4*d^6*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx=\int \frac{(a+b\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

output
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

3.116 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx$

Optimal result 1080
Mathematica [A] (verified) 1081
Rubi [A] (verified) 1081
Maple [B] (verified) 1086
Fricas [B] (verification not implemented) 1087
Sympy [F] 1088
Maxima [F(-2)] 1089
Giac [B] (verification not implemented) 1089
Mupad [F(-1)] 1090
Reduce [F] 1091

Optimal result

Integrand size = 34, antiderivative size = 300

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx)^{5/2}}{7d^3(bc-ad)(c+dx)^{7/2}} + \frac{2(7ad(2cCd-Bd^2-3c^2D)-b(12c^2Cd-5Bcd^2-2Ad^3-19c^3D))(a+bx)^{5/2}}{35d^3(bc-ad)^2(c+dx)^{5/2}} - \frac{2(Cd-3cD)(a+bx)^{3/2}}{3d^4(c+dx)^{3/2}} - \frac{2(bCd-4bcD+adD)\sqrt{a+bx}}{d^5\sqrt{c+dx}} + \frac{bD\sqrt{a+bx}\sqrt{c+dx}}{d^5} + \frac{\sqrt{b}(2bCd-9bcD+3adD)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{11/2}}$$

output
```
2/7*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(5/2)/d^3/(-a*d+b*c)/(d*x+c)^(7/
2)+2/35*(7*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-2*A*d^3-5*B*c*d^2+12*C*c^2*d-1
9*D*c^3))*(b*x+a)^(5/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(5/2)-2/3*(C*d-3*D*c)*(b*
x+a)^(3/2)/d^4/(d*x+c)^(3/2)-2*(C*b*d+D*a*d-4*D*b*c)*(b*x+a)^(1/2)/d^5/(d*
x+c)^(1/2)+b*D*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^5+b^(1/2)*(2*C*b*d+3*D*a*d-9*
D*b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(11/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx =$$
$$-\frac{\sqrt{a+bx}(-b^3(945c^6D+12Ad^6x^3+6cd^5x^2(7A+5Bx)-210c^5d(C-15Dx)+c^2d^4x^3(-352C+105Dx)}{}$$
$$+\frac{\sqrt{b}(2bCd-9bcD+3adD)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{d^{11/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(9/2),x]
```

output
```
-1/105*(Sqrt[a + b*x]*(-(b^3*(945*c^6*D + 12*A*d^6*x^3 + 6*c*d^5*x^2*(7*A
+ 5*B*x) - 210*c^5*d*(C - 15*D*x) + c^2*d^4*x^3*(-352*C + 105*D*x) + 14*c^
4*d^2*x*(-50*C + 261*D*x) + 4*c^3*d^3*x^2*(-203*C + 396*D*x))) + a^2*b*d^2
*(-1029*c^4*D + 4*c^3*d*(28*C - 873*D*x) + 6*c^2*d^2*x*(64*C - 693*D*x) -
6*c*d^3*(7*A + x*(B - 77*C*x + 315*D*x^2)) + d^4*x*(48*A + 7*x*(12*B + 5*x
*(8*C - 3*D*x)))) + 2*a^3*d^3*(48*c^3*D + 8*c^2*d*(C + 21*D*x) + 2*c*d^2*(
3*B + 7*x*(2*C + 15*D*x)) + d^3*(15*A + 7*x*(3*B + 5*x*(C + 3*D*x)))) + 2*
a*b^2*d*(945*c^5*D + 3*d^5*x^2*(A + 7*B*x) - 7*c^4*d*(25*C - 453*D*x) + 6*
c^2*d^3*x^2*(-115*C + 273*D*x) + 6*c^3*d^2*x*(-98*C + 619*D*x) + c*d^4*x*(
-42*A + x*(-24*B + 7*x*(-46*C + 15*D*x))))))/(d^5*(b*c - a*d)^2*(c + d*x)^
(7/2)) + (Sqrt[b]*(2*b*C*d - 9*b*c*D + 3*a*d*D)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x])/(Sqrt[d]*Sqrt[a + b*x])])/d^(11/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2124, 27, 1193, 27, 87, 57, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$\frac{2\int \frac{(a+bx)^{3/2}\left(-7\left(a-\frac{bc}{d}\right)Dx^2+\frac{7(bc-ad)(Cd-cD)x}{d^2}+\frac{7ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c-2Ad^3\right)}{d^3}\right)}{2(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+bx)^{3/2}\left(-7\left(a-\frac{bc}{d}\right)Dx^2+\frac{7(bc-ad)(Cd-cD)x}{d^2}+\frac{7ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c-2Ad^3\right)}{d^3}\right)}{(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \text{1193}$$

$$\frac{\frac{2\int \frac{35(bc-ad)^2(a+bx)^{3/2}(Cd+Dxd-2cD)}{2d^3(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2(a+bx)^{5/2}(7ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-5Bcd^2-19c^3D+12c^2Cd))}{5d^3(c+dx)^{5/2}(bc-ad)}}{7(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{7(bc-ad)\int \frac{(a+bx)^{3/2}(Cd+Dxd-2cD)}{(c+dx)^{5/2}}dx}{d^3}+\frac{2(a+bx)^{5/2}(7ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-5Bcd^2-19c^3D+12c^2Cd))}{5d^3(c+dx)^{5/2}(bc-ad)}}{7(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{7(bc-ad)\left(\frac{2(a+bx)^{5/2}(Cd-3cD)}{3(c+dx)^{3/2}(bc-ad)}-\frac{(3adD-9bcD+2bCd)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}dx}{3(bc-ad)}\right)}{d^3}+\frac{2(a+bx)^{5/2}(7ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-5Bcd^2-19c^3D+1}{5d^3(c+dx)^{5/2}(bc-ad)}}{7(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\downarrow$ 57

$$\frac{\frac{7(bc-ad)\left(\frac{2(a+bx)^{5/2}(Cd-3cD)}{3(c+dx)^{3/2}(bc-ad)}-\frac{(3adD-9bcD+2bCd)\left(\frac{3b\int\frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{d}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}\right)}{3(bc-ad)}\right)}{d^3}+\frac{2(a+bx)^{5/2}\left(7ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-5B\cdots\right.\right.}{5d^3(c+dx)^{5/2}(bc-ad)}}{7(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{7(bc-ad)\left(\frac{2(a+bx)^{5/2}(Cd-3cD)}{3(c+dx)^{3/2}(bc-ad)}-\frac{(3adD-9bcD+2bCd)\left(\frac{3b\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2d}\right)}{d}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}\right)}{3(bc-ad)}\right)}{d^3}+\frac{2(a+bx)^{5/2}\left(7ad\left(-B\cdots\right.\right.}{\cdots}}{7(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\downarrow$ 66

$$\frac{\frac{7(bc-ad)\left(\frac{2(a+bx)^{5/2}(Cd-3cD)}{3(c+dx)^{3/2}(bc-ad)}-\frac{(3adD-9bcD+2bCd)\left(\frac{3b\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}\right)}{d}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}\right)}{3(bc-ad)}\right)}{d^3}+\frac{2(a+bx)^{5/2}\left(7ad\left(\cdots\right.\right.}{\cdots}}{7(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{2(a+bx)^{5/2}(7ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-5Bcd^2-19c^3D+12c^2Cd))}{5d^3(c+dx)^{5/2}(bc-ad)}+\frac{7(bc-ad)\left(\frac{2(a+bx)^{5/2}(Cd-3cD)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\left(3b\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad}{}\right.\right.}{}\right.}{a}}{7(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(9/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(5/2))/(7*(b*c - a*d)*(
c + d*x)^(7/2)) + ((2*(7*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(12*c^2*C*d -
 5*B*c*d^2 - 2*A*d^3 - 19*c^3*D))*(a + b*x)^(5/2))/(5*d^3*(b*c - a*d)*(c +
 d*x)^(5/2)) + (7*(b*c - a*d)*((2*(C*d - 3*c*D)*(a + b*x)^(5/2))/(3*(b*c -
 a*d)*(c + d*x)^(3/2)) - ((2*b*C*d - 9*b*c*D + 3*a*d*D)*((-2*(a + b*x)^(3/
2))/(d*Sqrt[c + d*x]) + (3*b*((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*
d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3
/2))))/d))/(3*(b*c - a*d))))/d^3)/(7*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
 + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3445 vs. $2(262) = 524$.

Time = 0.53 (sec) , antiderivative size = 3446, normalized size of antiderivative = 11.49

method	result	size
default	Expression too large to display	3446

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/210*(b*x+a)^(1/2)*(-840*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^5*d^2*x+672*D*a^3*c^2*d^4*x*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)-700*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^2
*c^4*d^2+1575*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a^2*b^2*c^5*d^2-2205*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^6*d-24*A*b^3*d^6*x^3*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+420*D*a^3*d^6*x^3*((b*x+a)*(d*x+c))^(1/2
)*(d*b)^(1/2)+140*C*a^3*d^6*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+84*B*a
^3*d^6*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-210*C*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^6*d-1260*C*ln(1
/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^
2*b^2*c^2*d^5*x^2+2520*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^3*d^4*x^2-210*C*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c^4*d^3+420*C
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2
))*a*b^3*c^5*d^2-210*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/
2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*d^7*x^4-210*C*ln(1/2*(2*b*d*x+2*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^2*d^5*x^4+84*B*a*b^
2*d^6*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-60*B*b^3*c*d^5*x^3*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)+560*C*a^2*b*d^6*x^3*((b*x+a)*(d*x+c))^(1/2)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1157 vs. $2(265) = 530$.

Time = 119.02 (sec) , antiderivative size = 2339, normalized size of antiderivative = 7.80

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="fr
icas")
```

output
```
[-1/420*(105*(9*D*b^3*c^7 - (21*D*a*b^2 + 2*C*b^3)*c^6*d + (15*D*a^2*b + 4
*C*a*b^2)*c^5*d^2 - (3*D*a^3 + 2*C*a^2*b)*c^4*d^3 + (9*D*b^3*c^3*d^4 - (21
*D*a*b^2 + 2*C*b^3)*c^2*d^5 + (15*D*a^2*b + 4*C*a*b^2)*c*d^6 - (3*D*a^3 +
2*C*a^2*b)*d^7)*x^4 + 4*(9*D*b^3*c^4*d^3 - (21*D*a*b^2 + 2*C*b^3)*c^3*d^4
+ (15*D*a^2*b + 4*C*a*b^2)*c^2*d^5 - (3*D*a^3 + 2*C*a^2*b)*c*d^6)*x^3 + 6*
(9*D*b^3*c^5*d^2 - (21*D*a*b^2 + 2*C*b^3)*c^4*d^3 + (15*D*a^2*b + 4*C*a*b^
2)*c^3*d^4 - (3*D*a^3 + 2*C*a^2*b)*c^2*d^5)*x^2 + 4*(9*D*b^3*c^6*d - (21*D
*a*b^2 + 2*C*b^3)*c^5*d^2 + (15*D*a^2*b + 4*C*a*b^2)*c^4*d^3 - (3*D*a^3 +
2*C*a^2*b)*c^3*d^4)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d +
 a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(
b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(945*D*b^3*c^6 - 16*C*a^3*c^2*d^4 - 30
*A*a^3*d^6 - 210*(9*D*a*b^2 + C*b^3)*c^5*d + 7*(147*D*a^2*b + 50*C*a*b^2)*
c^4*d^2 - 16*(6*D*a^3 + 7*C*a^2*b)*c^3*d^3 - 6*(2*B*a^3 - 7*A*a^2*b)*c*d^5
 + 105*(D*b^3*c^2*d^4 - 2*D*a*b^2*c*d^5 + D*a^2*b*d^6)*x^4 + 2*(792*D*b^3*
c^3*d^3 - 2*(819*D*a*b^2 + 88*C*b^3)*c^2*d^4 + (945*D*a^2*b + 322*C*a*b^2
+ 15*B*b^3)*c*d^5 - (105*D*a^3 + 140*C*a^2*b + 21*B*a*b^2 - 6*A*b^3)*d^6)*
x^3 + 2*(1827*D*b^3*c^4*d^2 - 2*(1857*D*a*b^2 + 203*C*b^3)*c^3*d^3 + 3*(69
3*D*a^2*b + 230*C*a*b^2)*c^2*d^4 - 3*(70*D*a^3 + 77*C*a^2*b - 8*B*a*b^2 -
7*A*b^3)*c*d^5 - (35*C*a^3 + 42*B*a^2*b + 3*A*a*b^2)*d^6)*x^2 + 2*(1575*D*
b^3*c^5*d - 7*(453*D*a*b^2 + 50*C*b^3)*c^4*d^2 + 6*(291*D*a^2*b + 98*C*...
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(9/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1325 vs. 2(265) = 530.

Time = 0.56 (sec) , antiderivative size = 1325, normalized size of antiderivative = 4.42

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="gi
ac")
```

output
```
1/105*((((b*x + a)*(105*(D*b^13*c^3*d^8*abs(b) - 3*D*a*b^12*c^2*d^9*abs(b)
 + 3*D*a^2*b^11*c*d^10*abs(b) - D*a^3*b^10*d^11*abs(b))*(b*x + a)/(b^10*c^
3*d^9 - 3*a*b^9*c^2*d^10 + 3*a^2*b^8*c*d^11 - a^3*b^7*d^12) + 2*(792*D*b^1
4*c^4*d^7*abs(b) - 2640*D*a*b^13*c^3*d^8*abs(b) - 176*C*b^14*c^3*d^8*abs(b
) + 3213*D*a^2*b^12*c^2*d^9*abs(b) + 498*C*a*b^13*c^2*d^9*abs(b) + 15*B*b^
14*c^2*d^9*abs(b) - 1680*D*a^3*b^11*c*d^10*abs(b) - 462*C*a^2*b^12*c*d^10*
abs(b) - 36*B*a*b^13*c*d^10*abs(b) + 6*A*b^14*c*d^10*abs(b) + 315*D*a^4*b^
10*d^11*abs(b) + 140*C*a^3*b^11*d^11*abs(b) + 21*B*a^2*b^12*d^11*abs(b) -
6*A*a*b^13*d^11*abs(b))/(b^10*c^3*d^9 - 3*a*b^9*c^2*d^10 + 3*a^2*b^8*c*d^1
1 - a^3*b^7*d^12)) + 14*(261*D*b^15*c^5*d^6*abs(b) - 1131*D*a*b^14*c^4*d^7
*abs(b) - 58*C*b^15*c^4*d^7*abs(b) + 1914*D*a^2*b^13*c^3*d^8*abs(b) + 232*
C*a*b^14*c^3*d^8*abs(b) - 1569*D*a^3*b^12*c^2*d^9*abs(b) - 345*C*a^2*b^13*
c^2*d^9*abs(b) - 3*B*a*b^14*c^2*d^9*abs(b) + 3*A*b^15*c^2*d^9*abs(b) + 615
*D*a^4*b^11*c*d^10*abs(b) + 226*C*a^3*b^12*c*d^10*abs(b) + 6*B*a^2*b^13*c*
d^10*abs(b) - 6*A*a*b^14*c*d^10*abs(b) - 90*D*a^5*b^10*d^11*abs(b) - 55*C*
a^4*b^11*d^11*abs(b) - 3*B*a^3*b^12*d^11*abs(b) + 3*A*a^2*b^13*d^11*abs(b)
)/(b^10*c^3*d^9 - 3*a*b^9*c^2*d^10 + 3*a^2*b^8*c*d^11 - a^3*b^7*d^12))*(b*
x + a) + 350*(9*D*b^16*c^6*d^5*abs(b) - 48*D*a*b^15*c^5*d^6*abs(b) - 2*C*b
^16*c^5*d^6*abs(b) + 105*D*a^2*b^14*c^4*d^7*abs(b) + 10*C*a*b^15*c^4*d^7*a
bs(b) - 120*D*a^3*b^13*c^3*d^8*abs(b) - 20*C*a^2*b^14*c^3*d^8*abs(b) + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx=\int \frac{(a+b\,x)^{3/2}\left(A+B\,x+C\,x^2+x^3\,D\right)}{(c+d\,x)^{9/2}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(9/2),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\, dx = \int \frac{(bx+a)^{\frac{3}{2}} \left(Dx^3+C\,x^2+Bx+A\right)}{(dx+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x)
```

output
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x)
```

3.117 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}} \, dx$

Optimal result . . . 1092
Mathematica [A] (verified) . . . 1093
Rubi [A] (verified) . . . 1093
Maple [B] (verified) . . . 1097
Fricas [F(-1)] . . . 1098
Sympy [F] . . . 1099
Maxima [F(-2)] . . . 1099
Giac [B] (verification not implemented) . . . 1099
Mupad [F(-1)] . . . 1100
Reduce [F] . . . 1101

Optimal result

Integrand size = 34, antiderivative size = 348

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}} \, dx = \frac{2(c^2Cd - Bcd^2 + Ad^3 - c^3D)(a+bx)^{5/2}}{9d^3(bc-ad)(c+dx)^{9/2}}$$
$$+ \frac{2(9ad(2cCd - Bd^2 - 3c^2D) - b(14c^2Cd - 5Bcd^2 - 4Ad^3 - 23c^3D))(a+bx)^{5/2}}{63d^3(bc-ad)^2(c+dx)^{7/2}}$$
$$+ \frac{2(63a^2d^2(Cd-3cD) - 18abd(5cCd + Bd^2 - 18c^2D) + b^2(35c^2Cd + 10Bcd^2 + 8Ad^3 - 143c^3D))(a+bx}{315d^3(bc-ad)^3(c+dx)^{5/2}}$$
$$- \frac{2D(a+bx)^{3/2}}{3d^4(c+dx)^{3/2}} - \frac{2bD\sqrt{a+bx}}{d^5\sqrt{c+dx}} + \frac{2b^{3/2}D\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{11/2}}$$

output
```
2/9*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(5/2)/d^3/(-a*d+b*c)/(d*x+c)^(9/
2)+2/63*(9*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-4*A*d^3-5*B*c*d^2+14*C*c^2*d-2
3*D*c^3))*(b*x+a)^(5/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(7/2)+2/315*(63*a^2*d^2*(
C*d-3*D*c)-18*a*b*d*(B*d^2+5*C*c*d-18*D*c^2)+b^2*(8*A*d^3+10*B*c*d^2+35*C*
c^2*d-143*D*c^3))*(b*x+a)^(5/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(5/2)-2/3*D*(b*x+
a)^(3/2)/d^4/(d*x+c)^(3/2)-2*b*D*(b*x+a)^(1/2)/d^5/(d*x+c)^(1/2)+2*b^(3/2)
*D*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(11/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \frac{2\sqrt{a+bx}(35c^2Cd^5(a+bx)^4-35Bcd^6(a+bx)^4+35Ad^7(a+$$
$$+\frac{2b^{3/2}D\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{d^{11/2}}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(11/2),x]
```

output
```
(2*Sqrt[a + b*x]*(35*c^2*C*d^5*(a + b*x)^4 - 35*B*c*d^6*(a + b*x)^4 + 35*A
*d^7*(a + b*x)^4 - 35*c^3*d^4*D*(a + b*x)^4 + 45*b*B*c*d^5*(a + b*x)^3*(c
+ d*x) - 90*a*c*C*d^5*(a + b*x)^3*(c + d*x) - 90*A*b*d^6*(a + b*x)^3*(c +
d*x) + 45*a*B*d^6*(a + b*x)^3*(c + d*x) - 45*b*c^3*d^3*D*(a + b*x)^3*(c +
d*x) + 135*a*c^2*d^4*D*(a + b*x)^3*(c + d*x) + 63*A*b^2*d^5*(a + b*x)^2*(c
 + d*x)^2 - 63*a*b*B*d^5*(a + b*x)^2*(c + d*x)^2 + 63*a^2*C*d^5*(a + b*x)^
2*(c + d*x)^2 - 63*b^2*c^3*d^2*D*(a + b*x)^2*(c + d*x)^2 + 189*a*b*c^2*d^3
*D*(a + b*x)^2*(c + d*x)^2 - 189*a^2*c*d^4*D*(a + b*x)^2*(c + d*x)^2 - 105
*b^3*c^3*d*D*(a + b*x)*(c + d*x)^3 + 315*a*b^2*c^2*d^2*D*(a + b*x)*(c + d*
x)^3 - 315*a^2*b*c*d^3*D*(a + b*x)*(c + d*x)^3 + 105*a^3*d^4*D*(a + b*x)*(
c + d*x)^3 - 315*b^4*c^3*D*(c + d*x)^4 + 945*a*b^3*c^2*d*D*(c + d*x)^4 - 9
45*a^2*b^2*c*d^2*D*(c + d*x)^4 + 315*a^3*b*d^3*D*(c + d*x)^4))/(315*d^5*(b
*c - a*d)^3*(c + d*x)^(9/2)) + (2*b^(3/2)*D*ArcTanh[(Sqrt[b]*Sqrt[c + d*x]
)/(Sqrt[d]*Sqrt[a + b*x])])/d^(11/2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2124, 27, 1193, 27, 87, 57, 57, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx$$

$$\Big\downarrow \textbf{2124}$$

$$\frac{2\int \frac{(a+bx)^{3/2}\left(-9\left(a-\frac{bc}{d}\right)Dx^2+\frac{9(bc-ad)(Cd-cD)x}{d^2}+\frac{9ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c-4Ad^3\right)}{d^3}\right)}{2(c+dx)^{9/2}}dx}{9(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{(a+bx)^{3/2}\left(-9\left(a-\frac{bc}{d}\right)Dx^2+\frac{9(bc-ad)(Cd-cD)x}{d^2}+\frac{9ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c-4Ad^3\right)}{d^3}\right)}{(c+dx)^{9/2}}dx}{9(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \textbf{1193}$$

$$\frac{\frac{2\int \frac{(a+bx)^{3/2}\left(\left(-80Dc^3+35Cdc^2+10Bd^2c+8Ad^3\right)b^2-18ad\left(-11Dc^2+5Cdc+Bd^2\right)b+63a^2d^2(Cd-2cD)+63d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2(a+bx)^{5/2}(9ad(-Bd^2-}{}}{9(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{\int \frac{(a+bx)^{3/2}\left(\left(-80Dc^3+35Cdc^2+10Bd^2c+8Ad^3\right)b^2-18ad\left(-11Dc^2+5Cdc+Bd^2\right)b+63a^2d^2(Cd-2cD)+63d(bc-ad)^2Dx\right)}{(c+dx)^{7/2}}dx}{7d^3(bc-ad)}+\frac{2(a+bx)^{5/2}(9ad(-Bd^2-3}{}}{9(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \textbf{87}$$

$$\frac{\frac{63D(bc-ad)^2\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}dx+\frac{2(a+bx)^{5/2}\left(63a^2d^2(Cd-3cD)-18abd\left(Bd^2-18c^2D+5cCd\right)+b^2\left(8Ad^3+10Bcd^2-143c^3D+35c^2Cd\right)\right)}{5(c+dx)^{5/2}(bc-ad)}}{7d^3(bc-ad)}+\frac{2(a+bx)^{5/2}(9ad(}{}}{9(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$\downarrow$ 57

$$\frac{\frac{63D(bc-ad)^2\left(\frac{b\int\frac{\sqrt{a+bx}}{(c+dx)^{3/2}}dx}{d}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}\right)+\frac{2(a+bx)^{5/2}\left(63a^2d^2(Cd-3cD)-18abd\left(Bd^2-18c^2D+5cCd\right)+b^2\left(8Ad^3+10Bcd^2-143c^3D+35c^2Cd\right)\right)}{5(c+dx)^{5/2}(bc-ad)}}{7d^3(bc-ad)}+}{9(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$\downarrow$ 57

$$\frac{\frac{63D(bc-ad)^2\left(\frac{b\left(\frac{b\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}\right)}{d}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}\right)+\frac{2(a+bx)^{5/2}\left(63a^2d^2(Cd-3cD)-18abd\left(Bd^2-18c^2D+5cCd\right)+b^2\left(8Ad^3+10Bcd^2-14\right.\right.}{5(c+dx)^{5/2}(bc-ad)}}{7d^3(bc-ad)}}{9(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$\downarrow$ 66

$$\frac{\frac{63D(bc-ad)^2\left(\frac{b\left(\frac{2b\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}\right)}{d}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}\right)+\frac{2(a+bx)^{5/2}\left(63a^2d^2(Cd-3cD)-18abd\left(Bd^2-18c^2D+5cCd\right)+b^2\left(8Ad^3+10Bcd\right.\right.}{5(c+dx)^{5/2}(bc-ad)}}{7d^3(bc-ad)}}{9(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{\frac{2(a+bx)^{5/2}\left(63a^2d^2(Cd-3cD)-18abd\left(Bd^2-18c^2D+5cCd\right)+b^2\left(8Ad^3+10Bcd^2-143c^3D+35c^2Cd\right)\right)}{5(c+dx)^{5/2}(bc-ad)}+63D(bc-ad)^2\left(\frac{b\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{}}{d\sqrt{}}\right.}{d}\right.}{7d^3(bc-ad)}}{9(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(11/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(5/2))/(9*(b*c - a*d)*(
c + d*x)^(9/2)) + ((2*(9*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(14*c^2*C*d -
 5*B*c*d^2 - 4*A*d^3 - 23*c^3*D))*(a + b*x)^(5/2))/(7*d^3*(b*c - a*d)*(c +
 d*x)^(7/2)) + ((2*(63*a^2*d^2*(C*d - 3*c*D) - 18*a*b*d*(5*c*C*d + B*d^2 -
 18*c^2*D) + b^2*(35*c^2*C*d + 10*B*c*d^2 + 8*A*d^3 - 143*c^3*D))*(a + b*x
)^(5/2))/(5*(b*c - a*d)*(c + d*x)^(5/2)) + 63*(b*c - a*d)^2*D*((-2*(a + b*
x)^(3/2))/(3*d*(c + d*x)^(3/2)) + (b*((-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x])
 + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^
(3/2)))/d))/(7*d^3*(b*c - a*d)))/(9*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3317 vs. $2(308) = 616$.

Time = 0.50 (sec) , antiderivative size = 3318, normalized size of antiderivative = 9.53

method	result	size
default	Expression too large to display	3318

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/315*(b*x+a)^(1/2)*(90*B*a^4*d^7*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1
26*C*a^4*d^7*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-945*D*ln(1/2*(2*b*d*x
+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^7*d+1
6*A*b^4*d^7*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+210*D*a^4*d^7*x^3*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+20*B*a^4*c*d^6*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+16*C*a^4*c^2*d^5*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+32*D*a^4*c^3
*d^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-9450*D*ln(1/2*(2*b*d*x+2*((b*x+a)
*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^4*d^4*x^3-3150*D
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2
))*a^3*b^2*c^3*d^5*x^2+9450*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^4*d^4*x^2+126*C*a^2*b^2*d^7*x^4*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+70*C*b^4*c^2*d^5*x^4*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+840*D*a^3*b*d^7*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
-1126*D*b^4*c^3*d^4*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-8*A*a*b^3*d^7*
x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+72*A*b^4*c*d^6*x^3*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+18*B*a^2*b^2*d^7*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)+90*B*b^4*c^2*d^5*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-2730*D*b^4*c^
6*d*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-180*A*a^3*b*c*d^6*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+3168*D*a*b^3*c^2*d^5*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+9464*D*a*b^3*c^3*d^4*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-3...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(11/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(11/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1433 vs. $2(309) = 618$.

Time = 0.87 (sec) , antiderivative size = 1433, normalized size of antiderivative = 4.12

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="g
iac")
```

output
```
-2*D*b*abs(b)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d^5) - 2/315*((((b*x + a)*((563*D*b^13*c^4*d^8*abs
(b) - 2147*D*a*b^12*c^3*d^9*abs(b) - 35*C*b^13*c^3*d^9*abs(b) + 3033*D*a^2
*b^11*c^2*d^10*abs(b) + 125*C*a*b^12*c^2*d^10*abs(b) - 10*B*b^13*c^2*d^10*
abs(b) - 1869*D*a^3*b^10*c*d^11*abs(b) - 153*C*a^2*b^11*c*d^11*abs(b) + 28
*B*a*b^12*c*d^11*abs(b) - 8*A*b^13*c*d^11*abs(b) + 420*D*a^4*b^9*d^12*abs(
b) + 63*C*a^3*b^10*d^12*abs(b) - 18*B*a^2*b^11*d^12*abs(b) + 8*A*a*b^12*d^
12*abs(b))*(b*x + a)/(b^8*c^4*d^9 - 4*a*b^7*c^3*d^10 + 6*a^2*b^6*c^2*d^11
- 4*a^3*b^5*c*d^12 + a^4*b^4*d^13) + 9*(194*D*b^14*c^5*d^7*abs(b) - 970*D*
a*b^13*c^4*d^8*abs(b) + 1925*D*a^2*b^12*c^3*d^9*abs(b) + 10*C*a*b^13*c^3*d
^9*abs(b) - 5*B*b^14*c^3*d^9*abs(b) - 1891*D*a^3*b^11*c^2*d^10*abs(b) - 34
*C*a^2*b^12*c^2*d^10*abs(b) + 19*B*a*b^13*c^2*d^10*abs(b) - 4*A*b^14*c^2*d
^10*abs(b) + 917*D*a^4*b^10*c*d^11*abs(b) + 38*C*a^3*b^11*c*d^11*abs(b) -
23*B*a^2*b^12*c*d^11*abs(b) + 8*A*a*b^13*c*d^11*abs(b) - 175*D*a^5*b^9*d^1
2*abs(b) - 14*C*a^4*b^10*d^12*abs(b) + 9*B*a^3*b^11*d^12*abs(b) - 4*A*a^2*
b^12*d^12*abs(b))/(b^8*c^4*d^9 - 4*a*b^7*c^3*d^10 + 6*a^2*b^6*c^2*d^11 - 4
*a^3*b^5*c*d^12 + a^4*b^4*d^13)) + 63*(36*D*b^15*c^6*d^6*abs(b) - 216*D*a*
b^14*c^5*d^7*abs(b) + 540*D*a^2*b^13*c^4*d^8*abs(b) - 719*D*a^3*b^12*c^3*d
^9*abs(b) - C*a^2*b^13*c^3*d^9*abs(b) + B*a*b^14*c^3*d^9*abs(b) - A*b^15*c
^3*d^9*abs(b) + 537*D*a^4*b^11*c^2*d^10*abs(b) + 3*C*a^3*b^12*c^2*d^10*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}\,dx=\int \frac{(a+b\,x)^{3/2}\left(A+B\,x+C\,x^2+x^3\,D\right)}{(c+d\,x)^{11/2}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(11/2),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}\,dx=\int \frac{(bx+a)^{\frac{3}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(dx+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x)
```

output
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x)
```

3.118 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx$

Optimal result 1102
Mathematica [A] (verified) 1103
Rubi [A] (verified) 1103
Maple [A] (verified) 1106
Fricas [F(-1)] 1107
Sympy [F(-1)] 1107
Maxima [F(-2)] 1108
Giac [B] (verification not implemented) 1108
Mupad [F(-1)] 1109
Reduce [F] 1110

Optimal result

Integrand size = 34, antiderivative size = 375

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{5/2}}{11d^3(bc-ad)(c+dx)^{11/2}}$$
$$+\frac{2(11ad(2cCd-Bd^2-3c^2D)-b(16c^2Cd-5Bcd^2-6Ad^3-27c^3D))\,(a+bx)^{5/2}}{99d^3(bc-ad)^2(c+dx)^{9/2}}$$
$$+\frac{2(99a^2d^2(Cd-3cD)-22abd(5cCd+2Bd^2-21c^2D)+b^2(35c^2Cd+20Bcd^2+24Ad^3-189c^3D))\,(a+}{693d^3(bc-ad)^3(c+dx)^{7/2}}$$
$$-\frac{2(693a^3d^3D-99a^2bd^2(2Cd+15cD)+11ab^2d(20cCd+8Bd^2+105c^2D)-b^3(70c^2Cd+40Bcd^2+48A}{3465d^3(bc-ad)^4(c+dx)^{5/2}}$$

output

```
2/11*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(5/2)/d^3/(-a*d+b*c)/(d*x+c)^(1
1/2)+2/99*(11*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-6*A*d^3-5*B*c*d^2+16*C*c^2*
d-27*D*c^3))*(b*x+a)^(5/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(9/2)+2/693*(99*a^2*d^
2*(C*d-3*D*c)-22*a*b*d*(2*B*d^2+5*C*c*d-21*D*c^2)+b^2*(24*A*d^3+20*B*c*d^2
+35*C*c^2*d-189*D*c^3))*(b*x+a)^(5/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(7/2)-2/346
5*(693*a^3*d^3*D-99*a^2*b*d^2*(2*C*d+15*D*c)+11*a*b^2*d*(8*B*d^2+20*C*c*d+
105*D*c^2)-b^3*(48*A*d^3+40*B*c*d^2+70*C*c^2*d+315*D*c^3))*(b*x+a)^(5/2)/d
^3/(-a*d+b*c)^4/(d*x+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx = \frac{2(a+bx)^{5/2}\left(-315c^2Cd(a+bx)^3+315Bcd^2(a+bx)^3-315A\right.}{}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(13/2),x]
```

output
```
(2*(a + b*x)^(5/2)*(-315*c^2*C*d*(a + b*x)^3 + 315*B*c*d^2*(a + b*x)^3 - 3
15*A*d^3*(a + b*x)^3 + 315*c^3*D*(a + b*x)^3 + 385*b*c^2*C*(a + b*x)^2*(c
+ d*x) - 770*b*B*c*d*(a + b*x)^2*(c + d*x) + 770*a*c*C*d*(a + b*x)^2*(c +
d*x) + 1155*A*b*d^2*(a + b*x)^2*(c + d*x) - 385*a*B*d^2*(a + b*x)^2*(c + d
*x) - 1155*a*c^2*D*(a + b*x)^2*(c + d*x) + 495*b^2*B*c*(a + b*x)*(c + d*x)
^2 - 990*a*b*c*C*(a + b*x)*(c + d*x)^2 - 1485*A*b^2*d*(a + b*x)*(c + d*x)^
2 + 990*a*b*B*d*(a + b*x)*(c + d*x)^2 - 495*a^2*C*d*(a + b*x)*(c + d*x)^2
+ 1485*a^2*c*D*(a + b*x)*(c + d*x)^2 + 693*A*b^3*(c + d*x)^3 - 693*a*b^2*B
*(c + d*x)^3 + 693*a^2*b*C*(c + d*x)^3 - 693*a^3*D*(c + d*x)^3))/(3465*(b*
c - a*d)^4*(c + d*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2124, 27, 1193, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$\frac{2\int \frac{(a+bx)^{3/2}\left(-11\left(a-\frac{bc}{d}\right)Dx^2+\frac{11(bc-ad)(Cd-cD)x}{d^2}+\frac{11ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c-6Ad^3\right)}{d^3}\right)}{2(c+dx)^{11/2}}dx}{11(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+bx)^{3/2}\left(-11\left(a-\frac{bc}{d}\right)Dx^2+\frac{11(bc-ad)(Cd-cD)x}{d^2}+\frac{11ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c-6Ad^3\right)}{d^3}\right)}{(c+dx)^{11/2}}dx}{11(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$\frac{\frac{2\int \frac{(a+bx)^{3/2}\left(\left(-90Dc^3+35Cdc^2+20Bd^2c+24Ad^3\right)b^2-22ad\left(-12Dc^2+5Cdc+2Bd^2\right)b+99a^2d^2(Cd-2cD)+99d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{9/2}}dx}{9(bc-ad)}+\frac{2(a+bx)^{5/2}(11ad(-B}{}}{11(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(a+bx)^{3/2}\left(\left(-90Dc^3+35Cdc^2+20Bd^2c+24Ad^3\right)b^2-22ad\left(-12Dc^2+5Cdc+2Bd^2\right)b+99a^2d^2(Cd-2cD)+99d(bc-ad)^2Dx\right)}{(c+dx)^{9/2}}dx}{9d^3(bc-ad)}+\frac{2(a+bx)^{5/2}(11ad(-Bd^2}{}}{11(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$\frac{\frac{\frac{2(a+bx)^{5/2}\left(99a^2d^2(Cd-3cD)-22abd\left(2Bd^2-21c^2D+5cCd\right)+b^2\left(24Ad^3+20Bcd^2-189c^3D+35c^2Cd\right)\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\left(693a^3d^3D-99a^2bd^2(15cD+2Cd)+11ab^2d\left(8Bd^2+10\right.\right.}{}}{9d^3(bc-ad)}}{11}+\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\Big\downarrow$ 48

$$\frac{\frac{\frac{2(a+bx)^{5/2}\left(99a^2d^2(Cd-3cD)-22abd\left(2Bd^2-21c^2D+5cCd\right)+b^2\left(24Ad^3+20Bcd^2-189c^3D+35c^2Cd\right)\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{2(a+bx)^{5/2}\left(693a^3d^3D-99a^2bd^2(15cD+2Cd)+11ab^2\cdots\right.}{35\cdots}}{9d^3(bc-ad)}}{11(b\cdots}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(13/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(5/2))/(11*(b*c - a*d)*
(c + d*x)^(11/2)) + ((2*(11*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(16*c^2*C*
d - 5*B*c*d^2 - 6*A*d^3 - 27*c^3*D))*(a + b*x)^(5/2))/(9*d^3*(b*c - a*d)*(
c + d*x)^(9/2)) + ((2*(99*a^2*d^2*(C*d - 3*c*D) - 22*a*b*d*(5*c*C*d + 2*B*
d^2 - 21*c^2*D) + b^2*(35*c^2*C*d + 20*B*c*d^2 + 24*A*d^3 - 189*c^3*D))*(a
 + b*x)^(5/2))/(7*(b*c - a*d)*(c + d*x)^(7/2)) - (2*(693*a^3*d^3*D - 99*a^
2*b*d^2*(2*C*d + 15*c*D) + 11*a*b^2*d*(20*c*C*d + 8*B*d^2 + 105*c^2*D) - b
^3*(70*c^2*C*d + 40*B*c*d^2 + 48*A*d^3 + 315*c^3*D))*(a + b*x)^(5/2))/(35*
(b*c - a*d)^2*(c + d*x)^(5/2)))/(9*d^3*(b*c - a*d)))/(11*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.58

method	result
gosper	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-48A\,b^{3}d^{3}x^{3}+88Ba\,b^{2}d^{3}x^{3}-40B\,b^{3}c\,d^{2}x^{3}-198C\,a^{2}b\,d^{3}x^{3}+220Ca\,b^{2}c\,d^{2}x^{3}-70C\,b^{3}c^{2}d\,x^{3}+693Da^{3}d^{3}x^{3}-1485Da\right.}{}$
orering	$-\frac{2(bx+a)^{\frac{5}{2}}\left(-48A\,b^{3}d^{3}x^{3}+88Ba\,b^{2}d^{3}x^{3}-40B\,b^{3}c\,d^{2}x^{3}-198C\,a^{2}b\,d^{3}x^{3}+220Ca\,b^{2}c\,d^{2}x^{3}-70C\,b^{3}c^{2}d\,x^{3}+693Da^{3}d^{3}x^{3}-1485Da\right.}{}$
default	$-\frac{2\left(-48A\,b^{4}d^{3}x^{4}+88Ba\,b^{3}d^{3}x^{4}-40B\,b^{4}c\,d^{2}x^{4}-198C\,a^{2}b^{2}d^{3}x^{4}+220Ca\,b^{3}c\,d^{2}x^{4}-70C\,b^{4}c^{2}d\,x^{4}+693Da^{3}b\,d^{3}x^{4}-1485Da^{2}b^{2}c\,d^{2}\right.}{}$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/3465*(b*x+a)^(5/2)*(-48*A*b^3*d^3*x^3+88*B*a*b^2*d^3*x^3-40*B*b^3*c*d^2
*x^3-198*C*a^2*b*d^3*x^3+220*C*a*b^2*c*d^2*x^3-70*C*b^3*c^2*d*x^3+693*D*a^
3*d^3*x^3-1485*D*a^2*b*c*d^2*x^3+1155*D*a*b^2*c^2*d*x^3-315*D*b^3*c^3*x^3+
120*A*a*b^2*d^3*x^2-264*A*b^3*c*d^2*x^2-220*B*a^2*b*d^3*x^2+584*B*a*b^2*c*
d^2*x^2-220*B*b^3*c^2*d*x^2+495*C*a^3*d^3*x^2-1639*C*a^2*b*c*d^2*x^2+1385*
C*a*b^2*c^2*d*x^2-385*C*b^3*c^3*x^2+594*D*a^3*c*d^2*x^2-660*D*a^2*b*c^2*d*
x^2+210*D*a*b^2*c^3*x^2-210*A*a^2*b*d^3*x+660*A*a*b^2*c*d^2*x-594*A*b^3*c^
2*d*x+385*B*a^3*d^3*x-1385*B*a^2*b*c*d^2*x+1639*B*a*b^2*c^2*d*x-495*B*b^3*
c^3*x+220*C*a^3*c*d^2*x-584*C*a^2*b*c^2*d*x+220*C*a*b^2*c^3*x+264*D*a^3*c^
2*d*x-120*D*a^2*b*c^3*x+315*A*a^3*d^3-1155*A*a^2*b*c*d^2+1485*A*a*b^2*c^2*
d-693*A*b^3*c^3+70*B*a^3*c*d^2-220*B*a^2*b*c^2*d+198*B*a*b^2*c^3+40*C*a^3*
c^2*d-88*C*a^2*b*c^3+48*D*a^3*c^3)/(d*x+c)^(11/2)/(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1415 vs. $2(351) = 702$.

Time = 0.81 (sec) , antiderivative size = 1415, normalized size of antiderivative = 3.77

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="g
iac")
```

output
```
2/3465*(((b*x + a)*((315*D*b^15*c^4*d^5*abs(b) - 1470*D*a*b^14*c^3*d^6*abs
(b) + 70*C*b^15*c^3*d^6*abs(b) + 2640*D*a^2*b^13*c^2*d^7*abs(b) - 290*C*a*
b^14*c^2*d^7*abs(b) + 40*B*b^15*c^2*d^7*abs(b) - 2178*D*a^3*b^12*c*d^8*abs
(b) + 418*C*a^2*b^13*c*d^8*abs(b) - 128*B*a*b^14*c*d^8*abs(b) + 48*A*b^15*
c*d^8*abs(b) + 693*D*a^4*b^11*d^9*abs(b) - 198*C*a^3*b^12*d^9*abs(b) + 88*
B*a^2*b^13*d^9*abs(b) - 48*A*a*b^14*d^9*abs(b))*(b*x + a)/(b^9*c^5*d^5 - 5
*a*b^8*c^4*d^6 + 10*a^2*b^7*c^3*d^7 - 10*a^3*b^6*c^2*d^8 + 5*a^4*b^5*c*d^9
 - a^5*b^4*d^10) - 11*(105*D*a*b^15*c^4*d^5*abs(b) - 35*C*b^16*c^4*d^5*abs
(b) - 480*D*a^2*b^14*c^3*d^6*abs(b) + 180*C*a*b^15*c^3*d^6*abs(b) - 20*B*b
^16*c^3*d^6*abs(b) + 834*D*a^3*b^13*c^2*d^7*abs(b) - 354*C*a^2*b^14*c^2*d^
7*abs(b) + 84*B*a*b^15*c^2*d^7*abs(b) - 24*A*b^16*c^2*d^7*abs(b) - 648*D*a
^4*b^12*c*d^8*abs(b) + 308*C*a^3*b^13*c*d^8*abs(b) - 108*B*a^2*b^14*c*d^8*
abs(b) + 48*A*a*b^15*c*d^8*abs(b) + 189*D*a^5*b^11*d^9*abs(b) - 99*C*a^4*b
^12*d^9*abs(b) + 44*B*a^3*b^13*d^9*abs(b) - 24*A*a^2*b^14*d^9*abs(b))/(b^9
*c^5*d^5 - 5*a*b^8*c^4*d^6 + 10*a^2*b^7*c^3*d^7 - 10*a^3*b^6*c^2*d^8 + 5*a
^4*b^5*c*d^9 - a^5*b^4*d^10)) + 99*(15*D*a^2*b^15*c^4*d^5*abs(b) - 10*C*a*
b^16*c^4*d^5*abs(b) + 5*B*b^17*c^4*d^5*abs(b) - 66*D*a^3*b^14*c^3*d^6*abs(
b) + 46*C*a^2*b^15*c^3*d^6*abs(b) - 26*B*a*b^16*c^3*d^6*abs(b) + 6*A*b^17*
c^3*d^6*abs(b) + 108*D*a^4*b^13*c^2*d^7*abs(b) - 78*C*a^3*b^14*c^2*d^7*abs
(b) + 48*B*a^2*b^15*c^2*d^7*abs(b) - 18*A*a*b^16*c^2*d^7*abs(b) - 78*D*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx=\int \frac{(a+b\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{13/2}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(13/2),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(13/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(dx+c)^{\frac{13}{2}}}dx$$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x)
```

output
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x)
```

3.119 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{15/2}}\,dx$

Optimal result 1111
Mathematica [A] (verified) 1112
Rubi [A] (verified) 1113
Maple [B] (verified) 1116
Fricas [F(-1)] 1117
Sympy [F(-1)] 1118
Maxima [F(-2)] 1118
Giac [B] (verification not implemented) 1118
Mupad [F(-1)] 1119
Reduce [F] 1120

Optimal result

Integrand size = 34, antiderivative size = 496

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{5/2}}{13d^3(bc-ad)(c+dx)^{13/2}}$$
$$+\frac{2(13ad(2cCd-Bd^2-3c^2D)-b(18c^2Cd-5Bcd^2-8Ad^3-31c^3D))\,(a+bx)^{5/2}}{143d^3(bc-ad)^2(c+dx)^{11/2}}$$
$$+\frac{2(143a^2d^2(Cd-3cD)-26abd(5cCd+3Bd^2-24c^2D)+b^2(35c^2Cd+30Bcd^2+48Ad^3-243c^3D))\,(a\ \cdots}{1287d^3(bc-ad)^3(c+dx)^{9/2}}$$
$$-\frac{2(1287a^3d^3D-143a^2bd^2(4Cd+15cD)+13ab^2d(40cCd+24Bd^2+105c^2D)-b^3(140c^2Cd+120Bcd^2\ \cdots}{9009d^3(bc-ad)^4(c+dx)^{7/2}}$$
$$-\frac{4b(1287a^3d^3D-143a^2bd^2(4Cd+15cD)+13ab^2d(40cCd+24Bd^2+105c^2D)-b^3(140c^2Cd+120Bcd^2\ \cdots}{45045d^3(bc-ad)^5(c+dx)^{5/2}}$$

output
```
2/13*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(5/2)/d^3/(-a*d+b*c)/(d*x+c)^(1
3/2)+2/143*(13*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-8*A*d^3-5*B*c*d^2+18*C*c^2
*d-31*D*c^3))*(b*x+a)^(5/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(11/2)+2/1287*(143*a^
2*d^2*(C*d-3*D*c)-26*a*b*d*(3*B*d^2+5*C*c*d-24*D*c^2)+b^2*(48*A*d^3+30*B*c
*d^2+35*C*c^2*d-243*D*c^3))*(b*x+a)^(5/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(9/2)-2
/9009*(1287*a^3*d^3*D-143*a^2*b*d^2*(4*C*d+15*D*c)+13*a*b^2*d*(24*B*d^2+40
*C*c*d+105*D*c^2)-b^3*(192*A*d^3+120*B*c*d^2+140*C*c^2*d+315*D*c^3))*(b*x+
a)^(5/2)/d^3/(-a*d+b*c)^4/(d*x+c)^(7/2)-4/45045*b*(1287*a^3*d^3*D-143*a^2*
b*d^2*(4*C*d+15*D*c)+13*a*b^2*d*(24*B*d^2+40*C*c*d+105*D*c^2)-b^3*(192*A*d
^3+120*B*c*d^2+140*C*c^2*d+315*D*c^3))*(b*x+a)^(5/2)/d^3/(-a*d+b*c)^5/(d*x
+c)^(5/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \frac{2(a+bx)^{5/2}\left(3465c^2Cd^2(a+bx)^4-3465Bcd^3(a+bx)^4+346\right.}{}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(15/2),x]
```

output
```
(2*(a + b*x)^(5/2)*(3465*c^2*C*d^2*(a + b*x)^4 - 3465*B*c*d^3*(a + b*x)^4
+ 3465*A*d^4*(a + b*x)^4 - 3465*c^3*d*D*(a + b*x)^4 - 8190*b*c^2*C*d*(a +
b*x)^3*(c + d*x) + 12285*b*B*c*d^2*(a + b*x)^3*(c + d*x) - 8190*a*c*C*d^2*
(a + b*x)^3*(c + d*x) - 16380*A*b*d^3*(a + b*x)^3*(c + d*x) + 4095*a*B*d^3
*(a + b*x)^3*(c + d*x) + 4095*b*c^3*D*(a + b*x)^3*(c + d*x) + 12285*a*c^2*
d*D*(a + b*x)^3*(c + d*x) + 5005*b^2*c^2*C*(a + b*x)^2*(c + d*x)^2 - 15015
*b^2*B*c*d*(a + b*x)^2*(c + d*x)^2 + 20020*a*b*c*C*d*(a + b*x)^2*(c + d*x)
^2 + 30030*A*b^2*d^2*(a + b*x)^2*(c + d*x)^2 - 15015*a*b*B*d^2*(a + b*x)^2
*(c + d*x)^2 + 5005*a^2*C*d^2*(a + b*x)^2*(c + d*x)^2 - 15015*a*b*c^2*D*(a
 + b*x)^2*(c + d*x)^2 - 15015*a^2*c*d*D*(a + b*x)^2*(c + d*x)^2 + 6435*b^3
*B*c*(a + b*x)*(c + d*x)^3 - 12870*a*b^2*c*C*(a + b*x)*(c + d*x)^3 - 25740
*A*b^3*d*(a + b*x)*(c + d*x)^3 + 19305*a*b^2*B*d*(a + b*x)*(c + d*x)^3 - 1
2870*a^2*b*C*d*(a + b*x)*(c + d*x)^3 + 19305*a^2*b*c*D*(a + b*x)*(c + d*x)
^3 + 6435*a^3*d*D*(a + b*x)*(c + d*x)^3 + 9009*A*b^4*(c + d*x)^4 - 9009*a*
b^3*B*(c + d*x)^4 + 9009*a^2*b^2*C*(c + d*x)^4 - 9009*a^3*b*D*(c + d*x)^4)
)/(45045*(b*c - a*d)^5*(c + d*x)^(13/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1193, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2} \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}} \, dx$$

$$\Big\downarrow \text{ 2124}$$

$$\frac{2\int \frac{(a+bx)^{3/2}\left(-13\left(a-\frac{bc}{d}\right)Dx^2+\frac{13(bc-ad)(Cd-cD)x}{d^2}+\frac{13ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c-8Ad^3\right)}{d^3}\right)}{2(c+dx)^{13/2}}dx}{13(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+bx)^{3/2}\left(-13\left(a-\frac{bc}{d}\right)Dx^2+\frac{13(bc-ad)(Cd-cD)x}{d^2}+\frac{13ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-5Dc^3+5Cdc^2-5Bd^2c-8Ad^3\right)}{d^3}\right)}{(c+dx)^{13/2}}dx}{13(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$$\Big\downarrow \text{ 1193}$$

$$\frac{\frac{2\int \frac{(a+bx)^{3/2}\left(\left(-100Dc^3+35Cdc^2+30Bd^2c+48Ad^3\right)b^2-26ad\left(-13Dc^2+5Cdc+3Bd^2\right)b+143a^2d^2(Cd-2cD)+143d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{11/2}}dx}{11(bc-ad)}+\frac{2(a+bx)^{5/2}(13ad(-}{}}{13(bc-ad)}+\frac{2(a+bx)^{5/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{(a+bx)^{3/2}\left(\left(-100Dc^3+35Cdc^2+30Bd^2c+48Ad^3\right)b^2-26ad\left(-13Dc^2+5Cdc+3Bd^2\right)b+143a^2d^2(Cd-2cD)+143d(bc-ad)^2Dx\right)}{(c+dx)^{11/2}}dx}{11d^3(bc-ad)}+\frac{2(a+bx)^{5/2}(13ad(-\ldots}{\ldots}}{13(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$\frac{\frac{2(a+bx)^{5/2}\left(143a^2d^2(Cd-3cD)-26abd\left(3Bd^2-24c^2D+5cCd\right)+b^2\left(48Ad^3+30Bcd^2-243c^3D+35c^2Cd\right)\right)}{9(c+dx)^{9/2}(bc-ad)}-\frac{\left(1287a^3d^3D-143a^2bd^2(15cD+4Cd)+13ab^2d\left(24Bd\ldots\right.\right.}{\ldots}}{11d^3(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$\Big\downarrow$ 55

$$\frac{\frac{2(a+bx)^{5/2}\left(143a^2d^2(Cd-3cD)-26abd\left(3Bd^2-24c^2D+5cCd\right)+b^2\left(48Ad^3+30Bcd^2-243c^3D+35c^2Cd\right)\right)}{9(c+dx)^{9/2}(bc-ad)}-\frac{\left(1287a^3d^3D-143a^2bd^2(15cD+4Cd)+13ab^2d\left(24Bd\ldots\right.\right.}{\ldots}}{11d^3(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$\Big\downarrow$ 48

$$\frac{\frac{2(a+bx)^{5/2}\left(143a^2d^2(Cd-3cD)-26abd\left(3Bd^2-24c^2D+5cCd\right)+b^2\left(48Ad^3+30Bcd^2-243c^3D+35c^2Cd\right)\right)}{9(c+dx)^{9/2}(bc-ad)}-\frac{\left(\frac{4b(a+bx)^{5/2}}{35(c+dx)^{5/2}(bc-ad)^2}+\frac{2(a+bx)^{5/2}}{7(c+dx)^{7/2}(bc-ad)}\right)(12\ldots}{\ldots}}{11d^3(bc-ad)}$$

$$\frac{2(a+bx)^{5/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(15/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(5/2))/(13*(b*c - a*d)*
(c + d*x)^(13/2)) + ((2*(13*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(18*c^2*C*
d - 5*B*c*d^2 - 8*A*d^3 - 31*c^3*D))*(a + b*x)^(5/2))/(11*d^3*(b*c - a*d)*
(c + d*x)^(11/2)) + ((2*(143*a^2*d^2*(C*d - 3*c*D) - 26*a*b*d*(5*c*C*d + 3
*B*d^2 - 24*c^2*D) + b^2*(35*c^2*C*d + 30*B*c*d^2 + 48*A*d^3 - 243*c^3*D))
*(a + b*x)^(5/2))/(9*(b*c - a*d)*(c + d*x)^(9/2)) - ((1287*a^3*d^3*D - 143
*a^2*b*d^2*(4*C*d + 15*c*D) + 13*a*b^2*d*(40*c*C*d + 24*B*d^2 + 105*c^2*D)
 - b^3*(140*c^2*C*d + 120*B*c*d^2 + 192*A*d^3 + 315*c^3*D))*((2*(a + b*x)^
(5/2))/(7*(b*c - a*d)*(c + d*x)^(7/2)) + (4*b*(a + b*x)^(5/2))/(35*(b*c -
a*d)^2*(c + d*x)^(5/2))))/(9*(b*c - a*d)))/(11*d^3*(b*c - a*d)))/(13*(b*c
- a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(466) = 932$.

Time = 0.52 (sec) , antiderivative size = 1006, normalized size of antiderivative = 2.03

method	result	size
gosper	Expression too large to display	1006
orering	Expression too large to display	1006
default	Expression too large to display	1266

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/45045*(b*x+a)^(5/2)*(384*A*b^4*d^4*x^4-624*B*a*b^3*d^4*x^4+240*B*b^4*c*
d^3*x^4+1144*C*a^2*b^2*d^4*x^4-1040*C*a*b^3*c*d^3*x^4+280*C*b^4*c^2*d^2*x^
4-2574*D*a^3*b*d^4*x^4+4290*D*a^2*b^2*c*d^3*x^4-2730*D*a*b^3*c^2*d^2*x^4+6
30*D*b^4*c^3*d*x^4-960*A*a*b^3*d^4*x^3+2496*A*b^4*c*d^3*x^3+1560*B*a^2*b^2
*d^4*x^3-4656*B*a*b^3*c*d^3*x^3+1560*B*b^4*c^2*d^2*x^3-2860*C*a^3*b*d^4*x^
3+10036*C*a^2*b^2*c*d^3*x^3-7460*C*a*b^3*c^2*d^2*x^3+1820*C*b^4*c^3*d*x^3+
6435*D*a^4*d^4*x^3-27456*D*a^3*b*c*d^3*x^3+34710*D*a^2*b^2*c^2*d^2*x^3-193
20*D*a*b^3*c^3*d*x^3+4095*D*b^4*c^4*x^3+1680*A*a^2*b^2*d^4*x^2-6240*A*a*b^
3*c*d^3*x^2+6864*A*b^4*c^2*d^2*x^2-2730*B*a^3*b*d^4*x^2+11190*B*a^2*b^2*c*
d^3*x^2-15054*B*a*b^3*c^2*d^2*x^2+4290*B*b^4*c^3*d*x^2+5005*C*a^4*d^4*x^2-
23140*C*a^3*b*c*d^3*x^2+38574*C*a^2*b^2*c^2*d^2*x^2-23140*C*a*b^3*c^3*d*x^
2+5005*C*b^4*c^4*x^2+4290*D*a^4*c*d^3*x^2-15054*D*a^3*b*c^2*d^2*x^2+11190*
D*a^2*b^2*c^3*d*x^2-2730*D*a*b^3*c^4*x^2-2520*A*a^3*b*d^4*x+10920*A*a^2*b^
2*c*d^3*x-17160*A*a*b^3*c^2*d^2*x+10296*A*b^4*c^3*d*x+4095*B*a^4*d^4*x-193
20*B*a^3*b*c*d^3*x+34710*B*a^2*b^2*c^2*d^2*x-27456*B*a*b^3*c^3*d*x+6435*B*
b^4*c^4*x+1820*C*a^4*c*d^3*x-7460*C*a^3*b*c^2*d^2*x+10036*C*a^2*b^2*c^3*d*
x-2860*C*a*b^3*c^4*x+1560*D*a^4*c^2*d^2*x-4656*D*a^3*b*c^3*d*x+1560*D*a^2*
b^2*c^4*x+3465*A*a^4*d^4-16380*A*a^3*b*c*d^3+30030*A*a^2*b^2*c^2*d^2-25740
*A*a*b^3*c^3*d+9009*A*b^4*c^4+630*B*a^4*c*d^3-2730*B*a^3*b*c^2*d^2+4290*B*
a^2*b^2*c^3*d-2574*B*a*b^3*c^4+280*C*a^4*c^2*d^2-1040*C*a^3*b*c^3*d+114...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(3/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(15/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. $2(466) = 932$.

Time = 1.14 (sec) , antiderivative size = 2037, normalized size of antiderivative = 4.11

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x, algorithm="g
iac")
```

output
```
2/45045*((((b*x + a)*(2*(315*D*b^17*c^4*d^7*abs(b) - 1680*D*a*b^16*c^3*d^8
*abs(b) + 140*C*b^17*c^3*d^8*abs(b) + 3510*D*a^2*b^15*c^2*d^9*abs(b) - 660
*C*a*b^16*c^2*d^9*abs(b) + 120*B*b^17*c^2*d^9*abs(b) - 3432*D*a^3*b^14*c*d
^10*abs(b) + 1092*C*a^2*b^15*c*d^10*abs(b) - 432*B*a*b^16*c*d^10*abs(b) +
192*A*b^17*c*d^10*abs(b) + 1287*D*a^4*b^13*d^11*abs(b) - 572*C*a^3*b^14*d^
11*abs(b) + 312*B*a^2*b^15*d^11*abs(b) - 192*A*a*b^16*d^11*abs(b))*(b*x +
a)/(b^10*c^6*d^6 - 6*a*b^9*c^5*d^7 + 15*a^2*b^8*c^4*d^8 - 20*a^3*b^7*c^3*d
^9 + 15*a^4*b^6*c^2*d^10 - 6*a^5*b^5*c*d^11 + a^6*b^4*d^12) + 13*(315*D*b^
18*c^5*d^6*abs(b) - 1995*D*a*b^17*c^4*d^7*abs(b) + 140*C*b^18*c^4*d^7*abs(
b) + 5190*D*a^2*b^16*c^3*d^8*abs(b) - 800*C*a*b^17*c^3*d^8*abs(b) + 120*B*
b^18*c^3*d^8*abs(b) - 6942*D*a^3*b^15*c^2*d^9*abs(b) + 1752*C*a^2*b^16*c^2
*d^9*abs(b) - 552*B*a*b^17*c^2*d^9*abs(b) + 192*A*b^18*c^2*d^9*abs(b) + 47
19*D*a^4*b^14*c*d^10*abs(b) - 1664*C*a^3*b^15*c*d^10*abs(b) + 744*B*a^2*b^
16*c*d^10*abs(b) - 384*A*a*b^17*c*d^10*abs(b) - 1287*D*a^5*b^13*d^11*abs(b
) + 572*C*a^4*b^14*d^11*abs(b) - 312*B*a^3*b^15*d^11*abs(b) + 192*A*a^2*b^
16*d^11*abs(b))/(b^10*c^6*d^6 - 6*a*b^9*c^5*d^7 + 15*a^2*b^8*c^4*d^8 - 20*
a^3*b^7*c^3*d^9 + 15*a^4*b^6*c^2*d^10 - 6*a^5*b^5*c*d^11 + a^6*b^4*d^12))
- 143*(105*D*a*b^18*c^5*d^6*abs(b) - 35*C*b^19*c^5*d^6*abs(b) - 615*D*a^2*
b^17*c^4*d^7*abs(b) + 235*C*a*b^18*c^4*d^7*abs(b) - 30*B*b^19*c^4*d^7*abs(
b) + 1458*D*a^3*b^16*c^3*d^8*abs(b) - 638*C*a^2*b^17*c^3*d^8*abs(b) + 1...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{15/2}}\,dx=\int \frac{(a+b\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{15/2}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(15/2),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(15/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{15/2}}\,dx=\int \frac{(bx+a)^{\frac{3}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{15}{2}}}dx$$

input
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x)
```

output
```
int((b*x+a)^(3/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x)
```

3.120 $\int (a+bx)^{5/2}\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result . . . 1121
Mathematica [A] (verified) . . . 1122
Rubi [A] (verified) . . . 1123
Maple [B] (verified) . . . 1129
Fricas [A] (verification not implemented) . . . 1130
Sympy [F] . . . 1131
Maxima [F(-2)] . . . 1132
Giac [B] (verification not implemented) . . . 1132
Mupad [F(-1)] . . . 1133
Reduce [B] (verification not implemented) . . . 1134

Optimal result

Integrand size = 34, antiderivative size = 762

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx =$$

$$-\frac{(bc-ad)^3\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d(28cCd-24Bd^2-27c^2D)-b^3(42c^2Cd-56Bcd^2+8\right.}{1024b^4d^6}$$

$$+\frac{(bc-ad)^2\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d(28cCd-24Bd^2-27c^2D)-b^3(42c^2Cd-56Bcd^2+80\right.}{1536b^4d^5}$$

$$-\frac{(bc-ad)\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d(28cCd-24Bd^2-27c^2D)-b^3(42c^2Cd-56Bcd^2+80\right.}{1920b^4d^4}$$

$$-\frac{\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d(28cCd-24Bd^2-27c^2D)-b^3(42c^2Cd-56Bcd^2+80Ad^3-33\right.}{320b^4d^3}$$

$$-\frac{\left(42bcC-56bBd+70aCd-60acD-\frac{33bc^2D}{d}-\frac{75a^2dD}{b}\right)(a+bx)^{7/2}(c+dx)^{3/2}}{280b^2d^2}$$

$$+\frac{(14bCd-11bcD-31adD)(a+bx)^{9/2}(c+dx)^{3/2}}{84b^3d^2}+\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$$+\frac{(bc-ad)^4\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d(28cCd-24Bd^2-27c^2D)-b^3(42c^2Cd-56Bcd^2+80\right.}{1024b^{9/2}d^{13/2}}$$

output
```
-1/1024*(-a*d+b*c)^3*(5*a^3*d^3*D-5*a^2*b*d^2*(2*C*d-3*D*c)-a*b^2*d*(-24*B
*d^2+28*C*c*d-27*D*c^2)-b^3*(80*A*d^3-56*B*c*d^2+42*C*c^2*d-33*D*c^3))*(b*
x+a)^(1/2)*(d*x+c)^(1/2)/b^4/d^6+1/1536*(-a*d+b*c)^2*(5*a^3*d^3*D-5*a^2*b*
d^2*(2*C*d-3*D*c)-a*b^2*d*(-24*B*d^2+28*C*c*d-27*D*c^2)-b^3*(80*A*d^3-56*B
*c*d^2+42*C*c^2*d-33*D*c^3))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^4/d^5-1/1920*(-
a*d+b*c)*(5*a^3*d^3*D-5*a^2*b*d^2*(2*C*d-3*D*c)-a*b^2*d*(-24*B*d^2+28*C*c*
d-27*D*c^2)-b^3*(80*A*d^3-56*B*c*d^2+42*C*c^2*d-33*D*c^3))*(b*x+a)^(5/2)*(
d*x+c)^(1/2)/b^4/d^4-1/320*(5*a^3*d^3*D-5*a^2*b*d^2*(2*C*d-3*D*c)-a*b^2*d*
(-24*B*d^2+28*C*c*d-27*D*c^2)-b^3*(80*A*d^3-56*B*c*d^2+42*C*c^2*d-33*D*c^3
))*(b*x+a)^(7/2)*(d*x+c)^(1/2)/b^4/d^3-1/280*(42*C*b*c-56*B*b*d+70*C*a*d-6
0*D*a*c-33*b*c^2*D/d-75*a^2*d*D/b)*(b*x+a)^(7/2)*(d*x+c)^(3/2)/b^2/d^2+1/8
4*(14*C*b*d-31*D*a*d-11*D*b*c)*(b*x+a)^(9/2)*(d*x+c)^(3/2)/b^3/d^2+1/7*D*(
b*x+a)^(11/2)*(d*x+c)^(3/2)/b^3/d+1/1024*(-a*d+b*c)^4*(5*a^3*d^3*D-5*a^2*b
*d^2*(2*C*d-3*D*c)-a*b^2*d*(-24*B*d^2+28*C*c*d-27*D*c^2)-b^3*(80*A*d^3-56*
B*c*d^2+42*C*c^2*d-33*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c
)^(1/2))/b^(9/2)/d^(13/2)
```

Mathematica [A] (verified)

Time = 2.98 (sec) , antiderivative size = 726, normalized size of antiderivative = 0.95

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{\sqrt{a+bx}\sqrt{c+dx}\left(-525a^6d^6D+350a^5bd^5(3Cd+D(c+dx))-35a^4b^2d^4\left(-11c^2D+c(26Cd\cdots\right.\right.}{\cdots}+\frac{(bc-ad)^4\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)+ab^2d\left(-28cCd+24Bd^2+27c^2D\right)+b^3\left(-42c^2Cd+56Bcd^2\cdots\right.\right.}{1024b^{9/2}d^{13/2}}$$

input
```
Integrate[(a + b*x)^(5/2)*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(-525*a^6*d^6*D + 350*a^5*b*d^5*(3*C*d + D*(c
 + d*x)) - 35*a^4*b^2*d^4*(-11*c^2*D + c*(26*C*d + 6*d*D*x) + 4*d^2*(18*B
+ x*(5*C + 2*D*x))) + 20*a^3*b^3*d^3*(33*c^3*D - c^2*d*(63*C + 13*D*x) + 4
*c*d^2*(42*B + x*(7*C + 2*D*x)) + 4*d^3*(105*A + x*(21*B + 7*C*x + 3*D*x^2
))) + 2*a*b^5*d*(4935*c^5*D - 189*c^4*d*(35*C + 17*D*x) + 8*c^3*d^2*(1190*
B + x*(539*C + 318*D*x)) + 32*c*d^4*x*(315*A + x*(154*B + 91*C*x + 60*D*x^
2)) - 8*c^2*d^3*(1925*A + x*(777*B + 427*C*x + 271*D*x^2)) + 64*d^5*x^2*(5
95*A + x*(441*B + 350*C*x + 290*D*x^2))) + a^2*b^4*d^2*(-8043*c^4*D + 4*c^
3*d*(2933*C + 1251*D*x) - 8*c^2*d^2*(2422*B + x*(917*C + 486*D*x)) + 16*c*
d^3*(2555*A + x*(763*B + 357*C*x + 205*D*x^2)) + 32*d^4*x*(2065*A + x*(130
2*B + 945*C*x + 740*D*x^2))) + b^6*(-3465*c^6*D + 210*c^5*d*(21*C + 11*D*x
) - 84*c^4*d^2*(70*B + x*(35*C + 22*D*x)) + 128*c*d^5*x^2*(35*A + x*(21*B
+ 2*x*(7*C + 5*D*x))) + 256*d^6*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x)
)) + 16*c^3*d^3*(525*A + x*(245*B + 3*x*(49*C + 33*D*x))) - 32*c^2*d^4*x*(
175*A + x*(98*B + x*(63*C + 44*D*x))))))/(107520*b^4*d^6) + ((b*c - a*d)^4
*(5*a^3*d^3*D - 5*a^2*b*d^2*(2*C*d - 3*c*D) + a*b^2*d*(-28*c*C*d + 24*B*d^
2 + 27*c^2*D) + b^3*(-42*c^2*C*d + 56*B*c*d^2 - 80*A*d^3 + 33*c^3*D))*ArcT
anh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(1024*b^(9/2)*d^(13/
2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.324, Rules used = {2125, 27, 1194, 27, 90, 60, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \; 2125$$

$$\frac{\int \frac{1}{2}(a+bx)^{5/2}\sqrt{c+dx}\left(14Adb^3+(14bCd-31aDd-11bcD)x^2b^2+2\left(-10dDa^2-11bcDa+7b^2Bd\right)xb-a^2(1\ldots}{7b^3d}$$
$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$$\Big\downarrow \; 27$$

$$\int (a+bx)^{5/2}\sqrt{c+dx}\,(14Adb^3+(14bCd-31aDd-11bcD)x^2b^2+2(-10dDa^2-11bcDa+7b^2Bd)\,xb-a^2(11b$$
$$14b^3d$$
$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 1194

$$\int \tfrac{3}{2}b^2(a+bx)^{5/2}\sqrt{c+dx}\,(19d^2Da^3-2bd(7Cd-30cD)a^2-3b^2c(14Cd-11cD)a+56Ab^3d^2+b(-((-33Dc^2+42Cdc-56Bd^2)b^2)-10ad(7Cd-6cD)b$$
$$6b^2d$$
$$14b^3d$$
$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 27

$$\int (a+bx)^{5/2}\sqrt{c+dx}\,(19d^2Da^3-2bd(7Cd-30cD)a^2-3b^2c(14Cd-11cD)a+56Ab^3d^2+b(-((-33Dc^2+42Cdc-56Bd^2)b^2)-10ad(7Cd-6cD)b+75$$
$$4d$$
$$14b^3d$$
$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 90

$$\frac{(a+bx)^{7/2}(c+dx)^{3/2}\left(75a^2d^2D-10abd(7Cd-6cD)-\left(b^2\left(-56Bd^2-33c^2D+42cCd\right)\right)\right)}{5d}-7(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d(-24Bd^2-27c^2D+28cCd)-(b^3($$
$$10d$$
$$4d$$
$$14b^3d$$
$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 60

$$\frac{(a+bx)^{7/2}(c+dx)^{3/2}\left(75a^2d^2D-10abd(7Cd-6cD)-\left(b^2\left(-56Bd^2-33c^2D+42cCd\right)\right)\right)}{5d}-7(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d(-24Bd^2-27c^2D+28cCd)-(b^3($$
$$4d$$
$$14b^3d$$
$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 60

$$7\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d\left(-24Bd^2-27c^2D+28cCd\right)-\left(b^3\left(\right.\right.$$

$$\frac{\frac{(a+bx)^{7/2}(c+dx)^{3/2}\left(75a^2d^2D-10abd(7Cd-6cD)-\left(b^2\left(-56Bd^2-33c^2D+42cCd\right)\right)\right)}{5d}-}{4d}$$

$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 60

$$7\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d\left(-24Bd^2-27c^2D+28cCd\right)-\left(b^3\left(\right.\right.$$

$$\frac{\frac{(a+bx)^{7/2}(c+dx)^{3/2}\left(75a^2d^2D-10abd(7Cd-6cD)-\left(b^2\left(-56Bd^2-33c^2D+42cCd\right)\right)\right)}{5d}-}{4d}$$

$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 60

$$7\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d\left(-24Bd^2-27c^2D+28cCd\right)-\left(b^3\left(\right.\right.$$

$$\frac{(a+bx)^{7/2}(c+dx)^{3/2}\left(75a^2d^2D-10abd(7Cd-6cD)-\left(b^2\left(-56Bd^2-33c^2D+42cCd\right)\right)\right)}{5d}-$$

$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 66

$$7\left(5a^3d^3D-5a^2bd^2(2Cd-3cD)-ab^2d\left(-24Bd^2-27c^2D+28cCd\right)-\left(b^3\left(\right.\right.\right.$$

$$\frac{\frac{(a+bx)^{7/2}(c+dx)^{3/2}\left(75a^2d^2D-10abd(7Cd-6cD)-\left(b^2\left(-56Bd^2-33c^2D+42cCd\right)\right)\right)}{5d}-}{}$$

$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

$\downarrow$ 221

$$3(bc-$$

$$5(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\right.$$

$$(bc-ad)\left(\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}-\right.$$

$$7 \qquad 8b$$

$$\frac{(a+bx)^{7/2}(c+dx)^{3/2}\left(75a^2d^2D-10abd(7Cd-6cD)-\left(b^2\left(-56Bd^2-33c^2D+42cCd\right)\right)\right)}{5d}-$$

$$\frac{D(a+bx)^{11/2}(c+dx)^{3/2}}{7b^3d}$$

input

```
Int[(a + b*x)^(5/2)*Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3),x]
```

output

```
(D*(a + b*x)^(11/2)*(c + d*x)^(3/2))/(7*b^3*d) + (((14*b*C*d - 11*b*c*D -
31*a*d*D)*(a + b*x)^(9/2)*(c + d*x)^(3/2))/(6*d) + (((75*a^2*d^2*D - 10*a*
b*d*(7*C*d - 6*c*D) - b^2*(42*c*C*d - 56*B*d^2 - 33*c^2*D))*(a + b*x)^(7/2
)*(c + d*x)^(3/2))/(5*d) - (7*(5*a^3*d^3*D - 5*a^2*b*d^2*(2*C*d - 3*c*D) -
 a*b^2*d*(28*c*C*d - 24*B*d^2 - 27*c^2*D) - b^3*(42*c^2*C*d - 56*B*c*d^2 +
 80*A*d^3 - 33*c^3*D))*(((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b) + ((b*c - a*
d)*(((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d) - (5*(b*c - a*d)*(((a + b*x)^(3/
2)*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d
- ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(
Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*b)))/(10*d))/(4*d))/(14*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3757 vs. $2(706) = 1412$.

Time = 0.52 (sec) , antiderivative size = 3758, normalized size of antiderivative = 4.93

method	result	size
default	Expression too large to display	3758

input
```
int((b*x+a)^(5/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOS
E)
```

output
```
-1/215040*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(-38080*B*a*b^5*c^3*d^3*((b*x+a)*(d*
x+c))^(1/2)*(d*b)^(1/2)-3360*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^3*b^3
*d^6*x-7840*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^6*c^3*d^3*x+5040*B*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^4*b^2*d^6+11760*B*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)*b^6*c^4*d^2-2100*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^5*
b*d^6-8820*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^6*c^5*d-33600*A*ln(1/2*
(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b
^4*c*d^6+50400*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d
+b*c)/(d*b)^(1/2))*a^2*b^5*c^2*d^5-33600*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^6*c^3*d^4+4200*B*ln(1/2*(
2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b^
3*c*d^6+8400*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b
*c)/(d*b)^(1/2))*a^3*b^4*c^2*d^5-25200*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^5*c^3*d^4+21000*B*ln(1/2*
(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^6
*c^4*d^3-1260*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a^5*b^2*c*d^6-1050*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b^3*c^2*d^5-4200*C*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^4*
c^3*d^4+15750*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a...
```

Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 2320, normalized size of antiderivative = 3.04

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="fr
icas")
```

output
```
[-1/430080*(105*(33*D*b^7*c^7 - 21*(5*D*a*b^6 + 2*C*b^7)*c^6*d + 7*(15*D*a
^2*b^5 + 20*C*a*b^6 + 8*B*b^7)*c^5*d^2 - 5*(5*D*a^3*b^4 + 30*C*a^2*b^5 + 4
0*B*a*b^6 + 16*A*b^7)*c^4*d^3 - 5*(D*a^4*b^3 - 8*C*a^3*b^4 - 48*B*a^2*b^5
- 64*A*a*b^6)*c^3*d^4 - (3*D*a^5*b^2 - 10*C*a^4*b^3 + 80*B*a^3*b^4 + 480*A
*a^2*b^5)*c^2*d^5 - (5*D*a^6*b - 12*C*a^5*b^2 + 40*B*a^4*b^3 - 320*A*a^3*b
^4)*c*d^6 + (5*D*a^7 - 10*C*a^6*b + 24*B*a^5*b^2 - 80*A*a^4*b^3)*d^7)*sqrt
(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c
 + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) -
 4*(15360*D*b^7*d^7*x^6 - 3465*D*b^7*c^6*d + 210*(47*D*a*b^6 + 21*C*b^7)*c
^5*d^2 - 21*(383*D*a^2*b^5 + 630*C*a*b^6 + 280*B*b^7)*c^4*d^3 + 4*(165*D*a
^3*b^4 + 2933*C*a^2*b^5 + 4760*B*a*b^6 + 2100*A*b^7)*c^3*d^4 + 7*(55*D*a^4
*b^3 - 180*C*a^3*b^4 - 2768*B*a^2*b^5 - 4400*A*a*b^6)*c^2*d^5 + 70*(5*D*a^
5*b^2 - 13*C*a^4*b^3 + 48*B*a^3*b^4 + 584*A*a^2*b^5)*c*d^6 - 105*(5*D*a^6*
b - 10*C*a^5*b^2 + 24*B*a^4*b^3 - 80*A*a^3*b^4)*d^7 + 1280*(D*b^7*c*d^6 +
(29*D*a*b^6 + 14*C*b^7)*d^7)*x^5 - 128*(11*D*b^7*c^2*d^5 - 2*(15*D*a*b^6 +
 7*C*b^7)*c*d^6 - (185*D*a^2*b^5 + 350*C*a*b^6 + 168*B*b^7)*d^7)*x^4 + 16*
(99*D*b^7*c^3*d^4 - (271*D*a*b^6 + 126*C*b^7)*c^2*d^5 + (205*D*a^2*b^5 + 3
64*C*a*b^6 + 168*B*b^7)*c*d^6 + 3*(5*D*a^3*b^4 + 630*C*a^2*b^5 + 1176*B*a*
b^6 + 560*A*b^7)*d^7)*x^3 - 8*(231*D*b^7*c^4*d^3 - 6*(106*D*a*b^6 + 49*C*b
^7)*c^3*d^4 + 2*(243*D*a^2*b^5 + 427*C*a*b^6 + 196*B*b^7)*c^2*d^5 - 2*(...
```

Sympy [F]

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+bx)^{\frac{5}{2}}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
integrate((b*x+a)**(5/2)*(d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral((a + b*x)**(5/2)*sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4652 vs. $2(696) = 1392$.

Time = 0.71 (sec) , antiderivative size = 4652, normalized size of antiderivative = 6.10

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x, algorithm="gi
ac")
```

output
```
1/107520*(1680*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x +
 a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*
c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*
d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*
sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*
b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*B*a*abs(b) + 42*(sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)*(2*(4*(2*(b*x + a)*(8*(b*x + a)*(10*(b*x + a)/b^5 + (
b^30*c*d^9 - 61*a*b^29*d^10)/(b^34*d^10)) - 3*(3*b^31*c^2*d^8 + 14*a*b^30*
c*d^9 - 417*a^2*b^29*d^10)/(b^34*d^10)) + (21*b^32*c^3*d^7 + 77*a*b^31*c^2
*d^8 + 183*a^2*b^30*c*d^9 - 3481*a^3*b^29*d^10)/(b^34*d^10))*(b*x + a) - 5
*(21*b^33*c^4*d^6 + 56*a*b^32*c^3*d^7 + 106*a^2*b^31*c^2*d^8 + 176*a^3*b^3
0*c*d^9 - 2279*a^4*b^29*d^10)/(b^34*d^10))*(b*x + a) + 15*(21*b^34*c^5*d^5
 + 35*a*b^33*c^4*d^6 + 50*a^2*b^32*c^3*d^7 + 70*a^3*b^31*c^2*d^8 + 105*a^4
*b^30*c*d^9 - 793*a^5*b^29*d^10)/(b^34*d^10))*sqrt(b*x + a) + 15*(21*b^6*c
^6 + 14*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 + 20*a^3*b^3*c^3*d^3 + 35*a^4*b^2
*c^2*d^4 + 126*a^5*b*c*d^5 - 231*a^6*d^6)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
 + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^4*d^5))*D*a*abs(b) -
 107520*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+b\,x)^{5/2}\,\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^(5/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^(5/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1752, normalized size of antiderivative = 2.30

$$\int (a+bx)^{5/2}\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(5/2)*(d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
( - 525*sqrt(c + d*x)*sqrt(a + b*x)*a**6*b*d**7 + 1400*sqrt(c + d*x)*sqrt(
a + b*x)*a**5*b**2*c*d**6 + 350*sqrt(c + d*x)*sqrt(a + b*x)*a**5*b**2*d**7
*x + 5880*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b**4*d**6 - 525*sqrt(c + d*x)*s
qrt(a + b*x)*a**4*b**3*c**2*d**5 - 910*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b*
*3*c*d**6*x - 280*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b**3*d**7*x**2 + 44240*
sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**5*c*d**5 + 67760*sqrt(c + d*x)*sqrt(a
+ b*x)*a**3*b**5*d**6*x - 600*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**4*c**3*d
**4 + 300*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**4*c**2*d**5*x + 720*sqrt(c +
 d*x)*sqrt(a + b*x)*a**3*b**4*c*d**6*x**2 + 240*sqrt(c + d*x)*sqrt(a + b*x
)*a**3*b**4*d**7*x**3 - 50176*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**6*c**2*d
**4 + 32368*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**6*c*d**5*x + 117824*sqrt(c
 + d*x)*sqrt(a + b*x)*a**2*b**6*d**6*x**2 + 3689*sqrt(c + d*x)*sqrt(a + b*
x)*a**2*b**5*c**4*d**3 - 2332*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**5*c**3*d
**4*x + 1824*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**5*c**2*d**5*x**2 + 33520*
sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**5*c*d**6*x**3 + 23680*sqrt(c + d*x)*sq
rt(a + b*x)*a**2*b**5*d**7*x**4 + 27440*sqrt(c + d*x)*sqrt(a + b*x)*a*b**7
*c**3*d**3 - 18032*sqrt(c + d*x)*sqrt(a + b*x)*a*b**7*c**2*d**4*x + 14336*
sqrt(c + d*x)*sqrt(a + b*x)*a*b**7*c*d**5*x**2 + 83328*sqrt(c + d*x)*sqrt(
a + b*x)*a*b**7*d**6*x**3 - 3360*sqrt(c + d*x)*sqrt(a + b*x)*a*b**6*c**5*d
**2 + 2198*sqrt(c + d*x)*sqrt(a + b*x)*a*b**6*c**4*d**3*x - 1744*sqrt(c...
```

3.121 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx$

Optimal result . . . 1135
Mathematica [A] (verified) . . . 1136
Rubi [A] (verified) . . . 1137
Maple [B] (verified) . . . 1143
Fricas [A] (verification not implemented) . . . 1144
Sympy [F] . . . 1145
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Giac [A] (verification not implemented) . . . 1146
Mupad [F(-1)] . . . 1147
Reduce [B] (verification not implemented) . . . 1148

Optimal result

Integrand size = 34, antiderivative size = 639

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx =$$

$$-\frac{(bc-ad)^2\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d(56cCd-40Bd^2-63c^2D)-b^3(252c^2Cd-280Bcd^2+\right.}{512b^3d^6}$$

$$+\frac{(bc-ad)\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d(56cCd-40Bd^2-63c^2D)-b^3(252c^2Cd-280Bcd^2+\right.}{768b^3d^5}$$

$$-\frac{(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d(56cCd-40Bd^2-63c^2D)-b^3(252c^2Cd-280Bcd^2+320Ad^3-}{960b^3d^4}$$

$$-\frac{\left(36bcC-40bBd+44aCd-42acD-\frac{33bc^2D}{d}-\frac{45a^2dD}{b}\right)(a+bx)^{7/2}\sqrt{c+dx}}{160b^2d^2}$$

$$+\frac{(12bCd-11bcD-25adD)(a+bx)^{9/2}\sqrt{c+dx}}{60b^3d^2}+\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$+\frac{(bc-ad)^3\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d(56cCd-40Bd^2-63c^2D)-b^3(252c^2Cd-280Bcd^2+\right.}{512b^{7/2}d^{13/2}}$$

output
```
-1/512*(-a*d+b*c)^2*(5*a^3*d^3*D-3*a^2*b*d^2*(4*C*d-7*D*c)-a*b^2*d*(-40*B*
d^2+56*C*c*d-63*D*c^2)-b^3*(320*A*d^3-280*B*c*d^2+252*C*c^2*d-231*D*c^3))*
(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d^6+1/768*(-a*d+b*c)*(5*a^3*d^3*D-3*a^2*b*
d^2*(4*C*d-7*D*c)-a*b^2*d*(-40*B*d^2+56*C*c*d-63*D*c^2)-b^3*(320*A*d^3-280
*B*c*d^2+252*C*c^2*d-231*D*c^3))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3/d^5-1/960
*(5*a^3*d^3*D-3*a^2*b*d^2*(4*C*d-7*D*c)-a*b^2*d*(-40*B*d^2+56*C*c*d-63*D*c
^2)-b^3*(320*A*d^3-280*B*c*d^2+252*C*c^2*d-231*D*c^3))*(b*x+a)^(5/2)*(d*x+
c)^(1/2)/b^3/d^4-1/160*(36*C*b*c-40*B*b*d+44*C*a*d-42*D*a*c-33*b*c^2*D/d-4
5*a^2*d*D/b)*(b*x+a)^(7/2)*(d*x+c)^(1/2)/b^2/d^2+1/60*(12*C*b*d-25*D*a*d-1
1*D*b*c)*(b*x+a)^(9/2)*(d*x+c)^(1/2)/b^3/d^2+1/6*D*(b*x+a)^(11/2)*(d*x+c)^
(1/2)/b^3/d+1/512*(-a*d+b*c)^3*(5*a^3*d^3*D-3*a^2*b*d^2*(4*C*d-7*D*c)-a*b^
2*d*(-40*B*d^2+56*C*c*d-63*D*c^2)-b^3*(320*A*d^3-280*B*c*d^2+252*C*c^2*d-2
31*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(7/2)/d^
(13/2)
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}(75a^5d^5D-5a^4bd^4(36Cd-23cD+10dDx)}{}$$

$$+\frac{(bc-ad)^3\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)+ab^2d(-56cCd+40Bd^2+63c^2D)+b^3(-252c^2Cd+280Bcd\right.}{512b^{7/2}d^{13/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(75*a^5*d^5*D - 5*a^4*b*d^4*(36*C*d - 23*c*D
+ 10*d*D*x) + 10*a^3*b^2*d^3*(27*c^2*D - 4*c*d*(9*C + 2*D*x) + 4*d^2*(15*B
 + x*(3*C + D*x))) + a*b^4*d*(8295*c^4*D - 168*c^3*d*(55*C + 32*D*x) + 8*c
^2*d^2*(1325*B + x*(749*C + 531*D*x)) + 64*d^4*x*(130*A + x*(85*B + 63*C*x
 + 50*D*x^2)) - 32*c*d^3*(400*A + x*(215*B + 148*C*x + 113*D*x^2))) + b^5*
(-3465*c^5*D + 210*c^4*d*(18*C + 11*D*x) - 168*c^3*d^2*(25*B + x*(15*C + 1
1*D*x)) + 128*d^5*x^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 16*c^2*d^3*(
300*A + x*(175*B + 9*x*(14*C + 11*D*x))) - 64*c*d^4*x*(50*A + x*(35*B + x*
(27*C + 22*D*x)))) + 2*a^2*b^3*d^2*(-2709*c^3*D + 2*c^2*d*(1564*C + 831*D*
x) - 4*c*d^2*(955*B + x*(481*C + 321*D*x)) + 8*d^3*(660*A + x*(295*B + 3*x
*(62*C + 45*D*x))))))/(7680*b^3*d^6) + ((b*c - a*d)^3*(5*a^3*d^3*D - 3*a^2
*b*d^2*(4*C*d - 7*c*D) + a*b^2*d*(-56*c*C*d + 40*B*d^2 + 63*c^2*D) + b^3*(
-252*c^2*C*d + 280*B*c*d^2 - 320*A*d^3 + 231*c^3*D))*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(512*b^(7/2)*d^(13/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2125, 27, 1194, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 2125$$

$$\frac{\int \frac{(a+bx)^{5/2}\left(12Adb^3+(12bCd-25aDd-11bcD)x^2b^2+2\left(-7dDa^2-11bcDa+6b^2Bd\right)xb-a^2(11bc+ad)D\right)}{2\sqrt{c+dx}}dx}{6b^3d}+\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)^{5/2}\left(12Adb^3+(12bCd-25aDd-11bcD)x^2b^2+2\left(-7dDa^2-11bcDa+6b^2Bd\right)xb-a^2(11bc+ad)D\right)}{\sqrt{c+dx}}dx}{12b^3d}+\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$\Big\downarrow \ 1194$$

$$\frac{\frac{\int \frac{3b^2(a+bx)^{5/2}\left(5d^2Da^3-2bd(2Cd-21cD)a^2-3b^2c(12Cd-11cD)a+40Ab^3d^2+b\left(-\left(\left(-33Dc^2+36Cdc-40Bd^2\right)b^2\right)-2ad(22Cd-21cD)b+45a^2d^2D\right)x\right)}{2\sqrt{c+dx}}dx}{5b^2d}+\frac{(\cdots}{\cdots}}{12b^3d}$$

$$\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{(a+bx)^{5/2}\left(5d^2Da^3-2bd(2Cd-21cD)a^2-3b^2c(12Cd-11cD)a+40Ab^3d^2+b\left(-\left(\left(-33Dc^2+36Cdc-40Bd^2\right)b^2\right)-2ad(22Cd-21cD)b+45a^2d^2D\right)x\right)}{\sqrt{c+dx}}dx}{10d}+\frac{(a\cdots}{\cdots}}{12b^3d}$$

$$\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$\Big\downarrow \ 90$$

$$\frac{\frac{3\left(\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(45a^2d^2D-2abd(22Cd-21cD)-\left(b^2\left(-40Bd^2-33c^2D+36cCd\right)\right)\right)}{4d}-\frac{\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d\left(-40Bd^2-63c^2D+56cCd\right)-\left(b^3\left(\cdots\right.\right.\right.}{8d}\cdots\right.}{10d}\cdots}{12b^3d}$$

$$\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{3\left(\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(45a^2d^2D-2abd(22Cd-21cD)-\left(b^2\left(-40Bd^2-33c^2D+36cCd\right)\right)\right)}{4d}-\frac{\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d\left(-40Bd^2-63c^2D+56cCd\right)-\left(b^3\left(\cdots\right.\right.\right.}{\cdots}\right.}{10d}\cdots}{12b^3d}$$

$$\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$\Big\downarrow \ 60$$

$$3\left(\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(45a^2d^2D-2abd(22Cd-21cD)-\left(b^2\left(-40Bd^2-33c^2D+36cCd\right)\right)\right)}{4d}-\frac{\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d\left(-40Bd^2-63c^2D+56cCd\right)-\left(b^3\left(\right.\right.\right.}{}\right.$$

$$10d$$

$$\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$\Big\downarrow \; 60$$

$$3\left(\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(45a^2d^2D-2abd(22Cd-21cD)-\left(b^2\left(-40Bd^2-33c^2D+36cCd\right)\right)\right)}{4d}-\frac{\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d\left(-40Bd^2-63c^2D+56cCd\right)-\left(b^3\left(\right.\right.\right.}{}\right.$$

$$1$$

$$\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$$\Big\downarrow \; 66$$

$$3\left(\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(45a^2d^2D-2abd(22Cd-21cD)-\left(b^2\left(-40Bd^2-33c^2D+36cCd\right)\right)\right)}{4d}-\frac{\left(5a^3d^3D-3a^2bd^2(4Cd-7cD)-ab^2d\left(-40Bd^2-63c^2D+56cCd\right)-\left(b^3\left(\right.\right.\right.}{}\right.$$

$$\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

$\downarrow$ 221

$$3\left(\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(45a^2d^2D-2abd(22Cd-21cD)-\left(b^2\left(-40Bd^2-33c^2D+36cCd\right)\right)\right)}{4d}-\frac{\left(\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{D(a+bx)^{11/2}\sqrt{c+dx}}{6b^3d}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[c + d*x],x]
```

output
```
(D*(a + b*x)^(11/2)*Sqrt[c + d*x])/(6*b^3*d) + (((12*b*C*d - 11*b*c*D - 25
*a*d*D)*(a + b*x)^(9/2)*Sqrt[c + d*x])/(5*d) + (3*(((45*a^2*d^2*D - 2*a*b*
d*(22*C*d - 21*c*D) - b^2*(36*c*C*d - 40*B*d^2 - 33*c^2*D))*(a + b*x)^(7/2
)*Sqrt[c + d*x])/(4*d) - ((5*a^3*d^3*D - 3*a^2*b*d^2*(4*C*d - 7*c*D) - a*b
^2*d*(56*c*C*d - 40*B*d^2 - 63*c^2*D) - b^3*(252*c^2*C*d - 280*B*c*d^2 + 3
20*A*d^3 - 231*c^3*D))*(((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d) - (5*(b*c -
a*d)*(((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*
x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*d)))/(10*d))/(12
*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2805 vs. $2(589) = 1178$.

Time = 0.50 (sec) , antiderivative size = 2806, normalized size of antiderivative = 4.39

method	result	size
default	Expression too large to display	2806

input

```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/15360*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(3840*B*b^5*d^5*x^3*((b*x+a)*(d*x+c))^
(1/2)*(d*b)^(1/2)+5120*A*b^5*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-1
0752*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c^3*d^2*x-13760*B*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*c*d^4*x-7696*C*((b*x+a)*(d*x+c))^(1/2)*
(d*b)^(1/2)*a^2*b^3*c*d^4*x+11984*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*
b^4*c^2*d^3*x+6648*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*c^2*d^3*x
+4800*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*
b)^(1/2))*a^3*b^3*d^6+16640*A*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*d^
5*x+230*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^4*b*c*d^4+540*D*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)*a^3*b^2*c^2*d^3-10836*D*((b*x+a)*(d*x+c))^(1/2)*
(d*b)^(1/2)*a^2*b^3*c^3*d^2+16590*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*
b^4*c^4*d-5040*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^3*d^2*x+5600*B*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^2*d^3*x-160*D*a^3*b^2*c*d^4*x*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+2560*D*b^5*d^5*x^5*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+3072*C*b^5*d^5*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+4620
*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^4*d*x+21200*B*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)*a*b^4*c^2*d^3-720*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/
2)*a^3*b^2*c*d^4+21120*A*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*d^5+9
600*A*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^2*d^3+1200*B*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)*a^3*b^2*d^5-8400*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)...
```

Fricas [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 1758, normalized size of antiderivative = 2.75

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fr
icas")
```

output
```
[1/30720*(15*(231*D*b^6*c^6 - 126*(5*D*a*b^5 + 2*C*b^6)*c^5*d + 35*(15*D*a
^2*b^4 + 20*C*a*b^5 + 8*B*b^6)*c^4*d^2 - 20*(5*D*a^3*b^3 + 30*C*a^2*b^4 +
40*B*a*b^5 + 16*A*b^6)*c^3*d^3 - 15*(D*a^4*b^2 - 8*C*a^3*b^3 - 48*B*a^2*b^
4 - 64*A*a*b^5)*c^2*d^4 - 2*(3*D*a^5*b - 10*C*a^4*b^2 + 80*B*a^3*b^3 + 480
*A*a^2*b^4)*c*d^5 - (5*D*a^6 - 12*C*a^5*b + 40*B*a^4*b^2 - 320*A*a^3*b^3)*
d^6)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*
d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(1280*D*b^6*d^6*x^5 - 3465*D*b^6*c^5*d + 105*(79*D*a*b^5 + 36*
C*b^6)*c^4*d^2 - 42*(129*D*a^2*b^4 + 220*C*a*b^5 + 100*B*b^6)*c^3*d^3 + 2*
(135*D*a^3*b^3 + 3128*C*a^2*b^4 + 5300*B*a*b^5 + 2400*A*b^6)*c^2*d^4 + 5*(
23*D*a^4*b^2 - 72*C*a^3*b^3 - 1528*B*a^2*b^4 - 2560*A*a*b^5)*c*d^5 + 15*(5
*D*a^5*b - 12*C*a^4*b^2 + 40*B*a^3*b^3 + 704*A*a^2*b^4)*d^6 - 128*(11*D*b^
6*c*d^5 - (25*D*a*b^5 + 12*C*b^6)*d^6)*x^4 + 16*(99*D*b^6*c^2*d^4 - 2*(113
*D*a*b^5 + 54*C*b^6)*c*d^5 + 3*(45*D*a^2*b^4 + 84*C*a*b^5 + 40*B*b^6)*d^6)
*x^3 - 8*(231*D*b^6*c^3*d^3 - 9*(59*D*a*b^5 + 28*C*b^6)*c^2*d^4 + (321*D*a
^2*b^4 + 592*C*a*b^5 + 280*B*b^6)*c*d^5 - (5*D*a^3*b^3 + 372*C*a^2*b^4 + 6
80*B*a*b^5 + 320*A*b^6)*d^6)*x^2 + 2*(1155*D*b^6*c^4*d^2 - 84*(32*D*a*b^5
+ 15*C*b^6)*c^3*d^3 + 2*(831*D*a^2*b^4 + 1498*C*a*b^5 + 700*B*b^6)*c^2*d^4
 - 4*(10*D*a^3*b^3 + 481*C*a^2*b^4 + 860*B*a*b^5 + 400*A*b^6)*c*d^5 - 5*(5
*D*a^4*b^2 - 12*C*a^3*b^3 - 472*B*a^2*b^4 - 832*A*a*b^5)*d^6)*x)*sqrt(b...
```

Sympy [F]

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx=\int \frac{(a+bx)^{\frac{5}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/sqrt(c + d*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="gi
ac")
```

output
```
1/7680*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(2*(b*x + a)*(8*(b*x + a
)*(10*(b*x + a)*D/(b^4*d) - (11*D*b^13*c*d^9 + 25*D*a*b^12*d^10 - 12*C*b^1
3*d^10)/(b^16*d^11)) + 3*(33*D*b^14*c^2*d^8 + 42*D*a*b^13*c*d^9 - 36*C*b^1
4*c*d^9 + 45*D*a^2*b^12*d^10 - 44*C*a*b^13*d^10 + 40*B*b^14*d^10)/(b^16*d^
11)) - (231*D*b^15*c^3*d^7 + 63*D*a*b^14*c^2*d^8 - 252*C*b^15*c^2*d^8 + 21
*D*a^2*b^13*c*d^9 - 56*C*a*b^14*c*d^9 + 280*B*b^15*c*d^9 + 5*D*a^3*b^12*d^
10 - 12*C*a^2*b^13*d^10 + 40*B*a*b^14*d^10 - 320*A*b^15*d^10)/(b^16*d^11))
*(b*x + a) + 5*(231*D*b^16*c^4*d^6 - 168*D*a*b^15*c^3*d^7 - 252*C*b^16*c^3
*d^7 - 42*D*a^2*b^14*c^2*d^8 + 196*C*a*b^15*c^2*d^8 + 280*B*b^16*c^2*d^8 -
 16*D*a^3*b^13*c*d^9 + 44*C*a^2*b^14*c*d^9 - 240*B*a*b^15*c*d^9 - 320*A*b^
16*c*d^9 - 5*D*a^4*b^12*d^10 + 12*C*a^3*b^13*d^10 - 40*B*a^2*b^14*d^10 + 3
20*A*a*b^15*d^10)/(b^16*d^11))*(b*x + a) - 15*(231*D*b^17*c^5*d^5 - 399*D*
a*b^16*c^4*d^6 - 252*C*b^17*c^4*d^6 + 126*D*a^2*b^15*c^3*d^7 + 448*C*a*b^1
6*c^3*d^7 + 280*B*b^17*c^3*d^7 + 26*D*a^3*b^14*c^2*d^8 - 152*C*a^2*b^15*c^
2*d^8 - 520*B*a*b^16*c^2*d^8 - 320*A*b^17*c^2*d^8 + 11*D*a^4*b^13*c*d^9 -
32*C*a^3*b^14*c*d^9 + 200*B*a^2*b^15*c*d^9 + 640*A*a*b^16*c*d^9 + 5*D*a^5*
b^12*d^10 - 12*C*a^4*b^13*d^10 + 40*B*a^3*b^14*d^10 - 320*A*a^2*b^15*d^10)
/(b^16*d^11))*sqrt(b*x + a) - 15*(231*D*b^6*c^6 - 630*D*a*b^5*c^5*d - 252*
C*b^6*c^5*d + 525*D*a^2*b^4*c^4*d^2 + 700*C*a*b^5*c^4*d^2 + 280*B*b^6*c^4*
d^2 - 100*D*a^3*b^3*c^3*d^3 - 600*C*a^2*b^4*c^3*d^3 - 800*B*a*b^5*c^3*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{c+dx}}\,dx=\int \frac{(a+b\,x)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1334, normalized size of antiderivative = 2.09

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
(75*sqrt(c + d*x)*sqrt(a + b*x)*a**5*b*d**6 - 65*sqrt(c + d*x)*sqrt(a + b*
x)*a**4*b**2*c*d**5 - 50*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b**2*d**6*x + 11
160*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**4*d**5 - 90*sqrt(c + d*x)*sqrt(a +
 b*x)*a**3*b**3*c**2*d**4 + 40*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**3*c*d**
5*x + 40*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**3*d**6*x**2 - 20440*sqrt(c +
d*x)*sqrt(a + b*x)*a**2*b**5*c*d**4 + 13040*sqrt(c + d*x)*sqrt(a + b*x)*a*
*2*b**5*d**5*x + 838*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*c**3*d**3 - 524
*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*c**2*d**4*x + 408*sqrt(c + d*x)*sqr
t(a + b*x)*a**2*b**4*c*d**5*x**2 + 2160*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b
**4*d**6*x**3 + 15400*sqrt(c + d*x)*sqrt(a + b*x)*a*b**6*c**2*d**3 - 10080
*sqrt(c + d*x)*sqrt(a + b*x)*a*b**6*c*d**4*x + 8000*sqrt(c + d*x)*sqrt(a +
 b*x)*a*b**6*d**5*x**2 - 945*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*c**4*d**2
+ 616*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*c**3*d**3*x - 488*sqrt(c + d*x)*s
qrt(a + b*x)*a*b**5*c**2*d**4*x**2 + 416*sqrt(c + d*x)*sqrt(a + b*x)*a*b**
5*c*d**5*x**3 + 3200*sqrt(c + d*x)*sqrt(a + b*x)*a*b**5*d**6*x**4 - 4200*s
qrt(c + d*x)*sqrt(a + b*x)*b**7*c**3*d**2 + 2800*sqrt(c + d*x)*sqrt(a + b*
x)*b**7*c**2*d**3*x - 2240*sqrt(c + d*x)*sqrt(a + b*x)*b**7*c*d**4*x**2 +
1920*sqrt(c + d*x)*sqrt(a + b*x)*b**7*d**5*x**3 + 315*sqrt(c + d*x)*sqrt(a
 + b*x)*b**6*c**5*d - 210*sqrt(c + d*x)*sqrt(a + b*x)*b**6*c**4*d**2*x + 1
68*sqrt(c + d*x)*sqrt(a + b*x)*b**6*c**3*d**3*x**2 - 144*sqrt(c + d*x)*...
```

3.122 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx$

Optimal result . . . 1149
Mathematica [A] (verified) . . . 1150
Rubi [A] (verified) . . . 1151
Maple [B] (verified) . . . 1156
Fricas [A] (verification not implemented) . . . 1157
Sympy [F] . . . 1158
Maxima [F(-2)] . . . 1158
Giac [B] (verification not implemented) . . . 1158
Mupad [F(-1)] . . . 1159
Reduce [F] . . . 1160

Optimal result

Integrand size = 34, antiderivative size = 567

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{5/2}}{d^4\sqrt{c+dx}}$$
$$-\frac{(bc-ad)\,(3a^3d^3D-a^2bd^2(10Cd-21cD)-ab^2d(140cCd-80Bd^2-189c^2D)+b^3(630c^2Cd-560Bcd}{128b^2d^6}$$
$$+\frac{(3a^3d^3D-a^2bd^2(10Cd-21cD)-ab^2d(140cCd-80Bd^2-189c^2D)+b^3(630c^2Cd-560Bcd^2+480Ad}{192b^2d^5}$$
$$+\frac{(3a^2d^2D-2abd(5Cd-12cD)-b^2(150cCd-80Bd^2-213c^2D))\,(a+bx)^{5/2}\sqrt{c+dx}}{240b^2d^4}$$
$$+\frac{(10bCd-19bcD-11adD)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2d^3}+\frac{D(a+bx)^{9/2}\sqrt{c+dx}}{5b^2d^2}$$
$$+\frac{(bc-ad)^2\,(3a^3d^3D-a^2bd^2(10Cd-21cD)-ab^2d(140cCd-80Bd^2-189c^2D)+b^3(630c^2Cd-560Bcd}{128b^{5/2}d^{13/2}}$$

output
```
-2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(5/2)/d^4/(d*x+c)^(1/2)-1/128*(-a
*d+b*c)*(3*a^3*d^3*D-a^2*b*d^2*(10*C*d-21*D*c)-a*b^2*d*(-80*B*d^2+140*C*c*
d-189*D*c^2)+b^3*(480*A*d^3-560*B*c*d^2+630*C*c^2*d-693*D*c^3))*(b*x+a)^(1
/2)*(d*x+c)^(1/2)/b^2/d^6+1/192*(3*a^3*d^3*D-a^2*b*d^2*(10*C*d-21*D*c)-a*b
^2*d*(-80*B*d^2+140*C*c*d-189*D*c^2)+b^3*(480*A*d^3-560*B*c*d^2+630*C*c^2*
d-693*D*c^3))*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^2/d^5+1/240*(3*a^2*d^2*D-2*a*b
*d*(5*C*d-12*D*c)-b^2*(-80*B*d^2+150*C*c*d-213*D*c^2))*(b*x+a)^(5/2)*(d*x+
c)^(1/2)/b^2/d^4+1/40*(10*C*b*d-11*D*a*d-19*D*b*c)*(b*x+a)^(7/2)*(d*x+c)^(
1/2)/b^2/d^3+1/5*D*(b*x+a)^(9/2)*(d*x+c)^(1/2)/b^2/d^2+1/128*(-a*d+b*c)^2*
(3*a^3*d^3*D-a^2*b*d^2*(10*C*d-21*D*c)-a*b^2*d*(-80*B*d^2+140*C*c*d-189*D*
c^2)+b^3*(480*A*d^3-560*B*c*d^2+630*C*c^2*d-693*D*c^3))*arctanh(d^(1/2)*(b
*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(13/2)
```

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 519, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{a+bx}(-45a^4d^4D(c+dx)+30a^3bd^3(c+dx)(5Cd-8cD+$$

$$+\frac{(bc-ad)^2\left(3a^3d^3D+a^2bd^2(-10Cd+21cD)+ab^2d(-140cCd+80Bd^2+189c^2D)+b^3(630c^2Cd-560\right.}{128b^{5/2}d^{13/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[a + b*x]*(-45*a^4*d^4*D*(c + d*x) + 30*a^3*b*d^3*(c + d*x)*(5*C*d -
8*c*D + d*D*x) + 2*a^2*b^2*d^2*(4977*c^3*D + c^2*(-4195*C*d + 1986*d*D*x)
+ c*d^2*(3240*B - x*(1685*C + 699*D*x)) + 2*d^3*(-960*A + x*(660*B + 295*C
*x + 186*D*x^2))) + 2*a*b^3*d*(-10080*c^4*D + 21*c^3*d*(425*C - 171*D*x) +
 c^2*d^2*(-7600*B + x*(3185*C + 1377*D*x)) + 8*d^4*x*(270*A + x*(130*B + 8
5*C*x + 63*D*x^2)) + 4*c*d^3*(1500*A - x*(680*B + 305*C*x + 192*D*x^2))) +
 b^4*(10395*c^5*D - 315*c^4*d*(30*C - 11*D*x) + 42*c^3*d^2*(200*B - 3*x*(2
5*C + 11*D*x)) - 16*c*d^4*x*(150*A + x*(70*B + 45*C*x + 33*D*x^2)) + 32*d^
5*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 4*c^2*d^3*(-1800*A + x*(700*
B + 9*x*(35*C + 22*D*x))))))/(1920*b^2*d^6*Sqrt[c + d*x]) + ((b*c - a*d)^2
*(3*a^3*d^3*D + a^2*b*d^2*(-10*C*d + 21*c*D) + a*b^2*d*(-140*c*C*d + 80*B*
d^2 + 189*c^2*D) + b^3*(630*c^2*C*d - 560*B*c*d^2 + 480*A*d^3 - 693*c^3*D)
)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d
^(13/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.73, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2124, 27, 1194, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 2124}$$

$$\frac{2\int \frac{(a+bx)^{5/2}\left(-\left(\left(a-\frac{bc}{d}\right)Dx^2\right)+\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c+6Ad^3\right)}{d^3}\right)}{2\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+bx)^{5/2}\left(-\left(\left(a-\frac{bc}{d}\right)Dx^2\right)+\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c+6Ad^3\right)}{d^3}\right)}{\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$\downarrow$ 1194

$$\frac{\frac{\int \frac{(a+bx)^{5/2}\left(d\left(dDa^3+8bcDa^2+b^2\left(-\frac{19Dc^2}{d}+10Cc-10Bd\right)a-\frac{10b^3\left(-7Dc^3+7Cdc^2-7Bd^2c+6Ad^3\right)}{d^2}\right)+b(bc-ad)(10bCd-11aDd-19bcD)x\right)}{2d\sqrt{c+dx}}dx}{5b^2d}+\frac{D(a+b}{}}{bc-ad}+\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{(a+bx)^{5/2}\left(d\left(dDa^3+8bcDa^2+b^2\left(-\frac{19Dc^2}{d}+10Cc-10Bd\right)a-\frac{10b^3\left(-7Dc^3+7Cdc^2-7Bd^2c+6Ad^3\right)}{d^2}\right)+b(bc-ad)(10bCd-11aDd-19bcD)x\right)}{\sqrt{c+dx}}dx}{10b^2d^2}+\frac{D(a+b}{}}{bc-ad}+\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$\downarrow$ 90

$$\frac{\frac{\frac{(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)(-11adD-19bcD+10bCd)}{4d}-\frac{\left(3a^3d^3D-a^2bd^2(10Cd-21cD)-ab^2d\left(-80Bd^2-189c^2D+140cCd\right)+b^3\left(480Ad^3-560Bcd^2-693c^3D+630\right.}{8d}}{10b^2d^2}}{bc-ad}+\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)(-11adD-19bcD+10bCd)}{4d}-\frac{\left(3a^3d^3D-a^2bd^2(10Cd-21cD)-ab^2d\left(-80Bd^2-189c^2D+140cCd\right)+b^3\left(480Ad^3-560Bcd^2-693c^3D+630\right.}{8d}}{10b^2d^2}}{bc-ad}+\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$\downarrow$ 60

$$\left(3a^3d^3D-a^2bd^2(10Cd-21cD)-ab^2d\left(-80Bd^2-189c^2D+140cCd\right)+b^3\left(480Ad^3-560Bcd^2-693c^3D+630\right.\right.$$

$$\frac{\frac{\frac{(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)(-11adD-19bcD+10bCd)}{4d}-\frac{\cdots}{8d}}{10b^2d^2}}{bc-ad}$$

$$\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$\downarrow$ 60

$$\left(3a^3d^3D-a^2bd^2(10Cd-21cD)-ab^2d\left(-80Bd^2-189c^2D+140cCd\right)+b^3\left(480Ad^3-560Bcd^2-693c^3D+630\right.\right.$$

$$\frac{\frac{\frac{(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)(-11adD-19bcD+10bCd)}{4d}-\cdots}{10b^2d^2}}{bc-\cdots}$$

$$\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$\downarrow$ 66

$$\left(3a^3d^3D-a^2bd^2(10Cd-21cD)-ab^2d\left(-80Bd^2-189c^2D+140cCd\right)+b^3\left(480Ad^3-560Bcd^2-693c^3D+630\right.\right.$$

$$\frac{\frac{\frac{(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)(-11adD-19bcD+10bCd)}{4d}-\cdots}{10b^2d^2}}{bc\cdots}$$

$$\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)(-11adD-19bcD+10bCd)}{4d}-\frac{\left(\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{b}d^{3/2}}\right.}{4d}\right)}{6d}\right.}{}}{10b^2d^2}$$

$$bc$$

$$\frac{2(a+bx)^{7/2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(3/2),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x)^(7/2))/(d^3*(b*c - a*d)*S
qrt[c + d*x]) + (((b*c - a*d)*D*(a + b*x)^(9/2)*Sqrt[c + d*x])/(5*b^2*d^2)
 + (((b*c - a*d)*(10*b*C*d - 19*b*c*D - 11*a*d*D)*(a + b*x)^(7/2)*Sqrt[c +
 d*x])/(4*d) - ((3*a^3*d^3*D - a^2*b*d^2*(10*C*d - 21*c*D) - a*b^2*d*(140*
c*C*d - 80*B*d^2 - 189*c^2*D) + b^3*(630*c^2*C*d - 560*B*c*d^2 + 480*A*d^3
 - 693*c^3*D))*(((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d) - (5*(b*c - a*d)*(((
a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[
c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(8*d))/(10*b^2*d^2))/(b*c -
 a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3459 vs. $2(519) = 1038$.

Time = 0.64 (sec) , antiderivative size = 3460, normalized size of antiderivative = 6.10

method	result	size
default	Expression too large to display	3460

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/3840*(b*x+a)^(1/2)*(-3072*D*a*b^3*c*d^4*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b
)^(1/2)-14400*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a*b^4*c*d^5*x-1800*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2*c*d^5*x+13500*C*ln(1/2*(2
*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3
*c^2*d^4*x-21000*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a
*d+b*c)/(d*b)^(1/2))*a*b^4*c^3*d^3*x+225*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*c*d^5*x+2250*D*ln(1/2*(
2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^
2*c^2*d^4*x-15750*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+
a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^3*d^3*x+23625*D*ln(1/2*(2*b*d*x+2*((b*x+a)
*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^4*d^2*x-8400*B*l
n(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))
*b^5*c^3*d^3*x-150*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+a*d+b*c)/(d*b)^(1/2))*a^4*b*d^6*x+9450*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^4*d^2*x-10395*D*ln(1/2*(
2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^
5*d*x+7200*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c
)/(d*b)^(1/2))*a^2*b^3*c*d^5+768*D*b^4*d^5*x^5*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+960*C*b^4*d^5*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1280*B*b...
```

Fricas [A] (verification not implemented)

Time = 8.28 (sec) , antiderivative size = 1994, normalized size of antiderivative = 3.52

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="fr
icas")
```

output
```
[-1/7680*(15*(693*D*b^5*c^6 - 315*(5*D*a*b^4 + 2*C*b^5)*c^5*d + 70*(15*D*a
^2*b^3 + 20*C*a*b^4 + 8*B*b^5)*c^4*d^2 - 30*(5*D*a^3*b^2 + 30*C*a^2*b^3 +
40*B*a*b^4 + 16*A*b^5)*c^3*d^3 - 15*(D*a^4*b - 8*C*a^3*b^2 - 48*B*a^2*b^3
- 64*A*a*b^4)*c^2*d^4 - (3*D*a^5 - 10*C*a^4*b + 80*B*a^3*b^2 + 480*A*a^2*b
^3)*c*d^5 + (693*D*b^5*c^5*d - 315*(5*D*a*b^4 + 2*C*b^5)*c^4*d^2 + 70*(15*
D*a^2*b^3 + 20*C*a*b^4 + 8*B*b^5)*c^3*d^3 - 30*(5*D*a^3*b^2 + 30*C*a^2*b^3
 + 40*B*a*b^4 + 16*A*b^5)*c^2*d^4 - 15*(D*a^4*b - 8*C*a^3*b^2 - 48*B*a^2*b
^3 - 64*A*a*b^4)*c*d^5 - (3*D*a^5 - 10*C*a^4*b + 80*B*a^3*b^2 + 480*A*a^2*
b^3)*d^6)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*
d + a*b*d^2)*x) - 4*(384*D*b^5*d^6*x^5 + 10395*D*b^5*c^5*d - 3840*A*a^2*b^
3*d^6 - 630*(32*D*a*b^4 + 15*C*b^5)*c^4*d^2 + 42*(237*D*a^2*b^3 + 425*C*a*
b^4 + 200*B*b^5)*c^3*d^3 - 10*(24*D*a^3*b^2 + 839*C*a^2*b^3 + 1520*B*a*b^4
 + 720*A*b^5)*c^2*d^4 - 15*(3*D*a^4*b - 10*C*a^3*b^2 - 432*B*a^2*b^3 - 800
*A*a*b^4)*c*d^5 - 48*(11*D*b^5*c*d^5 - (21*D*a*b^4 + 10*C*b^5)*d^6)*x^4 +
8*(99*D*b^5*c^2*d^4 - 6*(32*D*a*b^4 + 15*C*b^5)*c*d^5 + (93*D*a^2*b^3 + 17
0*C*a*b^4 + 80*B*b^5)*d^6)*x^3 - 2*(693*D*b^5*c^3*d^3 - 9*(153*D*a*b^4 + 7
0*C*b^5)*c^2*d^4 + (699*D*a^2*b^3 + 1220*C*a*b^4 + 560*B*b^5)*c*d^5 - 5*(3
*D*a^3*b^2 + 118*C*a^2*b^3 + 208*B*a*b^4 + 96*A*b^5)*d^6)*x^2 + (3465*D*b^
5*c^4*d^2 - 126*(57*D*a*b^4 + 25*C*b^5)*c^3*d^3 + 2*(1986*D*a^2*b^3 + 3...
```

Sympy [F]

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \int \frac{(a+bx)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(3/2),x)
```

output
```
Integral((a + b*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. $2(515) = 1030$.

Time = 0.32 (sec) , antiderivative size = 1136, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x, algorithm="gi
ac")
```

output
```
1/1920*((2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)*D*abs(b)/(b^3*d) - (11*D
*b^6*c*d^9*abs(b) + 19*D*a*b^5*d^10*abs(b) - 10*C*b^6*d^10*abs(b))/(b^8*d^
11)) + (99*D*b^7*c^2*d^8*abs(b) + 72*D*a*b^6*c*d^9*abs(b) - 90*C*b^7*c*d^9
*abs(b) + 69*D*a^2*b^5*d^10*abs(b) - 70*C*a*b^6*d^10*abs(b) + 80*B*b^7*d^1
0*abs(b))/(b^8*d^11)) - (693*D*b^8*c^3*d^7*abs(b) - 189*D*a*b^7*c^2*d^8*ab
s(b) - 630*C*b^8*c^2*d^8*abs(b) - 21*D*a^2*b^6*c*d^9*abs(b) + 140*C*a*b^7*
c*d^9*abs(b) + 560*B*b^8*c*d^9*abs(b) - 3*D*a^3*b^5*d^10*abs(b) + 10*C*a^2
*b^6*d^10*abs(b) - 80*B*a*b^7*d^10*abs(b) - 480*A*b^8*d^10*abs(b))/(b^8*d^
11))*(b*x + a) + 5*(693*D*b^9*c^4*d^6*abs(b) - 882*D*a*b^8*c^3*d^7*abs(b)
- 630*C*b^9*c^3*d^7*abs(b) + 168*D*a^2*b^7*c^2*d^8*abs(b) + 770*C*a*b^8*c^
2*d^8*abs(b) + 560*B*b^9*c^2*d^8*abs(b) + 18*D*a^3*b^6*c*d^9*abs(b) - 130*
C*a^2*b^7*c*d^9*abs(b) - 640*B*a*b^8*c*d^9*abs(b) - 480*A*b^9*c*d^9*abs(b)
 + 3*D*a^4*b^5*d^10*abs(b) - 10*C*a^3*b^6*d^10*abs(b) + 80*B*a^2*b^7*d^10*
abs(b) + 480*A*a*b^8*d^10*abs(b))/(b^8*d^11))*(b*x + a) + 15*(693*D*b^10*c
^5*d^5*abs(b) - 1575*D*a*b^9*c^4*d^6*abs(b) - 630*C*b^10*c^4*d^6*abs(b) +
1050*D*a^2*b^8*c^3*d^7*abs(b) + 1400*C*a*b^9*c^3*d^7*abs(b) + 560*B*b^10*c
^3*d^7*abs(b) - 150*D*a^3*b^7*c^2*d^8*abs(b) - 900*C*a^2*b^8*c^2*d^8*abs(b
) - 1200*B*a*b^9*c^2*d^8*abs(b) - 480*A*b^10*c^2*d^8*abs(b) - 15*D*a^4*b^6
*c*d^9*abs(b) + 120*C*a^3*b^7*c*d^9*abs(b) + 720*B*a^2*b^8*c*d^9*abs(b) +
960*A*a*b^9*c*d^9*abs(b) - 3*D*a^5*b^5*d^10*abs(b) + 10*C*a^4*b^6*d^10*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx=\int \frac{(a+b\,x)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{3/2}}\,dx=\int \frac{(bx+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}}dx$$

input

```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

output

```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(3/2),x)
```

3.123 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx$

Optimal result 1161
Mathematica [C] (verified) 1162
Rubi [A] (verified) 1163
Maple [B] (verified) 1170
Fricas [B] (verification not implemented) 1171
Sympy [F] 1172
Maxima [F(-2)] 1173
Giac [B] (verification not implemented) 1173
Mupad [F(-1)] 1174
Reduce [F] 1175

Optimal result

Integrand size = 34, antiderivative size = 522

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{5/2}}{3d^4(c+dx)^{3/2}}$$
$$+\frac{2(3ad(2cCd-Bd^2-3c^2D)-b(11c^2Cd-8Bcd^2+5Ad^3-14c^3D))\,(a+bx)^{3/2}}{3d^5\sqrt{c+dx}}$$
$$-\frac{5(a^3d^3D-a^2bd^2(8Cd-21cD)+ab^2d(112cCd-48Bd^2-189c^2D)-b^3(168c^2Cd-112Bcd^2+64Ad^3}{64bd^6}$$
$$-\frac{(5a^2d^2D-10abd(4Cd-11cD)+b^2(136cCd-48Bd^2-259c^2D))\,(a+bx)^{3/2}\sqrt{c+dx}}{96bd^5}$$
$$+\frac{(8bCd-23bcD-adD)(a+bx)^{5/2}\sqrt{c+dx}}{24bd^4}+\frac{D(a+bx)^{7/2}\sqrt{c+dx}}{4bd^3}$$
$$+\frac{5(bc-ad)\,(a^3d^3D-a^2bd^2(8Cd-21cD)+ab^2d(112cCd-48Bd^2-189c^2D)-b^3(168c^2Cd-112Bcd^2}{64b^{3/2}d^{13/2}}$$

output
```
-2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(5/2)/d^4/(d*x+c)^(3/2)+2/3*(3*
a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(5*A*d^3-8*B*c*d^2+11*C*c^2*d-14*D*c^3))*(b
*x+a)^(3/2)/d^5/(d*x+c)^(1/2)-5/64*(a^3*d^3*D-a^2*b*d^2*(8*C*d-21*D*c)+a*b
^2*d*(-48*B*d^2+112*C*c*d-189*D*c^2)-b^3*(64*A*d^3-112*B*c*d^2+168*C*c^2*d
-231*D*c^3))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b/d^6-1/96*(5*a^2*d^2*D-10*a*b*d*
(4*C*d-11*D*c)+b^2*(-48*B*d^2+136*C*c*d-259*D*c^2))*(b*x+a)^(3/2)*(d*x+c)^
(1/2)/b/d^5+1/24*(8*C*b*d-D*a*d-23*D*b*c)*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b/d^
4+1/4*D*(b*x+a)^(7/2)*(d*x+c)^(1/2)/b/d^3+5/64*(-a*d+b*c)*(a^3*d^3*D-a^2*b
*d^2*(8*C*d-21*D*c)+a*b^2*d*(-48*B*d^2+112*C*c*d-189*D*c^2)-b^3*(64*A*d^3-
112*B*c*d^2+168*C*c^2*d-231*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/
(d*x+c)^(1/2))/b^(3/2)/d^(13/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 12.54 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \frac{\sqrt{\frac{b(c+dx)}{bc-ad}}\left(\frac{14\sqrt{bc-ad}(Cd-3cD)\left(15b^3d(bc-ad)^{5/2}(a+bx)\sqrt{\frac{b(c+dx)}{bc-ad}}-10b^3d^2(bc\right.}{}\right.}{}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(Sqrt[(b*(c + d*x))/(b*c - a*d)]*((14*Sqrt[b*c - a*d]*(C*d - 3*c*D)*(15*b^
3*d*(b*c - a*d)^(5/2)*(a + b*x)*Sqrt[(b*(c + d*x))/(b*c - a*d)] - 10*b^3*d
^2*(b*c - a*d)^(3/2)*(a + b*x)^2*Sqrt[(b*(c + d*x))/(b*c - a*d)] + 8*b^3*d
^3*Sqrt[b*c - a*d]*(a + b*x)^3*Sqrt[(b*(c + d*x))/(b*c - a*d)] - 15*b^3*Sq
rt[d]*(b*c - a*d)^3*Sqrt[a + b*x]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c
 - a*d]]))/(3*b^4) + (7*Sqrt[d]*D*(15*b*Sqrt[d]*(b*c - a*d)^3*(a + b*x)*(c
 + d*x) - 10*b*d^(3/2)*(b*c - a*d)^2*(a + b*x)^2*(c + d*x) + 8*b*d^(5/2)*(
b*c - a*d)*(a + b*x)^3*(c + d*x) + 48*b*d^(7/2)*(a + b*x)^4*(c + d*x) - 15
*(b*c - a*d)^(9/2)*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(
Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(12*b^2*Sqrt[(b*(c + d*x))/(b*c
- a*d)]) - (32*d^4*(-2*c*C*d + B*d^2 + 3*c^2*D)*(a + b*x)^4*Hypergeometric
2F1[3/2, 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(-(b*c) + a*d) + (32*b*d
^4*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(a + b*x)^4*Hypergeometric2F1[5/2,
7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(b*c - a*d)^2))/(112*d^7*Sqrt[a +
 b*x]*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.87, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2124, 27, 1193, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{2124}$$

$$\frac{2\int \frac{(a+bx)^{5/2}\left(-3\left(a-\frac{bc}{d}\right)Dx^2+\frac{3(bc-ad)(Cd-cD)x}{d^2}+\frac{3ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c+4Ad^3\right)}{d^3}\right)}{2(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+bx)^{5/2}\left(-3\left(a-\frac{bc}{d}\right)Dx^2+\frac{3(bc-ad)(Cd-cD)x}{d^2}+\frac{3ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c+4Ad^3\right)}{d^3}\right)}{(c+dx)^{3/2}}dx}{3(bc-ad)}+$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{\frac{2\int \frac{3(a+bx)^{5/2}\left(\left(-28Dc^3+21Cdc^2-14Bd^2c+8Ad^3\right)b^2-2ad\left(-11Dc^2+7Cdc-3Bd^2\right)b+a^2d^2(Cd-2cD)+d(bc-ad)^2Dx\right)}{2d^3\sqrt{c+dx}}dx}{bc-ad}+\frac{2(a+bx)^{7/2}\left(3ad\left(-Bd^2-3c\right.\right.}{}}{3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{(a+bx)^{5/2}\left(\left(-28Dc^3+21Cdc^2-14Bd^2c+8Ad^3\right)b^2-2ad\left(-11Dc^2+7Cdc-3Bd^2\right)b+a^2d^2(Cd-2cD)+d(bc-ad)^2Dx\right)}{\sqrt{c+dx}}dx}{d^3(bc-ad)}+\frac{2(a+bx)^{7/2}\left(3ad\left(-Bd^2-3c^2\right.\right.}{}}{3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{3\left(\frac{D(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)^2}{4b}-\frac{\left(a^3d^3D-a^2bd^2(8Cd-21cD)+ab^2d\left(-48Bd^2-189c^2D+112cCd\right)-\left(b^3\left(64Ad^3-112Bcd^2-231c^3D+168c^2Cd\right)\right)\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}}{8b}\right.}{d^3(bc-ad)}}{3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{3\left(\frac{\frac{D(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)^2}{4b}-\frac{\left(a^3d^3D-a^2bd^2(8Cd-21cD)+ab^2d\left(-48Bd^2-189c^2D+112cCd\right)-\left(b^3\left(64Ad^3-112Bcd^2-231c^3D+168c^2Cd\right)\right)\right)\left(\frac{(a+bx)^{5/2}}{3d}\cdots\right.}{8b}}{d^3(bc-ad)}\cdots\right.}{3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{3\left(\frac{\frac{D(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)^2}{4b}-\frac{\left(a^3d^3D-a^2bd^2(8Cd-21cD)+ab^2d\left(-48Bd^2-189c^2D+112cCd\right)-\left(b^3\left(64Ad^3-112Bcd^2-231c^3D+168c^2Cd\right)\right)\right)\left(\frac{(a+bx)^{5/2}}{3d}\cdots\right.}{8b}}{d^3(bc-ad)}\cdots\right.}{3(bc\cdots}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$3\left(\frac{D(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)^2}{4b}-\frac{\left(a^3d^3D-a^2bd^2(8Cd-21cD)+ab^2d\left(-48Bd^2-189c^2D+112cCd\right)-\left(b^3\left(64Ad^3-112Bcd^2-231c^3D+168c^2Cd\right)\right)\right)\left(\frac{(a+bx)^{5/2}}{3d}\right.}{8b}\right.$$

$$d^3(bc-ad)$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 66$$

$$\frac{3\left(\frac{D(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)^2}{4b}-\frac{\left(a^3d^3D-a^2bd^2(8Cd-21cD)+ab^2d\left(-48Bd^2-189c^2D+112cCd\right)-\left(b^3\left(64Ad^3-112Bcd^2-231c^3D+168c^2Cd\right)\right)\right)\left(\frac{(a+bx)^{5/2}}{3d}\right.}{8b}\right.}{d^3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$\downarrow$ 221

$$\frac{3\left(\frac{D(a+bx)^{7/2}\sqrt{c+dx}(bc-ad)^2}{4b}-\frac{\left(\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)}{6d}\right)(\cdots}{8b}\cdots\right)}{d^3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(5/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(7/2))/(3*(b*c - a*d)*(
c + d*x)^(3/2)) + ((2*(3*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(10*c^2*C*d -
 7*B*c*d^2 + 4*A*d^3 - 13*c^3*D))*(a + b*x)^(7/2))/(d^3*(b*c - a*d)*Sqrt[c
 + d*x]) + (3*(((b*c - a*d)^2*D*(a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b) - ((a
^3*d^3*D - a^2*b*d^2*(8*C*d - 21*c*D) + a*b^2*d*(112*c*C*d - 48*B*d^2 - 18
9*c^2*D) - b^3*(168*c^2*C*d - 112*B*c*d^2 + 64*A*d^3 - 231*c^3*D))*(((a +
b*x)^(5/2)*Sqrt[c + d*x])/(3*d) - (5*(b*c - a*d)*(((a + b*x)^(3/2)*Sqrt[c
+ d*x])/(2*d) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c -
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^
(3/2))))/(4*d)))/(6*d)))/(8*b)))/(d^3*(b*c - a*d)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3663 vs. $2(472) = 944$.

Time = 0.54 (sec) , antiderivative size = 3664, normalized size of antiderivative = 7.02

method	result	size
default	Expression too large to display	3664

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
1/384*(b*x+a)^(1/2)*(-966*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b*c*d^
4*x^2-3600*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c
)/(d*b)^(1/2))*a^2*b^2*c^2*d^4*x+8400*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^3*d^3*x-1280*A*a*b^2*c*d
^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-512*B*a^2*b*c*d^4*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)-1792*A*a*b^2*d^5*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+
1680*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b
)^(1/2))*b^4*c^2*d^4*x^2-2520*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*
(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^3*d^3*x^2+3360*B*ln(1/2*(2*b*d*x+2
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^3*d^3*x-9
240*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^3*c^4*d*x+3680*B*(d*b)^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)*a*b^2*c^2*d^3-6720*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^
(1/2)*a*b^2*c^3*d^2-960*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^3*d^3+30*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)*a^3*d^5*x^2-30*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)+a*d+b*c)/(d*b)^(1/2))*a^4*c*d^5*x+120*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*
x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b*c^2*d^4-2400*B*ln(1/2*
(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3
*c*d^5*x^2+2592*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b*c*d^4*x-1536*C
*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^2*c*d^4*x^2+2322*D*(d*b)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(469) = 938$.

Time = 21.14 (sec) , antiderivative size = 2058, normalized size of antiderivative = 3.94

$$\int \frac{(a+bx)^{5/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="fr
icas")
```

output
```
[1/768*(15*(231*D*b^4*c^6 - 84*(5*D*a*b^3 + 2*C*b^4)*c^5*d + 14*(15*D*a^2*
b^2 + 20*C*a*b^3 + 8*B*b^4)*c^4*d^2 - 4*(5*D*a^3*b + 30*C*a^2*b^2 + 40*B*a
*b^3 + 16*A*b^4)*c^3*d^3 - (D*a^4 - 8*C*a^3*b - 48*B*a^2*b^2 - 64*A*a*b^3)
*c^2*d^4 + (231*D*b^4*c^4*d^2 - 84*(5*D*a*b^3 + 2*C*b^4)*c^3*d^3 + 14*(15*
D*a^2*b^2 + 20*C*a*b^3 + 8*B*b^4)*c^2*d^4 - 4*(5*D*a^3*b + 30*C*a^2*b^2 +
40*B*a*b^3 + 16*A*b^4)*c*d^5 - (D*a^4 - 8*C*a^3*b - 48*B*a^2*b^2 - 64*A*a*
b^3)*d^6)*x^2 + 2*(231*D*b^4*c^5*d - 84*(5*D*a*b^3 + 2*C*b^4)*c^4*d^2 + 14
*(15*D*a^2*b^2 + 20*C*a*b^3 + 8*B*b^4)*c^3*d^3 - 4*(5*D*a^3*b + 30*C*a^2*b
^2 + 40*B*a*b^3 + 16*A*b^4)*c^2*d^4 - (D*a^4 - 8*C*a^3*b - 48*B*a^2*b^2 -
64*A*a*b^3)*c*d^5)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d +
a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) +
8*(b^2*c*d + a*b*d^2)*x) + 4*(48*D*b^4*d^6*x^5 - 3465*D*b^4*c^5*d - 128*A*
a^2*b^2*d^6 + 105*(49*D*a*b^3 + 24*C*b^4)*c^4*d^2 - 21*(83*D*a^2*b^2 + 160
*C*a*b^3 + 80*B*b^4)*c^3*d^3 + (15*D*a^3*b + 904*C*a^2*b^2 + 1840*B*a*b^3
+ 960*A*b^4)*c^2*d^4 - 128*(2*B*a^2*b^2 + 5*A*a*b^3)*c*d^5 - 8*(11*D*b^4*c
*d^5 - (17*D*a*b^3 + 8*C*b^4)*d^6)*x^4 + 2*(99*D*b^4*c^2*d^4 - 2*(79*D*a*b
^3 + 36*C*b^4)*c*d^5 + (59*D*a^2*b^2 + 104*C*a*b^3 + 48*B*b^4)*d^6)*x^3 -
3*(231*D*b^4*c^3*d^3 - 3*(129*D*a*b^3 + 56*C*b^4)*c^2*d^4 + (161*D*a^2*b^2
 + 256*C*a*b^3 + 112*B*b^4)*c*d^5 - (5*D*a^3*b + 88*C*a^2*b^2 + 144*B*a*b^
3 + 64*A*b^4)*d^6)*x^2 - 2*(2310*D*b^4*c^4*d^2 - 21*(167*D*a*b^3 + 80*C...
```

Sympy [F]

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx=\int \frac{(a+bx)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(5/2),x)
```

output
```
Integral((a + b*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1449 vs. $2(469) = 938$.

Time = 0.39 (sec) , antiderivative size = 1449, normalized size of antiderivative = 2.78

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x, algorithm="gi
ac")
```

output
```
1/192*(((2*(4*(b*x + a)*(6*(D*b^5*c*d^10*abs(b) - D*a*b^4*d^11*abs(b))*(b*
x + a)/(b^6*c*d^11 - a*b^5*d^12) - (11*D*b^6*c^2*d^9*abs(b) + 2*D*a*b^5*c*
d^10*abs(b) - 8*C*b^6*c*d^10*abs(b) - 13*D*a^2*b^4*d^11*abs(b) + 8*C*a*b^5
*d^11*abs(b))/(b^6*c*d^11 - a*b^5*d^12)) + 3*(33*D*b^7*c^3*d^8*abs(b) - 27
*D*a*b^6*c^2*d^9*abs(b) - 24*C*b^7*c^2*d^9*abs(b) + 3*D*a^2*b^5*c*d^10*abs
(b) + 16*C*a*b^6*c*d^10*abs(b) + 16*B*b^7*c*d^10*abs(b) - 9*D*a^3*b^4*d^11
*abs(b) + 8*C*a^2*b^5*d^11*abs(b) - 16*B*a*b^6*d^11*abs(b))/(b^6*c*d^11 -
a*b^5*d^12))*(b*x + a) - 3*(231*D*b^8*c^4*d^7*abs(b) - 420*D*a*b^7*c^3*d^8
*abs(b) - 168*C*b^8*c^3*d^8*abs(b) + 210*D*a^2*b^6*c^2*d^9*abs(b) + 280*C*
a*b^7*c^2*d^9*abs(b) + 112*B*b^8*c^2*d^9*abs(b) - 20*D*a^3*b^5*c*d^10*abs(
b) - 120*C*a^2*b^6*c*d^10*abs(b) - 160*B*a*b^7*c*d^10*abs(b) - 64*A*b^8*c*
d^10*abs(b) - D*a^4*b^4*d^11*abs(b) + 8*C*a^3*b^5*d^11*abs(b) + 48*B*a^2*b
^6*d^11*abs(b) + 64*A*a*b^7*d^11*abs(b))/(b^6*c*d^11 - a*b^5*d^12))*(b*x +
 a) - 20*(231*D*b^9*c^5*d^6*abs(b) - 651*D*a*b^8*c^4*d^7*abs(b) - 168*C*b^
9*c^4*d^7*abs(b) + 630*D*a^2*b^7*c^3*d^8*abs(b) + 448*C*a*b^8*c^3*d^8*abs(
b) + 112*B*b^9*c^3*d^8*abs(b) - 230*D*a^3*b^6*c^2*d^9*abs(b) - 400*C*a^2*b
^7*c^2*d^9*abs(b) - 272*B*a*b^8*c^2*d^9*abs(b) - 64*A*b^9*c^2*d^9*abs(b) +
 19*D*a^4*b^5*c*d^10*abs(b) + 128*C*a^3*b^6*c*d^10*abs(b) + 208*B*a^2*b^7*
c*d^10*abs(b) + 128*A*a*b^8*c*d^10*abs(b) + D*a^5*b^4*d^11*abs(b) - 8*C*a^
4*b^5*d^11*abs(b) - 48*B*a^3*b^6*d^11*abs(b) - 64*A*a^2*b^7*d^11*abs(b)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx=\int \frac{(a+b\,x)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{5/2}}\,dx = \int \frac{(bx+a)^{\frac{5}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

output
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(5/2),x)
```

3.124 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}} \, dx$

Optimal result 1176
Mathematica [A] (verified) 1177
Rubi [A] (verified) 1178
Maple [B] (verified) 1183
Fricas [B] (verification not implemented) 1184
Sympy [F] 1185
Maxima [F(-2)] 1186
Giac [B] (verification not implemented) 1186
Mupad [F(-1)] 1187
Reduce [F] 1188

Optimal result

Integrand size = 34, antiderivative size = 507

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{5/2}}{5d^4(c+dx)^{5/2}}$$

$$+\frac{2(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,(a+bx)^{3/2}}{3d^5(c+dx)^{3/2}}$$

$$-\frac{(a^3d^3D+3a^2bd^2(2Cd-7cD)-3ab^2d(12cCd-4Bd^2-25c^2D)+b^3(36c^2Cd-18Bcd^2+6Ad^3-61c^3}{3bd^6\sqrt{c+dx}}$$

$$+\frac{D(a+bx)^{7/2}}{3bd^3\sqrt{c+dx}}$$

$$+\frac{(7a^2d^2D+14abd(3Cd-10cD)-b^2(90cCd-24Bd^2-205c^2D))\,\sqrt{a+bx}\sqrt{c+dx}}{24d^6}$$

$$+\frac{(6bCd-19bcD+adD)(a+bx)^{3/2}\sqrt{c+dx}}{12d^5}$$

$$+\frac{(5a^3d^3D+15a^2bd^2(2Cd-7cD)-5ab^2d(28cCd-8Bd^2-63c^2D)+b^3(126c^2Cd-56Bcd^2+16Ad^3-2}{8\sqrt{b}d^{13/2}}$$

output
```
-2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(5/2)/d^4/(d*x+c)^(5/2)+2/3*(a*
d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(b*x+a)^
(3/2)/d^5/(d*x+c)^(3/2)-1/3*(a^3*d^3*D+3*a^2*b*d^2*(2*C*d-7*D*c)-3*a*b^2*d
*(-4*B*d^2+12*C*c*d-25*D*c^2)+b^3*(6*A*d^3-18*B*c*d^2+36*C*c^2*d-61*D*c^3)
)*(b*x+a)^(1/2)/b/d^6/(d*x+c)^(1/2)+1/3*D*(b*x+a)^(7/2)/b/d^3/(d*x+c)^(1/2
)+1/24*(7*a^2*d^2*D+14*a*b*d*(3*C*d-10*D*c)-b^2*(-24*B*d^2+90*C*c*d-205*D*
c^2))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^6+1/12*(6*C*b*d+D*a*d-19*D*b*c)*(b*x+a
)^(3/2)*(d*x+c)^(1/2)/d^5+1/8*(5*a^3*d^3*D+15*a^2*b*d^2*(2*C*d-7*D*c)-5*a*
b^2*d*(-8*B*d^2+28*C*c*d-63*D*c^2)+b^3*(16*A*d^3-56*B*c*d^2+126*C*c^2*d-23
1*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/d^(
13/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 459, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}\,dx = \frac{\sqrt{a+bx}(-a^2d^2(-693c^3D+c^2d(128C-1695Dx)+cd^2(32B$$

$$+\frac{(5a^3d^3D+15a^2bd^2(2Cd-7cD)+5ab^2d(-28cCd+8Bd^2+63c^2D)+b^3(126c^2Cd-56Bcd^2+16Ad^3-}{8\sqrt{b}d^{13/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
(Sqrt[a + b*x]*(-(a^2*d^2*(-693*c^3*D + c^2*d*(128*C - 1695*D*x) + c*d^2*(
32*B + 5*x*(64*C - 243*D*x)) + d^3*(48*A + 5*x*(16*B + 48*C*x - 33*D*x^2))
)) + b^2*(3465*c^5*D - 105*c^4*d*(18*C - 77*D*x) + 21*c^3*d^2*(40*B + x*(-
210*C + 253*D*x)) + 4*d^5*x^2*(-92*A + 5*x*(6*B + x*(3*C + 2*D*x))) - 2*c*
d^4*x*(280*A + x*(-644*B + 5*x*(27*C + 11*D*x))) + c^2*d^3*(-240*A + x*(19
60*B + 9*x*(-322*C + 55*D*x)))) - 2*a*b*d*(1785*c^4*D + c^3*(-735*C*d + 42
42*d*D*x) + c^2*d^2*(160*B + x*(-1757*C + 2880*D*x)) + d^4*x*(88*A - 5*x*(
-56*B + x*(27*C + 13*D*x))) + c*d^3*(40*A + x*(392*B + 5*x*(-241*C + 62*D*
x))))))/(120*d^6*(c + d*x)^(5/2)) + ((5*a^3*d^3*D + 15*a^2*b*d^2*(2*C*d -
7*c*D) + 5*a*b^2*d*(-28*c*C*d + 8*B*d^2 + 63*c^2*D) + b^3*(126*c^2*C*d - 5
6*B*c*d^2 + 16*A*d^3 - 231*c^3*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d
]*Sqrt[a + b*x])])/(8*Sqrt[b]*d^(13/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2124, 27, 1193, 27, 87, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}} \, dx$$

$$\Big\downarrow \text{2124}$$

$$\frac{2\int \frac{(a+bx)^{5/2}\left(-5\left(a-\frac{bc}{d}\right)Dx^2+\frac{5(bc-ad)(Cd-cD)x}{d^2}+\frac{5ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c+2Ad^3\right)}{d^3}\right)}{2(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+bx)^{5/2}\left(-5\left(a-\frac{bc}{d}\right)Dx^2+\frac{5(bc-ad)(Cd-cD)x}{d^2}+\frac{5ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c+2Ad^3\right)}{d^3}\right)}{(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \text{1193}$$

$$\frac{\frac{2\int \frac{(a+bx)^{5/2}\left(\left(-98Dc^3+63Cdc^2-28Bd^2c+8Ad^3\right)b^2-10ad\left(-12Dc^2+7Cdc-2Bd^2\right)b+15a^2d^2(Cd-2cD)+15d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2(a+bx)^{7/2}(5ad(-Bd^2 \cdots}{\cdots}}{5(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(a+bx)^{5/2}\left(\left(-98Dc^3+63Cdc^2-28Bd^2c+8Ad^3\right)b^2-10ad\left(-12Dc^2+7Cdc-2Bd^2\right)b+15a^2d^2(Cd-2cD)+15d(bc-ad)^2Dx\right)}{(c+dx)^{3/2}}dx}{3d^3(bc-ad)}+\frac{2(a+bx)^{7/2}\left(5ad\left(-Bd^2-\right.\right.}{}}{5(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{2(a+bx)^{7/2}\left(15a^2d^2(Cd-3cD)-10abd\left(-2Bd^2-15c^2D+7cCd\right)+b^2\left(8Ad^3-28Bcd^2-113c^3D+63c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{3\left(5a^3d^3D+15a^2bd^2(2Cd-7cD)-5ab^2d\left(-8Bd^2-63\right.\right.}{}}{3d^3(bc-ad)}}{5(bc}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{2(a+bx)^{7/2}\left(15a^2d^2(Cd-3cD)-10abd\left(-2Bd^2-15c^2D+7cCd\right)+b^2\left(8Ad^3-28Bcd^2-113c^3D+63c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{3\left(5a^3d^3D+15a^2bd^2(2Cd-7cD)-5ab^2d\left(-8Bd^2-63\right.\right.}{}}{3d^3(bc-ad)}}{}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{2(a+bx)^{7/2}\left(15a^2d^2(Cd-3cD)-10abd\left(-2Bd^2-15c^2D+7cCd\right)+b^2\left(8Ad^3-28Bcd^2-113c^3D+63c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{3\left(5a^3d^3D+15a^2bd^2(2Cd-7cD)-5ab^2d\left(-8Bd^2-63\right.\right.}{}}{3d^3(bc-ad)}}{}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{2(a+bx)^{7/2}\left(15a^2d^2(Cd-3cD)-10abd\left(-2Bd^2-15c^2D+7cCd\right)+b^2\left(8Ad^3-28Bcd^2-113c^3D+63c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{3\left(5a^3d^3D+15a^2bd^2(2Cd-7cD)-5ab^2d\left(-8Bd^2-63\right.\right.}{}}{}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 66

$$\frac{\frac{2(a+bx)^{7/2}\left(15a^2d^2(Cd-3cD)-10abd\left(-2Bd^2-15c^2D+7cCd\right)+b^2\left(8Ad^3-28Bcd^2-113c^3D+63c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{3\left(5a^3d^3D+15a^2bd^2(2Cd-7cD)-5ab^2d\left(-8Bd^2-63\right.\right.}{}}{}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 221

$$\frac{\frac{2(a+bx)^{7/2}\left(15a^2d^2(Cd-3cD)-10abd\left(-2Bd^2-15c^2D+7cCd\right)+b^2\left(8Ad^3-28Bcd^2-113c^3D+63c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}-\frac{3\left(\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}-\frac{5(bc-ad)\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(7/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(7/2))/(5*(b*c - a*d)*(
c + d*x)^(5/2)) + ((2*(5*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(12*c^2*C*d -
 7*B*c*d^2 + 2*A*d^3 - 17*c^3*D))*(a + b*x)^(7/2))/(3*d^3*(b*c - a*d)*(c +
 d*x)^(3/2)) + ((2*(15*a^2*d^2*(C*d - 3*c*D) - 10*a*b*d*(7*c*C*d - 2*B*d^2
 - 15*c^2*D) + b^2*(63*c^2*C*d - 28*B*c*d^2 + 8*A*d^3 - 113*c^3*D))*(a + b
*x)^(7/2))/((b*c - a*d)*Sqrt[c + d*x]) - (3*(5*a^3*d^3*D + 15*a^2*b*d^2*(2
*C*d - 7*c*D) - 5*a*b^2*d*(28*c*C*d - 8*B*d^2 - 63*c^2*D) + b^3*(126*c^2*C
*d - 56*B*c*d^2 + 16*A*d^3 - 231*c^3*D))*(((a + b*x)^(5/2)*Sqrt[c + d*x])/
(3*d) - (5*(b*c - a*d)*(((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d) - (3*(b*c -
a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt
[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*d)))/(6*d)))/(
b*c - a*d))/(3*d^3*(b*c - a*d)))/(5*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3420 vs. $2(457) = 914$.

Time = 0.57 (sec) , antiderivative size = 3421, normalized size of antiderivative = 6.75

method	result	size
default	Expression too large to display	3421

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
1/240*(16170*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^4*d*x+2940*C*(d*b
)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^3*d^2-7140*D*(d*b)^(1/2)*((b*x+a)*(d
*x+c))^(1/2)*a*b*c^4*d-1575*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c*d^5*x^3-352*A*a*b*d^5*x*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)-1120*A*b^2*c*d^4*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+6930*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^5+990*D*(d*b)^(1/2
)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^2*d^3*x^3-4725*D*ln(1/2*(2*b*d*x+2*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c^2*d^4*x^2+14175
*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1
/2))*a*b^2*c^3*d^3*x^2-64*B*a^2*c*d^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-
736*A*b^2*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-480*C*a^2*d^5*x^2*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+75*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*d^6*x^3-640*C*a^2*c*d^4*x*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+600*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^3*d^3+450*C*ln(1/2*(2*b*d
*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c^3*d
^3-10395*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*b^3*c^5*d*x-1240*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c*
d^4*x^3+4820*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c*d^4*x^2-11520*D*(
d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^2*d^3*x^2+7028*C*(d*b)^(1/2)*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(459) = 918$.

Time = 63.06 (sec) , antiderivative size = 1942, normalized size of antiderivative = 3.83

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="fr
icas")
```

output
```
[-1/480*(15*(231*D*b^3*c^6 - 63*(5*D*a*b^2 + 2*C*b^3)*c^5*d + 7*(15*D*a^2*
b + 20*C*a*b^2 + 8*B*b^3)*c^4*d^2 - (5*D*a^3 + 30*C*a^2*b + 40*B*a*b^2 + 1
6*A*b^3)*c^3*d^3 + (231*D*b^3*c^3*d^3 - 63*(5*D*a*b^2 + 2*C*b^3)*c^2*d^4 +
 7*(15*D*a^2*b + 20*C*a*b^2 + 8*B*b^3)*c*d^5 - (5*D*a^3 + 30*C*a^2*b + 40*
B*a*b^2 + 16*A*b^3)*d^6)*x^3 + 3*(231*D*b^3*c^4*d^2 - 63*(5*D*a*b^2 + 2*C*
b^3)*c^3*d^3 + 7*(15*D*a^2*b + 20*C*a*b^2 + 8*B*b^3)*c^2*d^4 - (5*D*a^3 +
30*C*a^2*b + 40*B*a*b^2 + 16*A*b^3)*c*d^5)*x^2 + 3*(231*D*b^3*c^5*d - 63*(
5*D*a*b^2 + 2*C*b^3)*c^4*d^2 + 7*(15*D*a^2*b + 20*C*a*b^2 + 8*B*b^3)*c^3*d
^3 - (5*D*a^3 + 30*C*a^2*b + 40*B*a*b^2 + 16*A*b^3)*c^2*d^4)*x)*sqrt(b*d)*
log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d
)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(40
*D*b^3*d^6*x^5 + 3465*D*b^3*c^5*d - 48*A*a^2*b*d^6 - 210*(17*D*a*b^2 + 9*C
*b^3)*c^4*d^2 + 21*(33*D*a^2*b + 70*C*a*b^2 + 40*B*b^3)*c^3*d^3 - 16*(8*C*
a^2*b + 20*B*a*b^2 + 15*A*b^3)*c^2*d^4 - 16*(2*B*a^2*b + 5*A*a*b^2)*c*d^5
- 10*(11*D*b^3*c*d^5 - (13*D*a*b^2 + 6*C*b^3)*d^6)*x^4 + 5*(99*D*b^3*c^2*d
^4 - 2*(62*D*a*b^2 + 27*C*b^3)*c*d^5 + 3*(11*D*a^2*b + 18*C*a*b^2 + 8*B*b^
3)*d^6)*x^3 + (5313*D*b^3*c^3*d^3 - 18*(320*D*a*b^2 + 161*C*b^3)*c^2*d^4 +
 (1215*D*a^2*b + 2410*C*a*b^2 + 1288*B*b^3)*c*d^5 - 16*(15*C*a^2*b + 35*B*
a*b^2 + 23*A*b^3)*d^6)*x^2 + (8085*D*b^3*c^4*d^2 - 42*(202*D*a*b^2 + 105*C
*b^3)*c^3*d^3 + (1695*D*a^2*b + 3514*C*a*b^2 + 1960*B*b^3)*c^2*d^4 - 16...
```

Sympy [F]

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx=\int \frac{(a+bx)^{\frac{5}{2}}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(7/2),x)
```

output
```
Integral((a + b*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(459) = 918$.

Time = 0.50 (sec) , antiderivative size = 1769, normalized size of antiderivative = 3.49

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x, algorithm="gi
ac")
```

output
```
1/120*(((5*(2*(b*x + a)*(4*(D*b^9*c^2*d^10*abs(b) - 2*D*a*b^8*c*d^11*abs(b
) + D*a^2*b^7*d^12*abs(b))*(b*x + a)/(b^8*c^2*d^11 - 2*a*b^7*c*d^12 + a^2*
b^6*d^13) - (11*D*b^10*c^3*d^9*abs(b) - 15*D*a*b^9*c^2*d^10*abs(b) - 6*C*b
^10*c^2*d^10*abs(b) - 3*D*a^2*b^8*c*d^11*abs(b) + 12*C*a*b^9*c*d^11*abs(b)
 + 7*D*a^3*b^7*d^12*abs(b) - 6*C*a^2*b^8*d^12*abs(b))/(b^8*c^2*d^11 - 2*a*
b^7*c*d^12 + a^2*b^6*d^13)) + 3*(33*D*b^11*c^4*d^8*abs(b) - 78*D*a*b^10*c^
3*d^9*abs(b) - 18*C*b^11*c^3*d^9*abs(b) + 60*D*a^2*b^9*c^2*d^10*abs(b) + 3
8*C*a*b^10*c^2*d^10*abs(b) + 8*B*b^11*c^2*d^10*abs(b) - 18*D*a^3*b^8*c*d^1
1*abs(b) - 22*C*a^2*b^9*c*d^11*abs(b) - 16*B*a*b^10*c*d^11*abs(b) + 3*D*a^
4*b^7*d^12*abs(b) + 2*C*a^3*b^8*d^12*abs(b) + 8*B*a^2*b^9*d^12*abs(b))/(b^
8*c^2*d^11 - 2*a*b^7*c*d^12 + a^2*b^6*d^13))*(b*x + a) + 23*(231*D*b^12*c^
5*d^7*abs(b) - 777*D*a*b^11*c^4*d^8*abs(b) - 126*C*b^12*c^4*d^8*abs(b) + 9
66*D*a^2*b^10*c^3*d^9*abs(b) + 392*C*a*b^11*c^3*d^9*abs(b) + 56*B*b^12*c^3
*d^9*abs(b) - 530*D*a^3*b^9*c^2*d^10*abs(b) - 436*C*a^2*b^10*c^2*d^10*abs(
b) - 152*B*a*b^11*c^2*d^10*abs(b) - 16*A*b^12*c^2*d^10*abs(b) + 115*D*a^4*
b^8*c*d^11*abs(b) + 200*C*a^3*b^9*c*d^11*abs(b) + 136*B*a^2*b^10*c*d^11*ab
s(b) + 32*A*a*b^11*c*d^11*abs(b) - 5*D*a^5*b^7*d^12*abs(b) - 30*C*a^4*b^8*
d^12*abs(b) - 40*B*a^3*b^9*d^12*abs(b) - 16*A*a^2*b^10*d^12*abs(b))/(b^8*c
^2*d^11 - 2*a*b^7*c*d^12 + a^2*b^6*d^13))*(b*x + a) + 35*(231*D*b^13*c^6*d
^6*abs(b) - 1008*D*a*b^12*c^5*d^7*abs(b) - 126*C*b^13*c^5*d^7*abs(b) + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx=\int \frac{{\left(a+b\,x\right)}^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{7/2}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{7/2}}\,dx = \int \frac{(bx+a)^{\frac{5}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

output
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(7/2),x)
```

3.125 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx$

Optimal result . . . 1189
Mathematica [A] (verified) . . . 1190
Rubi [A] (verified) . . . 1191
Maple [B] (verified) . . . 1195
Fricas [B] (verification not implemented) . . . 1196
Sympy [F] . . . 1197
Maxima [F(-2)] . . . 1198
Giac [B] (verification not implemented) . . . 1198
Mupad [F(-1)] . . . 1199
Reduce [F] . . . 1200

Optimal result

Integrand size = 34, antiderivative size = 411

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx)^{7/2}}{7d^3(bc-ad)(c+dx)^{7/2}}$$
$$+\frac{2(2cCd-Bd^2-3c^2D)(a+bx)^{5/2}}{5d^4(c+dx)^{5/2}}$$
$$-\frac{(3a^2d^2D+2abd(2Cd-9cD)-b^2(12cCd-4Bd^2-27c^2D))(a+bx)^{3/2}}{6bd^5(c+dx)^{3/2}}$$
$$+\frac{D(a+bx)^{7/2}}{2bd^3(c+dx)^{3/2}}$$
$$-\frac{(3a^2d^2D+2abd(2Cd-9cD)-b^2(8cCd-2Bd^2-21c^2D))\sqrt{a+bx}}{d^6\sqrt{c+dx}}$$
$$+\frac{b(4bCd-15bcD+3adD)\sqrt{a+bx}\sqrt{c+dx}}{4d^6}$$
$$+\frac{\sqrt{b}(15a^2d^2D+10abd(2Cd-9cD)-b^2(36cCd-8Bd^2-99c^2D))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{13/2}}$$

output
```
2/7*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(7/2)/d^3/(-a*d+b*c)/(d*x+c)^(7/
2)+2/5*(-B*d^2+2*C*c*d-3*D*c^2)*(b*x+a)^(5/2)/d^4/(d*x+c)^(5/2)-1/6*(3*a^2
*d^2*D+2*a*b*d*(2*C*d-9*D*c)-b^2*(-4*B*d^2+12*C*c*d-27*D*c^2))*(b*x+a)^(3/
2)/b/d^5/(d*x+c)^(3/2)+1/2*D*(b*x+a)^(7/2)/b/d^3/(d*x+c)^(3/2)-(3*a^2*d^2*
D+2*a*b*d*(2*C*d-9*D*c)-b^2*(-2*B*d^2+8*C*c*d-21*D*c^2))*(b*x+a)^(1/2)/d^6
/(d*x+c)^(1/2)+1/4*b*(4*C*b*d+3*D*a*d-15*D*b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2
)/d^6+1/4*b^(1/2)*(15*a^2*d^2*D+10*a*b*d*(2*C*d-9*D*c)-b^2*(-8*B*d^2+36*C*
c*d-99*D*c^2))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(13/
2)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx =$$

$$-\frac{\sqrt{a+bx}(-b^3(10395c^6D-120Ad^6x^3-630c^5d(6C-55Dx)+42c^4d^2(20B-300Cx+957Dx^2)+2cd^5x}{}$$

$$+\frac{\sqrt{b}(15a^2d^2D+10abd(2Cd-9cD)+b^2(-36cCd+8Bd^2+99c^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4d^{13/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(9/2),x]
```

output
```
-1/420*(Sqrt[a + b*x]*(-(b^3*(10395*c^6*D - 120*A*d^6*x^3 - 630*c^5*d*(6*C
 - 55*D*x) + 42*c^4*d^2*(20*B - 300*C*x + 957*D*x^2) + 2*c*d^5*x^3*(704*B
- 105*x*(2*C + D*x)) + c^2*d^4*x^2*(3248*B + 33*x*(-192*C + 35*D*x)) + 8*c
^3*d^3*x*(350*B + 9*x*(-203*C + 242*D*x)))) + a^2*b*d^2*(-6489*c^4*D + 4*c
^3*d*(224*C - 5529*D*x) + 2*c^2*d^2*(56*B + x*(1552*C - 13251*D*x)) + 4*c*
d^3*x*(92*B + 7*x*(136*C - 435*D*x)) + d^4*x*(360*A + 7*x*(88*B + 5*x*(56*
C - 27*D*x)))) + 2*a*b^2*d*(8190*c^5*D + c^4*(-2310*C*d + 27531*d*D*x) + 2
*c*d^4*x^2*(568*B + 7*x*(-292*C + 75*D*x)) + 8*c^3*d^2*(35*B + x*(-973*C +
 4041*D*x)) + 2*c^2*d^3*x*(476*B + x*(-4586*C + 7161*D*x)) + d^5*x^2*(180*
A + 7*x*(92*B - 15*x*(2*C + D*x)))) + 8*a^3*d^3*(48*c^3*D + 8*c^2*d*(C + 2
1*D*x) + 2*c*d^2*(3*B + 7*x*(2*C + 15*D*x)) + d^3*(15*A + 7*x*(3*B + 5*x*(
C + 3*D*x))))))/(d^6*(-(b*c) + a*d)*(c + d*x)^(7/2)) + (Sqrt[b]*(15*a^2*d^
2*D + 10*a*b*d*(2*C*d - 9*c*D) + b^2*(-36*c*C*d + 8*B*d^2 + 99*c^2*D))*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*d^(13/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2124, 27, 1193, 27, 87, 57, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}} \, dx$$

$$\Big\downarrow \ 2124$$

$$\frac{2\int -\frac{7(a+bx)^{5/2}\left(-\frac{(bc-ad)Dx^2}{d}-\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{(bc-ad)\left(-Dc^2+Cdc-Bd^2\right)}{d^3}\right)}{2(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\int \frac{(a+bx)^{5/2}\left(-\frac{(bc-ad)Dx^2}{d}-\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{(bc-ad)\left(-Dc^2+Cdc-Bd^2\right)}{d^3}\right)}{(c+dx)^{7/2}}dx}{bc-ad}$$

$$\Big\downarrow \ 1193$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\frac{2\int \frac{(bc-ad)(a+bx)^{5/2}\left(5ad(Cd-2cD)-b\left(-16Dc^2+9Cdc-2Bd^2\right)-5d(bc-ad)Dx\right)}{2d^3(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2(a+bx)^{7/2}\left(-Bd^2-3c^2D+2cCd\right)}{5d^3(c+dx)^{5/2}}}{bc-ad}$$

$$\Big\downarrow \ 27$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\frac{\int \frac{(a+bx)^{5/2}\left(5ad(Cd-2cD)-b\left(-16Dc^2+9Cdc-2Bd^2\right)-5d(bc-ad)Dx\right)}{(c+dx)^{5/2}}dx}{5d^3}+\frac{2(a+bx)^{7/2}(-Bd^2-3c^2D+2cCd)}{5d^3(c+dx)^{5/2}}}{bc-ad}$$

$$\Big\downarrow \; 87$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{7/2}\left(5ad(Cd-3cD)-b\left(-2Bd^2-21c^2D+9cCd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\left(15a^2d^2D+10abd(2Cd-9cD)-\left(b^2\left(-8Bd^2-99c^2D+36cCd\right)\right)\right)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}dx}{3(bc-ad)}}{5d^3}+\frac{2(a+bx)^{7/2}(-\cdots}{5d^3(\cdots}}{bc-ad}$$

$$\Big\downarrow \; 57$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{7/2}\left(5ad(Cd-3cD)-b\left(-2Bd^2-21c^2D+9cCd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\left(15a^2d^2D+10abd(2Cd-9cD)-\left(b^2\left(-8Bd^2-99c^2D+36cCd\right)\right)\right)\left(\frac{5b\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}dx}{d}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}\right)}{3(bc-ad)}\cdots}{5d^3}\cdots}{bc-ad}$$

$$\Big\downarrow \; 60$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{7/2}\left(5ad(Cd-3cD)-b\left(-2Bd^2-21c^2D+9cCd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\left(15a^2d^2D+10abd(2Cd-9cD)-\left(b^2\left(-8Bd^2-99c^2D+36cCd\right)\right)\right)\left(\frac{5b\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\int\cdots}{4\cdots}\right)}{d}\cdots\right)}{3(bc-ad)}\cdots}{5d^3}\cdots}{bc-ad}$$

$$\Big\downarrow \; 60$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{7/2}\left(5ad(Cd-3cD)-b\left(-2Bd^2-21c^2D+9cCd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\left(15a^2d^2D+10abd(2Cd-9cD)-\left(b^2\left(-8Bd^2-99c^2D+36cCd\right)\right)\right)\left(\frac{5b\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)}{3(bc-ad)}}{5d^3}}{bc-ad}$$

$\downarrow$ 66

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{7/2}\left(5ad(Cd-3cD)-b\left(-2Bd^2-21c^2D+9cCd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\left(15a^2d^2D+10abd(2Cd-9cD)-\left(b^2\left(-8Bd^2-99c^2D+36cCd\right)\right)\right)\left(\frac{5b\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)}{3(bc-ad)}}{5d^3}}{bc-ad}$$

$\downarrow$ 221

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}-\frac{\frac{\frac{2(a+bx)^{7/2}\left(5ad(Cd-3cD)-b\left(-2Bd^2-21c^2D+9cCd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\left(\frac{5b\left(\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}-\frac{3(bc-ad)\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{3/2}}\right)}{4d}\right)}{d}-\frac{2(a+b\cdots}{d\sqrt{c\cdots}}\cdots\right)\cdots}{3(bc-ad)}}{5d^3}}{bc-ad}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(9/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(7/2))/(7*(b*c - a*d)*(
c + d*x)^(7/2)) - ((2*(2*c*C*d - B*d^2 - 3*c^2*D)*(a + b*x)^(7/2))/(5*d^3*
(c + d*x)^(5/2)) + ((2*(5*a*d*(C*d - 3*c*D) - b*(9*c*C*d - 2*B*d^2 - 21*c^
2*D))*(a + b*x)^(7/2))/(3*(b*c - a*d)*(c + d*x)^(3/2)) - ((15*a^2*d^2*D +
10*a*b*d*(2*C*d - 9*c*D) - b^2*(36*c*C*d - 8*B*d^2 - 99*c^2*D))*((-2*(a +
b*x)^(5/2))/(d*Sqrt[c + d*x]) + (5*b*(((a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d
) - (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh
[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/(4*
d)))/d))/(3*(b*c - a*d)))/(5*d^3))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4205 vs. $2(363) = 726$.

Time = 0.52 (sec) , antiderivative size = 4206, normalized size of antiderivative = 10.23

method	result	size
default	Expression too large to display	4206

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/840*(b*x+a)^(1/2)*(3360*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^4*d^3*x-15120*C*ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^5*d^2*x-840*B
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2
))*a*b^3*c^4*d^3-420*D*a*b^2*d^6*x^5*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+4
20*D*b^3*c*d^5*x^5*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+2688*D*a^3*c^2*d^4*
x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-9240*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))
^(1/2)*a*b^2*c^4*d^2-5040*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b
)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^2*d^5*x^2+11025*D*ln(1/2*(2*b*d*x+2*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c^5*d^2-
19845*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*
b)^(1/2))*a*b^3*c^6*d+240*A*b^3*d^6*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)+1680*D*a^3*d^6*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+560*C*a^3*d^6*x^2
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+336*B*a^3*d^6*x*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+840*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+a*d+b*c)/(d*b)^(1/2))*b^4*c^5*d^2-3780*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^6*d-12600*C*ln(1/2*(2*b*
d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c^
2*d^5*x^2+35280*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*
d+b*c)/(d*b)^(1/2))*a*b^3*c^3*d^4*x^2-2100*C*ln(1/2*(2*b*d*x+2*((b*x+a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1220 vs. $2(359) = 718$.

Time = 147.45 (sec) , antiderivative size = 2465, normalized size of antiderivative = 6.00

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="fr
icas")
```

output
```
[1/1680*(105*(99*D*b^3*c^7 - 9*(21*D*a*b^2 + 4*C*b^3)*c^6*d + (105*D*a^2*b
 + 56*C*a*b^2 + 8*B*b^3)*c^5*d^2 - (15*D*a^3 + 20*C*a^2*b + 8*B*a*b^2)*c^4
*d^3 + (99*D*b^3*c^3*d^4 - 9*(21*D*a*b^2 + 4*C*b^3)*c^2*d^5 + (105*D*a^2*b
 + 56*C*a*b^2 + 8*B*b^3)*c*d^6 - (15*D*a^3 + 20*C*a^2*b + 8*B*a*b^2)*d^7)*
x^4 + 4*(99*D*b^3*c^4*d^3 - 9*(21*D*a*b^2 + 4*C*b^3)*c^3*d^4 + (105*D*a^2*
b + 56*C*a*b^2 + 8*B*b^3)*c^2*d^5 - (15*D*a^3 + 20*C*a^2*b + 8*B*a*b^2)*c*
d^6)*x^3 + 6*(99*D*b^3*c^5*d^2 - 9*(21*D*a*b^2 + 4*C*b^3)*c^4*d^3 + (105*D
*a^2*b + 56*C*a*b^2 + 8*B*b^3)*c^3*d^4 - (15*D*a^3 + 20*C*a^2*b + 8*B*a*b^
2)*c^2*d^5)*x^2 + 4*(99*D*b^3*c^6*d - 9*(21*D*a*b^2 + 4*C*b^3)*c^5*d^2 + (
105*D*a^2*b + 56*C*a*b^2 + 8*B*b^3)*c^4*d^3 - (15*D*a^3 + 20*C*a^2*b + 8*B
*a*b^2)*c^3*d^4)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^
2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d
) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(10395*D*b^3*c^6 - 48*B*a^3*c*d^5 - 120*A
*a^3*d^6 - 1260*(13*D*a*b^2 + 3*C*b^3)*c^5*d + 21*(309*D*a^2*b + 220*C*a*b
^2 + 40*B*b^3)*c^4*d^2 - 16*(24*D*a^3 + 56*C*a^2*b + 35*B*a*b^2)*c^3*d^3 -
 16*(4*C*a^3 + 7*B*a^2*b)*c^2*d^4 - 210*(D*b^3*c*d^5 - D*a*b^2*d^6)*x^5 +
105*(11*D*b^3*c^2*d^4 - 4*(5*D*a*b^2 + C*b^3)*c*d^5 + (9*D*a^2*b + 4*C*a*b
^2)*d^6)*x^4 + 4*(4356*D*b^3*c^3*d^3 - 33*(217*D*a*b^2 + 48*C*b^3)*c^2*d^4
 + (3045*D*a^2*b + 2044*C*a*b^2 + 352*B*b^3)*c*d^5 - 2*(105*D*a^3 + 245*C*
a^2*b + 161*B*a*b^2 + 15*A*b^3)*d^6)*x^3 + 2*(20097*D*b^3*c^4*d^2 - 36*...
```

Sympy [F]

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx=\int \frac{(a+bx)^{\frac{5}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(9/2),x)
```

output
```
Integral((a + b*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(9/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1920 vs. $2(359) = 718$.

Time = 0.75 (sec) , antiderivative size = 1920, normalized size of antiderivative = 4.67

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x, algorithm="gi
ac")
```

output
```
1/420*((((105*(b*x + a)*(2*(D*b^13*c^3*d^10*abs(b) - 3*D*a*b^12*c^2*d^11*a
bs(b) + 3*D*a^2*b^11*c*d^12*abs(b) - D*a^3*b^10*d^13*abs(b))*(b*x + a)/(b^
10*c^3*d^11 - 3*a*b^9*c^2*d^12 + 3*a^2*b^8*c*d^13 - a^3*b^7*d^14) - (11*D*
b^14*c^4*d^9*abs(b) - 32*D*a*b^13*c^3*d^10*abs(b) - 4*C*b^14*c^3*d^10*abs(
b) + 30*D*a^2*b^12*c^2*d^11*abs(b) + 12*C*a*b^13*c^2*d^11*abs(b) - 8*D*a^3
*b^11*c*d^12*abs(b) - 12*C*a^2*b^12*c*d^12*abs(b) - D*a^4*b^10*d^13*abs(b)
 + 4*C*a^3*b^11*d^13*abs(b))/(b^10*c^3*d^11 - 3*a*b^9*c^2*d^12 + 3*a^2*b^8
*c*d^13 - a^3*b^7*d^14)) - 8*(2178*D*b^15*c^5*d^8*abs(b) - 8514*D*a*b^14*c
^4*d^9*abs(b) - 792*C*b^15*c^4*d^9*abs(b) + 12804*D*a^2*b^13*c^3*d^10*abs(
b) + 2816*C*a*b^14*c^3*d^10*abs(b) + 176*B*b^15*c^3*d^10*abs(b) - 9093*D*a
^3*b^12*c^2*d^11*abs(b) - 3711*C*a^2*b^13*c^2*d^11*abs(b) - 513*B*a*b^14*c
^2*d^11*abs(b) - 15*A*b^15*c^2*d^11*abs(b) + 2940*D*a^4*b^11*c*d^12*abs(b)
 + 2142*C*a^3*b^12*c*d^12*abs(b) + 498*B*a^2*b^13*c*d^12*abs(b) + 30*A*a*b
^14*c*d^12*abs(b) - 315*D*a^5*b^10*d^13*abs(b) - 455*C*a^4*b^11*d^13*abs(b
) - 161*B*a^3*b^12*d^13*abs(b) - 15*A*a^2*b^13*d^13*abs(b))/(b^10*c^3*d^11
 - 3*a*b^9*c^2*d^12 + 3*a^2*b^8*c*d^13 - a^3*b^7*d^14))*(b*x + a) - 406*(9
9*D*b^16*c^6*d^7*abs(b) - 486*D*a*b^15*c^5*d^8*abs(b) - 36*C*b^16*c^5*d^8*
abs(b) + 969*D*a^2*b^14*c^4*d^9*abs(b) + 164*C*a*b^15*c^4*d^9*abs(b) + 8*B
*b^16*c^4*d^9*abs(b) - 996*D*a^3*b^13*c^3*d^10*abs(b) - 296*C*a^2*b^14*c^3
*d^10*abs(b) - 32*B*a*b^15*c^3*d^10*abs(b) + 549*D*a^4*b^12*c^2*d^11*ab...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx=\int \frac{{\left(a+b\,x\right)}^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{9/2}} \,d x$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(9/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{9/2}}\,dx=\int \frac{(bx+a)^{\frac{5}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(dx+c)^{\frac{9}{2}}}dx$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x)
```

output
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(9/2),x)
```

3.126 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx$

Optimal result . . . 1201
Mathematica [A] (verified) . . . 1202
Rubi [A] (verified) . . . 1202
Maple [B] (verified) . . . 1209
Fricas [F(-1)] . . . 1210
Sympy [F] . . . 1211
Maxima [F(-2)] . . . 1211
Giac [B] (verification not implemented) . . . 1211
Mupad [F(-1)] . . . 1212
Reduce [F] . . . 1213

Optimal result

Integrand size = 34, antiderivative size = 343

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx &= \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx)^{7/2}}{9d^3(bc-ad)(c+dx)^{9/2}} \\
&+ \frac{2(9ad(2cCd-Bd^2-3c^2D)-b(16c^2Cd-7Bcd^2-2Ad^3-25c^3D))(a+bx)^{7/2}}{63d^3(bc-ad)^2(c+dx)^{7/2}} \\
&- \frac{2(Cd-3cD)(a+bx)^{5/2}}{5d^4(c+dx)^{5/2}} - \frac{2(bCd-4bcD+adD)(a+bx)^{3/2}}{3d^5(c+dx)^{3/2}} \\
&- \frac{2b(bCd-5bcD+2adD)\sqrt{a+bx}}{d^6\sqrt{c+dx}} + \frac{b^2D\sqrt{a+bx}\sqrt{c+dx}}{d^6} \\
&+ \frac{b^{3/2}(2bCd-11bcD+5adD)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{13/2}}
\end{aligned}
$$

output
```
2/9*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(7/2)/d^3/(-a*d+b*c)/(d*x+c)^(9/
2)+2/63*(9*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-2*A*d^3-7*B*c*d^2+16*C*c^2*d-2
5*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(7/2)-2/5*(C*d-3*D*c)*(b*
x+a)^(5/2)/d^4/(d*x+c)^(5/2)-2/3*(C*b*d+D*a*d-4*D*b*c)*(b*x+a)^(3/2)/d^5/(
d*x+c)^(3/2)-2*b*(C*b*d+2*D*a*d-5*D*b*c)*(b*x+a)^(1/2)/d^6/(d*x+c)^(1/2)+b
^2*D*(b*x+a)^(1/2)*(d*x+c)^(1/2)/d^6+b^(3/2)*(2*C*b*d+5*D*a*d-11*D*b*c)*ar
ctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/d^(13/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \frac{\sqrt{a+bx}(b^4(3465c^7D+20Ad^7x^4+10cd^6x^3(9A+7Bx)-105}{}$$
$$+\frac{b^{3/2}(2bCd-11bcD+5adD)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{d^{13/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(11/2),x]
```

output
```
(Sqrt[a + b*x]*(b^4*(3465*c^7*D + 20*A*d^7*x^4 + 10*c*d^6*x^3*(9*A + 7*B*x
) - 105*c^6*d*(6*C - 143*D*x) + c^2*d^5*x^4*(-1126*C + 315*D*x) + 42*c^5*d
^2*x*(-65*C + 594*D*x) + 18*c^4*d^3*x^2*(-252*C + 1067*D*x) + c^3*d^4*x^3*
(-3492*C + 6193*D*x)) - 2*a*b^3*d*(3675*c^6*D + 5*d^6*x^3*(A + 9*B*x) - 21
*c^5*d*(25*C - 762*D*x) + c^2*d^4*x^3*(-2986*C + 6813*D*x) + 3*c^4*d^2*x*(
-763*C + 8919*D*x) + 2*c^3*d^3*x^2*(-1917*C + 10394*D*x) + c*d^5*x^2*(-135
*A + x*(-95*B - 1056*C*x + 315*D*x^2))) - 2*a^4*d^4*(16*c^3*D + 8*c^2*d*(C
 + 9*D*x) + 2*c*d^2*(5*B + 9*x*(2*C + 7*D*x)) + d^3*(35*A + 3*x*(15*B + 7*
x*(3*C + 5*D*x)))) - 2*a^3*b*d^3*(288*c^4*D + 8*c^3*d*(3*C + 161*D*x) + 8*
c^2*d^2*x*(13*C + 279*D*x) + d^4*x*(95*A + 3*x*(45*B + 7*x*(11*C + 35*D*x)
)) + c*d^3*(-45*A + x*(-5*B + 9*x*(19*C + 203*D*x)))) + 3*a^2*b^2*d^2*(149
1*c^5*D + 2*c^2*d^3*x^2*(-424*C + 4359*D*x) + 2*c^3*d^2*x*(-248*C + 5527*D
*x) + c^4*(-112*C*d + 6543*d*D*x) + d^5*x^2*(-50*A + x*(-90*B + 7*x*(-46*C
 + 15*D*x))) + c*d^4*x*(90*A + x*(50*B + x*(-646*C + 2961*D*x))))))/(315*d
^6*(b*c - a*d)^2*(c + d*x)^(9/2)) + (b^(3/2)*(2*b*C*d - 11*b*c*D + 5*a*d*D
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/d^(13/2)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2124, 27, 1193, 27, 87, 57, 57, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}dx$$

$$\Big\downarrow \; 2124$$

$$\frac{2\int \frac{(a+bx)^{5/2}\left(-9\left(a-\frac{bc}{d}\right)Dx^2+\frac{9(bc-ad)(Cd-cD)x}{d^2}+\frac{9ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-2Ad^3\right)}{d^3}\right)}{2(c+dx)^{9/2}}dx}{9(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a+bx)^{5/2}\left(-9\left(a-\frac{bc}{d}\right)Dx^2+\frac{9(bc-ad)(Cd-cD)x}{d^2}+\frac{9ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-2Ad^3\right)}{d^3}\right)}{(c+dx)^{9/2}}dx}{9(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{\frac{2\int \frac{63(bc-ad)^2(a+bx)^{5/2}(Cd+Dxd-2cD)}{2d^3(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2(a+bx)^{7/2}\left(9ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-7Bcd^2-25c^3D+16c^2Cd\right)\right)}{7d^3(c+dx)^{7/2}(bc-ad)}}{9(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{9(bc-ad)\int \frac{(a+bx)^{5/2}(Cd+Dxd-2cD)}{(c+dx)^{7/2}}dx}{d^3}+\frac{2(a+bx)^{7/2}\left(9ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-7Bcd^2-25c^3D+16c^2Cd\right)\right)}{7d^3(c+dx)^{7/2}(bc-ad)}}{9(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{9(bc-ad)\left(\frac{2(a+bx)^{7/2}(Cd-3cD)}{5(c+dx)^{5/2}(bc-ad)}-\frac{(5adD-11bcD+2bCd)\int\frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}dx}{5(bc-ad)}\right)}{d^3}+\frac{2(a+bx)^{7/2}(9ad(-Bd^2-3c^2D+2cCd)-b(-2Ad^3-7Bcd^2-25c^3D+}{7d^3(c+dx)^{7/2}(bc-ad)}}{9(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 57$$

$$\frac{\frac{9(bc-ad)\left(\frac{2(a+bx)^{7/2}(Cd-3cD)}{5(c+dx)^{5/2}(bc-ad)}-\frac{(5adD-11bcD+2bCd)\left(\frac{5b\int\frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}dx}{3d}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}\right)}{5(bc-ad)}\right)}{d^3}+\frac{2(a+bx)^{7/2}(9ad(-Bd^2-3c^2D+2cCd)-b(-2Ad}{7d^3(c+dx)^{7/2}(bc-ac}}{9(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 57$$

$$\frac{\frac{9(bc-ad)\left(\frac{2(a+bx)^{7/2}(Cd-3cD)}{5(c+dx)^{5/2}(bc-ad)}-\frac{(5adD-11bcD+2bCd)\left(\frac{5b\left(\frac{3b\int\frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{d}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}\right)}{3d}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}\right)}{5(bc-ad)}\right)}{d^3}+\frac{2(a+bx)^{7/2}(9ad(-Bd^2-3c^2D+}{7d^3(}}{9(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{9(bc-ad)\left(\frac{2(a+bx)^{7/2}(Cd-3cD)}{5(c+dx)^{5/2}(bc-ad)}-\frac{(5adD-11bcD+2bCd)\left(\frac{5b\left(\frac{3b\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2d}\right)}{d}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}\right)}{3d}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}\right)}{5(bc-ad)}\right)}{d^3}+\cdots}{9(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{9(bc-ad)\left(\frac{2(a+bx)^{7/2}(Cd-3cD)}{5(c+dx)^{5/2}(bc-ad)}-\frac{(5adD-11bcD+2bCd)\left(\frac{5b\left(\frac{3b\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc-ad)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}\right)}{d}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}\right)}{3d}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}\right)}{5(bc-ad)}\right)}{d^3}}{9(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{2(a+bx)^{7/2}\left(9ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-7Bcd^2-25c^3D+16c^2Cd\right)\right)}{7d^3(c+dx)^{7/2}(bc-ad)}+\frac{9(bc-ad)\left(\frac{2(a+bx)^{7/2}(Cd-3cD)}{5(c+dx)^{5/2}(bc-ad)}-\frac{\left(\cdots 5b\left(\cdots 3b\left(\frac{\sqrt{a+bx}\sqrt{c+dx}}{d}-\frac{(bc\cdots}{\cdots}\right)\cdots\right)\cdots\right)}{\cdots}\right)}{\cdots}}{9(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{9(c+dx)^{9/2}(bc-ad)}$$

input

```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(11/2),x]
```

output

```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(7/2))/(9*(b*c - a*d)*(
c + d*x)^(9/2)) + ((2*(9*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(16*c^2*C*d -
 7*B*c*d^2 - 2*A*d^3 - 25*c^3*D))*(a + b*x)^(7/2))/(7*d^3*(b*c - a*d)*(c +
 d*x)^(7/2)) + (9*(b*c - a*d)*((2*(C*d - 3*c*D)*(a + b*x)^(7/2))/(5*(b*c -
 a*d)*(c + d*x)^(5/2)) - ((2*b*C*d - 11*b*c*D + 5*a*d*D)*((-2*(a + b*x)^(5
/2))/(3*d*(c + d*x)^(3/2)) + (5*b*((-2*(a + b*x)^(3/2))/(d*Sqrt[c + d*x])
+ (3*b*((Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sq
rt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))/d))/(3*d)))/(5*
(b*c - a*d))))/d^3)/(9*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4495 vs. $2(299) = 598$.

Time = 0.55 (sec) , antiderivative size = 4496, normalized size of antiderivative = 13.11

method	result	size
default	Expression too large to display	4496

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/630*(b*x+a)^(1/2)*(180*B*a^4*d^7*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+
252*C*a^4*d^7*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-8505*D*ln(1/2*(2*b*d
*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^7*d
-40*A*b^4*d^7*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+420*D*a^4*d^7*x^3*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+40*B*a^4*c*d^6*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+32*C*a^4*c^2*d^5*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+64*D*a^4*c
^3*d^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-85050*D*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^4*d^4*x^3-157
50*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^
(1/2))*a^3*b^2*c^3*d^5*x^2+66150*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^4*d^4*x^2+1932*C*a^2*b^2*d^
7*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+2252*C*b^4*c^2*d^5*x^4*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)+2940*D*a^3*b*d^7*x^4*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)-12386*D*b^4*c^3*d^4*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+20*A
*a*b^3*d^7*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-180*A*b^4*c*d^6*x^3*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+540*B*a^2*b^2*d^7*x^3*((b*x+a)*(d*x+c))^(
1/2)*(d*b)^(1/2)-6300*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^2*d^6*x^3+12600*C*ln(1/2*(2*b*d*x+2*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^3*d^5*x^3-
6300*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \int \frac{(a+bx)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(11/2),x)
```

output
```
Integral((a + b*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(c + d*x)**(11/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1988 vs. $2(303) = 606$.

Time = 1.16 (sec) , antiderivative size = 1988, normalized size of antiderivative = 5.80

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x, algorithm="g
iac")
```

output
```
1/315*(((((b*x + a)*(315*(D*b^17*c^4*d^10*abs(b) - 4*D*a*b^16*c^3*d^11*abs
(b) + 6*D*a^2*b^15*c^2*d^12*abs(b) - 4*D*a^3*b^14*c*d^13*abs(b) + D*a^4*b^
13*d^14*abs(b))*(b*x + a)/(b^12*c^4*d^11 - 4*a*b^11*c^3*d^12 + 6*a^2*b^10*
c^2*d^13 - 4*a^3*b^9*c*d^14 + a^4*b^8*d^15) + (6193*D*b^18*c^5*d^9*abs(b)
- 27587*D*a*b^17*c^4*d^10*abs(b) - 1126*C*b^18*c^4*d^10*abs(b) + 48628*D*a
^2*b^16*c^3*d^11*abs(b) + 4364*C*a*b^17*c^3*d^11*abs(b) + 70*B*b^18*c^3*d^
11*abs(b) - 42312*D*a^3*b^15*c^2*d^12*abs(b) - 6316*C*a^2*b^16*c^2*d^12*ab
s(b) - 230*B*a*b^17*c^2*d^12*abs(b) + 20*A*b^18*c^2*d^12*abs(b) + 18123*D*
a^4*b^14*c*d^13*abs(b) + 4044*C*a^3*b^15*c*d^13*abs(b) + 250*B*a^2*b^16*c*
d^13*abs(b) - 40*A*a*b^17*c*d^13*abs(b) - 3045*D*a^5*b^13*d^14*abs(b) - 96
6*C*a^4*b^14*d^14*abs(b) - 90*B*a^3*b^15*d^14*abs(b) + 20*A*a^2*b^16*d^14*
abs(b))/(b^12*c^4*d^11 - 4*a*b^11*c^3*d^12 + 6*a^2*b^10*c^2*d^13 - 4*a^3*b
^9*c*d^14 + a^4*b^8*d^15)) + 18*(1067*D*b^19*c^6*d^8*abs(b) - 5820*D*a*b^1
8*c^5*d^9*abs(b) - 194*C*b^19*c^5*d^9*abs(b) + 13095*D*a^2*b^17*c^4*d^10*a
bs(b) + 970*C*a*b^18*c^4*d^10*abs(b) - 15525*D*a^3*b^16*c^3*d^11*abs(b) -
1935*C*a^2*b^17*c^3*d^11*abs(b) - 5*B*a*b^18*c^3*d^11*abs(b) + 5*A*b^19*c^
3*d^11*abs(b) + 10200*D*a^4*b^15*c^2*d^12*abs(b) + 1925*C*a^3*b^16*c^2*d^1
2*abs(b) + 15*B*a^2*b^17*c^2*d^12*abs(b) - 15*A*a*b^18*c^2*d^12*abs(b) - 3
507*D*a^5*b^14*c*d^13*abs(b) - 955*C*a^4*b^15*c*d^13*abs(b) - 15*B*a^3*b^1
6*c*d^13*abs(b) + 15*A*a^2*b^17*c*d^13*abs(b) + 490*D*a^6*b^13*d^14*abs...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{11/2}}\,dx=\int \frac{(a+b\,x)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{(c+d\,x)^{11/2}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(11/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{11/2}}\,dx = \int \frac{(bx+a)^{\frac{5}{2}}\,(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{11}{2}}}dx$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x)
```

output
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(11/2),x)
```

3.127 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx$

Optimal result 1214
Mathematica [A] (verified) 1215
Rubi [A] (verified) 1215
Maple [B] (verified) 1220
Fricas [F(-1)] 1221
Sympy [F(-1)] 1222
Maxima [F(-2)] 1222
Giac [B] (verification not implemented) 1222
Mupad [F(-1)] 1223
Reduce [F] 1224

Optimal result

Integrand size = 34, antiderivative size = 377

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{7/2}}{11d^3(bc-ad)(c+dx)^{11/2}}$$
$$+\frac{2(11ad(2cCd-Bd^2-3c^2D)-b(18c^2Cd-7Bcd^2-4Ad^3-29c^3D))\,(a+bx)^{7/2}}{99d^3(bc-ad)^2(c+dx)^{9/2}}$$
$$+\frac{2(99a^2d^2(Cd-3cD)-22abd(7cCd+Bd^2-24c^2D)+b^2(63c^2Cd+14Bcd^2+8Ad^3-239c^3D))\,(a+bx}{693d^3(bc-ad)^3(c+dx)^{7/2}}$$
$$-\frac{2D(a+bx)^{5/2}}{5d^4(c+dx)^{5/2}}-\frac{2bD(a+bx)^{3/2}}{3d^5(c+dx)^{3/2}}-\frac{2b^2D\sqrt{a+bx}}{d^6\sqrt{c+dx}}+\frac{2b^{5/2}D\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{13/2}}$$

output
```
2/11*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(7/2)/d^3/(-a*d+b*c)/(d*x+c)^(1
1/2)+2/99*(11*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-4*A*d^3-7*B*c*d^2+18*C*c^2*
d-29*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(9/2)+2/693*(99*a^2*d^
2*(C*d-3*D*c)-22*a*b*d*(B*d^2+7*C*c*d-24*D*c^2)+b^2*(8*A*d^3+14*B*c*d^2+63
*C*c^2*d-239*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(7/2)-2/5*D*(b
*x+a)^(5/2)/d^4/(d*x+c)^(5/2)-2/3*b*D*(b*x+a)^(3/2)/d^5/(d*x+c)^(3/2)-2*b^
2*D*(b*x+a)^(1/2)/d^6/(d*x+c)^(1/2)+2*b^(5/2)*D*arctanh(d^(1/2)*(b*x+a)^(1
/2)/b^(1/2)/(d*x+c)^(1/2))/d^(13/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx = \frac{2\sqrt{a+bx}(315c^2Cd^6(a+bx)^5-315Bcd^7(a+bx)^5+315Ad^8(}{} \\ +\frac{2b^{5/2}D\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{d^{13/2}}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(13/2),x]
```

output
```
(2*Sqrt[a + b*x]*(315*c^2*C*d^6*(a + b*x)^5 - 315*B*c*d^7*(a + b*x)^5 + 31
5*A*d^8*(a + b*x)^5 - 315*c^3*d^5*D*(a + b*x)^5 + 385*b*B*c*d^6*(a + b*x)^
4*(c + d*x) - 770*a*c*C*d^6*(a + b*x)^4*(c + d*x) - 770*A*b*d^7*(a + b*x)^
4*(c + d*x) + 385*a*B*d^7*(a + b*x)^4*(c + d*x) - 385*b*c^3*d^4*D*(a + b*x
)^4*(c + d*x) + 1155*a*c^2*d^5*D*(a + b*x)^4*(c + d*x) + 495*A*b^2*d^6*(a
+ b*x)^3*(c + d*x)^2 - 495*a*b*B*d^6*(a + b*x)^3*(c + d*x)^2 + 495*a^2*C*d
^6*(a + b*x)^3*(c + d*x)^2 - 495*b^2*c^3*d^3*D*(a + b*x)^3*(c + d*x)^2 + 1
485*a*b*c^2*d^4*D*(a + b*x)^3*(c + d*x)^2 - 1485*a^2*c*d^5*D*(a + b*x)^3*(
c + d*x)^2 - 693*b^3*c^3*d^2*D*(a + b*x)^2*(c + d*x)^3 + 2079*a*b^2*c^2*d^
3*D*(a + b*x)^2*(c + d*x)^3 - 2079*a^2*b*c*d^4*D*(a + b*x)^2*(c + d*x)^3 +
 693*a^3*d^5*D*(a + b*x)^2*(c + d*x)^3 - 1155*b^4*c^3*d*D*(a + b*x)*(c + d
*x)^4 + 3465*a*b^3*c^2*d^2*D*(a + b*x)*(c + d*x)^4 - 3465*a^2*b^2*c*d^3*D*
(a + b*x)*(c + d*x)^4 + 1155*a^3*b*d^4*D*(a + b*x)*(c + d*x)^4 - 3465*b^5*
c^3*D*(c + d*x)^5 + 10395*a*b^4*c^2*d*D*(c + d*x)^5 - 10395*a^2*b^3*c*d^2*
D*(c + d*x)^5 + 3465*a^3*b^2*d^3*D*(c + d*x)^5))/(3465*d^6*(b*c - a*d)^3*(
c + d*x)^(11/2)) + (2*b^(5/2)*D*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*S
qrt[a + b*x])])/d^(13/2)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2124, 27, 1193, 27, 87, 57, 57, 57, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx$$

$\downarrow$ 2124

$$\frac{2\int \frac{(a+bx)^{5/2}\left(-11\left(a-\frac{bc}{d}\right)Dx^2+\frac{11(bc-ad)(Cd-cD)x}{d^2}+\frac{11ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-4Ad^3\right)}{d^3}\right)}{2(c+dx)^{11/2}}dx}{11(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\int \frac{(a+bx)^{5/2}\left(-11\left(a-\frac{bc}{d}\right)Dx^2+\frac{11(bc-ad)(Cd-cD)x}{d^2}+\frac{11ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-4Ad^3\right)}{d^3}\right)}{(c+dx)^{11/2}}dx}{11(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\downarrow$ 1193

$$\frac{\frac{2\int \frac{(a+bx)^{5/2}\left(\left(-140Dc^3+63Cdc^2+14Bd^2c+8Ad^3\right)b^2-22ad\left(-15Dc^2+7Cdc+Bd^2\right)b+99a^2d^2(Cd-2cD)+99d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{9/2}}dx}{9(bc-ad)}+\frac{2(a+bx)^{7/2}(11ad(-Bd}{}}{11(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{(a+bx)^{5/2}\left(\left(-140Dc^3+63Cdc^2+14Bd^2c+8Ad^3\right)b^2-22ad\left(-15Dc^2+7Cdc+Bd^2\right)b+99a^2d^2(Cd-2cD)+99d(bc-ad)^2Dx\right)}{(c+dx)^{9/2}}dx}{9d^3(bc-ad)}+\frac{2(a+bx)^{7/2}(11ad(-Bd^2}{}}{11(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$\downarrow$ 87

$$\frac{\frac{99D(bc-ad)^2\int\frac{(a+bx)^{5/2}}{(c+dx)^{7/2}}dx+\frac{2(a+bx)^{7/2}\left(99a^2d^2(Cd-3cD)-22abd\left(Bd^2-24c^2D+7cCd\right)+b^2\left(8Ad^3+14Bcd^2-239c^3D+63c^2Cd\right)\right)}{7(c+dx)^{7/2}(bc-ad)}}{9d^3(bc-ad)}+\frac{2(a+bx)^{7/2}(11ac\cdots}{\cdots}}{11(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$$\Big\downarrow \ 57$$

$$\frac{\frac{99D(bc-ad)^2\left(\frac{b\int\frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}dx}{d}-\frac{2(a+bx)^{5/2}}{5d(c+dx)^{5/2}}\right)+\frac{2(a+bx)^{7/2}\left(99a^2d^2(Cd-3cD)-22abd\left(Bd^2-24c^2D+7cCd\right)+b^2\left(8Ad^3+14Bcd^2-239c^3D+63c^2Cd\right)\right)}{7(c+dx)^{7/2}(bc-ad)}}{9d^3(bc-ad)}+\cdots}{11(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$$\Big\downarrow \ 57$$

$$\frac{\frac{99D(bc-ad)^2\left(\frac{b\left(\frac{b\int\frac{\sqrt{a+bx}}{(c+dx)^{3/2}}dx}{d}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}\right)}{d}-\frac{2(a+bx)^{5/2}}{5d(c+dx)^{5/2}}\right)+\frac{2(a+bx)^{7/2}\left(99a^2d^2(Cd-3cD)-22abd\left(Bd^2-24c^2D+7cCd\right)+b^2\left(8Ad^3+14Bcd^2-2\cdots\right.\right.}{7(c+dx)^{7/2}(bc-ad)}}{9d^3(bc-ad)}\cdots}{11(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$$\Big\downarrow \ 57$$

$$\frac{\frac{99D(bc-ad)^2\left(\frac{b\left(\frac{b\left(\frac{b\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{d}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}\right)}{d}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}\right)}{d}-\frac{2(a+bx)^{5/2}}{5d(c+dx)^{5/2}}\right)+\frac{2(a+bx)^{7/2}\left(99a^2d^2(Cd-3cD)-22abd\left(Bd^2-24c^2D+7cCd\right)+b^2\cdots\right.}{7(c+dx)^{7/2}(bc-ad)}}{9d^3(bc-ad)}\cdots}{11(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$$\Big\downarrow \ 66$$

$$\frac{\frac{99D(bc-ad)^2\left(\frac{b\left(\frac{b\left(\frac{2b\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}\right)}{d}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}\right)}{d}-\frac{2(a+bx)^{5/2}}{5d(c+dx)^{5/2}}\right)+\frac{2(a+bx)^{7/2}\left(99a^2d^2(Cd-3cD)-22abd\left(Bd^2-24c^2D+7cCd\right)\ldots\right)}{7(c+dx)^{7/2}(bc-a\ldots)}}{9d^3(bc-ad)}}{11(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{2(a+bx)^{7/2}\left(99a^2d^2(Cd-3cD)-22abd\left(Bd^2-24c^2D+7cCd\right)+b^2\left(8Ad^3+14Bcd^2-239c^3D+63c^2Cd\right)\right)}{7(c+dx)^{7/2}(bc-ad)}+99D(bc-ad)^2\left(\frac{b\left(\frac{b\left(\frac{2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\ldots\right)}{d}\ldots\right)}{d}\ldots\right)}{9d^3(bc-ad)}}{11(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{11(c+dx)^{11/2}(bc-ad)}$$

input

```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(13/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(7/2))/(11*(b*c - a*d)*
(c + d*x)^(11/2)) + ((2*(11*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(18*c^2*C*
d - 7*B*c*d^2 - 4*A*d^3 - 29*c^3*D))*(a + b*x)^(7/2))/(9*d^3*(b*c - a*d)*(
c + d*x)^(9/2)) + ((2*(99*a^2*d^2*(C*d - 3*c*D) - 22*a*b*d*(7*c*C*d + B*d^
2 - 24*c^2*D) + b^2*(63*c^2*C*d + 14*B*c*d^2 + 8*A*d^3 - 239*c^3*D))*(a +
b*x)^(7/2))/(7*(b*c - a*d)*(c + d*x)^(7/2)) + 99*(b*c - a*d)^2*D*((-2*(a +
 b*x)^(5/2))/(5*d*(c + d*x)^(5/2)) + (b*((-2*(a + b*x)^(3/2))/(3*d*(c + d*
x)^(3/2)) + (b*((-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)))/d))/d))/(9*d^3
*(b*c - a*d)))/(11*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 57
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !IntegerQ[m]) &&  !(ILeQ[m
+ n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c
, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4162 vs. $2(331) = 662$.

Time = 0.54 (sec) , antiderivative size = 4163, normalized size of antiderivative = 11.04

method	result	size
default	Expression too large to display	4163

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/3465*(b*x+a)^(1/2)*(-3300*A*a^2*b^3*c*d^7*x^2*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+2970*A*a*b^4*c^2*d^6*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-45
40*B*a^3*b^2*c*d^7*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1650*B*a^2*b^3*
c^2*d^6*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-220*C*a^4*b*c*d^7*x^2*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+30*C*a^3*b^2*c^2*d^6*x^2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+4092*D*a^4*b*c^2*d^6*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+17248*D*a^3*b^2*c^4*d^4*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+630*A*
a^5*d^8*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-6930*D*b^5*c^8*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)-4180*A*a^3*b^2*c*d^7*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+2970*A*a^2*b^3*c^2*d^6*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-1280*B*
a^4*b*c*d^7*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+110*B*a^3*b^2*c^2*d^6*x*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-40*C*a^4*b*c^2*d^6*x*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+1888*D*a^4*b*c^3*d^5*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)-1280*B*a*b^4*c*d^7*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-4180*C*a^2*
b^3*c*d^7*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1610*C*a*b^4*c^2*d^6*x^4
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+23892*D*a^3*b^2*c*d^7*x^4*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)-116688*D*a^2*b^3*c^2*d^6*x^4*((b*x+a)*(d*x+c))^(1
/2)*(d*b)^(1/2)+138112*D*a*b^4*c^3*d^5*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)-220*A*a*b^4*c*d^7*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-4540*B*a^2*
b^3*c*d^7*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+2090*B*a*b^4*c^2*d^6*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(13/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2012 vs. $2(332) = 664$.

Time $= 1.12$ (sec) , antiderivative size $= 2012$, normalized size of antiderivative $= 5.34$

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{13/2}}\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x, algorithm="g
iac")
```

output
```
-2*D*b^2*abs(b)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*
b*d - a*b*d)))/(sqrt(b*d)*d^6) - 2/3465*(((((b*x + a)*((6508*D*b^16*c^5*d^
10*abs(b) - 31595*D*a*b^15*c^4*d^11*abs(b) - 315*C*b^16*c^4*d^11*abs(b) +
61090*D*a^2*b^14*c^3*d^12*abs(b) + 1400*C*a*b^15*c^3*d^12*abs(b) - 70*B*b^
16*c^3*d^12*abs(b) - 58740*D*a^3*b^13*c^2*d^13*abs(b) - 2350*C*a^2*b^14*c^
2*d^13*abs(b) + 250*B*a*b^15*c^2*d^13*abs(b) - 40*A*b^16*c^2*d^13*abs(b) +
 28050*D*a^4*b^12*c*d^14*abs(b) + 1760*C*a^3*b^13*c*d^14*abs(b) - 290*B*a^
2*b^14*c*d^14*abs(b) + 80*A*a*b^15*c*d^14*abs(b) - 5313*D*a^5*b^11*d^15*ab
s(b) - 495*C*a^4*b^12*d^15*abs(b) + 110*B*a^3*b^13*d^15*abs(b) - 40*A*a^2*
b^14*d^15*abs(b))*(b*x + a)/(b^9*c^5*d^11 - 5*a*b^8*c^4*d^12 + 10*a^2*b^7*
c^3*d^13 - 10*a^3*b^6*c^2*d^14 + 5*a^4*b^5*c*d^15 - a^5*b^4*d^16) + 11*(23
09*D*b^17*c^6*d^9*abs(b) - 13854*D*a*b^16*c^5*d^10*abs(b) + 34530*D*a^2*b^
15*c^4*d^11*abs(b) + 70*C*a*b^16*c^4*d^11*abs(b) - 35*B*b^17*c^4*d^11*abs(
b) - 45740*D*a^3*b^14*c^3*d^12*abs(b) - 300*C*a^2*b^15*c^3*d^12*abs(b) + 1
60*B*a*b^16*c^3*d^12*abs(b) - 20*A*b^17*c^3*d^12*abs(b) + 33945*D*a^4*b^13
*c^2*d^13*abs(b) + 480*C*a^3*b^14*c^2*d^13*abs(b) - 270*B*a^2*b^15*c^2*d^1
3*abs(b) + 60*A*a*b^16*c^2*d^13*abs(b) - 13374*D*a^5*b^12*c*d^14*abs(b) -
340*C*a^4*b^13*c*d^14*abs(b) + 200*B*a^3*b^14*c*d^14*abs(b) - 60*A*a^2*b^1
5*c*d^14*abs(b) + 2184*D*a^6*b^11*d^15*abs(b) + 90*C*a^5*b^12*d^15*abs(b)
- 55*B*a^4*b^13*d^15*abs(b) + 20*A*a^3*b^14*d^15*abs(b))/(b^9*c^5*d^11 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx=\int \frac{(a+b\,x)^{5/2}\left(A+B\,x+C\,x^2+x^3\,D\right)}{(c+d\,x)^{13/2}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(13/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(13/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{13/2}}\,dx=\int \frac{(bx+a)^{\frac{5}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(dx+c)^{\frac{13}{2}}}dx$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x)
```

output
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(13/2),x)
```

3.128 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{15/2}}\,dx$

Optimal result 1225
Mathematica [A] (verified) 1226
Rubi [A] (verified) 1226
Maple [A] (verified) 1229
Fricas [F(-1)] 1230
Sympy [F(-1)] 1230
Maxima [F(-2)] 1231
Giac [B] (verification not implemented) 1231
Mupad [F(-1)] 1232
Reduce [F] 1233

Optimal result

Integrand size = 34, antiderivative size = 375

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{7/2}}{13d^3(bc-ad)(c+dx)^{13/2}}$$
$$+\frac{2(13ad(2cCd-Bd^2-3c^2D)-b(20c^2Cd-7Bcd^2-6Ad^3-33c^3D))\,(a+bx)^{7/2}}{143d^3(bc-ad)^2(c+dx)^{11/2}}$$
$$+\frac{2(143a^2d^2(Cd-3cD)-26abd(7cCd+2Bd^2-27c^2D)+b^2(63c^2Cd+28Bcd^2+24Ad^3-297c^3D))\,(a -}{1287d^3(bc-ad)^3(c+dx)^{9/2}}$$
$$-\frac{2(1287a^3d^3D-143a^2bd^2(2Cd+21cD)+13ab^2d(28cCd+8Bd^2+189c^2D)-b^3(126c^2Cd+56Bcd^2+}{9009d^3(bc-ad)^4(c+dx)^{7/2}}$$

output
```
2/13*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(7/2)/d^3/(-a*d+b*c)/(d*x+c)^(1
3/2)+2/143*(13*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-6*A*d^3-7*B*c*d^2+20*C*c^2
*d-33*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(11/2)+2/1287*(143*a^
2*d^2*(C*d-3*D*c)-26*a*b*d*(2*B*d^2+7*C*c*d-27*D*c^2)+b^2*(24*A*d^3+28*B*c
*d^2+63*C*c^2*d-297*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(9/2)-2
/9009*(1287*a^3*d^3*D-143*a^2*b*d^2*(2*C*d+21*D*c)+13*a*b^2*d*(8*B*d^2+28*
C*c*d+189*D*c^2)-b^3*(48*A*d^3+56*B*c*d^2+126*C*c^2*d+693*D*c^3))*(b*x+a)^
(7/2)/d^3/(-a*d+b*c)^4/(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \frac{2(a+bx)^{13/2}\left(-693c^2Cd+693Bcd^2-693Ad^3+693c^3D+\right.}{}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(15/2),x]
```

output
```
(2*(a + b*x)^(13/2)*(-693*c^2*C*d + 693*B*c*d^2 - 693*A*d^3 + 693*c^3*D +
(819*b*c^2*C*(c + d*x))/(a + b*x) - (1638*b*B*c*d*(c + d*x))/(a + b*x) + (
1638*a*c*C*d*(c + d*x))/(a + b*x) + (2457*A*b*d^2*(c + d*x))/(a + b*x) - (
819*a*B*d^2*(c + d*x))/(a + b*x) - (2457*a*c^2*D*(c + d*x))/(a + b*x) + (1
001*b^2*B*c*(c + d*x)^2)/(a + b*x)^2 - (2002*a*b*c*C*(c + d*x)^2)/(a + b*x
)^2 - (3003*A*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (2002*a*b*B*d*(c + d*x)^2)/
(a + b*x)^2 - (1001*a^2*C*d*(c + d*x)^2)/(a + b*x)^2 + (3003*a^2*c*D*(c +
d*x)^2)/(a + b*x)^2 + (1287*A*b^3*(c + d*x)^3)/(a + b*x)^3 - (1287*a*b^2*B
*(c + d*x)^3)/(a + b*x)^3 + (1287*a^2*b*C*(c + d*x)^3)/(a + b*x)^3 - (1287
*a^3*D*(c + d*x)^3)/(a + b*x)^3))/(9009*(b*c - a*d)^4*(c + d*x)^(13/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2124, 27, 1193, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$\frac{2\int \frac{(a+bx)^{5/2}\left(-13\left(a-\frac{bc}{d}\right)Dx^2+\frac{13(bc-ad)(Cd-cD)x}{d^2}+\frac{13ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-6Ad^3\right)}{d^3}\right)}{2(c+dx)^{13/2}}dx}{13(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a+bx)^{5/2}\left(-13\left(a-\frac{bc}{d}\right)Dx^2+\frac{13(bc-ad)(Cd-cD)x}{d^2}+\frac{13ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-6Ad^3\right)}{d^3}\right)}{(c+dx)^{13/2}}dx}{13(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$\frac{\frac{2\int \frac{(a+bx)^{5/2}\left(\left(-154Dc^3+63Cdc^2+28Bd^2c+24Ad^3\right)b^2-26ad\left(-16Dc^2+7Cdc+2Bd^2\right)b+143a^2d^2(Cd-2cD)+143d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{11/2}}dx}{11(bc-ad)}+\frac{2(a+bx)^{7/2}(13ad(-}{}}{13(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{(a+bx)^{5/2}\left(\left(-154Dc^3+63Cdc^2+28Bd^2c+24Ad^3\right)b^2-26ad\left(-16Dc^2+7Cdc+2Bd^2\right)b+143a^2d^2(Cd-2cD)+143d(bc-ad)^2Dx\right)}{(c+dx)^{11/2}}dx}{11d^3(bc-ad)}+\frac{2(a+bx)^{7/2}(13ad(-}{}}{13(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$\frac{\frac{\frac{2(a+bx)^{7/2}\left(143a^2d^2(Cd-3cD)-26abd\left(2Bd^2-27c^2D+7cCd\right)+b^2\left(24Ad^3+28Bcd^2-297c^3D+63c^2Cd\right)\right)}{9(c+dx)^{9/2}(bc-ad)}-\frac{\left(1287a^3d^3D-143a^2bd^2(21cD+2Cd)+13ab^2d\left(8Bd^2\right.\right.}{}}{11d^3(bc-ad)}}{1}+\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

$\Big\downarrow$ 48

$$\frac{\frac{\frac{2(a+bx)^{7/2}\left(143a^2d^2(Cd-3cD)-26abd\left(2Bd^2-27c^2D+7cCd\right)+b^2\left(24Ad^3+28Bcd^2-297c^3D+63c^2Cd\right)\right)}{9(c+dx)^{9/2}(bc-ad)}-\frac{2(a+bx)^{7/2}\left(1287a^3d^3D-143a^2bd^2(21cD+2Cd)+13\cdots\right.}{\cdots}}{11d^3(bc-ad)}}{13\cdots}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{13(c+dx)^{13/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(15/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(7/2))/(13*(b*c - a*d)*
(c + d*x)^(13/2)) + ((2*(13*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(20*c^2*C*
d - 7*B*c*d^2 - 6*A*d^3 - 33*c^3*D))*(a + b*x)^(7/2))/(11*d^3*(b*c - a*d)*
(c + d*x)^(11/2)) + ((2*(143*a^2*d^2*(C*d - 3*c*D) - 26*a*b*d*(7*c*C*d + 2
*B*d^2 - 27*c^2*D) + b^2*(63*c^2*C*d + 28*B*c*d^2 + 24*A*d^3 - 297*c^3*D))
*(a + b*x)^(7/2))/(9*(b*c - a*d)*(c + d*x)^(9/2)) - (2*(1287*a^3*d^3*D - 1
43*a^2*b*d^2*(2*C*d + 21*c*D) + 13*a*b^2*d*(28*c*C*d + 8*B*d^2 + 189*c^2*D
) - b^3*(126*c^2*C*d + 56*B*c*d^2 + 48*A*d^3 + 693*c^3*D))*(a + b*x)^(7/2)
)/(63*(b*c - a*d)^2*(c + d*x)^(7/2)))/(11*d^3*(b*c - a*d)))/(13*(b*c - a*d
))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.58

method	result
gosper	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-48A\,b^{3}d^{3}x^{3}+104Ba\,b^{2}d^{3}x^{3}-56B\,b^{3}c\,d^{2}x^{3}-286C\,a^{2}b\,d^{3}x^{3}+364Ca\,b^{2}c\,d^{2}x^{3}-126C\,b^{3}c^{2}d\,x^{3}+1287Da^{3}d^{3}x^{3}-3003\right.}{}$
orering	$-\frac{2(bx+a)^{\frac{7}{2}}\left(-48A\,b^{3}d^{3}x^{3}+104Ba\,b^{2}d^{3}x^{3}-56B\,b^{3}c\,d^{2}x^{3}-286C\,a^{2}b\,d^{3}x^{3}+364Ca\,b^{2}c\,d^{2}x^{3}-126C\,b^{3}c^{2}d\,x^{3}+1287Da^{3}d^{3}x^{3}-3003\right.}{}$
default	Expression too large to display

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/9009*(b*x+a)^(7/2)*(-48*A*b^3*d^3*x^3+104*B*a*b^2*d^3*x^3-56*B*b^3*c*d^
2*x^3-286*C*a^2*b*d^3*x^3+364*C*a*b^2*c*d^2*x^3-126*C*b^3*c^2*d*x^3+1287*D
*a^3*d^3*x^3-3003*D*a^2*b*c*d^2*x^3+2457*D*a*b^2*c^2*d*x^3-693*D*b^3*c^3*x
^3+168*A*a*b^2*d^3*x^2-312*A*b^3*c*d^2*x^2-364*B*a^2*b*d^3*x^2+872*B*a*b^2
*c*d^2*x^2-364*B*b^3*c^2*d*x^2+1001*C*a^3*d^3*x^2-3133*C*a^2*b*c*d^2*x^2+2
807*C*a*b^2*c^2*d*x^2-819*C*b^3*c^3*x^2+858*D*a^3*c*d^2*x^2-1092*D*a^2*b*c
^2*d*x^2+378*D*a*b^2*c^3*x^2-378*A*a^2*b*d^3*x+1092*A*a*b^2*c*d^2*x-858*A*
b^3*c^2*d*x+819*B*a^3*d^3*x-2807*B*a^2*b*c*d^2*x+3133*B*a*b^2*c^2*d*x-1001
*B*b^3*c^3*x+364*C*a^3*c*d^2*x-872*C*a^2*b*c^2*d*x+364*C*a*b^2*c^3*x+312*D
*a^3*c^2*d*x-168*D*a^2*b*c^3*x+693*A*a^3*d^3-2457*A*a^2*b*c*d^2+3003*A*a*b
^2*c^2*d-1287*A*b^3*c^3+126*B*a^3*c*d^2-364*B*a^2*b*c^2*d+286*B*a*b^2*c^3+
56*C*a^3*c^2*d-104*C*a^2*b*c^3+48*D*a^3*c^3)/(d*x+c)^(13/2)/(a^4*d^4-4*a^3
*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(15/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1739 vs. 2(351) = 702.

Time = 1.44 (sec) , antiderivative size = 1739, normalized size of antiderivative = 4.64

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x, algorithm="g
iac")
```

output
```
2/9009*(((b*x + a)*((693*D*b^18*c^5*d^6*abs(b) - 3843*D*a*b^17*c^4*d^7*abs
(b) + 126*C*b^18*c^4*d^7*abs(b) + 8610*D*a^2*b^16*c^3*d^8*abs(b) - 616*C*a
*b^17*c^3*d^8*abs(b) + 56*B*b^18*c^3*d^8*abs(b) - 9750*D*a^3*b^15*c^2*d^9*
abs(b) + 1140*C*a^2*b^16*c^2*d^9*abs(b) - 216*B*a*b^17*c^2*d^9*abs(b) + 48
*A*b^18*c^2*d^9*abs(b) + 5577*D*a^4*b^14*c*d^10*abs(b) - 936*C*a^3*b^15*c*
d^10*abs(b) + 264*B*a^2*b^16*c*d^10*abs(b) - 96*A*a*b^17*c*d^10*abs(b) - 1
287*D*a^5*b^13*d^11*abs(b) + 286*C*a^4*b^14*d^11*abs(b) - 104*B*a^3*b^15*d
^11*abs(b) + 48*A*a^2*b^16*d^11*abs(b))*(b*x + a)/(b^10*c^6*d^6 - 6*a*b^9*
c^5*d^7 + 15*a^2*b^8*c^4*d^8 - 20*a^3*b^7*c^3*d^9 + 15*a^4*b^6*c^2*d^10 -
6*a^5*b^5*c*d^11 + a^6*b^4*d^12) - 13*(189*D*a*b^18*c^5*d^6*abs(b) - 63*C*
b^19*c^5*d^6*abs(b) - 1029*D*a^2*b^17*c^4*d^7*abs(b) + 371*C*a*b^18*c^4*d^
7*abs(b) - 28*B*b^19*c^4*d^7*abs(b) + 2250*D*a^3*b^16*c^3*d^8*abs(b) - 878
*C*a^2*b^17*c^3*d^8*abs(b) + 136*B*a*b^18*c^3*d^8*abs(b) - 24*A*b^19*c^3*d
^8*abs(b) - 2466*D*a^4*b^15*c^2*d^9*abs(b) + 1038*C*a^3*b^16*c^2*d^9*abs(b
) - 240*B*a^2*b^17*c^2*d^9*abs(b) + 72*A*a*b^18*c^2*d^9*abs(b) + 1353*D*a^
5*b^14*c*d^10*abs(b) - 611*C*a^4*b^15*c*d^10*abs(b) + 184*B*a^3*b^16*c*d^1
0*abs(b) - 72*A*a^2*b^17*c*d^10*abs(b) - 297*D*a^6*b^13*d^11*abs(b) + 143*
C*a^5*b^14*d^11*abs(b) - 52*B*a^4*b^15*d^11*abs(b) + 24*A*a^3*b^16*d^11*ab
s(b))/(b^10*c^6*d^6 - 6*a*b^9*c^5*d^7 + 15*a^2*b^8*c^4*d^8 - 20*a^3*b^7*c^
3*d^9 + 15*a^4*b^6*c^2*d^10 - 6*a^5*b^5*c*d^11 + a^6*b^4*d^12)) + 143*(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx=\int \frac{(a+b\,x)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{(c+d\,x)^{15/2}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(15/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(15/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{15/2}}\,dx=\int \frac{(bx+a)^{\frac{5}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(dx+c)^{\frac{15}{2}}}dx$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x)
```

output
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(15/2),x)
```

3.129 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{17/2}}\,dx$

Optimal result 1234
Mathematica [A] (verified) 1235
Rubi [A] (verified) 1236
Maple [B] (verified) 1239
Fricas [F(-1)] 1240
Sympy [F(-1)] 1241
Maxima [F(-2)] 1241
Giac [B] (verification not implemented) 1241
Mupad [F(-1)] 1242
Reduce [F] 1243

Optimal result

Integrand size = 34, antiderivative size = 496

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{17/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\,(a+bx)^{7/2}}{15d^3(bc-ad)(c+dx)^{15/2}}$$

$$+\frac{2(15ad(2cCd-Bd^2-3c^2D)-b(22c^2Cd-7Bcd^2-8Ad^3-37c^3D))\,(a+bx)^{7/2}}{195d^3(bc-ad)^2(c+dx)^{13/2}}$$

$$+\frac{2(65a^2d^2(Cd-3cD)-10abd(7cCd+3Bd^2-30c^2D)+b^2(21c^2Cd+14Bcd^2+16Ad^3-121c^3D))\,(a+}{715d^3(bc-ad)^3(c+dx)^{11/2}}$$

$$-\frac{2(715a^3d^3D-65a^2bd^2(4Cd+21cD)+5ab^2d(56cCd+24Bd^2+189c^2D)-b^3(84c^2Cd+56Bcd^2+64A}{6435d^3(bc-ad)^4(c+dx)^{9/2}}$$

$$-\frac{4b(715a^3d^3D-65a^2bd^2(4Cd+21cD)+5ab^2d(56cCd+24Bd^2+189c^2D)-b^3(84c^2Cd+56Bcd^2+64}{45045d^3(bc-ad)^5(c+dx)^{7/2}}$$

output
```
2/15*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(7/2)/d^3/(-a*d+b*c)/(d*x+c)^(1
5/2)+2/195*(15*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-8*A*d^3-7*B*c*d^2+22*C*c^2
*d-37*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(13/2)+2/715*(65*a^2*
d^2*(C*d-3*D*c)-10*a*b*d*(3*B*d^2+7*C*c*d-30*D*c^2)+b^2*(16*A*d^3+14*B*c*d
^2+21*C*c^2*d-121*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(11/2)-2/
6435*(715*a^3*d^3*D-65*a^2*b*d^2*(4*C*d+21*D*c)+5*a*b^2*d*(24*B*d^2+56*C*c
*d+189*D*c^2)-b^3*(64*A*d^3+56*B*c*d^2+84*C*c^2*d+231*D*c^3))*(b*x+a)^(7/2
)/d^3/(-a*d+b*c)^4/(d*x+c)^(9/2)-4/45045*b*(715*a^3*d^3*D-65*a^2*b*d^2*(4*
C*d+21*D*c)+5*a*b^2*d*(24*B*d^2+56*C*c*d+189*D*c^2)-b^3*(64*A*d^3+56*B*c*d
^2+84*C*c^2*d+231*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^5/(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{17/2}}\,dx = \frac{2(a+bx)^{7/2}\left(3003c^2Cd^2(a+bx)^4-3003Bcd^3(a+bx)^4+300\right.}{}$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(17/2),x]
```

output
```
(2*(a + b*x)^(7/2)*(3003*c^2*C*d^2*(a + b*x)^4 - 3003*B*c*d^3*(a + b*x)^4
+ 3003*A*d^4*(a + b*x)^4 - 3003*c^3*d*D*(a + b*x)^4 - 6930*b*c^2*C*d*(a +
b*x)^3*(c + d*x) + 10395*b*B*c*d^2*(a + b*x)^3*(c + d*x) - 6930*a*c*C*d^2*
(a + b*x)^3*(c + d*x) - 13860*A*b*d^3*(a + b*x)^3*(c + d*x) + 3465*a*B*d^3
*(a + b*x)^3*(c + d*x) + 3465*b*c^3*D*(a + b*x)^3*(c + d*x) + 10395*a*c^2*
d*D*(a + b*x)^3*(c + d*x) + 4095*b^2*c^2*C*(a + b*x)^2*(c + d*x)^2 - 12285
*b^2*B*c*d*(a + b*x)^2*(c + d*x)^2 + 16380*a*b*c*C*d*(a + b*x)^2*(c + d*x)
^2 + 24570*A*b^2*d^2*(a + b*x)^2*(c + d*x)^2 - 12285*a*b*B*d^2*(a + b*x)^2
*(c + d*x)^2 + 4095*a^2*C*d^2*(a + b*x)^2*(c + d*x)^2 - 12285*a*b*c^2*D*(a
 + b*x)^2*(c + d*x)^2 - 12285*a^2*c*d*D*(a + b*x)^2*(c + d*x)^2 + 5005*b^3
*B*c*(a + b*x)*(c + d*x)^3 - 10010*a*b^2*c*C*(a + b*x)*(c + d*x)^3 - 20020
*A*b^3*d*(a + b*x)*(c + d*x)^3 + 15015*a*b^2*B*d*(a + b*x)*(c + d*x)^3 - 1
0010*a^2*b*C*d*(a + b*x)*(c + d*x)^3 + 15015*a^2*b*c*D*(a + b*x)*(c + d*x)
^3 + 5005*a^3*d*D*(a + b*x)*(c + d*x)^3 + 6435*A*b^4*(c + d*x)^4 - 6435*a*
b^3*B*(c + d*x)^4 + 6435*a^2*b^2*C*(c + d*x)^4 - 6435*a^3*b*D*(c + d*x)^4)
)/(45045*(b*c - a*d)^5*(c + d*x)^(15/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {2124, 27, 1193, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{17/2}}\,dx$$

$$\Big\downarrow \ 2124$$

$$\frac{2\int \frac{(a+bx)^{5/2}\left(-15\left(a-\frac{bc}{d}\right)Dx^2+\frac{15(bc-ad)(Cd-cD)x}{d^2}+\frac{15ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-8Ad^3\right)}{d^3}\right)}{2(c+dx)^{15/2}}dx}{15(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{15(c+dx)^{15/2}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(a+bx)^{5/2}\left(-15\left(a-\frac{bc}{d}\right)Dx^2+\frac{15(bc-ad)(Cd-cD)x}{d^2}+\frac{15ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-8Ad^3\right)}{d^3}\right)}{(c+dx)^{15/2}}dx}{15(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{15(c+dx)^{15/2}(bc-ad)}$$

$$\Big\downarrow \ 1193$$

$$\frac{\frac{2\int \frac{3(a+bx)^{5/2}\left(\left(-56Dc^3+21Cdc^2+14Bd^2c+16Ad^3\right)b^2-10ad\left(-17Dc^2+7Cdc+3Bd^2\right)b+65a^2d^2(Cd-2cD)+65d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{13/2}}dx}{13(bc-ad)}+\frac{2(a+bx)^{7/2}(15ad(-B}{}}{15(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{15(c+dx)^{15/2}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{(a+bx)^{5/2}\left(\left(-56Dc^3+21Cdc^2+14Bd^2c+16Ad^3\right)b^2-10ad\left(-17Dc^2+7Cdc+3Bd^2\right)b+65a^2d^2(Cd-2cD)+65d(bc-ad)^2Dx\right)}{(c+dx)^{13/2}}dx}{13d^3(bc-ad)}+\frac{2(a+bx)^{7/2}(15ad(-B}{}}{15(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{15(c+dx)^{15/2}(bc-ad)}$$

$\downarrow$ 87

$$\frac{3\left(\frac{2(a+bx)^{7/2}\left(65a^2d^2(Cd-3cD)-10abd\left(3Bd^2-30c^2D+7cCd\right)+b^2\left(16Ad^3+14Bcd^2-121c^3D+21c^2Cd\right)\right)}{11(c+dx)^{11/2}(bc-ad)}-\frac{\left(715a^3d^3D-65a^2bd^2(21cD+4Cd)+5ab^2d\left(24Bd^2\right.\right.}{}\right.}{13d^3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{15(c+dx)^{15/2}(bc-ad)}$$

$\downarrow$ 55

$$\frac{3\left(\frac{2(a+bx)^{7/2}\left(65a^2d^2(Cd-3cD)-10abd\left(3Bd^2-30c^2D+7cCd\right)+b^2\left(16Ad^3+14Bcd^2-121c^3D+21c^2Cd\right)\right)}{11(c+dx)^{11/2}(bc-ad)}-\frac{\left(715a^3d^3D-65a^2bd^2(21cD+4Cd)+5ab^2d\left(24Bd^2\right.\right.}{}\right.}{13d^3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{15(c+dx)^{15/2}(bc-ad)}$$

$\downarrow$ 48

$$\frac{3\left(\frac{2(a+bx)^{7/2}\left(65a^2d^2(Cd-3cD)-10abd\left(3Bd^2-30c^2D+7cCd\right)+b^2\left(16Ad^3+14Bcd^2-121c^3D+21c^2Cd\right)\right)}{11(c+dx)^{11/2}(bc-ad)}-\frac{\left(\frac{4b(a+bx)^{7/2}}{63(c+dx)^{7/2}(bc-ad)^2}+\frac{2(a+bx)^{7/2}}{9(c+dx)^{9/2}(bc-ad)}\right)\left(\right.}{}\right.}{13d^3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{15(c+dx)^{15/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(17/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(7/2))/(15*(b*c - a*d)*
(c + d*x)^(15/2)) + ((2*(15*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(22*c^2*C*
d - 7*B*c*d^2 - 8*A*d^3 - 37*c^3*D))*(a + b*x)^(7/2))/(13*d^3*(b*c - a*d)*
(c + d*x)^(13/2)) + (3*((2*(65*a^2*d^2*(C*d - 3*c*D) - 10*a*b*d*(7*c*C*d +
 3*B*d^2 - 30*c^2*D) + b^2*(21*c^2*C*d + 14*B*c*d^2 + 16*A*d^3 - 121*c^3*D
))*(a + b*x)^(7/2))/(11*(b*c - a*d)*(c + d*x)^(11/2)) - ((715*a^3*d^3*D -
65*a^2*b*d^2*(4*C*d + 21*c*D) + 5*a*b^2*d*(56*c*C*d + 24*B*d^2 + 189*c^2*D
) - b^3*(84*c^2*C*d + 56*B*c*d^2 + 64*A*d^3 + 231*c^3*D))*((2*(a + b*x)^(7
/2))/(9*(b*c - a*d)*(c + d*x)^(9/2)) + (4*b*(a + b*x)^(7/2))/(63*(b*c - a*
d)^2*(c + d*x)^(7/2))))/(11*(b*c - a*d))))/(13*d^3*(b*c - a*d)))/(15*(b*c
- a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(466) = 932$.

Time = 0.54 (sec) , antiderivative size = 1006, normalized size of antiderivative = 2.03

method	result	size
gosper	Expression too large to display	1006
orering	Expression too large to display	1006
default	Expression too large to display	1586

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(17/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/45045*(b*x+a)^(7/2)*(128*A*b^4*d^4*x^4-240*B*a*b^3*d^4*x^4+112*B*b^4*c*
d^3*x^4+520*C*a^2*b^2*d^4*x^4-560*C*a*b^3*c*d^3*x^4+168*C*b^4*c^2*d^2*x^4-
1430*D*a^3*b*d^4*x^4+2730*D*a^2*b^2*c*d^3*x^4-1890*D*a*b^3*c^2*d^2*x^4+462
*D*b^4*c^3*d*x^4-448*A*a*b^3*d^4*x^3+960*A*b^4*c*d^3*x^3+840*B*a^2*b^2*d^4
*x^3-2192*B*a*b^3*c*d^3*x^3+840*B*b^4*c^2*d^2*x^3-1820*C*a^3*b*d^4*x^3+586
0*C*a^2*b^2*c*d^3*x^3-4788*C*a*b^3*c^2*d^2*x^3+1260*C*b^4*c^3*d*x^3+5005*D
*a^4*d^4*x^3-20280*D*a^3*b*c*d^3*x^3+27090*D*a^2*b^2*c^2*d^2*x^3-15792*D*a
*b^3*c^3*d*x^3+3465*D*b^4*c^4*x^3+1008*A*a^2*b^2*d^4*x^2-3360*A*a*b^3*c*d^
3*x^2+3120*A*b^4*c^2*d^2*x^2-1890*B*a^3*b*d^4*x^2+7182*B*a^2*b^2*c*d^3*x^2
-8790*B*a*b^3*c^2*d^2*x^2+2730*B*b^4*c^3*d*x^2+4095*C*a^4*d^4*x^2-18060*C*
a^3*b*c*d^3*x^2+28698*C*a^2*b^2*c^2*d^2*x^2-18060*C*a*b^3*c^3*d*x^2+4095*C
*b^4*c^4*x^2+2730*D*a^4*c*d^3*x^2-8790*D*a^3*b*c^2*d^2*x^2+7182*D*a^2*b^2*
c^3*d*x^2-1890*D*a*b^3*c^4*x^2-1848*A*a^3*b*d^4*x+7560*A*a^2*b^2*c*d^3*x-1
0920*A*a*b^3*c^2*d^2*x+5720*A*b^4*c^3*d*x+3465*B*a^4*d^4*x-15792*B*a^3*b*c
*d^3*x+27090*B*a^2*b^2*c^2*d^2*x-20280*B*a*b^3*c^3*d*x+5005*B*b^4*c^4*x+12
60*C*a^4*c*d^3*x-4788*C*a^3*b*c^2*d^2*x+5860*C*a^2*b^2*c^3*d*x-1820*C*a*b^
3*c^4*x+840*D*a^4*c^2*d^2*x-2192*D*a^3*b*c^3*d*x+840*D*a^2*b^2*c^4*x+3003*
A*a^4*d^4-13860*A*a^3*b*c*d^3+24570*A*a^2*b^2*c^2*d^2-20020*A*a*b^3*c^3*d+
6435*A*b^4*c^4+462*B*a^4*c*d^3-1890*B*a^3*b*c^2*d^2+2730*B*a^2*b^2*c^3*d-1
430*B*a*b^3*c^4+168*C*a^4*c^2*d^2-560*C*a^3*b*c^3*d+520*C*a^2*b^2*c^4+1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(17/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{17/2}} \, dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(17/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{17/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(17/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2452 vs. $2(466) = 932$.

Time = 1.93 (sec) , antiderivative size = 2452, normalized size of antiderivative = 4.94

$$\int \frac{(a+bx)^{5/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{17/2}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(17/2),x, algorithm="g
iac")
```

output
```
2/45045*((((b*x + a)*(2*(231*D*b^20*c^5*d^8*abs(b) - 1407*D*a*b^19*c^4*d^9
*abs(b) + 84*C*b^20*c^4*d^9*abs(b) + 3486*D*a^2*b^18*c^3*d^10*abs(b) - 448
*C*a*b^19*c^3*d^10*abs(b) + 56*B*b^20*c^3*d^10*abs(b) - 4390*D*a^3*b^17*c^
2*d^11*abs(b) + 904*C*a^2*b^18*c^2*d^11*abs(b) - 232*B*a*b^19*c^2*d^11*abs
(b) + 64*A*b^20*c^2*d^11*abs(b) + 2795*D*a^4*b^16*c*d^12*abs(b) - 800*C*a^
3*b^17*c*d^12*abs(b) + 296*B*a^2*b^18*c*d^12*abs(b) - 128*A*a*b^19*c*d^12*
abs(b) - 715*D*a^5*b^15*d^13*abs(b) + 260*C*a^4*b^16*d^13*abs(b) - 120*B*a
^3*b^17*d^13*abs(b) + 64*A*a^2*b^18*d^13*abs(b))*(b*x + a)/(b^11*c^7*d^7 -
 7*a*b^10*c^6*d^8 + 21*a^2*b^9*c^5*d^9 - 35*a^3*b^8*c^4*d^10 + 35*a^4*b^7*
c^3*d^11 - 21*a^5*b^6*c^2*d^12 + 7*a^6*b^5*c*d^13 - a^7*b^4*d^14) + 15*(23
1*D*b^21*c^6*d^7*abs(b) - 1638*D*a*b^20*c^5*d^8*abs(b) + 84*C*b^21*c^5*d^8
*abs(b) + 4893*D*a^2*b^19*c^4*d^9*abs(b) - 532*C*a*b^20*c^4*d^9*abs(b) + 5
6*B*b^21*c^4*d^9*abs(b) - 7876*D*a^3*b^18*c^3*d^10*abs(b) + 1352*C*a^2*b^1
9*c^3*d^10*abs(b) - 288*B*a*b^20*c^3*d^10*abs(b) + 64*A*b^21*c^3*d^10*abs(
b) + 7185*D*a^4*b^17*c^2*d^11*abs(b) - 1704*C*a^3*b^18*c^2*d^11*abs(b) + 5
28*B*a^2*b^19*c^2*d^11*abs(b) - 192*A*a*b^20*c^2*d^11*abs(b) - 3510*D*a^5*
b^16*c*d^12*abs(b) + 1060*C*a^4*b^17*c*d^12*abs(b) - 416*B*a^3*b^18*c*d^12
*abs(b) + 192*A*a^2*b^19*c*d^12*abs(b) + 715*D*a^6*b^15*d^13*abs(b) - 260*
C*a^5*b^16*d^13*abs(b) + 120*B*a^4*b^17*d^13*abs(b) - 64*A*a^3*b^18*d^13*a
bs(b))/(b^11*c^7*d^7 - 7*a*b^10*c^6*d^8 + 21*a^2*b^9*c^5*d^9 - 35*a^3*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{17/2}}\,dx=\int \frac{(a+b\,x)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{17/2}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(17/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(17/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{17/2}}\,dx = \int \frac{(bx+a)^{\frac{5}{2}}(Dx^3+C\,x^2+Bx+A)}{(dx+c)^{\frac{17}{2}}}dx$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(17/2),x)
```

output
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(17/2),x)
```

3.130 $\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{19/2}} dx$

Optimal result 1244
Mathematica [A] (verified) 1245
Rubi [A] (verified) 1246
Maple [B] (verified) 1250
Fricas [F(-1)] 1251
Sympy [F(-1)] 1252
Maxima [F(-2)] 1252
Giac [B] (verification not implemented) 1252
Mupad [F(-1)] 1253
Reduce [F] 1254

Optimal result

Integrand size = 34, antiderivative size = 619

$$\int \frac{(a+bx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(c+dx)^{19/2}} dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)(a+bx)^{7/2}}{17d^3(bc-ad)(c+dx)^{17/2}}$$
$$+\frac{2(17ad(2cCd-Bd^2-3c^2D)-b(24c^2Cd-7Bcd^2-10Ad^3-41c^3D))(a+bx)^{7/2}}{255d^3(bc-ad)^2(c+dx)^{15/2}}$$
$$+\frac{2(255a^2d^2(Cd-3cD)-34abd(7cCd+4Bd^2-33c^2D)+b^2(63c^2Cd+56Bcd^2+80Ad^3-437c^3D))(a-}{3315d^3(bc-ad)^3(c+dx)^{13/2}}$$
$$-\frac{2(1105a^3d^3D-255a^2bd^2(2Cd+7cD)+17ab^2d(28cCd+16Bd^2+63c^2D)-b^3(126c^2Cd+112Bcd^2+}{12155d^3(bc-ad)^4(c+dx)^{11/2}}$$
$$-\frac{8b(1105a^3d^3D-255a^2bd^2(2Cd+7cD)+17ab^2d(28cCd+16Bd^2+63c^2D)-b^3(126c^2Cd+112Bcd^2+}{109395d^3(bc-ad)^5(c+dx)^{9/2}}$$
$$-\frac{16b^2(1105a^3d^3D-255a^2bd^2(2Cd+7cD)+17ab^2d(28cCd+16Bd^2+63c^2D)-b^3(126c^2Cd+112Bcd^2}{765765d^3(bc-ad)^6(c+dx)^{7/2}}$$

output
```
2/17*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(7/2)/d^3/(-a*d+b*c)/(d*x+c)^(1
7/2)+2/255*(17*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-10*A*d^3-7*B*c*d^2+24*C*c^
2*d-41*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(15/2)+2/3315*(255*a
^2*d^2*(C*d-3*D*c)-34*a*b*d*(4*B*d^2+7*C*c*d-33*D*c^2)+b^2*(80*A*d^3+56*B*
c*d^2+63*C*c^2*d-437*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(13/2)
-2/12155*(1105*a^3*d^3*D-255*a^2*b*d^2*(2*C*d+7*D*c)+17*a*b^2*d*(16*B*d^2+
28*C*c*d+63*D*c^2)-b^3*(160*A*d^3+112*B*c*d^2+126*C*c^2*d+231*D*c^3))*(b*x
+a)^(7/2)/d^3/(-a*d+b*c)^4/(d*x+c)^(11/2)-8/109395*b*(1105*a^3*d^3*D-255*a
^2*b*d^2*(2*C*d+7*D*c)+17*a*b^2*d*(16*B*d^2+28*C*c*d+63*D*c^2)-b^3*(160*A*
d^3+112*B*c*d^2+126*C*c^2*d+231*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^5/(d*
x+c)^(9/2)-16/765765*b^2*(1105*a^3*d^3*D-255*a^2*b*d^2*(2*C*d+7*D*c)+17*a*
b^2*d*(16*B*d^2+28*C*c*d+63*D*c^2)-b^3*(160*A*d^3+112*B*c*d^2+126*C*c^2*d+
231*D*c^3))*(b*x+a)^(7/2)/d^3/(-a*d+b*c)^6/(d*x+c)^(7/2)
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 853, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{19/2}}\,dx = \frac{2(a+bx)^{7/2}\,(-45045c^2Cd^3(a+bx)^5+45045Bcd^4(a+bx)^5-$$

input
```
Integrate[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(19/2),x]
```

output
```
(2*(a + b*x)^(7/2)*(-45045*c^2*C*d^3*(a + b*x)^5 + 45045*B*c*d^4*(a + b*x)
^5 - 45045*A*d^5*(a + b*x)^5 + 45045*c^3*d^2*D*(a + b*x)^5 + 153153*b*c^2*
C*d^2*(a + b*x)^4*(c + d*x) - 204204*b*B*c*d^3*(a + b*x)^4*(c + d*x) + 102
102*a*c*C*d^3*(a + b*x)^4*(c + d*x) + 255255*A*b*d^4*(a + b*x)^4*(c + d*x)
 - 51051*a*B*d^4*(a + b*x)^4*(c + d*x) - 102102*b*c^3*d*D*(a + b*x)^4*(c +
 d*x) - 153153*a*c^2*d^2*D*(a + b*x)^4*(c + d*x) - 176715*b^2*c^2*C*d*(a +
 b*x)^3*(c + d*x)^2 + 353430*b^2*B*c*d^2*(a + b*x)^3*(c + d*x)^2 - 353430*
a*b*c*C*d^2*(a + b*x)^3*(c + d*x)^2 - 589050*A*b^2*d^3*(a + b*x)^3*(c + d*
x)^2 + 235620*a*b*B*d^3*(a + b*x)^3*(c + d*x)^2 - 58905*a^2*C*d^3*(a + b*x
)^3*(c + d*x)^2 + 58905*b^2*c^3*D*(a + b*x)^3*(c + d*x)^2 + 353430*a*b*c^2
*d*D*(a + b*x)^3*(c + d*x)^2 + 176715*a^2*c*d^2*D*(a + b*x)^3*(c + d*x)^2
+ 69615*b^3*c^2*C*(a + b*x)^2*(c + d*x)^3 - 278460*b^3*B*c*d*(a + b*x)^2*(
c + d*x)^3 + 417690*a*b^2*c*C*d*(a + b*x)^2*(c + d*x)^3 + 696150*A*b^3*d^2
*(a + b*x)^2*(c + d*x)^3 - 417690*a*b^2*B*d^2*(a + b*x)^2*(c + d*x)^3 + 20
8845*a^2*b*C*d^2*(a + b*x)^2*(c + d*x)^3 - 208845*a*b^2*c^2*D*(a + b*x)^2*
(c + d*x)^3 - 417690*a^2*b*c*d*D*(a + b*x)^2*(c + d*x)^3 - 69615*a^3*d^2*D
*(a + b*x)^2*(c + d*x)^3 + 85085*b^4*B*c*(a + b*x)*(c + d*x)^4 - 170170*a*
b^3*c*C*(a + b*x)*(c + d*x)^4 - 425425*A*b^4*d*(a + b*x)*(c + d*x)^4 + 340
340*a*b^3*B*d*(a + b*x)*(c + d*x)^4 - 255255*a^2*b^2*C*d*(a + b*x)*(c + d*
x)^4 + 255255*a^2*b^2*c*D*(a + b*x)*(c + d*x)^4 + 170170*a^3*b*d*D*(a +...
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2124, 27, 1193, 27, 87, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{19/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$\frac{2\int \frac{(a+bx)^{5/2}\left(-17\left(a-\frac{bc}{d}\right)Dx^2+\frac{17(bc-ad)(Cd-cD)x}{d^2}+\frac{17ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-10Ad^3\right)}{d^3}\right)}{2(c+dx)^{17/2}}dx}{17(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{17(c+dx)^{17/2}(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{(a+bx)^{5/2}\left(-17\left(a-\frac{bc}{d}\right)Dx^2+\frac{17(bc-ad)(Cd-cD)x}{d^2}+\frac{17ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-7Dc^3+7Cdc^2-7Bd^2c-10Ad^3\right)}{d^3}\right)}{(c+dx)^{17/2}}dx}{17(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{17(c+dx)^{17/2}(bc-ad)}$$

$\big\downarrow$ 1193

$$\frac{\frac{2\int \frac{(a+bx)^{5/2}\left(\left(-182Dc^3+63Cdc^2+56Bd^2c+80Ad^3\right)b^2-34ad\left(-18Dc^2+7Cdc+4Bd^2\right)b+255a^2d^2(Cd-2cD)+255d(bc-ad)^2Dx\right)}{2d^3(c+dx)^{15/2}}dx}{15(bc-ad)}+\frac{2(a+bx)^{7/2}(17ad(-}{}}{17(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{17(c+dx)^{17/2}(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{(a+bx)^{5/2}\left(\left(-182Dc^3+63Cdc^2+56Bd^2c+80Ad^3\right)b^2-34ad\left(-18Dc^2+7Cdc+4Bd^2\right)b+255a^2d^2(Cd-2cD)+255d(bc-ad)^2Dx\right)}{(c+dx)^{15/2}}dx}{15d^3(bc-ad)}+\frac{2(a+bx)^{7/2}(17ad(-}{}}{17(bc-ad)}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{17(c+dx)^{17/2}(bc-ad)}$$

$\big\downarrow$ 87

$$\frac{\frac{\frac{2(a+bx)^{7/2}\left(255a^2d^2(Cd-3cD)-34abd\left(4Bd^2-33c^2D+7cCd\right)+b^2\left(80Ad^3+56Bcd^2-437c^3D+63c^2Cd\right)\right)}{13(c+dx)^{13/2}(bc-ad)}-\frac{3\left(1105a^3d^3D-255a^2bd^2(7cD+2Cd)+17ab^2d\left(16Bd\right.\right.}{}}{15d^3(bc-ad)}}{}+\frac{2(a+bx)^{7/2}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{17(c+dx)^{17/2}(bc-ad)}$$

$\big\downarrow$ 55

$$\frac{\frac{2(a+bx)^{7/2}\left(255a^2d^2(Cd-3cD)-34abd\left(4Bd^2-33c^2D+7cCd\right)+b^2\left(80Ad^3+56Bcd^2-437c^3D+63c^2Cd\right)\right)}{13(c+dx)^{13/2}(bc-ad)}-\frac{3\left(1105a^3d^3D-255a^2bd^2(7cD+2Cd)+17ab^2d\left(16Bd\right.\right.}{}}{15d^3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{17(c+dx)^{17/2}(bc-ad)}$$

$\Big\downarrow$ 55

$$\frac{\frac{2(a+bx)^{7/2}\left(255a^2d^2(Cd-3cD)-34abd\left(4Bd^2-33c^2D+7cCd\right)+b^2\left(80Ad^3+56Bcd^2-437c^3D+63c^2Cd\right)\right)}{13(c+dx)^{13/2}(bc-ad)}-\frac{3\left(1105a^3d^3D-255a^2bd^2(7cD+2Cd)+17ab^2d\left(16Bd\right.\right.}{}}{15d^3(bc-ad)}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{17(c+dx)^{17/2}(bc-ad)}$$

$\Big\downarrow$ 48

$$\frac{\frac{2(a+bx)^{7/2}\left(255a^2d^2(Cd-3cD)-34abd\left(4Bd^2-33c^2D+7cCd\right)+b^2\left(80Ad^3+56Bcd^2-437c^3D+63c^2Cd\right)\right)}{13(c+dx)^{13/2}(bc-ad)}-\frac{3\left(\frac{2(a+bx)^{7/2}}{11(c+dx)^{11/2}(bc-ad)}+\frac{4b\left(\frac{4b(a+bx)^{7/2}}{63(c+dx)^{7/2}(bc-ad}\right.}{11(b}\right.}{}}{15d^3(bc-ad}$$

$$\frac{2(a+bx)^{7/2}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{17(c+dx)^{17/2}(bc-ad)}$$

input
```
Int[((a + b*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(19/2),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*(a + b*x)^(7/2))/(17*(b*c - a*d)*
(c + d*x)^(17/2)) + ((2*(17*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(24*c^2*C*
d - 7*B*c*d^2 - 10*A*d^3 - 41*c^3*D))*(a + b*x)^(7/2))/(15*d^3*(b*c - a*d)
*(c + d*x)^(15/2)) + ((2*(255*a^2*d^2*(C*d - 3*c*D) - 34*a*b*d*(7*c*C*d +
4*B*d^2 - 33*c^2*D) + b^2*(63*c^2*C*d + 56*B*c*d^2 + 80*A*d^3 - 437*c^3*D)
)*(a + b*x)^(7/2))/(13*(b*c - a*d)*(c + d*x)^(13/2)) - (3*(1105*a^3*d^3*D
- 255*a^2*b*d^2*(2*C*d + 7*c*D) + 17*a*b^2*d*(28*c*C*d + 16*B*d^2 + 63*c^2
*D) - b^3*(126*c^2*C*d + 112*B*c*d^2 + 160*A*d^3 + 231*c^3*D))*((2*(a + b*
x)^(7/2))/(11*(b*c - a*d)*(c + d*x)^(11/2)) + (4*b*((2*(a + b*x)^(7/2))/(9
*(b*c - a*d)*(c + d*x)^(9/2)) + (4*b*(a + b*x)^(7/2))/(63*(b*c - a*d)^2*(c
 + d*x)^(7/2))))/(11*(b*c - a*d))))/(13*(b*c - a*d)))/(15*d^3*(b*c - a*d))
)/(17*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1498 vs. 2(583) = 1166.

Time = 0.59 (sec) , antiderivative size = 1499, normalized size of antiderivative = 2.42

method	result	size
gosper	Expression too large to display	1499
orering	Expression too large to display	1499
default	Expression too large to display	2210

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(19/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/765765*(b*x+a)^(7/2)*(-1280*A*b^5*d^5*x^5+2176*B*a*b^4*d^5*x^5-896*B*b^
5*c*d^4*x^5-4080*C*a^2*b^3*d^5*x^5+3808*C*a*b^4*c*d^4*x^5-1008*C*b^5*c^2*d
^3*x^5+8840*D*a^3*b^2*d^5*x^5-14280*D*a^2*b^3*c*d^4*x^5+8568*D*a*b^4*c^2*d
^3*x^5-1848*D*b^5*c^3*d^2*x^5+4480*A*a*b^4*d^5*x^4-10880*A*b^5*c*d^4*x^4-7
616*B*a^2*b^3*d^5*x^4+21632*B*a*b^4*c*d^4*x^4-7616*B*b^5*c^2*d^3*x^4+14280
*C*a^3*b^2*d^5*x^4-48008*C*a^2*b^3*c*d^4*x^4+35896*C*a*b^4*c^2*d^3*x^4-856
8*C*b^5*c^3*d^2*x^4-30940*D*a^4*b*d^5*x^4+125120*D*a^3*b^2*c*d^4*x^4-15136
8*D*a^2*b^3*c^2*d^3*x^4+79296*D*a*b^4*c^3*d^2*x^4-15708*D*b^5*c^4*d*x^4-10
080*A*a^2*b^3*d^5*x^3+38080*A*a*b^4*c*d^4*x^3-40800*A*b^5*c^2*d^3*x^3+1713
6*B*a^3*b^2*d^5*x^3-71792*B*a^2*b^3*c*d^4*x^3+96016*B*a*b^4*c^2*d^3*x^3-28
560*B*b^5*c^3*d^2*x^3-32130*C*a^4*b*d^5*x^3+151368*C*a^3*b^2*c*d^4*x^3-251
276*C*a^2*b^3*c^2*d^3*x^3+151368*C*a*b^4*c^3*d^2*x^3-32130*C*b^5*c^4*d*x^3
+69615*D*a^5*d^5*x^3-375445*D*a^4*b*c*d^4*x^3+774078*D*a^3*b^2*c^2*d^3*x^3
-724626*D*a^2*b^3*c^3*d^2*x^3+328083*D*a*b^4*c^4*d*x^3-58905*D*b^5*c^5*x^3
+18480*A*a^3*b^2*d^5*x^2-85680*A*a^2*b^3*c*d^4*x^2+142800*A*a*b^4*c^2*d^3*
x^2-88400*A*b^5*c^3*d^2*x^2-31416*B*a^4*b*d^5*x^2+158592*B*a^3*b^2*c*d^4*x
^2-302736*B*a^2*b^3*c^2*d^3*x^2+250240*B*a*b^4*c^3*d^2*x^2-61880*B*b^5*c^4
*d*x^2+58905*C*a^5*d^5*x^2-328083*C*a^4*b*c*d^4*x^2+724626*C*a^3*b^2*c^2*d
^3*x^2-774078*C*a^2*b^3*c^3*d^2*x^2+375445*C*a*b^4*c^4*d*x^2-69615*C*b^5*c
^5*x^2+32130*D*a^5*c*d^4*x^2-151368*D*a^4*b*c^2*d^3*x^2+251276*D*a^3*b^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2} (A+Bx+Cx^2+Dx^3)}{(c+dx)^{19/2}} dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(19/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{19/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**(5/2)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(19/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{19/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(19/2),x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3251 vs. $2(583) = 1166$.

Time = 2.69 (sec) , antiderivative size = 3251, normalized size of antiderivative = 5.25

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{19/2}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(19/2),x, algorithm="g
iac")
```

output
```
2/765765*((((4*(b*x + a)*(2*(231*D*b^22*c^5*d^10*abs(b) - 1533*D*a*b^21*c^
4*d^11*abs(b) + 126*C*b^22*c^4*d^11*abs(b) + 4158*D*a^2*b^20*c^3*d^12*abs(
b) - 728*C*a*b^21*c^3*d^12*abs(b) + 112*B*b^22*c^3*d^12*abs(b) - 5746*D*a^
3*b^19*c^2*d^13*abs(b) + 1588*C*a^2*b^20*c^2*d^13*abs(b) - 496*B*a*b^21*c^
2*d^13*abs(b) + 160*A*b^22*c^2*d^13*abs(b) + 3995*D*a^4*b^18*c*d^14*abs(b)
 - 1496*C*a^3*b^19*c*d^14*abs(b) + 656*B*a^2*b^20*c*d^14*abs(b) - 320*A*a*
b^21*c*d^14*abs(b) - 1105*D*a^5*b^17*d^15*abs(b) + 510*C*a^4*b^18*d^15*abs
(b) - 272*B*a^3*b^19*d^15*abs(b) + 160*A*a^2*b^20*d^15*abs(b))*(b*x + a)/(
b^12*c^8*d^8 - 8*a*b^11*c^7*d^9 + 28*a^2*b^10*c^6*d^10 - 56*a^3*b^9*c^5*d^
11 + 70*a^4*b^8*c^4*d^12 - 56*a^5*b^7*c^3*d^13 + 28*a^6*b^6*c^2*d^14 - 8*a
^7*b^5*c*d^15 + a^8*b^4*d^16) + 17*(231*D*b^23*c^6*d^9*abs(b) - 1764*D*a*b
^22*c^5*d^10*abs(b) + 126*C*b^23*c^5*d^10*abs(b) + 5691*D*a^2*b^21*c^4*d^1
1*abs(b) - 854*C*a*b^22*c^4*d^11*abs(b) + 112*B*b^23*c^4*d^11*abs(b) - 990
4*D*a^3*b^20*c^3*d^12*abs(b) + 2316*C*a^2*b^21*c^3*d^12*abs(b) - 608*B*a*b
^22*c^3*d^12*abs(b) + 160*A*b^23*c^3*d^12*abs(b) + 9741*D*a^4*b^19*c^2*d^1
3*abs(b) - 3084*C*a^3*b^20*c^2*d^13*abs(b) + 1152*B*a^2*b^21*c^2*d^13*abs(
b) - 480*A*a*b^22*c^2*d^13*abs(b) - 5100*D*a^5*b^18*c*d^14*abs(b) + 2006*C
*a^4*b^19*c*d^14*abs(b) - 928*B*a^3*b^20*c*d^14*abs(b) + 480*A*a^2*b^21*c*
d^14*abs(b) + 1105*D*a^6*b^17*d^15*abs(b) - 510*C*a^5*b^18*d^15*abs(b) + 2
72*B*a^4*b^19*d^15*abs(b) - 160*A*a^3*b^20*d^15*abs(b))/(b^12*c^8*d^8 -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{19/2}}\,dx=\int \frac{(a+b\,x)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{(c+d\,x)^{19/2}}\,dx$$

input
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(19/2),x)
```

output
```
int(((a + b*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(19/2), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{19/2}}\,dx = \int \frac{(bx+a)^{\frac{5}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(dx+c)^{\frac{19}{2}}}dx$$

input
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(19/2),x)
```

output
```
int((b*x+a)^(5/2)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(19/2),x)
```

3.131 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$

Optimal result 1255
Mathematica [A] (verified) 1256
Rubi [A] (verified) 1257
Maple [B] (verified) 1261
Fricas [A] (verification not implemented) 1262
Sympy [F] 1263
Maxima [F(-2)] 1263
Giac [B] (verification not implemented) 1263
Mupad [F(-1)] 1264
Reduce [B] (verification not implemented) 1265

Optimal result

Integrand size = 34, antiderivative size = 516

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}}\,dx =$$
$$-\frac{(bc-ad)\left(63a^3d^3D-7a^2bd^2(10Cd-3cD)-ab^2d(20cCd-80Bd^2-9c^2D)-b^3(6c^2Cd-16Bcd^2+96\right.}{128b^5d^3}$$
$$-\frac{(63a^3d^3D-7a^2bd^2(10Cd-3cD)-ab^2d(20cCd-80Bd^2-9c^2D)-b^3(6c^2Cd-16Bcd^2+96Ad^3-3c}{192b^4d^3}$$
$$-\frac{\left(6bcC-16bBd+26aCd-12acD-\frac{3bc^2D}{d}-\frac{33a^2dD}{b}\right)\sqrt{a+bx}(c+dx)^{5/2}}{48b^2d^2}$$
$$+\frac{(2bCd-bcD-5adD)(a+bx)^{3/2}(c+dx)^{5/2}}{8b^3d^2}+\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$
$$-\frac{(bc-ad)^2\left(63a^3d^3D-7a^2bd^2(10Cd-3cD)-ab^2d(20cCd-80Bd^2-9c^2D)-b^3(6c^2Cd-16Bcd^2+9\right.}{128b^{11/2}d^{7/2}}$$

output
```
-1/128*(-a*d+b*c)*(63*a^3*d^3*D-7*a^2*b*d^2*(10*C*d-3*D*c)-a*b^2*d*(-80*B*
d^2+20*C*c*d-9*D*c^2)-b^3*(96*A*d^3-16*B*c*d^2+6*C*c^2*d-3*D*c^3))*(b*x+a)
^(1/2)*(d*x+c)^(1/2)/b^5/d^3-1/192*(63*a^3*d^3*D-7*a^2*b*d^2*(10*C*d-3*D*c
)-a*b^2*d*(-80*B*d^2+20*C*c*d-9*D*c^2)-b^3*(96*A*d^3-16*B*c*d^2+6*C*c^2*d-
3*D*c^3))*(b*x+a)^(1/2)*(d*x+c)^(3/2)/b^4/d^3-1/48*(6*C*b*c-16*B*b*d+26*C*
a*d-12*D*a*c-3*b*c^2*D/d-33*a^2*d*D/b)*(b*x+a)^(1/2)*(d*x+c)^(5/2)/b^2/d^2
+1/8*(2*C*b*d-5*D*a*d-D*b*c)*(b*x+a)^(3/2)*(d*x+c)^(5/2)/b^3/d^2+1/5*D*(b*
x+a)^(5/2)*(d*x+c)^(5/2)/b^3/d-1/128*(-a*d+b*c)^2*(63*a^3*d^3*D-7*a^2*b*d^
2*(10*C*d-3*D*c)-a*b^2*d*(-80*B*d^2+20*C*c*d-9*D*c^2)-b^3*(96*A*d^3-16*B*c
*d^2+6*C*c^2*d-3*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/
2))/b^(11/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\sqrt{c+dx}(945a^4d^4D-210a^3bd^3(5Cd+6cD+3dDx)}{} \\ -\frac{(bc-ad)^2\,(63a^3d^3D+7a^2bd^2(-10Cd+3cD)+ab^2d(-20cCd+80Bd^2+9c^2D)+b^3(-6c^2Cd+16Bcd}{128b^{11/2}d^{7/2}}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x],x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(945*a^4*d^4*D - 210*a^3*b*d^3*(5*C*d + 6*c*D
 + 3*d*D*x) + 2*a^2*b^2*d^2*(57*c^2*D + c*d*(725*C + 399*D*x) + 2*d^2*(300
*B + 7*x*(25*C + 18*D*x))) - 2*a*b^3*d*(-30*c^3*D + 3*c^2*d*(25*C + 7*D*x)
 + 4*c*d^2*(220*B + x*(115*C + 78*D*x)) + 8*d^3*(90*A + x*(50*B + 35*C*x +
 27*D*x^2))) + b^4*(45*c^4*D - 30*c^3*d*(3*C + D*x) + 12*c^2*d^2*(20*B + x
*(5*C + 2*D*x)) + 16*c*d^3*(150*A + x*(70*B + 45*C*x + 33*D*x^2)) + 32*d^4
*x*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))))/(1920*b^5*d^3) - ((b*c - a*d)^
2*(63*a^3*d^3*D + 7*a^2*b*d^2*(-10*C*d + 3*c*D) + a*b^2*d*(-20*c*C*d + 80*
B*d^2 + 9*c^2*D) + b^3*(-6*c^2*C*d + 16*B*c*d^2 - 96*A*d^3 + 3*c^3*D))*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(11/2)*d^(7/
2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2125, 27, 1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{ 2125}$$

$$\frac{\int \frac{5(c+dx)^{3/2}\left(2Adb^3+(2bCd-5aDd-bcD)x^2b^2+2\left(-2dDa^2-bcDa+b^2Bd\right)xb-a^2(bc+ad)D\right)}{2\sqrt{a+bx}}dx}{5b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(c+dx)^{3/2}\left(2Adb^3+(2bCd-5aDd-bcD)x^2b^2+2\left(-2dDa^2-bcDa+b^2Bd\right)xb-a^2(bc+ad)D\right)}{\sqrt{a+bx}}dx}{2b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

$$\Big\downarrow \text{ 1194}$$

$$\frac{\frac{\int \frac{b^2(c+dx)^{3/2}\left(17d^2Da^3-2bd(5Cd-6cD)a^2-3b^2c(2Cd-cD)a+16Ab^3d^2+b\left(-\left(\left(-3Dc^2+6Cdc-16Bd^2\right)b^2\right)-2ad(13Cd-6cD)b+33a^2d^2D\right)x\right)}{2\sqrt{a+bx}}dx}{4b^2d}+\frac{(a+bx)^3 \cdots}{\cdots}}{2b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{(c+dx)^{3/2}\left(17d^2Da^3-2bd(5Cd-6cD)a^2-3b^2c(2Cd-cD)a+16Ab^3d^2+b\left(-\left(\left(-3Dc^2+6Cdc-16Bd^2\right)b^2\right)-2ad(13Cd-6cD)b+33a^2d^2D\right)x\right)}{\sqrt{a+bx}}dx}{8d}+\frac{(a+bx)^{3/2} \cdots}{\cdots}}{2b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}\left(33a^2d^2D-2abd(13Cd-6cD)-\left(b^2\left(-16Bd^2-3c^2D+6cCd\right)\right)\right)}{3d}-\frac{\left(63a^3d^3D-7a^2bd^2(10Cd-3cD)-ab^2d\left(-80Bd^2-9c^2D+20cCd\right)-\left(b^3\left(96Ad\cdots\right.\right.\right.}{6d}\cdots}{8d}}{2b^3d}$$

$$\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

$\big\downarrow$ 60

$$\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}\left(33a^2d^2D-2abd(13Cd-6cD)-\left(b^2\left(-16Bd^2-3c^2D+6cCd\right)\right)\right)}{3d}-\frac{\frac{\left(63a^3d^3D-7a^2bd^2(10Cd-3cD)-ab^2d\left(-80Bd^2-9c^2D+20cCd\right)-\left(b^3\left(96Ad\cdots\right.\right.\right.}{6\cdots}\cdots}{8d}}{2b^3d}$$

$$\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

$\big\downarrow$ 60

$$\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}\left(33a^2d^2D-2abd(13Cd-6cD)-\left(b^2\left(-16Bd^2-3c^2D+6cCd\right)\right)\right)}{3d}-\frac{\frac{\left(63a^3d^3D-7a^2bd^2(10Cd-3cD)-ab^2d\left(-80Bd^2-9c^2D+20cCd\right)-\left(b^3\left(96Ad\cdots\right.\right.\right.}{\cdots}}{8d}}{2\cdots}$$

$$\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

$\big\downarrow$ 66

$$\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}\left(33a^2d^2D-2abd(13Cd-6cD)-\left(b^2\left(-16Bd^2-3c^2D+6cCd\right)\right)\right)}{3d}-\frac{\frac{\left(63a^3d^3D-7a^2bd^2(10Cd-3cD)-ab^2d\left(-80Bd^2-9c^2D+20cCd\right)-\left(b^3\left(96Ad\cdots\right.\right.\right.}{\cdots}}{8d}}{\cdots}$$

$$\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

$\big\downarrow$ 221

$$\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}\left(33a^2d^2D-2abd(13Cd-6cD)-\left(b^2\left(-16Bd^2-3c^2D+6cCd\right)\right)\right)}{3d}-\frac{\left(\frac{3(bc-ad)\left(\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4b}+\frac{\sqrt{a+bx}(c}{2}\right.}{}}{8d}$$

$$\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^3d}$$

input
```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x],x]
```

output
```
(D*(a + b*x)^(5/2)*(c + d*x)^(5/2))/(5*b^3*d) + (((2*b*C*d - b*c*D - 5*a*d
*D)*(a + b*x)^(3/2)*(c + d*x)^(5/2))/(4*d) + (((33*a^2*d^2*D - 2*a*b*d*(13
*C*d - 6*c*D) - b^2*(6*c*C*d - 16*B*d^2 - 3*c^2*D))*Sqrt[a + b*x]*(c + d*x
)^(5/2))/(3*d) - ((63*a^3*d^3*D - 7*a^2*b*d^2*(10*C*d - 3*c*D) - a*b^2*d*(
20*c*C*d - 80*B*d^2 - 9*c^2*D) - b^3*(6*c^2*C*d - 16*B*c*d^2 + 96*A*d^3 -
3*c^3*D))*((Sqrt[a + b*x]*(c + d*x)^(3/2))/(2*b) + (3*(b*c - a*d)*((Sqrt[a
 + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(S
qrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])))/(4*b)))/(6*d))/(8*d))/(2*b^3*d
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(472) = 944$.

Time = 0.54 (sec) , antiderivative size = 1998, normalized size of antiderivative = 3.87

method	result	size
default	Expression too large to display	1998

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/3840*(d*x+c)^(1/2)*(b*x+a)^(1/2)*(1440*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*d^5+1440*A*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^2*
d^3+2900*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*c*d^3-300*C*(d*b)^(
1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^2*d^2-2520*D*(d*b)^(1/2)*((b*x+a)*(d*
x+c))^(1/2)*a^3*b*c*d^3+228*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*
c^2*d^2+2240*B*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^4*c*d^3*x+1400*C*(d*b
)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b^2*d^4*x+120*C*(d*b)^(1/2)*((b*x+a)*(
d*x+c))^(1/2)*b^4*c^2*d^2*x+768*D*b^4*d^4*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b
)^(1/2)+960*C*b^4*d^4*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1280*B*b^4*d
^4*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-1840*C*(d*b)^(1/2)*((b*x+a)*(d*
x+c))^(1/2)*a*b^3*c*d^3*x+1596*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*b
^2*c*d^3*x-84*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^2*d^2*x+120*D*
(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b^3*c^3*d-1248*D*a*b^3*c*d^3*x^2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-3520*B*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2
)*a*b^3*c*d^3+2160*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c*d^4-720*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d^3-1800*C*ln(1/2*(
2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^
2*c*d^4+540*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d...
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1278, normalized size of antiderivative = 2.48

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
[-1/7680*(15*(3*D*b^5*c^5 + 3*(D*a*b^4 - 2*C*b^5)*c^4*d + 2*(3*D*a^2*b^3 -
 4*C*a*b^4 + 8*B*b^5)*c^3*d^2 + 6*(5*D*a^3*b^2 - 6*C*a^2*b^3 + 8*B*a*b^4 -
 16*A*b^5)*c^2*d^3 - 3*(35*D*a^4*b - 40*C*a^3*b^2 + 48*B*a^2*b^3 - 64*A*a*
b^4)*c*d^4 + (63*D*a^5 - 70*C*a^4*b + 80*B*a^3*b^2 - 96*A*a^2*b^3)*d^5)*sq
rt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b
*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x)
 - 4*(384*D*b^5*d^5*x^4 + 45*D*b^5*c^4*d + 30*(2*D*a*b^4 - 3*C*b^5)*c^3*d^
2 + 6*(19*D*a^2*b^3 - 25*C*a*b^4 + 40*B*b^5)*c^2*d^3 - 10*(126*D*a^3*b^2 -
 145*C*a^2*b^3 + 176*B*a*b^4 - 240*A*b^5)*c*d^4 + 15*(63*D*a^4*b - 70*C*a^
3*b^2 + 80*B*a^2*b^3 - 96*A*a*b^4)*d^5 + 48*(11*D*b^5*c*d^4 - (9*D*a*b^4 -
 10*C*b^5)*d^5)*x^3 + 8*(3*D*b^5*c^2*d^3 - 6*(13*D*a*b^4 - 15*C*b^5)*c*d^4
 + (63*D*a^2*b^3 - 70*C*a*b^4 + 80*B*b^5)*d^5)*x^2 - 2*(15*D*b^5*c^3*d^2 +
 3*(7*D*a*b^4 - 10*C*b^5)*c^2*d^3 - (399*D*a^2*b^3 - 460*C*a*b^4 + 560*B*b
^5)*c*d^4 + 5*(63*D*a^3*b^2 - 70*C*a^2*b^3 + 80*B*a*b^4 - 96*A*b^5)*d^5)*x
)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*d^4), 1/3840*(15*(3*D*b^5*c^5 + 3*(D*a
*b^4 - 2*C*b^5)*c^4*d + 2*(3*D*a^2*b^3 - 4*C*a*b^4 + 8*B*b^5)*c^3*d^2 + 6*
(5*D*a^3*b^2 - 6*C*a^2*b^3 + 8*B*a*b^4 - 16*A*b^5)*c^2*d^3 - 3*(35*D*a^4*b
 - 40*C*a^3*b^2 + 48*B*a^2*b^3 - 64*A*a*b^4)*c*d^4 + (63*D*a^5 - 70*C*a^4*
b + 80*B*a^3*b^2 - 96*A*a^2*b^3)*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c
 + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +...
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Integral((c + d*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/sqrt(a + b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1672 vs. $2(465) = 930$.

Time = 0.35 (sec) , antiderivative size = 1672, normalized size of antiderivative = 3.24

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
-1/1920*(1920*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2
*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*sqrt(b*x + a))*A*c*abs(b)/b^2 - 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6
)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^1
4*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^
3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x
+ a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*D*c*abs(
b)/b^2 - 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)
*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2
*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3
 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqr
t(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c
*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*C*d*abs(b)/b^2 - 480*(sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a)
 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
+ sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*B*c*abs(b)/b^3 - 48
0*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}}\,dx=\int \frac{(c+d\,x)^{3/2}\left(A+B\,x+C\,x^2+x^3\,D\right)}{\sqrt{a+b\,x}}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(1/2),x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 974, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(945*sqrt(c + d*x)*sqrt(a + b*x)*a**4*b*d**5 - 2310*sqrt(c + d*x)*sqrt(a +
 b*x)*a**3*b**2*c*d**4 - 630*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b**2*d**5*x
- 240*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**4*d**4 + 1564*sqrt(c + d*x)*sqrt
(a + b*x)*a**2*b**3*c**2*d**3 + 1498*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**3
*c*d**4*x + 504*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**3*d**5*x**2 + 640*sqrt
(c + d*x)*sqrt(a + b*x)*a*b**5*c*d**3 + 160*sqrt(c + d*x)*sqrt(a + b*x)*a*
b**5*d**4*x - 90*sqrt(c + d*x)*sqrt(a + b*x)*a*b**4*c**3*d**2 - 962*sqrt(c
 + d*x)*sqrt(a + b*x)*a*b**4*c**2*d**3*x - 1184*sqrt(c + d*x)*sqrt(a + b*x
)*a*b**4*c*d**4*x**2 - 432*sqrt(c + d*x)*sqrt(a + b*x)*a*b**4*d**5*x**3 +
240*sqrt(c + d*x)*sqrt(a + b*x)*b**6*c**2*d**2 + 1120*sqrt(c + d*x)*sqrt(a
 + b*x)*b**6*c*d**3*x + 640*sqrt(c + d*x)*sqrt(a + b*x)*b**6*d**4*x**2 - 4
5*sqrt(c + d*x)*sqrt(a + b*x)*b**5*c**4*d + 30*sqrt(c + d*x)*sqrt(a + b*x)
*b**5*c**3*d**2*x + 744*sqrt(c + d*x)*sqrt(a + b*x)*b**5*c**2*d**3*x**2 +
1008*sqrt(c + d*x)*sqrt(a + b*x)*b**5*c*d**4*x**3 + 384*sqrt(c + d*x)*sqrt
(a + b*x)*b**5*d**5*x**4 - 945*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x)
+ sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**5*d**5 + 2625*sqrt(d)*sqrt(b)
*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**4
*b*c*d**4 + 240*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(
c + d*x))/sqrt(a*d - b*c))*a**3*b**3*d**4 - 2250*sqrt(d)*sqrt(b)*log((sqrt
(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**3*b**2*c...
```

3.132 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}}\,dx$

Optimal result 1266
Mathematica [A] (verified) 1267
Rubi [A] (verified) 1267
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Fricas [A] (verification not implemented) 1272
Sympy [F] 1273
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Giac [B] (verification not implemented) 1274
Mupad [F(-1)] 1275
Reduce [B] (verification not implemented) 1275

Optimal result

Integrand size = 34, antiderivative size = 393

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}}\,dx = -\frac{(35a^3d^3D-5a^2bd^2(8Cd-3cD)-ab^2d(16cCd-48Bd^2-9c^2D)-b^3(8c^2Cd-16Bcd^2+64Ad^3-5}{64b^4d^3}$$
$$-\frac{\left(8bcC-16bBd+24aCd-14acD-\frac{5bc^2D}{d}-\frac{29a^2dD}{b}\right)\sqrt{a+bx}(c+dx)^{3/2}}{32b^2d^2}$$
$$+\frac{(8bCd-5bcD-19adD)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^3d^2}+\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$
$$-\frac{(bc-ad)\left(35a^3d^3D-5a^2bd^2(8Cd-3cD)-ab^2d(16cCd-48Bd^2-9c^2D)-b^3(8c^2Cd-16Bcd^2+6\right.}{64b^{9/2}d^{7/2}}$$

output
```
-1/64*(35*a^3*d^3*D-5*a^2*b*d^2*(8*C*d-3*D*c)-a*b^2*d*(-48*B*d^2+16*C*c*d-
9*D*c^2)-b^3*(64*A*d^3-16*B*c*d^2+8*C*c^2*d-5*D*c^3))*(b*x+a)^(1/2)*(d*x+c
)^(1/2)/b^4/d^3-1/32*(8*C*b*c-16*B*b*d+24*C*a*d-14*D*a*c-5*b*c^2*D/d-29*a^
2*d*D/b)*(b*x+a)^(1/2)*(d*x+c)^(3/2)/b^2/d^2+1/24*(8*C*b*d-19*D*a*d-5*D*b*
c)*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b^3/d^2+1/4*D*(b*x+a)^(5/2)*(d*x+c)^(3/2)/b
^3/d-1/64*(-a*d+b*c)*(35*a^3*d^3*D-5*a^2*b*d^2*(8*C*d-3*D*c)-a*b^2*d*(-48*
B*d^2+16*C*c*d-9*D*c^2)-b^3*(64*A*d^3-16*B*c*d^2+8*C*c^2*d-5*D*c^3))*arcta
nh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(9/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 11.05 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$$
$$= \frac{\sqrt{c+dx}\Big(\sqrt{d}\sqrt{a+bx}(-105a^3d^3D+5a^2bd^2(24Cd+5cD+14dDx)-ab^2d(-17c^2D+4cd(8C+3Dx)}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x],x]
```

output
```
(Sqrt[c + d*x]*(Sqrt[d]*Sqrt[a + b*x]*(-105*a^3*d^3*D + 5*a^2*b*d^2*(24*C*
d + 5*c*D + 14*d*D*x) - a*b^2*d*(-17*c^2*D + 4*c*d*(8*C + 3*D*x) + 8*d^2*(
18*B + 10*C*x + 7*D*x^2)) + b^3*(15*c^3*D - 2*c^2*d*(12*C + 5*D*x) + 8*c*d
^2*(6*B + 2*C*x + D*x^2) + 16*d^3*(12*A + 6*B*x + 4*C*x^2 + 3*D*x^3))) - (
3*Sqrt[b*c - a*d]*(35*a^3*d^3*D - 5*a^2*b*d^2*(8*C*d - 3*c*D) + a*b^2*d*(-
16*c*C*d + 48*B*d^2 + 9*c^2*D) + b^3*(-8*c^2*C*d + 16*B*c*d^2 - 64*A*d^3 +
 5*c^3*D))*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/Sqrt[(b*(c +
d*x))/(b*c - a*d)]))/(192*b^4*d^(7/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2125, 27, 1194, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$$
$$\Big\downarrow \text{2125}$$
$$\frac{\int \frac{\sqrt{c+dx}(8Adb^3+(8bCd-19aDd-5bcD)x^2b^2+2(-7dDa^2-5bcDa+4b^2Bd)xb-a^2(5bc+3ad)D)}{2\sqrt{a+bx}}dx}{4b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\left(8Adb^3+(8bCd-19aDd-5bcD)x^2b^2+2\left(-7dDa^2-5bcDa+4b^2Bd\right)xb-a^2(5bc+3ad)D\right)}{\sqrt{a+bx}}dx}{8b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$

$$\Big\downarrow \; 1194$$

$$\frac{\frac{\int \frac{3b^2\sqrt{c+dx}\left(13d^2Da^3-2bd(4Cd-7cD)a^2-b^2c(8Cd-5cD)a+16Ab^3d^2+b\left(-\left(\left(-5Dc^2+8Cdc-16Bd^2\right)b^2\right)-2ad(12Cd-7cD)b+29a^2d^2D\right)x\right)}{2\sqrt{a+bx}}dx}{3b^2d}+\frac{(a+bx)^{3/2}\cdots}{\cdots}}{8b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{c+dx}\left(13d^2Da^3-2bd(4Cd-7cD)a^2-b^2c(8Cd-5cD)a+16Ab^3d^2+b\left(-\left(\left(-5Dc^2+8Cdc-16Bd^2\right)b^2\right)-2ad(12Cd-7cD)b+29a^2d^2D\right)x\right)}{\sqrt{a+bx}}dx}{2d}+\frac{(a+bx)^{3/2}(c\cdots}{\cdots}}{8b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{3/2}\left(29a^2d^2D-2abd(12Cd-7cD)-\left(b^2\left(-16Bd^2-5c^2D+8cCd\right)\right)\right)}{2d}-\frac{\left(35a^3d^3D-5a^2bd^2(8Cd-3cD)-ab^2d\left(-48Bd^2-9c^2D+16cCd\right)-\left(b^3\left(64Ad^3\cdots\right.\right.\right.}{4d}\cdots}{2d}\cdots}{8b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{3/2}\left(29a^2d^2D-2abd(12Cd-7cD)-\left(b^2\left(-16Bd^2-5c^2D+8cCd\right)\right)\right)}{2d}-\frac{\left(35a^3d^3D-5a^2bd^2(8Cd-3cD)-ab^2d\left(-48Bd^2-9c^2D+16cCd\right)-\left(b^3\left(64Ad^3\cdots\right.\right.\right.}{4\cdots}\cdots}{2d}\cdots}{8b^3d}+\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{3/2}\left(29a^2d^2D-2abd(12Cd-7cD)-\left(b^2\left(-16Bd^2-5c^2D+8cCd\right)\right)\right)}{2d}-\frac{\left(35a^3d^3D-5a^2bd^2(8Cd-3cD)-ab^2d\left(-48Bd^2-9c^2D+16cCd\right)-\left(b^3\left(64Ad^3\right.\right.\right.}{}}{2d}}{8b^3d}$$

$$\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{3/2}\left(29a^2d^2D-2abd(12Cd-7cD)-\left(b^2\left(-16Bd^2-5c^2D+8cCd\right)\right)\right)}{2d}-\frac{\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)\left(35a^3d^3D-5a^2bd^2(8C\right.}{}}{2d}}{8b^3d}$$

$$\frac{D(a+bx)^{5/2}(c+dx)^{3/2}}{4b^3d}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x],x]
```

output
```
(D*(a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*b^3*d) + (((8*b*C*d - 5*b*c*D - 19*
a*d*D)*(a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*d) + (((29*a^2*d^2*D - 2*a*b*d*
(12*C*d - 7*c*D) - b^2*(8*c*C*d - 16*B*d^2 - 5*c^2*D))*Sqrt[a + b*x]*(c +
d*x)^(3/2))/(2*d) - ((35*a^3*d^3*D - 5*a^2*b*d^2*(8*C*d - 3*c*D) - a*b^2*d
*(16*c*C*d - 48*B*d^2 - 9*c^2*D) - b^3*(8*c^2*C*d - 16*B*c*d^2 + 64*A*d^3
- 5*c^3*D))*((Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])))/(4*d))/(2*
d))/(8*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1333 vs. $2(355) = 710$.

Time = 0.53 (sec) , antiderivative size = 1334, normalized size of antiderivative = 3.39

method	result	size
default	Expression too large to display	1334

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/384*(d*x+c)^(1/2)*(b*x+a)^(1/2)*(48*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^2*d^2-24*C*ln(1/2*(2*b*d*
x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^3*d+48
*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^3*c^2*d-384*A*(d*b)^(1/2)*((b*x+a
)*(d*x+c))^(1/2)*b^3*d^3+24*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^2*c*
d^2*x+120*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)
/(d*b)^(1/2))*a^3*b*d^4+12*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^3*d+210*D*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)*a^3*d^3-144*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*d^4-30*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^
(1/2)*b^3*c^3-240*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b*d^3+60*D*ln(
1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a
^3*b*c*d^3+18*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a^2*b^2*c^2*d^2+288*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)*a*b^2*d^3-72*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d
+b*c)/(d*b)^(1/2))*a^2*b^2*c*d^3-192*B*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
*b^3*d^3*x+192*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d
+b*c)/(d*b)^(1/2))*a*b^3*d^4-192*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c*d^3+112*D*a*b^2*d^3*x^2*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)-16*D*b^3*c*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 888, normalized size of antiderivative = 2.26

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
[1/768*(3*(5*D*b^4*c^4 + 4*(D*a*b^3 - 2*C*b^4)*c^3*d + 2*(3*D*a^2*b^2 - 4*
C*a*b^3 + 8*B*b^4)*c^2*d^2 + 4*(5*D*a^3*b - 6*C*a^2*b^2 + 8*B*a*b^3 - 16*A
*b^4)*c*d^3 - (35*D*a^4 - 40*C*a^3*b + 48*B*a^2*b^2 - 64*A*a*b^3)*d^4)*sqr
t(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*
c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x)
+ 4*(48*D*b^4*d^4*x^3 + 15*D*b^4*c^3*d + (17*D*a*b^3 - 24*C*b^4)*c^2*d^2 +
 (25*D*a^2*b^2 - 32*C*a*b^3 + 48*B*b^4)*c*d^3 - 3*(35*D*a^3*b - 40*C*a^2*b
^2 + 48*B*a*b^3 - 64*A*b^4)*d^4 + 8*(D*b^4*c*d^3 - (7*D*a*b^3 - 8*C*b^4)*d
^4)*x^2 - 2*(5*D*b^4*c^2*d^2 + 2*(3*D*a*b^3 - 4*C*b^4)*c*d^3 - (35*D*a^2*b
^2 - 40*C*a*b^3 + 48*B*b^4)*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*d^4)
, 1/384*(3*(5*D*b^4*c^4 + 4*(D*a*b^3 - 2*C*b^4)*c^3*d + 2*(3*D*a^2*b^2 - 4
*C*a*b^3 + 8*B*b^4)*c^2*d^2 + 4*(5*D*a^3*b - 6*C*a^2*b^2 + 8*B*a*b^3 - 16*
A*b^4)*c*d^3 - (35*D*a^4 - 40*C*a^3*b + 48*B*a^2*b^2 - 64*A*a*b^3)*d^4)*sq
rt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*
x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48*D*b^4*d^4*
x^3 + 15*D*b^4*c^3*d + (17*D*a*b^3 - 24*C*b^4)*c^2*d^2 + (25*D*a^2*b^2 - 3
2*C*a*b^3 + 48*B*b^4)*c*d^3 - 3*(35*D*a^3*b - 40*C*a^2*b^2 + 48*B*a*b^3 -
64*A*b^4)*d^4 + 8*(D*b^4*c*d^3 - (7*D*a*b^3 - 8*C*b^4)*d^4)*x^2 - 2*(5*D*b
^4*c^2*d^2 + 2*(3*D*a*b^3 - 4*C*b^4)*c*d^3 - (35*D*a^2*b^2 - 40*C*a*b^3 +
48*B*b^4)*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*d^4)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3)/sqrt(a + b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(351) = 702.

Time = 0.22 (sec) , antiderivative size = 710, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
-1/192*(192*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*sqrt(b*x + a))*A*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b
*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14
*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6))
 + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*
d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) +
sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*D*abs(b)/b^2 -
48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d
^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*
d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*B*
abs(b)/b^3 - 8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*b*x + 4*a + (b*c
*d^3 - 13*a*d^4)/d^4)*(b*x + a) - 3*(b^2*c^2*d^2 + 2*a*b*c*d^3 - 11*a^2*d^
4)/d^4)*sqrt(b*x + a) - 3*(b^4*c^3 + a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - 5*a^3
*b*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*
b*d)))/(sqrt(b*d)*d^2))*C*abs(b)/b^4)/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{c+d\,x}\,(A+B\,x+C\,x^2+x^3\,D)}{\sqrt{a+b\,x}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(1/2),x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$$
$$= \frac{-105\sqrt{dx+c}\,\sqrt{bx+a}\,a^3b\,d^4+145\sqrt{dx+c}\,\sqrt{bx+a}\,a^2b^2c\,d^3+70\sqrt{dx+c}\,\sqrt{bx+a}\,a^2b^2d^4x+48\sqrt{dx+}}{}$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x)
```

output
```
( - 105*sqrt(c + d*x)*sqrt(a + b*x)*a**3*b*d**4 + 145*sqrt(c + d*x)*sqrt(a
 + b*x)*a**2*b**2*c*d**3 + 70*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b**2*d**4*x
 + 48*sqrt(c + d*x)*sqrt(a + b*x)*a*b**4*d**3 - 15*sqrt(c + d*x)*sqrt(a +
b*x)*a*b**3*c**2*d**2 - 92*sqrt(c + d*x)*sqrt(a + b*x)*a*b**3*c*d**3*x - 5
6*sqrt(c + d*x)*sqrt(a + b*x)*a*b**3*d**4*x**2 + 48*sqrt(c + d*x)*sqrt(a +
 b*x)*b**5*c*d**2 + 96*sqrt(c + d*x)*sqrt(a + b*x)*b**5*d**3*x - 9*sqrt(c
+ d*x)*sqrt(a + b*x)*b**4*c**3*d + 6*sqrt(c + d*x)*sqrt(a + b*x)*b**4*c**2
*d**2*x + 72*sqrt(c + d*x)*sqrt(a + b*x)*b**4*c*d**3*x**2 + 48*sqrt(c + d*
x)*sqrt(a + b*x)*b**4*d**4*x**3 + 105*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a
+ b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**4*d**4 - 180*sqrt(d)*s
qrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c)
)*a**3*b*c*d**3 - 48*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*
sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b**3*d**3 + 54*sqrt(d)*sqrt(b)*log((s
qrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b**2*c
**2*d**2 + 96*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c
+ d*x))/sqrt(a*d - b*c))*a*b**4*c*d**2 + 12*sqrt(d)*sqrt(b)*log((sqrt(d)*s
qrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**3*c**3*d - 48*
sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a
*d - b*c))*b**5*c**2*d + 9*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sq
rt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**4*c**4)/(192*b**5*d**3)
```

3.133 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$

Optimal result 1277
Mathematica [A] (verified) 1278
Rubi [A] (verified) 1278
Maple [B] (verified) 1281
Fricas [A] (verification not implemented) 1282
Sympy [F] 1283
Maxima [F(-2)] 1283
Giac [A] (verification not implemented) 1284
Mupad [F(-1)] 1284
Reduce [B] (verification not implemented) 1285

Optimal result

Integrand size = 34, antiderivative size = 272

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx = -\frac{\left(6bcC-8bBd+10aCd-8acD-\frac{5bc^2D}{d}-\frac{11a^2dD}{b}\right)\sqrt{a+bx}\sqrt{c+dx}}{8b^2d^2} + \frac{(6bCd-5bcD-13adD)(a+bx)^{3/2}\sqrt{c+dx}}{12b^3d^2} + \frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3b^3d} - \frac{(5a^3d^3D-3a^2bd^2(2Cd-cD)-ab^2d(4cCd-8Bd^2-3c^2D)-b^3(6c^2Cd-8Bcd^2+16Ad^3-5c^3D))}{8b^{7/2}d^{7/2}}$$

output
```
-1/8*(6*C*b*c-8*B*b*d+10*C*a*d-8*D*a*c-5*b*c^2*D/d-11*a^2*d*D/b)*(b*x+a)^(
1/2)*(d*x+c)^(1/2)/b^2/d^2+1/12*(6*C*b*d-13*D*a*d-5*D*b*c)*(b*x+a)^(3/2)*(
d*x+c)^(1/2)/b^3/d^2+1/3*D*(b*x+a)^(5/2)*(d*x+c)^(1/2)/b^3/d-1/8*(5*a^3*d^
3*D-3*a^2*b*d^2*(2*C*d-D*c)-a*b^2*d*(-8*B*d^2+4*C*c*d-3*D*c^2)-b^3*(16*A*d
^3-8*B*c*d^2+6*C*c^2*d-5*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*
x+c)^(1/2))/b^(7/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$$
$$= \frac{\sqrt{a+bx}\sqrt{c+dx}(15a^2d^2D-2abd(9Cd-7cD+5dDx)+b^2(15c^2D-2cd(9C+5Dx)+4d^2(6B+3C}{24b^3d^3}$$
$$-\frac{(5a^3d^3D+3a^2bd^2(-2Cd+cD)+ab^2d(-4cCd+8Bd^2+3c^2D)+b^3(-6c^2Cd+8Bcd^2-16Ad^3+5c}{8b^{7/2}d^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[c + d*x]*(15*a^2*d^2*D - 2*a*b*d*(9*C*d - 7*c*D + 5*d*
D*x) + b^2*(15*c^2*D - 2*c*d*(9*C + 5*D*x) + 4*d^2*(6*B + 3*C*x + 2*D*x^2)
)))/(24*b^3*d^3) - ((5*a^3*d^3*D + 3*a^2*b*d^2*(-2*C*d + c*D) + a*b^2*d*(-
4*c*C*d + 8*B*d^2 + 3*c^2*D) + b^3*(-6*c^2*C*d + 8*B*c*d^2 - 16*A*d^3 + 5*
c^3*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*b^(7/
2)*d^(7/2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2125, 27, 1194, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{ 2125}$$
$$\frac{\int \frac{6Adb^3+(6bCd-13aDd-5bcD)x^2b^2+2\left(-4dDa^2-5bcDa+3b^2Bd\right)xb-a^2(5bc+ad)D}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{3b^3d}+\frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3b^3d}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{6Adb^3+(6bCd-13aDd-5bcD)x^2b^2+2\left(-4dDa^2-5bcDa+3b^2Bd\right)xb-a^2(5bc+ad)D}{\sqrt{a+bx}\sqrt{c+dx}}dx}{6b^3d}+\frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3b^3d}$$

$\big\downarrow$ 1194

$$\frac{\frac{\int \frac{3b^2\left(3d^2Da^3-2bd(Cd-4cD)a^2-b^2c(6Cd-5cD)a+8Ab^3d^2+b\left(-\left(\left(-5Dc^2+6Cdc-8Bd^2\right)b^2\right)-2ad(5Cd-4cD)b+11a^2d^2D\right)x\right)}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{2b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(-1\ldots}{2\ldots}}{6b^3d}$$

$$\frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3b^3d}$$

$\big\downarrow$ 27

$$\frac{\frac{3\int \frac{3d^2Da^3-2bd(Cd-4cD)a^2-b^2c(6Cd-5cD)a+8Ab^3d^2+b\left(-\left(\left(-5Dc^2+6Cdc-8Bd^2\right)b^2\right)-2ad(5Cd-4cD)b+11a^2d^2D\right)x}{\sqrt{a+bx}\sqrt{c+dx}}dx}{4d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}(-13ad\ldots}{2d}}{6b^3d}$$

$$\frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3b^3d}$$

$\big\downarrow$ 90

$$\frac{\frac{3\left(\frac{\sqrt{a+bx}\sqrt{c+dx}\left(11a^2d^2D-2abd(5Cd-4cD)-\left(b^2\left(-8Bd^2-5c^2D+6cCd\right)\right)\right)}{d}-\frac{\left(5a^3d^3D-3a^2bd^2(2Cd-cD)-ab^2d\left(-8Bd^2-3c^2D+4cCd\right)-\left(b^3\left(16Ad^3-8Bcd\ldots\right.\right.\right.}{2d}\ldots\right.}{4d}\ldots}{6b^3d}$$

$$\frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3b^3d}$$

$\big\downarrow$ 66

$$\frac{\frac{3\left(\frac{\sqrt{a+bx}\sqrt{c+dx}\left(11a^2d^2D-2abd(5Cd-4cD)-\left(b^2\left(-8Bd^2-5c^2D+6cCd\right)\right)\right)}{d}-\frac{\left(5a^3d^3D-3a^2bd^2(2Cd-cD)-ab^2d\left(-8Bd^2-3c^2D+4cCd\right)-\left(b^3\left(16Ad^3-8Bcd\ldots\right.\right.\right.}{d}\ldots\right.}{4d}\ldots}{6b^3d}$$

$$\frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3b^3d}$$

$\big\downarrow$ 221

$$\frac{\frac{3\left(\frac{\sqrt{a+bx}\sqrt{c+dx}\left(11a^2d^2D-2abd(5Cd-4cD)-\left(b^2\left(-8Bd^2-5c^2D+6cCd\right)\right)\right)}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)\left(5a^3d^3D-3a^2bd^2(2Cd-cD)-ab^2d\left(-8Bd^2-3c^2D+4\ldots\right.\right.}{\sqrt{b}d^{3/2}}\ldots\right.}{4d}\ldots}{6b^3d}$$

$$\frac{D(a+bx)^{5/2}\sqrt{c+dx}}{3b^3d}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

output
```
(D*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b^3*d) + (((6*b*C*d - 5*b*c*D - 13*a*
d*D)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d) + (3*(((11*a^2*d^2*D - 2*a*b*d*(
5*C*d - 4*c*D) - b^2*(6*c*C*d - 8*B*d^2 - 5*c^2*D))*Sqrt[a + b*x]*Sqrt[c +
 d*x])/d - ((5*a^3*d^3*D - 3*a^2*b*d^2*(2*C*d - c*D) - a*b^2*d*(4*c*C*d -
8*B*d^2 - 3*c^2*D) - b^3*(6*c^2*C*d - 8*B*c*d^2 + 16*A*d^3 - 5*c^3*D))*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))))
/(4*d))/(6*b^3*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 811 vs. $2(240) = 480$.

Time = 0.53 (sec) , antiderivative size = 812, normalized size of antiderivative = 2.99

method	result
default	$\frac{\left(16Db^2d^2x^2\sqrt{db}\,\sqrt{(bx+a)(xd+c)}+48A\ln\left(\frac{2bdx+2\sqrt{(bx+a)(xd+c)}\,\sqrt{db}+ad+bc}{2\sqrt{db}}\right)b^3d^3-24B\ln\left(\frac{2bdx+2\sqrt{(bx+a)(xd+c)}\,\sqrt{db}+ad+bc}{2\sqrt{db}}\right)a\right.}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/48*(16*D*b^2*d^2*x^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+48*A*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*d^3-
24*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^
(1/2))*a*b^2*d^3-24*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)+a*d+b*c)/(d*b)^(1/2))*b^3*c*d^2+18*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*d^3+12*C*ln(1/2*(2*b*d*x+2*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d^2+18*C
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2
))*b^3*c^2*d+24*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*d^2*x-15*D*ln(1/
2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3
*d^3-9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d
*b)^(1/2))*a^2*b*c*d^2-9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^2*d-15*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^3-20*D*(d*b)^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)*a*b*d^2*x-20*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*
b^2*c*d*x+48*B*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*d^2-36*C*(d*b)^(1/2
)*((b*x+a)*(d*x+c))^(1/2)*a*b*d^2-36*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)
*b^2*c*d+30*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a^2*d^2+28*D*(d*b)^(1/2)
*((b*x+a)*(d*x+c))^(1/2)*a*b*c*d+30*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*
b^2*c^2)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d^3/(d*b)^(1/2)/((b*x+a)*(d*x+...
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$$
$$= \left[-\frac{3\,(5\,Db^3c^3+3\,(Dab^2-2\,Cb^3)c^2d+(3\,Da^2b-4\,Cab^2+8\,Bb^3)cd^2+(5\,Da^3-6\,Ca^2b+8\,Bab^2-16}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(5*D*b^3*c^3 + 3*(D*a*b^2 - 2*C*b^3)*c^2*d + (3*D*a^2*b - 4*C*a*
b^2 + 8*B*b^3)*c*d^2 + (5*D*a^3 - 6*C*a^2*b + 8*B*a*b^2 - 16*A*b^3)*d^3)*s
qrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x +
b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x
) - 4*(8*D*b^3*d^3*x^2 + 15*D*b^3*c^2*d + 2*(7*D*a*b^2 - 9*C*b^3)*c*d^2 +
3*(5*D*a^2*b - 6*C*a*b^2 + 8*B*b^3)*d^3 - 2*(5*D*b^3*c*d^2 + (5*D*a*b^2 -
6*C*b^3)*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4), 1/48*(3*(5*D*b^3*
c^3 + 3*(D*a*b^2 - 2*C*b^3)*c^2*d + (3*D*a^2*b - 4*C*a*b^2 + 8*B*b^3)*c*d^
2 + (5*D*a^3 - 6*C*a^2*b + 8*B*a*b^2 - 16*A*b^3)*d^3)*sqrt(-b*d)*arctan(1/
2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^
2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(8*D*b^3*d^3*x^2 + 15*D*b^3*c^2*
d + 2*(7*D*a*b^2 - 9*C*b^3)*c*d^2 + 3*(5*D*a^2*b - 6*C*a*b^2 + 8*B*b^3)*d^
3 - 2*(5*D*b^3*c*d^2 + (5*D*a*b^2 - 6*C*b^3)*d^3)*x)*sqrt(b*x + a)*sqrt(d*
x + c))/(b^4*d^4)]
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}\sqrt{c + dx}}\,dx = \int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}\sqrt{c + dx}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x)*sqrt(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}\sqrt{c + dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx = \frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)D}{b^4d}-\frac{5\,Db^{12}cd^3+13\,Dab^{11}d^4-6\,Cb^{12}d^4}{b^{15}d^5}\right)+\frac{3\,(5\,Db^{13}c^2d^2+8\,D\cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="gi
ac")
```

output
```
1/24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)*D/(b^4*d) - (5*D*b^12*c*d^3 + 13*D*a*b^11*d^4 - 6*C*b^12*d^4)/(b^1
5*d^5)) + 3*(5*D*b^13*c^2*d^2 + 8*D*a*b^12*c*d^3 - 6*C*b^13*c*d^3 + 11*D*a
^2*b^11*d^4 - 10*C*a*b^12*d^4 + 8*B*b^13*d^4)/(b^15*d^5)) + 3*(5*D*b^3*c^3
 + 3*D*a*b^2*c^2*d - 6*C*b^3*c^2*d + 3*D*a^2*b*c*d^2 - 4*C*a*b^2*c*d^2 + 8
*B*b^3*c*d^2 + 5*D*a^3*d^3 - 6*C*a^2*b*d^3 + 8*B*a*b^2*d^3 - 16*A*b^3*d^3)
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*b^3*d^3))*b/abs(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx = \text{Hanged}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\sqrt{c+dx}}\,dx$$
$$=\frac{15\sqrt{dx+c}\,\sqrt{bx+a}\,a^2b\,d^3-4\sqrt{dx+c}\,\sqrt{bx+a}\,a\,b^2c\,d^2-10\sqrt{dx+c}\,\sqrt{bx+a}\,a\,b^2d^3x+24\sqrt{dx+c}\,\sqrt{bx}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

output
```
(15*sqrt(c + d*x)*sqrt(a + b*x)*a**2*b*d**3 - 4*sqrt(c + d*x)*sqrt(a + b*x
)*a*b**2*c*d**2 - 10*sqrt(c + d*x)*sqrt(a + b*x)*a*b**2*d**3*x + 24*sqrt(c
 + d*x)*sqrt(a + b*x)*b**4*d**2 - 3*sqrt(c + d*x)*sqrt(a + b*x)*b**3*c**2*
d + 2*sqrt(c + d*x)*sqrt(a + b*x)*b**3*c*d**2*x + 8*sqrt(c + d*x)*sqrt(a +
 b*x)*b**3*d**3*x**2 - 15*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqr
t(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**3*d**3 + 9*sqrt(d)*sqrt(b)*log((sq
rt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b*c*d**
2 + 24*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))
/sqrt(a*d - b*c))*a*b**3*d**2 + 3*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*
x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**2*c**2*d - 24*sqrt(d)*sq
rt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))
*b**4*c*d + 3*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c
+ d*x))/sqrt(a*d - b*c))*b**3*c**3)/(24*b**4*d**3)
```

3.134 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx$

Optimal result . . . 1286
Mathematica [A] (verified) . . . 1287
Rubi [A] (verified) . . . 1287
Maple [B] (verified) . . . 1290
Fricas [B] (verification not implemented) . . . 1291
Sympy [F] . . . 1292
Maxima [F(-2)] . . . 1293
Giac [B] (verification not implemented) . . . 1293
Mupad [F(-1)] . . . 1294
Reduce [B] (verification not implemented) . . . 1294

Optimal result

Integrand size = 34, antiderivative size = 223

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx=\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{d^3(bc-ad)\sqrt{c+dx}}+\frac{(4bCd-9bcD-3adD)\sqrt{a+bx}\sqrt{c+dx}}{4b^2d^3}+\frac{D\sqrt{a+bx}(c+dx)^{3/2}}{2bd^3}+\frac{(3a^2d^2D-2abd(2Cd-3cD)-b^2(12cCd-8Bd^2-15c^2D))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{5/2}d^{7/2}}$$

output
```
2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d^3/(-a*d+b*c)/(d*x+c)^(1/2)
+1/4*(4*C*b*d-3*D*a*d-9*D*b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d^3+1/2*D*(
b*x+a)^(1/2)*(d*x+c)^(3/2)/b/d^3+1/4*(3*a^2*d^2*D-2*a*b*d*(2*C*d-3*D*c)-b^
2*(-8*B*d^2+12*C*c*d-15*D*c^2))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x
+c)^(1/2))/b^(5/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx = \frac{-\sqrt{b}\sqrt{d}\sqrt{a+bx}(3a^2d^2D(c+dx)-2abd(c+dx)(2Cd-2cD+dDx)+b^2}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

output
```
(-(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x]*(3*a^2*d^2*D*(c + d*x) - 2*a*b*d*(c + d*x
)*(2*C*d - 2*c*D + d*D*x) + b^2*(8*A*d^3 - 15*c^3*D + c^2*d*(12*C - 5*D*x)
 + 2*c*d^2*(-4*B + 2*C*x + D*x^2)))) - (b*c - a*d)*(3*a^2*d^2*D + 2*a*b*d*
(-2*C*d + 3*c*D) + b^2*(-12*c*C*d + 8*B*d^2 + 15*c^2*D))*Sqrt[c + d*x]*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(5/2)*d^(7/2)*
(-(b*c) + a*d)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1194, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \ 2124$$

$$\frac{2\int -\frac{-\frac{(bc-ad)Dx^2}{d}-\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{(bc-ad)\left(-Dc^2+Cdc-Bd^2\right)}{d^3}}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{bc-ad}+\frac{2\sqrt{a+bx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{2\sqrt{a+bx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}-\frac{\int \frac{-\frac{(bc-ad)Dx^2}{d}-\frac{(bc-ad)(Cd-cD)x}{d^2}+\frac{(bc-ad)\left(-Dc^2+Cdc-Bd^2\right)}{d^3}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{bc-ad}$$

$\big\downarrow$ 1194

$$\frac{2\sqrt{a+bx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}-\frac{\frac{\int \frac{(bc-ad)\left(4\left(-Dc^2+Cdc-Bd^2\right)b^2+3acdDb-d(4bCd-5aDd-7bcD)xb+a^2d^2D\right)}{2d^2\sqrt{a+bx}\sqrt{c+dx}}dx}{2b^2d}-\frac{D(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{2b^2d^2}}{bc-ad}$$

$\big\downarrow$ 27

$$\frac{2\sqrt{a+bx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}-\frac{\frac{(bc-ad)\int \frac{4\left(-Dc^2+Cdc-Bd^2\right)b^2+3acdDb-d(4bCd-5aDd-7bcD)xb+a^2d^2D}{\sqrt{a+bx}\sqrt{c+dx}}dx}{4b^2d^3}-\frac{D(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{2b^2d^2}}{bc-ad}$$

$\big\downarrow$ 90

$$\frac{2\sqrt{a+bx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}-\frac{\frac{(bc-ad)\left(-\frac{1}{2}\left(3a^2d^2D-2abd(2Cd-3cD)-\left(b^2\left(-8Bd^2-15c^2D+12cCd\right)\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx-\sqrt{a+bx}\sqrt{c+dx}(-5adD-7bcD+4bCd)\right)}{4b^2d^3}-D(a+}{bc-ad}$$

$\big\downarrow$ 66

$$\frac{2\sqrt{a+bx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}-\frac{\frac{(bc-ad)\left(-\left(3a^2d^2D-2abd(2Cd-3cD)-\left(b^2\left(-8Bd^2-15c^2D+12cCd\right)\right)\right)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}-\sqrt{a+bx}\sqrt{c+dx}(-5adD-7bcD+4bCd)\right)}{4b^2d^3}-D(}{bc-ad}$$

$\big\downarrow$ 221

$$\frac{2\sqrt{a+bx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3\sqrt{c+dx}(bc-ad)}-\frac{\frac{(bc-ad)\left(-\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)\left(3a^2d^2D-2abd(2Cd-3cD)-\left(b^2\left(-8Bd^2-15c^2D+12cCd\right)\right)\right)}{\sqrt{b}\sqrt{d}}-\sqrt{a+bx}\sqrt{c+dx}(-5adD-7bcD+4bCd)\right)}{4b^2d^3}-D(a+b}{bc-ad}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

output
```
(2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[a + b*x])/(d^3*(b*c - a*d)*Sqr
t[c + d*x]) - (-1/2*((b*c - a*d)*D*(a + b*x)^(3/2)*Sqrt[c + d*x])/(b^2*d^2
) + ((b*c - a*d)*(-((4*b*C*d - 7*b*c*D - 5*a*d*D)*Sqrt[a + b*x]*Sqrt[c + d
*x]) - ((3*a^2*d^2*D - 2*a*b*d*(2*C*d - 3*c*D) - b^2*(12*c*C*d - 8*B*d^2 -
 15*c^2*D))*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqr
t[b]*Sqrt[d])))/(4*b^2*d^3))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. $2(193) = 386$.

Time = 0.61 (sec) , antiderivative size = 1370, normalized size of antiderivative = 6.14

method	result	size
default	Expression too large to display	1370

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/8*(b*x+a)^(1/2)*(6*D*a^2*d^3*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-16*B
*b^2*c*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+6*D*a^2*c*d^2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)-8*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*d^4*x+8*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c*d^3*x+24*C*b^2*c^2*d*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-8*C*a*b*d^3*x*((b*x+a)*(d*x+c))^(1/2)*
(d*b)^(1/2)-30*D*b^2*c^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-3*D*ln(1/2*(2
*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*d^4
*x+8*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b
)^(1/2))*b^3*c^2*d^2-12*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^3*d-3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*c*d^3+16*A*b^2*d^3*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+4*D*a*b*c*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)-9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*a*b^2*c^2*d^2*x-4*D*a*b*d^3*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+4*D*b^2*c*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+8*C*b^2*c*d^2
*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-10*D*b^2*c^2*d*x*((b*x+a)*(d*x+c))^
(1/2)*(d*b)^(1/2)-8*C*a*b*c*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+8*D*a*
b*c^2*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+8*C*ln(1/2*(2*b*d*x+2*((b*x+a)
*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c*d^3*x-3*D*ln(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(192) = 384$.

Time = 3.12 (sec) , antiderivative size = 930, normalized size of antiderivative = 4.17

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}(c + dx)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fr
icas")
```

output
```
[1/16*((15*D*b^3*c^4 - 3*(3*D*a*b^2 + 4*C*b^3)*c^3*d - (3*D*a^2*b - 8*C*a*
b^2 - 8*B*b^3)*c^2*d^2 - (3*D*a^3 - 4*C*a^2*b + 8*B*a*b^2)*c*d^3 + (15*D*b
^3*c^3*d - 3*(3*D*a*b^2 + 4*C*b^3)*c^2*d^2 - (3*D*a^2*b - 8*C*a*b^2 - 8*B*
b^3)*c*d^3 - (3*D*a^3 - 4*C*a^2*b + 8*B*a*b^2)*d^4)*x)*sqrt(b*d)*log(8*b^2
*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*
d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(15*D*b^3*c^
3*d - 8*A*b^3*d^4 - 4*(D*a*b^2 + 3*C*b^3)*c^2*d^2 - (3*D*a^2*b - 4*C*a*b^2
 - 8*B*b^3)*c*d^3 - 2*(D*b^3*c*d^3 - D*a*b^2*d^4)*x^2 + (5*D*b^3*c^2*d^2 -
 2*(D*a*b^2 + 2*C*b^3)*c*d^3 - (3*D*a^2*b - 4*C*a*b^2)*d^4)*x)*sqrt(b*x +
a)*sqrt(d*x + c))/(b^4*c^2*d^4 - a*b^3*c*d^5 + (b^4*c*d^5 - a*b^3*d^6)*x),
 -1/8*((15*D*b^3*c^4 - 3*(3*D*a*b^2 + 4*C*b^3)*c^3*d - (3*D*a^2*b - 8*C*a*
b^2 - 8*B*b^3)*c^2*d^2 - (3*D*a^3 - 4*C*a^2*b + 8*B*a*b^2)*c*d^3 + (15*D*b
^3*c^3*d - 3*(3*D*a*b^2 + 4*C*b^3)*c^2*d^2 - (3*D*a^2*b - 8*C*a*b^2 - 8*B*
b^3)*c*d^3 - (3*D*a^3 - 4*C*a^2*b + 8*B*a*b^2)*d^4)*x)*sqrt(-b*d)*arctan(1
/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x
^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(15*D*b^3*c^3*d - 8*A*b^3*d^4 -
 4*(D*a*b^2 + 3*C*b^3)*c^2*d^2 - (3*D*a^2*b - 4*C*a*b^2 - 8*B*b^3)*c*d^3 -
 2*(D*b^3*c*d^3 - D*a*b^2*d^4)*x^2 + (5*D*b^3*c^2*d^2 - 2*(D*a*b^2 + 2*C*b
^3)*c*d^3 - (3*D*a^2*b - 4*C*a*b^2)*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(
b^4*c^2*d^4 - a*b^3*c*d^5 + (b^4*c*d^5 - a*b^3*d^6)*x)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x)*(c + d*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(192) = 384$.

Time = 0.19 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx=\frac{\left((bx+a)\left(\frac{2\,(Db^6cd^4|b|-Dab^5d^5|b|)(bx+a)}{b^9cd^5-ab^8d^6}-\frac{5\,Db^7c^2d^3|b|+2\,Dab^6cd^4|b|-4\,Cb^7cd^4|b|-7\,Da^2b^5}{b^9cd^5-ab^8d^6}\right.\right.}{}$$

$$-\frac{(15\,Db^2c^2|b|+6\,Dabcd|b|-12\,Cb^2cd|b|+3\,Da^2d^2|b|-4\,Cabd^2|b|+8\,Bb^2d^2|b|)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\right.\right.}{4\,\sqrt{bd}b^3d^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="gi
ac")
```

output
```
1/4*((b*x + a)*(2*(D*b^6*c*d^4*abs(b) - D*a*b^5*d^5*abs(b))*(b*x + a)/(b^9
*c*d^5 - a*b^8*d^6) - (5*D*b^7*c^2*d^3*abs(b) + 2*D*a*b^6*c*d^4*abs(b) - 4
*C*b^7*c*d^4*abs(b) - 7*D*a^2*b^5*d^5*abs(b) + 4*C*a*b^6*d^5*abs(b))/(b^9*
c*d^5 - a*b^8*d^6)) - (15*D*b^8*c^3*d^2*abs(b) - 9*D*a*b^7*c^2*d^3*abs(b)
- 12*C*b^8*c^2*d^3*abs(b) - 3*D*a^2*b^6*c*d^4*abs(b) + 8*C*a*b^7*c*d^4*abs
(b) + 8*B*b^8*c*d^4*abs(b) + 5*D*a^3*b^5*d^5*abs(b) - 4*C*a^2*b^6*d^5*abs(
b) - 8*A*b^8*d^5*abs(b))/(b^9*c*d^5 - a*b^8*d^6))*sqrt(b*x + a)/sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d) - 1/4*(15*D*b^2*c^2*abs(b) + 6*D*a*b*c*d*abs(b)
- 12*C*b^2*c*d*abs(b) + 3*D*a^2*d^2*abs(b) - 4*C*a*b*d^2*abs(b) + 8*B*b^2*
d^2*abs(b))*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
- a*b*d)))/(sqrt(b*d)*b^3*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{3/2}}\,dx=\frac{-3\sqrt{dx+c}\,\sqrt{bx+a}\,abc\,d^2-3\sqrt{dx+c}\,\sqrt{bx+a}\,ab\,d^3x-8\sqrt{dx+c}\,\sqrt{bx+}}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

output

```
( - 3*sqrt(c + d*x)*sqrt(a + b*x)*a*b*c*d**2 - 3*sqrt(c + d*x)*sqrt(a + b*
x)*a*b*d**3*x - 8*sqrt(c + d*x)*sqrt(a + b*x)*b**3*d**2 - 3*sqrt(c + d*x)*
sqrt(a + b*x)*b**2*c**2*d - sqrt(c + d*x)*sqrt(a + b*x)*b**2*c*d**2*x + 2*
sqrt(c + d*x)*sqrt(a + b*x)*b**2*d**3*x**2 + 3*sqrt(d)*sqrt(b)*log((sqrt(d
)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*c*d**2 + 3*
sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a
*d - b*c))*a**2*d**3*x + 2*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sq
rt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b*c**2*d + 2*sqrt(d)*sqrt(b)*log((
sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b*c*d**2
*x + 8*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))
/sqrt(a*d - b*c))*b**3*c*d + 8*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x)
+ sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**3*d**2*x + 3*sqrt(d)*sqrt(b)*
log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**2*
c**3 + 3*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x
))/sqrt(a*d - b*c))*b**2*c**2*d*x + sqrt(d)*sqrt(b)*a**2*c*d**2 + sqrt(d)*
sqrt(b)*a**2*d**3*x - 8*sqrt(d)*sqrt(b)*b**3*c*d - 8*sqrt(d)*sqrt(b)*b**3*
d**2*x - sqrt(d)*sqrt(b)*b**2*c**3 - sqrt(d)*sqrt(b)*b**2*c**2*d*x)/(4*b**
3*d**3*(c + d*x))
```

3.135 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx$

Optimal result . . . 1296
Mathematica [A] (verified) . . . 1297
Rubi [A] (verified) . . . 1297
Maple [B] (verified) . . . 1300
Fricas [B] (verification not implemented) . . . 1301
Sympy [F] . . . 1302
Maxima [F(-2)] . . . 1303
Giac [B] (verification not implemented) . . . 1303
Mupad [F(-1)] . . . 1304
Reduce [F] . . . 1304

Optimal result

Integrand size = 34, antiderivative size = 232

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx &= \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{3d^3(bc-ad)(c+dx)^{3/2}}\\ &+\frac{2(3ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-Bcd^2-2Ad^3-7c^3D))\sqrt{a+bx}}{3d^3(bc-ad)^2\sqrt{c+dx}}\\ &+\frac{D\sqrt{a+bx}\sqrt{c+dx}}{bd^3}+\frac{(2bCd-5bcD-adD)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}d^{7/2}}\end{aligned}$$

output
```
2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d^3/(-a*d+b*c)/(d*x+c)^(3/
2)+2/3*(3*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-2*A*d^3-B*c*d^2+4*C*c^2*d-7*D*c
^3))*(b*x+a)^(1/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(1/2)+D*(b*x+a)^(1/2)*(d*x+c)^
(1/2)/b/d^3+(2*C*b*d-D*a*d-5*D*b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/
(d*x+c)^(1/2))/b^(3/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx = \frac{\sqrt{a+bx}(3a^2d^2D(c+dx)^2+b^2(15c^4D+4Ad^4x+2cd^3(3A+Bx)+c^2d^2x}{}$$
$$+\frac{(2bCd-5bcD-adD)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}d^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

output
```
(Sqrt[a + b*x]*(3*a^2*d^2*D*(c + d*x)^2 + b^2*(15*c^4*D + 4*A*d^4*x + 2*c*
d^3*(3*A + B*x) + c^2*d^2*x*(-8*C + 3*D*x) + c^3*(-6*C*d + 20*d*D*x)) - 2*
a*b*d*(11*c^3*D + d^3*(A + 3*B*x) - 5*c^2*d*(C - 3*D*x) + c*d^2*(2*B - 6*C
*x + 3*D*x^2))))/(3*b*d^3*(b*c - a*d)^2*(c + d*x)^(3/2)) + ((2*b*C*d - 5*b
*c*D - a*d*D)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b
^(3/2)*d^(7/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1193, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx$$
$$\Big\downarrow \ 2124$$
$$\frac{2\int \frac{-3\left(a-\frac{bc}{d}\right)Dx^2+\frac{3(bc-ad)(Cd-cD)x}{d^2}+\frac{3ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-Dc^3+Cdc^2-Bd^2c-2Ad^3\right)}{d^3}}{2\sqrt{a+bx}(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 27$$

$$\frac{\int \frac{-3\left(a-\frac{bc}{d}\right)Dx^2+\frac{3(bc-ad)(Cd-cD)x}{d^2}+\frac{3ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-Dc^3+Cdc^2-Bd^2c-2Ad^3\right)}{d^3}}{\sqrt{a+bx}(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 1193$$

$$\frac{\frac{2\int \frac{3(bc-ad)^2(Cd+Dxd-2cD)}{2d^3\sqrt{a+bx}\sqrt{c+dx}}dx}{bc-ad}+\frac{2\sqrt{a+bx}\left(3ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-Bcd^2-7c^3D+4c^2Cd\right)\right)}{d^3\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 27$$

$$\frac{\frac{3(bc-ad)\int \frac{Cd+Dxd-2cD}{\sqrt{a+bx}\sqrt{c+dx}}dx}{d^3}+\frac{2\sqrt{a+bx}\left(3ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-Bcd^2-7c^3D+4c^2Cd\right)\right)}{d^3\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 90$$

$$\frac{\frac{3(bc-ad)\left(\frac{(-adD-5bcD+2bCd)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2b}+\frac{D\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{d^3}+\frac{2\sqrt{a+bx}\left(3ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-Bcd^2-7c^3D+4c^2Cd\right)\right)}{d^3\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 66$$

$$\frac{\frac{3(bc-ad)\left(\frac{(-adD-5bcD+2bCd)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{b}+\frac{D\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{d^3}+\frac{2\sqrt{a+bx}\left(3ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-Bcd^2-7c^3D+4c^2C\right)\right)}{d^3\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 221$$

$$\frac{\frac{2\sqrt{a+bx}\left(3ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(-2Ad^3-Bcd^2-7c^3D+4c^2Cd\right)\right)}{d^3\sqrt{c+dx}(bc-ad)}+\frac{3(bc-ad)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-adD-5bcD+2bCd)}{b^{3/2}\sqrt{d}}+\frac{D\sqrt{a+bx}\sqrt{c+}}{b}\right.}{d^3}}{3(bc-ad)}$$

$$\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*Sqrt[a + b*x])/(3*(b*c - a*d)*(c
+ d*x)^(3/2)) + ((2*(3*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - B*
c*d^2 - 2*A*d^3 - 7*c^3*D))*Sqrt[a + b*x])/(d^3*(b*c - a*d)*Sqrt[c + d*x])
 + (3*(b*c - a*d)*((D*Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((2*b*C*d - 5*b*c*D
 - a*d*D)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/
2)*Sqrt[d])))/d^3)/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1706 vs. $2(204) = 408$.

Time = 0.57 (sec) , antiderivative size = 1707, normalized size of antiderivative = 7.36

method	result	size
default	Expression too large to display	1707

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/6*(b*x+a)^(1/2)*(6*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)+a*d+b*c)/(d*b)^(1/2))*a^3*c*d^4*x-6*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c^2*d^3+12*C*ln(1/2*(2*b
*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^3
*d^2+9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d
*b)^(1/2))*a^2*b*c^3*d^2+44*D*a*b*c^3*d*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2
)+12*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b
)^(1/2))*a*b^2*c*d^4*x^2+9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c*d^4*x^2-27*D*ln(1/2*(2*b*d*x+2*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^2*d^3*x^2+12
*C*b^2*c^3*d*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-6*D*a^2*c^2*d^2*(d*b)^(1/
2)*((b*x+a)*(d*x+c))^(1/2)-6*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*d^5*x^2-12*C*ln(1/2*(2*b*d*x+2*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^3*d^2*x-6*D*b^
2*c^2*d^2*x^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+30*D*ln(1/2*(2*b*d*x+2*(
(b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*c^4*d*x-6*D*a
^2*d^4*x^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-8*A*b^2*d^4*x*(d*b)^(1/2)*(
(b*x+a)*(d*x+c))^(1/2)+4*A*a*b*d^4*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-30*
D*b^2*c^4*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)-12*A*b^2*c*d^3*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+12*D*a*b*c*d^3*x^2*(d*b)^(1/2)*((b*x+a)*(d*x+c))...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(202) = 404$.

Time = 8.80 (sec) , antiderivative size = 1280, normalized size of antiderivative = 5.52

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}(c + dx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fr
icas")
```

output
```
[-1/12*(3*(5*D*b^3*c^5 - (9*D*a*b^2 + 2*C*b^3)*c^4*d + (3*D*a^2*b + 4*C*a*
b^2)*c^3*d^2 + (D*a^3 - 2*C*a^2*b)*c^2*d^3 + (5*D*b^3*c^3*d^2 - (9*D*a*b^2
 + 2*C*b^3)*c^2*d^3 + (3*D*a^2*b + 4*C*a*b^2)*c*d^4 + (D*a^3 - 2*C*a^2*b)*
d^5)*x^2 + 2*(5*D*b^3*c^4*d - (9*D*a*b^2 + 2*C*b^3)*c^3*d^2 + (3*D*a^2*b +
 4*C*a*b^2)*c^2*d^3 + (D*a^3 - 2*C*a^2*b)*c*d^4)*x)*sqrt(b*d)*log(8*b^2*d^
2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(15*D*b^3*c^4*d
 - 2*A*a*b^2*d^5 - 2*(11*D*a*b^2 + 3*C*b^3)*c^3*d^2 + (3*D*a^2*b + 10*C*a*
b^2)*c^2*d^3 - 2*(2*B*a*b^2 - 3*A*b^3)*c*d^4 + 3*(D*b^3*c^2*d^3 - 2*D*a*b^
2*c*d^4 + D*a^2*b*d^5)*x^2 + 2*(10*D*b^3*c^3*d^2 - (15*D*a*b^2 + 4*C*b^3)*
c^2*d^3 + (3*D*a^2*b + 6*C*a*b^2 + B*b^3)*c*d^4 - (3*B*a*b^2 - 2*A*b^3)*d^
5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*c^4*d^4 - 2*a*b^3*c^3*d^5 + a^2*b^
2*c^2*d^6 + (b^4*c^2*d^6 - 2*a*b^3*c*d^7 + a^2*b^2*d^8)*x^2 + 2*(b^4*c^3*d
^5 - 2*a*b^3*c^2*d^6 + a^2*b^2*c*d^7)*x), 1/6*(3*(5*D*b^3*c^5 - (9*D*a*b^2
 + 2*C*b^3)*c^4*d + (3*D*a^2*b + 4*C*a*b^2)*c^3*d^2 + (D*a^3 - 2*C*a^2*b)*
c^2*d^3 + (5*D*b^3*c^3*d^2 - (9*D*a*b^2 + 2*C*b^3)*c^2*d^3 + (3*D*a^2*b +
4*C*a*b^2)*c*d^4 + (D*a^3 - 2*C*a^2*b)*d^5)*x^2 + 2*(5*D*b^3*c^4*d - (9*D*
a*b^2 + 2*C*b^3)*c^3*d^2 + (3*D*a^2*b + 4*C*a*b^2)*c^2*d^3 + (D*a^3 - 2*C*
a^2*b)*c*d^4)*x)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sq
rt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x)*(c + d*x)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(202) = 404.

Time = 0.22 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.37

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx = \frac{\left((bx+a)\left(\frac{3\left(Db^6c^2d^4|b|-2\,Dab^5cd^5|b|+Da^2b^4d^6|b|\right)(bx+a)}{b^7c^2d^5-2\,ab^6cd^6+a^2b^5d^7} + \frac{2\left(10\,Db^7c^3d^3|b|-18\,Dab^6c^2d^4|b\right.}{}\right.\right.}{}$$

$$+ \frac{(5\,Dbc|b|+Dad|b|-2\,Cbd|b|)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}b^2d^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="gi
ac")
```

output
```
1/3*((b*x + a)*(3*(D*b^6*c^2*d^4*abs(b) - 2*D*a*b^5*c*d^5*abs(b) + D*a^2*b
^4*d^6*abs(b))*(b*x + a)/(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7) + 2*(
10*D*b^7*c^3*d^3*abs(b) - 18*D*a*b^6*c^2*d^4*abs(b) - 4*C*b^7*c^2*d^4*abs(
b) + 9*D*a^2*b^5*c*d^5*abs(b) + 6*C*a*b^6*c*d^5*abs(b) + B*b^7*c*d^5*abs(b
) - 3*D*a^3*b^4*d^6*abs(b) - 3*B*a*b^6*d^6*abs(b) + 2*A*b^7*d^6*abs(b))/(b
^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)) + 3*(5*D*b^8*c^4*d^2*abs(b) - 1
4*D*a*b^7*c^3*d^3*abs(b) - 2*C*b^8*c^3*d^3*abs(b) + 12*D*a^2*b^6*c^2*d^4*a
bs(b) + 6*C*a*b^7*c^2*d^4*abs(b) - 4*D*a^3*b^5*c*d^5*abs(b) - 4*C*a^2*b^6*
c*d^5*abs(b) - 2*B*a*b^7*c*d^5*abs(b) + 2*A*b^8*c*d^5*abs(b) + D*a^4*b^4*d
^6*abs(b) + 2*B*a^2*b^6*d^6*abs(b) - 2*A*a*b^7*d^6*abs(b))/(b^7*c^2*d^5 -
2*a*b^6*c*d^6 + a^2*b^5*d^7))*sqrt(b*x + a)/(b^2*c + (b*x + a)*b*d - a*b*d
)^(3/2) + (5*D*b*c*abs(b) + D*a*d*abs(b) - 2*C*b*d*abs(b))*log(abs(-sqrt(b
*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d
^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{a+b\,x}\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{5/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

3.136 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx$

Optimal result 1305
Mathematica [A] (verified) 1306
Rubi [A] (verified) 1306
Maple [B] (verified) 1309
Fricas [B] (verification not implemented) 1310
Sympy [F] 1311
Maxima [F(-2)] 1312
Giac [B] (verification not implemented) 1312
Mupad [F(-1)] 1313
Reduce [F] 1314

Optimal result

Integrand size = 34, antiderivative size = 296

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{5d^3(bc-ad)(c+dx)^{5/2}}$$
$$+\frac{2(5ad(2cCd-Bd^2-3c^2D)-b(6c^2Cd-Bcd^2-4Ad^3-11c^3D))\sqrt{a+bx}}{15d^3(bc-ad)^2(c+dx)^{3/2}}$$
$$+\frac{2(15a^2d^2(Cd-3cD)-10abd(cCd+Bd^2-6c^2D)+b^2(3c^2Cd+2Bcd^2+8Ad^3-23c^3D))\sqrt{a+bx}}{15d^3(bc-ad)^3\sqrt{c+dx}}$$
$$+\frac{2D\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{7/2}}$$

output
```
2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d^3/(-a*d+b*c)/(d*x+c)^(5/
2)+2/15*(5*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-4*A*d^3-B*c*d^2+6*C*c^2*d-11*D
*c^3))*(b*x+a)^(1/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(3/2)+2/15*(15*a^2*d^2*(C*d-
3*D*c)-10*a*b*d*(B*d^2+C*c*d-6*D*c^2)+b^2*(8*A*d^3+2*B*c*d^2+3*C*c^2*d-23*
D*c^3))*(b*x+a)^(1/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(1/2)+2*D*arctanh(d^(1/2)*(
b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(1/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx =$$
$$-\frac{2\sqrt{a+bx}(b^2c(-15c^4D-35c^3dDx+2Bd^4x^2-23c^2d^2Dx^2+cd^3x(5B+3Cx))+Ad^3(3a^2d^2-2abd(5c$$
$$+\frac{2D\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}d^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*(c + d*x)^(7/2)),x]
```

output
```
(-2*Sqrt[a + b*x]*(b^2*c*(-15*c^4*D - 35*c^3*d*D*x + 2*B*d^4*x^2 - 23*c^2*
d^2*D*x^2 + c*d^3*x*(5*B + 3*C*x)) + A*d^3*(3*a^2*d^2 - 2*a*b*d*(5*c + 2*d
*x) + b^2*(15*c^2 + 20*c*d*x + 8*d^2*x^2)) - 2*a*b*d*(B*d^2*(5*c^2 + 13*c*
d*x + 5*d^2*x^2) + c*(-20*c^3*D - 47*c^2*d*D*x + 5*C*d^3*x^2 + 2*c*d^2*x*(
C - 15*D*x))) + a^2*d^2*(-33*c^3*D + 5*d^3*x*(B + 3*C*x) + c^2*d*(8*C - 75
*D*x) + c*d^2*(2*B + 5*x*(4*C - 9*D*x)))))/(15*d^3*(-(b*c) + a*d)^3*(c + d
*x)^(5/2)) + (2*D*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(Sqrt[b]*d^(7/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1193, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}dx$$
$$\Big\downarrow \; 2124$$

$$\frac{2\int \frac{-5\left(a-\frac{bc}{d}\right)Dx^2+\frac{5(bc-ad)(Cd-cD)x}{d^2}+\frac{5ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-Dc^3+Cdc^2-Bd^2c-4Ad^3\right)}{d^3}}{2\sqrt{a+bx}(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{-5\left(a-\frac{bc}{d}\right)Dx^2+\frac{5(bc-ad)(Cd-cD)x}{d^2}+\frac{5ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-Dc^3+Cdc^2-Bd^2c-4Ad^3\right)}{d^3}}{\sqrt{a+bx}(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$\frac{\frac{2\int \frac{\left(-8Dc^3+3Cdc^2+2Bd^2c+8Ad^3\right)b^2-10ad\left(-3Dc^2+Cdc+Bd^2\right)b+15a^2d^2(Cd-2cD)+15d(bc-ad)^2Dx}{2d^3\sqrt{a+bx}(c+dx)^{3/2}}dx}{3(bc-ad)}+\frac{2\sqrt{a+bx}(5ad(-Bd^2-3c^2D+2cCd)-b(-}{3d^3(c+dx)^{3/2}(b}}{5(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(-8Dc^3+3Cdc^2+2Bd^2c+8Ad^3\right)b^2-10ad\left(-3Dc^2+Cdc+Bd^2\right)b+15a^2d^2(Cd-2cD)+15d(bc-ad)^2Dx}{\sqrt{a+bx}(c+dx)^{3/2}}dx}{3d^3(bc-ad)}+\frac{2\sqrt{a+bx}(5ad(-Bd^2-3c^2D+2cCd)-b(-4}{3d^3(c+dx)^{3/2}(bc-}}{5(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$\frac{\frac{15D(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx+\frac{2\sqrt{a+bx}\left(15a^2d^2(Cd-3cD)-10abd\left(Bd^2-6c^2D+cCd\right)+b^2\left(8Ad^3+2Bcd^2-23c^3D+3c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}}{3d^3(bc-ad)}+\frac{2\sqrt{a+bx}(5ad(-Bd^2-}{}}{5(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 66

$$\frac{\frac{30D(bc-ad)^2\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}+\frac{2\sqrt{a+bx}\left(15a^2d^2(Cd-3cD)-10abd\left(Bd^2-6c^2D+cCd\right)+b^2\left(8Ad^3+2Bcd^2-23c^3D+3c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}}{3d^3(bc-ad)}+\frac{2\sqrt{a+bx}(5ad(-B\ldots}{}}{5(bc-ad)}$$

$$\frac{2\sqrt{a+bx}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

$\big\downarrow$ 221

$$\frac{\frac{\frac{2\sqrt{a+bx}\left(15a^2d^2(Cd-3cD)-10abd\left(Bd^2-6c^2D+cCd\right)+b^2\left(8Ad^3+2Bcd^2-23c^3D+3c^2Cd\right)\right)}{\sqrt{c+dx}(bc-ad)}+\frac{30D(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}}{3d^3(bc-ad)}+\frac{2\sqrt{a+bx}(5ad(-B\ldots}{}}{5(bc-ad)}$$

$$\frac{2\sqrt{a+bx}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{5(c+dx)^{5/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*(c + d*x)^(7/2)),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*Sqrt[a + b*x])/(5*(b*c - a*d)*(c
+ d*x)^(5/2)) + ((2*(5*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(6*c^2*C*d - B*
c*d^2 - 4*A*d^3 - 11*c^3*D))*Sqrt[a + b*x])/(3*d^3*(b*c - a*d)*(c + d*x)^(
3/2)) + ((2*(15*a^2*d^2*(C*d - 3*c*D) - 10*a*b*d*(c*C*d + B*d^2 - 6*c^2*D)
 + b^2*(3*c^2*C*d + 2*B*c*d^2 + 8*A*d^3 - 23*c^3*D))*Sqrt[a + b*x])/((b*c
- a*d)*Sqrt[c + d*x]) + (30*(b*c - a*d)^2*D*ArcTanh[(Sqrt[d]*Sqrt[a + b*x]
)/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[d]))/(3*d^3*(b*c - a*d)))/(5*(b*
c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1728 vs. $2(266) = 532$.

Time = 0.60 (sec) , antiderivative size = 1729, normalized size of antiderivative = 5.84

method	result	size
default	Expression too large to display	1729

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/15*(b*x+a)^(1/2)*(-70*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^4*d*x
+80*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^4*d+45*D*ln(1/2*(2*b*d*x+2
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c*d^5*x^3
-8*A*a*b*d^5*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+40*A*b^2*c*d^4*x*((b*x+
a)*(d*x+c))^(1/2)*(d*b)^(1/2)-30*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2
*c^5+135*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*a^2*b*c^2*d^4*x^2-135*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(
1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^2*c^3*d^3*x^2+4*B*a^2*c*d^4*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+16*A*b^2*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*
(d*b)^(1/2)+30*C*a^2*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-15*D*ln(1
/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^
3*d^6*x^3+40*C*a^2*c*d^4*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+45*D*ln(1/2
*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^3*
c^5*d*x-20*C*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c*d^4*x^2+120*D*(d*b)
^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c^2*d^3*x^2-8*C*(d*b)^(1/2)*((b*x+a)*(d
*x+c))^(1/2)*a*b*c^2*d^3*x+188*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a*b*c
^3*d^2*x+4*B*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c*d^4*x^2+6*C*(d*b)^(
1/2)*((b*x+a)*(d*x+c))^(1/2)*b^2*c^2*d^3*x^2-52*B*a*b*c*d^4*x*((b*x+a)*(d*
x+c))^(1/2)*(d*b)^(1/2)+45*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b*c^4*d^2-45*D*ln(1/2*(2*b*d*x+2*((b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(267) = 534$.

Time = 29.80 (sec) , antiderivative size = 1584, normalized size of antiderivative = 5.35

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}(c + dx)^{7/2}}\, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(7/2),x, algorithm="fr
icas")
```

output
```
[1/30*(15*(D*b^3*c^6 - 3*D*a*b^2*c^5*d + 3*D*a^2*b*c^4*d^2 - D*a^3*c^3*d^3
 + (D*b^3*c^3*d^3 - 3*D*a*b^2*c^2*d^4 + 3*D*a^2*b*c*d^5 - D*a^3*d^6)*x^3 +
 3*(D*b^3*c^4*d^2 - 3*D*a*b^2*c^3*d^3 + 3*D*a^2*b*c^2*d^4 - D*a^3*c*d^5)*x
^2 + 3*(D*b^3*c^5*d - 3*D*a*b^2*c^4*d^2 + 3*D*a^2*b*c^3*d^3 - D*a^3*c^2*d^
4)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b
*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) - 4*(15*D*b^3*c^5*d - 40*D*a*b^2*c^4*d^2 + 33*D*a^2*b*c^3*d^3 - 3
*A*a^2*b*d^6 - (8*C*a^2*b - 10*B*a*b^2 + 15*A*b^3)*c^2*d^4 - 2*(B*a^2*b -
5*A*a*b^2)*c*d^5 + (23*D*b^3*c^3*d^3 - 3*(20*D*a*b^2 + C*b^3)*c^2*d^4 + (4
5*D*a^2*b + 10*C*a*b^2 - 2*B*b^3)*c*d^5 - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b
^3)*d^6)*x^2 + (35*D*b^3*c^4*d^2 - 94*D*a*b^2*c^3*d^3 + (75*D*a^2*b + 4*C*
a*b^2 - 5*B*b^3)*c^2*d^4 - 2*(10*C*a^2*b - 13*B*a*b^2 + 10*A*b^3)*c*d^5 -
(5*B*a^2*b - 4*A*a*b^2)*d^6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*c^6*d^4
- 3*a*b^3*c^5*d^5 + 3*a^2*b^2*c^4*d^6 - a^3*b*c^3*d^7 + (b^4*c^3*d^7 - 3*a
*b^3*c^2*d^8 + 3*a^2*b^2*c*d^9 - a^3*b*d^10)*x^3 + 3*(b^4*c^4*d^6 - 3*a*b^
3*c^3*d^7 + 3*a^2*b^2*c^2*d^8 - a^3*b*c*d^9)*x^2 + 3*(b^4*c^5*d^5 - 3*a*b^
3*c^4*d^6 + 3*a^2*b^2*c^3*d^7 - a^3*b*c^2*d^8)*x), -1/15*(15*(D*b^3*c^6 -
3*D*a*b^2*c^5*d + 3*D*a^2*b*c^4*d^2 - D*a^3*c^3*d^3 + (D*b^3*c^3*d^3 - 3*D
*a*b^2*c^2*d^4 + 3*D*a^2*b*c*d^5 - D*a^3*d^6)*x^3 + 3*(D*b^3*c^4*d^2 - 3*D
*a*b^2*c^3*d^3 + 3*D*a^2*b*c^2*d^4 - D*a^3*c*d^5)*x^2 + 3*(D*b^3*c^5*d ...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}\,(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(7/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x)*(c + d*x)**(7/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(7/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. $2(267) = 534$.

Time = 1.50 (sec) , antiderivative size = 746, normalized size of antiderivative = 2.52

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(7/2),x, algorithm="gi
ac")
```

output
```
-2/15*((b*x + a)*((23*D*b^8*c^3*d^4*abs(b) - 60*D*a*b^7*c^2*d^5*abs(b) - 3
*C*b^8*c^2*d^5*abs(b) + 45*D*a^2*b^6*c*d^6*abs(b) + 10*C*a*b^7*c*d^6*abs(b
) - 2*B*b^8*c*d^6*abs(b) - 15*C*a^2*b^6*d^7*abs(b) + 10*B*a*b^7*d^7*abs(b)
 - 8*A*b^8*d^7*abs(b))*(b*x + a)/(b^7*c^3*d^5 - 3*a*b^6*c^2*d^6 + 3*a^2*b^
5*c*d^7 - a^3*b^4*d^8) + 5*(7*D*b^9*c^4*d^3*abs(b) - 28*D*a*b^8*c^3*d^4*ab
s(b) + 39*D*a^2*b^7*c^2*d^5*abs(b) + 2*C*a*b^8*c^2*d^5*abs(b) - B*b^9*c^2*
d^5*abs(b) - 18*D*a^3*b^6*c*d^6*abs(b) - 8*C*a^2*b^7*c*d^6*abs(b) + 6*B*a*
b^8*c*d^6*abs(b) - 4*A*b^9*c*d^6*abs(b) + 6*C*a^3*b^6*d^7*abs(b) - 5*B*a^2
*b^7*d^7*abs(b) + 4*A*a*b^8*d^7*abs(b))/(b^7*c^3*d^5 - 3*a*b^6*c^2*d^6 + 3
*a^2*b^5*c*d^7 - a^3*b^4*d^8)) + 15*(D*b^10*c^5*d^2*abs(b) - 5*D*a*b^9*c^4
*d^3*abs(b) + 10*D*a^2*b^8*c^3*d^4*abs(b) - 9*D*a^3*b^7*c^2*d^5*abs(b) - C
*a^2*b^8*c^2*d^5*abs(b) + B*a*b^9*c^2*d^5*abs(b) - A*b^10*c^2*d^5*abs(b) +
 3*D*a^4*b^6*c*d^6*abs(b) + 2*C*a^3*b^7*c*d^6*abs(b) - 2*B*a^2*b^8*c*d^6*a
bs(b) + 2*A*a*b^9*c*d^6*abs(b) - C*a^4*b^6*d^7*abs(b) + B*a^3*b^7*d^7*abs(
b) - A*a^2*b^8*d^7*abs(b))/(b^7*c^3*d^5 - 3*a*b^6*c^2*d^6 + 3*a^2*b^5*c*d^
7 - a^3*b^4*d^8))*sqrt(b*x + a)/(b^2*c + (b*x + a)*b*d - a*b*d)^(5/2) - 2*
D*abs(b)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a
*b*d)))/(sqrt(b*d)*b*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{7/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(7/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(7/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{7/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(7/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(7/2),x)
```

3.137 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{9/2}} \, dx$

Optimal result 1315
Mathematica [A] (verified) 1316
Rubi [A] (verified) 1316
Maple [A] (verified) 1319
Fricas [B] (verification not implemented) 1320
Sympy [F] 1321
Maxima [F(-2)] 1321
Giac [B] (verification not implemented) 1322
Mupad [F(-1)] 1323
Reduce [F] 1323

Optimal result

Integrand size = 34, antiderivative size = 374

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{9/2}} \, dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{7d^3(bc-ad)(c+dx)^{7/2}}$$
$$+\frac{2(7ad(2cCd-Bd^2-3c^2D)-b(8c^2Cd-Bcd^2-6Ad^3-15c^3D))\sqrt{a+bx}}{35d^3(bc-ad)^2(c+dx)^{5/2}}$$
$$+\frac{2(35a^2d^2(Cd-3cD)-14abd(cCd+2Bd^2-9c^2D)+b^2(3c^2Cd+4Bcd^2+24Ad^3-45c^3D))\sqrt{a+bx}}{105d^3(bc-ad)^3(c+dx)^{3/2}}$$
$$-\frac{2(105a^3d^3D-35a^2bd^2(2Cd+3cD)+7ab^2d(4cCd+8Bd^2+9c^2D)-b^3(6c^2Cd+8Bcd^2+48Ad^3+15}{105d^3(bc-ad)^4\sqrt{c+dx}}$$

output
```
2/7*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d^3/(-a*d+b*c)/(d*x+c)^(7/
2)+2/35*(7*a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(-6*A*d^3-B*c*d^2+8*C*c^2*d-15*D
*c^3))*(b*x+a)^(1/2)/d^3/(-a*d+b*c)^2/(d*x+c)^(5/2)+2/105*(35*a^2*d^2*(C*d
-3*D*c)-14*a*b*d*(2*B*d^2+C*c*d-9*D*c^2)+b^2*(24*A*d^3+4*B*c*d^2+3*C*c^2*d
-45*D*c^3))*(b*x+a)^(1/2)/d^3/(-a*d+b*c)^3/(d*x+c)^(3/2)-2/105*(105*a^3*d^
3*D-35*a^2*b*d^2*(2*C*d+3*D*c)+7*a*b^2*d*(8*B*d^2+4*C*c*d+9*D*c^2)-b^3*(48
*A*d^3+8*B*c*d^2+6*C*c^2*d+15*D*c^3))*(b*x+a)^(1/2)/d^3/(-a*d+b*c)^4/(d*x+
c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{9/2}}\,dx = \frac{2\sqrt{a+bx}\left(105Ab^3-105ab^2B+105a^2bC-105a^3D-\frac{15c^2Cd(a+bx)^3}{(c+dx)^3}+\frac{15Bcd^2(a+bx)^3}{(c+dx)^3}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*(c + d*x)^(9/2)),x]
```

output
```
(2*Sqrt[a + b*x]*(105*A*b^3 - 105*a*b^2*B + 105*a^2*b*C - 105*a^3*D - (15*
c^2*C*d*(a + b*x)^3)/(c + d*x)^3 + (15*B*c*d^2*(a + b*x)^3)/(c + d*x)^3 -
(15*A*d^3*(a + b*x)^3)/(c + d*x)^3 + (15*c^3*D*(a + b*x)^3)/(c + d*x)^3 +
(21*b*c^2*C*(a + b*x)^2)/(c + d*x)^2 - (42*b*B*c*d*(a + b*x)^2)/(c + d*x)^
2 + (42*a*c*C*d*(a + b*x)^2)/(c + d*x)^2 + (63*A*b*d^2*(a + b*x)^2)/(c + d
*x)^2 - (21*a*B*d^2*(a + b*x)^2)/(c + d*x)^2 - (63*a*c^2*D*(a + b*x)^2)/(c
 + d*x)^2 + (35*b^2*B*c*(a + b*x))/(c + d*x) - (70*a*b*c*C*(a + b*x))/(c +
 d*x) - (105*A*b^2*d*(a + b*x))/(c + d*x) + (70*a*b*B*d*(a + b*x))/(c + d*
x) - (35*a^2*C*d*(a + b*x))/(c + d*x) + (105*a^2*c*D*(a + b*x))/(c + d*x))
)/(105*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2124, 27, 1193, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{9/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$\frac{2\int \frac{-7\left(a-\frac{bc}{d}\right)Dx^2+\frac{7(bc-ad)(Cd-cD)x}{d^2}+\frac{7ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-Dc^3+Cdc^2-Bd^2c-6Ad^3\right)}{d^3}}{2\sqrt{a+bx}(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-7\left(a-\frac{bc}{d}\right)Dx^2+\frac{7(bc-ad)(Cd-cD)x}{d^2}+\frac{7ad\left(-Dc^2+Cdc-Bd^2\right)-b\left(-Dc^3+Cdc^2-Bd^2c-6Ad^3\right)}{d^3}}{\sqrt{a+bx}(c+dx)^{7/2}}dx}{7(bc-ad)}+\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \ 1193$$

$$\frac{\frac{2\int \frac{\left(-10Dc^3+3Cdc^2+4Bd^2c+24Ad^3\right)b^2-14ad\left(-4Dc^2+Cdc+2Bd^2\right)b+35a^2d^2(Cd-2cD)+35d(bc-ad)^2Dx}{2d^3\sqrt{a+bx}(c+dx)^{5/2}}dx}{5(bc-ad)}+\frac{2\sqrt{a+bx}\left(7ad\left(-Bd^2-3c^2D+2cCd\right)-b\cdots\right.}{5d^3(c+dx)^{5/2}\cdots}}{7(bc-ad)}$$

$$\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(-10Dc^3+3Cdc^2+4Bd^2c+24Ad^3\right)b^2-14ad\left(-4Dc^2+Cdc+2Bd^2\right)b+35a^2d^2(Cd-2cD)+35d(bc-ad)^2Dx}{\sqrt{a+bx}(c+dx)^{5/2}}dx}{5d^3(bc-ad)}+\frac{2\sqrt{a+bx}\left(7ad\left(-Bd^2-3c^2D+2cCd\right)-b(\cdots\right.}{5d^3(c+dx)^{5/2}(\cdots}}{7(bc-ad)}$$

$$\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \ 87$$

$$\frac{\frac{\frac{2\sqrt{a+bx}\left(35a^2d^2(Cd-3cD)-14abd\left(2Bd^2-9c^2D+cCd\right)+b^2\left(24Ad^3+4Bcd^2-45c^3D+3c^2Cd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\left(105a^3d^3D-35a^2bd^2(3cD+2Cd)+7ab^2d\left(8Bd^2+9c^2D+4cCd\right)\cdots\right.}{3(b\cdots}}{5d^3(bc-ad)}}{7(bc-a\cdots}$$

$$\frac{2\sqrt{a+bx}\left(A+\frac{c\left(-Bd^2+c^2(-D)+cCd\right)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \ 48$$

$$\frac{\frac{\frac{2\sqrt{a+bx}\left(35a^2d^2(Cd-3cD)-14abd\left(2Bd^2-9c^2D+cCd\right)+b^2\left(24Ad^3+4Bcd^2-45c^3D+3c^2Cd\right)\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{2\sqrt{a+bx}\left(105a^3d^3D-35a^2bd^2(3cD+2Cd)+7ab^2d\left(8Bd^2+9c^2D\right.\right.}{3\sqrt{c+dx}(bc-ad}}{5d^3(bc-ad)}}{7(bc-ad)}$$

$$\frac{2\sqrt{a+bx}\left(A+\frac{c(-Bd^2+c^2(-D)+cCd)}{d^3}\right)}{7(c+dx)^{7/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[a + b*x]*(c + d*x)^(9/2)),x]
```

output
```
(2*(A + (c*(c*C*d - B*d^2 - c^2*D))/d^3)*Sqrt[a + b*x])/(7*(b*c - a*d)*(c
+ d*x)^(7/2)) + ((2*(7*a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(8*c^2*C*d - B*
c*d^2 - 6*A*d^3 - 15*c^3*D))*Sqrt[a + b*x])/(5*d^3*(b*c - a*d)*(c + d*x)^(
5/2)) + ((2*(35*a^2*d^2*(C*d - 3*c*D) - 14*a*b*d*(c*C*d + 2*B*d^2 - 9*c^2*
D) + b^2*(3*c^2*C*d + 4*B*c*d^2 + 24*A*d^3 - 45*c^3*D))*Sqrt[a + b*x])/(3*
(b*c - a*d)*(c + d*x)^(3/2)) - (2*(105*a^3*d^3*D - 35*a^2*b*d^2*(2*C*d + 3
*c*D) + 7*a*b^2*d*(4*c*C*d + 8*B*d^2 + 9*c^2*D) - b^3*(6*c^2*C*d + 8*B*c*d
^2 + 48*A*d^3 + 15*c^3*D))*Sqrt[a + b*x])/(3*(b*c - a*d)^2*Sqrt[c + d*x]))
/(5*d^3*(b*c - a*d)))/(7*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.47

method	result
default	$-\frac{2\sqrt{bx+a}\,(-48A\,b^3d^3x^3+56Ba\,b^2d^3x^3-8B\,b^3c\,d^2x^3-70C\,a^2b\,d^3x^3+28Ca\,b^2c\,d^2x^3-6C\,b^3c^2d\,x^3+105Da^3d^3x^3-105Da^2bc\,d^2x^{\cdots}}{}$
gosper	$-\frac{2\sqrt{bx+a}\,(-48A\,b^3d^3x^3+56Ba\,b^2d^3x^3-8B\,b^3c\,d^2x^3-70C\,a^2b\,d^3x^3+28Ca\,b^2c\,d^2x^3-6C\,b^3c^2d\,x^3+105Da^3d^3x^3-105Da^2bc\,d^2x^{\cdots}}{}$
orering	$-\frac{2\sqrt{bx+a}\,(-48A\,b^3d^3x^3+56Ba\,b^2d^3x^3-8B\,b^3c\,d^2x^3-70C\,a^2b\,d^3x^3+28Ca\,b^2c\,d^2x^3-6C\,b^3c^2d\,x^3+105Da^3d^3x^3-105Da^2bc\,d^2x^{\cdots}}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/105*(b*x+a)^(1/2)*(-48*A*b^3*d^3*x^3+56*B*a*b^2*d^3*x^3-8*B*b^3*c*d^2*x
^3-70*C*a^2*b*d^3*x^3+28*C*a*b^2*c*d^2*x^3-6*C*b^3*c^2*d*x^3+105*D*a^3*d^3
*x^3-105*D*a^2*b*c*d^2*x^3+63*D*a*b^2*c^2*d*x^3-15*D*b^3*c^3*x^3+24*A*a*b^
2*d^3*x^2-168*A*b^3*c*d^2*x^2-28*B*a^2*b*d^3*x^2+200*B*a*b^2*c*d^2*x^2-28*
B*b^3*c^2*d*x^2+35*C*a^3*d^3*x^2-259*C*a^2*b*c*d^2*x^2+101*C*a*b^2*c^2*d*x
^2-21*C*b^3*c^3*x^2+210*D*a^3*c*d^2*x^2-84*D*a^2*b*c^2*d*x^2+18*D*a*b^2*c^
3*x^2-18*A*a^2*b*d^3*x+84*A*a*b^2*c*d^2*x-210*A*b^3*c^2*d*x+21*B*a^3*d^3*x
-101*B*a^2*b*c*d^2*x+259*B*a*b^2*c^2*d*x-35*B*b^3*c^3*x+28*C*a^3*c*d^2*x-2
00*C*a^2*b*c^2*d*x+28*C*a*b^2*c^3*x+168*D*a^3*c^2*d*x-24*D*a^2*b*c^3*x+15*
A*a^3*d^3-63*A*a^2*b*c*d^2+105*A*a*b^2*c^2*d-105*A*b^3*c^3+6*B*a^3*c*d^2-2
8*B*a^2*b*c^2*d+70*B*a*b^2*c^3+8*C*a^3*c^2*d-56*C*a^2*b*c^3+48*D*a^3*c^3)/
(d*x+c)^(7/2)/(a*d-b*c)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(350) = 700$.

Time = 73.91 (sec) , antiderivative size = 712, normalized size of antiderivative = 1.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{9/2}}\,dx = -\frac{2\,(15\,Aa^3d^3+(48\,Da^3-56\,Ca^2b+70\,Bab^2-105\,Ab^3)c^3+(8\,Ca^3-28\,Ba^2b+105\,Aab^2)c^2d+3\,(2\,Ba^3}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(9/2),x, algorithm="fr
icas")
```

output
```
-2/105*(15*A*a^3*d^3 + (48*D*a^3 - 56*C*a^2*b + 70*B*a*b^2 - 105*A*b^3)*c^
3 + (8*C*a^3 - 28*B*a^2*b + 105*A*a*b^2)*c^2*d + 3*(2*B*a^3 - 21*A*a^2*b)*
c*d^2 - (15*D*b^3*c^3 - 3*(21*D*a*b^2 - 2*C*b^3)*c^2*d + (105*D*a^2*b - 28
*C*a*b^2 + 8*B*b^3)*c*d^2 - (105*D*a^3 - 70*C*a^2*b + 56*B*a*b^2 - 48*A*b^
3)*d^3)*x^3 + (3*(6*D*a*b^2 - 7*C*b^3)*c^3 - (84*D*a^2*b - 101*C*a*b^2 + 2
8*B*b^3)*c^2*d + (210*D*a^3 - 259*C*a^2*b + 200*B*a*b^2 - 168*A*b^3)*c*d^2
 + (35*C*a^3 - 28*B*a^2*b + 24*A*a*b^2)*d^3)*x^2 - ((24*D*a^2*b - 28*C*a*b
^2 + 35*B*b^3)*c^3 - (168*D*a^3 - 200*C*a^2*b + 259*B*a*b^2 - 210*A*b^3)*c
^2*d - (28*C*a^3 - 101*B*a^2*b + 84*A*a*b^2)*c*d^2 - 3*(7*B*a^3 - 6*A*a^2*
b)*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(b^4*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^
2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (b^4*c^4*d^4 - 4*a*b^3*c^3*d^5
 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*x^4 + 4*(b^4*c^5*d^3 - 4*a
*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2*d^6 + a^4*c*d^7)*x^3 + 6*(b
^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^4 - 4*a^3*b*c^3*d^5 + a^4*c
^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6*a^2*b^2*c^5*d^3 - 4*a^3*b
*c^4*d^4 + a^4*c^3*d^5)*x)
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}(c + dx)^{9/2}}\, dx = \int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}\,(c + dx)^{\frac{9}{2}}}\, dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(9/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x)*(c + d*x)**(9/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}(c + dx)^{9/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(9/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1086 vs. $2(350) = 700$.

Time = 0.33 (sec) , antiderivative size = 1086, normalized size of antiderivative = 2.90

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{a + bx}(c + dx)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(9/2),x, algorithm="gi
ac")
```

output
```
2/105*(((b*x + a)*((15*D*b^10*c^3*d^3*abs(b) - 63*D*a*b^9*c^2*d^4*abs(b) +
 6*C*b^10*c^2*d^4*abs(b) + 105*D*a^2*b^8*c*d^5*abs(b) - 28*C*a*b^9*c*d^5*a
bs(b) + 8*B*b^10*c*d^5*abs(b) - 105*D*a^3*b^7*d^6*abs(b) + 70*C*a^2*b^8*d^
6*abs(b) - 56*B*a*b^9*d^6*abs(b) + 48*A*b^10*d^6*abs(b))*(b*x + a)/(b^8*c^
4*d^3 - 4*a*b^7*c^3*d^4 + 6*a^2*b^6*c^2*d^5 - 4*a^3*b^5*c*d^6 + a^4*b^4*d^
7) - 7*(9*D*a*b^10*c^3*d^3*abs(b) - 3*C*b^11*c^3*d^3*abs(b) - 39*D*a^2*b^9
*c^2*d^4*abs(b) + 17*C*a*b^10*c^2*d^4*abs(b) - 4*B*b^11*c^2*d^4*abs(b) + 7
5*D*a^3*b^8*c*d^5*abs(b) - 49*C*a^2*b^9*c*d^5*abs(b) + 32*B*a*b^10*c*d^5*a
bs(b) - 24*A*b^11*c*d^5*abs(b) - 45*D*a^4*b^7*d^6*abs(b) + 35*C*a^3*b^8*d^
6*abs(b) - 28*B*a^2*b^9*d^6*abs(b) + 24*A*a*b^10*d^6*abs(b))/(b^8*c^4*d^3
- 4*a*b^7*c^3*d^4 + 6*a^2*b^6*c^2*d^5 - 4*a^3*b^5*c*d^6 + a^4*b^4*d^7)) +
35*(3*D*a^2*b^10*c^3*d^3*abs(b) - 2*C*a*b^11*c^3*d^3*abs(b) + B*b^12*c^3*d
^3*abs(b) - 15*D*a^3*b^9*c^2*d^4*abs(b) + 12*C*a^2*b^10*c^2*d^4*abs(b) - 9
*B*a*b^11*c^2*d^4*abs(b) + 6*A*b^12*c^2*d^4*abs(b) + 21*D*a^4*b^8*c*d^5*ab
s(b) - 18*C*a^3*b^9*c*d^5*abs(b) + 15*B*a^2*b^10*c*d^5*abs(b) - 12*A*a*b^1
1*c*d^5*abs(b) - 9*D*a^5*b^7*d^6*abs(b) + 8*C*a^4*b^8*d^6*abs(b) - 7*B*a^3
*b^9*d^6*abs(b) + 6*A*a^2*b^10*d^6*abs(b))/(b^8*c^4*d^3 - 4*a*b^7*c^3*d^4
+ 6*a^2*b^6*c^2*d^5 - 4*a^3*b^5*c*d^6 + a^4*b^4*d^7))*(b*x + a) - 105*(D*a
^3*b^10*c^3*d^3*abs(b) - C*a^2*b^11*c^3*d^3*abs(b) + B*a*b^12*c^3*d^3*abs(
b) - A*b^13*c^3*d^3*abs(b) - 3*D*a^4*b^9*c^2*d^4*abs(b) + 3*C*a^3*b^10*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{9/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{9/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(9/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(1/2)*(c + d*x)^(9/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx}(c+dx)^{9/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{\sqrt{bx+a}\,\left(dx+c\right)^{\frac{9}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(9/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(9/2),x)
```

3.138 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx$

Optimal result 1324
Mathematica [A] (verified) 1325
Rubi [A] (verified) 1326
Maple [B] (verified) 1331
Fricas [A] (verification not implemented) 1332
Sympy [F] 1333
Maxima [F(-2)] 1334
Giac [B] (verification not implemented) 1334
Mupad [F(-1)] 1335
Reduce [F] 1336

Optimal result

Integrand size = 34, antiderivative size = 562

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx =$$

$$-\frac{(bc-ad)\left(693a^3d^3D-63a^2bd^2(10Cd+3cD)+7ab^2d(20cCd+80Bd^2-3c^2D)+b^3(10c^2Cd-80Bcd^2\right.}{128b^6d^2}$$

$$-\frac{(693a^3d^3D-63a^2bd^2(10Cd+3cD)+7ab^2d(20cCd+80Bd^2-3c^2D)+b^3(10c^2Cd-80Bcd^2-480Ad^3}{192b^5d^2}$$

$$-\frac{2(Ab^3-a(b^2B-abC+a^2D))\,(c+dx)^{5/2}}{b^4\sqrt{a+bx}}$$

$$+\frac{(75a^2d^2D-6abd(7Cd+4cD)+b^2(10cCd+16Bd^2-3c^2D))\,\sqrt{a+bx}(c+dx)^{5/2}}{48b^4d^2}$$

$$+\frac{(2bCd+bcD-7adD)(a+bx)^{3/2}(c+dx)^{5/2}}{8b^4d}+\frac{D(a+bx)^{5/2}(c+dx)^{5/2}}{5b^4}$$

$$-\frac{(bc-ad)^2\left(693a^3d^3D-63a^2bd^2(10Cd+3cD)+7ab^2d(20cCd+80Bd^2-3c^2D)+b^3(10c^2Cd-80Bcd\right.}{128b^{13/2}d^{5/2}}$$

output
```
-1/128*(-a*d+b*c)*(693*a^3*d^3*D-63*a^2*b*d^2*(10*C*d+3*D*c)+7*a*b^2*d*(80
*B*d^2+20*C*c*d-3*D*c^2)+b^3*(-480*A*d^3-80*B*c*d^2+10*C*c^2*d-3*D*c^3))*(
b*x+a)^(1/2)*(d*x+c)^(1/2)/b^6/d^2-1/192*(693*a^3*d^3*D-63*a^2*b*d^2*(10*C
*d+3*D*c)+7*a*b^2*d*(80*B*d^2+20*C*c*d-3*D*c^2)+b^3*(-480*A*d^3-80*B*c*d^2
+10*C*c^2*d-3*D*c^3))*(b*x+a)^(1/2)*(d*x+c)^(3/2)/b^5/d^2-2*(A*b^3-a*(B*b^
2-C*a*b+D*a^2))*(d*x+c)^(5/2)/b^4/(b*x+a)^(1/2)+1/48*(75*a^2*d^2*D-6*a*b*d
*(7*C*d+4*D*c)+b^2*(16*B*d^2+10*C*c*d-3*D*c^2))*(b*x+a)^(1/2)*(d*x+c)^(5/2
)/b^4/d^2+1/8*(2*C*b*d-7*D*a*d+D*b*c)*(b*x+a)^(3/2)*(d*x+c)^(5/2)/b^4/d+1/
5*D*(b*x+a)^(5/2)*(d*x+c)^(5/2)/b^4-1/128*(-a*d+b*c)^2*(693*a^3*d^3*D-63*a
^2*b*d^2*(10*C*d+3*D*c)+7*a*b^2*d*(80*B*d^2+20*C*c*d-3*D*c^2)+b^3*(-480*A*
d^3-80*B*c*d^2+10*C*c^2*d-3*D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/
(d*x+c)^(1/2))/b^(13/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 543, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \frac{\sqrt{c+dx}(10395a^5d^4D-315a^4bd^3(30Cd+64cD-11dDx)+$$

$$+\frac{(bc-ad)^2(-693a^3d^3D+63a^2bd^2(10Cd+3cD)-7ab^2d(20cCd+80Bd^2-3c^2D)+b^3(-10c^2Cd+80B}{128b^{13/2}d^{5/2}}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(3/2),x]
```

output
```
(Sqrt[c + d*x]*(10395*a^5*d^4*D - 315*a^4*b*d^3*(30*C*d + 64*c*D - 11*d*D*
x) + 42*a^3*b^2*d^2*(237*c^2*D + c*d*(425*C - 171*D*x) + d^2*(200*B - 3*x*
(25*C + 11*D*x))) - a*b^4*(45*c^4*D - 30*c^3*d*(5*C - 7*D*x) + 2*c^2*d^2*(
-3240*B + x*(1685*C + 699*D*x)) + 16*d^4*x*(150*A + x*(70*B + 45*C*x + 33*
D*x^2)) - 8*c*d^3*(1500*A - x*(680*B + 305*C*x + 192*D*x^2))) + 2*a^2*b^3*
d*(-120*c^3*D + c^2*(-4195*C*d + 1986*d*D*x) + c*d^2*(-7600*B + x*(3185*C
+ 1377*D*x)) + 2*d^3*(-1800*A + x*(700*B + 9*x*(35*C + 22*D*x)))) + b^5*(4
80*A*d^2*(-8*c^2 + 9*c*d*x + 2*d^2*x^2) + x*(-45*c^4*D + 30*c^3*d*(5*C + D
*x) + 32*d^4*x^2*(20*B + 3*x*(5*C + 4*D*x)) + 16*c*d^3*x*(130*B + x*(85*C
+ 63*D*x)) + 4*c^2*d^2*(660*B + x*(295*C + 186*D*x))))))/(1920*b^6*d^2*Sqr
t[a + b*x]) + ((b*c - a*d)^2*(-693*a^3*d^3*D + 63*a^2*b*d^2*(10*C*d + 3*c*
D) - 7*a*b^2*d*(20*c*C*d + 80*B*d^2 - 3*c^2*D) + b^3*(-10*c^2*C*d + 80*B*c
*d^2 + 480*A*d^3 + 3*c^3*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt
[a + b*x])])/(128*b^(13/2)*d^(5/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.74, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2124, 27, 1194, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2} \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}} \, dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{(c+dx)^{5/2}\left(\left(c-\frac{ad}{b}\right)Dx^2+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{-7dDa^3+b(7Cd+cD)a^2-b^2(cC+7Bd)a+b^3(Bc+6Ad)}{b^3}\right)}{2\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^{5/2}\left(-\frac{7dDa^3}{b^3}+\frac{(7Cd+cD)a^2}{b^2}-\frac{(cC+7Bd)a}{b}+\left(c-\frac{ad}{b}\right)Dx^2+Bc+6Ad+\frac{(bc-ad)(bC-aD)x}{b^2}\right)}{\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 1194

$$\frac{\frac{\int \frac{(c+dx)^{5/2}\left(-\frac{63d^2Da^3}{b}+2d(35Cd+3cD)a^2-b\left(3Dc^2+10Cdc+70Bd^2\right)a+10b^2d(Bc+6Ad)+(bc-ad)(10bCd-27aDd-3bcD)x\right)}{2\sqrt{a+bx}}dx}{5b^2d}+\frac{D(a+bx)^{3/2}(c+dx)^{7/}}{5b^3d}}{bc-ad}-\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{(c+dx)^{5/2}\left(-\frac{63d^2Da^3}{b}+2d(35Cd+3cD)a^2-b\left(3Dc^2+10Cdc+70Bd^2\right)a+10b^2d(Bc+6Ad)+(bc-ad)(10bCd-27aDd-3bcD)x\right)}{\sqrt{a+bx}}dx}{10b^2d}+\frac{D(a+bx)^{3/2}(c+dx)^{7/}}{5b^3d}}{bc-ad}-\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 90

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)(-27adD-3bcD+10bCd)}{4bd}-\frac{\left(693a^3d^3D-63a^2bd^2(3cD+10Cd)+7ab^2d\left(80Bd^2-3c^2D+20cCd\right)+b^3\left(-480Ad^3-80Bcd^2-3c^3D+10c^2Cd\right.}{8bd}}{10b^2d}}{bc-ad}-\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)(-27adD-3bcD+10bCd)}{4bd}-\frac{\left(693a^3d^3D-63a^2bd^2(3cD+10Cd)+7ab^2d\left(80Bd^2-3c^2D+20cCd\right)+b^3\left(-480Ad^3-80Bcd^2-3c^3D+10c^2Cd\right.}{8bd}}{10b^2d}}{bc-ad}-\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)(-27adD-3bcD+10bCd)}{4bd}-\frac{\frac{\left(693a^3d^3D-63a^2bd^2(3cD+10Cd)+7ab^2d\left(80Bd^2-3c^2D+20cCd\right)+b^3\left(-480Ad^3-80Bcd^2-3c^3D+10c^2Cd\right.\right.}{8bd}}{10b^2d}}{bc-ad}$$

$$\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)(-27adD-3bcD+10bCd)}{4bd}-\frac{\left(693a^3d^3D-63a^2bd^2(3cD+10Cd)+7ab^2d\left(80Bd^2-3c^2D+20cCd\right)+b^3\left(-480Ad^3-80Bcd^2-3c^3D+10c^2Cd\right.\right.}{10b^2d}}{bc-}$$

$$\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 66

$$\frac{\frac{\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)(-27adD-3bcD+10bCd)}{4bd}-\frac{\left(693a^3d^3D-63a^2bd^2(3cD+10Cd)+7ab^2d\left(80Bd^2-3c^2D+20cCd\right)+b^3\left(-480Ad^3-80Bcd^2-3c^3D+10c^2Cd\right.\right.}{10b^2d}}{bc}$$

$$\frac{2(c+dx)^{7/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)(-27adD-3bcD+10bCd)}{4bd}-\frac{\left(\frac{5(bc-ad)\left(\frac{3(bc-ad)\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4b}+\frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b}\right)}{6b}+\frac{\sqrt{a+\cdots}}{\cdots}\right)\cdots}{10b^2d}\cdots}{bc\cdots}$$

$$\frac{2(c+dx)^{7/2}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

input
```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(3/2),x]
```

output
```
(-2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(7/2))/((b*c - a*d)*Sq
rt[a + b*x]) + (((b*c - a*d)*D*(a + b*x)^(3/2)*(c + d*x)^(7/2))/(5*b^3*d)
+ (((b*c - a*d)*(10*b*C*d - 3*b*c*D - 27*a*d*D)*Sqrt[a + b*x]*(c + d*x)^(7
/2))/(4*b*d) - ((693*a^3*d^3*D - 63*a^2*b*d^2*(10*C*d + 3*c*D) + 7*a*b^2*d
*(20*c*C*d + 80*B*d^2 - 3*c^2*D) + b^3*(10*c^2*C*d - 80*B*c*d^2 - 480*A*d^
3 - 3*c^3*D))*((Sqrt[a + b*x]*(c + d*x)^(5/2))/(3*b) + (5*(b*c - a*d)*((Sq
rt[a + b*x]*(c + d*x)^(3/2))/(2*b) + (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c
 + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
+ d*x])])/(b^(3/2)*Sqrt[d])))/(4*b)))/(6*b)))/(8*b*d))/(10*b^2*d))/(b*c -
a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3459 vs. 2(514) = 1028.

Time = 0.54 (sec) , antiderivative size = 3460, normalized size of antiderivative = 6.16

method	result	size
default	Expression too large to display	3460

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/3840*(d*x+c)^(1/2)*(768*D*b^5*d^4*x^5*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)+960*C*b^5*d^4*x^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1280*B*b^5*d^4*x^3
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+12740*C*a^2*b^3*c*d^3*x*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)-6740*C*a*b^4*c^2*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b
)^(1/2)-14364*D*a^3*b^2*c*d^3*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1920*A
*b^5*d^4*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-90*D*b^5*c^4*x*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)-14400*A*a^2*b^3*d^4*((b*x+a)*(d*x+c))^(1/2)*(d*b
)^(1/2)-7680*A*b^5*c^2*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+16800*B*a^3
*b^2*d^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-18900*C*a^4*b*d^4*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)-90*D*a*b^4*c^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)+2250*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d
*b)^(1/2))*a^3*b^3*c^3*d^2+225*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^4*c^4*d+7200*A*ln(1/2*(2*b*d*x+2*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^4*d^5*x+72
00*A*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^
(1/2))*b^6*c^2*d^3*x-8400*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b
)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^3*d^5*x+1200*B*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^6*c^3*d^2*x+9450*C*
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2)
)*a^4*b^2*d^5*x-150*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(...
```

Fricas [A] (verification not implemented)

Time = 6.76 (sec) , antiderivative size = 2028, normalized size of antiderivative = 3.61

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="fr
icas")
```

output
```
[1/7680*(15*(3*D*a*b^5*c^5 + 5*(3*D*a^2*b^4 - 2*C*a*b^5)*c^4*d + 10*(15*D*
a^3*b^3 - 12*C*a^2*b^4 + 8*B*a*b^5)*c^3*d^2 - 30*(35*D*a^4*b^2 - 30*C*a^3*
b^3 + 24*B*a^2*b^4 - 16*A*a*b^5)*c^2*d^3 + 5*(315*D*a^5*b - 280*C*a^4*b^2
+ 240*B*a^3*b^3 - 192*A*a^2*b^4)*c*d^4 - (693*D*a^6 - 630*C*a^5*b + 560*B*
a^4*b^2 - 480*A*a^3*b^3)*d^5 + (3*D*b^6*c^5 + 5*(3*D*a*b^5 - 2*C*b^6)*c^4*
d + 10*(15*D*a^2*b^4 - 12*C*a*b^5 + 8*B*b^6)*c^3*d^2 - 30*(35*D*a^3*b^3 -
30*C*a^2*b^4 + 24*B*a*b^5 - 16*A*b^6)*c^2*d^3 + 5*(315*D*a^4*b^2 - 280*C*a
^3*b^3 + 240*B*a^2*b^4 - 192*A*a*b^5)*c*d^4 - (693*D*a^5*b - 630*C*a^4*b^2
 + 560*B*a^3*b^3 - 480*A*a^2*b^4)*d^5)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^
2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x +
 a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(384*D*b^6*d^5*x^5 - 45*D
*a*b^5*c^4*d - 30*(8*D*a^2*b^4 - 5*C*a*b^5)*c^3*d^2 + 2*(4977*D*a^3*b^3 -
4195*C*a^2*b^4 + 3240*B*a*b^5 - 1920*A*b^6)*c^2*d^3 - 10*(2016*D*a^4*b^2 -
 1785*C*a^3*b^3 + 1520*B*a^2*b^4 - 1200*A*a*b^5)*c*d^4 + 15*(693*D*a^5*b -
 630*C*a^4*b^2 + 560*B*a^3*b^3 - 480*A*a^2*b^4)*d^5 + 48*(21*D*b^6*c*d^4 -
 (11*D*a*b^5 - 10*C*b^6)*d^5)*x^4 + 8*(93*D*b^6*c^2*d^3 - 2*(96*D*a*b^5 -
85*C*b^6)*c*d^4 + (99*D*a^2*b^4 - 90*C*a*b^5 + 80*B*b^6)*d^5)*x^3 + 2*(15*
D*b^6*c^3*d^2 - (699*D*a*b^5 - 590*C*b^6)*c^2*d^3 + (1377*D*a^2*b^4 - 1220
*C*a*b^5 + 1040*B*b^6)*c*d^4 - (693*D*a^3*b^3 - 630*C*a^2*b^4 + 560*B*a*b^
5 - 480*A*b^6)*d^5)*x^2 - (45*D*b^6*c^4*d + 30*(7*D*a*b^5 - 5*C*b^6)*c^...
```

Sympy [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx=\int \frac{(c+dx)^{\frac{5}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Integral((c + d*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(a + b*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(518) = 1036$.

Time = 0.81 (sec) , antiderivative size = 1146, normalized size of antiderivative = 2.04

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="gi
ac")
```

output
```
1/1920*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8
*(b*x + a)*D*d^2*abs(b)/b^8 + (21*D*b^40*c*d^9*abs(b) - 51*D*a*b^39*d^10*a
bs(b) + 10*C*b^40*d^10*abs(b))/(b^47*d^8)) + (93*D*b^41*c^2*d^8*abs(b) - 6
96*D*a*b^40*c*d^9*abs(b) + 170*C*b^41*c*d^9*abs(b) + 843*D*a^2*b^39*d^10*a
bs(b) - 330*C*a*b^40*d^10*abs(b) + 80*B*b^41*d^10*abs(b))/(b^47*d^8)) + 5*
(3*D*b^42*c^3*d^7*abs(b) - 363*D*a*b^41*c^2*d^8*abs(b) + 118*C*b^42*c^2*d^
8*abs(b) + 1341*D*a^2*b^40*c*d^9*abs(b) - 652*C*a*b^41*c*d^9*abs(b) + 208*
B*b^42*c*d^9*abs(b) - 1077*D*a^3*b^39*d^10*abs(b) + 630*C*a^2*b^40*d^10*ab
s(b) - 304*B*a*b^41*d^10*abs(b) + 96*A*b^42*d^10*abs(b))/(b^47*d^8))*(b*x
+ a) - 15*(3*D*b^43*c^4*d^6*abs(b) + 18*D*a*b^42*c^3*d^7*abs(b) - 10*C*b^4
3*c^3*d^7*abs(b) - 600*D*a^2*b^41*c^2*d^8*abs(b) + 382*C*a*b^42*c^2*d^8*ab
s(b) - 176*B*b^43*c^2*d^8*abs(b) + 1422*D*a^3*b^40*c*d^9*abs(b) - 1022*C*a
^2*b^41*c*d^9*abs(b) + 640*B*a*b^42*c*d^9*abs(b) - 288*A*b^43*c*d^9*abs(b)
 - 843*D*a^4*b^39*d^10*abs(b) + 650*C*a^3*b^40*d^10*abs(b) - 464*B*a^2*b^4
1*d^10*abs(b) + 288*A*a*b^42*d^10*abs(b))/(b^47*d^8))*sqrt(b*x + a) + 4*(D
*a^3*b^3*c^3*d*abs(b) - C*a^2*b^4*c^3*d*abs(b) + B*a*b^5*c^3*d*abs(b) - A*
b^6*c^3*d*abs(b) - 3*D*a^4*b^2*c^2*d^2*abs(b) + 3*C*a^3*b^3*c^2*d^2*abs(b)
 - 3*B*a^2*b^4*c^2*d^2*abs(b) + 3*A*a*b^5*c^2*d^2*abs(b) + 3*D*a^5*b*c*d^3
*abs(b) - 3*C*a^4*b^2*c*d^3*abs(b) + 3*B*a^3*b^3*c*d^3*abs(b) - 3*A*a^2*b^
4*c*d^3*abs(b) - D*a^6*d^4*abs(b) + C*a^5*b*d^4*abs(b) - B*a^4*b^2*d^4*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx=\int \frac{(c+d\,x)^{5/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^{3/2}}\,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(3/2),x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx = \int \frac{(dx+c)^{\frac{5}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(bx+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

output
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

3.139 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx$

Optimal result 1337
Mathematica [C] (verified) 1338
Rubi [A] (verified) 1339
Maple [B] (verified) 1343
Fricas [A] (verification not implemented) 1344
Sympy [F] 1345
Maxima [F(-2)] 1346
Giac [A] (verification not implemented) 1346
Mupad [F(-1)] 1347
Reduce [F] 1348

Optimal result

Integrand size = 34, antiderivative size = 441

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = -\frac{(315a^3d^3D-35a^2bd^2(8Cd+3cD)+5ab^2d(16cCd+48Bd^2-3c^2D)+b^3(8c^2Cd-48Bcd^2-192Ad^3-}{64b^5d^2}$$
$$-\frac{2(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{3/2}}{b^4\sqrt{a+bx}}$$
$$+\frac{\left(\frac{69a^2dD}{b}-2a(20Cd+9cD)+b\left(8cC+16Bd-\frac{3c^2D}{d}\right)\right)\sqrt{a+bx}(c+dx)^{3/2}}{32b^3d}$$
$$+\frac{(8bCd-3bcD-21adD)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^4d}+\frac{D(a+bx)^{3/2}(c+dx)^{5/2}}{4b^3d}$$
$$-\frac{(bc-ad)(315a^3d^3D-35a^2bd^2(8Cd+3cD)+5ab^2d(16cCd+48Bd^2-3c^2D)+b^3(8c^2Cd-48Bcd^2-}{64b^{11/2}d^{5/2}}$$

output
```
-1/64*(315*a^3*d^3*D-35*a^2*b*d^2*(8*C*d+3*D*c)+5*a*b^2*d*(48*B*d^2+16*C*c
*d-3*D*c^2)+b^3*(-192*A*d^3-48*B*c*d^2+8*C*c^2*d-3*D*c^3))*(b*x+a)^(1/2)*(
d*x+c)^(1/2)/b^5/d^2-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(3/2)/b^4/(b*
x+a)^(1/2)+1/32*(69*a^2*d*D/b-2*a*(20*C*d+9*D*c)+b*(8*C*c+16*B*d-3*c^2*D/d
))*(b*x+a)^(1/2)*(d*x+c)^(3/2)/b^3/d+1/24*(8*C*b*d-21*D*a*d-3*D*b*c)*(b*x+
a)^(3/2)*(d*x+c)^(3/2)/b^4/d+1/4*D*(b*x+a)^(3/2)*(d*x+c)^(5/2)/b^3/d-1/64*
(-a*d+b*c)*(315*a^3*d^3*D-35*a^2*b*d^2*(8*C*d+3*D*c)+5*a*b^2*d*(48*B*d^2+1
6*C*c*d-3*D*c^2)+b^3*(-192*A*d^3-48*B*c*d^2+8*C*c^2*d-3*D*c^3))*arctanh(d^
(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(11/2)/d^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \frac{2(c+dx)^{3/2}\left(-(bc-ad)^3 D\,\text{Hypergeometric2F1}\left(-\frac{9}{2},-\frac{1}{2},\frac{1}{2},\right.\right.}{}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(3/2),x]
```

output
```
(2*(c + d*x)^(3/2)*(-((b*c - a*d)^3*D*Hypergeometric2F1[-9/2, -1/2, 1/2, (
d*(a + b*x))/(-(b*c) + a*d)]) + b*(b*c - a*d)^2*(-(C*d) + 3*c*D)*Hypergeom
etric2F1[-7/2, -1/2, 1/2, (d*(a + b*x))/(-(b*c) + a*d)] - b^2*(b*c - a*d)*
(-2*c*C*d + B*d^2 + 3*c^2*D)*Hypergeometric2F1[-5/2, -1/2, 1/2, (d*(a + b*
x))/(-(b*c) + a*d)] + b^3*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*Hypergeom
etric2F1[-3/2, -1/2, 1/2, (d*(a + b*x))/(-(b*c) + a*d)]))/(b^4*d^3*Sqrt[a
+ b*x]*((b*(c + d*x))/(b*c - a*d))^(3/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2124, 27, 1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{(c+dx)^{3/2}\left(\left(c-\frac{ad}{b}\right)Dx^2+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{-5dDa^3+b(5Cd+cD)a^2-b^2(cC+5Bd)a+b^3(Bc+4Ad)}{b^3}\right)}{2\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^{3/2}\left(-\frac{5dDa^3}{b^3}+\frac{(5Cd+cD)a^2}{b^2}-\frac{(cC+5Bd)a}{b}+\left(c-\frac{ad}{b}\right)Dx^2+Bc+4Ad+\frac{(bc-ad)(bC-aD)x}{b^2}\right)}{\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \text{1194}$$

$$\frac{\frac{\int \frac{(c+dx)^{3/2}\left(-\frac{35d^2Da^3}{b}+2d(20Cd+3cD)a^2-b\left(3Dc^2+8Cdc+40Bd^2\right)a+8b^2d(Bc+4Ad)+(bc-ad)(8bCd-21aDd-3bcD)x\right)}{2\sqrt{a+bx}}dx}{4b^2d}+\frac{D(a+bx)^{3/2}(c+dx)^{5/2}(b}{4b^3d}}{bc-ad}-\frac{2(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(c+dx)^{3/2}\left(-\frac{35d^2Da^3}{b}+2d(20Cd+3cD)a^2-b\left(3Dc^2+8Cdc+40Bd^2\right)a+8b^2d(Bc+4Ad)+(bc-ad)(8bCd-21aDd-3bcD)x\right)}{\sqrt{a+bx}}dx}{8b^2d}+\frac{D(a+bx)^{3/2}(c+dx)^{5/2}(b}{4b^3d}}{bc-ad}$$

$$\frac{2(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 90

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)(-21adD-3bcD+8bCd)}{3bd}-\frac{\left(315a^3d^3D-35a^2bd^2(3cD+8Cd)+5ab^2d\left(48Bd^2-3c^2D+16cCd\right)+b^3\left(-192Ad^3-48Bcd^2-3c^3D+8c^2Cd\right)\right)}{6bd}}{8b^2d}}{bc-ad}$$

$$\frac{2(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)(-21adD-3bcD+8bCd)}{3bd}-\frac{\left(315a^3d^3D-35a^2bd^2(3cD+8Cd)+5ab^2d\left(48Bd^2-3c^2D+16cCd\right)+b^3\left(-192Ad^3-48Bcd^2-3c^3D+8c^2Cd\right)\right)}{6bd}}{8b^2d}}{bc-ad}$$

$$\frac{2(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)(-21adD-3bcD+8bCd)}{3bd}-\frac{\left(315a^3d^3D-35a^2bd^2(3cD+8Cd)+5ab^2d\left(48Bd^2-3c^2D+16cCd\right)+b^3\left(-192Ad^3-48Bcd^2-3c^3D+8c^2Cd\right)\right)}{6bd}}{8b^2d}}{bc-ad}$$

$$\frac{2(c+dx)^{5/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 66

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)(-21adD-3bcD+8bCd)}{3bd}-\frac{\left(315a^3d^3D-35a^2bd^2(3cD+8Cd)+5ab^2d\left(48Bd^2-3c^2D+16cCd\right)+b^3\left(-192Ad^3-48Bcd^2-3c^3D+8c^2Cd\right)\right)}{6bd}}{8b^2d}}{bc-ad}$$

$$\frac{2(c+dx)^{5/2}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)(-21adD-3bcD+8bCd)}{3bd}-\frac{\left(\frac{3(bc-ad)\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4b}+\frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b}\right)\left(315a^3d^3D-35a^2b\right.}{6bd}}{8b^2d}}{bc-ad}$$

$$\frac{2(c+dx)^{5/2}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

input
```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(3/2),x]
```

output
```
(-2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(5/2))/((b*c - a*d)*Sq
rt[a + b*x]) + (((b*c - a*d)*D*(a + b*x)^(3/2)*(c + d*x)^(5/2))/(4*b^3*d)
+ (((b*c - a*d)*(8*b*C*d - 3*b*c*D - 21*a*d*D)*Sqrt[a + b*x]*(c + d*x)^(5/
2))/(3*b*d) - ((315*a^3*d^3*D - 35*a^2*b*d^2*(8*C*d + 3*c*D) + 5*a*b^2*d*(
16*c*C*d + 48*B*d^2 - 3*c^2*D) + b^3*(8*c^2*C*d - 48*B*c*d^2 - 192*A*d^3 -
 3*c^3*D))*((Sqrt[a + b*x]*(c + d*x)^(3/2))/(2*b) + (3*(b*c - a*d)*((Sqrt[
a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(
Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])))/(4*b)))/(6*b*d))/(8*b^2*d))/(
b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2427 vs. $2(399) = 798$.

Time = 0.51 (sec) , antiderivative size = 2428, normalized size of antiderivative = 5.51

method	result	size
default	Expression too large to display	2428

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/384*(d*x+c)^(1/2)*(264*D*a*b^3*c*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+24*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*b^5*c^3*d*x-945*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*d^4*x-576*A*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c*d^3+1890*D*a^
4*d^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^4*x+576*A*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*
d^4-720*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(
d*b)^(1/2))*a^3*b^2*d^4+840*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*d^4-9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(
d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^4+864*B*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*
c*d^3-144*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)
/(d*b)^(1/2))*a*b^4*c^2*d^2-1080*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2*c*d^3+216*C*ln(1/2*(2*b*d*x+2
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^2*d^2
+24*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)
^(1/2))*a*b^4*c^3*d+1260*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*c*d^3-270*D*ln(1/2*(2*b*d*x+2*((b*x+...
```

Fricas [A] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 1454, normalized size of antiderivative = 3.30

$$\int \frac{(c+dx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="fr
icas")
```

output
```
[-1/768*(3*(3*D*a*b^4*c^4 + 4*(3*D*a^2*b^3 - 2*C*a*b^4)*c^3*d + 6*(15*D*a^
3*b^2 - 12*C*a^2*b^3 + 8*B*a*b^4)*c^2*d^2 - 12*(35*D*a^4*b - 30*C*a^3*b^2
+ 24*B*a^2*b^3 - 16*A*a*b^4)*c*d^3 + (315*D*a^5 - 280*C*a^4*b + 240*B*a^3*
b^2 - 192*A*a^2*b^3)*d^4 + (3*D*b^5*c^4 + 4*(3*D*a*b^4 - 2*C*b^5)*c^3*d +
6*(15*D*a^2*b^3 - 12*C*a*b^4 + 8*B*b^5)*c^2*d^2 - 12*(35*D*a^3*b^2 - 30*C*
a^2*b^3 + 24*B*a*b^4 - 16*A*b^5)*c*d^3 + (315*D*a^4*b - 280*C*a^3*b^2 + 24
0*B*a^2*b^3 - 192*A*a*b^4)*d^4)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 +
 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqr
t(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(48*D*b^5*d^4*x^4 - 9*D*a*b^4*c^
3*d - 3*(13*D*a^2*b^3 - 8*C*a*b^4)*c^2*d^2 + (945*D*a^3*b^2 - 800*C*a^2*b^
3 + 624*B*a*b^4 - 384*A*b^5)*c*d^3 - 3*(315*D*a^4*b - 280*C*a^3*b^2 + 240*
B*a^2*b^3 - 192*A*a*b^4)*d^4 + 8*(9*D*b^5*c*d^3 - (9*D*a*b^4 - 8*C*b^5)*d^
4)*x^3 + 2*(3*D*b^5*c^2*d^2 - 2*(33*D*a*b^4 - 28*C*b^5)*c*d^3 + (63*D*a^2*
b^3 - 56*C*a*b^4 + 48*B*b^5)*d^4)*x^2 - (9*D*b^5*c^3*d + 3*(11*D*a*b^4 - 8
*C*b^5)*c^2*d^2 - (357*D*a^2*b^3 - 304*C*a*b^4 + 240*B*b^5)*c*d^3 + (315*D
*a^3*b^2 - 280*C*a^2*b^3 + 240*B*a*b^4 - 192*A*b^5)*d^4)*x)*sqrt(b*x + a)*
sqrt(d*x + c))/(b^7*d^3*x + a*b^6*d^3), -1/384*(3*(3*D*a*b^4*c^4 + 4*(3*D*
a^2*b^3 - 2*C*a*b^4)*c^3*d + 6*(15*D*a^3*b^2 - 12*C*a^2*b^3 + 8*B*a*b^4)*c
^2*d^2 - 12*(35*D*a^4*b - 30*C*a^3*b^2 + 24*B*a^2*b^3 - 16*A*a*b^4)*c*d^3
+ (315*D*a^5 - 280*C*a^4*b + 240*B*a^3*b^2 - 192*A*a^2*b^3)*d^4 + (3*D*...
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Integral((c + d*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(a + b*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 793, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="gi
ac")
```

output
```
1/192*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*
x + a)*D*d*abs(b)/b^7 + (9*D*b^28*c*d^6*abs(b) - 33*D*a*b^27*d^7*abs(b) +
8*C*b^28*d^7*abs(b))/(b^34*d^6)) + (3*D*b^29*c^2*d^5*abs(b) - 174*D*a*b^28
*c*d^6*abs(b) + 56*C*b^29*c*d^6*abs(b) + 315*D*a^2*b^27*d^7*abs(b) - 152*C
*a*b^28*d^7*abs(b) + 48*B*b^29*d^7*abs(b))/(b^34*d^6)) - 3*(3*D*b^30*c^3*d
^4*abs(b) + 15*D*a*b^29*c^2*d^5*abs(b) - 8*C*b^30*c^2*d^5*abs(b) - 279*D*a
^2*b^28*c*d^6*abs(b) + 176*C*a*b^29*c*d^6*abs(b) - 80*B*b^30*c*d^6*abs(b)
+ 325*D*a^3*b^27*d^7*abs(b) - 232*C*a^2*b^28*d^7*abs(b) + 144*B*a*b^29*d^7
*abs(b) - 64*A*b^30*d^7*abs(b))/(b^34*d^6))*sqrt(b*x + a) + 4*(D*a^3*b^2*c
^2*d*abs(b) - C*a^2*b^3*c^2*d*abs(b) + B*a*b^4*c^2*d*abs(b) - A*b^5*c^2*d*
abs(b) - 2*D*a^4*b*c*d^2*abs(b) + 2*C*a^3*b^2*c*d^2*abs(b) - 2*B*a^2*b^3*c
*d^2*abs(b) + 2*A*a*b^4*c*d^2*abs(b) + D*a^5*d^3*abs(b) - C*a^4*b*d^3*abs(
b) + B*a^3*b^2*d^3*abs(b) - A*a^2*b^3*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*sqrt(b*d)*b^5
) - 1/128*(3*D*b^4*c^4*abs(b) + 12*D*a*b^3*c^3*d*abs(b) - 8*C*b^4*c^3*d*ab
s(b) + 90*D*a^2*b^2*c^2*d^2*abs(b) - 72*C*a*b^3*c^2*d^2*abs(b) + 48*B*b^4*
c^2*d^2*abs(b) - 420*D*a^3*b*c*d^3*abs(b) + 360*C*a^2*b^2*c*d^3*abs(b) - 2
88*B*a*b^3*c*d^3*abs(b) + 192*A*b^4*c*d^3*abs(b) + 315*D*a^4*d^4*abs(b) -
280*C*a^3*b*d^4*abs(b) + 240*B*a^2*b^2*d^4*abs(b) - 192*A*a*b^3*d^4*abs(b)
)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx=\int \frac{(c+d\,x)^{3/2}\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^{3/2}}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(3/2),x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2} (A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}} \, dx = \int \frac{(dx+c)^{\frac{3}{2}} (Dx^3+C\,x^2+Bx+A)}{(bx+a)^{\frac{3}{2}}} dx$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

output
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

3.140 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx$

Optimal result 1349
Mathematica [A] (verified) 1350
Rubi [A] (verified) 1350
Maple [B] (verified) 1354
Fricas [A] (verification not implemented) 1355
Sympy [F] 1356
Maxima [F(-2)] 1357
Giac [A] (verification not implemented) 1357
Mupad [F(-1)] 1358
Reduce [F] 1358

Optimal result

Integrand size = 34, antiderivative size = 320

$$\begin{aligned}
\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = &-\frac{2(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b^4\sqrt{a+bx}} \\
&+\frac{\left(\frac{29a^2dD}{b}-2a(9Cd+2cD)+b\left(2cC+8Bd-\frac{c^2D}{d}\right)\right)\sqrt{a+bx}\sqrt{c+dx}}{8b^3d} \\
&+\frac{(2bCd-bcD-5adD)(a+bx)^{3/2}\sqrt{c+dx}}{4b^4d}+\frac{D(a+bx)^{3/2}(c+dx)^{3/2}}{3b^3d} \\
&-\frac{(35a^3d^3D-15a^2bd^2(2Cd+cD)+3ab^2d(4cCd+8Bd^2-c^2D)+b^3(2c^2Cd-8Bcd^2-16Ad^3-c^3D))\,a}{8b^{9/2}d^{5/2}}
\end{aligned}$$

output

```
-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^4/(b*x+a)^(1/2)+1/8*(29*a
^2*d*D/b-2*a*(9*C*d+2*D*c)+b*(2*C*c+8*B*d-c^2*D/d))*(b*x+a)^(1/2)*(d*x+c)^
(1/2)/b^3/d+1/4*(2*C*b*d-5*D*a*d-D*b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^4/d+
1/3*D*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b^3/d-1/8*(35*a^3*d^3*D-15*a^2*b*d^2*(2*
C*d+D*c)+3*a*b^2*d*(8*B*d^2+4*C*c*d-D*c^2)+b^3*(-16*A*d^3-8*B*c*d^2+2*C*c^
2*d-D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(9/2)/d
^(5/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \frac{\sqrt{c+dx}(105a^3d^2D-5a^2bd(18Cd+2cD-7dDx)-ab^2(3c^2D}{} + \frac{(-35a^3d^3D+15a^2bd^2(2Cd+cD)+3ab^2d(-4cCd-8Bd^2+c^2D)+b^3(-2c^2Cd+8Bcd^2+16Ad^3+c^3}{8b^{9/2}d^{5/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(3/2),x]
```

output
```
(Sqrt[c + d*x]*(105*a^3*d^2*D - 5*a^2*b*d*(18*C*d + 2*c*D - 7*d*D*x) - a*b
^2*(3*c^2*D + c*(-6*C*d + 8*d*D*x) + 2*d^2*(-36*B + 15*C*x + 7*D*x^2)) + b
^3*(-48*A*d^2 + x*(-3*c^2*D + 2*c*d*(3*C + D*x) + 4*d^2*(6*B + 3*C*x + 2*D
*x^2)))))/(24*b^4*d^2*Sqrt[a + b*x]) + ((-35*a^3*d^3*D + 15*a^2*b*d^2*(2*C
*d + c*D) + 3*a*b^2*d*(-4*c*C*d - 8*B*d^2 + c^2*D) + b^3*(-2*c^2*C*d + 8*B
*c*d^2 + 16*A*d^3 + c^3*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
a + b*x])])/(8*b^(9/2)*d^(5/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2124, 27, 1194, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(\left(c-\frac{ad}{b}\right)Dx^2+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{-3dDa^3+b(3Cd+cD)a^2-b^2(cC+3Bd)a+b^3(Bc+2Ad)}{b^3}\right)}{2\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{3/2}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\left(-\frac{3dDa^3}{b^3}+\frac{(3Cd+cD)a^2}{b^2}-\frac{(cC+3Bd)a}{b}+\left(c-\frac{ad}{b}\right)Dx^2+Bc+2Ad+\frac{(bc-ad)(bC-aD)x}{b^2}\right)}{\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \; 1194$$

$$\frac{\frac{\int \frac{3\sqrt{c+dx}\left(-\frac{5d^2Da^3}{b}+2d(3Cd+cD)a^2-b\left(Dc^2+2Cdc+6Bd^2\right)a+2b^2d(Bc+2Ad)+(bc-ad)(2bCd-5aDd-bcD)x\right)}{2\sqrt{a+bx}}dx}{3b^2d}+\frac{D(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}{3b^3d}}{bc-ad}-\frac{2(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{c+dx}\left(-\frac{5d^2Da^3}{b}+2d(3Cd+cD)a^2-b\left(Dc^2+2Cdc+6Bd^2\right)a+2b^2d(Bc+2Ad)+(bc-ad)(2bCd-5aDd-bcD)x\right)}{\sqrt{a+bx}}dx}{2b^2d}+\frac{D(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}{3b^3d}}{bc-ad}-\frac{2(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)(-5adD-bcD+2bCd)}{2bd}-\frac{\left(35a^3d^3D-15a^2bd^2(cD+2Cd)+3ab^2d\left(8Bd^2+c^2(-D)+4cCd\right)+b^3\left(-16Ad^3-8Bcd^2+c^3(-D)+2c^2Cd\right)\right)\int \frac{\sqrt{\cdots}}{\sqrt{\cdots}}\cdots}{4bd}}{2b^2d}\cdots}{bc-ad}\cdots-\frac{2(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \; 60$$

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)(-5adD-bcD+2bCd)}{2bd}-\frac{\left(35a^3d^3D-15a^2bd^2(cD+2Cd)+3ab^2d\left(8Bd^2+c^2(-D)+4cCd\right)+b^3\left(-16Ad^3-8Bcd^2+c^3(-D)+2c^2Cd\right)\right)\left(\frac{(b\cdots}{\cdots}\cdots\right)}{4bd}}{2b^2d}\cdots}{bc-ad}\cdots-\frac{2(c+dx)^{3/2}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)(-5adD-bcD+2bCd)}{2bd}-\frac{\left(35a^3d^3D-15a^2bd^2(cD+2Cd)+3ab^2d\left(8Bd^2+c^2(-D)+4cCd\right)+b^3\left(-16Ad^3-8Bcd^2+c^3(-D)+2c^2Cd\right)\right)\left(\frac{(bc\ldots}{\ldots}\right.}{4bd}}{2b^2d}}{bc-ad}$$

$$\frac{2(c+dx)^{3/2}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{\frac{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)(-5adD-bcD+2bCd)}{2bd}-\frac{\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)\left(35a^3d^3D-15a^2bd^2(cD+2Cd)+3ab^2d\left(8Bd^2+c^2(-D)\ldots\right.\right.}{4bd}}{2b^2d}}{bc-ad}$$

$$\frac{2(c+dx)^{3/2}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(3/2),x]
```

output

```
(-2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(3/2))/((b*c - a*d)*Sq
rt[a + b*x]) + (((b*c - a*d)*D*(a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*b^3*d)
+ (((b*c - a*d)*(2*b*C*d - b*c*D - 5*a*d*D)*Sqrt[a + b*x]*(c + d*x)^(3/2))
/(2*b*d) - ((35*a^3*d^3*D - 15*a^2*b*d^2*(2*C*d + c*D) + 3*a*b^2*d*(4*c*C*
d + 8*B*d^2 - c^2*D) + b^3*(2*c^2*C*d - 8*B*c*d^2 - 16*A*d^3 - c^3*D))*((S
qrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x
])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])))/(4*b*d))/(2*b^2*d))/(b*c
- a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1547 vs. $2(284) = 568$.

Time = 0.52 (sec) , antiderivative size = 1548, normalized size of antiderivative = 4.84

method	result	size
default	Expression too large to display	1548

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/48*(d*x+c)^(1/2)*(-16*D*a*b^2*c*d*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-
105*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)
^(1/2))*a^4*d^3+16*D*b^3*d^2*x^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+24*C*
b^3*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+48*B*b^3*d^2*x*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)-6*D*b^3*c^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+144*B*a*b^2*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-180*C*a^2*b*d^2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-6*D*a*b^2*c^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)-72*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)
/(d*b)^(1/2))*a*b^3*d^3*x+24*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c*d^2*x+90*C*ln(1/2*(2*b*d*x+2*((b*x+
a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*d^3*x-6*C*ln(1
/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^
4*c^2*d*x-105*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a^3*b*d^3*x+24*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c*d^2-36*C*ln(1/2*(2*b*d*x+2*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c*d^2-6*C*
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2)
)*a*b^3*c^2*d+45*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a
*d+b*c)/(d*b)^(1/2))*a^3*b*c*d^2+9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(
1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c^2*d-96*A*b^3*d^2*((b*x...
```

Fricas [A] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 976, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="fr
icas")
```

output
```
[1/96*(3*(D*a*b^3*c^3 + (3*D*a^2*b^2 - 2*C*a*b^3)*c^2*d + (15*D*a^3*b - 12
*C*a^2*b^2 + 8*B*a*b^3)*c*d^2 - (35*D*a^4 - 30*C*a^3*b + 24*B*a^2*b^2 - 16
*A*a*b^3)*d^3 + (D*b^4*c^3 + (3*D*a*b^3 - 2*C*b^4)*c^2*d + (15*D*a^2*b^2 -
 12*C*a*b^3 + 8*B*b^4)*c*d^2 - (35*D*a^3*b - 30*C*a^2*b^2 + 24*B*a*b^3 - 1
6*A*b^4)*d^3)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d
^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^
2*c*d + a*b*d^2)*x) + 4*(8*D*b^4*d^3*x^3 - 3*D*a*b^3*c^2*d - 2*(5*D*a^2*b^
2 - 3*C*a*b^3)*c*d^2 + 3*(35*D*a^3*b - 30*C*a^2*b^2 + 24*B*a*b^3 - 16*A*b^
4)*d^3 + 2*(D*b^4*c*d^2 - (7*D*a*b^3 - 6*C*b^4)*d^3)*x^2 - (3*D*b^4*c^2*d
+ 2*(4*D*a*b^3 - 3*C*b^4)*c*d^2 - (35*D*a^2*b^2 - 30*C*a*b^3 + 24*B*b^4)*d
^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*d^3*x + a*b^5*d^3), -1/48*(3*(D*a
*b^3*c^3 + (3*D*a^2*b^2 - 2*C*a*b^3)*c^2*d + (15*D*a^3*b - 12*C*a^2*b^2 +
8*B*a*b^3)*c*d^2 - (35*D*a^4 - 30*C*a^3*b + 24*B*a^2*b^2 - 16*A*a*b^3)*d^3
 + (D*b^4*c^3 + (3*D*a*b^3 - 2*C*b^4)*c^2*d + (15*D*a^2*b^2 - 12*C*a*b^3 +
 8*B*b^4)*c*d^2 - (35*D*a^3*b - 30*C*a^2*b^2 + 24*B*a*b^3 - 16*A*b^4)*d^3)
*x)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(8*D*b^4
*d^3*x^3 - 3*D*a*b^3*c^2*d - 2*(5*D*a^2*b^2 - 3*C*a*b^3)*c*d^2 + 3*(35*D*a
^3*b - 30*C*a^2*b^2 + 24*B*a*b^3 - 16*A*b^4)*d^3 + 2*(D*b^4*c*d^2 - (7*D*a
*b^3 - 6*C*b^4)*d^3)*x^2 - (3*D*b^4*c^2*d + 2*(4*D*a*b^3 - 3*C*b^4)*c*d...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3)/(a + b*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \frac{1}{24}\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)D|b}{b^6}\right.\right.$$
$$+\frac{4\,(Da^3bcd|b|-Ca^2b^2cd|b|+Bab^3cd|b|-Ab^4cd|b|-Da^4d^2|b|+Ca^3bd^2|b|-Ba^2b^2d^2|b|+Aab^3d^2|b|)}{\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)\sqrt{bd}b^4}$$
$$-\frac{(Db^3c^3|b|+3\,Dab^2c^2d|b|-2\,Cb^3c^2d|b|+15\,Da^2bcd^2|b|-12\,Cab^2cd^2|b|+8\,Bb^3cd^2|b|-35\,Da^3d^3|b|+3}{16\,\sqrt{bd}b^5d^2}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="gi
ac")
```

output
```
1/24*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*
x + a)*D*abs(b)/b^6 + (D*b^18*c*d^3*abs(b) - 19*D*a*b^17*d^4*abs(b) + 6*C*
b^18*d^4*abs(b))/(b^23*d^4)) - 3*(D*b^19*c^2*d^2*abs(b) + 4*D*a*b^18*c*d^3
*abs(b) - 2*C*b^19*c*d^3*abs(b) - 29*D*a^2*b^17*d^4*abs(b) + 18*C*a*b^18*d
^4*abs(b) - 8*B*b^19*d^4*abs(b))/(b^23*d^4)) + 4*(D*a^3*b*c*d*abs(b) - C*a
^2*b^2*c*d*abs(b) + B*a*b^3*c*d*abs(b) - A*b^4*c*d*abs(b) - D*a^4*d^2*abs(
b) + C*a^3*b*d^2*abs(b) - B*a^2*b^2*d^2*abs(b) + A*a*b^3*d^2*abs(b))/((b^2
*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2)*sqrt(b*d)*b^4) - 1/16*(D*b^3*c^3*abs(b) + 3*D*a*b^2*c^2*d*abs(b) - 2
*C*b^3*c^2*d*abs(b) + 15*D*a^2*b*c*d^2*abs(b) - 12*C*a*b^2*c*d^2*abs(b) +
8*B*b^3*c*d^2*abs(b) - 35*D*a^3*d^3*abs(b) + 30*C*a^2*b*d^3*abs(b) - 24*B*
a*b^2*d^3*abs(b) + 16*A*b^3*d^3*abs(b))*log((sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sqrt(b*d)*b^5*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx=\int \frac{\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(3/2),x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx=\int \frac{\sqrt{dx+c}\,(Dx^3+C\,x^2+Bx+A)}{(bx+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

output
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

3.141 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx$

Optimal result . . . 1359
Mathematica [A] (verified) . . . 1360
Rubi [A] (verified) . . . 1360
Maple [B] (verified) . . . 1363
Fricas [B] (verification not implemented) . . . 1364
Sympy [F] . . . 1365
Maxima [F(-2)] . . . 1366
Giac [A] (verification not implemented) . . . 1366
Mupad [F(-1)] . . . 1367
Reduce [B] (verification not implemented) . . . 1367

Optimal result

Integrand size = 34, antiderivative size = 223

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx &= -\frac{2(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b^3(bc-ad)\sqrt{a+bx}} \\ &+\frac{(4bCd-3bcD-9adD)\sqrt{a+bx}\sqrt{c+dx}}{4b^3d^2}+\frac{D(a+bx)^{3/2}\sqrt{c+dx}}{2b^3d} \\ &+\frac{(15a^2d^2D-6abd(2Cd-cD)-b^2(4cCd-8Bd^2-3c^2D))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{7/2}d^{5/2}}\end{aligned}$$

output
```
-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^3/(-a*d+b*c)/(b*x+a)^(1/2
)+1/4*(4*C*b*d-9*D*a*d-3*D*b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^3/d^2+1/2*D*
(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^3/d+1/4*(15*a^2*d^2*D-6*a*b*d*(2*C*d-D*c)-b^
2*(-8*B*d^2+4*C*c*d-3*D*c^2))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c
)^(1/2))/b^(7/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx =$$
$$-\frac{\sqrt{c+dx}(-15a^3d^2D+a^2bd(12Cd+4cD-5dDx)+b^3(8Ad^2+cx(-4Cd+3cD-2dDx))+ab^2(3c^2D}{4b^3d^2(bc-ad)\sqrt{a+bx}}$$
$$+\frac{(15a^2d^2D+6abd(-2Cd+cD)+b^2(-4cCd+8Bd^2+3c^2D))\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4b^{7/2}d^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

output
```
-1/4*(Sqrt[c + d*x]*(-15*a^3*d^2*D + a^2*b*d*(12*C*d + 4*c*D - 5*d*D*x) +
b^3*(8*A*d^2 + c*x*(-4*C*d + 3*c*D - 2*d*D*x)) + a*b^2*(3*c^2*D + c*(-4*C*
d + 2*d*D*x) + 2*d^2*(-4*B + 2*C*x + D*x^2))))/(b^3*d^2*(b*c - a*d)*Sqrt[a
 + b*x]) + ((15*a^2*d^2*D + 6*a*b*d*(-2*C*d + c*D) + b^2*(-4*c*C*d + 8*B*d
^2 + 3*c^2*D))*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(
4*b^(7/2)*d^(5/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1194, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \; 2124$$
$$-\frac{2\int -\frac{\frac{(bc-ad)Dx^2}{b}+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{(bc-ad)\left(Da^2-bCa+b^2B\right)}{b^3}}{2\sqrt{a+bx}\sqrt{c+dx}}dx}{bc-ad}-\frac{2\sqrt{c+dx}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\frac{(bc-ad)Dx^2}{b}+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{(bc-ad)\left(Da^2-bCa+b^2B\right)}{b^3}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{bc-ad}-\frac{2\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \ 1194$$

$$\frac{\frac{\int \frac{(bc-ad)\left(3dDa^2-4bCda-3bcDa+4b^2Bd+b(4bCd-9aDd-3bcD)x\right)}{2b\sqrt{a+bx}\sqrt{c+dx}}dx}{2b^2d}+\frac{D(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{2b^3d}}{bc-ad}-\frac{2\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(bc-ad)\int \frac{3dDa^2-4bCda-3bcDa+4b^2Bd+b(4bCd-9aDd-3bcD)x}{\sqrt{a+bx}\sqrt{c+dx}}dx}{4b^3d}+\frac{D(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}{2b^3d}}{bc-ad}-\frac{2\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \ 90$$

$$\frac{\frac{(bc-ad)\left(\frac{\left(15a^2d^2D-6abd(2Cd-cD)-\left(b^2\left(-8Bd^2-3c^2D+4cCd\right)\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2d}+\frac{\sqrt{a+bx}\sqrt{c+dx}(-9adD-3bcD+4bCd)}{d}\right)}{4b^3d}+\frac{D(a+bx)^{3/2}\sqrt{c+dx}(}{2b^3d}}{bc-ad}$$

$$\frac{2\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \ 66$$

$$\frac{\frac{(bc-ad)\left(\frac{\left(15a^2d^2D-6abd(2Cd-cD)-\left(b^2\left(-8Bd^2-3c^2D+4cCd\right)\right)\right)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}+\frac{\sqrt{a+bx}\sqrt{c+dx}(-9adD-3bcD+4bCd)}{d}\right)}{4b^3d}+\frac{D(a+bx)^{3/2}\sqrt{c+}}{2b^3d}}{bc-ad}$$

$$\frac{2\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{(bc-ad)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)\left(15a^2d^2D-6abd(2Cd-cD)-\left(b^2\left(-8Bd^2-3c^2D+4cCd\right)\right)\right)}{\sqrt{b}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(-9adD-3bcD+4bCd)}{d}\right)}{4b^3d}+\frac{D(a+bx)^{3/2}\sqrt{c}}{2b^3}}{bc-ad}$$

$$\frac{2\sqrt{c+dx}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

output
```
(-2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*Sqrt[c + d*x])/((b*c - a*d)*Sqrt
[a + b*x]) + (((b*c - a*d)*D*(a + b*x)^(3/2)*Sqrt[c + d*x])/(2*b^3*d) + ((
b*c - a*d)*(((4*b*C*d - 3*b*c*D - 9*a*d*D)*Sqrt[a + b*x]*Sqrt[c + d*x])/d
+ ((15*a^2*d^2*D - 6*a*b*d*(2*C*d - c*D) - b^2*(4*c*C*d - 8*B*d^2 - 3*c^2*
D))*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(
3/2))))/(4*b^3*d))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1369 vs. $2(193) = 386$.

Time = 0.60 (sec) , antiderivative size = 1370, normalized size of antiderivative = 6.14

method	result	size
default	Expression too large to display	1370

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/8*(d*x+c)^(1/2)*(4*D*a*b^2*c*d*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+15*
D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/
2))*a^4*d^3+6*D*b^3*c^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-16*B*a*b^2*d
^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+24*C*a^2*b*d^2*((b*x+a)*(d*x+c))^(1
/2)*(d*b)^(1/2)+6*D*a*b^2*c^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+8*B*ln(1
/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*
b^3*d^3*x-8*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*
c)/(d*b)^(1/2))*b^4*c*d^2*x-12*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*d^3*x+4*C*ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^2*d*x+15*D*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*
a^3*b*d^3*x-8*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a*b^3*c*d^2+8*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2
)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c*d^2+4*C*ln(1/2*(2*b*d*x+2*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^2*d-9*D*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*
a^3*b*c*d^2-3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+
b*c)/(d*b)^(1/2))*a^2*b^2*c^2*d+16*A*b^3*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)-30*D*a^3*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-3*D*ln(1/2*(2*b*d*
x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^3*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(191) = 382$.

Time = 2.91 (sec) , antiderivative size = 940, normalized size of antiderivative = 4.22

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{3/2}\sqrt{c + dx}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fr
icas")
```

output
```
[1/16*((3*D*a*b^3*c^3 + (3*D*a^2*b^2 - 4*C*a*b^3)*c^2*d + (9*D*a^3*b - 8*C
*a^2*b^2 + 8*B*a*b^3)*c*d^2 - (15*D*a^4 - 12*C*a^3*b + 8*B*a^2*b^2)*d^3 +
(3*D*b^4*c^3 + (3*D*a*b^3 - 4*C*b^4)*c^2*d + (9*D*a^2*b^2 - 8*C*a*b^3 + 8*
B*b^4)*c*d^2 - (15*D*a^3*b - 12*C*a^2*b^2 + 8*B*a*b^3)*d^3)*x)*sqrt(b*d)*l
og(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)
*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(3*D
*a*b^3*c^2*d + 4*(D*a^2*b^2 - C*a*b^3)*c*d^2 - (15*D*a^3*b - 12*C*a^2*b^2
+ 8*B*a*b^3 - 8*A*b^4)*d^3 - 2*(D*b^4*c*d^2 - D*a*b^3*d^3)*x^2 + (3*D*b^4*
c^2*d + 2*(D*a*b^3 - 2*C*b^4)*c*d^2 - (5*D*a^2*b^2 - 4*C*a*b^3)*d^3)*x)*sq
rt(b*x + a)*sqrt(d*x + c))/(a*b^5*c*d^3 - a^2*b^4*d^4 + (b^6*c*d^3 - a*b^5
*d^4)*x), -1/8*((3*D*a*b^3*c^3 + (3*D*a^2*b^2 - 4*C*a*b^3)*c^2*d + (9*D*a^
3*b - 8*C*a^2*b^2 + 8*B*a*b^3)*c*d^2 - (15*D*a^4 - 12*C*a^3*b + 8*B*a^2*b^
2)*d^3 + (3*D*b^4*c^3 + (3*D*a*b^3 - 4*C*b^4)*c^2*d + (9*D*a^2*b^2 - 8*C*a
*b^3 + 8*B*b^4)*c*d^2 - (15*D*a^3*b - 12*C*a^2*b^2 + 8*B*a*b^3)*d^3)*x)*sq
rt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*
x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(3*D*a*b^3*c^2
*d + 4*(D*a^2*b^2 - C*a*b^3)*c*d^2 - (15*D*a^3*b - 12*C*a^2*b^2 + 8*B*a*b^
3 - 8*A*b^4)*d^3 - 2*(D*b^4*c*d^2 - D*a*b^3*d^3)*x^2 + (3*D*b^4*c^2*d + 2*
(D*a*b^3 - 2*C*b^4)*c*d^2 - (5*D*a^2*b^2 - 4*C*a*b^3)*d^3)*x)*sqrt(b*x + a
)*sqrt(d*x + c))/(a*b^5*c*d^3 - a^2*b^4*d^4 + (b^6*c*d^3 - a*b^5*d^4)*x...
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{3/2}\sqrt{c + dx}}\, dx = \int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{\frac{3}{2}}\sqrt{c + dx}}\, dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = \frac{1}{4}\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2(bx+a)D}{b^3d|b|} - \frac{3Db^8cd+9Dab^7d^2-}{b^{10}d^3|b|}\right.$$
$$+\frac{4(Da^3d-Ca^2bd+Bab^2d-Ab^3d)}{\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)\sqrt{bd}b|b|}$$
$$-\frac{(3Db^2c^2+6Dabcd-4Cb^2cd+15Da^2d^2-12Cabd^2+8Bb^2d^2)\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a\right.\right.}{8\sqrt{bd}b^2d^2|b|}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="gi
ac")
```

output
```
1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*D/(b^3*
d*abs(b)) - (3*D*b^8*c*d + 9*D*a*b^7*d^2 - 4*C*b^8*d^2)/(b^10*d^3*abs(b)))
 + 4*(D*a^3*d - C*a^2*b*d + B*a*b^2*d - A*b^3*d)/((b^2*c - a*b*d - (sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*sqrt(b*d)*b*ab
s(b)) - 1/8*(3*D*b^2*c^2 + 6*D*a*b*c*d - 4*C*b^2*c*d + 15*D*a^2*d^2 - 12*C
*a*b*d^2 + 8*B*b^2*d^2)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^2)/(sqrt(b*d)*b^2*d^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = \frac{15\sqrt{d}\,\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{d}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{dx+c}}{\sqrt{ad-bc}}\right)a^2d^2-6\sqrt{d}\,\sqrt{b}\,\sqrt{bx+a}\,\log\left(\sqrt{d}\cdots\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2),x)
```

output
```
(15*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqr
t(c + d*x))/sqrt(a*d - b*c))*a**2*d**2 - 6*sqrt(d)*sqrt(b)*sqrt(a + b*x)*l
og((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b*c*
d + 8*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*s
qrt(c + d*x))/sqrt(a*d - b*c))*b**3*d - sqrt(d)*sqrt(b)*sqrt(a + b*x)*log(
(sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**2*c**2
 - 10*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*d**2 + 2*sqrt(d)*sqrt(b)*sqrt(a +
 b*x)*a*b*c*d - 15*sqrt(c + d*x)*a**2*b*d**2 + sqrt(c + d*x)*a*b**2*c*d -
5*sqrt(c + d*x)*a*b**2*d**2*x + sqrt(c + d*x)*b**3*c*d*x + 2*sqrt(c + d*x)
*b**3*d**2*x**2)/(4*sqrt(a + b*x)*b**4*d**2)
```

3.142 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal result 1369
Mathematica [A] (verified) 1370
Rubi [A] (verified) 1370
Maple [B] (verified) 1373
Fricas [B] (verification not implemented) 1374
Sympy [F] 1375
Maxima [F(-2)] 1376
Giac [B] (verification not implemented) 1376
Mupad [F(-1)] 1377
Reduce [B] (verification not implemented) 1377

Optimal result

Integrand size = 34, antiderivative size = 200

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{d^2(bc-ad)^2\sqrt{c+dx}} - \frac{2(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{b^2(bc-ad)^2\sqrt{a+bx}} + \frac{D\sqrt{a+bx}\sqrt{c+dx}}{b^2d^2} + \frac{(2bCd-3bcD-3adD)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}d^{5/2}}$$

output
```
-2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d^2/(-a*d+b*c)^2/(d*x+c)^(1
/2)-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)
^(1/2)+D*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d^2+(2*C*b*d-3*D*a*d-3*D*b*c)*arc
tanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{3a^3d^2D(c+dx)+a^2bd(c+dx)(-2Cd-2cD+dDx)+b^3(-2Ad^2(c+2dx)\cdots}{}$$
$$+\frac{(2bCd-3bcD-3adD)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b^{5/2}d^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(3*a^3*d^2*D*(c + d*x) + a^2*b*d*(c + d*x)*(-2*C*d - 2*c*D + d*D*x) + b^3*
(-2*A*d^2*(c + 2*d*x) + c*x*(-2*c*C*d + 2*B*d^2 + 3*c^2*D + c*d*D*x)) + a*
b^2*(3*c^3*D - 2*d^3*(A - B*x) - c^2*d*(2*C + D*x) + 2*c*d^2*(2*B - D*x^2)
))/(b^2*d^2*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x]) + ((2*b*C*d - 3*b*c
*D - 3*a*d*D)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(b
^(5/2)*d^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1193, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{\left(c-\frac{ad}{b}\right)Dx^2+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{dDa^3-b(Cd-cD)a^2-b^2(cC-Bd)a+b^3(Bc-2Ad)}{b^3}}{2\sqrt{a+bx}(c+dx)^{3/2}}dx}{bc-ad}-\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{dDa^3}{b^3}-\frac{(Cd-cD)a^2}{b^2}-\frac{(cC-Bd)a}{b}+\left(c-\frac{ad}{b}\right)Dx^2+Bc-2Ad+\frac{(bc-ad)(bC-aD)x}{b^2}}{\sqrt{a+bx}(c+dx)^{3/2}}dx}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{\frac{2\int \frac{(bc-ad)^2(bCd-aDd+bDxd-bcD)}{2b^2d^2\sqrt{a+bx}\sqrt{c+dx}}dx}{bc-ad}-\frac{2\sqrt{a+bx}\left(\frac{a^2d(bC-aD)}{b^3}-B\left(\frac{ad}{b}+c\right)+2Ad-\frac{c^3D}{d^2}+\frac{c^2C}{d}\right)}{\sqrt{c+dx}(bc-ad)}}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(bc-ad)\int \frac{bCd-aDd+bDxd-bcD}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b^2d^2}-\frac{2\sqrt{a+bx}\left(\frac{a^2d(bC-aD)}{b^3}-B\left(\frac{ad}{b}+c\right)+2Ad-\frac{c^3D}{d^2}+\frac{c^2C}{d}\right)}{\sqrt{c+dx}(bc-ad)}}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{(bc-ad)\left(\frac{1}{2}(-3adD-3bcD+2bCd)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx+D\sqrt{a+bx}\sqrt{c+dx}\right)}{b^2d^2}-\frac{2\sqrt{a+bx}\left(\frac{a^2d(bC-aD)}{b^3}-B\left(\frac{ad}{b}+c\right)+2Ad-\frac{c^3D}{d^2}+\frac{c^2C}{d}\right)}{\sqrt{c+dx}(bc-ad)}}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{(bc-ad)\left((-3adD-3bcD+2bCd)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}+D\sqrt{a+bx}\sqrt{c+dx}\right)}{b^2d^2}-\frac{2\sqrt{a+bx}\left(\frac{a^2d(bC-aD)}{b^3}-B\left(\frac{ad}{b}+c\right)+2Ad-\frac{c^3D}{d^2}+\frac{c^2C}{d}\right)}{\sqrt{c+dx}(bc-ad)}}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(bc-ad)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-3adD-3bcD+2bCd)}{\sqrt{b}\sqrt{d}}+D\sqrt{a+bx}\sqrt{c+dx}\right)}{b^2d^2}-\frac{2\sqrt{a+bx}\left(\frac{a^2d(bC-aD)}{b^3}-B\left(\frac{ad}{b}+c\right)+2Ad-\frac{c^3D}{d^2}+\frac{c^2C}{d}\right)}{\sqrt{c+dx}(bc-ad)}}{bc-ad}-\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

output
```
(-2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3))/((b*c - a*d)*Sqrt[a + b*x]*Sqrt
[c + d*x]) + ((-2*((c^2*C)/d + 2*A*d - B*(c + (a*d)/b) - (c^3*D)/d^2 + (a^
2*d*(b*C - a*D))/b^3)*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x]) + ((b*c -
 a*d)*(D*Sqrt[a + b*x]*Sqrt[c + d*x] + ((2*b*C*d - 3*b*c*D - 3*a*d*D)*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[d])))/
(b^2*d^2))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1744 vs. $2(176) = 352$.

Time = 0.61 (sec) , antiderivative size = 1745, normalized size of antiderivative = 8.72

method	result	size
default	Expression too large to display	1745

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/2*(-4*B*a*b^2*d^3*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-4*B*b^3*c*d^2*x
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+4*C*a^2*b*d^3*x*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+2*D*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b*c*d^2*x+2*D*(
(b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^2*c^2*d*x-2*C*ln(1/2*(2*b*d*x+2*((b
*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*d^4*x^2+4*A
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^2*d^3+8*A*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)*b^3*d^3*x+4*A*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^3*c*d^2-2
*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1
/2))*b^4*c^2*d^2*x^2+3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b*d^4*x^2+3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^4*c^3*d*x^2-2*C*ln(1/2*(2
*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b*d
^4*x-2*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d
*b)^(1/2))*b^4*c^3*d*x-2*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b*c*d^3+4*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d
*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c^2*d^2-2*C*ln(1/2*
(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3
*c^3*d-3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*a^3*b*c^2*d^2-3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c^3*d+4*C*b^3*c^2*d*x*((b*x+a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(177) = 354$.

Time = 4.17 (sec) , antiderivative size = 1332, normalized size of antiderivative = 6.66

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{3/2}(c + dx)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fr
icas")
```

output
```
[-1/4*((3*D*a*b^3*c^4 - (3*D*a^2*b^2 + 2*C*a*b^3)*c^3*d - (3*D*a^3*b - 4*C
*a^2*b^2)*c^2*d^2 + (3*D*a^4 - 2*C*a^3*b)*c*d^3 + (3*D*b^4*c^3*d - (3*D*a*
b^3 + 2*C*b^4)*c^2*d^2 - (3*D*a^2*b^2 - 4*C*a*b^3)*c*d^3 + (3*D*a^3*b - 2*
C*a^2*b^2)*d^4)*x^2 + (3*D*b^4*c^4 - 2*C*b^4*c^3*d + 2*C*a^2*b^2*c*d^3 - 2
*(3*D*a^2*b^2 - C*a*b^3)*c^2*d^2 + (3*D*a^4 - 2*C*a^3*b)*d^4)*x)*sqrt(b*d)
*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*
d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(3
*D*a*b^3*c^3*d - 2*A*a*b^3*d^4 - 2*(D*a^2*b^2 + C*a*b^3)*c^2*d^2 + (3*D*a^
3*b - 2*C*a^2*b^2 + 4*B*a*b^3 - 2*A*b^4)*c*d^3 + (D*b^4*c^2*d^2 - 2*D*a*b^
3*c*d^3 + D*a^2*b^2*d^4)*x^2 + (3*D*b^4*c^3*d - (D*a*b^3 + 2*C*b^4)*c^2*d^
2 - (D*a^2*b^2 - 2*B*b^4)*c*d^3 + (3*D*a^3*b - 2*C*a^2*b^2 + 2*B*a*b^3 - 4
*A*b^4)*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(a*b^5*c^3*d^3 - 2*a^2*b^4*c^
2*d^4 + a^3*b^3*c*d^5 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^2 +
(b^6*c^3*d^3 - a*b^5*c^2*d^4 - a^2*b^4*c*d^5 + a^3*b^3*d^6)*x), 1/2*((3*D*
a*b^3*c^4 - (3*D*a^2*b^2 + 2*C*a*b^3)*c^3*d - (3*D*a^3*b - 4*C*a^2*b^2)*c^
2*d^2 + (3*D*a^4 - 2*C*a^3*b)*c*d^3 + (3*D*b^4*c^3*d - (3*D*a*b^3 + 2*C*b^
4)*c^2*d^2 - (3*D*a^2*b^2 - 4*C*a*b^3)*c*d^3 + (3*D*a^3*b - 2*C*a^2*b^2)*d
^4)*x^2 + (3*D*b^4*c^4 - 2*C*b^4*c^3*d + 2*C*a^2*b^2*c*d^3 - 2*(3*D*a^2*b^
2 - C*a*b^3)*c^2*d^2 + (3*D*a^4 - 2*C*a^3*b)*d^4)*x)*sqrt(-b*d)*arctan(1/2
*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)**(3/2)*(c + d*x)**(3/2)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{3/2}(c + dx)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(177) = 354$.

Time = 0.28 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.06

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{3/2}(c + dx)^{3/2}}\, dx = \frac{\sqrt{bx+a}\left(\frac{(Db^6c^2d^2 - 2\,Dab^5cd^3 + Da^2b^4d^4)(bx+a)}{b^7c^2d^3|b| - 2\,ab^6cd^4|b| + a^2b^5d^5|b|} + \frac{3\,Db^7c^3d - 3\,Dab^6c^2d^2 - 2\,Cb^7c^2d^2 + 3\,Da^2b}{b^7c^2d^3|b| - 2\,ab^6cd^4|}\right.}{\sqrt{b^2c + (bx+a)bd - abd}}$$
$$+ \frac{4\,(Da^3d - Ca^2bd + Bab^2d - Ab^3d)}{\left(\sqrt{bd}bc|b| - \sqrt{bd}ad|b|\right)\left(b^2c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2\right)}$$
$$+ \frac{(3\,Dbc + 3\,Dad - 2\,Cbd)\log\left(\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2\right)}{2\,\sqrt{bd}bd^2|b|}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="gi
ac")
```

output
```
sqrt(b*x + a)*((D*b^6*c^2*d^2 - 2*D*a*b^5*c*d^3 + D*a^2*b^4*d^4)*(b*x + a)
/(b^7*c^2*d^3*abs(b) - 2*a*b^6*c*d^4*abs(b) + a^2*b^5*d^5*abs(b)) + (3*D*b
^7*c^3*d - 3*D*a*b^6*c^2*d^2 - 2*C*b^7*c^2*d^2 + 3*D*a^2*b^5*c*d^3 + 2*B*b
^7*c*d^3 - D*a^3*b^4*d^4 - 2*A*b^7*d^4)/(b^7*c^2*d^3*abs(b) - 2*a*b^6*c*d^
4*abs(b) + a^2*b^5*d^5*abs(b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 4*(D
*a^3*d - C*a^2*b*d + B*a*b^2*d - A*b^3*d)/((sqrt(b*d)*b*c*abs(b) - sqrt(b*
d)*a*d*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)) + 1/2*(3*D*b*c + 3*D*a*d - 2*C*b*d)*log((sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sqrt(b*d)*b*d^
2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^{3/2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.74

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx=\frac{-3\sqrt{d}\,\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{d}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{dx+c}}{\sqrt{ad-bc}}\right)a^2c\,d^2-3\sqrt{d}\,\sqrt{b}\,\sqrt{bx+a}\,\log\left(\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

output
```
( - 3*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*s
qrt(c + d*x))/sqrt(a*d - b*c))*a**2*c*d**2 - 3*sqrt(d)*sqrt(b)*sqrt(a + b*
x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*
*2*d**3*x + 2*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + s
qrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b*c**2*d + 2*sqrt(d)*sqrt(b)*sqrt
(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b
*c))*a*b*c*d**2*x + sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*
x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**2*c**3 + sqrt(d)*sqrt(b)*s
qrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d
- b*c))*b**2*c**2*d*x + 2*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*c*d**2 + 2*sq
rt(d)*sqrt(b)*sqrt(a + b*x)*a**2*d**3*x - 2*sqrt(d)*sqrt(b)*sqrt(a + b*x)*
b**3*c*d - 2*sqrt(d)*sqrt(b)*sqrt(a + b*x)*b**3*d**2*x + 3*sqrt(c + d*x)*a
**2*b*c*d**2 + 3*sqrt(c + d*x)*a**2*b*d**3*x - 2*sqrt(c + d*x)*a*b**3*d**2
 - sqrt(c + d*x)*a*b**2*c**2*d + sqrt(c + d*x)*a*b**2*d**3*x**2 - 2*sqrt(c
 + d*x)*b**4*d**2*x - sqrt(c + d*x)*b**3*c**2*d*x - sqrt(c + d*x)*b**3*c*d
**2*x**2)/(sqrt(a + b*x)*b**3*d**2*(a*c*d + a*d**2*x - b*c**2 - b*c*d*x))
```

3.143 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$

Optimal result . . . 1379
Mathematica [A] (verified) . . . 1380
Rubi [A] (verified) . . . 1380
Maple [B] (verified) . . . 1383
Fricas [B] (verification not implemented) . . . 1384
Sympy [F] . . . 1385
Maxima [F(-2)] . . . 1386
Giac [B] (verification not implemented) . . . 1386
Mupad [F(-1)] . . . 1387
Reduce [F] . . . 1388

Optimal result

Integrand size = 34, antiderivative size = 276

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{3d^2(bc-ad)^2(c+dx)^{3/2}}-\frac{2(Ab^3-a(b^2B-abC+a^2D))}{b^2(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}-\frac{2(3a^2bCd^3-3a^3d^3D+3ab^2d(2cCd-2Bd^2-3c^2D)-b^3(c^2Cd+2Bcd^2-8Ad^3-4c^3D))\sqrt{a+bx}}{3b^2d^2(bc-ad)^3\sqrt{c+dx}}+\frac{2D\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}d^{5/2}}$$

output

```
-2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d^2/(-a*d+b*c)^2/(d*x+c)^
(3/2)-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^2/(-a*d+b*c)^2/(b*x+a)^(1/2)/(d*x+
c)^(1/2)-2/3*(3*a^2*b*C*d^3-3*a^3*d^3*D+3*a*b^2*d*(-2*B*d^2+2*C*c*d-3*D*c^
2)-b^3*(-8*A*d^3+2*B*c*d^2+C*c^2*d-4*D*c^3))*(b*x+a)^(1/2)/b^2/d^2/(-a*d+b
*c)^3/(d*x+c)^(1/2)+2*D*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2
))/b^(3/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = \frac{2(3a^3d^2D(c+dx)^2+a^2bd(8c^3D+2cd^2(B-6Cx)+3d^3x(B-Cx)+c^2(-$$

$$+\frac{2D\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b^{3/2}d^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

output
```
(2*(3*a^3*d^2*D*(c + d*x)^2 + a^2*b*d*(8*c^3*D + 2*c*d^2*(B - 6*C*x) + 3*d
^3*x*(B - C*x) + c^2*(-8*C*d + 9*d*D*x)) + b^3*c*x*(B*d^2*(3*c + 2*d*x) +
c*(-3*c^2*D + C*d^2*x - 4*c*d*D*x)) - A*b*d^2*(-(a^2*d^2) + 2*a*b*d*(3*c +
 2*d*x) + b^2*(3*c^2 + 12*c*d*x + 8*d^2*x^2)) + a*b^2*(2*B*d^2*(3*c^2 + 5*
c*d*x + 3*d^2*x^2) + c*(-3*c^3*D + 4*c^2*d*D*x - 6*C*d^3*x^2 + c*d^2*x*(-4
*C + 9*D*x)))))/(3*b*d^2*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(3/2)) + (2
*D*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(b^(3/2)*d^(5
/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1193, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \ 2124$$

$$-\frac{2\int -\frac{\left(c-\frac{ad}{b}\right)Dx^2+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{3dDa^3-b(3Cd-cD)a^2-b^2(cC-3Bd)a+b^3(Bc-4Ad)}{b^3}}{2\sqrt{a+bx}(c+dx)^{5/2}}dx}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{3dDa^3}{b^3}-\frac{(3Cd-cD)a^2}{b^2}-\frac{(cC-3Bd)a}{b}+\left(c-\frac{ad}{b}\right)Dx^2+Bc-4Ad+\frac{(bc-ad)(bC-aD)x}{b^2}}{\sqrt{a+bx}(c+dx)^{5/2}}dx}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{\frac{2\int \frac{\left(-Dc^3+Cdc^2+2Bd^2c-8Ad^3\right)b^3-3ad\left(-Dc^2+2Cdc-2Bd^2\right)b^2-3a^2d^2(Cd-cD)b+3d(bc-ad)^2Dxb+3a^3d^3D}{2b^2d^2\sqrt{a+bx}(c+dx)^{3/2}}dx}{3(bc-ad)}-\frac{2\sqrt{a+bx}\left(\frac{3a^2d(bC-aD)}{b^3}-B\left(\frac{3ad}{b}+\cdots\right.\right.}{3(c+dx)^{3/2}(bc-\cdots}}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(-Dc^3+Cdc^2+2Bd^2c-8Ad^3\right)b^3-3ad\left(-Dc^2+2Cdc-2Bd^2\right)b^2-3a^2d^2(Cd-cD)b+3d(bc-ad)^2Dxb+3a^3d^3D}{\sqrt{a+bx}(c+dx)^{3/2}}dx}{3b^2d^2(bc-ad)}-\frac{2\sqrt{a+bx}\left(\frac{3a^2d(bC-aD)}{b^3}-B\left(\frac{3ad}{b}+c\cdots\right.\right.}{3(c+dx)^{3/2}(bc-a\cdots}}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{3bD(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx-\frac{2\sqrt{a+bx}\left(-3a^3d^3D+3a^2bCd^3+3ab^2d\left(-2Bd^2-3c^2D+2cCd\right)-\left(b^3\left(-8Ad^3+2Bcd^2-4c^3D+c^2Cd\right)\right)\right)}{\sqrt{c+dx}(bc-ad)}}{3b^2d^2(bc-ad)}-\frac{2\sqrt{a+bx}\left(\cdots\right.}{\cdots}}{bc-ad}-\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{6bD(bc-ad)^2\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}-\frac{2\sqrt{a+bx}\left(-3a^3d^3D+3a^2bCd^3+3ab^2d\left(-2Bd^2-3c^2D+2cCd\right)-\left(b^3\left(-8Ad^3+2Bcd^2-4c^3D+c^2Cd\right)\right)\right)}{\sqrt{c+dx}(bc-ad)}}{3b^2d^2(bc-ad)} - \frac{2\sqrt{a+bx}\Big(}{}}{bc-ad}$$

$$\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\frac{6\sqrt{b}D(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{d}}-\frac{2\sqrt{a+bx}\left(-3a^3d^3D+3a^2bCd^3+3ab^2d\left(-2Bd^2-3c^2D+2cCd\right)-\left(b^3\left(-8Ad^3+2Bcd^2-4c^3D+c^2Cd\right)\right)\right)}{\sqrt{c+dx}(bc-ad)}}{3b^2d^2(bc-ad)} - \frac{2\sqrt{a+bx}}{}}{bc-ad}$$

$$\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

output
```
(-2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3))/((b*c - a*d)*Sqrt[a + b*x]*(c +
 d*x)^(3/2)) + ((-2*((c^2*C)/d + 4*A*d - B*(c + (3*a*d)/b) - (c^3*D)/d^2 +
 (3*a^2*d*(b*C - a*D))/b^3)*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2))
 + ((-2*(3*a^2*b*C*d^3 - 3*a^3*d^3*D + 3*a*b^2*d*(2*c*C*d - 2*B*d^2 - 3*c^
2*D) - b^3*(c^2*C*d + 2*B*c*d^2 - 8*A*d^3 - 4*c^3*D))*Sqrt[a + b*x])/((b*c
 - a*d)*Sqrt[c + d*x]) + (6*Sqrt[b]*(b*c - a*d)^2*D*ArcTanh[(Sqrt[d]*Sqrt[
a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/Sqrt[d])/(3*b^2*d^2*(b*c - a*d)))/(b*c
 - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1892 vs. $2(248) = 496$.

Time = 0.64 (sec) , antiderivative size = 1893, normalized size of antiderivative = 6.86

method	result	size
default	Expression too large to display	1893

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/3*(-12*C*a*b^2*c*d^3*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+18*D*a*b^2
*c^2*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+20*B*a*b^2*c*d^3*x*((b*x+
a)*(d*x+c))^(1/2)*(d*b)^(1/2)-24*C*a^2*b*c*d^3*x*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)-8*C*a*b^2*c^2*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+18*D*a^
2*b*c^2*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+8*D*a*b^2*c^3*d*x*((b*x+
a)*(d*x+c))^(1/2)*(d*b)^(1/2)+12*B*a*b^2*d^4*x^2*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+4*B*b^3*c*d^3*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-6*C*a^2*b
*d^4*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+2*C*b^3*c^2*d^2*x^2*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)-8*D*b^3*c^3*d*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)-8*A*a*b^2*d^4*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-24*A*b^3*c*d^3*
x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+6*B*a^2*b*d^4*x*((b*x+a)*(d*x+c))^(1
/2)*(d*b)^(1/2)+6*B*b^3*c^2*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+12*D
*a^3*c*d^3*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-12*A*a*b^2*c*d^3*((b*x+a)
*(d*x+c))^(1/2)*(d*b)^(1/2)+4*B*a^2*b*c*d^3*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+12*B*a*b^2*c^2*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-16*C*a^2*b*c^
2*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+16*D*a^2*b*c^3*d*((b*x+a)*(d*x+c
))^(1/2)*(d*b)^(1/2)+9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(
1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^2*c*d^4*x^3-9*D*ln(1/2*(2*b*d*x+2*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^3*c^2*d^3*x^3+3*D*l
n(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(249) = 498$.

Time = 14.67 (sec) , antiderivative size = 1732, normalized size of antiderivative = 6.28

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{3/2}(c + dx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fr
icas")
```

output
```
[1/6*(3*(D*a*b^3*c^5 - 3*D*a^2*b^2*c^4*d + 3*D*a^3*b*c^3*d^2 - D*a^4*c^2*d
^3 + (D*b^4*c^3*d^2 - 3*D*a*b^3*c^2*d^3 + 3*D*a^2*b^2*c*d^4 - D*a^3*b*d^5)
*x^3 + (2*D*b^4*c^4*d - 5*D*a*b^3*c^3*d^2 + 3*D*a^2*b^2*c^2*d^3 + D*a^3*b*
c*d^4 - D*a^4*d^5)*x^2 + (D*b^4*c^5 - D*a*b^3*c^4*d - 3*D*a^2*b^2*c^3*d^2
+ 5*D*a^3*b*c^2*d^3 - 2*D*a^4*c*d^4)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a
)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(3*D*a*b^3*c^4*d - 8*D*a^2*
b^2*c^3*d^2 - A*a^2*b^2*d^5 - (3*D*a^3*b - 8*C*a^2*b^2 + 6*B*a*b^3 - 3*A*b
^4)*c^2*d^3 - 2*(B*a^2*b^2 - 3*A*a*b^3)*c*d^4 + (4*D*b^4*c^3*d^2 - (9*D*a*
b^3 + C*b^4)*c^2*d^3 + 2*(3*C*a*b^3 - B*b^4)*c*d^4 - (3*D*a^3*b - 3*C*a^2*
b^2 + 6*B*a*b^3 - 8*A*b^4)*d^5)*x^2 + (3*D*b^4*c^4*d - 4*D*a*b^3*c^3*d^2 -
 (9*D*a^2*b^2 - 4*C*a*b^3 + 3*B*b^4)*c^2*d^3 - 2*(3*D*a^3*b - 6*C*a^2*b^2
+ 5*B*a*b^3 - 6*A*b^4)*c*d^4 - (3*B*a^2*b^2 - 4*A*a*b^3)*d^5)*x)*sqrt(b*x
+ a)*sqrt(d*x + c))/(a*b^5*c^5*d^3 - 3*a^2*b^4*c^4*d^4 + 3*a^3*b^3*c^3*d^5
 - a^4*b^2*c^2*d^6 + (b^6*c^3*d^5 - 3*a*b^5*c^2*d^6 + 3*a^2*b^4*c*d^7 - a^
3*b^3*d^8)*x^3 + (2*b^6*c^4*d^4 - 5*a*b^5*c^3*d^5 + 3*a^2*b^4*c^2*d^6 + a^
3*b^3*c*d^7 - a^4*b^2*d^8)*x^2 + (b^6*c^5*d^3 - a*b^5*c^4*d^4 - 3*a^2*b^4*
c^3*d^5 + 5*a^3*b^3*c^2*d^6 - 2*a^4*b^2*c*d^7)*x), -1/3*(3*(D*a*b^3*c^5 -
3*D*a^2*b^2*c^4*d + 3*D*a^3*b*c^3*d^2 - D*a^4*c^2*d^3 + (D*b^4*c^3*d^2 - 3
*D*a*b^3*c^2*d^3 + 3*D*a^2*b^2*c*d^4 - D*a^3*b*d^5)*x^3 + (2*D*b^4*c^4*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)**(3/2)*(c + d*x)**(5/2)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(249) = 498$.

Time = 0.37 (sec) , antiderivative size = 817, normalized size of antiderivative = 2.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="gi
ac")
```

output
```
-2/3*sqrt(b*x + a)*((4*D*b^8*c^5*d^2 - 17*D*a*b^7*c^4*d^3 - C*b^8*c^4*d^3
+ 22*D*a^2*b^6*c^3*d^4 + 8*C*a*b^7*c^3*d^4 - 2*B*b^8*c^3*d^4 - 9*D*a^3*b^5
*c^2*d^5 - 13*C*a^2*b^6*c^2*d^5 + B*a*b^7*c^2*d^5 + 5*A*b^8*c^2*d^5 + 6*C*
a^3*b^5*c*d^6 + 4*B*a^2*b^6*c*d^6 - 10*A*a*b^7*c*d^6 - 3*B*a^3*b^5*d^7 + 5
*A*a^2*b^6*d^7)*(b*x + a)/(b^7*c^5*d^3*abs(b) - 5*a*b^6*c^4*d^4*abs(b) + 1
0*a^2*b^5*c^3*d^5*abs(b) - 10*a^3*b^4*c^2*d^6*abs(b) + 5*a^4*b^3*c*d^7*abs
(b) - a^5*b^2*d^8*abs(b)) + 3*(D*b^9*c^6*d - 6*D*a*b^8*c^5*d^2 + 12*D*a^2*
b^7*c^4*d^3 + 2*C*a*b^8*c^4*d^3 - B*b^9*c^4*d^3 - 10*D*a^3*b^6*c^3*d^4 - 6
*C*a^2*b^7*c^3*d^4 + 2*B*a*b^8*c^3*d^4 + 2*A*b^9*c^3*d^4 + 3*D*a^4*b^5*c^2
*d^5 + 6*C*a^3*b^6*c^2*d^5 - 6*A*a*b^8*c^2*d^5 - 2*C*a^4*b^5*c*d^6 - 2*B*a
^3*b^6*c*d^6 + 6*A*a^2*b^7*c*d^6 + B*a^4*b^5*d^7 - 2*A*a^3*b^6*d^7)/(b^7*c
^5*d^3*abs(b) - 5*a*b^6*c^4*d^4*abs(b) + 10*a^2*b^5*c^3*d^5*abs(b) - 10*a^
3*b^4*c^2*d^6*abs(b) + 5*a^4*b^3*c*d^7*abs(b) - a^5*b^2*d^8*abs(b)))/(b^2*
c + (b*x + a)*b*d - a*b*d)^(3/2) + 4*(sqrt(b*d)*D*a^3 - sqrt(b*d)*C*a^2*b
+ sqrt(b*d)*B*a*b^2 - sqrt(b*d)*A*b^3)/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b)
 + a^2*d^2*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2)) - D*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^2)/(sqrt(b*d)*d^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{3/2}(c + dx)^{5/2}}\,dx = \int \frac{A + B\,x + C\,x^2 + x^3\,D}{(a + b\,x)^{3/2}\,(c + d\,x)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

3.144 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx$

Optimal result 1389
Mathematica [A] (verified) 1390
Rubi [A] (verified) 1390
Maple [A] (verified) 1393
Fricas [B] (verification not implemented) 1394
Sympy [F] 1395
Maxima [F(-2)] 1395
Giac [B] (verification not implemented) 1396
Mupad [F(-1)] 1397
Reduce [F] 1397

Optimal result

Integrand size = 34, antiderivative size = 355

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx = -\frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{5d^2(bc-ad)^2(c+dx)^{5/2}} - \frac{2(Ab^3-a(b^2B-abC+a^2D))}{b^2(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/2}} - \frac{2(15a^2bCd^3-15a^3d^3D+5ab^2d(2cCd-4Bd^2-3c^2D)-b^3(c^2Cd+4Bcd^2-24Ad^3-6c^3D))\sqrt{a+bx}}{15b^2d^2(bc-ad)^3(c+dx)^{3/2}} + \frac{2(15a^3d^3D-15a^2bd^2(2Cd-3cD)-5ab^2d(4cCd-8Bd^2+3c^2D)+b^3(2c^2Cd+8Bcd^2-48Ad^3+3c^3I}{15bd^2(bc-ad)^4\sqrt{c+dx}}$$

output
```
-2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d^2/(-a*d+b*c)^2/(d*x+c)^
(5/2)-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^2/(-a*d+b*c)^2/(b*x+a)^(1/2)/(d*x+
c)^(3/2)-2/15*(15*a^2*b*C*d^3-15*a^3*d^3*D+5*a*b^2*d*(-4*B*d^2+2*C*c*d-3*D
*c^2)-b^3*(-24*A*d^3+4*B*c*d^2+C*c^2*d-6*D*c^3))*(b*x+a)^(1/2)/b^2/d^2/(-a
*d+b*c)^3/(d*x+c)^(3/2)+2/15*(15*a^3*d^3*D-15*a^2*b*d^2*(2*C*d-3*D*c)-5*a*
b^2*d*(-8*B*d^2+4*C*c*d+3*D*c^2)+b^3*(-48*A*d^3+8*B*c*d^2+2*C*c^2*d+3*D*c^
3))*(b*x+a)^(1/2)/b/d^2/(-a*d+b*c)^4/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx =$$
$$-\frac{2(a+bx)^{5/2}\left(3c^2Cd-3Bcd^2+3Ad^3-3c^3D-\frac{5bc^2C(c+dx)}{a+bx}+\frac{10bBcd(c+dx)}{a+bx}-\frac{10acCd(c+dx)}{a+bx}-\frac{15Abd^2(c+dx)}{a+bx}+\cdots\right.}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(3/2)*(c + d*x)^(7/2)),x]
```

output
```
(-2*(a + b*x)^(5/2)*(3*c^2*C*d - 3*B*c*d^2 + 3*A*d^3 - 3*c^3*D - (5*b*c^2*
C*(c + d*x))/(a + b*x) + (10*b*B*c*d*(c + d*x))/(a + b*x) - (10*a*c*C*d*(c
 + d*x))/(a + b*x) - (15*A*b*d^2*(c + d*x))/(a + b*x) + (5*a*B*d^2*(c + d*
x))/(a + b*x) + (15*a*c^2*D*(c + d*x))/(a + b*x) - (15*b^2*B*c*(c + d*x)^2
)/(a + b*x)^2 + (30*a*b*c*C*(c + d*x)^2)/(a + b*x)^2 + (45*A*b^2*d*(c + d*
x)^2)/(a + b*x)^2 - (30*a*b*B*d*(c + d*x)^2)/(a + b*x)^2 + (15*a^2*C*d*(c
+ d*x)^2)/(a + b*x)^2 - (45*a^2*c*D*(c + d*x)^2)/(a + b*x)^2 + (15*A*b^3*(
c + d*x)^3)/(a + b*x)^3 - (15*a*b^2*B*(c + d*x)^3)/(a + b*x)^3 + (15*a^2*b
*C*(c + d*x)^3)/(a + b*x)^3 - (15*a^3*D*(c + d*x)^3)/(a + b*x)^3))/(15*(b*
c - a*d)^4*(c + d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2124, 27, 1193, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx$$
$$\Big\downarrow \; 2124$$

$$-\frac{2\int -\frac{\left(c-\frac{ad}{b}\right)Dx^2+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{5dDa^3-b(5Cd-cD)a^2-b^2(cC-5Bd)a+b^3(Bc-6Ad)}{b^3}}{2\sqrt{a+bx}(c+dx)^{7/2}}dx}{bc-ad}-\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\frac{5dDa^3}{b^3}-\frac{(5Cd-cD)a^2}{b^2}-\frac{(cC-5Bd)a}{b}+\left(c-\frac{ad}{b}\right)Dx^2+Bc-6Ad+\frac{(bc-ad)(bC-aD)x}{b^2}}{\sqrt{a+bx}(c+dx)^{7/2}}dx}{bc-ad}-\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$\frac{\frac{2\int \frac{\left(-Dc^3+Cdc^2+4Bd^2c-24Ad^3\right)b^3-5ad\left(-Dc^2+2Cdc-4Bd^2\right)b^2-5a^2d^2(3Cd-cD)b+5d(bc-ad)^2Dxb+15a^3d^3D}{2b^2d^2\sqrt{a+bx}(c+dx)^{5/2}}dx}{5(bc-ad)}-\frac{2\sqrt{a+bx}\left(\frac{5a^2d(bC-aD)}{b^3}-B\left(\frac{5a}{b}\right.\right.}{5(c+dx)^{5/2}(\ldots}}{bc-ad}-\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(-Dc^3+Cdc^2+4Bd^2c-24Ad^3\right)b^3-5ad\left(-Dc^2+2Cdc-4Bd^2\right)b^2-5a^2d^2(3Cd-cD)b+5d(bc-ad)^2Dxb+15a^3d^3D}{\sqrt{a+bx}(c+dx)^{5/2}}dx}{5b^2d^2(bc-ad)}-\frac{2\sqrt{a+bx}\left(\frac{5a^2d(bC-aD)}{b^3}-B\left(\frac{5ad}{b}\right.\right.}{5(c+dx)^{5/2}(bc\ldots}}{bc-ad}-\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$\frac{\frac{\frac{b\left(15a^3d^3D-15a^2bd^2(2Cd-3cD)-5ab^2d\left(-8Bd^2+3c^2D+4cCd\right)+b^3\left(-48Ad^3+8Bcd^2+3c^3D+2c^2Cd\right)\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}}dx}{3(bc-ad)}-\frac{2\sqrt{a+bx}\left(-15a^3d^3D+15a^2bC\ldots\right.}{\ldots}}{5b^2d^2(bc-ad)}}{bc-ad}-\frac{2\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)}$$

$\Big\downarrow$ 48

$$\frac{\frac{\frac{2b\sqrt{a+bx}\left(15a^3d^3D-15a^2bd^2(2Cd-3cD)-5ab^2d\left(-8Bd^2+3c^2D+4cCd\right)+b^3\left(-48Ad^3+8Bcd^2+3c^3D+2c^2Cd\right)\right)}{3\sqrt{c+dx}(bc-ad)^2}-\frac{2\sqrt{a+bx}\left(-15a^3d^3D+15a^2bCd^3+5ab^2d\left(-4\right.\right.}{3(}}{5b^2d^2(bc-ad)}}{bc-ad}$$

$$\frac{2\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(3/2)*(c + d*x)^(7/2)),x]
```

output
```
(-2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3))/((b*c - a*d)*Sqrt[a + b*x]*(c +
 d*x)^(5/2)) + ((-2*((c^2*C)/d + 6*A*d - B*(c + (5*a*d)/b) - (c^3*D)/d^2 +
 (5*a^2*d*(b*C - a*D))/b^3)*Sqrt[a + b*x])/(5*(b*c - a*d)*(c + d*x)^(5/2))
 + ((-2*(15*a^2*b*C*d^3 - 15*a^3*d^3*D + 5*a*b^2*d*(2*c*C*d - 4*B*d^2 - 3*
c^2*D) - b^3*(c^2*C*d + 4*B*c*d^2 - 24*A*d^3 - 6*c^3*D))*Sqrt[a + b*x])/(3
*(b*c - a*d)*(c + d*x)^(3/2)) + (2*b*(15*a^3*d^3*D - 15*a^2*b*d^2*(2*C*d -
 3*c*D) - 5*a*b^2*d*(4*c*C*d - 8*B*d^2 + 3*c^2*D) + b^3*(2*c^2*C*d + 8*B*c
*d^2 - 48*A*d^3 + 3*c^3*D))*Sqrt[a + b*x])/(3*(b*c - a*d)^2*Sqrt[c + d*x])
)/(5*b^2*d^2*(b*c - a*d)))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.55

method	result
default	$-\frac{2(48Ab^3d^3x^3-40Bab^2d^3x^3-8Bb^3cd^2x^3+30Ca^2bd^3x^3+20Cab^2cd^2x^3-2Cb^3c^2dx^3-15Da^3d^3x^3-45Da^2bcd^2x^3+15Dab^2c}{}$
gosper	$-\frac{2(48Ab^3d^3x^3-40Bab^2d^3x^3-8Bb^3cd^2x^3+30Ca^2bd^3x^3+20Cab^2cd^2x^3-2Cb^3c^2dx^3-15Da^3d^3x^3-45Da^2bcd^2x^3+15Dab^2c}{}$
orering	$-\frac{2(48Ab^3d^3x^3-40Bab^2d^3x^3-8Bb^3cd^2x^3+30Ca^2bd^3x^3+20Cab^2cd^2x^3-2Cb^3c^2dx^3-15Da^3d^3x^3-45Da^2bcd^2x^3+15Dab^2c}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/15*(48*A*b^3*d^3*x^3-40*B*a*b^2*d^3*x^3-8*B*b^3*c*d^2*x^3+30*C*a^2*b*d^
3*x^3+20*C*a*b^2*c*d^2*x^3-2*C*b^3*c^2*d*x^3-15*D*a^3*d^3*x^3-45*D*a^2*b*c
*d^2*x^3+15*D*a*b^2*c^2*d*x^3-3*D*b^3*c^3*x^3+24*A*a*b^2*d^3*x^2+120*A*b^3
*c*d^2*x^2-20*B*a^2*b*d^3*x^2-104*B*a*b^2*c*d^2*x^2-20*B*b^3*c^2*d*x^2+15*
C*a^3*d^3*x^2+85*C*a^2*b*c*d^2*x^2+49*C*a*b^2*c^2*d*x^2-5*C*b^3*c^3*x^2-90
*D*a^3*c*d^2*x^2-60*D*a^2*b*c^2*d*x^2+6*D*a*b^2*c^3*x^2-6*A*a^2*b*d^3*x+60
*A*a*b^2*c*d^2*x+90*A*b^3*c^2*d*x+5*B*a^3*d^3*x-49*B*a^2*b*c*d^2*x-85*B*a*
b^2*c^2*d*x-15*B*b^3*c^3*x+20*C*a^3*c*d^2*x+104*C*a^2*b*c^2*d*x+20*C*a*b^2
*c^3*x-120*D*a^3*c^2*d*x-24*D*a^2*b*c^3*x+3*A*a^3*d^3-15*A*a^2*b*c*d^2+45*
A*a*b^2*c^2*d+15*A*b^3*c^3+2*B*a^3*c*d^2-20*B*a^2*b*c^2*d-30*B*a*b^2*c^3+8
*C*a^3*c^2*d+40*C*a^2*b*c^3-48*D*a^3*c^3)/(d*x+c)^(5/2)/(b*x+a)^(1/2)/(a*d
-b*c)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 748 vs. $2(334) = 668$.

Time = 33.65 (sec) , antiderivative size = 748, normalized size of antiderivative = 2.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx =$$
$$-\frac{2\,(3\,Aa^3d^3-(48\,Da^3-40\,Ca^2b+30\,Bab^2-15\,Ab^3)c^3+(8\,Ca^3-20\,Ba^2b+45\,Aab^2)c^2d+(2\,Ba^3-15\,\cdots}{\cdots}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(7/2),x, algorithm="fr
icas")
```

output
```
-2/15*(3*A*a^3*d^3 - (48*D*a^3 - 40*C*a^2*b + 30*B*a*b^2 - 15*A*b^3)*c^3 +
 (8*C*a^3 - 20*B*a^2*b + 45*A*a*b^2)*c^2*d + (2*B*a^3 - 15*A*a^2*b)*c*d^2
- (3*D*b^3*c^3 - (15*D*a*b^2 - 2*C*b^3)*c^2*d + (45*D*a^2*b - 20*C*a*b^2 +
 8*B*b^3)*c*d^2 + (15*D*a^3 - 30*C*a^2*b + 40*B*a*b^2 - 48*A*b^3)*d^3)*x^3
 + ((6*D*a*b^2 - 5*C*b^3)*c^3 - (60*D*a^2*b - 49*C*a*b^2 + 20*B*b^3)*c^2*d
 - (90*D*a^3 - 85*C*a^2*b + 104*B*a*b^2 - 120*A*b^3)*c*d^2 + (15*C*a^3 - 2
0*B*a^2*b + 24*A*a*b^2)*d^3)*x^2 - ((24*D*a^2*b - 20*C*a*b^2 + 15*B*b^3)*c
^3 + (120*D*a^3 - 104*C*a^2*b + 85*B*a*b^2 - 90*A*b^3)*c^2*d - (20*C*a^3 -
 49*B*a^2*b + 60*A*a*b^2)*c*d^2 - (5*B*a^3 - 6*A*a^2*b)*d^3)*x)*sqrt(b*x +
 a)*sqrt(d*x + c)/(a*b^4*c^7 - 4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4
*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*c^4*d^3 - 4*a*b^4*c^3*d^4 + 6*a^2*b^3*c^2*
d^5 - 4*a^3*b^2*c*d^6 + a^4*b*d^7)*x^4 + (3*b^5*c^5*d^2 - 11*a*b^4*c^4*d^3
 + 14*a^2*b^3*c^3*d^4 - 6*a^3*b^2*c^2*d^5 - a^4*b*c*d^6 + a^5*d^7)*x^3 + 3
*(b^5*c^6*d - 3*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3 + 2*a^3*b^2*c^3*d^4 - 3*
a^4*b*c^2*d^5 + a^5*c*d^6)*x^2 + (b^5*c^7 - a*b^4*c^6*d - 6*a^2*b^3*c^5*d^
2 + 14*a^3*b^2*c^4*d^3 - 11*a^4*b*c^3*d^4 + 3*a^5*c^2*d^5)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(7/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)**(3/2)*(c + d*x)**(7/2)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(7/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2300 vs. $2(334) = 668$.

Time = 0.53 (sec) , antiderivative size = 2300, normalized size of antiderivative = 6.48

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{3/2}(c + dx)^{7/2}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(7/2),x, algorithm="gi
ac")
```

output
```
4*(D^2*a^6*b^3*d - 2*C*D*a^5*b^4*d + C^2*a^4*b^5*d + 2*B*D*a^4*b^5*d - 2*B
*C*a^3*b^6*d - 2*A*D*a^3*b^6*d + B^2*a^2*b^7*d + 2*A*C*a^2*b^7*d - 2*A*B*a
*b^8*d + A^2*b^9*d)/((sqrt(b*d)*D*a^3*b^3*c - sqrt(b*d)*C*a^2*b^4*c + sqrt
(b*d)*B*a*b^5*c - sqrt(b*d)*A*b^6*c - sqrt(b*d)*D*a^4*b^2*d + sqrt(b*d)*C*
a^3*b^3*d - sqrt(b*d)*B*a^2*b^4*d + sqrt(b*d)*A*a*b^5*d - sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*D*a^3*b + sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*C*
a^2*b^2 - sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^2*B*a*b^3 + sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2*A*b^4)*(b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*
a^2*b*c*d^2*abs(b) - a^3*d^3*abs(b))) + 2/15*((b*x + a)*((3*D*b^13*c^8*d^2
 - 30*D*a*b^12*c^7*d^3 + 2*C*b^13*c^7*d^3 + 150*D*a^2*b^11*c^6*d^4 - 30*C*
a*b^12*c^6*d^4 + 8*B*b^13*c^6*d^4 - 405*D*a^3*b^10*c^5*d^5 + 105*C*a^2*b^1
1*c^5*d^5 - 15*B*a*b^12*c^5*d^5 - 33*A*b^13*c^5*d^5 + 615*D*a^4*b^9*c^4*d^
6 - 145*C*a^3*b^10*c^4*d^6 - 45*B*a^2*b^11*c^4*d^6 + 165*A*a*b^12*c^4*d^6
 - 528*D*a^5*b^8*c^3*d^7 + 60*C*a^4*b^9*c^3*d^7 + 170*B*a^3*b^10*c^3*d^7 -
330*A*a^2*b^11*c^3*d^7 + 240*D*a^6*b^7*c^2*d^8 + 48*C*a^5*b^8*c^2*d^8 - 21
0*B*a^4*b^9*c^2*d^8 + 330*A*a^3*b^10*c^2*d^8 - 45*D*a^7*b^6*c*d^9 - 55*C*a
^6*b^7*c*d^9 + 117*B*a^5*b^8*c*d^9 - 165*A*a^4*b^9*c*d^9 + 15*C*a^7*b^6*d^
10 - 25*B*a^6*b^7*d^10 + 33*A*a^5*b^8*d^10)*(b*x + a)/(b^11*c^9*d^2*abs...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{7/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(7/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(7/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{3/2}(c+dx)^{7/2}}\,dx=\int \frac{D x^3+C\,x^2+Bx+A}{(bx+a)^{\frac{3}{2}}\,(dx+c)^{\frac{7}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(7/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(7/2),x)
```

3.145 $\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx$

Optimal result . . . 1398
Mathematica [C] (verified) . . . 1399
Rubi [A] (verified) . . . 1400
Maple [B] (verified) . . . 1406
Fricas [B] (verification not implemented) . . . 1407
Sympy [F] . . . 1408
Maxima [F(-2)] . . . 1409
Giac [B] (verification not implemented) . . . 1409
Mupad [F(-1)] . . . 1410
Reduce [F] . . . 1411

Optimal result

Integrand size = 34, antiderivative size = 520

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx =$$

$$-\frac{5(231a^3d^3D-21a^2bd^2(8Cd+9cD)+7ab^2d(16cCd+16Bd^2+3c^2D)-b^3(8c^2Cd+48Bcd^2+64Ad^3-$$

$$64b^6d$$

$$-\frac{2(b^3(3Bc+5Ad)-2ab^2(3cC+4Bd)-14a^3dD+a^2b(11Cd+9cD))(c+dx)^{3/2}}{3b^5\sqrt{a+bx}}$$

$$+\frac{(117a^2d^2D-2abd(28Cd+33cD)+b^2(24cCd+16Bd^2-3c^2D))\sqrt{a+bx}(c+dx)^{3/2}}{32b^5d}$$

$$+\frac{(8bCd-bcD-23adD)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^5}$$

$$-\frac{2(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{5/2}}{3b^4(a+bx)^{3/2}}+\frac{D\sqrt{a+bx}(c+dx)^{7/2}}{4b^3d}$$

$$-\frac{5(bc-ad)(231a^3d^3D-21a^2bd^2(8Cd+9cD)+7ab^2d(16cCd+16Bd^2+3c^2D)-b^3(8c^2Cd+48Bcd^2+$$

$$64b^{13/2}d^{3/2}$$

output
```
-5/64*(231*a^3*d^3*D-21*a^2*b*d^2*(8*C*d+9*D*c)+7*a*b^2*d*(16*B*d^2+16*C*c
*d+3*D*c^2)-b^3*(64*A*d^3+48*B*c*d^2+8*C*c^2*d-D*c^3))*(b*x+a)^(1/2)*(d*x+
c)^(1/2)/b^6/d-2/3*(b^3*(5*A*d+3*B*c)-2*a*b^2*(4*B*d+3*C*c)-14*a^3*d*D+a^2
*b*(11*C*d+9*D*c))*(d*x+c)^(3/2)/b^5/(b*x+a)^(1/2)+1/32*(117*a^2*d^2*D-2*a
*b*d*(28*C*d+33*D*c)+b^2*(16*B*d^2+24*C*c*d-3*D*c^2))*(b*x+a)^(1/2)*(d*x+c
)^(3/2)/b^5/d+1/24*(8*C*b*d-23*D*a*d-D*b*c)*(b*x+a)^(3/2)*(d*x+c)^(3/2)/b^
5-2/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(5/2)/b^4/(b*x+a)^(3/2)+1/4*D*
(b*x+a)^(1/2)*(d*x+c)^(7/2)/b^3/d-5/64*(-a*d+b*c)*(231*a^3*d^3*D-21*a^2*b*
d^2*(8*C*d+9*D*c)+7*a*b^2*d*(16*B*d^2+16*C*c*d+3*D*c^2)-b^3*(64*A*d^3+48*B
*c*d^2+8*C*c^2*d-D*c^3))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/
2))/b^(13/2)/d^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.32 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}\,dx = \frac{2(c+dx)^{5/2}\left(-(bc-ad)^3D\,\text{Hypergeometric2F1}\left(-\frac{11}{2},-\frac{3}{2},-\right.\right.}{}$$

input
```
Integrate[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(5/2),x]
```

output
```
(2*(c + d*x)^(5/2)*(-((b*c - a*d)^3*D*Hypergeometric2F1[-11/2, -3/2, -1/2,
 (d*(a + b*x))/(-(b*c) + a*d)]) + b*(b*c - a*d)^2*(-(C*d) + 3*c*D)*Hyperge
ometric2F1[-9/2, -3/2, -1/2, (d*(a + b*x))/(-(b*c) + a*d)] - b^2*(b*c - a*
d)*(-2*c*C*d + B*d^2 + 3*c^2*D)*Hypergeometric2F1[-7/2, -3/2, -1/2, (d*(a
+ b*x))/(-(b*c) + a*d)] + b^3*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*Hyper
geometric2F1[-5/2, -3/2, -1/2, (d*(a + b*x))/(-(b*c) + a*d)]))/(3*b^4*d^3*
(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2124, 27, 1193, 27, 90, 60, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \ 2124$$

$$-\frac{2\int -\frac{(c+dx)^{5/2}\left(3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{-7dDa^3+b(7Cd+3cD)a^2-b^2(3cC+7Bd)a+b^3(3Bc+4Ad)}{b^3}\right)}{2(a+bx)^{3/2}}dx}{3(bc-ad)}-\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(c+dx)^{5/2}\left(-\frac{7dDa^3}{b^3}+\frac{(7Cd+3cD)a^2}{b^2}-\frac{(3cC+7Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc+4Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^{3/2}}dx}{3(bc-ad)}-\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 1193$$

$$\frac{-\frac{2\int -\frac{3(c+dx)^{5/2}\left(-28d^2Da^3+bd(21Cd+22cD)a^2-2b^2\left(Dc^2+7Cdc+7Bd^2\right)a+b^3\left(Cc^2+6Bdc+8Ad^2\right)+b(bc-ad)^2Dx\right)}{2b^3\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{7/2}(-13a^3dD+a^2b\cdots}{\cdots}}{3(bc-ad)}-\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{3\int \frac{(c+dx)^{5/2}\left(-28d^2Da^3+bd(21Cd+22cD)a^2-2b^2\left(Dc^2+7Cdc+7Bd^2\right)a+b^3\left(Cc^2+6Bdc+8Ad^2\right)+b(bc-ad)^2Dx\right)}{\sqrt{a+bx}}dx}{b^3(bc-ad)}-\frac{2(c+dx)^{7/2}(-13a^3dD+a^2b(9cD\cdots}{b^3\sqrt{\cdots}}}{3(bc-ad)}-\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 90

$$\frac{3\left(\frac{D\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)^2}{4d}-\frac{\left(231a^3d^3D-21a^2bd^2(9cD+8Cd)+7ab^2d\left(16Bd^2+3c^2D+16cCd\right)-\left(b^3\left(64Ad^3+48Bcd^2+c^3(-D)+8c^2Cd\right)\right)\right)\int\frac{(c+dx)^{5/2}}{\sqrt{a+bx}}dx}{8d}}{b^3(bc-ad)}}{3(bc-ad)}$$

$$\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{3\left(\frac{D\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)^2}{4d}-\frac{\left(231a^3d^3D-21a^2bd^2(9cD+8Cd)+7ab^2d\left(16Bd^2+3c^2D+16cCd\right)-\left(b^3\left(64Ad^3+48Bcd^2+c^3(-D)+8c^2Cd\right)\right)\right)\left(\frac{5(bc-ad)\int\frac{(c}{}}{6b}\right.}{8d}}{b^3(bc-ad)}}{3(bc-ad)}$$

$$\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{3\left(\frac{D\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)^2}{4d}-\frac{\left(231a^3d^3D-21a^2bd^2(9cD+8Cd)+7ab^2d\left(16Bd^2+3c^2D+16cCd\right)-\left(b^3\left(64Ad^3+48Bcd^2+c^3(-D)+8c^2Cd\right)\right)\right)\left(\frac{5(bc-ad)\left(\frac{3(}{}\right.}{}\right.}{8d}}{b^3(bc-ad)}}{3(bc-}$$

$$\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{3\left(\frac{D\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)^2}{4d}-\frac{\left(231a^3d^3D-21a^2bd^2(9cD+8Cd)+7ab^2d\left(16Bd^2+3c^2D+16cCd\right)-\left(b^3\left(64Ad^3+48Bcd^2+c^3(-D)+8c^2Cd\right)\right)\right)\left(\frac{5(bc-ad)\left(3(\cdots\right.}{8d}\cdots\right.}{\cdots}\right.}{b^3(bc-ad)\cdots}\cdots$$

$$\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 66$$

$$\frac{3\left(\frac{D\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)^2}{4d}-\frac{\left(231a^3d^3D-21a^2bd^2(9cD+8Cd)+7ab^2d\left(16Bd^2+3c^2D+16cCd\right)-\left(b^3\left(64Ad^3+48Bcd^2+c^3(-D)+8c^2Cd\right)\right)\right)\left(5(bc-ad)\left(3(\ldots\right.\right.}{8d\ldots}\right.}{b^3(bc-ad)\ldots}$$

$$\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 221

$$\frac{3\left(\frac{D\sqrt{a+bx}(c+dx)^{7/2}(bc-ad)^2}{4d}-\frac{\left(\frac{5(bc-ad)\left(\frac{3(bc-ad)\left(\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4b}+\frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b}\right)}{6b}+\frac{\sqrt{a+bx}(c+dx)^{5/2}}{3b}\right)(2}{8d}}{b^3(bc-ad)}$$

$$\frac{2(c+dx)^{7/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

input
```
Int[((c + d*x)^(5/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(5/2),x]
```

output
```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(7/2))/(3*b^3*(b*c - a*d
)*(a + b*x)^(3/2)) + ((-2*(b^3*(3*B*c + 4*A*d) - a*b^2*(6*c*C + 7*B*d) - 1
3*a^3*d*D + a^2*b*(10*C*d + 9*c*D))*(c + d*x)^(7/2))/(b^3*(b*c - a*d)*Sqrt
[a + b*x]) + (3*(((b*c - a*d)^2*D*Sqrt[a + b*x]*(c + d*x)^(7/2))/(4*d) - (
(231*a^3*d^3*D - 21*a^2*b*d^2*(8*C*d + 9*c*D) + 7*a*b^2*d*(16*c*C*d + 16*B
*d^2 + 3*c^2*D) - b^3*(8*c^2*C*d + 48*B*c*d^2 + 64*A*d^3 - c^3*D))*((Sqrt[
a + b*x]*(c + d*x)^(5/2))/(3*b) + (5*(b*c - a*d)*((Sqrt[a + b*x]*(c + d*x)
^(3/2))/(2*b) + (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c -
a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sq
rt[d])))/(4*b)))/(6*b)))/(8*d)))/(b^3*(b*c - a*d)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3663 vs. $2(470) = 940$.

Time = 0.57 (sec) , antiderivative size = 3664, normalized size of antiderivative = 7.05

method	result	size
default	Expression too large to display	3664

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOS
E)
```

output

```
-1/384*(d*x+c)^(1/2)*(5040*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b^2*d^4*x-6930*D*ln(1/2*(2*b*d*x+2*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^5*b*d^4*x-960*A*ln
(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*
a^2*b^4*c*d^3-3150*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)
+a*d+b*c)/(d*b)^(1/2))*a^4*b^2*c^2*d^2-1920*A*(d*b)^(1/2)*((b*x+a)*(d*x+c)
)^(1/2)*a^2*b^3*d^3-1808*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*c^2
*d+288*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a*b^4*d^3*x^3-396*D*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*d^3*x^3-236*D*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)*b^5*c^2*d*x^3-96*D*b^5*d^3*x^5*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1
/2)+2400*B*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/
(d*b)^(1/2))*a*b^5*c*d^3*x^2-10290*D*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)*a
^4*b*c*d^2-528*C*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*b^5*c^2*d*x^2-30*D*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)*a^2*b^3*c^3-120*C*ln(1/2*(2*b*d*x+2*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^4*c^3*d+300*D*
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2)
)*a^3*b^3*c^3*d-128*C*b^5*d^3*x^4*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+256*
A*b^5*c^2*d*(d*b)^(1/2)*((b*x+a)*(d*x+c))^(1/2)+966*D*((b*x+a)*(d*x+c))^(1
/2)*(d*b)^(1/2)*a*b^4*c^2*d*x^2+15*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(
1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^6*c^4*x^2+15*D*ln(1/2*(2*b*d*x...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(468) = 936.

Time = 18.86 (sec) , antiderivative size = 2106, normalized size of antiderivative = 4.05

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="fr
icas")
```

output
```
[1/768*(15*(D*a^2*b^4*c^4 + 4*(5*D*a^3*b^3 - 2*C*a^2*b^4)*c^3*d - 6*(35*D*
a^4*b^2 - 20*C*a^3*b^3 + 8*B*a^2*b^4)*c^2*d^2 + 4*(105*D*a^5*b - 70*C*a^4*
b^2 + 40*B*a^3*b^3 - 16*A*a^2*b^4)*c*d^3 - (231*D*a^6 - 168*C*a^5*b + 112*
B*a^4*b^2 - 64*A*a^3*b^3)*d^4 + (D*b^6*c^4 + 4*(5*D*a*b^5 - 2*C*b^6)*c^3*d
 - 6*(35*D*a^2*b^4 - 20*C*a*b^5 + 8*B*b^6)*c^2*d^2 + 4*(105*D*a^3*b^3 - 70
*C*a^2*b^4 + 40*B*a*b^5 - 16*A*b^6)*c*d^3 - (231*D*a^4*b^2 - 168*C*a^3*b^3
 + 112*B*a^2*b^4 - 64*A*a*b^5)*d^4)*x^2 + 2*(D*a*b^5*c^4 + 4*(5*D*a^2*b^4
- 2*C*a*b^5)*c^3*d - 6*(35*D*a^3*b^3 - 20*C*a^2*b^4 + 8*B*a*b^5)*c^2*d^2 +
 4*(105*D*a^4*b^2 - 70*C*a^3*b^3 + 40*B*a^2*b^4 - 16*A*a*b^5)*c*d^3 - (231
*D*a^5*b - 168*C*a^4*b^2 + 112*B*a^3*b^3 - 64*A*a^2*b^4)*d^4)*x)*sqrt(b*d)
*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*
d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(4
8*D*b^6*d^4*x^5 + 15*D*a^2*b^4*c^3*d - (1743*D*a^3*b^3 - 904*C*a^2*b^4 + 2
56*B*a*b^5 + 128*A*b^6)*c^2*d^2 + 5*(1029*D*a^4*b^2 - 672*C*a^3*b^3 + 368*
B*a^2*b^4 - 128*A*a*b^5)*c*d^3 - 15*(231*D*a^5*b - 168*C*a^4*b^2 + 112*B*a
^3*b^3 - 64*A*a^2*b^4)*d^4 + 8*(17*D*b^6*c*d^3 - (11*D*a*b^5 - 8*C*b^6)*d^
4)*x^4 + 2*(59*D*b^6*c^2*d^2 - 2*(79*D*a*b^5 - 52*C*b^6)*c*d^3 + 3*(33*D*a
^2*b^4 - 24*C*a*b^5 + 16*B*b^6)*d^4)*x^3 + 3*(5*D*b^6*c^3*d - (161*D*a*b^5
 - 88*C*b^6)*c^2*d^2 + (387*D*a^2*b^4 - 256*C*a*b^5 + 144*B*b^6)*c*d^3 - (
231*D*a^3*b^3 - 168*C*a^2*b^4 + 112*B*a*b^5 - 64*A*b^6)*d^4)*x^2 + 2*(1...
```

Sympy [F]

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx=\int \frac{(c+dx)^{\frac{5}{2}}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(5/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Integral((c + d*x)**(5/2)*(A + B*x + C*x**2 + D*x**3)/(a + b*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2808 vs. 2(468) = 936.

Time = 1.04 (sec) , antiderivative size = 2808, normalized size of antiderivative = 5.40

$$\int \frac{(c+dx)^{5/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="gi
ac")
```

output
```
1/192*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*
x + a)*D*d^2*abs(b)/b^8 + (17*D*b^32*c*d^7*abs(b) - 41*D*a*b^31*d^8*abs(b)
 + 8*C*b^32*d^8*abs(b))/(b^39*d^6)) + (59*D*b^33*c^2*d^6*abs(b) - 430*D*a*
b^32*c*d^7*abs(b) + 104*C*b^33*c*d^7*abs(b) + 515*D*a^2*b^31*d^8*abs(b) -
200*C*a*b^32*d^8*abs(b) + 48*B*b^33*d^8*abs(b))/(b^39*d^6)) + 3*(5*D*b^34*
c^3*d^5*abs(b) - 279*D*a*b^33*c^2*d^6*abs(b) + 88*C*b^34*c^2*d^6*abs(b) +
975*D*a^2*b^32*c*d^7*abs(b) - 464*C*a*b^33*c*d^7*abs(b) + 144*B*b^34*c*d^7
*abs(b) - 765*D*a^3*b^31*d^8*abs(b) + 440*C*a^2*b^32*d^8*abs(b) - 208*B*a*
b^33*d^8*abs(b) + 64*A*b^34*d^8*abs(b))/(b^39*d^6))*sqrt(b*x + a) + 5/128*
(D*b^4*c^4*abs(b) + 20*D*a*b^3*c^3*d*abs(b) - 8*C*b^4*c^3*d*abs(b) - 210*D
*a^2*b^2*c^2*d^2*abs(b) + 120*C*a*b^3*c^2*d^2*abs(b) - 48*B*b^4*c^2*d^2*ab
s(b) + 420*D*a^3*b*c*d^3*abs(b) - 280*C*a^2*b^2*c*d^3*abs(b) + 160*B*a*b^3
*c*d^3*abs(b) - 64*A*b^4*c*d^3*abs(b) - 231*D*a^4*d^4*abs(b) + 168*C*a^3*b
*d^4*abs(b) - 112*B*a^2*b^2*d^4*abs(b) + 64*A*a*b^3*d^4*abs(b))*log((sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sqrt(b*d)*b^
7*d) - 4/3*(9*D*a^2*b^7*c^5*d*abs(b) - 6*C*a*b^8*c^5*d*abs(b) + 3*B*b^9*c^
5*d*abs(b) - 52*D*a^3*b^6*c^4*d^2*abs(b) + 37*C*a^2*b^7*c^4*d^2*abs(b) - 2
2*B*a*b^8*c^4*d^2*abs(b) + 7*A*b^9*c^4*d^2*abs(b) + 118*D*a^4*b^5*c^3*d^3*
abs(b) - 88*C*a^3*b^6*c^3*d^3*abs(b) + 58*B*a^2*b^7*c^3*d^3*abs(b) - 28*A*
a*b^8*c^3*d^3*abs(b) - 132*D*a^5*b^4*c^2*d^4*abs(b) + 102*C*a^4*b^5*c^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{5/2}\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx=\int \frac{(c+d\,x)^{5/2}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{(a+b\,x)^{5/2}}\,dx$$

input
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(5/2),x)
```

output
```
int(((c + d*x)^(5/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{5/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}\,dx=\int \frac{(dx+c)^{\frac{5}{2}}\left(Dx^3+C\,x^2+Bx+A\right)}{(bx+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

output
```
int((d*x+c)^(5/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

3.146 $\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}} dx$

Optimal result 1412
Mathematica [A] (verified) 1413
Rubi [A] (verified) 1414
Maple [B] (verified) 1418
Fricas [B] (verification not implemented) 1419
Sympy [F] 1420
Maxima [F(-2)] 1420
Giac [B] (verification not implemented) 1420
Mupad [F(-1)] 1421
Reduce [F] 1422

Optimal result

Integrand size = 34, antiderivative size = 392

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}} dx = -\frac{2(b^3(Bc+Ad)-2ab^2(cC+Bd)-4a^3dD+3a^2b(Cd+cD))\sqrt{c+dx}}{b^5\sqrt{a+bx}} + \frac{(157a^2d^2D-2abd(39Cd+40cD)+b^2(30cCd+24Bd^2-5c^2D))\sqrt{a+bx}\sqrt{c+dx}}{24b^5d} + \frac{(6bCd-bcD-17adD)(a+bx)^{3/2}\sqrt{c+dx}}{12b^5} - \frac{2(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{3/2}}{3b^4(a+bx)^{3/2}} + \frac{D\sqrt{a+bx}(c+dx)^{5/2}}{3b^3d} - \frac{(105a^3d^3D-35a^2bd^2(2Cd+3cD)+5ab^2d(12cCd+8Bd^2+3c^2D)-b^3(6c^2Cd+24Bcd^2+16Ad^3-c}{8b^{11/2}d^{3/2}}$$

output
```
-2*(b^3*(A*d+B*c)-2*a*b^2*(B*d+C*c)-4*a^3*d*D+3*a^2*b*(C*d+D*c))*(d*x+c)^(
1/2)/b^5/(b*x+a)^(1/2)+1/24*(157*a^2*d^2*D-2*a*b*d*(39*C*d+40*D*c)+b^2*(24
*B*d^2+30*C*c*d-5*D*c^2))*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^5/d+1/12*(6*C*b*d-
17*D*a*d-D*b*c)*(b*x+a)^(3/2)*(d*x+c)^(1/2)/b^5-2/3*(A*b^3-a*(B*b^2-C*a*b+
D*a^2))*(d*x+c)^(3/2)/b^4/(b*x+a)^(3/2)+1/3*D*(b*x+a)^(1/2)*(d*x+c)^(5/2)/
b^3/d-1/8*(105*a^3*d^3*D-35*a^2*b*d^2*(2*C*d+3*D*c)+5*a*b^2*d*(8*B*d^2+12*
C*c*d+3*D*c^2)-b^3*(16*A*d^3+24*B*c*d^2+6*C*c^2*d-D*c^3))*arctanh(d^(1/2)*
(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(11/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{c+dx}(315a^4d^2D-210a^3bd(Cd+D(c-2dx))-16Ab^3d(3}{} + \frac{(-105a^3d^3D+35a^2bd^2(2Cd+3cD)-5ab^2d(12cCd+8Bd^2+3c^2D)+b^3(6c^2Cd+24Bcd^2+16Ad^3-}{8b^{11/2}d^{3/2}}$$

input
```
Integrate[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[c + d*x]*(315*a^4*d^2*D - 210*a^3*b*d*(C*d + D*(c - 2*d*x)) - 16*A*b
^3*d*(3*a*d + b*(c + 4*d*x)) + b^4*x*(24*B*d*(-2*c + d*x) + x*(3*c^2*D + 4
*d^2*x*(3*C + 2*D*x) + 2*c*d*(15*C + 7*D*x))) + a^2*b^2*(3*c^2*D + 2*c*d*(
55*C - 147*D*x) + d^2*(120*B + 7*x*(-40*C + 9*D*x))) - 2*a*b^3*(16*B*d*(c
- 5*d*x) + 3*x*(-(c^2*D) + d^2*x*(7*C + 3*D*x) + c*(-26*C*d + 9*d*D*x)))))
/(24*b^5*d*(a + b*x)^(3/2)) + ((-105*a^3*d^3*D + 35*a^2*b*d^2*(2*C*d + 3*c
*D) - 5*a*b^2*d*(12*c*C*d + 8*B*d^2 + 3*c^2*D) + b^3*(6*c^2*C*d + 24*B*c*d
^2 + 16*A*d^3 - c^3*D))*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c +
d*x])])/(8*b^(11/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2124, 27, 1193, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{(c+dx)^{3/2}\left(3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{-5dDa^3+b(5Cd+3cD)a^2-b^2(3cC+5Bd)a+b^3(3Bc+2Ad)}{b^3}\right)}{2(a+bx)^{3/2}}dx}{3(bc-ad)}-\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^{3/2}\left(-\frac{5dDa^3}{b^3}+\frac{(5Cd+3cD)a^2}{b^2}-\frac{(3cC+5Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc+2Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^{3/2}}dx}{3(bc-ad)}-\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{1193}$$

$$\frac{-\frac{2\int -\frac{(c+dx)^{3/2}\left(-50d^2Da^3+bd(35Cd+48cD)a^2-2b^2\left(3Dc^2+15Cdc+10Bd^2\right)a+b^3\left(3Cc^2+12Bdc+8Ad^2\right)+3b(bc-ad)^2Dx\right)}{2b^3\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{5/2}\left(-11a^3dD\cdots\right.}{\cdots}}{3(bc-ad)}-\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{(c+dx)^{3/2}\left(-50d^2Da^3+bd(35Cd+48cD)a^2-2b^2\left(3Dc^2+15Cdc+10Bd^2\right)a+b^3\left(3Cc^2+12Bdc+8Ad^2\right)+3b(bc-ad)^2Dx\right)}{\sqrt{a+bx}}dx}{b^3(bc-ad)}-\frac{2(c+dx)^{5/2}\left(-11a^3dD+a^2b\cdots\right.}{\cdots}}{3(bc-ad)}-\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 90

$$\frac{\frac{\frac{D\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)^2}{d}-\frac{\left(105a^3d^3D-35a^2bd^2(3cD+2Cd)+5ab^2d\left(8Bd^2+3c^2D+12cCd\right)-\left(b^3\left(16Ad^3+24Bcd^2+c^3(-D)+6c^2Cd\right)\right)\right)\int\frac{(c+dx)^{3/2}}{\sqrt{a+bx}}dx}{2d}}{b^3(bc-ad)}-\cdots}{3(bc-ad)}$$

$$\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{D\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)^2}{d}-\frac{\left(105a^3d^3D-35a^2bd^2(3cD+2Cd)+5ab^2d\left(8Bd^2+3c^2D+12cCd\right)-\left(b^3\left(16Ad^3+24Bcd^2+c^3(-D)+6c^2Cd\right)\right)\right)\left(\frac{3(bc-ad)\int\frac{\sqrt{c+dx}}{\sqrt{a+bx}}\cdots}{4b}\cdots\right)}{2d}}{b^3(bc-ad)}\cdots}{3(bc-ad)}$$

$$\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\frac{D\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)^2}{d}-\frac{\left(105a^3d^3D-35a^2bd^2(3cD+2Cd)+5ab^2d\left(8Bd^2+3c^2D+12cCd\right)-\left(b^3\left(16Ad^3+24Bcd^2+c^3(-D)+6c^2Cd\right)\right)\right)\left(\frac{3(bc-ad)\left(\frac{(bc-a\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{2d}}{b^3(bc-ad)}\cdots}{3(bc-a\cdots}$$

$$\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 66

$$\frac{\frac{\frac{D\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)^2}{d}-\frac{\left(105a^3d^3D-35a^2bd^2(3cD+2Cd)+5ab^2d\left(8Bd^2+3c^2D+12cCd\right)-\left(b^3\left(16Ad^3+24Bcd^2+c^3(-D)+6c^2Cd\right)\right)\right)\left(\frac{3(bc-ad)\left(\frac{(bc-a\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{2d}}{b^3(bc-ad)}\cdots}{3(bc-\cdots}$$

$$\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\downarrow$ 221

$$\frac{\frac{D\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)^2}{d}-\frac{\left(\frac{3(bc-ad)\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4b}+\frac{\sqrt{a+bx}(c+dx)^{3/2}}{2b}\right)\left(105a^3d^3D-35a^2bd^2(3cD+2Cd)+5ab^2\right.}{2d}}{b^3(bc-ad)}}{3(bc-}$$

$$\frac{2(c+dx)^{5/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

input

```
Int[((c + d*x)^(3/2)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(5/2),x]
```

output

```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(5/2))/(3*b^3*(b*c - a*d
)*(a + b*x)^(3/2)) + ((-2*(b^3*(3*B*c + 2*A*d) - a*b^2*(6*c*C + 5*B*d) - 1
1*a^3*d*D + a^2*b*(8*C*d + 9*c*D))*(c + d*x)^(5/2))/(b^3*(b*c - a*d)*Sqrt[
a + b*x]) + (((b*c - a*d)^2*D*Sqrt[a + b*x]*(c + d*x)^(5/2))/d - ((105*a^3
*d^3*D - 35*a^2*b*d^2*(2*C*d + 3*c*D) + 5*a*b^2*d*(12*c*C*d + 8*B*d^2 + 3*
c^2*D) - b^3*(6*c^2*C*d + 24*B*c*d^2 + 16*A*d^3 - c^3*D))*((Sqrt[a + b*x]*
(c + d*x)^(3/2))/(2*b) + (3*(b*c - a*d)*((Sqrt[a + b*x]*Sqrt[c + d*x])/b +
 ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b
^(3/2)*Sqrt[d])))/(4*b)))/(2*d))/(b^3*(b*c - a*d)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2407 vs. $2(350) = 700$.

Time = 0.56 (sec) , antiderivative size = 2408, normalized size of antiderivative = 6.14

method	result	size
default	Expression too large to display	2408

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
1/48*(d*x+c)^(1/2)*(220*C*a^2*b^2*c*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-
420*D*a^3*b*c*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-180*C*ln(1/2*(2*b*d*x+
2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c*d^2*x^
2+315*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*
b)^(1/2))*a^2*b^3*c*d^2*x^2+12*D*a*b^3*c^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)-64*B*a*b^3*c*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-315*D*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^5*d^3+
126*D*a^2*b^2*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+320*B*a*b^3*d^2*
x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-96*B*b^4*c*d*x*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)-560*C*a^2*b^2*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+840
*D*a^3*b*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-45*D*ln(1/2*(2*b*d*x+2*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d*x^2-
360*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)
^(1/2))*a^2*b^3*c*d^2*x+36*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*
b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d*x+630*D*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2*c*d^2*x-90*D*
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2)
)*a^2*b^3*c^2*d*x+630*D*a^4*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+48*A*l
n(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))
*b^5*d^3*x^2-3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(349) = 698$.

Time = 15.43 (sec) , antiderivative size = 1446, normalized size of antiderivative = 3.69

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="fr
icas")
```

output
```
[-1/96*(3*(D*a^2*b^3*c^3 + 3*(5*D*a^3*b^2 - 2*C*a^2*b^3)*c^2*d - 3*(35*D*a
^4*b - 20*C*a^3*b^2 + 8*B*a^2*b^3)*c*d^2 + (105*D*a^5 - 70*C*a^4*b + 40*B*
a^3*b^2 - 16*A*a^2*b^3)*d^3 + (D*b^5*c^3 + 3*(5*D*a*b^4 - 2*C*b^5)*c^2*d -
 3*(35*D*a^2*b^3 - 20*C*a*b^4 + 8*B*b^5)*c*d^2 + (105*D*a^3*b^2 - 70*C*a^2
*b^3 + 40*B*a*b^4 - 16*A*b^5)*d^3)*x^2 + 2*(D*a*b^4*c^3 + 3*(5*D*a^2*b^3 -
 2*C*a*b^4)*c^2*d - 3*(35*D*a^3*b^2 - 20*C*a^2*b^3 + 8*B*a*b^4)*c*d^2 + (1
05*D*a^4*b - 70*C*a^3*b^2 + 40*B*a^2*b^3 - 16*A*a*b^4)*d^3)*x)*sqrt(b*d)*l
og(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)
*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*D
*b^5*d^3*x^4 + 3*D*a^2*b^3*c^2*d - 2*(105*D*a^3*b^2 - 55*C*a^2*b^3 + 16*B*
a*b^4 + 8*A*b^5)*c*d^2 + 3*(105*D*a^4*b - 70*C*a^3*b^2 + 40*B*a^2*b^3 - 16
*A*a*b^4)*d^3 + 2*(7*D*b^5*c*d^2 - 3*(3*D*a*b^4 - 2*C*b^5)*d^3)*x^3 + 3*(D
*b^5*c^2*d - 2*(9*D*a*b^4 - 5*C*b^5)*c*d^2 + (21*D*a^2*b^3 - 14*C*a*b^4 +
8*B*b^5)*d^3)*x^2 + 2*(3*D*a*b^4*c^2*d - 3*(49*D*a^2*b^3 - 26*C*a*b^4 + 8*
B*b^5)*c*d^2 + 2*(105*D*a^3*b^2 - 70*C*a^2*b^3 + 40*B*a*b^4 - 16*A*b^5)*d^
3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^8*d^2*x^2 + 2*a*b^7*d^2*x + a^2*b^6*
d^2), 1/48*(3*(D*a^2*b^3*c^3 + 3*(5*D*a^3*b^2 - 2*C*a^2*b^3)*c^2*d - 3*(35
*D*a^4*b - 20*C*a^3*b^2 + 8*B*a^2*b^3)*c*d^2 + (105*D*a^5 - 70*C*a^4*b + 4
0*B*a^3*b^2 - 16*A*a^2*b^3)*d^3 + (D*b^5*c^3 + 3*(5*D*a*b^4 - 2*C*b^5)*c^2
*d - 3*(35*D*a^2*b^3 - 20*C*a*b^4 + 8*B*b^5)*c*d^2 + (105*D*a^3*b^2 - 7...
```

Sympy [F]

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \int \frac{(c+dx)^{\frac{3}{2}}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(3/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Integral((c + d*x)**(3/2)*(A + B*x + C*x**2 + D*x**3)/(a + b*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2075 vs. $2(349) = 698$.

Time = 0.71 (sec) , antiderivative size = 2075, normalized size of antiderivative = 5.29

$$\int \frac{(c+dx)^{3/2}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="gi
ac")
```

output
```
1/24*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*
x + a)*D*d*abs(b)/b^7 + (7*D*b^21*c*d^4*abs(b) - 25*D*a*b^20*d^5*abs(b) +
6*C*b^21*d^5*abs(b))/(b^27*d^4)) + 3*(D*b^22*c^2*d^3*abs(b) - 32*D*a*b^21*
c*d^4*abs(b) + 10*C*b^22*c*d^4*abs(b) + 55*D*a^2*b^20*d^5*abs(b) - 26*C*a*
b^21*d^5*abs(b) + 8*B*b^22*d^5*abs(b))/(b^27*d^4)) + 1/16*(D*b^3*c^3*abs(b
) + 15*D*a*b^2*c^2*d*abs(b) - 6*C*b^3*c^2*d*abs(b) - 105*D*a^2*b*c*d^2*abs
(b) + 60*C*a*b^2*c*d^2*abs(b) - 24*B*b^3*c*d^2*abs(b) + 105*D*a^3*d^3*abs(
b) - 70*C*a^2*b*d^3*abs(b) + 40*B*a*b^2*d^3*abs(b) - 16*A*b^3*d^3*abs(b))*
log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sq
rt(b*d)*b^6*d) - 4/3*(9*D*a^2*b^6*c^4*d*abs(b) - 6*C*a*b^7*c^4*d*abs(b) +
3*B*b^8*c^4*d*abs(b) - 40*D*a^3*b^5*c^3*d^2*abs(b) + 28*C*a^2*b^6*c^3*d^2*
abs(b) - 16*B*a*b^7*c^3*d^2*abs(b) + 4*A*b^8*c^3*d^2*abs(b) + 66*D*a^4*b^4
*c^2*d^3*abs(b) - 48*C*a^3*b^5*c^2*d^3*abs(b) + 30*B*a^2*b^6*c^2*d^3*abs(b
) - 12*A*a*b^7*c^2*d^3*abs(b) - 48*D*a^5*b^3*c*d^4*abs(b) + 36*C*a^4*b^4*c
*d^4*abs(b) - 24*B*a^3*b^5*c*d^4*abs(b) + 12*A*a^2*b^6*c*d^4*abs(b) + 13*D
*a^6*b^2*d^5*abs(b) - 10*C*a^5*b^3*d^5*abs(b) + 7*B*a^4*b^4*d^5*abs(b) - 4
*A*a^3*b^5*d^5*abs(b) - 18*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^2*D*a^2*b^4*c^3*d*abs(b) + 12*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*C*a*b^5*c^3*d*abs(b) - 6*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*B*b^6*c^3*d*abs...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}\,dx=\int \frac{(c+d\,x)^{3/2}\left(A+B\,x+C\,x^2+x^3\,D\right)}{(a+b\,x)^{5/2}}\,dx$$

input
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(5/2),x)
```

output
```
int(((c + d*x)^(3/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^{3/2} (A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}} \, dx = \int \frac{(dx+c)^{\frac{3}{2}} (Dx^3+C\,x^2+Bx+A)}{(bx+a)^{\frac{5}{2}}} dx$$

input
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

output
```
int((d*x+c)^(3/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

3.147 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}\,dx$

Optimal result . . . 1423
Mathematica [A] (verified) . . . 1424
Rubi [A] (verified) . . . 1424
Maple [B] (verified) . . . 1428
Fricas [B] (verification not implemented) . . . 1429
Sympy [F] . . . 1430
Maxima [F(-2)] . . . 1431
Giac [B] (verification not implemented) . . . 1431
Mupad [F(-1)] . . . 1432
Reduce [F] . . . 1433

Optimal result

Integrand size = 34, antiderivative size = 263

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = -\frac{2(b^2B-2abC+3a^2D)\sqrt{c+dx}}{b^4\sqrt{a+bx}} + \frac{(4bCd-bcD-11adD)\sqrt{a+bx}\sqrt{c+dx}}{4b^4d} - \frac{2(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{3/2}}{3b^3(bc-ad)(a+bx)^{3/2}} + \frac{D\sqrt{a+bx}(c+dx)^{3/2}}{2b^3d} + \frac{(35a^2d^2D-10abd(2Cd+cD)+b^2(4cCd+8Bd^2-c^2D))\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{9/2}d^{3/2}}$$

output
```
-2*(B*b^2-2*C*a*b+3*D*a^2)*(d*x+c)^(1/2)/b^4/(b*x+a)^(1/2)+1/4*(4*C*b*d-11
*D*a*d-D*b*c)*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^4/d-2/3*(A*b^3-a*(B*b^2-C*a*b+
D*a^2))*(d*x+c)^(3/2)/b^3/(-a*d+b*c)/(b*x+a)^(3/2)+1/2*D*(b*x+a)^(1/2)*(d*
x+c)^(3/2)/b^3/d+1/4*(35*a^2*d^2*D-10*a*b*d*(2*C*d+D*c)+b^2*(8*B*d^2+4*C*c
*d-D*c^2))*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(9/2)/d^
(3/2)
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \frac{\sqrt{c+dx}(105a^4d^2D-8Ab^4d(c+dx)-20a^3bd(3Cd+5cD-7d}{}$$
$$+\frac{(35a^2d^2D-10abd(2Cd+cD)+b^2(4cCd+8Bd^2-c^2D))\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4b^{9/2}d^{3/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(5/2),x]
```

output
```
(Sqrt[c + d*x]*(105*a^4*d^2*D - 8*A*b^4*d*(c + d*x) - 20*a^3*b*d*(3*C*d +
5*c*D - 7*d*D*x) + 3*b^4*c*x*(-8*B*d + 4*C*d*x + D*x*(c + 2*d*x)) - 2*a*b^
3*(8*B*d*(c - 2*d*x) + 3*x*(-(c^2*D) + 4*c*d*(-3*C + D*x) + d^2*x*(2*C + D
*x))) + a^2*b^2*(3*c^2*D + 2*c*d*(26*C - 69*D*x) + d^2*(24*B + x*(-80*C +
21*D*x)))))/(12*b^4*d*(b*c - a*d)*(a + b*x)^(3/2)) + ((35*a^2*d^2*D - 10*a
*b*d*(2*C*d + c*D) + b^2*(4*c*C*d + 8*B*d^2 - c^2*D))*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(9/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2124, 27, 1193, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}dx$$
$$\Big\downarrow \ 2124$$
$$-\frac{2\int -\frac{3\sqrt{c+dx}\left(\frac{(bc-ad)Dx^2}{b}+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{(bc-ad)\left(Da^2-bCa+b^2B\right)}{b^3}\right)}{2(a+bx)^{3/2}}dx}{3(bc-ad)}-\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\sqrt{c+dx}\left(\frac{(bc-ad)Dx^2}{b}+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{(bc-ad)\left(Da^2-bCa+b^2B\right)}{b^3}\right)}{(a+bx)^{3/2}}dx}{bc-ad}-\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow 1193$$

$$\frac{-\frac{2\int -\frac{(bc-ad)\sqrt{c+dx}\left(8dDa^2-b(5Cd+2cD)a+b^2(cC+2Bd)+b(bc-ad)Dx\right)}{2b^3\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{3/2}\left(3a^2D-2abC+b^2B\right)}{b^3\sqrt{a+bx}}}{bc-ad}-\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\int \frac{\sqrt{c+dx}\left(8dDa^2-b(5Cd+2cD)a+b^2(cC+2Bd)+b(bc-ad)Dx\right)}{\sqrt{a+bx}}dx}{b^3}-\frac{2(c+dx)^{3/2}\left(3a^2D-2abC+b^2B\right)}{b^3\sqrt{a+bx}}}{bc-ad}-\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow 90$$

$$\frac{\frac{\frac{\left(35a^2d^2D-10abd(cD+2Cd)+b^2\left(8Bd^2+c^2(-D)+4cCd\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}dx}{4d}+\frac{D\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{2d}}{b^3}-\frac{2(c+dx)^{3/2}\left(3a^2D-2abC+b^2B\right)}{b^3\sqrt{a+bx}}}{bc-ad}-\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow 60$$

$$\frac{\frac{\frac{\left(35a^2d^2D-10abd(cD+2Cd)+b^2\left(8Bd^2+c^2(-D)+4cCd\right)\right)\left(\frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2b}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4d}+\frac{D\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{2d}}{b^3}-\frac{2(c+dx)^{3/2}(3a}{b^3\sqrt{}}}{bc-ad}-\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow 66$$

$$\frac{\frac{\frac{\left(35a^2d^2D-10abd(cD+2Cd)+b^2\left(8Bd^2+c^2(-D)+4cCd\right)\right)\left(\frac{(bc-ad)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{b}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)}{4d}+\frac{D\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{2d}}{b^3}-\frac{2(c+dx)^{3/2}\cdots}{\cdots}}{bc-ad}$$

$$\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\left(\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}\right)\left(35a^2d^2D-10abd(cD+2Cd)+b^2\left(8Bd^2+c^2(-D)+4cCd\right)\right)}{4d}+\frac{D\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}{2d}}{b^3}-\frac{2(c+dx)^{3/\cdots}}{\cdots}}{bc-ad}$$

$$\frac{2(c+dx)^{3/2}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(5/2),x]
```

output

```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(3/2))/(3*b^3*(b*c - a*d
)*(a + b*x)^(3/2)) + ((-2*(b^2*B - 2*a*b*C + 3*a^2*D)*(c + d*x)^(3/2))/(b^
3*Sqrt[a + b*x]) + (((b*c - a*d)*D*Sqrt[a + b*x]*(c + d*x)^(3/2))/(2*d) +
((35*a^2*d^2*D - 10*a*b*d*(2*C*d + c*D) + b^2*(4*c*C*d + 8*B*d^2 - c^2*D))
*((Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])))/(4*d))/b^3)/(b*c - a*
d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. $2(227) = 454$.

Time = 0.54 (sec) , antiderivative size = 2145, normalized size of antiderivative = 8.16

method	result	size
default	Expression too large to display	2145

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
1/24*(-104*C*a^2*b^2*c*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+200*D*a^3*b*c
*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+72*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*
x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c*d^2*x^2-135*D*ln(1/2
*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*
b^3*c*d^2*x^2-12*D*a*b^3*c^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+32*B*a*
b^3*c*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+105*D*ln(1/2*(2*b*d*x+2*((b*x+
a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^5*d^3-42*D*a^2*b^2*d
^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-64*B*a*b^3*d^2*x*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+48*B*b^4*c*d*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+1
60*C*a^2*b^2*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-280*D*a^3*b*d^2*x*(
(b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+27*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c)
)^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d*x^2+144*C*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*
c*d^2*x-24*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c
)/(d*b)^(1/2))*a*b^4*c^2*d*x-270*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2*c*d^2*x+54*D*ln(1/2*(2*b*d*x+
2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^2*d*
x-210*D*a^4*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+3*D*ln(1/2*(2*b*d*x+2*
((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^3*x^2+24*B
*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(226) = 452$.

Time = 11.67 (sec) , antiderivative size = 1402, normalized size of antiderivative = 5.33

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="fr
icas")
```

output
```
[-1/48*(3*(D*a^2*b^3*c^3 + (9*D*a^3*b^2 - 4*C*a^2*b^3)*c^2*d - (45*D*a^4*b
 - 24*C*a^3*b^2 + 8*B*a^2*b^3)*c*d^2 + (35*D*a^5 - 20*C*a^4*b + 8*B*a^3*b^
2)*d^3 + (D*b^5*c^3 + (9*D*a*b^4 - 4*C*b^5)*c^2*d - (45*D*a^2*b^3 - 24*C*a
*b^4 + 8*B*b^5)*c*d^2 + (35*D*a^3*b^2 - 20*C*a^2*b^3 + 8*B*a*b^4)*d^3)*x^2
 + 2*(D*a*b^4*c^3 + (9*D*a^2*b^3 - 4*C*a*b^4)*c^2*d - (45*D*a^3*b^2 - 24*C
*a^2*b^3 + 8*B*a*b^4)*c*d^2 + (35*D*a^4*b - 20*C*a^3*b^2 + 8*B*a^2*b^3)*d^
3)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b
*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) - 4*(3*D*a^2*b^3*c^2*d - 4*(25*D*a^3*b^2 - 13*C*a^2*b^3 + 4*B*a*b
^4 + 2*A*b^5)*c*d^2 + 3*(35*D*a^4*b - 20*C*a^3*b^2 + 8*B*a^2*b^3)*d^3 + 6*
(D*b^5*c*d^2 - D*a*b^4*d^3)*x^3 + 3*(D*b^5*c^2*d - 4*(2*D*a*b^4 - C*b^5)*c
*d^2 + (7*D*a^2*b^3 - 4*C*a*b^4)*d^3)*x^2 + 2*(3*D*a*b^4*c^2*d - 3*(23*D*a
^2*b^3 - 12*C*a*b^4 + 4*B*b^5)*c*d^2 + 2*(35*D*a^3*b^2 - 20*C*a^2*b^3 + 8*
B*a*b^4 - 2*A*b^5)*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(a^2*b^6*c*d^2 - a
^3*b^5*d^3 + (b^8*c*d^2 - a*b^7*d^3)*x^2 + 2*(a*b^7*c*d^2 - a^2*b^6*d^3)*x
), 1/24*(3*(D*a^2*b^3*c^3 + (9*D*a^3*b^2 - 4*C*a^2*b^3)*c^2*d - (45*D*a^4*
b - 24*C*a^3*b^2 + 8*B*a^2*b^3)*c*d^2 + (35*D*a^5 - 20*C*a^4*b + 8*B*a^3*b
^2)*d^3 + (D*b^5*c^3 + (9*D*a*b^4 - 4*C*b^5)*c^2*d - (45*D*a^2*b^3 - 24*C*
a*b^4 + 8*B*b^5)*c*d^2 + (35*D*a^3*b^2 - 20*C*a^2*b^3 + 8*B*a*b^4)*d^3)*x^
2 + 2*(D*a*b^4*c^3 + (9*D*a^2*b^3 - 4*C*a*b^4)*c^2*d - (45*D*a^3*b^2 - ...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx=\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2 + D*x**3)/(a + b*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. 2(226) = 452.

Time = 0.42 (sec) , antiderivative size = 1305, normalized size of antiderivative = 4.96

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="gi
ac")
```

output
```
1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*D*abs(b
)/b^6 + (D*b^12*c*d*abs(b) - 13*D*a*b^11*d^2*abs(b) + 4*C*b^12*d^2*abs(b))
/(b^17*d^2)) + 1/8*(D*b^2*c^2*abs(b) + 10*D*a*b*c*d*abs(b) - 4*C*b^2*c*d*a
bs(b) - 35*D*a^2*d^2*abs(b) + 20*C*a*b*d^2*abs(b) - 8*B*b^2*d^2*abs(b))*lo
g((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sqrt
(b*d)*b^5*d) - 4/3*(9*D*a^2*b^5*c^3*d*abs(b) - 6*C*a*b^6*c^3*d*abs(b) + 3*
B*b^7*c^3*d*abs(b) - 28*D*a^3*b^4*c^2*d^2*abs(b) + 19*C*a^2*b^5*c^2*d^2*ab
s(b) - 10*B*a*b^6*c^2*d^2*abs(b) + A*b^7*c^2*d^2*abs(b) + 29*D*a^4*b^3*c*d
^3*abs(b) - 20*C*a^3*b^4*c*d^3*abs(b) + 11*B*a^2*b^5*c*d^3*abs(b) - 2*A*a*
b^6*c*d^3*abs(b) - 10*D*a^5*b^2*d^4*abs(b) + 7*C*a^4*b^3*d^4*abs(b) - 4*B*
a^3*b^4*d^4*abs(b) + A*a^2*b^5*d^4*abs(b) - 18*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*D*a^2*b^3*c^2*d*abs(b) + 12*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*C*a*b^4*c^2*d*a
bs(b) - 6*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2*B*b^5*c^2*d*abs(b) + 36*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*D*a^3*b^2*c*d^2*abs(b) - 24*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*C*a^2*b^3*c*d^2*abs(b) + 12*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*B*a*b^4*c*d^2*ab
s(b) - 18*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2*D*a^4*b*d^3*abs(b) + 12*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx=\int \frac{\sqrt{c+d\,x}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(5/2),x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}\,(Dx^3+C\,x^2+Bx+A)}{(bx+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

output
```
int((d*x+c)^(1/2)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

3.148 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx$

Optimal result 1434
Mathematica [A] (verified) 1435
Rubi [A] (verified) 1435
Maple [B] (verified) 1438
Fricas [B] (verification not implemented) 1439
Sympy [F] 1440
Maxima [F(-2)] 1441
Giac [B] (verification not implemented) 1441
Mupad [F(-1)] 1442
Reduce [F] 1443

Optimal result

Integrand size = 34, antiderivative size = 228

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx = -\frac{2(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{3b^3(bc-ad)(a+bx)^{3/2}} - \frac{2(b^3(3Bc-2Ad)-ab^2(6cC+Bd)-7a^3dD+a^2b(4Cd+9cD))\sqrt{c+dx}}{3b^3(bc-ad)^2\sqrt{a+bx}} + \frac{D\sqrt{a+bx}\sqrt{c+dx}}{b^3d} + \frac{(2bCd-bcD-5adD)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}d^{3/2}}$$

output

```
-2/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^3/(-a*d+b*c)/(b*x+a)^(3
/2)-2/3*(b^3*(-2*A*d+3*B*c)-a*b^2*(B*d+6*C*c)-7*a^3*d*D+a^2*b*(4*C*d+9*D*c
))*(d*x+c)^(1/2)/b^3/(-a*d+b*c)^2/(b*x+a)^(1/2)+D*(b*x+a)^(1/2)*(d*x+c)^(1
/2)/b^3/d+(2*C*b*d-5*D*a*d-D*b*c)*arctanh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d
*x+c)^(1/2))/b^(7/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx = \frac{\sqrt{c+dx}(15a^4d^2D+2Ab^3d(-bc+3ad+2bdx)+3b^4cx(-2Bd+cDx)-2$$
$$+\frac{(2bCd-bcD-5adD)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b^{7/2}d^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[c + d*x]*(15*a^4*d^2*D + 2*A*b^3*d*(-(b*c) + 3*a*d + 2*b*d*x) + 3*b^
4*c*x*(-2*B*d + c*D*x) - 2*a^3*b*d*(3*C*d + 11*c*D - 10*d*D*x) + a^2*b^2*(
3*c^2*D + 10*c*d*(C - 3*D*x) + d^2*x*(-8*C + 3*D*x)) + 2*a*b^3*(B*d*(-2*c
+ d*x) + 3*c*x*(2*C*d + c*D - d*D*x))))/(3*b^3*d*(b*c - a*d)^2*(a + b*x)^(
3/2)) + ((2*b*C*d - b*c*D - 5*a*d*D)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt
[d]*Sqrt[a + b*x])])/(b^(7/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1193, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{-dDa^3+b(Cd+3cD)a^2-b^2(3cC+Bd)a+b^3(3Bc-2Ad)}{b^3}}{2(a+bx)^{3/2}\sqrt{c+dx}}dx}{3(bc-ad)}-\frac{2\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-\frac{dDa^3}{b^3}+\frac{(Cd+3cD)a^2}{b^2}-\frac{(3cC+Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-2Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^{3/2}\sqrt{c+dx}}dx}{3(bc-ad)}-\frac{2\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$\frac{-\frac{2\int -\frac{3(bc-ad)^2(bC-2aD+bDx)}{2b^3\sqrt{a+bx}\sqrt{c+dx}}dx}{bc-ad}-\frac{2\sqrt{c+dx}(-7a^3dD+a^2b(9cD+4Cd)-ab^2(Bd+6cC)+b^3(3Bc-2Ad))}{b^3\sqrt{a+bx}(bc-ad)}}{3(bc-ad)}-\frac{2\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{3(bc-ad)\int \frac{bC-2aD+bDx}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b^3}-\frac{2\sqrt{c+dx}(-7a^3dD+a^2b(9cD+4Cd)-ab^2(Bd+6cC)+b^3(3Bc-2Ad))}{b^3\sqrt{a+bx}(bc-ad)}}{3(bc-ad)}-\frac{2\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 90

$$\frac{\frac{3(bc-ad)\left(\frac{(-5adD-bcD+2bCd)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{2d}+\frac{D\sqrt{a+bx}\sqrt{c+dx}}{d}\right)}{b^3}-\frac{2\sqrt{c+dx}(-7a^3dD+a^2b(9cD+4Cd)-ab^2(Bd+6cC)+b^3(3Bc-2Ad))}{b^3\sqrt{a+bx}(bc-ad)}}{3(bc-ad)}-\frac{2\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 66

$$\frac{\frac{3(bc-ad)\left(\frac{(-5adD-bcD+2bCd)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}+\frac{D\sqrt{a+bx}\sqrt{c+dx}}{d}\right)}{b^3}-\frac{2\sqrt{c+dx}(-7a^3dD+a^2b(9cD+4Cd)-ab^2(Bd+6cC)+b^3(3Bc-2Ad)}{b^3\sqrt{a+bx}(bc-ad)}}{3(bc-ad)}-\frac{2\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 221

$$\frac{\frac{3(bc-ad)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)(-5adD-bcD+2bCd)}{\sqrt{b}d^{3/2}}+\frac{D\sqrt{a+bx}\sqrt{c+dx}}{d}\right)}{b^3}-\frac{2\sqrt{c+dx}(-7a^3dD+a^2b(9cD+4Cd)-ab^2(Bd+6cC)+b^3(3Bc-2Aa}{b^3\sqrt{a+bx}(bc-ad)}}{3(bc-ad)}$$

$$\frac{2\sqrt{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

output
```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Sqrt[c + d*x])/(3*b^3*(b*c - a*d)*
(a + b*x)^(3/2)) + ((-2*(b^3*(3*B*c - 2*A*d) - a*b^2*(6*c*C + B*d) - 7*a^3
*d*D + a^2*b*(4*C*d + 9*c*D))*Sqrt[c + d*x])/(b^3*(b*c - a*d)*Sqrt[a + b*x
]) + (3*(b*c - a*d)*((D*Sqrt[a + b*x]*Sqrt[c + d*x])/d + ((2*b*C*d - b*c*D
 - 5*a*d*D)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqr
t[b]*d^(3/2))))/b^3)/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1706 vs. $2(200) = 400$.

Time = 0.63 (sec) , antiderivative size = 1707, normalized size of antiderivative = 7.49

method	result	size
default	Expression too large to display	1707

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/6*(d*x+c)^(1/2)*(20*C*a^2*b^2*c*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-44
*D*a^3*b*c*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-12*C*ln(1/2*(2*b*d*x+2*((
b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c*d^2*x^2+27
*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1
/2))*a^2*b^3*c*d^2*x^2+12*D*a*b^3*c^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2
)-8*B*a*b^3*c*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-15*D*ln(1/2*(2*b*d*x+2
*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^5*d^3+6*D*a^2
*b^2*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+4*B*a*b^3*d^2*x*((b*x+a)*
(d*x+c))^(1/2)*(d*b)^(1/2)-12*B*b^4*c*d*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1
/2)-16*C*a^2*b^2*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+40*D*a^3*b*d^2*
x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+
c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d*x^2-24*C*ln(1/2*(2
*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3
*c*d^2*x+12*C*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*
c)/(d*b)^(1/2))*a*b^4*c^2*d*x+54*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/
2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2*c*d^2*x-18*D*ln(1/2*(2*b*d*x+
2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^2*d*
x+30*D*a^4*d^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-3*D*ln(1/2*(2*b*d*x+2*(
(b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^3*x^2+6*C*l
n(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(201) = 402$.

Time = 8.57 (sec) , antiderivative size = 1286, normalized size of antiderivative = 5.64

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{5/2}\sqrt{c + dx}}\, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fr
icas")
```

output
```
[-1/12*(3*(D*a^2*b^3*c^3 + (3*D*a^3*b^2 - 2*C*a^2*b^3)*c^2*d - (9*D*a^4*b
- 4*C*a^3*b^2)*c*d^2 + (5*D*a^5 - 2*C*a^4*b)*d^3 + (D*b^5*c^3 + (3*D*a*b^4
 - 2*C*b^5)*c^2*d - (9*D*a^2*b^3 - 4*C*a*b^4)*c*d^2 + (5*D*a^3*b^2 - 2*C*a
^2*b^3)*d^3)*x^2 + 2*(D*a*b^4*c^3 + (3*D*a^2*b^3 - 2*C*a*b^4)*c^2*d - (9*D
*a^3*b^2 - 4*C*a^2*b^3)*c*d^2 + (5*D*a^4*b - 2*C*a^3*b^2)*d^3)*x)*sqrt(b*d
)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a
*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(
3*D*a^2*b^3*c^2*d - 2*(11*D*a^3*b^2 - 5*C*a^2*b^3 + 2*B*a*b^4 + A*b^5)*c*d
^2 + 3*(5*D*a^4*b - 2*C*a^3*b^2 + 2*A*a*b^4)*d^3 + 3*(D*b^5*c^2*d - 2*D*a*
b^4*c*d^2 + D*a^2*b^3*d^3)*x^2 + 2*(3*D*a*b^4*c^2*d - 3*(5*D*a^2*b^3 - 2*C
*a*b^4 + B*b^5)*c*d^2 + (10*D*a^3*b^2 - 4*C*a^2*b^3 + B*a*b^4 + 2*A*b^5)*d
^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(a^2*b^6*c^2*d^2 - 2*a^3*b^5*c*d^3 + a
^4*b^4*d^4 + (b^8*c^2*d^2 - 2*a*b^7*c*d^3 + a^2*b^6*d^4)*x^2 + 2*(a*b^7*c^
2*d^2 - 2*a^2*b^6*c*d^3 + a^3*b^5*d^4)*x), 1/6*(3*(D*a^2*b^3*c^3 + (3*D*a^
3*b^2 - 2*C*a^2*b^3)*c^2*d - (9*D*a^4*b - 4*C*a^3*b^2)*c*d^2 + (5*D*a^5 -
2*C*a^4*b)*d^3 + (D*b^5*c^3 + (3*D*a*b^4 - 2*C*b^5)*c^2*d - (9*D*a^2*b^3 -
 4*C*a*b^4)*c*d^2 + (5*D*a^3*b^2 - 2*C*a^2*b^3)*d^3)*x^2 + 2*(D*a*b^4*c^3
+ (3*D*a^2*b^3 - 2*C*a*b^4)*c^2*d - (9*D*a^3*b^2 - 4*C*a^2*b^3)*c*d^2 + (5
*D*a^4*b - 2*C*a^3*b^2)*d^3)*x)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d
)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{\frac{5}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)**(5/2)*sqrt(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(201) = 402.

Time = 0.29 (sec) , antiderivative size = 736, normalized size of antiderivative = 3.23

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx = \frac{(Dbc+5\,Dad-2\,Cbd)\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{2\,\sqrt{bd}b^2d|b|} + \frac{\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}D|b|}{b^5d} - \frac{4\left(9\,Da^2b^4c^2d-6\,Cab^5c^2d+3\,Bb^6c^2d-16\,Da^3b^3cd^2+10\,Ca^2b^4cd^2-4\,Bab^5cd^2-2\,Ab^6cd^2+7\,Da^4b^2\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="gi
ac")
```

output
```
1/2*(D*b*c + 5*D*a*d - 2*C*b*d)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2)/(sqrt(b*d)*b^2*d*abs(b)) + sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d)*sqrt(b*x + a)*D*abs(b)/(b^5*d) - 4/3*(9*D*a^2*b^4*c^2*d
- 6*C*a*b^5*c^2*d + 3*B*b^6*c^2*d - 16*D*a^3*b^3*c*d^2 + 10*C*a^2*b^4*c*d^
2 - 4*B*a*b^5*c*d^2 - 2*A*b^6*c*d^2 + 7*D*a^4*b^2*d^3 - 4*C*a^3*b^3*d^3 +
B*a^2*b^4*d^3 + 2*A*a*b^5*d^3 - 18*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*D*a^2*b^2*c*d + 12*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*C*a*b^3*c*d - 6*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*B*b^4*c*d + 12*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*D*a^3*b*d^2 - 6*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*C*a^2*b^2*d^2
 + 6*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*A*b
^4*d^2 + 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^4*D*a^2*d - 6*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*C*a*b*d + 3*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^4*B*b^2*d)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^2)^3*sqrt(b*d)*b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

3.149 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx$

Optimal result 1444
Mathematica [A] (verified) 1445
Rubi [A] (verified) 1445
Maple [B] (verified) 1448
Fricas [B] (verification not implemented) 1449
Sympy [F] 1450
Maxima [F(-2)] 1451
Giac [B] (verification not implemented) 1451
Mupad [F(-1)] 1452
Reduce [B] (verification not implemented) 1453

Optimal result

Integrand size = 34, antiderivative size = 246

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{d(bc-ad)^3\sqrt{c+dx}} - \frac{2(Ab^3-a(b^2B-abC+a^2D))\sqrt{c+dx}}{3b^2(bc-ad)^2(a+bx)^{3/2}} - \frac{2(b^3(3Bc-5Ad)-ab^2(6cC-2Bd)-4a^3dD+a^2b(Cd+9cD))\sqrt{c+dx}}{3b^2(bc-ad)^3\sqrt{a+bx}} + \frac{2D\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}d^{3/2}}$$

output
```
2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d/(-a*d+b*c)^3/(d*x+c)^(1/2)
-2/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/2)/b^2/(-a*d+b*c)^2/(b*x+a)^
(3/2)-2/3*(b^3*(-5*A*d+3*B*c)-a*b^2*(-2*B*d+6*C*c)-4*a^3*d*D+a^2*b*(C*d+9*
D*c))*(d*x+c)^(1/2)/b^2/(-a*d+b*c)^3/(b*x+a)^(1/2)+2*D*arctanh(d^(1/2)*(b*
x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx = \frac{6a^4d^2D(c+dx)+8a^3bdD(-2c^2-cdx+d^2x^2)-6b^4cx(c(-Cd+cD)x+}{}$$
$$+\frac{2D\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}d^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

output
```
(6*a^4*d^2*D*(c + d*x) + 8*a^3*b*d*D*(-2*c^2 - c*d*x + d^2*x^2) - 6*b^4*c*
x*(c*(-(C*d) + c*D)*x + B*d*(c + 2*d*x)) - 4*a*b^3*(3*c*x*(-2*c*C*d + c^2*
D - C*d^2*x) + B*d*(c^2 + 5*c*d*x + d^2*x^2)) + 2*A*b^2*d*(3*a^2*d^2 + 6*a
*b*d*(c + 2*d*x) + b^2*(-c^2 + 4*c*d*x + 8*d^2*x^2)) - 2*a^2*b^2*(3*c^3*D
+ d^3*x*(3*B + C*x) + c^2*(-8*C*d + 9*d*D*x) + c*d^2*(6*B - 4*C*x + 9*D*x^
2)))/(3*b^2*d*(b*c - a*d)^3*(a + b*x)^(3/2)*Sqrt[c + d*x]) + (2*D*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(5/2)*d^(3/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.37, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {2124, 27, 1193, 27, 87, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2124}$$

$$-\frac{2\int -\frac{3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{dDa^3-b(Cd-3cD)a^2-b^2(3cC-Bd)a+b^3(3Bc-4Ad)}{b^3}}{2(a+bx)^{3/2}(c+dx)^{3/2}}dx}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{dDa^3}{b^3}-\frac{(Cd-3cD)a^2}{b^2}-\frac{(3cC-Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-4Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^{3/2}(c+dx)^{3/2}}dx}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{-\frac{2\int -\frac{4d^2Da^3-bd(Cd+6cD)a^2+2b^2\left(-3Dc^2+3Cdc-Bd^2\right)a+b^3\left(3Cc^2-6Bdc+8Ad^2\right)+3b(bc-ad)^2Dx}{2b^3\sqrt{a+bx}(c+dx)^{3/2}}dx}{bc-ad}-\frac{2\left(-5a^3dD+a^2b(9cD+2Cd)-ab^2(6cC-B\ldots\right.}{b^3\sqrt{a+bx}\sqrt{c+dx}(bc-ad\ldots}}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{4d^2Da^3-bd(Cd+6cD)a^2+2b^2\left(-3Dc^2+3Cdc-Bd^2\right)a+b^3\left(3Cc^2-6Bdc+8Ad^2\right)+3b(bc-ad)^2Dx}{\sqrt{a+bx}(c+dx)^{3/2}}dx}{b^3(bc-ad)}-\frac{2\left(-5a^3dD+a^2b(9cD+2Cd)-ab^2(6cC-Bd)+b^3\ldots\right.}{b^3\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{\frac{3bD(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx}{d}+\frac{2\sqrt{a+bx}\left(4a^3d^3D-a^2bd^2(9cD+Cd)+2ab^2d^2(3cC-Bd)+b^3\left(8Ad^3-6Bcd^2-3c^3D+3c^2Cd\right)\right)}{d\sqrt{c+dx}(bc-ad)}}{b^3(bc-ad)}-\frac{2\left(-5a^3dD+a^2b(9cD+2\ldots\right.}{b^3\sqrt{a\ldots}}}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 66$$

$$\frac{\frac{\frac{6bD(bc-ad)^2\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}}{d}+\frac{2\sqrt{a+bx}\left(4a^3d^3D-a^2bd^2(9cD+Cd)+2ab^2d^2(3cC-Bd)+b^3\left(8Ad^3-6Bcd^2-3c^3D+3c^2Cd\right)\right)}{d\sqrt{c+dx}(bc-ad)}}{b^3(bc-ad)}-\frac{2\left(-5a^3dD+a^2b(9c\ldots\right.}{b^3\ldots}}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2\sqrt{a+bx}\left(4a^3d^3D-a^2bd^2(9cD+Cd)+2ab^2d^2(3cC-Bd)+b^3\left(8Ad^3-6Bcd^2-3c^3D+3c^2Cd\right)\right)}{d\sqrt{c+dx}(bc-ad)}+\frac{6\sqrt{b}D(bc-ad)^2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}}{b^3(bc-ad)}-\frac{2(-5a^3dD+a^2b(9\cdots}{}}{3(bc-ad)}$$

$$\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}\sqrt{c+dx}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

output
```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(3*b^3*(b*c - a*d)*(a + b*x)^(3/2
)*Sqrt[c + d*x]) + ((-2*(b^3*(3*B*c - 4*A*d) - a*b^2*(6*c*C - B*d) - 5*a^3
*d*D + a^2*b*(2*C*d + 9*c*D)))/(b^3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x
]) + ((2*(2*a*b^2*d^2*(3*c*C - B*d) + 4*a^3*d^3*D - a^2*b*d^2*(C*d + 9*c*D
) + b^3*(3*c^2*C*d - 6*B*c*d^2 + 8*A*d^3 - 3*c^3*D))*Sqrt[a + b*x])/(d*(b*
c - a*d)*Sqrt[c + d*x]) + (6*Sqrt[b]*(b*c - a*d)^2*D*ArcTanh[(Sqrt[d]*Sqrt
[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2))/(b^3*(b*c - a*d)))/(3*(b*c -
 a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1892 vs. $2(218) = 436$.

Time = 0.76 (sec) , antiderivative size = 1893, normalized size of antiderivative = 7.70

method	result	size
default	Expression too large to display	1893

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/3*(12*C*a*b^3*c*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-18*D*a^2*b^
2*c*d^2*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-20*B*a*b^3*c*d^2*x*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+8*C*a^2*b^2*c*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)+24*C*a*b^3*c^2*d*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-8*D*a^3*
b*c*d^2*x*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-18*D*a^2*b^2*c^2*d*x*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)-3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)
*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^5*d^4*x-3*D*ln(1/2*(2*b*d*x+2*((b*x+a
)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^5*c*d^3+3*D*ln(1/2*(2
*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3
*c^4+3*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d
*b)^(1/2))*b^5*c^4*x^2+9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)
^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c*d^3*x^3-9*D*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d^2*x^3+15*
D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/
2))*a^3*b^2*c*d^3*x^2-9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^
(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b^3*c^2*d^2*x^2-3*D*ln(1/2*(2*b*d*x+2*((b*
x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^3*d*x^2+3*D*
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2)
)*a^4*b*c*d^3*x+9*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+
a*d+b*c)/(d*b)^(1/2))*a^3*b^2*c^2*d^2*x-15*D*ln(1/2*(2*b*d*x+2*((b*x+a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(220) = 440$.

Time = 13.41 (sec) , antiderivative size = 1710, normalized size of antiderivative = 6.95

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{5/2}(c + dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fr
icas")
```

output
```
[1/6*(3*(D*a^2*b^3*c^4 - 3*D*a^3*b^2*c^3*d + 3*D*a^4*b*c^2*d^2 - D*a^5*c*d
^3 + (D*b^5*c^3*d - 3*D*a*b^4*c^2*d^2 + 3*D*a^2*b^3*c*d^3 - D*a^3*b^2*d^4)
*x^3 + (D*b^5*c^4 - D*a*b^4*c^3*d - 3*D*a^2*b^3*c^2*d^2 + 5*D*a^3*b^2*c*d^
3 - 2*D*a^4*b*d^4)*x^2 + (2*D*a*b^4*c^4 - 5*D*a^2*b^3*c^3*d + 3*D*a^3*b^2*
c^2*d^2 + D*a^4*b*c*d^3 - D*a^5*d^4)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a
)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(3*D*a^2*b^3*c^3*d - 3*A*a^
2*b^3*d^4 + (8*D*a^3*b^2 - 8*C*a^2*b^3 + 2*B*a*b^4 + A*b^5)*c^2*d^2 - 3*(D
*a^4*b - 2*B*a^2*b^3 + 2*A*a*b^4)*c*d^3 + (3*D*b^5*c^3*d - 3*C*b^5*c^2*d^2
 + 3*(3*D*a^2*b^3 - 2*C*a*b^4 + 2*B*b^5)*c*d^3 - (4*D*a^3*b^2 - C*a^2*b^3
- 2*B*a*b^4 + 8*A*b^5)*d^4)*x^2 + (6*D*a*b^4*c^3*d + 3*(3*D*a^2*b^3 - 4*C*
a*b^4 + B*b^5)*c^2*d^2 + 2*(2*D*a^3*b^2 - 2*C*a^2*b^3 + 5*B*a*b^4 - 2*A*b^
5)*c*d^3 - 3*(D*a^4*b - B*a^2*b^3 + 4*A*a*b^4)*d^4)*x)*sqrt(b*x + a)*sqrt(
d*x + c))/(a^2*b^6*c^4*d^2 - 3*a^3*b^5*c^3*d^3 + 3*a^4*b^4*c^2*d^4 - a^5*b
^3*c*d^5 + (b^8*c^3*d^3 - 3*a*b^7*c^2*d^4 + 3*a^2*b^6*c*d^5 - a^3*b^5*d^6)
*x^3 + (b^8*c^4*d^2 - a*b^7*c^3*d^3 - 3*a^2*b^6*c^2*d^4 + 5*a^3*b^5*c*d^5
- 2*a^4*b^4*d^6)*x^2 + (2*a*b^7*c^4*d^2 - 5*a^2*b^6*c^3*d^3 + 3*a^3*b^5*c^
2*d^4 + a^4*b^4*c*d^5 - a^5*b^3*d^6)*x), -1/3*(3*(D*a^2*b^3*c^4 - 3*D*a^3*
b^2*c^3*d + 3*D*a^4*b*c^2*d^2 - D*a^5*c*d^3 + (D*b^5*c^3*d - 3*D*a*b^4*c^2
*d^2 + 3*D*a^2*b^3*c*d^3 - D*a^3*b^2*d^4)*x^3 + (D*b^5*c^4 - D*a*b^4*c^...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)**(5/2)*(c + d*x)**(3/2)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1208 vs. 2(220) = 440.

Time = 0.60 (sec) , antiderivative size = 1208, normalized size of antiderivative = 4.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="gi
ac")
```

output
```
-2*(D*b^4*c^3*abs(b) - C*b^4*c^2*d*abs(b) + B*b^4*c*d^2*abs(b) - A*b^4*d^3
*abs(b))*sqrt(b*x + a)/((b^7*c^3*d - 3*a*b^6*c^2*d^2 + 3*a^2*b^5*c*d^3 - a
^3*b^4*d^4)*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) - D*log((sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/(sqrt(b*d)*b*d*abs(b)) -
 4/3*(9*D*a^2*b^5*c^3*d - 6*C*a*b^6*c^3*d + 3*B*b^7*c^3*d - 22*D*a^3*b^4*c
^2*d^2 + 13*C*a^2*b^5*c^2*d^2 - 4*B*a*b^6*c^2*d^2 - 5*A*b^7*c^2*d^2 + 17*D
*a^4*b^3*c*d^3 - 8*C*a^3*b^4*c*d^3 - B*a^2*b^5*c*d^3 + 10*A*a*b^6*c*d^3 -
4*D*a^5*b^2*d^4 + C*a^4*b^3*d^4 + 2*B*a^3*b^4*d^4 - 5*A*a^2*b^5*d^4 - 18*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*D*a^2*b^3
*c^2*d + 12*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2*C*a*b^4*c^2*d - 6*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*B*b^5*c^2*d + 24*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^2*D*a^3*b^2*c*d^2 - 12*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*C*a^2*b^3*c*d^2 + 12*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*A*b^5*c*d^2 - 6*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*D*a^4*b*d^3 + 6*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*B*a^2*b^3
*d^3 - 12*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2*A*a*b^4*d^3 + 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*D*a^2*b*c*d - 6*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^{5/2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(5/2)*(c + d*x)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

output
```
(2*(3*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*s
qrt(c + d*x))/sqrt(a*d - b*c))*a**3*c*d**2 + 3*sqrt(d)*sqrt(b)*sqrt(a + b*
x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*
*3*d**3*x - 6*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + s
qrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b*c**2*d - 3*sqrt(d)*sqrt(b)*s
qrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d
- b*c))*a**2*b*c*d**2*x + 3*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqr
t(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a**2*b*d**3*x**2 + 3*
sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c
+ d*x))/sqrt(a*d - b*c))*a*b**2*c**3 - 3*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log
((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**2*c
**2*d*x - 6*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqr
t(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*b**2*c*d**2*x**2 + 3*sqrt(d)*sqrt(b
)*sqrt(a + b*x)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a
*d - b*c))*b**3*c**3*x + 3*sqrt(d)*sqrt(b)*sqrt(a + b*x)*log((sqrt(d)*sqrt
(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b**3*c**2*d*x**2 - 2*s
qrt(d)*sqrt(b)*sqrt(a + b*x)*a**3*c*d**2 - 2*sqrt(d)*sqrt(b)*sqrt(a + b*x)
*a**3*d**3*x - 2*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a**2*b*c*d**2*x - 2*sqrt(d)
*sqrt(b)*sqrt(a + b*x)*a**2*b*d**3*x**2 + 6*sqrt(d)*sqrt(b)*sqrt(a + b*x)*
a*b**3*c*d + 6*sqrt(d)*sqrt(b)*sqrt(a + b*x)*a*b**3*d**2*x + 6*sqrt(d)*...
```

3.150 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx$

Optimal result 1454
Mathematica [A] (verified) 1455
Rubi [A] (verified) 1455
Maple [A] (verified) 1458
Fricas [B] (verification not implemented) 1459
Sympy [F] 1460
Maxima [F(-2)] 1460
Giac [B] (verification not implemented) 1461
Mupad [F(-1)] 1462
Reduce [F] 1462

Optimal result

Integrand size = 34, antiderivative size = 324

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{3d(bc-ad)^3(c+dx)^{3/2}} - \frac{2(Ab^3-a(b^2B-abC+a^2D))}{3b^2(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}} - \frac{2(b^3(3Bc-7Ad)-ab^2(6cC-4Bd)-2a^3dD-a^2b(Cd-9cD))}{3b^2(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}} + \frac{2(a^3d^3D+a^2bd^2(2Cd-9cD)+ab^2d(12cCd-8Bd^2-9c^2D)+b^3(2c^2Cd-8Bcd^2+16Ad^3+c^3D))\sqrt{a+bx}}{3b^2d(bc-ad)^4\sqrt{c+dx}}$$

output
```
2/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d/(-a*d+b*c)^3/(d*x+c)^(3/
2)-2/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^2/(-a*d+b*c)^2/(b*x+a)^(3/2)/(d*x+c
)^(1/2)-2/3*(b^3*(-7*A*d+3*B*c)-a*b^2*(-4*B*d+6*C*c)-2*a^3*d*D-a^2*b*(C*d-
9*D*c))/b^2/(-a*d+b*c)^3/(b*x+a)^(1/2)/(d*x+c)^(1/2)+2/3*(a^3*d^3*D+a^2*b*
d^2*(2*C*d-9*D*c)+a*b^2*d*(-8*B*d^2+12*C*c*d-9*D*c^2)+b^3*(16*A*d^3-8*B*c*
d^2+2*C*c^2*d+D*c^3))*(b*x+a)^(1/2)/b^2/d/(-a*d+b*c)^4/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx =$$
$$-\frac{2(c^2Cd(a+bx)^3-Bcd^2(a+bx)^3+Ad^3(a+bx)^3-c^3D(a+bx)^3-3bc^2C(a+bx)^2(c+dx)+6bBcd(a$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

output
```
(-2*(c^2*C*d*(a + b*x)^3 - B*c*d^2*(a + b*x)^3 + A*d^3*(a + b*x)^3 - c^3*D
*(a + b*x)^3 - 3*b*c^2*C*(a + b*x)^2*(c + d*x) + 6*b*B*c*d*(a + b*x)^2*(c
+ d*x) - 6*a*c*C*d*(a + b*x)^2*(c + d*x) - 9*A*b*d^2*(a + b*x)^2*(c + d*x)
 + 3*a*B*d^2*(a + b*x)^2*(c + d*x) + 9*a*c^2*D*(a + b*x)^2*(c + d*x) + 3*b
^2*B*c*(a + b*x)*(c + d*x)^2 - 6*a*b*c*C*(a + b*x)*(c + d*x)^2 - 9*A*b^2*d
*(a + b*x)*(c + d*x)^2 + 6*a*b*B*d*(a + b*x)*(c + d*x)^2 - 3*a^2*C*d*(a +
b*x)*(c + d*x)^2 + 9*a^2*c*D*(a + b*x)*(c + d*x)^2 + A*b^3*(c + d*x)^3 - a
*b^2*B*(c + d*x)^3 + a^2*b*C*(c + d*x)^3 - a^3*D*(c + d*x)^3))/(3*(b*c - a
*d)^4*(a + b*x)^(3/2)*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2124, 27, 1193, 27, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}dx$$
$$\Big\downarrow \text{ 2124}$$

$$-\frac{2\int -\frac{3\left(\left(c-\frac{ad}{b}\right)Dx^2+\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{dDa^3-b(Cd-cD)a^2-b^2(cC-Bd)a+b^3(Bc-2Ad)}{b^3}\right)}{2(a+bx)^{3/2}(c+dx)^{5/2}}dx}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{\frac{dDa^3}{b^3}-\frac{(Cd-cD)a^2}{b^2}-\frac{(cC-Bd)a}{b}+\left(c-\frac{ad}{b}\right)Dx^2+Bc-2Ad+\frac{(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^{3/2}(c+dx)^{5/2}}dx}{bc-ad}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

$\big\downarrow$ 1193

$$\frac{-\frac{2\int -\frac{2d^2Da^3+bd(Cd-8cD)a^2+2b^2\left(-Dc^2+3Cdc-2Bd^2\right)a+b^3\left(Cc^2-4Bdc+8Ad^2\right)+b(bc-ad)^2Dx}{2b^3\sqrt{a+bx}(c+dx)^{5/2}}dx}{bc-ad}-\frac{2\left(-\frac{a^3dD}{b^3}+\frac{3a^2cD}{b^2}-\frac{a(2cC-Bd)}{b}-2Ad+Bc\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}}{bc-ad}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{2d^2Da^3+bd(Cd-8cD)a^2+2b^2\left(-Dc^2+3Cdc-2Bd^2\right)a+b^3\left(Cc^2-4Bdc+8Ad^2\right)+b(bc-ad)^2Dx}{\sqrt{a+bx}(c+dx)^{5/2}}dx}{b^3(bc-ad)}-\frac{2\left(-\frac{a^3dD}{b^3}+\frac{3a^2cD}{b^2}-\frac{a(2cC-Bd)}{b}-2Ad+Bc\right)}{\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}}{bc-ad}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

$\big\downarrow$ 87

$$\frac{\frac{\frac{b\left(a^3d^3D+a^2bd^2(2Cd-9cD)+ab^2d\left(-8Bd^2-9c^2D+12cCd\right)+b^3\left(16Ad^3-8Bcd^2+c^3D+2c^2Cd\right)\right)\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/2}}dx}{3d(bc-ad)}+\frac{2\sqrt{a+bx}\left(2a^3d^3D+a^2bd^2(Cd-9cD)+\ldots\right.}{\ldots}}{b^3(bc-ad)}\ldots}{bc-ad}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

$\big\downarrow$ 48

$$\frac{\frac{\frac{2\sqrt{a+bx}\left(2a^3d^3D+a^2bd^2(Cd-9cD)+2ab^2d^2(3cC-2Bd)+b^3\left(8Ad^3-4Bcd^2+c^3(-D)+c^2Cd\right)\right)}{3d(c+dx)^{3/2}(bc-ad)}+\frac{2b\sqrt{a+bx}\left(a^3d^3D+a^2bd^2(2Cd-9cD)+ab^2d\left(-8Bd^2-9c^2D+12\right.\right.}{3d\sqrt{c+dx}(bc-ad)^2}}{b^3(bc-ad)}}{bc-ad}$$

$$\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

output

```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(3*b^3*(b*c - a*d)*(a + b*x)^(3/2
)*(c + d*x)^(3/2)) + ((-2*(B*c - 2*A*d - (a*(2*c*C - B*d))/b + (3*a^2*c*D)
/b^2 - (a^3*d*D)/b^3))/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/2)) + ((2*(
2*a*b^2*d^2*(3*c*C - 2*B*d) + 2*a^3*d^3*D + a^2*b*d^2*(C*d - 9*c*D) + b^3*
(c^2*C*d - 4*B*c*d^2 + 8*A*d^3 - c^3*D))*Sqrt[a + b*x])/(3*d*(b*c - a*d)*(
c + d*x)^(3/2)) + (2*b*(a^3*d^3*D + a^2*b*d^2*(2*C*d - 9*c*D) + a*b^2*d*(1
2*c*C*d - 8*B*d^2 - 9*c^2*D) + b^3*(2*c^2*C*d - 8*B*c*d^2 + 16*A*d^3 + c^3
*D))*Sqrt[a + b*x])/(3*d*(b*c - a*d)^2*Sqrt[c + d*x]))/(b^3*(b*c - a*d)))/
(b*c - a*d)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.69

method	result
default	$-\frac{2(-16A\,b^3d^3x^3+8Ba\,b^2d^3x^3+8B\,b^3c\,d^2x^3-2C\,a^2b\,d^3x^3-12Ca\,b^2c\,d^2x^3-2C\,b^3c^2d\,x^3-Da^3d^3x^3+9Da^2bc\,d^2x^3+9Da\,b^2c^2d\,x^3}{}$
gosper	$-\frac{2(-16A\,b^3d^3x^3+8Ba\,b^2d^3x^3+8B\,b^3c\,d^2x^3-2C\,a^2b\,d^3x^3-12Ca\,b^2c\,d^2x^3-2C\,b^3c^2d\,x^3-Da^3d^3x^3+9Da^2bc\,d^2x^3+9Da\,b^2c^2d\,x^3}{}$
orering	$-\frac{2(-16A\,b^3d^3x^3+8Ba\,b^2d^3x^3+8B\,b^3c\,d^2x^3-2C\,a^2b\,d^3x^3-12Ca\,b^2c\,d^2x^3-2C\,b^3c^2d\,x^3-Da^3d^3x^3+9Da^2bc\,d^2x^3+9Da\,b^2c^2d\,x^3}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/3*(-16*A*b^3*d^3*x^3+8*B*a*b^2*d^3*x^3+8*B*b^3*c*d^2*x^3-2*C*a^2*b*d^3*
x^3-12*C*a*b^2*c*d^2*x^3-2*C*b^3*c^2*d*x^3-D*a^3*d^3*x^3+9*D*a^2*b*c*d^2*x
^3+9*D*a*b^2*c^2*d*x^3-D*b^3*c^3*x^3-24*A*a*b^2*d^3*x^2-24*A*b^3*c*d^2*x^2
+12*B*a^2*b*d^3*x^2+24*B*a*b^2*c*d^2*x^2+12*B*b^3*c^2*d*x^2-3*C*a^3*d^3*x^
2-21*C*a^2*b*c*d^2*x^2-21*C*a*b^2*c^2*d*x^2-3*C*b^3*c^3*x^2+6*D*a^3*c*d^2*
x^2+36*D*a^2*b*c^2*d*x^2+6*D*a*b^2*c^3*x^2-6*A*a^2*b*d^3*x-36*A*a*b^2*c*d^
2*x-6*A*b^3*c^2*d*x+3*B*a^3*d^3*x+21*B*a^2*b*c*d^2*x+21*B*a*b^2*c^2*d*x+3*
B*b^3*c^3*x-12*C*a^3*c*d^2*x-24*C*a^2*b*c^2*d*x-12*C*a*b^2*c^3*x+24*D*a^3*
c^2*d*x+24*D*a^2*b*c^3*x+A*a^3*d^3-9*A*a^2*b*c*d^2-9*A*a*b^2*c^2*d+A*b^3*c
^3+2*B*a^3*c*d^2+12*B*a^2*b*c^2*d+2*B*a*b^2*c^3-8*C*a^3*c^2*d-8*C*a^2*b*c^
3+16*D*a^3*c^3)/(a*d-b*c)^4/(b*x+a)^(3/2)/(d*x+c)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 734 vs. $2(304) = 608$.

Time = 40.18 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.27

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{5/2}(c + dx)^{5/2}}\, dx = -\frac{2\,(Aa^3d^3 + (16\,Da^3 - 8\,Ca^2b + 2\,Bab^2 + Ab^3)c^3 - (8\,Ca^3 - 12\,Ba^2b + 9\,Aab^2)c^2d + (2\,Ba^3 - 9\,Aa^2b)cd}{3\,(a^2b^4c^6 - 4\,a}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fr
icas")
```

output
```
-2/3*(A*a^3*d^3 + (16*D*a^3 - 8*C*a^2*b + 2*B*a*b^2 + A*b^3)*c^3 - (8*C*a^
3 - 12*B*a^2*b + 9*A*a*b^2)*c^2*d + (2*B*a^3 - 9*A*a^2*b)*c*d^2 - (D*b^3*c
^3 - (9*D*a*b^2 - 2*C*b^3)*c^2*d - (9*D*a^2*b - 12*C*a*b^2 + 8*B*b^3)*c*d^
2 + (D*a^3 + 2*C*a^2*b - 8*B*a*b^2 + 16*A*b^3)*d^3)*x^3 + 3*((2*D*a*b^2 -
C*b^3)*c^3 + (12*D*a^2*b - 7*C*a*b^2 + 4*B*b^3)*c^2*d + (2*D*a^3 - 7*C*a^2
*b + 8*B*a*b^2 - 8*A*b^3)*c*d^2 - (C*a^3 - 4*B*a^2*b + 8*A*a*b^2)*d^3)*x^2
 + 3*((8*D*a^2*b - 4*C*a*b^2 + B*b^3)*c^3 + (8*D*a^3 - 8*C*a^2*b + 7*B*a*b
^2 - 2*A*b^3)*c^2*d - (4*C*a^3 - 7*B*a^2*b + 12*A*a*b^2)*c*d^2 + (B*a^3 -
2*A*a^2*b)*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^4*c^6 - 4*a^3*b^3*c^
5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4
*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 +
2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3
*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^
3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^
5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)
*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)**(5/2)*(c + d*x)**(5/2)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2066 vs. $2(304) = 608$.

Time = 1.06 (sec) , antiderivative size = 2066, normalized size of antiderivative = 6.38

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{5/2}(c + dx)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="gi
ac")
```

output
```
2/3*sqrt(b*x + a)*((D*b^7*c^6*d*abs(b) - 12*D*a*b^6*c^5*d^2*abs(b) + 2*C*b
^7*c^5*d^2*abs(b) + 30*D*a^2*b^5*c^4*d^3*abs(b) - 5*B*b^7*c^4*d^3*abs(b) -
 28*D*a^3*b^4*c^3*d^4*abs(b) - 12*C*a^2*b^5*c^3*d^4*abs(b) + 12*B*a*b^6*c^
3*d^4*abs(b) + 8*A*b^7*c^3*d^4*abs(b) + 9*D*a^4*b^3*c^2*d^5*abs(b) + 16*C*
a^3*b^4*c^2*d^5*abs(b) - 6*B*a^2*b^5*c^2*d^5*abs(b) - 24*A*a*b^6*c^2*d^5*a
bs(b) - 6*C*a^4*b^3*c*d^6*abs(b) - 4*B*a^3*b^4*c*d^6*abs(b) + 24*A*a^2*b^5
*c*d^6*abs(b) + 3*B*a^4*b^3*d^7*abs(b) - 8*A*a^3*b^4*d^7*abs(b))*(b*x + a)
/(b^9*c^7*d - 7*a*b^8*c^6*d^2 + 21*a^2*b^7*c^5*d^3 - 35*a^3*b^6*c^4*d^4 +
35*a^4*b^5*c^3*d^5 - 21*a^5*b^4*c^2*d^6 + 7*a^6*b^3*c*d^7 - a^7*b^2*d^8) -
 3*(3*D*a*b^7*c^6*d*abs(b) - C*b^8*c^6*d*abs(b) - 12*D*a^2*b^6*c^5*d^2*abs
(b) + 2*C*a*b^7*c^5*d^2*abs(b) + 2*B*b^8*c^5*d^2*abs(b) + 18*D*a^3*b^5*c^4
*d^3*abs(b) + 2*C*a^2*b^6*c^4*d^3*abs(b) - 7*B*a*b^7*c^4*d^3*abs(b) - 3*A*
b^8*c^4*d^3*abs(b) - 12*D*a^4*b^4*c^3*d^4*abs(b) - 8*C*a^3*b^5*c^3*d^4*abs
(b) + 8*B*a^2*b^6*c^3*d^4*abs(b) + 12*A*a*b^7*c^3*d^4*abs(b) + 3*D*a^5*b^3
*c^2*d^5*abs(b) + 7*C*a^4*b^4*c^2*d^5*abs(b) - 2*B*a^3*b^5*c^2*d^5*abs(b)
- 18*A*a^2*b^6*c^2*d^5*abs(b) - 2*C*a^5*b^3*c*d^6*abs(b) - 2*B*a^4*b^4*c*d
^6*abs(b) + 12*A*a^3*b^5*c*d^6*abs(b) + B*a^5*b^3*d^7*abs(b) - 3*A*a^4*b^4
*d^7*abs(b))/(b^9*c^7*d - 7*a*b^8*c^6*d^2 + 21*a^2*b^7*c^5*d^3 - 35*a^3*b^
6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b^4*c^2*d^6 + 7*a^6*b^3*c*d^7 - a^
7*b^2*d^8))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) - 4/3*(9*sqrt(b*d)*D*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(5/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{(bx+a)^{\frac{5}{2}}\,(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

3.151 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx$

Optimal result 1463
Mathematica [A] (verified) 1464
Rubi [A] (verified) 1464
Maple [B] (verified) 1468
Fricas [F(-1)] 1469
Sympy [F] 1470
Maxima [F(-2)] 1470
Giac [B] (verification not implemented) 1470
Mupad [F(-1)] 1471
Reduce [F] 1472

Optimal result

Integrand size = 34, antiderivative size = 437

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx = \frac{2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt{a+bx}}{5d(bc-ad)^3(c+dx)^{5/2}}$$
$$-\frac{2(Ab^3-a(b^2B-abC+a^2D))}{3b^2(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}}$$
$$-\frac{2(b^2(Bc-3Ad)-ab(2cC-2Bd)-a^2(Cd-3cD))}{b(bc-ad)^3\sqrt{a+bx}(c+dx)^{3/2}}$$
$$-\frac{2(5a^3d^3D-5a^2bd^2(4Cd-9cD)-5ab^2d(8cCd-8Bd^2-3c^2D)-b^3(4c^2Cd-24Bcd^2+64Ad^3+c^3D)}{15b^2d(bc-ad)^4(c+dx)^{3/2}}$$
$$-\frac{4(5a^3d^3D-5a^2bd^2(4Cd-9cD)-5ab^2d(8cCd-8Bd^2-3c^2D)-b^3(4c^2Cd-24Bcd^2+64Ad^3+c^3D)}{15bd(bc-ad)^5\sqrt{c+dx}}$$

output
```
2/5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(b*x+a)^(1/2)/d/(-a*d+b*c)^3/(d*x+c)^(5/
2)-2/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))/b^2/(-a*d+b*c)^2/(b*x+a)^(3/2)/(d*x+c
)^(3/2)-2*(b^2*(-3*A*d+B*c)-a*b*(-2*B*d+2*C*c)-a^2*(C*d-3*D*c))/b/(-a*d+b*
c)^3/(b*x+a)^(1/2)/(d*x+c)^(3/2)-2/15*(5*a^3*d^3*D-5*a^2*b*d^2*(4*C*d-9*D*
c)-5*a*b^2*d*(-8*B*d^2+8*C*c*d-3*D*c^2)-b^3*(64*A*d^3-24*B*c*d^2+4*C*c^2*d
+D*c^3))*(b*x+a)^(1/2)/b^2/d/(-a*d+b*c)^4/(d*x+c)^(3/2)-4/15*(5*a^3*d^3*D-
5*a^2*b*d^2*(4*C*d-9*D*c)-5*a*b^2*d*(-8*B*d^2+8*C*c*d-3*D*c^2)-b^3*(64*A*d
^3-24*B*c*d^2+4*C*c^2*d+D*c^3))*(b*x+a)^(1/2)/b/d/(-a*d+b*c)^5/(d*x+c)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx = -\frac{2(-3c^2Cd^2(a+bx)^4+3Bcd^3(a+bx)^4-3Ad^4(a+bx)^4+3c^3dD(a+bx)^4+10bc^2Cd(a+bx)^3(c+dx)}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(5/2)*(c + d*x)^(7/2)),x]
```

output
```
(-2*(-3*c^2*C*d^2*(a + b*x)^4 + 3*B*c*d^3*(a + b*x)^4 - 3*A*d^4*(a + b*x)^
4 + 3*c^3*d*D*(a + b*x)^4 + 10*b*c^2*C*d*(a + b*x)^3*(c + d*x) - 15*b*B*c*
d^2*(a + b*x)^3*(c + d*x) + 10*a*c*C*d^2*(a + b*x)^3*(c + d*x) + 20*A*b*d^
3*(a + b*x)^3*(c + d*x) - 5*a*B*d^3*(a + b*x)^3*(c + d*x) - 5*b*c^3*D*(a +
 b*x)^3*(c + d*x) - 15*a*c^2*d*D*(a + b*x)^3*(c + d*x) - 15*b^2*c^2*C*(a +
 b*x)^2*(c + d*x)^2 + 45*b^2*B*c*d*(a + b*x)^2*(c + d*x)^2 - 60*a*b*c*C*d*
(a + b*x)^2*(c + d*x)^2 - 90*A*b^2*d^2*(a + b*x)^2*(c + d*x)^2 + 45*a*b*B*
d^2*(a + b*x)^2*(c + d*x)^2 - 15*a^2*C*d^2*(a + b*x)^2*(c + d*x)^2 + 45*a*
b*c^2*D*(a + b*x)^2*(c + d*x)^2 + 45*a^2*c*d*D*(a + b*x)^2*(c + d*x)^2 + 1
5*b^3*B*c*(a + b*x)*(c + d*x)^3 - 30*a*b^2*c*C*(a + b*x)*(c + d*x)^3 - 60*
A*b^3*d*(a + b*x)*(c + d*x)^3 + 45*a*b^2*B*d*(a + b*x)*(c + d*x)^3 - 30*a^
2*b*C*d*(a + b*x)*(c + d*x)^3 + 45*a^2*b*c*D*(a + b*x)*(c + d*x)^3 + 15*a^
3*d*D*(a + b*x)*(c + d*x)^3 + 5*A*b^4*(c + d*x)^4 - 5*a*b^3*B*(c + d*x)^4
+ 5*a^2*b^2*C*(c + d*x)^4 - 5*a^3*b*D*(c + d*x)^4))/(15*(b*c - a*d)^5*(a +
 b*x)^(3/2)*(c + d*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2124, 27, 1193, 27, 87, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{2\int -\frac{3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{5dDa^3-b(5Cd-3cD)a^2-b^2(3cC-5Bd)a+b^3(3Bc-8Ad)}{b^3}}{2(a+bx)^{3/2}(c+dx)^{7/2}}dx}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{5dDa^3}{b^3}-\frac{(5Cd-3cD)a^2}{b^2}-\frac{(3cC-5Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-8Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^{3/2}(c+dx)^{7/2}}dx}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{-\frac{2\int -\frac{3\left(d(5Cd-14cD)a^2+2b\left(-Dc^2+5Cdc-5Bd^2\right)a+b^2\left(Cc^2-6Bdc+16Ad^2\right)+(bc-ad)^2Dx\right)}{2b^2\sqrt{a+bx}(c+dx)^{7/2}}dx}{bc-ad}-\frac{2\left(a^3(-d)D-a^2b(2Cd-9cD)-ab^2(6cC-5Bd)+b^3 \cdots\right.}{b^3\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)}}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3\int \frac{d(5Cd-14cD)a^2+2b\left(-Dc^2+5Cdc-5Bd^2\right)a+b^2\left(Cc^2-6Bdc+16Ad^2\right)+(bc-ad)^2Dx}{\sqrt{a+bx}(c+dx)^{7/2}}dx}{b^2(bc-ad)}-\frac{2\left(a^3(-d)D-a^2b(2Cd-9cD)-ab^2(6cC-5Bd)+b^3(3Bc- \cdots\right.}{b^3\sqrt{a+bx}(c+dx)^{5/2}(bc-ad)}}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 87$$

$$\frac{\frac{3\left(\frac{2\sqrt{a+bx}\left(5a^2d^2(Cd-3cD)+10abd^2(cC-Bd)+b^2\left(16Ad^3-6Bcd^2+c^3(-D)+c^2Cd\right)\right)}{5d(c+dx)^{5/2}(bc-ad)}-\frac{\left(5a^3d^3D-5a^2bd^2(4Cd-9cD)-5ab^2d\left(-8Bd^2-3c^2D+8cCd\right)-\left(b^3\left(64 \cdots\right.\right.\right.}{5d(bc-ad)}\cdots}{b^2(bc-ad)}\cdots}{3(bc-ad)}-\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \; 55$$

$$\frac{3\left(\frac{2\sqrt{a+bx}\left(5a^2d^2(Cd-3cD)+10abd^2(cC-Bd)+b^2\left(16Ad^3-6Bcd^2+c^3(-D)+c^2Cd\right)\right)}{5d(c+dx)^{5/2}(bc-ad)}-\frac{\left(5a^3d^3D-5a^2bd^2(4Cd-9cD)-5ab^2d\left(-8Bd^2-3c^2D+8cCd\right)-\left(b^3\left(64\right.\right.\right.}{}\right.}{b^2(bc-ad)}}{3(bc}$$

$$\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \text{48}$$

$$\frac{3\left(\frac{2\sqrt{a+bx}\left(5a^2d^2(Cd-3cD)+10abd^2(cC-Bd)+b^2\left(16Ad^3-6Bcd^2+c^3(-D)+c^2Cd\right)\right)}{5d(c+dx)^{5/2}(bc-ad)}-\frac{\left(\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}\right)\left(5a^3d^3D-5a^2bd^2(4Cd-9\right.}{5d}\right)}{b^2(bc-ad)}}{3(bc-a}$$

$$\frac{2\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(c+dx)^{5/2}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^(5/2)*(c + d*x)^(7/2)),x]
```

output
```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(3*b^3*(b*c - a*d)*(a + b*x)^(3/2
)*(c + d*x)^(5/2)) + ((-2*(b^3*(3*B*c - 8*A*d) - a*b^2*(6*c*C - 5*B*d) - a
^3*d*D - a^2*b*(2*C*d - 9*c*D)))/(b^3*(b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^
(5/2)) + (3*((2*(10*a*b*d^2*(c*C - B*d) + 5*a^2*d^2*(C*d - 3*c*D) + b^2*(c
^2*C*d - 6*B*c*d^2 + 16*A*d^3 - c^3*D))*Sqrt[a + b*x])/(5*d*(b*c - a*d)*(c
 + d*x)^(5/2)) - ((5*a^3*d^3*D - 5*a^2*b*d^2*(4*C*d - 9*c*D) - 5*a*b^2*d*(
8*c*C*d - 8*B*d^2 - 3*c^2*D) - b^3*(4*c^2*C*d - 24*B*c*d^2 + 64*A*d^3 + c^
3*D))*((2*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (4*b*Sqrt[a + b
*x])/(3*(b*c - a*d)^2*Sqrt[c + d*x])))/(5*d*(b*c - a*d))))/(b^2*(b*c - a*d
)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(409) = 818$.

Time = 0.67 (sec) , antiderivative size = 950, normalized size of antiderivative = 2.17

method	result
default	$-\frac{2(128A\,b^4d^4x^4-80Ba\,b^3d^4x^4-48B\,b^4c\,d^3x^4+40C\,a^2b^2d^4x^4+80Ca\,b^3c\,d^3x^4+8C\,b^4c^2d^2x^4-10Da^3b\,d^4x^4-90Da^2b^2c\,d^3x^4-30}{}$
gosper	Expression too large to display
orering	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
-2/15*(128*A*b^4*d^4*x^4-80*B*a*b^3*d^4*x^4-48*B*b^4*c*d^3*x^4+40*C*a^2*b^
2*d^4*x^4+80*C*a*b^3*c*d^3*x^4+8*C*b^4*c^2*d^2*x^4-10*D*a^3*b*d^4*x^4-90*D
*a^2*b^2*c*d^3*x^4-30*D*a*b^3*c^2*d^2*x^4+2*D*b^4*c^3*d*x^4+192*A*a*b^3*d^
4*x^3+320*A*b^4*c*d^3*x^3-120*B*a^2*b^2*d^4*x^3-272*B*a*b^3*c*d^3*x^3-120*
B*b^4*c^2*d^2*x^3+60*C*a^3*b*d^4*x^3+220*C*a^2*b^2*c*d^3*x^3+212*C*a*b^3*c
^2*d^2*x^3+20*C*b^4*c^3*d*x^3-15*D*a^4*d^4*x^3-160*D*a^3*b*c*d^3*x^3-270*D
*a^2*b^2*c^2*d^2*x^3-72*D*a*b^3*c^3*d*x^3+5*D*b^4*c^4*x^3+48*A*a^2*b^2*d^4
*x^2+480*A*a*b^3*c*d^3*x^2+240*A*b^4*c^2*d^2*x^2-30*B*a^3*b*d^4*x^2-318*B*
a^2*b^2*c*d^3*x^2-330*B*a*b^3*c^2*d^2*x^2-90*B*b^4*c^3*d*x^2+15*C*a^4*d^4*
x^2+180*C*a^3*b*c*d^3*x^2+378*C*a^2*b^2*c^2*d^2*x^2+180*C*a*b^3*c^3*d*x^2+
15*C*b^4*c^4*x^2-90*D*a^4*c*d^3*x^2-330*D*a^3*b*c^2*d^2*x^2-318*D*a^2*b^2*
c^3*d*x^2-30*D*a*b^3*c^4*x^2-8*A*a^3*b*d^4*x+120*A*a^2*b^2*c*d^3*x+360*A*a
*b^3*c^2*d^2*x+40*A*b^4*c^3*d*x+5*B*a^4*d^4*x-72*B*a^3*b*c*d^3*x-270*B*a^2
*b^2*c^2*d^2*x-160*B*a*b^3*c^3*d*x-15*B*b^4*c^4*x+20*C*a^4*c*d^3*x+212*C*a
^3*b*c^2*d^2*x+220*C*a^2*b^2*c^3*d*x+60*C*a*b^3*c^4*x-120*D*a^4*c^2*d^2*x-
272*D*a^3*b*c^3*d*x-120*D*a^2*b^2*c^4*x+3*A*a^4*d^4-20*A*a^3*b*c*d^3+90*A*
a^2*b^2*c^2*d^2+60*A*a*b^3*c^3*d-5*A*b^4*c^4+2*B*a^4*c*d^3-30*B*a^3*b*c^2*
d^2-90*B*a^2*b^2*c^3*d-10*B*a*b^3*c^4+8*C*a^4*c^2*d^2+80*C*a^3*b*c^3*d+40*
C*a^2*b^2*c^4-48*D*a^4*c^3*d-80*D*a^3*b*c^4)/(d*x+c)^(5/2)/(b*x+a)^(3/2)/(
a*d-b*c)^5
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(7/2),x, algorithm="fr
icas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{\frac{5}{2}}\,(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(7/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)**(5/2)*(c + d*x)**(7/2)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(7/2),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3566 vs. $2(408) = 816$.

Time = 1.48 (sec) , antiderivative size = 3566, normalized size of antiderivative = 8.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(7/2),x, algorithm="gi
ac")
```

output
```
2/15*((b*x + a)*((2*D*b^15*c^10*d^3 - 44*D*a*b^14*c^9*d^4 + 8*C*b^15*c^9*d
^4 + 207*D*a^2*b^13*c^8*d^5 - 6*C*a*b^14*c^8*d^5 - 33*B*b^15*c^8*d^5 - 385
*D*a^3*b^12*c^7*d^6 - 167*C*a^2*b^13*c^7*d^6 + 191*B*a*b^14*c^7*d^6 + 73*A
*b^15*c^7*d^6 + 175*D*a^4*b^11*c^6*d^7 + 665*C*a^3*b^12*c^6*d^7 - 413*B*a^
2*b^13*c^6*d^7 - 511*A*a*b^14*c^6*d^7 + 483*D*a^5*b^10*c^5*d^8 - 1155*C*a^
4*b^11*c^5*d^8 + 315*B*a^3*b^12*c^5*d^8 + 1533*A*a^2*b^13*c^5*d^8 - 931*D*
a^6*b^9*c^4*d^9 + 1057*C*a^5*b^10*c^4*d^9 + 245*B*a^4*b^11*c^4*d^9 - 2555*
A*a^3*b^12*c^4*d^9 + 733*D*a^7*b^8*c^3*d^10 - 469*C*a^6*b^9*c^3*d^10 - 707
*B*a^5*b^10*c^3*d^10 + 2555*A*a^4*b^11*c^3*d^10 - 285*D*a^8*b^7*c^2*d^11 +
 27*C*a^7*b^8*c^2*d^11 + 609*B*a^6*b^9*c^2*d^11 - 1533*A*a^5*b^10*c^2*d^11
 + 45*D*a^9*b^6*c*d^12 + 55*C*a^8*b^7*c*d^12 - 247*B*a^7*b^8*c*d^12 + 511*
A*a^6*b^9*c*d^12 - 15*C*a^9*b^6*d^13 + 40*B*a^8*b^7*d^13 - 73*A*a^7*b^8*d^
13)*(b*x + a)/(b^14*c^12*d^2*abs(b) - 12*a*b^13*c^11*d^3*abs(b) + 66*a^2*b
^12*c^10*d^4*abs(b) - 220*a^3*b^11*c^9*d^5*abs(b) + 495*a^4*b^10*c^8*d^6*a
bs(b) - 792*a^5*b^9*c^7*d^7*abs(b) + 924*a^6*b^8*c^6*d^8*abs(b) - 792*a^7*
b^7*c^5*d^9*abs(b) + 495*a^8*b^6*c^4*d^10*abs(b) - 220*a^9*b^5*c^3*d^11*ab
s(b) + 66*a^10*b^4*c^2*d^12*abs(b) - 12*a^11*b^3*c*d^13*abs(b) + a^12*b^2*
d^14*abs(b)) + 5*(D*b^16*c^11*d^2 - 23*D*a*b^15*c^10*d^3 + 4*C*b^16*c^10*d
^3 + 130*D*a^2*b^14*c^9*d^4 - 10*C*a*b^15*c^9*d^4 - 15*B*b^16*c^9*d^4 - 33
2*D*a^3*b^13*c^8*d^5 - 58*C*a^2*b^14*c^8*d^5 + 103*B*a*b^15*c^8*d^5 + 3...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^{5/2}(c+dx)^{7/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^{5/2}\,{\left(c+d\,x\right)}^{7/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(5/2)*(c + d*x)^(7/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^(5/2)*(c + d*x)^(7/2)), x)
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^{5/2}(c + dx)^{7/2}}\, dx = \int \frac{Dx^3 + C\, x^2 + Bx + A}{(bx + a)^{\frac{5}{2}}\, (dx + c)^{\frac{7}{2}}} dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(7/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(7/2),x)
```

3.152 $\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$

Optimal result 1473
Mathematica [A] (verified) 1474
Rubi [A] (verified) 1474
Maple [B] (verified) 1478
Fricas [B] (verification not implemented) 1479
Sympy [F] 1480
Maxima [F(-2)] 1481
Giac [B] (verification not implemented) 1481
Mupad [F(-1)] 1482
Reduce [F] 1483

Optimal result

Integrand size = 39, antiderivative size = 363

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = -\frac{2(c^2Cd^2-Bcd^3+Ad^4-c^3dD+c^4F)\sqrt{a+bx}}{3d^3(bc-ad)^2(c+dx)^{3/2}}$$
$$-\frac{2(Ab^4-a(b^3B-ab^2C+a^2bD-a^3F))}{b^3(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}$$
$$-\frac{2(3a^2b^2Cd^4-3a^3bd^4D+3a^4d^4F+3ab^3d(2cCd^2-2Bd^3-3c^2dD+4c^3F)-b^4(c^2Cd^2+2Bcd^3-8Ad}{3b^3d^3(bc-ad)^3\sqrt{c+dx}}$$
$$+\frac{F\sqrt{a+bx}\sqrt{c+dx}}{b^2d^3}+\frac{(2bdD-5bcF-3adF)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}d^{7/2}}$$

output
```
-2/3*(A*d^4-B*c*d^3+C*c^2*d^2-D*c^3*d+F*c^4)*(b*x+a)^(1/2)/d^3/(-a*d+b*c)^
2/(d*x+c)^(3/2)-2*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))/b^3/(-a*d+b*c)^2
/(b*x+a)^(1/2)/(d*x+c)^(1/2)-2/3*(3*a^2*b^2*C*d^4-3*a^3*b*d^4*D+3*a^4*d^4*
F+3*a*b^3*d*(-2*B*d^3+2*C*c*d^2-3*D*c^2*d+4*F*c^3)-b^4*(-8*A*d^4+2*B*c*d^3
+C*c^2*d^2-4*D*c^3*d+7*F*c^4))*(b*x+a)^(1/2)/b^3/d^3/(-a*d+b*c)^3/(d*x+c)^
(1/2)+F*(b*x+a)^(1/2)*(d*x+c)^(1/2)/b^2/d^3+(2*D*b*d-3*F*a*d-5*F*b*c)*arct
anh(d^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(d*x+c)^(1/2))/b^(5/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx = \frac{-9a^4d^3F(c+dx)^2+3a^3bd^2(c+dx)^2(2dD+3cF-dFx)-2Ab^2d^3}{}$$

$$+\frac{(2bdD-5bcF-3adF)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b^{5/2}d^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3 + F*x^4)/((a + b*x)^(3/2)*(c + d*x)^(5/
2)),x]
```

output
```
(-9*a^4*d^3*F*(c + d*x)^2 + 3*a^3*b*d^2*(c + d*x)^2*(2*d*D + 3*c*F - d*F*x
) - 2*A*b^2*d^3*(-(a^2*d^2) + 2*a*b*d*(3*c + 2*d*x) + b^2*(3*c^2 + 12*c*d*
x + 8*d^2*x^2)) + a^2*b^2*d*(-31*c^4*F + 6*d^4*x*(B - C*x) + c^3*d*(16*D -
 33*F*x) + c*d^3*(4*B - 24*C*x + 9*F*x^3) + c^2*d^2*(-16*C + 9*x*(2*D + F*
x))) + b^4*c*x*(2*B*d^3*(3*c + 2*d*x) + c*(15*c^3*F + 2*C*d^3*x + c*d^2*x*
(-8*D + 3*F*x) + c^2*d*(-6*D + 20*F*x))) + a*b^3*(4*B*d^3*(3*c^2 + 5*c*d*x
 + 3*d^2*x^2) + c*(15*c^4*F - 12*C*d^4*x^2 + c^2*d^2*x*(8*D - 39*F*x) - c^
3*d*(6*D + 11*F*x) + c*d^3*x*(-8*C + 18*D*x - 9*F*x^2))))/(3*b^2*d^3*(b*c
- a*d)^3*Sqrt[a + b*x]*(c + d*x)^(3/2)) + ((2*b*d*D - 5*b*c*F - 3*a*d*F)*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(b^(5/2)*d^(7/2))
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2124, 27, 2124, 27, 25, 1193, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{(a+bx)^{3/2}(c+dx)^{5/2}}dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{2\int -\frac{\left(c-\frac{ad}{b}\right)Fx^3+\frac{(bc-ad)(bD-aF)x^2}{b^2}+\frac{(bc-ad)\left(Fa^2-bDa+b^2C\right)x}{b^3}+\frac{-3dFa^4+b(3dD-cF)a^3-b^2(3Cd-cD)a^2-b^3(cC-3Bd)a+b^4(Bc-4Ad)}{b^4}}{2\sqrt{a+bx}(c+dx)^{5/2}}dx}{bc-ad}-\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{-\frac{3dFa^4}{b^4}+\frac{(3dD-cF)a^3}{b^3}-\frac{(3Cd-cD)a^2}{b^2}-\frac{(cC-3Bd)a}{b}+\left(c-\frac{ad}{b}\right)Fx^3+\frac{(bc-ad)(bD-aF)x^2}{b^2}+Bc-4Ad+\frac{(bc-ad)\left(Fa^2-bDa+b^2C\right)x}{b^3}}{\sqrt{a+bx}(c+dx)^{5/2}}dx}{bc-ad}-\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 2124

$$\frac{\frac{2\int -\frac{-\frac{3Fx^2(bc-ad)^2}{bd}-\frac{3(bdD-bcF-adF)x(bc-ad)^2}{b^2d^2}+\frac{-\left(\left(Fc^4-dDc^3+Cd^2c^2+2Bd^3c-8Ad^4\right)b^4\right)+3ad\left(Fc^3-dDc^2+2Cd^2c-2Bd^3\right)b^3+3a^2d^3(Cd-cD)b^2-\cdots}{b^3d^3}}{2\sqrt{a+bx}(c+dx)^{3/2}}\cdots}{3(bc-ad)}\cdots}{bc-ad}-\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\frac{2\sqrt{a+bx}\left(B\left(\frac{3ad}{b}+c\right)-\frac{d^4\left(\frac{3a^2\left(a^2F-abD+b^2C\right)}{b^4}+4A\right)+c^4F-c^3dD+c^2Cd^2}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}-\frac{\int -\frac{-\frac{3dFa^4}{b^3}+\frac{3(dD-cF)a^3}{b^2}-\frac{3(Cd-cD)a^2}{b}-3\left(\frac{Fc^3}{d^2}-\frac{Dc^2}{d}+2Cc-2B\cdots\right.}{\cdots}}{\cdots}}{bc-ad}-\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-\frac{3dFa^4}{b^3}+\frac{3(dD-cF)a^3}{b^2}-\frac{3(Cd-cD)a^2}{b}-3\left(\frac{Fc^3}{d^2}-\frac{Dc^2}{d}+2Cc-2Bd\right)a+\frac{3(bc-ad)^2Fx^2}{bd}+2bBc+\frac{b\left(Fc^4-dDc^3+Cd^2c^2-8Ad^4\right)}{d^3}+\frac{3(bc-ad)^2(bdD-bcF-adF)\cdots}{b^2d^2}}{\sqrt{a+bx}(c+dx)^{3/2}}\cdots}{3(bc-ad)}\cdots}{bc-ad}-\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 1193$$

$$\frac{\frac{\frac{2\int \frac{3(bc-ad)^3(bdD-2bcF-adF+bdFx)}{2b^2d^3\sqrt{a+bx}\sqrt{c+dx}}dx}{bc-ad}+\frac{2\sqrt{a+bx}\left(-\frac{3a^4dF}{b^3}+\frac{3a^3dD}{b^2}-\frac{3a^2Cd}{b}-3a\left(-2Bd+\frac{4c^3F}{d^2}-\frac{3c^2D}{d}+2cC\right)+\frac{b\left(-8Ad^4+7c^4F-4c^3dD+c^2Cd^2\right)}{d^3}+2bB\cdots\right.}{\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}\cdots}{bc-ad}$$

$$\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{3(bc-ad)^2\int \frac{bdD-2bcF-adF+bdFx}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b^2d^3}+\frac{2\sqrt{a+bx}\left(-\frac{3a^4dF}{b^3}+\frac{3a^3dD}{b^2}-\frac{3a^2Cd}{b}-3a\left(-2Bd+\frac{4c^3F}{d^2}-\frac{3c^2D}{d}+2cC\right)+\frac{b\left(-8Ad^4+7c^4F-4c^3dD+c^2Cd^2\right)}{d^3}+2bBc\right)}{\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}\cdots}{bc-ad}$$

$$\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 90$$

$$\frac{\frac{\frac{3(bc-ad)^2\left(\frac{1}{2}(-3adF-5bcF+2bdD)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}dx+F\sqrt{a+bx}\sqrt{c+dx}\right)}{b^2d^3}+\frac{2\sqrt{a+bx}\left(-\frac{3a^4dF}{b^3}+\frac{3a^3dD}{b^2}-\frac{3a^2Cd}{b}-3a\left(-2Bd+\frac{4c^3F}{d^2}-\frac{3c^2D}{d}+2cC\right)+\frac{b\left(-8Ad\cdots\right.}{\cdots}\cdots\right.}{\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}\cdots}{bc-ad}$$

$$\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 66$$

$$\frac{\frac{\frac{3(bc-ad)^2\left((-3adF-5bcF+2bdD)\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{\sqrt{a+bx}}{\sqrt{c+dx}}+F\sqrt{a+bx}\sqrt{c+dx}\right)}{b^2d^3}+\frac{2\sqrt{a+bx}\left(-\frac{3a^4dF}{b^3}+\frac{3a^3dD}{b^2}-\frac{3a^2Cd}{b}-3a\left(-2Bd+\frac{4c^3F}{d^2}-\frac{3c^2D}{d}+2cC\right)+\frac{b\left(-8\cdots\right.}{\cdots}\cdots\right.}{\sqrt{c+dx}(bc-ad)}}{3(bc-ad)}\cdots}{bc-ad}$$

$$\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

$$\downarrow 221$$

$$\frac{\frac{2\sqrt{a+bx}\left(B\left(\frac{3ad}{b}+c\right)-\frac{d^4\left(\frac{3a^2\left(a^2F-abD+b^2C\right)}{b^4}+4A\right)+c^4F-c^3dD+c^2Cd^2}{d^3}\right)}{3(c+dx)^{3/2}(bc-ad)}+\frac{\frac{2\sqrt{a+bx}\left(-\frac{3a^4dF}{b^3}+\frac{3a^3dD}{b^2}-\frac{3a^2Cd}{b}-3a\left(-2Bd+\frac{4c^3F}{d^2}-\frac{3c^2D}{d}+2cC\cdots\right.\right.}{\sqrt{c+dx}(bc-ad)}\cdots}{bc-ad}$$

$$\frac{2\left(Ab^4-a\left(a^3(-F)+a^2bD-ab^2C+b^3B\right)\right)}{b^4\sqrt{a+bx}(c+dx)^{3/2}(bc-ad)}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3 + F*x^4)/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

output

```
(-2*(A*b^4 - a*(b^3*B - a*b^2*C + a^2*b*D - a^3*F)))/(b^4*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(3/2)) + ((2*(B*(c + (3*a*d)/b) - (c^2*C*d^2 - c^3*d*D
 + c^4*F + d^4*(4*A + (3*a^2*(b^2*C - a*b*D + a^2*F))/b^4))/d^3)*Sqrt[a +
b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2)) + ((2*(2*b*B*c - (3*a^2*C*d)/b + (3*
a^3*d*D)/b^2 - (3*a^4*d*F)/b^3 + (b*(c^2*C*d^2 - 8*A*d^4 - 4*c^3*d*D + 7*c
^4*F))/d^3 - 3*a*(2*c*C - 2*B*d - (3*c^2*D)/d + (4*c^3*F)/d^2))*Sqrt[a + b
*x])/((b*c - a*d)*Sqrt[c + d*x]) + (3*(b*c - a*d)^2*(F*Sqrt[a + b*x]*Sqrt[
c + d*x] + ((2*b*d*D - 5*b*c*F - 3*a*d*F)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/
(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[d])))/(b^2*d^3))/(3*(b*c - a*d)))/
(b*c - a*d)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3639 vs. $2(331) = 662$.

Time = 0.66 (sec) , antiderivative size = 3640, normalized size of antiderivative = 10.03

method	result	size
default	Expression too large to display	3640

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
-1/6*(-62*F*a^2*b^2*c^4*d*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+18*D*ln(1/2*
(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^2*b
^3*c*d^5*x^3-18*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*
d+b*c)/(d*b)^(1/2))*a*b^4*c^2*d^4*x^3-12*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*c*d^5*x+30*D*ln(1/2*(2*
b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^3*b^2*
c^2*d^4*x-18*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b
*c)/(d*b)^(1/2))*a^2*b^3*c^3*d^3*x-6*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))
^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a*b^4*c^4*d^2*x+9*F*ln(1/2*(2*b*d
*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*d^6*x
^3-15*F*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*
b)^(1/2))*b^5*c^4*d^2*x^3-6*D*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d
*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*a^4*b*d^6*x^2+12*D*ln(1/2*(2*b*d*x+2*((b*x
+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b^5*c^4*d^2*x^2+6*D*l
n(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))
*b^5*c^5*d*x-32*A*b^4*d^5*x^2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)-30*F*ln(
1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*d+b*c)/(d*b)^(1/2))*b
^5*c^5*d*x^2+18*F*ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+a*
d+b*c)/(d*b)^(1/2))*a^5*c*d^5*x-12*D*b^4*c^4*d*x*((b*x+a)*(d*x+c))^(1/2)*(
d*b)^(1/2)-24*A*a*b^3*c*d^4*((b*x+a)*(d*x+c))^(1/2)*(d*b)^(1/2)+8*B*a^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(331) = 662$.

Time = 71.30 (sec) , antiderivative size = 2598, normalized size of antiderivative = 7.16

$$\int \frac{A + Bx + Cx^2 + Dx^3 + Fx^4}{(a + bx)^{3/2}(c + dx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorit
hm="fricas")
```

output

```
[-1/12*(3*(5*F*a*b^4*c^6 - 2*(6*F*a^2*b^3 + D*a*b^4)*c^5*d + 6*(F*a^3*b^2
+ D*a^2*b^3)*c^4*d^2 + 2*(2*F*a^4*b - 3*D*a^3*b^2)*c^3*d^3 - (3*F*a^5 - 2*
D*a^4*b)*c^2*d^4 + (5*F*b^5*c^4*d^2 - 2*(6*F*a*b^4 + D*b^5)*c^3*d^3 + 6*(F
*a^2*b^3 + D*a*b^4)*c^2*d^4 + 2*(2*F*a^3*b^2 - 3*D*a^2*b^3)*c*d^5 - (3*F*a
^4*b - 2*D*a^3*b^2)*d^6)*x^3 + (10*F*b^5*c^5*d + 10*D*a*b^4*c^3*d^3 - (19*
F*a*b^4 + 4*D*b^5)*c^4*d^2 + 2*(7*F*a^3*b^2 - 3*D*a^2*b^3)*c^2*d^4 - 2*(F*
a^4*b + D*a^3*b^2)*c*d^5 - (3*F*a^5 - 2*D*a^4*b)*d^6)*x^2 + (5*F*b^5*c^6 -
 2*(F*a*b^4 + D*b^5)*c^5*d - 2*(9*F*a^2*b^3 - D*a*b^4)*c^4*d^2 + 2*(8*F*a^
3*b^2 + 3*D*a^2*b^3)*c^3*d^3 + 5*(F*a^4*b - 2*D*a^3*b^2)*c^2*d^4 - 2*(3*F*
a^5 - 2*D*a^4*b)*c*d^5)*x)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c
*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x +
c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(15*F*a*b^4*c^5*d + 2*A*a^2*b^3*d^6 - (3
1*F*a^2*b^3 + 6*D*a*b^4)*c^4*d^2 + (9*F*a^3*b^2 + 16*D*a^2*b^3)*c^3*d^3 -
(9*F*a^4*b - 6*D*a^3*b^2 + 16*C*a^2*b^3 - 12*B*a*b^4 + 6*A*b^5)*c^2*d^4 +
4*(B*a^2*b^3 - 3*A*a*b^4)*c*d^5 + 3*(F*b^5*c^3*d^3 - 3*F*a*b^4*c^2*d^4 + 3
*F*a^2*b^3*c*d^5 - F*a^3*b^2*d^6)*x^3 + (20*F*b^5*c^4*d^2 - (39*F*a*b^4 +
8*D*b^5)*c^3*d^3 + (9*F*a^2*b^3 + 18*D*a*b^4 + 2*C*b^5)*c^2*d^4 + (3*F*a^3
*b^2 - 12*C*a*b^4 + 4*B*b^5)*c*d^5 - (9*F*a^4*b - 6*D*a^3*b^2 + 6*C*a^2*b^
3 - 12*B*a*b^4 + 16*A*b^5)*d^6)*x^2 + (15*F*b^5*c^5*d - (11*F*a*b^4 + 6*D*
b^5)*c^4*d^2 - (33*F*a^2*b^3 - 8*D*a*b^4)*c^3*d^3 + (15*F*a^3*b^2 + 18*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input

```
integrate((F*x**4+D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

output

```
Integral((A + B*x + C*x**2 + D*x**3 + F*x**4)/((a + b*x)**(3/2)*(c + d*x)*
*(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3 + Fx^4}{(a + bx)^{3/2}(c + dx)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. $2(331) = 662$.

Time = 0.51 (sec) , antiderivative size = 1300, normalized size of antiderivative = 3.58

$$\int \frac{A + Bx + Cx^2 + Dx^3 + Fx^4}{(a + bx)^{3/2}(c + dx)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorit
hm="giac")
```

output
```
1/3*((b*x + a)*(3*(F*b^8*c^5*d^4*abs(b) - 5*F*a*b^7*c^4*d^5*abs(b) + 10*F*
a^2*b^6*c^3*d^6*abs(b) - 10*F*a^3*b^5*c^2*d^7*abs(b) + 5*F*a^4*b^4*c*d^8*a
bs(b) - F*a^5*b^3*d^9*abs(b))*(b*x + a)/(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 +
10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)
+ 2*(10*F*b^9*c^6*d^3*abs(b) - 44*F*a*b^8*c^5*d^4*abs(b) - 4*D*b^9*c^5*d^4
*abs(b) + 76*F*a^2*b^7*c^4*d^5*abs(b) + 17*D*a*b^8*c^4*d^5*abs(b) + C*b^9*
c^4*d^5*abs(b) - 72*F*a^3*b^6*c^3*d^6*abs(b) - 22*D*a^2*b^7*c^3*d^6*abs(b)
 - 8*C*a*b^8*c^3*d^6*abs(b) + 2*B*b^9*c^3*d^6*abs(b) + 45*F*a^4*b^5*c^2*d^
7*abs(b) + 9*D*a^3*b^6*c^2*d^7*abs(b) + 13*C*a^2*b^7*c^2*d^7*abs(b) - B*a*
b^8*c^2*d^7*abs(b) - 5*A*b^9*c^2*d^7*abs(b) - 18*F*a^5*b^4*c*d^8*abs(b) -
6*C*a^3*b^6*c*d^8*abs(b) - 4*B*a^2*b^7*c*d^8*abs(b) + 10*A*a*b^8*c*d^8*abs
(b) + 3*F*a^6*b^3*d^9*abs(b) + 3*B*a^3*b^6*d^9*abs(b) - 5*A*a^2*b^7*d^9*ab
s(b))/(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*c^
2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)) + 3*(5*F*b^10*c^7*d^2*abs(b) - 27
*F*a*b^9*c^6*d^3*abs(b) - 2*D*b^10*c^6*d^3*abs(b) + 57*F*a^2*b^8*c^5*d^4*a
bs(b) + 12*D*a*b^9*c^5*d^4*abs(b) - 63*F*a^3*b^7*c^4*d^5*abs(b) - 24*D*a^2
*b^8*c^4*d^5*abs(b) - 4*C*a*b^9*c^4*d^5*abs(b) + 2*B*b^10*c^4*d^5*abs(b) +
 43*F*a^4*b^6*c^3*d^6*abs(b) + 20*D*a^3*b^7*c^3*d^6*abs(b) + 12*C*a^2*b^8*
c^3*d^6*abs(b) - 4*B*a*b^9*c^3*d^6*abs(b) - 4*A*b^10*c^3*d^6*abs(b) - 21*F
*a^5*b^5*c^2*d^7*abs(b) - 6*D*a^4*b^6*c^2*d^7*abs(b) - 12*C*a^3*b^7*c^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx=\int \frac{A+B\,x+C\,x^2+F\,x^4+x^3\,D}{{\left(a+b\,x\right)}^{3/2}\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/((a + b*x)^(3/2)*(c + d*x)^(5/2)), x
)
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3 + Fx^4}{(a + bx)^{3/2}(c + dx)^{5/2}} \, dx = \int \frac{F\,x^4 + Dx^3 + C\,x^2 + Bx + A}{(bx + a)^{\frac{3}{2}}\,(dx + c)^{\frac{5}{2}}} dx$$

input

```
int((F*x^4+D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

output

```
int((F*x^4+D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

3.153 $\int (a+bx)^3 \sqrt[3]{c+dx} \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result 1484
Mathematica [A] (verified) 1485
Rubi [A] (verified) 1486
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Optimal result

Integrand size = 32, antiderivative size = 438

$$\begin{aligned}
&\int (a+bx)^3 \sqrt[3]{c+dx} \left(A+Bx+Cx^2+Dx^3\right) \, dx \\
&= -\frac{3(bc-ad)^3 \left(c^2Cd-Bcd^2+Ad^3-c^3D\right) (c+dx)^{4/3}}{4d^7} \\
&\quad -\frac{3(bc-ad)^2 \left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right) (c+dx)^{7/3}}{7d^7} \\
&\quad -\frac{3(bc-ad) \left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)}{10d^7} \\
&\quad +\frac{3(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D)}{13d^7} \\
&\quad +\frac{3b(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D)) (c+dx)^{16/3}}{16d^7} \\
&\quad +\frac{3b^2(bCd-6bcD+3adD)(c+dx)^{19/3}}{19d^7}+\frac{3b^3D(c+dx)^{22/3}}{22d^7}
\end{aligned}$$

output
```
-3/4*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(4/3)/d^7-3/7*(-a*
d+b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*
c^3))*(d*x+c)^(7/3)/d^7-3/10*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d
^2+8*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^
(10/3)/d^7+3/13*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c
*d-10*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(13/3)/d^7
+3/16*b*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d
*x+c)^(16/3)/d^7+3/19*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(19/3)/d^7+3/22*
b^3*D*(d*x+c)^(22/3)/d^7
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.14

$$\int (a+bx)^3 \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{3(c+dx)^{4/3}\left(836a^3d^3(-81c^3D+9c^2d(13C+12Dx)-3cd^2(65B+52Cx+42Dx^2)+d^3(455A+2x(13\right.}{}$$

input
```
Integrate[(a + b*x)^3*(c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(3*(c + d*x)^(4/3)*(836*a^3*d^3*(-81*c^3*D + 9*c^2*d*(13*C + 12*D*x) - 3*c
*d^2*(65*B + 52*C*x + 42*D*x^2) + d^3*(455*A + 2*x*(130*B + 91*C*x + 70*D*
x^2))) + 627*a^2*b*d^2*(243*c^4*D - 324*c^3*d*(C + D*x) + 18*c^2*d^2*(26*B
 + 3*x*(8*C + 7*D*x)) + d^4*x*(1040*A + 7*x*(104*B + 80*C*x + 65*D*x^2)) -
 12*c*d^3*(65*A + x*(52*B + 7*x*(6*C + 5*D*x)))) + b^3*(32805*c^6*D - 3645
*c^5*d*(11*C + 12*D*x) + 243*c^4*d^2*(209*B + 10*x*(22*C + 21*D*x)) - 162*
c^3*d^3*(418*A + x*(418*B + 35*x*(11*C + 10*D*x))) + 35*d^6*x^3*(3344*A +
13*x*(209*B + 8*x*(22*C + 19*D*x))) + 9*c^2*d^4*x*(10032*A + 7*x*(1254*B +
 25*x*(44*C + 39*D*x))) - 21*c*d^5*x^2*(5016*A + 5*x*(836*B + 13*x*(55*C +
 48*D*x)))) + 33*a*b^2*d*(-3645*c^5*D + 243*c^4*d*(19*C + 20*D*x) - 162*c^
3*d^2*(38*B + x*(38*C + 35*D*x)) + 7*d^5*x^2*(1976*A + 5*x*(304*B + 13*x*(
19*C + 16*D*x))) + 18*c^2*d^3*(494*A + x*(456*B + 7*x*(57*C + 50*D*x))) -
3*c*d^4*x*(3952*A + 7*x*(456*B + 5*x*(76*C + 65*D*x))))))/(1521520*d^7)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{7/3}(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3(c+dx)^{10/3}(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2\right.\right.}{10d^7}$$
$$\frac{3b(c+dx)^{16/3}\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{16d^7}+$$
$$\frac{3(c+dx)^{13/3}\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c\right.\right.}{13d^7}$$
$$\frac{3(c+dx)^{7/3}(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{7d^7}-$$
$$\frac{3(c+dx)^{4/3}(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^7}+$$
$$\frac{3b^2(c+dx)^{19/3}(3adD-6bcD+bCd)}{19d^7}+\frac{3b^3D(c+dx)^{22/3}}{22d^7}$$

input
```
Int[(a + b*x)^3*(c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-3*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(4/3))/(4*
d^7) - (3*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d -
4*B*c*d^2 + 3*A*d^3 - 6*c^3*D))*(c + d*x)^(7/3))/(7*d^7) - (3*(b*c - a*d)*
(a^2*d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^
2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(10/3))/(10*d^7) + (3*(
a^3*d^3*D + 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^
2*D) + b^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(13/3))/
(13*d^7) + (3*b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^
2 - 15*c^2*D))*(c + d*x)^(16/3))/(16*d^7) + (3*b^2*(b*C*d - 6*b*c*D + 3*a*
d*D)*(c + d*x)^(19/3))/(19*d^7) + (3*b^3*D*(c + d*x)^(22/3))/(22*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$3\left(\left(\frac{4x^3\left(\frac{13}{22}Dx^3+\frac{13}{19}Cx^2+\frac{13}{16}Bx+A\right)b^3}{13}+\frac{6x^2\left(\frac{10}{19}Dx^3+\frac{5}{8}Cx^2+\frac{10}{13}Bx+A\right)ab^2}{5}+\frac{12x\left(\frac{7}{16}Dx^3+\frac{7}{13}Cx^2+\frac{7}{10}Bx+A\right)a^2b}{7}+a^3(\right.\right.$
derivativedivides	$\frac{3b^3D(xd+c)^{\frac{22}{3}}}{22}+\frac{3\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{19}{3}}}{19}+\frac{3\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(Bd^2-2Ccd+3Dc^2\right)\right)(}{16}$
default	$\frac{3b^3D(xd+c)^{\frac{22}{3}}}{22}+\frac{3\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{19}{3}}}{19}+\frac{3\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(Bd^2-2Ccd+3Dc^2\right)\right)(}{16}$
gosper	$3(xd+c)^{\frac{4}{3}}(69160Dx^6b^3d^6+80080Cb^3d^6x^5+240240Dx^5ab^2d^6-65520Dx^5b^3cd^5+95095Bx^4b^3d^6+285285Cx^4ab^2d^6$
orering	$3(xd+c)^{\frac{4}{3}}(69160Dx^6b^3d^6+80080Cb^3d^6x^5+240240Dx^5ab^2d^6-65520Dx^5b^3cd^5+95095Bx^4b^3d^6+285285Cx^4ab^2d^6$
trager	Expression too large to display

input
```
int((b*x+a)^3*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
3/4*((4/13*x^3*(13/22*D*x^3+13/19*C*x^2+13/16*B*x+A)*b^3+6/5*x^2*(10/19*D*
x^3+5/8*C*x^2+10/13*B*x+A)*a*b^2+12/7*x*(7/16*D*x^3+7/13*C*x^2+7/10*B*x+A)
*a^2*b+a^3*(4/13*D*x^3+2/5*C*x^2+4/7*B*x+A))*d^6-9/7*(14/65*x^2*(130/209*D
*x^3+325/456*C*x^2+5/6*B*x+A)*b^3+4/5*x*(175/304*D*x^3+35/52*C*x^2+21/26*B
*x+A)*a*b^2+a^2*(7/13*D*x^3+42/65*C*x^2+4/5*B*x+A)*b+1/3*(42/65*D*x^2+4/5*
C*x+B)*a^3)*c*d^5+27/35*(4/13*x*(2275/3344*D*x^3+175/228*C*x^2+7/8*B*x+A)*
b^3+a*(175/247*D*x^3+21/26*C*x^2+12/13*B*x+A)*b^2+a^2*(21/26*D*x^2+12/13*C
*x+B)*b+1/3*a^3*(12/13*D*x+C))*c^2*d^4-81/455*((175/209*D*x^3+35/38*C*x^2+
B*x+A)*b^3+3*(35/38*D*x^2+C*x+B)*a*b^2+3*a^2*(D*x+C)*b+a^3*D)*c^3*d^3+243/
1820*c^4*((210/209*D*x^2+20/19*C*x+B)*b^2+3*(20/19*D*x+C)*a*b+3*D*a^2)*b*d
^2-729/6916*((12/11*D*x+C)*b+3*D*a)*c^5*b^2*d+6561/76076*D*b^3*c^6)*(d*x+c
)^(4/3)/d^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.79

$$\int (a+bx)^3\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
3/1521520*(69160*D*b^3*d^7*x^7 + 32805*D*b^3*c^7 + 380380*A*a^3*c*d^6 - 40
095*(3*D*a*b^2 + C*b^3)*c^6*d + 50787*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^5*
d^2 - 67716*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^4*d^3 + 97812*(C*a^3
 + 3*B*a^2*b + 3*A*a*b^2)*c^3*d^4 - 163020*(B*a^3 + 3*A*a^2*b)*c^2*d^5 + 3
640*(D*b^3*c*d^6 + 22*(3*D*a*b^2 + C*b^3)*d^7)*x^6 - 455*(9*D*b^3*c^2*d^5
 - 11*(3*D*a*b^2 + C*b^3)*c*d^6 - 209*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^7)*
x^5 + 35*(135*D*b^3*c^3*d^4 - 165*(3*D*a*b^2 + C*b^3)*c^2*d^5 + 209*(3*D*a
^2*b + 3*C*a*b^2 + B*b^3)*c*d^6 + 3344*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*
b^3)*d^7)*x^4 - 14*(405*D*b^3*c^4*d^3 - 495*(3*D*a*b^2 + C*b^3)*c^3*d^4 +
627*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^5 - 836*(D*a^3 + 3*C*a^2*b + 3*B
*a*b^2 + A*b^3)*c*d^6 - 10868*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^7)*x^3 + 2
*(3645*D*b^3*c^5*d^2 - 4455*(3*D*a*b^2 + C*b^3)*c^4*d^3 + 5643*(3*D*a^2*b
+ 3*C*a*b^2 + B*b^3)*c^3*d^4 - 7524*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3
)*c^2*d^5 + 10868*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^6 + 108680*(B*a^3 +
3*A*a^2*b)*d^7)*x^2 - (10935*D*b^3*c^6*d - 380380*A*a^3*d^7 - 13365*(3*D*a
*b^2 + C*b^3)*c^5*d^2 + 16929*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^3 - 22
572*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^4 + 32604*(C*a^3 + 3*B*a
^2*b + 3*A*a*b^2)*c^2*d^5 - 54340*(B*a^3 + 3*A*a^2*b)*c*d^6)*x)*(d*x + c)^
(1/3)/d^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(456) = 912$.

Time = 1.95 (sec) , antiderivative size = 1028, normalized size of antiderivative = 2.35

$$\int (a+bx)^3\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)**(1/3)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((3*(D*b**3*(c + d*x)**(22/3)/(22*d**6) + (c + d*x)**(19/3)*(C*b*
*3*d + 3*D*a*b**2*d - 6*D*b**3*c)/(19*d**6) + (c + d*x)**(16/3)*(B*b**3*d*
*2 + 3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d +
15*D*b**3*c**2)/(16*d**6) + (c + d*x)**(13/3)*(A*b**3*d**3 + 3*B*a*b**2*d*
*3 - 4*B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c*
*2*d + D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c
**3)/(13*d**6) + (c + d*x)**(10/3)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*
B*a**2*b*d**4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C
*a**2*b*c*d**3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**
3 + 18*D*a**2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/(10*d**6)
 + (c + d*x)**(7/3)*(3*A*a**2*b*d**5 - 6*A*a*b**2*c*d**4 + 3*A*b**3*c**2*d
**3 + B*a**3*d**5 - 6*B*a**2*b*c*d**4 + 9*B*a*b**2*c**2*d**3 - 4*B*b**3*c*
*3*d**2 - 2*C*a**3*c*d**4 + 9*C*a**2*b*c**2*d**3 - 12*C*a*b**2*c**3*d**2 +
 5*C*b**3*c**4*d + 3*D*a**3*c**2*d**3 - 12*D*a**2*b*c**3*d**2 + 15*D*a*b**
2*c**4*d - 6*D*b**3*c**5)/(7*d**6) + (c + d*x)**(4/3)*(A*a**3*d**6 - 3*A*a
**2*b*c*d**5 + 3*A*a*b**2*c**2*d**4 - A*b**3*c**3*d**3 - B*a**3*c*d**5 + 3
*B*a**2*b*c**2*d**4 - 3*B*a*b**2*c**3*d**3 + B*b**3*c**4*d**2 + C*a**3*c**
2*d**4 - 3*C*a**2*b*c**3*d**3 + 3*C*a*b**2*c**4*d**2 - C*b**3*c**5*d - D*a
**3*c**3*d**3 + 3*D*a**2*b*c**4*d**2 - 3*D*a*b**2*c**5*d + D*b**3*c**6)/(4
*d**6))/d, Ne(d, 0)), (c**(1/3)*(A*a**3*x + D*b**3*x**7/7 + x**6*(C*b**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.42

$$\int (a+bx)^3\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{3\left(69160\,(dx+c)^{\frac{22}{3}}Db^3 - 80080\,(6\,Db^3c - (3\,Dab^2+Cb^3)d)(dx+c)^{\frac{19}{3}} + 95095\,(15\,Db^3c^2 - 5\,(3\,Dab^2 \cdots\right.}{}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
3/1521520*(69160*(d*x + c)^(22/3)*D*b^3 - 80080*(6*D*b^3*c - (3*D*a*b^2 +
C*b^3)*d)*(d*x + c)^(19/3) + 95095*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^3)*c
*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(16/3) - 117040*(20*D*
b^3*c^3 - 10*(3*D*a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)
*c*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(13/3) + 1
52152*(15*D*b^3*c^4 - 10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*
b^2 + B*b^3)*c^2*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (
C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^4)*(d*x + c)^(10/3) - 217360*(6*D*b^3*c^5
 - 5*(3*D*a*b^2 + C*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2
 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3*B*a^2*
b + 3*A*a*b^2)*c*d^4 - (B*a^3 + 3*A*a^2*b)*d^5)*(d*x + c)^(7/3) + 380380*(
D*b^3*c^6 + A*a^3*d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2
 + B*b^3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a
^3 + 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)*(d*x + c)
^(4/3))/d^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1913 vs. $2(412) = 824$.

Time = 0.15 (sec) , antiderivative size = 1913, normalized size of antiderivative = 4.37

$$\int (a+bx)^3 \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
3/1521520*(1521520*(d*x + c)^(1/3)*A*a^3*c + 380380*((d*x + c)^(4/3) - 4*(
d*x + c)^(1/3)*c)*A*a^3 + 380380*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*B
*a^3*c/d + 1141140*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*A*a^2*b*c/d + 1
08680*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*C
*a^3*c/d^2 + 326040*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c
)^(1/3)*c^2)*B*a^2*b*c/d^2 + 326040*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)
*c + 14*(d*x + c)^(1/3)*c^2)*A*a*b^2*c/d^2 + 108680*(2*(d*x + c)^(7/3) - 7
*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*B*a^3/d + 326040*(2*(d*x + c)
^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*A*a^2*b/d + 10868*(
14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140
*(d*x + c)^(1/3)*c^3)*D*a^3*c/d^3 + 32604*(14*(d*x + c)^(10/3) - 60*(d*x +
 c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*C*a^2*b*c
/d^3 + 32604*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(
4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*B*a*b^2*c/d^3 + 10868*(14*(d*x + c)^(1
0/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3
)*c^3)*A*b^3*c/d^3 + 10868*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 1
05*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*C*a^3/d^2 + 32604*(14*(d
*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x
 + c)^(1/3)*c^3)*B*a^2*b/d^2 + 32604*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(
7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*A*a*b^2/d^2...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (a+b\,x)^3\,(c+d\,x)^{1/3}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^3*(c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^3*(c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.35

$$\int (a+bx)^3\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$=\frac{3(dx+c)^{\frac{1}{3}}\left(69160b^3d^7x^7+240240a\,b^2d^7x^6+83720b^3c\,d^6x^6+285285a^2b\,d^7x^5+300300a\,b^2c\,d^6x^5+950\right.}{}$$

input
```
int((b*x+a)^3*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(3*(c + d*x)**(1/3)*(380380*a**4*c*d**5 + 380380*a**4*d**6*x - 652080*a**3
*b*c**2*d**4 + 217360*a**3*b*c*d**5*x + 869440*a**3*b*d**6*x**2 + 30096*a*
*3*c**4*d**3 - 10032*a**3*c**3*d**4*x + 6688*a**3*c**2*d**5*x**2 + 163856*
a**3*c*d**6*x**3 + 117040*a**3*d**7*x**4 + 586872*a**2*b**2*c**3*d**3 - 19
5624*a**2*b**2*c**2*d**4*x + 130416*a**2*b**2*c*d**5*x**2 + 912912*a**2*b*
*2*d**6*x**3 - 50787*a**2*b*c**5*d**2 + 16929*a**2*b*c**4*d**3*x - 11286*a
**2*b*c**3*d**4*x**2 + 8778*a**2*b*c**2*d**5*x**3 + 373065*a**2*b*c*d**6*x
**4 + 285285*a**2*b*d**7*x**5 - 270864*a*b**3*c**4*d**2 + 90288*a*b**3*c**
3*d**3*x - 60192*a*b**3*c**2*d**4*x**2 + 46816*a*b**3*c*d**5*x**3 + 468160
*a*b**3*d**6*x**4 + 32076*a*b**2*c**6*d - 10692*a*b**2*c**5*d**2*x + 7128*
a*b**2*c**4*d**3*x**2 - 5544*a*b**2*c**3*d**4*x**3 + 4620*a*b**2*c**2*d**5
*x**4 + 300300*a*b**2*c*d**6*x**5 + 240240*a*b**2*d**7*x**6 + 50787*b**4*c
**5*d - 16929*b**4*c**4*d**2*x + 11286*b**4*c**3*d**3*x**2 - 8778*b**4*c**
2*d**4*x**3 + 7315*b**4*c*d**5*x**4 + 95095*b**4*d**6*x**5 - 7290*b**3*c**
7 + 2430*b**3*c**6*d*x - 1620*b**3*c**5*d**2*x**2 + 1260*b**3*c**4*d**3*x*
*3 - 1050*b**3*c**3*d**4*x**4 + 910*b**3*c**2*d**5*x**5 + 83720*b**3*c*d**
6*x**6 + 69160*b**3*d**7*x**7))/(1521520*d**6)
```

3.154 $\int (a+bx)^2 \sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result . . . 1494
Mathematica [A] (verified) . . . 1495
Rubi [A] (verified) . . . 1495
Maple [A] (verified) . . . 1497
Fricas [A] (verification not implemented) . . . 1497
Sympy [A] (verification not implemented) . . . 1498
Maxima [A] (verification not implemented) . . . 1499
Giac [B] (verification not implemented) . . . 1500
Mupad [F(-1)] . . . 1501
Reduce [B] (verification not implemented) . . . 1501

Optimal result

Integrand size = 32, antiderivative size = 326

$$\begin{aligned}\int (a+bx)^2 \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{3(bc-ad)^2\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{4/3}}{4d^6}\\ &+\frac{3(bc-ad)\left(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D)\right)(c+dx)^{7/3}}{7d^6}\\ &+\frac{3(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D))(c+dx)^{10/3}}{10d^6}\\ &+\frac{3(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))(c+dx)^{13/3}}{13d^6}\\ &+\frac{3b(bCd-5bcD+2adD)(c+dx)^{16/3}}{16d^6}+\frac{3b^2D(c+dx)^{19/3}}{19d^6}\end{aligned}$$

output
```
3/4*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(4/3)/d^6+3/7*(-a*d
+b*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3
))*(d*x+c)^(7/3)/d^6+3/10*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D
*c^2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(10/3)/d^6+3/13*(a
^2*d^2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(13/3)
/d^6+3/16*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(16/3)/d^6+3/19*b^2*D*(d*x+c)^
(19/3)/d^6
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00

$$\int (a+bx)^2\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{3(c+dx)^{4/3}\left(76a^2d^2(-81c^3D+9c^2d(13C+12Dx)-3cd^2(65B+52Cx+42Dx^2)+d^3(455A+2x(130\right.}{}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(3*(c + d*x)^(4/3)*(76*a^2*d^2*(-81*c^3*D + 9*c^2*d*(13*C + 12*D*x) - 3*c*
d^2*(65*B + 52*C*x + 42*D*x^2) + d^3*(455*A + 2*x*(130*B + 91*C*x + 70*D*x
^2))) + 38*a*b*d*(243*c^4*D - 324*c^3*d*(C + D*x) + 18*c^2*d^2*(26*B + 3*x
*(8*C + 7*D*x)) + d^4*x*(1040*A + 7*x*(104*B + 80*C*x + 65*D*x^2)) - 12*c*
d^3*(65*A + x*(52*B + 7*x*(6*C + 5*D*x)))) + b^2*(-3645*c^5*D + 243*c^4*d*
(19*C + 20*D*x) - 162*c^3*d^2*(38*B + x*(38*C + 35*D*x)) + 7*d^5*x^2*(1976
*A + 5*x*(304*B + 13*x*(19*C + 16*D*x))) + 18*c^2*d^3*(494*A + x*(456*B +
7*x*(57*C + 50*D*x))) - 3*c*d^4*x*(3952*A + 7*x*(456*B + 5*x*(76*C + 65*D*
x))))))/(138320*d^6)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{(c+dx)^{7/3}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3(c+dx)^{10/3}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{10d^6}+\frac{3(c+dx)^{13/3}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{13d^6}+\frac{3(c+dx)^{7/3}(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{7d^6}+\frac{3(c+dx)^{4/3}(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^6}+\frac{3b(c+dx)^{16/3}(2adD-5bcD+bCd)}{16d^6}+\frac{3b^2D(c+dx)^{19/3}}{19d^6}$$

input
```
Int[(a + b*x)^2*(c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(3*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(4/3))/(4*d
^6) + (3*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B
*c*d^2 + 2*A*d^3 - 5*c^3*D))*(c + d*x)^(7/3))/(7*d^6) + (3*(a^2*d^2*(C*d -
 3*c*D) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2
 + A*d^3 - 10*c^3*D))*(c + d*x)^(10/3))/(10*d^6) + (3*(a^2*d^2*D + 2*a*b*d
*(C*d - 4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(13/3))/(13*d
^6) + (3*b*(b*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(16/3))/(16*d^6) + (3*b^2
*D*(c + d*x)^(19/3))/(19*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{3(xd+c)^{\frac{4}{3}}\left(\left(\frac{2x^2\left(\frac{10}{19}Dx^3+\frac{5}{8}Cx^2+\frac{10}{13}Bx+A\right)b^2}{5}+\frac{8x\left(\frac{7}{16}Dx^3+\frac{7}{13}Cx^2+\frac{7}{10}Bx+A\right)ab}{7}+a^2\left(\frac{4}{13}Dx^3+\frac{2}{5}Cx^2+\frac{4}{7}Bx+A\right)\right)d^5\right.}{}$
derivativedivides	$\frac{3b^2D(xd+c)^{\frac{19}{3}}}{19}+\frac{3\left(2b(ad-bc)D+b^2(Cd-3Dc)\right)(xd+c)^{\frac{16}{3}}}{16}+\frac{3\left((ad-bc)^2D+2b(ad-bc)(Cd-3Dc)+b^2\left(Bd^2-2Ccd+3Dc^2\right)\right)(xd+c)}{13}$
default	$\frac{3b^2D(xd+c)^{\frac{19}{3}}}{19}+\frac{3\left(2b(ad-bc)D+b^2(Cd-3Dc)\right)(xd+c)^{\frac{16}{3}}}{16}+\frac{3\left((ad-bc)^2D+2b(ad-bc)(Cd-3Dc)+b^2\left(Bd^2-2Ccd+3Dc^2\right)\right)(xd+c)}{13}$
gosper	$3(xd+c)^{\frac{4}{3}}(7280Dx^5b^2d^5+8645Cx^4b^2d^5+17290Dx^4abd^5-6825Dx^4b^2cd^4+10640Bx^3b^2d^5+21280Cx^3abd^5-7980C$
orering	$3(xd+c)^{\frac{4}{3}}(7280Dx^5b^2d^5+8645Cx^4b^2d^5+17290Dx^4abd^5-6825Dx^4b^2cd^4+10640Bx^3b^2d^5+21280Cx^3abd^5-7980C$
trager	$3\left(7280b^2Dd^6x^6+8645Cb^2d^6x^5+17290Dabd^6x^5+455Db^2cd^5x^5+10640Bb^2d^6x^4+21280Cabd^6x^4+665Cb^2cd^5x^4+\right.$

input
```
int((b*x+a)^2*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
3/4*(d*x+c)^(4/3)*((2/5*x^2*(10/19*D*x^3+5/8*C*x^2+10/13*B*x+A)*b^2+8/7*x*
(7/16*D*x^3+7/13*C*x^2+7/10*B*x+A)*a*b+a^2*(4/13*D*x^3+2/5*C*x^2+4/7*B*x+A
))*d^5-6/7*(2/5*x*(175/304*D*x^3+35/52*C*x^2+21/26*B*x+A)*b^2+a*(7/13*D*x^
3+42/65*C*x^2+4/5*B*x+A)*b+1/2*(42/65*D*x^2+4/5*C*x+B)*a^2)*c*d^4+9/35*((1
75/247*D*x^3+21/26*C*x^2+12/13*B*x+A)*b^2+2*(21/26*D*x^2+12/13*C*x+B)*a*b+
a^2*(12/13*D*x+C))*c^2*d^3-81/455*((35/38*D*x^2+C*x+B)*b^2+2*a*(D*x+C)*b+D
*a^2)*c^3*d^2+243/1820*((20/19*D*x+C)*b+2*D*a)*c^4*b*d-729/6916*D*b^2*c^5)
/d^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.53

$$\int (a+bx)^2\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$=\frac{3\left(7280\,Db^2d^6x^6-3645\,Db^2c^6+34580\,Aa^2cd^5+4617\,(2\,Dab+Cb^2)c^5d-6156\,(Da^2+2\,Cab+Bb^2)c^4\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
3/138320*(7280*D*b^2*d^6*x^6 - 3645*D*b^2*c^6 + 34580*A*a^2*c*d^5 + 4617*(
2*D*a*b + C*b^2)*c^5*d - 6156*(D*a^2 + 2*C*a*b + B*b^2)*c^4*d^2 + 8892*(C*
a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 14820*(B*a^2 + 2*A*a*b)*c^2*d^4 + 455*(D*
b^2*c*d^5 + 19*(2*D*a*b + C*b^2)*d^6)*x^5 - 35*(15*D*b^2*c^2*d^4 - 19*(2*D
*a*b + C*b^2)*c*d^5 - 304*(D*a^2 + 2*C*a*b + B*b^2)*d^6)*x^4 + 14*(45*D*b^
2*c^3*d^3 - 57*(2*D*a*b + C*b^2)*c^2*d^4 + 76*(D*a^2 + 2*C*a*b + B*b^2)*c*
d^5 + 988*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*x^3 - 2*(405*D*b^2*c^4*d^2 - 513*
(2*D*a*b + C*b^2)*c^3*d^3 + 684*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^4 - 988*(C
*a^2 + 2*B*a*b + A*b^2)*c*d^5 - 9880*(B*a^2 + 2*A*a*b)*d^6)*x^2 + (1215*D*
b^2*c^5*d + 34580*A*a^2*d^6 - 1539*(2*D*a*b + C*b^2)*c^4*d^2 + 2052*(D*a^2
 + 2*C*a*b + B*b^2)*c^3*d^3 - 2964*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 + 494
0*(B*a^2 + 2*A*a*b)*c*d^5)*x)*(d*x + c)^(1/3)/d^6
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.97

$$\int (a+bx)^2\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \begin{cases} \frac{3\left(\frac{Db^2(c+dx)^{\frac{19}{3}}}{19d^5}+\frac{(c+dx)^{\frac{16}{3}}\left(Cb^2d+2Dabd-5Db^2c\right)}{16d^5}+\frac{(c+dx)^{\frac{13}{3}}\left(Bb^2d^2+2Cabd^2-4Cb^2cd+Da^2d^2-8Dabcd+10Db^2c^2\right)}{13d^5}+\frac{(c+dx)^{\frac{10}{3}}\left(Ab^2d^3+2Babd^3-\right.}{}\right.}{} \\ \sqrt[3]{c}\left(Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5(Cb^2+2Dab)}{5}+\frac{x^4(Bb^2+2Cab+Da^2)}{4}+\frac{x^3(Ab^2+2Bab+Ca^2)}{3}+\frac{x^2\cdot(2Aab+Ba^2)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)**2*(d*x+c)**(1/3)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((3*(D*b**2*(c + d*x)**(19/3)/(19*d**5) + (c + d*x)**(16/3)*(C*b*
*2*d + 2*D*a*b*d - 5*D*b**2*c)/(16*d**5) + (c + d*x)**(13/3)*(B*b**2*d**2
+ 2*C*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2
)/(13*d**5) + (c + d*x)**(10/3)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d
**2 + C*a**2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 1
2*D*a*b*c**2*d - 10*D*b**2*c**3)/(10*d**5) + (c + d*x)**(7/3)*(2*A*a*b*d**
4 - 2*A*b**2*c*d**3 + B*a**2*d**4 - 4*B*a*b*c*d**3 + 3*B*b**2*c**2*d**2 -
2*C*a**2*c*d**3 + 6*C*a*b*c**2*d**2 - 4*C*b**2*c**3*d + 3*D*a**2*c**2*d**2
 - 8*D*a*b*c**3*d + 5*D*b**2*c**4)/(7*d**5) + (c + d*x)**(4/3)*(A*a**2*d**
5 - 2*A*a*b*c*d**4 + A*b**2*c**2*d**3 - B*a**2*c*d**4 + 2*B*a*b*c**2*d**3
- B*b**2*c**3*d**2 + C*a**2*c**2*d**3 - 2*C*a*b*c**3*d**2 + C*b**2*c**4*d
- D*a**2*c**3*d**2 + 2*D*a*b*c**4*d - D*b**2*c**5)/(4*d**5))/d, Ne(d, 0)),
 (c**(1/3)*(A*a**2*x + D*b**2*x**6/6 + x**5*(C*b**2 + 2*D*a*b)/5 + x**4*(B
*b**2 + 2*C*a*b + D*a**2)/4 + x**3*(A*b**2 + 2*B*a*b + C*a**2)/3 + x**2*(2
*A*a*b + B*a**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.19

$$\int (a+bx)^2 \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$= \frac{3\left(7280\,(dx+c)^{\frac{19}{3}}Db^2 - 8645\,(5\,Db^2c-(2\,Dab+Cb^2)d)(dx+c)^{\frac{16}{3}} + 10640\,(10\,Db^2c^2-4\,(2\,Dab+Cb\right.}{}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
3/138320*(7280*(d*x + c)^(19/3)*D*b^2 - 8645*(5*D*b^2*c - (2*D*a*b + C*b^2
)*d)*(d*x + c)^(16/3) + 10640*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d + (D
*a^2 + 2*C*a*b + B*b^2)*d^2)*(d*x + c)^(13/3) - 13832*(10*D*b^2*c^3 - 6*(2
*D*a*b + C*b^2)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a
*b + A*b^2)*d^3)*(d*x + c)^(10/3) + 19760*(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^
2)*c^3*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^
2)*c*d^3 + (B*a^2 + 2*A*a*b)*d^4)*(d*x + c)^(7/3) - 34580*(D*b^2*c^5 - A*a
^2*d^5 - (2*D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*
a^2 + 2*B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)*(d*x + c)^(4/3))
/d^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1269 vs. $2(302) = 604$.

Time = 0.14 (sec) , antiderivative size = 1269, normalized size of antiderivative = 3.89

$$\int (a+bx)^2\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
3/138320*(138320*(d*x + c)^(1/3)*A*a^2*c + 34580*((d*x + c)^(4/3) - 4*(d*x
 + c)^(1/3)*c)*A*a^2 + 34580*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*B*a^2
*c/d + 69160*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*A*a*b*c/d + 9880*(2*(
d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*C*a^2*c/d^2
 + 19760*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2
)*B*a*b*c/d^2 + 9880*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x +
c)^(1/3)*c^2)*A*b^2*c/d^2 + 9880*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c
+ 14*(d*x + c)^(1/3)*c^2)*B*a^2/d + 19760*(2*(d*x + c)^(7/3) - 7*(d*x + c)
^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*A*a*b/d + 988*(14*(d*x + c)^(10/3) - 60
*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*D*
a^2*c/d^3 + 1976*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x +
c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*C*a*b*c/d^3 + 988*(14*(d*x + c)^(1
0/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3
)*c^3)*B*b^2*c/d^3 + 988*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105
*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*C*a^2/d^2 + 1976*(14*(d*x
+ c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x +
c)^(1/3)*c^3)*B*a*b/d^2 + 988*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c
+ 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*A*b^2/d^2 + 608*(35*(
d*x + c)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 455*(
d*x + c)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*D*a*b*c/d^4 + 304*(35*(d*...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int (a+b\,x)^2\,(c+d\,x)^{1/3}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^2*(c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^2*(c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.17

$$\int (a+bx)^2\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{3(dx+c)^{\frac{1}{3}}\left(3640b^2d^6x^6+8645ab\,d^6x^5+4550b^2c\,d^5x^5+5320a^2d^6x^4+11305abc\,d^5x^4+5320b^3d^5x^4+70\right.}{}$$

input
```
int((b*x+a)^2*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(3*(c + d*x)**(1/3)*(17290*a**3*c*d**4 + 17290*a**3*d**5*x - 22230*a**2*b*
c**2*d**3 + 7410*a**2*b*c*d**4*x + 29640*a**2*b*d**5*x**2 + 1368*a**2*c**4
*d**2 - 456*a**2*c**3*d**3*x + 304*a**2*c**2*d**4*x**2 + 7448*a**2*c*d**5*
x**3 + 5320*a**2*d**6*x**4 + 13338*a*b**2*c**3*d**2 - 4446*a*b**2*c**2*d**
3*x + 2964*a*b**2*c*d**4*x**2 + 20748*a*b**2*d**5*x**3 - 1539*a*b*c**5*d +
 513*a*b*c**4*d**2*x - 342*a*b*c**3*d**3*x**2 + 266*a*b*c**2*d**4*x**3 + 1
1305*a*b*c*d**5*x**4 + 8645*a*b*d**6*x**5 - 3078*b**3*c**4*d + 1026*b**3*c
**3*d**2*x - 684*b**3*c**2*d**3*x**2 + 532*b**3*c*d**4*x**3 + 5320*b**3*d*
*5*x**4 + 486*b**2*c**6 - 162*b**2*c**5*d*x + 108*b**2*c**4*d**2*x**2 - 84
*b**2*c**3*d**3*x**3 + 70*b**2*c**2*d**4*x**4 + 4550*b**2*c*d**5*x**5 + 36
40*b**2*d**6*x**6))/(69160*d**5)
```

3.155 $\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result . . . 1502
Mathematica [A] (verified) . . . 1503
Rubi [A] (verified) . . . 1503
Maple [A] (verified) . . . 1505
Fricas [A] (verification not implemented) . . . 1505
Sympy [A] (verification not implemented) . . . 1506
Maxima [A] (verification not implemented) . . . 1507
Giac [B] (verification not implemented) . . . 1507
Mupad [F(-1)] . . . 1508
Reduce [B] (verification not implemented) . . . 1509

Optimal result

Integrand size = 30, antiderivative size = 214

$$\begin{aligned}\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx &= -\frac{3(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{4/3}}{4d^5}\\ &\quad-\frac{3(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))(c+dx)^{7/3}}{7d^5}\\ &\quad+\frac{3(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))(c+dx)^{10/3}}{10d^5}\\ &\quad+\frac{3(bCd-4bcD+adD)(c+dx)^{13/3}}{13d^5}+\frac{3bD(c+dx)^{16/3}}{16d^5}\end{aligned}$$

output
```
-3/4*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(4/3)/d^5-3/7*(a*d*(
-B*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(7/
3)/d^5+3/10*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(10/3)/d^
5+3/13*(C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(13/3)/d^5+3/16*b*D*(d*x+c)^(16/3)/d^
5
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.86

$$\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{3(c+dx)^{4/3}\left(4ad(-81c^3D+9c^2d(13C+12Dx)-3cd^2(65B+52Cx+42Dx^2)+d^3(455A+2x(130B\right.}{}$$

input
```
Integrate[(a + b*x)*(c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(3*(c + d*x)^(4/3)*(4*a*d*(-81*c^3*D + 9*c^2*d*(13*C + 12*D*x) - 3*c*d^2*(
65*B + 52*C*x + 42*D*x^2) + d^3*(455*A + 2*x*(130*B + 91*C*x + 70*D*x^2)))
 + b*(243*c^4*D - 324*c^3*d*(C + D*x) + 18*c^2*d^2*(26*B + 3*x*(8*C + 7*D*
x)) + d^4*x*(1040*A + 7*x*(104*B + 80*C*x + 65*D*x^2)) - 12*c*d^3*(65*A +
x*(52*B + 7*x*(6*C + 5*D*x))))))/(7280*d^5)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \ 2123$$

$$\int \left(\frac{(c+dx)^{4/3}\left(b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)}{d^4}+\frac{\sqrt[3]{c+dx}(ad-bc)\left(Ad^3-\right.}{d^4}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{3(c+dx)^{7/3}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{7d^5}-\frac{3(c+dx)^{4/3}(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^5}+\frac{3(c+dx)^{10/3}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{10d^5}+\frac{3(c+dx)^{13/3}(adD-4bcD+bCd)}{13d^5}+\frac{3bD(c+dx)^{16/3}}{16d^5}$$

input
```
Int[(a + b*x)*(c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(-3*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(4/3))/(4*d^
5) - (3*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^
3 - 4*c^3*D))*(c + d*x)^(7/3))/(7*d^5) + (3*(a*d*(C*d - 3*c*D) - b*(3*c*C*
d - B*d^2 - 6*c^2*D))*(c + d*x)^(10/3))/(10*d^5) + (3*(b*C*d - 4*b*c*D + a
*d*D)*(c + d*x)^(13/3))/(13*d^5) + (3*b*D*(c + d*x)^(16/3))/(16*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{3\left(\left(\frac{Db\,x^4}{4}+\frac{4(Cb+Da)x^3}{13}+\frac{2(Bb+Ca)x^2}{5}+\frac{4(Ab+Ba)x}{7}+Aa\right)d^4-\frac{3\left(\frac{7Db\,x^3}{13}+\frac{42(Cb+Da)x^2}{65}+\frac{4(Bb+Ca)x}{5}+Ab+Ba\right)c\,d^3}{7}+}{4d^5}$
derivativedivides	$\frac{\frac{3bD(xd+c)^{\frac{16}{3}}}{16}+\frac{3((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{13}{3}}}{13}+\frac{3\left((ad-bc)(Cd-3Dc)+b\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{10}{3}}}{10}+\frac{3\left((ad-bc)\left(B\right.\right.}{}}{d^5}$
default	$\frac{\frac{3bD(xd+c)^{\frac{16}{3}}}{16}+\frac{3((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{13}{3}}}{13}+\frac{3\left((ad-bc)(Cd-3Dc)+b\left(B\,d^2-2Ccd+3Dc^2\right)\right)(xd+c)^{\frac{10}{3}}}{10}+\frac{3\left((ad-bc)\left(B\right.\right.}{}}{d^5}$
gosper	$\frac{3(xd+c)^{\frac{4}{3}}\left(455Db\,x^4d^4+560C\,x^3b\,d^4+560Dx^3a\,d^4-420Dx^3bc\,d^3+728B\,x^2b\,d^4+728C\,x^2a\,d^4-504C\,x^2bc\,d^3-504Dx\right.}{}$
orering	$\frac{3(xd+c)^{\frac{4}{3}}\left(455Db\,x^4d^4+560C\,x^3b\,d^4+560Dx^3a\,d^4-420Dx^3bc\,d^3+728B\,x^2b\,d^4+728C\,x^2a\,d^4-504C\,x^2bc\,d^3-504Dx\right.}{}$
trager	$\frac{3\left(455bDd^5x^5+560Cb\,d^5x^4+560Da\,d^5x^4+35Dbc\,d^4x^4+728Bb\,d^5x^3+728Ca\,d^5x^3+56Cbc\,d^4x^3+56Dac\,d^4x^3-42Db\,c\right.}{}$

input
```
int((b*x+a)*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
3/4*((1/4*D*b*x^4+4/13*(C*b+D*a)*x^3+2/5*(B*b+C*a)*x^2+4/7*(A*b+B*a)*x+A*a
)*d^4-3/7*(7/13*D*b*x^3+42/65*(C*b+D*a)*x^2+4/5*(B*b+C*a)*x+A*b+B*a)*c*d^3
+9/35*(21/26*D*b*x^2+12/13*(C*b+D*a)*x+B*b+C*a)*c^2*d^2-81/455*c^3*(D*b*x+
C*b+D*a)*d+243/1820*D*b*c^4)*(d*x+c)^(4/3)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.24

$$\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{3\left(455\,Dbd^5x^5+243\,Dbc^5+1820\,Aacd^4-324\,(Da+Cb)c^4d+468\,(Ca+Bb)c^3d^2-780\,(Ba+Ab)c^2d\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
3/7280*(455*D*b*d^5*x^5 + 243*D*b*c^5 + 1820*A*a*c*d^4 - 324*(D*a + C*b)*c
^4*d + 468*(C*a + B*b)*c^3*d^2 - 780*(B*a + A*b)*c^2*d^3 + 35*(D*b*c*d^4 +
 16*(D*a + C*b)*d^5)*x^4 - 14*(3*D*b*c^2*d^3 - 4*(D*a + C*b)*c*d^4 - 52*(C
*a + B*b)*d^5)*x^3 + 2*(27*D*b*c^3*d^2 - 36*(D*a + C*b)*c^2*d^3 + 52*(C*a
+ B*b)*c*d^4 + 520*(B*a + A*b)*d^5)*x^2 - (81*D*b*c^4*d - 1820*A*a*d^5 - 1
08*(D*a + C*b)*c^3*d^2 + 156*(C*a + B*b)*c^2*d^3 - 260*(B*a + A*b)*c*d^4)*
x)*(d*x + c)^(1/3)/d^5
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.50

$$\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\begin{cases} \frac{3\left(\frac{Db(c+dx)^{\frac{16}{3}}}{16d^4}+\frac{(c+dx)^{\frac{13}{3}}(Cbd+Dad-4Dbc)}{13d^4}+\frac{(c+dx)^{\frac{10}{3}}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6Dbc^2\right)}{10d^4}+\frac{(c+dx)^{\frac{7}{3}}\left(Abd^3+Bad^3-2Bbcd^2-2Cacd^2+3Cbc^2d+\right.}{7d^4}}{d} \\ \sqrt[3]{c}\left(Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}\right) \end{cases}$$

input
```
integrate((b*x+a)*(d*x+c)**(1/3)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((3*(D*b*(c + d*x)**(16/3)/(16*d**4) + (c + d*x)**(13/3)*(C*b*d +
 D*a*d - 4*D*b*c)/(13*d**4) + (c + d*x)**(10/3)*(B*b*d**2 + C*a*d**2 - 3*C
*b*c*d - 3*D*a*c*d + 6*D*b*c**2)/(10*d**4) + (c + d*x)**(7/3)*(A*b*d**3 +
B*a*d**3 - 2*B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D
*b*c**3)/(7*d**4) + (c + d*x)**(4/3)*(A*a*d**4 - A*b*c*d**3 - B*a*c*d**3 +
 B*b*c**2*d**2 + C*a*c**2*d**2 - C*b*c**3*d - D*a*c**3*d + D*b*c**4)/(4*d*
*4))/d, Ne(d, 0)), (c**(1/3)*(A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/4 + x*
*3*(B*b + C*a)/3 + x**2*(A*b + B*a)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{3\left(455\,(dx+c)^{\frac{16}{3}}Db - 560\,(4\,Dbc - (Da+Cb)d)(dx+c)^{\frac{13}{3}} + 728\,(6\,Dbc^2 - 3\,(Da+Cb)cd + (Ca+B\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
3/7280*(455*(d*x + c)^(16/3)*D*b - 560*(4*D*b*c - (D*a + C*b)*d)*(d*x + c)
^(13/3) + 728*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*(d*x + c)^
(10/3) - 1040*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 - (B*
a + A*b)*d^3)*(d*x + c)^(7/3) + 1820*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c^3*
d + (C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)*(d*x + c)^(4/3))/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(196) = 392.

Time = 0.13 (sec) , antiderivative size = 721, normalized size of antiderivative = 3.37

$$\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
3/7280*(7280*(d*x + c)^(1/3)*A*a*c + 1820*((d*x + c)^(4/3) - 4*(d*x + c)^(
1/3)*c)*A*a + 1820*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*B*a*c/d + 1820*
((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*A*b*c/d + 520*(2*(d*x + c)^(7/3) -
 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*C*a*c/d^2 + 520*(2*(d*x + c
)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*B*b*c/d^2 + 520*(2
*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*B*a/d + 5
20*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*A*b/
d + 52*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c
^2 - 140*(d*x + c)^(1/3)*c^3)*D*a*c/d^3 + 52*(14*(d*x + c)^(10/3) - 60*(d*
x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*C*b*c/
d^3 + 52*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)
*c^2 - 140*(d*x + c)^(1/3)*c^3)*C*a/d^2 + 52*(14*(d*x + c)^(10/3) - 60*(d*
x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*B*b/d^
2 + 16*(35*(d*x + c)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)
*c^2 - 455*(d*x + c)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*D*b*c/d^4 + 16*(
35*(d*x + c)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 4
55*(d*x + c)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*D*a/d^3 + 16*(35*(d*x +
c)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 455*(d*x +
c)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*C*b/d^3 + 5*(91*(d*x + c)^(16/3) -
 560*(d*x + c)^(13/3)*c + 1456*(d*x + c)^(10/3)*c^2 - 2080*(d*x + c)^(7...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int (a+b\,x)\,\left(c+d\,x\right)^{1/3}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)*(c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)*(c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98

$$\int (a+bx)\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{3(dx+c)^{\frac{1}{3}}\left(455b\,d^5x^5+560a\,d^5x^4+595bc\,d^4x^4+784ac\,d^4x^3+728b^2d^4x^3+14b\,c^2d^3x^3+2080ab\,d^4x^2+\right.}{}$$

input
```
int((b*x+a)*(d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(3*(c + d*x)**(1/3)*(1820*a**2*c*d**3 + 1820*a**2*d**4*x - 1560*a*b*c**2*d
**2 + 520*a*b*c*d**3*x + 2080*a*b*d**4*x**2 + 144*a*c**4*d - 48*a*c**3*d**
2*x + 32*a*c**2*d**3*x**2 + 784*a*c*d**4*x**3 + 560*a*d**5*x**4 + 468*b**2
*c**3*d - 156*b**2*c**2*d**2*x + 104*b**2*c*d**3*x**2 + 728*b**2*d**4*x**3
 - 81*b*c**5 + 27*b*c**4*d*x - 18*b*c**3*d**2*x**2 + 14*b*c**2*d**3*x**3 +
 595*b*c*d**4*x**4 + 455*b*d**5*x**5))/(7280*d**4)
```

3.156 $\int \sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)\,dx$

Optimal result . . . 1510
Mathematica [A] (verified) . . . 1510
Rubi [A] (verified) . . . 1511
Maple [A] (verified) . . . 1512
Fricas [A] (verification not implemented) . . . 1513
Sympy [A] (verification not implemented) . . . 1513
Maxima [A] (verification not implemented) . . . 1514
Giac [B] (verification not implemented) . . . 1514
Mupad [F(-1)] . . . 1515
Reduce [B] (verification not implemented) . . . 1515

Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{3(c^2Cd-Bcd^2+Ad^3-c^3D)\,(c+dx)^{4/3}}{4d^4}-\frac{3(2cCd-Bd^2-3c^2D)\,(c+dx)^{7/3}}{7d^4}+\frac{3(Cd-3cD)(c+dx)^{10/3}}{10d^4}+\frac{3D(c+dx)^{13/3}}{13d^4}$$

output
```
3/4*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(4/3)/d^4-3/7*(-B*d^2+2*C*c*d-3*
D*c^2)*(d*x+c)^(7/3)/d^4+3/10*(C*d-3*D*c)*(d*x+c)^(10/3)/d^4+3/13*D*(d*x+c
)^(13/3)/d^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{3(c+dx)^{4/3}\left(-81c^3D+9c^2d(13C+12Dx)-3cd^2(65B+52Cx+42Dx^2)+d^3(455A+2x(130B+91\right.}{1820d^4}$$

input
```
Integrate[(c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(3*(c + d*x)^(4/3)*(-81*c^3*D + 9*c^2*d*(13*C + 12*D*x) - 3*c*d^2*(65*B +
52*C*x + 42*D*x^2) + d^3*(455*A + 2*x*(130*B + 91*C*x + 70*D*x^2))))/(1820
*d^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{\sqrt[3]{c+dx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3}+\frac{(c+dx)^{4/3}\left(Bd^2+3c^2D-2cCd\right)}{d^3}+\frac{(c+dx)^{7/3}(Cd-3cD)}{d^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3(c+dx)^{4/3}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{4d^4}-\frac{3(c+dx)^{7/3}\left(-Bd^2-3c^2D+2cCd\right)}{7d^4}+\frac{3(c+dx)^{10/3}(Cd-3cD)}{10d^4}+\frac{3D(c+dx)^{13/3}}{13d^4}$$

input
```
Int[(c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(4/3))/(4*d^4) - (3*(2*c*
C*d - B*d^2 - 3*c^2*D)*(c + d*x)^(7/3))/(7*d^4) + (3*(C*d - 3*c*D)*(c + d*
x)^(10/3))/(10*d^4) + (3*D*(c + d*x)^(13/3))/(13*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{3\left(\left(\frac{4}{13}Dx^3+\frac{2}{5}Cx^2+\frac{4}{7}Bx+A\right)d^3-\frac{3\left(\frac{42}{65}Dx^2+\frac{4}{5}Cx+B\right)c\,d^2}{7}+\frac{9\left(\frac{12Dx}{13}+C\right)c^2d}{35}-\frac{81Dc^3}{455}\right)(xd+c)^{\frac{4}{3}}}{4d^4}$
gosper	$\frac{3(xd+c)^{\frac{4}{3}}\left(140Dx^3d^3+182Cx^2d^3-126Dx^2c\,d^2+260Bx\,d^3-156Cxc\,d^2+108Dx\,c^2d+455A\,d^3-195Bc\,d^2+117C\,c^2d-\right.}{1820d^4}$
orering	$\frac{3(xd+c)^{\frac{4}{3}}\left(140Dx^3d^3+182Cx^2d^3-126Dx^2c\,d^2+260Bx\,d^3-156Cxc\,d^2+108Dx\,c^2d+455A\,d^3-195Bc\,d^2+117C\,c^2d-\right.}{1820d^4}$
derivativedivides	$\frac{\frac{3D(xd+c)^{\frac{13}{3}}}{13}+\frac{3(Cd-3Dc)(xd+c)^{\frac{10}{3}}}{10}+\frac{3\left(B\,d^2-2Ccd+3Dc^2\right)(xd+c)^{\frac{7}{3}}}{7}+\frac{3\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)(xd+c)^{\frac{4}{3}}}{4}}{d^4}$
default	$\frac{\frac{3D(xd+c)^{\frac{13}{3}}}{13}+\frac{3(Cd-3Dc)(xd+c)^{\frac{10}{3}}}{10}+\frac{3\left(B\,d^2-2Ccd+3Dc^2\right)(xd+c)^{\frac{7}{3}}}{7}+\frac{3\left(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3\right)(xd+c)^{\frac{4}{3}}}{4}}{d^4}$
trager	$\frac{3\left(140Dd^4x^4+182C\,d^4x^3+14Dc\,d^3x^3+260B\,d^4x^2+26Cc\,d^3x^2-18Dc^2d^2x^2+455A\,d^4x+65Bc\,d^3x-39C\,c^2d^2x+27xD\right.}{1820d^4}$

input
```
int((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
3/4*((4/13*D*x^3+2/5*C*x^2+4/7*B*x+A)*d^3-3/7*(42/65*D*x^2+4/5*C*x+B)*c*d^
2+9/35*(12/13*D*x+C)*c^2*d-81/455*D*c^3)*(d*x+c)^(4/3)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.10

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{3\,(140\,Dd^4x^4 - 81\,Dc^4 + 117\,Cc^3d - 195\,Bc^2d^2 + 455\,Acd^3 + 14\,(Dcd^3 + 13\,Cd^4)x^3 - 2\,(9\,Dc^2d^2 - 13\,C}{1820\,d^4}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
3/1820*(140*D*d^4*x^4 - 81*D*c^4 + 117*C*c^3*d - 195*B*c^2*d^2 + 455*A*c*d
^3 + 14*(D*c*d^3 + 13*C*d^4)*x^3 - 2*(9*D*c^2*d^2 - 13*C*c*d^3 - 130*B*d^4
)*x^2 + (27*D*c^3*d - 39*C*c^2*d^2 + 65*B*c*d^3 + 455*A*d^4)*x)*(d*x + c)^
(1/3)/d^4
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \begin{cases} \frac{3\left(\frac{D(c+dx)^{\frac{13}{3}}}{13d^3} + \frac{(c+dx)^{\frac{10}{3}}(Cd-3Dc)}{10d^3} + \frac{(c+dx)^{\frac{7}{3}}\left(Bd^2-2Ccd+3Dc^2\right)}{7d^3} + \frac{(c+dx)^{\frac{4}{3}}\left(Ad^3-Bcd^2+Cc^2d-Dc^3\right)}{4d^3}\right)}{d} & \text{for } d \neq 0 \\ \sqrt[3]{c}\left(Ax + \frac{Bx^2}{2} + \frac{Cx^3}{3} + \frac{Dx^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**(1/3)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((3*(D*(c + d*x)**(13/3)/(13*d**3) + (c + d*x)**(10/3)*(C*d - 3*D
*c)/(10*d**3) + (c + d*x)**(7/3)*(B*d**2 - 2*C*c*d + 3*D*c**2)/(7*d**3) +
(c + d*x)**(4/3)*(A*d**3 - B*c*d**2 + C*c**2*d - D*c**3)/(4*d**3))/d, Ne(d
, 0)), (c**(1/3)*(A*x + B*x**2/2 + C*x**3/3 + D*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{3\left(140\,(dx+c)^{\frac{13}{3}}D-182\,(3\,Dc-Cd)(dx+c)^{\frac{10}{3}}+260\,(3\,Dc^2-2\,Ccd+Bd^2)(dx+c)^{\frac{7}{3}}-455\,(Dc^3-\right.}{1820\,d^4}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
3/1820*(140*(d*x + c)^(13/3)*D - 182*(3*D*c - C*d)*(d*x + c)^(10/3) + 260*
(3*D*c^2 - 2*C*c*d + B*d^2)*(d*x + c)^(7/3) - 455*(D*c^3 - C*c^2*d + B*c*d
^2 - A*d^3)*(d*x + c)^(4/3))/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(101) = 202.

Time = 0.13 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.59

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{3\left(1820\,(dx+c)^{\frac{1}{3}}Ac+455\left((dx+c)^{\frac{4}{3}}-4\,(dx+c)^{\frac{1}{3}}c\right)A+\frac{455\left((dx+c)^{\frac{4}{3}}-4\,(dx+c)^{\frac{1}{3}}c\right)Bc}{d}+\frac{130\left(2\,(dx+c)^{\frac{7}{3}}-7\,(dx\right.}{}\right.}{}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
3/1820*(1820*(d*x + c)^(1/3)*A*c + 455*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3
)*c)*A + 455*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*B*c/d + 130*(2*(d*x +
 c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*C*c/d^2 + 130*(2
*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*B/d + 13*
(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 14
0*(d*x + c)^(1/3)*c^3)*D*c/d^3 + 13*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7
/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*C/d^2 + 4*(35*(
d*x + c)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 455*(
d*x + c)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*D/d^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (c+d\,x)^{1/3}\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{3(dx+c)^{\frac{1}{3}}\left(140d^4x^4+196c\,d^3x^3+260b\,d^3x^2+8c^2d^2x^2+455a\,d^3x+65bc\,d^2x-12c^3dx+455ac\,d^2-1\right.}{1820d^3}$$

input
```
int((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(3*(c + d*x)**(1/3)*(455*a*c*d**2 + 455*a*d**3*x - 195*b*c**2*d + 65*b*c*d
**2*x + 260*b*d**3*x**2 + 36*c**4 - 12*c**3*d*x + 8*c**2*d**2*x**2 + 196*c
*d**3*x**3 + 140*d**4*x**4))/(1820*d**3)
```

3.157 $\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$

Optimal result 1516
Mathematica [A] (verified) 1517
Rubi [A] (verified) 1518
Maple [A] (verified) 1519
Fricas [A] (verification not implemented) 1521
Sympy [F] 1521
Maxima [F(-2)] 1522
Giac [B] (verification not implemented) 1522
Mupad [F(-1)] 1523
Reduce [B] (verification not implemented) 1523

Optimal result

Integrand size = 32, antiderivative size = 371

$$\begin{aligned}
\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx &= \frac{3(Ab^3-a(b^2B-abC+a^2D))\sqrt[3]{c+dx}}{b^4} \\
&+ \frac{3(a^2d^2D-abd(Cd-cD)-b^2(cCd-Bd^2-c^2D))(c+dx)^{4/3}}{4b^3d^3} \\
&+ \frac{3(bCd-2bcD-adD)(c+dx)^{7/3}}{7b^2d^3} + \frac{3D(c+dx)^{10/3}}{10bd^3} \\
&- \frac{\sqrt{3}\sqrt[3]{bc-ad}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{13/3}} \\
&- \frac{\sqrt[3]{bc-ad}(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{2b^{13/3}} \\
&+ \frac{3\sqrt[3]{bc-ad}(Ab^3-a(b^2B-abC+a^2D))\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{13/3}}
\end{aligned}$$

output
```
3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/3)/b^4+3/4*(a^2*d^2*D-a*b*d*(C*
d-D*c)-b^2*(-B*d^2+C*c*d-D*c^2))*(d*x+c)^(4/3)/b^3/d^3+3/7*(C*b*d-D*a*d-2*
D*b*c)*(d*x+c)^(7/3)/b^2/d^3+3/10*D*(d*x+c)^(10/3)/b/d^3-3^(1/2)*(-a*d+b*c
)^(1/3)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3
)/(-a*d+b*c)^(1/3))*3^(1/2))/b^(13/3)-1/2*(-a*d+b*c)^(1/3)*(A*b^3-a*(B*b^2
-C*a*b+D*a^2))*ln(b*x+a)/b^(13/3)+3/2*(-a*d+b*c)^(1/3)*(A*b^3-a*(B*b^2-C*a
*b+D*a^2))*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/b^(13/3)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$

$$= \frac{\frac{3\sqrt[3]{b}\sqrt[3]{c+dx}(-140a^3d^3D+35a^2bd^2(4Cd+D(c+dx))-5ab^2d(-3c^2D+cd(7C+Dx)+d^2(28B+7Cx+4Dx^2))+b^3(9c^3D-3c^2d(5C+Dx)+c}{d^3}}{}$$

input
```
Integrate[((c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
((3*b^(1/3)*(c + d*x)^(1/3)*(-140*a^3*d^3*D + 35*a^2*b*d^2*(4*C*d + D*(c +
 d*x)) - 5*a*b^2*d*(-3*c^2*D + c*d*(7*C + D*x) + d^2*(28*B + 7*C*x + 4*D*x
^2)) + b^3*(9*c^3*D - 3*c^2*d*(5*C + D*x) + c*d^2*(35*B + 5*C*x + 2*D*x^2)
 + d^3*(140*A + 35*B*x + 20*C*x^2 + 14*D*x^3))))/d^3 + 140*Sqrt[3]*(-(b*c)
 + a*d)^(1/3)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(1 - (2*b^(1/3)*(
c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3))/Sqrt[3]] - 140*(-(b*c) + a*d)^(1/3)*
(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Log[(-(b*c) + a*d)^(1/3) + b^(1/3)*(c
+ d*x)^(1/3)] + 70*(-(b*c) + a*d)^(1/3)*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)
)*Log[(-(b*c) + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3)
+ b^(2/3)*(c + d*x)^(2/3)])/(140*b^(13/3))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}dx$$

$$\Big\downarrow \ 2123$$

$$\int \left(\frac{\sqrt[3]{c+dx}(Ab^3-a(a^2D-abC+b^2B))}{b^3(a+bx)}+\frac{\sqrt[3]{c+dx}(a^2d^2D-abd(Cd-cD)-(b^2(-Bd^2+c^2(-D)+cCd)}{b^3d^2}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{3}\sqrt[3]{bc-ad}(Ab^3-a(a^2D-abC+b^2B))\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{13/3}}-\frac{\sqrt[3]{bc-ad}\log(a+bx)\left(Ab^3-a(a^2D-abC+b^2B)\right)}{2b^{13/3}}+\frac{3\sqrt[3]{bc-ad}(Ab^3-a(a^2D-abC+b^2B))\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{13/3}}+\frac{3\sqrt[3]{c+dx}(Ab^3-a(a^2D-abC+b^2B))}{b^4}+\frac{3(c+dx)^{4/3}\left(a^2d^2D-abd(Cd-cD)-(b^2(-Bd^2+c^2(-D)+cCd))\right)}{4b^3d^3}+\frac{3(c+dx)^{7/3}(-adD-2bcD+bCd)}{7b^2d^3}+\frac{3D(c+dx)^{10/3}}{10bd^3}$$

input
```
Int[((c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
(3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(1/3))/b^4 + (3*(a^2*d^2*
D - a*b*d*(C*d - c*D) - b^2*(c*C*d - B*d^2 - c^2*D))*(c + d*x)^(4/3))/(4*b
^3*d^3) + (3*(b*C*d - 2*b*c*D - a*d*D)*(c + d*x)^(7/3))/(7*b^2*d^3) + (3*D
*(c + d*x)^(10/3))/(10*b*d^3) - (Sqrt[3]*(b*c - a*d)^(1/3)*(A*b^3 - a*(b^2
*B - a*b*C + a^2*D))*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(
1/3))/Sqrt[3]])/b^(13/3) - ((b*c - a*d)^(1/3)*(A*b^3 - a*(b^2*B - a*b*C +
a^2*D))*Log[a + b*x])/(2*b^(13/3)) + (3*(b*c - a*d)^(1/3)*(A*b^3 - a*(b^2*
B - a*b*C + a^2*D))*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b
^(13/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{-3\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\left(\left(\left(\left(\frac{1}{10}Dx^3+\frac{1}{7}Cx^2+\frac{1}{4}Bx+A\right)b^3-\left(\frac{1}{7}Dx^2+\frac{1}{4}Cx+B\right)ab^2+a^2\left(\frac{Dx}{4}+C\right)b-a^3D\right)d^3+\frac{\left(\left(\frac{2}{35}Dx^2+\frac{1}{7}Cx\cdots\right.\right.}{\cdots}\right.\right.}{\cdots}$
derivativedivides	$\frac{3\left(\frac{D(xd+c)^{\frac{10}{3}}b^3}{10}+\frac{Cb^3d(xd+c)^{\frac{7}{3}}}{7}-\frac{Dab^2d(xd+c)^{\frac{7}{3}}}{7}-\frac{2Db^3c(xd+c)^{\frac{7}{3}}}{7}+\frac{Bb^3d^2(xd+c)^{\frac{4}{3}}}{4}-\frac{Cab^2d^2(xd+c)^{\frac{4}{3}}}{4}-\frac{Cb^3cd(xd+c)^{\frac{4}{3}}}{4}+\cdots\right.}{\cdots}$
default	$\frac{3\left(\frac{D(xd+c)^{\frac{10}{3}}b^3}{10}+\frac{Cb^3d(xd+c)^{\frac{7}{3}}}{7}-\frac{Dab^2d(xd+c)^{\frac{7}{3}}}{7}-\frac{2Db^3c(xd+c)^{\frac{7}{3}}}{7}+\frac{Bb^3d^2(xd+c)^{\frac{4}{3}}}{4}-\frac{Cab^2d^2(xd+c)^{\frac{4}{3}}}{4}-\frac{Cb^3cd(xd+c)^{\frac{4}{3}}}{4}+\cdots\right.}{\cdots}$

input
```
int((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/((a*d-b*c)/b)^(2/3)*(-3*((a*d-b*c)/b)^(2/3)*(((1/10*D*x^3+1/7*C*x^2+1/4
*B*x+A)*b^3-(1/7*D*x^2+1/4*C*x+B)*a*b^2+a^2*(1/4*D*x+C)*b-a^3*D)*d^3+1/4*(
(2/35*D*x^2+1/7*C*x+B)*b^2-(1/7*D*x+C)*a*b+D*a^2)*c*b*d^2-3/28*((1/5*D*x+C
)*b-D*a)*c^2*b^2*d+9/140*D*b^3*c^3)*(d*x+c)^(1/3)*b+(arctan(1/3*3^(1/2)*(2
*(d*x+c)^(1/3)-((a*d-b*c)/b)^(1/3))/((a*d-b*c)/b)^(1/3))*3^(1/2)+ln((d*x+c
)^(1/3)+((a*d-b*c)/b)^(1/3))-1/2*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x
+c)^(1/3)+((a*d-b*c)/b)^(2/3)))*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)*(a*d-b*c)*d^
3)/d^3/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx =$$

$$-\frac{140\sqrt{3}(Da^3-Ca^2b+Bab^2-Ab^3)d^3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(dx+c)^{\frac{1}{3}}b\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}+\sqrt{3}(bc-ad)}{3(bc-ad)}\right)-70\,(Da^3-}{}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
-1/140*(140*sqrt(3)*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^3*((b*c - a*d)/b
)^(1/3)*arctan(-1/3*(2*sqrt(3)*(d*x + c)^(1/3)*b*((b*c - a*d)/b)^(2/3) + s
qrt(3)*(b*c - a*d))/(b*c - a*d)) - 70*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*
d^3*((b*c - a*d)/b)^(1/3)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*
d)/b)^(1/3) + ((b*c - a*d)/b)^(2/3)) + 140*(D*a^3 - C*a^2*b + B*a*b^2 - A*
b^3)*d^3*((b*c - a*d)/b)^(1/3)*log((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)
) - 3*(14*D*b^3*d^3*x^3 + 9*D*b^3*c^3 + 15*(D*a*b^2 - C*b^3)*c^2*d + 35*(D
*a^2*b - C*a*b^2 + B*b^3)*c*d^2 - 140*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*
d^3 + 2*(D*b^3*c*d^2 - 10*(D*a*b^2 - C*b^3)*d^3)*x^2 - (3*D*b^3*c^2*d + 5*
(D*a*b^2 - C*b^3)*c*d^2 - 35*(D*a^2*b - C*a*b^2 + B*b^3)*d^3)*x)*(d*x + c)
^(1/3))/(b^4*d^3)
```

Sympy [F]

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$

input
```
integrate((d*x+c)**(1/3)*(D*x**3+C*x**2+B*x+A)/(b*x+a),x)
```

output
```
Integral((c + d*x)**(1/3)*(A + B*x + C*x**2 + D*x**3)/(a + b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(323) = 646$.

Time = 0.18 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
-(D*a^3*b^7*c*d^33 - C*a^2*b^8*c*d^33 + B*a*b^9*c*d^33 - A*b^10*c*d^33 - D
*a^4*b^6*d^34 + C*a^3*b^7*d^34 - B*a^2*b^8*d^34 + A*a*b^9*d^34)*((b*c - a*
d)/b)^(1/3)*log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b^11*c*d^33
 - a*b^10*d^34) + (sqrt(3)*(b^3*c - a*b^2*d)^(1/3)*D*a^3 - sqrt(3)*(b^3*c
- a*b^2*d)^(1/3)*C*a^2*b + sqrt(3)*(b^3*c - a*b^2*d)^(1/3)*B*a*b^2 - sqrt(
3)*(b^3*c - a*b^2*d)^(1/3)*A*b^3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) +
((b*c - a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/b^5 + 1/2*((b^3*c - a*b^2*d)
^(1/3)*D*a^3 - (b^3*c - a*b^2*d)^(1/3)*C*a^2*b + (b^3*c - a*b^2*d)^(1/3)*B
*a*b^2 - (b^3*c - a*b^2*d)^(1/3)*A*b^3)*log((d*x + c)^(2/3) + (d*x + c)^(1
/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d)/b)^(2/3))/b^5 + 3/140*(14*(d*x +
c)^(10/3)*D*b^9*d^27 - 40*(d*x + c)^(7/3)*D*b^9*c*d^27 + 35*(d*x + c)^(4/3
)*D*b^9*c^2*d^27 - 20*(d*x + c)^(7/3)*D*a*b^8*d^28 + 20*(d*x + c)^(7/3)*C*
b^9*d^28 + 35*(d*x + c)^(4/3)*D*a*b^8*c*d^28 - 35*(d*x + c)^(4/3)*C*b^9*c*
d^28 + 35*(d*x + c)^(4/3)*D*a^2*b^7*d^29 - 35*(d*x + c)^(4/3)*C*a*b^8*d^29
 + 35*(d*x + c)^(4/3)*B*b^9*d^29 - 140*(d*x + c)^(1/3)*D*a^3*b^6*d^30 + 14
0*(d*x + c)^(1/3)*C*a^2*b^7*d^30 - 140*(d*x + c)^(1/3)*B*a*b^8*d^30 + 140*
(d*x + c)^(1/3)*A*b^9*d^30)/(b^10*d^30)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx=\int \frac{(c+d\,x)^{1/3}\,(A+B\,x+C\,x^2+x^3\,D)}{a+b\,x}\,dx$$

input
```
int(((c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D))/(a + b*x),x)
```

output
```
int(((c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D))/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1132, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

output

```
( - 140*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c
+ d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**4*d**4 + 280*sqrt(3)*atan
((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(
1/6)*(a*d - b*c)**(1/6)))*a**3*b*c*d**3 - 140*sqrt(3)*atan((b**(1/6)*(a*d
- b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)
**(1/6)))*a**2*b**2*c**2*d**2 - 140*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1
/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*
a**4*d**4 + 280*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(
1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**3*b*c*d**3 - 140*
sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**
(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**2*c**2*d**2 - 420*b**(1/3)*(
c + d*x)**(1/3)*(a*d - b*c)**(2/3)*a**3*d**3 + 525*b**(1/3)*(c + d*x)**(1/
3)*(a*d - b*c)**(2/3)*a**2*b*c*d**2 + 105*b**(1/3)*(c + d*x)**(1/3)*(a*d -
 b*c)**(2/3)*a**2*b*d**3*x - 60*b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2/
3)*a*b**2*c**2*d - 120*b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2/3)*a*b**2
*c*d**2*x - 60*b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2/3)*a*b**2*d**3*x*
*2 + 105*b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2/3)*b**4*c*d + 105*b**(1
/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2/3)*b**4*d**2*x - 18*b**(1/3)*(c + d*x
)**(1/3)*(a*d - b*c)**(2/3)*b**3*c**3 + 6*b**(1/3)*(c + d*x)**(1/3)*(a*d -
 b*c)**(2/3)*b**3*c**2*d*x + 66*b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**...
```

3.158 $\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$

Optimal result . . . 1525
Mathematica [A] (verified) . . . 1526
Rubi [A] (verified) . . . 1527
Maple [A] (verified) . . . 1529
Fricas [B] (verification not implemented) . . . 1531
Sympy [F(-1)] . . . 1532
Maxima [F(-2)] . . . 1533
Giac [A] (verification not implemented) . . . 1533
Mupad [F(-1)] . . . 1534
Reduce [B] (verification not implemented) . . . 1534

Optimal result

Integrand size = 32, antiderivative size = 418

$$\begin{aligned}
&\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx \\
&= \frac{3(b^2B-2abC+3a^2D)\sqrt[3]{c+dx}}{b^4}-\frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt[3]{c+dx}}{b^4(a+bx)} \\
&\quad+\frac{3(bCd-bcD-2adD)(c+dx)^{4/3}}{4b^3d^2}+\frac{3D(c+dx)^{7/3}}{7b^2d^2} \\
&\quad-\frac{(b^3(3Bc+Ad)-2ab^2(3cC+2Bd)-10a^3dD+a^2b(7Cd+9cD))\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}b^{13/3}(bc-ad)^{2/3}} \\
&\quad-\frac{(b^3(3Bc+Ad)-2ab^2(3cC+2Bd)-10a^3dD+a^2b(7Cd+9cD))\log(a+bx)}{6b^{13/3}(bc-ad)^{2/3}} \\
&\quad+\frac{(b^3(3Bc+Ad)-2ab^2(3cC+2Bd)-10a^3dD+a^2b(7Cd+9cD))\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{13/3}(bc-ad)^{2/3}}
\end{aligned}$$

output
```
3*(B*b^2-2*C*a*b+3*D*a^2)*(d*x+c)^(1/3)/b^4-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*
(d*x+c)^(1/3)/b^4/(b*x+a)+3/4*(C*b*d-2*D*a*d-D*b*c)*(d*x+c)^(4/3)/b^3/d^2+
3/7*D*(d*x+c)^(7/3)/b^2/d^2-1/3*(b^3*(A*d+3*B*c)-2*a*b^2*(2*B*d+3*C*c)-10*
a^3*d*D+a^2*b*(7*C*d+9*D*c))*arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/(-a*d+b
*c)^(1/3))*3^(1/2))*3^(1/2)/b^(13/3)/(-a*d+b*c)^(2/3)-1/6*(b^3*(A*d+3*B*c)
-2*a*b^2*(2*B*d+3*C*c)-10*a^3*d*D+a^2*b*(7*C*d+9*D*c))*ln(b*x+a)/b^(13/3)/
(-a*d+b*c)^(2/3)+1/2*(b^3*(A*d+3*B*c)-2*a*b^2*(2*B*d+3*C*c)-10*a^3*d*D+a^2
*b*(7*C*d+9*D*c))*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/b^(13/3)/(-a*
d+b*c)^(2/3)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$

$$= \frac{\frac{3\sqrt[3]{b}\sqrt[3]{c+dx}(280a^3d^2D-14a^2bd(14Cd+3D(c-5dx))-ab^2(9c^2D+c(-21Cd+39dDx)+d^2(-112B+147Cx+30Dx^2))+b^3(-28Ad^2+3x(-}{d^2(a+bx)}}{}$$

input
```
Integrate[((c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
((3*b^(1/3)*(c + d*x)^(1/3)*(280*a^3*d^2*D - 14*a^2*b*d*(14*C*d + 3*D*(c -
 5*d*x)) - a*b^2*(9*c^2*D + c*(-21*C*d + 39*d*D*x) + d^2*(-112*B + 147*C*x
 + 30*D*x^2)) + b^3*(-28*A*d^2 + 3*x*(-3*c^2*D + c*d*(7*C + D*x) + d^2*(28
*B + 7*C*x + 4*D*x^2)))))/(d^2*(a + b*x)) - (28*Sqrt[3]*(b^3*(3*B*c + A*d)
 - 2*a*b^2*(3*c*C + 2*B*d) - 10*a^3*d*D + a^2*b*(7*C*d + 9*c*D))*ArcTan[(1
 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3))/Sqrt[3]])/(-(b*c) + a
*d)^(2/3) + (28*(b^3*(3*B*c + A*d) - 2*a*b^2*(3*c*C + 2*B*d) - 10*a^3*d*D
+ a^2*b*(7*C*d + 9*c*D))*Log[(-(b*c) + a*d)^(1/3) + b^(1/3)*(c + d*x)^(1/3
)]])/(-(b*c) + a*d)^(2/3) - (14*(b^3*(3*B*c + A*d) - 2*a*b^2*(3*c*C + 2*B*d
) - 10*a^3*d*D + a^2*b*(7*C*d + 9*c*D))*Log[(-(b*c) + a*d)^(2/3) - b^(1/3)
*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)])/(-(b*c)
+ a*d)^(2/3))/(84*b^(13/3))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2124, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c+dx}\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{ 2124}$$

$$-\frac{\int -\frac{\sqrt[3]{c+dx}\left(3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{-4dDa^3+b(4Cd+3cD)a^2-b^2(3cC+4Bd)a+b^3(3Bc+Ad)}{b^3}\right)}{3(a+bx)}dx}{bc-ad}-\frac{(c+dx)^{4/3}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt[3]{c+dx}\left(-\frac{4dDa^3}{b^3}+\frac{(4Cd+3cD)a^2}{b^2}-\frac{(3cC+4Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc+Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}\right)}{a+bx}dx}{3(bc-ad)}-\frac{(c+dx)^{4/3}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 1195}$$

$$\frac{\int \left(\frac{3(bc-ad)D(c+dx)^{4/3}}{b^2d}+\frac{3(bc-ad)(bCd-2aDd-bcD)\sqrt[3]{c+dx}}{b^3d}+\frac{\left(-10dDa^3+b(7Cd+9cD)a^2-2b^2(3cC+2Bd)a+b^3(3Bc+Ad)\right)\sqrt[3]{c+}}{b^3(a+bx)}\right.}{3(bc-ad)}$$
$$-\frac{(c+dx)^{4/3}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{3}\sqrt[3]{bc-ad}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)(-10a^3dD+a^2b(9cD+7Cd)-2ab^2(2Bd+3cC)+b^3(Ad+3Bc))}{b^{13/3}} - \frac{\sqrt[3]{bc-ad}\log(a+bx)(-1(}{}$$

$$\frac{(c+dx)^{4/3}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{(a+bx)(bc-ad)}$$

input
```
Int[((c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
-(((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(4/3))/((b*c - a*d)*(a
+ b*x))) + ((3*(b^3*(3*B*c + A*d) - 2*a*b^2*(3*c*C + 2*B*d) - 10*a^3*d*D +
 a^2*b*(7*C*d + 9*c*D))*(c + d*x)^(1/3))/b^4 + (9*(b*c - a*d)*(b*C*d - b*c
*D - 2*a*d*D)*(c + d*x)^(4/3))/(4*b^3*d^2) + (9*(b*c - a*d)*D*(c + d*x)^(7
/3))/(7*b^2*d^2) - (Sqrt[3]*(b*c - a*d)^(1/3)*(b^3*(3*B*c + A*d) - 2*a*b^2
*(3*c*C + 2*B*d) - 10*a^3*d*D + a^2*b*(7*C*d + 9*c*D))*ArcTan[(1 + (2*b^(1
/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/b^(13/3) - ((b*c - a*d)^
(1/3)*(b^3*(3*B*c + A*d) - 2*a*b^2*(3*c*C + 2*B*d) - 10*a^3*d*D + a^2*b*(7
*C*d + 9*c*D))*Log[a + b*x])/(2*b^(13/3)) + (3*(b*c - a*d)^(1/3)*(b^3*(3*B
*c + A*d) - 2*a*b^2*(3*c*C + 2*B*d) - 10*a^3*d*D + a^2*b*(7*C*d + 9*c*D))*
Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(13/3)))/(3*(b*c -
a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{6\left(\left(\left(-\frac{3}{7}Dx^{3}-\frac{3}{4}Cx^{2}-3Bx+A\right)b^{3}-4\left(-\frac{15}{56}Dx^{2}-\frac{21}{16}Cx+B\right)ab^{2}+7\left(-\frac{15Dx}{14}+C\right)a^{2}b-10a^{3}D\right)d^{2}-\frac{3\left(\left(\frac{Dx}{7}+C\right)b-2I\right.}{4}\right.}{}$ … $3a$
derivativedivides	$\frac{3\left(\frac{D(xd+c)^{\frac{7}{3}}b^{2}}{7}+\frac{Cb^{2}d(xd+c)^{\frac{4}{3}}}{4}-\frac{Dabd(xd+c)^{\frac{4}{3}}}{2}-\frac{Db^{2}c(xd+c)^{\frac{4}{3}}}{4}+Bd^{2}b^{2}(xd+c)^{\frac{1}{3}}-2Cad^{2}b(xd+c)^{\frac{1}{3}}+3Da^{2}d^{2}(xd+c)^{\frac{1}{3}}\right)}{b^{4}}+-$ … $3a$
default	$\frac{3\left(\frac{D(xd+c)^{\frac{7}{3}}b^{2}}{7}+\frac{Cb^{2}d(xd+c)^{\frac{4}{3}}}{4}-\frac{Dabd(xd+c)^{\frac{4}{3}}}{2}-\frac{Db^{2}c(xd+c)^{\frac{4}{3}}}{4}+Bd^{2}b^{2}(xd+c)^{\frac{1}{3}}-2Cad^{2}b(xd+c)^{\frac{1}{3}}+3Da^{2}d^{2}(xd+c)^{\frac{1}{3}}\right)}{b^{4}}+-$

input
```
int((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(6*(((-3/7*D*x^3-3/4*C*x^2-3*B*x+A)*b^3-4*(-15/56*D*x^2-21/16*C*x+B)*
a*b^2+7*(-15/14*D*x+C)*a^2*b-10*a^3*D)*d^2-3/4*((1/7*D*x+C)*b-2*D*a)*(b*x+
a)*c*b*d+9/28*D*b^2*c^2*(b*x+a))*((a*d-b*c)/b)^(2/3)*b*(d*x+c)^(1/3)+((A*b
^3-4*B*a*b^2+7*C*a^2*b-10*D*a^3)*d+3*b*c*(B*b^2-2*C*a*b+3*D*a^2))*d^2*(b*x
+a)*(-2*arctan(1/3*3^(1/2)*(2*(d*x+c)^(1/3)-((a*d-b*c)/b)^(1/3))/((a*d-b*c
)/b)^(1/3))*3^(1/2)+ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a
*d-b*c)/b)^(2/3))-2*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))))/((a*d-b*c)/b)^
(2/3)/d^2/b^5/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1337 vs. $2(381) = 762$.

Time = 0.44 (sec) , antiderivative size = 2767, normalized size of antiderivative = 6.62

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="fricas
")
```

output
```
[-1/84*(42*sqrt(1/3)*(3*(3*D*a^3*b^3 - 2*C*a^2*b^4 + B*a*b^5)*c^2*d^2 - (1
9*D*a^4*b^2 - 13*C*a^3*b^3 + 7*B*a^2*b^4 - A*a*b^5)*c*d^3 + (10*D*a^5*b -
7*C*a^4*b^2 + 4*B*a^3*b^3 - A*a^2*b^4)*d^4 + (3*(3*D*a^2*b^4 - 2*C*a*b^5 +
 B*b^6)*c^2*d^2 - (19*D*a^3*b^3 - 13*C*a^2*b^4 + 7*B*a*b^5 - A*b^6)*c*d^3
+ (10*D*a^4*b^2 - 7*C*a^3*b^3 + 4*B*a^2*b^4 - A*a*b^5)*d^4)*x)*sqrt(-(b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b)*log(-(3*b^2*c^2 - 4*a*b*c*d + a^2*
d^2 + 2*(b^2*c*d - a*b*d^2)*x + 3*sqrt(1/3)*(2*(b^2*c - a*b*d)*(d*x + c)^(
2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 -
2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3))*sqrt(-(b^3*c^2 - 2*a*b^2*c
*d + a^2*b*d^2)^(1/3)/b) - 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*
c - a*d)*(d*x + c)^(1/3))/(b*x + a)) + 14*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2)^(2/3)*(3*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c*d^2 - (10*D*a^4 - 7*C*a
^3*b + 4*B*a^2*b^2 - A*a*b^3)*d^3 + (3*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c
*d^2 - (10*D*a^3*b - 7*C*a^2*b^2 + 4*B*a*b^3 - A*b^4)*d^3)*x)*log(-(b^2*c
- a*b*d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c
- a*d) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 28*(
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(3*(3*D*a^3*b - 2*C*a^2*b^2 + B*a
*b^3)*c*d^2 - (10*D*a^4 - 7*C*a^3*b + 4*B*a^2*b^2 - A*a*b^3)*d^3 + (3*(3*D
*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c*d^2 - (10*D*a^3*b - 7*C*a^2*b^2 + 4*B*a*b^
3 - A*b^4)*d^3)*x)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/3)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/3*(9*D*a^2*b*c - 6*C*a*b^2*c + 3*B*b^3*c - 10*D*a^3*d + 7*C*a^2*b*d - 4*
B*a*b^2*d + A*b^3*d)*((b*c - a*d)/b)^(1/3)*log(abs((d*x + c)^(1/3) - ((b*c
 - a*d)/b)^(1/3)))/(b^5*c - a*b^4*d) - ((b^3*c - a*b^2*d)^(1/3)*A*b^3*d +
(3*b^3*c - 4*a*b^2*d)*(b^3*c - a*b^2*d)^(1/3)*B - (6*a*b^2*c - 7*a^2*b*d)*
(b^3*c - a*b^2*d)^(1/3)*C + (9*a^2*b*c - 10*a^3*d)*(b^3*c - a*b^2*d)^(1/3)
*D)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a*d)/b)^(1/3))/((b*c -
 a*d)/b)^(1/3))/(sqrt(3)*b^6*c - sqrt(3)*a*b^5*d) - 1/6*(A*b^3*d + (3*b^3*
c - 4*a*b^2*d)*B - (6*a*b^2*c - 7*a^2*b*d)*C + (9*a^2*b*c - 10*a^3*d)*D)*l
og((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d)/
b)^(2/3))/((b^3*c - a*b^2*d)^(2/3)*b^3) + ((d*x + c)^(1/3)*D*a^3*d - (d*x
+ c)^(1/3)*C*a^2*b*d + (d*x + c)^(1/3)*B*a*b^2*d - (d*x + c)^(1/3)*A*b^3*d
)/(((d*x + c)*b - b*c + a*d)*b^4) + 3/28*(4*(d*x + c)^(7/3)*D*b^12*d^12 -
7*(d*x + c)^(4/3)*D*b^12*c*d^12 - 14*(d*x + c)^(4/3)*D*a*b^11*d^13 + 7*(d*
x + c)^(4/3)*C*b^12*d^13 + 84*(d*x + c)^(1/3)*D*a^2*b^10*d^14 - 56*(d*x +
c)^(1/3)*C*a*b^11*d^14 + 28*(d*x + c)^(1/3)*B*b^12*d^14)/(b^14*d^14)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx=\int \frac{{\left(c+d\,x\right)}^{1/3}\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(((c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2,x)
```

output
```
int(((c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2905, normalized size of antiderivative = 6.95

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

output
```
(140*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d
*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**4*d**3 - 224*sqrt(3)*atan((b
**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6
)*(a*d - b*c)**(1/6)))*a**3*b*c*d**2 + 140*sqrt(3)*atan((b**(1/6)*(a*d - b
*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(
1/6)))*a**3*b*d**3*x + 42*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3
) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**3*
d**2 + 84*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(
c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**2*c**2*d - 224*sqr
t(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/
6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**2*c*d**2*x - 42*sqrt(3)*atan((b
**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6
)*(a*d - b*c)**(1/6)))*a*b**4*c*d + 42*sqrt(3)*atan((b**(1/6)*(a*d - b*c)*
*(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)
))*a*b**4*d**2*x + 84*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) -
2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*c**2*d*
x - 42*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c +
 d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*b**5*c*d*x + 140*sqrt(3)*atan
((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/6))/(b**(
1/6)*(a*d - b*c)**(1/6)))*a**4*d**3 - 224*sqrt(3)*atan((b**(1/6)*(a*d -...
```

3.159 $\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$

Optimal result 1536
Mathematica [A] (verified) 1537
Rubi [A] (verified) 1538
Maple [A] (verified) 1543
Fricas [B] (verification not implemented) 1545
Sympy [F(-1)] 1545
Maxima [F(-2)] 1546
Giac [A] (verification not implemented) 1546
Mupad [F(-1)] 1547
Reduce [B] (verification not implemented) 1548

Optimal result

Integrand size = 32, antiderivative size = 530

$$\begin{aligned}
&\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx \\
&= \frac{3(bC-3aD)\sqrt[3]{c+dx}}{b^4} - \frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt[3]{c+dx}}{2b^4(a+bx)^2} \\
&\quad - \frac{(b^3(6Bc+Ad)-ab^2(12cC+7Bd)-19a^3dD+a^2b(13Cd+18cD))\sqrt[3]{c+dx}}{6b^4(bc-ad)(a+bx)} \\
&\quad + \frac{3D(c+dx)^{4/3}}{4b^3d} \\
&\quad - \frac{(b^3(9c^2C+3Bcd-Ad^2)-35a^3d^2D+7a^2bd(2Cd+9cD)-ab^2(24cCd+2Bd^2+27c^2D))\arctan\left(\cdots\right.}{3\sqrt{3}b^{13/3}(bc-ad)^{5/3}} \\
&\quad - \frac{(b^3(9c^2C+3Bcd-Ad^2)-35a^3d^2D+7a^2bd(2Cd+9cD)-ab^2(24cCd+2Bd^2+27c^2D))\log(a+b\cdots}{18b^{13/3}(bc-ad)^{5/3}} \\
&\quad + \frac{(b^3(9c^2C+3Bcd-Ad^2)-35a^3d^2D+7a^2bd(2Cd+9cD)-ab^2(24cCd+2Bd^2+27c^2D))\log\left(\sqrt[3]{bc}\cdots\right.}{6b^{13/3}(bc-ad)^{5/3}}
\end{aligned}$$

output
```
3*(C*b-3*D*a)*(d*x+c)^(1/3)/b^4-1/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^
(1/3)/b^4/(b*x+a)^2-1/6*(b^3*(A*d+6*B*c)-a*b^2*(7*B*d+12*C*c)-19*a^3*d*D+a
^2*b*(13*C*d+18*D*c))*(d*x+c)^(1/3)/b^4/(-a*d+b*c)/(b*x+a)+3/4*D*(d*x+c)^(
4/3)/b^3/d-1/9*(b^3*(-A*d^2+3*B*c*d+9*C*c^2)-35*a^3*d^2*D+7*a^2*b*d*(2*C*d
+9*D*c)-a*b^2*(2*B*d^2+24*C*c*d+27*D*c^2))*arctan(1/3*(1+2*b^(1/3)*(d*x+c)
^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/b^(13/3)/(-a*d+b*c)^(5/3)-1/18*(
b^3*(-A*d^2+3*B*c*d+9*C*c^2)-35*a^3*d^2*D+7*a^2*b*d*(2*C*d+9*D*c)-a*b^2*(2
*B*d^2+24*C*c*d+27*D*c^2))*ln(b*x+a)/b^(13/3)/(-a*d+b*c)^(5/3)+1/6*(b^3*(-
A*d^2+3*B*c*d+9*C*c^2)-35*a^3*d^2*D+7*a^2*b*d*(2*C*d+9*D*c)-a*b^2*(2*B*d^2
+24*C*c*d+27*D*c^2))*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/b^(13/3)/(
-a*d+b*c)^(5/3)
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$$

$$= \frac{-\frac{3\sqrt[3]{b}\sqrt[3]{c+dx}(140a^4d^2D-2Ab^3d(3bc-2ad+bdx)-7a^3bd(8Cd+7D(3c-5dx))+3b^4cx(-4Bd+3x(4Cd+cD+dDx))+a^2b^2(9c^2D+9cd(6C}{d(-bc+ad)(a+bx)^2}}{}$$

input
```
Integrate[((c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
((-3*b^(1/3)*(c + d*x)^(1/3)*(140*a^4*d^2*D - 2*A*b^3*d*(3*b*c - 2*a*d + b
*d*x) - 7*a^3*b*d*(8*C*d + 7*D*(3*c - 5*d*x)) + 3*b^4*c*x*(-4*B*d + 3*x*(4
*C*d + c*D + d*D*x)) + a^2*b^2*(9*c^2*D + 9*c*d*(6*C - 29*D*x) + 2*d^2*(4*
B - 49*C*x + 45*D*x^2)) + a*b^3*(2*B*d*(-3*c + 7*d*x) - 3*x*(-6*c^2*D + 3*
d^2*x*(4*C + D*x) + c*(-32*C*d + 33*d*D*x)))))/(d*(-(b*c) + a*d)*(a + b*x)
^2) + (4*Sqrt[3]*(b^3*(9*c^2*C + 3*B*c*d - A*d^2) - 35*a^3*d^2*D + 7*a^2*b
*d*(2*C*d + 9*c*D) - a*b^2*(24*c*C*d + 2*B*d^2 + 27*c^2*D))*ArcTan[(1 - (2
*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3))/Sqrt[3]])/(-(b*c) + a*d)^(
5/3) - (4*(b^3*(9*c^2*C + 3*B*c*d - A*d^2) - 35*a^3*d^2*D + 7*a^2*b*d*(2*C
*d + 9*c*D) - a*b^2*(24*c*C*d + 2*B*d^2 + 27*c^2*D))*Log[(-(b*c) + a*d)^(1
/3) + b^(1/3)*(c + d*x)^(1/3)])/(-(b*c) + a*d)^(5/3) + (2*(b^3*(9*c^2*C +
3*B*c*d - A*d^2) - 35*a^3*d^2*D + 7*a^2*b*d*(2*C*d + 9*c*D) - a*b^2*(24*c*
C*d + 2*B*d^2 + 27*c^2*D))*Log[(-(b*c) + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*
d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)])/(-(b*c) + a*d)^(5/3))
/(36*b^(13/3))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2124, 27, 1193, 27, 90, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{\int -\frac{2\sqrt[3]{c+dx}\left(3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{-2dDa^3+b(2Cd+3cD)a^2-\frac{1}{2}b^2(6cC+4Bd)a+\frac{1}{2}b^3(6Bc-2Ad)}{b^3}\right)}{3(a+bx)^2}dx}{2(bc-ad)}-\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt[3]{c+dx}\left(-\frac{2dDa^3}{b^3}+\frac{(2Cd+3cD)a^2}{b^2}-\frac{(3cC+2Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^2}dx}{3(bc-ad)}-\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 1193

$$\frac{-\frac{\int -\frac{\sqrt[3]{c+dx}\left(-26d^2Da^3+bd(14Cd+45cD)a^2-2b^2\left(9Dc^2+12Cdc+Bd^2\right)a+b^3\left(9Cc^2+3Bdc-Ad^2\right)+9b(bc-ad)^2Dx\right)}{3b^3(a+bx)}dx}{bc-ad}-\frac{(c+dx)^{4/3}\left(-8a^3dD+a^2b(\cdots\right.}{\cdots}}{3(bc-ad)}-\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt[3]{c+dx}\left(-26d^2Da^3+bd(14Cd+45cD)a^2-2b^2\left(9Dc^2+12Cdc+Bd^2\right)a+b^3\left(9Cc^2+3Bdc-Ad^2\right)+9b(bc-ad)^2Dx\right)}{a+bx}dx}{3b^3(bc-ad)}-\frac{(c+dx)^{4/3}\left(-8a^3dD+a^2b(9cD\cdots\right.}{b^3(\cdots}}{3(bc-ad)}-\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 90

$$\frac{\frac{\left(-35a^3d^2D+7a^2bd(9cD+2Cd)-ab^2\left(2Bd^2+27c^2D+24cCd\right)+b^3\left(-Ad^2+3Bcd+9c^2C\right)\right)\int \frac{\sqrt[3]{c+dx}}{a+bx}dx+\frac{27D(c+dx)^{4/3}(bc-ad)^2}{4d}}{3b^3(bc-ad)}-\frac{(c+dx)^{4/3}\cdots}{\cdots}}{3(bc-ad)}-\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 60

$$\frac{\frac{\left(-35a^3d^2D+7a^2bd(9cD+2Cd)-ab^2\left(2Bd^2+27c^2D+24cCd\right)+b^3\left(-Ad^2+3Bcd+9c^2C\right)\right)\left(\frac{(bc-ad)\int \frac{1}{(a+bx)(c+dx)^{2/3}}dx}{b}+\frac{3\sqrt[3]{c+dx}}{b}\right)+\frac{27D(c\cdots}{\cdots}}{3b^3(bc-ad)}\cdots}{3(bc-ad)}-\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 69

$$\frac{\left(-35a^3d^2D+7a^2bd(9cD+2Cd)-ab^2\left(2Bd^2+27c^2D+24cCd\right)+b^3\left(-Ad^2+3Bcd+9c^2C\right)\right)\left(\frac{(bc-ad)\left(-\frac{3\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{c+dx}\sqrt[3]{bc-ad}}{\sqrt[3]{b}}+\cdots}}{2b^{2/3}\sqrt[3]{bc-ad}}\cdots\right)}{\cdots}\cdots\right)}{3b^3(bc-ad)}$$

$$\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\left(-35a^3d^2D+7a^2bd(9cD+2Cd)-ab^2\left(2Bd^2+27c^2D+24cCd\right)+b^3\left(-Ad^2+3Bcd+9c^2C\right)\right)\left(\frac{(bc-ad)\left(-\frac{3\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{c+dx}\sqrt[3]{bc-ad}}{\sqrt[3]{b}}+\cdots}}{2b^{2/3}\sqrt[3]{bc-ad}}\cdots\right)}{\cdots}\cdots\right)}{3b^3(bc-ad)}$$

$$\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\left(-35a^3d^2D+7a^2bd(9cD+2Cd)-ab^2\left(2Bd^2+27c^2D+24cCd\right)+b^3\left(-Ad^2+3Bcd+9c^2C\right)\right)\left(\frac{(bc-ad)\left(\frac{3\int \frac{1}{-(c+dx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}-\cdots\right)}{\cdots}\cdots\right)}{3b^3(bc-ad)}$$

$$\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 217$$

$$\frac{\left(\frac{(bc-ad)\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}\right)}{b}+\frac{3\sqrt[3]{c+dx}}{b}\right)\left(-35a^3d^2D+7a^2bd(9c\right.}{3b^3(bc-ad)}$$

$$\frac{(c+dx)^{4/3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

input
```
Int[((c + d*x)^(1/3)*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
-1/2*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(4/3))/(b^3*(b*c - a*d
)*(a + b*x)^2) + (-(((b^3*(3*B*c - A*d) - 2*a*b^2*(3*c*C + B*d) - 8*a^3*d*
D + a^2*b*(5*C*d + 9*c*D))*(c + d*x)^(4/3))/(b^3*(b*c - a*d)*(a + b*x))) +
 ((27*(b*c - a*d)^2*D*(c + d*x)^(4/3))/(4*d) + (b^3*(9*c^2*C + 3*B*c*d - A
*d^2) - 35*a^3*d^2*D + 7*a^2*b*d*(2*C*d + 9*c*D) - a*b^2*(24*c*C*d + 2*B*d
^2 + 27*c^2*D))*((3*(c + d*x)^(1/3))/b + ((b*c - a*d)*(-((Sqrt[3]*ArcTan[(
1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b^(1/3)*(b*c
 - a*d)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c
 - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(1/3)*(b*c - a*d)^(2/3))))/
b))/(3*b^3*(b*c - a*d)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\left(\left(-\frac{A\,b^{4}x}{2}+a\left(-\frac{9}{4}Dx^{3}-9C\,x^{2}+\frac{7}{2}Bx+A\right)b^{3}+2\left(\frac{45}{4}Dx^{2}-\frac{49}{4}Cx+B\right)a^{2}b^{2}-14\left(-\frac{35Dx}{8}+C\right)a^{3}b+35Da^{4}\right)\cdots\right.}{\cdots}$
derivativedivides	$\frac{3\left(\frac{D(xd+c)^{\frac{4}{3}}b}{4}+Cdb(xd+c)^{\frac{1}{3}}-3Dad(xd+c)^{\frac{1}{3}}\right)}{b^{4}}+\frac{3d\left(\frac{\frac{bd\left(b^{3}dA-7Ba\,b^{2}d+6B\,b^{3}c+13C\,a^{2}bd-12a\,b^{2}cC-19a^{3}dD+18a^{2}bcD\right)(xd\cdots}{18ad-18bc}}{((xd+\cdots}\cdots\right)}{\cdots}$
default	$\frac{3\left(\frac{D(xd+c)^{\frac{4}{3}}b}{4}+Cdb(xd+c)^{\frac{1}{3}}-3Dad(xd+c)^{\frac{1}{3}}\right)}{b^{4}}+\frac{3d\left(\frac{\frac{bd\left(b^{3}dA-7Ba\,b^{2}d+6B\,b^{3}c+13C\,a^{2}bd-12a\,b^{2}cC-19a^{3}dD+18a^{2}bcD\right)(xd\cdots}{18ad-18bc}}{((xd+\cdots}\cdots\right)}{\cdots}$

input
```
int((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(((a*d-b*c)/b)^(2/3)*((-1/2*A*b^4*x+a*(-9/4*D*x^3-9*C*x^2+7/2*B*x+A)*
b^3+2*(45/4*D*x^2-49/4*C*x+B)*a^2*b^2-14*(-35/8*D*x+C)*a^3*b+35*D*a^4)*d^2
-3/2*c*b*((-3/2*D*x^3-6*C*x^2+2*B*x+A)*b^3+a*(33/2*D*x^2-16*C*x+B)*b^2-9*(
-29/6*D*x+C)*a^2*b+49/2*a^3*D)*d+9/4*D*b^2*c^2*(b*x+a)^2)*b*(d*x+c)^(1/3)+
1/6*((A*b^3+2*B*a*b^2-14*C*a^2*b+35*D*a^3)*d^2-3*b*c*(B*b^2-8*C*a*b+21*D*a
^2)*d-9*b^2*c^2*(C*b-3*D*a))*d*(b*x+a)^2*(-2*arctan(2/3*3^(1/2)/((a*d-b*c)
/b)^(1/3)*(d*x+c)^(1/3)-1/3*3^(1/2))*3^(1/2)+ln((d*x+c)^(2/3)-((a*d-b*c)/b
)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/b)^(2/3))-2*ln((d*x+c)^(1/3)+((a*d-b*c)/b
)^(1/3))))/((a*d-b*c)/b)^(2/3)/b^5/(a*d-b*c)/(b*x+a)^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1984 vs. $2(485) = 970$.

Time = 1.02 (sec) , antiderivative size = 4061, normalized size of antiderivative = 7.66

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/3)*(D*x**3+C*x**2+B*x+A)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/9*(27*D*a*b^2*c^2 - 9*C*b^3*c^2 - 63*D*a^2*b*c*d + 24*C*a*b^2*c*d - 3*B
*b^3*c*d + 35*D*a^3*d^2 - 14*C*a^2*b*d^2 + 2*B*a*b^2*d^2 + A*b^3*d^2)*((b*
c - a*d)/b)^(1/3)*log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b^6*c
^2 - 2*a*b^5*c*d + a^2*b^4*d^2) + 1/3*(A*b^3*d^2 - (3*b^3*c*d - 2*a*b^2*d^
2)*B - (9*b^3*c^2 - 24*a*b^2*c*d + 14*a^2*b*d^2)*C + (27*a*b^2*c^2 - 63*a^
2*b*c*d + 35*a^3*d^2)*D)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a
*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/((sqrt(3)*b^4*c - sqrt(3)*a*b^3*d)*(b
^3*c - a*b^2*d)^(2/3)) + 1/18*(A*b^3*d^2 - (3*b^3*c*d - 2*a*b^2*d^2)*B - (
9*b^3*c^2 - 24*a*b^2*c*d + 14*a^2*b*d^2)*C + (27*a*b^2*c^2 - 63*a^2*b*c*d
+ 35*a^3*d^2)*D)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/
3) + ((b*c - a*d)/b)^(2/3))/((b^4*c - a*b^3*d)*(b^3*c - a*b^2*d)^(2/3)) -
1/6*(18*(d*x + c)^(4/3)*D*a^2*b^2*c*d - 12*(d*x + c)^(4/3)*C*a*b^3*c*d + 6
*(d*x + c)^(4/3)*B*b^4*c*d - 18*(d*x + c)^(1/3)*D*a^2*b^2*c^2*d + 12*(d*x
+ c)^(1/3)*C*a*b^3*c^2*d - 6*(d*x + c)^(1/3)*B*b^4*c^2*d - 19*(d*x + c)^(4
/3)*D*a^3*b*d^2 + 13*(d*x + c)^(4/3)*C*a^2*b^2*d^2 - 7*(d*x + c)^(4/3)*B*a
*b^3*d^2 + (d*x + c)^(4/3)*A*b^4*d^2 + 34*(d*x + c)^(1/3)*D*a^3*b*c*d^2 -
22*(d*x + c)^(1/3)*C*a^2*b^2*c*d^2 + 10*(d*x + c)^(1/3)*B*a*b^3*c*d^2 + 2*
(d*x + c)^(1/3)*A*b^4*c*d^2 - 16*(d*x + c)^(1/3)*D*a^4*d^3 + 10*(d*x + c)^
(1/3)*C*a^3*b*d^3 - 4*(d*x + c)^(1/3)*B*a^2*b^2*d^3 - 2*(d*x + c)^(1/3)*A*
a*b^3*d^3)/((b^5*c - a*b^4*d)*((d*x + c)*b - b*c + a*d)^2) + 3/4*((d*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx=\int \frac{(c+d\,x)^{1/3}\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^3}\,dx$$

input
```
int(((c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3,x)
```

output
```
int(((c + d*x)^(1/3)*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3359, normalized size of antiderivative = 6.34

$$\int \frac{\sqrt[3]{c+dx}(A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^(1/3)*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
( - 140*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c
+ d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**4*d**2 + 168*sqrt(3)*atan
((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(
1/6)*(a*d - b*c)**(1/6)))*a**3*b*c*d - 280*sqrt(3)*atan((b**(1/6)*(a*d - b
*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(
1/6)))*a**3*b*d**2*x - 12*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3
) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**3*
d - 36*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c +
 d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**2*c**2 + 336*sqrt(3)*
atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(
b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**2*c*d*x - 140*sqrt(3)*atan((b**(1/6)
*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d
- b*c)**(1/6)))*a**2*b**2*d**2*x**2 - 24*sqrt(3)*atan((b**(1/6)*(a*d - b*c
)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/
6)))*a*b**4*d*x - 72*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2
*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*c**2*x +
 168*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d
*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*c*d*x**2 - 12*sqrt(3)*at
an((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b*
*(1/6)*(a*d - b*c)**(1/6)))*b**5*d*x**2 - 36*sqrt(3)*atan((b**(1/6)*(a*...
```

3.160 $\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} \, dx$

Optimal result . . . 1549
Mathematica [A] (verified) . . . 1550
Rubi [A] (verified) . . . 1551
Maple [A] (verified) . . . 1552
Fricas [A] (verification not implemented) . . . 1553
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Maxima [A] (verification not implemented) . . . 1555
Giac [B] (verification not implemented) . . . 1556
Mupad [F(-1)] . . . 1557
Reduce [B] (verification not implemented) . . . 1558

Optimal result

Integrand size = 32, antiderivative size = 436

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} \, dx = -\frac{3(bc-ad)^3 (c^2Cd-Bcd^2+Ad^3-c^3D) \sqrt[3]{c+dx}}{d^7} - \frac{3(bc-ad)^2 (ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)) (c+dx)^{4/3}}{4d^7} - \frac{3(bc-ad) (a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)) (c}{7d^7} + \frac{3(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D)) (}{10d^7} + \frac{3b(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D)) (c+dx)^{13/3}}{13d^7} + \frac{3b^2(bCd-6bcD+3adD)(c+dx)^{16/3}}{16d^7} + \frac{3b^3D(c+dx)^{19/3}}{19d^7}$$

output
```
-3*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1/3)/d^7-3/4*(-a*d+
b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^
3))*(d*x+c)^(4/3)/d^7-3/7*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^2+
8*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^(7/
3)/d^7+3/10*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d-1
0*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(10/3)/d^7+3/1
3*b*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*x+c
)^(13/3)/d^7+3/16*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(16/3)/d^7+3/19*b^3*
D*(d*x+c)^(19/3)/d^7
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} dx = \frac{3\sqrt[3]{c+dx}(988a^3d^3(-81c^3D+9c^2d(10C+3Dx)-3cd^2(35B+$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(2/3),x]
```

output
```
(3*(c + d*x)^(1/3)*(988*a^3*d^3*(-81*c^3*D + 9*c^2*d*(10*C + 3*D*x) - 3*c*
d^2*(35*B + 2*x*(5*C + 3*D*x)) + d^3*(140*A + x*(35*B + 20*C*x + 14*D*x^2)
)) + 228*a^2*b*d^2*(972*c^4*D - 81*c^3*d*(13*C + 4*D*x) + 9*c^2*d^2*(130*B
 + 3*x*(13*C + 8*D*x)) - 3*c*d^3*(455*A + 2*x*(65*B + 39*C*x + 28*D*x^2))
 + d^4*x*(455*A + 2*x*(130*B + 91*C*x + 70*D*x^2))) + 57*a*b^2*d*(-3645*c^5
*D + 243*c^4*d*(16*C + 5*D*x) - 162*c^3*d^2*(26*B + x*(8*C + 5*D*x)) + d^5
*x^2*(1040*A + 7*x*(104*B + 80*C*x + 65*D*x^2)) - 3*c*d^4*x*(520*A + x*(31
2*B + 7*x*(32*C + 25*D*x))) + 18*c^2*d^3*(260*A + x*(78*B + x*(48*C + 35*D
*x)))) + b^3*(65610*c^6*D - 3645*c^5*d*(19*C + 6*D*x) + 243*c^4*d^2*(304*B
 + 95*C*x + 60*D*x^2) - 162*c^3*d^3*(494*A + x*(152*B + 95*C*x + 70*D*x^2)
) + 7*d^6*x^3*(1976*A + 5*x*(304*B + 247*C*x + 208*D*x^2)) + 18*c^2*d^4*x*
(1482*A + x*(912*B + 665*C*x + 525*D*x^2)) - 3*c*d^5*x^2*(5928*A + 7*x*(60
8*B + 5*x*(95*C + 78*D*x))))))/(138320*d^7)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}} \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{4/3}(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3(c+dx)^{7/3}(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2C\right.\right.}{7d^7}$$

$$\frac{3b(c+dx)^{13/3}\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{13d^7}+$$

$$\frac{3(c+dx)^{10/3}\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c\right.\right.}{10d^7}$$

$$\frac{3(c+dx)^{4/3}(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{4d^7}-$$

$$\frac{3\sqrt[3]{c+dx}(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^7}+$$

$$\frac{3b^2(c+dx)^{16/3}(3adD-6bcD+bCd)}{16d^7}+\frac{3b^3D(c+dx)^{19/3}}{19d^7}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(2/3),x]
```

output
```
(-3*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(1/3))/d^7
 - (3*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d - 4*B*
c*d^2 + 3*A*d^3 - 6*c^3*D))*(c + d*x)^(4/3))/(4*d^7) - (3*(b*c - a*d)*(a^2
*d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^2*C*
d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(7/3))/(7*d^7) + (3*(a^3*d^
3*D + 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*D) +
 b^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(10/3))/(10*d^
7) + (3*b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^2 - 15
*c^2*D))*(c + d*x)^(13/3))/(13*d^7) + (3*b^2*(b*C*d - 6*b*c*D + 3*a*d*D)*(
c + d*x)^(16/3))/(16*d^7) + (3*b^3*D*(c + d*x)^(19/3))/(19*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{3b^3D(xd+c)^{\frac{19}{3}}}{19}+\frac{3\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{16}{3}}}{16}+\frac{3\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(B\,d^2-2Ccd+3Dc^2\right)\right)(}{13}$
default	$\frac{3b^3D(xd+c)^{\frac{19}{3}}}{19}+\frac{3\left(3(ad-bc)b^2D+b^3(Cd-3Dc)\right)(xd+c)^{\frac{16}{3}}}{16}+\frac{3\left(3(ad-bc)^2bD+3(ad-bc)b^2(Cd-3Dc)+b^3\left(B\,d^2-2Ccd+3Dc^2\right)\right)(}{13}$
pseudoelliptic	$3\left(\left(\frac{x^3\left(\frac{10}{19}Dx^3+\frac{5}{8}C\,x^2+\frac{10}{13}Bx+A\right)b^3}{10}+\frac{3x^2\left(\frac{7}{16}Dx^3+\frac{7}{13}C\,x^2+\frac{7}{10}Bx+A\right)a\,b^2}{7}+\frac{3x\left(\frac{4}{13}Dx^3+\frac{2}{5}C\,x^2+\frac{4}{7}Bx+A\right)a^2b}{4}+a^3\left(\frac{1}{10}D\right.\right.\right.$
gosper	$3(xd+c)^{\frac{1}{3}}\left(7280Dx^6b^3d^6+8645C\,b^3d^6x^5+25935Dx^5a\,b^2d^6-8190Dx^5b^3c\,d^5+10640B\,x^4b^3d^6+31920C\,x^4a\,b^2d^6-997\right.$
trager	$3(xd+c)^{\frac{1}{3}}\left(7280Dx^6b^3d^6+8645C\,b^3d^6x^5+25935Dx^5a\,b^2d^6-8190Dx^5b^3c\,d^5+10640B\,x^4b^3d^6+31920C\,x^4a\,b^2d^6-997\right.$
orering	$3(xd+c)^{\frac{1}{3}}\left(7280Dx^6b^3d^6+8645C\,b^3d^6x^5+25935Dx^5a\,b^2d^6-8190Dx^5b^3c\,d^5+10640B\,x^4b^3d^6+31920C\,x^4a\,b^2d^6-997\right.$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/d^7*(1/19*b^3*D*(d*x+c)^(19/3)+1/16*(3*(a*d-b*c)*b^2*D+b^3*(C*d-3*D*c))*
(d*x+c)^(16/3)+1/13*(3*(a*d-b*c)^2*b*D+3*(a*d-b*c)*b^2*(C*d-3*D*c)+b^3*(B*
d^2-2*C*c*d+3*D*c^2))*(d*x+c)^(13/3)+1/10*((a*d-b*c)^3*D+3*(a*d-b*c)^2*b*(
C*d-3*D*c)+3*(a*d-b*c)*b^2*(B*d^2-2*C*c*d+3*D*c^2)+b^3*(A*d^3-B*c*d^2+C*c^
2*d-D*c^3))*(d*x+c)^(10/3)+1/7*((a*d-b*c)^3*(C*d-3*D*c)+3*(a*d-b*c)^2*b*(B
*d^2-2*C*c*d+3*D*c^2)+3*(a*d-b*c)*b^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3))*(d*x+
c)^(7/3)+1/4*((a*d-b*c)^3*(B*d^2-2*C*c*d+3*D*c^2)+3*(a*d-b*c)^2*b*(A*d^3-B
*c*d^2+C*c^2*d-D*c^3))*(d*x+c)^(4/3)+(a*d-b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*
c^3)*(d*x+c)^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx=\frac{3\left(7280\,Db^3d^6x^6+65610\,Db^3c^6+138320\,Aa^3d^6-69255\,(3\,Dab\right.}{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="fricas
")
```

output
```
3/138320*(7280*D*b^3*d^6*x^6 + 65610*D*b^3*c^6 + 138320*A*a^3*d^6 - 69255*
(3*D*a*b^2 + C*b^3)*c^5*d + 73872*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2
- 80028*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + 88920*(C*a^3 + 3
*B*a^2*b + 3*A*a*b^2)*c^2*d^4 - 103740*(B*a^3 + 3*A*a^2*b)*c*d^5 - 455*(18
*D*b^3*c*d^5 - 19*(3*D*a*b^2 + C*b^3)*d^6)*x^5 + 35*(270*D*b^3*c^2*d^4 - 2
85*(3*D*a*b^2 + C*b^3)*c*d^5 + 304*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^6)*x^
4 - 14*(810*D*b^3*c^3*d^3 - 855*(3*D*a*b^2 + C*b^3)*c^2*d^4 + 912*(3*D*a^2
*b + 3*C*a*b^2 + B*b^3)*c*d^5 - 988*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3
)*d^6)*x^3 + 2*(7290*D*b^3*c^4*d^2 - 7695*(3*D*a*b^2 + C*b^3)*c^3*d^3 + 82
08*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^4 - 8892*(D*a^3 + 3*C*a^2*b + 3*B
*a*b^2 + A*b^3)*c*d^5 + 9880*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^6)*x^2 - (2
1870*D*b^3*c^5*d - 23085*(3*D*a*b^2 + C*b^3)*c^4*d^2 + 24624*(3*D*a^2*b +
3*C*a*b^2 + B*b^3)*c^3*d^3 - 26676*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)
*c^2*d^4 + 29640*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^5 - 34580*(B*a^3 + 3*
A*a^2*b)*d^6)*x)*(d*x + c)^(1/3)/d^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(454) = 908$.

Time = 1.74 (sec) , antiderivative size = 1027, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(2/3),x)
```

output
```
Piecewise((3*(D*b**3*(c + d*x)**(19/3)/(19*d**6) + (c + d*x)**(16/3)*(C*b*
*3*d + 3*D*a*b**2*d - 6*D*b**3*c)/(16*d**6) + (c + d*x)**(13/3)*(B*b**3*d*
*2 + 3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d +
15*D*b**3*c**2)/(13*d**6) + (c + d*x)**(10/3)*(A*b**3*d**3 + 3*B*a*b**2*d*
*3 - 4*B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c*
*2*d + D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c
**3)/(10*d**6) + (c + d*x)**(7/3)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*B
*a**2*b*d**4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C*
a**2*b*c*d**3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**3
 + 18*D*a**2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/(7*d**6) +
 (c + d*x)**(4/3)*(3*A*a**2*b*d**5 - 6*A*a*b**2*c*d**4 + 3*A*b**3*c**2*d**
3 + B*a**3*d**5 - 6*B*a**2*b*c*d**4 + 9*B*a*b**2*c**2*d**3 - 4*B*b**3*c**3
*d**2 - 2*C*a**3*c*d**4 + 9*C*a**2*b*c**2*d**3 - 12*C*a*b**2*c**3*d**2 + 5
*C*b**3*c**4*d + 3*D*a**3*c**2*d**3 - 12*D*a**2*b*c**3*d**2 + 15*D*a*b**2*
c**4*d - 6*D*b**3*c**5)/(4*d**6) + (c + d*x)**(1/3)*(A*a**3*d**6 - 3*A*a**
2*b*c*d**5 + 3*A*a*b**2*c**2*d**4 - A*b**3*c**3*d**3 - B*a**3*c*d**5 + 3*B
*a**2*b*c**2*d**4 - 3*B*a*b**2*c**3*d**3 + B*b**3*c**4*d**2 + C*a**3*c**2*
d**4 - 3*C*a**2*b*c**3*d**3 + 3*C*a*b**2*c**4*d**2 - C*b**3*c**5*d - D*a**
3*c**3*d**3 + 3*D*a**2*b*c**4*d**2 - 3*D*a*b**2*c**5*d + D*b**3*c**6)/d**6
)/d, Ne(d, 0)), ((A*a**3*x + D*b**3*x**7/7 + x**6*(C*b**3 + 3*D*a*b**2)...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx = \frac{3\left(7280\,(dx+c)^{\frac{19}{3}}Db^3 - 8645\,(6\,Db^3c-(3\,Dab^2+Cb^3)d)(dx+\right.}{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="maxima
")
```

output

```
3/138320*(7280*(d*x + c)^(19/3)*D*b^3 - 8645*(6*D*b^3*c - (3*D*a*b^2 + C*b
^3)*d)*(d*x + c)^(16/3) + 10640*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^3)*c*d
+ (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(13/3) - 13832*(20*D*b^3*
c^3 - 10*(3*D*a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d
^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(10/3) + 19760
*(15*D*b^3*c^4 - 10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*b^2 +
 B*b^3)*c^2*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (C*a^3
 + 3*B*a^2*b + 3*A*a*b^2)*d^4)*(d*x + c)^(7/3) - 34580*(6*D*b^3*c^5 - 5*(3
*D*a*b^2 + C*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2 - 3*(D
*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3*B*a^2*b + 3*A
*a*b^2)*c*d^4 - (B*a^3 + 3*A*a^2*b)*d^5)*(d*x + c)^(4/3) + 138320*(D*b^3*c
^6 + A*a^3*d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B*b^
3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3*
B*a^2*b + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)*(d*x + c)^(1/3))
/d^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(412) = 824$.

Time = 0.13 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
3/138320*(138320*(d*x + c)^(1/3)*A*a^3 + 34580*((d*x + c)^(4/3) - 4*(d*x +
 c)^(1/3)*c)*B*a^3/d + 103740*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*A*a^
2*b/d + 9880*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)
*c^2)*C*a^3/d^2 + 29640*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x
 + c)^(1/3)*c^2)*B*a^2*b/d^2 + 29640*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3
)*c + 14*(d*x + c)^(1/3)*c^2)*A*a*b^2/d^2 + 988*(14*(d*x + c)^(10/3) - 60*
(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*D*a
^3/d^3 + 2964*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^
(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*C*a^2*b/d^3 + 2964*(14*(d*x + c)^(10/
3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*
c^3)*B*a*b^2/d^3 + 988*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(
d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*A*b^3/d^3 + 912*(35*(d*x + c
)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 455*(d*x + c
)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*D*a^2*b/d^4 + 912*(35*(d*x + c)^(13
/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 455*(d*x + c)^(4/
3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*C*a*b^2/d^4 + 304*(35*(d*x + c)^(13/3) -
 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 455*(d*x + c)^(4/3)*c^
3 + 455*(d*x + c)^(1/3)*c^4)*B*b^3/d^4 + 285*(91*(d*x + c)^(16/3) - 560*(d
*x + c)^(13/3)*c + 1456*(d*x + c)^(10/3)*c^2 - 2080*(d*x + c)^(7/3)*c^3 +
1820*(d*x + c)^(4/3)*c^4 - 1456*(d*x + c)^(1/3)*c^5)*D*a*b^2/d^5 + 95*(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx = \int \frac{(a+b\,x)^3\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{2/3}}\,dx$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(2/3),x)
```

output
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} \, dx = \frac{3(dx+c)^{\frac{1}{3}} (7280b^3d^6x^6 + 25935a\,b^2d^6x^5 + 455b^3c\,d^5x^5 + 31920$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x)
```

output
```
(3*(c + d*x)**(1/3)*(138320*a**4*d**5 - 414960*a**3*b*c*d**4 + 138320*a**3
*b*d**5*x + 8892*a**3*c**3*d**3 - 2964*a**3*c**2*d**4*x + 1976*a**3*c*d**5
*x**2 + 13832*a**3*d**6*x**3 + 533520*a**2*b**2*c**2*d**3 - 177840*a**2*b*
*2*c*d**4*x + 118560*a**2*b**2*d**5*x**2 - 18468*a**2*b*c**4*d**2 + 6156*a
**2*b*c**3*d**3*x - 4104*a**2*b*c**2*d**4*x**2 + 3192*a**2*b*c*d**5*x**3 +
 31920*a**2*b*d**6*x**4 - 320112*a*b**3*c**3*d**2 + 106704*a*b**3*c**2*d**
3*x - 71136*a*b**3*c*d**4*x**2 + 55328*a*b**3*d**5*x**3 + 13851*a*b**2*c**
5*d - 4617*a*b**2*c**4*d**2*x + 3078*a*b**2*c**3*d**3*x**2 - 2394*a*b**2*c
**2*d**4*x**3 + 1995*a*b**2*c*d**5*x**4 + 25935*a*b**2*d**6*x**5 + 73872*b
**4*c**4*d - 24624*b**4*c**3*d**2*x + 16416*b**4*c**2*d**3*x**2 - 12768*b*
*4*c*d**4*x**3 + 10640*b**4*d**5*x**4 - 3645*b**3*c**6 + 1215*b**3*c**5*d*
x - 810*b**3*c**4*d**2*x**2 + 630*b**3*c**3*d**3*x**3 - 525*b**3*c**2*d**4
*x**4 + 455*b**3*c*d**5*x**5 + 7280*b**3*d**6*x**6))/(138320*d**6)
```

3.161 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} dx$

Optimal result . . . 1559
Mathematica [A] (verified) . . . 1560
Rubi [A] (verified) . . . 1560
Maple [A] (verified) . . . 1562
Fricas [A] (verification not implemented) . . . 1562
Sympy [A] (verification not implemented) . . . 1563
Maxima [A] (verification not implemented) . . . 1564
Giac [A] (verification not implemented) . . . 1565
Mupad [F(-1)] . . . 1565
Reduce [B] (verification not implemented) . . . 1566

Optimal result

Integrand size = 32, antiderivative size = 324

$$\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} dx = \frac{3(bc-ad)^2(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt[3]{c+dx}}{d^6} + \frac{3(bc-ad)(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))(c+dx)^{4/3}}{4d^6} + \frac{3(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D))(c+dx)^{7/3}}{7d^6} + \frac{3(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))(c+dx)^{10/3}}{10d^6} + \frac{3b(bCd-5bcD+2adD)(c+dx)^{13/3}}{13d^6} + \frac{3b^2D(c+dx)^{16/3}}{16d^6}$$

output
```
3*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1/3)/d^6+3/4*(-a*d+b
*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))
*(d*x+c)^(4/3)/d^6+3/7*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*c^
2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(7/3)/d^6+3/10*(a^2*d
^2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(10/3)/d^6
+3/13*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(13/3)/d^6+3/16*b^2*D*(d*x+c)^(16/
3)/d^6
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx = \frac{3\sqrt[3]{c+dx}(52a^2d^2(-81c^3D+9c^2d(10C+3Dx)-3cd^2(35B+2$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(2/3),x]
```

output
```
(3*(c + d*x)^(1/3)*(52*a^2*d^2*(-81*c^3*D + 9*c^2*d*(10*C + 3*D*x) - 3*c*d
^2*(35*B + 2*x*(5*C + 3*D*x)) + d^3*(140*A + x*(35*B + 20*C*x + 14*D*x^2))
) + 8*a*b*d*(972*c^4*D - 81*c^3*d*(13*C + 4*D*x) + 9*c^2*d^2*(130*B + 3*x*
(13*C + 8*D*x)) - 3*c*d^3*(455*A + 2*x*(65*B + 39*C*x + 28*D*x^2)) + d^4*x
*(455*A + 2*x*(130*B + 91*C*x + 70*D*x^2))) + b^2*(-3645*c^5*D + 243*c^4*d
*(16*C + 5*D*x) - 162*c^3*d^2*(26*B + x*(8*C + 5*D*x)) + d^5*x^2*(1040*A +
 7*x*(104*B + 80*C*x + 65*D*x^2)) - 3*c*d^4*x*(520*A + x*(312*B + 7*x*(32*
C + 25*D*x))) + 18*c^2*d^3*(260*A + x*(78*B + x*(48*C + 35*D*x))))))/(7280
*d^6)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{4/3} \left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c-}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3(c+dx)^{7/3}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{7d^6}+\frac{3(c+dx)^{10/3}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{10d^6}+\frac{3(c+dx)^{4/3}(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{4d^6}+\frac{3\sqrt[3]{c+dx}(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^6}+\frac{3b(c+dx)^{13/3}(2adD-5bcD+bCd)}{13d^6}+\frac{3b^2D(c+dx)^{16/3}}{16d^6}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(2/3),x]
```

output
```
(3*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(1/3))/d^6
+ (3*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B*c*d
^2 + 2*A*d^3 - 5*c^3*D))*(c + d*x)^(4/3))/(4*d^6) + (3*(a^2*d^2*(C*d - 3*c
*D) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 + A
*d^3 - 10*c^3*D))*(c + d*x)^(7/3))/(7*d^6) + (3*(a^2*d^2*D + 2*a*b*d*(C*d
- 4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(10/3))/(10*d^6) +
(3*b*(b*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(13/3))/(13*d^6) + (3*b^2*D*(c
+ d*x)^(16/3))/(16*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$3\left(\left(\frac{x^{2}\left(\frac{7}{16}Dx^{3}+\frac{7}{13}Cx^{2}+\frac{7}{10}Bx+A\right)b^{2}}{7}+\frac{x\left(\frac{4}{13}Dx^{3}+\frac{2}{5}Cx^{2}+\frac{4}{7}Bx+A\right)ab}{2}+a^{2}\left(\frac{1}{10}Dx^{3}+\frac{1}{7}Cx^{2}+\frac{1}{4}Bx+A\right)\right)d^{5}-\frac{3\left(\left(\frac{35}{104}D\right.\right.}{}\right.$
derivativedivides	$\frac{3b^{2}D(xd+c)^{\frac{16}{3}}}{16}+\frac{3\left(2b(ad-bc)D+b^{2}(Cd-3Dc)\right)(xd+c)^{\frac{13}{3}}}{13}+\frac{3\left((ad-bc)^{2}D+2b(ad-bc)(Cd-3Dc)+b^{2}\left(Bd^{2}-2Ccd+3Dc^{2}\right)\right)(xd+c}{10}$
default	$\frac{3b^{2}D(xd+c)^{\frac{16}{3}}}{16}+\frac{3\left(2b(ad-bc)D+b^{2}(Cd-3Dc)\right)(xd+c)^{\frac{13}{3}}}{13}+\frac{3\left((ad-bc)^{2}D+2b(ad-bc)(Cd-3Dc)+b^{2}\left(Bd^{2}-2Ccd+3Dc^{2}\right)\right)(xd+c}{10}$
gosper	$3(xd+c)^{\frac{1}{3}}\left(455Dx^{5}b^{2}d^{5}+560Cx^{4}b^{2}d^{5}+1120Dx^{4}ab\,d^{5}-525Dx^{4}b^{2}c\,d^{4}+728Bx^{3}b^{2}d^{5}+1456Cx^{3}ab\,d^{5}-672Cx^{3}b^{2}c\,d^{4}\right.$
trager	$3(xd+c)^{\frac{1}{3}}\left(455Dx^{5}b^{2}d^{5}+560Cx^{4}b^{2}d^{5}+1120Dx^{4}ab\,d^{5}-525Dx^{4}b^{2}c\,d^{4}+728Bx^{3}b^{2}d^{5}+1456Cx^{3}ab\,d^{5}-672Cx^{3}b^{2}c\,d^{4}\right.$
orering	$3(xd+c)^{\frac{1}{3}}\left(455Dx^{5}b^{2}d^{5}+560Cx^{4}b^{2}d^{5}+1120Dx^{4}ab\,d^{5}-525Dx^{4}b^{2}c\,d^{4}+728Bx^{3}b^{2}d^{5}+1456Cx^{3}ab\,d^{5}-672Cx^{3}b^{2}c\,d^{4}\right.$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3*((1/7*x^2*(7/16*D*x^3+7/13*C*x^2+7/10*B*x+A)*b^2+1/2*x*(4/13*D*x^3+2/5*C
*x^2+4/7*B*x+A)*a*b+a^2*(1/10*D*x^3+1/7*C*x^2+1/4*B*x+A))*d^5-3/2*(1/7*(35
/104*D*x^3+28/65*C*x^2+3/5*B*x+A)*x*b^2+a*(8/65*D*x^3+6/35*C*x^2+2/7*B*x+A
)*b+1/2*(6/35*D*x^2+2/7*C*x+B)*a^2)*c*d^4+9/14*((7/52*D*x^3+12/65*C*x^2+3/
10*B*x+A)*b^2+2*(12/65*D*x^2+3/10*C*x+B)*a*b+a^2*(3/10*D*x+C))*c^2*d^3-81/
140*((5/26*D*x^2+4/13*C*x+B)*b^2+2*(4/13*D*x+C)*a*b+D*a^2)*c^3*d^2+243/455
*((5/16*D*x+C)*b+2*D*a)*c^4*b*d-729/1456*D*b^2*c^5)*(d*x+c)^(1/3)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx=\frac{3\left(455\,Db^2d^5x^5-3645\,Db^2c^5+7280\,Aa^2d^5+3888\left(2\,Dab+Cb\right.\right.}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="fricas
")
```

output
```
3/7280*(455*D*b^2*d^5*x^5 - 3645*D*b^2*c^5 + 7280*A*a^2*d^5 + 3888*(2*D*a*
b + C*b^2)*c^4*d - 4212*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 + 4680*(C*a^2 +
2*B*a*b + A*b^2)*c^2*d^3 - 5460*(B*a^2 + 2*A*a*b)*c*d^4 - 35*(15*D*b^2*c*d
^4 - 16*(2*D*a*b + C*b^2)*d^5)*x^4 + 14*(45*D*b^2*c^2*d^3 - 48*(2*D*a*b +
C*b^2)*c*d^4 + 52*(D*a^2 + 2*C*a*b + B*b^2)*d^5)*x^3 - 2*(405*D*b^2*c^3*d^
2 - 432*(2*D*a*b + C*b^2)*c^2*d^3 + 468*(D*a^2 + 2*C*a*b + B*b^2)*c*d^4 -
520*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*x^2 + (1215*D*b^2*c^4*d - 1296*(2*D*a*b
 + C*b^2)*c^3*d^2 + 1404*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 - 1560*(C*a^2 +
 2*B*a*b + A*b^2)*c*d^4 + 1820*(B*a^2 + 2*A*a*b)*d^5)*x)*(d*x + c)^(1/3)/d
^6
```

Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.97

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx = \begin{cases} \frac{3\left(\frac{Db^2(c+dx)^{\frac{16}{3}}}{16d^5}+\frac{(c+dx)^{\frac{13}{3}}\left(Cb^2d+2Dabd-5Db^2c\right)}{13d^5}+\frac{(c+dx)^{\frac{10}{3}}\left(Bb^2d^2+2Cabd^2-4Cb^2c\right.}{10d^5}\right.}{} \\ \frac{Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5\left(Cb^2+2Dab\right)}{5}+\frac{x^4\left(Bb^2+2Cab+Da^2\right)}{4}+\frac{x^3\left(Ab^2+2Bab+Ca^2\right)}{3}+\frac{x^2\cdot\left(2A\right.}{}}{c^{\frac{2}{3}}} \end{cases}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(2/3),x)
```

output
```
Piecewise((3*(D*b**2*(c + d*x)**(16/3)/(16*d**5) + (c + d*x)**(13/3)*(C*b*
*2*d + 2*D*a*b*d - 5*D*b**2*c)/(13*d**5) + (c + d*x)**(10/3)*(B*b**2*d**2
+ 2*C*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2
)/(10*d**5) + (c + d*x)**(7/3)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d*
*2 + C*a**2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 12
*D*a*b*c**2*d - 10*D*b**2*c**3)/(7*d**5) + (c + d*x)**(4/3)*(2*A*a*b*d**4
- 2*A*b**2*c*d**3 + B*a**2*d**4 - 4*B*a*b*c*d**3 + 3*B*b**2*c**2*d**2 - 2*
C*a**2*c*d**3 + 6*C*a*b*c**2*d**2 - 4*C*b**2*c**3*d + 3*D*a**2*c**2*d**2 -
 8*D*a*b*c**3*d + 5*D*b**2*c**4)/(4*d**5) + (c + d*x)**(1/3)*(A*a**2*d**5
- 2*A*a*b*c*d**4 + A*b**2*c**2*d**3 - B*a**2*c*d**4 + 2*B*a*b*c**2*d**3 -
B*b**2*c**3*d**2 + C*a**2*c**2*d**3 - 2*C*a*b*c**3*d**2 + C*b**2*c**4*d -
D*a**2*c**3*d**2 + 2*D*a*b*c**4*d - D*b**2*c**5)/d**5)/d, Ne(d, 0)), ((A*a
**2*x + D*b**2*x**6/6 + x**5*(C*b**2 + 2*D*a*b)/5 + x**4*(B*b**2 + 2*C*a*b
 + D*a**2)/4 + x**3*(A*b**2 + 2*B*a*b + C*a**2)/3 + x**2*(2*A*a*b + B*a**2
)/2)/c**(2/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx = \frac{3\left(455\,(dx+c)^{\frac{16}{3}}Db^2-560\,(5\,Db^2c-(2\,Dab+Cb^2)d)(dx+c\right.}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="maxima
")
```

output
```
3/7280*(455*(d*x + c)^(16/3)*D*b^2 - 560*(5*D*b^2*c - (2*D*a*b + C*b^2)*d)
*(d*x + c)^(13/3) + 728*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d + (D*a^2 +
 2*C*a*b + B*b^2)*d^2)*(d*x + c)^(10/3) - 1040*(10*D*b^2*c^3 - 6*(2*D*a*b
+ C*b^2)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*b + A*
b^2)*d^3)*(d*x + c)^(7/3) + 1820*(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^2)*c^3*d
+ 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3
+ (B*a^2 + 2*A*a*b)*d^4)*(d*x + c)^(4/3) - 7280*(D*b^2*c^5 - A*a^2*d^5 - (
2*D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 + 2*B*
a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)*(d*x + c)^(1/3))/d^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
3/7280*(7280*(d*x + c)^(1/3)*A*a^2 + 1820*((d*x + c)^(4/3) - 4*(d*x + c)^(
1/3)*c)*B*a^2/d + 3640*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*A*a*b/d + 5
20*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*C*a^
2/d^2 + 1040*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)
*c^2)*B*a*b/d^2 + 520*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x +
 c)^(1/3)*c^2)*A*b^2/d^2 + 52*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c
+ 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*D*a^2/d^3 + 104*(14*(
d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*
x + c)^(1/3)*c^3)*C*a*b/d^3 + 52*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)
*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*B*b^2/d^3 + 32*(35
*(d*x + c)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 455
*(d*x + c)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*D*a*b/d^4 + 16*(35*(d*x +
c)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c^2 - 455*(d*x +
c)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*C*b^2/d^4 + 5*(91*(d*x + c)^(16/3)
 - 560*(d*x + c)^(13/3)*c + 1456*(d*x + c)^(10/3)*c^2 - 2080*(d*x + c)^(7/
3)*c^3 + 1820*(d*x + c)^(4/3)*c^4 - 1456*(d*x + c)^(1/3)*c^5)*D*b^2/d^5)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} dx = \int \frac{(a+b\,x)^2\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{2/3}} \,dx$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(2/3),x)
```

output
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}} \, dx = \frac{3(dx+c)^{\frac{1}{3}} (455b^2d^5x^5 + 1120ab\,d^5x^4 + 35b^2c\,d^4x^4 + 728a^2d^5x^3}{}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x)
```

output
```
(3*(c + d*x)**(1/3)*(7280*a**3*d**4 - 16380*a**2*b*c*d**3 + 5460*a**2*b*d*
*4*x + 468*a**2*c**3*d**2 - 156*a**2*c**2*d**3*x + 104*a**2*c*d**4*x**2 +
728*a**2*d**5*x**3 + 14040*a*b**2*c**2*d**2 - 4680*a*b**2*c*d**3*x + 3120*
a*b**2*d**4*x**2 - 648*a*b*c**4*d + 216*a*b*c**3*d**2*x - 144*a*b*c**2*d**
3*x**2 + 112*a*b*c*d**4*x**3 + 1120*a*b*d**5*x**4 - 4212*b**3*c**3*d + 140
4*b**3*c**2*d**2*x - 936*b**3*c*d**3*x**2 + 728*b**3*d**4*x**3 + 243*b**2*
c**5 - 81*b**2*c**4*d*x + 54*b**2*c**3*d**2*x**2 - 42*b**2*c**2*d**3*x**3
+ 35*b**2*c*d**4*x**4 + 455*b**2*d**5*x**5))/(7280*d**5)
```

3.162 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx$

Optimal result . . . 1567
Mathematica [A] (verified) . . . 1568
Rubi [A] (verified) . . . 1568
Maple [A] (verified) . . . 1570
Fricas [A] (verification not implemented) . . . 1570
Sympy [A] (verification not implemented) . . . 1571
Maxima [A] (verification not implemented) . . . 1571
Giac [A] (verification not implemented) . . . 1572
Mupad [F(-1)] . . . 1572
Reduce [B] (verification not implemented) . . . 1573

Optimal result

Integrand size = 30, antiderivative size = 212

$$\begin{aligned}\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx =& -\frac{3(bc-ad)(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt[3]{c+dx}}{d^5} \\ &- \frac{3(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))(c+dx)^{4/3}}{4d^5} \\ &+ \frac{3(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))(c+dx)^{7/3}}{7d^5} \\ &+ \frac{3(bCd-4bcD+adD)(c+dx)^{10/3}}{10d^5} + \frac{3bD(c+dx)^{13/3}}{13d^5}\end{aligned}$$

output
```
-3*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1/3)/d^5-3/4*(a*d*(-B
*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(4/3)
/d^5+3/7*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(7/3)/d^5+3/
10*(C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(10/3)/d^5+3/13*b*D*(d*x+c)^(13/3)/d^5
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx = \frac{3\sqrt[3]{c+dx}(13ad(-81c^3D+9c^2d(10C+3Dx)-3cd^2(35B+10C$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(2/3),x]
```

output
```
(3*(c + d*x)^(1/3)*(13*a*d*(-81*c^3*D + 9*c^2*d*(10*C + 3*D*x) - 3*c*d^2*(
35*B + 10*C*x + 6*D*x^2) + d^3*(140*A + 35*B*x + 20*C*x^2 + 14*D*x^3)) + b
*(972*c^4*D - 81*c^3*d*(13*C + 4*D*x) + 9*c^2*d^2*(130*B + 3*x*(13*C + 8*D
*x)) - 3*c*d^3*(455*A + 2*x*(65*B + 39*C*x + 28*D*x^2)) + d^4*x*(455*A + 2
*x*(130*B + 91*C*x + 70*D*x^2)))))/(1820*d^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{\sqrt[3]{c+dx}\left(b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)\right)}{d^4}+\frac{(ad-bc)\left(Ad^3-Bcd^2+c^3(-\right.}{d^4(c+dx)^{2/3}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3(c+dx)^{4/3}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{4d^5}-\frac{3\sqrt[3]{c+dx}(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^5}+\frac{3(c+dx)^{7/3}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{7d^5}+\frac{3(c+dx)^{10/3}(adD-4bcD+bCd)}{10d^5}+\frac{3bD(c+dx)^{13/3}}{13d^5}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(2/3),x]
```

output
```
(-3*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(1/3))/d^5 -
 (3*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 -
4*c^3*D))*(c + d*x)^(4/3))/(4*d^5) + (3*(a*d*(C*d - 3*c*D) - b*(3*c*C*d -
B*d^2 - 6*c^2*D))*(c + d*x)^(7/3))/(7*d^5) + (3*(b*C*d - 4*b*c*D + a*d*D)*
(c + d*x)^(10/3))/(10*d^5) + (3*b*D*(c + d*x)^(13/3))/(13*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{3\left(\left(\left(\frac{Db\,x^{4}}{13}+\frac{(Cb+Da)x^{3}}{10}+\frac{(Bb+Ca)x^{2}}{7}+\frac{(Ab+Ba)x}{4}+Aa\right)d^{4}-\frac{3\left(\frac{8Db\,x^{3}}{65}+\frac{6(Cb+Da)x^{2}}{35}+\frac{2(Bb+Ca)x}{7}+Ab+Ba\right)c\,d^{3}}{4}+\frac{9\left(12\right.}{}\right.\right.}{d^{5}}$
derivativedivides	$\frac{\frac{3bD(xd+c)^{\frac{13}{3}}}{13}+\frac{3((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{10}{3}}}{10}+\frac{3\left((ad-bc)(Cd-3Dc)+b\left(B\,d^{2}-2Ccd+3Dc^{2}\right)\right)(xd+c)^{\frac{7}{3}}}{7}+\frac{3\left((ad-bc)\left(B\,d\right.\right.}{}}{d^{5}}$
default	$\frac{\frac{3bD(xd+c)^{\frac{13}{3}}}{13}+\frac{3((ad-bc)D+b(Cd-3Dc))(xd+c)^{\frac{10}{3}}}{10}+\frac{3\left((ad-bc)(Cd-3Dc)+b\left(B\,d^{2}-2Ccd+3Dc^{2}\right)\right)(xd+c)^{\frac{7}{3}}}{7}+\frac{3\left((ad-bc)\left(B\,d\right.\right.}{}}{d^{5}}$
gosper	$\frac{3(xd+c)^{\frac{1}{3}}\left(140Db\,x^{4}d^{4}+182C\,x^{3}b\,d^{4}+182Dx^{3}a\,d^{4}-168Dx^{3}bc\,d^{3}+260B\,x^{2}b\,d^{4}+260C\,x^{2}a\,d^{4}-234C\,x^{2}bc\,d^{3}-234Dx\right.}{}$
trager	$\frac{3(xd+c)^{\frac{1}{3}}\left(140Db\,x^{4}d^{4}+182C\,x^{3}b\,d^{4}+182Dx^{3}a\,d^{4}-168Dx^{3}bc\,d^{3}+260B\,x^{2}b\,d^{4}+260C\,x^{2}a\,d^{4}-234C\,x^{2}bc\,d^{3}-234Dx\right.}{}$
orering	$\frac{3(xd+c)^{\frac{1}{3}}\left(140Db\,x^{4}d^{4}+182C\,x^{3}b\,d^{4}+182Dx^{3}a\,d^{4}-168Dx^{3}bc\,d^{3}+260B\,x^{2}b\,d^{4}+260C\,x^{2}a\,d^{4}-234C\,x^{2}bc\,d^{3}-234Dx\right.}{}$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3*((1/13*D*b*x^4+1/10*(C*b+D*a)*x^3+1/7*(B*b+C*a)*x^2+1/4*(A*b+B*a)*x+A*a)
*d^4-3/4*(8/65*D*b*x^3+6/35*(C*b+D*a)*x^2+2/7*(B*b+C*a)*x+A*b+B*a)*c*d^3+9
/14*(12/65*D*b*x^2+3/10*(C*b+D*a)*x+B*b+C*a)*c^2*d^2-81/140*(4/13*D*b*x+C*
b+D*a)*c^3*d+243/455*D*b*c^4)*(d*x+c)^(1/3)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx = \frac{3\,(140\,Dbd^4x^4+972\,Dbc^4+1820\,Aad^4-1053\,(Da+Cb)c^3d+\cdots}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
3/1820*(140*D*b*d^4*x^4 + 972*D*b*c^4 + 1820*A*a*d^4 - 1053*(D*a + C*b)*c^
3*d + 1170*(C*a + B*b)*c^2*d^2 - 1365*(B*a + A*b)*c*d^3 - 14*(12*D*b*c*d^3
 - 13*(D*a + C*b)*d^4)*x^3 + 2*(108*D*b*c^2*d^2 - 117*(D*a + C*b)*c*d^3 +
130*(C*a + B*b)*d^4)*x^2 - (324*D*b*c^3*d - 351*(D*a + C*b)*c^2*d^2 + 390*
(C*a + B*b)*c*d^3 - 455*(B*a + A*b)*d^4)*x)*(d*x + c)^(1/3)/d^5
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx = \begin{cases} \frac{3\left(\frac{Db(c+dx)^{\frac{13}{3}}}{13d^4} + \frac{(c+dx)^{\frac{10}{3}}(Cbd+Dad-4Dbc)}{10d^4} + \frac{(c+dx)^{\frac{7}{3}}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6D\right.}{7d^4}\right.}{} \\ \frac{Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}}{c^{\frac{2}{3}}} \end{cases}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(2/3),x)
```

output
```
Piecewise((3*(D*b*(c + d*x)**(13/3)/(13*d**4) + (c + d*x)**(10/3)*(C*b*d +
 D*a*d - 4*D*b*c)/(10*d**4) + (c + d*x)**(7/3)*(B*b*d**2 + C*a*d**2 - 3*C*
b*c*d - 3*D*a*c*d + 6*D*b*c**2)/(7*d**4) + (c + d*x)**(4/3)*(A*b*d**3 + B*
a*d**3 - 2*B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D*b
*c**3)/(4*d**4) + (c + d*x)**(1/3)*(A*a*d**4 - A*b*c*d**3 - B*a*c*d**3 + B
*b*c**2*d**2 + C*a*c**2*d**2 - C*b*c**3*d - D*a*c**3*d + D*b*c**4)/d**4)/d
, Ne(d, 0)), ((A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/4 + x**3*(B*b + C*a)/
3 + x**2*(A*b + B*a)/2)/c**(2/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx = \frac{3\left(140\,(dx+c)^{\frac{13}{3}}Db - 182\,(4\,Dbc-(Da+Cb)d)(dx+c)^{\frac{10}{3}} + 2\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
3/1820*(140*(d*x + c)^(13/3)*D*b - 182*(4*D*b*c - (D*a + C*b)*d)*(d*x + c)
^(10/3) + 260*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*(d*x + c)^
(7/3) - 455*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 - (B*a
+ A*b)*d^3)*(d*x + c)^(4/3) + 1820*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c^3*d
+ (C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)*(d*x + c)^(1/3))/d^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx = \frac{3\left(1820\,(dx+c)^{\frac{1}{3}}Aa + \frac{455\left((dx+c)^{\frac{4}{3}}-4\,(dx+c)^{\frac{1}{3}}c\right)Ba}{d} + \frac{455\left((dx+c)^{\frac{4}{3}}-4\right.}{d}\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
3/1820*(1820*(d*x + c)^(1/3)*A*a + 455*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3
)*c)*B*a/d + 455*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*A*b/d + 130*(2*(d
*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*C*a/d^2 + 13
0*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2)*B*b/d
^2 + 13*(14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3)*
c^2 - 140*(d*x + c)^(1/3)*c^3)*D*a/d^3 + 13*(14*(d*x + c)^(10/3) - 60*(d*x
 + c)^(7/3)*c + 105*(d*x + c)^(4/3)*c^2 - 140*(d*x + c)^(1/3)*c^3)*C*b/d^3
 + 4*(35*(d*x + c)^(13/3) - 182*(d*x + c)^(10/3)*c + 390*(d*x + c)^(7/3)*c
^2 - 455*(d*x + c)^(4/3)*c^3 + 455*(d*x + c)^(1/3)*c^4)*D*b/d^4)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{2/3}}\,dx = \int \frac{(a+b\,x)\,(A+B\,x+C\,x^2+x^3\,D)}{(c+d\,x)^{2/3}}\,dx$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(2/3),x)
```

output
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{2/3}}\,dx = \frac{3(dx+c)^{\frac{1}{3}}\left(140b\,d^4x^4+182a\,d^4x^3+14bc\,d^3x^3+26ac\,d^3x^2+26\right.}{}$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x)
```

output
```
(3*(c + d*x)**(1/3)*(1820*a**2*d**3 - 2730*a*b*c*d**2 + 910*a*b*d**3*x + 1
17*a*c**3*d - 39*a*c**2*d**2*x + 26*a*c*d**3*x**2 + 182*a*d**4*x**3 + 1170
*b**2*c**2*d - 390*b**2*c*d**2*x + 260*b**2*d**3*x**2 - 81*b*c**4 + 27*b*c
**3*d*x - 18*b*c**2*d**2*x**2 + 14*b*c*d**3*x**3 + 140*b*d**4*x**4))/(1820
*d**4)
```

3.163 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx$

Optimal result 1574
Mathematica [A] (verified) 1574
Rubi [A] (verified) 1575
Maple [A] (verified) 1576
Fricas [A] (verification not implemented) 1577
Sympy [A] (verification not implemented) 1577
Maxima [A] (verification not implemented) 1578
Giac [A] (verification not implemented) 1578
Mupad [F(-1)] 1579
Reduce [B] (verification not implemented) 1579

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx=\frac{3(c^2Cd-Bcd^2+Ad^3-c^3D)\sqrt[3]{c+dx}}{d^4}-\frac{3(2cCd-Bd^2-3c^2D)\,(c+dx)^{4/3}}{4d^4}+\frac{3(Cd-3cD)(c+dx)^{7/3}}{7d^4}+\frac{3D(c+dx)^{10/3}}{10d^4}$$

output
```
3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1/3)/d^4-3/4*(-B*d^2+2*C*c*d-3*D*
c^2)*(d*x+c)^(4/3)/d^4+3/7*(C*d-3*D*c)*(d*x+c)^(7/3)/d^4+3/10*D*(d*x+c)^(1
0/3)/d^4
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx=\frac{3\sqrt[3]{c+dx}(-81c^3D+9c^2d(10C+3Dx)-3cd^2(35B+2x(5C+3Dx))+a}{140d^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^(2/3),x]
```

output
```
(3*(c + d*x)^(1/3)*(-81*c^3*D + 9*c^2*d*(10*C + 3*D*x) - 3*c*d^2*(35*B + 2
*x*(5*C + 3*D*x)) + d^3*(140*A + x*(35*B + 20*C*x + 14*D*x^2))))/(140*d^4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)^{2/3}}+\frac{\sqrt[3]{c+dx}\left(Bd^2+3c^2D-2cCd\right)}{d^3}+\frac{(c+dx)^{4/3}(Cd-3cD)}{d^3}+\frac{D(c+dx)^{7/}}{d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt[3]{c+dx}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^4}-\frac{3(c+dx)^{4/3}\left(-Bd^2-3c^2D+2cCd\right)}{4d^4}+\frac{3(c+dx)^{7/3}(Cd-3cD)}{7d^4}+\frac{3D(c+dx)^{10/3}}{10d^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^(2/3),x]
```

output
```
(3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(1/3))/d^4 - (3*(2*c*C*d
- B*d^2 - 3*c^2*D)*(c + d*x)^(4/3))/(4*d^4) + (3*(C*d - 3*c*D)*(c + d*x)^(
7/3))/(7*d^4) + (3*D*(c + d*x)^(10/3))/(10*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{3\left(\left(\frac{1}{10}Dx^3+\frac{1}{7}Cx^2+\frac{1}{4}Bx+A\right)d^3-\frac{3\left(\frac{6}{35}Dx^2+\frac{2}{7}Cx+B\right)c\,d^2}{4}+\frac{9\left(\frac{3Dx}{10}+C\right)c^2d}{14}-\frac{81Dc^3}{140}\right)(xd+c)^{\frac{1}{3}}}{d^4}$
gosper	$\frac{3(xd+c)^{\frac{1}{3}}\left(14Dx^3d^3+20Cx^2d^3-18Dx^2c\,d^2+35Bx\,d^3-30Cxc\,d^2+27Dx\,c^2d+140A\,d^3-105Bc\,d^2+90C\,c^2d-81Dc^3\right)}{140d^4}$
trager	$\frac{3(xd+c)^{\frac{1}{3}}\left(14Dx^3d^3+20Cx^2d^3-18Dx^2c\,d^2+35Bx\,d^3-30Cxc\,d^2+27Dx\,c^2d+140A\,d^3-105Bc\,d^2+90C\,c^2d-81Dc^3\right)}{140d^4}$
orering	$\frac{3(xd+c)^{\frac{1}{3}}\left(14Dx^3d^3+20Cx^2d^3-18Dx^2c\,d^2+35Bx\,d^3-30Cxc\,d^2+27Dx\,c^2d+140A\,d^3-105Bc\,d^2+90C\,c^2d-81Dc^3\right)}{140d^4}$
derivativedivides	$\frac{\frac{3D(xd+c)^{\frac{10}{3}}}{10}+\frac{3Cd(xd+c)^{\frac{7}{3}}}{7}-\frac{9Dc(xd+c)^{\frac{7}{3}}}{7}+\frac{3B\,d^2(xd+c)^{\frac{4}{3}}}{4}-\frac{3Ccd(xd+c)^{\frac{4}{3}}}{2}+\frac{9Dc^2(xd+c)^{\frac{4}{3}}}{4}+3A\,d^3(xd+c)^{\frac{1}{3}}-3Bc\,d^2(xd-}{d^4}$
default	$\frac{\frac{3D(xd+c)^{\frac{10}{3}}}{10}+\frac{3Cd(xd+c)^{\frac{7}{3}}}{7}-\frac{9Dc(xd+c)^{\frac{7}{3}}}{7}+\frac{3B\,d^2(xd+c)^{\frac{4}{3}}}{4}-\frac{3Ccd(xd+c)^{\frac{4}{3}}}{2}+\frac{9Dc^2(xd+c)^{\frac{4}{3}}}{4}+3A\,d^3(xd+c)^{\frac{1}{3}}-3Bc\,d^2(xd-}{d^4}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3*((1/10*D*x^3+1/7*C*x^2+1/4*B*x+A)*d^3-3/4*(6/35*D*x^2+2/7*C*x+B)*c*d^2+9
/14*(3/10*D*x+C)*c^2*d-81/140*D*c^3)*(d*x+c)^(1/3)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx = \frac{3\,(14\,Dd^3x^3-81\,Dc^3+90\,Cc^2d-105\,Bcd^2+140\,Ad^3-2\,(9\,Dcd^2-10\,C}{140\,d^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
3/140*(14*D*d^3*x^3 - 81*D*c^3 + 90*C*c^2*d - 105*B*c*d^2 + 140*A*d^3 - 2*
(9*D*c*d^2 - 10*C*d^3)*x^2 + (27*D*c^2*d - 30*C*c*d^2 + 35*B*d^3)*x)*(d*x
+ c)^(1/3)/d^4
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx = \begin{cases} \frac{3A\sqrt[3]{c+dx}+\frac{3B\left(-c\sqrt[3]{c+dx}+\frac{(c+dx)^{\frac{4}{3}}}{4}\right)}{d}+\frac{3C\left(c^2\sqrt[3]{c+dx}-\frac{c(c+dx)^{\frac{4}{3}}}{2}+\frac{(c+dx)^{\frac{7}{3}}}{7}\right)}{d^2}+\frac{3D\left(-\right.}{}}{d} \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}}{c^{\frac{2}{3}}} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(2/3),x)
```

output
```
Piecewise(((3*A*(c + d*x)**(1/3) + 3*B*(-c*(c + d*x)**(1/3) + (c + d*x)**(
4/3)/4)/d + 3*C*(c**2*(c + d*x)**(1/3) - c*(c + d*x)**(4/3)/2 + (c + d*x)*
*(7/3)/7)/d**2 + 3*D*(-c**3*(c + d*x)**(1/3) + 3*c**2*(c + d*x)**(4/3)/4 -
 3*c*(c + d*x)**(7/3)/7 + (c + d*x)**(10/3)/10)/d**3)/d, Ne(d, 0)), ((A*x
+ B*x**2/2 + C*x**3/3 + D*x**4/4)/c**(2/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx = \frac{3\left(140\,(dx+c)^{\frac{1}{3}}A+\frac{35\left((dx+c)^{\frac{4}{3}}-4\,(dx+c)^{\frac{1}{3}}c\right)B}{d}+\frac{10\left(2\,(dx+c)^{\frac{7}{3}}-7\,(dx+c)^{\frac{4}{3}}c+14\,(dx+\right.}{d^2}\right.}{140\,d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
3/140*(140*(d*x + c)^(1/3)*A + 35*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*
B/d + 10*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2
)*C/d^2 + (14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3
)*c^2 - 140*(d*x + c)^(1/3)*c^3)*D/d^3)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx = \frac{3\left(140\,(dx+c)^{\frac{1}{3}}A+\frac{35\left((dx+c)^{\frac{4}{3}}-4\,(dx+c)^{\frac{1}{3}}c\right)B}{d}+\frac{10\left(2\,(dx+c)^{\frac{7}{3}}-7\,(dx+c)^{\frac{4}{3}}c+14\,(dx+\right.}{d^2}\right.}{140\,d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
3/140*(140*(d*x + c)^(1/3)*A + 35*((d*x + c)^(4/3) - 4*(d*x + c)^(1/3)*c)*
B/d + 10*(2*(d*x + c)^(7/3) - 7*(d*x + c)^(4/3)*c + 14*(d*x + c)^(1/3)*c^2
)*C/d^2 + (14*(d*x + c)^(10/3) - 60*(d*x + c)^(7/3)*c + 105*(d*x + c)^(4/3
)*c^2 - 140*(d*x + c)^(1/3)*c^3)*D/d^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(c+d\,x)^{2/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(2/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(2/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{2/3}}\,dx=\frac{3(dx+c)^{\frac{1}{3}}\,(14d^3x^3+2c\,d^2x^2+35b\,d^2x-3c^2dx+140a\,d^2-105bcd+9c^3)}{140d^3}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(2/3),x)
```

output
```
(3*(c + d*x)**(1/3)*(140*a*d**2 - 105*b*c*d + 35*b*d**2*x + 9*c**3 - 3*c**
2*d*x + 2*c*d**2*x**2 + 14*d**3*x**3))/(140*d**3)
```

3.164 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx$

Optimal result 1580
Mathematica [A] (verified) 1581
Rubi [A] (verified) 1581
Maple [A] (verified) 1583
Fricas [B] (verification not implemented) 1584
Sympy [F] 1585
Maxima [F(-2)] 1586
Giac [A] (verification not implemented) 1586
Mupad [F(-1)] 1587
Reduce [B] (verification not implemented) 1587

Optimal result

Integrand size = 32, antiderivative size = 330

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx = \frac{3(a^2d^2D-abd(Cd-cD)-b^2(cCd-Bd^2-c^2D))\sqrt[3]{c+dx}}{b^3d^3} + \frac{3(bCd-2bcD-adD)(c+dx)^{4/3}}{4b^2d^3} + \frac{3D(c+dx)^{7/3}}{7bd^3} - \frac{\sqrt{3}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{10/3}(bc-ad)^{2/3}} - \frac{(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{2b^{10/3}(bc-ad)^{2/3}} + \frac{3(Ab^3-a(b^2B-abC+a^2D))\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{10/3}(bc-ad)^{2/3}}$$

output
```
3*(a^2*d^2*D-a*b*d*(C*d-D*c)-b^2*(-B*d^2+C*c*d-D*c^2))*(d*x+c)^(1/3)/b^3/d
^3+3/4*(C*b*d-D*a*d-2*D*b*c)*(d*x+c)^(4/3)/b^2/d^3+3/7*D*(d*x+c)^(7/3)/b/d
^3-3^(1/2)*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(
1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/b^(10/3)/(-a*d+b*c)^(2/3)-1/2*(A*b^3-a*(B*
b^2-C*a*b+D*a^2))*ln(b*x+a)/b^(10/3)/(-a*d+b*c)^(2/3)+3/2*(A*b^3-a*(B*b^2-
C*a*b+D*a^2))*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/b^(10/3)/(-a*d+b*
c)^(2/3)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx = \frac{3\sqrt[3]{b}(-bc+ad)^{2/3}\sqrt[3]{c+dx}(28a^2d^2D-7abd(4Cd-3cD+dDx)+b^2(18c^2}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^(2/3)),x]
```

output
```
(3*b^(1/3)*(-(b*c) + a*d)^(2/3)*(c + d*x)^(1/3)*(28*a^2*d^2*D - 7*a*b*d*(4
*C*d - 3*c*D + d*D*x) + b^2*(18*c^2*D - 3*c*d*(7*C + 2*D*x) + d^2*(28*B +
7*C*x + 4*D*x^2))) - 28*Sqrt[3]*d^3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Ar
cTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3))/Sqrt[3]] + 28*
d^3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Log[(-(b*c) + a*d)^(1/3) + b^(1/3)
*(c + d*x)^(1/3)] - 14*d^3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Log[(-(b*c)
 + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c
+ d*x)^(2/3)])/(28*b^(10/3)*d^3*(-(b*c) + a*d)^(2/3))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{b^3(a+bx)(c+dx)^{2/3}}+\frac{a^2d^2D-abd(Cd-cD)-\left(b^2\left(-Bd^2+c^2(-D)+cCd\right)\right)}{b^3d^2(c+dx)^{2/3}}+\frac{\sqrt[3]{c+dx}(-adD}{b^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{3}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{10/3}(bc-ad)^{2/3}}-\frac{\log(a+bx)\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^{10/3}(bc-ad)^{2/3}}+\frac{3\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{10/3}(bc-ad)^{2/3}}+\frac{3\sqrt[3]{c+dx}\left(a^2d^2D-abd(Cd-cD)-\left(b^2\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{b^3d^3}+\frac{3(c+dx)^{4/3}(-adD-2bcD+bCd)}{4b^2d^3}+\frac{3D(c+dx)^{7/3}}{7bd^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^(2/3)),x]
```

output
```
(3*(a^2*d^2*D - a*b*d*(C*d - c*D) - b^2*(c*C*d - B*d^2 - c^2*D))*(c + d*x)
^(1/3))/(b^3*d^3) + (3*(b*C*d - 2*b*c*D - a*d*D)*(c + d*x)^(4/3))/(4*b^2*d
^3) + (3*D*(c + d*x)^(7/3))/(7*b*d^3) - (Sqrt[3]*(A*b^3 - a*(b^2*B - a*b*C
 + a^2*D))*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(b^(10/3)*(b*c - a*d)^(2/3)) - ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*
Log[a + b*x])/(2*b^(10/3)*(b*c - a*d)^(2/3)) + (3*(A*b^3 - a*(b^2*B - a*b*
C + a^2*D))*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(10/3)*
(b*c - a*d)^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{3\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\left(\left(\left(\frac{1}{7}Dx^2+\frac{1}{4}Cx+B\right)b^2-\left(\frac{Dx}{4}+C\right)ab+Da^2\right)d^2-\frac{3\left(\left(\frac{2Dx}{7}+C\right)b-Da\right)cbd}{4}+\frac{9Db^2c^2}{14}\right)(xd+c)^{\frac{1}{3}}b-\frac{\left(b^3A-a\,b^2\right.}{}}{}$
derivativedivides	$\frac{3\left(\frac{D(xd+c)^{\frac{7}{3}}b^2}{7}+\frac{C\,b^2d(xd+c)^{\frac{4}{3}}}{4}-\frac{Dabd(xd+c)^{\frac{4}{3}}}{4}-\frac{Db^2c(xd+c)^{\frac{4}{3}}}{2}+B\,d^2b^2(xd+c)^{\frac{1}{3}}-Ca\,d^2b(xd+c)^{\frac{1}{3}}-C\,b^2cd(xd+c)^{\frac{1}{3}}+Da^2d^2\right.}{b^3}$
default	$\frac{3\left(\frac{D(xd+c)^{\frac{7}{3}}b^2}{7}+\frac{C\,b^2d(xd+c)^{\frac{4}{3}}}{4}-\frac{Dabd(xd+c)^{\frac{4}{3}}}{4}-\frac{Db^2c(xd+c)^{\frac{4}{3}}}{2}+B\,d^2b^2(xd+c)^{\frac{1}{3}}-Ca\,d^2b(xd+c)^{\frac{1}{3}}-C\,b^2cd(xd+c)^{\frac{1}{3}}+Da^2d^2\right.}{b^3}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3*(((a*d-b*c)/b)^(2/3)*((((1/7*D*x^2+1/4*C*x+B)*b^2-(1/4*D*x+C)*a*b+D*a^2)*
d^2-3/4*((2/7*D*x+C)*b-D*a)*c*b*d+9/14*D*b^2*c^2)*(d*x+c)^(1/3)*b-1/6*(A*b
^3-B*a*b^2+C*a^2*b-D*a^3)*(-2*arctan(1/3*3^(1/2)*(2*(d*x+c)^(1/3)-((a*d-b*
c)/b)^(1/3))/((a*d-b*c)/b)^(1/3))*3^(1/2)+ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(
1/3)*(d*x+c)^(1/3)+((a*d-b*c)/b)^(2/3))-2*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(
1/3)))*d^3)/((a*d-b*c)/b)^(2/3)/d^3/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(286) = 572$.

Time = 0.12 (sec) , antiderivative size = 1656, normalized size of antiderivative = 5.02

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)(c + dx)^{2/3}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
[1/28*(14*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(D*a^3 - C*a^2*b + B*a
*b^2 - A*b^3)*d^3*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)
^(2/3)*(d*x + c)^(1/3)) - 28*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(D*
a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^3*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3)
+ (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)) - 14*sqrt(3)*((D*a^3*b^2 - C*
a^2*b^3 + B*a*b^4 - A*b^5)*c*d^3 - (D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*
b^4)*d^4)*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b)*log(-(3*b^2*c
^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x - sqrt(3)*(2*(b^2*c - a
*b*d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a
*d) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3))*sqrt(-(b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b) - 3*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)^(1/3)*(b*c - a*d)*(d*x + c)^(1/3))/(b*x + a)) + 3*(18*D*b^5*c^4 -
3*(5*D*a*b^4 + 7*C*b^5)*c^3*d + 2*(2*D*a^2*b^3 + 7*C*a*b^4 + 14*B*b^5)*c^2
*d^2 - 7*(5*D*a^3*b^2 - 5*C*a^2*b^3 + 8*B*a*b^4)*c*d^3 + 28*(D*a^4*b - C*a
^3*b^2 + B*a^2*b^3)*d^4 + 4*(D*b^5*c^2*d^2 - 2*D*a*b^4*c*d^3 + D*a^2*b^3*d
^4)*x^2 - (6*D*b^5*c^3*d - (5*D*a*b^4 + 7*C*b^5)*c^2*d^2 - 2*(4*D*a^2*b^3
- 7*C*a*b^4)*c*d^3 + 7*(D*a^3*b^2 - C*a^2*b^3)*d^4)*x)*(d*x + c)^(1/3))/(b
^6*c^2*d^3 - 2*a*b^5*c*d^4 + a^2*b^4*d^5), 1/28*(14*(b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)^(2/3)*(D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^3*log(-(b^2*c...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\left(c+dx\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(2/3),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)*(c + d*x)**(2/3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx =$$
$$-\frac{(Da^3b^4d^{24} - Ca^2b^5d^{24} + Bab^6d^{24} - Ab^7d^{24})\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\log\left(\left|(dx+c)^{\frac{1}{3}} - \left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right|\right)}{b^8cd^{24} - ab^7d^{25}}$$
$$+\frac{(\sqrt{3}Da^3 - \sqrt{3}Ca^2b + \sqrt{3}Bab^2 - \sqrt{3}Ab^3)\arctan\left(\frac{\sqrt{3}\left(2\,(dx+c)^{\frac{1}{3}} + \left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{(b^3c - ab^2d)^{\frac{2}{3}}b^2}$$
$$+\frac{(Da^3 - Ca^2b + Bab^2 - Ab^3)\log\left((dx+c)^{\frac{2}{3}} + (dx+c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\,(b^3c - ab^2d)^{\frac{2}{3}}b^2}$$
$$+\frac{3\left(4\,(dx+c)^{\frac{7}{3}}Db^6d^{18} - 14\,(dx+c)^{\frac{4}{3}}Db^6cd^{18} + 28\,(dx+c)^{\frac{1}{3}}Db^6c^2d^{18} - 7\,(dx+c)^{\frac{4}{3}}Dab^5d^{19} + 7\,(dx+c)\right.}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
-(D*a^3*b^4*d^24 - C*a^2*b^5*d^24 + B*a*b^6*d^24 - A*b^7*d^24)*((b*c - a*d
)/b)^(1/3)*log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b^8*c*d^24 -
 a*b^7*d^25) + (sqrt(3)*D*a^3 - sqrt(3)*C*a^2*b + sqrt(3)*B*a*b^2 - sqrt(3
)*A*b^3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a*d)/b)^(1/3))/((
b*c - a*d)/b)^(1/3))/((b^3*c - a*b^2*d)^(2/3)*b^2) + 1/2*(D*a^3 - C*a^2*b
+ B*a*b^2 - A*b^3)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(
1/3) + ((b*c - a*d)/b)^(2/3))/((b^3*c - a*b^2*d)^(2/3)*b^2) + 3/28*(4*(d*x
 + c)^(7/3)*D*b^6*d^18 - 14*(d*x + c)^(4/3)*D*b^6*c*d^18 + 28*(d*x + c)^(1
/3)*D*b^6*c^2*d^18 - 7*(d*x + c)^(4/3)*D*a*b^5*d^19 + 7*(d*x + c)^(4/3)*C*
b^6*d^19 + 28*(d*x + c)^(1/3)*D*a*b^5*c*d^19 - 28*(d*x + c)^(1/3)*C*b^6*c*
d^19 + 28*(d*x + c)^(1/3)*D*a^2*b^4*d^20 - 28*(d*x + c)^(1/3)*C*a*b^5*d^20
 + 28*(d*x + c)^(1/3)*B*b^6*d^20)/(b^7*d^21)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,\left(c+d\,x\right)^{2/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(2/3)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(2/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{2/3}}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(2/3),x)
```

output

```
(28*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*
x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**3*d**3 - 28*sqrt(3)*atan((b**
(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*
(a*d - b*c)**(1/6)))*a**2*b*c*d**2 + 28*sqrt(3)*atan((b**(1/6)*(a*d - b*c)
**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6
)))*a**3*d**3 - 28*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b
**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b*c*d**2 + 8
4*b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2/3)*a**2*d**2 - 21*b**(1/3)*(c
+ d*x)**(1/3)*(a*d - b*c)**(2/3)*a*b*c*d - 21*b**(1/3)*(c + d*x)**(1/3)*(a
*d - b*c)**(2/3)*a*b*d**2*x + 84*b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2
/3)*b**3*d - 9*b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2/3)*b**2*c**2 + 3*
b**(1/3)*(c + d*x)**(1/3)*(a*d - b*c)**(2/3)*b**2*c*d*x + 12*b**(1/3)*(c +
 d*x)**(1/3)*(a*d - b*c)**(2/3)*b**2*d**2*x**2 - 28*log((a*d - b*c)**(1/3)
 + b**(1/3)*(c + d*x)**(1/3))*a**3*d**3 + 28*log((a*d - b*c)**(1/3) + b**(
1/3)*(c + d*x)**(1/3))*a**2*b*c*d**2 + 14*log( - b**(1/6)*(c + d*x)**(1/6)
*(a*d - b*c)**(1/6)*sqrt(3) + (a*d - b*c)**(1/3) + b**(1/3)*(c + d*x)**(1/
3))*a**3*d**3 - 14*log( - b**(1/6)*(c + d*x)**(1/6)*(a*d - b*c)**(1/6)*sqr
t(3) + (a*d - b*c)**(1/3) + b**(1/3)*(c + d*x)**(1/3))*a**2*b*c*d**2 + 14*
log(b**(1/6)*(c + d*x)**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + (a*d - b*c)**(1
/3) + b**(1/3)*(c + d*x)**(1/3))*a**3*d**3 - 14*log(b**(1/6)*(c + d*x)*...
```

3.165 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{2/3}}\,dx$

Optimal result 1589
Mathematica [A] (verified) 1590
Rubi [A] (verified) 1591
Maple [A] (verified) 1593
Fricas [B] (verification not implemented) 1595
Sympy [F(-1)] 1596
Maxima [F(-2)] 1597
Giac [A] (verification not implemented) 1597
Mupad [F(-1)] 1598
Reduce [B] (verification not implemented) 1598

Optimal result

Integrand size = 32, antiderivative size = 395

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{2/3}}\,dx = \frac{3(bCd-bcD-2adD)\sqrt[3]{c+dx}}{b^3d^2} - \frac{\left(Ab^3-a(b^2B-abC+a^2D)\right)\sqrt[3]{c+dx}}{b^3(bc-ad)(a+bx)} + \frac{3D(c+dx)^{4/3}}{4b^2d^2} - \frac{\left(b^3(3Bc-2Ad)-ab^2(6cC+Bd)-7a^3dD+a^2b(4Cd+9cD)\right)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}b^{10/3}(bc-ad)^{5/3}} - \frac{\left(b^3(3Bc-2Ad)-ab^2(6cC+Bd)-7a^3dD+a^2b(4Cd+9cD)\right)\log(a+bx)}{6b^{10/3}(bc-ad)^{5/3}} + \frac{\left(b^3(3Bc-2Ad)-ab^2(6cC+Bd)-7a^3dD+a^2b(4Cd+9cD)\right)\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{10/3}(bc-ad)^{5/3}}$$

output
```
3*(C*b*d-2*D*a*d-D*b*c)*(d*x+c)^(1/3)/b^3/d^2-(A*b^3-a*(B*b^2-C*a*b+D*a^2)
)*(d*x+c)^(1/3)/b^3/(-a*d+b*c)/(b*x+a)+3/4*D*(d*x+c)^(4/3)/b^2/d^2-1/3*(b^
3*(-2*A*d+3*B*c)-a*b^2*(B*d+6*C*c)-7*a^3*d*D+a^2*b*(4*C*d+9*D*c))*arctan(1
/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/b^(10/3)/
(-a*d+b*c)^(5/3)-1/6*(b^3*(-2*A*d+3*B*c)-a*b^2*(B*d+6*C*c)-7*a^3*d*D+a^2*b
*(4*C*d+9*D*c))*ln(b*x+a)/b^(10/3)/(-a*d+b*c)^(5/3)+1/2*(b^3*(-2*A*d+3*B*c
)-a*b^2*(B*d+6*C*c)-7*a^3*d*D+a^2*b*(4*C*d+9*D*c))*ln((-a*d+b*c)^(1/3)-b^(
1/3)*(d*x+c)^(1/3))/b^(10/3)/(-a*d+b*c)^(5/3)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{2/3}}\,dx = \frac{\frac{3\sqrt[3]{b}\sqrt[3]{c+dx}(-28a^3d^2D+a^2bd(16Cd+3D(5c-7dx))+b^3(4Ad^2-3cx(4Cd-3cD+dDx))+ab^2(9c^2}{d^2(-bc+ad)(a+bx)}}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^(2/3)),x]
```

output
```
((3*b^(1/3)*(c + d*x)^(1/3)*(-28*a^3*d^2*D + a^2*b*d*(16*C*d + 3*D*(5*c -
7*d*x)) + b^3*(4*A*d^2 - 3*c*x*(4*C*d - 3*c*D + d*D*x)) + a*b^2*(9*c^2*D -
 12*c*d*(C - D*x) + d^2*(-4*B + 3*x*(4*C + D*x)))))/(d^2*(-(b*c) + a*d)*(a
 + b*x)) + (4*Sqrt[3]*(b^3*(3*B*c - 2*A*d) - a*b^2*(6*c*C + B*d) - 7*a^3*d
*D + a^2*b*(4*C*d + 9*c*D))*ArcTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c
) + a*d)^(1/3))/Sqrt[3]])/(-(b*c) + a*d)^(5/3) - (4*(b^3*(3*B*c - 2*A*d) -
 a*b^2*(6*c*C + B*d) - 7*a^3*d*D + a^2*b*(4*C*d + 9*c*D))*Log[(-(b*c) + a*
d)^(1/3) + b^(1/3)*(c + d*x)^(1/3)])/(-(b*c) + a*d)^(5/3) + (2*(b^3*(3*B*c
 - 2*A*d) - a*b^2*(6*c*C + B*d) - 7*a^3*d*D + a^2*b*(4*C*d + 9*c*D))*Log[(
-(b*c) + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/
3)*(c + d*x)^(2/3)])/(-(b*c) + a*d)^(5/3))/(12*b^(10/3))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2124, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{2/3}}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{-dDa^3+b(Cd+3cD)a^2-b^2(3cC+Bd)a+b^3(3Bc-2Ad)}{b^3}}{3(a+bx)(c+dx)^{2/3}}dx}{bc-ad}-\frac{\sqrt[3]{c+dx}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-\frac{dDa^3}{b^3}+\frac{(Cd+3cD)a^2}{b^2}-\frac{(3cC+Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-2Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}}{(a+bx)(c+dx)^{2/3}}dx}{3(bc-ad)}-\frac{\sqrt[3]{c+dx}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 1195$$

$$\frac{\int \left(\frac{3(bc-ad)\sqrt[3]{c+dx}D}{b^2d}+\frac{3(bc-ad)(bCd-2aDd-bcD)}{b^3d(c+dx)^{2/3}}+\frac{-7dDa^3+b(4Cd+9cD)a^2-b^2(6cC+Bd)a+b^3(3Bc-2Ad)}{b^3(a+bx)(c+dx)^{2/3}}\right)dx}{3(bc-ad)}-\frac{\sqrt[3]{c+dx}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)\left(-7a^3dD+a^2b(9cD+4Cd)-ab^2(Bd+6cC)+b^3(3Bc-2Ad)\right)}{b^{10/3}(bc-ad)^{2/3}}-\frac{\log(a+bx)\left(-7a^3dD+a^2b(9cD+4Cd)-ab^2\right.}{2b^{10/3}(bc-ad)^{2/}}$$

$$\frac{\sqrt[3]{c+dx}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{(a+bx)(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^(2/3)),x]
```

output
```
-(((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(1/3))/((b*c - a*d)*(a
+ b*x))) + ((9*(b*c - a*d)*(b*C*d - b*c*D - 2*a*d*D)*(c + d*x)^(1/3))/(b^3
*d^2) + (9*(b*c - a*d)*D*(c + d*x)^(4/3))/(4*b^2*d^2) - (Sqrt[3]*(b^3*(3*B
*c - 2*A*d) - a*b^2*(6*c*C + B*d) - 7*a^3*d*D + a^2*b*(4*C*d + 9*c*D))*Arc
Tan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b^(10/3
)*(b*c - a*d)^(2/3)) - ((b^3*(3*B*c - 2*A*d) - a*b^2*(6*c*C + B*d) - 7*a^3
*d*D + a^2*b*(4*C*d + 9*c*D))*Log[a + b*x])/(2*b^(10/3)*(b*c - a*d)^(2/3))
 + (3*(b^3*(3*B*c - 2*A*d) - a*b^2*(6*c*C + B*d) - 7*a^3*d*D + a^2*b*(4*C*
d + 9*c*D))*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(10/3)*
(b*c - a*d)^(2/3)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\left(\left(\left(b^3A-(-\frac{3}{4}Dx^2-3Cx+B)a\,b^2+4\left(-\frac{21Dx}{16}+C\right)a^2b-7a^3D\right)d^2-3\left(\left(\frac{Dx}{4}+C\right)b-\frac{5Da}{4}\right)(bx+a)cbd+\frac{9Db^2c^2\cdots}{\cdots}\cdots\right)\cdots}{\cdots}$
derivativedivides	$\frac{\frac{3\left(\frac{D(xd+c)^{\frac{4}{3}}b}{4}+Cdb(xd+c)^{\frac{1}{3}}-2Dad(xd+c)^{\frac{1}{3}}-Dbc(xd+c)^{\frac{1}{3}}\right)}{b^3}+\frac{3d^2\left(\frac{d\left(b^3A-a\,b^2B+a^2bC-a^3D\right)(xd+c)^{\frac{1}{3}}}{3(ad-bc)((xd+c)b+ad-bc)}+\frac{\left(2b^3dA+Ba\,b^2d-\cdots\right)\cdots}{\cdots}\right)}{\cdots}}{\cdots}$
default	$\frac{\frac{3\left(\frac{D(xd+c)^{\frac{4}{3}}b}{4}+Cdb(xd+c)^{\frac{1}{3}}-2Dad(xd+c)^{\frac{1}{3}}-Dbc(xd+c)^{\frac{1}{3}}\right)}{b^3}+\frac{3d^2\left(\frac{d\left(b^3A-a\,b^2B+a^2bC-a^3D\right)(xd+c)^{\frac{1}{3}}}{3(ad-bc)((xd+c)b+ad-bc)}+\frac{\left(2b^3dA+Ba\,b^2d-\cdots\right)\cdots}{\cdots}\right)}{\cdots}}{\cdots}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
(((a*d-b*c)/b)^(2/3)*((b^3*A-(-3/4*D*x^2-3*C*x+B)*a*b^2+4*(-21/16*D*x+C)*a
^2*b-7*a^3*D)*d^2-3*((1/4*D*x+C)*b-5/4*D*a)*(b*x+a)*c*b*d+9/4*D*b^2*c^2*(b
*x+a))*b*(d*x+c)^(1/3)-1/3*((b^3*A+1/2*a*b^2*B-2*a^2*b*C+7/2*a^3*D)*d-3/2*
b*c*(B*b^2-2*C*a*b+3*D*a^2))*(-2*arctan(2/3*3^(1/2)/((a*d-b*c)/b)^(1/3)*(d
*x+c)^(1/3)-1/3*3^(1/2))*3^(1/2)+ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x
+c)^(1/3)+((a*d-b*c)/b)^(2/3))-2*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3)))*d^
2*(b*x+a))/((a*d-b*c)/b)^(2/3)/d^2/b^4/(a*d-b*c)/(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1264 vs. $2(359) = 718$.

Time = 1.11 (sec) , antiderivative size = 2621, normalized size of antiderivative = 6.64

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(2/3),x, algorithm="fricas
")
```

output
```
[-1/12*(6*sqrt(1/3)*(3*(3*D*a^3*b^3 - 2*C*a^2*b^4 + B*a*b^5)*c^2*d^2 - 2*(
8*D*a^4*b^2 - 5*C*a^3*b^3 + 2*B*a^2*b^4 + A*a*b^5)*c*d^3 + (7*D*a^5*b - 4*
C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*d^4 + (3*(3*D*a^2*b^4 - 2*C*a*b^5 + B
*b^6)*c^2*d^2 - 2*(8*D*a^3*b^3 - 5*C*a^2*b^4 + 2*B*a*b^5 + A*b^6)*c*d^3 +
(7*D*a^4*b^2 - 4*C*a^3*b^3 + B*a^2*b^4 + 2*A*a*b^5)*d^4)*x)*sqrt(-(b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b)*log(-(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2
 + 2*(b^2*c*d - a*b*d^2)*x + 3*sqrt(1/3)*(2*(b^2*c - a*b*d)*(d*x + c)^(2/3
) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a
*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3))*sqrt(-(b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2)^(1/3)/b) - 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c -
 a*d)*(d*x + c)^(1/3))/(b*x + a)) + 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^
(2/3)*(3*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c*d^2 - (7*D*a^4 - 4*C*a^3*b
+ B*a^2*b^2 + 2*A*a*b^3)*d^3 + (3*(3*D*a^2*b^2 - 2*C*a*b^3 + B*b^4)*c*d^2
- (7*D*a^3*b - 4*C*a^2*b^2 + B*a*b^3 + 2*A*b^4)*d^3)*x)*log(-(b^2*c - a*b*
d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d)
 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 4*(b^3*c^2
 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(3*(3*D*a^3*b - 2*C*a^2*b^2 + B*a*b^3)*c
*d^2 - (7*D*a^4 - 4*C*a^3*b + B*a^2*b^2 + 2*A*a*b^3)*d^3 + (3*(3*D*a^2*b^2
 - 2*C*a*b^3 + B*b^4)*c*d^2 - (7*D*a^3*b - 4*C*a^2*b^2 + B*a*b^3 + 2*A*b^4
)*d^3)*x)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^{2/3}} \, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(2/3),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^{2/3}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(2/3),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.40

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^{2/3}}\, dx = \frac{(9\,Da^2bc - 6\,Cab^2c + 3\,Bb^3c - 7\,Da^3d + 4\,Ca^2bd - Bab^2d - 2\,Ab^3d)\left(\frac{bc-ad}{b}\right.}{3\,(b^5c^2 - 2\,ab^4cd + a^2b^3d^2)}$$
$$+ \frac{(2\,Ab^3d - (3\,b^3c - ab^2d)B + 2\,(3\,ab^2c - 2\,a^2bd)C - (9\,a^2bc - 7\,a^3d)D)\arctan\left(\frac{\sqrt{3}\left(2\,(dx+c)^{\frac{1}{3}} + \left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\left(\sqrt{3}b^3c - \sqrt{3}ab^2d\right)(b^3c - ab^2d)^{\frac{2}{3}}}$$
$$+ \frac{(2\,Ab^3d - (3\,b^3c - ab^2d)B + 2\,(3\,ab^2c - 2\,a^2bd)C - (9\,a^2bc - 7\,a^3d)D)\log\left((dx + c)^{\frac{2}{3}} + (dx + c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right.\right.}{6\,(b^3c - ab^2d)^{\frac{5}{3}}}$$
$$+ \frac{(dx + c)^{\frac{1}{3}}Da^3d - (dx + c)^{\frac{1}{3}}Ca^2bd + (dx + c)^{\frac{1}{3}}Bab^2d - (dx + c)^{\frac{1}{3}}Ab^3d}{(b^4c - ab^3d)((dx + c)b - bc + ad)}$$
$$+ \frac{3\left((dx + c)^{\frac{4}{3}}Db^6d^6 - 4\,(dx + c)^{\frac{1}{3}}Db^6cd^6 - 8\,(dx + c)^{\frac{1}{3}}Dab^5d^7 + 4\,(dx + c)^{\frac{1}{3}}Cb^6d^7\right)}{4\,b^8d^8}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
1/3*(9*D*a^2*b*c - 6*C*a*b^2*c + 3*B*b^3*c - 7*D*a^3*d + 4*C*a^2*b*d - B*a
*b^2*d - 2*A*b^3*d)*((b*c - a*d)/b)^(1/3)*log(abs((d*x + c)^(1/3) - ((b*c
- a*d)/b)^(1/3)))/(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2) + (2*A*b^3*d - (3*
b^3*c - a*b^2*d)*B + 2*(3*a*b^2*c - 2*a^2*b*d)*C - (9*a^2*b*c - 7*a^3*d)*D
)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a*d)/b)^(1/3))/((b*c - a
*d)/b)^(1/3))/((sqrt(3)*b^3*c - sqrt(3)*a*b^2*d)*(b^3*c - a*b^2*d)^(2/3))
+ 1/6*(2*A*b^3*d - (3*b^3*c - a*b^2*d)*B + 2*(3*a*b^2*c - 2*a^2*b*d)*C - (
9*a^2*b*c - 7*a^3*d)*D)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)
/b)^(1/3) + ((b*c - a*d)/b)^(2/3))/(b^3*c - a*b^2*d)^(5/3) + ((d*x + c)^(1
/3)*D*a^3*d - (d*x + c)^(1/3)*C*a^2*b*d + (d*x + c)^(1/3)*B*a*b^2*d - (d*x
 + c)^(1/3)*A*b^3*d)/((b^4*c - a*b^3*d)*((d*x + c)*b - b*c + a*d)) + 3/4*(
(d*x + c)^(4/3)*D*b^6*d^6 - 4*(d*x + c)^(1/3)*D*b^6*c*d^6 - 8*(d*x + c)^(1
/3)*D*a*b^5*d^7 + 4*(d*x + c)^(1/3)*C*b^6*d^7)/(b^8*d^8)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{2/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^{2/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(2/3)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(2/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1644, normalized size of antiderivative = 4.16

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{2/3}}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(2/3),x)
```

output

```
( - 28*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c +
 d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**3*d**2 + 24*sqrt(3)*atan((
b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/
6)*(a*d - b*c)**(1/6)))*a**2*b*c*d - 28*sqrt(3)*atan((b**(1/6)*(a*d - b*c)
**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6
)))*a**2*b*d**2*x - 12*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) -
 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*d + 24
*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)*
*(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**2*c*d*x - 12*sqrt(3)*atan((b**
(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*
(a*d - b*c)**(1/6)))*b**4*d*x - 28*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/
6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a
**3*d**2 + 24*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/
3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b*c*d - 28*sqrt(3
)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/6))
/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b*d**2*x - 12*sqrt(3)*atan((b**(1/6)*
(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d -
 b*c)**(1/6)))*a*b**3*d + 24*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqr
t(3) + 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**2*
c*d*x - 12*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/...
```

3.166 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{2/3}}\,dx$

Optimal result 1600
Mathematica [A] (verified) 1601
Rubi [A] (verified) 1602
Maple [A] (verified) 1606
Fricas [B] (verification not implemented) 1608
Sympy [F(-1)] 1608
Maxima [F(-2)] 1609
Giac [A] (verification not implemented) 1609
Mupad [F(-1)] 1610
Reduce [B] (verification not implemented) 1611

Optimal result

Integrand size = 32, antiderivative size = 513

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{2/3}}\,dx = \frac{3D\sqrt[3]{c+dx}}{b^3d} - \frac{(Ab^3-a(b^2B-abC+a^2D))\sqrt[3]{c+dx}}{2b^3(bc-ad)(a+bx)^2}$$

$$-\frac{(b^3(6Bc-5Ad)-ab^2(12cC+Bd)-13a^3dD+a^2b(7Cd+18cD))\sqrt[3]{c+dx}}{6b^3(bc-ad)^2(a+bx)}$$

$$-\frac{(b^3(9c^2C-6Bcd+5Ad^2)-14a^3d^2D+2a^2bd(Cd+18cD)-ab^2(6cCd-Bd^2+27c^2D))\arctan\left(\frac{1+2\sqrt[3]{\cdots}}{\cdots}\right)}{3\sqrt{3}b^{10/3}(bc-ad)^{8/3}}$$

$$-\frac{(b^3(9c^2C-6Bcd+5Ad^2)-14a^3d^2D+2a^2bd(Cd+18cD)-ab^2(6cCd-Bd^2+27c^2D))\log(a+bx)}{18b^{10/3}(bc-ad)^{8/3}}$$

$$+\frac{(b^3(9c^2C-6Bcd+5Ad^2)-14a^3d^2D+2a^2bd(Cd+18cD)-ab^2(6cCd-Bd^2+27c^2D))\log\left(\sqrt[3]{bc-ad}\cdots\right)}{6b^{10/3}(bc-ad)^{8/3}}$$

output
```
3*D*(d*x+c)^(1/3)/b^3/d-1/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1/3)/b^
3/(-a*d+b*c)/(b*x+a)^2-1/6*(b^3*(-5*A*d+6*B*c)-a*b^2*(B*d+12*C*c)-13*a^3*d
*D+a^2*b*(7*C*d+18*D*c))*(d*x+c)^(1/3)/b^3/(-a*d+b*c)^2/(b*x+a)-1/9*(b^3*(
5*A*d^2-6*B*c*d+9*C*c^2)-14*a^3*d^2*D+2*a^2*b*d*(C*d+18*D*c)-a*b^2*(-B*d^2
+6*C*c*d+27*D*c^2))*arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/(-a*d+b*c)^(1/3)
)*3^(1/2))*3^(1/2)/b^(10/3)/(-a*d+b*c)^(8/3)-1/18*(b^3*(5*A*d^2-6*B*c*d+9*
C*c^2)-14*a^3*d^2*D+2*a^2*b*d*(C*d+18*D*c)-a*b^2*(-B*d^2+6*C*c*d+27*D*c^2)
)*ln(b*x+a)/b^(10/3)/(-a*d+b*c)^(8/3)+1/6*(b^3*(5*A*d^2-6*B*c*d+9*C*c^2)-1
4*a^3*d^2*D+2*a^2*b*d*(C*d+18*D*c)-a*b^2*(-B*d^2+6*C*c*d+27*D*c^2))*ln((-a
*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/b^(10/3)/(-a*d+b*c)^(8/3)
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{2/3}}\,dx = \frac{3\sqrt[3]{b}\sqrt[3]{c+dx}(28a^4d^2D+Ab^3d(-3bc+8ad+5bdx)+6b^4cx(-Bd+3cDx)+a^3bd(-4Cd-51cD+49dD\cdots}{d(bc-a\cdots}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^(2/3)),x]
```

output
```
((3*b^(1/3)*(c + d*x)^(1/3)*(28*a^4*d^2*D + A*b^3*d*(-3*b*c + 8*a*d + 5*b*
d*x) + 6*b^4*c*x*(-(B*d) + 3*c*D*x) + a^3*b*d*(-4*C*d - 51*c*D + 49*d*D*x)
 + a*b^3*(B*d*(-3*c + d*x) + 12*c*x*(C*d + 3*c*D - 3*d*D*x)) + a^2*b^2*(18
*c^2*D + 9*c*d*(C - 10*D*x) + d^2*(-2*B - 7*C*x + 18*D*x^2))))/(d*(b*c - a
*d)^2*(a + b*x)^2) - (2*Sqrt[3]*(b^3*(9*c^2*C - 6*B*c*d + 5*A*d^2) - 14*a^
3*d^2*D + 2*a^2*b*d*(C*d + 18*c*D) + a*b^2*(-6*c*C*d + B*d^2 - 27*c^2*D))*
ArcTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3))/Sqrt[3]])/(-
(b*c) + a*d)^(8/3) + (2*(b^3*(9*c^2*C - 6*B*c*d + 5*A*d^2) - 14*a^3*d^2*D
+ 2*a^2*b*d*(C*d + 18*c*D) + a*b^2*(-6*c*C*d + B*d^2 - 27*c^2*D))*Log[(-(b
*c) + a*d)^(1/3) + b^(1/3)*(c + d*x)^(1/3)])/(-(b*c) + a*d)^(8/3) - ((b^3*
(9*c^2*C - 6*B*c*d + 5*A*d^2) - 14*a^3*d^2*D + 2*a^2*b*d*(C*d + 18*c*D) +
a*b^2*(-6*c*C*d + B*d^2 - 27*c^2*D))*Log[(-(b*c) + a*d)^(2/3) - b^(1/3)*(-
(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)])/(-(b*c) + a
*d)^(8/3))/(18*b^(10/3))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2124, 27, 1193, 27, 90, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{2/3}}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{6\left(c-\frac{ad}{b}\right)Dx^2+\frac{6(bc-ad)(bC-aD)x}{b^2}+\frac{-dDa^3+b(Cd+6cD)a^2-b^2(6cC+Bd)a+b^3(6Bc-5Ad)}{b^3}}{3(a+bx)^2(c+dx)^{2/3}}dx}{2(bc-ad)}-\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-\frac{dDa^3}{b^3}+\frac{(Cd+6cD)a^2}{b^2}-\frac{(6cC+Bd)a}{b}+6\left(c-\frac{ad}{b}\right)Dx^2+6Bc-5Ad+\frac{6(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^2(c+dx)^{2/3}}dx}{6(bc-ad)}-\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{-\frac{\int -\frac{2\left(-5d^2Da^3+2bd(Cd+9cD)a^2-b^2\left(18Dc^2+6Cdc-Bd^2\right)a+b^3\left(9Cc^2-6Bdc+5Ad^2\right)+9b(bc-ad)^2Dx\right)}{3b^3(a+bx)(c+dx)^{2/3}}dx}{bc-ad}-\frac{\sqrt[3]{c+dx}(-13a^3dD+a^2b(18cD+7Cd \ldots}{b^3(a+bx)(\ldots}}{6(bc-ad)}-\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\int \frac{-5d^2Da^3+2bd(Cd+9cD)a^2-b^2\left(18Dc^2+6Cdc-Bd^2\right)a+b^3\left(9Cc^2-6Bdc+5Ad^2\right)+9b(bc-ad)^2Dx}{(a+bx)(c+dx)^{2/3}}dx}{3b^3(bc-ad)}-\frac{\sqrt[3]{c+dx}(-13a^3dD+a^2b(18cD+7Cd)-ab^2 \ldots}{b^3(a+bx)(bc-ad \ldots}}{6(bc-ad)}-\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 90

$$\frac{\frac{2\left(\left(-14a^3d^2D+2a^2bd(18cD+Cd)-ab^2\left(-Bd^2+27c^2D+6cCd\right)+b^3\left(5Ad^2-6Bcd+9c^2C\right)\right)\int \frac{1}{(a+bx)(c+dx)^{2/3}}dx+\frac{27D\sqrt[3]{c+dx}(bc-ad)^2}{d}\right)}{3b^3(bc-ad)}-\cdots}{6(bc-ad)}-$$

$$\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 69

$$\frac{2\left(\left(-14a^3d^2D+2a^2bd(18cD+Cd)-ab^2\left(-Bd^2+27c^2D+6cCd\right)+b^3\left(5Ad^2-6Bcd+9c^2C\right)\right)\left(-\frac{3\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{c+dx}\sqrt[3]{bc-ad}}{\sqrt[3]{b}}+(c+dx)^{2\cdots}}\cdots}{2b^{2/3}\sqrt[3]{bc-ad}}\cdots\right.\cdots}{3b^3(bc-ad)}\cdots$$

$$\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 16

$$\frac{2\left(\left(-14a^3d^2D+2a^2bd(18cD+Cd)-ab^2\left(-Bd^2+27c^2D+6cCd\right)+b^3\left(5Ad^2-6Bcd+9c^2C\right)\right)\left(-\frac{3\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{c+dx}\sqrt[3]{bc-ad}}{\sqrt[3]{b}}+(c+dx)^{2\cdots}}\cdots}{2b^{2/3}\sqrt[3]{bc-ad}}\cdots\right.\cdots}{3b^3(bc-ad)}\cdots$$

$$\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 1082

$$\frac{2\left(\left(-14a^3d^2D+2a^2bd(18cD+Cd)-ab^2\left(-Bd^2+27c^2D+6cCd\right)+b^3\left(5Ad^2-6Bcd+9c^2C\right)\right)\left(\frac{3\int \frac{1}{-(c+dx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\log(\cdots}{2\sqrt[3]{b}(\cdots}\cdots\right.\cdots}{3b^3(bc-ad)}\cdots$$

$$\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$\downarrow$ 217

$$\frac{\frac{2\left(\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}\right)(-14a^3d^2D+2a^2bd(18cD+Cd)-ab^2(-Bd^2}{3b^3(bc-ad)}}{6(b}$$

$$\frac{\sqrt[3]{c+dx}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^(2/3)),x]
```

output
```
-1/2*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(1/3))/(b^3*(b*c - a*d
)*(a + b*x)^2) + (-(((b^3*(6*B*c - 5*A*d) - a*b^2*(12*c*C + B*d) - 13*a^3*
d*D + a^2*b*(7*C*d + 18*c*D))*(c + d*x)^(1/3))/(b^3*(b*c - a*d)*(a + b*x))
) + (2*((27*(b*c - a*d)^2*D*(c + d*x)^(1/3))/d + (b^3*(9*c^2*C - 6*B*c*d +
 5*A*d^2) - 14*a^3*d^2*D + 2*a^2*b*d*(C*d + 18*c*D) - a*b^2*(6*c*C*d - B*d
^2 + 27*c^2*D))*(-((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c -
 a*d)^(1/3))/Sqrt[3]])/(b^(1/3)*(b*c - a*d)^(2/3))) - Log[a + b*x]/(2*b^(1
/3)*(b*c - a*d)^(2/3)) + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3
)])/(2*b^(1/3)*(b*c - a*d)^(2/3)))))/(3*b^3*(b*c - a*d)))/(6*(b*c - a*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{5\left(-\frac{24\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\left(\left(\left(\frac{5A\,b^{4}x}{8}+a\left(\frac{Bx}{8}+A\right)b^{3}-\frac{\left(-9Dx^{2}+\frac{7}{2}Cx+B\right)a^{2}b^{2}}{4}-\frac{\left(-\frac{49Dx}{4}+C\right)a^{3}b}{2}+\frac{7Da^{4}}{2}\right)d^{2}-\frac{3cb\left(\left(2Bx+A\right)b^{3}+a\right.}{}\right.\right.}{5}\right.}{}$
derivativedivides	$\frac{\frac{3D(xd+c)^{\frac{1}{3}}}{b^{3}}+\frac{3d\left(\frac{\frac{bd\left(5b^{3}dA+Ba\,b^{2}d-6B\,b^{3}c-7C\,a^{2}bd+12a\,b^{2}cC+13a^{3}dD-18a^{2}bcD\right)(xd+c)^{\frac{4}{3}}}{18a^{2}d^{2}-36abcd+18b^{2}c^{2}}+\frac{\left(4b^{3}dA-Ba\,b^{2}d-3B\,b^{3}c-2C\right.}{}}{((xd+c)b+ad-bc)^{2}}\right.}{}}{}$
default	$\frac{\frac{3D(xd+c)^{\frac{1}{3}}}{b^{3}}+\frac{3d\left(\frac{\frac{bd\left(5b^{3}dA+Ba\,b^{2}d-6B\,b^{3}c-7C\,a^{2}bd+12a\,b^{2}cC+13a^{3}dD-18a^{2}bcD\right)(xd+c)^{\frac{4}{3}}}{18a^{2}d^{2}-36abcd+18b^{2}c^{2}}+\frac{\left(4b^{3}dA-Ba\,b^{2}d-3B\,b^{3}c-2C\right.}{}}{((xd+c)b+ad-bc)^{2}}\right.}{}}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-5/18*(-24/5*((a*d-b*c)/b)^(2/3)*((5/8*A*b^4*x+a*(1/8*B*x+A)*b^3-1/4*(-9*D
*x^2+7/2*C*x+B)*a^2*b^2-1/2*(-49/4*D*x+C)*a^3*b+7/2*D*a^4)*d^2-3/8*c*b*((2
*B*x+A)*b^3+a*(12*D*x^2-4*C*x+B)*b^2-3*a^2*(-10*D*x+C)*b+17*a^3*D)*d+9/4*D
*b^2*c^2*(b*x+a)^2)*b*(d*x+c)^(1/3)+((b^3*A+1/5*a*b^2*B+2/5*a^2*b*C-14/5*a
^3*D)*d^2-6/5*b*c*(B*b^2+C*a*b-6*D*a^2)*d+9/5*b^2*c^2*(C*b-3*D*a))*(-2*arc
tan(2/3*3^(1/2)/((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)-1/3*3^(1/2))*3^(1/2)+ln(
(d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/b)^(2/3))-2*ln(
(d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3)))*d*(b*x+a)^2)/((a*d-b*c)/b)^(2/3)/(a*d-
b*c)^2/b^4/(b*x+a)^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1944 vs. $2(471) = 942$.

Time = 2.21 (sec) , antiderivative size = 3981, normalized size of antiderivative = 7.76

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(2/3),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(2/3),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)^{2/3}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(2/3),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.78

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3(c + dx)^{2/3}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
-1/9*(27*D*a*b^2*c^2 - 9*C*b^3*c^2 - 36*D*a^2*b*c*d + 6*C*a*b^2*c*d + 6*B*
b^3*c*d + 14*D*a^3*d^2 - 2*C*a^2*b*d^2 - B*a*b^2*d^2 - 5*A*b^3*d^2)*((b*c
- a*d)/b)^(1/3)*log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b^6*c^3
 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3) - 1/3*(5*A*b^3*d^2 - (6*
b^3*c*d - a*b^2*d^2)*B + (9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*C - (27*a
*b^2*c^2 - 36*a^2*b*c*d + 14*a^3*d^2)*D)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(
1/3) + ((b*c - a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/((sqrt(3)*b^4*c^2 - 2
*sqrt(3)*a*b^3*c*d + sqrt(3)*a^2*b^2*d^2)*(b^3*c - a*b^2*d)^(2/3)) - 1/18*
(5*A*b^3*d^2 - (6*b^3*c*d - a*b^2*d^2)*B + (9*b^3*c^2 - 6*a*b^2*c*d + 2*a^
2*b*d^2)*C - (27*a*b^2*c^2 - 36*a^2*b*c*d + 14*a^3*d^2)*D)*log((d*x + c)^(
2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d)/b)^(2/3))/((b^
4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(b^3*c - a*b^2*d)^(2/3)) - 1/6*(18*(d*x
 + c)^(4/3)*D*a^2*b^2*c*d - 12*(d*x + c)^(4/3)*C*a*b^3*c*d + 6*(d*x + c)^(
4/3)*B*b^4*c*d - 18*(d*x + c)^(1/3)*D*a^2*b^2*c^2*d + 12*(d*x + c)^(1/3)*C
*a*b^3*c^2*d - 6*(d*x + c)^(1/3)*B*b^4*c^2*d - 13*(d*x + c)^(4/3)*D*a^3*b*
d^2 + 7*(d*x + c)^(4/3)*C*a^2*b^2*d^2 - (d*x + c)^(4/3)*B*a*b^3*d^2 - 5*(d
*x + c)^(4/3)*A*b^4*d^2 + 28*(d*x + c)^(1/3)*D*a^3*b*c*d^2 - 16*(d*x + c)^
(1/3)*C*a^2*b^2*c*d^2 + 4*(d*x + c)^(1/3)*B*a*b^3*c*d^2 + 8*(d*x + c)^(1/3
)*A*b^4*c*d^2 - 10*(d*x + c)^(1/3)*D*a^4*d^3 + 4*(d*x + c)^(1/3)*C*a^3*b*d
^3 + 2*(d*x + c)^(1/3)*B*a^2*b^2*d^3 - 8*(d*x + c)^(1/3)*A*a*b^3*d^3)/(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{2/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^{2/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(2/3)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(2/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3364, normalized size of antiderivative = 6.56

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{2/3}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(2/3),x)
```

output
```
(28*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*
x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**4*d**2 - 48*sqrt(3)*atan((b**
(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*
(a*d - b*c)**(1/6)))*a**3*b*c*d + 56*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(
1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))
*a**3*b*d**2*x - 12*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*
b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**3*d + 18
*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)*
*(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**2*c**2 - 96*sqrt(3)*atan((b
**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6
)*(a*d - b*c)**(1/6)))*a**2*b**2*c*d*x + 28*sqrt(3)*atan((b**(1/6)*(a*d -
b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**
(1/6)))*a**2*b**2*d**2*x**2 - 24*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)
*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b
**4*d*x + 36*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3
)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*c**2*x - 48*sqrt
(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6
))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*c*d*x**2 - 12*sqrt(3)*atan((b**(1
/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a
*d - b*c)**(1/6)))*b**5*d*x**2 + 18*sqrt(3)*atan((b**(1/6)*(a*d - b*c)*...
```

3.167 $\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}} \, dx$

Optimal result 1612
Mathematica [A] (verified) 1613
Rubi [A] (verified) 1614
Maple [A] (verified) 1615
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Giac [B] (verification not implemented) 1619
Mupad [F(-1)] 1620
Reduce [B] (verification not implemented) 1621

Optimal result

Integrand size = 32, antiderivative size = 436

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}} \, dx = \frac{3(bc-ad)^3 (c^2Cd - Bcd^2 + Ad^3 - c^3D)}{d^7 \sqrt[3]{c+dx}}$$
$$- \frac{3(bc-ad)^2 (ad(2cCd - Bd^2 - 3c^2D) - b(5c^2Cd - 4Bcd^2 + 3Ad^3 - 6c^3D)) (c+dx)^{2/3}}{2d^7}$$
$$- \frac{3(bc-ad) (a^2d^2(Cd - 3cD) - abd(8cCd - 3Bd^2 - 15c^2D) + b^2(10c^2Cd - 6Bcd^2 + 3Ad^3 - 15c^3D)) (c}{5d^7}$$
$$+ \frac{3(a^3d^3D + 3a^2bd^2(Cd - 4cD) - 3ab^2d(4cCd - Bd^2 - 10c^2D) + b^3(10c^2Cd - 4Bcd^2 + Ad^3 - 20c^3D)) (c}{8d^7}$$
$$+ \frac{3b(3a^2d^2D + 3abd(Cd - 5cD) - b^2(5cCd - Bd^2 - 15c^2D)) (c+dx)^{11/3}}{11d^7}$$
$$+ \frac{3b^2(bCd - 6bcD + 3adD)(c+dx)^{14/3}}{14d^7} + \frac{3b^3D(c+dx)^{17/3}}{17d^7}$$

output
```
3*(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^7/(d*x+c)^(1/3)-3/2*(-a*d+b
*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^3
))*(d*x+c)^(2/3)/d^7-3/5*(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^2+8
*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^(5/3
)/d^7+3/8*(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*d-10*
D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(8/3)/d^7+3/11*b
*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*x+c)^(
11/3)/d^7+3/14*b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(14/3)/d^7+3/17*b^3*D*(
d*x+c)^(17/3)/d^7
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx)^3\,(A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx = \frac{3(1309a^3d^3(81c^3D-9c^2d(8C-3Dx)+3cd^2(20B-x(8C+3$$

input
```
Integrate[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(4/3),x]
```

output
```
(3*(1309*a^3*d^3*(81*c^3*D - 9*c^2*d*(8*C - 3*D*x) + 3*c*d^2*(20*B - x*(8*
C + 3*D*x)) + d^3*(-40*A + x*(20*B + 8*C*x + 5*D*x^2))) + 357*a^2*b*d^2*(-
972*c^4*D + 81*c^3*d*(11*C - 4*D*x) - 9*c^2*d^2*(88*B - 3*x*(11*C + 4*D*x)
) + 3*c*d^3*(220*A - x*(88*B + 33*C*x + 20*D*x^2)) + d^4*x*(220*A + x*(88*
B + 55*C*x + 40*D*x^2))) + b^3*(-131220*c^6*D + 7290*c^5*d*(17*C - 6*D*x)
+ 5*d^6*x^3*(1309*A + 952*B*x + 748*C*x^2 + 616*D*x^3) - 486*c^4*d^2*(238*
B - 5*x*(17*C + 6*D*x)) + 81*c^3*d^3*(1309*A - 2*x*(238*B + 85*C*x + 50*D*
x^2)) - 3*c*d^5*x^2*(3927*A + 20*x*(119*B + 85*C*x + 66*D*x^2)) + 9*c^2*d^
4*x*(3927*A + 2*x*(714*B + 425*C*x + 300*D*x^2))) + 51*a*b^2*d*(7290*c^5*D
 - 486*c^4*d*(14*C - 5*D*x) + 81*c^3*d^2*(77*B - 2*x*(14*C + 5*D*x)) + d^5
*x^2*(616*A + 5*x*(77*B + 56*C*x + 44*D*x^2)) - 9*c^2*d^3*(616*A - x*(231*
B + 84*C*x + 50*D*x^2)) - 3*c*d^4*x*(616*A + x*(231*B + 20*x*(7*C + 5*D*x)
)))))/(52360*d^7*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{2/3}(bc-ad)\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3(c+dx)^{5/3}(bc-ad)\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2C\right.\right.}{5d^7}$$

$$\frac{3b(c+dx)^{11/3}\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{11d^7}+$$

$$\frac{3(c+dx)^{8/3}\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2\right.\right.}{8d^7}$$

$$\frac{3(c+dx)^{2/3}(bc-ad)^2\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{2d^7}+$$

$$\frac{3(bc-ad)^3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^7\sqrt[3]{c+dx}}+\frac{3b^2(c+dx)^{14/3}(3adD-6bcD+bCd)}{14d^7}+$$

$$\frac{3b^3D(c+dx)^{17/3}}{17d^7}$$

input
```
Int[((a + b*x)^3*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(4/3),x]
```

output
```
(3*(b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^7*(c + d*x)^(1/3)
) - (3*(b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*C*d - 4*B
*c*d^2 + 3*A*d^3 - 6*c^3*D))*(c + d*x)^(2/3))/(2*d^7) - (3*(b*c - a*d)*(a^
2*d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(10*c^2*C
*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(5/3))/(5*d^7) + (3*(a^3*d
^3*D + 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d - B*d^2 - 10*c^2*D)
+ b^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^(8/3))/(8*d^7
) + (3*b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) - b^2*(5*c*C*d - B*d^2 - 15*
c^2*D))*(c + d*x)^(11/3))/(11*d^7) + (3*b^2*(b*C*d - 6*b*c*D + 3*a*d*D)*(c
 + d*x)^(14/3))/(14*d^7) + (3*b^3*D*(c + d*x)^(17/3))/(17*d^7)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{3\left(\left(-\frac{\left(\frac{8}{17}Dx^3+\frac{4}{7}Cx^2+\frac{8}{11}Bx+A\right)x^3b^3}{8}-\frac{3x^2\left(\frac{5}{14}Dx^3+\frac{5}{11}Cx^2+\frac{5}{8}Bx+A\right)ab^2}{5}-\frac{3\left(\frac{2}{11}Dx^3+\frac{1}{4}Cx^2+\frac{2}{5}Bx+A\right)xa^2b}{2}+a^3(\cdots\right.\right.}{\cdots}$
gosper	$-\frac{3(-3080Dx^6b^3d^6-3740Cb^3d^6x^5-11220Dx^5ab^2d^6+3960Dx^5b^3cd^5-4760Bx^4b^3d^6-14280Cx^4ab^2d^6+5100Cx^4\cdots}{\cdots}$
trager	$-\frac{3(-3080Dx^6b^3d^6-3740Cb^3d^6x^5-11220Dx^5ab^2d^6+3960Dx^5b^3cd^5-4760Bx^4b^3d^6-14280Cx^4ab^2d^6+5100Cx^4\cdots}{\cdots}$
orering	$-\frac{3(-3080Dx^6b^3d^6-3740Cb^3d^6x^5-11220Dx^5ab^2d^6+3960Dx^5b^3cd^5-4760Bx^4b^3d^6-14280Cx^4ab^2d^6+5100Cx^4\cdots}{\cdots}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/(d*x+c)^(1/3)*((-1/8*(8/17*D*x^3+4/7*C*x^2+8/11*B*x+A)*x^3*b^3-3/5*x^2*
(5/14*D*x^3+5/11*C*x^2+5/8*B*x+A)*a*b^2-3/2*(2/11*D*x^3+1/4*C*x^2+2/5*B*x+
A)*x*a^2*b+a^3*(-1/8*D*x^3-1/5*C*x^2-1/2*B*x+A))*d^6-9/2*(-1/20*x^2*(40/11
9*D*x^3+100/231*C*x^2+20/33*B*x+A)*b^3-2/5*(25/154*D*x^3+5/22*C*x^2+3/8*B*
x+A)*x*a*b^2+a^2*(-1/11*D*x^3-3/20*C*x^2-2/5*B*x+A)*b+1/3*(-3/20*D*x^2-2/5
*C*x+B)*a^3)*c*d^5+27/5*(-1/8*x*(200/1309*D*x^3+50/231*C*x^2+4/11*B*x+A)*b
^3+a*(-25/308*D*x^3-3/22*C*x^2-3/8*B*x+A)*b^2+a^2*(-3/22*D*x^2-3/8*C*x+B)*
b+1/3*(-3/8*D*x+C)*a^3)*c^2*d^4-81/40*c^3*((-100/1309*D*x^3-10/77*C*x^2-4/
11*B*x+A)*b^3+3*(-10/77*D*x^2-4/11*C*x+B)*a*b^2+3*(-4/11*D*x+C)*a^2*b+a^3*
D)*d^3+243/110*c^4*b*((-15/119*D*x^2-5/14*C*x+B)*b^2+3*(-5/14*D*x+C)*a*b+3
*D*a^2)*d^2-729/308*((-6/17*D*x+C)*b+3*D*a)*c^5*b^2*d+6561/2618*D*b^3*c^6)
/d^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx = \frac{3\,(3080\,Db^3d^6x^6 - 131220\,Db^3c^6 - 52360\,Aa^3d^6 + 123930\,(3\,D$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="fricas
")
```

output
```
3/52360*(3080*D*b^3*d^6*x^6 - 131220*D*b^3*c^6 - 52360*A*a^3*d^6 + 123930*
(3*D*a*b^2 + C*b^3)*c^5*d - 115668*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^4*d^2
 + 106029*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 - 94248*(C*a^3 +
 3*B*a^2*b + 3*A*a*b^2)*c^2*d^4 + 78540*(B*a^3 + 3*A*a^2*b)*c*d^5 - 220*(1
8*D*b^3*c*d^5 - 17*(3*D*a*b^2 + C*b^3)*d^6)*x^5 + 20*(270*D*b^3*c^2*d^4 -
255*(3*D*a*b^2 + C*b^3)*c*d^5 + 238*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^6)*x
^4 - 5*(1620*D*b^3*c^3*d^3 - 1530*(3*D*a*b^2 + C*b^3)*c^2*d^4 + 1428*(3*D*
a^2*b + 3*C*a*b^2 + B*b^3)*c*d^5 - 1309*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A
*b^3)*d^6)*x^3 + (14580*D*b^3*c^4*d^2 - 13770*(3*D*a*b^2 + C*b^3)*c^3*d^3
+ 12852*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^2*d^4 - 11781*(D*a^3 + 3*C*a^2*b
 + 3*B*a*b^2 + A*b^3)*c*d^5 + 10472*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*d^6)*x
^2 - (43740*D*b^3*c^5*d - 41310*(3*D*a*b^2 + C*b^3)*c^4*d^2 + 38556*(3*D*a
^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^3 - 35343*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 +
 A*b^3)*c^2*d^4 + 31416*(C*a^3 + 3*B*a^2*b + 3*A*a*b^2)*c*d^5 - 26180*(B*a
^3 + 3*A*a^2*b)*d^6)*x)*(d*x + c)^(2/3)/(d^8*x + c*d^7)
```

Sympy [A] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx)^3 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx = \begin{cases} \frac{3\left(\frac{Db^3(c+dx)^{\frac{17}{3}}}{17d^6}+\frac{(c+dx)^{\frac{14}{3}}\left(Cb^3d+3Dab^2d-6Db^3c\right)}{14d^6}+\frac{(c+dx)^{\frac{11}{3}}\left(Bb^3d^2+3Cab^2d^2-5Cb\right.}{}\right.}{} \\ \frac{Aa^3x+\frac{Db^3x^7}{7}+\frac{x^6\left(Cb^3+3Dab^2\right)}{6}+\frac{x^5\left(Bb^3+3Cab^2+3Da^2b\right)}{5}+\frac{x^4\left(Ab^3+3Bab^2+3Ca^2b+\right.}{4}}{c^{\frac{4}{3}}} \end{cases}$$

input
```
integrate((b*x+a)**3*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(4/3),x)
```

output
```
Piecewise((3*(D*b**3*(c + d*x)**(17/3)/(17*d**6) + (c + d*x)**(14/3)*(C*b*
*3*d + 3*D*a*b**2*d - 6*D*b**3*c)/(14*d**6) + (c + d*x)**(11/3)*(B*b**3*d*
*2 + 3*C*a*b**2*d**2 - 5*C*b**3*c*d + 3*D*a**2*b*d**2 - 15*D*a*b**2*c*d +
15*D*b**3*c**2)/(11*d**6) + (c + d*x)**(8/3)*(A*b**3*d**3 + 3*B*a*b**2*d**
3 - 4*B*b**3*c*d**2 + 3*C*a**2*b*d**3 - 12*C*a*b**2*c*d**2 + 10*C*b**3*c**
2*d + D*a**3*d**3 - 12*D*a**2*b*c*d**2 + 30*D*a*b**2*c**2*d - 20*D*b**3*c*
*3)/(8*d**6) + (c + d*x)**(5/3)*(3*A*a*b**2*d**4 - 3*A*b**3*c*d**3 + 3*B*a
**2*b*d**4 - 9*B*a*b**2*c*d**3 + 6*B*b**3*c**2*d**2 + C*a**3*d**4 - 9*C*a*
*2*b*c*d**3 + 18*C*a*b**2*c**2*d**2 - 10*C*b**3*c**3*d - 3*D*a**3*c*d**3 +
 18*D*a**2*b*c**2*d**2 - 30*D*a*b**2*c**3*d + 15*D*b**3*c**4)/(5*d**6) + (
c + d*x)**(2/3)*(3*A*a**2*b*d**5 - 6*A*a*b**2*c*d**4 + 3*A*b**3*c**2*d**3
+ B*a**3*d**5 - 6*B*a**2*b*c*d**4 + 9*B*a*b**2*c**2*d**3 - 4*B*b**3*c**3*d
**2 - 2*C*a**3*c*d**4 + 9*C*a**2*b*c**2*d**3 - 12*C*a*b**2*c**3*d**2 + 5*C
*b**3*c**4*d + 3*D*a**3*c**2*d**3 - 12*D*a**2*b*c**3*d**2 + 15*D*a*b**2*c*
*4*d - 6*D*b**3*c**5)/(2*d**6) + (a*d - b*c)**3*(-A*d**3 + B*c*d**2 - C*c*
*2*d + D*c**3)/(d**6*(c + d*x)**(1/3)))/d, Ne(d, 0)), ((A*a**3*x + D*b**3*
x**7/7 + x**6*(C*b**3 + 3*D*a*b**2)/6 + x**5*(B*b**3 + 3*C*a*b**2 + 3*D*a*
*2*b)/5 + x**4*(A*b**3 + 3*B*a*b**2 + 3*C*a**2*b + D*a**3)/4 + x**3*(3*A*a
*b**2 + 3*B*a**2*b + C*a**3)/3 + x**2*(3*A*a**2*b + B*a**3)/2)/c**(4/3), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx)^3(A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx = \frac{3\left(\frac{3080\,(dx+c)^{\frac{17}{3}}Db^3 - 3740\,(6\,Db^3c - (3\,Dab^2+Cb^3)d)(dx+c)^{\frac{14}{3}} + 4760\,(15\,Db^3c^2 -}{}\right.}{}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="maxima
")
```

output
```
3/52360*((3080*(d*x + c)^(17/3)*D*b^3 - 3740*(6*D*b^3*c - (3*D*a*b^2 + C*b
^3)*d)*(d*x + c)^(14/3) + 4760*(15*D*b^3*c^2 - 5*(3*D*a*b^2 + C*b^3)*c*d +
 (3*D*a^2*b + 3*C*a*b^2 + B*b^3)*d^2)*(d*x + c)^(11/3) - 6545*(20*D*b^3*c^
3 - 10*(3*D*a*b^2 + C*b^3)*c^2*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c*d^2
 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*d^3)*(d*x + c)^(8/3) + 10472*(1
5*D*b^3*c^4 - 10*(3*D*a*b^2 + C*b^3)*c^3*d + 6*(3*D*a^2*b + 3*C*a*b^2 + B*
b^3)*c^2*d^2 - 3*(D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c*d^3 + (C*a^3 +
3*B*a^2*b + 3*A*a*b^2)*d^4)*(d*x + c)^(5/3) - 26180*(6*D*b^3*c^5 - 5*(3*D*
a*b^2 + C*b^3)*c^4*d + 4*(3*D*a^2*b + 3*C*a*b^2 + B*b^3)*c^3*d^2 - 3*(D*a^
3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^2*d^3 + 2*(C*a^3 + 3*B*a^2*b + 3*A*a*
b^2)*c*d^4 - (B*a^3 + 3*A*a^2*b)*d^5)*(d*x + c)^(2/3))/d^6 - 52360*(D*b^3*
c^6 + A*a^3*d^6 - (3*D*a*b^2 + C*b^3)*c^5*d + (3*D*a^2*b + 3*C*a*b^2 + B*b
^3)*c^4*d^2 - (D*a^3 + 3*C*a^2*b + 3*B*a*b^2 + A*b^3)*c^3*d^3 + (C*a^3 + 3
*B*a^2*b + 3*A*a*b^2)*c^2*d^4 - (B*a^3 + 3*A*a^2*b)*c*d^5)/((d*x + c)^(1/3
)*d^6))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(412) = 824$.

Time = 0.15 (sec) , antiderivative size = 1067, normalized size of antiderivative = 2.45

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
-3*(D*b^3*c^6 - 3*D*a*b^2*c^5*d - C*b^3*c^5*d + 3*D*a^2*b*c^4*d^2 + 3*C*a*
b^2*c^4*d^2 + B*b^3*c^4*d^2 - D*a^3*c^3*d^3 - 3*C*a^2*b*c^3*d^3 - 3*B*a*b^
2*c^3*d^3 - A*b^3*c^3*d^3 + C*a^3*c^2*d^4 + 3*B*a^2*b*c^2*d^4 + 3*A*a*b^2*
c^2*d^4 - B*a^3*c*d^5 - 3*A*a^2*b*c*d^5 + A*a^3*d^6)/((d*x + c)^(1/3)*d^7)
 + 3/52360*(3080*(d*x + c)^(17/3)*D*b^3*d^112 - 22440*(d*x + c)^(14/3)*D*b
^3*c*d^112 + 71400*(d*x + c)^(11/3)*D*b^3*c^2*d^112 - 130900*(d*x + c)^(8/
3)*D*b^3*c^3*d^112 + 157080*(d*x + c)^(5/3)*D*b^3*c^4*d^112 - 157080*(d*x
+ c)^(2/3)*D*b^3*c^5*d^112 + 11220*(d*x + c)^(14/3)*D*a*b^2*d^113 + 3740*(
d*x + c)^(14/3)*C*b^3*d^113 - 71400*(d*x + c)^(11/3)*D*a*b^2*c*d^113 - 238
00*(d*x + c)^(11/3)*C*b^3*c*d^113 + 196350*(d*x + c)^(8/3)*D*a*b^2*c^2*d^1
13 + 65450*(d*x + c)^(8/3)*C*b^3*c^2*d^113 - 314160*(d*x + c)^(5/3)*D*a*b^
2*c^3*d^113 - 104720*(d*x + c)^(5/3)*C*b^3*c^3*d^113 + 392700*(d*x + c)^(2
/3)*D*a*b^2*c^4*d^113 + 130900*(d*x + c)^(2/3)*C*b^3*c^4*d^113 + 14280*(d*
x + c)^(11/3)*D*a^2*b*d^114 + 14280*(d*x + c)^(11/3)*C*a*b^2*d^114 + 4760*
(d*x + c)^(11/3)*B*b^3*d^114 - 78540*(d*x + c)^(8/3)*D*a^2*b*c*d^114 - 785
40*(d*x + c)^(8/3)*C*a*b^2*c*d^114 - 26180*(d*x + c)^(8/3)*B*b^3*c*d^114 +
 188496*(d*x + c)^(5/3)*D*a^2*b*c^2*d^114 + 188496*(d*x + c)^(5/3)*C*a*b^2
*c^2*d^114 + 62832*(d*x + c)^(5/3)*B*b^3*c^2*d^114 - 314160*(d*x + c)^(2/3
)*D*a^2*b*c^3*d^114 - 314160*(d*x + c)^(2/3)*C*a*b^2*c^3*d^114 - 104720*(d
*x + c)^(2/3)*B*b^3*c^3*d^114 + 6545*(d*x + c)^(8/3)*D*a^3*d^115 + 1963...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx=\int \frac{(a+b\,x)^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{(c+d\,x)^{4/3}}\,dx$$

input
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(4/3),x)
```

output
```
int(((a + b*x)^3*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(4/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx)^3 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx = \frac{\frac{81}{10}a\,b^3c^2d^3x-\frac{729}{440}a^2b\,c^4d^2-3a^4d^5-\frac{2187}{5236}b^3c^6+18a^3bc\,d^4+6a^3b}{}$$

input
```
int((b*x+a)^3*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x)
```

output
```
(3*( - 52360*a**4*d**5 + 314160*a**3*b*c*d**4 + 104720*a**3*b*d**5*x + 117
81*a**3*c**3*d**3 + 3927*a**3*c**2*d**4*x - 1309*a**3*c*d**5*x**2 + 6545*a
**3*d**6*x**3 - 565488*a**2*b**2*c**2*d**3 - 188496*a**2*b**2*c*d**4*x + 6
2832*a**2*b**2*d**5*x**2 - 28917*a**2*b*c**4*d**2 - 9639*a**2*b*c**3*d**3*
x + 3213*a**2*b*c**2*d**4*x**2 - 1785*a**2*b*c*d**5*x**3 + 14280*a**2*b*d*
*6*x**4 + 424116*a*b**3*c**3*d**2 + 141372*a*b**3*c**2*d**3*x - 47124*a*b*
*3*c*d**4*x**2 + 26180*a*b**3*d**5*x**3 + 24786*a*b**2*c**5*d + 8262*a*b**
2*c**4*d**2*x - 2754*a*b**2*c**3*d**3*x**2 + 1530*a*b**2*c**2*d**4*x**3 -
1020*a*b**2*c*d**5*x**4 + 11220*a*b**2*d**6*x**5 - 115668*b**4*c**4*d - 38
556*b**4*c**3*d**2*x + 12852*b**4*c**2*d**3*x**2 - 7140*b**4*c*d**4*x**3 +
 4760*b**4*d**5*x**4 - 7290*b**3*c**6 - 2430*b**3*c**5*d*x + 810*b**3*c**4
*d**2*x**2 - 450*b**3*c**3*d**3*x**3 + 300*b**3*c**2*d**4*x**4 - 220*b**3*
c*d**5*x**5 + 3080*b**3*d**6*x**6))/(52360*(c + d*x)**(1/3)*d**6)
```

3.168 $\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx$

Optimal result 1622
Mathematica [A] (verified) 1623
Rubi [A] (verified) 1623
Maple [A] (verified) 1625
Fricas [A] (verification not implemented) 1625
Sympy [A] (verification not implemented) 1626
Maxima [A] (verification not implemented) 1627
Giac [B] (verification not implemented) 1628
Mupad [F(-1)] 1628
Reduce [B] (verification not implemented) 1629

Optimal result

Integrand size = 32, antiderivative size = 324

$$\begin{aligned}\int \frac{(a+bx)^2(A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx &= -\frac{3(bc-ad)^2(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^6\sqrt[3]{c+dx}}\\ &+\frac{3(bc-ad)(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D))(c+dx)^{2/3}}{2d^6}\\ &+\frac{3(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D))(c+dx)^{5/3}}{5d^6}\\ &+\frac{3(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D))(c+dx)^{8/3}}{8d^6}\\ &+\frac{3b(bCd-5bcD+2adD)(c+dx)^{11/3}}{11d^6}+\frac{3b^2D(c+dx)^{14/3}}{14d^6}\end{aligned}$$

output
```
-3*(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^6/(d*x+c)^(1/3)+3/2*(-a*d+
b*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3)
)*(d*x+c)^(2/3)/d^6+3/5*(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*c
^2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(5/3)/d^6+3/8*(a^2*d
^2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(8/3)/d^6+
3/11*b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(11/3)/d^6+3/14*b^2*D*(d*x+c)^(14/3
)/d^6
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx = \frac{3(77a^2d^2(81c^3D-9c^2d(8C-3Dx)+3cd^2(20B-x(8C+3Dx$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(4/3),x]
```

output
```
(3*(77*a^2*d^2*(81*c^3*D - 9*c^2*d*(8*C - 3*D*x) + 3*c*d^2*(20*B - x*(8*C
+ 3*D*x)) + d^3*(-40*A + x*(20*B + 8*C*x + 5*D*x^2))) + 14*a*b*d*(-972*c^4
*D + 81*c^3*d*(11*C - 4*D*x) - 9*c^2*d^2*(88*B - 3*x*(11*C + 4*D*x)) + 3*c
*d^3*(220*A - x*(88*B + 33*C*x + 20*D*x^2)) + d^4*x*(220*A + x*(88*B + 55*
C*x + 40*D*x^2))) + b^2*(7290*c^5*D - 486*c^4*d*(14*C - 5*D*x) + 81*c^3*d^
2*(77*B - 2*x*(14*C + 5*D*x)) + d^5*x^2*(616*A + 5*x*(77*B + 56*C*x + 44*D
*x^2)) - 9*c^2*d^3*(616*A - x*(231*B + 84*C*x + 50*D*x^2)) - 3*c*d^4*x*(61
6*A + x*(231*B + 20*x*(7*C + 5*D*x))))))/(3080*d^6*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 \left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{2/3}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c$$

$$\Big\downarrow \text{2009}$$

$$\frac{3(c+dx)^{5/3}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{5d^6}+\frac{3(c+dx)^{8/3}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{8d^6}+\frac{3(c+dx)^{2/3}(bc-ad)\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{2d^6}-\frac{3(bc-ad)^2\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^6\sqrt[3]{c+dx}}+\frac{3b(c+dx)^{11/3}(2adD-5bcD+bCd)}{11d^6}+\frac{3b^2D(c+dx)^{14/3}}{14d^6}$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(4/3),x]
```

output
```
(-3*(b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^6*(c + d*x)^(1/3
)) + (3*(b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d - 3*B*
c*d^2 + 2*A*d^3 - 5*c^3*D))*(c + d*x)^(2/3))/(2*d^6) + (3*(a^2*d^2*(C*d -
3*c*D) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2
+ A*d^3 - 10*c^3*D))*(c + d*x)^(5/3))/(5*d^6) + (3*(a^2*d^2*D + 2*a*b*d*(C
*d - 4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)^(8/3))/(8*d^6) +
 (3*b*(b*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(11/3))/(11*d^6) + (3*b^2*D*(c
 + d*x)^(14/3))/(14*d^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{3\left(\left(\left(-\frac{x^{2}\left(\frac{5}{14} D x^{3}+\frac{5}{11} C\,x^{2}+\frac{5}{8} B x+A\right) b^{2}}{5}-\left(\frac{2}{11} D x^{3}+\frac{1}{4} C\,x^{2}+\frac{2}{5} B x+A\right) x a b+a^{2}\left(-\frac{1}{8} D x^{3}-\frac{1}{5} C\,x^{2}-\frac{1}{2} B x+A\right)\right) d^{5}-3 c\left(\right.\right.}{}$
gosper	$-\frac{3\left(-220 D x^{5} b^{2} d^{5}-280 C\,x^{4} b^{2} d^{5}-560 D x^{4} a b\,d^{5}+300 D x^{4} b^{2} c\,d^{4}-385 B\,x^{3} b^{2} d^{5}-770 C\,x^{3} a b\,d^{5}+420 C\,x^{3} b^{2} c\,d^{4}-385 D\right.}{}$
trager	$-\frac{3\left(-220 D x^{5} b^{2} d^{5}-280 C\,x^{4} b^{2} d^{5}-560 D x^{4} a b\,d^{5}+300 D x^{4} b^{2} c\,d^{4}-385 B\,x^{3} b^{2} d^{5}-770 C\,x^{3} a b\,d^{5}+420 C\,x^{3} b^{2} c\,d^{4}-385 D\right.}{}$
orering	$-\frac{3\left(-220 D x^{5} b^{2} d^{5}-280 C\,x^{4} b^{2} d^{5}-560 D x^{4} a b\,d^{5}+300 D x^{4} b^{2} c\,d^{4}-385 B\,x^{3} b^{2} d^{5}-770 C\,x^{3} a b\,d^{5}+420 C\,x^{3} b^{2} c\,d^{4}-385 D\right.}{}$
derivativedivides	$\frac{\frac{3 C\,a^{2} d^{3} (x d+c)^{\frac{5}{3}}}{5}-\frac{3 C\,b^{2} c d (x d+c)^{\frac{8}{3}}}{2}+\frac{6 B a b\,d^{3} (x d+c)^{\frac{5}{3}}}{5}+\frac{9 B\,b^{2} c^{2} d^{2} (x d+c)^{\frac{2}{3}}}{2}-\frac{9 B\,b^{2} c\,d^{2} (x d+c)^{\frac{5}{3}}}{5}-3 C\,a^{2} c\,d^{3} (x d+c)^{\frac{2}{3}}+\frac{6 D a}{}}{}$
default	$\frac{\frac{3 C\,a^{2} d^{3} (x d+c)^{\frac{5}{3}}}{5}-\frac{3 C\,b^{2} c d (x d+c)^{\frac{8}{3}}}{2}+\frac{6 B a b\,d^{3} (x d+c)^{\frac{5}{3}}}{5}+\frac{9 B\,b^{2} c^{2} d^{2} (x d+c)^{\frac{2}{3}}}{2}-\frac{9 B\,b^{2} c\,d^{2} (x d+c)^{\frac{5}{3}}}{5}-3 C\,a^{2} c\,d^{3} (x d+c)^{\frac{2}{3}}+\frac{6 D a}{}}{}$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3*((-1/5*x^2*(5/14*D*x^3+5/11*C*x^2+5/8*B*x+A)*b^2-(2/11*D*x^3+1/4*C*x^2+
2/5*B*x+A)*x*a*b+a^2*(-1/8*D*x^3-1/5*C*x^2-1/2*B*x+A))*d^5-3*c*(-1/5*(25/1
54*D*x^3+5/22*C*x^2+3/8*B*x+A)*x*b^2+a*(-1/11*D*x^3-3/20*C*x^2-2/5*B*x+A)*
b+1/2*(-3/20*D*x^2-2/5*C*x+B)*a^2)*d^4+9/5*((-25/308*D*x^3-3/22*C*x^2-3/8*
B*x+A)*b^2+2*a*(-3/22*D*x^2-3/8*C*x+B)*b+a^2*(-3/8*D*x+C))*c^2*d^3-81/40*(
(-10/77*D*x^2-4/11*C*x+B)*b^2+2*(-4/11*D*x+C)*a*b+D*a^2)*c^3*d^2+243/110*(
(-5/14*D*x+C)*b+2*D*a)*c^4*b*d-729/308*D*b^2*c^5)/(d*x+c)^(1/3)/d^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx = \frac{3\left(220\,Db^2d^5x^5+7290\,Db^2c^5-3080\,Aa^2d^5-6804\,(2\,Dab+Cb\right.}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="fricas
")
```

output
```
3/3080*(220*D*b^2*d^5*x^5 + 7290*D*b^2*c^5 - 3080*A*a^2*d^5 - 6804*(2*D*a*
b + C*b^2)*c^4*d + 6237*(D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - 5544*(C*a^2 +
2*B*a*b + A*b^2)*c^2*d^3 + 4620*(B*a^2 + 2*A*a*b)*c*d^4 - 20*(15*D*b^2*c*d
^4 - 14*(2*D*a*b + C*b^2)*d^5)*x^4 + 5*(90*D*b^2*c^2*d^3 - 84*(2*D*a*b + C
*b^2)*c*d^4 + 77*(D*a^2 + 2*C*a*b + B*b^2)*d^5)*x^3 - (810*D*b^2*c^3*d^2 -
 756*(2*D*a*b + C*b^2)*c^2*d^3 + 693*(D*a^2 + 2*C*a*b + B*b^2)*c*d^4 - 616
*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*x^2 + (2430*D*b^2*c^4*d - 2268*(2*D*a*b +
C*b^2)*c^3*d^2 + 2079*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^3 - 1848*(C*a^2 + 2*
B*a*b + A*b^2)*c*d^4 + 1540*(B*a^2 + 2*A*a*b)*d^5)*x)*(d*x + c)^(2/3)/(d^7
*x + c*d^6)
```

Sympy [A] (verification not implemented)

Time = 9.08 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx = \begin{cases} \frac{3\left(\frac{Db^2(c+dx)^{\frac{14}{3}}}{14d^5} + \frac{(c+dx)^{\frac{11}{3}}\left(Cb^2d+2Dabd-5Db^2c\right)}{11d^5} + \frac{(c+dx)^{\frac{8}{3}}\left(Bb^2d^2+2Cabd^2-4Cb^2cd\right.}{8d^5}}{} \\ \frac{Aa^2x+\frac{Db^2x^6}{6}+\frac{x^5\left(Cb^2+2Dab\right)}{5}+\frac{x^4\left(Bb^2+2Cab+Da^2\right)}{4}+\frac{x^3\left(Ab^2+2Bab+Ca^2\right)}{3}+\frac{x^2\cdot\left(2A\right.}{}}{c^{\frac{4}{3}}} \end{cases}$$

input
```
integrate((b*x+a)**2*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(4/3),x)
```

output
```
Piecewise((3*(D*b**2*(c + d*x)**(14/3)/(14*d**5) + (c + d*x)**(11/3)*(C*b*
*2*d + 2*D*a*b*d - 5*D*b**2*c)/(11*d**5) + (c + d*x)**(8/3)*(B*b**2*d**2 +
 2*C*a*b*d**2 - 4*C*b**2*c*d + D*a**2*d**2 - 8*D*a*b*c*d + 10*D*b**2*c**2)
/(8*d**5) + (c + d*x)**(5/3)*(A*b**2*d**3 + 2*B*a*b*d**3 - 3*B*b**2*c*d**2
 + C*a**2*d**3 - 6*C*a*b*c*d**2 + 6*C*b**2*c**2*d - 3*D*a**2*c*d**2 + 12*D
*a*b*c**2*d - 10*D*b**2*c**3)/(5*d**5) + (c + d*x)**(2/3)*(2*A*a*b*d**4 -
2*A*b**2*c*d**3 + B*a**2*d**4 - 4*B*a*b*c*d**3 + 3*B*b**2*c**2*d**2 - 2*C*
a**2*c*d**3 + 6*C*a*b*c**2*d**2 - 4*C*b**2*c**3*d + 3*D*a**2*c**2*d**2 - 8
*D*a*b*c**3*d + 5*D*b**2*c**4)/(2*d**5) + (a*d - b*c)**2*(-A*d**3 + B*c*d*
*2 - C*c**2*d + D*c**3)/(d**5*(c + d*x)**(1/3)))/d, Ne(d, 0)), ((A*a**2*x
+ D*b**2*x**6/6 + x**5*(C*b**2 + 2*D*a*b)/5 + x**4*(B*b**2 + 2*C*a*b + D*a
**2)/4 + x**3*(A*b**2 + 2*B*a*b + C*a**2)/3 + x**2*(2*A*a*b + B*a**2)/2)/c
**(4/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx = \frac{3\left(\frac{220\,(dx+c)^{\frac{14}{3}}Db^2-280\left(5\,Db^2c-\left(2\,Dab+Cb^2\right)d\right)(dx+c)^{\frac{11}{3}}+385\left(10\,Db^2c^2-4\left(2\right.\right.}{}\right.}{}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="maxima
")
```

output
```
3/3080*((220*(d*x + c)^(14/3)*D*b^2 - 280*(5*D*b^2*c - (2*D*a*b + C*b^2)*d
)*(d*x + c)^(11/3) + 385*(10*D*b^2*c^2 - 4*(2*D*a*b + C*b^2)*c*d + (D*a^2
+ 2*C*a*b + B*b^2)*d^2)*(d*x + c)^(8/3) - 616*(10*D*b^2*c^3 - 6*(2*D*a*b +
 C*b^2)*c^2*d + 3*(D*a^2 + 2*C*a*b + B*b^2)*c*d^2 - (C*a^2 + 2*B*a*b + A*b
^2)*d^3)*(d*x + c)^(5/3) + 1540*(5*D*b^2*c^4 - 4*(2*D*a*b + C*b^2)*c^3*d +
 3*(D*a^2 + 2*C*a*b + B*b^2)*c^2*d^2 - 2*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 +
 (B*a^2 + 2*A*a*b)*d^4)*(d*x + c)^(2/3))/d^5 + 3080*(D*b^2*c^5 - A*a^2*d^5
 - (2*D*a*b + C*b^2)*c^4*d + (D*a^2 + 2*C*a*b + B*b^2)*c^3*d^2 - (C*a^2 +
2*B*a*b + A*b^2)*c^2*d^3 + (B*a^2 + 2*A*a*b)*c*d^4)/((d*x + c)^(1/3)*d^5))
/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(302) = 604$.

Time = 0.14 (sec) , antiderivative size = 651, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
3*(D*b^2*c^5 - 2*D*a*b*c^4*d - C*b^2*c^4*d + D*a^2*c^3*d^2 + 2*C*a*b*c^3*d
^2 + B*b^2*c^3*d^2 - C*a^2*c^2*d^3 - 2*B*a*b*c^2*d^3 - A*b^2*c^2*d^3 + B*a
^2*c*d^4 + 2*A*a*b*c*d^4 - A*a^2*d^5)/((d*x + c)^(1/3)*d^6) + 3/3080*(220*
(d*x + c)^(14/3)*D*b^2*d^78 - 1400*(d*x + c)^(11/3)*D*b^2*c*d^78 + 3850*(d
*x + c)^(8/3)*D*b^2*c^2*d^78 - 6160*(d*x + c)^(5/3)*D*b^2*c^3*d^78 + 7700*
(d*x + c)^(2/3)*D*b^2*c^4*d^78 + 560*(d*x + c)^(11/3)*D*a*b*d^79 + 280*(d*
x + c)^(11/3)*C*b^2*d^79 - 3080*(d*x + c)^(8/3)*D*a*b*c*d^79 - 1540*(d*x +
 c)^(8/3)*C*b^2*c*d^79 + 7392*(d*x + c)^(5/3)*D*a*b*c^2*d^79 + 3696*(d*x +
 c)^(5/3)*C*b^2*c^2*d^79 - 12320*(d*x + c)^(2/3)*D*a*b*c^3*d^79 - 6160*(d*
x + c)^(2/3)*C*b^2*c^3*d^79 + 385*(d*x + c)^(8/3)*D*a^2*d^80 + 770*(d*x +
c)^(8/3)*C*a*b*d^80 + 385*(d*x + c)^(8/3)*B*b^2*d^80 - 1848*(d*x + c)^(5/3
)*D*a^2*c*d^80 - 3696*(d*x + c)^(5/3)*C*a*b*c*d^80 - 1848*(d*x + c)^(5/3)*
B*b^2*c*d^80 + 4620*(d*x + c)^(2/3)*D*a^2*c^2*d^80 + 9240*(d*x + c)^(2/3)*
C*a*b*c^2*d^80 + 4620*(d*x + c)^(2/3)*B*b^2*c^2*d^80 + 616*(d*x + c)^(5/3)
*C*a^2*d^81 + 1232*(d*x + c)^(5/3)*B*a*b*d^81 + 616*(d*x + c)^(5/3)*A*b^2*
d^81 - 3080*(d*x + c)^(2/3)*C*a^2*c*d^81 - 6160*(d*x + c)^(2/3)*B*a*b*c*d^
81 - 3080*(d*x + c)^(2/3)*A*b^2*c*d^81 + 1540*(d*x + c)^(2/3)*B*a^2*d^82 +
 3080*(d*x + c)^(2/3)*A*a*b*d^82)/d^84
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\, dx = \int \frac{{\left(a+b\,x\right)}^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c+d\,x\right)}^{4/3}} \,d x$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(4/3),x)
```

output
```
int(((a + b*x)^2*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(4/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx = \frac{\frac{27}{2}a^2bc\,d^3+\frac{9}{40}a^2c^2d^3x-\frac{81}{5}a\,b^2c^2d^2-\frac{243}{220}ab\,c^4d+\frac{243}{1540}b^2c^4dx+\ldots}{}$$

input
```
int((b*x+a)^2*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x)
```

output
```
(3*( - 3080*a**3*d**4 + 13860*a**2*b*c*d**3 + 4620*a**2*b*d**4*x + 693*a**
2*c**3*d**2 + 231*a**2*c**2*d**3*x - 77*a**2*c*d**4*x**2 + 385*a**2*d**5*x
**3 - 16632*a*b**2*c**2*d**2 - 5544*a*b**2*c*d**3*x + 1848*a*b**2*d**4*x**
2 - 1134*a*b*c**4*d - 378*a*b*c**3*d**2*x + 126*a*b*c**2*d**3*x**2 - 70*a*
b*c*d**4*x**3 + 560*a*b*d**5*x**4 + 6237*b**3*c**3*d + 2079*b**3*c**2*d**2
*x - 693*b**3*c*d**3*x**2 + 385*b**3*d**4*x**3 + 486*b**2*c**5 + 162*b**2*
c**4*d*x - 54*b**2*c**3*d**2*x**2 + 30*b**2*c**2*d**3*x**3 - 20*b**2*c*d**
4*x**4 + 220*b**2*d**5*x**5))/(3080*(c + d*x)**(1/3)*d**5)
```

3.169 $\int \frac{(a+bx)(A+Bx+Cx^2+Dx^3)}{(c+dx)^{4/3}}\,dx$

Optimal result . . . 1630
Mathematica [A] (verified) . . . 1631
Rubi [A] (verified) . . . 1631
Maple [A] (verified) . . . 1633
Fricas [A] (verification not implemented) . . . 1633
Sympy [A] (verification not implemented) . . . 1634
Maxima [A] (verification not implemented) . . . 1634
Giac [A] (verification not implemented) . . . 1635
Mupad [F(-1)] . . . 1636
Reduce [B] (verification not implemented) . . . 1636

Optimal result

Integrand size = 30, antiderivative size = 212

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx = \frac{3(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)}{d^5\sqrt[3]{c+dx}} - \frac{3(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))\,(c+dx)^{2/3}}{2d^5} + \frac{3(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))\,(c+dx)^{5/3}}{5d^5} + \frac{3(bCd-4bcD+adD)(c+dx)^{8/3}}{8d^5} + \frac{3bD(c+dx)^{11/3}}{11d^5}$$

output
```
3*(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^5/(d*x+c)^(1/3)-3/2*(a*d*(-B*
d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(2/3)/
d^5+3/5*(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(5/3)/d^5+3/8
*(C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(8/3)/d^5+3/11*b*D*(d*x+c)^(11/3)/d^5
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx = \frac{33ad(81c^3D-9c^2d(8C-3Dx)+3cd^2(20B-x(8C+3Dx))+$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(4/3),x]
```

output
```
(33*a*d*(81*c^3*D - 9*c^2*d*(8*C - 3*D*x) + 3*c*d^2*(20*B - x*(8*C + 3*D*x
)) + d^3*(-40*A + x*(20*B + 8*C*x + 5*D*x^2))) + 3*b*(-972*c^4*D + 81*c^3*
d*(11*C - 4*D*x) - 9*c^2*d^2*(88*B - 3*x*(11*C + 4*D*x)) + 3*c*d^3*(220*A
- x*(88*B + 33*C*x + 20*D*x^2)) + d^4*x*(220*A + x*(88*B + 55*C*x + 40*D*x
^2))))/(440*d^5*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\left(-Bd^2-3c^2D+2cCd\right)}{d^4\sqrt[3]{c+dx}}+\frac{(ad-bc)\left(Ad^3-Bcd^2+c^3(-D)+c^2C\right.}{d^4(c+dx)^{4/3}}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{3(c+dx)^{2/3}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{2d^5}+\frac{3(bc-ad)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^5\sqrt[3]{c+dx}}+\frac{3(c+dx)^{5/3}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{5d^5}+\frac{3(c+dx)^{8/3}(adD-4bcD+bCd)}{8d^5}+\frac{3bD(c+dx)^{11/3}}{11d^5}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2 + D*x^3))/(c + d*x)^(4/3),x]
```

output
```
(3*(b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^5*(c + d*x)^(1/3))
- (3*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2 + A*d^3 -
 4*c^3*D))*(c + d*x)^(2/3))/(2*d^5) + (3*(a*d*(C*d - 3*c*D) - b*(3*c*C*d -
 B*d^2 - 6*c^2*D))*(c + d*x)^(5/3))/(5*d^5) + (3*(b*C*d - 4*b*c*D + a*d*D)
*(c + d*x)^(8/3))/(8*d^5) + (3*b*D*(c + d*x)^(11/3))/(11*d^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$-\frac{3\left(\left(-\frac{D b\, x^{4}}{11}+\frac{(-C b-D a) x^{3}}{8}+\frac{(-B b-C a) x^{2}}{5}+\frac{(-A b-B a) x}{2}+A a\right) d^{4}-\frac{3 c\left(-\frac{D b\, x^{3}}{11}+\frac{3(-C b-D a) x^{2}}{20}+\frac{2(-B b-C a) x}{5}+A b+\cdots\right.}{2}\cdots}{(x d+c)^{\frac{1}{3}} d^{5}}$
gosper	$-\frac{3\left(-40 D b\, x^{4} d^{4}-55 C\, x^{3} b\, d^{4}-55 D x^{3} a\, d^{4}+60 D x^{3} b c\, d^{3}-88 B\, x^{2} b\, d^{4}-88 C\, x^{2} a\, d^{4}+99 C\, x^{2} b c\, d^{3}+99 D x^{2} a c\, d^{3}-108 D\cdots\right.}{\cdots}$
trager	$-\frac{3\left(-40 D b\, x^{4} d^{4}-55 C\, x^{3} b\, d^{4}-55 D x^{3} a\, d^{4}+60 D x^{3} b c\, d^{3}-88 B\, x^{2} b\, d^{4}-88 C\, x^{2} a\, d^{4}+99 C\, x^{2} b c\, d^{3}+99 D x^{2} a c\, d^{3}-108 D\cdots\right.}{\cdots}$
orering	$-\frac{3\left(-40 D b\, x^{4} d^{4}-55 C\, x^{3} b\, d^{4}-55 D x^{3} a\, d^{4}+60 D x^{3} b c\, d^{3}-88 B\, x^{2} b\, d^{4}-88 C\, x^{2} a\, d^{4}+99 C\, x^{2} b c\, d^{3}+99 D x^{2} a c\, d^{3}-108 D\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{3 b D (x d+c)^{\frac{11}{3}}}{11}+\frac{3 C b d (x d+c)^{\frac{8}{3}}}{8}+\frac{3 D a d (x d+c)^{\frac{8}{3}}}{8}-\frac{3 D b c (x d+c)^{\frac{8}{3}}}{2}+\frac{3 B b\, d^{2} (x d+c)^{\frac{5}{3}}}{5}+\frac{3 C a\, d^{2} (x d+c)^{\frac{5}{3}}}{5}-\frac{9 C b c d (x d+c)^{\frac{5}{3}}}{5}-\frac{9 D a c\cdots}{\cdots}}{\cdots}$
default	$\frac{\frac{3 b D (x d+c)^{\frac{11}{3}}}{11}+\frac{3 C b d (x d+c)^{\frac{8}{3}}}{8}+\frac{3 D a d (x d+c)^{\frac{8}{3}}}{8}-\frac{3 D b c (x d+c)^{\frac{8}{3}}}{2}+\frac{3 B b\, d^{2} (x d+c)^{\frac{5}{3}}}{5}+\frac{3 C a\, d^{2} (x d+c)^{\frac{5}{3}}}{5}-\frac{9 C b c d (x d+c)^{\frac{5}{3}}}{5}-\frac{9 D a c\cdots}{\cdots}}{\cdots}$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/(d*x+c)^(1/3)*((-1/11*D*b*x^4+1/8*(-C*b-D*a)*x^3+1/5*(-B*b-C*a)*x^2+1/2
*(-A*b-B*a)*x+A*a)*d^4-3/2*c*(-1/11*D*b*x^3+3/20*(-C*b-D*a)*x^2+2/5*(-B*b-
C*a)*x+A*b+B*a)*d^3+9/5*c^2*(-3/22*D*b*x^2+3/8*(-C*b-D*a)*x+B*b+C*a)*d^2-8
1/40*c^3*(-4/11*D*b*x+C*b+D*a)*d+243/110*D*b*c^4)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx=\frac{3\left(40\,Dbd^4x^4-972\,Dbc^4-440\,Aad^4+891\,(Da+Cb)c^3d-792\cdots\right.}{\cdots}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
3/440*(40*D*b*d^4*x^4 - 972*D*b*c^4 - 440*A*a*d^4 + 891*(D*a + C*b)*c^3*d
- 792*(C*a + B*b)*c^2*d^2 + 660*(B*a + A*b)*c*d^3 - 5*(12*D*b*c*d^3 - 11*(
D*a + C*b)*d^4)*x^3 + (108*D*b*c^2*d^2 - 99*(D*a + C*b)*c*d^3 + 88*(C*a +
B*b)*d^4)*x^2 - (324*D*b*c^3*d - 297*(D*a + C*b)*c^2*d^2 + 264*(C*a + B*b)
*c*d^3 - 220*(B*a + A*b)*d^4)*x)*(d*x + c)^(2/3)/(d^6*x + c*d^5)
```

Sympy [A] (verification not implemented)

Time = 3.86 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx=\begin{cases} \frac{3\left(\frac{Db(c+dx)^{\frac{11}{3}}}{11d^4}+\frac{(c+dx)^{\frac{8}{3}}(Cbd+Dad-4Dbc)}{8d^4}+\frac{(c+dx)^{\frac{5}{3}}\left(Bbd^2+Cad^2-3Cbcd-3Dacd+6Db\right.}{5d^4}\right.}{} \\ \frac{Aax+\frac{Dbx^5}{5}+\frac{x^4(Cb+Da)}{4}+\frac{x^3(Bb+Ca)}{3}+\frac{x^2(Ab+Ba)}{2}}{c^{\frac{4}{3}}} \end{cases}$$

input
```
integrate((b*x+a)*(D*x**3+C*x**2+B*x+A)/(d*x+c)**(4/3),x)
```

output
```
Piecewise((3*(D*b*(c + d*x)**(11/3)/(11*d**4) + (c + d*x)**(8/3)*(C*b*d +
D*a*d - 4*D*b*c)/(8*d**4) + (c + d*x)**(5/3)*(B*b*d**2 + C*a*d**2 - 3*C*b*
c*d - 3*D*a*c*d + 6*D*b*c**2)/(5*d**4) + (c + d*x)**(2/3)*(A*b*d**3 + B*a*
d**3 - 2*B*b*c*d**2 - 2*C*a*c*d**2 + 3*C*b*c**2*d + 3*D*a*c**2*d - 4*D*b*c
**3)/(2*d**4) + (a*d - b*c)*(-A*d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(d**4
*(c + d*x)**(1/3)))/d, Ne(d, 0)), ((A*a*x + D*b*x**5/5 + x**4*(C*b + D*a)/
4 + x**3*(B*b + C*a)/3 + x**2*(A*b + B*a)/2)/c**(4/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx=\frac{3\left(\frac{40\,(dx+c)^{\frac{11}{3}}Db-55\,(4\,Dbc-(Da+Cb)d)(dx+c)^{\frac{8}{3}}+88\,(6\,Dbc^2-3\,(Da+Cb)cd+(Ca\right.}{}\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
3/440*((40*(d*x + c)^(11/3)*D*b - 55*(4*D*b*c - (D*a + C*b)*d)*(d*x + c)^(
8/3) + 88*(6*D*b*c^2 - 3*(D*a + C*b)*c*d + (C*a + B*b)*d^2)*(d*x + c)^(5/3
) - 220*(4*D*b*c^3 - 3*(D*a + C*b)*c^2*d + 2*(C*a + B*b)*c*d^2 - (B*a + A*
b)*d^3)*(d*x + c)^(2/3))/d^4 - 440*(D*b*c^4 + A*a*d^4 - (D*a + C*b)*c^3*d
+ (C*a + B*b)*c^2*d^2 - (B*a + A*b)*c*d^3)/((d*x + c)^(1/3)*d^4))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx = -\frac{3\left(Dbc^4-Dac^3d-Cbc^3d+Cac^2d^2+Bbc^2d^2-Bacd^3-Abcd^3+Aad^4\right)}{(dx+c)^{\frac{1}{3}}d^5} + \frac{3\left(40\,(dx+c)^{\frac{11}{3}}Dbd^{50}-220\,(dx+c)^{\frac{8}{3}}Dbcd^{50}+528\,(dx+c)^{\frac{5}{3}}Dbc^2d^{50}-880\,(dx+c)^{\frac{2}{3}}Dbc^3d^{50}+55\,(dx\right.}{}$$

input
```
integrate((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
-3*(D*b*c^4 - D*a*c^3*d - C*b*c^3*d + C*a*c^2*d^2 + B*b*c^2*d^2 - B*a*c*d^
3 - A*b*c*d^3 + A*a*d^4)/((d*x + c)^(1/3)*d^5) + 3/440*(40*(d*x + c)^(11/3
)*D*b*d^50 - 220*(d*x + c)^(8/3)*D*b*c*d^50 + 528*(d*x + c)^(5/3)*D*b*c^2*
d^50 - 880*(d*x + c)^(2/3)*D*b*c^3*d^50 + 55*(d*x + c)^(8/3)*D*a*d^51 + 55
*(d*x + c)^(8/3)*C*b*d^51 - 264*(d*x + c)^(5/3)*D*a*c*d^51 - 264*(d*x + c)
^(5/3)*C*b*c*d^51 + 660*(d*x + c)^(2/3)*D*a*c^2*d^51 + 660*(d*x + c)^(2/3)
*C*b*c^2*d^51 + 88*(d*x + c)^(5/3)*C*a*d^52 + 88*(d*x + c)^(5/3)*B*b*d^52
- 440*(d*x + c)^(2/3)*C*a*c*d^52 - 440*(d*x + c)^(2/3)*B*b*c*d^52 + 220*(d
*x + c)^(2/3)*B*a*d^53 + 220*(d*x + c)^(2/3)*A*b*d^53)/d^55
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx=\int \frac{(a+b\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{(c+d\,x)}^{4/3}}\,dx$$

input
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(4/3),x)
```

output
```
int(((a + b*x)*(A + B*x + C*x^2 + x^3*D))/(c + d*x)^(4/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx)\left(A+Bx+Cx^2+Dx^3\right)}{(c+dx)^{4/3}}\,dx=\frac{\frac{3}{11}b\,d^4x^4+\frac{3}{8}a\,d^4x^3-\frac{3}{88}bc\,d^3x^3-\frac{3}{40}ac\,d^3x^2+\frac{3}{5}b^2d^3x^2+\frac{27}{440}b\,c^2d$$

input
```
int((b*x+a)*(D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x)
```

output
```
(3*( - 440*a**2*d**3 + 1320*a*b*c*d**2 + 440*a*b*d**3*x + 99*a*c**3*d + 33
*a*c**2*d**2*x - 11*a*c*d**3*x**2 + 55*a*d**4*x**3 - 792*b**2*c**2*d - 264
*b**2*c*d**2*x + 88*b**2*d**3*x**2 - 81*b*c**4 - 27*b*c**3*d*x + 9*b*c**2*
d**2*x**2 - 5*b*c*d**3*x**3 + 40*b*d**4*x**4))/(440*(c + d*x)**(1/3)*d**4)
```

3.170 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx$

Optimal result 1637
Mathematica [A] (verified) 1637
Rubi [A] (verified) 1638
Maple [A] (verified) 1639
Fricas [A] (verification not implemented) 1640
Sympy [A] (verification not implemented) 1640
Maxima [A] (verification not implemented) 1641
Giac [A] (verification not implemented) 1641
Mupad [F(-1)] 1642
Reduce [B] (verification not implemented) 1642

Optimal result

Integrand size = 25, antiderivative size = 115

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx = -\frac{3(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^4\sqrt[3]{c+dx}} - \frac{3(2cCd-Bd^2-3c^2D)(c+dx)^{2/3}}{2d^4} + \frac{3(Cd-3cD)(c+dx)^{5/3}}{5d^4} + \frac{3D(c+dx)^{8/3}}{8d^4}$$

output
```
(-3*A*d^3+3*B*c*d^2-3*C*c^2*d+3*D*c^3)/d^4/(d*x+c)^(1/3)-3/2*(-B*d^2+2*C*c
*d-3*D*c^2)*(d*x+c)^(2/3)/d^4+3/5*(C*d-3*D*c)*(d*x+c)^(5/3)/d^4+3/8*D*(d*x
+c)^(8/3)/d^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx = \frac{3(81c^3D-9c^2d(8C-3Dx)+3cd^2(20B-x(8C+3Dx))+d^3(-40A+x}{40d^4\sqrt[3]{c+dx}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^(4/3),x]
```

output
```
(3*(81*c^3*D - 9*c^2*d*(8*C - 3*D*x) + 3*c*d^2*(20*B - x*(8*C + 3*D*x)) +
d^3*(-40*A + x*(20*B + 8*C*x + 5*D*x^2))))/(40*d^4*(c + d*x)^(1/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx$$
$$\Big\downarrow \text{ 2389}$$
$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)^{4/3}}+\frac{Bd^2+3c^2D-2cCd}{d^3\sqrt[3]{c+dx}}+\frac{(c+dx)^{2/3}(Cd-3cD)}{d^3}+\frac{D(c+dx)^{5/3}}{d^3}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{3\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^4\sqrt[3]{c+dx}}-\frac{3(c+dx)^{2/3}\left(-Bd^2-3c^2D+2cCd\right)}{2d^4}+\frac{3(c+dx)^{5/3}(Cd-3cD)}{5d^4}+\frac{3D(c+dx)^{8/3}}{8d^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(c + d*x)^(4/3),x]
```

output
```
(-3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(d^4*(c + d*x)^(1/3)) - (3*(2*c*C
*d - B*d^2 - 3*c^2*D)*(c + d*x)^(2/3))/(2*d^4) + (3*(C*d - 3*c*D)*(c + d*x
)^(5/3))/(5*d^4) + (3*D*(c + d*x)^(8/3))/(8*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\left(15D x^{3}+24C x^{2}+60B x -120A \right) d^{3}+180\left(-\frac{3}{20}D x^{2}-\frac{2}{5}C x +B \right) c \,d^{2}-216\left(-\frac{3 D x}{8}+C \right) c^{2} d +243 D c^{3}}{40 \left(x d +c \right)^{\frac{1}{3}} d^{4}}$
gosper	$-\frac{3\left(-5D x^{3} d^{3}-8C \,x^{2} d^{3}+9D c \,d^{2} x^{2}-20B \,d^{3} x +24C c \,d^{2} x -27D x \,c^{2} d +40A \,d^{3}-60B c \,d^{2}+72C \,c^{2} d -81D c^{3}\right)}{40 \left(x d +c \right)^{\frac{1}{3}} d^{4}}$
trager	$-\frac{3\left(-5D x^{3} d^{3}-8C \,x^{2} d^{3}+9D c \,d^{2} x^{2}-20B \,d^{3} x +24C c \,d^{2} x -27D x \,c^{2} d +40A \,d^{3}-60B c \,d^{2}+72C \,c^{2} d -81D c^{3}\right)}{40 \left(x d +c \right)^{\frac{1}{3}} d^{4}}$
orering	$-\frac{3\left(-5D x^{3} d^{3}-8C \,x^{2} d^{3}+9D c \,d^{2} x^{2}-20B \,d^{3} x +24C c \,d^{2} x -27D x \,c^{2} d +40A \,d^{3}-60B c \,d^{2}+72C \,c^{2} d -81D c^{3}\right)}{40 \left(x d +c \right)^{\frac{1}{3}} d^{4}}$
derivativedivides	$\frac{\frac{3 D \left(x d +c \right)^{\frac{8}{3}}}{8}+\frac{3 C d \left(x d +c \right)^{\frac{5}{3}}}{5}-\frac{9 D c \left(x d +c \right)^{\frac{5}{3}}}{5}+\frac{3 B \,d^{2} \left(x d +c \right)^{\frac{2}{3}}}{2}-3 C c d \left(x d +c \right)^{\frac{2}{3}}+\frac{9 D \,c^{2} \left(x d +c \right)^{\frac{2}{3}}}{2}-\frac{3 \left(A \,d^{3}-B c \,d^{2}+C \,c^{2} d -D c^{3}\right)}{\left(x d +c \right)^{\frac{1}{3}}}}{d^{4}}$
default	$\frac{\frac{3 D \left(x d +c \right)^{\frac{8}{3}}}{8}+\frac{3 C d \left(x d +c \right)^{\frac{5}{3}}}{5}-\frac{9 D c \left(x d +c \right)^{\frac{5}{3}}}{5}+\frac{3 B \,d^{2} \left(x d +c \right)^{\frac{2}{3}}}{2}-3 C c d \left(x d +c \right)^{\frac{2}{3}}+\frac{9 D \,c^{2} \left(x d +c \right)^{\frac{2}{3}}}{2}-\frac{3 \left(A \,d^{3}-B c \,d^{2}+C \,c^{2} d -D c^{3}\right)}{\left(x d +c \right)^{\frac{1}{3}}}}{d^{4}}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/40*((15*D*x^3+24*C*x^2+60*B*x-120*A)*d^3+180*(-3/20*D*x^2-2/5*C*x+B)*c*d
^2-216*(-3/8*D*x+C)*c^2*d+243*D*c^3)/(d*x+c)^(1/3)/d^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx = \frac{3\,(5\,Dd^3x^3+81\,Dc^3-72\,Cc^2d+60\,Bcd^2-40\,Ad^3-(9\,Dcd^2-8\,Cd^3)x^2+\cdots}{40\,(d^5x+cd^4)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
3/40*(5*D*d^3*x^3 + 81*D*c^3 - 72*C*c^2*d + 60*B*c*d^2 - 40*A*d^3 - (9*D*c
*d^2 - 8*C*d^3)*x^2 + (27*D*c^2*d - 24*C*c*d^2 + 20*B*d^3)*x)*(d*x + c)^(2
/3)/(d^5*x + c*d^4)
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx = \begin{cases} \frac{3\left(\frac{D(c+dx)^{\frac{8}{3}}}{8d^3}+\frac{(c+dx)^{\frac{5}{3}}(Cd-3Dc)}{5d^3}+\frac{(c+dx)^{\frac{2}{3}}\left(Bd^2-2Ccd+3Dc^2\right)}{2d^3}+\frac{-Ad^3+Bcd^2-Cc^2d+Dc^3}{d^3\sqrt[3]{c+dx}}\right)}{d} & \text{for } d \neq 0 \\ \frac{Ax+\frac{Bx^2}{2}+\frac{Cx^3}{3}+\frac{Dx^4}{4}}{c^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(4/3),x)
```

output
```
Piecewise((3*(D*(c + d*x)**(8/3)/(8*d**3) + (c + d*x)**(5/3)*(C*d - 3*D*c)
/(5*d**3) + (c + d*x)**(2/3)*(B*d**2 - 2*C*c*d + 3*D*c**2)/(2*d**3) + (-A*
d**3 + B*c*d**2 - C*c**2*d + D*c**3)/(d**3*(c + d*x)**(1/3)))/d, Ne(d, 0))
, ((A*x + B*x**2/2 + C*x**3/3 + D*x**4/4)/c**(4/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx = \frac{3\left(\frac{5\,(dx+c)^{\frac{8}{3}}D-8\,(3\,Dc-Cd)(dx+c)^{\frac{5}{3}}+20\,(3\,Dc^2-2\,Ccd+Bd^2)(dx+c)^{\frac{2}{3}}}{d^3}+\frac{40\,(Dc^3-Cc^2d+Bcd^2}{(dx+c)^{\frac{1}{3}}d^3}\right.}{40\,d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
3/40*((5*(d*x + c)^(8/3)*D - 8*(3*D*c - C*d)*(d*x + c)^(5/3) + 20*(3*D*c^2
 - 2*C*c*d + B*d^2)*(d*x + c)^(2/3))/d^3 + 40*(D*c^3 - C*c^2*d + B*c*d^2 -
 A*d^3)/((d*x + c)^(1/3)*d^3))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx = \frac{3\,(Dc^3-Cc^2d+Bcd^2-Ad^3)}{(dx+c)^{\frac{1}{3}}d^4} + \frac{3\left(5\,(dx+c)^{\frac{8}{3}}Dd^{28}-24\,(dx+c)^{\frac{5}{3}}Dcd^{28}+60\,(dx+c)^{\frac{2}{3}}Dc^2d^{28}+8\,(dx+c)^{\frac{5}{3}}Cd^{29}-40\,(dx+c)^{\frac{2}{3}}Ccd^{29}\right.}{40\,d^{32}}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
3*(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)/((d*x + c)^(1/3)*d^4) + 3/40*(5*(d*x
 + c)^(8/3)*D*d^28 - 24*(d*x + c)^(5/3)*D*c*d^28 + 60*(d*x + c)^(2/3)*D*c^
2*d^28 + 8*(d*x + c)^(5/3)*C*d^29 - 40*(d*x + c)^(2/3)*C*c*d^29 + 20*(d*x
+ c)^(2/3)*B*d^30)/d^32
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(c+d\,x)^{4/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(4/3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(c + d*x)^(4/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)^{4/3}}\,dx=\frac{\frac{3}{8}d^3x^3-\frac{3}{40}c\,d^2x^2+\frac{3}{2}b\,d^2x+\frac{9}{40}c^2dx-3a\,d^2+\frac{9}{2}bcd+\frac{27}{40}c^3}{(dx+c)^{\frac{1}{3}}\,d^3}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(4/3),x)
```

output
```
(3*( - 40*a*d**2 + 60*b*c*d + 20*b*d**2*x + 9*c**3 + 3*c**2*d*x - c*d**2*x
**2 + 5*d**3*x**3))/(40*(c + d*x)**(1/3)*d**3)
```

3.171 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx$

Optimal result 1643
Mathematica [A] (verified) 1644
Rubi [A] (verified) 1644
Maple [A] (verified) 1646
Fricas [B] (verification not implemented) 1647
Sympy [F] 1648
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Giac [A] (verification not implemented) 1649
Mupad [F(-1)] 1650
Reduce [B] (verification not implemented) 1650

Optimal result

Integrand size = 32, antiderivative size = 317

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx = \frac{3(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^3(bc-ad)\sqrt[3]{c+dx}} + \frac{3(bCd-2bcD-adD)(c+dx)^{2/3}}{2b^2d^3} + \frac{3D(c+dx)^{5/3}}{5bd^3} + \frac{\sqrt{3}(Ab^3-a(b^2B-abC+a^2D))\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{8/3}(bc-ad)^{4/3}} - \frac{(Ab^3-a(b^2B-abC+a^2D))\log(a+bx)}{2b^{8/3}(bc-ad)^{4/3}} + \frac{3(Ab^3-a(b^2B-abC+a^2D))\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{8/3}(bc-ad)^{4/3}}$$

output
```
3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^3/(-a*d+b*c)/(d*x+c)^(1/3)+3/2*(C*b*d-D*
a*d-2*D*b*c)*(d*x+c)^(2/3)/b^2/d^3+3/5*D*(d*x+c)^(5/3)/b/d^3+3^(1/2)*(A*b^
3-a*(B*b^2-C*a*b+D*a^2))*arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/(-a*d+b*c)^
(1/3))*3^(1/2))/b^(8/3)/(-a*d+b*c)^(4/3)-1/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))
*ln(b*x+a)/b^(8/3)/(-a*d+b*c)^(4/3)+3/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*ln((
-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/b^(8/3)/(-a*d+b*c)^(4/3)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx = \frac{\frac{3b^{2/3}\left(-5a^2d^2D(c+dx)+abd(c+dx)(5Cd-3cD+2dDx)+b^2\left(-10Ad^3+18c^3D+c^2(-15Cd+6dDx)+cd^2\cdots\right.\right.}{d^3(-bc+ad)\sqrt[3]{c+dx}}}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^(4/3)),x]
```

output
```
((3*b^(2/3)*(-5*a^2*d^2*D*(c + d*x) + a*b*d*(c + d*x)*(5*C*d - 3*c*D + 2*d
*D*x) + b^2*(-10*A*d^3 + 18*c^3*D + c^2*(-15*C*d + 6*d*D*x) + c*d^2*(10*B
- 5*C*x - 2*D*x^2))))/(d^3*(-(b*c) + a*d)*(c + d*x)^(1/3)) + (10*Sqrt[3]*(
A*b^3 - a*(b^2*B - a*b*C + a^2*D))*ArcTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))
/(-(b*c) + a*d)^(1/3))/Sqrt[3]])/(-(b*c) + a*d)^(4/3) + (10*(A*b^3 - a*(b^
2*B - a*b*C + a^2*D))*Log[(-(b*c) + a*d)^(1/3) + b^(1/3)*(c + d*x)^(1/3)])
/(-(b*c) + a*d)^(4/3) + (5*(-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))*Log[(-(b
*c) + a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*
(c + d*x)^(2/3)])/(-(b*c) + a*d)^(4/3))/(10*b^(8/3))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{Ab^3 - a\left(a^2D - abC + b^2B\right)}{b^3(a+bx)(c+dx)^{4/3}} + \frac{a^2d^2D - abd(Cd - cD) - \left(b^2\left(-Bd^2 + c^2(-D) + cCd\right)\right)}{b^3d^2(c+dx)^{4/3}} + \frac{-adD - 2bcD + \cdots}{b^2d^2\sqrt[3]{c+d\cdots}} \right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{3}\left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right)\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{8/3}(bc-ad)^{4/3}} + \frac{3\left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right)}{b^3\sqrt[3]{c+dx}(bc-ad)} - \frac{\log(a+bx)\left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right)}{2b^{8/3}(bc-ad)^{4/3}} + \frac{3\left(Ab^3 - a\left(a^2D - abC + b^2B\right)\right)\log\left(\sqrt[3]{bc-ad} - \sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{8/3}(bc-ad)^{4/3}} - \frac{3\left(a^2d^2D - abd(Cd - cD) - \left(b^2\left(-Bd^2 + c^2(-D) + cCd\right)\right)\right)}{b^3d^3\sqrt[3]{c+dx}} + \frac{3(c+dx)^{2/3}(-adD - 2bcD + bCd)}{2b^2d^3} + \frac{3D(c+dx)^{5/3}}{5bd^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)*(c + d*x)^(4/3)),x]
```

output
```
(3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D)))/(b^3*(b*c - a*d)*(c + d*x)^(1/3))
- (3*(a^2*d^2*D - a*b*d*(C*d - c*D) - b^2*(c*C*d - B*d^2 - c^2*D)))/(b^3*d
^3*(c + d*x)^(1/3)) + (3*(b*C*d - 2*b*c*D - a*d*D)*(c + d*x)^(2/3))/(2*b^2
*d^3) + (3*D*(c + d*x)^(5/3))/(5*b*d^3) + (Sqrt[3]*(A*b^3 - a*(b^2*B - a*b
*C + a^2*D))*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]])/(b^(8/3)*(b*c - a*d)^(4/3)) - ((A*b^3 - a*(b^2*B - a*b*C + a^2*D))
*Log[a + b*x])/(2*b^(8/3)*(b*c - a*d)^(4/3)) + (3*(A*b^3 - a*(b^2*B - a*b*
C + a^2*D))*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(8/3)*(
b*c - a*d)^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{3\left(\frac{D(xd+c)^{\frac{5}{3}}b}{5}+\frac{(Cbd-Dad-2Dbc)(xd+c)^{\frac{2}{3}}}{2}\right)}{b^2}-\frac{3\left(A d^3-Bc d^2+C c^2 d-Dc^3\right)}{(ad-bc)(xd+c)^{\frac{1}{3}}}-3\left(-\frac{\ln\left((xd+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((xd+c)^{\frac{2}{3}}\cdots\right.}{\cdots}\cdots\right)}{d^3}$
default	$\frac{\frac{3\left(\frac{D(xd+c)^{\frac{5}{3}}b}{5}+\frac{(Cbd-Dad-2Dbc)(xd+c)^{\frac{2}{3}}}{2}\right)}{b^2}-\frac{3\left(A d^3-Bc d^2+C c^2 d-Dc^3\right)}{(ad-bc)(xd+c)^{\frac{1}{3}}}-3\left(-\frac{\ln\left((xd+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((xd+c)^{\frac{2}{3}}\cdots\right.}{\cdots}\cdots\right)}{d^3}$
pseudoelliptic	$-\frac{\frac{\ln\left((xd+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(xd+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)\left(b^3A-a\,b^2B+a^2bC-a^3D\right)d^3(xd+c)^{\frac{1}{3}}}{2}+\sqrt{3}\,(xd+c)^{\frac{1}{3}}d^3\left(b^3A-a\,b^2B+a^2\cdots\right.}{\cdots}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/d^3*(1/b^2*(1/5*D*(d*x+c)^(5/3)*b+1/2*(C*b*d-D*a*d-2*D*b*c)*(d*x+c)^(2/3
))-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)/(d*x+c)^(1/3)-(-1/3/b/((a*d-b*c
)/b)^(1/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))+1/6/b/((a*d-b*c)/b)^(1/3)
*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/b)^(2/3))+1
/3*3^(1/2)/b/((a*d-b*c)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/b)^(1/3)
*(d*x+c)^(1/3)-1)))*d^3*(A*b^3-B*a*b^2+C*a^2*b-D*a^3)/(a*d-b*c)/b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(276) = 552$.

Time = 0.13 (sec) , antiderivative size = 1671, normalized size of antiderivative = 5.27

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)(c + dx)^{4/3}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
[-1/10*(5*sqrt(3)*((D*a^3*b^2 - C*a^2*b^3 + B*a*b^4 - A*b^5)*c^2*d^3 - (D*
a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*c*d^4 + ((D*a^3*b^2 - C*a^2*b^3 +
 B*a*b^4 - A*b^5)*c*d^4 - (D*a^4*b - C*a^3*b^2 + B*a^2*b^3 - A*a*b^4)*d^5)
*x)*sqrt((-b^3*c + a*b^2*d)^(1/3)/(b*c - a*d))*log((2*b^2*d*x + 3*b^2*c -
a*b*d + sqrt(3)*((-b^3*c + a*b^2*d)^(1/3)*(b*c - a*d) - (b^2*c - a*b*d)*(d
*x + c)^(1/3) + 2*(-b^3*c + a*b^2*d)^(2/3)*(d*x + c)^(2/3))*sqrt((-b^3*c +
 a*b^2*d)^(1/3)/(b*c - a*d)) - 3*(-b^3*c + a*b^2*d)^(2/3)*(d*x + c)^(1/3))
/(b*x + a)) - 5*((D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^4*x + (D*a^3 - C*a^
2*b + B*a*b^2 - A*b^3)*c*d^3)*(-b^3*c + a*b^2*d)^(2/3)*log((d*x + c)^(2/3)
*b^2 - (-b^3*c + a*b^2*d)^(1/3)*(d*x + c)^(1/3)*b + (-b^3*c + a*b^2*d)^(2/
3)) + 10*((D*a^3 - C*a^2*b + B*a*b^2 - A*b^3)*d^4*x + (D*a^3 - C*a^2*b + B
*a*b^2 - A*b^3)*c*d^3)*(-b^3*c + a*b^2*d)^(2/3)*log((d*x + c)^(1/3)*b + (-
b^3*c + a*b^2*d)^(1/3)) + 3*(18*D*b^5*c^4 + 10*A*a*b^4*d^4 - 3*(7*D*a*b^4
+ 5*C*b^5)*c^3*d - 2*(D*a^2*b^3 - 10*C*a*b^4 - 5*B*b^5)*c^2*d^2 + 5*(D*a^3
*b^2 - C*a^2*b^3 - 2*B*a*b^4 - 2*A*b^5)*c*d^3 - 2*(D*b^5*c^2*d^2 - 2*D*a*b
^4*c*d^3 + D*a^2*b^3*d^4)*x^2 + (6*D*b^5*c^3*d - (7*D*a*b^4 + 5*C*b^5)*c^2
*d^2 - 2*(2*D*a^2*b^3 - 5*C*a*b^4)*c*d^3 + 5*(D*a^3*b^2 - C*a^2*b^3)*d^4)*
x)*(d*x + c)^(2/3))/(b^6*c^3*d^3 - 2*a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + (b^6*
c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x), -1/10*(10*sqrt(3)*((D*a^3*b^2 -
 C*a^2*b^3 + B*a*b^4 - A*b^5)*c^2*d^3 - (D*a^4*b - C*a^3*b^2 + B*a^2*b^...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)\,(c+dx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(4/3),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((a + b*x)*(c + d*x)**(4/3)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx = -\frac{\left(Da^3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}} - Ca^2b\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}} + Bab^2\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}} - Ab^3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\log\left(\left|(dx+c)^{\frac{1}{3}} - \left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right|\right)}{b^4c^2 - 2\,ab^3cd + a^2b^2d^2}$$
$$- \frac{\left(\sqrt{3}Da^3 - \sqrt{3}Ca^2b + \sqrt{3}Bab^2 - \sqrt{3}Ab^3\right)\arctan\left(\frac{\sqrt{3}\left(2\,(dx+c)^{\frac{1}{3}} + \left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{(b^3c-ab^2d)^{\frac{4}{3}}}$$
$$+ \frac{(Da^3 - Ca^2b + Bab^2 - Ab^3)\log\left((dx+c)^{\frac{2}{3}} + (dx+c)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}} + \left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\,(b^3c-ab^2d)^{\frac{4}{3}}}$$
$$- \frac{3\,(Dc^3 - Cc^2d + Bcd^2 - Ad^3)}{(bcd^3 - ad^4)(dx+c)^{\frac{1}{3}}}$$
$$+ \frac{3\left(2\,(dx+c)^{\frac{5}{3}}Db^4d^{12} - 10\,(dx+c)^{\frac{2}{3}}Db^4cd^{12} - 5\,(dx+c)^{\frac{2}{3}}Dab^3d^{13} + 5\,(dx+c)^{\frac{2}{3}}Cb^4d^{13}\right)}{10\,b^5d^{15}}$$

input

```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(4/3),x, algorithm="giac")
```

output

```
-(D*a^3*((b*c - a*d)/b)^(1/3) - C*a^2*b*((b*c - a*d)/b)^(1/3) + B*a*b^2*((
b*c - a*d)/b)^(1/3) - A*b^3*((b*c - a*d)/b)^(1/3))*((b*c - a*d)/b)^(1/3)*l
og(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b^4*c^2 - 2*a*b^3*c*d +
a^2*b^2*d^2) - (sqrt(3)*D*a^3 - sqrt(3)*C*a^2*b + sqrt(3)*B*a*b^2 - sqrt(3
)*A*b^3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a*d)/b)^(1/3))/((
b*c - a*d)/b)^(1/3))/(b^3*c - a*b^2*d)^(4/3) + 1/2*(D*a^3 - C*a^2*b + B*a*
b^2 - A*b^3)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) +
 ((b*c - a*d)/b)^(2/3))/(b^3*c - a*b^2*d)^(4/3) - 3*(D*c^3 - C*c^2*d + B*c
*d^2 - A*d^3)/((b*c*d^3 - a*d^4)*(d*x + c)^(1/3)) + 3/10*(2*(d*x + c)^(5/3
)*D*b^4*d^12 - 10*(d*x + c)^(2/3)*D*b^4*c*d^12 - 5*(d*x + c)^(2/3)*D*a*b^3
*d^13 + 5*(d*x + c)^(2/3)*C*b^4*d^13)/(b^5*d^15)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)\,\left(c+d\,x\right)^{4/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(4/3)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)*(c + d*x)^(4/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)(c+dx)^{4/3}}\,dx=\frac{-10(dx+c)^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}(ad-bc)^{\frac{1}{6}}\sqrt{3}-2b^{\frac{1}{3}}(dx+c)^{\frac{1}{6}}}{b^{\frac{1}{6}}(ad-bc)^{\frac{1}{6}}}\right)a^2d^2-10(dx+c)^{\frac{1}{3}}\sqrt{3}\,at}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(4/3),x)
```

output
```
( - 10*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3)
 - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*d**2 -
10*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*
b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*d**2 - 15*b
**(2/3)*(a*d - b*c)**(1/3)*a*c*d - 15*b**(2/3)*(a*d - b*c)**(1/3)*a*d**2*x
 - 30*b**(2/3)*(a*d - b*c)**(1/3)*b**2*d - 9*b**(2/3)*(a*d - b*c)**(1/3)*b
*c**2 - 3*b**(2/3)*(a*d - b*c)**(1/3)*b*c*d*x + 6*b**(2/3)*(a*d - b*c)**(1
/3)*b*d**2*x**2 - 10*(c + d*x)**(1/3)*log((a*d - b*c)**(1/3) + b**(1/3)*(c
 + d*x)**(1/3))*a**2*d**2 + 5*(c + d*x)**(1/3)*log( - b**(1/6)*(c + d*x)**
(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + (a*d - b*c)**(1/3) + b**(1/3)*(c + d*x)
**(1/3))*a**2*d**2 + 5*(c + d*x)**(1/3)*log(b**(1/6)*(c + d*x)**(1/6)*(a*d
 - b*c)**(1/6)*sqrt(3) + (a*d - b*c)**(1/3) + b**(1/3)*(c + d*x)**(1/3))*a
**2*d**2)/(10*b**(2/3)*(c + d*x)**(1/3)*(a*d - b*c)**(1/3)*b**2*d**2)
```

3.172 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx$

Optimal result . . . 1651
Mathematica [A] (verified) . . . 1652
Rubi [A] (verified) . . . 1653
Maple [A] (verified) . . . 1655
Fricas [B] (verification not implemented) . . . 1656
Sympy [F(-1)] . . . 1656
Maxima [F(-2)] . . . 1657
Giac [B] (verification not implemented) . . . 1657
Mupad [F(-1)] . . . 1658
Reduce [B] (verification not implemented) . . . 1659

Optimal result

Integrand size = 32, antiderivative size = 414

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx &= -\frac{3(c^2Cd-Bcd^2+Ad^3-c^3D)}{d^2(bc-ad)^2\sqrt[3]{c+dx}} \\ &+ \frac{3D(c+dx)^{2/3}}{2b^2d^2} - \frac{(Ab^3-a(b^2B-abC+a^2D))\,(c+dx)^{2/3}}{b^2(bc-ad)^2(a+bx)} \\ &+ \frac{(b^3(3Bc-4Ad)-ab^2(6cC-Bd)-5a^3dD+a^2b(2Cd+9cD))\arctan\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}b^{8/3}(bc-ad)^{7/3}} \\ &- \frac{(b^3(3Bc-4Ad)-ab^2(6cC-Bd)-5a^3dD+a^2b(2Cd+9cD))\log(a+bx)}{6b^{8/3}(bc-ad)^{7/3}} \\ &+ \frac{(b^3(3Bc-4Ad)-ab^2(6cC-Bd)-5a^3dD+a^2b(2Cd+9cD))\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{8/3}(bc-ad)^{7/3}}\end{aligned}$$

output
```
(-3*A*d^3+3*B*c*d^2-3*C*c^2*d+3*D*c^3)/d^2/(-a*d+b*c)^2/(d*x+c)^(1/3)+3/2*
D*(d*x+c)^(2/3)/b^2/d^2-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(2/3)/b^2/(-
a*d+b*c)^2/(b*x+a)+1/3*(b^3*(-4*A*d+3*B*c)-a*b^2*(-B*d+6*C*c)-5*a^3*d*D+a^
2*b*(2*C*d+9*D*c))*arctan(1/3*(1+2*b^(1/3)*(d*x+c)^(1/3)/(-a*d+b*c)^(1/3))
*3^(1/2))*3^(1/2)/b^(8/3)/(-a*d+b*c)^(7/3)-1/6*(b^3*(-4*A*d+3*B*c)-a*b^2*(
-B*d+6*C*c)-5*a^3*d*D+a^2*b*(2*C*d+9*D*c))*ln(b*x+a)/b^(8/3)/(-a*d+b*c)^(7
/3)+1/2*(b^3*(-4*A*d+3*B*c)-a*b^2*(-B*d+6*C*c)-5*a^3*d*D+a^2*b*(2*C*d+9*D*
c))*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3))/b^(8/3)/(-a*d+b*c)^(7/3)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx=\frac{\frac{3b^{2/3}\left(5a^3d^2D(c+dx)+a^2bd(c+dx)(-2Cd-6cD+3dDx)+b^3\left(-2Ad^2(c+4dx)+3cx\left(-2cCd+2Bd^2+3c\right.\right.\right.}{d^2(bc-ad)^2(a+bx)\sqrt[3]{c+}}}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^(4/3)),x]
```

output
```
((3*b^(2/3)*(5*a^3*d^2*D*(c + d*x) + a^2*b*d*(c + d*x)*(-2*C*d - 6*c*D + 3
*d*D*x) + b^3*(-2*A*d^2*(c + 4*d*x) + 3*c*x*(-2*c*C*d + 2*B*d^2 + 3*c^2*D
+ c*d*D*x)) + a*b^2*(9*c^3*D + 2*d^3*(-3*A + B*x) - 3*c^2*d*(2*C + D*x) +
2*c*d^2*(4*B - 3*D*x^2))))/(d^2*(b*c - a*d)^2*(a + b*x)*(c + d*x)^(1/3)) -
 (2*Sqrt[3]*(b^3*(3*B*c - 4*A*d) + a*b^2*(-6*c*C + B*d) - 5*a^3*d*D + a^2*
b*(2*C*d + 9*c*D))*ArcTan[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^
(1/3))/Sqrt[3]])/(-(b*c) + a*d)^(7/3) - (2*(b^3*(3*B*c - 4*A*d) + a*b^2*(-
6*c*C + B*d) - 5*a^3*d*D + a^2*b*(2*C*d + 9*c*D))*Log[(-(b*c) + a*d)^(1/3)
 + b^(1/3)*(c + d*x)^(1/3)])/(-(b*c) + a*d)^(7/3) + ((b^3*(3*B*c - 4*A*d)
+ a*b^2*(-6*c*C + B*d) - 5*a^3*d*D + a^2*b*(2*C*d + 9*c*D))*Log[(-(b*c) +
a*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d
*x)^(2/3)])/(-(b*c) + a*d)^(7/3))/(6*b^(8/3))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2124, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{dDa^3-b(Cd-3cD)a^2-b^2(3cC-Bd)a+b^3(3Bc-4Ad)}{b^3}}{3(a+bx)(c+dx)^{4/3}}dx}{bc-ad}-\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt[3]{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\frac{dDa^3}{b^3}-\frac{(Cd-3cD)a^2}{b^2}-\frac{(3cC-Bd)a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-4Ad+\frac{3(bc-ad)(bC-aD)x}{b^2}}{(a+bx)(c+dx)^{4/3}}dx}{3(bc-ad)}-\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt[3]{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 1195$$

$$\frac{\int \left(\frac{3(bc-ad)D}{b^2d\sqrt[3]{c+dx}}+\frac{3(bc-ad)(bCd-2aDd-bcD)}{b^3d(c+dx)^{4/3}}+\frac{-5dDa^3+b(2Cd+9cD)a^2-b^2(6cC-Bd)a+b^3(3Bc-4Ad)}{b^3(a+bx)(c+dx)^{4/3}}\right)dx}{3(bc-ad)}-\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt[3]{c+dx}(bc-ad)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)(-5a^3dD+a^2b(9cD+2Cd)-ab^2(6cC-Bd)+b^3(3Bc-4Ad))}{b^{8/3}(bc-ad)^{4/3}}+\frac{3(-5a^3dD+a^2b(9cD+2Cd)-ab^2(6cC-Bd)\cdots}{b^3\sqrt[3]{c+dx}(bc-ad)}}{}$$

$$\frac{A-\frac{a(a^2D-abC+b^2B)}{b^3}}{(a+bx)\sqrt[3]{c+dx}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^2*(c + d*x)^(4/3)),x]
```

output
```
-((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)/((b*c - a*d)*(a + b*x)*(c + d*x)^(
1/3))) + ((-9*(b*c - a*d)*(b*C*d - b*c*D - 2*a*d*D))/(b^3*d^2*(c + d*x)^(1
/3)) + (3*(b^3*(3*B*c - 4*A*d) - a*b^2*(6*c*C - B*d) - 5*a^3*d*D + a^2*b*(
2*C*d + 9*c*D)))/(b^3*(b*c - a*d)*(c + d*x)^(1/3)) + (9*(b*c - a*d)*D*(c +
 d*x)^(2/3))/(2*b^2*d^2) + (Sqrt[3]*(b^3*(3*B*c - 4*A*d) - a*b^2*(6*c*C -
B*d) - 5*a^3*d*D + a^2*b*(2*C*d + 9*c*D))*ArcTan[(1 + (2*b^(1/3)*(c + d*x)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b^(8/3)*(b*c - a*d)^(4/3)) - ((b^3*(
3*B*c - 4*A*d) - a*b^2*(6*c*C - B*d) - 5*a^3*d*D + a^2*b*(2*C*d + 9*c*D))*
Log[a + b*x])/(2*b^(8/3)*(b*c - a*d)^(4/3)) + (3*(b^3*(3*B*c - 4*A*d) - a*
b^2*(6*c*C - B*d) - 5*a^3*d*D + a^2*b*(2*C*d + 9*c*D))*Log[(b*c - a*d)^(1/
3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(8/3)*(b*c - a*d)^(4/3)))/(3*(b*c - a*
d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{3D(xd+c)^{\frac{2}{3}}}{2b^2}-\frac{3d^2\left(\frac{\left(\frac{1}{3}b^3dA-\frac{1}{3}Ba\,b^2d+\frac{1}{3}C\,a^2bd-\frac{1}{3}a^3dD\right)(xd+c)^{\frac{2}{3}}}{(xd+c)b+ad-bc}+\left(\frac{4}{3}b^3dA-B\,b^3c-\frac{2}{3}C\,a^2bd+2a\,b^2cC+\frac{5}{3}a^3dD-3a^2bcD-\frac{1}{3}\right.\right.}{}}{}$
default	$\frac{\frac{3D(xd+c)^{\frac{2}{3}}}{2b^2}-\frac{3d^2\left(\frac{\left(\frac{1}{3}b^3dA-\frac{1}{3}Ba\,b^2d+\frac{1}{3}C\,a^2bd-\frac{1}{3}a^3dD\right)(xd+c)^{\frac{2}{3}}}{(xd+c)b+ad-bc}+\left(\frac{4}{3}b^3dA-B\,b^3c-\frac{2}{3}C\,a^2bd+2a\,b^2cC+\frac{5}{3}a^3dD-3a^2bcD-\frac{1}{3}\right.\right.}{}}{}$
pseudoelliptic	$-\frac{2\left(\frac{9\left(\left(\frac{4Ab^3x}{3}+a\left(-\frac{Bx}{3}+A\right)b^2+\frac{x\,a^2\left(-\frac{3Dx}{2}+C\right)b}{3}-\frac{5Da^3x}{6}\right)d^3+\frac{\left((-3Bx+A)b^3-4\left(-\frac{3Dx^2}{4}+B\right)a\,b^2+a^2\left(\frac{3Dx}{2}+C\right)b-\frac{5a}{}\right.}{3}\right.}{2}\right.}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/d^2*(1/2*D*(d*x+c)^(2/3)/b^2-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/(a*d-b*c)^2/(
d*x+c)^(1/3)-1/(a*d-b*c)^2*d^2/b^2*((1/3*b^3*d*A-1/3*B*a*b^2*d+1/3*C*a^2*b
*d-1/3*a^3*d*D)*(d*x+c)^(2/3)/((d*x+c)*b+a*d-b*c)+(4/3*b^3*d*A-B*b^3*c-2/3
*C*a^2*b*d+2*a*b^2*c*C+5/3*a^3*d*D-3*a^2*b*c*D-1/3*B*a*b^2*d)*(-1/3/b/((a*
d-b*c)/b)^(1/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))+1/6/b/((a*d-b*c)/b)^
(1/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/b)^(2/
3))+1/3*3^(1/2)/b/((a*d-b*c)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/b)^
(1/3)*(d*x+c)^(1/3)-1)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1532 vs. $2(379) = 758$.

Time = 0.58 (sec) , antiderivative size = 3152, normalized size of antiderivative = 7.61

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(4/3),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(4/3),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^{4/3}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(4/3),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(379) = 758.

Time = 0.22 (sec) , antiderivative size = 813, normalized size of antiderivative = 1.96

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^2(c + dx)^{4/3}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
1/3*(9*D*a^2*b*c*((b*c - a*d)/b)^(1/3) - 6*C*a*b^2*c*((b*c - a*d)/b)^(1/3)
 + 3*B*b^3*c*((b*c - a*d)/b)^(1/3) - 5*D*a^3*d*((b*c - a*d)/b)^(1/3) + 2*C
*a^2*b*d*((b*c - a*d)/b)^(1/3) + B*a*b^2*d*((b*c - a*d)/b)^(1/3) - 4*A*b^3
*d*((b*c - a*d)/b)^(1/3))*((b*c - a*d)/b)^(1/3)*log(abs((d*x + c)^(1/3) -
((b*c - a*d)/b)^(1/3)))/(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b
^2*d^3) - (4*A*b^3*d - (3*b^3*c + a*b^2*d)*B + 2*(3*a*b^2*c - a^2*b*d)*C -
 (9*a^2*b*c - 5*a^3*d)*D)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c -
a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/((sqrt(3)*b^4*c^2 - 2*sqrt(3)*a*b^3*
c*d + sqrt(3)*a^2*b^2*d^2)*(b^3*c - a*b^2*d)^(1/3)) + 1/6*(4*A*b^3*d - (3*
b^3*c + a*b^2*d)*B + 2*(3*a*b^2*c - a^2*b*d)*C - (9*a^2*b*c - 5*a^3*d)*D)*
log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d)
/b)^(2/3))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(b^3*c - a*b^2*d)^(1/3))
 + (3*(d*x + c)*D*b^3*c^3 - 3*D*b^3*c^4 - 3*(d*x + c)*C*b^3*c^2*d + 3*D*a*
b^2*c^3*d + 3*C*b^3*c^3*d + 3*(d*x + c)*B*b^3*c*d^2 - 3*C*a*b^2*c^2*d^2 -
3*B*b^3*c^2*d^2 + (d*x + c)*D*a^3*d^3 - (d*x + c)*C*a^2*b*d^3 + (d*x + c)*
B*a*b^2*d^3 - 4*(d*x + c)*A*b^3*d^3 + 3*B*a*b^2*c*d^3 + 3*A*b^3*c*d^3 - 3*
A*a*b^2*d^4)/((b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*((d*x + c)^(4/3)
*b - (d*x + c)^(1/3)*b*c + (d*x + c)^(1/3)*a*d)) + 3/2*(d*x + c)^(2/3)*D/(
b^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{{\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^{4/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(4/3)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^2*(c + d*x)^(4/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1910, normalized size of antiderivative = 4.61

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^2(c+dx)^{4/3}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(4/3),x)
```

output
```
(10*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2
*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**3*d**2 - 12*
(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**
(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b*c*d + 10*(c
+ d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/
3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b*d**2*x + 6*(c +
 d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3
)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*d - 12*(c + d*x)
**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c
+ d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**2*c*d*x + 6*(c + d*x)**
(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c +
d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*b**4*d*x + 10*(c + d*x)**(1/3)
*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)*
*(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**3*d**2 - 12*(c + d*x)**(1/3)*sqr
t(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/
6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b*c*d + 10*(c + d*x)**(1/3)*sqrt(3
)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/6))
/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b*d**2*x + 6*(c + d*x)**(1/3)*sqrt(3)
*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) + 2*b**(1/3)*(c + d*x)**(1/6))/
(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*d - 12*(c + d*x)**(1/3)*sqrt(3)*a...
```

3.173 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{4/3}}\,dx$

Optimal result 1660
Mathematica [A] (verified) 1661
Rubi [A] (verified) 1662
Maple [A] (verified) 1668
Fricas [B] (verification not implemented) 1670
Sympy [F(-1)] 1670
Maxima [F(-2)] 1671
Giac [B] (verification not implemented) 1671
Mupad [F(-1)] 1672
Reduce [B] (verification not implemented) 1673

Optimal result

Integrand size = 32, antiderivative size = 541

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{4/3}}\,dx = \frac{3(c^2Cd-Bcd^2+Ad^3-c^3D)}{d(bc-ad)^3\sqrt[3]{c+dx}}$$
$$-\frac{(Ab^3-a(b^2B-abC+a^2D))\,(c+dx)^{2/3}}{2b^2(bc-ad)^2(a+bx)^2}$$
$$-\frac{(b^3(3Bc-5Ad)-ab^2(6cC-2Bd)-4a^3dD+a^2b(Cd+9cD))\,(c+dx)^{2/3}}{3b^2(bc-ad)^3(a+bx)}$$
$$+\frac{(b^3(9c^2C-12Bcd+14Ad^2)-5a^3d^2D-a^2bd(Cd-18cD)+ab^2(6cCd-2Bd^2-27c^2D))\arctan\left(\frac{1+^2}{}\right.}{3\sqrt{3}b^{8/3}(bc-ad)^{10/3}}$$
$$-\frac{(b^3(9c^2C-12Bcd+14Ad^2)-5a^3d^2D-a^2bd(Cd-18cD)+ab^2(6cCd-2Bd^2-27c^2D))\log(a+bx)}{18b^{8/3}(bc-ad)^{10/3}}$$
$$+\frac{(b^3(9c^2C-12Bcd+14Ad^2)-5a^3d^2D-a^2bd(Cd-18cD)+ab^2(6cCd-2Bd^2-27c^2D))\log\left(\sqrt[3]{bc-a}\right.}{6b^{8/3}(bc-ad)^{10/3}}$$

output
```
3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d/(-a*d+b*c)^3/(d*x+c)^(1/3)-1/2*(A*b^3-a*
(B*b^2-C*a*b+D*a^2))*(d*x+c)^(2/3)/b^2/(-a*d+b*c)^2/(b*x+a)^2-1/3*(b^3*(-5
*A*d+3*B*c)-a*b^2*(-2*B*d+6*C*c)-4*a^3*d*D+a^2*b*(C*d+9*D*c))*(d*x+c)^(2/3
)/b^2/(-a*d+b*c)^3/(b*x+a)+1/9*(b^3*(14*A*d^2-12*B*c*d+9*C*c^2)-5*a^3*d^2*
D-a^2*b*d*(C*d-18*D*c)+a*b^2*(-2*B*d^2+6*C*c*d-27*D*c^2))*arctan(1/3*(1+2*
b^(1/3)*(d*x+c)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))*3^(1/2)/b^(8/3)/(-a*d+b*c
)^(10/3)-1/18*(b^3*(14*A*d^2-12*B*c*d+9*C*c^2)-5*a^3*d^2*D-a^2*b*d*(C*d-18
*D*c)+a*b^2*(-2*B*d^2+6*C*c*d-27*D*c^2))*ln(b*x+a)/b^(8/3)/(-a*d+b*c)^(10/
3)+1/6*(b^3*(14*A*d^2-12*B*c*d+9*C*c^2)-5*a^3*d^2*D-a^2*b*d*(C*d-18*D*c)+a
*b^2*(-2*B*d^2+6*C*c*d-27*D*c^2))*ln((-a*d+b*c)^(1/3)-b^(1/3)*(d*x+c)^(1/3
))/b^(8/3)/(-a*d+b*c)^(10/3)
```

Mathematica [A] (verified)

Time = 4.07 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{4/3}}\,dx = \frac{\frac{3b^{2/3}(-5a^4d^2D(c+dx)-a^3bd(c+dx)(Cd-15cD+8dDx)+6b^4cx(3c(-Cd+cD)x+Bd(c+4dx))+ab^3(-}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^(4/3)),x]
```

output
```
((3*b^(2/3)*(-5*a^4*d^2*D*(c + d*x) - a^3*b*d*(c + d*x)*(C*d - 15*c*D + 8*
d*D*x) + 6*b^4*c*x*(3*c*(-(C*d) + c*D)*x + B*d*(c + 4*d*x)) + a*b^3*(-12*c
*x*(4*c*C*d - 3*c^2*D + C*d^2*x) + B*d*(3*c^2 + 43*c*d*x + 4*d^2*x^2)) - A
*b^2*d*(18*a^2*d^2 + a*b*d*(13*c + 49*d*x) + b^2*(-3*c^2 + 7*c*d*x + 28*d^
2*x^2)) + a^2*b^2*(18*c^3*D + d^3*x*(7*B + 2*C*x) - 9*c^2*d*(3*C - 2*D*x)
+ c*d^2*(25*B - 7*C*x + 18*D*x^2))))/(d*(-(b*c) + a*d)^3*(a + b*x)^2*(c +
d*x)^(1/3)) + (2*Sqrt[3]*(b^3*(9*c^2*C - 12*B*c*d + 14*A*d^2) - 5*a^3*d^2*
D + a^2*b*d*(-(C*d) + 18*c*D) + a*b^2*(6*c*C*d - 2*B*d^2 - 27*c^2*D))*ArcT
an[(1 - (2*b^(1/3)*(c + d*x)^(1/3))/(-(b*c) + a*d)^(1/3))/Sqrt[3]])/(-(b*c
) + a*d)^(10/3) + (2*(b^3*(9*c^2*C - 12*B*c*d + 14*A*d^2) - 5*a^3*d^2*D +
a^2*b*d*(-(C*d) + 18*c*D) + a*b^2*(6*c*C*d - 2*B*d^2 - 27*c^2*D))*Log[(-(b
*c) + a*d)^(1/3) + b^(1/3)*(c + d*x)^(1/3)])/(-(b*c) + a*d)^(10/3) - ((b^3
*(9*c^2*C - 12*B*c*d + 14*A*d^2) - 5*a^3*d^2*D + a^2*b*d*(-(C*d) + 18*c*D)
 + a*b^2*(6*c*C*d - 2*B*d^2 - 27*c^2*D))*Log[(-(b*c) + a*d)^(2/3) - b^(1/3
)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^(2/3)])/(-(b*c)
 + a*d)^(10/3))/(18*b^(8/3))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2124, 27, 1193, 27, 87, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{4/3}}dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{\int -\frac{6\left(c-\frac{ad}{b}\right)Dx^2+\frac{6(bc-ad)(bC-aD)x}{b^2}+\frac{dDa^3-b(Cd-6cD)a^2-b^2(6cC-Bd)a+b^3(6Bc-7Ad)}{b^3}}{3(a+bx)^2(c+dx)^{4/3}}dx}{2(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{dDa^3}{b^3}-\frac{(Cd-6cD)a^2}{b^2}-\frac{(6cC-Bd)a}{b}+6\left(c-\frac{ad}{b}\right)Dx^2+6Bc-7Ad+\frac{6(bc-ad)(bC-aD)x}{b^2}}{(a+bx)^2(c+dx)^{4/3}}dx}{6(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

↓ 1193

$$-\frac{-\frac{\int \frac{2\left(-4d^2Da^3+bCd^2a^2-2b^2\left(-9Dc^2+3Cdc-Bd^2\right)a-b^3\left(9Cc^2-12Bdc+14Ad^2\right)-9b(bc-ad)^2Dx\right)}{3b^3(a+bx)(c+dx)^{4/3}}dx}{bc-ad}-\frac{-11a^3dD+a^2b(18cD+5Cd)-ab^2(12cC-Bd)}{b^3(a+bx)\sqrt[3]{c+dx}(bc-ad)}}{6(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

↓ 27

$$-\frac{-\frac{2\int \frac{-4d^2Da^3+bCd^2a^2-2b^2\left(-9Dc^2+3Cdc-Bd^2\right)a-b^3\left(9Cc^2-12Bdc+14Ad^2\right)-9b(bc-ad)^2Dx}{(a+bx)(c+dx)^{4/3}}dx}{3b^3(bc-ad)}-\frac{-11a^3dD+a^2b(18cD+5Cd)-ab^2(12cC-Bd)+}{b^3(a+bx)\sqrt[3]{c+dx}(bc-ad)}}{6(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

↓ 87

$$-\frac{-\frac{2\left(-\frac{b\left(-5a^3d^2D-a^2bd(Cd-18cD)+ab^2\left(-2Bd^2-27c^2D+6cCd\right)+b^3\left(14Ad^2-12Bcd+9c^2C\right)\right)\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}dx}{bc-ad}-\frac{3\left(4a^3d^3D-a^2bd^2(9cD+Cd)+2ab\right.}{d}\right.}{3b^3(bc-ad)}}{6(bc-ad)}-\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

↓ 67

$$-\frac{2\left(-\frac{b\left(-5a^3d^2D-a^2bd(Cd-18cD)+ab^2\left(-2Bd^2-27c^2D+6cCd\right)+b^3\left(14Ad^2-12Bcd+9c^2C\right)\right)\left(-\frac{3\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{b}}-\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\int \frac{}{\frac{(bc-}{b}}}{}\right.}{bc-ad}\right.}{}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

$\downarrow$ 16

$$-\frac{2\left(-\frac{b\left(-5a^3d^2D-a^2bd(Cd-18cD)+ab^2\left(-2Bd^2-27c^2D+6cCd\right)+b^3\left(14Ad^2-12Bcd+9c^2C\right)\right)\left(\frac{3\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{c+dx}\sqrt[3]{bc-ad}}{\sqrt[3]{b}}+(c+dx)^{2/3}}d}{2b}\right.}{bc-ad}\right.}{}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

$\downarrow$ 1082

$$-\frac{2\left(-\frac{b\left(-5a^3d^2D-a^2bd(Cd-18cD)+ab^2\left(-2Bd^2-27c^2D+6cCd\right)+b^3\left(14Ad^2-12Bcd+9c^2C\right)\right)\left(-\frac{3\int\frac{1}{-(c+dx)^{2/3}-3}d\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1\right)}{b^{2/3}\sqrt[3]{bc-ad}}-\frac{\log(\cdots}{2b^{2/3}\sqrt[3]{\cdots}}\right.}{bc-ad}\cdots\right)}{3b^3(bc-a\cdots}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

$\Big\downarrow$ 217

$$-\frac{2\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}\right)\left(-5a^3d^2D-a^2bd(Cd-18cD)+ab^2\left(-2Bd^2-\cdots\right.\right.}{bc-ad}\cdots\right)}{3b^3(bc-ad)\cdots}$$

$$\frac{Ab^3-a\left(a^2D-abC+b^2B\right)}{2b^3(a+bx)^2\sqrt[3]{c+dx}(bc-ad)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((a + b*x)^3*(c + d*x)^(4/3)),x]
```

output
```
-1/2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))/(b^3*(b*c - a*d)*(a + b*x)^2*(c +
 d*x)^(1/3)) + (-((b^3*(6*B*c - 7*A*d) - a*b^2*(12*c*C - B*d) - 11*a^3*d*D
 + a^2*b*(5*C*d + 18*c*D))/(b^3*(b*c - a*d)*(a + b*x)*(c + d*x)^(1/3))) -
(2*((-3*(2*a*b^2*d^2*(3*c*C - B*d) + 4*a^3*d^3*D - a^2*b*d^2*(C*d + 9*c*D)
 + b^3*(9*c^2*C*d - 12*B*c*d^2 + 14*A*d^3 - 9*c^3*D)))/(d*(b*c - a*d)*(c +
 d*x)^(1/3)) - (b*(b^3*(9*c^2*C - 12*B*c*d + 14*A*d^2) - 5*a^3*d^2*D - a^2
*b*d*(C*d - 18*c*D) + a*b^2*(6*c*C*d - 2*B*d^2 - 27*c^2*D))*((Sqrt[3]*ArcT
an[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(b^(2/3)*
(b*c - a*d)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*c - a*d)^(1/3)) + (3*Log[(
b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(2/3)*(b*c - a*d)^(1/3))
))/(b*c - a*d)))/(3*b^3*(b*c - a*d)))/(6*(b*c - a*d))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{14\left(\frac{27\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\left(\left(\frac{14A\,b^4x^2}{9}+\frac{49x\left(-\frac{4Bx}{49}+A\right)a\,b^3}{18}+a^2\left(A-\frac{1}{9}C\,x^2-\frac{7}{18}Bx\right)b^2+\frac{a^3x(8Dx+C)b}{18}+\frac{5Da^4x}{18}\right)d^3+\frac{13\left(\frac{7\left(-\frac{24B}{7}\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$
derivativedivides	$-\frac{3d\left(\frac{d\left(5b^3dA-2Ba\,b^2d-3B\,b^3c-C\,a^2bd+6a\,b^2cC+4a^3dD-9a^2bcD\right)(xd+c)^{\frac{5}{3}}}{9b}+\frac{d\left(13Aa\,b^3d^2-13A\,b^4cd-7B\,a^2b^2d^2+Ba\,b^3cd+\cdots\right)}{((xd+c)b+ad-bc)^2}\cdots\right)}{\cdots}$
default	$-\frac{3d\left(\frac{d\left(5b^3dA-2Ba\,b^2d-3B\,b^3c-C\,a^2bd+6a\,b^2cC+4a^3dD-9a^2bcD\right)(xd+c)^{\frac{5}{3}}}{9b}+\frac{d\left(13Aa\,b^3d^2-13A\,b^4cd-7B\,a^2b^2d^2+Ba\,b^3cd+\cdots\right)}{((xd+c)b+ad-bc)^2}\cdots\right)}{\cdots}$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-14/9/((a*d-b*c)/b)^(1/3)*(27/14*((a*d-b*c)/b)^(1/3)*((14/9*A*b^4*x^2+49/1
8*x*(-4/49*B*x+A)*a*b^3+a^2*(A-1/9*C*x^2-7/18*B*x)*b^2+1/18*a^3*x*(8*D*x+C
)*b+5/18*D*a^4*x)*d^3+13/18*(7/13*(-24/7*B*x+A)*x*b^4+a*(12/13*C*x^2-43/13
*B*x+A)*b^3-25/13*(18/25*D*x^2-7/25*C*x+B)*a^2*b^2+1/13*a^3*(-7*D*x+C)*b+5
/13*D*a^4)*c*d^2-1/6*c^2*b*((-6*C*x^2+2*B*x+A)*b^3+a*(-16*C*x+B)*b^2-9*(-2
/3*D*x+C)*a^2*b+5*a^3*D)*d-D*b^2*c^3*(b*x+a)^2)*b+(d*x+c)^(1/3)*((b^3*A-1/
7*a*b^2*B-1/14*a^2*b*C-5/14*a^3*D)*d^2-6/7*(B*b^2-1/2*C*a*b-3/2*D*a^2)*c*b
*d+9/14*b^2*c^2*(C*b-3*D*a))*d*(b*x+a)^2*(arctan(1/3*3^(1/2)*(2/((a*d-b*c)
/b)^(1/3)*(d*x+c)^(1/3)-1))*3^(1/2)+1/2*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/
3)*(d*x+c)^(1/3)+((a*d-b*c)/b)^(2/3))-ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3)
)))/(d*x+c)^(1/3)/(a*d-b*c)^3/(b*x+a)^2/b^3/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2490 vs. $2(506) = 1012$.

Time = 1.03 (sec) , antiderivative size = 5068, normalized size of antiderivative = 9.37

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{4/3}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(4/3),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{4/3}} \, dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(4/3),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{4/3}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(4/3),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(506) = 1012$.

Time = 0.24 (sec) , antiderivative size = 1154, normalized size of antiderivative = 2.13

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{4/3}} \, dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
-1/9*(27*D*a*b^2*c^2*((b*c - a*d)/b)^(1/3) - 9*C*b^3*c^2*((b*c - a*d)/b)^(
1/3) - 18*D*a^2*b*c*d*((b*c - a*d)/b)^(1/3) - 6*C*a*b^2*c*d*((b*c - a*d)/b
)^(1/3) + 12*B*b^3*c*d*((b*c - a*d)/b)^(1/3) + 5*D*a^3*d^2*((b*c - a*d)/b)
^(1/3) + C*a^2*b*d^2*((b*c - a*d)/b)^(1/3) + 2*B*a*b^2*d^2*((b*c - a*d)/b)
^(1/3) - 14*A*b^3*d^2*((b*c - a*d)/b)^(1/3))*((b*c - a*d)/b)^(1/3)*log(abs
((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^
2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4) + 1/3*(14*A*b^3*d^2 - 2*(6*
b^3*c*d + a*b^2*d^2)*B + (9*b^3*c^2 + 6*a*b^2*c*d - a^2*b*d^2)*C - (27*a*b
^2*c^2 - 18*a^2*b*c*d + 5*a^3*d^2)*D)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3
) + ((b*c - a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/((sqrt(3)*b^5*c^3 - 3*sq
rt(3)*a*b^4*c^2*d + 3*sqrt(3)*a^2*b^3*c*d^2 - sqrt(3)*a^3*b^2*d^3)*(b^3*c
- a*b^2*d)^(1/3)) - 1/18*(14*A*b^3*d^2 - 2*(6*b^3*c*d + a*b^2*d^2)*B + (9*
b^3*c^2 + 6*a*b^2*c*d - a^2*b*d^2)*C - (27*a*b^2*c^2 - 18*a^2*b*c*d + 5*a^
3*d^2)*D)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((
b*c - a*d)/b)^(2/3))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2
*d^3)*(b^3*c - a*b^2*d)^(1/3)) - 3*(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)/((b
^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(d*x + c)^(1/3)) - 1
/6*(18*(d*x + c)^(5/3)*D*a^2*b^2*c*d - 12*(d*x + c)^(5/3)*C*a*b^3*c*d + 6*
(d*x + c)^(5/3)*B*b^4*c*d - 18*(d*x + c)^(2/3)*D*a^2*b^2*c^2*d + 12*(d*x +
 c)^(2/3)*C*a*b^3*c^2*d - 6*(d*x + c)^(2/3)*B*b^4*c^2*d - 8*(d*x + c)^(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx)^3(c+dx)^{4/3}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(a+b\,x)^3\,(c+d\,x)^{4/3}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(4/3)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((a + b*x)^3*(c + d*x)^(4/3)), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 3902, normalized size of antiderivative = 7.21

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(a + bx)^3 (c + dx)^{4/3}} \, dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(4/3),x)
```

output
```
( - 10*b**(1/3)*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)
*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**
4*d**2 + 24*b**(1/3)*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**
(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6))
)*a**3*b*c*d - 20*b**(1/3)*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d -
b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**
(1/6)))*a**3*b*d**2*x + 24*b**(1/3)*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6
)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d
 - b*c)**(1/6)))*a**2*b**3*d - 18*b**(1/3)*(c + d*x)**(1/3)*sqrt(3)*atan((
b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/
6)*(a*d - b*c)**(1/6)))*a**2*b**2*c**2 + 48*b**(1/3)*(c + d*x)**(1/3)*sqrt
(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c + d*x)**(1/6
))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**2*c*d*x - 10*b**(1/3)*(c + d*x)*
*(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) - 2*b**(1/3)*(c +
 d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a**2*b**2*d**2*x**2 + 48*b**(
1/3)*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt(3) -
2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**4*d*x - 3
6*b**(1/3)*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)**(1/6)*sqrt
(3) - 2*b**(1/3)*(c + d*x)**(1/6))/(b**(1/6)*(a*d - b*c)**(1/6)))*a*b**3*c
**2*x + 24*b**(1/3)*(c + d*x)**(1/3)*sqrt(3)*atan((b**(1/6)*(a*d - b*c)...
```

3.174 $\int (a+bx)^3(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal result . . . 1674
Mathematica [A] (verified) . . . 1675
Rubi [A] (verified) . . . 1676
Maple [B] (verified) . . . 1677
Fricas [B] (verification not implemented) . . . 1678
Sympy [B] (verification not implemented) . . . 1679
Maxima [B] (verification not implemented) . . . 1680
Giac [B] (verification not implemented) . . . 1681
Mupad [F(-1)] . . . 1682
Reduce [B] (verification not implemented) . . . 1682

Optimal result

Integrand size = 30, antiderivative size = 455

$$\int (a+bx)^3(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= -\frac{(bc-ad)^3\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{1+n}}{d^7(1+n)}$$
$$-\frac{(bc-ad)^2\left(ad(2cCd-Bd^2-3c^2D)-b(5c^2Cd-4Bcd^2+3Ad^3-6c^3D)\right)(c+dx)^{2+n}}{d^7(2+n)}$$
$$-\frac{(bc-ad)\left(a^2d^2(Cd-3cD)-abd(8cCd-3Bd^2-15c^2D)+b^2(10c^2Cd-6Bcd^2+3Ad^3-15c^3D)\right)(}{d^7(3+n)}$$
$$+\frac{(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d(4cCd-Bd^2-10c^2D)+b^3(10c^2Cd-4Bcd^2+Ad^3-20c^3D))}{d^7(4+n)}$$
$$+\frac{b\left(3a^2d^2D+3abd(Cd-5cD)-b^2(5cCd-Bd^2-15c^2D)\right)(c+dx)^{5+n}}{d^7(5+n)}$$
$$+\frac{b^2(bCd-6bcD+3adD)(c+dx)^{6+n}}{d^7(6+n)}+\frac{b^3D(c+dx)^{7+n}}{d^7(7+n)}$$

output
```
-(-a*d+b*c)^3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1+n)/d^7/(1+n)-(-a*d+
b*c)^2*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(3*A*d^3-4*B*c*d^2+5*C*c^2*d-6*D*c^
3))*(d*x+c)^(2+n)/d^7/(2+n)-(-a*d+b*c)*(a^2*d^2*(C*d-3*D*c)-a*b*d*(-3*B*d^
2+8*C*c*d-15*D*c^2)+b^2*(3*A*d^3-6*B*c*d^2+10*C*c^2*d-15*D*c^3))*(d*x+c)^(
3+n)/d^7/(3+n)+(a^3*d^3*D+3*a^2*b*d^2*(C*d-4*D*c)-3*a*b^2*d*(-B*d^2+4*C*c*
d-10*D*c^2)+b^3*(A*d^3-4*B*c*d^2+10*C*c^2*d-20*D*c^3))*(d*x+c)^(4+n)/d^7/(
4+n)+b*(3*a^2*d^2*D+3*a*b*d*(C*d-5*D*c)-b^2*(-B*d^2+5*C*c*d-15*D*c^2))*(d*
x+c)^(5+n)/d^7/(5+n)+b^2*(C*b*d+3*D*a*d-6*D*b*c)*(d*x+c)^(6+n)/d^7/(6+n)+b
^3*D*(d*x+c)^(7+n)/d^7/(7+n)
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.92

$$\int (a+bx)^3(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{(c+dx)^{1+n}\left(\frac{(bc-ad)^3(-c^2Cd+Bcd^2-Ad^3+c^3D)}{1+n}-\frac{(bc-ad)^2(-ad(-2cCd+Bd^2+3c^2D)+b(-5c^2Cd+4Bcd^2-3Ad^3+6c^3D))(c+dx)}{2+n}\right.}{}$$

input
```
Integrate[(a + b*x)^3*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((c + d*x)^(1 + n)*(((b*c - a*d)^3*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))
/(1 + n) - ((b*c - a*d)^2*(-(a*d*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-5*c^2
*C*d + 4*B*c*d^2 - 3*A*d^3 + 6*c^3*D))*(c + d*x))/(2 + n) + ((b*c - a*d)*(
a^2*d^2*(-(C*d) + 3*c*D) + a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D) + b^2*(-10
*c^2*C*d + 6*B*c*d^2 - 3*A*d^3 + 15*c^3*D))*(c + d*x)^2)/(3 + n) + ((a^3*d
^3*D + 3*a^2*b*d^2*(C*d - 4*c*D) + 3*a*b^2*d*(-4*c*C*d + B*d^2 + 10*c^2*D)
 + b^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c + d*x)^3)/(4 + n) +
 (b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) + b^2*(-5*c*C*d + B*d^2 + 15*c^2*
D))*(c + d*x)^4)/(5 + n) + (b^2*(b*C*d - 6*b*c*D + 3*a*d*D)*(c + d*x)^5)/(
6 + n) + (b^3*D*(c + d*x)^6)/(7 + n)))/d^7
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)dx$$

$$\Big\downarrow \text{ 2123}$$

$$\int \left(\frac{(bc-ad)(c+dx)^{n+2}\left(-a^2d^2(Cd-3cD)+abd\left(-3Bd^2-15c^2D+8cCd\right)-\left(b^2\left(3Ad^3-6Bcd^2-15c^3D+\right.\right.\right.}{d^6}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(bc-ad)(c+dx)^{n+3}\left(a^2d^2(Cd-3cD)-abd\left(-3Bd^2-15c^2D+8cCd\right)+b^2\left(3Ad^3-6Bcd^2-15c^3D+10c^2C\right.\right.}{d^7(n+3)}$$
$$\frac{b(c+dx)^{n+5}\left(3a^2d^2D+3abd(Cd-5cD)-\left(b^2\left(-Bd^2-15c^2D+5cCd\right)\right)\right)}{d^7(n+5)}+$$
$$\frac{(c+dx)^{n+4}\left(a^3d^3D+3a^2bd^2(Cd-4cD)-3ab^2d\left(-Bd^2-10c^2D+4cCd\right)+b^3\left(Ad^3-4Bcd^2-20c^3D+10c^2C\right.\right.}{d^7(n+4)}$$
$$\frac{(bc-ad)^3(c+dx)^{n+1}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^7(n+1)}-$$
$$\frac{(bc-ad)^2(c+dx)^{n+2}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(3Ad^3-4Bcd^2-6c^3D+5c^2Cd\right)\right)}{d^7(n+2)}+$$
$$\frac{b^2(c+dx)^{n+6}(3adD-6bcD+bCd)}{d^7(n+6)}+\frac{b^3D(c+dx)^{n+7}}{d^7(n+7)}$$

input
```
Int[(a + b*x)^3*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
-(((b*c - a*d)^3*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(1 + n))/(d
^7*(1 + n))) - ((b*c - a*d)^2*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(5*c^2*
C*d - 4*B*c*d^2 + 3*A*d^3 - 6*c^3*D))*(c + d*x)^(2 + n))/(d^7*(2 + n)) - (
(b*c - a*d)*(a^2*d^2*(C*d - 3*c*D) - a*b*d*(8*c*C*d - 3*B*d^2 - 15*c^2*D)
+ b^2*(10*c^2*C*d - 6*B*c*d^2 + 3*A*d^3 - 15*c^3*D))*(c + d*x)^(3 + n))/(d
^7*(3 + n)) + ((a^3*d^3*D + 3*a^2*b*d^2*(C*d - 4*c*D) - 3*a*b^2*d*(4*c*C*d
 - B*d^2 - 10*c^2*D) + b^3*(10*c^2*C*d - 4*B*c*d^2 + A*d^3 - 20*c^3*D))*(c
 + d*x)^(4 + n))/(d^7*(4 + n)) + (b*(3*a^2*d^2*D + 3*a*b*d*(C*d - 5*c*D) -
 b^2*(5*c*C*d - B*d^2 - 15*c^2*D))*(c + d*x)^(5 + n))/(d^7*(5 + n)) + (b^2
*(b*C*d - 6*b*c*D + 3*a*d*D)*(c + d*x)^(6 + n))/(d^7*(6 + n)) + (b^3*D*(c
+ d*x)^(7 + n))/(d^7*(7 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3931 vs. $2(455) = 910$.

Time = 0.61 (sec) , antiderivative size = 3932, normalized size of antiderivative = 8.64

method	result	size
norman	Expression too large to display	3932
gosper	Expression too large to display	5003
orering	Expression too large to display	5006
parallelrisch	Expression too large to display	9319

input
```
int((b*x+a)^3*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output

```
b^3*D/(7+n)*x^7*exp(n*ln(d*x+c))+c*(A*a^3*d^6*n^6+27*A*a^3*d^6*n^5-3*A*a^2
*b*c*d^5*n^5-B*a^3*c*d^5*n^5+295*A*a^3*d^6*n^4-75*A*a^2*b*c*d^5*n^4+6*A*a*
b^2*c^2*d^4*n^4-25*B*a^3*c*d^5*n^4+6*B*a^2*b*c^2*d^4*n^4+2*C*a^3*c^2*d^4*n
^4+1665*A*a^3*d^6*n^3-735*A*a^2*b*c*d^5*n^3+132*A*a*b^2*c^2*d^4*n^3-6*A*b^
3*c^3*d^3*n^3-245*B*a^3*c*d^5*n^3+132*B*a^2*b*c^2*d^4*n^3-18*B*a*b^2*c^3*d
^3*n^3+44*C*a^3*c^2*d^4*n^3-18*C*a^2*b*c^3*d^3*n^3-6*D*a^3*c^3*d^3*n^3+510
4*A*a^3*d^6*n^2-3525*A*a^2*b*c*d^5*n^2+1074*A*a*b^2*c^2*d^4*n^2-108*A*b^3*
c^3*d^3*n^2-1175*B*a^3*c*d^5*n^2+1074*B*a^2*b*c^2*d^4*n^2-324*B*a*b^2*c^3*
d^3*n^2+24*B*b^3*c^4*d^2*n^2+358*C*a^3*c^2*d^4*n^2-324*C*a^2*b*c^3*d^3*n^2
+72*C*a*b^2*c^4*d^2*n^2-108*D*a^3*c^3*d^3*n^2+72*D*a^2*b*c^4*d^2*n^2+8028*
A*a^3*d^6*n-8262*A*a^2*b*c*d^5*n+3828*A*a*b^2*c^2*d^4*n-642*A*b^3*c^3*d^3*
n-2754*B*a^3*c*d^5*n+3828*B*a^2*b*c^2*d^4*n-1926*B*a*b^2*c^3*d^3*n+312*B*b
^3*c^4*d^2*n+1276*C*a^3*c^2*d^4*n-1926*C*a^2*b*c^3*d^3*n+936*C*a*b^2*c^4*d
^2*n-120*C*b^3*c^5*d*n-642*D*a^3*c^3*d^3*n+936*D*a^2*b*c^4*d^2*n-360*D*a*b
^2*c^5*d*n+5040*A*a^3*d^6-7560*A*a^2*b*c*d^5+5040*A*a*b^2*c^2*d^4-1260*A*b
^3*c^3*d^3-2520*B*a^3*c*d^5+5040*B*a^2*b*c^2*d^4-3780*B*a*b^2*c^3*d^3+1008
*B*b^3*c^4*d^2+1680*C*a^3*c^2*d^4-3780*C*a^2*b*c^3*d^3+3024*C*a*b^2*c^4*d^
2-840*C*b^3*c^5*d-1260*D*a^3*c^3*d^3+3024*D*a^2*b*c^4*d^2-2520*D*a*b^2*c^5
*d+720*D*b^3*c^6)/d^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+1306
8*n+5040)*exp(n*ln(d*x+c))+(A*b^3*d^3*n^3+3*B*a*b^2*d^3*n^3+B*b^3*c*d^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3889 vs. $2(455) = 910$.

Time = 0.14 (sec) , antiderivative size = 3889, normalized size of antiderivative = 8.55

$$\int (a+bx)^3(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)^3*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65321 vs. $2(445) = 890$.

Time = 11.29 (sec) , antiderivative size = 65321, normalized size of antiderivative = 143.56

$$\int (a+bx)^3(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**3*(d*x+c)**n*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((c**n*(A*a**3*x + 3*A*a**2*b*x**2/2 + A*a*b**2*x**3 + A*b**3*x**
4/4 + B*a**3*x**2/2 + B*a**2*b*x**3 + 3*B*a*b**2*x**4/4 + B*b**3*x**5/5 +
C*a**3*x**3/3 + 3*C*a**2*b*x**4/4 + 3*C*a*b**2*x**5/5 + C*b**3*x**6/6 + D*
a**3*x**4/4 + 3*D*a**2*b*x**5/5 + D*a*b**2*x**6/2 + D*b**3*x**7/7), Eq(d,
0)), (-10*A*a**3*d**6/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c**4*d**9*x**2
 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*c*d**12*x**5 + 60*d**1
3*x**6) - 6*A*a**2*b*c*d**5/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c**4*d**
9*x**2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*c*d**12*x**5 + 6
0*d**13*x**6) - 36*A*a**2*b*d**6*x/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c
**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*c*d**12*x
**5 + 60*d**13*x**6) - 3*A*a*b**2*c**2*d**4/(60*c**6*d**7 + 360*c**5*d**8*
x + 900*c**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x**4 + 360*
c*d**12*x**5 + 60*d**13*x**6) - 18*A*a*b**2*c*d**5*x/(60*c**6*d**7 + 360*c
**5*d**8*x + 900*c**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**2*d**11*x*
*4 + 360*c*d**12*x**5 + 60*d**13*x**6) - 45*A*a*b**2*d**6*x**2/(60*c**6*d*
*7 + 360*c**5*d**8*x + 900*c**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 900*c**
2*d**11*x**4 + 360*c*d**12*x**5 + 60*d**13*x**6) - A*b**3*c**3*d**3/(60*c*
*6*d**7 + 360*c**5*d**8*x + 900*c**4*d**9*x**2 + 1200*c**3*d**10*x**3 + 90
0*c**2*d**11*x**4 + 360*c*d**12*x**5 + 60*d**13*x**6) - 6*A*b**3*c**2*d**4
*x/(60*c**6*d**7 + 360*c**5*d**8*x + 900*c**4*d**9*x**2 + 1200*c**3*d**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. $2(455) = 910$.

Time = 0.10 (sec) , antiderivative size = 1802, normalized size of antiderivative = 3.96

$$\int (a+bx)^3(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*B*a^3/((n^2 + 3*n + 2)*d^2)
+ 3*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*A*a^2*b/((n^2 + 3*n + 2)
*d^2) + (d*x + c)^(n + 1)*A*a^3/(d*(n + 1)) + ((n^2 + 3*n + 2)*d^3*x^3 + (
n^2 + n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*C*a^3/((n^3 + 6*n^2
+ 11*n + 6)*d^3) + 3*((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c*d^2*x^2 - 2*c^
2*d*n*x + 2*c^3)*(d*x + c)^n*B*a^2*b/((n^3 + 6*n^2 + 11*n + 6)*d^3) + 3*((
n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x +
 c)^n*A*a*b^2/((n^3 + 6*n^2 + 11*n + 6)*d^3) + ((n^3 + 6*n^2 + 11*n + 6)*d
^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d
*n*x - 6*c^4)*(d*x + c)^n*D*a^3/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^4)
+ 3*((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3*
(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*C*a^2*b/((n^4 + 1
0*n^3 + 35*n^2 + 50*n + 24)*d^4) + 3*((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (
n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c
^4)*(d*x + c)^n*B*a*b^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^4) + ((n^3
+ 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3*(n^2 + n)*
c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*A*b^3/((n^4 + 10*n^3 + 35*n
^2 + 50*n + 24)*d^4) + 3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^5*x^5 + (n
^4 + 6*n^3 + 11*n^2 + 6*n)*c*d^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*c^2*d^3*x^3 +
 12*(n^2 + n)*c^3*d^2*x^2 - 24*c^4*d*n*x + 24*c^5)*(d*x + c)^n*D*a^2*b/...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9032 vs. 2(455) = 910.

Time = 0.20 (sec) , antiderivative size = 9032, normalized size of antiderivative = 19.85

$$\int (a+bx)^3(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^3*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
((d*x + c)^n*D*b^3*d^7*n^6*x^7 + (d*x + c)^n*D*b^3*c*d^6*n^6*x^6 + 3*(d*x
+ c)^n*D*a*b^2*d^7*n^6*x^6 + (d*x + c)^n*C*b^3*d^7*n^6*x^6 + 21*(d*x + c)^
n*D*b^3*d^7*n^5*x^7 + 3*(d*x + c)^n*D*a*b^2*c*d^6*n^6*x^5 + (d*x + c)^n*C*
b^3*c*d^6*n^6*x^5 + 3*(d*x + c)^n*D*a^2*b*d^7*n^6*x^5 + 3*(d*x + c)^n*C*a*
b^2*d^7*n^6*x^5 + (d*x + c)^n*B*b^3*d^7*n^6*x^5 + 15*(d*x + c)^n*D*b^3*c*d
^6*n^5*x^6 + 66*(d*x + c)^n*D*a*b^2*d^7*n^5*x^6 + 22*(d*x + c)^n*C*b^3*d^7
*n^5*x^6 + 175*(d*x + c)^n*D*b^3*d^7*n^4*x^7 + 3*(d*x + c)^n*D*a^2*b*c*d^6
*n^6*x^4 + 3*(d*x + c)^n*C*a*b^2*c*d^6*n^6*x^4 + (d*x + c)^n*B*b^3*c*d^6*n
^6*x^4 + (d*x + c)^n*D*a^3*d^7*n^6*x^4 + 3*(d*x + c)^n*C*a^2*b*d^7*n^6*x^4
 + 3*(d*x + c)^n*B*a*b^2*d^7*n^6*x^4 + (d*x + c)^n*A*b^3*d^7*n^6*x^4 - 6*(
d*x + c)^n*D*b^3*c^2*d^5*n^5*x^5 + 51*(d*x + c)^n*D*a*b^2*c*d^6*n^5*x^5 +
17*(d*x + c)^n*C*b^3*c*d^6*n^5*x^5 + 69*(d*x + c)^n*D*a^2*b*d^7*n^5*x^5 +
69*(d*x + c)^n*C*a*b^2*d^7*n^5*x^5 + 23*(d*x + c)^n*B*b^3*d^7*n^5*x^5 + 85
*(d*x + c)^n*D*b^3*c*d^6*n^4*x^6 + 570*(d*x + c)^n*D*a*b^2*d^7*n^4*x^6 + 1
90*(d*x + c)^n*C*b^3*d^7*n^4*x^6 + 735*(d*x + c)^n*D*b^3*d^7*n^3*x^7 + (d*
x + c)^n*D*a^3*c*d^6*n^6*x^3 + 3*(d*x + c)^n*C*a^2*b*c*d^6*n^6*x^3 + 3*(d*
x + c)^n*B*a*b^2*c*d^6*n^6*x^3 + (d*x + c)^n*A*b^3*c*d^6*n^6*x^3 + (d*x +
c)^n*C*a^3*d^7*n^6*x^3 + 3*(d*x + c)^n*B*a^2*b*d^7*n^6*x^3 + 3*(d*x + c)^n
*A*a*b^2*d^7*n^6*x^3 - 15*(d*x + c)^n*D*a*b^2*c^2*d^5*n^5*x^4 - 5*(d*x + c
)^n*C*b^3*c^2*d^5*n^5*x^4 + 57*(d*x + c)^n*D*a^2*b*c*d^6*n^5*x^4 + 57*(...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^3(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx = \int (a+b\,x)^3\,(c+d\,x)^n\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^3*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^3*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 3687, normalized size of antiderivative = 8.10

$$\int (a+bx)^3(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^3*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output

```
((c + d*x)**n*(a**4*c*d**5*n**6 + 27*a**4*c*d**5*n**5 + 295*a**4*c*d**5*n*
*4 + 1665*a**4*c*d**5*n**3 + 5104*a**4*c*d**5*n**2 + 8028*a**4*c*d**5*n +
5040*a**4*c*d**5 + a**4*d**6*n**6*x + 27*a**4*d**6*n**5*x + 295*a**4*d**6*
n**4*x + 1665*a**4*d**6*n**3*x + 5104*a**4*d**6*n**2*x + 8028*a**4*d**6*n*
x + 5040*a**4*d**6*x - 4*a**3*b*c**2*d**4*n**5 - 100*a**3*b*c**2*d**4*n**4
 - 980*a**3*b*c**2*d**4*n**3 - 4700*a**3*b*c**2*d**4*n**2 - 11016*a**3*b*c
**2*d**4*n - 10080*a**3*b*c**2*d**4 + 4*a**3*b*c*d**5*n**6*x + 100*a**3*b*
c*d**5*n**5*x + 980*a**3*b*c*d**5*n**4*x + 4700*a**3*b*c*d**5*n**3*x + 110
16*a**3*b*c*d**5*n**2*x + 10080*a**3*b*c*d**5*n*x + 4*a**3*b*d**6*n**6*x**
2 + 104*a**3*b*d**6*n**5*x**2 + 1080*a**3*b*d**6*n**4*x**2 + 5680*a**3*b*d
**6*n**3*x**2 + 15716*a**3*b*d**6*n**2*x**2 + 21096*a**3*b*d**6*n*x**2 + 1
0080*a**3*b*d**6*x**2 + 2*a**3*c**4*d**3*n**4 + 38*a**3*c**4*d**3*n**3 + 2
50*a**3*c**4*d**3*n**2 + 634*a**3*c**4*d**3*n + 420*a**3*c**4*d**3 - 2*a**
3*c**3*d**4*n**5*x - 38*a**3*c**3*d**4*n**4*x - 250*a**3*c**3*d**4*n**3*x
- 634*a**3*c**3*d**4*n**2*x - 420*a**3*c**3*d**4*n*x + a**3*c**2*d**5*n**6
*x**2 + 20*a**3*c**2*d**5*n**5*x**2 + 144*a**3*c**2*d**5*n**4*x**2 + 442*a
**3*c**2*d**5*n**3*x**2 + 527*a**3*c**2*d**5*n**2*x**2 + 210*a**3*c**2*d**
5*n*x**2 + 2*a**3*c*d**6*n**6*x**3 + 46*a**3*c*d**6*n**5*x**3 + 410*a**3*c
*d**6*n**4*x**3 + 1786*a**3*c*d**6*n**3*x**3 + 3956*a**3*c*d**6*n**2*x**3
+ 4216*a**3*c*d**6*n*x**3 + 1680*a**3*c*d**6*x**3 + a**3*d**7*n**6*x**4...
```

3.175 $\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result . . . 1684
Mathematica [A] (verified) . . . 1685
Rubi [A] (verified) . . . 1685
Maple [B] (verified) . . . 1687
Fricas [B] (verification not implemented) . . . 1688
Sympy [B] (verification not implemented) . . . 1689
Maxima [B] (verification not implemented) . . . 1690
Giac [B] (verification not implemented) . . . 1691
Mupad [F(-1)] . . . 1692
Reduce [B] (verification not implemented) . . . 1692

Optimal result

Integrand size = 30, antiderivative size = 338

$$\begin{aligned}\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx &= \frac{(bc-ad)^2 \left(c^2Cd-Bcd^2+Ad^3-c^3D\right) (c+dx)^{1+n}}{d^6(1+n)}\\
&+\frac{(bc-ad) \left(ad(2cCd-Bd^2-3c^2D)-b(4c^2Cd-3Bcd^2+2Ad^3-5c^3D)\right) (c+dx)^{2+n}}{d^6(2+n)}\\
&+\frac{\left(a^2d^2(Cd-3cD)-2abd(3cCd-Bd^2-6c^2D)+b^2(6c^2Cd-3Bcd^2+Ad^3-10c^3D)\right) (c+dx)^{3+n}}{d^6(3+n)}\\
&+\frac{\left(a^2d^2D+2abd(Cd-4cD)-b^2(4cCd-Bd^2-10c^2D)\right) (c+dx)^{4+n}}{d^6(4+n)}\\
&+\frac{b(bCd-5bcD+2adD)(c+dx)^{5+n}}{d^6(5+n)}+\frac{b^2D(c+dx)^{6+n}}{d^6(6+n)}\end{aligned}$$

output
```
(-a*d+b*c)^2*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1+n)/d^6/(1+n)+(-a*d+b
*c)*(a*d*(-B*d^2+2*C*c*d-3*D*c^2)-b*(2*A*d^3-3*B*c*d^2+4*C*c^2*d-5*D*c^3))
*(d*x+c)^(2+n)/d^6/(2+n)+(a^2*d^2*(C*d-3*D*c)-2*a*b*d*(-B*d^2+3*C*c*d-6*D*
c^2)+b^2*(A*d^3-3*B*c*d^2+6*C*c^2*d-10*D*c^3))*(d*x+c)^(3+n)/d^6/(3+n)+(a^
2*d^2*D+2*a*b*d*(C*d-4*D*c)-b^2*(-B*d^2+4*C*c*d-10*D*c^2))*(d*x+c)^(4+n)/d
^6/(4+n)+b*(C*b*d+2*D*a*d-5*D*b*c)*(d*x+c)^(5+n)/d^6/(5+n)+b^2*D*(d*x+c)^(
6+n)/d^6/(6+n)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.91

$$\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= \frac{(c+dx)^{1+n}\left(\frac{(bc-ad)^2(c^2Cd-Bcd^2+Ad^3-c^3D)}{1+n}+\frac{(bc-ad)(-ad(-2cCd+Bd^2+3c^2D)+b(-4c^2Cd+3Bcd^2-2Ad^3+5c^3D))(c+dx)}{2+n}+\right.}{}$$

input
```
Integrate[(a + b*x)^2*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((c + d*x)^(1 + n)*(((b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D))/(1
 + n) + ((b*c - a*d)*(-(a*d*(-2*c*C*d + B*d^2 + 3*c^2*D)) + b*(-4*c^2*C*d
+ 3*B*c*d^2 - 2*A*d^3 + 5*c^3*D))*(c + d*x))/(2 + n) + ((a^2*d^2*(C*d - 3*
c*D) + 2*a*b*d*(-3*c*C*d + B*d^2 + 6*c^2*D) + b^2*(6*c^2*C*d - 3*B*c*d^2 +
 A*d^3 - 10*c^3*D))*(c + d*x)^2)/(3 + n) + ((a^2*d^2*D + 2*a*b*d*(C*d - 4*
c*D) + b^2*(-4*c*C*d + B*d^2 + 10*c^2*D))*(c + d*x)^3)/(4 + n) + (b*(b*C*d
 - 5*b*c*D + 2*a*d*D)*(c + d*x)^4)/(5 + n) + (b^2*D*(c + d*x)^5)/(6 + n)))
/d^6
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(c+dx)^{n+2}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^5}+\frac{(c\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^{n+3}\left(a^2d^2(Cd-3cD)-2abd\left(-Bd^2-6c^2D+3cCd\right)+b^2\left(Ad^3-3Bcd^2-10c^3D+6c^2Cd\right)\right)}{d^6(n+3)}+\frac{(c+dx)^{n+4}\left(a^2d^2D+2abd(Cd-4cD)-\left(b^2\left(-Bd^2-10c^2D+4cCd\right)\right)\right)}{d^6(n+4)}+\frac{(bc-ad)^2(c+dx)^{n+1}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^6(n+1)}+\frac{(bc-ad)(c+dx)^{n+2}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(2Ad^3-3Bcd^2-5c^3D+4c^2Cd\right)\right)}{d^6(n+2)}+\frac{b(c+dx)^{n+5}(2adD-5bcD+bCd)}{d^6(n+5)}+\frac{b^2D(c+dx)^{n+6}}{d^6(n+6)}$$

input
```
Int[(a + b*x)^2*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((b*c - a*d)^2*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(1 + n))/(d^6
*(1 + n)) + ((b*c - a*d)*(a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(4*c^2*C*d -
 3*B*c*d^2 + 2*A*d^3 - 5*c^3*D))*(c + d*x)^(2 + n))/(d^6*(2 + n)) + ((a^2*
d^2*(C*d - 3*c*D) - 2*a*b*d*(3*c*C*d - B*d^2 - 6*c^2*D) + b^2*(6*c^2*C*d -
 3*B*c*d^2 + A*d^3 - 10*c^3*D))*(c + d*x)^(3 + n))/(d^6*(3 + n)) + ((a^2*d
^2*D + 2*a*b*d*(C*d - 4*c*D) - b^2*(4*c*C*d - B*d^2 - 10*c^2*D))*(c + d*x)
^(4 + n))/(d^6*(4 + n)) + (b*(b*C*d - 5*b*c*D + 2*a*d*D)*(c + d*x)^(5 + n)
)/(d^6*(5 + n)) + (b^2*D*(c + d*x)^(6 + n))/(d^6*(6 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2174 vs. 2(338) = 676.

Time = 0.52 (sec) , antiderivative size = 2175, normalized size of antiderivative = 6.43

method	result	size
norman	Expression too large to display	2175
gosper	Expression too large to display	2588
orering	Expression too large to display	2591
parallelrisch	Expression too large to display	5150

input
```
int((b*x+a)^2*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
b^2*D/(n+6)*x^6*exp(n*ln(d*x+c))+c*(A*a^2*d^5*n^5+20*A*a^2*d^5*n^4-2*A*a*b
*c*d^4*n^4-B*a^2*c*d^4*n^4+155*A*a^2*d^5*n^3-36*A*a*b*c*d^4*n^3+2*A*b^2*c^
2*d^3*n^3-18*B*a^2*c*d^4*n^3+4*B*a*b*c^2*d^3*n^3+2*C*a^2*c^2*d^3*n^3+580*A
*a^2*d^5*n^2-238*A*a*b*c*d^4*n^2+30*A*b^2*c^2*d^3*n^2-119*B*a^2*c*d^4*n^2+
60*B*a*b*c^2*d^3*n^2-6*B*b^2*c^3*d^2*n^2+30*C*a^2*c^2*d^3*n^2-12*C*a*b*c^3
*d^2*n^2-6*D*a^2*c^3*d^2*n^2+1044*A*a^2*d^5*n-684*A*a*b*c*d^4*n+148*A*b^2*
c^2*d^3*n-342*B*a^2*c*d^4*n+296*B*a*b*c^2*d^3*n-66*B*b^2*c^3*d^2*n+148*C*a
^2*c^2*d^3*n-132*C*a*b*c^3*d^2*n+24*C*b^2*c^4*d*n-66*D*a^2*c^3*d^2*n+48*D*
a*b*c^4*d*n+720*A*a^2*d^5-720*A*a*b*c*d^4+240*A*b^2*c^2*d^3-360*B*a^2*c*d^
4+480*B*a*b*c^2*d^3-180*B*b^2*c^3*d^2+240*C*a^2*c^2*d^3-360*C*a*b*c^3*d^2+
144*C*b^2*c^4*d-180*D*a^2*c^3*d^2+288*D*a*b*c^4*d-120*D*b^2*c^5)/d^6/(n^6+
21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*exp(n*ln(d*x+c))+(B*b^2*d^2*n^
2+2*C*a*b*d^2*n^2+C*b^2*c*d*n^2+D*a^2*d^2*n^2+2*D*a*b*c*d*n^2+11*B*b^2*d^2
*n+22*C*a*b*d^2*n+6*C*b^2*c*d*n+11*D*a^2*d^2*n+12*D*a*b*c*d*n-5*D*b^2*c^2*
n+30*B*b^2*d^2+60*C*a*b*d^2+30*D*a^2*d^2)/d^2/(n^3+15*n^2+74*n+120)*x^4*ex
p(n*ln(d*x+c))+(A*b^2*d^3*n^3+2*B*a*b*d^3*n^3+B*b^2*c*d^2*n^3+C*a^2*d^3*n^
3+2*C*a*b*c*d^2*n^3+D*a^2*c*d^2*n^3+15*A*b^2*d^3*n^2+30*B*a*b*d^3*n^2+11*B
*b^2*c*d^2*n^2+15*C*a^2*d^3*n^2+22*C*a*b*c*d^2*n^2-4*C*b^2*c^2*d*n^2+11*D*
a^2*c*d^2*n^2-8*D*a*b*c^2*d*n^2+74*A*b^2*d^3*n+148*B*a*b*d^3*n+30*B*b^2*c*
d^2*n+74*C*a^2*d^3*n+60*C*a*b*c*d^2*n-24*C*b^2*c^2*d*n+30*D*a^2*c*d^2*n...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2156 vs. $2(342) = 684$.

Time = 0.12 (sec) , antiderivative size = 2156, normalized size of antiderivative = 6.38

$$\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
(A*a^2*c*d^5*n^5 - 120*D*b^2*c^6 + 720*A*a^2*c*d^5 + 144*(2*D*a*b + C*b^2)
*c^5*d - 180*(D*a^2 + 2*C*a*b + B*b^2)*c^4*d^2 + 240*(C*a^2 + 2*B*a*b + A*
b^2)*c^3*d^3 - 360*(B*a^2 + 2*A*a*b)*c^2*d^4 + (D*b^2*d^6*n^5 + 15*D*b^2*d
^6*n^4 + 85*D*b^2*d^6*n^3 + 225*D*b^2*d^6*n^2 + 274*D*b^2*d^6*n + 120*D*b^
2*d^6)*x^6 + (144*(2*D*a*b + C*b^2)*d^6 + (D*b^2*c*d^5 + (2*D*a*b + C*b^2)
*d^6)*n^5 + 2*(5*D*b^2*c*d^5 + 8*(2*D*a*b + C*b^2)*d^6)*n^4 + 5*(7*D*b^2*c
*d^5 + 19*(2*D*a*b + C*b^2)*d^6)*n^3 + 10*(5*D*b^2*c*d^5 + 26*(2*D*a*b + C
*b^2)*d^6)*n^2 + 12*(2*D*b^2*c*d^5 + 27*(2*D*a*b + C*b^2)*d^6)*n)*x^5 + (2
0*A*a^2*c*d^5 - (B*a^2 + 2*A*a*b)*c^2*d^4)*n^4 + (180*(D*a^2 + 2*C*a*b + B
*b^2)*d^6 + ((2*D*a*b + C*b^2)*c*d^5 + (D*a^2 + 2*C*a*b + B*b^2)*d^6)*n^5
- (5*D*b^2*c^2*d^4 - 12*(2*D*a*b + C*b^2)*c*d^5 - 17*(D*a^2 + 2*C*a*b + B*
b^2)*d^6)*n^4 - (30*D*b^2*c^2*d^4 - 47*(2*D*a*b + C*b^2)*c*d^5 - 107*(D*a^
2 + 2*C*a*b + B*b^2)*d^6)*n^3 - (55*D*b^2*c^2*d^4 - 72*(2*D*a*b + C*b^2)*c
*d^5 - 307*(D*a^2 + 2*C*a*b + B*b^2)*d^6)*n^2 - 6*(5*D*b^2*c^2*d^4 - 6*(2*
D*a*b + C*b^2)*c*d^5 - 66*(D*a^2 + 2*C*a*b + B*b^2)*d^6)*n)*x^4 + (155*A*a
^2*c*d^5 + 2*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 18*(B*a^2 + 2*A*a*b)*c^2*
d^4)*n^3 + (240*(C*a^2 + 2*B*a*b + A*b^2)*d^6 + ((D*a^2 + 2*C*a*b + B*b^2)
*c*d^5 + (C*a^2 + 2*B*a*b + A*b^2)*d^6)*n^5 - 2*(2*(2*D*a*b + C*b^2)*c^2*d
^4 - 7*(D*a^2 + 2*C*a*b + B*b^2)*c*d^5 - 9*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*
n^4 + (20*D*b^2*c^3*d^3 - 36*(2*D*a*b + C*b^2)*c^2*d^4 + 65*(D*a^2 + 2*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32849 vs. $2(328) = 656$.

Time = 6.97 (sec) , antiderivative size = 32849, normalized size of antiderivative = 97.19

$$\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**2*(d*x+c)**n*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((c**n*(A*a**2*x + A*a*b*x**2 + A*b**2*x**3/3 + B*a**2*x**2/2 + 2
*B*a*b*x**3/3 + B*b**2*x**4/4 + C*a**2*x**3/3 + C*a*b*x**4/2 + C*b**2*x**5
/5 + D*a**2*x**4/4 + 2*D*a*b*x**5/5 + D*b**2*x**6/6), Eq(d, 0)), (-12*A*a*
*2*d**5/(60*c**5*d**6 + 300*c**4*d**7*x + 600*c**3*d**8*x**2 + 600*c**2*d*
*9*x**3 + 300*c*d**10*x**4 + 60*d**11*x**5) - 6*A*a*b*c*d**4/(60*c**5*d**6
 + 300*c**4*d**7*x + 600*c**3*d**8*x**2 + 600*c**2*d**9*x**3 + 300*c*d**10
*x**4 + 60*d**11*x**5) - 30*A*a*b*d**5*x/(60*c**5*d**6 + 300*c**4*d**7*x +
 600*c**3*d**8*x**2 + 600*c**2*d**9*x**3 + 300*c*d**10*x**4 + 60*d**11*x**
5) - 2*A*b**2*c**2*d**3/(60*c**5*d**6 + 300*c**4*d**7*x + 600*c**3*d**8*x*
*2 + 600*c**2*d**9*x**3 + 300*c*d**10*x**4 + 60*d**11*x**5) - 10*A*b**2*c*
d**4*x/(60*c**5*d**6 + 300*c**4*d**7*x + 600*c**3*d**8*x**2 + 600*c**2*d**
9*x**3 + 300*c*d**10*x**4 + 60*d**11*x**5) - 20*A*b**2*d**5*x**2/(60*c**5*
d**6 + 300*c**4*d**7*x + 600*c**3*d**8*x**2 + 600*c**2*d**9*x**3 + 300*c*d
**10*x**4 + 60*d**11*x**5) - 3*B*a**2*c*d**4/(60*c**5*d**6 + 300*c**4*d**7
*x + 600*c**3*d**8*x**2 + 600*c**2*d**9*x**3 + 300*c*d**10*x**4 + 60*d**11
*x**5) - 15*B*a**2*d**5*x/(60*c**5*d**6 + 300*c**4*d**7*x + 600*c**3*d**8*
x**2 + 600*c**2*d**9*x**3 + 300*c*d**10*x**4 + 60*d**11*x**5) - 4*B*a*b*c*
*2*d**3/(60*c**5*d**6 + 300*c**4*d**7*x + 600*c**3*d**8*x**2 + 600*c**2*d*
*9*x**3 + 300*c*d**10*x**4 + 60*d**11*x**5) - 20*B*a*b*c*d**4*x/(60*c**5*d
**6 + 300*c**4*d**7*x + 600*c**3*d**8*x**2 + 600*c**2*d**9*x**3 + 300*c...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. 2(342) = 684.

Time = 0.09 (sec) , antiderivative size = 1118, normalized size of antiderivative = 3.31

$$\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*B*a^2/((n^2 + 3*n + 2)*d^2)
+ 2*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*A*a*b/((n^2 + 3*n + 2)*d
^2) + (d*x + c)^(n + 1)*A*a^2/(d*(n + 1)) + ((n^2 + 3*n + 2)*d^3*x^3 + (n^
2 + n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*C*a^2/((n^3 + 6*n^2 +
11*n + 6)*d^3) + 2*((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c*d^2*x^2 - 2*c^2*
d*n*x + 2*c^3)*(d*x + c)^n*B*a*b/((n^3 + 6*n^2 + 11*n + 6)*d^3) + ((n^2 +
3*n + 2)*d^3*x^3 + (n^2 + n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*
A*b^2/((n^3 + 6*n^2 + 11*n + 6)*d^3) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 +
 (n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6
*c^4)*(d*x + c)^n*D*a^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^4) + 2*((n^
3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3*(n^2 + n
)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*C*a*b/((n^4 + 10*n^3 + 35
*n^2 + 50*n + 24)*d^4) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2
+ 2*n)*c*d^3*x^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c
)^n*B*b^2/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^4) + 2*((n^4 + 10*n^3 + 3
5*n^2 + 50*n + 24)*d^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*c*d^4*x^4 - 4*(n
^3 + 3*n^2 + 2*n)*c^2*d^3*x^3 + 12*(n^2 + n)*c^3*d^2*x^2 - 24*c^4*d*n*x +
24*c^5)*(d*x + c)^n*D*a*b/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)
*d^5) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^5*x^5 + (n^4 + 6*n^3 + 11*n
^2 + 6*n)*c*d^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*c^2*d^3*x^3 + 12*(n^2 + n)*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4972 vs. $2(342) = 684$.

Time = 0.16 (sec) , antiderivative size = 4972, normalized size of antiderivative = 14.71

$$\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
((d*x + c)^n*D*b^2*d^6*n^5*x^6 + (d*x + c)^n*D*b^2*c*d^5*n^5*x^5 + 2*(d*x
+ c)^n*D*a*b*d^6*n^5*x^5 + (d*x + c)^n*C*b^2*d^6*n^5*x^5 + 15*(d*x + c)^n*
D*b^2*d^6*n^4*x^6 + 2*(d*x + c)^n*D*a*b*c*d^5*n^5*x^4 + (d*x + c)^n*C*b^2*
c*d^5*n^5*x^4 + (d*x + c)^n*D*a^2*d^6*n^5*x^4 + 2*(d*x + c)^n*C*a*b*d^6*n^
5*x^4 + (d*x + c)^n*B*b^2*d^6*n^5*x^4 + 10*(d*x + c)^n*D*b^2*c*d^5*n^4*x^5
 + 32*(d*x + c)^n*D*a*b*d^6*n^4*x^5 + 16*(d*x + c)^n*C*b^2*d^6*n^4*x^5 + 8
5*(d*x + c)^n*D*b^2*d^6*n^3*x^6 + (d*x + c)^n*D*a^2*c*d^5*n^5*x^3 + 2*(d*x
 + c)^n*C*a*b*c*d^5*n^5*x^3 + (d*x + c)^n*B*b^2*c*d^5*n^5*x^3 + (d*x + c)^
n*C*a^2*d^6*n^5*x^3 + 2*(d*x + c)^n*B*a*b*d^6*n^5*x^3 + (d*x + c)^n*A*b^2*
d^6*n^5*x^3 - 5*(d*x + c)^n*D*b^2*c^2*d^4*n^4*x^4 + 24*(d*x + c)^n*D*a*b*c
*d^5*n^4*x^4 + 12*(d*x + c)^n*C*b^2*c*d^5*n^4*x^4 + 17*(d*x + c)^n*D*a^2*d
^6*n^4*x^4 + 34*(d*x + c)^n*C*a*b*d^6*n^4*x^4 + 17*(d*x + c)^n*B*b^2*d^6*n
^4*x^4 + 35*(d*x + c)^n*D*b^2*c*d^5*n^3*x^5 + 190*(d*x + c)^n*D*a*b*d^6*n^
3*x^5 + 95*(d*x + c)^n*C*b^2*d^6*n^3*x^5 + 225*(d*x + c)^n*D*b^2*d^6*n^2*x
^6 + (d*x + c)^n*C*a^2*c*d^5*n^5*x^2 + 2*(d*x + c)^n*B*a*b*c*d^5*n^5*x^2 +
 (d*x + c)^n*A*b^2*c*d^5*n^5*x^2 + (d*x + c)^n*B*a^2*d^6*n^5*x^2 + 2*(d*x
+ c)^n*A*a*b*d^6*n^5*x^2 - 8*(d*x + c)^n*D*a*b*c^2*d^4*n^4*x^3 - 4*(d*x +
c)^n*C*b^2*c^2*d^4*n^4*x^3 + 14*(d*x + c)^n*D*a^2*c*d^5*n^4*x^3 + 28*(d*x
+ c)^n*C*a*b*c*d^5*n^4*x^3 + 14*(d*x + c)^n*B*b^2*c*d^5*n^4*x^3 + 18*(d*x
+ c)^n*C*a^2*d^6*n^4*x^3 + 36*(d*x + c)^n*B*a*b*d^6*n^4*x^3 + 18*(d*x +...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int (a+b\,x)^2\,(c+d\,x)^n\,\left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((a + b*x)^2*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^2*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 2082, normalized size of antiderivative = 6.16

$$\int (a+bx)^2(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
int((b*x+a)^2*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output
```
((c + d*x)**n*(a**3*c*d**4*n**5 + 20*a**3*c*d**4*n**4 + 155*a**3*c*d**4*n*
*3 + 580*a**3*c*d**4*n**2 + 1044*a**3*c*d**4*n + 720*a**3*c*d**4 + a**3*d*
*5*n**5*x + 20*a**3*d**5*n**4*x + 155*a**3*d**5*n**3*x + 580*a**3*d**5*n**
2*x + 1044*a**3*d**5*n*x + 720*a**3*d**5*x - 3*a**2*b*c**2*d**3*n**4 - 54*
a**2*b*c**2*d**3*n**3 - 357*a**2*b*c**2*d**3*n**2 - 1026*a**2*b*c**2*d**3*
n - 1080*a**2*b*c**2*d**3 + 3*a**2*b*c*d**4*n**5*x + 54*a**2*b*c*d**4*n**4
*x + 357*a**2*b*c*d**4*n**3*x + 1026*a**2*b*c*d**4*n**2*x + 1080*a**2*b*c*
d**4*n*x + 3*a**2*b*d**5*n**5*x**2 + 57*a**2*b*d**5*n**4*x**2 + 411*a**2*b
*d**5*n**3*x**2 + 1383*a**2*b*d**5*n**2*x**2 + 2106*a**2*b*d**5*n*x**2 + 1
080*a**2*b*d**5*x**2 + 2*a**2*c**4*d**2*n**3 + 24*a**2*c**4*d**2*n**2 + 82
*a**2*c**4*d**2*n + 60*a**2*c**4*d**2 - 2*a**2*c**3*d**3*n**4*x - 24*a**2*
c**3*d**3*n**3*x - 82*a**2*c**3*d**3*n**2*x - 60*a**2*c**3*d**3*n*x + a**2
*c**2*d**4*n**5*x**2 + 13*a**2*c**2*d**4*n**4*x**2 + 53*a**2*c**2*d**4*n**
3*x**2 + 71*a**2*c**2*d**4*n**2*x**2 + 30*a**2*c**2*d**4*n*x**2 + 2*a**2*c
*d**5*n**5*x**3 + 32*a**2*c*d**5*n**4*x**3 + 186*a**2*c*d**5*n**3*x**3 + 4
84*a**2*c*d**5*n**2*x**3 + 568*a**2*c*d**5*n*x**3 + 240*a**2*c*d**5*x**3 +
 a**2*d**6*n**5*x**4 + 17*a**2*d**6*n**4*x**4 + 107*a**2*d**6*n**3*x**4 +
307*a**2*d**6*n**2*x**4 + 396*a**2*d**6*n*x**4 + 180*a**2*d**6*x**4 + 6*a*
b**2*c**3*d**2*n**3 + 90*a*b**2*c**3*d**2*n**2 + 444*a*b**2*c**3*d**2*n +
720*a*b**2*c**3*d**2 - 6*a*b**2*c**2*d**3*n**4*x - 90*a*b**2*c**2*d**3*...
```

3.176 $\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result 1694
Mathematica [A] (verified) 1695
Rubi [A] (verified) 1695
Maple [B] (verified) 1696
Fricas [B] (verification not implemented) 1697
Sympy [B] (verification not implemented) 1698
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Giac [B] (verification not implemented) 1701
Mupad [F(-1)] 1702
Reduce [B] (verification not implemented) 1702

Optimal result

Integrand size = 28, antiderivative size = 226

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = -\frac{(bc-ad)\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{1+n}}{d^5(1+n)} - \frac{(ad(2cCd-Bd^2-3c^2D)-b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))(c+dx)^{2+n}}{d^5(2+n)} + \frac{(ad(Cd-3cD)-b(3cCd-Bd^2-6c^2D))(c+dx)^{3+n}}{d^5(3+n)} + \frac{(bCd-4bcD+adD)(c+dx)^{4+n}}{d^5(4+n)} + \frac{bD(c+dx)^{5+n}}{d^5(5+n)}$$

output
```
-(-a*d+b*c)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1+n)/d^5/(1+n)-(a*d*(-B
*d^2+2*C*c*d-3*D*c^2)-b*(A*d^3-2*B*c*d^2+3*C*c^2*d-4*D*c^3))*(d*x+c)^(2+n)
/d^5/(2+n)+(a*d*(C*d-3*D*c)-b*(-B*d^2+3*C*c*d-6*D*c^2))*(d*x+c)^(3+n)/d^5/
(3+n)+(C*b*d+D*a*d-4*D*b*c)*(d*x+c)^(4+n)/d^5/(4+n)+b*D*(d*x+c)^(5+n)/d^5/
(5+n)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{(c+dx)^{1+n}\left(\frac{(bc-ad)(-c^2Cd+Bcd^2-Ad^3+c^3D)}{1+n}+\frac{(ad(-2cCd+Bd^2+3c^2D)+b(3c^2Cd-2Bcd^2+Ad^3-4c^3D))(c+dx)}{2+n}+\frac{(ad(Cd-3c}{d^5}$$

input
```
Integrate[(a + b*x)*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((c + d*x)^(1 + n)*(((b*c - a*d)*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D))/(
1 + n) + ((a*d*(-2*c*C*d + B*d^2 + 3*c^2*D) + b*(3*c^2*C*d - 2*B*c*d^2 + A
*d^3 - 4*c^3*D))*(c + d*x))/(2 + n) + ((a*d*(C*d - 3*c*D) + b*(-3*c*C*d +
B*d^2 + 6*c^2*D))*(c + d*x)^2)/(3 + n) + ((b*C*d - 4*b*c*D + a*d*D)*(c + d
*x)^3)/(4 + n) + (b*D*(c + d*x)^4)/(5 + n)))/d^5
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{(ad-bc)(c+dx)^n \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^4}+\frac{(c+dx)^{n+1}\left(b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)-ad\right.}{d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)(c+dx)^{n+1}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^5(n+1)}-\frac{(c+dx)^{n+2}\left(ad\left(-Bd^2-3c^2D+2cCd\right)-b\left(Ad^3-2Bcd^2-4c^3D+3c^2Cd\right)\right)}{d^5(n+2)}+\frac{(c+dx)^{n+3}\left(ad(Cd-3cD)-b\left(-Bd^2-6c^2D+3cCd\right)\right)}{d^5(n+3)}+\frac{(c+dx)^{n+4}(adD-4bcD+bCd)}{d^5(n+4)}+\frac{bD(c+dx)^{n+5}}{d^5(n+5)}$$

input
```
Int[(a + b*x)*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
-(((b*c - a*d)*(c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(1 + n))/(d^5
*(1 + n))) - ((a*d*(2*c*C*d - B*d^2 - 3*c^2*D) - b*(3*c^2*C*d - 2*B*c*d^2
+ A*d^3 - 4*c^3*D))*(c + d*x)^(2 + n))/(d^5*(2 + n)) + ((a*d*(C*d - 3*c*D)
 - b*(3*c*C*d - B*d^2 - 6*c^2*D))*(c + d*x)^(3 + n))/(d^5*(3 + n)) + ((b*C
*d - 4*b*c*D + a*d*D)*(c + d*x)^(4 + n))/(d^5*(4 + n)) + (b*D*(c + d*x)^(5
 + n))/(d^5*(5 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(226) = 452$.

Time = 0.42 (sec) , antiderivative size = 964, normalized size of antiderivative = 4.27

method	result
norman	$\frac{bDx^5 e^{n\ln(xd+c)}}{n+5}+\frac{c(Aa\,d^4n^4+14Aa\,d^4n^3-Abc\,d^3n^3-Bac\,d^3n^3+71Aa\,d^4n^2-12Abc\,d^3n^2-12Bac\,d^3n^2+2Bb\,c^2d^2n^2+2C}{}$
gosper	Expression too large to display
orering	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((b*x+a)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
b*D/(n+5)*x^5*exp(n*ln(d*x+c))+c*(A*a*d^4*n^4+14*A*a*d^4*n^3-A*b*c*d^3*n^3
-B*a*c*d^3*n^3+71*A*a*d^4*n^2-12*A*b*c*d^3*n^2-12*B*a*c*d^3*n^2+2*B*b*c^2*
d^2*n^2+2*C*a*c^2*d^2*n^2+154*A*a*d^4*n-47*A*b*c*d^3*n-47*B*a*c*d^3*n+18*B
*b*c^2*d^2*n+18*C*a*c^2*d^2*n-6*C*b*c^3*d*n-6*D*a*c^3*d*n+120*A*a*d^4-60*A
*b*c*d^3-60*B*a*c*d^3+40*B*b*c^2*d^2+40*C*a*c^2*d^2-30*C*b*c^3*d-30*D*a*c^
3*d+24*D*b*c^4)/d^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*exp(n*ln(d*x+c))
+(C*b*d*n+D*a*d*n+D*b*c*n+5*C*b*d+5*D*a*d)/d/(n^2+9*n+20)*x^4*exp(n*ln(d*x
+c))+(B*b*d^2*n^2+C*a*d^2*n^2+C*b*c*d*n^2+D*a*c*d*n^2+9*B*b*d^2*n+9*C*a*d^
2*n+5*C*b*c*d*n+5*D*a*c*d*n-4*D*b*c^2*n+20*B*b*d^2+20*C*a*d^2)/d^2/(n^3+12
*n^2+47*n+60)*x^3*exp(n*ln(d*x+c))+(A*b*d^3*n^3+B*a*d^3*n^3+B*b*c*d^2*n^3+
C*a*c*d^2*n^3+12*A*b*d^3*n^2+12*B*a*d^3*n^2+9*B*b*c*d^2*n^2+9*C*a*c*d^2*n^
2-3*C*b*c^2*d*n^2-3*D*a*c^2*d*n^2+47*A*b*d^3*n+47*B*a*d^3*n+20*B*b*c*d^2*n
+20*C*a*c*d^2*n-15*C*b*c^2*d*n-15*D*a*c^2*d*n+12*D*b*c^3*n+60*A*b*d^3+60*B
*a*d^3)/d^3/(n^4+14*n^3+71*n^2+154*n+120)*x^2*exp(n*ln(d*x+c))+(A*a*d^4*n^
4+A*b*c*d^3*n^4+B*a*c*d^3*n^4+14*A*a*d^4*n^3+12*A*b*c*d^3*n^3+12*B*a*c*d^3
*n^3-2*B*b*c^2*d^2*n^3-2*C*a*c^2*d^2*n^3+71*A*a*d^4*n^2+47*A*b*c*d^3*n^2+4
7*B*a*c*d^3*n^2-18*B*b*c^2*d^2*n^2-18*C*a*c^2*d^2*n^2+6*C*b*c^3*d*n^2+6*D*
a*c^3*d*n^2+154*A*a*d^4*n+60*A*b*c*d^3*n+60*B*a*c*d^3*n-40*B*b*c^2*d^2*n-4
0*C*a*c^2*d^2*n+30*C*b*c^3*d*n+30*D*a*c^3*d*n-24*D*b*c^4*n+120*A*a*d^4)/d^
4/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*x*exp(n*ln(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(228) = 456$.

Time = 0.10 (sec) , antiderivative size = 964, normalized size of antiderivative = 4.27

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
(A*a*c*d^4*n^4 + 24*D*b*c^5 + 120*A*a*c*d^4 - 30*(D*a + C*b)*c^4*d + 40*(C
*a + B*b)*c^3*d^2 - 60*(B*a + A*b)*c^2*d^3 + (D*b*d^5*n^4 + 10*D*b*d^5*n^3
 + 35*D*b*d^5*n^2 + 50*D*b*d^5*n + 24*D*b*d^5)*x^5 + (30*(D*a + C*b)*d^5 +
 (D*b*c*d^4 + (D*a + C*b)*d^5)*n^4 + (6*D*b*c*d^4 + 11*(D*a + C*b)*d^5)*n^
3 + (11*D*b*c*d^4 + 41*(D*a + C*b)*d^5)*n^2 + (6*D*b*c*d^4 + 61*(D*a + C*b
)*d^5)*n)*x^4 + (14*A*a*c*d^4 - (B*a + A*b)*c^2*d^3)*n^3 + (40*(C*a + B*b)
*d^5 + ((D*a + C*b)*c*d^4 + (C*a + B*b)*d^5)*n^4 - 4*(D*b*c^2*d^3 - 2*(D*a
 + C*b)*c*d^4 - 3*(C*a + B*b)*d^5)*n^3 - (12*D*b*c^2*d^3 - 17*(D*a + C*b)*
c*d^4 - 49*(C*a + B*b)*d^5)*n^2 - 2*(4*D*b*c^2*d^3 - 5*(D*a + C*b)*c*d^4 -
 39*(C*a + B*b)*d^5)*n)*x^3 + (71*A*a*c*d^4 + 2*(C*a + B*b)*c^3*d^2 - 12*(
B*a + A*b)*c^2*d^3)*n^2 + (60*(B*a + A*b)*d^5 + ((C*a + B*b)*c*d^4 + (B*a
+ A*b)*d^5)*n^4 - (3*(D*a + C*b)*c^2*d^3 - 10*(C*a + B*b)*c*d^4 - 13*(B*a
+ A*b)*d^5)*n^3 + (12*D*b*c^3*d^2 - 18*(D*a + C*b)*c^2*d^3 + 29*(C*a + B*b
)*c*d^4 + 59*(B*a + A*b)*d^5)*n^2 + (12*D*b*c^3*d^2 - 15*(D*a + C*b)*c^2*d
^3 + 20*(C*a + B*b)*c*d^4 + 107*(B*a + A*b)*d^5)*n)*x^2 + (154*A*a*c*d^4 -
 6*(D*a + C*b)*c^4*d + 18*(C*a + B*b)*c^3*d^2 - 47*(B*a + A*b)*c^2*d^3)*n
+ (120*A*a*d^5 + (A*a*d^5 + (B*a + A*b)*c*d^4)*n^4 + 2*(7*A*a*d^5 - (C*a +
 B*b)*c^2*d^3 + 6*(B*a + A*b)*c*d^4)*n^3 + (71*A*a*d^5 + 6*(D*a + C*b)*c^3
*d^2 - 18*(C*a + B*b)*c^2*d^3 + 47*(B*a + A*b)*c*d^4)*n^2 - 2*(12*D*b*c^4*
d - 77*A*a*d^5 - 15*(D*a + C*b)*c^3*d^2 + 20*(C*a + B*b)*c^2*d^3 - 30*(...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13522 vs. $2(211) = 422$.

Time = 2.66 (sec) , antiderivative size = 13522, normalized size of antiderivative = 59.83

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)**n*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((c**n*(A*a*x + A*b*x**2/2 + B*a*x**2/2 + B*b*x**3/3 + C*a*x**3/3
 + C*b*x**4/4 + D*a*x**4/4 + D*b*x**5/5), Eq(d, 0)), (-3*A*a*d**4/(12*c**4
*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4
) - A*b*c*d**3/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d
**8*x**3 + 12*d**9*x**4) - 4*A*b*d**4*x/(12*c**4*d**5 + 48*c**3*d**6*x + 7
2*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) - B*a*c*d**3/(12*c**4*d*
*5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) -
 4*B*a*d**4*x/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d*
*8*x**3 + 12*d**9*x**4) - B*b*c**2*d**2/(12*c**4*d**5 + 48*c**3*d**6*x + 7
2*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) - 4*B*b*c*d**3*x/(12*c**
4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**
4) - 6*B*b*d**4*x**2/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 +
48*c*d**8*x**3 + 12*d**9*x**4) - C*a*c**2*d**2/(12*c**4*d**5 + 48*c**3*d**
6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) - 4*C*a*c*d**3*x/
(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d
**9*x**4) - 6*C*a*d**4*x**2/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*
x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) - 3*C*b*c**3*d/(12*c**4*d**5 + 48*c*
*3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x**3 + 12*d**9*x**4) - 12*C*b*c*
*2*d**2*x/(12*c**4*d**5 + 48*c**3*d**6*x + 72*c**2*d**7*x**2 + 48*c*d**8*x
**3 + 12*d**9*x**4) - 18*C*b*c*d**3*x**2/(12*c**4*d**5 + 48*c**3*d**6*x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(228) = 456$.

Time = 0.07 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.64

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{(d^2(n+1)x^2+cdnx-c^2)(dx+c)^n Ba}{(n^2+3n+2)d^2} + \frac{(d^2(n+1)x^2+cdnx-c^2)(dx+c)^n Ab}{(n^2+3n+2)d^2} + \frac{(dx+c)^{n+1}Aa}{d(n+1)} + \frac{((n^2+3n+2)d^3x^3+(n^2+n)cd^2x^2-2c^2dnx+2c^3)(dx+c)^n Ca}{(n^3+6n^2+11n+6)d^3} + \frac{((n^2+3n+2)d^3x^3+(n^2+n)cd^2x^2-2c^2dnx+2c^3)(dx+c)^n Bb}{(n^3+6n^2+11n+6)d^3} + \frac{((n^3+6n^2+11n+6)d^4x^4+(n^3+3n^2+2n)cd^3x^3-3(n^2+n)c^2d^2x^2+6c^3dnx-6c^4)(dx+c)^n D}{(n^4+10n^3+35n^2+50n+24)d^4} + \frac{((n^3+6n^2+11n+6)d^4x^4+(n^3+3n^2+2n)cd^3x^3-3(n^2+n)c^2d^2x^2+6c^3dnx-6c^4)(dx+c)^n C}{(n^4+10n^3+35n^2+50n+24)d^4} + \frac{((n^4+10n^3+35n^2+50n+24)d^5x^5+(n^4+6n^3+11n^2+6n)cd^4x^4-4(n^3+3n^2+2n)c^2d^3x^3+}{(n^5+15n^4+85n^3+225n^2+274n+120)d^5}$$

input
```
integrate((b*x+a)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*B*a/((n^2 + 3*n + 2)*d^2) +
(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*A*b/((n^2 + 3*n + 2)*d^2) +
(d*x + c)^(n + 1)*A*a/(d*(n + 1)) + ((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c
*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*C*a/((n^3 + 6*n^2 + 11*n + 6)*
d^3) + ((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^
3)*(d*x + c)^n*B*b/((n^3 + 6*n^2 + 11*n + 6)*d^3) + ((n^3 + 6*n^2 + 11*n +
 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*
c^3*d*n*x - 6*c^4)*(d*x + c)^n*D*a/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^
4) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3
*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*C*b/((n^4 + 10*n
^3 + 35*n^2 + 50*n + 24)*d^4) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*d^5*x
^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*c*d^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*c^2*d^
3*x^3 + 12*(n^2 + n)*c^3*d^2*x^2 - 24*c^4*d*n*x + 24*c^5)*(d*x + c)^n*D*b/
((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*d^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2224 vs. $2(228) = 456$.

Time = 0.15 (sec) , antiderivative size = 2224, normalized size of antiderivative = 9.84

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
((d*x + c)^n*D*b*d^5*n^4*x^5 + (d*x + c)^n*D*b*c*d^4*n^4*x^4 + (d*x + c)^n
*D*a*d^5*n^4*x^4 + (d*x + c)^n*C*b*d^5*n^4*x^4 + 10*(d*x + c)^n*D*b*d^5*n^
3*x^5 + (d*x + c)^n*D*a*c*d^4*n^4*x^3 + (d*x + c)^n*C*b*c*d^4*n^4*x^3 + (d
*x + c)^n*C*a*d^5*n^4*x^3 + (d*x + c)^n*B*b*d^5*n^4*x^3 + 6*(d*x + c)^n*D*
b*c*d^4*n^3*x^4 + 11*(d*x + c)^n*D*a*d^5*n^3*x^4 + 11*(d*x + c)^n*C*b*d^5*
n^3*x^4 + 35*(d*x + c)^n*D*b*d^5*n^2*x^5 + (d*x + c)^n*C*a*c*d^4*n^4*x^2 +
 (d*x + c)^n*B*b*c*d^4*n^4*x^2 + (d*x + c)^n*B*a*d^5*n^4*x^2 + (d*x + c)^n
*A*b*d^5*n^4*x^2 - 4*(d*x + c)^n*D*b*c^2*d^3*n^3*x^3 + 8*(d*x + c)^n*D*a*c
*d^4*n^3*x^3 + 8*(d*x + c)^n*C*b*c*d^4*n^3*x^3 + 12*(d*x + c)^n*C*a*d^5*n^
3*x^3 + 12*(d*x + c)^n*B*b*d^5*n^3*x^3 + 11*(d*x + c)^n*D*b*c*d^4*n^2*x^4
+ 41*(d*x + c)^n*D*a*d^5*n^2*x^4 + 41*(d*x + c)^n*C*b*d^5*n^2*x^4 + 50*(d*
x + c)^n*D*b*d^5*n*x^5 + (d*x + c)^n*B*a*c*d^4*n^4*x + (d*x + c)^n*A*b*c*d
^4*n^4*x + (d*x + c)^n*A*a*d^5*n^4*x - 3*(d*x + c)^n*D*a*c^2*d^3*n^3*x^2 -
 3*(d*x + c)^n*C*b*c^2*d^3*n^3*x^2 + 10*(d*x + c)^n*C*a*c*d^4*n^3*x^2 + 10
*(d*x + c)^n*B*b*c*d^4*n^3*x^2 + 13*(d*x + c)^n*B*a*d^5*n^3*x^2 + 13*(d*x
+ c)^n*A*b*d^5*n^3*x^2 - 12*(d*x + c)^n*D*b*c^2*d^3*n^2*x^3 + 17*(d*x + c)
^n*D*a*c*d^4*n^2*x^3 + 17*(d*x + c)^n*C*b*c*d^4*n^2*x^3 + 49*(d*x + c)^n*C
*a*d^5*n^2*x^3 + 49*(d*x + c)^n*B*b*d^5*n^2*x^3 + 6*(d*x + c)^n*D*b*c*d^4*
n*x^4 + 61*(d*x + c)^n*D*a*d^5*n*x^4 + 61*(d*x + c)^n*C*b*d^5*n*x^4 + 24*(
d*x + c)^n*D*b*d^5*x^5 + (d*x + c)^n*A*a*c*d^4*n^4 - 2*(d*x + c)^n*C*a*...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \int (a+b\,x)\,\left(c+d\,x\right)^n\,\left(A+B\,x+C\,x^2+x^3\,D\right) \,dx$$

input
```
int((a + b*x)*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 973, normalized size of antiderivative = 4.31

$$\int (a+bx)(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{Too large to display}$$

input
```
int((b*x+a)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output

```
((c + d*x)**n*(a**2*c*d**3*n**4 + 14*a**2*c*d**3*n**3 + 71*a**2*c*d**3*n**
2 + 154*a**2*c*d**3*n + 120*a**2*c*d**3 + a**2*d**4*n**4*x + 14*a**2*d**4*
n**3*x + 71*a**2*d**4*n**2*x + 154*a**2*d**4*n*x + 120*a**2*d**4*x - 2*a*b
*c**2*d**2*n**3 - 24*a*b*c**2*d**2*n**2 - 94*a*b*c**2*d**2*n - 120*a*b*c**
2*d**2 + 2*a*b*c*d**3*n**4*x + 24*a*b*c*d**3*n**3*x + 94*a*b*c*d**3*n**2*x
 + 120*a*b*c*d**3*n*x + 2*a*b*d**4*n**4*x**2 + 26*a*b*d**4*n**3*x**2 + 118
*a*b*d**4*n**2*x**2 + 214*a*b*d**4*n*x**2 + 120*a*b*d**4*x**2 + 2*a*c**4*d
*n**2 + 12*a*c**4*d*n + 10*a*c**4*d - 2*a*c**3*d**2*n**3*x - 12*a*c**3*d**
2*n**2*x - 10*a*c**3*d**2*n*x + a*c**2*d**3*n**4*x**2 + 7*a*c**2*d**3*n**3
*x**2 + 11*a*c**2*d**3*n**2*x**2 + 5*a*c**2*d**3*n*x**2 + 2*a*c*d**4*n**4*
x**3 + 20*a*c*d**4*n**3*x**3 + 66*a*c*d**4*n**2*x**3 + 88*a*c*d**4*n*x**3
+ 40*a*c*d**4*x**3 + a*d**5*n**4*x**4 + 11*a*d**5*n**3*x**4 + 41*a*d**5*n*
*2*x**4 + 61*a*d**5*n*x**4 + 30*a*d**5*x**4 + 2*b**2*c**3*d*n**2 + 18*b**2
*c**3*d*n + 40*b**2*c**3*d - 2*b**2*c**2*d**2*n**3*x - 18*b**2*c**2*d**2*n
**2*x - 40*b**2*c**2*d**2*n*x + b**2*c*d**3*n**4*x**2 + 10*b**2*c*d**3*n**
3*x**2 + 29*b**2*c*d**3*n**2*x**2 + 20*b**2*c*d**3*n*x**2 + b**2*d**4*n**4
*x**3 + 12*b**2*d**4*n**3*x**3 + 49*b**2*d**4*n**2*x**3 + 78*b**2*d**4*n*x
**3 + 40*b**2*d**4*x**3 - 6*b*c**5*n - 6*b*c**5 + 6*b*c**4*d*n**2*x + 6*b*
c**4*d*n*x - 3*b*c**3*d**2*n**3*x**2 - 6*b*c**3*d**2*n**2*x**2 - 3*b*c**3*
d**2*n*x**2 + b*c**2*d**3*n**4*x**3 + 4*b*c**2*d**3*n**3*x**3 + 5*b*c**...
```

3.177 $\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal result . . . 1704
Mathematica [A] (verified) . . . 1704
Rubi [A] (verified) . . . 1705
Maple [B] (verified) . . . 1706
Fricas [B] (verification not implemented) . . . 1707
Sympy [B] (verification not implemented) . . . 1707
Maxima [A] (verification not implemented) . . . 1708
Giac [B] (verification not implemented) . . . 1709
Mupad [F(-1)] . . . 1710
Reduce [B] (verification not implemented) . . . 1710

Optimal result

Integrand size = 23, antiderivative size = 126

$$\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c+dx)^{1+n}}{d^4(1+n)} - \frac{\left(2cCd-Bd^2-3c^2D\right)(c+dx)^{2+n}}{d^4(2+n)} + \frac{(Cd-3cD)(c+dx)^{3+n}}{d^4(3+n)} + \frac{D(c+dx)^{4+n}}{d^4(4+n)}$$

output
```
(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(d*x+c)^(1+n)/d^4/(1+n)-(-B*d^2+2*C*c*d-3*D*
c^2)*(d*x+c)^(2+n)/d^4/(2+n)+(C*d-3*D*c)*(d*x+c)^(3+n)/d^4/(3+n)+D*(d*x+c)
^(4+n)/d^4/(4+n)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

$$\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{(c+dx)^{1+n}\left(\frac{c^2Cd-Bcd^2+Ad^3-c^3D}{1+n} + \frac{(-2cCd+Bd^2+3c^2D)(c+dx)}{2+n} + \frac{(Cd-3cD)(c+dx)^2}{3+n} + \frac{D(c+dx)^3}{4+n}\right)}{d^4}$$

input
```
Integrate[(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((c + d*x)^(1 + n)*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)/(1 + n) + ((-2*c*C
*d + B*d^2 + 3*c^2*D)*(c + d*x))/(2 + n) + ((C*d - 3*c*D)*(c + d*x)^2)/(3
+ n) + (D*(c + d*x)^3)/(4 + n)))/d^4
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{(c+dx)^n \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^3}+\frac{(c+dx)^{n+1} \left(Bd^2+3c^2D-2cCd\right)}{d^3}+\frac{(Cd-3cD)(c+dx)^{n+2}}{d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^{n+1} \left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{d^4(n+1)}-\frac{(c+dx)^{n+2} \left(-Bd^2-3c^2D+2cCd\right)}{d^4(n+2)}+\frac{(Cd-3cD)(c+dx)^{n+3}}{d^4(n+3)}+\frac{D(c+dx)^{n+4}}{d^4(n+4)}$$

input
```
Int[(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c + d*x)^(1 + n))/(d^4*(1 + n)) - ((
2*c*C*d - B*d^2 - 3*c^2*D)*(c + d*x)^(2 + n))/(d^4*(2 + n)) + ((C*d - 3*c*
D)*(c + d*x)^(3 + n))/(d^4*(3 + n)) + (D*(c + d*x)^(4 + n))/(d^4*(4 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(126) = 252.

Time = 0.43 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.44

method	result
gosper	$(xd+c)^{1+n}\left(Dd^3n^3x^3+C\,d^3n^3x^2+6Dd^3n^2x^3+B\,d^3n^3x+7C\,d^3n^2x^2-3Dc\,d^2n^2x^2+11Dd^3n\,x^3+A\,d^3n^3+8B\,d^3n^2x-2Cc\,d\right.$
orering	$(xd+c)^{n}\left(Dd^3n^3x^3+C\,d^3n^3x^2+6Dd^3n^2x^3+B\,d^3n^3x+7C\,d^3n^2x^2-3Dc\,d^2n^2x^2+11Dd^3n\,x^3+A\,d^3n^3+8B\,d^3n^2x-2Cc\,d^2n\right.$
norman	$\frac{Dx^4e^{n\ln(xd+c)}}{4+n}+\frac{c\left(A\,d^3n^3+9A\,d^3n^2-Bc\,d^2n^2+26A\,d^3n-7Bc\,d^2n+2c^2Cdn+24A\,d^3-12Bc\,d^2+8C\,c^2d-6Dc^3\right)e^{n\ln(xd+}}{d^4(n^4+10n^3+35n^2+50n+24)}$
parallelrisch	$-12B(xd+c)^nc^3d^2+8C(xd+c)^nc^4d+2Dx^3(xd+c)^nc^2d^3n-3Dx^2(xd+c)^nc^3d^2n^2+9Ax(xd+c)^nc\,d^4n^2+19B\,x^2(xd+c)^nc\,d^4$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(d*x+c)^(1+n)/(n^4+10*n^3+35*n^2+50*n+24)*(D*d^3*n^3*x^3+C*d^3*n^3*x
^2+6*D*d^3*n^2*x^3+B*d^3*n^3*x+7*C*d^3*n^2*x^2-3*D*c*d^2*n^2*x^2+11*D*d^3*
n*x^3+A*d^3*n^3+8*B*d^3*n^2*x-2*C*c*d^2*n^2*x+14*C*d^3*n*x^2-9*D*c*d^2*n*x
^2+6*D*d^3*x^3+9*A*d^3*n^2-B*c*d^2*n^2+19*B*d^3*n*x-10*C*c*d^2*n*x+8*C*d^3
*x^2+6*D*c^2*d*n*x-6*D*c*d^2*x^2+26*A*d^3*n-7*B*c*d^2*n+12*B*d^3*x+2*C*c^2
*d*n-8*C*c*d^2*x+6*D*c^2*d*x+24*A*d^3-12*B*c*d^2+8*C*c^2*d-6*D*c^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(127) = 254.

Time = 0.09 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.13

$$\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{(Acd^3n^3 - 6\,Dc^4 + 8\,Cc^3d - 12\,Bc^2d^2 + 24\,Acd^3 + (Dd^4n^3 + 6\,Dd^4n^2 + 11\,Dd^4n + 6\,Dd^4)x^4 + (8\,Cd^4}{}$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
(A*c*d^3*n^3 - 6*D*c^4 + 8*C*c^3*d - 12*B*c^2*d^2 + 24*A*c*d^3 + (D*d^4*n^
3 + 6*D*d^4*n^2 + 11*D*d^4*n + 6*D*d^4)*x^4 + (8*C*d^4 + (D*c*d^3 + C*d^4)
*n^3 + (3*D*c*d^3 + 7*C*d^4)*n^2 + 2*(D*c*d^3 + 7*C*d^4)*n)*x^3 - (B*c^2*d
^2 - 9*A*c*d^3)*n^2 + (12*B*d^4 + (C*c*d^3 + B*d^4)*n^3 - (3*D*c^2*d^2 - 5
*C*c*d^3 - 8*B*d^4)*n^2 - (3*D*c^2*d^2 - 4*C*c*d^3 - 19*B*d^4)*n)*x^2 + (2
*C*c^3*d - 7*B*c^2*d^2 + 26*A*c*d^3)*n + (24*A*d^4 + (B*c*d^3 + A*d^4)*n^3
 - (2*C*c^2*d^2 - 7*B*c*d^3 - 9*A*d^4)*n^2 + 2*(3*D*c^3*d - 4*C*c^2*d^2 +
6*B*c*d^3 + 13*A*d^4)*n)*x)*(d*x + c)^n/(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2
 + 50*d^4*n + 24*d^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3798 vs. 2(112) = 224.

Time = 1.07 (sec) , antiderivative size = 3798, normalized size of antiderivative = 30.14

$$\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Piecewise((c**n*(A*x + B*x**2/2 + C*x**3/3 + D*x**4/4), Eq(d, 0)), (-2*A*d
**3/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) - B*c*d*
*2/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) - 3*B*d**
3*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) - 2*C*c*
*2*d/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) - 6*C*c
*d**2*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) - 6*
C*d**3*x**2/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3)
 + 6*D*c**3*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6
*d**7*x**3) + 11*D*c**3/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6
*d**7*x**3) + 18*D*c**2*d*x*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 1
8*c*d**6*x**2 + 6*d**7*x**3) + 27*D*c**2*d*x/(6*c**3*d**4 + 18*c**2*d**5*x
 + 18*c*d**6*x**2 + 6*d**7*x**3) + 18*D*c*d**2*x**2*log(c/d + x)/(6*c**3*d
**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 18*D*c*d**2*x**2/(6
*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*D*d**3*x**
3*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**
3), Eq(n, -4)), (-A*d**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - B*c*d*
*2/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 2*B*d**3*x/(2*c**2*d**4 + 4*
c*d**5*x + 2*d**6*x**2) + 2*C*c**2*d*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*
x + 2*d**6*x**2) + 3*C*c**2*d/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 4
*C*c*d**2*x*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 4*C...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.86

$$\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \frac{(d^2(n+1)x^2+cdnx-c^2)(dx+c)^n B}{(n^2+3\,n+2)d^2} + \frac{(dx+c)^{n+1}A}{d(n+1)} + \frac{((n^2+3\,n+2)d^3x^3+(n^2+n)cd^2x^2-2\,c^2dnx+2\,c^3)(dx+c)^n C}{(n^3+6\,n^2+11\,n+6)d^3} + \frac{((n^3+6\,n^2+11\,n+6)d^4x^4+(n^3+3\,n^2+2\,n)cd^3x^3-3\,(n^2+n)c^2d^2x^2+6\,c^3dnx-6\,c^4)(dx+c)^n D}{(n^4+10\,n^3+35\,n^2+50\,n+24)d^4}$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*B/((n^2 + 3*n + 2)*d^2) + (d
*x + c)^(n + 1)*A/(d*(n + 1)) + ((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c*d^2
*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*C/((n^3 + 6*n^2 + 11*n + 6)*d^3) +
 ((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x^3 - 3*(n^
2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*D/((n^4 + 10*n^3 + 3
5*n^2 + 50*n + 24)*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(127) = 254$.

Time = 0.13 (sec) , antiderivative size = 728, normalized size of antiderivative = 5.78

$$\int (c + dx)^n \left(A + Bx + Cx^2 + Dx^3\right) dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
((d*x + c)^n*D*d^4*n^3*x^4 + (d*x + c)^n*D*c*d^3*n^3*x^3 + (d*x + c)^n*C*d
^4*n^3*x^3 + 6*(d*x + c)^n*D*d^4*n^2*x^4 + (d*x + c)^n*C*c*d^3*n^3*x^2 + (
d*x + c)^n*B*d^4*n^3*x^2 + 3*(d*x + c)^n*D*c*d^3*n^2*x^3 + 7*(d*x + c)^n*C
*d^4*n^2*x^3 + 11*(d*x + c)^n*D*d^4*n*x^4 + (d*x + c)^n*B*c*d^3*n^3*x + (d
*x + c)^n*A*d^4*n^3*x - 3*(d*x + c)^n*D*c^2*d^2*n^2*x^2 + 5*(d*x + c)^n*C*
c*d^3*n^2*x^2 + 8*(d*x + c)^n*B*d^4*n^2*x^2 + 2*(d*x + c)^n*D*c*d^3*n*x^3
+ 14*(d*x + c)^n*C*d^4*n*x^3 + 6*(d*x + c)^n*D*d^4*x^4 + (d*x + c)^n*A*c*d
^3*n^3 - 2*(d*x + c)^n*C*c^2*d^2*n^2*x + 7*(d*x + c)^n*B*c*d^3*n^2*x + 9*(
d*x + c)^n*A*d^4*n^2*x - 3*(d*x + c)^n*D*c^2*d^2*n*x^2 + 4*(d*x + c)^n*C*c
*d^3*n*x^2 + 19*(d*x + c)^n*B*d^4*n*x^2 + 8*(d*x + c)^n*C*d^4*x^3 - (d*x +
 c)^n*B*c^2*d^2*n^2 + 9*(d*x + c)^n*A*c*d^3*n^2 + 6*(d*x + c)^n*D*c^3*d*n*
x - 8*(d*x + c)^n*C*c^2*d^2*n*x + 12*(d*x + c)^n*B*c*d^3*n*x + 26*(d*x + c
)^n*A*d^4*n*x + 12*(d*x + c)^n*B*d^4*x^2 + 2*(d*x + c)^n*C*c^3*d*n - 7*(d*
x + c)^n*B*c^2*d^2*n + 26*(d*x + c)^n*A*c*d^3*n + 24*(d*x + c)^n*A*d^4*x -
 6*(d*x + c)^n*D*c^4 + 8*(d*x + c)^n*C*c^3*d - 12*(d*x + c)^n*B*c^2*d^2 +
24*(d*x + c)^n*A*c*d^3)/(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24
*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \int (c+d\,x)^n \left(A+B\,x+C\,x^2+x^3\,D\right) dx$$

input
```
int((c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.74

$$\int (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= \frac{(dx+c)^n \left(d^4n^3x^4+2c\,d^3n^3x^3+6d^4n^2x^4+b\,d^3n^3x^2+c^2d^2n^3x^2+10c\,d^3n^2x^3+11d^4n\,x^4+a\,d^3n^3x+bc\right.}{}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output
```
((c + d*x)**n*(a*c*d**2*n**3 + 9*a*c*d**2*n**2 + 26*a*c*d**2*n + 24*a*c*d*
*2 + a*d**3*n**3*x + 9*a*d**3*n**2*x + 26*a*d**3*n*x + 24*a*d**3*x - b*c**
2*d*n**2 - 7*b*c**2*d*n - 12*b*c**2*d + b*c*d**2*n**3*x + 7*b*c*d**2*n**2*
x + 12*b*c*d**2*n*x + b*d**3*n**3*x**2 + 8*b*d**3*n**2*x**2 + 19*b*d**3*n*
x**2 + 12*b*d**3*x**2 + 2*c**4*n + 2*c**4 - 2*c**3*d*n**2*x - 2*c**3*d*n*x
 + c**2*d**2*n**3*x**2 + 2*c**2*d**2*n**2*x**2 + c**2*d**2*n*x**2 + 2*c*d*
*3*n**3*x**3 + 10*c*d**3*n**2*x**3 + 16*c*d**3*n*x**3 + 8*c*d**3*x**3 + d*
*4*n**3*x**4 + 6*d**4*n**2*x**4 + 11*d**4*n*x**4 + 6*d**4*x**4))/(d**3*(n*
*4 + 10*n**3 + 35*n**2 + 50*n + 24))
```

3.178 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$

Optimal result 1711
Mathematica [A] (verified) 1712
Rubi [A] (verified) 1712
Maple [F] 1713
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Sympy [F] 1714
Maxima [F] 1714
Giac [F] 1715
Mupad [F(-1)] 1715
Reduce [F] 1715

Optimal result

Integrand size = 30, antiderivative size = 203

$$\begin{aligned}\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{a+bx}\,dx &= \frac{(a^2d^2D-abd(Cd-cD)-b^2(cCd-Bd^2-c^2D))(c+dx)^{1+n}}{b^3d^3(1+n)} \\ &\quad+\frac{(bCd-2bcD-adD)(c+dx)^{2+n}}{b^2d^3(2+n)}+\frac{D(c+dx)^{3+n}}{bd^3(3+n)} \\ &\quad-\frac{(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(c+dx)}{bc-ad}\right)}{b^3(bc-ad)(1+n)}\end{aligned}$$

output
```
(a^2*d^2*D-a*b*d*(C*d-D*c)-b^2*(-B*d^2+C*c*d-D*c^2))*(d*x+c)^(1+n)/b^3/d^3
/(1+n)+(C*b*d-D*a*d-2*D*b*c)*(d*x+c)^(2+n)/b^2/d^3/(2+n)+D*(d*x+c)^(3+n)/b
/d^3/(3+n)-(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1+n)*hypergeom([1, 1+n],
[2+n],b*(d*x+c)/(-a*d+b*c))/b^3/(-a*d+b*c)/(1+n)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$
$$= \frac{(c+dx)^{1+n}\left(\frac{a^2d^2D+abd(-Cd+cD)+b^2(-cCd+Bd^2+c^2D)}{d^3(1+n)}+\frac{b(bCd-2bcD-adD)(c+dx)}{d^3(2+n)}+\frac{b^2D(c+dx)^2}{d^3(3+n)}-\frac{(Ab^3-a(b^2B-abC+\cdots}{\cdots}\right.}{b^3}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
((c + d*x)^(1 + n)*((a^2*d^2*D + a*b*d*(-(C*d) + c*D) + b^2*(-(c*C*d) + B*
d^2 + c^2*D))/(d^3*(1 + n)) + (b*(b*C*d - 2*b*c*D - a*d*D)*(c + d*x))/(d^3
*(2 + n)) + (b^2*D*(c + d*x)^2)/(d^3*(3 + n)) - ((A*b^3 - a*(b^2*B - a*b*C
 + a^2*D))*Hypergeometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/
((b*c - a*d)*(1 + n))))/b^3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{(c+dx)^n (Ab^3-a(a^2D-abC+b^2B))}{b^3(a+bx)}+\frac{(c+dx)^n (a^2d^2D-abd(Cd-cD)-(b^2(-Bd^2+c^2(-D)+cC\cdots}{b^3d^2}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(c+dx)}{bc-ad}\right)}{b^3(n+1)(bc-ad)}+\frac{(c+dx)^{n+1}\left(a^2d^2D-abd(Cd-cD)-\left(b^2\left(-Bd^2+c^2(-D)+cCd\right)\right)\right)}{b^3d^3(n+1)}+\frac{(c+dx)^{n+2}(-adD-2bcD+bCd)}{b^2d^3(n+2)}+\frac{D(c+dx)^{n+3}}{bd^3(n+3)}$$

input
```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x),x]
```

output
```
((a^2*d^2*D - a*b*d*(C*d - c*D) - b^2*(c*C*d - B*d^2 - c^2*D))*(c + d*x)^(
1 + n))/(b^3*d^3*(1 + n)) + ((b*C*d - 2*b*c*D - a*d*D)*(c + d*x)^(2 + n))/
(b^2*d^3*(2 + n)) + (D*(c + d*x)^(3 + n))/(b*d^3*(3 + n)) - ((A*b^3 - a*(b
^2*B - a*b*C + a^2*D))*(c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n
, (b*(c + d*x))/(b*c - a*d)])/(b^3*(b*c - a*d)*(1 + n))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

Maple [F]

$$\int \frac{(xd+c)^n\left(Dx^3+C\,x^2+Bx+A\right)}{bx+a}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{bx+a}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a),x)
```

output
```
Integral((c + d*x)**n*(A + B*x + C*x**2 + D*x**3)/(a + b*x), x)
```

Maxima [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{bx+a}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{bx+a}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{a+b\,x}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x),x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{a+bx}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a),x)
```

output
```
( - (c + d*x)**n*a**2*c*d**2*n**2 - 5*(c + d*x)**n*a**2*c*d**2*n - 6*(c +
d*x)**n*a**2*c*d**2 + (c + d*x)**n*a**2*d**3*n**3*x + 5*(c + d*x)**n*a**2*
d**3*n**2*x + 6*(c + d*x)**n*a**2*d**3*n*x + 2*(c + d*x)**n*a*b*c**2*d*n**
2 + 8*(c + d*x)**n*a*b*c**2*d*n + 6*(c + d*x)**n*a*b*c**2*d - 2*(c + d*x)*
*n*a*b*c*d**2*n**3*x - 8*(c + d*x)**n*a*b*c*d**2*n**2*x - 6*(c + d*x)**n*a
*b*c*d**2*n*x - (c + d*x)**n*a*b*d**3*n**3*x**2 - 4*(c + d*x)**n*a*b*d**3*
n**2*x**2 - 3*(c + d*x)**n*a*b*d**3*n*x**2 + (c + d*x)**n*b**3*c*d*n**3 +
5*(c + d*x)**n*b**3*c*d*n**2 + 6*(c + d*x)**n*b**3*c*d*n + (c + d*x)**n*b*
*3*d**2*n**3*x + 5*(c + d*x)**n*b**3*d**2*n**2*x + 6*(c + d*x)**n*b**3*d**
2*n*x - (c + d*x)**n*b**2*c**3*n**2 - (c + d*x)**n*b**2*c**3*n + (c + d*x)
**n*b**2*c**2*d*n**3*x + (c + d*x)**n*b**2*c**2*d*n**2*x + 2*(c + d*x)**n*
b**2*c*d**2*n**3*x**2 + 5*(c + d*x)**n*b**2*c*d**2*n**2*x**2 + 3*(c + d*x)
**n*b**2*c*d**2*n*x**2 + (c + d*x)**n*b**2*d**3*n**3*x**3 + 3*(c + d*x)**n
*b**2*d**3*n**2*x**3 + 2*(c + d*x)**n*b**2*d**3*n*x**3 - int(((c + d*x)**n
*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*d**4*n**4 - 6*int(((c + d*x)*
*n*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*d**4*n**3 - 11*int(((c + d*
x)**n*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*d**4*n**2 - 6*int(((c +
d*x)**n*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**3*d**4*n + 2*int(((c + d
*x)**n*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b*c*d**3*n**4 + 12*int(
((c + d*x)**n*x)/(a*c + a*d*x + b*c*x + b*d*x**2),x)*a**2*b*c*d**3*n**3...
```

3.179 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$

Optimal result . . . 1717
Mathematica [A] (verified) . . . 1718
Rubi [A] (verified) . . . 1718
Maple [F] . . . 1720
Fricas [F] . . . 1720
Sympy [F(-2)] . . . 1721
Maxima [F] . . . 1721
Giac [F] . . . 1721
Mupad [F(-1)] . . . 1722
Reduce [F] . . . 1722

Optimal result

Integrand size = 30, antiderivative size = 215

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx = \frac{(bCd-bcD-2adD)(c+dx)^{1+n}}{b^3d^2(1+n)} + \frac{\left(b^2B-2abC+3a^2D\right)(c+dx)^{1+n}}{b^3dn(a+bx)} + \frac{D(c+dx)^{2+n}}{b^2d^2(2+n)} - \frac{\left(a^3dD(3+n)-b^3(Bc+Adn)+ab^2(2cC+Bd(1+n))-a^2b(3cD+Cd(2+n))\right)(c+dx)^{1+n}\,\mathrm{Hyper}}{b^3(bc-ad)^2n(1+n)}$$

output
```
(C*b*d-2*D*a*d-D*b*c)*(d*x+c)^(1+n)/b^3/d^2/(1+n)+(B*b^2-2*C*a*b+3*D*a^2)*
(d*x+c)^(1+n)/b^3/d/n/(b*x+a)+D*(d*x+c)^(2+n)/b^2/d^2/(2+n)-(a^3*d*D*(3+n)
-b^3*(A*d*n+B*c)+a*b^2*(2*C*c+B*d*(1+n))-a^2*b*(3*D*c+C*d*(2+n)))*(d*x+c)^
(1+n)*hypergeom([2, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/b^3/(-a*d+b*c)^2/n/(1
+n)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$= \frac{(c+dx)^{1+n}\left(\frac{bCd-bcD-2adD}{d^2(1+n)}+\frac{bD(c+dx)}{d^2(2+n)}-\frac{(b^2B-2abC+3a^2D)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(c+dx)}{bc-ad}\right)}{(bc-ad)(1+n)}+\frac{d(Ab^3-a(b^2B-a\right.}{}}{b^3}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
((c + d*x)^(1 + n)*((b*C*d - b*c*D - 2*a*d*D)/(d^2*(1 + n)) + (b*D*(c + d*
x))/(d^2*(2 + n)) - ((b^2*B - 2*a*b*C + 3*a^2*D)*Hypergeometric2F1[1, 1 +
n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/((b*c - a*d)*(1 + n)) + (d*(A*b^3 -
a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[2, 1 + n, 2 + n, (b*(c + d*x)
)/(b*c - a*d)])/((b*c - a*d)^2*(1 + n))))/b^3
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2124, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \ 2124$$
$$-\frac{\int \frac{(c+dx)^n\left(-\left(\left(c-\frac{ad}{b}\right)Dx^2\right)-\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{dD(n+1)a^3-b(cD+Cd(n+1))a^2+b^2(cC+Bd(n+1))a-b^3(Bc+Adn)}{b^3}\right)}{a+bx}dx}{bc-ad}-\frac{(c+dx)^{n+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{1195}$$

$$-\frac{\int \left(-\frac{(bc-ad)(bCd-2aDd-bcD)(c+dx)^n}{b^3d}+\frac{(dD(n+3)a^3-b(3cD+Cd(n+2))a^2+b^2(2cC+Bd(n+1))a-b^3(Bc+Adn))(c+dx)^n}{b^3(a+bx)}-\frac{(bc-ad)}{}\right.}{bc-ad}$$

$$\frac{(c+dx)^{n+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(c+dx)^{n+1}\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{(a+bx)(bc-ad)}-$$

$$-\frac{\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(c+dx)}{bc-ad}\right)(a^3dD(n+3)-a^2b(3cD+Cd(n+2))+ab^2(Bd(n+1)+2cC)-b^3(Adn+Bc))}{b^3(n+1)(bc-ad)}-\frac{(bc-ad)(}{}}{bc-ad}$$

input
```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^2,x]
```

output
```
-(((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(1 + n))/((b*c - a*d)*(
a + b*x))) - (-(((b*c - a*d)*(b*C*d - b*c*D - 2*a*d*D)*(c + d*x)^(1 + n))/
(b^3*d^2*(1 + n))) - ((b*c - a*d)*D*(c + d*x)^(2 + n))/(b^2*d^2*(2 + n)) -
 ((a^3*d*D*(3 + n) - b^3*(B*c + A*d*n) + a*b^2*(2*c*C + B*d*(1 + n)) - a^2
*b*(3*c*D + C*d*(2 + n)))*(c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, (b*(c + d*x))/(b*c - a*d)])/(b^3*(b*c - a*d)*(1 + n)))/(b*c - a*d)
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(xd+c)^n \left(Dx^3+C\,x^2+Bx+A\right)}{(bx+a)^2}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

Fricas [F]

$$\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^2}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^2}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b^2*x^2 + 2*a*b*x + a^2),
x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} \, dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^2} \, dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} \, dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^2} \, dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} \, dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^2} \, dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2,x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^2} \, dx = \text{too large to display}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^2,x)
```

output

```
( - (c + d*x)**n*a**3*c*d**2*n**2 - 5*(c + d*x)**n*a**3*c*d**2*n - 6*(c +
d*x)**n*a**3*c*d**2 + (c + d*x)**n*a**3*d**3*n**3*x + 5*(c + d*x)**n*a**3*
d**3*n**2*x + 6*(c + d*x)**n*a**3*d**3*n*x + 2*(c + d*x)**n*a**2*b*c**2*d*
n**2 + 8*(c + d*x)**n*a**2*b*c**2*d*n + 4*(c + d*x)**n*a**2*b*c**2*d - 2*(
c + d*x)**n*a**2*b*c*d**2*n**3*x - 9*(c + d*x)**n*a**2*b*c*d**2*n**2*x - 9
*(c + d*x)**n*a**2*b*c*d**2*n*x - 6*(c + d*x)**n*a**2*b*c*d**2*x - (c + d*
x)**n*a**2*b*d**3*n**3*x**2 - 3*(c + d*x)**n*a**2*b*d**3*n**2*x**2 + (c +
d*x)**n*a*b**3*c*d*n**3 + 2*(c + d*x)**n*a*b**3*c*d*n**2 - (c + d*x)**n*a*
b**3*c*d*n - 2*(c + d*x)**n*a*b**3*c*d + (c + d*x)**n*a*b**3*d**2*n**3*x +
 3*(c + d*x)**n*a*b**3*d**2*n**2*x + 2*(c + d*x)**n*a*b**3*d**2*n*x - (c +
 d*x)**n*a*b**2*c**3*n**2 - (c + d*x)**n*a*b**2*c**3*n + (c + d*x)**n*a*b*
*2*c**2*d*n**3*x + 3*(c + d*x)**n*a*b**2*c**2*d*n**2*x + 8*(c + d*x)**n*a*
b**2*c**2*d*n*x + 4*(c + d*x)**n*a*b**2*c**2*d*x + 2*(c + d*x)**n*a*b**2*c
*d**2*n**3*x**2 + 3*(c + d*x)**n*a*b**2*c*d**2*n**2*x**2 + 3*(c + d*x)**n*
a*b**2*c*d**2*n*x**2 + (c + d*x)**n*a*b**2*d**3*n**3*x**3 + (c + d*x)**n*a
*b**2*d**3*n**2*x**3 - (c + d*x)**n*b**4*c*d*n**2*x - 3*(c + d*x)**n*b**4*
c*d*n*x - 2*(c + d*x)**n*b**4*c*d*x - (c + d*x)**n*b**3*c**3*n**2*x - (c +
 d*x)**n*b**3*c**3*n*x - 2*(c + d*x)**n*b**3*c**2*d*n**2*x**2 - 2*(c + d*x
)**n*b**3*c**2*d*n*x**2 - (c + d*x)**n*b**3*c*d**2*n**2*x**3 - (c + d*x)**
n*b**3*c*d**2*n*x**3 - int(((c + d*x)**n*x)/(a**3*c*d*n + a**3*d**2*n*x...
```

3.180 $\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3} \, dx$

Optimal result . . . 1724
Mathematica [A] (verified) . . . 1725
Rubi [A] (verified) . . . 1725
Maple [F] . . . 1728
Fricas [F] . . . 1729
Sympy [F] . . . 1729
Maxima [F] . . . 1729
Giac [F] . . . 1730
Mupad [F(-1)] . . . 1730
Reduce [F] . . . 1730

Optimal result

Integrand size = 30, antiderivative size = 271

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3} \, dx$$
$$= \frac{D(c+dx)^{1+n}}{b^3d(1+n)} - \frac{(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{1+n}}{2b^3(bc-ad)(a+bx)^2} + \frac{(bC-3aD)(c+dx)^{1+n}}{b^3dn(a+bx)}$$
$$+ \frac{(b^3(2c^2C+2Bcdn-Ad^2(1-n)n) - a^3d^2D(6+5n+n^2) + a^2bd(2+n)(6cD+Cd(1+n)) - ab^2(6c}{2b^3(bc-ad)^3n}$$

output
```
D*(d*x+c)^(1+n)/b^3/d/(1+n)-1/2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1+n
)/b^3/(-a*d+b*c)/(b*x+a)^2+(C*b-3*D*a)*(d*x+c)^(1+n)/b^3/d/n/(b*x+a)+1/2*(
b^3*(2*C*c^2+2*B*c*d*n-A*d^2*(1-n)*n)-a^3*d^2*D*(n^2+5*n+6)+a^2*b*d*(2+n)*
(6*D*c+C*d*(1+n))-a*b^2*(6*D*c^2+4*c*C*d*(1+n)+B*d^2*n*(1+n)))*(d*x+c)^(1+
n)*hypergeom([2, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/b^3/(-a*d+b*c)^3/n/(1+n)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$
$$=\frac{(c+dx)^{1+n}\left(\frac{D}{d}-\frac{(bC-3aD)\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(c+dx)}{bc-ad}\right)}{bc-ad}+\frac{d\left(b^2B-2abC+3a^2D\right)\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{b(}{b}\right.}{(bc-ad)^2}\right.}{b^3(1+n)}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
((c + d*x)^(1 + n)*(D/d - ((b*C - 3*a*D)*Hypergeometric2F1[1, 1 + n, 2 + n
, (b*(c + d*x))/(b*c - a*d)])/(b*c - a*d) + (d*(b^2*B - 2*a*b*C + 3*a^2*D)
*Hypergeometric2F1[2, 1 + n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/(b*c - a*d
)^2 - (d^2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[3, 1 + n,
 2 + n, (b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^3))/(b^3*(1 + n))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.33, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2124, 25, 1193, 25, 27, 90, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^3}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{\int -\frac{(c+dx)^n\left(-\frac{dD(n+1)a^3}{b^3}+\frac{(2cD+Cd(n+1))a^2}{b^2}-\frac{(2cC+Bd(n+1))a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc-Ad(1-n)+\frac{2(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^2}dx}{2(bc-ad)}-\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(c+dx)^n\left(-\frac{dD(n+1)a^3}{b^3}+\frac{(2cD+Cd(n+1))a^2}{b^2}-\frac{(2cC+Bd(n+1))a}{b}+2\left(c-\frac{ad}{b}\right)Dx^2+2Bc-Ad(1-n)+\frac{2(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^2}dx}{2(bc-ad)}-\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{-\frac{\int -\frac{(c+dx)^n\left(-d^2D\left(n^2+5n+4\right)a^3+bd\left(2cD(3n+4)+Cd\left(n^2+3n+2\right)\right)a^2-b^2\left(4Dc^2+4Cd(n+1)c+Bd^2n(n+1)\right)a+b^3\left(2Cc^2+2Bdnc-Ad^2(1-n)n\right)+2b(bc-ad}{b^3(a+bx)}}{bc-ad}}{2(bc-ad)}$$

$$\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{(c+dx)^n\left(-d^2D\left(n^2+5n+4\right)a^3+bd\left(2cD(3n+4)+Cd\left(n^2+3n+2\right)\right)a^2-b^2\left(4Dc^2+4Cd(n+1)c+Bd^2n(n+1)\right)a+b^3\left(2Cc^2+2Bdnc-Ad^2(1-n)n\right)+2b(bc-ad)^2Dx}{b^3(a+bx)}}{bc-ad}}{2(bc-ad)}$$

$$\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(c+dx)^n\left(-d^2D\left(n^2+5n+4\right)a^3+bd\left(2cD(3n+4)+Cd\left(n^2+3n+2\right)\right)a^2-b^2\left(4Dc^2+4Cd(n+1)c+Bd^2n(n+1)\right)a+b^3\left(2Cc^2+2Bdnc-Ad^2(1-n)n\right)+2b(bc-ad)^2Dx}{a+bx}}{b^3(bc-ad)}}{2(bc-ad)}$$

$$\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 90$$

$$\frac{\frac{\left(a^3\left(-d^2\right)D\left(n^2+5n+6\right)+a^2bd(n+2)(6cD+Cd(n+1))-ab^2\left(Bd^2n(n+1)+6c^2D+4cCd(n+1)\right)+b^3\left(-Ad^2(1-n)n+2Bcdn+2c^2C\right)\right)\int \frac{(c+dx)^n}{a+bx}dx}{b^3(bc-ad)}}{2(bc-ad)}$$

$$\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 78$$

$$\frac{\frac{\frac{2D(bc-ad)^2(c+dx)^{n+1}}{d(n+1)}-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{b(c+dx)}{bc-ad}\right)\left(a^3\left(-d^2\right)D\left(n^2+5n+6\right)+a^2bd(n+2)(6cD+Cd(n+1))-ab^2\left(Bd^2n(n+1)+6c^2\right.\right.}{(n+1)(bc-ad)}}{b^3(bc-ad)}}{2(bc}$$

$$\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{2b^3(a+bx)^2(bc-ad)}$$

input
```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^3,x]
```

output
```
-1/2*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(1 + n))/(b^3*(b*c - a
*d)*(a + b*x)^2) + (-(((b^3*(2*B*c - A*d*(1 - n)) - a^3*d*D*(5 + n) - a*b^
2*(4*c*C + B*d*(1 + n)) + a^2*b*(6*c*D + C*d*(3 + n)))*(c + d*x)^(1 + n))/
(b^3*(b*c - a*d)*(a + b*x))) + ((2*(b*c - a*d)^2*D*(c + d*x)^(1 + n))/(d*(
1 + n)) - ((b^3*(2*c^2*C + 2*B*c*d*n - A*d^2*(1 - n)*n) - a^3*d^2*D*(6 + 5
*n + n^2) + a^2*b*d*(2 + n)*(6*c*D + C*d*(1 + n)) - a*b^2*(6*c^2*D + 4*c*C
*d*(1 + n) + B*d^2*n*(1 + n)))*(c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 +
n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/((b*c - a*d)*(1 + n)))/(b^3*(b*c - a
*d)))/(2*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(xd+c)^n (Dx^3+C x^2+Bx+A)}{(bx+a)^3}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

Fricas [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^3}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx=\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**n*(A + B*x + C*x**2 + D*x**3)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^3}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^3, x)
```

Giac [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^3}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^3}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3,x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^3}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
( - (c + d*x)**n*a**3*c*d**2*n**2 - 5*(c + d*x)**n*a**3*c*d**2*n - 6*(c +
d*x)**n*a**3*c*d**2 + (c + d*x)**n*a**3*d**3*n**3*x + 5*(c + d*x)**n*a**3*
d**3*n**2*x + 6*(c + d*x)**n*a**3*d**3*n*x + 2*(c + d*x)**n*a**2*b*c**2*d*
n**2 + 8*(c + d*x)**n*a**2*b*c**2*d*n + 2*(c + d*x)**n*a**2*b*c**2*d - 2*(
c + d*x)**n*a**2*b*c*d**2*n**3*x - 10*(c + d*x)**n*a**2*b*c*d**2*n**2*x -
12*(c + d*x)**n*a**2*b*c*d**2*n*x - 12*(c + d*x)**n*a**2*b*c*d**2*x - (c +
 d*x)**n*a**2*b*d**3*n**3*x**2 - 2*(c + d*x)**n*a**2*b*d**3*n**2*x**2 + 3*
(c + d*x)**n*a**2*b*d**3*n*x**2 + (c + d*x)**n*a*b**3*c*d*n**3 - (c + d*x)
**n*a*b**3*c*d*n**2 - 2*(c + d*x)**n*a*b**3*c*d*n + (c + d*x)**n*a*b**3*d*
*2*n**3*x + (c + d*x)**n*a*b**3*d**2*n**2*x - (c + d*x)**n*a*b**2*c**3*n**
2 - (c + d*x)**n*a*b**2*c**3*n + (c + d*x)**n*a*b**2*c**2*d*n**3*x + 5*(c
+ d*x)**n*a*b**2*c**2*d*n**2*x + 16*(c + d*x)**n*a*b**2*c**2*d*n*x + 4*(c
+ d*x)**n*a*b**2*c**2*d*x + 2*(c + d*x)**n*a*b**2*c*d**2*n**3*x**2 + (c +
d*x)**n*a*b**2*c*d**2*n**2*x**2 + 3*(c + d*x)**n*a*b**2*c*d**2*n*x**2 - 6*
(c + d*x)**n*a*b**2*c*d**2*x**2 + (c + d*x)**n*a*b**2*d**3*n**3*x**3 - (c
+ d*x)**n*a*b**2*d**3*n**2*x**3 - 2*(c + d*x)**n*b**4*c*d*n**2*x - 2*(c +
d*x)**n*b**4*c*d*n*x - 2*(c + d*x)**n*b**3*c**3*n**2*x - 2*(c + d*x)**n*b*
*3*c**3*n*x - 4*(c + d*x)**n*b**3*c**2*d*n**2*x**2 + 2*(c + d*x)**n*b**3*c
**2*d*n*x**2 + 2*(c + d*x)**n*b**3*c**2*d*x**2 - 2*(c + d*x)**n*b**3*c*d**
2*n**2*x**3 + 2*(c + d*x)**n*b**3*c*d**2*n*x**3 - int(((c + d*x)**n*x)/...
```

3.181 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$

Optimal result . . . 1732
Mathematica [A] (verified) . . . 1733
Rubi [A] (verified) . . . 1733
Maple [F] . . . 1736
Fricas [F] . . . 1736
Sympy [F] . . . 1737
Maxima [F] . . . 1737
Giac [F] . . . 1737
Mupad [F(-1)] . . . 1738
Reduce [F] . . . 1738

Optimal result

Integrand size = 30, antiderivative size = 332

$$\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = -\frac{(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{1+n}}{3b^3(bc-ad)(a+bx)^3} + \frac{(adD(1+3n)-b(cD+Cdn))(c+dx)^{1+n}}{b^3d^2(1-n)n(a+bx)^2} + \frac{D(c+dx)^{1+n}}{b^3dn(a+bx)} - \frac{(ab^2d(1+n)(18c^2D+6cCdn-Bd^2(1-n)n)-a^2bd^2(9cD+Cdn)(2+3n+n^2)+a^3d^3D(6+11n$$

output
```
-1/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1+n)/b^3/(-a*d+b*c)/(b*x+a)^3+
(a*d*D*(1+3*n)-b*(C*d*n+D*c))*(d*x+c)^(1+n)/b^3/d^2/(1-n)/n/(b*x+a)^2+D*(d
*x+c)^(1+n)/b^3/d/n/(b*x+a)-1/3*(a*b^2*d*(1+n)*(18*D*c^2+6*c*C*d*n-B*d^2*(
1-n)*n)-a^2*b*d^2*(C*d*n+9*D*c)*(n^2+3*n+2)+a^3*d^3*D*(n^3+6*n^2+11*n+6)-b
^3*(6*D*c^3+6*c^2*C*d*n-3*B*c*d^2*(1-n)*n+A*d^3*n*(n^2-3*n+2)))*(d*x+c)^(1
+n)*hypergeom([3, 1+n],[2+n],b*(d*x+c)/(-a*d+b*c))/b^3/(-a*d+b*c)^4/n/(-n^
2+1)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4} dx$$
$$= \frac{(c+dx)^{1+n} \left(-(bc-ad)^3 D \,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{b(c+dx)}{bc-ad}\right)+d(bc-ad)^2(bC-3aD)\,\text{Hyp}\right.}{}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
((c + d*x)^(1 + n)*(-((b*c - a*d)^3*D*Hypergeometric2F1[1, 1 + n, 2 + n, (
b*(c + d*x))/(b*c - a*d)]) + d*(b*c - a*d)^2*(b*C - 3*a*D)*Hypergeometric2
F1[2, 1 + n, 2 + n, (b*(c + d*x))/(b*c - a*d)] + d^2*(-(b*c) + a*d)*(b^2*B
 - 2*a*b*C + 3*a^2*D)*Hypergeometric2F1[3, 1 + n, 2 + n, (b*(c + d*x))/(b*
c - a*d)] + d^3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[4, 1
 + n, 2 + n, (b*(c + d*x))/(b*c - a*d)]))/(b^3*(b*c - a*d)^4*(1 + n))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.52, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2124, 25, 1193, 27, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4} dx$$

$$\Big\downarrow \ 2124$$

$$-\frac{\int -\frac{(c+dx)^n \left(-\frac{dD(n+1)a^3}{b^3}+\frac{(3cD+Cd(n+1))a^2}{b^2}-\frac{(3cC+Bd(n+1))a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-Ad(2-n)+\frac{3(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^3}dx}{3(bc-ad)}-\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{(c+dx)^n\left(-\frac{dD(n+1)a^3}{b^3}+\frac{(3cD+Cd(n+1))a^2}{b^2}-\frac{(3cC+Bd(n+1))a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-Ad(2-n)+\frac{3(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^3}dx}{3(bc-ad)}-\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 1193

$$-\frac{\frac{\int \frac{(c+dx)^n\left(d^2D\left(n^2+6n+5\right)a^3-bd\left(3cD(3n+5)+Cd\left(n^2+3n+2\right)\right)a^2+b^2\left(12Dc^2+6Cd(n+1)c-Bd^2\left(1-n^2\right)\right)a-b^3\left(6Cc^2-3Bd(1-n)c+Ad^2\left(n^2-3n+2\right)\right)-6b}{b^3(a+bx)^2}}{2(bc-ad)}}{3(bc-ad)}\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{(c+dx)^n\left(d^2D\left(n^2+6n+5\right)a^3-bd\left(3cD(3n+5)+Cd\left(n^2+3n+2\right)\right)a^2+b^2\left(12Dc^2+6Cd(n+1)c-Bd^2\left(1-n^2\right)\right)a-b^3\left(6Cc^2-3Bd(1-n)c+Ad^2\left(n^2-3n+2\right)\right)-6b}{(a+bx)^2}}{2b^3(bc-ad)}}{3(bc-ad)}\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 87

$$-\frac{\frac{\left(a^3d^3D\left(n^3+6n^2+11n+6\right)-a^2bd^2\left(n^2+3n+2\right)(9cD+Cdn)+ab^2d(n+1)\left(-Bd^2(1-n)n+18c^2D+6cCdn\right)-\left(b^3\left(Ad^3n\left(n^2-3n+2\right)-3Bcd^2(1-n)n+6c^3D+6c^2\right.\right.\right.}{bc-ad}}{}$$

$$\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

$\Big\downarrow$ 78

$$-\frac{-\frac{(c+dx)^{n+1}\left(a^3d^2D\left(n^2+6n+11\right)-a^2bd\left(9cD(n+3)+Cd\left(n^2+3n+2\right)\right)+ab^2\left(-Bd^2\left(1-n^2\right)+18c^2D+6cCd(n+1)\right)-b^3\left(Ad^2\left(n^2-3n+2\right)-3Bcd(1-n)+6c^2C\right.\right.}{(a+bx)(bc-ad)}}{}$$

$$\frac{(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^3(bc-ad)}$$

input
```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^4,x]
```

output
```
-1/3*((A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(1 + n))/(b^3*(b*c - a
*d)*(a + b*x)^3) + (-1/2*((b^3*(3*B*c - A*d*(2 - n)) - a^3*d*D*(7 + n) - a
*b^2*(6*c*C + B*d*(1 + n)) + a^2*b*(9*c*D + C*d*(4 + n)))*(c + d*x)^(1 + n
))/(b^3*(b*c - a*d)*(a + b*x)^2) - (-(((a^3*d^2*D*(11 + 6*n + n^2) + a*b^2
*(18*c^2*D + 6*c*C*d*(1 + n) - B*d^2*(1 - n^2)) - b^3*(6*c^2*C - 3*B*c*d*(
1 - n) + A*d^2*(2 - 3*n + n^2)) - a^2*b*d*(9*c*D*(3 + n) + C*d*(2 + 3*n +
n^2)))*(c + d*x)^(1 + n))/((b*c - a*d)*(a + b*x))) - ((a*b^2*d*(1 + n)*(18
*c^2*D + 6*c*C*d*n - B*d^2*(1 - n)*n) - a^2*b*d^2*(9*c*D + C*d*n)*(2 + 3*n
 + n^2) + a^3*d^3*D*(6 + 11*n + 6*n^2 + n^3) - b^3*(6*c^3*D + 6*c^2*C*d*n
- 3*B*c*d^2*(1 - n)*n + A*d^3*n*(2 - 3*n + n^2)))*(c + d*x)^(1 + n)*Hyperg
eometric2F1[1, 1 + n, 2 + n, (b*(c + d*x))/(b*c - a*d)])/((b*c - a*d)^2*(1
 + n)))/(2*b^3*(b*c - a*d)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(xd+c)^n (Dx^3+C\,x^2+Bx+A)}{(bx+a)^4}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x)
```

Fricas [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^4}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b^4*x^4 + 4*a*b^3*x^3 + 6*
a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**4,x)
```

output
```
Integral((c + d*x)**n*(A + B*x + C*x**2 + D*x**3)/(a + b*x)**4, x)
```

Maxima [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^4}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^4, x)
```

Giac [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^4}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^4}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4,x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^4, x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^4}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^4,x)
```

output

```
( - (c + d*x)**n*a**3*c*d**2*n**2 - 5*(c + d*x)**n*a**3*c*d**2*n - 6*(c +
d*x)**n*a**3*c*d**2 + (c + d*x)**n*a**3*d**3*n**3*x + 5*(c + d*x)**n*a**3*
d**3*n**2*x + 6*(c + d*x)**n*a**3*d**3*n*x + 2*(c + d*x)**n*a**2*b*c**2*d*
n**2 + 8*(c + d*x)**n*a**2*b*c**2*d*n - 2*(c + d*x)**n*a**2*b*c*d**2*n**3*
x - 11*(c + d*x)**n*a**2*b*c*d**2*n**2*x - 15*(c + d*x)**n*a**2*b*c*d**2*n
*x - 18*(c + d*x)**n*a**2*b*c*d**2*x - (c + d*x)**n*a**2*b*d**3*n**3*x**2
- (c + d*x)**n*a**2*b*d**3*n**2*x**2 + 6*(c + d*x)**n*a**2*b*d**3*n*x**2 +
 (c + d*x)**n*a*b**3*c*d*n**3 - 4*(c + d*x)**n*a*b**3*c*d*n**2 + 3*(c + d*
x)**n*a*b**3*c*d*n + (c + d*x)**n*a*b**3*d**2*n**3*x - (c + d*x)**n*a*b**3
*d**2*n**2*x - (c + d*x)**n*a*b**2*c**3*n**2 - (c + d*x)**n*a*b**2*c**3*n
+ (c + d*x)**n*a*b**2*c**2*d*n**3*x + 7*(c + d*x)**n*a*b**2*c**2*d*n**2*x
+ 24*(c + d*x)**n*a*b**2*c**2*d*n*x + 2*(c + d*x)**n*a*b**2*c*d**2*n**3*x*
*2 - (c + d*x)**n*a*b**2*c*d**2*n**2*x**2 + 3*(c + d*x)**n*a*b**2*c*d**2*n
*x**2 - 18*(c + d*x)**n*a*b**2*c*d**2*x**2 + (c + d*x)**n*a*b**2*d**3*n**3
*x**3 - 3*(c + d*x)**n*a*b**2*d**3*n**2*x**3 + 2*(c + d*x)**n*a*b**2*d**3*
n*x**3 - 3*(c + d*x)**n*b**4*c*d*n**2*x + 3*(c + d*x)**n*b**4*c*d*n*x - 3*
(c + d*x)**n*b**3*c**3*n**2*x - 3*(c + d*x)**n*b**3*c**3*n*x - 6*(c + d*x)
**n*b**3*c**2*d*n**2*x**2 + 12*(c + d*x)**n*b**3*c**2*d*n*x**2 - 3*(c + d*
x)**n*b**3*c*d**2*n**2*x**3 + 9*(c + d*x)**n*b**3*c*d**2*n*x**3 - 6*(c + d
*x)**n*b**3*c*d**2*x**3 - int(((c + d*x)**n*x)/(a**5*c*d*n**3 - 3*a**5*...
```

3.182 $\int (a+bx)^{3/2}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result . . . 1740
Mathematica [A] (verified) . . . 1741
Rubi [A] (verified) . . . 1742
Maple [F] . . . 1745
Fricas [F] . . . 1745
Sympy [F] . . . 1746
Maxima [F] . . . 1746
Giac [F] . . . 1746
Mupad [F(-1)] . . . 1747
Reduce [F] . . . 1747

Optimal result

Integrand size = 32, antiderivative size = 483

$$\int (a+bx)^{3/2}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{2(3a^2d^2D(43+26n+4n^2)+b^2(63c^2D-7cCd(11+2n)+Bd^2(99+40n+4n^2))+abd(21c}{b^3d^3(7+2n)(9+2n)(11+2n)}$$
$$-\frac{2(9bcD+6adD(4+n)-bCd(11+2n))(a+bx)^{7/2}(c+dx)^{1+n}}{b^3d^2(9+2n)(11+2n)}$$
$$+\frac{2D(a+bx)^{9/2}(c+dx)^{1+n}}{b^3d(11+2n)}$$
$$-\frac{2(8a^3d^3D(6+11n+6n^2+n^3)+4a^2bd^2(2+3n+n^2)(15cD-Cd(11+2n))+2ab^2d(1+n)(105c^2D-}{}$$

output
```
2*(3*a^2*d^2*D*(4*n^2+26*n+43)+b^2*(63*D*c^2-7*c*C*d*(11+2*n)+B*d^2*(4*n^2
+40*n+99))+a*b*d*(21*c*D*(5+2*n)-C*d*(8*n^2+66*n+121)))*(b*x+a)^(5/2)*(d*x
+c)^(1+n)/b^3/d^3/(7+2*n)/(9+2*n)/(11+2*n)-2*(9*D*b*c+6*a*d*D*(4+n)-b*C*d*
(11+2*n))*(b*x+a)^(7/2)*(d*x+c)^(1+n)/b^3/d^2/(9+2*n)/(11+2*n)+2*D*(b*x+a)
^(9/2)*(d*x+c)^(1+n)/b^3/d/(11+2*n)-2/5*(8*a^3*d^3*D*(n^3+6*n^2+11*n+6)+4*
a^2*b*d^2*(n^2+3*n+2)*(15*D*c-C*d*(11+2*n))+2*a*b^2*d*(1+n)*(105*D*c^2-10*
c*C*d*(11+2*n)+B*d^2*(4*n^2+40*n+99))+b^3*(315*D*c^3-35*c^2*C*d*(11+2*n)+5
*B*c*d^2*(4*n^2+40*n+99)-A*d^3*(8*n^3+108*n^2+478*n+693)))*(b*x+a)^(5/2)*(
d*x+c)^n*hypergeom([5/2, -n],[7/2],-d*(b*x+a)/(-a*d+b*c))/b^4/d^3/(7+2*n)/
(9+2*n)/(11+2*n)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.48

$$\int (a+bx)^{3/2}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{2(a+bx)^{5/2}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(693(Ab^3-a(b^2B-abC+a^2D)) \text{Hypergeometric2F1}\left(\frac{5}{2}\right.\right.}{}$$

input
```
Integrate[(a + b*x)^(3/2)*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(a + b*x)^(5/2)*(c + d*x)^n*(693*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hy
pergeometric2F1[5/2, -n, 7/2, (d*(a + b*x))/(-(b*c) + a*d)] + 5*(a + b*x)*
(99*(b^2*B - 2*a*b*C + 3*a^2*D)*Hypergeometric2F1[7/2, -n, 9/2, (d*(a + b*
x))/(-(b*c) + a*d)] - 7*(a + b*x)*(-11*(b*C - 3*a*D)*Hypergeometric2F1[9/2
, -n, 11/2, (d*(a + b*x))/(-(b*c) + a*d)] - 9*D*(a + b*x)*Hypergeometric2F
1[11/2, -n, 13/2, (d*(a + b*x))/(-(b*c) + a*d)]))))/(3465*b^4*((b*(c + d*x
))/(b*c - a*d))^n)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2125, 27, 1194, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{3/2}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \ 2125$$

$$\frac{2\int \frac{1}{2}(a+bx)^{3/2}(c+dx)^n \left(Ad(2n+11)b^3-(9bcD+6ad(n+4)D-bCd(2n+11))x^2b^2-\left(3dD(2n+5)a^2+1\right.\right.}{b^3d(2n+11)}$$

$$\frac{2D(a+bx)^{9/2}(c+dx)^{n+1}}{b^3d(2n+11)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (a+bx)^{3/2}(c+dx)^n \left(Ad(2n+11)b^3-(9bcD+6ad(n+4)D-bCd(2n+11))x^2b^2-\left(3dD(2n+5)a^2+18bc\right.\right.}{b^3d(2n+11)}$$

$$\frac{2D(a+bx)^{9/2}(c+dx)^{n+1}}{b^3d(2n+11)}$$

$$\Big\downarrow \ 1194$$

$$\frac{2\int \frac{1}{2}b^2(a+bx)^{3/2}(c+dx)^n\left(2d^2D\left(4n^2+19n+15\right)a^3+bd\left(21cD(2n+5)-2Cd\left(2n^2+13n+11\right)\right)a^2+7b^2c(9cD-Cd(2n+11))a+Ab^3d^2\left(4n^2+40n+99\right.\right.}{b^2d(2n+9)}$$

$$\frac{2D(a+bx)^{9/2}(c+dx)^{n+1}}{b^3d(2n+11)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (a+bx)^{3/2}(c+dx)^n\left(2d^2D\left(4n^2+19n+15\right)a^3+bd\left(21cD(2n+5)-2Cd\left(2n^2+13n+11\right)\right)a^2+7b^2c(9cD-Cd(2n+11))a+Ab^3d^2\left(4n^2+40n+99\right)+b(\right.}{d(2n+9)}$$

$$\frac{2D(a+bx)^{9/2}(c+dx)^{n+1}}{b^3d(2n+11)}$$

$$\Big\downarrow \ 90$$

$$\frac{\frac{2(a+bx)^{5/2}(c+dx)^{n+1}\left(3a^2d^2D\left(4n^2+26n+43\right)+abd\left(21cD(2n+5)-Cd\left(8n^2+66n+121\right)\right)+b^2\left(Bd^2\left(4n^2+40n+99\right)+63c^2D-7cCd(2n+11)\right)\right)}{d(2n+7)}-\frac{\left(8a^3d^3D\left(n^3+\right.\right.}{}}{}$$

$$\frac{2D(a+bx)^{9/2}(c+dx)^{n+1}}{b^3d(2n+11)}$$

$$\Big\downarrow \text{ 80}$$

$$\frac{\frac{2(a+bx)^{5/2}(c+dx)^{n+1}\left(3a^2d^2D\left(4n^2+26n+43\right)+abd\left(21cD(2n+5)-Cd\left(8n^2+66n+121\right)\right)+b^2\left(Bd^2\left(4n^2+40n+99\right)+63c^2D-7cCd(2n+11)\right)\right)}{d(2n+7)}-\frac{(c+dx)^n\left(\frac{b(c+}{bc-}\right.}{}}{}$$

$$\frac{2D(a+bx)^{9/2}(c+dx)^{n+1}}{b^3d(2n+11)}$$

$$\Big\downarrow \text{ 79}$$

$$\frac{\frac{2(a+bx)^{5/2}(c+dx)^{n+1}\left(3a^2d^2D\left(4n^2+26n+43\right)+abd\left(21cD(2n+5)-Cd\left(8n^2+66n+121\right)\right)+b^2\left(Bd^2\left(4n^2+40n+99\right)+63c^2D-7cCd(2n+11)\right)\right)}{d(2n+7)}-\frac{2(a+bx)^{5/2}(c+}{}}{}$$

$$\frac{2D(a+bx)^{9/2}(c+dx)^{n+1}}{b^3d(2n+11)}$$

input
```
Int[(a + b*x)^(3/2)*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*D*(a + b*x)^(9/2)*(c + d*x)^(1 + n))/(b^3*d*(11 + 2*n)) + ((-2*(9*b*c*D
 + 6*a*d*D*(4 + n) - b*C*d*(11 + 2*n))*(a + b*x)^(7/2)*(c + d*x)^(1 + n))/
(d*(9 + 2*n)) + ((2*(3*a^2*d^2*D*(43 + 26*n + 4*n^2) + b^2*(63*c^2*D - 7*c
*C*d*(11 + 2*n) + B*d^2*(99 + 40*n + 4*n^2)) + a*b*d*(21*c*D*(5 + 2*n) - C
*d*(121 + 66*n + 8*n^2)))*(a + b*x)^(5/2)*(c + d*x)^(1 + n))/(d*(7 + 2*n))
 - (2*(8*a^3*d^3*D*(6 + 11*n + 6*n^2 + n^3) + 4*a^2*b*d^2*(2 + 3*n + n^2)*
(15*c*D - C*d*(11 + 2*n)) + 2*a*b^2*d*(1 + n)*(105*c^2*D - 10*c*C*d*(11 +
2*n) + B*d^2*(99 + 40*n + 4*n^2)) + b^3*(315*c^3*D - 35*c^2*C*d*(11 + 2*n)
 + 5*B*c*d^2*(99 + 40*n + 4*n^2) - A*d^3*(693 + 478*n + 108*n^2 + 8*n^3)))
*(a + b*x)^(5/2)*(c + d*x)^n*Hypergeometric2F1[5/2, -n, 7/2, -((d*(a + b*x
))/(b*c - a*d))])/(5*b*d*(7 + 2*n)*((b*(c + d*x))/(b*c - a*d))^n))/(d*(9 +
 2*n)))/(b^3*d*(11 + 2*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [F]

$$\int (bx+a)^{\frac{3}{2}} (xd+c)^n \left(Dx^3+C\,x^2+Bx+A\right) dx$$

input
```
int((b*x+a)^(3/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x+a)^(3/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int (a+bx)^{3/2}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(Dx^3+Cx^2+Bx+A\right)(bx+a)^{\frac{3}{2}}(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
integral((D*b*x^4 + (D*a + C*b)*x^3 + (C*a + B*b)*x^2 + A*a + (B*a + A*b)*
x)*sqrt(b*x + a)*(d*x + c)^n, x)
```

Sympy [F]

$$\int (a+bx)^{3/2}(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int (a+bx)^{\frac{3}{2}}\,(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

input
```
integrate((b*x+a)**(3/2)*(d*x+c)**n*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral((a + b*x)**(3/2)*(c + d*x)**n*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F]

$$\int (a+bx)^{3/2}(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)(bx+a)^{\frac{3}{2}}(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x + a)^(3/2)*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^{3/2}(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx=\int \left(Dx^3+Cx^2+Bx+A\right)(bx+a)^{\frac{3}{2}}(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x + a)^(3/2)*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^{3/2}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \int (a+b\,x)^{3/2}\,(c+d\,x)^n\,\left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((a + b*x)^(3/2)*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^(3/2)*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (a+bx)^{3/2}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{too large to display}$$

input
```
int((b*x+a)^(3/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output
```
(2*( - 24*(c + d*x)**n*sqrt(a + b*x)*a**5*c*d**4*n**2 - 120*(c + d*x)**n*s
qrt(a + b*x)*a**5*c*d**4*n - 144*(c + d*x)**n*sqrt(a + b*x)*a**5*c*d**4 +
24*(c + d*x)**n*sqrt(a + b*x)*a**5*d**5*n**3*x + 120*(c + d*x)**n*sqrt(a +
 b*x)*a**5*d**5*n**2*x + 144*(c + d*x)**n*sqrt(a + b*x)*a**5*d**5*n*x + 96
*(c + d*x)**n*sqrt(a + b*x)*a**4*b*c**2*d**3*n**2 + 336*(c + d*x)**n*sqrt(
a + b*x)*a**4*b*c**2*d**3*n + 264*(c + d*x)**n*sqrt(a + b*x)*a**4*b*c**2*d
**3 - 96*(c + d*x)**n*sqrt(a + b*x)*a**4*b*c*d**4*n**3*x - 324*(c + d*x)**
n*sqrt(a + b*x)*a**4*b*c*d**4*n**2*x - 204*(c + d*x)**n*sqrt(a + b*x)*a**4
*b*c*d**4*n*x + 72*(c + d*x)**n*sqrt(a + b*x)*a**4*b*c*d**4*x - 24*(c + d*
x)**n*sqrt(a + b*x)*a**4*b*d**5*n**3*x**2 - 108*(c + d*x)**n*sqrt(a + b*x)
*a**4*b*d**5*n**2*x**2 - 108*(c + d*x)**n*sqrt(a + b*x)*a**4*b*d**5*n*x**2
 + 32*(c + d*x)**n*sqrt(a + b*x)*a**3*b**3*c*d**3*n**5 + 528*(c + d*x)**n*
sqrt(a + b*x)*a**3*b**3*c*d**3*n**4 + 3232*(c + d*x)**n*sqrt(a + b*x)*a**3
*b**3*c*d**3*n**3 + 8808*(c + d*x)**n*sqrt(a + b*x)*a**3*b**3*c*d**3*n**2
+ 9510*(c + d*x)**n*sqrt(a + b*x)*a**3*b**3*c*d**3*n + 1485*(c + d*x)**n*s
qrt(a + b*x)*a**3*b**3*c*d**3 + 32*(c + d*x)**n*sqrt(a + b*x)*a**3*b**3*d*
*4*n**5*x + 528*(c + d*x)**n*sqrt(a + b*x)*a**3*b**3*d**4*n**4*x + 3232*(c
 + d*x)**n*sqrt(a + b*x)*a**3*b**3*d**4*n**3*x + 8748*(c + d*x)**n*sqrt(a
+ b*x)*a**3*b**3*d**4*n**2*x + 8910*(c + d*x)**n*sqrt(a + b*x)*a**3*b**3*d
**4*n*x - 144*(c + d*x)**n*sqrt(a + b*x)*a**3*b**2*c**3*d**2*n**2 - 144...
```

3.183 $\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result 1749
Mathematica [A] (verified) 1750
Rubi [A] (verified) 1751
Maple [F] 1754
Fricas [F] 1754
Sympy [F] 1755
Maxima [F] 1755
Giac [F] 1755
Mupad [F(-1)] 1756
Reduce [F] 1756

Optimal result

Integrand size = 32, antiderivative size = 484

$$\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$
$$= \frac{2(a^2d^2D(89+66n+12n^2)+b^2(35c^2D-5cCd(9+2n)+Bd^2(63+32n+4n^2))+abd(5cD(13+6n)-}{b^3d^3(5+2n)(7+2n)(9+2n)}$$
$$-\frac{2(7bcD-bCd(9+2n)+2adD(10+3n))(a+bx)^{5/2}(c+dx)^{1+n}}{b^3d^2(7+2n)(9+2n)}$$
$$+\frac{2D(a+bx)^{7/2}(c+dx)^{1+n}}{b^3d(9+2n)}$$
$$-\frac{2(8a^3d^3D(6+11n+6n^2+n^3)+4a^2bd^2(2+3n+n^2)(9cD-Cd(9+2n))+2ab^2d(1+n)(45c^2D-}{}$$

output
```
2*(a^2*d^2*D*(12*n^2+66*n+89)+b^2*(35*D*c^2-5*c*C*d*(9+2*n)+B*d^2*(4*n^2+3
2*n+63))+a*b*d*(5*c*D*(13+6*n)-C*d*(8*n^2+54*n+81)))*(b*x+a)^(3/2)*(d*x+c)
^(1+n)/b^3/d^3/(5+2*n)/(7+2*n)/(9+2*n)-2*(7*D*b*c-b*C*d*(9+2*n)+2*a*d*D*(1
0+3*n))*(b*x+a)^(5/2)*(d*x+c)^(1+n)/b^3/d^2/(7+2*n)/(9+2*n)+2*D*(b*x+a)^(7
/2)*(d*x+c)^(1+n)/b^3/d/(9+2*n)-2/3*(8*a^3*d^3*D*(n^3+6*n^2+11*n+6)+4*a^2*
b*d^2*(n^2+3*n+2)*(9*D*c-C*d*(9+2*n))+2*a*b^2*d*(1+n)*(45*D*c^2-6*c*C*d*(9
+2*n)+B*d^2*(4*n^2+32*n+63))+b^3*(105*D*c^3-15*c^2*C*d*(9+2*n)+3*B*c*d^2*(
4*n^2+32*n+63)-A*d^3*(8*n^3+84*n^2+286*n+315)))*(b*x+a)^(3/2)*(d*x+c)^n*hy
pergeom([3/2, -n],[5/2],-d*(b*x+a)/(-a*d+b*c))/b^4/d^3/(5+2*n)/(7+2*n)/(9+
2*n)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.48

$$\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \frac{2(a+bx)^{3/2}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(105(Ab^3-a(b^2B-abC+a^2D)) \text{Hypergeometric2F1}\left(\frac{3}{2},-n,\frac{5}{2},\frac{d(a-}{-bc}\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b*x]*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*(a + b*x)^(3/2)*(c + d*x)^n*(105*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hy
pergeometric2F1[3/2, -n, 5/2, (d*(a + b*x))/(-(b*c) + a*d)] + (a + b*x)*(6
3*(b^2*B - 2*a*b*C + 3*a^2*D)*Hypergeometric2F1[5/2, -n, 7/2, (d*(a + b*x)
)/(-(b*c) + a*d)] - 5*(a + b*x)*(-9*(b*C - 3*a*D)*Hypergeometric2F1[7/2, -
n, 9/2, (d*(a + b*x))/(-(b*c) + a*d)] - 7*D*(a + b*x)*Hypergeometric2F1[9/
2, -n, 11/2, (d*(a + b*x))/(-(b*c) + a*d)]))))/(315*b^4*((b*(c + d*x))/(b*
c - a*d))^n)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2125, 27, 1194, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$

$$\Big\downarrow \text{2125}$$

$$2\int \tfrac{1}{2}\sqrt{a+bx}(c+dx)^n \left(Ad(2n+9)b^3-(7bcD+2ad(3n+10)D-bCd(2n+9))x^2b^2-\left(dD(6n+13)a^2+14bc\right.\right.$$

$$b^3d(2n+9)$$

$$\frac{2D(a+bx)^{7/2}(c+dx)^{n+1}}{b^3d(2n+9)}$$

$$\Big\downarrow \text{27}$$

$$\int \sqrt{a+bx}(c+dx)^n \left(Ad(2n+9)b^3-(7bcD+2ad(3n+10)D-bCd(2n+9))x^2b^2-\left(dD(6n+13)a^2+14bcDa\right.\right.$$

$$b^3d(2n+9)$$

$$\frac{2D(a+bx)^{7/2}(c+dx)^{n+1}}{b^3d(2n+9)}$$

$$\Big\downarrow \text{1194}$$

$$2\int \tfrac{1}{2}b^2\sqrt{a+bx}(c+dx)^n\left(2d^2D\left(4n^2+17n+13\right)a^3+bd\left(5cD(6n+13)-2Cd\left(2n^2+11n+9\right)\right)a^2+5b^2c(7cD-Cd(2n+9))a+Ab^3d^2\left(4n^2+32n+63\right)+b(\right.$$

$$b^2d(2n+7)$$

$$\frac{2D(a+bx)^{7/2}(c+dx)^{n+1}}{b^3d(2n+9)}$$

$$\Big\downarrow \text{27}$$

$$\int \sqrt{a+bx}(c+dx)^n\left(2d^2D\left(4n^2+17n+13\right)a^3+bd\left(5cD(6n+13)-2Cd\left(2n^2+11n+9\right)\right)a^2+5b^2c(7cD-Cd(2n+9))a+Ab^3d^2\left(4n^2+32n+63\right)+b((35D\right.$$

$$d(2n+7)$$

$$\frac{2D(a+bx)^{7/2}(c+dx)^{n+1}}{b^3d(2n+9)}$$

$$\Big\downarrow \text{90}$$

$$\frac{\frac{2(a+bx)^{3/2}(c+dx)^{n+1}\left(a^2d^2D\left(12n^2+66n+89\right)+abd\left(5cD(6n+13)-Cd\left(8n^2+54n+81\right)\right)+b^2\left(Bd^2\left(4n^2+32n+63\right)+35c^2D-5cCd(2n+9)\right)\right)}{d(2n+5)}-\frac{\left(8a^3d^3D\left(n^3+6n\right.\right.}{}}{}$$

$$\frac{2D(a+bx)^{7/2}(c+dx)^{n+1}}{b^3d(2n+9)}$$

$$\Big\downarrow \; 80$$

$$\frac{\frac{2(a+bx)^{3/2}(c+dx)^{n+1}\left(a^2d^2D\left(12n^2+66n+89\right)+abd\left(5cD(6n+13)-Cd\left(8n^2+54n+81\right)\right)+b^2\left(Bd^2\left(4n^2+32n+63\right)+35c^2D-5cCd(2n+9)\right)\right)}{d(2n+5)}-\frac{(c+dx)^n\left(\frac{b(c+dx}{bc-ad}\right.}{}}{}$$

$$\frac{2D(a+bx)^{7/2}(c+dx)^{n+1}}{b^3d(2n+9)}$$

$$\Big\downarrow \; 79$$

$$\frac{\frac{2(a+bx)^{3/2}(c+dx)^{n+1}\left(a^2d^2D\left(12n^2+66n+89\right)+abd\left(5cD(6n+13)-Cd\left(8n^2+54n+81\right)\right)+b^2\left(Bd^2\left(4n^2+32n+63\right)+35c^2D-5cCd(2n+9)\right)\right)}{d(2n+5)}-\frac{2(a+bx)^{3/2}(c+dx}{}}{}$$

$$\frac{2D(a+bx)^{7/2}(c+dx)^{n+1}}{b^3d(2n+9)}$$

input
```
Int[Sqrt[a + b*x]*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(2*D*(a + b*x)^(7/2)*(c + d*x)^(1 + n))/(b^3*d*(9 + 2*n)) + ((-2*(7*b*c*D
 - b*C*d*(9 + 2*n) + 2*a*d*D*(10 + 3*n))*(a + b*x)^(5/2)*(c + d*x)^(1 + n))
/(d*(7 + 2*n)) + ((2*(a^2*d^2*D*(89 + 66*n + 12*n^2) + b^2*(35*c^2*D - 5*c
*C*d*(9 + 2*n) + B*d^2*(63 + 32*n + 4*n^2)) + a*b*d*(5*c*D*(13 + 6*n) - C*
d*(81 + 54*n + 8*n^2)))*(a + b*x)^(3/2)*(c + d*x)^(1 + n))/(d*(5 + 2*n)) -
 (2*(8*a^3*d^3*D*(6 + 11*n + 6*n^2 + n^3) + 4*a^2*b*d^2*(2 + 3*n + n^2)*(9
*c*D - C*d*(9 + 2*n)) + 2*a*b^2*d*(1 + n)*(45*c^2*D - 6*c*C*d*(9 + 2*n) +
B*d^2*(63 + 32*n + 4*n^2)) + b^3*(105*c^3*D - 15*c^2*C*d*(9 + 2*n) + 3*B*c
*d^2*(63 + 32*n + 4*n^2) - A*d^3*(315 + 286*n + 84*n^2 + 8*n^3)))*(a + b*x
)^(3/2)*(c + d*x)^n*Hypergeometric2F1[3/2, -n, 5/2, -((d*(a + b*x))/(b*c -
 a*d))])/(3*b*d*(5 + 2*n)*((b*(c + d*x))/(b*c - a*d))^n))/(d*(7 + 2*n)))/(
b^3*d*(9 + 2*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [F]

$$\int \sqrt{bx+a}\,(xd+c)^{n}\left(Dx^{3}+C\,x^{2}+Bx+A\right)dx$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x+a)^(1/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int \sqrt{a+bx}(c+dx)^{n}\left(A+Bx+Cx^{2}+Dx^{3}\right)\,dx$$
$$=\int \left(Dx^{3}+Cx^{2}+Bx+A\right)\sqrt{bx+a}(dx+c)^{n}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x + a)*(d*x + c)^n, x)
```

Sympy [F]

$$\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**n*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(a + b*x)*(c + d*x)**n*(A + B*x + C*x**2 + D*x**3), x)
```

Maxima [F]

$$\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(Dx^3+Cx^2+Bx+A\right)\sqrt{bx+a}(dx+c)^n dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x + a)*(d*x + c)^n, x)
```

Giac [F]

$$\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(Dx^3+Cx^2+Bx+A\right)\sqrt{bx+a}(dx+c)^n dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x + a)*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx$$
$$= \int \sqrt{a+b\,x}\,(c+d\,x)^n \left(A+B\,x+C\,x^2+x^3\,D\right) \, dx$$

input
```
int((a + b*x)^(1/2)*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^(1/2)*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int \sqrt{a+bx}(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx = \text{too large to display}$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output

```
(2*( - 8*(c + d*x)**n*sqrt(a + b*x)*a**4*c*d**3*n**2 - 40*(c + d*x)**n*sqr
t(a + b*x)*a**4*c*d**3*n - 48*(c + d*x)**n*sqrt(a + b*x)*a**4*c*d**3 + 8*(
c + d*x)**n*sqrt(a + b*x)*a**4*d**4*n**3*x + 40*(c + d*x)**n*sqrt(a + b*x)
*a**4*d**4*n**2*x + 48*(c + d*x)**n*sqrt(a + b*x)*a**4*d**4*n*x + 24*(c +
d*x)**n*sqrt(a + b*x)*a**3*b*c**2*d**2*n**2 + 88*(c + d*x)**n*sqrt(a + b*x
)*a**3*b*c**2*d**2*n + 72*(c + d*x)**n*sqrt(a + b*x)*a**3*b*c**2*d**2 - 24
*(c + d*x)**n*sqrt(a + b*x)*a**3*b*c*d**3*n**3*x - 84*(c + d*x)**n*sqrt(a
+ b*x)*a**3*b*c*d**3*n**2*x - 52*(c + d*x)**n*sqrt(a + b*x)*a**3*b*c*d**3*
n*x + 24*(c + d*x)**n*sqrt(a + b*x)*a**3*b*c*d**3*x - 8*(c + d*x)**n*sqrt(
a + b*x)*a**3*b*d**4*n**3*x**2 - 36*(c + d*x)**n*sqrt(a + b*x)*a**3*b*d**4
*n**2*x**2 - 36*(c + d*x)**n*sqrt(a + b*x)*a**3*b*d**4*n*x**2 + 16*(c + d*
x)**n*sqrt(a + b*x)*a**2*b**3*c*d**2*n**4 + 176*(c + d*x)**n*sqrt(a + b*x)
*a**2*b**3*c*d**2*n**3 + 648*(c + d*x)**n*sqrt(a + b*x)*a**2*b**3*c*d**2*n
**2 + 852*(c + d*x)**n*sqrt(a + b*x)*a**2*b**3*c*d**2*n + 189*(c + d*x)**n
*sqrt(a + b*x)*a**2*b**3*c*d**2 + 16*(c + d*x)**n*sqrt(a + b*x)*a**2*b**3*
d**3*n**4*x + 176*(c + d*x)**n*sqrt(a + b*x)*a**2*b**3*d**3*n**3*x + 636*(
c + d*x)**n*sqrt(a + b*x)*a**2*b**3*d**3*n**2*x + 756*(c + d*x)**n*sqrt(a
+ b*x)*a**2*b**3*d**3*n*x - 24*(c + d*x)**n*sqrt(a + b*x)*a**2*b**2*c**3*d
*n**2 - 24*(c + d*x)**n*sqrt(a + b*x)*a**2*b**2*c**3*d*n + 24*(c + d*x)**n
*sqrt(a + b*x)*a**2*b**2*c**2*d**2*n**3*x + 12*(c + d*x)**n*sqrt(a + b*...
```

3.184 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$

Optimal result . . . 1758
Mathematica [A] (verified) . . . 1759
Rubi [A] (verified) . . . 1760
Maple [F] . . . 1763
Fricas [F] . . . 1763
Sympy [F] . . . 1764
Maxima [F] . . . 1764
Giac [F] . . . 1764
Mupad [F(-1)] . . . 1765
Reduce [F] . . . 1765

Optimal result

Integrand size = 32, antiderivative size = 482

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}}\,dx = \frac{2(3a^2d^2D(19+18n+4n^2)+b^2(15c^2D-3cCd(7+2n)+Bd^2(35+24n+4n^2))+abd(3cD(11+6n)-}{b^3d^3(3+2n)(5+2n)(7+2n)}$$
$$-\frac{2(5bcD-bCd(7+2n)+2adD(8+3n))(a+bx)^{3/2}(c+dx)^{1+n}}{b^3d^2(5+2n)(7+2n)}$$
$$+\frac{2D(a+bx)^{5/2}(c+dx)^{1+n}}{b^3d(7+2n)}$$
$$-\frac{2(8a^3d^3D(6+11n+6n^2+n^3)+4a^2bd^2(2+3n+n^2)(3cD-Cd(7+2n))+2ab^2d(1+n)(9c^2D-2}{}$$

output
```
2*(3*a^2*d^2*D*(4*n^2+18*n+19)+b^2*(15*D*c^2-3*c*C*d*(7+2*n)+B*d^2*(4*n^2+
24*n+35))+a*b*d*(3*c*D*(11+6*n)-C*d*(8*n^2+42*n+49)))*(b*x+a)^(1/2)*(d*x+c
)^(1+n)/b^3/d^3/(3+2*n)/(5+2*n)/(7+2*n)-2*(5*D*b*c-b*C*d*(7+2*n)+2*a*d*D*(
8+3*n))*(b*x+a)^(3/2)*(d*x+c)^(1+n)/b^3/d^2/(5+2*n)/(7+2*n)+2*D*(b*x+a)^(5
/2)*(d*x+c)^(1+n)/b^3/d/(7+2*n)-2*(8*a^3*d^3*D*(n^3+6*n^2+11*n+6)+4*a^2*b*
d^2*(n^2+3*n+2)*(3*D*c-C*d*(7+2*n))+2*a*b^2*d*(1+n)*(9*D*c^2-2*c*C*d*(7+2*
n)+B*d^2*(4*n^2+24*n+35))+b^3*(15*D*c^3-3*c^2*C*d*(7+2*n)+B*c*d^2*(4*n^2+2
4*n+35)-A*d^3*(8*n^3+60*n^2+142*n+105)))*(b*x+a)^(1/2)*(d*x+c)^n*hypergeom
([1/2, -n],[3/2],-d*(b*x+a)/(-a*d+b*c))/b^4/d^3/(3+2*n)/(5+2*n)/(7+2*n)/((
b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.48

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}}\,dx$$

$$= \frac{2\sqrt{a+bx}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(105(Ab^3-a(b^2B-abC+a^2D))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},\frac{d(a+bx}{-bc+ad}\right.\right.}{}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(c + d*x)^n*(105*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hype
rgeometric2F1[1/2, -n, 3/2, (d*(a + b*x))/(-(b*c) + a*d)] + 35*(b^2*B - 2*
a*b*C + 3*a^2*D)*(a + b*x)*Hypergeometric2F1[3/2, -n, 5/2, (d*(a + b*x))/(
-(b*c) + a*d)] + 21*(b*C - 3*a*D)*(a + b*x)^2*Hypergeometric2F1[5/2, -n, 7
/2, (d*(a + b*x))/(-(b*c) + a*d)] + 15*D*(a + b*x)^3*Hypergeometric2F1[7/2
, -n, 9/2, (d*(a + b*x))/(-(b*c) + a*d)]))/(105*b^4*((b*(c + d*x))/(b*c -
a*d))^n)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2125, 27, 1194, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{\sqrt{a+bx}} \, dx$$

$$\Big\downarrow \; 2125$$

$$\frac{2\int \frac{(c+dx)^n\left(Ad(2n+7)b^3-(5bcD+2ad(3n+8)D-bCd(2n+7))x^2b^2-\left(dD(6n+11)a^2+10bcDa-b^2Bd(2n+7)\right)xb-a^2D(5bc+2ad(n+1))\right)}{2\sqrt{a+bx}}dx}{b^3d(2n+7)}+\frac{2D(a+bx)^{5/2}(c+dx)^{n+1}}{b^3d(2n+7)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)^n\left(Ad(2n+7)b^3-(5bcD+2ad(3n+8)D-bCd(2n+7))x^2b^2-\left(dD(6n+11)a^2+10bcDa-b^2Bd(2n+7)\right)xb-a^2D(5bc+2ad(n+1))\right)}{\sqrt{a+bx}}dx}{b^3d(2n+7)}+\frac{2D(a+bx)^{5/2}(c+dx)^{n+1}}{b^3d(2n+7)}$$

$$\Big\downarrow \; 1194$$

$$\frac{\frac{2\int \frac{b^2(c+dx)^n\left(2d^2D\left(4n^2+15n+11\right)a^3+bd\left(3cD(6n+11)-2Cd\left(2n^2+9n+7\right)\right)a^2+3b^2c(5cD-Cd(2n+7))a+Ab^3d^2\left(4n^2+24n+35\right)+b\left(\left(15Dc^2-3Cd(2n+7)c+B\right.\right.\right.}{2\sqrt{a+bx}}}{b^2d(2n+5)}}{b^3d(}$$

$$\frac{2D(a+bx)^{5/2}(c+dx)^{n+1}}{b^3d(2n+7)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{(c+dx)^n\left(2d^2D\left(4n^2+15n+11\right)a^3+bd\left(3cD(6n+11)-2Cd\left(2n^2+9n+7\right)\right)a^2+3b^2c(5cD-Cd(2n+7))a+Ab^3d^2\left(4n^2+24n+35\right)+b\left(\left(15Dc^2-3Cd(2n+7)c+Bd^2\left(4\right.\right.\right.\right.}{\sqrt{a+bx}}}{d(2n+5)}}{b^3d(2n}$$

$$\frac{2D(a+bx)^{5/2}(c+dx)^{n+1}}{b^3d(2n+7)}$$

$\Big\downarrow$ 90

$$\frac{\frac{2\sqrt{a+bx}(c+dx)^{n+1}\left(3a^2d^2D\left(4n^2+18n+19\right)+abd\left(3cD(6n+11)-Cd\left(8n^2+42n+49\right)\right)+b^2\left(Bd^2\left(4n^2+24n+35\right)+15c^2D-3cCd(2n+7)\right)\right)}{d(2n+3)}-\frac{\left(8a^3d^3D\left(n^3+6n^2+\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{2D(a+bx)^{5/2}(c+dx)^{n+1}}{b^3d(2n+7)}$$

$\Big\downarrow$ 80

$$\frac{\frac{2\sqrt{a+bx}(c+dx)^{n+1}\left(3a^2d^2D\left(4n^2+18n+19\right)+abd\left(3cD(6n+11)-Cd\left(8n^2+42n+49\right)\right)+b^2\left(Bd^2\left(4n^2+24n+35\right)+15c^2D-3cCd(2n+7)\right)\right)}{d(2n+3)}-\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2D(a+bx)^{5/2}(c+dx)^{n+1}}{b^3d(2n+7)}$$

$\Big\downarrow$ 79

$$\frac{\frac{2\sqrt{a+bx}(c+dx)^{n+1}\left(3a^2d^2D\left(4n^2+18n+19\right)+abd\left(3cD(6n+11)-Cd\left(8n^2+42n+49\right)\right)+b^2\left(Bd^2\left(4n^2+24n+35\right)+15c^2D-3cCd(2n+7)\right)\right)}{d(2n+3)}-\frac{2\sqrt{a+bx}(c+dx)^n\left(\frac{b(\cdots}{b\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2D(a+bx)^{5/2}(c+dx)^{n+1}}{b^3d(2n+7)}$$

input
```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/Sqrt[a + b*x],x]
```

output
```
(2*D*(a + b*x)^(5/2)*(c + d*x)^(1 + n))/(b^3*d*(7 + 2*n)) + ((-2*(5*b*c*D
 - b*C*d*(7 + 2*n) + 2*a*d*D*(8 + 3*n))*(a + b*x)^(3/2)*(c + d*x)^(1 + n))/
(d*(5 + 2*n)) + ((2*(3*a^2*d^2*D*(19 + 18*n + 4*n^2) + b^2*(15*c^2*D - 3*c
*C*d*(7 + 2*n) + B*d^2*(35 + 24*n + 4*n^2)) + a*b*d*(3*c*D*(11 + 6*n) - C*
d*(49 + 42*n + 8*n^2)))*Sqrt[a + b*x]*(c + d*x)^(1 + n))/(d*(3 + 2*n)) - (
2*(8*a^3*d^3*D*(6 + 11*n + 6*n^2 + n^3) + 4*a^2*b*d^2*(2 + 3*n + n^2)*(3*c
*D - C*d*(7 + 2*n)) + 2*a*b^2*d*(1 + n)*(9*c^2*D - 2*c*C*d*(7 + 2*n) + B*d
^2*(35 + 24*n + 4*n^2)) + b^3*(15*c^3*D - 3*c^2*C*d*(7 + 2*n) + B*c*d^2*(3
5 + 24*n + 4*n^2) - A*d^3*(105 + 142*n + 60*n^2 + 8*n^3)))*Sqrt[a + b*x]*(
c + d*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((d*(a + b*x))/(b*c - a*d))])/
(b*d*(3 + 2*n)*((b*(c + d*x))/(b*c - a*d))^n))/(d*(5 + 2*n)))/(b^3*d*(7 +
2*n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [F]

$$\int \frac{(xd+c)^{n} \left(Dx^{3}+C\,x^{2}+Bx+A\right)}{\sqrt{bx+a}}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{\sqrt{bx+a}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/sqrt(b*x + a), x)
```

Sympy [F]

$$\int \frac{(c+dx)^n\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \int \frac{(c+dx)^n\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Integral((c + d*x)**n*(A + B*x + C*x**2 + D*x**3)/sqrt(a + b*x), x)
```

Maxima [F]

$$\int \frac{(c+dx)^n\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{\sqrt{bx+a}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^n\,(A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{\sqrt{bx+a}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{\sqrt{a+b\,x}}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(1/2),x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{\sqrt{a+bx}}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(2*( - 8*(c + d*x)**n*sqrt(a + b*x)*a**3*c*d**2*n**2 - 40*(c + d*x)**n*sqr
t(a + b*x)*a**3*c*d**2*n - 48*(c + d*x)**n*sqrt(a + b*x)*a**3*c*d**2 + 8*(
c + d*x)**n*sqrt(a + b*x)*a**3*d**3*n**3*x + 40*(c + d*x)**n*sqrt(a + b*x)
*a**3*d**3*n**2*x + 48*(c + d*x)**n*sqrt(a + b*x)*a**3*d**3*n*x + 16*(c +
d*x)**n*sqrt(a + b*x)*a**2*b*c**2*d*n**2 + 64*(c + d*x)**n*sqrt(a + b*x)*a
**2*b*c**2*d*n + 56*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c**2*d - 16*(c + d*x
)**n*sqrt(a + b*x)*a**2*b*c*d**2*n**3*x - 60*(c + d*x)**n*sqrt(a + b*x)*a*
*2*b*c*d**2*n**2*x - 36*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c*d**2*n*x + 24*
(c + d*x)**n*sqrt(a + b*x)*a**2*b*c*d**2*x - 8*(c + d*x)**n*sqrt(a + b*x)*
a**2*b*d**3*n**3*x**2 - 36*(c + d*x)**n*sqrt(a + b*x)*a**2*b*d**3*n**2*x**
2 - 36*(c + d*x)**n*sqrt(a + b*x)*a**2*b*d**3*n*x**2 + 8*(c + d*x)**n*sqrt
(a + b*x)*a*b**3*c*d*n**3 + 52*(c + d*x)**n*sqrt(a + b*x)*a*b**3*c*d*n**2
+ 94*(c + d*x)**n*sqrt(a + b*x)*a*b**3*c*d*n + 35*(c + d*x)**n*sqrt(a + b*
x)*a*b**3*c*d + 8*(c + d*x)**n*sqrt(a + b*x)*a*b**3*d**2*n**3*x + 48*(c +
d*x)**n*sqrt(a + b*x)*a*b**3*d**2*n**2*x + 70*(c + d*x)**n*sqrt(a + b*x)*a
*b**3*d**2*n*x - 8*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**3*n**2 - 8*(c + d*
x)**n*sqrt(a + b*x)*a*b**2*c**3*n + 8*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c*
*2*d*n**3*x - 32*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**2*d*n*x - 28*(c + d*
x)**n*sqrt(a + b*x)*a*b**2*c**2*d*x + 16*(c + d*x)**n*sqrt(a + b*x)*a*b**2
*c*d**2*n**3*x**2 + 48*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c*d**2*n**2*x*...
```

3.185 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx$

Optimal result 1767
Mathematica [A] (verified) 1768
Rubi [A] (verified) 1768
Maple [F] 1772
Fricas [F] 1772
Sympy [F(-2)] 1773
Maxima [F] 1773
Giac [F] 1773
Mupad [F(-1)] 1774
Reduce [F] 1774

Optimal result

Integrand size = 32, antiderivative size = 413

$$\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = -\frac{2(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{1+n}}{b^3(bc-ad)\sqrt{a+bx}}$$
$$-\frac{2(3bcD+6adD(2+n)-bCd(5+2n))\sqrt{a+bx}(c+dx)^{1+n}}{b^3d^2(3+2n)(5+2n)}$$
$$+\frac{2D(a+bx)^{3/2}(c+dx)^{1+n}}{b^3d(5+2n)}$$
$$+\frac{2\left(2(bc-ad)\left(\frac{bc}{2}+ad(1+n)\right)(3bcD+6adD(2+n)-bCd(5+2n))-2d\left(\frac{3}{2}+n\right)\left(4a^3d^2D(2+3n+n^2\right)\right.}{}$$

output
```
-2*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1+n)/b^3/(-a*d+b*c)/(b*x+a)^(1/2
)-2*(3*D*b*c+6*a*d*D*(2+n)-b*C*d*(5+2*n))*(b*x+a)^(1/2)*(d*x+c)^(1+n)/b^3/
d^2/(3+2*n)/(5+2*n)+2*D*(b*x+a)^(3/2)*(d*x+c)^(1+n)/b^3/d/(5+2*n)+2*(2*(-a
*d+b*c)*(1/2*b*c+a*d*(1+n))*(3*D*b*c+6*a*d*D*(2+n)-b*C*d*(5+2*n))-2*d*(3/2
+n)*(4*a^3*d^2*D*(n^2+3*n+2)-b^3*d*(5+2*n)*(B*c+A*d*(1+2*n))-2*a^2*b*d*(3*
D*c+C*d*(2*n^2+7*n+5))+a*b^2*(3*D*c^2+c*C*d*(5+2*n)+2*B*d^2*(2*n^2+7*n+5))
))*(b*x+a)^(1/2)*(d*x+c)^n*hypergeom([1/2, -n],[3/2],-d*(b*x+a)/(-a*d+b*c)
)/b^4/d^2/(-a*d+b*c)/(3+2*n)/(5+2*n)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx = \frac{2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(-15(Ab^3-a(b^2B-abC+a^2D))\,\text{Hyp}\right.}{}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(3/2),x]
```

output
```
(2*(c + d*x)^n*(-15*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[
-1/2, -n, 1/2, (d*(a + b*x))/(-(b*c) + a*d)] + (a + b*x)*(15*(b^2*B - 2*a*
b*C + 3*a^2*D)*Hypergeometric2F1[1/2, -n, 3/2, (d*(a + b*x))/(-(b*c) + a*d
)] - (a + b*x)*(-5*(b*C - 3*a*D)*Hypergeometric2F1[3/2, -n, 5/2, (d*(a + b
*x))/(-(b*c) + a*d)] - 3*D*(a + b*x)*Hypergeometric2F1[5/2, -n, 7/2, (d*(a
 + b*x))/(-(b*c) + a*d)]))))/(15*b^4*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a
*d))^n)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1194, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int \frac{(c+dx)^n\left(-\left(\left(c-\frac{ad}{b}\right)Dx^2\right)-\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{2dD(n+1)a^3-b(cD+2Cd(n+1))a^2+b^2(cC+2Bd(n+1))a-b^3(Bc+Ad(2n+1))}{b^3}\right)}{2\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{n+1}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow 27$$

$$-\frac{\int \frac{(c+dx)^n\left(-\left(\left(c-\frac{ad}{b}\right)Dx^2\right)-\frac{(bc-ad)(bC-aD)x}{b^2}+\frac{2dD(n+1)a^3-b(cD+2Cd(n+1))a^2+b^2(cC+2Bd(n+1))a-b^3(Bc+Ad(2n+1))}{b^3}\right)}{\sqrt{a+bx}}dx}{bc-ad}-\frac{2(c+dx)^{n+1}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow 1194$$

$$-\frac{\frac{2\int \frac{(c+dx)^n\left(\frac{4d^2D\left(n^2+3n+2\right)a^3}{b}-2d\left(3cD+Cd\left(2n^2+7n+5\right)\right)a^2+b\left(3Dc^2+Cd(2n+5)c+2Bd^2\left(2n^2+7n+5\right)\right)a-b^2d(2n+5)(Bc+Ad(2n+1))+(bc-ad)(3bcD \cdots}{2\sqrt{a+bx}}}{b^2d(2n+5)} \cdots}{bc-ad}$$

$$-\frac{2(c+dx)^{n+1}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow 27$$

$$-\frac{\frac{\int \frac{(c+dx)^n\left(\frac{4d^2D\left(n^2+3n+2\right)a^3}{b}-2d\left(3cD+Cd\left(2n^2+7n+5\right)\right)a^2+b\left(3Dc^2+Cd(2n+5)c+2Bd^2\left(2n^2+7n+5\right)\right)a-b^2d(2n+5)(Bc+Ad(2n+1))+(bc-ad)(3bcD+ \cdots}{\sqrt{a+bx}}}{b^2d(2n+5)} \cdots}{bc-ad}$$

$$-\frac{2(c+dx)^{n+1}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow 90$$

$$-\frac{\frac{\left(\frac{4a^3d^2D\left(n^2+3n+2\right)}{b}-2a^2d\left(3cD+Cd\left(2n^2+7n+5\right)\right)+ab\left(2Bd^2\left(2n^2+7n+5\right)+3c^2D+cCd(2n+5)\right)-\frac{(bc-ad)(2ad(n+1)+bc)(6adD(n+2)+3bcD-bC \cdots}{bd(2n+3)} \cdots}{b^2d(2n+5)} \cdots}{b \cdots}$$

$$-\frac{2(c+dx)^{n+1}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow 80$$

$$-\frac{\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{4a^3d^2D\left(n^2+3n+2\right)}{b}-2a^2d(3cD+Cd(2n^2+7n+5))+ab(2Bd^2(2n^2+7n+5)+3c^2D+cCd(2n+5))-\frac{(bc-ad)(2ad(n+1)+b \cdots}{\cdots}\right.}{\cdots}}{b^2d(\cdots}$$

$$\frac{2(c+dx)^{n+1}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}$$

$$\Big\downarrow \; 79$$

$$-\frac{2(c+dx)^{n+1}\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{\sqrt{a+bx}(bc-ad)}-$$

$$\frac{\frac{2\sqrt{a+bx}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},-\frac{d(a+bx)}{bc-ad}\right)\left(\frac{4a^3d^2D\left(n^2+3n+2\right)}{b}-2a^2d\left(3cD+Cd\left(2n^2+7n+5\right)\right)+ab\left(2Bd^2\left(2n^2+7n+5\right)+3c^2 \cdots\right.\right.}{b}}{b^2 \cdots}$$

input
```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(3/2),x]
```

output
```
(-2*(A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*(c + d*x)^(1 + n))/((b*c - a*d)*
Sqrt[a + b*x]) - ((-2*(b*c - a*d)*D*(a + b*x)^(3/2)*(c + d*x)^(1 + n))/(b^
3*d*(5 + 2*n)) + ((2*(b*c - a*d)*(3*b*c*D + 6*a*d*D*(2 + n) - b*C*d*(5 + 2
*n))*Sqrt[a + b*x]*(c + d*x)^(1 + n))/(b*d*(3 + 2*n)) + (2*((4*a^3*d^2*D*(
2 + 3*n + n^2))/b - b^2*d*(5 + 2*n)*(B*c + A*d*(1 + 2*n)) - ((b*c - a*d)*(
b*c + 2*a*d*(1 + n))*(3*b*c*D + 6*a*d*D*(2 + n) - b*C*d*(5 + 2*n)))/(b*d*(
3 + 2*n)) - 2*a^2*d*(3*c*D + C*d*(5 + 7*n + 2*n^2)) + a*b*(3*c^2*D + c*C*d
*(5 + 2*n) + 2*B*d^2*(5 + 7*n + 2*n^2)))*Sqrt[a + b*x]*(c + d*x)^n*Hyperge
ometric2F1[1/2, -n, 3/2, -((d*(a + b*x))/(b*c - a*d))]])/(b*((b*(c + d*x))/
(b*c - a*d))^n))/(b^2*d*(5 + 2*n)))/(b*c - a*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(xd+c)^{n} \left(D x^{3}+C\,x^{2}+Bx+A\right)}{(bx+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{3/2}}\,dx=\int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x + a)*(d*x + c)^n/(b^2*x^2 + 2*
a*b*x + a^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^{3/2}}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(3/2),x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

output

```
(2*( - 8*(c + d*x)**n*sqrt(a + b*x)*a**3*c*d**2*n**2 - 40*(c + d*x)**n*sqr
t(a + b*x)*a**3*c*d**2*n - 48*(c + d*x)**n*sqrt(a + b*x)*a**3*c*d**2 + 8*(
c + d*x)**n*sqrt(a + b*x)*a**3*d**3*n**3*x + 40*(c + d*x)**n*sqrt(a + b*x)
*a**3*d**3*n**2*x + 48*(c + d*x)**n*sqrt(a + b*x)*a**3*d**3*n*x + 16*(c +
d*x)**n*sqrt(a + b*x)*a**2*b*c**2*d*n**2 + 64*(c + d*x)**n*sqrt(a + b*x)*a
**2*b*c**2*d*n + 40*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c**2*d - 16*(c + d*x
)**n*sqrt(a + b*x)*a**2*b*c*d**2*n**3*x - 68*(c + d*x)**n*sqrt(a + b*x)*a*
*2*b*c*d**2*n**2*x - 60*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c*d**2*n*x - 24*
(c + d*x)**n*sqrt(a + b*x)*a**2*b*c*d**2*x - 8*(c + d*x)**n*sqrt(a + b*x)*
a**2*b*d**3*n**3*x**2 - 28*(c + d*x)**n*sqrt(a + b*x)*a**2*b*d**3*n**2*x**
2 - 12*(c + d*x)**n*sqrt(a + b*x)*a**2*b*d**3*n*x**2 + 8*(c + d*x)**n*sqrt
(a + b*x)*a*b**3*c*d*n**3 + 28*(c + d*x)**n*sqrt(a + b*x)*a*b**3*c*d*n**2
+ 14*(c + d*x)**n*sqrt(a + b*x)*a*b**3*c*d*n - 15*(c + d*x)**n*sqrt(a + b*
x)*a*b**3*c*d + 8*(c + d*x)**n*sqrt(a + b*x)*a*b**3*d**2*n**3*x + 32*(c +
d*x)**n*sqrt(a + b*x)*a*b**3*d**2*n**2*x + 30*(c + d*x)**n*sqrt(a + b*x)*a
*b**3*d**2*n*x - 8*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**3*n**2 - 8*(c + d*
x)**n*sqrt(a + b*x)*a*b**2*c**3*n + 8*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c*
*2*d*n**3*x + 16*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**2*d*n**2*x + 32*(c +
 d*x)**n*sqrt(a + b*x)*a*b**2*c**2*d*n*x + 20*(c + d*x)**n*sqrt(a + b*x)*a
*b**2*c**2*d*x + 16*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c*d**2*n**3*x**2 ...
```

3.186 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx$

Optimal result 1776
Mathematica [A] (verified) 1777
Rubi [A] (warning: unable to verify) 1777
Maple [F] 1781
Fricas [F] 1781
Sympy [F(-2)] 1781
Maxima [F] 1782
Giac [F] 1782
Mupad [F(-1)] 1782
Reduce [F] 1783

Optimal result

Integrand size = 32, antiderivative size = 419

$$\int \frac{(c+dx)^n\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = -\frac{2(Ab^3-a(b^2B-abC+a^2D))\,(c+dx)^{1+n}}{3b^3(bc-ad)(a+bx)^{3/2}} - \frac{2(bcD-bCd(3+2n)+2adD(4+3n))(c+dx)^{1+n}}{b^3d^2(1+2n)(3+2n)\sqrt{a+bx}} + \frac{2D\sqrt{a+bx}(c+dx)^{1+n}}{b^3d(3+2n)} + \frac{2\left(6(bc-ad)\left(\frac{bc}{2}-ad(1+n)\right)(bcD-bCd(3+2n)+2adD(4+3n))+2d\left(\frac{1}{2}+n\right)(4a^3d^2Dn(1+n)-b^3\right.}{}$$

output
```
-2/3*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1+n)/b^3/(-a*d+b*c)/(b*x+a)^(3
/2)-2*(D*b*c-b*C*d*(3+2*n)+2*a*d*D*(4+3*n))*(d*x+c)^(1+n)/b^3/d^2/(1+2*n)/
(3+2*n)/(b*x+a)^(1/2)+2*D*(b*x+a)^(1/2)*(d*x+c)^(1+n)/b^3/d/(3+2*n)+2/3*(6
*(-a*d+b*c)*(1/2*b*c-a*d*(1+n))*(D*b*c-b*C*d*(3+2*n)+2*a*d*D*(4+3*n))+2*d*
(1/2+n)*(4*a^3*d^2*D*n*(1+n)-b^3*d*(3*B*c-A*d*(1-2*n))*(3+2*n)-2*a^2*b*d*(
3*D*c+C*d*(2*n^2+5*n+3))+a*b^2*(3*D*c^2+3*c*C*d*(3+2*n)+2*B*d^2*(2*n^2+5*n
+3))))*(d*x+c)^n*hypergeom([-1/2, -n],[1/2],-d*(b*x+a)/(-a*d+b*c))/b^4/d^2
/(-a*d+b*c)/(1+2*n)/(3+2*n)/(b*x+a)^(1/2)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [A] (verified)

Time = 2.48 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.55

$$\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}\,dx = \frac{2(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(\left(-Ab^3+a(b^2B-abC+a^2D)\right) \text{Hyper}\right.}{}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(5/2),x]
```

output
```
(2*(c + d*x)^n*((-(A*b^3) + a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[-
3/2, -n, -1/2, (d*(a + b*x))/(-(b*c) + a*d)] - (a + b*x)*(3*(b^2*B - 2*a*b
*C + 3*a^2*D)*Hypergeometric2F1[-1/2, -n, 1/2, (d*(a + b*x))/(-(b*c) + a*d
)] + (a + b*x)*((-3*b*C + 9*a*D)*Hypergeometric2F1[1/2, -n, 3/2, (d*(a + b
*x))/(-(b*c) + a*d)] - D*(a + b*x)*Hypergeometric2F1[3/2, -n, 5/2, (d*(a +
 b*x))/(-(b*c) + a*d)]))))/(3*b^4*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*
d))^n)
```

Rubi [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1193, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{2\int -\frac{(c+dx)^n\left(3\left(c-\frac{ad}{b}\right)Dx^2+\frac{3(bc-ad)(bC-aD)x}{b^2}+\frac{-2dD(n+1)a^3+b(3cD+2Cd(n+1))a^2-b^2(3cC+2Bd(n+1))a+b^3(3Bc-Ad(1-2n))}{b^3}\right)}{2(a+bx)^{3/2}}dx}{3(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(c+dx)^n\left(-\frac{2dD(n+1)a^3}{b^3}+\frac{(3cD+2Cd(n+1))a^2}{b^2}-\frac{(3cC+2Bd(n+1))a}{b}+3\left(c-\frac{ad}{b}\right)Dx^2+3Bc-Ad(1-2n)+\frac{3(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^{3/2}}dx}{3(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 1193$$

$$\frac{-\frac{2\int \frac{(c+dx)^n\left(2d^2D\left(2n^2+9n+7\right)a^3-bd\left(3cD(6n+7)+4Cd\left(n^2+3n+2\right)\right)a^2+2b^2\left(3Dc^2+6Cd(n+1)c+Bd^2\left(2n^2+3n+1\right)\right)a-b^3\left(3Cc^2+3Bd(2n+1)c-Ad^2\left(1-4\ldots\right)\right)\right)}{2b^3\sqrt{a+bx}}}{bc-ad}}{3(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{(c+dx)^n\left(2d^2D\left(2n^2+9n+7\right)a^3-bd\left(3cD(6n+7)+4Cd\left(n^2+3n+2\right)\right)a^2+2b^2\left(3Dc^2+6Cd(n+1)c+Bd^2\left(2n^2+3n+1\right)\right)a-b^3\left(3Cc^2+3Bd(2n+1)c-Ad^2\left(1-4n^{\ldots}\right)\right)\right)}{\sqrt{a+bx}}}{b^3(bc-ad)}}{3(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 90$$

$$-\frac{\frac{\left(2a^3d^2D(2n^2+9n+7)-a^2bd(3cD(6n+7)+4Cd(n^2+3n+2))+2ab^2\left(Bd^2\left(2n^2+3n+1\right)+3c^2D+6cCd(n+1)\right)+\frac{3D(bc-ad)^2(2ad(n+1)+bc)}{d(2n+3)}-b^3(-\ldots\right)}{b^3(bc-ad)}}{\ldots}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 80$$

$$-\frac{\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(2a^3d^2D(2n^2+9n+7)-a^2bd(3cD(6n+7)+4Cd(n^2+3n+2))+2ab^2\left(Bd^2\left(2n^2+3n+1\right)+3c^2D+6cCd(n+1)\right)+\frac{3D(bc-a\ldots}{\ldots}\right)}{b^3(bc-ad)}}{\ldots}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

$$\Big\downarrow \; 79$$

$$-\frac{\frac{\frac{2\sqrt{a+bx}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},-\frac{d(a+bx)}{bc-ad}\right)\left(2a^3d^2D\left(2n^2+9n+7\right)-a^2bd\left(3cD(6n+7)+4Cd\left(n^2+3n+2\right)\right)+2ab^2\left(Bd^2\left(2n^2+3\right.\right.\right.}{b}}{b^3(bc-ad)}}{}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{3b^3(a+bx)^{3/2}(bc-ad)}$$

input
```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(5/2),x]
```

output
```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(1 + n))/(3*b^3*(b*c - a
*d)*(a + b*x)^(3/2)) + ((-2*(b^3*(3*B*c - A*d*(1 - 2*n)) - 2*a^3*d*D*(4 +
n) - 2*a*b^2*(3*c*C + B*d*(1 + n)) + a^2*b*(9*c*D + C*d*(5 + 2*n)))*(c + d
*x)^(1 + n))/(b^3*(b*c - a*d)*Sqrt[a + b*x]) - ((-6*(b*c - a*d)^2*D*Sqrt[a
 + b*x]*(c + d*x)^(1 + n))/(d*(3 + 2*n)) + (2*(2*a^3*d^2*D*(7 + 9*n + 2*n^
2) + (3*(b*c - a*d)^2*D*(b*c + 2*a*d*(1 + n)))/(d*(3 + 2*n)) - b^3*(3*c^2*
C + 3*B*c*d*(1 + 2*n) - A*d^2*(1 - 4*n^2)) - a^2*b*d*(3*c*D*(7 + 6*n) + 4*
C*d*(2 + 3*n + n^2)) + 2*a*b^2*(3*c^2*D + 6*c*C*d*(1 + n) + B*d^2*(1 + 3*n
 + 2*n^2)))*Sqrt[a + b*x]*(c + d*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((d
*(a + b*x))/(b*c - a*d))])/(b*((b*(c + d*x))/(b*c - a*d))^n))/(b^3*(b*c -
a*d)))/(3*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(xd+c)^n (Dx^3 + C\,x^2 + Bx + A)}{(bx+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x + a)*(d*x + c)^n/(b^3*x^3 + 3*
a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx)^n\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n\,(A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}}\,dx = \int \frac{{\left(c+d\,x\right)}^n\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(a+b\,x\right)}^{5/2}}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(5/2),x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{5/2}} \, dx = \text{too large to display}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

output
```
(2*( - 8*(c + d*x)**n*sqrt(a + b*x)*a**3*c*d**2*n**2 - 40*(c + d*x)**n*sqr
t(a + b*x)*a**3*c*d**2*n - 48*(c + d*x)**n*sqrt(a + b*x)*a**3*c*d**2 + 8*(
c + d*x)**n*sqrt(a + b*x)*a**3*d**3*n**3*x + 40*(c + d*x)**n*sqrt(a + b*x)
*a**3*d**3*n**2*x + 48*(c + d*x)**n*sqrt(a + b*x)*a**3*d**3*n*x + 16*(c +
d*x)**n*sqrt(a + b*x)*a**2*b*c**2*d*n**2 + 64*(c + d*x)**n*sqrt(a + b*x)*a
**2*b*c**2*d*n + 24*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c**2*d - 16*(c + d*x
)**n*sqrt(a + b*x)*a**2*b*c*d**2*n**3*x - 76*(c + d*x)**n*sqrt(a + b*x)*a*
*2*b*c*d**2*n**2*x - 84*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c*d**2*n*x - 72*
(c + d*x)**n*sqrt(a + b*x)*a**2*b*c*d**2*x - 8*(c + d*x)**n*sqrt(a + b*x)*
a**2*b*d**3*n**3*x**2 - 20*(c + d*x)**n*sqrt(a + b*x)*a**2*b*d**3*n**2*x**
2 + 12*(c + d*x)**n*sqrt(a + b*x)*a**2*b*d**3*n*x**2 + 8*(c + d*x)**n*sqrt
(a + b*x)*a*b**3*c*d*n**3 + 4*(c + d*x)**n*sqrt(a + b*x)*a*b**3*c*d*n**2 -
 18*(c + d*x)**n*sqrt(a + b*x)*a*b**3*c*d*n - 9*(c + d*x)**n*sqrt(a + b*x)
*a*b**3*c*d + 8*(c + d*x)**n*sqrt(a + b*x)*a*b**3*d**2*n**3*x + 16*(c + d*
x)**n*sqrt(a + b*x)*a*b**3*d**2*n**2*x + 6*(c + d*x)**n*sqrt(a + b*x)*a*b*
*3*d**2*n*x - 8*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**3*n**2 - 8*(c + d*x)*
*n*sqrt(a + b*x)*a*b**2*c**3*n + 8*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**2*
d*n**3*x + 32*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**2*d*n**2*x + 96*(c + d*
x)**n*sqrt(a + b*x)*a*b**2*c**2*d*n*x + 36*(c + d*x)**n*sqrt(a + b*x)*a*b*
*2*c**2*d*x + 16*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c*d**2*n**3*x**2 + 1...
```

3.187 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}}\,dx$

Optimal result . . . 1784
Mathematica [A] (verified) . . . 1785
Rubi [A] (warning: unable to verify) . . . 1785
Maple [F] . . . 1789
Fricas [F] . . . 1789
Sympy [F(-1)] . . . 1789
Maxima [F] . . . 1790
Giac [F] . . . 1790
Mupad [F(-1)] . . . 1790
Reduce [F] . . . 1791

Optimal result

Integrand size = 32, antiderivative size = 404

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{7/2}}\,dx = -\frac{2(Ab^3-a(b^2B-abC+a^2D))\,(c+dx)^{1+n}}{5b^3(bc-ad)(a+bx)^{5/2}} - \frac{2(bcD-2adD(2+3n)+bC(d+2dn))(c+dx)^{1+n}}{b^3d^2\left(1-4n^2\right)(a+bx)^{3/2}} + \frac{2D(c+dx)^{1+n}}{b^3d(1+2n)\sqrt{a+bx}} - \frac{2(8a^3d^3D(6+11n+6n^2+n^3)-4a^2bd^2(2+3n+n^2)\left(15cD+C(d+2dn)\right)+2ab^2d(1+n)\left(45c^2D+1\right.}{}$$

output
```
-2/5*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1+n)/b^3/(-a*d+b*c)/(b*x+a)^(5
/2)-2*(D*b*c-2*a*d*D*(2+3*n)+b*C*(2*d*n+d))*(d*x+c)^(1+n)/b^3/d^2/(-4*n^2+
1)/(b*x+a)^(3/2)+2*D*(d*x+c)^(1+n)/b^3/d/(1+2*n)/(b*x+a)^(1/2)-2/15*(8*a^3
*d^3*D*(n^3+6*n^2+11*n+6)-4*a^2*b*d^2*(n^2+3*n+2)*(15*D*c+C*(2*d*n+d))+2*a
*b^2*d*(1+n)*(45*D*c^2+10*c*C*d*(1+2*n)-B*d^2*(-4*n^2+1))-b^3*(15*D*c^3+15
*c^2*C*d*(1+2*n)-5*B*c*d^2*(-4*n^2+1)+A*d^3*(8*n^3-12*n^2-2*n+3)))*(d*x+c)
^n*hypergeom([-3/2, -n],[-1/2],-d*(b*x+a)/(-a*d+b*c))/b^4/d^2/(-a*d+b*c)/(
1-2*n)/(1+2*n)/(b*x+a)^(3/2)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}} \, dx = \frac{2(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(-3(Ab^3-a(b^2B-abC+a^2D)) \text{Hype}\right.}{}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(7/2),x]
```

output
```
(2*(c + d*x)^n*(-3*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[-
5/2, -n, -3/2, (d*(a + b*x))/(-(b*c) + a*d)] - 5*(a + b*x)*((b^2*B - 2*a*b
*C + 3*a^2*D)*Hypergeometric2F1[-3/2, -n, -1/2, (d*(a + b*x))/(-(b*c) + a*
d)] - 3*(a + b*x)*((-(b*C) + 3*a*D)*Hypergeometric2F1[-1/2, -n, 1/2, (d*(a
 + b*x))/(-(b*c) + a*d)] + D*(a + b*x)*Hypergeometric2F1[1/2, -n, 3/2, (d*
(a + b*x))/(-(b*c) + a*d)]))))/(15*b^4*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c
 - a*d))^n)
```

Rubi [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1193, 27, 87, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}} \, dx$$

$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{(c+dx)^n \left(5\left(c-\frac{ad}{b}\right)Dx^2+\frac{5(bc-ad)(bC-aD)x}{b^2}+\frac{-2dD(n+1)a^3+b(5cD+2Cd(n+1))a^2-b^2(5cC+2Bd(n+1))a+b^3(5Bc-Ad(3-2n))}{b^3}\right)}{2(a+bx)^{5/2}}dx}{5(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^3(a+bx)^{5/2}(bc-ad)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)^n\left(-\frac{2dD(n+1)a^3}{b^3}+\frac{(5cD+2Cd(n+1))a^2}{b^2}-\frac{(5cC+2Bd(n+1))a}{b}+5\left(c-\frac{ad}{b}\right)Dx^2+5Bc-Ad(3-2n)+\frac{5(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^{5/2}}dx}{5(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^3(a+bx)^{5/2}(bc-ad)}$$

$\downarrow$ 1193

$$-\frac{\frac{2\int \frac{(c+dx)^n\left(2d^2D\left(2n^2+11n+9\right)a^3-bd\left(15cD(2n+3)+4Cd\left(n^2+3n+2\right)\right)a^2+2b^2\left(15Dc^2+10Cd(n+1)c-Bd^2\left(-2n^2-n+1\right)\right)a-b^3\left(15Cc^2-5Bd(1-2n)c+Ad^2\cdots\right.\right.}{2b^3(a+bx)^{3/2}}}{3(bc-ad)}\cdots}{5(bc-\cdots}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^3(a+bx)^{5/2}(bc-ad)}$$

$\downarrow$ 27

$$-\frac{\frac{\int \frac{(c+dx)^n\left(2d^2D\left(2n^2+11n+9\right)a^3-bd\left(15cD(2n+3)+4Cd\left(n^2+3n+2\right)\right)a^2+2b^2\left(15Dc^2+10Cd(n+1)c-Bd^2\left(-2n^2-n+1\right)\right)a-b^3\left(15Cc^2-5Bd(1-2n)c+Ad^2\cdots\right.\right.}{(a+bx)^{3/2}}}{3b^3(bc-ad)}\cdots}{5(bc-a\cdots}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^3(a+bx)^{5/2}(bc-ad)}$$

$\downarrow$ 87

$$-\frac{\frac{\left(8a^3d^3D\left(n^3+6n^2+11n+6\right)-4a^2bd^2\left(n^2+3n+2\right)(15cD+C(2dn+d))+2ab^2d(n+1)\left(-Bd^2\left(1-4n^2\right)+45c^2D+10cCd(2n+1)\right)-\left(b^3\left(Ad^3\left(8n^3-12n^2-2n+3\right)-\cdots\right.\right.\right.}{bc-ad}\cdots}{\cdots}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^3(a+bx)^{5/2}(bc-ad)}$$

$\downarrow$ 80

$$-\frac{\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(8a^3d^3D\left(n^3+6n^2+11n+6\right)-4a^2bd^2\left(n^2+3n+2\right)(15cD+C(2dn+d))+2ab^2d(n+1)\left(-Bd^2\left(1-4n^2\right)+45c^2D+10cCd(2n+1)\right)-\left(b^3\left(Ad\cdots\right.\right.\right.}{bc-ad}\cdots}{\cdots}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^3(a+bx)^{5/2}(bc-ad)}$$

$\downarrow$ 79

$$-\frac{\frac{2\sqrt{a+bx}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(\frac{1}{2},-n,\frac{3}{2},-\frac{d(a+bx)}{bc-ad}\right)\left(8a^3d^3D\left(n^3+6n^2+11n+6\right)-4a^2bd^2\left(n^2+3n+2\right)(15cD+C(2dn+d))+2ab^2d(n+1)\right.}{b(bc-ad)}}{}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{5b^3(a+bx)^{5/2}(bc-ad)}$$

input

```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(7/2),x]
```

output

```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(1 + n))/(5*b^3*(b*c - a
*d)*(a + b*x)^(5/2)) + ((-2*(b^3*(5*B*c - A*d*(3 - 2*n)) - 2*a^3*d*D*(6 +
n) - 2*a*b^2*(5*c*C + B*d*(1 + n)) + a^2*b*(15*c*D + C*d*(7 + 2*n)))*(c +
d*x)^(1 + n))/(3*b^3*(b*c - a*d)*(a + b*x)^(3/2)) - ((-2*(a^3*d^2*D*(33 +
22*n + 4*n^2) + a*b^2*(45*c^2*D + 20*c*C*d*(1 + n) - 2*B*d^2*(1 - n - 2*n^
2)) - a^2*b*d*(15*c*D*(5 + 2*n) + 4*C*d*(2 + 3*n + n^2)) - b^3*(15*c^2*C -
 5*B*c*d*(1 - 2*n) + A*d^2*(3 - 8*n + 4*n^2)))*(c + d*x)^(1 + n))/((b*c -
a*d)*Sqrt[a + b*x]) + (2*(8*a^3*d^3*D*(6 + 11*n + 6*n^2 + n^3) - 4*a^2*b*d
^2*(2 + 3*n + n^2)*(15*c*D + C*(d + 2*d*n)) + 2*a*b^2*d*(1 + n)*(45*c^2*D
+ 10*c*C*d*(1 + 2*n) - B*d^2*(1 - 4*n^2)) - b^3*(15*c^3*D + 15*c^2*C*d*(1
+ 2*n) - 5*B*c*d^2*(1 - 4*n^2) + A*d^3*(3 - 2*n - 12*n^2 + 8*n^3)))*Sqrt[a
 + b*x]*(c + d*x)^n*Hypergeometric2F1[1/2, -n, 3/2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(b*c - a*d)*((b*(c + d*x))/(b*c - a*d))^n))/(3*b^3*(b*c - a*d)
))/(5*(b*c - a*d))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(xd+c)^n (Dx^3+C x^2+Bx+A)}{(bx+a)^{\frac{7}{2}}}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(7/2),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(7/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(7/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x + a)*(d*x + c)^n/(b^4*x^4 + 4*
a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(7/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}}\,dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^{7/2}}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(7/2),x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(7/2),x)
```

output
```
(2*( - 8*(c + d*x)**n*sqrt(a + b*x)*a**3*c*d**2*n**2 - 40*(c + d*x)**n*sqr
t(a + b*x)*a**3*c*d**2*n - 48*(c + d*x)**n*sqrt(a + b*x)*a**3*c*d**2 + 8*(
c + d*x)**n*sqrt(a + b*x)*a**3*d**3*n**3*x + 40*(c + d*x)**n*sqrt(a + b*x)
*a**3*d**3*n**2*x + 48*(c + d*x)**n*sqrt(a + b*x)*a**3*d**3*n*x + 16*(c +
d*x)**n*sqrt(a + b*x)*a**2*b*c**2*d*n**2 + 64*(c + d*x)**n*sqrt(a + b*x)*a
**2*b*c**2*d*n + 8*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c**2*d - 16*(c + d*x)
**n*sqrt(a + b*x)*a**2*b*c*d**2*n**3*x - 84*(c + d*x)**n*sqrt(a + b*x)*a**
2*b*c*d**2*n**2*x - 108*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c*d**2*n*x - 120
*(c + d*x)**n*sqrt(a + b*x)*a**2*b*c*d**2*x - 8*(c + d*x)**n*sqrt(a + b*x)
*a**2*b*d**3*n**3*x**2 - 12*(c + d*x)**n*sqrt(a + b*x)*a**2*b*d**3*n**2*x*
*2 + 36*(c + d*x)**n*sqrt(a + b*x)*a**2*b*d**3*n*x**2 + 8*(c + d*x)**n*sqr
t(a + b*x)*a*b**3*c*d*n**3 - 20*(c + d*x)**n*sqrt(a + b*x)*a*b**3*c*d*n**2
 - 2*(c + d*x)**n*sqrt(a + b*x)*a*b**3*c*d*n + 5*(c + d*x)**n*sqrt(a + b*x
)*a*b**3*c*d + 8*(c + d*x)**n*sqrt(a + b*x)*a*b**3*d**2*n**3*x - 2*(c + d*
x)**n*sqrt(a + b*x)*a*b**3*d**2*n*x - 8*(c + d*x)**n*sqrt(a + b*x)*a*b**2*
c**3*n**2 - 8*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**3*n + 8*(c + d*x)**n*sq
rt(a + b*x)*a*b**2*c**2*d*n**3*x + 48*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c*
*2*d*n**2*x + 160*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c**2*d*n*x + 20*(c + d
*x)**n*sqrt(a + b*x)*a*b**2*c**2*d*x + 16*(c + d*x)**n*sqrt(a + b*x)*a*b**
2*c*d**2*n**3*x**2 + 24*(c + d*x)**n*sqrt(a + b*x)*a*b**2*c*d**2*n*x**2...
```

3.188 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx$

Optimal result 1792
Mathematica [A] (verified) 1793
Rubi [A] (warning: unable to verify) 1793
Maple [F] 1797
Fricas [F] 1797
Sympy [F(-2)] 1798
Maxima [F] 1798
Giac [F] 1798
Mupad [F(-1)] 1799
Reduce [F] 1799

Optimal result

Integrand size = 32, antiderivative size = 411

$$\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx = -\frac{2(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{1+n}}{7b^3(bc-ad)(a+bx)^{7/2}} + \frac{2(3bcD-bCd(1-2n)-6adDn)(c+dx)^{1+n}}{b^3d^2(1-2n)(3-2n)(a+bx)^{5/2}} - \frac{2D(c+dx)^{1+n}}{b^3d(1-2n)(a+bx)^{3/2}} - \frac{2\Big(b^3d(7Bc-Ad(5-2n))(1-2n)-4a^3d^2D(4+3n-n^2)+\frac{7(bc-ad)(3bcD-bCd(1-2n)-6adDn)(5bc-2ad(1+n))}{d(3-2n)}$$

output

```
-2/7*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1+n)/b^3/(-a*d+b*c)/(b*x+a)^(7
/2)+2*(3*D*b*c-b*C*d*(1-2*n)-6*a*d*D*n)*(d*x+c)^(1+n)/b^3/d^2/(1-2*n)/(3-2
*n)/(b*x+a)^(5/2)-2*D*(d*x+c)^(1+n)/b^3/d/(1-2*n)/(b*x+a)^(3/2)-2/35*(b^3*
d*(7*B*c-A*d*(5-2*n))*(1-2*n)-4*a^3*d^2*D*(-n^2+3*n+4)+7*(-a*d+b*c)*(3*D*b
*c-b*C*d*(1-2*n)-6*a*d*D*n)*(5*b*c-2*a*d*(1+n))/d/(3-2*n)+2*a^2*b*d*(21*D*
c+C*d*(-2*n^2-n+1))-a*b^2*(21*D*c^2+7*c*C*d*(1-2*n)+2*B*d^2*(-2*n^2-n+1)))
*(d*x+c)^n*hypergeom([-5/2, -n],[-3/2],-d*(b*x+a)/(-a*d+b*c))/b^4/d/(-a*d+
b*c)/(1-2*n)/(b*x+a)^(5/2)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [A] (verified)

Time = 3.27 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx =$$

$$-\frac{2(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(15(Ab^3-a(b^2B-abC+a^2D))\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-n,-\frac{5}{2},\frac{d(a+bx)}{-bc+ad}\right)+7\right.}{}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(9/2),x]
```

output
```
(-2*(c + d*x)^n*(15*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[
-7/2, -n, -5/2, (d*(a + b*x))/(-(b*c) + a*d)] + 7*(a + b*x)*(3*(b^2*B - 2*
a*b*C + 3*a^2*D)*Hypergeometric2F1[-5/2, -n, -3/2, (d*(a + b*x))/(-(b*c) +
 a*d)] - 5*(a + b*x)*((-(b*C) + 3*a*D)*Hypergeometric2F1[-3/2, -n, -1/2, (
d*(a + b*x))/(-(b*c) + a*d)] - 3*D*(a + b*x)*Hypergeometric2F1[-1/2, -n, 1
/2, (d*(a + b*x))/(-(b*c) + a*d)]))))/(105*b^4*(a + b*x)^(7/2)*((b*(c + d*
x))/(b*c - a*d))^n)
```

Rubi [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.41, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1193, 27, 87, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx$$

$$\Big\downarrow \; 2124$$

$$-\frac{2\int -\frac{(c+dx)^n\left(7\left(c-\frac{ad}{b}\right)Dx^2+\frac{7(bc-ad)(bC-aD)x}{b^2}+\frac{-2dD(n+1)a^3+b(7cD+2Cd(n+1))a^2-b^2(7cC+2Bd(n+1))a+b^3(7Bc-Ad(5-2n))}{b^3}\right)}{2(a+bx)^{7/2}}dx}{7(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^3(a+bx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(c+dx)^n\left(-\frac{2dD(n+1)a^3}{b^3}+\frac{(7cD+2Cd(n+1))a^2}{b^2}-\frac{(7cC+2Bd(n+1))a}{b}+7\left(c-\frac{ad}{b}\right)Dx^2+7Bc-Ad(5-2n)+\frac{7(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^{7/2}}dx}{7(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^3(a+bx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$-\frac{\frac{2\int \frac{(c+dx)^n\left(2d^2D\left(2n^2+13n+11\right)a^3-bd\left(7cD(6n+11)+4Cd\left(n^2+3n+2\right)\right)a^2+2b^2\left(35Dc^2+14Cd(n+1)c-Bd^2\left(-2n^2+n+3\right)\right)a-b^3\left(35Cc^2-7Bd(3-2n)c+Ac\right.}{2b^3(a+bx)^{5/2}}}{5(bc-ad)}}{7(bc-}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^3(a+bx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{(c+dx)^n\left(2d^2D\left(2n^2+13n+11\right)a^3-bd\left(7cD(6n+11)+4Cd\left(n^2+3n+2\right)\right)a^2+2b^2\left(35Dc^2+14Cd(n+1)c-Bd^2\left(-2n^2+n+3\right)\right)a-b^3\left(35Cc^2-7Bd(3-2n)c+Ad^2\right.}{(a+bx)^{5/2}}}{5b^3(bc-ad)}}{7(bc-}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^3(a+bx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$-\frac{-\frac{1}{3}\left(\frac{d(1-2n)\left(2a^3d^2D\left(2n^2+13n+11\right)-a^2bd\left(7cD(6n+11)+4Cd\left(n^2+3n+2\right)\right)+2ab^2\left(-Bd^2\left(-2n^2+n+3\right)+35c^2D+14cCd(n+1)\right)-b^3\left(Ad^2\left(4n^2-16n+15\right)-\right.}{bc-ad}\right.}{}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^3(a+bx)^{7/2}(bc-ad)}$$

$\Big\downarrow$ 80

$$-\frac{-\frac{1}{3}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(\frac{d(1-2n)\left(2a^3d^2D\left(2n^2+13n+11\right)-a^2bd\left(7cD(6n+11)+4Cd\left(n^2+3n+2\right)\right)+2ab^2\left(-Bd^2\left(-2n^2+n+3\right)+35c^2D+14cCd(n+1)\right)-\right.}{bc-ad}\right.}{}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^3(a+bx)^{7/2}(bc-ad)}$$

$$\Big\downarrow \text{ 79}$$

$$-\frac{\frac{2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{1}{2},-n,\frac{1}{2},-\frac{d(a+bx)}{bc-ad}\right)\left(\frac{d(1-2n)\left(2a^3d^2D\left(2n^2+13n+11\right)-a^2bd\left(7cD(6n+11)+4Cd\left(n^2+3n+2\right)\right)+2ab^2\left(-Ba\right.\right.}{b}\right.}{3b\sqrt{a+bx}}}{}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{7b^3(a+bx)^{7/2}(bc-ad)}$$

input

```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(9/2),x]
```

output

```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(1 + n))/(7*b^3*(b*c - a
*d)*(a + b*x)^(7/2)) + ((-2*(b^3*(7*B*c - A*d*(5 - 2*n)) - 2*a^3*d*D*(8 +
n) - 2*a*b^2*(7*c*C + B*d*(1 + n)) + a^2*b*(21*c*D + C*d*(9 + 2*n)))*(c +
d*x)^(1 + n))/(5*b^3*(b*c - a*d)*(a + b*x)^(5/2)) - ((-2*(a^3*d^2*D*(57 +
26*n + 4*n^2) + a*b^2*(105*c^2*D + 28*c*C*d*(1 + n) - 2*B*d^2*(3 + n - 2*n
^2)) - a^2*b*d*(21*c*D*(7 + 2*n) + 4*C*d*(2 + 3*n + n^2)) - b^3*(35*c^2*C
- 7*B*c*d*(3 - 2*n) + A*d^2*(15 - 16*n + 4*n^2)))*(c + d*x)^(1 + n))/(3*(b
*c - a*d)*(a + b*x)^(3/2)) + (2*(70*(b*c - a*d)*D*((3*b*c)/2 - a*d*(1 + n)
) + (d*(1 - 2*n)*(2*a^3*d^2*D*(11 + 13*n + 2*n^2) + 2*a*b^2*(35*c^2*D + 14
*c*C*d*(1 + n) - B*d^2*(3 + n - 2*n^2)) - a^2*b*d*(7*c*D*(11 + 6*n) + 4*C*
d*(2 + 3*n + n^2)) - b^3*(35*c^2*C - 7*B*c*d*(3 - 2*n) + A*d^2*(15 - 16*n
+ 4*n^2))))/(b*c - a*d))*(c + d*x)^n*Hypergeometric2F1[-1/2, -n, 1/2, -((d
*(a + b*x))/(b*c - a*d))])/(3*b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^
n))/(5*b^3*(b*c - a*d)))/(7*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(xd+c)^{n} (D x^{3}+C\,x^{2}+B x+A)}{(b x+a)^{\frac{9}{2}}}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(9/2),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(9/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(9/2),x, algorithm="fricas
")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x + a)*(d*x + c)^n/(b^5*x^5 + 5*
a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(9/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(9/2),x, algorithm="maxima
")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(9/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^{9/2}}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(9/2),x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{9/2}}\,dx = \int \frac{(dx+c)^n\,(Dx^3+C\,x^2+Bx+A)}{(bx+a)^{\frac{9}{2}}}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(9/2),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(9/2),x)
```

3.189 $\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{11/2}}\,dx$

Optimal result 1800
Mathematica [A] (verified) 1801
Rubi [A] (warning: unable to verify) 1801
Maple [F] 1805
Fricas [F] 1805
Sympy [F(-2)] 1806
Maxima [F] 1806
Giac [F] 1806
Mupad [F(-1)] 1807
Reduce [F] 1807

Optimal result

Integrand size = 32, antiderivative size = 415

$$\int \frac{(c+dx)^n(A+Bx+Cx^2+Dx^3)}{(a+bx)^{11/2}}\,dx = -\frac{2(Ab^3-a(b^2B-abC+a^2D))(c+dx)^{1+n}}{9b^3(bc-ad)(a+bx)^{9/2}} + \frac{2(5bcD+2adD(2-3n)-bCd(3-2n))(c+dx)^{1+n}}{b^3d^2(3-2n)(5-2n)(a+bx)^{7/2}} - \frac{2D(c+dx)^{1+n}}{b^3d(3-2n)(a+bx)^{5/2}} - \frac{2\left(b^3d(9Bc-Ad(7-2n))(3-2n)-4a^3d^2D(6+5n-n^2)+\frac{9(bc-ad)(5bcD+2adD(2-3n)-bCd(3-2n))(7bc-2ad(1+}{d(5-2n)}\right.}{}$$

output
```
-2/9*(A*b^3-a*(B*b^2-C*a*b+D*a^2))*(d*x+c)^(1+n)/b^3/(-a*d+b*c)/(b*x+a)^(9
/2)+2*(5*D*b*c+2*a*d*D*(2-3*n)-b*C*d*(3-2*n))*(d*x+c)^(1+n)/b^3/d^2/(3-2*n
)/(5-2*n)/(b*x+a)^(7/2)-2*D*(d*x+c)^(1+n)/b^3/d/(3-2*n)/(b*x+a)^(5/2)-2/63
*(b^3*d*(9*B*c-A*d*(7-2*n))*(3-2*n)-4*a^3*d^2*D*(-n^2+5*n+6)+9*(-a*d+b*c)*
(5*D*b*c+2*a*d*D*(2-3*n)-b*C*d*(3-2*n))*(7*b*c-2*a*d*(1+n))/d/(5-2*n)+2*a^
2*b*d*(45*D*c+C*d*(-2*n^2+n+3))-a*b^2*(45*D*c^2+9*c*C*d*(3-2*n)+2*B*d^2*(-
2*n^2+n+3)))*(d*x+c)^n*hypergeom([-7/2, -n],[-5/2],-d*(b*x+a)/(-a*d+b*c))/
b^4/d/(-a*d+b*c)/(3-2*n)/(b*x+a)^(7/2)/((b*(d*x+c)/(-a*d+b*c))^n)
```

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{11/2}} dx =$$
$$-\frac{2(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(35(Ab^3-a(b^2B-abC+a^2D)) \text{Hypergeometric2F1}\left(-\frac{9}{2},-n,-\frac{7}{2},\frac{d(a+bx)}{-bc+ad}\right)+3\right.}{}$$

input
```
Integrate[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(11/2),x]
```

output
```
(-2*(c + d*x)^n*(35*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*Hypergeometric2F1[
-9/2, -n, -7/2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(a + b*x)*(15*(b^2*B - 2
*a*b*C + 3*a^2*D)*Hypergeometric2F1[-7/2, -n, -5/2, (d*(a + b*x))/(-(b*c)
+ a*d)] - 7*(a + b*x)*((-3*b*C + 9*a*D)*Hypergeometric2F1[-5/2, -n, -3/2,
(d*(a + b*x))/(-(b*c) + a*d)] - 5*D*(a + b*x)*Hypergeometric2F1[-3/2, -n,
-1/2, (d*(a + b*x))/(-(b*c) + a*d)]))))/(315*b^4*(a + b*x)^(9/2)*((b*(c +
d*x))/(b*c - a*d))^n)
```

Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2124, 27, 1193, 27, 87, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{11/2}} dx$$
$$\Big\downarrow \text{2124}$$

$$-\frac{2\int -\frac{(c+dx)^n\left(9\left(c-\frac{ad}{b}\right)Dx^2+\frac{9(bc-ad)(bC-aD)x}{b^2}+\frac{-2dD(n+1)a^3+b(9cD+2Cd(n+1))a^2-b^2(9cC+2Bd(n+1))a+b^3(9Bc-Ad(7-2n))}{b^3}\right)}{2(a+bx)^{9/2}}dx}{9(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{9b^3(a+bx)^{9/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(c+dx)^n\left(-\frac{2dD(n+1)a^3}{b^3}+\frac{(9cD+2Cd(n+1))a^2}{b^2}-\frac{(9cC+2Bd(n+1))a}{b}+9\left(c-\frac{ad}{b}\right)Dx^2+9Bc-Ad(7-2n)+\frac{9(bc-ad)(bC-aD)x}{b^2}\right)}{(a+bx)^{9/2}}dx}{9(bc-ad)}-\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{9b^3(a+bx)^{9/2}(bc-ad)}$$

$\Big\downarrow$ 1193

$$-\frac{\frac{2\int \frac{(c+dx)^n\left(2d^2D\left(2n^2+15n+13\right)a^3-bd\left(9cD(6n+13)+4Cd\left(n^2+3n+2\right)\right)a^2+2b^2\left(63Dc^2+18Cd(n+1)c-Bd^2\left(-2n^2+3n+5\right)\right)a-b^3\left(63Cc^2-9Bd(5-2n)c+A\right.}{2b^3(a+bx)^{7/2}}}{7(bc-ad)}}{9(bc}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{9b^3(a+bx)^{9/2}(bc-ad)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{(c+dx)^n\left(2d^2D\left(2n^2+15n+13\right)a^3-bd\left(9cD(6n+13)+4Cd\left(n^2+3n+2\right)\right)a^2+2b^2\left(63Dc^2+18Cd(n+1)c-Bd^2\left(-2n^2+3n+5\right)\right)a-b^3\left(63Cc^2-9Bd(5-2n)c+Ad\right.}{(a+bx)^{7/2}}}{7b^3(bc-ad)}}{9(bc-}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{9b^3(a+bx)^{9/2}(bc-ad)}$$

$\Big\downarrow$ 87

$$-\frac{-\frac{\left(2d\left(\frac{3}{2}-n\right)\left(2a^3d^2D\left(2n^2+15n+13\right)-a^2bd\left(9cD(6n+13)+4Cd\left(n^2+3n+2\right)\right)+2ab^2\left(-Bd^2\left(-2n^2+3n+5\right)+63c^2D+18cCd(n+1)\right)-b^3\left(Ad^2\left(4n^2-24n+35\right)-\right.\right.\right.}{5(bc-ad)}}{}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{9b^3(a+bx)^{9/2}(bc-ad)}$$

$\Big\downarrow$ 80

$$-\frac{-\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(2d\left(\frac{3}{2}-n\right)\left(2a^3d^2D\left(2n^2+15n+13\right)-a^2bd\left(9cD(6n+13)+4Cd\left(n^2+3n+2\right)\right)+2ab^2\left(-Bd^2\left(-2n^2+3n+5\right)+63c^2D+18cCd(n+1)\right)-b\right.\right.}{5(bc-ad)}}{}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{9b^3(a+bx)^{9/2}(bc-ad)}$$

$$\Big\downarrow \; 79$$

$$-\frac{\frac{2(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(-\frac{3}{2},-n,-\frac{1}{2},-\frac{d(a+bx)}{bc-ad}\right)\left(2d\left(\frac{3}{2}-n\right)\left(2a^3d^2D\left(2n^2+15n+13\right)-a^2bd\left(9cD(6n+13)+4Cd\left(n^2+3n+2\right)\right)+2ab^2\left(-\right.\right.\right.}{15b(a+bx)^{3/2}(bc-ad)}}{}$$

$$\frac{2(c+dx)^{n+1}\left(Ab^3-a\left(a^2D-abC+b^2B\right)\right)}{9b^3(a+bx)^{9/2}(bc-ad)}$$

input
```
Int[((c + d*x)^n*(A + B*x + C*x^2 + D*x^3))/(a + b*x)^(11/2),x]
```

output
```
(-2*(A*b^3 - a*(b^2*B - a*b*C + a^2*D))*(c + d*x)^(1 + n))/(9*b^3*(b*c - a
*d)*(a + b*x)^(9/2)) + ((-2*(b^3*(9*B*c - A*d*(7 - 2*n)) - 2*a^3*d*D*(10 +
 n) - 2*a*b^2*(9*c*C + B*d*(1 + n)) + a^2*b*(27*c*D + C*d*(11 + 2*n)))*(c
+ d*x)^(1 + n))/(7*b^3*(b*c - a*d)*(a + b*x)^(7/2)) - ((-2*(a^3*d^2*D*(89
+ 30*n + 4*n^2) + a*b^2*(189*c^2*D + 36*c*C*d*(1 + n) - 2*B*d^2*(5 + 3*n -
 2*n^2)) - a^2*b*d*(27*c*D*(9 + 2*n) + 4*C*d*(2 + 3*n + n^2)) - b^3*(63*c^
2*C - 9*B*c*d*(5 - 2*n) + A*d^2*(35 - 24*n + 4*n^2)))*(c + d*x)^(1 + n))/(
5*(b*c - a*d)*(a + b*x)^(5/2)) + (2*(126*(b*c - a*d)^2*D*((5*b*c)/2 - a*d*
(1 + n)) + 2*d*(3/2 - n)*(2*a^3*d^2*D*(13 + 15*n + 2*n^2) + 2*a*b^2*(63*c^
2*D + 18*c*C*d*(1 + n) - B*d^2*(5 + 3*n - 2*n^2)) - a^2*b*d*(9*c*D*(13 + 6
*n) + 4*C*d*(2 + 3*n + n^2)) - b^3*(63*c^2*C - 9*B*c*d*(5 - 2*n) + A*d^2*(
35 - 24*n + 4*n^2))))*(c + d*x)^n*Hypergeometric2F1[-3/2, -n, -1/2, -((d*(
a + b*x))/(b*c - a*d))])/(15*b*(b*c - a*d)*(a + b*x)^(3/2)*((b*(c + d*x))/
(b*c - a*d))^n))/(7*b^3*(b*c - a*d)))/(9*(b*c - a*d))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1193
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[R*(d + e*x)^(m + 1)*((f + g*x)^(n + 1)/((m + 1)*(e*f - d*g))
), x] + Simp[1/((m + 1)*(e*f - d*g))   Int[(d + e*x)^(m + 1)*(f + g*x)^n*Ex
pandToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a
, b, c, d, e, f, g, n}, x] && IGtQ[p, 0] && ILtQ[2*m, -2] &&  !IntegerQ[n]
&&  !(EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0])
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

Maple [F]

$$\int \frac{(xd+c)^n (Dx^3+C x^2+Bx+A)}{(bx+a)^{\frac{11}{2}}}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(11/2),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(11/2),x)
```

Fricas [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{11/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(11/2),x, algorithm="frica
s")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*sqrt(b*x + a)*(d*x + c)^n/(b^6*x^6 + 6*
a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x +
 a^6), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{11/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**n*(D*x**3+C*x**2+B*x+A)/(b*x+a)**(11/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{11/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(11/2),x, algorithm="maxim
a")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(11/2), x)
```

Giac [F]

$$\int \frac{(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx)^{11/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)(dx+c)^n}{(bx+a)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(11/2),x, algorithm="giac"
)
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(d*x + c)^n/(b*x + a)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{11/2}}\,dx = \int \frac{(c+d\,x)^n\,(A+B\,x+C\,x^2+x^3\,D)}{(a+b\,x)^{11/2}}\,dx$$

input
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(11/2),x)
```

output
```
int(((c + d*x)^n*(A + B*x + C*x^2 + x^3*D))/(a + b*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^n (A+Bx+Cx^2+Dx^3)}{(a+bx)^{11/2}}\,dx = \int \frac{(dx+c)^n\,(Dx^3+C\,x^2+Bx+A)}{(bx+a)^{\frac{11}{2}}}dx$$

input
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(11/2),x)
```

output
```
int((d*x+c)^n*(D*x^3+C*x^2+B*x+A)/(b*x+a)^(11/2),x)
```

3.190 $\int (a+bx)^m(A+Bx)(c+dx)^n\,dx$

Optimal result 1808
Mathematica [A] (verified) 1808
Rubi [A] (verified) 1809
Maple [F] 1810
Fricas [F] 1811
Sympy [F(-2)] 1811
Maxima [F] 1811
Giac [F] 1812
Mupad [F(-1)] 1812
Reduce [F] 1812

Optimal result

Integrand size = 20, antiderivative size = 131

$$\int (a+bx)^m(A+Bx)(c+dx)^n\,dx=\frac{B(a+bx)^{1+m}(c+dx)^{1+n}}{bd(2+m+n)} - \frac{(bBc(1+m)+aBd(1+n)-Abd(2+m+n))(a+bx)^{1+m}(c+dx)^{1+n}\,\text{Hypergeometric2F1}\left(1,2+n\ldots\right.}{bd(bc-ad)(1+m)(2+m+n)}$$

output
```
B*(b*x+a)^(1+m)*(d*x+c)^(1+n)/b/d/(2+m+n)-(b*B*c*(1+m)+a*B*d*(1+n)-A*b*d*(
2+m+n))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hypergeom([1, 2+m+n],[2+m],-d*(b*x+a)/
(-a*d+b*c))/b/d/(-a*d+b*c)/(1+m)/(2+m+n)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int (a+bx)^m(A+Bx)(c+dx)^n\,dx = \frac{(a+bx)^{1+m}(c+dx)^n\left(bB(c+dx)-\frac{(bBc(1+m)+aBd(1+n)-Abd(2+m+n))\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(1+m,-n,2+\ldots\right.}{1+m}\right.}{b^2d(2+m+n)}$$

input
```
Integrate[(a + b*x)^m*(A + B*x)*(c + d*x)^n,x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*(b*B*(c + d*x) - ((b*B*c*(1 + m) + a*B*d*(1
 + n) - A*b*d*(2 + m + n))*Hypergeometric2F1[1 + m, -n, 2 + m, (d*(a + b*x
))/(-(b*c) + a*d)])/((1 + m)*((b*(c + d*x))/(b*c - a*d))^n)))/(b^2*d*(2 +
m + n))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)(a+bx)^m(c+dx)^n\,dx$$

$$\Big\downarrow \text{90}$$

$$\left(A-\frac{B(ad(n+1)+bc(m+1))}{bd(m+n+2)}\right)\int (a+bx)^m(c+dx)^n dx+\frac{B(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+2)}$$

$$\Big\downarrow \text{80}$$

$$(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(A-\frac{B(ad(n+1)+bc(m+1))}{bd(m+n+2)}\right)\int (a+bx)^m\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^n dx+\frac{B(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+2)}$$

$$\Big\downarrow \text{79}$$

$$\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(A-\frac{B(ad(n+1)+bc(m+1))}{bd(m+n+2)}\right)\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)}{b(m+1)}$$
$$\frac{B(a+bx)^{m+1}(c+dx)^{n+1}}{bd(m+n+2)}$$

input
```
Int[(a + b*x)^m*(A + B*x)*(c + d*x)^n,x]
```

output
```
(B*(a + b*x)^(1 + m)*(c + d*x)^(1 + n))/(b*d*(2 + m + n)) + ((A - (B*(b*c*
(1 + m) + a*d*(1 + n)))/(b*d*(2 + m + n)))*(a + b*x)^(1 + m)*(c + d*x)^n*H
ypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*(1 +
m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

Maple [F]

$$\int (bx+a)^m (Bx+A) (xd+c)^n \, dx$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n,x)
```

output
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n,x)
```

Fricas [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n,x, algorithm="fricas")
```

output
```
integral((B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(A+Bx)(c+dx)^n\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(B*x+A)*(d*x+c)**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n,x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n\,dx = \int (Bx+A)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(B*x+A)*(d*x+c)^n,x, algorithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(A+Bx)(c+dx)^n\,dx = \int (A+B\,x)\,\left(a+b\,x\right)^m\,\left(c+d\,x\right)^n\,dx$$

input
```
int((A + B*x)*(a + b*x)^m*(c + d*x)^n,x)
```

output
```
int((A + B*x)*(a + b*x)^m*(c + d*x)^n, x)
```

Reduce [F]

$$\int (a+bx)^m(A+Bx)(c+dx)^n\,dx = \text{too large to display}$$

input
```
int((b*x+a)^m*(B*x+A)*(d*x+c)^n,x)
```

output
```
((c + d*x)**n*(a + b*x)**m*a**2*c*d*m**2 + 2*(c + d*x)**n*(a + b*x)**m*a**
2*c*d*m*n + (c + d*x)**n*(a + b*x)**m*a**2*c*d*m + (c + d*x)**n*(a + b*x)*
*m*a**2*c*d*n**2 + 2*(c + d*x)**n*(a + b*x)**m*a**2*c*d*n + 2*(c + d*x)**n
*(a + b*x)**m*a**2*d**2*m*n*x + (c + d*x)**n*(a + b*x)**m*a**2*d**2*n**2*x
 + 2*(c + d*x)**n*(a + b*x)**m*a**2*d**2*n*x - (c + d*x)**n*(a + b*x)**m*a
*b*c**2*n + 2*(c + d*x)**n*(a + b*x)**m*a*b*c*d*m**2*x + (c + d*x)**n*(a +
 b*x)**m*a*b*c*d*m*n*x + 2*(c + d*x)**n*(a + b*x)**m*a*b*c*d*m*x + (c + d*
x)**n*(a + b*x)**m*a*b*c*d*n**2*x + (c + d*x)**n*(a + b*x)**m*a*b*d**2*m*n
*x**2 + (c + d*x)**n*(a + b*x)**m*a*b*d**2*n**2*x**2 + (c + d*x)**n*(a + b
*x)**m*a*b*d**2*n*x**2 + (c + d*x)**n*(a + b*x)**m*b**2*c**2*m*n*x + (c +
d*x)**n*(a + b*x)**m*b**2*c*d*m**2*x**2 + (c + d*x)**n*(a + b*x)**m*b**2*c
*d*m*n*x**2 + (c + d*x)**n*(a + b*x)**m*b**2*c*d*m*x**2 + int(((c + d*x)**
n*(a + b*x)**m*x)/(a**2*c*d*m**2*n + 2*a**2*c*d*m*n**2 + 3*a**2*c*d*m*n +
a**2*c*d*n**3 + 3*a**2*c*d*n**2 + 2*a**2*c*d*n + a**2*d**2*m**2*n*x + 2*a*
*2*d**2*m*n**2*x + 3*a**2*d**2*m*n*x + a**2*d**2*n**3*x + 3*a**2*d**2*n**2
*x + 2*a**2*d**2*n*x + a*b*c**2*m**3 + 2*a*b*c**2*m**2*n + 3*a*b*c**2*m**2
 + a*b*c**2*m*n**2 + 3*a*b*c**2*m*n + 2*a*b*c**2*m + a*b*c*d*m**3*x + 3*a*
b*c*d*m**2*n*x + 3*a*b*c*d*m**2*x + 3*a*b*c*d*m*n**2*x + 6*a*b*c*d*m*n*x +
 2*a*b*c*d*m*x + a*b*c*d*n**3*x + 3*a*b*c*d*n**2*x + 2*a*b*c*d*n*x + a*b*d
**2*m**2*n*x**2 + 2*a*b*d**2*m*n**2*x**2 + 3*a*b*d**2*m*n*x**2 + a*b*d*...
```

3.191 $\int (a+bx)^m(c+dx)^n\left(A+Bx+Cx^2\right)\,dx$

Optimal result . . . 1814
Mathematica [A] (verified) . . . 1815
Rubi [A] (warning: unable to verify) . . . 1815
Maple [F] . . . 1818
Fricas [F] . . . 1818
Sympy [F(-2)] . . . 1818
Maxima [F] . . . 1819
Giac [F] . . . 1819
Mupad [F(-1)] . . . 1819
Reduce [F] . . . 1820

Optimal result

Integrand size = 25, antiderivative size = 256

$$\int (a+bx)^m(c+dx)^n\left(A+Bx+Cx^2\right)\,dx$$
$$= -\frac{(bcC(2+m)-bBd(3+m+n)+aCd(4+m+2n))(a+bx)^{1+m}(c+dx)^{1+n}}{b^2d^2(2+m+n)(3+m+n)}$$
$$+\frac{C(a+bx)^{2+m}(c+dx)^{1+n}}{b^2d(3+m+n)}$$
$$-\frac{\left(d(2+m+n)\left(abcC(2+m)+a^2Cd(1+n)-Ab^2d(3+m+n)\right)-(bc(1+m)+ad(1+n))(bcC(2-\right.}{b^2d^2(bc-a}$$

output
```
-(b*c*C*(2+m)-b*B*d*(3+m+n)+a*C*d*(4+m+2*n))*(b*x+a)^(1+m)*(d*x+c)^(1+n)/b
^2/d^2/(2+m+n)/(3+m+n)+C*(b*x+a)^(2+m)*(d*x+c)^(1+n)/b^2/d/(3+m+n)-(d*(2+m
+n)*(a*b*c*C*(2+m)+a^2*C*d*(1+n)-A*b^2*d*(3+m+n))-(b*c*(1+m)+a*d*(1+n))*(b
*c*C*(2+m)-b*B*d*(3+m+n)+a*C*d*(4+m+2*n)))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*hyp
ergeom([1, 2+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^2/d^2/(-a*d+b*c)/(1+m)/(2
+m+n)/(3+m+n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.73

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2\right) dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(C(bc-ad)^2 \text{Hypergeometric2F1}\left(1+m,-2-n,2+m,\frac{d(a+bx)}{-bc+ad}\right)+\right.}{}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(A + B*x + C*x^2),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*(C*(b*c - a*d)^2*Hypergeometric2F1[1 + m, -
2 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(-((b*c - a*d)*(2*c*C - B*
d)*Hypergeometric2F1[1 + m, -1 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)])
+ b*(c^2*C - B*c*d + A*d^2)*Hypergeometric2F1[1 + m, -n, 2 + m, (d*(a + b*
x))/(-(b*c) + a*d)])))/(b^3*d^2*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1194, 25, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^m \left(A+Bx+Cx^2\right)(c+dx)^n dx$$

$$\Big\downarrow \text{ 1194}$$

$$\frac{\int -(a+bx)^m(c+dx)^n \left(Cd(n+1)a^2+bcC(m+2)a-Ab^2d(m+n+3)+b(bcC(m+2)-bBd(m+n+3)+\right.}{b^2d(m+n+3)}$$
$$\frac{C(a+bx)^{m+2}(c+dx)^{n+1}}{b^2d(m+n+3)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C(a+bx)^{m+2}(c+dx)^{n+1}}{b^2d(m+n+3)} - \frac{\int (a+bx)^m(c+dx)^n \left(Cd(n+1)a^2 + bcC(m+2)a - Ab^2d(m+n+3) + b(bcC(m+2) - bBd(m+n+3) + aC\right.}{b^2d(m+n+3)}$$

$\downarrow$ 90

$$\frac{C(a+bx)^{m+2}(c+dx)^{n+1}}{b^2d(m+n+3)} - \frac{\left(a^2Cd(n+1) - \frac{(ad(n+1)+bc(m+1))(aCd(m+2n+4)-bBd(m+n+3)+bcC(m+2))}{d(m+n+2)} + abcC(m+2) - Ab^2d(m+n+3)\right) \int (a}{b^2d(m+n+3)}$$

$\downarrow$ 80

$$\frac{C(a+bx)^{m+2}(c+dx)^{n+1}}{b^2d(m+n+3)} - \frac{(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left(a^2Cd(n+1) - \frac{(ad(n+1)+bc(m+1))(aCd(m+2n+4)-bBd(m+n+3)+bcC(m+2))}{d(m+n+2)} + abcC(m+2) - \right.}{b^2d(m+}$$

$\downarrow$ 79

$$\frac{C(a+bx)^{m+2}(c+dx)^{n+1}}{b^2d(m+n+3)} - \frac{\frac{(a+bx)^{m+1}(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\text{Hypergeometric2F1}\left(m+1,-n,m+2,-\frac{d(a+bx)}{bc-ad}\right)\left(a^2Cd(n+1)-\frac{(ad(n+1)+bc(m+1))(aCd(m+2n+4)-bBd(m+n+3}{d(m+n+2)}\right.}{b(m+1)}}{b^2d(m+n+3)}$$

input

```
Int[(a + b*x)^m*(c + d*x)^n*(A + B*x + C*x^2),x]
```

output

```
(C*(a + b*x)^(2 + m)*(c + d*x)^(1 + n))/(b^2*d*(3 + m + n)) - (((b*c*C*(2
+ m) - b*B*d*(3 + m + n) + a*C*d*(4 + m + 2*n))*(a + b*x)^(1 + m)*(c + d*x
)^(1 + n))/(d*(2 + m + n)) + ((a*b*c*C*(2 + m) + a^2*C*d*(1 + n) - A*b^2*d
*(3 + m + n) - ((b*c*(1 + m) + a*d*(1 + n))*(b*c*C*(2 + m) - b*B*d*(3 + m
+ n) + a*C*d*(4 + m + 2*n)))/(d*(2 + m + n)))*(a + b*x)^(1 + m)*(c + d*x)^
n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b*x))/(b*c - a*d))])/(b*(1
 + m)*((b*(c + d*x))/(b*c - a*d))^n))/(b^2*d*(3 + m + n))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

Maple [F]

$$\int (bx+a)^m (xd+c)^n \left(C\,x^2+Bx+A\right) dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(C*x^2+B*x+A),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(C*x^2+B*x+A),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2\right)\,dx = \int \left(Cx^2+Bx+A\right)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2\right)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(C*x**2+B*x+A),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2\right) \, dx = \int \left(Cx^2+Bx+A\right)(bx+a)^m(dx+c)^n \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2\right) \, dx = \int \left(Cx^2+Bx+A\right)(bx+a)^m(dx+c)^n \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2\right) \, dx = \int (a+b\,x)^m\,(c+d\,x)^n\,\left(C\,x^2+B\,x+A\right) \, dx$$

input
```
int((a + b*x)^m*(c + d*x)^n*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x)^m*(c + d*x)^n*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2\right)\,dx = \int (bx+a)^m\,(dx+c)^n\,\left(C\,x^2+Bx+A\right)dx$$

input

```
int((b*x+a)^m*(d*x+c)^n*(C*x^2+B*x+A),x)
```

output

```
int((b*x+a)^m*(d*x+c)^n*(C*x^2+B*x+A),x)
```

3.192 $\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal result . . . 1821
Mathematica [A] (verified) . . . 1822
Rubi [A] (warning: unable to verify) . . . 1823
Maple [F] . . . 1826
Fricas [F] . . . 1826
Sympy [F(-2)] . . . 1827
Maxima [F] . . . 1827
Giac [F] . . . 1827
Mupad [F(-1)] . . . 1828
Reduce [F] . . . 1828

Optimal result

Integrand size = 30, antiderivative size = 505

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx =$$

$$-\frac{(d(3+m+n)\left(2abcD(3+m)-b^2Bd(4+m+n)+a^2dD(6+m+3n)\right)-(bc(2+m)+ad(4+m+}{b^3d^3(2+m+n)(3+m+n}$$

$$-\frac{(bcD(3+m)-bCd(4+m+n)+adD(9+2m+3n))(a+bx)^{2+m}(c+dx)^{1+n}}{b^3d^2(3+m+n)(4+m+n)}$$

$$+\frac{D(a+bx)^{3+m}(c+dx)^{1+n}}{b^3d(4+m+n)}$$

$$+\frac{((bc(1+m)+ad(1+n))\left(d(3+m+n)\left(2abcD(3+m)-b^2Bd(4+m+n)+a^2dD(6+m+3n)\right)-\right.}{}$$

output
```
-(d*(3+m+n)*(2*a*b*c*D*(3+m)-b^2*B*d*(4+m+n)+a^2*d*D*(6+m+3*n))-(b*c*(2+m)
+a*d*(4+m+2*n))*(b*c*D*(3+m)-b*C*d*(4+m+n)+a*d*D*(9+2*m+3*n)))*(b*x+a)^(1+
m)*(d*x+c)^(1+n)/b^3/d^3/(2+m+n)/(3+m+n)/(4+m+n)-(b*c*D*(3+m)-b*C*d*(4+m+n
)+a*d*D*(9+2*m+3*n))*(b*x+a)^(2+m)*(d*x+c)^(1+n)/b^3/d^2/(3+m+n)/(4+m+n)+D
*(b*x+a)^(3+m)*(d*x+c)^(1+n)/b^3/d/(4+m+n)+((b*c*(1+m)+a*d*(1+n))*(d*(3+m+
n)*(2*a*b*c*D*(3+m)-b^2*B*d*(4+m+n)+a^2*d*D*(6+m+3*n))-(b*c*(2+m)+a*d*(4+m
+2*n))*(b*c*D*(3+m)-b*C*d*(4+m+n)+a*d*D*(9+2*m+3*n)))+d*(2+m+n)*(a*(b*c*(2
+m)+a*d*(1+n))*(b*c*D*(3+m)-b*C*d*(4+m+n)+a*d*D*(9+2*m+3*n))+d*(3+m+n)*(A*
b^3*d*(4+m+n)-a^2*D*(b*c*(3+m)+a*d*(1+n)))))*(b*x+a)^(1+m)*(d*x+c)^(1+n)*h
ypergeom([1, 2+m+n],[2+m],-d*(b*x+a)/(-a*d+b*c))/b^3/d^3/(-a*d+b*c)/(1+m)/
(2+m+n)/(3+m+n)/(4+m+n)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.50

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$= \frac{(a+bx)^{1+m}(c+dx)^n \left(\frac{b(c+dx)}{bc-ad}\right)^{-n} \left((bc-ad)^3 D\,\text{Hypergeometric2F1}\left(1+m,-3-n,2+m,\frac{d(a+bx)}{-bc+ad}\right)+\right.}{}$$

input
```
Integrate[(a + b*x)^m*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^n*((b*c - a*d)^3*D*Hypergeometric2F1[1 + m, -
3 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b*(b*c - a*d)^2*(C*d - 3*c*D
)*Hypergeometric2F1[1 + m, -2 - n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] +
b^2*(b*c - a*d)*(-2*c*C*d + B*d^2 + 3*c^2*D)*Hypergeometric2F1[1 + m, -1 -
 n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)] + b^3*(c^2*C*d - B*c*d^2 + A*d^3
- c^3*D)*Hypergeometric2F1[1 + m, -n, 2 + m, (d*(a + b*x))/(-(b*c) + a*d)]
))/(b^4*d^3*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n)
```

Rubi [A] (warning: unable to verify)

Time = 1.11 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2125, 1194, 27, 90, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2125}$$

$$\frac{\int (a+bx)^m(c+dx)^n \left(Ad(m+n+4)b^3-(bcD(m+3)-bCd(m+n+4)+adD(2m+3n+9))x^2b^2-(dD(m$$
$$b^3d(m+n+4)$$

$$\frac{D(a+bx)^{m+3}(c+dx)^{n+1}}{b^3d(m+n+4)}$$

$$\Big\downarrow \text{1194}$$

$$\int b^2(a+bx)^m(c+dx)^n \left(d^2D(n+1)(m+2n+6)a^3-bd(Cd(n+1)(m+n+4)-cD(m+2)(m+3n+6))a^2+b^2c(m+2)(cD(m+3)-Cd(m+n+4))a+Ab^3$$

$$\frac{D(a+bx)^{m+3}(c+dx)^{n+1}}{b^3d(m+n+4)}$$

$$\Big\downarrow \text{27}$$

$$\int (a+bx)^m(c+dx)^n \left(d^2D(n+1)(m+2n+6)a^3-bd(Cd(n+1)(m+n+4)-cD(m+2)(m+3n+6))a^2+b^2c(m+2)(cD(m+3)-Cd(m+n+4))a+Ab^3d^2$$

$$\frac{D(a+bx)^{m+3}(c+dx)^{n+1}}{b^3d(m+n+4)}$$

$$\Big\downarrow \text{90}$$

$$\Bigg(a^3d^2D(n+1)(m+2n+6)-\frac{(ad(n+1)+bc(m+1))\left(a^2d^2D\left(m^2+m(3n+8)+3\left(n^2+5n+6\right)\right)+abd\left(cD(m+2)(m+3n+6)-Cd\left(m^2+m(3n+8)+2\left(n^2+6n+8\right)\right)\right)+b}{d(m+n+2)}$$

$$\frac{D(a+bx)^{m+3}(c+dx)^{n+1}}{b^3d(m+n+4)}$$

$$\Big\downarrow \text{80}$$

$$\frac{(c+dx)^n\left(\frac{b(c+dx)}{bc-ad}\right)^{-n}\left(a^3d^2D(n+1)(m+2n+6)-\frac{(ad(n+1)+bc(m+1))\left(a^2d^2D\left(m^2+m(3n+8)+3\left(n^2+5n+6\right)\right)+abd\left(cD(m+2)(m+3n+6)-Cd\left(m^2+m(3n\right.\right.\right.}{d(m}}{}$$

$$\frac{D(a+bx)^{m+3}(c+dx)^{n+1}}{b^3d(m+n+4)}$$

$\big\downarrow$ 79

$$\frac{\frac{(a+bx)^{m+1}(c+dx)^{n+1}\left(a^2d^2D\left(m^2+m(3n+8)+3\left(n^2+5n+6\right)\right)+abd\left(cD(m+2)(m+3n+6)-Cd\left(m^2+m(3n+8)+2\left(n^2+6n+8\right)\right)\right)+b^2\left(Bd^2\left(m^2+m(2n+7)+n^2+\right.\right.\right.}{d(m+n+2)}}{}$$

$$\frac{D(a+bx)^{m+3}(c+dx)^{n+1}}{b^3d(m+n+4)}$$

input
```
Int[(a + b*x)^m*(c + d*x)^n*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(D*(a + b*x)^(3 + m)*(c + d*x)^(1 + n))/(b^3*d*(4 + m + n)) + (-(((b*c*D*(
3 + m) - b*C*d*(4 + m + n) + a*d*D*(9 + 2*m + 3*n))*(a + b*x)^(2 + m)*(c +
 d*x)^(1 + n))/(d*(3 + m + n))) + (((a^2*d^2*D*(m^2 + m*(8 + 3*n) + 3*(6 +
 5*n + n^2)) + b^2*(c^2*D*(6 + 5*m + m^2) - c*C*d*(2 + m)*(4 + m + n) + B*
d^2*(12 + m^2 + 7*n + n^2 + m*(7 + 2*n))) + a*b*d*(c*D*(2 + m)*(6 + m + 3*
n) - C*d*(m^2 + m*(8 + 3*n) + 2*(8 + 6*n + n^2))))*(a + b*x)^(1 + m)*(c +
d*x)^(1 + n))/(d*(2 + m + n)) + ((a^3*d^2*D*(1 + n)*(6 + m + 2*n) + a*b^2*
c*(2 + m)*(c*D*(3 + m) - C*d*(4 + m + n)) + A*b^3*d^2*(12 + m^2 + 7*n + n^
2 + m*(7 + 2*n)) - a^2*b*d*(C*d*(1 + n)*(4 + m + n) - c*D*(2 + m)*(6 + m +
 3*n)) - ((b*c*(1 + m) + a*d*(1 + n))*(a^2*d^2*D*(m^2 + m*(8 + 3*n) + 3*(6
 + 5*n + n^2)) + b^2*(c^2*D*(6 + 5*m + m^2) - c*C*d*(2 + m)*(4 + m + n) +
B*d^2*(12 + m^2 + 7*n + n^2 + m*(7 + 2*n))) + a*b*d*(c*D*(2 + m)*(6 + m +
3*n) - C*d*(m^2 + m*(8 + 3*n) + 2*(8 + 6*n + n^2)))))/(d*(2 + m + n)))*(a
+ b*x)^(1 + m)*(c + d*x)^n*Hypergeometric2F1[1 + m, -n, 2 + m, -((d*(a + b
*x))/(b*c - a*d))])/(b*(1 + m)*((b*(c + d*x))/(b*c - a*d))^n))/(d*(3 + m +
 n)))/(b^3*d*(4 + m + n))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2125
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[k*(a + b*x
)^(m + q)*((c + d*x)^(n + 1)/(d*b^q*(m + n + q + 1))), x] + Simp[1/(d*b^q*(
m + n + q + 1))   Int[(a + b*x)^m*(c + d*x)^n*ExpandToSum[d*b^q*(m + n + q
+ 1)*Px - d*k*(m + n + q + 1)*(a + b*x)^q - k*(b*c - a*d)*(m + q)*(a + b*x)
^(q - 1), x], x], x] /; NeQ[m + n + q + 1, 0]] /; FreeQ[{a, b, c, d, m, n},
 x] && PolyQ[Px, x]
```

Maple [F]

$$\int \left(bx+a\right)^{m} \left(xd+c\right)^{n} \left(D x^{3}+C\,x^{2}+Bx+A\right) dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

Fricas [F]

$$\int (a+bx)^m(c+dx)^n \left(A+Bx+Cx^2+Dx^3\right)\,dx = \int \left(Dx^3+Cx^2+Bx+A\right)(bx+a)^m(dx+c)^n\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
integral((D*x^3 + C*x^2 + B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (a+bx)^m (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*x+a)**m*(d*x+c)**n*(D*x**3+C*x**2+B*x+A),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (a+bx)^m (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(Dx^3+Cx^2+Bx+A\right)(bx+a)^m (dx+c)^n \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Giac [F]

$$\int (a+bx)^m (c+dx)^n \left(A+Bx+Cx^2+Dx^3\right) dx = \int \left(Dx^3+Cx^2+Bx+A\right)(bx+a)^m (dx+c)^n \, dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*(b*x + a)^m*(d*x + c)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^m(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int (a+b\,x)^m\,(c+d\,x)^n\,\left(A+B\,x+C\,x^2+x^3\,D\right)\,dx$$

input
```
int((a + b*x)^m*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
int((a + b*x)^m*(c + d*x)^n*(A + B*x + C*x^2 + x^3*D), x)
```

Reduce [F]

$$\int (a+bx)^m(c+dx)^n\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\int (bx+a)^m\,(dx+c)^n\left(Dx^3+C\,x^2+Bx+A\right)dx$$

input
```
int((b*x+a)^m*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

output
```
int((b*x+a)^m*(d*x+c)^n*(D*x^3+C*x^2+B*x+A),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 1829
4.2 Links to plain text integration problems used in this report for each CAS . 1847

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file