Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/27-1.1.1.7

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**100**]. This is test number [27].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (100)	0.00 (0)
Mathematica	100.00 (100)	0.00 (0)
Maple	100.00 (100)	0.00 (0)
Fricas	84.00 (84)	16.00 (16)
Giac	71.00 (71)	29.00 (29)
Reduce	68.00 (68)	32.00 (32)
Mupad	64.00 (64)	36.00 (36)
Maxima	49.00 (49)	51.00 (51)
Sympy	11.00 (11)	89.00 (89)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

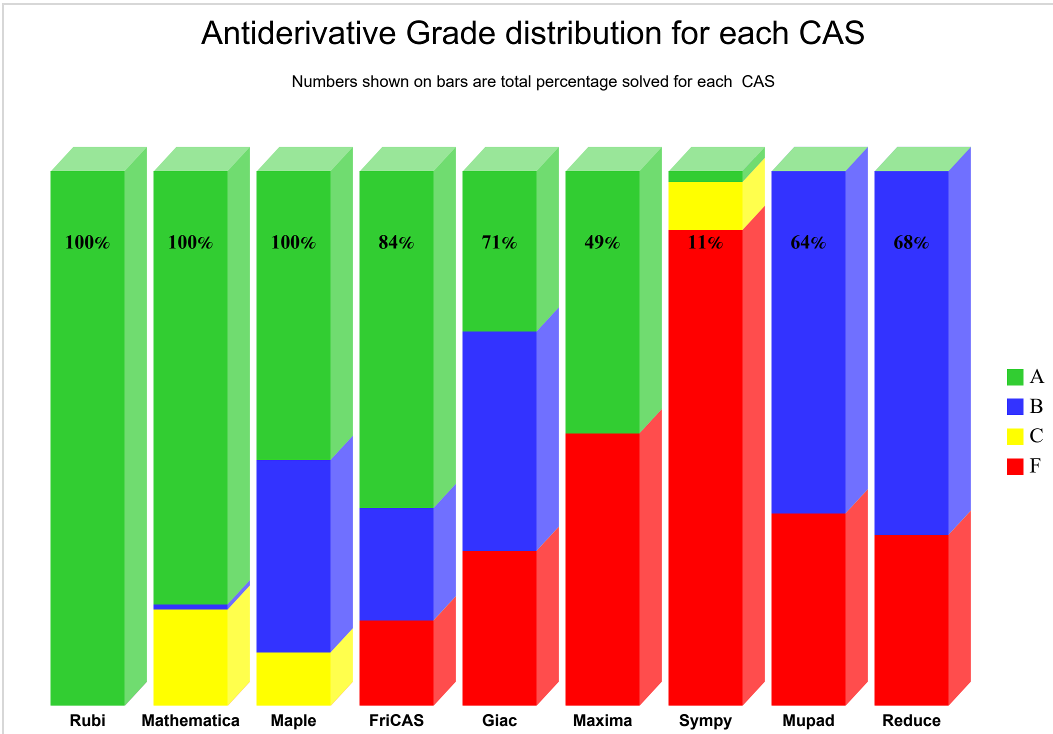
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

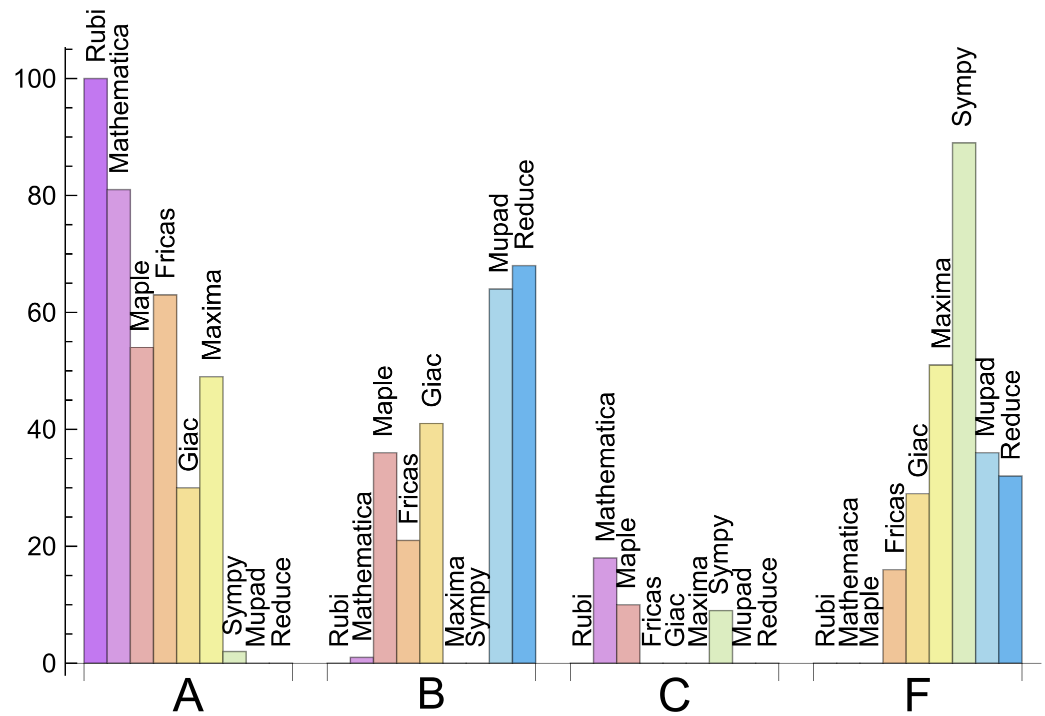
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	81.000	1.000	18.000	0.000
Fricas	63.000	21.000	0.000	16.000
Maple	54.000	36.000	10.000	0.000
Maxima	49.000	0.000	0.000	51.000
Giac	30.000	41.000	0.000	29.000
Sympy	2.000	0.000	9.000	89.000
Mupad	0.000	64.000	0.000	36.000
Reduce	0.000	68.000	0.000	32.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	16	0.00	100.00	0.00
Giac	29	62.07	0.00	37.93
Reduce	32	100.00	0.00	0.00
Mupad	36	0.00	100.00	0.00
Maxima	51	35.29	0.00	64.71
Sympy	89	57.30	42.70	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.11
Rubi	0.86
Reduce	1.41
Maple	2.22
Giac	3.34
Fricas	3.91
Mathematica	6.77
Sympy	18.49
Mupad	27.34

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	180.00	1.07	140.00	1.05
Sympy	282.09	3.06	257.00	3.04
Rubi	374.86	1.07	258.50	1.05
Mathematica	535.02	1.05	226.50	0.90
Fricas	961.86	2.06	374.00	1.56
Reduce	1431.63	5.32	305.00	1.69
Maple	1485.71	3.19	424.00	1.60
Giac	1703.85	4.33	475.00	2.24
Mupad	6142.00	27.34	689.50	4.67

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

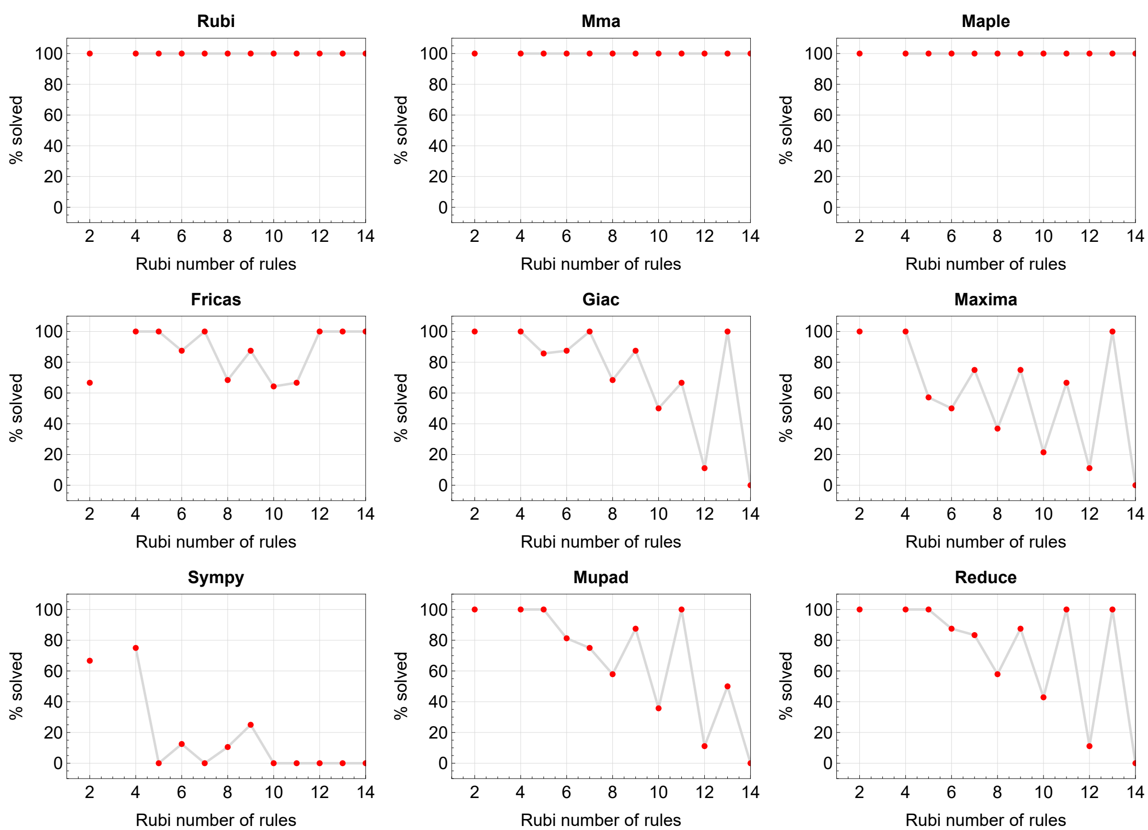


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

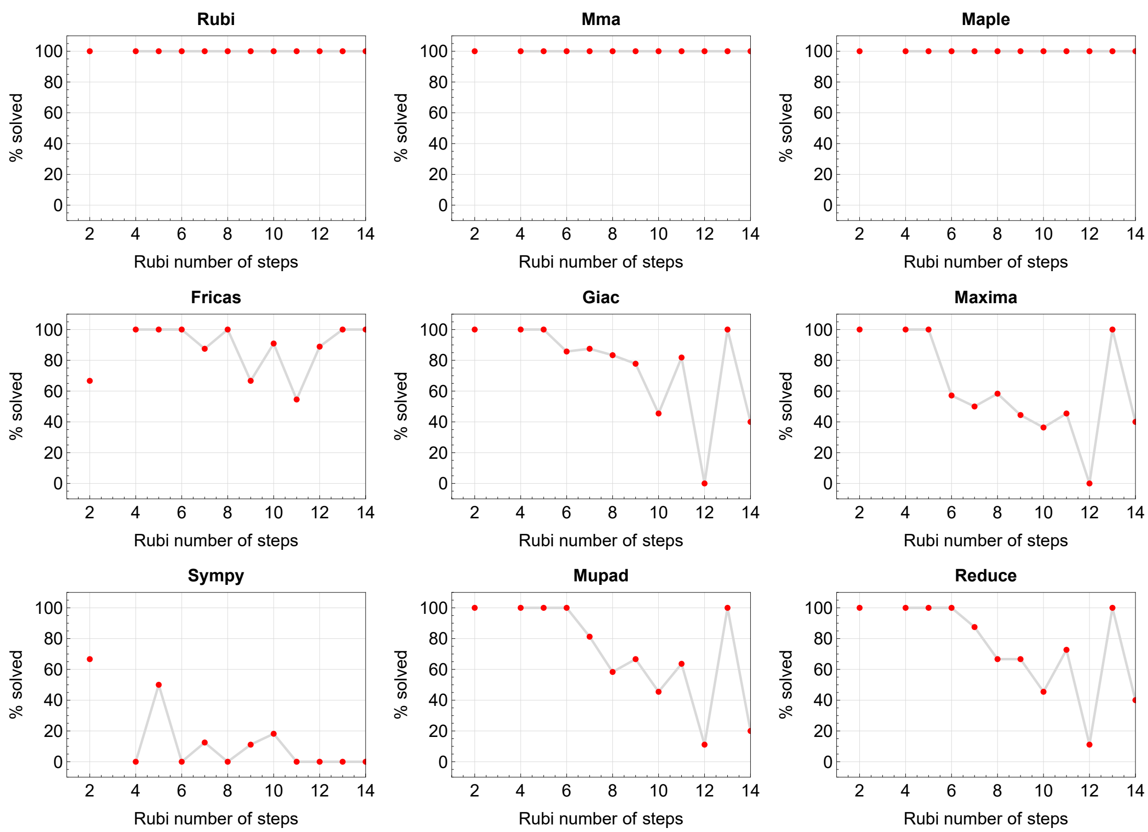


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

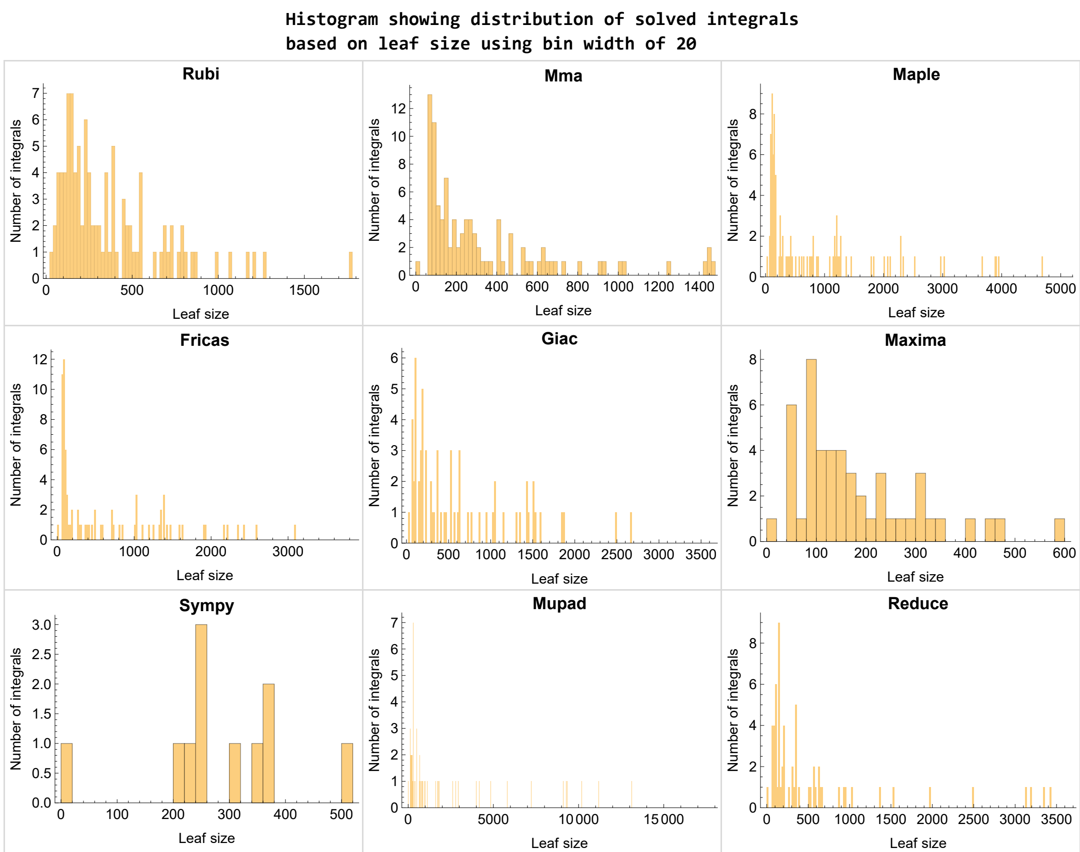


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

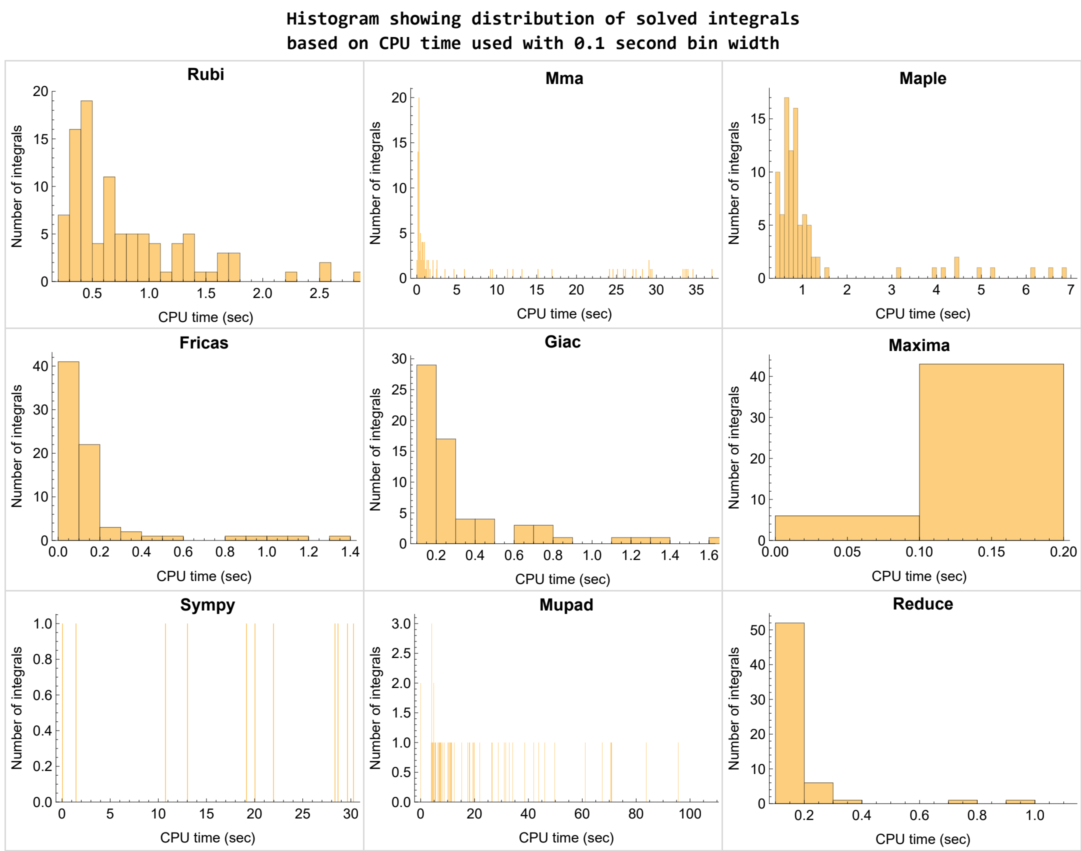


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

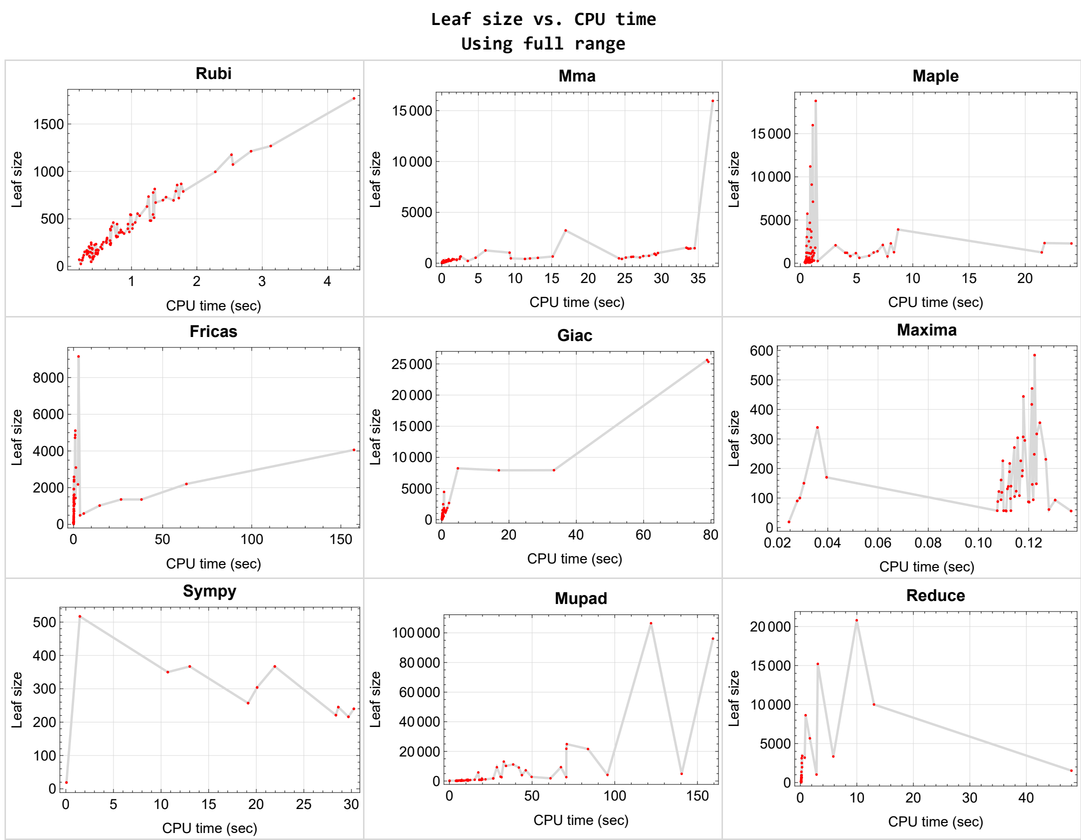


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{20, 21, 22, 23\}$

Mathematica $\{9, 11, 12, 13, 14, 15, 16\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

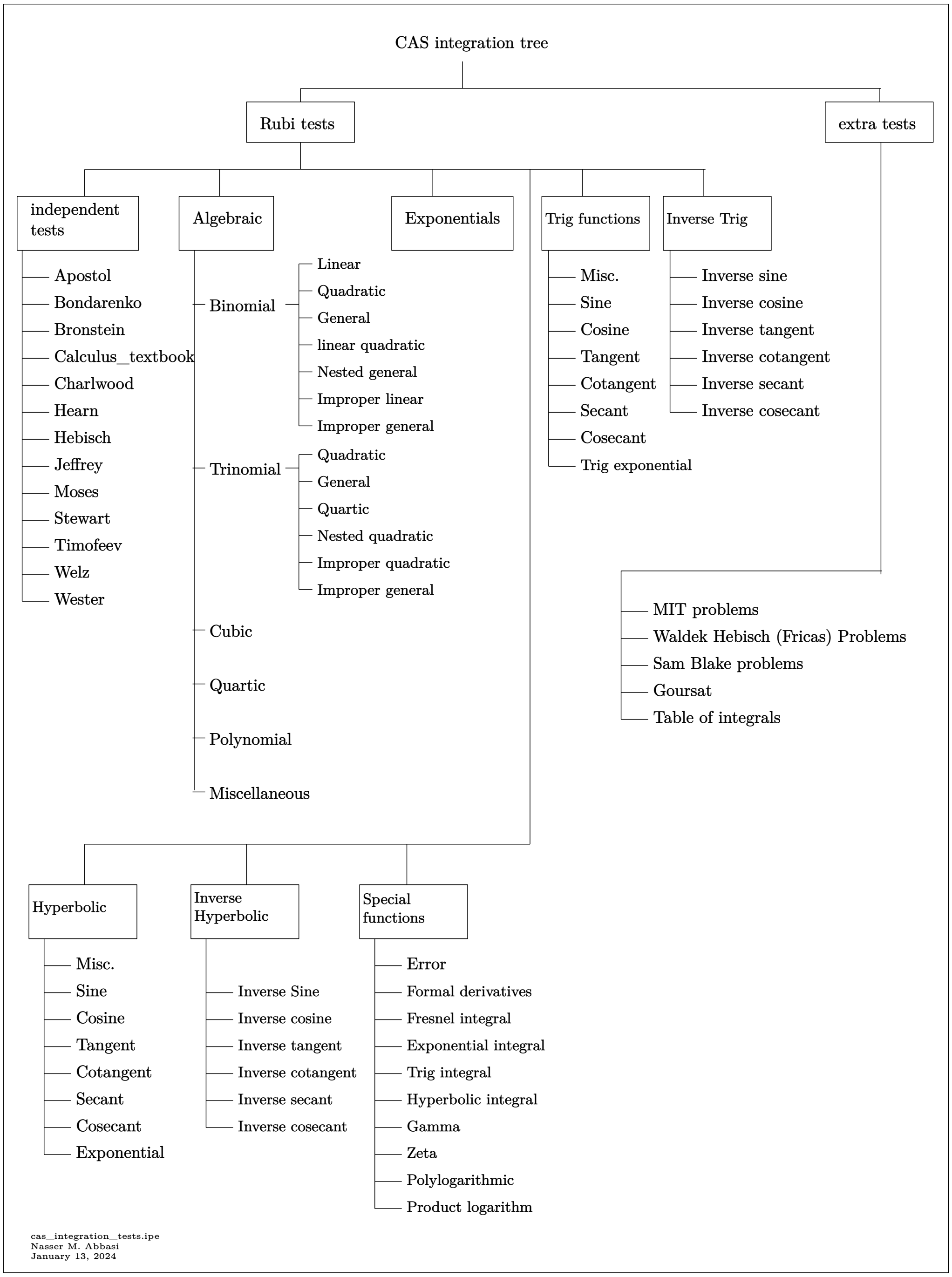
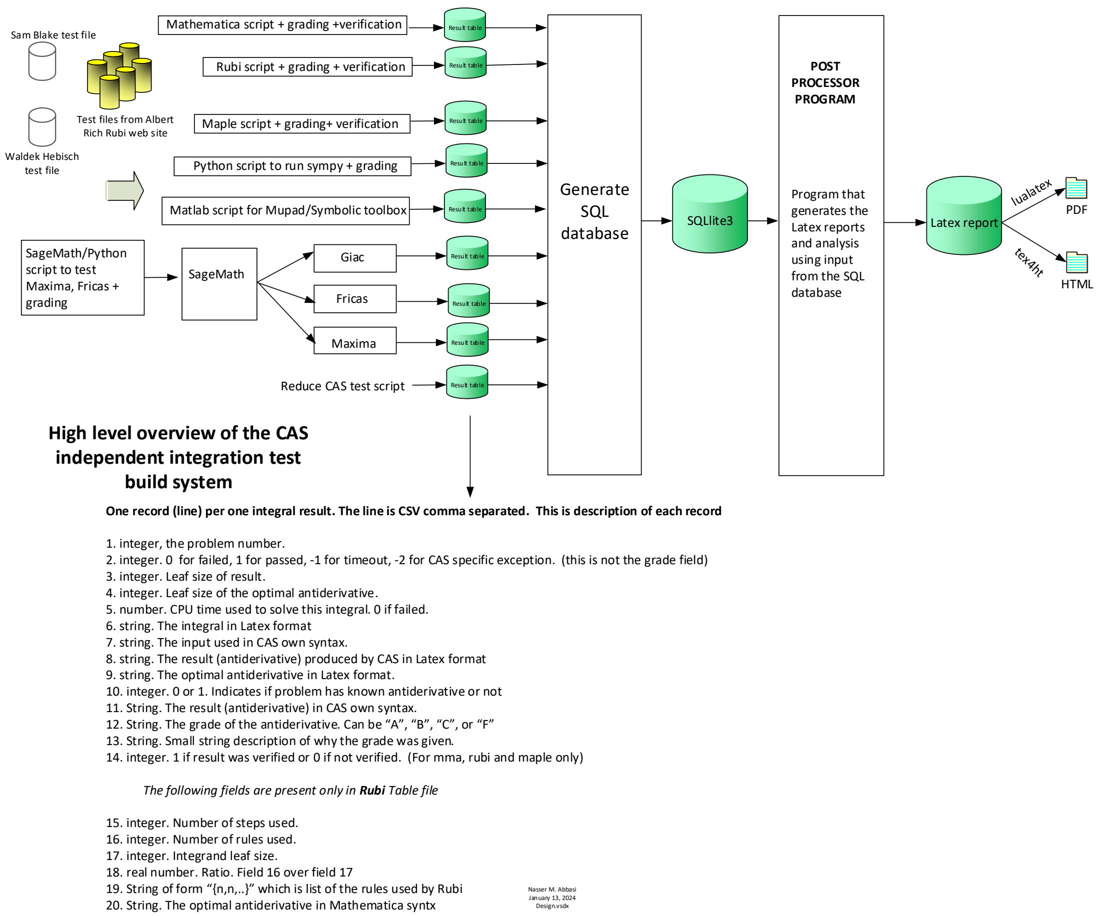


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82 }

B grade { 69 }

C grade { 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 60, 61, 62, 83, 84, 85, 89, 90, 91, 92, 95, 96, 97 }

B grade { 35, 36, 37, 42, 51, 57, 58, 59, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 86, 87, 88, 93, 94, 98, 99, 100 }

C grade { 2, 3, 4, 5, 6, 12, 20, 41, 43, 44 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 45, 46, 47, 48, 52, 53, 54, 55, 60, 62, 63, 64, 65, 69, 70, 71, 76, 77, 78, 83, 84, 89, 90, 95 }

B grade { 36, 37, 42, 43, 44, 58, 59, 81, 85, 86, 87, 88, 91, 92, 93, 94, 96, 97, 98, 99, 100 }

C grade { }

F normal fail { }

F(-1) timedout fail { 49, 50, 51, 56, 57, 61, 66, 67, 68, 72, 73, 74, 75, 79, 80, 82 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 45, 46, 47, 48, 52, 53, 54, 55, 60, 61, 62 }

B grade { }

C grade { }

F normal fail { 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

F(-1) timedout fail { }

F(-2) exception fail { 35, 36, 37, 42, 43, 44, 49, 50, 51, 56, 57, 58, 59, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82 }

Giac

A grade { 1, 2, 3, 9, 10, 11, 12, 13, 17, 18, 19, 24, 25, 38, 39, 40, 41, 50, 52, 53, 54, 55, 60, 61, 69, 70, 71, 76, 77, 78 }

B grade { 4, 5, 6, 7, 8, 14, 15, 16, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 46, 47, 48, 51, 57, 58, 59, 62, 63, 64, 65, 67, 68, 73, 74, 75, 80, 81, 82 }

C grade { }

F normal fail { 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

F(-1) timedout fail { }

F(-2) exception fail { 35, 36, 37, 42, 43, 44, 49, 56, 66, 72, 79 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 71, 77, 78, 79 }

C grade { }

F normal fail { }

F(-1) timedout fail { 45, 51, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

F(-2) exception fail { }

Sympy

A grade { 60, 62 }

B grade { }

C grade { 4, 5, 12, 13, 17, 18, 19, 20, 27 }

F normal fail { 31, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 56, 57, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99 }

F(-1) timedout fail { 1, 2, 3, 6, 7, 8, 9, 10, 11, 14, 15, 16, 21, 22, 23, 24, 25, 26, 28, 29, 30, 38, 39, 40, 41, 52, 53, 54, 55, 58, 59, 61, 75, 81, 82, 88, 94, 100 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 70, 71, 77, 78, 80 }

C grade { }

F normal fail { 63, 64, 66, 67, 68, 69, 72, 73, 74, 75, 76, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	134	94	157	119	97	0	105	140	485
N.S.	1	1.11	0.78	1.30	0.98	0.80	0.00	0.87	1.16	4.01
time (sec)	N/A	0.453	0.321	0.678	0.109	0.081	0.000	0.133	0.145	10.639

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	95	75	139	87	78	0	76	100	244
N.S.	1	1.03	0.82	1.51	0.95	0.85	0.00	0.83	1.09	2.65
time (sec)	N/A	0.365	0.275	0.607	0.120	0.082	0.000	0.128	0.143	6.927

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	64	117	57	67	0	60	76	232
N.S.	1	1.03	1.02	1.86	0.90	1.06	0.00	0.95	1.21	3.68
time (sec)	N/A	0.240	0.240	0.601	0.107	0.071	0.000	0.129	0.144	6.752

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	73	96	57	81	245	196	148	122
N.S.	1	1.00	1.52	2.00	1.19	1.69	5.10	4.08	3.08	2.54
time (sec)	N/A	0.384	0.181	0.604	0.110	0.078	28.622	0.182	0.148	4.801

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	73	97	57	84	221	282	148	114
N.S.	1	1.00	1.52	2.02	1.19	1.75	4.60	5.88	3.08	2.38
time (sec)	N/A	0.382	0.205	0.678	0.111	0.076	28.367	0.206	0.146	4.532

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	74	70	108	98	65	0	407	273	312
N.S.	1	1.04	0.99	1.52	1.38	0.92	0.00	5.73	3.85	4.39
time (sec)	N/A	0.408	0.201	0.639	0.113	0.074	0.000	0.211	0.151	5.587

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	105	77	126	108	78	0	619	204	304
N.S.	1	1.06	0.78	1.27	1.09	0.79	0.00	6.25	2.06	3.07
time (sec)	N/A	0.428	0.245	0.700	0.116	0.073	0.000	0.259	0.152	5.343

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	141	107	144	161	96	0	861	350	684
N.S.	1	1.06	0.80	1.08	1.21	0.72	0.00	6.47	2.63	5.14
time (sec)	N/A	0.484	0.280	0.746	0.109	0.079	0.000	0.282	0.159	7.394

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	175	93	124	150	91	0	145	147	767
N.S.	1	1.10	0.58	0.78	0.94	0.57	0.00	0.91	0.92	4.82
time (sec)	N/A	0.477	0.201	0.636	0.030	0.075	0.000	0.136	0.144	18.037

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	137	74	108	100	73	0	105	102	318
N.S.	1	1.26	0.68	0.99	0.92	0.67	0.00	0.96	0.94	2.92
time (sec)	N/A	0.400	0.173	0.408	0.029	0.079	0.000	0.138	0.144	11.084

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	70	63	96	90	61	0	80	85	312
N.S.	1	0.91	0.82	1.25	1.17	0.79	0.00	1.04	1.10	4.05
time (sec)	N/A	0.201	0.134	0.413	0.028	0.073	0.000	0.127	0.144	11.322

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	89	69	95	56	73	240	71	84	118
N.S.	1	1.62	1.25	1.73	1.02	1.33	4.36	1.29	1.53	2.15
time (sec)	N/A	0.405	0.141	0.701	0.111	0.086	30.253	0.133	0.141	4.336

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	89	69	95	56	82	216	83	92	118
N.S.	1	1.62	1.25	1.73	1.02	1.49	3.93	1.51	1.67	2.15
time (sec)	N/A	0.411	0.137	0.401	0.137	0.085	29.683	0.133	0.142	4.139

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	102	60	76	61	69	0	145	126	316
N.S.	1	1.23	0.72	0.92	0.73	0.83	0.00	1.75	1.52	3.81
time (sec)	N/A	0.410	0.128	0.405	0.128	0.082	0.000	0.143	0.145	8.837

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	133	71	89	86	90	0	187	140	304
N.S.	1	1.15	0.61	0.77	0.74	0.78	0.00	1.61	1.21	2.62
time (sec)	N/A	0.453	0.173	0.413	0.120	0.079	0.000	0.141	0.145	8.122

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	168	93	107	122	101	0	325	202	695
N.S.	1	1.09	0.60	0.69	0.79	0.66	0.00	2.11	1.31	4.51
time (sec)	N/A	0.493	0.230	0.419	0.108	0.079	0.000	0.154	0.151	19.236

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	234	149	145	295	138	367	228	142	287
N.S.	1	1.11	0.71	0.69	1.40	0.66	1.75	1.09	0.68	1.37
time (sec)	N/A	0.457	0.290	0.915	0.118	0.083	21.950	0.175	0.157	4.135

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	186	116	109	217	104	367	164	106	215
N.S.	1	1.17	0.73	0.69	1.36	0.65	2.31	1.03	0.67	1.35
time (sec)	N/A	0.401	0.231	0.779	0.112	0.080	13.016	0.172	0.151	5.112

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	139	80	73	139	71	350	102	70	143
N.S.	1	1.28	0.73	0.67	1.28	0.65	3.21	0.94	0.64	1.31
time (sec)	N/A	0.334	0.167	0.790	0.112	0.080	10.705	0.164	0.145	3.924

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	C	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	107	106	143	105	80	304	189	199	161
N.S.	1	1.07	1.06	1.43	1.05	0.80	3.04	1.89	1.99	1.61
time (sec)	N/A	0.355	0.220	0.838	0.114	0.083	20.090	0.278	0.156	4.826

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	149	85	137	123	98	0	374	344	422
N.S.	1	1.51	0.86	1.38	1.24	0.99	0.00	3.78	3.47	4.26
time (sec)	N/A	0.373	0.245	0.876	0.115	0.081	0.000	0.317	0.159	6.333

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	180	102	143	193	102	0	767	505	932
N.S.	1	1.43	0.81	1.13	1.53	0.81	0.00	6.09	4.01	7.40
time (sec)	N/A	0.400	0.242	0.875	0.118	0.083	0.000	0.430	0.167	11.168

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	226	141	179	271	137	0	1434	589	1621
N.S.	1	1.26	0.78	0.99	1.51	0.76	0.00	7.97	3.27	9.01
time (sec)	N/A	0.435	0.351	0.880	0.114	0.096	0.000	0.612	0.182	19.642

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	192	134	161	226	134	0	177	214	1132
N.S.	1	0.89	0.62	0.75	1.05	0.62	0.00	0.82	0.99	5.24
time (sec)	N/A	0.393	0.284	0.857	0.117	0.095	0.000	0.163	0.150	21.807

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	155	98	128	146	100	0	115	141	651
N.S.	1	1.28	0.81	1.06	1.21	0.83	0.00	0.95	1.17	5.38
time (sec)	N/A	0.311	0.197	0.823	0.121	0.085	0.000	0.171	0.147	12.556

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	126	86	116	94	90	0	239	105	306
N.S.	1	1.34	0.91	1.23	1.00	0.96	0.00	2.54	1.12	3.26
time (sec)	N/A	0.318	0.179	0.892	0.122	0.083	0.000	0.209	0.157	7.610

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	135	81	107	94	90	257	530	103	138
N.S.	1	1.35	0.81	1.07	0.94	0.90	2.57	5.30	1.03	1.38
time (sec)	N/A	0.325	0.167	0.847	0.109	0.081	19.145	0.259	0.160	4.497

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	156	87	82	148	76	0	1055	79	146
N.S.	1	1.26	0.70	0.66	1.19	0.61	0.00	8.51	0.64	1.18
time (sec)	N/A	0.338	0.140	0.882	0.123	0.085	0.000	0.359	0.163	4.197

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	193	124	118	226	110	0	1451	115	218
N.S.	1	1.08	0.70	0.66	1.27	0.62	0.00	8.15	0.65	1.22
time (sec)	N/A	0.362	0.182	0.888	0.110	0.105	0.000	0.494	0.173	4.258

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	230	158	154	304	144	0	1847	151	290
N.S.	1	0.99	0.68	0.66	1.31	0.62	0.00	7.96	0.65	1.25
time (sec)	N/A	0.385	0.234	1.013	0.116	0.149	0.000	0.721	0.184	4.661

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	383	373	508	444	406	0	1539	869	3993
N.S.	1	0.95	0.92	1.26	1.10	1.00	0.00	3.81	2.15	9.88
time (sec)	N/A	0.836	1.682	0.528	0.118	0.095	0.000	0.296	0.179	43.844

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	271	262	358	307	279	0	1059	577	2920
N.S.	1	0.95	0.92	1.26	1.08	0.98	0.00	3.73	2.03	10.28
time (sec)	N/A	0.672	1.267	0.482	0.118	0.083	0.000	0.240	0.160	30.933

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	169	159	231	174	170	0	631	320	736
N.S.	1	1.03	0.97	1.41	1.06	1.04	0.00	3.85	1.95	4.49
time (sec)	N/A	0.457	0.696	0.440	0.117	0.081	0.000	0.201	0.159	10.332

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	91	89	155	93	95	0	284	140	361
N.S.	1	0.96	0.94	1.63	0.98	1.00	0.00	2.99	1.47	3.80
time (sec)	N/A	0.257	0.286	0.444	0.131	0.073	0.000	0.156	0.150	7.482

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	227	225	423	0	591	0	0	662	13115
N.S.	1	1.11	1.10	2.06	0.00	2.88	0.00	0.00	3.23	63.98
time (sec)	N/A	0.679	0.754	0.859	0.000	5.752	0.000	0.000	0.229	32.886

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	356	244	648	0	1358	0	0	1966	11177
N.S.	1	1.11	0.76	2.01	0.00	4.22	0.00	0.00	6.11	34.71
time (sec)	N/A	0.810	1.180	0.785	0.000	26.661	0.000	0.000	0.298	38.554

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	320	310	1156	0	2201	0	0	3422	21781
N.S.	1	1.06	1.03	3.84	0.00	7.31	0.00	0.00	11.37	72.36
time (sec)	N/A	0.764	2.030	0.812	0.000	63.351	0.000	0.000	0.332	70.716

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	359	259	361	355	286	0	374	565	2606
N.S.	1	1.08	0.78	1.09	1.07	0.86	0.00	1.13	1.70	7.85
time (sec)	N/A	0.858	0.999	0.793	0.124	0.084	0.000	0.171	0.161	31.508

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	244	178	256	231	192	0	232	357	1732
N.S.	1	1.07	0.78	1.12	1.01	0.84	0.00	1.02	1.57	7.60
time (sec)	N/A	0.684	0.659	0.741	0.127	0.088	0.000	0.156	0.151	26.441

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	140	107	173	131	114	0	112	184	492
N.S.	1	1.10	0.84	1.36	1.03	0.90	0.00	0.88	1.45	3.87
time (sec)	N/A	0.458	0.459	0.661	0.112	0.084	0.000	0.140	0.148	10.187

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	64	117	57	67	0	60	76	232
N.S.	1	1.03	1.02	1.86	0.90	1.06	0.00	0.95	1.21	3.68
time (sec)	N/A	0.249	0.242	0.702	0.113	0.080	0.000	0.132	0.146	7.099

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	156	289	0	493	0	0	535	5803
N.S.	1	1.04	1.28	2.37	0.00	4.04	0.00	0.00	4.39	47.57
time (sec)	N/A	0.478	0.591	1.130	0.000	3.660	0.000	0.000	0.173	17.472

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	184	197	899	0	1025	0	0	1375	10198
N.S.	1	1.13	1.21	5.52	0.00	6.29	0.00	0.00	8.44	62.56
time (sec)	N/A	0.550	0.894	1.155	0.000	14.648	0.000	0.000	0.188	34.200

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	263	228	1449	0	1580	0	0	2499	9097
N.S.	1	1.06	0.92	5.84	0.00	6.37	0.00	0.00	10.08	36.68
time (sec)	N/A	0.613	1.226	1.108	0.000	0.141	0.000	0.000	0.257	42.008

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	512	402	575	584	1001	0	2671	951	0
N.S.	1	0.92	0.73	1.04	1.05	1.81	0.00	4.82	1.72	0.00
time (sec)	N/A	1.345	0.936	0.687	0.122	0.134	0.000	2.190	0.204	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	398	286	401	417	703	0	1868	637	4853
N.S.	1	0.96	0.69	0.97	1.00	1.69	0.00	4.50	1.53	11.69
time (sec)	N/A	1.014	0.667	0.649	0.121	0.122	0.000	1.684	0.165	140.458

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	284	183	253	248	441	0	1142	356	1765
N.S.	1	1.11	0.72	0.99	0.97	1.73	0.00	4.48	1.40	6.92
time (sec)	N/A	0.622	0.412	0.607	0.122	0.098	0.000	1.130	0.157	26.594

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	153	123	163	140	265	0	527	161	876
N.S.	1	0.90	0.72	0.96	0.82	1.56	0.00	3.10	0.95	5.15
time (sec)	N/A	0.283	0.210	0.598	0.113	0.091	0.000	0.611	0.155	15.290

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	345	271	452	0	0	0	0	3341	24910
N.S.	1	1.09	0.86	1.43	0.00	0.00	0.00	0.00	10.57	78.83
time (sec)	N/A	0.886	0.782	1.046	0.000	0.000	0.000	0.000	5.833	70.997

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	462	260	707	0	0	0	631	10007	21612
N.S.	1	1.04	0.59	1.59	0.00	0.00	0.00	1.42	22.54	48.68
time (sec)	N/A	1.057	0.800	1.050	0.000	0.000	0.000	0.459	13.029	83.799

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	441	326	1278	0	0	0	1592	20805	0
N.S.	1	1.13	0.84	3.28	0.00	0.00	0.00	4.08	53.35	0.00
time (sec)	N/A	1.017	1.429	1.050	0.000	0.000	0.000	0.741	9.974	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	482	283	390	471	700	0	571	622	4167
N.S.	1	1.04	0.61	0.84	1.01	1.51	0.00	1.23	1.34	8.96
time (sec)	N/A	1.283	0.601	0.766	0.121	0.121	0.000	0.263	0.168	95.584

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	362	200	270	317	482	0	363	399	2799
N.S.	1	1.08	0.60	0.81	0.95	1.44	0.00	1.08	1.19	8.36
time (sec)	N/A	0.967	0.420	0.746	0.123	0.110	0.000	0.240	0.162	49.721

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	248	128	175	189	302	0	191	209	1011
N.S.	1	1.22	0.63	0.86	0.93	1.49	0.00	0.94	1.03	4.98
time (sec)	N/A	0.617	0.231	0.678	0.112	0.104	0.000	0.222	0.153	19.951

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	116	90	126	88	196	0	106	93	489
N.S.	1	0.93	0.72	1.01	0.70	1.57	0.00	0.85	0.74	3.91
time (sec)	N/A	0.269	0.137	0.682	0.108	0.096	0.000	0.202	0.147	11.521

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	233	178	299	0	0	0	0	3194	9298
N.S.	1	1.25	0.96	1.61	0.00	0.00	0.00	0.00	17.17	49.99
time (sec)	N/A	0.671	0.405	1.220	0.000	0.000	0.000	0.000	0.768	28.702

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	308	229	1166	0	0	0	526	8629	106511
N.S.	1	1.31	0.97	4.96	0.00	0.00	0.00	2.24	36.72	453.24
time (sec)	N/A	0.772	0.708	1.234	0.000	0.000	0.000	0.304	0.928	121.924

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	357	252	1794	0	1355	0	1425	15212	9344
N.S.	1	1.19	0.84	5.96	0.00	4.50	0.00	4.73	50.54	31.04
time (sec)	N/A	0.829	0.991	1.321	0.000	38.127	0.000	0.636	3.089	67.450

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	230	185	1095	0	1186	0	627	3120	7235
N.S.	1	1.10	0.88	5.21	0.00	5.65	0.00	2.99	14.86	34.45
time (sec)	N/A	0.547	0.547	0.942	0.000	0.128	0.000	0.363	0.229	46.166

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	19	20	19	19	19	22	19	19
N.S.	1	1.00	0.76	0.80	0.76	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.224	0.010	0.476	0.025	0.075	0.063	0.128	0.147	0.042

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	141	170	0	0	170	300	166
N.S.	1	1.00	1.00	1.00	1.21	0.00	0.00	1.21	2.13	1.18
time (sec)	N/A	0.454	0.059	0.853	0.040	0.000	0.000	0.130	0.149	6.856

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	278	253	339	368	517	475	310	279
N.S.	1	1.00	1.09	1.00	1.33	1.45	2.04	1.87	1.22	1.10
time (sec)	N/A	0.569	0.242	1.541	0.036	0.078	1.471	0.128	0.148	0.076

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1353	815	1253	5734	0	3096	0	4456	34	0
N.S.	1	0.60	0.93	4.24	0.00	2.29	0.00	3.29	0.03	0.00
time (sec)	N/A	1.355	5.964	0.623	0.000	1.312	0.000	0.709	200.021	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	715	461	662	3025	0	1620	0	2481	32	0
N.S.	1	0.64	0.93	4.23	0.00	2.27	0.00	3.47	0.04	0.00
time (sec)	N/A	0.721	2.586	0.571	0.000	0.413	0.000	0.452	200.023	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	249	283	1207	0	840	0	1035	1024	0
N.S.	1	0.75	0.86	3.66	0.00	2.55	0.00	3.14	3.10	0.00
time (sec)	N/A	0.383	0.879	0.535	0.000	0.137	0.000	0.274	2.861	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	481	404	3898	0	0	0	0	34	0
N.S.	1	1.06	0.89	8.60	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.297	1.482	0.845	0.000	0.000	0.000	0.000	200.024	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	546	358	4680	0	0	0	1507	34	0
N.S.	1	1.04	0.68	8.93	0.00	0.00	0.00	2.88	0.06	0.00
time (sec)	N/A	1.329	2.052	0.844	0.000	0.000	0.000	1.388	200.023	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	695	536	11204	0	0	0	8241	34	0
N.S.	1	1.05	0.81	16.98	0.00	0.00	0.00	12.49	0.05	0.00
time (sec)	N/A	1.643	4.622	0.882	0.000	0.000	0.000	4.816	200.026	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1035	778	3220	3958	0	2176	0	1509	34	0
N.S.	1	0.75	3.11	3.82	0.00	2.10	0.00	1.46	0.03	0.00
time (sec)	N/A	1.331	16.908	0.635	0.000	2.491	0.000	0.270	200.022	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	535	419	474	2002	0	1114	0	733	1525	0
N.S.	1	0.78	0.89	3.74	0.00	2.08	0.00	1.37	2.85	0.00
time (sec)	N/A	0.696	9.424	0.546	0.000	0.592	0.000	0.193	47.982	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	208	217	763	0	576	0	313	648	1832
N.S.	1	0.85	0.88	3.10	0.00	2.34	0.00	1.27	2.63	7.45
time (sec)	N/A	0.349	3.551	0.533	0.000	0.201	0.000	0.159	0.194	61.137

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	312	465	1822	0	0	0	0	34	0
N.S.	1	1.07	1.59	6.24	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.789	12.036	0.923	0.000	0.000	0.000	0.000	200.018	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	396	417	3670	0	0	0	1354	34	0
N.S.	1	1.09	1.15	10.08	0.00	0.00	0.00	3.72	0.09	0.00
time (sec)	N/A	0.944	11.353	0.977	0.000	0.000	0.000	1.234	200.027	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	533	523	9100	0	0	0	7922	34	0
N.S.	1	1.20	1.17	20.40	0.00	0.00	0.00	17.76	0.08	0.00
time (sec)	N/A	1.125	13.113	1.010	0.000	0.000	0.000	16.992	200.016	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	719	657	15990	0	0	0	25338	34	0
N.S.	1	1.05	0.96	23.34	0.00	0.00	0.00	36.99	0.05	0.00
time (sec)	N/A	1.724	15.142	1.102	0.000	0.000	0.000	79.236	200.028	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	735	632	2528	0	1436	0	946	34	0
N.S.	1	1.02	0.87	3.50	0.00	1.99	0.00	1.31	0.05	0.00
time (sec)	N/A	1.260	2.516	0.758	0.000	1.103	0.000	0.210	200.025	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	381	314	1199	0	720	0	441	932	2621
N.S.	1	1.04	0.86	3.28	0.00	1.97	0.00	1.21	2.55	7.18
time (sec)	N/A	0.680	0.933	0.648	0.000	0.306	0.000	0.166	0.205	70.691

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	167	141	425	0	380	0	190	359	833
N.S.	1	1.02	0.86	2.59	0.00	2.32	0.00	1.16	2.19	5.08
time (sec)	N/A	0.316	0.327	0.685	0.000	0.155	0.000	0.146	0.149	18.246

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	204	183	746	0	0	0	0	88	96118
N.S.	1	1.09	0.97	3.97	0.00	0.00	0.00	0.00	0.47	511.27
time (sec)	N/A	0.528	0.498	1.062	0.000	0.000	0.000	0.000	1.345	159.396

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	299	249	2973	0	0	0	1319	5671	0
N.S.	1	1.18	0.98	11.70	0.00	0.00	0.00	5.19	22.33	0.00
time (sec)	N/A	0.626	1.086	0.967	0.000	0.000	0.000	0.868	1.686	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	445	420	7119	0	4058	0	7939	34	0
N.S.	1	1.10	1.04	17.67	0.00	10.07	0.00	19.70	0.08	0.00
time (sec)	N/A	0.946	2.438	1.119	0.000	157.369	0.000	33.364	200.025	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	770	869	1036	18802	0	0	0	25632	34	0
N.S.	1	1.13	1.35	24.42	0.00	0.00	0.00	33.29	0.04	0.00
time (sec)	N/A	1.760	9.260	1.368	0.000	0.000	0.000	78.859	200.018	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1183	1213	1422	2077	0	1916	0	0	34	0
N.S.	1	1.03	1.20	1.76	0.00	1.62	0.00	0.00	0.03	0.00
time (sec)	N/A	2.829	33.717	3.129	0.000	0.158	0.000	0.000	200.019	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	762	789	917	1205	0	1393	0	0	34	0
N.S.	1	1.04	1.20	1.58	0.00	1.83	0.00	0.00	0.04	0.00
time (sec)	N/A	1.791	29.022	4.142	0.000	0.142	0.000	0.000	200.025	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	708	729	633	1163	0	1463	0	0	34	0
N.S.	1	1.03	0.89	1.64	0.00	2.07	0.00	0.00	0.05	0.00
time (sec)	N/A	1.528	26.108	4.939	0.000	0.159	0.000	0.000	200.021	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	697	815	1378	0	2588	0	0	34	0
N.S.	1	1.01	1.19	2.01	0.00	3.77	0.00	0.00	0.05	0.00
time (sec)	N/A	1.480	29.223	6.862	0.000	0.271	0.000	0.000	200.027	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	961	996	1444	2292	0	4721	0	0	34	0
N.S.	1	1.04	1.50	2.39	0.00	4.91	0.00	0.00	0.04	0.00
time (sec)	N/A	2.282	33.932	8.046	0.000	0.846	0.000	0.000	200.024	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1714	1770	15963	3900	0	9150	0	0	34	0
N.S.	1	1.03	9.31	2.28	0.00	5.34	0.00	0.00	0.02	0.00
time (sec)	N/A	4.404	36.985	8.695	0.000	2.801	0.000	0.000	200.018	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1233	1268	1470	2108	0	1931	0	0	34	0
N.S.	1	1.03	1.19	1.71	0.00	1.57	0.00	0.00	0.03	0.00
time (sec)	N/A	3.130	34.507	7.332	0.000	0.154	0.000	0.000	200.018	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	792	922	1205	0	1392	0	0	34	0
N.S.	1	1.03	1.20	1.57	0.00	1.81	0.00	0.00	0.04	0.00
time (sec)	N/A	1.678	29.063	3.990	0.000	0.179	0.000	0.000	200.018	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	545	562	812	0	1036	0	0	0	0
N.S.	1	1.03	1.06	1.53	0.00	1.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.983	27.042	4.427	0.000	0.124	0.000	0.000	108.086	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	540	555	551	861	0	1336	0	0	34	0
N.S.	1	1.03	1.02	1.59	0.00	2.47	0.00	0.00	0.06	0.00
time (sec)	N/A	1.092	25.117	6.114	0.000	0.147	0.000	0.000	200.028	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	630	724	1269	0	2429	0	0	34	0
N.S.	1	1.06	1.21	2.13	0.00	4.08	0.00	0.00	0.06	0.00
time (sec)	N/A	1.236	28.269	8.326	0.000	0.311	0.000	0.000	200.026	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1034	1073	1449	2330	0	4867	0	0	34	0
N.S.	1	1.04	1.40	2.25	0.00	4.71	0.00	0.00	0.03	0.00
time (sec)	N/A	2.552	33.577	21.716	0.000	0.961	0.000	0.000	200.028	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	825	857	1000	1233	0	1388	0	0	34	0
N.S.	1	1.04	1.21	1.49	0.00	1.68	0.00	0.00	0.04	0.00
time (sec)	N/A	1.698	29.492	6.516	0.000	0.139	0.000	0.000	200.018	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	542	615	812	0	1036	0	0	0	0
N.S.	1	1.03	1.17	1.55	0.00	1.98	0.00	0.00	0.00	0.00
time (sec)	N/A	0.992	25.859	4.455	0.000	0.132	0.000	0.000	109.348	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	394	418	615	0	807	0	0	0	0
N.S.	1	1.03	1.09	1.60	0.00	2.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.676	24.575	5.247	0.000	0.126	0.000	0.000	32.353	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	445	477	784	0	1240	0	0	0	0
N.S.	1	1.05	1.13	1.86	0.00	2.94	0.00	0.00	0.00	0.00
time (sec)	N/A	0.779	24.178	7.748	0.000	0.141	0.000	0.000	38.619	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	672	699	1249	0	2344	0	0	0	0
N.S.	1	1.05	1.09	1.95	0.00	3.65	0.00	0.00	0.00	0.00
time (sec)	N/A	1.369	27.496	21.469	0.000	0.284	0.000	0.000	141.002	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1116	1176	1520	2283	0	5108	0	0	34	0
N.S.	1	1.05	1.36	2.05	0.00	4.58	0.00	0.00	0.03	0.00
time (sec)	N/A	2.529	33.378	24.109	0.000	1.005	0.000	0.000	200.026	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [83] had the largest ratio of [.3684209999999999998]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	8	1.11	33	0.242
2	A	7	7	1.03	31	0.226
3	A	5	5	1.03	30	0.167
4	A	10	9	1.00	33	0.273
5	A	9	8	1.00	33	0.242
6	A	8	7	1.04	33	0.212
7	A	10	9	1.06	33	0.273
8	A	13	12	1.06	33	0.364
9	A	11	10	1.10	32	0.312
10	A	9	8	1.26	30	0.267
11	A	4	4	0.91	29	0.138
12	A	10	9	1.62	32	0.281
13	A	9	8	1.62	32	0.250
14	A	7	6	1.23	32	0.188
15	A	8	7	1.15	32	0.219
16	A	10	9	1.09	32	0.281
17	A	5	4	1.11	35	0.114
18	A	5	4	1.17	35	0.114
19	A	5	4	1.28	33	0.121
20	A	7	6	1.07	35	0.171
21	A	8	7	1.51	35	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
22	A	9	8	1.43	35	0.229
23	A	9	8	1.26	35	0.229
24	A	8	7	0.89	35	0.200
25	A	7	6	1.28	32	0.188
26	A	8	7	1.34	35	0.200
27	A	7	6	1.35	35	0.171
28	A	5	5	1.26	35	0.143
29	A	6	6	1.08	35	0.171
30	A	7	7	0.99	35	0.200
31	A	11	11	0.95	37	0.297
32	A	9	9	0.95	37	0.243
33	A	7	7	1.03	35	0.200
34	A	6	6	0.96	30	0.200
35	A	11	10	1.11	37	0.270
36	A	9	8	1.11	37	0.216
37	A	9	8	1.06	37	0.216
38	A	11	11	1.08	37	0.297
39	A	9	9	1.07	37	0.243
40	A	6	6	1.10	35	0.171
41	A	5	5	1.03	30	0.167
42	A	9	8	1.04	37	0.216
43	A	7	6	1.13	37	0.162
44	A	6	5	1.06	37	0.135
45	A	14	13	0.92	40	0.325
46	A	11	10	0.96	40	0.250
47	A	9	8	1.11	38	0.211
48	A	7	6	0.90	33	0.182
49	A	12	11	1.09	40	0.275
50	A	11	10	1.04	40	0.250
51	A	11	10	1.13	40	0.250
52	A	14	13	1.04	40	0.325
53	A	11	10	1.08	40	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	8	7	1.22	38	0.184
55	A	6	5	0.93	33	0.152
56	A	10	9	1.25	40	0.225
57	A	9	8	1.31	40	0.200
58	A	7	6	1.19	40	0.150
59	A	7	6	1.10	32	0.188
60	A	2	2	1.00	24	0.083
61	A	2	2	1.00	32	0.062
62	A	2	2	1.00	30	0.067
63	A	10	9	0.60	36	0.250
64	A	8	7	0.64	34	0.206
65	A	8	7	0.75	29	0.241
66	A	11	10	1.06	36	0.278
67	A	11	10	1.04	36	0.278
68	A	11	10	1.05	36	0.278
69	A	9	8	0.75	36	0.222
70	A	7	6	0.78	34	0.176
71	A	7	6	0.85	29	0.207
72	A	9	8	1.07	36	0.222
73	A	9	8	1.09	36	0.222
74	A	9	8	1.20	36	0.222
75	A	9	8	1.05	36	0.222
76	A	8	7	1.02	36	0.194
77	A	6	5	1.04	34	0.147
78	A	6	5	1.02	29	0.172
79	A	7	6	1.09	36	0.167
80	A	7	6	1.18	36	0.167
81	A	7	6	1.10	36	0.167
82	A	9	8	1.13	36	0.222
83	A	14	14	1.03	38	0.368
84	A	12	12	1.04	38	0.316
85	A	12	12	1.03	38	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	12	12	1.01	38	0.316
87	A	12	12	1.04	38	0.316
88	A	14	14	1.03	38	0.368
89	A	14	14	1.03	38	0.368
90	A	12	12	1.03	38	0.316
91	A	10	10	1.03	38	0.263
92	A	10	10	1.03	38	0.263
93	A	10	10	1.06	38	0.263
94	A	12	12	1.04	38	0.316
95	A	12	12	1.04	38	0.316
96	A	10	10	1.03	38	0.263
97	A	8	8	1.03	38	0.211
98	A	8	8	1.05	38	0.211
99	A	10	10	1.05	38	0.263
100	A	12	12	1.05	38	0.316

CHAPTER 3

Listing of integrals

3.1 $\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 121

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{b\sqrt{1-d^2x^2}}{d^4} - \frac{(3c+4ad^2)\,x\sqrt{1-d^2x^2}}{8d^4} - \frac{cx^3\sqrt{1-d^2x^2}}{4d^2} + \frac{b(1-d^2x^2)^{3/2}}{3d^4} + \frac{(3c+4ad^2)\arcsin(dx)}{8d^5}$$

output
```
-b*(-d^2*x^2+1)^(1/2)/d^4-1/8*(4*a*d^2+3*c)*x*(-d^2*x^2+1)^(1/2)/d^4-1/4*c
*x^3*(-d^2*x^2+1)^(1/2)/d^2+1/3*b*(-d^2*x^2+1)^(3/2)/d^4+1/8*(4*a*d^2+3*c)
*arcsin(d*x)/d^5
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{1-d^2x^2}(-16b-9cx-12ad^2x-8bd^2x^2-6cd^2x^3)}{24d^4} + \frac{(3c+4ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{4d^5}$$

input
```
Integrate[(x^2*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(-16*b - 9*c*x - 12*a*d^2*x - 8*b*d^2*x^2 - 6*c*d^2*x^3
))/(24*d^4) + ((3*c + 4*a*d^2)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(4*
d^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2112, 2340, 25, 533, 27, 533, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2340}$$

$$-\frac{\int -\frac{x^2(4ad^2+4bxd^2+3c)}{\sqrt{1-d^2x^2}}dx}{4d^2}-\frac{cx^3\sqrt{1-d^2x^2}}{4d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2(4ad^2+4bxd^2+3c)}{\sqrt{1-d^2x^2}}dx}{4d^2}-\frac{cx^3\sqrt{1-d^2x^2}}{4d^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{\int \frac{d^2x(8b+3(4ad^2+3c)x)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{4}{3}bx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{cx^3\sqrt{1-d^2x^2}}{4d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}\int \frac{x(8b+3(4ad^2+3c)x)}{\sqrt{1-d^2x^2}}dx-\frac{4}{3}bx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{cx^3\sqrt{1-d^2x^2}}{4d^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{1}{3}\left(\frac{\int \frac{16bxd^2+3\left(4ad^2+3c\right)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{3}{2}x\sqrt{1-d^2x^2}\left(4a+\frac{3c}{d^2}\right)\right)-\frac{4}{3}bx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{cx^3\sqrt{1-d^2x^2}}{4d^2}$$

$\Big\downarrow$ 455

$$\frac{\frac{1}{3}\left(\frac{3\left(4ad^2+3c\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx-16b\sqrt{1-d^2x^2}}{2d^2}-\frac{3}{2}x\sqrt{1-d^2x^2}\left(4a+\frac{3c}{d^2}\right)\right)-\frac{4}{3}bx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{cx^3\sqrt{1-d^2x^2}}{4d^2}$$

$\Big\downarrow$ 223

$$\frac{\frac{1}{3}\left(\frac{\frac{3\left(4ad^2+3c\right)\arcsin(dx)}{d}-16b\sqrt{1-d^2x^2}}{2d^2}-\frac{3}{2}x\sqrt{1-d^2x^2}\left(4a+\frac{3c}{d^2}\right)\right)-\frac{4}{3}bx^2\sqrt{1-d^2x^2}}{4d^2}-\frac{cx^3\sqrt{1-d^2x^2}}{4d^2}$$

input
```
Int[(x^2*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/4*(c*x^3*Sqrt[1 - d^2*x^2])/d^2 + ((-4*b*x^2*Sqrt[1 - d^2*x^2])/3 + ((-
3*(4*a + (3*c)/d^2)*x*Sqrt[1 - d^2*x^2])/2 + (-16*b*Sqrt[1 - d^2*x^2] + (3
*(3*c + 4*a*d^2)*ArcSin[d*x])/d)/(2*d^2))/3)/(4*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.30

method	result
risch	$\frac{(6c\,x^3d^2+8b\,d^2x^2+12xa\,d^2+9cx+16b)\sqrt{xd+1}\,(xd-1)\sqrt{(-xd+1)(xd+1)}}{24d^4\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}+\frac{(4a\,d^2+3c)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-xd+1)(xd+1)}}{8d^4\sqrt{d^2}\,\sqrt{-xd+1}\,\sqrt{xd+1}}$
default	$-\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\left(6\,\mathrm{csgn}(d)c\,d^3x^3\sqrt{-d^2x^2+1}+8\,\mathrm{csgn}(d)b\,d^3x^2\sqrt{-d^2x^2+1}+12\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^3ax+9\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)\right.}{24d^5\sqrt{-d^2x^2+1}}$

input
```
int(x^2*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/24*(6*c*d^2*x^3+8*b*d^2*x^2+12*a*d^2*x+9*c*x+16*b)*(d*x+1)^(1/2)*(d*x-1)
/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8*
(4*a*d^2+3*c)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-
d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(6\,cd^3x^3+8\,bd^3x^2+16\,bd+3\,(4\,ad^3+3\,cd)x)\sqrt{dx+1}\sqrt{-dx+1}+6\,(4\,ad^2+3\,c)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx}}{dx}\right.}{24\,d^5}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/24*((6*c*d^3*x^3 + 8*b*d^3*x^2 + 16*b*d + 3*(4*a*d^3 + 3*c*d)*x)*sqrt(d
*x + 1)*sqrt(-d*x + 1) + 6*(4*a*d^2 + 3*c)*arctan((sqrt(d*x + 1)*sqrt(-d*x
 + 1) - 1)/(d*x)))/d^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}cx^3}{4\,d^2} - \frac{\sqrt{-d^2x^2+1}bx^2}{3\,d^2} - \frac{\sqrt{-d^2x^2+1}ax}{2\,d^2} + \frac{a\arcsin{(dx)}}{2\,d^3} - \frac{3\,\sqrt{-d^2x^2+1}cx}{8\,d^4} - \frac{2\,\sqrt{-d^2x^2+1}b}{3\,d^4} + \frac{3\,c\arcsin{(dx)}}{8\,d^5}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="max
ima")
```

output
```
-1/4*sqrt(-d^2*x^2 + 1)*c*x^3/d^2 - 1/3*sqrt(-d^2*x^2 + 1)*b*x^2/d^2 - 1/2
*sqrt(-d^2*x^2 + 1)*a*x/d^2 + 1/2*a*arcsin(d*x)/d^3 - 3/8*sqrt(-d^2*x^2 +
1)*c*x/d^4 - 2/3*sqrt(-d^2*x^2 + 1)*b/d^4 + 3/8*c*arcsin(d*x)/d^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(12\,ad^2 - (12\,ad^2 + 2\,(3\,(dx+1)c + 4\,bd - 9\,c)(dx+1) - 16\,bd + 27\,c)(dx+1) - 24\,bd + 15\,c)\sqrt{dx+1}}{24\,d^5}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="gia
c")
```

output
```
1/24*((12*a*d^2 - (12*a*d^2 + 2*(3*(d*x + 1)*c + 4*b*d - 9*c)*(d*x + 1) -
16*b*d + 27*c)*(d*x + 1) - 24*b*d + 15*c)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6
*(4*a*d^2 + 3*c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^5
```

Mupad [B] (verification not implemented)

Time = 10.64 (sec) , antiderivative size = 485, normalized size of antiderivative = 4.01

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$

$$= \frac{\frac{23\,c\,\left(\sqrt{1-d\,x}-1\right)^3}{2\,\left(\sqrt{d\,x+1}-1\right)^3}-\frac{333\,c\,\left(\sqrt{1-d\,x}-1\right)^5}{2\,\left(\sqrt{d\,x+1}-1\right)^5}+\frac{671\,c\,\left(\sqrt{1-d\,x}-1\right)^7}{2\,\left(\sqrt{d\,x+1}-1\right)^7}-\frac{671\,c\,\left(\sqrt{1-d\,x}-1\right)^9}{2\,\left(\sqrt{d\,x+1}-1\right)^9}+\frac{333\,c\,\left(\sqrt{1-d\,x}-1\right)^{11}}{2\,\left(\sqrt{d\,x+1}-1\right)^{11}}-\frac{23\,c\,\left(\sqrt{1-d\,x}-1\right)^{13}}{2\,\left(\sqrt{d\,x+1}-1\right)^{13}}}{d^5\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^8}$$

$$-\frac{3\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2\,d^5}-\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}$$

$$-\frac{\frac{14\,a\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,a\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,a\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}-\frac{2\,a\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}$$

$$-\frac{\sqrt{1-d\,x}\left(\frac{2\,b}{3\,d^4}+\frac{b\,x^3}{3\,d}+\frac{b\,x^2}{3\,d^2}+\frac{2\,b\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}$$

input
```
int((x^2*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((23*c*((1 - d*x)^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) - (333*c*((1 -
 d*x)^(1/2) - 1)^5)/(2*((d*x + 1)^(1/2) - 1)^5) + (671*c*((1 - d*x)^(1/2)
- 1)^7)/(2*((d*x + 1)^(1/2) - 1)^7) - (671*c*((1 - d*x)^(1/2) - 1)^9)/(2*(
(d*x + 1)^(1/2) - 1)^9) + (333*c*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(
1/2) - 1)^11) - (23*c*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^1
3) - (3*c*((1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15) + (3*c*((
1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)))/(d^5*(((1 - d*x)^(1/2) - 1
)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (3*c*atan(((1 - d*x)^(1/2) - 1)/((d*
x + 1)^(1/2) - 1)))/(2*d^5) - (2*a*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(
1/2) - 1)))/d^3 - ((14*a*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3
- (14*a*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*a*((1 - d*x)
^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*a*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 +
1)^4) - ((1 - d*x)^(1/2)*((2*b)/(3*d^4) + (b*x^3)/(3*d) + (b*x^2)/(3*d^2)
+ (2*b*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.16

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-24asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)a\,d^2-18asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)c-12\sqrt{dx+1}\,\sqrt{-dx+1}\,a\,d^3x-8\sqrt{dx+1}\,\sqrt{-dx+1}\,b\,d^3x}{24d^5}$$

input
```
int(x^2*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 24*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**2 - 18*asin(sqrt( - d*x + 1)/sq
rt(2))*c - 12*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**3*x - 8*sqrt(d*x + 1)*sq
rt( - d*x + 1)*b*d**3*x**2 - 16*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d - 6*sqr
t(d*x + 1)*sqrt( - d*x + 1)*c*d**3*x**3 - 9*sqrt(d*x + 1)*sqrt( - d*x + 1)
*c*d*x)/(24*d**5)
```

3.2 $\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 92

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(2c+3ad^2)\sqrt{1-d^2x^2}}{3d^4} - \frac{bx\sqrt{1-d^2x^2}}{2d^2} - \frac{cx^2\sqrt{1-d^2x^2}}{3d^2} + \frac{b\arcsin(dx)}{2d^3}$$

output
```
-1/3*(3*a*d^2+2*c)*(-d^2*x^2+1)^(1/2)/d^4-1/2*b*x*(-d^2*x^2+1)^(1/2)/d^2-1
/3*c*x^2*(-d^2*x^2+1)^(1/2)/d^2+1/2*b*arcsin(d*x)/d^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{1-d^2x^2}(-4c-6ad^2-3bd^2x-2cd^2x^2)}{6d^4} + \frac{b\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(-4*c - 6*a*d^2 - 3*b*d^2*x - 2*c*d^2*x^2))/(6*d^4) + (
b*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d^3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {2112, 2340, 25, 533, 27, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2340}$$

$$-\frac{\int -\frac{x(3ad^2+3bxd^2+2c)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x(3ad^2+3bxd^2+2c)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{533}$$

$$\frac{\frac{\int \frac{d^2(3b+2(3ad^2+2c)x)}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{3}{2}bx\sqrt{1-d^2x^2}}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{3b+2(3ad^2+2c)x}{\sqrt{1-d^2x^2}}dx-\frac{3}{2}bx\sqrt{1-d^2x^2}}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{1}{2}\left(3b\int \frac{1}{\sqrt{1-d^2x^2}}dx-2\sqrt{1-d^2x^2}\left(3a+\frac{2c}{d^2}\right)\right)-\frac{3}{2}bx\sqrt{1-d^2x^2}}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{1}{2}\left(\frac{3b\arcsin(dx)}{d}-2\sqrt{1-d^2x^2}\left(3a+\frac{2c}{d^2}\right)\right)-\frac{3}{2}bx\sqrt{1-d^2x^2}}{3d^2}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

input
```
Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/3*(c*x^2*Sqrt[1 - d^2*x^2])/d^2 + ((-3*b*x*Sqrt[1 - d^2*x^2])/2 + (-2*(
3*a + (2*c)/d^2)*Sqrt[1 - d^2*x^2] + (3*b*ArcSin[d*x])/d)/2)/(3*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.51

method	result
default	$-\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\left(2\,\mathrm{csgn}(d) c\,d^{2} x^{2} \sqrt{-d^{2} x^{2}+1}+3 \sqrt{-d^{2} x^{2}+1}\,\mathrm{csgn}(d) b\,d^{2} x+6 \sqrt{-d^{2} x^{2}+1}\,\mathrm{csgn}(d) a\,d^{2}+4 \sqrt{-d^{2} x^{2}+1}\,\mathrm{csgn}(d) c-\right.}{6 d^{4} \sqrt{-d^{2} x^{2}+1}}$
risch	$\frac{\left(2 c\,d^{2} x^{2}+3 b\,d^{2} x+6 a\,d^{2}+4 c\right) \sqrt{xd+1}\,\left(xd-1\right) \sqrt{\left(-xd+1\right)\left(xd+1\right)}}{6 d^{4} \sqrt{-\left(xd+1\right)\left(xd-1\right)}\,\sqrt{-xd+1}}+\frac{b \arctan\left(\frac{\sqrt{d^{2}}\,x}{\sqrt{-d^{2} x^{2}+1}}\right) \sqrt{\left(-xd+1\right)\left(xd+1\right)}}{2 d^{2} \sqrt{d^{2}}\,\sqrt{-xd+1}\,\sqrt{xd+1}}$

input
```
int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*c*d^2*x^2*(-d^2*x^2+1)^(1/2)+
3*(-d^2*x^2+1)^(1/2)*csgn(d)*b*d^2*x+6*(-d^2*x^2+1)^(1/2)*csgn(d)*a*d^2+4*
(-d^2*x^2+1)^(1/2)*csgn(d)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)
*csgn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)+(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

output
```
-1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^
2 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}cx^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}bx}{2\,d^2} -\frac{\sqrt{-d^2x^2+1}a}{d^2}+\frac{b\arcsin(dx)}{2\,d^3}-\frac{2\,\sqrt{-d^2x^2+1}c}{3\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(
-d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{6\,bd\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)-(6\,ad^2+(2\,(dx+1)c+3\,bd-4\,c)(dx+1)-3\,bd+6\,c)\sqrt{dx+1}\sqrt{-dx+}}{6\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

output
```
1/6*(6*b*d*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - (6*a*d^2 + (2*(d*x + 1)*c +
 3*b*d - 4*c)*(d*x + 1) - 3*b*d + 6*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

Mupad [B] (verification not implemented)

Time = 6.93 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.65

$$\begin{aligned}\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= -\frac{\sqrt{1-d\,x}\,\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}\\ &\quad-\frac{\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}\\ &\quad-\frac{\sqrt{1-d\,x}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}\end{aligned}$$

input
```
int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*
x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3
)/((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2
) - 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((
1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/
((d*x + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3
)/(3*d) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{-6asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)bd-6\sqrt{dx+1}\,\sqrt{-dx+1}\,a\,d^2-3\sqrt{dx+1}\,\sqrt{-dx+1}\,b\,d^2x-2\sqrt{dx+1}\,\sqrt{-dx+1}\,c\,d}{6d^4}$$

input
```
int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 6*asin(sqrt( - d*x + 1)/sqrt(2))*b*d - 6*sqrt(d*x + 1)*sqrt( - d*x + 1
)*a*d**2 - 3*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**2*x - 2*sqrt(d*x + 1)*sqr
t( - d*x + 1)*c*d**2*x**2 - 4*sqrt(d*x + 1)*sqrt( - d*x + 1)*c)/(6*d**4)
```

3.3 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{(c+2ad^2)\arcsin(dx)}{2d^3}$$

output
```
-b*(-d^2*x^2+1)^(1/2)/d^2-1/2*c*x*(-d^2*x^2+1)^(1/2)/d^2+1/2*(2*a*d^2+c)*a
rcsin(d*x)/d^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(-2b-cx)\sqrt{1-d^2x^2}}{2d^2} + \frac{(c+2ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
((-2*b - c*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*d^2)*ArcTan[(d*x)/(-1
 + Sqrt[1 - d^2*x^2])])/d^3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1188, 2346, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{1188}$$

$$\int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\frac{2ad^2+2bxd^2+c}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2ad^2+2bxd^2+c}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{(2ad^2+c)\int \frac{1}{\sqrt{1-d^2x^2}}dx-2b\sqrt{1-d^2x^2}}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{(2ad^2+c)\arcsin(dx)}{d}-2b\sqrt{1-d^2x^2}}{2d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/2*(c*x*Sqrt[1 - d^2*x^2])/d^2 + (-2*b*Sqrt[1 - d^2*x^2] + ((c + 2*a*d^2
)*ArcSin[d*x])/d)/(2*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$-\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\left(\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)dcx-2\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)a\,d^2+2\,\operatorname{csgn}(d)d\sqrt{-d^2x^2+1}\,b-\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)c\right)\operatorname{csgn}}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(cx+2b)\sqrt{xd+1}\,(xd-1)\sqrt{(-xd+1)(xd+1)}}{2d^2\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}+\frac{(2a\,d^2+c)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-xd+1)(xd+1)}}{2d^2\sqrt{d^2}\,\sqrt{-xd+1}\,\sqrt{xd+1}}$

input
```
int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*((-d^2*x^2+1)^(1/2)*csgn(d)*d*c*x-2*
arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2
)*b-arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,ad^2+c)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arcta
n((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{a\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}b}{d^2} + \frac{c\arcsin(dx)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

output
```
a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^
2 + 1/2*c*arcsin(d*x)/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{((dx+1)c+2\,bd-c)\sqrt{dx+1}\sqrt{-dx+1}-2\,(2\,ad^2+c)\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{dx+1}\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(((d*x + 1)*c + 2*b*d - c)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*a*d^2
+ c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= -\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,c\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3} - \frac{14\,c\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5} + \frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7} - \frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}$$

input
```
int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
 d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c
*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x
)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/(
(d*x + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) -
1)^7 - (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)
^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-4asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)a\,d^2 - 2asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)c - 2\sqrt{dx+1}\,\sqrt{-dx+1}\,bd - \sqrt{dx+1}\,\sqrt{-dx+1}\,cdx}{2d^3}$$

input
```
int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output

```
( - 4*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**2 - 2*asin(sqrt( - d*x + 1)/sqrt
(2))*c - 2*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d - sqrt(d*x + 1)*sqrt( - d*x
+ 1)*c*d*x)/(2*d**3)
```

3.4 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 48

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\arcsin(dx)}{d}-a\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

output
```
-c*(-d^2*x^2+1)^(1/2)/d^2+b*arcsin(d*x)/d-a*arctanh((-d^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{2b\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d} - a\log(x)+a\log\left(-1+\sqrt{1-d^2x^2}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-((c*Sqrt[1 - d^2*x^2])/d^2) + (2*b*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])]
)/d - a*Log[x] + a*Log[-1 + Sqrt[1 - d^2*x^2]]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2112, 2340, 25, 27, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{ 2340}$$

$$-\frac{\int -\frac{d^2(a+bx)}{x\sqrt{1-d^2x^2}}dx}{d^2}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d^2(a+bx)}{x\sqrt{1-d^2x^2}}dx}{d^2}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{a+bx}{x\sqrt{1-d^2x^2}}dx-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 538}$$

$$a\int \frac{1}{x\sqrt{1-d^2x^2}}dx+b\int \frac{1}{\sqrt{1-d^2x^2}}dx-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 223}$$

$$a\int \frac{1}{x\sqrt{1-d^2x^2}}dx+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{1-d^2x^2}}dx^2+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{a\int \frac{1}{\frac{1}{d^2}-\frac{x^4}{d^2}}d\sqrt{1-d^2x^2}}{d^2}+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

$$\Big\downarrow \text{221}$$

$$-a\text{arctanh}\left(\sqrt{1-d^2x^2}\right)+\frac{b\arcsin(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

input
```
Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*
x^2]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\left(-\operatorname{csgn}(d)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^2-\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)c+\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-(xd+1)(xd-1)}}\right)bd\right)\operatorname{csgn}(d)}{d^2\sqrt{-d^2x^2+1}}$	96

input
```
int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^2*(-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a
*d^2-(-d^2*x^2+1)^(1/2)*csgn(d)*c+arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1
/2))*b*d)*csgn(d)/(-d^2*x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{ad^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\sqrt{-dx+1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

output
```
(a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x
+ 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.62 (sec) , antiderivative size = 245, normalized size of antiderivative = 5.10

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

input
```
integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()),
((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/
2)) - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2
))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1
/4, 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((
-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp
_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = -a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

output
```
-a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d
^2*x^2 + 1)*c/d^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(44) = 88.

Time = 0.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 4.08

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{ad^2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-ad^2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-2\right|\right)-\left(\pi+2\,\mathrm{ar}\right.}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

output
```
-(a*d^2*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/
(sqrt(2) - sqrt(-d*x + 1)) + 2)) - a*d^2*log(abs(-(sqrt(2) - sqrt(-d*x + 1
))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) - 2)) - (pi +
2*arctan(1/2*sqrt(d*x + 1)*((sqrt(2) - sqrt(-d*x + 1))^2/(d*x + 1) - 1)/(s
qrt(2) - sqrt(-d*x + 1))))*b*d + sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

Mupad [B] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.54

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{\sqrt{1-d\,x}\,\left(\frac{c}{d^2}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}} - \frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}$$

input
```
int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*
x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d)
)/(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
- 1)*(d^2)^(1/2))))/(d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.08

$$\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{-2asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)bd-\sqrt{dx+1}\,\sqrt{-dx+1}\,c-\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)}{2}\right)-1\right)a\,d^2+\log\left(-\sqrt{2}-\right.}{}$$

input
```
int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 2*asin(sqrt( - d*x + 1)/sqrt(2))*b*d - sqrt(d*x + 1)*sqrt( - d*x + 1)*
c - log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*a*d**2 + l
og( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*a*d**2 - log(sq
rt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*a*d**2 + log(sqrt(2) +
tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*a*d**2)/d**2
```

3.5 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 48

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}-b\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

output
```
-a*(-d^2*x^2+1)^(1/2)/x+c*arcsin(d*x)/d-b*arctanh((-d^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx=-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{2c\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d}-b\log(x)+b\log\left(-1+\sqrt{1-d^2x^2}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-((a*Sqrt[1 - d^2*x^2])/x) + (2*c*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/
d - b*Log[x] + b*Log[-1 + Sqrt[1 - d^2*x^2]]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {2112, 2338, 25, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\int -\frac{b+cx}{x\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{x}$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{b+cx}{x\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{x}$$

$$\Big\downarrow \text{ 538}$$

$$b\int \frac{1}{x\sqrt{1-d^2x^2}}dx+c\int \frac{1}{\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{x}$$

$$\Big\downarrow \text{ 223}$$

$$b\int \frac{1}{x\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}b\int \frac{1}{x^2\sqrt{1-d^2x^2}}dx^2-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{b\int \frac{1}{\frac{1}{d^2}-\frac{x^4}{d^2}}d\sqrt{1-d^2x^2}}{d^2}-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{a\sqrt{1-d^2x^2}}{x}+\frac{c\arcsin(dx)}{d}-b\text{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

input
```
Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^
2]]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.02

method	result	size
default	$\frac{\left(-\operatorname{csgn}(d) d \operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right) b x-\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d) d a+\arctan\left(\frac{\operatorname{csgn}(d) d x}{\sqrt{-d^2x^2+1}}\right) c x\right)\sqrt{-x d+1}\,\sqrt{x d+1}\,\operatorname{csgn}(d)}{\sqrt{-d^2x^2+1}\,x d}$	97
risch	$\frac{a\sqrt{x d+1}\,(x d-1)\sqrt{(-x d+1)(x d+1)}}{x\sqrt{-(x d+1)(x d-1)}\,\sqrt{-x d+1}}+\frac{\left(\frac{c\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{\sqrt{d^2}}-b\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\right)\sqrt{(-x d+1)(x d+1)}}{\sqrt{-x d+1}\,\sqrt{x d+1}}$	129

input
```
int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
(-csgn(d)*d*arctanh(1/(-d^2*x^2+1)^(1/2))*b*x-(-d^2*x^2+1)^(1/2)*csgn(d)*d
*a+arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*c*x)*(-d*x+1)^(1/2)*(d*x+1)^(1/2
)*csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.75

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-\sqrt{dx+1}\sqrt{-dx+1}ad-2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fri
cas")
```

output
```
(b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
 + 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 28.37 (sec) , antiderivative size = 221, normalized size of antiderivative = 4.60

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{iadG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{adG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

input
```
integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)
), 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
 ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/
2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1,
 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/
(4*pi**(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2,
 3/4, 1, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4,
 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/
(d**2*x**2))/(4*pi**(3/2)*d)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}a}{x}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="max
ima")
```

output
```
-b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d
^2*x^2 + 1)*a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(44) = 88.

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 5.88

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\frac{4ad^2\left(\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}-\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}\right)^2-4}+bd\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-bd\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{\cdots}}{\sqrt{2}-\cdots}\right.\right.}{d}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="gia
c")
```

output
```
-(4*a*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(
2) - sqrt(-d*x + 1)))/(((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*
x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4) + b*d*log(abs(-(sqrt(2) - sqrt(-
d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) -
 b*d*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sq
rt(2) - sqrt(-d*x + 1)) - 2)) - (pi + 2*arctan(1/2*sqrt(d*x + 1)*((sqrt(2)
 - sqrt(-d*x + 1))^2/(d*x + 1) - 1)/(sqrt(2) - sqrt(-d*x + 1))))*c)/d
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{4\,c\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

input
```
int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*
x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1)
)/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(
d*x + 1)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.08

$$\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{-2asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)cx-\sqrt{dx+1}\,\sqrt{-dx+1}\,ad-\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)}{2}\right)-1\right)bdx+\log\left(-\sqrt{2}\right.}{}$$

input
```
int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 2*asin(sqrt( - d*x + 1)/sqrt(2))*c*x - sqrt(d*x + 1)*sqrt( - d*x + 1)*
a*d - log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*b*d*x +
log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*b*d*x - log(sq
rt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*b*d*x + log(sqrt(2) + t
an(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*b*d*x)/(d*x)
```

3.6 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 71

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x} -\frac{1}{2}\left(2c+ad^2\right)\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

output
```
-1/2*a*(-d^2*x^2+1)^(1/2)/x^2-b*(-d^2*x^2+1)^(1/2)/x-1/2*(a*d^2+2*c)*arcta
nh((-d^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{1}{2}\left(-\frac{(a+2bx)\sqrt{1-d^2x^2}}{x^2}-\left(2c+ad^2\right)\log(x) +\left(2c+ad^2\right)\log\left(-1+\sqrt{1-d^2x^2}\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-(((a + 2*b*x)*Sqrt[1 - d^2*x^2])/x^2) - (2*c + a*d^2)*Log[x] + (2*c + a*
d^2)*Log[-1 + Sqrt[1 - d^2*x^2]])/2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2112, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{1}{2}\int -\frac{2b+\left(ad^2+2c\right)x}{x^2\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{2b+\left(ad^2+2c\right)x}{x^2\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{2}\left(\left(ad^2+2c\right)\int \frac{1}{x\sqrt{1-d^2x^2}}dx-\frac{2b\sqrt{1-d^2x^2}}{x}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(ad^2+2c\right)\int \frac{1}{x^2\sqrt{1-d^2x^2}}dx^2-\frac{2b\sqrt{1-d^2x^2}}{x}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\left(ad^2+2c\right)\int \frac{1}{\frac{1}{d^2}-\frac{x^4}{d^2}}d\sqrt{1-d^2x^2}}{d^2}-\frac{2b\sqrt{1-d^2x^2}}{x}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(-\left(ad^2+2c\right)\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)-\frac{2b\sqrt{1-d^2x^2}}{x}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}$$

input
```
Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/2*(a*Sqrt[1 - d^2*x^2])/x^2 + ((-2*b*Sqrt[1 - d^2*x^2])/x - (2*c + a*d^
2)*ArcTanh[Sqrt[1 - d^2*x^2]])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.52

method	result	si
default	$-\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\operatorname{csgn}(d)^{2}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)a\,d^{2}x^{2}+2\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)c\,x^{2}+2\sqrt{-d^{2}x^{2}+1}\,bx+\sqrt{-d^{2}x^{2}+1}\,a\right)}{2\sqrt{-d^{2}x^{2}+1}\,x^{2}}$	1
risch	$\frac{\sqrt{xd+1}\,(xd-1)(2bx+a)\sqrt{(-xd+1)(xd+1)}}{2x^{2}\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}-\frac{\left(c+\frac{a\,d^{2}}{2}\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)\sqrt{(-xd+1)(xd+1)}}{\sqrt{-xd+1}\,\sqrt{xd+1}}$	1

input
```
int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))
*a*d^2*x^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*c*x^2+2*(-d^2*x^2+1)^(1/2)*b*x+
(-d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(ad^2+2\,c)x^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\sqrt{-dx+1}}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x
+ a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,ad^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - c\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - \frac{\sqrt{-d^2x^2+1}b}{x} - \frac{\sqrt{-d^2x^2+1}a}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="max
ima")
```

output
```
-1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2
*x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2
+ 1)*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(61) = 122$.

Time = 0.21 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.73

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{(ad^3+2\,cd)\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-(ad^3+2\,cd)\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-\right.\right.}{}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="gia
c")
```

output
```
-1/2*((a*d^3 + 2*c*d)*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) +
sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) - (a*d^3 + 2*c*d)*log(abs(-
(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-
d*x + 1)) - 2)) - 4*(a*d^3*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqr
t(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 - 2*b*d^2*((sqrt(2) - sqrt(-d*x +
 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 + 4*a*d^3
*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt
(-d*x + 1))) + 8*b*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*
x + 1)/(sqrt(2) - sqrt(-d*x + 1))))/(((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
+ 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.39

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right) - \frac{\frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2}{2}+\frac{15\,a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^4}{2\,\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\,\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\,\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}} + \frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)}{2} - \frac{a\,d^2\,\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2} - \frac{b\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x} + \frac{a\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{32\,\left(\sqrt{d\,x+1}-1\right)^2}$$

input
```
int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
c*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*
x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/(
(d*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2
*((d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 -
d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log(((1 - d*x)^(1/2)
- 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log(((1 - d*x)^(1/2) - 1)/
((d*x + 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2
*((1 - d*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.85

$$\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{-\sqrt{dx+1}\,\sqrt{-dx+1}\,a-2\sqrt{dx+1}\,\sqrt{-dx+1}\,bx-\log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)}{2}\right)-1\right)a\,d^2x^2-2\,\mathrm{lo}}{}$$

input
```
int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - sqrt(d*x + 1)*sqrt( - d*x + 1)*a - 2*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*
x - log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*a*d**2*x**
2 - 2*log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*c*x**2 +
 log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*a*d**2*x**2 +
 2*log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*c*x**2 - lo
g(sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*a*d**2*x**2 - 2*log
(sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*c*x**2 + log(sqrt(2)
 + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*a*d**2*x**2 + 2*log(sqrt(2)
+ tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*c*x**2)/(2*x**2)
```

3.7 $\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 99

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{3x^3}-\frac{b\sqrt{1-d^2x^2}}{2x^2}-\frac{(3c+2ad^2)\sqrt{1-d^2x^2}}{3x}-\frac{1}{2}bd^2\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

output
```
-1/3*a*(-d^2*x^2+1)^(1/2)/x^3-1/2*b*(-d^2*x^2+1)^(1/2)/x^2-1/3*(2*a*d^2+3*
c)*(-d^2*x^2+1)^(1/2)/x-1/2*b*d^2*arctanh((-d^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{1}{6}\left(-\frac{\sqrt{1-d^2x^2}(3x(b+2cx)+a(2+4d^2x^2))}{x^3}-3bd^2\log(x)+3bd^2\log\left(-1+\sqrt{1-d^2x^2}\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-((Sqrt[1 - d^2*x^2]*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)))/x^3) - 3*b*d^
2*Log[x] + 3*b*d^2*Log[-1 + Sqrt[1 - d^2*x^2]])/6
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2112, 2338, 25, 539, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{1}{3}\int -\frac{3b+\left(2ad^2+3c\right)x}{x^3\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{3b+\left(2ad^2+3c\right)x}{x^3\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{3x^3}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int -\frac{3bxd^2+2\left(2ad^2+3c\right)}{x^2\sqrt{1-d^2x^2}}dx-\frac{3b\sqrt{1-d^2x^2}}{2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{1}{2}\int \frac{3bxd^2+2\left(2ad^2+3c\right)}{x^2\sqrt{1-d^2x^2}}dx-\frac{3b\sqrt{1-d^2x^2}}{2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{3x^3}$$

$$\Big\downarrow \text{534}$$

$$\frac{1}{3}\left(\frac{1}{2}\left(3bd^2\int \frac{1}{x\sqrt{1-d^2x^2}}dx-\frac{2\sqrt{1-d^2x^2}\left(2ad^2+3c\right)}{x}\right)-\frac{3b\sqrt{1-d^2x^2}}{2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{3x^3}$$

$$\Big\downarrow \ 243$$

$$\frac{1}{3}\left(\frac{1}{2}\left(\frac{3}{2}bd^2\int\frac{1}{x^2\sqrt{1-d^2x^2}}dx^2-\frac{2\sqrt{1-d^2x^2}(2ad^2+3c)}{x}\right)-\frac{3b\sqrt{1-d^2x^2}}{2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{3x^3}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-3b\int\frac{1}{\frac{1}{d^2}-\frac{x^4}{d^2}}d\sqrt{1-d^2x^2}-\frac{2\sqrt{1-d^2x^2}(2ad^2+3c)}{x}\right)-\frac{3b\sqrt{1-d^2x^2}}{2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{3x^3}$$

$$\Big\downarrow \ 221$$

$$\frac{1}{3}\left(\frac{1}{2}\left(-\frac{2\sqrt{1-d^2x^2}(2ad^2+3c)}{x}-3bd^2\text{arctanh}\left(\sqrt{1-d^2x^2}\right)\right)-\frac{3b\sqrt{1-d^2x^2}}{2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{3x^3}$$

input
```
Int[(a + b*x + c*x^2)/(x^4*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/3*(a*Sqrt[1 - d^2*x^2])/x^3 + ((-3*b*Sqrt[1 - d^2*x^2])/(2*x^2) + ((-2*
(3*c + 2*a*d^2)*Sqrt[1 - d^2*x^2])/x - 3*b*d^2*ArcTanh[Sqrt[1 - d^2*x^2]])
/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

method	result
risch	$\frac{\sqrt{xd+1}\,(xd-1)\,(4a\,d^2x^2+6c\,x^2+3bx+2a)\,\sqrt{(-xd+1)\,(xd+1)}}{6x^3\sqrt{-(xd+1)\,(xd-1)}\,\sqrt{-xd+1}}-\frac{b\,d^2\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-xd+1)\,(xd+1)}}{2\sqrt{-xd+1}\,\sqrt{xd+1}}$
default	$-\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\operatorname{csgn}(d)^2\left(3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)b\,d^2x^3+4\sqrt{-d^2x^2+1}\,a\,d^2x^2+6\sqrt{-d^2x^2+1}\,c\,x^2+3\sqrt{-d^2x^2+1}\,bx+2\sqrt{-d^2x}\right.}{6\sqrt{-d^2x^2+1}\,x^3}$

input
```
int((c*x^2+b*x+a)/x^4/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/6*(d*x+1)^(1/2)*(d*x-1)*(4*a*d^2*x^2+6*c*x^2+3*b*x+2*a)/x^3/(-(d*x+1)*(d
*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)-1/2*b*d^2*arctanh(1/(
-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$=\frac{3\,bd^2x^3\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-(2\,(2\,ad^2+3\,c)x^2+3\,bx+2\,a)\sqrt{dx+1}\sqrt{-dx+1}}{6\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/6*(3*b*d^2*x^3*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*(2*a*d^2 +
 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**4/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,bd^2\log\left(\frac{2\,\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{2\,\sqrt{-d^2x^2+1}ad^2}{3\,x} - \frac{\sqrt{-d^2x^2+1}c}{x}-\frac{\sqrt{-d^2x^2+1}b}{2\,x^2}-\frac{\sqrt{-d^2x^2+1}a}{3\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="max
ima")
```

output
```
-1/2*b*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-d^2*x^2
 + 1)*a*d^2/x - sqrt(-d^2*x^2 + 1)*c/x - 1/2*sqrt(-d^2*x^2 + 1)*b/x^2 - 1/
3*sqrt(-d^2*x^2 + 1)*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(83) = 166.

Time = 0.26 (sec) , antiderivative size = 619, normalized size of antiderivative = 6.25

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{3\,bd^3\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}+2\right|\right)-3\,bd^3\log\left(\left|-\frac{\sqrt{2}-\sqrt{-dx+1}}{\sqrt{dx+1}}+\frac{\sqrt{dx+1}}{\sqrt{2}-\sqrt{-dx+1}}-2\right|\right)+\frac{4\left(6\,ad^4\left(\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="gia
c")
```

output
```
-1/6*(3*b*d^3*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x
 + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) - 3*b*d^3*log(abs(-(sqrt(2) - sqrt(
-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) - 2))
+ 4*(6*a*d^4*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sq
rt(2) - sqrt(-d*x + 1)))^5 - 3*b*d^3*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
+ 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^5 - 16*a*d^4*((sqrt(2) -
sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^
3 + 6*c*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqr
t(2) - sqrt(-d*x + 1)))^5 + 96*a*d^4*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
+ 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1))) - 48*c*d^2*((sqrt(2) - sq
rt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3
+ 48*b*d^3*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt
(2) - sqrt(-d*x + 1))) + 96*c*d^2*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1
) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1))))/(((sqrt(2) - sqrt(-d*x + 1)
)/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^2 - 4)^3)/d
```

Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.07

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{b\,d^2\,\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)}{2} - \frac{\frac{b\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{b\,d^2}{2}+\frac{15\,b\,d^2\,\left(\sqrt{1-d\,x}-1\right)^4}{2\,\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\,\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\,\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\,\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}} - \frac{b\,d^2\,\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2} - \frac{c\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x} - \frac{\sqrt{1-d\,x}\,\left(\frac{2\,a\,d^3\,x^3}{3}+\frac{2\,a\,d^2\,x^2}{3}+\frac{a\,d\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{d\,x+1}} + \frac{b\,d^2\,\left(\sqrt{1-d\,x}-1\right)^2}{32\,\left(\sqrt{d\,x+1}-1\right)^2}$$

input
```
int((a + b*x + c*x^2)/(x^4*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(b*d^2*log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - ((b*d
^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (b*d^2)/2 + (15*b*d^
2*((1 - d*x)^(1/2) - 1)^4)/(2*((d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1
/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x +
 1)^(1/2) - 1)^4 + (16*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) -
 (b*d^2*log(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/2 - (c*(1 - d*x)
^(1/2)*(d*x + 1)^(1/2))/x - ((1 - d*x)^(1/2)*(a/3 + (2*a*d^2*x^2)/3 + (2*a
*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*((1 - d*x)^(1/2)
- 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.06

$$\int \frac{a+bx+cx^2}{x^4\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-4\sqrt{dx+1}\,\sqrt{-dx+1}\,a\,d^2x^2-2\sqrt{dx+1}\,\sqrt{-dx+1}\,a-3\sqrt{dx+1}\,\sqrt{-dx+1}\,bx-6\sqrt{dx+1}\,\sqrt{-dx+}}{}$$

input
```
int((c*x^2+b*x+a)/x^4/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 4*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**2*x**2 - 2*sqrt(d*x + 1)*sqrt( -
 d*x + 1)*a - 3*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*x - 6*sqrt(d*x + 1)*sqrt(
 - d*x + 1)*c*x**2 - 3*log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))
/2) - 1)*b*d**2*x**3 + 3*log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2
))/2) + 1)*b*d**2*x**3 - 3*log(sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2)
)/2) - 1)*b*d**2*x**3 + 3*log(sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))
/2) + 1)*b*d**2*x**3)/(6*x**3)
```

3.8 $\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 133

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{a\sqrt{1-d^2x^2}}{4x^4}-\frac{b\sqrt{1-d^2x^2}}{3x^3}-\frac{(4c+3ad^2)\sqrt{1-d^2x^2}}{8x^2} -\frac{2bd^2\sqrt{1-d^2x^2}}{3x}-\frac{1}{8}d^2(4c+3ad^2)\operatorname{arctanh}\left(\sqrt{1-d^2x^2}\right)$$

output
```
-1/4*a*(-d^2*x^2+1)^(1/2)/x^4-1/3*b*(-d^2*x^2+1)^(1/2)/x^3-1/8*(3*a*d^2+4*
c)*(-d^2*x^2+1)^(1/2)/x^2-2/3*b*d^2*(-d^2*x^2+1)^(1/2)/x-1/8*d^2*(3*a*d^2+
4*c)*arctanh((-d^2*x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{1-d^2x^2}(-6a-8bx-12cx^2-9ad^2x^2-16bd^2x^3)}{24x^4} -\frac{1}{8}d^2(4c+3ad^2)\log(x) +\frac{1}{8}d^2(4c+3ad^2)\log\left(-1+\sqrt{1-d^2x^2}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^5*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(-6*a - 8*b*x - 12*c*x^2 - 9*a*d^2*x^2 - 16*b*d^2*x^3))
/(24*x^4) - (d^2*(4*c + 3*a*d^2)*Log[x])/8 + (d^2*(4*c + 3*a*d^2)*Log[-1 +
 Sqrt[1 - d^2*x^2]])/8
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2112, 2338, 25, 539, 25, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{1}{4}\int -\frac{4b+\left(3ad^2+4c\right)x}{x^4\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{4b+\left(3ad^2+4c\right)x}{x^4\sqrt{1-d^2x^2}}dx-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \text{539}$$

$$\frac{1}{4}\left(-\frac{1}{3}\int -\frac{8bxd^2+3\left(3ad^2+4c\right)}{x^3\sqrt{1-d^2x^2}}dx-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right)-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{8bxd^2+3\left(3ad^2+4c\right)}{x^3\sqrt{1-d^2x^2}}dx-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right)-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \ 539$$

$$\frac{1}{4}\left(\frac{1}{3}\left(-\frac{1}{2}\int -\frac{d^2\left(16b+3\left(3ad^2+4c\right)x\right)}{x^2\sqrt{1-d^2x^2}}dx-\frac{3\sqrt{1-d^2x^2}\left(3ad^2+4c\right)}{2x^2}\right)-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right)-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{d^2\left(16b+3\left(3ad^2+4c\right)x\right)}{x^2\sqrt{1-d^2x^2}}dx-\frac{3\sqrt{1-d^2x^2}\left(3ad^2+4c\right)}{2x^2}\right)-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right)-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\int \frac{16b+3\left(3ad^2+4c\right)x}{x^2\sqrt{1-d^2x^2}}dx-\frac{3\sqrt{1-d^2x^2}\left(3ad^2+4c\right)}{2x^2}\right)-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right)-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \ 534$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\left(3\left(3ad^2+4c\right)\int \frac{1}{x\sqrt{1-d^2x^2}}dx-\frac{16b\sqrt{1-d^2x^2}}{x}\right)-\frac{3\sqrt{1-d^2x^2}\left(3ad^2+4c\right)}{2x^2}\right)-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right)-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \ 243$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\left(\frac{3}{2}\left(3ad^2+4c\right)\int \frac{1}{x^2\sqrt{1-d^2x^2}}dx^2-\frac{16b\sqrt{1-d^2x^2}}{x}\right)-\frac{3\sqrt{1-d^2x^2}\left(3ad^2+4c\right)}{2x^2}\right)-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right)-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \ 73$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\left(-\frac{3\left(3ad^2+4c\right)\int \frac{1}{\frac{1}{d^2}-\frac{x^4}{d^2}}d\sqrt{1-d^2x^2}}{d^2}-\frac{16b\sqrt{1-d^2x^2}}{x}\right)-\frac{3\sqrt{1-d^2x^2}\left(3ad^2+4c\right)}{2x^2}\right)-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right)-\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

$$\Big\downarrow \ 221$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\left(-3(3ad^2+4c)\,\text{arctanh}\left(\sqrt{1-d^2x^2}\right)-\frac{16b\sqrt{1-d^2x^2}}{x}\right)-\frac{3\sqrt{1-d^2x^2}(3ad^2+4c)}{2x^2}\right)-\frac{4b\sqrt{1-d^2x^2}}{3x^3}\right.$$

$$\frac{a\sqrt{1-d^2x^2}}{4x^4}$$

input
```
Int[(a + b*x + c*x^2)/(x^5*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/4*(a*Sqrt[1 - d^2*x^2])/x^4 + ((-4*b*Sqrt[1 - d^2*x^2])/(3*x^3) + ((-3*
(4*c + 3*a*d^2)*Sqrt[1 - d^2*x^2])/(2*x^2) + (d^2*((-16*b*Sqrt[1 - d^2*x^2
])/x - 3*(4*c + 3*a*d^2)*ArcTanh[Sqrt[1 - d^2*x^2]]))/2)/3)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\sqrt{xd+1}\,(xd-1)\left(16b\,d^2x^3+9a\,d^2x^2+12c\,x^2+8bx+6a\right)\sqrt{(-xd+1)(xd+1)}}{24x^4\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}-\frac{d^2\left(3a\,d^2+4c\right)\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-xd+1)(xd+}}{8\sqrt{-xd+1}\,\sqrt{xd+1}}$
default	$-\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\operatorname{csgn}(d)^2\left(9\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)a\,d^4x^4+12\operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)c\,d^2x^4+16\sqrt{-d^2x^2+1}\,b\,d^2x^3+9\sqrt{-d^2x^2}\right.}{24\sqrt{-d^2x^2+1}\,x^4}$

input
```
int((c*x^2+b*x+a)/x^5/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/24*(d*x+1)^(1/2)*(d*x-1)*(16*b*d^2*x^3+9*a*d^2*x^2+12*c*x^2+8*b*x+6*a)/x
^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)-1/8*d^
2*(3*a*d^2+4*c)*arctanh(1/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d
*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{3\,(3\,ad^4+4\,cd^2)x^4\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right) - (16\,bd^2x^3+3\,(3\,ad^2+4\,c)x^2+8\,bx+6\,a)\sqrt{dx+1}\sqrt{-dx+1}}{24\,x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/24*(3*(3*a*d^4 + 4*c*d^2)*x^4*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x)
- (16*b*d^2*x^3 + 3*(3*a*d^2 + 4*c)*x^2 + 8*b*x + 6*a)*sqrt(d*x + 1)*sqrt(
-d*x + 1))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**5/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.21

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{3}{8}\,ad^4\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - \frac{1}{2}\,cd^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right) - \frac{2\sqrt{-d^2x^2+1}bd^2}{3\,x} - \frac{3\sqrt{-d^2x^2+1}ad^2}{8\,x^2} - \frac{\sqrt{-d^2x^2+1}c}{2\,x^2} - \frac{\sqrt{-d^2x^2+1}b}{3\,x^3} - \frac{\sqrt{-d^2x^2+1}a}{4\,x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="max
ima")
```

output
```
-3/8*a*d^4*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - 1/2*c*d^2*log(2*s
qrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - 2/3*sqrt(-d^2*x^2 + 1)*b*d^2/x - 3/
8*sqrt(-d^2*x^2 + 1)*a*d^2/x^2 - 1/2*sqrt(-d^2*x^2 + 1)*c/x^2 - 1/3*sqrt(-
d^2*x^2 + 1)*b/x^3 - 1/4*sqrt(-d^2*x^2 + 1)*a/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(113) = 226.

Time = 0.28 (sec) , antiderivative size = 861, normalized size of antiderivative = 6.47

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="gia
c")
```

output
```
-1/24*(3*(3*a*d^5 + 4*c*d^3)*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x
+ 1) + sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)) + 2)) - 3*(3*a*d^5 + 4*c*d
^3)*log(abs(-(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + sqrt(d*x + 1)/(sqr
t(2) - sqrt(-d*x + 1)) - 2)) - 4*(15*a*d^5*((sqrt(2) - sqrt(-d*x + 1))/sqr
t(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^7 - 24*b*d^4*((sqrt
(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x +
 1)))^7 + 36*a*d^5*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x +
1)/(sqrt(2) - sqrt(-d*x + 1)))^5 + 12*c*d^3*((sqrt(2) - sqrt(-d*x + 1))/sq
rt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^7 + 160*b*d^4*((sq
rt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x
 + 1)))^5 + 144*a*d^5*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x
 + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 - 48*c*d^3*((sqrt(2) - sqrt(-d*x + 1))
/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^5 - 640*b*d^4*(
(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-
d*x + 1)))^3 + 960*a*d^5*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(
d*x + 1)/(sqrt(2) - sqrt(-d*x + 1))) - 192*c*d^3*((sqrt(2) - sqrt(-d*x + 1
))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1)))^3 + 1536*b*d^
4*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt(d*x + 1)/(sqrt(2) - sqr
t(-d*x + 1))) + 768*c*d^3*((sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - sqrt
(d*x + 1)/(sqrt(2) - sqrt(-d*x + 1))))/(((sqrt(2) - sqrt(-d*x + 1))/sqr...
```

Mupad [B] (verification not implemented)

Time = 7.39 (sec) , antiderivative size = 684, normalized size of antiderivative = 5.14

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)/(x^5*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((a*d^4)/4 + (6*a*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (
53*a*d^4*((1 - d*x)^(1/2) - 1)^4)/(2*((d*x + 1)^(1/2) - 1)^4) - (87*a*d^4*
((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (657*a*d^4*((1 - d*x)^(
1/2) - 1)^8)/(4*((d*x + 1)^(1/2) - 1)^8) - (121*a*d^4*((1 - d*x)^(1/2) - 1
)^10)/((d*x + 1)^(1/2) - 1)^10)/((256*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^
(1/2) - 1)^4 - (1024*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (1
536*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (1024*((1 - d*x)^(1
/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (256*((1 - d*x)^(1/2) - 1)^12)/((d
*x + 1)^(1/2) - 1)^12) - ((c*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
 - 1)^2 - (c*d^2)/2 + (15*c*d^2*((1 - d*x)^(1/2) - 1)^4)/(2*((d*x + 1)^(1/
2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (32*((
1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 - d*x)^(1/2) - 1)
^6)/((d*x + 1)^(1/2) - 1)^6) + (3*a*d^4*log(((1 - d*x)^(1/2) - 1)^2/((d*x
+ 1)^(1/2) - 1)^2 - 1))/8 + (c*d^2*log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^
(1/2) - 1)^2 - 1))/2 - (3*a*d^4*log(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2)
 - 1)))/8 - (c*d^2*log(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/2 - (
(1 - d*x)^(1/2)*(b/3 + (2*b*d^2*x^2)/3 + (2*b*d^3*x^3)/3 + (b*d*x)/3))/(x^
3*(d*x + 1)^(1/2)) + (7*a*d^4*((1 - d*x)^(1/2) - 1)^2)/(256*((d*x + 1)^(1/
2) - 1)^2) + (a*d^4*((1 - d*x)^(1/2) - 1)^4)/(1024*((d*x + 1)^(1/2) - 1)^4
) + (c*d^2*((1 - d*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.63

$$\int \frac{a+bx+cx^2}{x^5\sqrt{1-dx}\sqrt{1+dx}}\,dx$$

$$= \frac{-9\sqrt{dx+1}\,\sqrt{-dx+1}\,a\,d^2x^2-6\sqrt{dx+1}\,\sqrt{-dx+1}\,a-16\sqrt{dx+1}\,\sqrt{-dx+1}\,b\,d^2x^3-8\sqrt{dx+1}\,\sqrt{-d}}{}$$

input
```
int((c*x^2+b*x+a)/x^5/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 9*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**2*x**2 - 6*sqrt(d*x + 1)*sqrt( -
 d*x + 1)*a - 16*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**2*x**3 - 8*sqrt(d*x +
 1)*sqrt( - d*x + 1)*b*x - 12*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*x**2 - 9*lo
g( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*a*d**4*x**4 - 12
*log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*c*d**2*x**4 +
 9*log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*a*d**4*x**4
 + 12*log( - sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*c*d**2*x
**4 - 9*log(sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*a*d**4*x*
*4 - 12*log(sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - 1)*c*d**2*x*
*4 + 9*log(sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*a*d**4*x**
4 + 12*log(sqrt(2) + tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + 1)*c*d**2*x**
4)/(24*x**4)
```

3.9 $\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 159

$$
\begin{aligned}
\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = & -\frac{(3c-4d(4b+3ad))\sqrt{-1+dx}\sqrt{1+dx}}{24d^5} \\
& +\frac{(9c+4bd)x^2\sqrt{-1+dx}\sqrt{1+dx}}{12d^3} \\
& -\frac{(3c-4ad^2)(-1+dx)^{3/2}\sqrt{1+dx}}{8d^5} \\
& +\frac{c(-1+dx)^{7/2}\sqrt{1+dx}}{4d^5}+\frac{(3c+4ad^2)\,\text{arccosh}(dx)}{8d^5}
\end{aligned}
$$

output

```
-1/24*(3*c-4*d*(3*a*d+4*b))*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^5+1/12*(4*b*d+9*
c)*x^2*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^3-1/8*(-4*a*d^2+3*c)*(d*x-1)^(3/2)*(d
*x+1)^(1/2)/d^5+1/4*c*(d*x-1)^(7/2)*(d*x+1)^(1/2)/d^5+1/8*(4*a*d^2+3*c)*ar
ccosh(d*x)/d^5
```

Mathematica [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.58

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{d\sqrt{-1+dx}\sqrt{1+dx}(8b(2+d^2x^2)+3x(4ad^2+c(3+2d^2x^2)))+6(3c+4ad^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{24d^5}$$

input
```
Integrate[(x^2*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(d*Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(8*b*(2 + d^2*x^2) + 3*x*(4*a*d^2 + c*(3 +
 2*d^2*x^2))) + 6*(3*c + 4*a*d^2)*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(24
*d^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2113, 2340, 533, 25, 27, 533, 25, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx+cx^2\right)}{\sqrt{dx-1}\sqrt{dx+1}}\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{x^2\left(cx^2+bx+a\right)}{\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\int \frac{x^2\left(4ad^2+4bxd^2+3c\right)}{\sqrt{d^2x^2-1}}dx}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{533}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{4}{3}bx^2\sqrt{d^2x^2-1}-\frac{\int -\frac{d^2x\left(8b+3\left(4ad^2+3c\right)x\right)}{\sqrt{d^2x^2-1}}dx}{3d^2}}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{\int \frac{d^2x\left(8b+3\left(4ad^2+3c\right)x\right)}{\sqrt{d^2x^2-1}}dx}{3d^2}+\frac{4}{3}bx^2\sqrt{d^2x^2-1}}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{3}\int \frac{x\left(8b+3\left(4ad^2+3c\right)x\right)}{\sqrt{d^2x^2-1}}dx+\frac{4}{3}bx^2\sqrt{d^2x^2-1}}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{533}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{3}\left(\frac{3}{2}x\sqrt{d^2x^2-1}\left(4a+\frac{3c}{d^2}\right)-\frac{\int -\frac{16bxd^2+3\left(4ad^2+3c\right)}{\sqrt{d^2x^2-1}}dx}{2d^2}\right)+\frac{4}{3}bx^2\sqrt{d^2x^2-1}}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{3}\left(\frac{\int \frac{16bxd^2+3\left(4ad^2+3c\right)}{\sqrt{d^2x^2-1}}dx}{2d^2}+\frac{3}{2}x\sqrt{d^2x^2-1}\left(4a+\frac{3c}{d^2}\right)\right)+\frac{4}{3}bx^2\sqrt{d^2x^2-1}}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{455}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{3}\left(\frac{3\left(4ad^2+3c\right)\int\frac{1}{\sqrt{d^2x^2-1}}dx+16b\sqrt{d^2x^2-1}}{2d^2}+\frac{3}{2}x\sqrt{d^2x^2-1}\left(4a+\frac{3c}{d^2}\right)\right)+\frac{4}{3}bx^2\sqrt{d^2x^2-1}}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{3}\left(\frac{3\left(4ad^2+3c\right)\int\frac{1}{1-\frac{d^2x^2}{d^2x^2-1}}d\frac{x}{\sqrt{d^2x^2-1}}+16b\sqrt{d^2x^2-1}}{2d^2}+\frac{3}{2}x\sqrt{d^2x^2-1}\left(4a+\frac{3c}{d^2}\right)\right)+\frac{4}{3}bx^2\sqrt{d^2x^2-1}}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{3}\left(\frac{\frac{3\left(4ad^2+3c\right)\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+16b\sqrt{d^2x^2-1}}{2d^2}+\frac{3}{2}x\sqrt{d^2x^2-1}\left(4a+\frac{3c}{d^2}\right)\right)+\frac{4}{3}bx^2\sqrt{d^2x^2-1}}{4d^2}+\frac{cx^3\sqrt{d^2x^2-1}}{4d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(x^2*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((c*x^3*Sqrt[-1 + d^2*x^2])/(4*d^2) + ((4*b*x^2*Sqrt[-
1 + d^2*x^2])/3 + ((3*(4*a + (3*c)/d^2)*x*Sqrt[-1 + d^2*x^2])/2 + (16*b*Sq
rt[-1 + d^2*x^2] + (3*(3*c + 4*a*d^2)*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/d
)/(2*d^2))/3)/(4*d^2)))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

method	result
risch	$\frac{(6c\,x^3d^2+8b\,d^2x^2+12xa\,d^2+9cx+16b)\sqrt{xd+1}\,\sqrt{xd-1}}{24d^4}+\frac{(4a\,d^2+3c)\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(xd+1)(xd-1)}}{8d^4\sqrt{d^2}\,\sqrt{xd-1}\,\sqrt{xd+1}}$
default	$\frac{\sqrt{xd-1}\,\sqrt{xd+1}\,\left(6\ \mathrm{csgn}(d)c\,d^3x^3\sqrt{d^2x^2-1}+8\ \mathrm{csgn}(d)b\,d^3x^2\sqrt{d^2x^2-1}+12\ \mathrm{csgn}(d)d^3\sqrt{d^2x^2-1}\,ax+9\ \mathrm{csgn}(d)d\sqrt{d^2x^2-1}\,cx+16\ \mathrm{cs}\right.}{24d^5\sqrt{d^2x^2-1}}$

input
```
int(x^2*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*c*d^2*x^3+8*b*d^2*x^2+12*a*d^2*x+9*c*x+16*b)*(d*x+1)^(1/2)*(d*x-1)
^(1/2)/d^4+1/8*(4*a*d^2+3*c)/d^4*ln(d^2*x/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(
d^2)^(1/2)*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$=\frac{(6\,cd^3x^3+8\,bd^3x^2+16\,bd+3\,(4\,ad^3+3\,cd)x)\sqrt{dx+1}\sqrt{dx-1}-3\,(4\,ad^2+3\,c)\log\left(-dx+\sqrt{dx+1}\sqrt{\right.}}{24\,d^5}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/24*((6*c*d^3*x^3 + 8*b*d^3*x^2 + 16*b*d + 3*(4*a*d^3 + 3*c*d)*x)*sqrt(d*
x + 1)*sqrt(d*x - 1) - 3*(4*a*d^2 + 3*c)*log(-d*x + sqrt(d*x + 1)*sqrt(d*x
 - 1)))/d^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d^2x^2-1}cx^3}{4\,d^2} + \frac{\sqrt{d^2x^2-1}bx^2}{3\,d^2} + \frac{\sqrt{d^2x^2-1}ax}{2\,d^2} + \frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3} + \frac{3\,\sqrt{d^2x^2-1}cx}{8\,d^4} + \frac{2\,\sqrt{d^2x^2-1}b}{3\,d^4} + \frac{3\,c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{8\,d^5}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
1/4*sqrt(d^2*x^2 - 1)*c*x^3/d^2 + 1/3*sqrt(d^2*x^2 - 1)*b*x^2/d^2 + 1/2*sq
rt(d^2*x^2 - 1)*a*x/d^2 + 1/2*a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 +
 3/8*sqrt(d^2*x^2 - 1)*c*x/d^4 + 2/3*sqrt(d^2*x^2 - 1)*b/d^4 + 3/8*c*log(2
*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\left((dx+1)\left(2\,(dx+1)\left(\frac{3\,(dx+1)c}{d^4}+\frac{4\,bd^{17}-9\,cd^{16}}{d^{20}}\right)+\frac{12\,ad^{18}-16\,bd^{17}+27\,cd^{16}}{d^{20}}\right)-\frac{3\,(4\,ad^{18}-8\,bd^{17}+5\,cd^{16})}{d^{20}}\right)\sqrt{dx+1}\sqrt{d}}{24\,d}$$

input
```
integrate(x^2*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

output
```
1/24*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)*c/d^4 + (4*b*d^17 - 9*c*d^16)/d
^20) + (12*a*d^18 - 16*b*d^17 + 27*c*d^16)/d^20) - 3*(4*a*d^18 - 8*b*d^17
+ 5*c*d^16)/d^20)*sqrt(d*x + 1)*sqrt(d*x - 1) - 6*(4*a*d^2 + 3*c)*log(sqrt
(d*x + 1) - sqrt(d*x - 1))/d^4)/d
```

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 767, normalized size of antiderivative = 4.82

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
int((x^2*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((23*c*((d*x - 1)^(1/2) - 1i)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (333*c*((d*
x - 1)^(1/2) - 1i)^5)/(2*((d*x + 1)^(1/2) - 1)^5) + (671*c*((d*x - 1)^(1/2
) - 1i)^7)/(2*((d*x + 1)^(1/2) - 1)^7) + (671*c*((d*x - 1)^(1/2) - 1i)^9)/
(2*((d*x + 1)^(1/2) - 1)^9) + (333*c*((d*x - 1)^(1/2) - 1i)^11)/(2*((d*x +
 1)^(1/2) - 1)^11) + (23*c*((d*x - 1)^(1/2) - 1i)^13)/(2*((d*x + 1)^(1/2)
- 1)^13) - (3*c*((d*x - 1)^(1/2) - 1i)^15)/(2*((d*x + 1)^(1/2) - 1)^15) -
(3*c*((d*x - 1)^(1/2) - 1i))/(2*((d*x + 1)^(1/2) - 1)))/(d^5 - (8*d^5*((d*
x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((d*x - 1)^(1/2) -
 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (56*d^5*((d*x - 1)^(1/2) - 1i)^6)/((d*x
+ 1)^(1/2) - 1)^6 + (70*d^5*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8 - (56*d^5*((d*x - 1)^(1/2) - 1i)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^
5*((d*x - 1)^(1/2) - 1i)^12)/((d*x + 1)^(1/2) - 1)^12 - (8*d^5*((d*x - 1)^
(1/2) - 1i)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^5*((d*x - 1)^(1/2) - 1i)^16)
/((d*x + 1)^(1/2) - 1)^16) - ((14*a*((d*x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(
1/2) - 1)^3 + (14*a*((d*x - 1)^(1/2) - 1i)^5)/((d*x + 1)^(1/2) - 1)^5 + (2
*a*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1/2) - 1)^7 + (2*a*((d*x - 1)^(1/
2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*d^3*((d*x - 1)^(1/2) - 1i)^2)/(
(d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(1/2) - 1i)^4)/((d*x + 1)^(1/2)
 - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((d*x + 1)^(1/2) - 1)^6 + (d^3*
((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1)^8) + (2*a*atanh(((d*x -...
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$= \frac{12\sqrt{dx+1}\,\sqrt{dx-1}\,a\,d^3x+8\sqrt{dx+1}\,\sqrt{dx-1}\,b\,d^3x^2+16\sqrt{dx+1}\,\sqrt{dx-1}\,bd+6\sqrt{dx+1}\,\sqrt{dx-1}\,c\,\ldots}{24d^5}$$

input
```
int(x^2*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
(12*sqrt(d*x + 1)*sqrt(d*x - 1)*a*d**3*x + 8*sqrt(d*x + 1)*sqrt(d*x - 1)*b
*d**3*x**2 + 16*sqrt(d*x + 1)*sqrt(d*x - 1)*b*d + 6*sqrt(d*x + 1)*sqrt(d*x
 - 1)*c*d**3*x**3 + 9*sqrt(d*x + 1)*sqrt(d*x - 1)*c*d*x + 24*log((sqrt(d*x
 - 1) + sqrt(d*x + 1))/sqrt(2))*a*d**2 + 18*log((sqrt(d*x - 1) + sqrt(d*x
+ 1))/sqrt(2))*c)/(24*d**5)
```

3.10 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 109

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(2c+d(b+2ad))\sqrt{-1+dx}\sqrt{1+dx}}{2d^4} + \frac{(4c+3bd)(-1+dx)^{3/2}\sqrt{1+dx}}{6d^4} + \frac{c(-1+dx)^{5/2}\sqrt{1+dx}}{3d^4} + \frac{b\operatorname{arccosh}(dx)}{2d^3}$$

output
```
1/2*(2*c+d*(2*a*d+b))*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^4+1/6*(3*b*d+4*c)*(d*x
-1)^(3/2)*(d*x+1)^(1/2)/d^4+1/3*c*(d*x-1)^(5/2)*(d*x+1)^(1/2)/d^4+1/2*b*ar
ccosh(d*x)/d^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+dx}\sqrt{1+dx}(3d^2(2a+bx)+2c(2+d^2x^2))+6b\,d\,\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{6d^4}$$

input
```
Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2)) + 6*
b*d*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*d^4)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2113, 2340, 533, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{dx-1}\sqrt{dx+1}}\,dx$$

$$\Big\downarrow \ 2113$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{x(cx^2+bx+a)}{\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 2340$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\int \frac{x(3ad^2+3bxd^2+2c)}{\sqrt{d^2x^2-1}}dx}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 533$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{3}{2}bx\sqrt{d^2x^2-1}-\frac{\int -\frac{d^2(3b+2(3ad^2+2c)x)}{\sqrt{d^2x^2-1}}dx}{2d^2}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{\int \frac{d^2\left(3b+2\left(3ad^2+2c\right)x\right)}{\sqrt{d^2x^2-1}}dx}{2d^2}+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{2}\int \frac{3b+2\left(3ad^2+2c\right)x}{\sqrt{d^2x^2-1}}dx+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{455}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{2}\left(3b\int \frac{1}{\sqrt{d^2x^2-1}}dx+2\sqrt{d^2x^2-1}\left(3a+\frac{2c}{d^2}\right)\right)+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{2}\left(3b\int \frac{1}{1-\frac{d^2x^2}{d^2x^2-1}}d\frac{x}{\sqrt{d^2x^2-1}}+2\sqrt{d^2x^2-1}\left(3a+\frac{2c}{d^2}\right)\right)+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\frac{1}{2}\left(2\sqrt{d^2x^2-1}\left(3a+\frac{2c}{d^2}\right)+\frac{3b\,\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)+\frac{3}{2}bx\sqrt{d^2x^2-1}}{3d^2}+\frac{cx^2\sqrt{d^2x^2-1}}{3d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((c*x^2*Sqrt[-1 + d^2*x^2])/(3*d^2) + ((3*b*x*Sqrt[-1
+ d^2*x^2])/2 + (2*(3*a + (2*c)/d^2)*Sqrt[-1 + d^2*x^2] + (3*b*ArcTanh[(d*
x)/Sqrt[-1 + d^2*x^2]])/d)/2)/(3*d^2)))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

method	result
risch	$\frac{(2c\,d^2x^2+3b\,d^2x+6a\,d^2+4c)\sqrt{xd+1}\,\sqrt{xd-1}}{6d^4}+\frac{b\ln\left(\frac{d^2x}{\sqrt{d^2}}+\sqrt{d^2x^2-1}\right)\sqrt{(xd+1)(xd-1)}}{2d^2\sqrt{d^2}\,\sqrt{xd-1}\,\sqrt{xd+1}}$
default	$\frac{\sqrt{xd-1}\,\sqrt{xd+1}\,\left(2\,\mathrm{csgn}(d)c\,d^2x^2\sqrt{d^2x^2-1}+3\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)b\,d^2x+6\,\mathrm{csgn}(d)\sqrt{d^2x^2-1}\,a\,d^2+4\,\mathrm{csgn}(d)\sqrt{d^2x^2-1}\,c+3\ln\left(\left(\sqrt{d^2\cdots\right.\right.\right.}{6d^4\sqrt{d^2x^2-1}}$

input
```
int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*c*d^2*x^2+3*b*d^2*x+6*a*d^2+4*c)*(d*x+1)^(1/2)*(d*x-1)^(1/2)/d^4+1/
2*b/d^2*ln(d^2*x/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(d^2)^(1/2)*((d*x+1)*(d*x-
1))^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c)\sqrt{dx+1}\sqrt{dx-1}}{6\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

output
```
-1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d
^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.92

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d^2x^2-1}cx^2}{3\,d^2} + \frac{\sqrt{d^2x^2-1}bx}{2\,d^2} + \frac{\sqrt{d^2x^2-1}a}{d^2} + \frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3} + \frac{2\,\sqrt{d^2x^2-1}c}{3\,d^4}$$

input
```
integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

output
```
1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2
*x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqr
t(d^2*x^2 - 1)*c/d^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{dx+1}\sqrt{dx-1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\,(2\,ad^{11}-bd^{10}+2\,cd^9)}{d^{12}}\right)-\frac{6\,b\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{6\,d}$$

input
```
integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
 - 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d
*x + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.92

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{d\,x-1}\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\frac{14\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}+\frac{14\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{6\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}+\frac{d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{d\,x+1}-1\right)^8}} + \frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}$$

input
```
int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d
*x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) -
1i)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)
^(1/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 -
 (4*d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x -
 1)^(1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)
^6)/((d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1
/2) - 1)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3
*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(
1/2))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$= \frac{6\sqrt{dx+1}\,\sqrt{dx-1}\,a\,d^2+3\sqrt{dx+1}\,\sqrt{dx-1}\,b\,d^2x+2\sqrt{dx+1}\,\sqrt{dx-1}\,c\,d^2x^2+4\sqrt{dx+1}\,\sqrt{dx-1}\,c+}{6d^4}$$

input
```
int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
(6*sqrt(d*x + 1)*sqrt(d*x - 1)*a*d**2 + 3*sqrt(d*x + 1)*sqrt(d*x - 1)*b*d*
*2*x + 2*sqrt(d*x + 1)*sqrt(d*x - 1)*c*d**2*x**2 + 4*sqrt(d*x + 1)*sqrt(d*
x - 1)*c + 6*log((sqrt(d*x - 1) + sqrt(d*x + 1))/sqrt(2))*b*d)/(6*d**4)
```

3.11 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 77

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(c+2bd)\sqrt{-1+dx}\sqrt{1+dx}}{2d^3} + \frac{c(-1+dx)^{3/2}\sqrt{1+dx}}{2d^3} + \frac{(c+2ad^2)\operatorname{arccosh}(dx)}{2d^3}$$

output
```
1/2*(2*b*d+c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^3+1/2*c*(d*x-1)^(3/2)*(d*x+1)^
(1/2)/d^3+1/2*(2*a*d^2+c)*arccosh(d*x)/d^3
```

Mathematica [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{d(2b+cx)\sqrt{-1+dx}\sqrt{1+dx}+2(c+2ad^2)\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{2d^3}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(d*(2*b + c*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x] + 2*(c + 2*a*d^2)*ArcTanh[Sqrt
[(-1 + d*x)/(1 + d*x)]])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1189, 83, 646, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{dx-1}\sqrt{dx+1}}\,dx$$

$$\Big\downarrow \text{1189}$$

$$\int \frac{cx^2+a}{\sqrt{dx-1}\sqrt{dx+1}}dx+b\int \frac{x}{\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{83}$$

$$\int \frac{cx^2+a}{\sqrt{dx-1}\sqrt{dx+1}}dx+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{d^2}$$

$$\Big\downarrow \text{646}$$

$$\frac{(2ad^2+c)\int \frac{1}{\sqrt{dx-1}\sqrt{dx+1}}dx}{2d^2}+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{d^2}+\frac{cx\sqrt{dx-1}\sqrt{dx+1}}{2d^2}$$

$$\Big\downarrow \text{43}$$

$$\frac{(2ad^2+c)\operatorname{arccosh}(dx)}{2d^3}+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{d^2}+\frac{cx\sqrt{dx-1}\sqrt{dx+1}}{2d^2}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(b*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + (c*x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/
(2*d^2) + ((c + 2*a*d^2)*ArcCosh[d*x])/(2*d^3)
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

rule 1189
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2), x_Symbol] :> Simp[b   Int[x*(d + e*x)^m*(f + g*x)^n, x],
x] + Int[(d + e*x)^m*(f + g*x)^n*(a + c*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, m, n}, x] && EqQ[m, n] && EqQ[e*f + d*g, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.25

method	result
risch	$\frac{(cx+2b)\sqrt{xd+1}\,\sqrt{xd-1}}{2d^{2}}+\frac{(2a\,d^{2}+c)\ln\left(\frac{d^{2}x}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-1}\right)\sqrt{(xd+1)(xd-1)}}{2d^{2}\sqrt{d^{2}}\,\sqrt{xd-1}\,\sqrt{xd+1}}$
default	$\frac{\sqrt{xd-1}\,\sqrt{xd+1}\,\left(\operatorname{csgn}(d)d\sqrt{d^{2}x^{2}-1}\,cx+2\ln\left(\left(\sqrt{d^{2}x^{2}-1}\,\operatorname{csgn}(d)+xd\right)\operatorname{csgn}(d)\right)a\,d^{2}+2\,\operatorname{csgn}(d)d\sqrt{d^{2}x^{2}-1}\,b+\ln\left(\left(\sqrt{d^{2}x^{2}-1}\,\operatorname{csgn}(\right.\right.\right.}{2d^{3}\sqrt{d^{2}x^{2}-1}}$

input
```
int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x+2*b)*(d*x+1)^(1/2)*(d*x-1)^(1/2)/d^2+1/2*(2*a*d^2+c)/d^2*ln(d^2*x
/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))/(d^2)^(1/2)*((d*x+1)*(d*x-1))^(1/2)/(d*x-1
)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{dx-1}-(2\,ad^2+c)\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x
+ sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.17

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d}+\frac{\sqrt{d^2x^2-1}cx}{2\,d^2} +\frac{\sqrt{d^2x^2-1}b}{d^2}+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{2\,d^3}$$

input
```
integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

output
```
a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
 sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{dx+1}\sqrt{dx-1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\,(2\,ad^2+c)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

input
```
integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
 - 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

Mupad [B] (verification not implemented)

Time = 11.32 (sec) , antiderivative size = 312, normalized size of antiderivative = 4.05

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$= \frac{b\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2} + \frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}}$$
$$-\frac{\frac{14\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}+\frac{14\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}+\frac{2\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{6\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}-\frac{4\,d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}+\frac{d^3\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{d\,x+1}-1\right)^8}}$$

input
```
int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d
*x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) -
1i)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)
^(1/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 -
 (4*d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x -
 1)^(1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)
^6)/((d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1
/2) - 1)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*
(-d^2)^(1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$= \frac{2\sqrt{dx+1}\,\sqrt{dx-1}\,bd+\sqrt{dx+1}\,\sqrt{dx-1}\,cdx+4\log\left(\frac{\sqrt{dx-1}+\sqrt{dx+1}}{\sqrt{2}}\right)a\,d^2+2\log\left(\frac{\sqrt{dx-1}+\sqrt{dx+1}}{\sqrt{2}}\right)c}{2d^3}$$

input
```
int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
(2*sqrt(d*x + 1)*sqrt(d*x - 1)*b*d + sqrt(d*x + 1)*sqrt(d*x - 1)*c*d*x + 4
*log((sqrt(d*x - 1) + sqrt(d*x + 1))/sqrt(2))*a*d**2 + 2*log((sqrt(d*x - 1
) + sqrt(d*x + 1))/sqrt(2))*c)/(2*d**3)
```

3.12 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+\frac{b\operatorname{arccosh}(dx)}{d}+a\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
c*(d*x-1)^(1/2)*(d*x+1)^(1/2)/d^2+b*arccosh(d*x)/d+a*arctan((d*x-1)^(1/2)*
(d*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{c\sqrt{-1+dx}\sqrt{1+dx}}{d^2}+2a\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)+\frac{2b\operatorname{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

input
```
Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(c*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/d^2 + 2*a*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x
)]] + (2*b*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2113, 2340, 27, 538, 224, 219, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{\int \frac{d^2(a+bx)}{x\sqrt{d^2x^2-1}}dx}{d^2}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2x^2-1}\left(\int \frac{a+bx}{x\sqrt{d^2x^2-1}}dx+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{538}$$

$$\frac{\sqrt{d^2x^2-1}\left(a\int \frac{1}{x\sqrt{d^2x^2-1}}dx+b\int \frac{1}{\sqrt{d^2x^2-1}}dx+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2x^2-1}\left(a\int \frac{1}{x\sqrt{d^2x^2-1}}dx+b\int \frac{1}{1-\frac{d^2x^2}{d^2x^2-1}}d\frac{x}{\sqrt{d^2x^2-1}}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d^2x^2-1}\left(a\int \frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{b\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{b\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{a\int \frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}}{d^2}+\frac{b\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{d^2x^2-1}\left(a\arctan\left(\sqrt{d^2x^2-1}\right)+\frac{b\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}+\frac{c\sqrt{d^2x^2-1}}{d^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((c*Sqrt[-1 + d^2*x^2])/d^2 + a*ArcTan[Sqrt[-1 + d^2*x
^2]] + (b*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/d))/(Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

method	result
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\operatorname{csgn}(d) a\, d^2+\operatorname{csgn}(d)\sqrt{d^2x^2-1}\, c+\ln\left(\left(\sqrt{(xd+1)(xd-1)}\,\operatorname{csgn}(d)+xd\right)\operatorname{csgn}(d)\right) bd\right)\sqrt{xd-1}\,\sqrt{xd+1}\,\operatorname{csgn}(d)}{d^2\sqrt{d^2x^2-1}}$

input
```
int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-arctan(1/(d^2*x^2-1)^(1/2))*csgn(d)*a*d^2+csgn(d)*(d^2*x^2-1)^(1/2)*c+ln
((((d*x+1)*(d*x-1))^(1/2)*csgn(d)+x*d)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)
^(1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$= \frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

output
```
(2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(
d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.36

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

input
```
integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
-a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()),
 ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/
2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
 ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4,
 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_
polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -a\arcsin\left(\frac{1}{d|x|}\right) + \frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d} + \frac{\sqrt{d^2x^2-1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

output
```
-a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(
d^2*x^2 - 1)*c/d^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right) - \frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d} + \frac{\sqrt{dx+1}\sqrt{dx-1}c}{d^2}$$

input
```
integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
-2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1)
- sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{c\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2} - \frac{4\,b\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - a\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

input
```
int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) -
1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x -
1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)
/((d*x + 1)^(1/2) - 1)))*1i
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2atan\left(\sqrt{dx-1}+\sqrt{dx+1}-1\right)a\,d^2-2atan\left(\sqrt{dx-1}+\sqrt{dx+1}+1\right)a\,d^2+\sqrt{dx+1}\,\sqrt{dx-1}\,c+2l}{d^2}$$

input
```
int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
(2*atan(sqrt(d*x - 1) + sqrt(d*x + 1) - 1)*a*d**2 - 2*atan(sqrt(d*x - 1) +
 sqrt(d*x + 1) + 1)*a*d**2 + sqrt(d*x + 1)*sqrt(d*x - 1)*c + 2*log((sqrt(d
*x - 1) + sqrt(d*x + 1))/sqrt(2))*b*d)/d**2
```

3.13 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{x} + \frac{c\,\mathrm{arccosh}(dx)}{d} + b\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
a*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x+c*arccosh(d*x)/d+b*arctan((d*x-1)^(1/2)*(d
*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{x} + 2b\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right) + \frac{2c\,\mathrm{arctanh}\left(\sqrt{\frac{-1+dx}{1+dx}}\right)}{d}$$

input
```
Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(a*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/x + 2*b*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]
] + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2113, 2338, 538, 224, 219, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^2\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x^2\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\sqrt{d^2x^2-1}\left(\int \frac{b+cx}{x\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{x}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{538}$$

$$\frac{\sqrt{d^2x^2-1}\left(b\int \frac{1}{x\sqrt{d^2x^2-1}}dx+c\int \frac{1}{\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{x}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2x^2-1}\left(b\int \frac{1}{x\sqrt{d^2x^2-1}}dx+c\int \frac{1}{1-\frac{d^2x^2}{d^2x^2-1}}d\frac{x}{\sqrt{d^2x^2-1}}+\frac{a\sqrt{d^2x^2-1}}{x}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d^2x^2-1}\left(b\int \frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{x}+\frac{c\,\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{a\sqrt{d^2x^2-1}}{x}+\frac{c\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{b\int \frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}}{d^2}+\frac{a\sqrt{d^2x^2-1}}{x}+\frac{c\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \; 218$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{a\sqrt{d^2x^2-1}}{x}+b\arctan\left(\sqrt{d^2x^2-1}\right)+\frac{c\text{arctanh}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((a*Sqrt[-1 + d^2*x^2])/x + b*ArcTan[Sqrt[-1 + d^2*x^2
]] + (c*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/d))/(Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

method	result	s
risch	$\frac{a\sqrt{xd-1}\,\sqrt{xd+1}}{x}+\frac{\left(\frac{c\ln\left(\frac{d^{2}x}{\sqrt{d^{2}}}+\sqrt{d^{2}x^{2}-1}\right)}{\sqrt{d^{2}}}-b\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)\right)\sqrt{(xd+1)(xd-1)}}{\sqrt{xd-1}\,\sqrt{xd+1}}$	9
default	$\frac{\left(-\operatorname{csgn}(d)d\arctan\left(\frac{1}{\sqrt{d^{2}x^{2}-1}}\right)bx+\sqrt{d^{2}x^{2}-1}\,\operatorname{csgn}(d)da+\ln\left(\left(\sqrt{d^{2}x^{2}-1}\,\operatorname{csgn}(d)+xd\right)\operatorname{csgn}(d)\right)cx\right)\sqrt{xd-1}\,\sqrt{xd+1}\,\operatorname{csgn}(d)}{\sqrt{d^{2}x^{2}-1}\,xd}$	9

input
```
int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
a*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x+(c*ln(d^2*x/(d^2)^(1/2)+(d^2*x^2-1)^(1/2))
/(d^2)^(1/2)-b*arctan(1/(d^2*x^2-1)^(1/2)))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1
)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.49

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$=\frac{ad^2x+2\,bdx\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}ad-cx\log\left(-dx+\sqrt{dx+1}\sqrt{dx-\right.}{dx}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

output
```
(a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
 1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 29.68 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.93

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{adG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{iadG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{cG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

input
```
integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
-a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1)
, ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2)
, (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1,
 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/
4, 1, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4,
0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d
**2*x**2))/(4*pi**(3/2)*d)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -b\arcsin\left(\frac{1}{d|x|}\right) + \frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}d\right)}{d} + \frac{\sqrt{d^2x^2-1}a}{x}$$

input
```
integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
-b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(
d^2*x^2 - 1)*a/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -d\left(\frac{2\,b\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d} - \frac{8\,a}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4} + \frac{c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-\right.\right.}{d^2}\right.$$

input
```
integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

output
```
-d*(2*b*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2)/d - 8*a/((sqrt(d*x +
 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2)/d^2
)
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.15

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x} - \frac{4\,c\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}} - b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right) - \ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)\,1\mathrm{i}$$

input
```
int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i
))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/(
(d*x + 1)^(1/2) - 1)))*1i
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2atan\left(\sqrt{dx-1}+\sqrt{dx+1}-1\right)bdx - 2atan\left(\sqrt{dx-1}+\sqrt{dx+1}+1\right)bdx + \sqrt{dx+1}\,\sqrt{dx-1}\,ad + 2}{dx}$$

input
```
int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
(2*atan(sqrt(d*x - 1) + sqrt(d*x + 1) - 1)*b*d*x - 2*atan(sqrt(d*x - 1) +
sqrt(d*x + 1) + 1)*b*d*x + sqrt(d*x + 1)*sqrt(d*x - 1)*a*d + 2*log((sqrt(d
*x - 1) + sqrt(d*x + 1))/sqrt(2))*c*x + a*d**2*x)/(d*x)
```

3.14 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{2x^2}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{x} + \frac{1}{2}\left(2c+ad^2\right)\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
1/2*a*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^2+b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x+1/2*
(a*d^2+2*c)*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{(a+2bx)\sqrt{-1+dx}\sqrt{1+dx}}{2x^2} + \left(2c+ad^2\right)\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
((a + 2*b*x)*Sqrt[-1 + d*x]*Sqrt[1 + d*x])/(2*x^2) + (2*c + a*d^2)*ArcTan[
Sqrt[(-1 + d*x)/(1 + d*x)]]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2113, 2338, 534, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{ 2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x^3\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\int \frac{2b+(ad^2+2c)x}{x^2\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\left((ad^2+2c)\int \frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{2b\sqrt{d^2x^2-1}}{x}\right)+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\left(\frac{1}{2}(ad^2+2c)\int \frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{2b\sqrt{d^2x^2-1}}{x}\right)+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\left(\frac{(ad^2+2c)\int \frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}}{d^2}+\frac{2b\sqrt{d^2x^2-1}}{x}\right)+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{2}\left((ad^2+2c)\arctan\left(\sqrt{d^2x^2-1}\right)+\frac{2b\sqrt{d^2x^2-1}}{x}\right)+\frac{a\sqrt{d^2x^2-1}}{2x^2}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((a*Sqrt[-1 + d^2*x^2])/(2*x^2) + ((2*b*Sqrt[-1 + d^2*
x^2])/x + (2*c + a*d^2)*ArcTan[Sqrt[-1 + d^2*x^2]])/2))/(Sqrt[-1 + d*x]*Sq
rt[1 + d*x])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\sqrt{xd+1}\,\sqrt{xd-1}\,(2bx+a)}{2x^2}-\frac{\left(c+\frac{a\,d^2}{2}\right)\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\sqrt{(xd+1)(xd-1)}}{\sqrt{xd-1}\,\sqrt{xd+1}}$	76
default	$-\frac{\sqrt{xd-1}\,\sqrt{xd+1}\,\mathrm{csgn}(d)^2\left(\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)a\,d^2x^2+2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)c\,x^2-2\sqrt{d^2x^2-1}\,bx-\sqrt{d^2x^2-1}\,a\right)}{2\sqrt{d^2x^2-1}\,x^2}$	103

input
```
int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(2*b*x+a)/x^2-(c+1/2*a*d^2)*arctan(1/(d^2*
x^2-1)^(1/2))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{2\,bdx^2+2\,(ad^2+2\,c)x^2\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+(2\,bx+a)\sqrt{dx+1}\sqrt{dx-1}}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x
- 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right)-c\arcsin\left(\frac{1}{d|x|}\right)+\frac{\sqrt{d^2x^2-1}b}{x}+\frac{\sqrt{d^2x^2-1}a}{2\,x^2}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 -
1)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(67) = 134$.

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.75

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx =$$
$$-\frac{(ad^3+2\,cd)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}\cdots\right.\right.}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

input
```
integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

output
```
-((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3
*(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1
))^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x +
1) - sqrt(d*x - 1))^4 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.81

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\frac{a\,d^2\,1\mathrm{i}}{32}+\frac{a\,d^2\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{16\,\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4\,15\mathrm{i}}{32\,\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{2\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}} - c\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right) - \ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i} - \frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)\,1\mathrm{i}}{2} + \frac{a\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2} + \frac{b\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x} + \frac{a\,d^2\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2\,1\mathrm{i}}{32\,\left(\sqrt{d\,x+1}-1\right)^2}$$

input
```
int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2)
- 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4
))/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2)
 - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1
/2) - 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)
 - log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*
x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x
 - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x +
 1)^(1/2))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) -
1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$= \frac{2atan\left(\sqrt{dx-1}+\sqrt{dx+1}-1\right)a\,d^2x^2+4atan\left(\sqrt{dx-1}+\sqrt{dx+1}-1\right)c\,x^2-2atan\left(\sqrt{dx-1}+\sqrt{d}\right.}{2x}$$

input
```
int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
(2*atan(sqrt(d*x - 1) + sqrt(d*x + 1) - 1)*a*d**2*x**2 + 4*atan(sqrt(d*x -
 1) + sqrt(d*x + 1) - 1)*c*x**2 - 2*atan(sqrt(d*x - 1) + sqrt(d*x + 1) + 1
)*a*d**2*x**2 - 4*atan(sqrt(d*x - 1) + sqrt(d*x + 1) + 1)*c*x**2 + sqrt(d*
x + 1)*sqrt(d*x - 1)*a + 2*sqrt(d*x + 1)*sqrt(d*x - 1)*b*x)/(2*x**2)
```

3.15 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 116

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{3x^3}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{2x^2}+\frac{(3c+2ad^2)\sqrt{-1+dx}\sqrt{1+dx}}{3x}+\frac{1}{2}bd^2\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
1/3*a*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^3+1/2*b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^
2+1/3*(2*a*d^2+3*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x+1/2*b*d^2*arctan((d*x-1)
^(1/2)*(d*x+1)^(1/2))
```

Mathematica [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.61

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+dx}\sqrt{1+dx}(3x(b+2cx)+a(2+4d^2x^2))}{6x^3}+bd^2\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)))/(6*x^
3) + b*d^2*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2113, 2338, 539, 534, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^4\sqrt{dx-1}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x^4\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\int \frac{3b+(2ad^2+3c)x}{x^3\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{539}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{3bxd^2+2(2ad^2+3c)}{x^2\sqrt{d^2x^2-1}}dx+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\left(3bd^2\int \frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{2\sqrt{d^2x^2-1}(2ad^2+3c)}{x}\right)+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{3}{2}bd^2\int \frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{2\sqrt{d^2x^2-1}(2ad^2+3c)}{x}\right)+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\left(3b\int \frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}+\frac{2\sqrt{d^2x^2-1}(2ad^2+3c)}{x}\right)+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{2\sqrt{d^2x^2-1}(2ad^2+3c)}{x}+3bd^2\arctan\left(\sqrt{d^2x^2-1}\right)\right)+\frac{3b\sqrt{d^2x^2-1}}{2x^2}\right)+\frac{a\sqrt{d^2x^2-1}}{3x^3}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((a*Sqrt[-1 + d^2*x^2])/(3*x^3) + ((3*b*Sqrt[-1 + d^2*
x^2])/(2*x^2) + ((2*(3*c + 2*a*d^2)*Sqrt[-1 + d^2*x^2])/x + 3*b*d^2*ArcTan
[Sqrt[-1 + d^2*x^2]])/2)/3))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

method	result	s
risch	$\frac{\sqrt{xd+1}\,\sqrt{xd-1}\,(4a\,d^2x^2+6c\,x^2+3bx+2a)}{6x^3}-\frac{b\,d^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\sqrt{(xd+1)(xd-1)}}{2\sqrt{xd-1}\,\sqrt{xd+1}}$	8
default	$-\frac{\sqrt{xd-1}\,\sqrt{xd+1}\,\operatorname{csgn}(d)^2\left(3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)b\,d^2x^3-4\sqrt{d^2x^2-1}\,a\,d^2x^2-6\sqrt{d^2x^2-1}\,c\,x^2-3\sqrt{d^2x^2-1}\,bx-2\sqrt{d^2x^2-1}\,a\right)}{6\sqrt{d^2x^2-1}\,x^3}$	1

input
```
int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(4*a*d^2*x^2+6*c*x^2+3*b*x+2*a)/x^3-1/2*b*
d^2*arctan(1/(d^2*x^2-1)^(1/2))*((d*x+1)*(d*x-1))^(1/2)/(d*x-1)^(1/2)/(d*x
+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$= \frac{6\,bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+2\,(2\,ad^3+3\,cd)x^3+(2\,(2\,ad^2+3\,c)x^2+3\,bx+2\,a)\sqrt{dx+1}{6\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
 3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x
 - 1))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{2}\,bd^2\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\sqrt{d^2x^2-1}ad^2}{3\,x}+\frac{\sqrt{d^2x^2-1}c}{x}+\frac{\sqrt{d^2x^2-1}b}{2\,x^2}+\frac{\sqrt{d^2x^2-1}a}{3\,x^3}$$

input
```
integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
-1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2
*x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(92) = 184.

Time = 0.14 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.61

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{1}{3}\,d^3\left(\frac{3\,b\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{2\left(3\,bd\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,c\left(\sqrt{dx+1}-\sqrt{c\right.\right.}{}\right.$$

input
```
integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

output
```
-1/3*d^3*(3*b*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2)/d + 2*(3*b*d*(
sqrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*(sqrt(d*x + 1) - sqrt(d*x - 1))^8
 - 96*a*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*(sqrt(d*x + 1) - sqrt
(d*x - 1))^4 - 48*b*d*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^2 - 192*
c)/(((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3*d^2))
```

Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.62

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\frac{b\,d^2\,1\mathrm{i}}{32}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}} - \frac{b\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2} + \frac{b\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2} + \frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x} + \frac{\sqrt{d\,x-1}\,\left(\frac{2\,a\,d^3\,x^3}{3}+\frac{2\,a\,d^2\,x^2}{3}+\frac{a\,d\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{d\,x+1}} + \frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

input
```
int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2)
- 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4
))/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2)
 - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1
/2) - 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2
+ 1)*1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/
2 + (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d
^2*x^2)/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*(
(d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.21

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$
$$= \frac{6atan\left(\sqrt{dx-1}+\sqrt{dx+1}-1\right)b\,d^2x^3-6atan\left(\sqrt{dx-1}+\sqrt{dx+1}+1\right)b\,d^2x^3+4\sqrt{dx+1}\,\sqrt{dx-1}\,a}{6x^3}$$

input
```
int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
(6*atan(sqrt(d*x - 1) + sqrt(d*x + 1) - 1)*b*d**2*x**3 - 6*atan(sqrt(d*x -
 1) + sqrt(d*x + 1) + 1)*b*d**2*x**3 + 4*sqrt(d*x + 1)*sqrt(d*x - 1)*a*d**
2*x**2 + 2*sqrt(d*x + 1)*sqrt(d*x - 1)*a + 3*sqrt(d*x + 1)*sqrt(d*x - 1)*b
*x + 6*sqrt(d*x + 1)*sqrt(d*x - 1)*c*x**2 - 4*a*d**3*x**3 - 2*c*d*x**3)/(6
*x**3)
```

3.16 $\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 154

$$\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{a\sqrt{-1+dx}\sqrt{1+dx}}{4x^4}+\frac{b\sqrt{-1+dx}\sqrt{1+dx}}{3x^3} + \frac{(4c+3ad^2)\sqrt{-1+dx}\sqrt{1+dx}}{8x^2} + \frac{2bd^2\sqrt{-1+dx}\sqrt{1+dx}}{3x} + \frac{1}{8}d^2(4c+3ad^2)\arctan\left(\sqrt{-1+dx}\sqrt{1+dx}\right)$$

output
```
1/4*a*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^4+1/3*b*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^
3+1/8*(3*a*d^2+4*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)/x^2+2/3*b*d^2*(d*x-1)^(1/2
)*(d*x+1)^(1/2)/x+1/8*d^2*(3*a*d^2+4*c)*arctan((d*x-1)^(1/2)*(d*x+1)^(1/2)
)
```

Mathematica [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.60

$$\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{1}{24}\left(\frac{\sqrt{-1+dx}\sqrt{1+dx}(a(6+9d^2x^2)+4x(3cx+b(2+4d^2x^2)))}{x^4} + 6d^2\left(4c+3ad^2\right)\arctan\left(\sqrt{\frac{-1+dx}{1+dx}}\right)\right)$$

input
```
Integrate[(a + b*x + c*x^2)/(x^5*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
((Sqrt[-1 + d*x]*Sqrt[1 + d*x]*(a*(6 + 9*d^2*x^2) + 4*x*(3*c*x + b*(2 + 4*
d^2*x^2))))/x^4 + 6*d^2*(4*c + 3*a*d^2)*ArcTan[Sqrt[(-1 + d*x)/(1 + d*x)]]
)/24
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2113, 2338, 539, 539, 27, 534, 243, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{x^5\sqrt{dx-1}\sqrt{dx+1}}\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{d^2x^2-1}\int \frac{cx^2+bx+a}{x^5\sqrt{d^2x^2-1}}dx}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{4}\int \frac{4b+(3ad^2+4c)x}{x^4\sqrt{d^2x^2-1}}dx+\frac{a\sqrt{d^2x^2-1}}{4x^4}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 539$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{4}\left(\frac{1}{3}\int\frac{8bxd^2+3(3ad^2+4c)}{x^3\sqrt{d^2x^2-1}}dx+\frac{4b\sqrt{d^2x^2-1}}{3x^3}\right)+\frac{a\sqrt{d^2x^2-1}}{4x^4}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 539$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int\frac{d^2(16b+3(3ad^2+4c)x)}{x^2\sqrt{d^2x^2-1}}dx+\frac{3\sqrt{d^2x^2-1}(3ad^2+4c)}{2x^2}\right)+\frac{4b\sqrt{d^2x^2-1}}{3x^3}\right)+\frac{a\sqrt{d^2x^2-1}}{4x^4}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\int\frac{16b+3(3ad^2+4c)x}{x^2\sqrt{d^2x^2-1}}dx+\frac{3\sqrt{d^2x^2-1}(3ad^2+4c)}{2x^2}\right)+\frac{4b\sqrt{d^2x^2-1}}{3x^3}\right)+\frac{a\sqrt{d^2x^2-1}}{4x^4}\right)}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 534$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\left(3(3ad^2+4c)\int\frac{1}{x\sqrt{d^2x^2-1}}dx+\frac{16b\sqrt{d^2x^2-1}}{x}\right)+\frac{3\sqrt{d^2x^2-1}(3ad^2+4c)}{2x^2}\right)+\frac{4b\sqrt{d^2x^2-1}}{3x^3}\right)+\frac{a\sqrt{d^2x^2-}}{4x^4}\right.}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 243$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\left(\frac{3}{2}(3ad^2+4c)\int\frac{1}{x^2\sqrt{d^2x^2-1}}dx^2+\frac{16b\sqrt{d^2x^2-1}}{x}\right)+\frac{3\sqrt{d^2x^2-1}(3ad^2+4c)}{2x^2}\right)+\frac{4b\sqrt{d^2x^2-1}}{3x^3}\right)+\frac{a\sqrt{d^2x}}{4x^4}\right.}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 73$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\left(\frac{3(3ad^2+4c)\int\frac{1}{\frac{x^4}{d^2}+\frac{1}{d^2}}d\sqrt{d^2x^2-1}}{d^2}+\frac{16b\sqrt{d^2x^2-1}}{x}\right)+\frac{3\sqrt{d^2x^2-1}(3ad^2+4c)}{2x^2}\right)+\frac{4b\sqrt{d^2x^2-1}}{3x^3}\right)+\frac{a\sqrt{d^2}}{4}\right.}{\sqrt{dx-1}\sqrt{dx+1}}$$

$$\Big\downarrow \ 218$$

$$\frac{\sqrt{d^2x^2-1}\left(\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}d^2\left(3(3ad^2+4c)\arctan\left(\sqrt{d^2x^2-1}\right)+\frac{16b\sqrt{d^2x^2-1}}{x}\right)+\frac{3\sqrt{d^2x^2-1}(3ad^2+4c)}{2x^2}\right)+\frac{4b\sqrt{d^2x^2-1}}{3x^3}\right)+a\right.}{\sqrt{dx-1}\sqrt{dx+1}}$$

input
```
Int[(a + b*x + c*x^2)/(x^5*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[-1 + d^2*x^2]*((a*Sqrt[-1 + d^2*x^2])/(4*x^4) + ((4*b*Sqrt[-1 + d^2*
x^2])/(3*x^3) + ((3*(4*c + 3*a*d^2)*Sqrt[-1 + d^2*x^2])/(2*x^2) + (d^2*((1
6*b*Sqrt[-1 + d^2*x^2])/x + 3*(4*c + 3*a*d^2)*ArcTan[Sqrt[-1 + d^2*x^2]]))
/2)/3)/4))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

method	result
risch	$\frac{\sqrt{xd+1}\,\sqrt{xd-1}\,\left(16 b\,d^{2} x^{3}+9 a\,d^{2} x^{2}+12 c\,x^{2}+8 b x+6 a\right)}{24 x^{4}}-\frac{d^{2} \left(3 a\,d^{2}+4 c\right) \arctan\left(\frac{1}{\sqrt{d^{2} x^{2}-1}}\right) \sqrt{\left(x d+1\right) \left(x d-1\right)}}{8 \sqrt{x d-1}\,\sqrt{x d+1}}$
default	$-\frac{\sqrt{x d-1}\,\sqrt{x d+1}\,\operatorname{csgn}\left(d\right)^{2} \left(9 \arctan\left(\frac{1}{\sqrt{d^{2} x^{2}-1}}\right) a\,d^{4} x^{4}+12 \arctan\left(\frac{1}{\sqrt{d^{2} x^{2}-1}}\right) c\,d^{2} x^{4}-16 \sqrt{d^{2} x^{2}-1}\,b\,d^{2} x^{3}-9 \sqrt{d^{2} x^{2}-1}\,a\,d^{2} x^{2}-\right.}{24 \sqrt{d^{2} x^{2}-1}\,x^{4}}$

input
```
int((c*x^2+b*x+a)/x^5/(d*x-1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(d*x+1)^(1/2)*(d*x-1)^(1/2)*(16*b*d^2*x^3+9*a*d^2*x^2+12*c*x^2+8*b*x+
6*a)/x^4-1/8*d^2*(3*a*d^2+4*c)*arctan(1/(d^2*x^2-1)^(1/2))*((d*x+1)*(d*x-1
))^(1/2)/(d*x-1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.66

$$\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{16\,bd^3x^4+6\,(3\,ad^4+4\,cd^2)x^4\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+(16\,bd^2x^3+3\,(3\,ad^2+4\,c)x^2+8\,bx+}{24\,x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

output
```
1/24*(16*b*d^3*x^4 + 6*(3*a*d^4 + 4*c*d^2)*x^4*arctan(-d*x + sqrt(d*x + 1)
*sqrt(d*x - 1)) + (16*b*d^2*x^3 + 3*(3*a*d^2 + 4*c)*x^2 + 8*b*x + 6*a)*sqr
t(d*x + 1)*sqrt(d*x - 1))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/x**5/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{3}{8}\,ad^4\arcsin\left(\frac{1}{d|x|}\right)-\frac{1}{2}\,cd^2\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\,\sqrt{d^2x^2-1}bd^2}{3\,x}+\frac{3\,\sqrt{d^2x^2-1}ad^2}{8\,x^2}+\frac{\sqrt{d^2x^2-1}c}{2\,x^2}+\frac{\sqrt{d^2x^2-1}b}{3\,x^3}+\frac{\sqrt{d^2x^2-1}a}{4\,x^4}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

output
```
-3/8*a*d^4*arcsin(1/(d*abs(x))) - 1/2*c*d^2*arcsin(1/(d*abs(x))) + 2/3*sqr
t(d^2*x^2 - 1)*b*d^2/x + 3/8*sqrt(d^2*x^2 - 1)*a*d^2/x^2 + 1/2*sqrt(d^2*x^
2 - 1)*c/x^2 + 1/3*sqrt(d^2*x^2 - 1)*b/x^3 + 1/4*sqrt(d^2*x^2 - 1)*a/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(124) = 248$.

Time = 0.15 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.11

$$\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx = -\frac{3\left(3\,ad^5+4\,cd^3\right)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(9\,ad^5\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{14}+12\,cd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{14}+132\cdots\right)}{\cdots}}{\cdots}$$

input
```
integrate((c*x^2+b*x+a)/x^5/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

output
```
-1/12*(3*(3*a*d^5 + 4*c*d^3)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2)
 + 2*(9*a*d^5*(sqrt(d*x + 1) - sqrt(d*x - 1))^14 + 12*c*d^3*(sqrt(d*x + 1)
 - sqrt(d*x - 1))^14 + 132*a*d^5*(sqrt(d*x + 1) - sqrt(d*x - 1))^10 + 48*c
*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^10 - 384*b*d^4*(sqrt(d*x + 1) - sqrt(
d*x - 1))^8 - 528*a*d^5*(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 192*c*d^3*(sqr
t(d*x + 1) - sqrt(d*x - 1))^6 - 2048*b*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 576*a*d^5*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 768*c*d^3*(sqrt(d*x + 1
) - sqrt(d*x - 1))^2 - 2048*b*d^4)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)
^4)/d
```

Mupad [B] (verification not implemented)

Time = 19.24 (sec) , antiderivative size = 695, normalized size of antiderivative = 4.51

$$\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)/(x^5*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((c*d^2*1i)/32 + (c*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2)
- 1)^2) - (c*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4
))/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2)
 - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1
/2) - 1)^6) - ((a*d^4*1i)/1024 - (a*d^4*((d*x - 1)^(1/2) - 1i)^2*3i)/(128*
((d*x + 1)^(1/2) - 1)^2) - (a*d^4*((d*x - 1)^(1/2) - 1i)^4*53i)/(512*((d*x
 + 1)^(1/2) - 1)^4) + (a*d^4*((d*x - 1)^(1/2) - 1i)^6*87i)/(256*((d*x + 1)
^(1/2) - 1)^6) + (a*d^4*((d*x - 1)^(1/2) - 1i)^8*657i)/(1024*((d*x + 1)^(1
/2) - 1)^8) + (a*d^4*((d*x - 1)^(1/2) - 1i)^10*121i)/(256*((d*x + 1)^(1/2)
 - 1)^10))/(((d*x - 1)^(1/2) - 1i)^4/((d*x + 1)^(1/2) - 1)^4 + (4*((d*x -
1)^(1/2) - 1i)^6)/((d*x + 1)^(1/2) - 1)^6 + (6*((d*x - 1)^(1/2) - 1i)^8)/(
(d*x + 1)^(1/2) - 1)^8 + (4*((d*x - 1)^(1/2) - 1i)^10)/((d*x + 1)^(1/2) -
1)^10 + ((d*x - 1)^(1/2) - 1i)^12/((d*x + 1)^(1/2) - 1)^12) - (a*d^4*log((
(d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*3i)/8 - (c*d^2*log(((
d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^4*log(((d
*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*3i)/8 + (c*d^2*log(((d*x - 1)^(
1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + ((d*x - 1)^(1/2)*(b/3 + (2*b*d^2
*x^2)/3 + (2*b*d^3*x^3)/3 + (b*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (a*d^4*((d
*x - 1)^(1/2) - 1i)^2*7i)/(256*((d*x + 1)^(1/2) - 1)^2) - (a*d^4*((d*x - 1
)^(1/2) - 1i)^4*1i)/(1024*((d*x + 1)^(1/2) - 1)^4) + (c*d^2*((d*x - 1)^...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.31

$$\int \frac{a+bx+cx^2}{x^5\sqrt{-1+dx}\sqrt{1+dx}}\,dx$$

$$= \frac{18atan\left(\sqrt{dx-1}+\sqrt{dx+1}-1\right)a\,d^4x^4+24atan\left(\sqrt{dx-1}+\sqrt{dx+1}-1\right)c\,d^2x^4-18atan\left(\sqrt{dx-1}+\cdots\right.}{}$$

input
```
int((c*x^2+b*x+a)/x^5/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
(18*atan(sqrt(d*x - 1) + sqrt(d*x + 1) - 1)*a*d**4*x**4 + 24*atan(sqrt(d*x
 - 1) + sqrt(d*x + 1) - 1)*c*d**2*x**4 - 18*atan(sqrt(d*x - 1) + sqrt(d*x
+ 1) + 1)*a*d**4*x**4 - 24*atan(sqrt(d*x - 1) + sqrt(d*x + 1) + 1)*c*d**2*
x**4 + 9*sqrt(d*x + 1)*sqrt(d*x - 1)*a*d**2*x**2 + 6*sqrt(d*x + 1)*sqrt(d*
x - 1)*a + 16*sqrt(d*x + 1)*sqrt(d*x - 1)*b*d**2*x**3 + 8*sqrt(d*x + 1)*sq
rt(d*x - 1)*b*x + 12*sqrt(d*x + 1)*sqrt(d*x - 1)*c*x**2 - 16*b*d**3*x**4)/
(24*x**4)
```

3.17 $\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 210

$$\begin{aligned}\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{d^4(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^{10}} \\ &+\frac{d^2(4cd^4+3bd^2e^2+2ae^4)\,(d-ex)^{3/2}(d+ex)^{3/2}}{3e^{10}} \\ &-\frac{(6cd^4+3bd^2e^2+ae^4)\,(d-ex)^{5/2}(d+ex)^{5/2}}{5e^{10}} \\ &+\frac{(4cd^2+be^2)\,(d-ex)^{7/2}(d+ex)^{7/2}}{7e^{10}} \\ &-\frac{c(d-ex)^{9/2}(d+ex)^{9/2}}{9e^{10}}\end{aligned}$$

output
```
-d^4*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^10+1/3*d^2*(2*
a*e^4+3*b*d^2*e^2+4*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^10-1/5*(a*e^4+3*
b*d^2*e^2+6*c*d^4)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^10+1/7*(b*e^2+4*c*d^2)*(
-e*x+d)^(7/2)*(e*x+d)^(7/2)/e^10-1/9*c*(-e*x+d)^(9/2)*(e*x+d)^(9/2)/e^10
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{\sqrt{d-ex}\sqrt{d+ex}(21ae^4(8d^4+4d^2e^2x^2+3e^4x^4)+9b(16d^6e^2+8d^4e^4x^2+6d^2e^6x^4+5e^8x^6)+c(128d^8}{315e^{10}}$$

input
```
Integrate[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*a*e^4*(8*d^4 + 4*d^2*e^2*x^2 + 3*e
^4*x^4) + 9*b*(16*d^6*e^2 + 8*d^4*e^4*x^2 + 6*d^2*e^6*x^4 + 5*e^8*x^6) + c
*(128*d^8 + 64*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 40*d^2*e^6*x^6 + 35*e^8*x^8)
))/e^10
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1905, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^5\left(cx^4+bx^2+a\right)}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^4\left(cx^4+bx^2+a\right)}{\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1195}$$

$$\frac{\sqrt{d^2-e^2x^2}\int\left(\frac{c\left(d^2-e^2x^2\right)^{7/2}}{e^8}+\frac{\left(-4cd^2-be^2\right)\left(d^2-e^2x^2\right)^{5/2}}{e^8}+\frac{\left(6cd^4+3be^2d^2+ae^4\right)\left(d^2-e^2x^2\right)^{3/2}}{e^8}+\frac{\left(-4cd^6-3be^2d^4-2ae^4d^2\right)\sqrt{d^2-}}{e^8}\right.}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{2\left(d^2-e^2x^2\right)^{5/2}\left(ae^4+3bd^2e^2+6cd^4\right)}{5e^{10}}+\frac{2d^2\left(d^2-e^2x^2\right)^{3/2}\left(2ae^4+3bd^2e^2+4cd^4\right)}{3e^{10}}-\frac{2d^4\sqrt{d^2-e^2x^2}\left(ae^4+bd^2e^2+cd^4\right)}{e^{10}}+\frac{2(}{}\right.}{2\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*((-2*d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d^2 - e^2*x
^2])/e^10 + (2*d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d^2 - e^2*x^2)^(3/2)
)/(3*e^10) - (2*(6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^(5/2))/(5*
e^10) + (2*(4*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^(7/2))/(7*e^10) - (2*c*(d^2 -
 e^2*x^2)^(9/2))/(9*e^10)))/(2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.69

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^{8}e^{8}+45b\,e^{8}x^{6}+40c\,d^{2}e^{6}x^{6}+63a\,e^{8}x^{4}+54b\,d^{2}e^{6}x^{4}+48c\,d^{4}e^{4}x^{4}+84a\,d^{2}e^{6}x^{2}+72b\,d^{4}e^{4}x^{2}+64c\,d^{6}e^{2}x^{2}+1}{315e^{10}}$
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^{8}e^{8}+45b\,e^{8}x^{6}+40c\,d^{2}e^{6}x^{6}+63a\,e^{8}x^{4}+54b\,d^{2}e^{6}x^{4}+48c\,d^{4}e^{4}x^{4}+84a\,d^{2}e^{6}x^{2}+72b\,d^{4}e^{4}x^{2}+64c\,d^{6}e^{2}x^{2}+1}{315e^{10}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^{8}e^{8}+45b\,e^{8}x^{6}+40c\,d^{2}e^{6}x^{6}+63a\,e^{8}x^{4}+54b\,d^{2}e^{6}x^{4}+48c\,d^{4}e^{4}x^{4}+84a\,d^{2}e^{6}x^{2}+72b\,d^{4}e^{4}x^{2}+64c\,d^{6}e^{2}x^{2}+1}{315e^{10}}$
orering	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(35c\,x^{8}e^{8}+45b\,e^{8}x^{6}+40c\,d^{2}e^{6}x^{6}+63a\,e^{8}x^{4}+54b\,d^{2}e^{6}x^{4}+48c\,d^{4}e^{4}x^{4}+84a\,d^{2}e^{6}x^{2}+72b\,d^{4}e^{4}x^{2}+64c\,d^{6}e^{2}x^{2}+1}{315e^{10}}$

input
```
int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(35*c*e^8*x^8+45*b*e^8*x^6+40*c*d^2*e^
6*x^6+63*a*e^8*x^4+54*b*d^2*e^6*x^4+48*c*d^4*e^4*x^4+84*a*d^2*e^6*x^2+72*b
*d^4*e^4*x^2+64*c*d^6*e^2*x^2+168*a*d^4*e^4+144*b*d^6*e^2+128*c*d^8)/e^10
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.66

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(35\,ce^8x^8+128\,cd^8+144\,bd^6e^2+168\,ad^4e^4+5\,(8\,cd^2e^6+9\,be^8)x^6+3\,(16\,cd^4e^4+18\,bd^2e^6+21\,ae^8)}{315\,e^{10}}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/315*(35*c*e^8*x^8 + 128*c*d^8 + 144*b*d^6*e^2 + 168*a*d^4*e^4 + 5*(8*c*
d^2*e^6 + 9*b*e^8)*x^6 + 3*(16*c*d^4*e^4 + 18*b*d^2*e^6 + 21*a*e^8)*x^4 +
4*(16*c*d^6*e^2 + 18*b*d^4*e^4 + 21*a*d^2*e^6)*x^2)*sqrt(e*x + d)*sqrt(-e*
x + d)/e^10
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.95 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.75

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iad^5G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{ad^5G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{ibd^7G_{6,6}^{6,2}\left(\begin{matrix} -\frac{13}{4},-\frac{11}{4} & -3,-3,-\frac{5}{2},1 \\ -\frac{7}{2},-\frac{13}{4},-3,-\frac{11}{4},-\frac{5}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{bd^7G_{6,6}^{2,6}\left(\begin{matrix} -4,-\frac{15}{4},-\frac{7}{2},-\frac{13}{4},-3,1 & \\ -\frac{15}{4},-\frac{13}{4} & -4,-\frac{7}{2},-\frac{7}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{icd^9G_{6,6}^{6,2}\left(\begin{matrix} -\frac{17}{4},-\frac{15}{4} & -4,-4,-\frac{7}{2},1 \\ -\frac{9}{2},-\frac{17}{4},-4,-\frac{15}{4},-\frac{7}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^{10}} - \frac{cd^9G_{6,6}^{2,6}\left(\begin{matrix} -5,-\frac{19}{4},-\frac{9}{2},-\frac{17}{4},-4,1 & \\ -\frac{19}{4},-\frac{17}{4} & -5,-\frac{9}{2},-\frac{9}{2},0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^{10}}$$

input
```
integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
-I*a*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/
4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - a*d**5*meijerg(((
-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)),
d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6) - I*b*d**7*meijerg
(((-13/4, -11/4), (-3, -3, -5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0),
()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**8) - b*d**7*meijerg(((-4, -15/4, -7
/2, -13/4, -3, 1), ()), ((-15/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_po
lar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**8) - I*c*d**9*meijerg(((-17/4, -
15/4), (-4, -4, -7/2, 1)), ((-9/2, -17/4, -4, -15/4, -7/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**10) - c*d**9*meijerg(((-5, -19/4, -9/2, -17/4,
 -4, 1), ()), ((-19/4, -17/4), (-5, -9/2, -9/2, 0)), d**2*exp_polar(-2*I*p
i)/(e**2*x**2))/(4*pi**(3/2)*e**10)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{\sqrt{-e^2x^2+d^2}cx^8}{9\,e^2}-\frac{8\,\sqrt{-e^2x^2+d^2}cd^2x^6}{63\,e^4}\\&-\frac{\sqrt{-e^2x^2+d^2}bx^6}{7\,e^2}-\frac{16\,\sqrt{-e^2x^2+d^2}cd^4x^4}{105\,e^6}\\&-\frac{6\,\sqrt{-e^2x^2+d^2}bd^2x^4}{35\,e^4}-\frac{\sqrt{-e^2x^2+d^2}ax^4}{5\,e^2}\\&-\frac{64\,\sqrt{-e^2x^2+d^2}cd^6x^2}{315\,e^8}-\frac{8\,\sqrt{-e^2x^2+d^2}bd^4x^2}{35\,e^6}\\&-\frac{4\,\sqrt{-e^2x^2+d^2}ad^2x^2}{15\,e^4}-\frac{128\,\sqrt{-e^2x^2+d^2}cd^8}{315\,e^{10}}\\&-\frac{16\,\sqrt{-e^2x^2+d^2}bd^6}{35\,e^8}-\frac{8\,\sqrt{-e^2x^2+d^2}ad^4}{15\,e^6}\end{aligned}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/9*sqrt(-e^2*x^2 + d^2)*c*x^8/e^2 - 8/63*sqrt(-e^2*x^2 + d^2)*c*d^2*x^6/
e^4 - 1/7*sqrt(-e^2*x^2 + d^2)*b*x^6/e^2 - 16/105*sqrt(-e^2*x^2 + d^2)*c*d
^4*x^4/e^6 - 6/35*sqrt(-e^2*x^2 + d^2)*b*d^2*x^4/e^4 - 1/5*sqrt(-e^2*x^2 +
 d^2)*a*x^4/e^2 - 64/315*sqrt(-e^2*x^2 + d^2)*c*d^6*x^2/e^8 - 8/35*sqrt(-e
^2*x^2 + d^2)*b*d^4*x^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*a*d^2*x^2/e^4 - 12
8/315*sqrt(-e^2*x^2 + d^2)*c*d^8/e^10 - 16/35*sqrt(-e^2*x^2 + d^2)*b*d^6/e
^8 - 8/15*sqrt(-e^2*x^2 + d^2)*a*d^4/e^6
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.09

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(315\,cd^8+315\,bd^6e^2+315\,ad^4e^4-(840\,cd^7+630\,bd^5e^2+420\,ad^3e^4-(1932\,cd^6+1071\,bd^4e^2+462\,a}{}$$

input
```
integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/315*(315*c*d^8 + 315*b*d^6*e^2 + 315*a*d^4*e^4 - (840*c*d^7 + 630*b*d^5
*e^2 + 420*a*d^3*e^4 - (1932*c*d^6 + 1071*b*d^4*e^2 + 462*a*d^2*e^4 - (295
2*c*d^5 + 1116*b*d^3*e^2 + 252*a*d*e^4 - (3098*c*d^4 + 729*b*d^2*e^2 + 63*
a*e^4 - 5*(440*c*d^3 + 54*b*d*e^2 - (204*c*d^2 + 9*b*e^2 + 7*((e*x + d)*c
- 8*c*d)*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))
*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d)/e^10
```

Mupad [B] (verification not implemented)

Time = 4.14 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.37

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\,\left(\frac{128\,c\,d^9+144\,b\,d^7\,e^2+168\,a\,d^5\,e^4}{315\,e^{10}}+\frac{x^7\,\left(40\,c\,d^2\,e^7+45\,b\,e^9\right)}{315\,e^{10}}+\frac{x^2\,\left(64\,c\,d^7\,e^2+72\,b\,d^5\,e^4+84\,a\,d^3\,e^6\right)}{315\,e^{10}}+\frac{x^3\,\left(64\,c\,d^6\,e^3+72\,b}{315\,e}\right.}{}$$

input
```
int((x^5*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*((128*c*d^9 + 168*a*d^5*e^4 + 144*b*d^7*e^2)/(315*e^10)
+ (x^7*(45*b*e^9 + 40*c*d^2*e^7))/(315*e^10) + (x^2*(84*a*d^3*e^6 + 72*b*d
^5*e^4 + 64*c*d^7*e^2))/(315*e^10) + (x^3*(84*a*d^2*e^7 + 72*b*d^4*e^5 + 6
4*c*d^6*e^3))/(315*e^10) + (c*x^9)/(9*e) + (x^5*(63*a*e^9 + 54*b*d^2*e^7 +
 48*c*d^4*e^5))/(315*e^10) + (x*(168*a*d^4*e^5 + 144*b*d^6*e^3 + 128*c*d^8
*e))/(315*e^10) + (x^6*(40*c*d^3*e^6 + 45*b*d*e^8))/(315*e^10) + (x^4*(54*
b*d^3*e^6 + 48*c*d^5*e^4 + 63*a*d*e^8))/(315*e^10) + (c*d*x^8)/(9*e^2)))/(
d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.68

$$\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(-35c\,e^8x^8-45b\,e^8x^6-40c\,d^2e^6x^6-63a\,e^8x^4-54b\,d^2e^6x^4-48c\,d^4e^4x^4-84a\,d^2e^6}{315e^{10}}$$

input
```
int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(d - e*x)*( - 168*a*d**4*e**4 - 84*a*d**2*e**6*x**2 - 6
3*a*e**8*x**4 - 144*b*d**6*e**2 - 72*b*d**4*e**4*x**2 - 54*b*d**2*e**6*x**
4 - 45*b*e**8*x**6 - 128*c*d**8 - 64*c*d**6*e**2*x**2 - 48*c*d**4*e**4*x**
4 - 40*c*d**2*e**6*x**6 - 35*c*e**8*x**8))/(315*e**10)
```

3.18 $\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 159

$$\begin{aligned}\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{d^2(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^8}\\ &+\frac{(3cd^4+2bd^2e^2+ae^4)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^8}\\ &-\frac{(3cd^2+be^2)(d-ex)^{5/2}(d+ex)^{5/2}}{5e^8}\\ &+\frac{c(d-ex)^{7/2}(d+ex)^{7/2}}{7e^8}\end{aligned}$$

output
```
-d^2*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^8+1/3*(a*e^4+2
*b*d^2*e^2+3*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^8-1/5*(b*e^2+3*c*d^2)*(
-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^8+1/7*c*(-e*x+d)^(7/2)*(e*x+d)^(7/2)/e^8
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.73

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(35ae^4(2d^2+e^2x^2)+7b(8d^4e^2+4d^2e^4x^2+3e^6x^4)+3c(16d^6+8d^4e^2x^2+6d^2e^4x^4+}{105e^8}$$

input
```
Integrate[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*a*e^4*(2*d^2 + e^2*x^2) + 7*b*(8*d
^4*e^2 + 4*d^2*e^4*x^2 + 3*e^6*x^4) + 3*c*(16*d^6 + 8*d^4*e^2*x^2 + 6*d^2*
e^4*x^4 + 5*e^6*x^6)))/e^8
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1905, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^3(cx^4+bx^2+a)}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^2(cx^4+bx^2+a)}{\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1195}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \left(-\frac{c\left(d^2-e^2x^2\right)^{5/2}}{e^6}+\frac{\left(3cd^2+be^2\right)\left(d^2-e^2x^2\right)^{3/2}}{e^6}+\frac{\left(-3cd^4-2be^2d^2-ae^4\right)\sqrt{d^2-e^2x^2}}{e^6}+\frac{cd^6+be^2d^4+ae^4d^2}{e^6\sqrt{d^2-e^2x^2}}\right)dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2\left(d^2-e^2x^2\right)^{3/2}\left(ae^4+2bd^2e^2+3cd^4\right)}{3e^8}-\frac{2d^2\sqrt{d^2-e^2x^2}\left(ae^4+bd^2e^2+cd^4\right)}{e^8}-\frac{2\left(d^2-e^2x^2\right)^{5/2}\left(be^2+3cd^2\right)}{5e^8}+\frac{2c\left(d^2-e^2x^2\right)^{7/2}}{7e^8}\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*((-2*d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d^2 - e^2*x
^2])/e^8 + (2*(3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^(3/2))/(3*e^
8) - (2*(3*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^(5/2))/(5*e^8) + (2*c*(d^2 - e^2
*x^2)^(7/2))/(7*e^8)))/(2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15c\,x^{6}e^{6}+21b\,e^{6}x^{4}+18c\,d^{2}e^{4}x^{4}+35a\,e^{6}x^{2}+28b\,d^{2}e^{4}x^{2}+24c\,d^{4}e^{2}x^{2}+70a\,d^{2}e^{4}+56b\,d^{4}e^{2}+48c\,d^{6}\right)}{105e^{8}}$	109
default	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15c\,x^{6}e^{6}+21b\,e^{6}x^{4}+18c\,d^{2}e^{4}x^{4}+35a\,e^{6}x^{2}+28b\,d^{2}e^{4}x^{2}+24c\,d^{4}e^{2}x^{2}+70a\,d^{2}e^{4}+56b\,d^{4}e^{2}+48c\,d^{6}\right)}{105e^{8}}$	109
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15c\,x^{6}e^{6}+21b\,e^{6}x^{4}+18c\,d^{2}e^{4}x^{4}+35a\,e^{6}x^{2}+28b\,d^{2}e^{4}x^{2}+24c\,d^{4}e^{2}x^{2}+70a\,d^{2}e^{4}+56b\,d^{4}e^{2}+48c\,d^{6}\right)}{105e^{8}}$	109
orering	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15c\,x^{6}e^{6}+21b\,e^{6}x^{4}+18c\,d^{2}e^{4}x^{4}+35a\,e^{6}x^{2}+28b\,d^{2}e^{4}x^{2}+24c\,d^{4}e^{2}x^{2}+70a\,d^{2}e^{4}+56b\,d^{4}e^{2}+48c\,d^{6}\right)}{105e^{8}}$	109

input
```
int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*c*e^6*x^6+21*b*e^6*x^4+18*c*d^2*e^
4*x^4+35*a*e^6*x^2+28*b*d^2*e^4*x^2+24*c*d^4*e^2*x^2+70*a*d^2*e^4+56*b*d^4
*e^2+48*c*d^6)/e^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(15\,ce^6x^6+48\,cd^6+56\,bd^4e^2+70\,ad^2e^4+3\,(6\,cd^2e^4+7\,be^6)x^4+(24\,cd^4e^2+28\,bd^2e^4+35\,ae^6)x^2)\sqrt{\cdots}}{105\,e^8}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/105*(15*c*e^6*x^6 + 48*c*d^6 + 56*b*d^4*e^2 + 70*a*d^2*e^4 + 3*(6*c*d^2
*e^4 + 7*b*e^6)*x^4 + (24*c*d^4*e^2 + 28*b*d^2*e^4 + 35*a*e^6)*x^2)*sqrt(e
*x + d)*sqrt(-e*x + d)/e^8
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.02 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.31

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iad^3 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{ad^3 G_{6,6}^{2,6}\left(\begin{matrix} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 & \\ -\frac{7}{4}, -\frac{5}{4} & -2, -\frac{3}{2}, -\frac{3}{2}, 0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{ibd^5 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4}, -\frac{7}{4} & -2, -2, -\frac{3}{2}, 1 \\ -\frac{5}{2}, -\frac{9}{4}, -2, -\frac{7}{4}, -\frac{3}{2}, 0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{bd^5 G_{6,6}^{2,6}\left(\begin{matrix} -3, -\frac{11}{4}, -\frac{5}{2}, -\frac{9}{4}, -2, 1 & \\ -\frac{11}{4}, -\frac{9}{4} & -3, -\frac{5}{2}, -\frac{5}{2}, 0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{icd^7 G_{6,6}^{6,2}\left(\begin{matrix} -\frac{13}{4}, -\frac{11}{4} & -3, -3, -\frac{5}{2}, 1 \\ -\frac{7}{2}, -\frac{13}{4}, -3, -\frac{11}{4}, -\frac{5}{2}, 0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8} - \frac{cd^7 G_{6,6}^{2,6}\left(\begin{matrix} -4, -\frac{15}{4}, -\frac{7}{2}, -\frac{13}{4}, -3, 1 & \\ -\frac{15}{4}, -\frac{13}{4} & -4, -\frac{7}{2}, -\frac{7}{2}, 0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^8}$$

input
```
integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
-I*a*d**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/
4, -1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - a*d**3*meijerg(((
-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d*
*2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4) - I*b*d**5*meijerg((
(-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), d
**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - b*d**5*meijerg(((-3, -11/4, -5/2, -9
/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), d**2*exp_polar(-2*I
*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6) - I*c*d**7*meijerg(((-13/4, -11/4), (
-3, -3, -5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0), ()), d**2/(e**2*x**
2))/(4*pi**(3/2)*e**8) - c*d**7*meijerg(((-4, -15/4, -7/2, -13/4, -3, 1),
()), ((-15/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*
x**2))/(4*pi**(3/2)*e**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^6}{7\,e^2} - \frac{6\,\sqrt{-e^2x^2+d^2}cd^2x^4}{35\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^4}{5\,e^2} - \frac{8\,\sqrt{-e^2x^2+d^2}cd^4x^2}{35\,e^6} - \frac{4\,\sqrt{-e^2x^2+d^2}bd^2x^2}{15\,e^4} - \frac{\sqrt{-e^2x^2+d^2}ax^2}{3\,e^2} - \frac{16\,\sqrt{-e^2x^2+d^2}cd^6}{35\,e^8} - \frac{8\,\sqrt{-e^2x^2+d^2}bd^4}{15\,e^6} - \frac{2\,\sqrt{-e^2x^2+d^2}ad^2}{3\,e^4}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/7*sqrt(-e^2*x^2 + d^2)*c*x^6/e^2 - 6/35*sqrt(-e^2*x^2 + d^2)*c*d^2*x^4/
e^4 - 1/5*sqrt(-e^2*x^2 + d^2)*b*x^4/e^2 - 8/35*sqrt(-e^2*x^2 + d^2)*c*d^4
*x^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*b*d^2*x^2/e^4 - 1/3*sqrt(-e^2*x^2 + d
^2)*a*x^2/e^2 - 16/35*sqrt(-e^2*x^2 + d^2)*c*d^6/e^8 - 8/15*sqrt(-e^2*x^2
+ d^2)*b*d^4/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*a*d^2/e^4
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{(105\,cd^6+105\,bd^4e^2+105\,ad^2e^4-(210\,cd^5+140\,bd^3e^2+70\,ade^4-(357\,cd^4+154\,bd^2e^2+35\,ae^4-3}{}$$

input
```
integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/105*(105*c*d^6 + 105*b*d^4*e^2 + 105*a*d^2*e^4 - (210*c*d^5 + 140*b*d^3
*e^2 + 70*a*d*e^4 - (357*c*d^4 + 154*b*d^2*e^2 + 35*a*e^4 - 3*(124*c*d^3 +
 28*b*d*e^2 - (81*c*d^2 + 7*b*e^2 + 5*((e*x + d)*c - 6*c*d)*(e*x + d))*(e*
x + d))*(e*x + d))*(e*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d)/e^8
```

Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.35

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{\sqrt{d-e\,x}\,\left(\frac{48\,c\,d^7+56\,b\,d^5\,e^2+70\,a\,d^3\,e^4}{105\,e^8}+\frac{x^5\,\left(18\,c\,d^2\,e^5+21\,b\,e^7\right)}{105\,e^8}+\frac{c\,x^7}{7\,e}+\frac{x^3\,\left(24\,c\,d^4\,e^3+28\,b\,d^2\,e^5+35\,a\,e^7\right)}{105\,e^8}+\frac{x\,(48\,c\,d^6\,e+56\,b}{105}\right.}{\sqrt{d+e\,x}}$$

input
```
int((x^3*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*((48*c*d^7 + 70*a*d^3*e^4 + 56*b*d^5*e^2)/(105*e^8) + (x
^5*(21*b*e^7 + 18*c*d^2*e^5))/(105*e^8) + (c*x^7)/(7*e) + (x^3*(35*a*e^7 +
 28*b*d^2*e^5 + 24*c*d^4*e^3))/(105*e^8) + (x*(70*a*d^2*e^5 + 56*b*d^4*e^3
 + 48*c*d^6*e))/(105*e^8) + (x^4*(18*c*d^3*e^4 + 21*b*d*e^6))/(105*e^8) +
(x^2*(28*b*d^3*e^4 + 24*c*d^5*e^2 + 35*a*d*e^6))/(105*e^8) + (c*d*x^6)/(7*
e^2)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

$$\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(-15c\,e^6x^6-21b\,e^6x^4-18c\,d^2e^4x^4-35a\,e^6x^2-28b\,d^2e^4x^2-24c\,d^4e^2x^2-70a\,d^2e^4}{105e^8}$$

input
```
int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(d - e*x)*( - 70*a*d**2*e**4 - 35*a*e**6*x**2 - 56*b*d*
*4*e**2 - 28*b*d**2*e**4*x**2 - 21*b*e**6*x**4 - 48*c*d**6 - 24*c*d**4*e**
2*x**2 - 18*c*d**2*e**4*x**4 - 15*c*e**6*x**6))/(105*e**8)
```

3.19 $\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 109

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(cd^4+bd^2e^2+ae^4)\sqrt{d-ex}\sqrt{d+ex}}{e^6} + \frac{(2cd^2+be^2)(d-ex)^{3/2}(d+ex)^{3/2}}{3e^6} - \frac{c(d-ex)^{5/2}(d+ex)^{5/2}}{5e^6}$$

output
```
-(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6+1/3*(b*e^2+2*c*d
^2)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^6-1/5*c*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^
6
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(5(2bd^2e^2+3ae^4+be^4x^2)+c(8d^4+4d^2e^2x^2+3e^4x^4))}{15e^6}$$

input
```
Integrate[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(5*(2*b*d^2*e^2 + 3*a*e^4 + b*e^4*x^2)
+ c*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)))/e^6
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1905, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x(cx^4+bx^2+a)}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1576}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1140}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \left(\frac{c(d^2-e^2x^2)^{3/2}}{e^4}+\frac{(-2cd^2-be^2)\sqrt{d^2-e^2x^2}}{e^4}+\frac{cd^4+be^2d^2+ae^4}{e^4\sqrt{d^2-e^2x^2}}\right)dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{2\sqrt{d^2-e^2x^2}(ae^4+bd^2e^2+cd^4)}{e^6}+\frac{2(d^2-e^2x^2)^{3/2}(be^2+2cd^2)}{3e^6}-\frac{2c(d^2-e^2x^2)^{5/2}}{5e^6}\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*((-2*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d^2 - e^2*x^2])
/e^6 + (2*(2*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^(3/2))/(3*e^6) - (2*c*(d^2 - e
^2*x^2)^(5/2))/(5*e^6)))/(2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{\sqrt{e x+d}\,\sqrt{-e x+d}\,\left(3 c\,x^{4} e^{4}+5 b\,e^{4} x^{2}+4 c\,d^{2} e^{2} x^{2}+15 a\,e^{4}+10 b\,d^{2} e^{2}+8 c\,d^{4}\right)}{15 e^{6}}$	73
default	$-\frac{\sqrt{e x+d}\,\sqrt{-e x+d}\,\left(3 c\,x^{4} e^{4}+5 b\,e^{4} x^{2}+4 c\,d^{2} e^{2} x^{2}+15 a\,e^{4}+10 b\,d^{2} e^{2}+8 c\,d^{4}\right)}{15 e^{6}}$	73
risch	$-\frac{\sqrt{e x+d}\,\sqrt{-e x+d}\,\left(3 c\,x^{4} e^{4}+5 b\,e^{4} x^{2}+4 c\,d^{2} e^{2} x^{2}+15 a\,e^{4}+10 b\,d^{2} e^{2}+8 c\,d^{4}\right)}{15 e^{6}}$	73
orering	$-\frac{\sqrt{e x+d}\,\sqrt{-e x+d}\,\left(3 c\,x^{4} e^{4}+5 b\,e^{4} x^{2}+4 c\,d^{2} e^{2} x^{2}+15 a\,e^{4}+10 b\,d^{2} e^{2}+8 c\,d^{4}\right)}{15 e^{6}}$	73

input
```
int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*c*e^4*x^4+5*b*e^4*x^2+4*c*d^2*e^2*x^
2+15*a*e^4+10*b*d^2*e^2+8*c*d^4)/e^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\left(3\,ce^4x^4+8\,cd^4+10\,bd^2e^2+15\,ae^4+\left(4\,cd^2e^2+5\,be^4\right)x^2\right)\sqrt{ex+d}\sqrt{-ex+d}}{15\,e^6}$$

input
```
integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/15*(3*c*e^4*x^4 + 8*c*d^4 + 10*b*d^2*e^2 + 15*a*e^4 + (4*c*d^2*e^2 + 5*
b*e^4)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/e^6
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.71 (sec) , antiderivative size = 350, normalized size of antiderivative = 3.21

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{iadG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\,\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{adG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\,\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{ibd^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\,\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{bd^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\,\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{icd^5G_{6,6}^{6,2}\left(\begin{matrix} -\frac{9}{4},-\frac{7}{4} & -2,-2,-\frac{3}{2},1 \\ -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},0 & \end{matrix}\middle|\,\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6} - \frac{cd^5G_{6,6}^{2,6}\left(\begin{matrix} -3,-\frac{11}{4},-\frac{5}{2},-\frac{9}{4},-2,1 & \\ -\frac{11}{4},-\frac{9}{4} & -3,-\frac{5}{2},-\frac{5}{2},0 \end{matrix}\middle|\,\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^6}$$

input
```
integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
-I*a*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0
), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - a*d*meijerg(((-1, -3/4, -1/
2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-
2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*b*d**3*meijerg(((-5/4, -3/4),
(-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2
))/(4*pi**(3/2)*e**4) - b*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ())
, ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2)
)/(4*pi**(3/2)*e**4) - I*c*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)),
 ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**
6) - c*d**5*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4),
(-3, -5/2, -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e*
*6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^4}{5\,e^2} - \frac{4\sqrt{-e^2x^2+d^2}cd^2x^2}{15\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^2}{3\,e^2} - \frac{8\sqrt{-e^2x^2+d^2}cd^4}{15\,e^6} - \frac{2\sqrt{-e^2x^2+d^2}bd^2}{3\,e^4} - \frac{\sqrt{-e^2x^2+d^2}a}{e^2}$$

input
```
integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

output
```
-1/5*sqrt(-e^2*x^2 + d^2)*c*x^4/e^2 - 4/15*sqrt(-e^2*x^2 + d^2)*c*d^2*x^2/
e^4 - 1/3*sqrt(-e^2*x^2 + d^2)*b*x^2/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*c*d^4
/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*b*d^2/e^4 - sqrt(-e^2*x^2 + d^2)*a/e^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(15\,cd^4+15\,bd^2e^2+15\,ae^4-(20\,cd^3+10\,bde^2-(22\,cd^2+5\,be^2+3\,((ex+d)c-4\,cd)(ex+d))(ex+}{15\,e^6}$$

input
```
integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

output
```
-1/15*(15*c*d^4 + 15*b*d^2*e^2 + 15*a*e^4 - (20*c*d^3 + 10*b*d*e^2 - (22*c
*d^2 + 5*b*e^2 + 3*((e*x + d)*c - 4*c*d)*(e*x + d))*(e*x + d))*(e*x + d))*
sqrt(e*x + d)*sqrt(-e*x + d)/e^6
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\left(\frac{8\,c\,d^5+10\,b\,d^3\,e^2+15\,a\,d\,e^4}{15\,e^6}+\frac{x^3\,\left(4\,c\,d^2\,e^3+5\,b\,e^5\right)}{15\,e^6}+\frac{c\,x^5}{5\,e}+\frac{x^2\,\left(4\,c\,d^3\,e^2+5\,b\,d\,e^4\right)}{15\,e^6}+\frac{x\,\left(8\,c\,d^4\,e+10\,b\,d^2\,e^3+15\,a\,e^5\right)}{15\,e^6}+\right.}{\sqrt{d+e\,x}}$$

input
```
int((x*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*((8*c*d^5 + 10*b*d^3*e^2 + 15*a*d*e^4)/(15*e^6) + (x^3*(
5*b*e^5 + 4*c*d^2*e^3))/(15*e^6) + (c*x^5)/(5*e) + (x^2*(4*c*d^3*e^2 + 5*b
*d*e^4))/(15*e^6) + (x*(15*a*e^5 + 10*b*d^2*e^3 + 8*c*d^4*e))/(15*e^6) + (
c*d*x^4)/(5*e^2)))/(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

$$\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(-3c\,e^4x^4-5b\,e^4x^2-4c\,d^2e^2x^2-15a\,e^4-10b\,d^2e^2-8c\,d^4)}{15e^6}$$

input
```
int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(d - e*x)*( - 15*a*e**4 - 10*b*d**2*e**2 - 5*b*e**4*x**
2 - 8*c*d**4 - 4*c*d**2*e**2*x**2 - 3*c*e**4*x**4))/(15*e**6)
```

3.20 $\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 100

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(2cd^2+3be^2)\sqrt{d-ex}\sqrt{d+ex}}{3e^4} - \frac{cx^2\sqrt{d-ex}\sqrt{d+ex}}{3e^2} - \frac{a\,\text{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{d}$$

output
```
-1/3*(3*b*e^2+2*c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4-1/3*c*x^2*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/e^2-a*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(2cd^2+3be^2+ce^2x^2)}{3e^4} + \frac{a\log\left(-1+\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{d} - \frac{a\log\left(d+\frac{d\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{d}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(2*c*d^2 + 3*b*e^2 + c*e^2*x^2))/e^4 + (
a*Log[-1 + Sqrt[d + e*x]/Sqrt[d - e*x]])/d - (a*Log[d + (d*Sqrt[d + e*x])/
Sqrt[d - e*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1905, 1578, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^2\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1192}$$

$$\frac{\sqrt{d^2-e^2x^2}\int -\frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1467}$$

$$-\frac{\sqrt{d^2-e^2x^2}\int \left(\frac{ae^4}{d^2-x^4}+be^2-cx^4+cd^2\right)d\sqrt{d^2-e^2x^2}}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{ae^4\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}-\sqrt{d^2-e^2x^2}\,(be^2+cd^2)+\frac{cx^6}{3}\right)}{e^4\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*((c*x^6)/3 - (c*d^2 + b*e^2)*Sqrt[d^2 - e^2*x^2] - (a
*e^4*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.84 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(\operatorname{csgn}(d) c d\, e^{2} x^{2} \sqrt{-e^{2}x^{2}+d^{2}}+3\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(d) b d\, e^{2}+2\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(d) c\, d^{3}+3\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\ \mathrm{c}\right.}{x}\right.\right.}{3d\sqrt{-e^{2}x^{2}+d^{2}}\,e^{4}}$

input
```
int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d*(csgn(d)*c*d*e^2*x^2*(-e^2*x^2+d^2)^(1
/2)+3*(-e^2*x^2+d^2)^(1/2)*csgn(d)*b*d*e^2+2*(-e^2*x^2+d^2)^(1/2)*csgn(d)*
c*d^3+3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4)*csgn(d)/(-e^2*x^
2+d^2)^(1/2)/e^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{3\,ae^4\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-\left(cde^2x^2+2\,cd^3+3\,bde^2\right)\sqrt{ex+d}\sqrt{-ex+d}}{3\,de^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fri
cas")
```

output
```
1/3*(3*a*e^4*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) - (c*d*e^2*x^2 + 2*
c*d^3 + 3*b*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d*e^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.04

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{ibdG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{bdG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^2} - \frac{icd^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4} - \frac{cd^3G_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}$$

input
```
integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
 d**2/(e**2*x**2))/(4*pi**(3/2)*d) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/
(4*pi**(3/2)*d) - I*b*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/
4, 0, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - b*d*meijer
g(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)),
 d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*c*d**3*meijer
g(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ())
, d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - c*d**3*meijerg(((-2, -7/4, -3/2,
-5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*
I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^2}{3\,e^2} - \frac{a\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{2\,\sqrt{-e^2x^2+d^2}cd^2}{3\,e^4} - \frac{\sqrt{-e^2x^2+d^2}b}{e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="max
ima")
```

output
```
-1/3*sqrt(-e^2*x^2 + d^2)*c*x^2/e^2 - a*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2
 + d^2)*d/abs(x))/d - 2/3*sqrt(-e^2*x^2 + d^2)*c*d^2/e^4 - sqrt(-e^2*x^2 +
 d^2)*b/e^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(84) = 168.

Time = 0.28 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.89

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{3\,ae^4\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d} - \frac{3\,ae^4\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}-2\right|\right)}{d} + (3\,cd^2+3\,be^2+((ex}{3\,e^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gia
c")
```

output
```
-1/3*(3*a*e^4*log(abs(-(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) +
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)) + 2))/d - 3*a*e^4*log(abs
(-(sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2
)*sqrt(d) - sqrt(-e*x + d)) - 2))/d + (3*c*d^2 + 3*b*e^2 + ((e*x + d)*c -
2*c*d)*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d))/e^4
```

Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.61

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{a\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d} - \frac{\sqrt{d-e\,x}\left(\frac{2\,c\,d^3}{3\,e^4}+\frac{c\,x^3}{3\,e}+\frac{c\,d\,x^2}{3\,e^2}+\frac{2\,c\,d^2\,x}{3\,e^3}\right)}{\sqrt{d+e\,x}} - \frac{\left(\frac{b\,d}{e^2}+\frac{b\,x}{e}\right)\sqrt{d-e\,x}}{\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
(a*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
 log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - ((d -
e*x)^(1/2)*((2*c*d^3)/(3*e^4) + (c*x^3)/(3*e) + (c*d*x^2)/(3*e^2) + (2*c*d
^2*x)/(3*e^3)))/(d + e*x)^(1/2) - (((b*d)/e^2 + (b*x)/e)*(d - e*x)^(1/2))/
(d + e*x)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.99

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{-3\sqrt{ex+d}\,\sqrt{-ex+d}\,bd\,e^2-2\sqrt{ex+d}\,\sqrt{-ex+d}\,c\,d^3-\sqrt{ex+d}\,\sqrt{-ex+d}\,cd\,e^2x^2-3\log\left(-\sqrt{2}+\text{ta}\right.}{}$$

input
```
int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 3*sqrt(d + e*x)*sqrt(d - e*x)*b*d*e**2 - 2*sqrt(d + e*x)*sqrt(d - e*x)
*c*d**3 - sqrt(d + e*x)*sqrt(d - e*x)*c*d*e**2*x**2 - 3*log( - sqrt(2) + t
an(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*a*e**4 + 3*log( - sqrt(2)
 + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*a*e**4 - 3*log(sqrt(2
) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*a*e**4 + 3*log(sqrt(
2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*a*e**4)/(3*d*e**4)
```

3.21 $\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 99

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{c\sqrt{d-ex}\sqrt{d+ex}}{e^2} - \frac{a\sqrt{d-ex}\sqrt{d+ex}}{2d^2x^2} - \frac{(2bd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{2d^3}$$

output
```
-c*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^2-1/2*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2
/x^2-1/2*(a*e^2+2*b*d^2)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{\sqrt{d-ex}\sqrt{d+ex}(ade^2+2cd^3x^2)}{e^2x^2} + 2(2bd^2+ae^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{2d^3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/2*((Sqrt[d - e*x]*Sqrt[d + e*x]*(a*d*e^2 + 2*c*d^3*x^2))/(e^2*x^2) + 2*
(2*b*d^2 + a*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e*x]])/d^3
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.51, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1905, 1578, 1192, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^3\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^4\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{(d^2-x^4)^2}d\sqrt{d^2-e^2x^2}}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1471}$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(\frac{ae^4\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}-\frac{\int -\frac{2cd^4-2cx^4d^2+2be^2d^2+ae^4}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{2d^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{2cd^4-2cx^4d^2+2be^2d^2+ae^4}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{2d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{299}$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(\frac{e^2\left(ae^2+2bd^2\right)\int \frac{1}{d^2-x^4}d\sqrt{d^2-e^2x^2}+2cd^2\sqrt{d^2-e^2x^2}}{2d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{e^2\left(ae^2+2bd^2\right)\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d}+2cd^2\sqrt{d^2-e^2x^2}}{2d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-((Sqrt[d^2 - e^2*x^2]*((a*e^4*Sqrt[d^2 - e^2*x^2])/(2*d^2*(d^2 - x^4)) +
(2*c*d^2*Sqrt[d^2 - e^2*x^2] + (e^2*(2*b*d^2 + a*e^2)*ArcTanh[Sqrt[d^2 - e
^2*x^2]/d])/d)/(2*d^2)))/(e^2*Sqrt[d - e*x]*Sqrt[d + e*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{a\sqrt{-ex+d}\,\sqrt{ex+d}}{2d^{2}x^{2}}+\frac{\left(-\frac{\left(a\,e^{2}+2b\,d^{2}\right)\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\sqrt{-e^{2}x^{2}+d^{2}}}{x}\right)}{\sqrt{d^{2}}}-\frac{2c\,d^{2}\sqrt{-(ex-d)(ex+d)}}{e^{2}}\right)\sqrt{(ex+d)(-ex+d)}}{2d^{2}\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(2\,\mathrm{csgn}(d)c\,d^{3}x^{2}\sqrt{-e^{2}x^{2}+d^{2}}+\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\,\mathrm{csgn}(d)+d\right)}{x}\right)a\,e^{4}x^{2}+2\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\,\mathrm{csgn}(d)+d\right)}{x}\right)b\,d^{2}}{2d^{3}\sqrt{-e^{2}x^{2}+d^{2}}\,x^{2}e^{2}}$

input
```
int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/2*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^2+1/2/d^2*(-(a*e^2+2*b*d^2)/(d^2
)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-2*c*d^2/e^2*(-(e*
x-d)*(e*x+d))^(1/2))*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\,cd^4x^2-(2\,bd^2e^2+ae^4)x^2\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)+(2\,cd^3x^2+ade^2)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^3e^2x^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/2*(2*c*d^4*x^2 - (2*b*d^2*e^2 + a*e^4)*x^2*log((sqrt(e*x + d)*sqrt(-e*x
 + d) - d)/x) + (2*c*d^3*x^2 + a*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^3
*e^2*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.24

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{b\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d} - \frac{ae^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^3} - \frac{\sqrt{-e^2x^2+d^2}c}{e^2} - \frac{\sqrt{-e^2x^2+d^2}a}{2\,d^2x^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-b*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*a*e^2*log(2
*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - sqrt(-e^2*x^2 + d^2)*
c/e^2 - 1/2*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(83) = 166$.

Time = 0.32 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.78

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\sqrt{ex+d}\sqrt{-ex+d}c + \frac{(2\,bd^2e^2+ae^4)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d^3} - \frac{(2\,bd^2e^2+ae^4)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\right.\right.}{d^3}}{2\,e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/2*(2*sqrt(e*x + d)*sqrt(-e*x + d)*c + (2*b*d^2*e^2 + a*e^4)*log(abs(-(s
qrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sq
rt(d) - sqrt(-e*x + d)) + 2))/d^3 - (2*b*d^2*e^2 + a*e^4)*log(abs(-(sqrt(2
)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d)
 - sqrt(-e*x + d)) - 2))/d^3 - 4*(a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d)
)/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 + 4*
a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(s
qrt(2)*sqrt(d) - sqrt(-e*x + d))))/((((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/s
qrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2 - 4)^2*
d^3))/e^2
```

Mupad [B] (verification not implemented)

Time = 6.33 (sec) , antiderivative size = 422, normalized size of antiderivative = 4.26

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{b\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d} - \frac{\left(\frac{c\,d}{e^2}+\frac{c\,x}{e}\right)\sqrt{d-e\,x}}{\sqrt{d+e\,x}} - \frac{\frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{a\,e^2}{2}+\frac{15\,a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{2\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}}{\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{32\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^6}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^6}} - \frac{a\,e^2\,\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,d^3} + \frac{a\,e^2\,\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)}{2\,d^3} + \frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{32\,d^3\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}$$

input
```
int((a + b*x^2 + c*x^4)/(x^3*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
(b*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
 log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - (((c*d
)/e^2 + (c*x)/e)*(d - e*x)^(1/2))/(d + e*x)^(1/2) - ((a*e^2*((d + e*x)^(1/
2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (a*e^2)/2 + (15*a*e^2*((d
 + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((
d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d +
 e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*
x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6) - (a*e^2*log(((d + e
*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (a*e^2*log(((
d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(2*d^3) +
(a*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^
2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.47

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$= \frac{-\sqrt{ex+d}\,\sqrt{-ex+d}\,a d e^2 - 2\sqrt{ex+d}\,\sqrt{-ex+d}\,c\,d^3x^2 - \log\left(-\sqrt{2}+\tan\left(\frac{asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\,\sqrt{2}}\right)}{2}\right)-1\right)a\,e^4x^2}{}$$

input
```
int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - sqrt(d + e*x)*sqrt(d - e*x)*a*d*e**2 - 2*sqrt(d + e*x)*sqrt(d - e*x)*c
*d**3*x**2 - log( - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2)
 - 1)*a*e**4*x**2 - 2*log( - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqr
t(2)))/2) - 1)*b*d**2*e**2*x**2 + log( - sqrt(2) + tan(asin(sqrt(d - e*x)/
(sqrt(d)*sqrt(2)))/2) + 1)*a*e**4*x**2 + 2*log( - sqrt(2) + tan(asin(sqrt(
d - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*b*d**2*e**2*x**2 - log(sqrt(2) + tan(a
sin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*a*e**4*x**2 - 2*log(sqrt(2) +
 tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*b*d**2*e**2*x**2 + log(
sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*a*e**4*x**2 +
2*log(sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*b*d**2*e
**2*x**2)/(2*d**3*e**2*x**2)
```

3.22 $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 126

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx=-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4}-\frac{(4bd^2+3ae^2)\sqrt{d-ex}\sqrt{d+ex}}{8d^4x^2}-\frac{(8cd^4+4bd^2e^2+3ae^4)\,\text{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{8d^5}$$

output
```
-1/4*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^4-1/8*(3*a*e^2+4*b*d^2)*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/d^4/x^2-1/8*(3*a*e^4+4*b*d^2*e^2+8*c*d^4)*arctanh((-e
*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx=-\frac{\frac{d\sqrt{d-ex}\sqrt{d+ex}(2ad^2+4bd^2x^2+3ae^2x^2)}{x^4}+2(8cd^4+4bd^2e^2+3ae^4)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{8d^5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/8*((d*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*d^2 + 4*b*d^2*x^2 + 3*a*e^2*x^2)
)/x^4 + 2*(8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*ArcTanh[Sqrt[d + e*x]/Sqrt[d -
 e*x]])/d^5
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.43, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1905, 1578, 1192, 25, 1471, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^5\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^6\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1192}$$

$$\frac{\sqrt{d^2-e^2x^2}\int -\frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{(d^2-x^4)^3}d\sqrt{d^2-e^2x^2}}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{(d^2-x^4)^3}d\sqrt{d^2-e^2x^2}}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int -\frac{4cd^4-4cx^4d^2+4be^2d^2+3ae^4}{\left(d^2-x^4\right)^2}d\sqrt{d^2-e^2x^2}}{4d^2}-\frac{ae^4\sqrt{d^2-e^2x^2}}{4d^2(d^2-x^4)^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int \frac{4cd^4-4cx^4d^2+4be^2d^2+3ae^4}{\left(d^2-x^4\right)^2}d\sqrt{d^2-e^2x^2}}{4d^2}-\frac{ae^4\sqrt{d^2-e^2x^2}}{4d^2\left(d^2-x^4\right)^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 298$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\frac{\left(3ae^4+4bd^2e^2+8cd^4\right)\int \frac{1}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{2d^2}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{3ae^2}{d^2}+4b\right)}{2\left(d^2-x^4\right)}}{4d^2}-\frac{ae^4\sqrt{d^2-e^2x^2}}{4d^2\left(d^2-x^4\right)^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(3ae^4+4bd^2e^2+8cd^4\right)}{2d^3}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{3ae^2}{d^2}+4b\right)}{2\left(d^2-x^4\right)}}{4d^2}-\frac{ae^4\sqrt{d^2-e^2x^2}}{4d^2\left(d^2-x^4\right)^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/4*(a*e^4*Sqrt[d^2 - e^2*x^2])/(d^2*(d^2 - x^4)^2)
 - ((e^2*(4*b + (3*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/(2*(d^2 - x^4)) + ((8*
c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^3))/(4
*d^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(3a\,e^2x^2+4b\,d^2x^2+2a\,d^2)}{8d^4x^4}-\frac{(3a\,e^4+4b\,d^2e^2+8c\,d^4)\ln\left(\frac{2d^2+2\sqrt{d^2}\,\sqrt{-e^2x^2+d^2}}{x}\right)\sqrt{(ex+d)(-ex+d)}}{8d^4\sqrt{d^2}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\left(3\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(d)+d\right)}{x}\right)a\,e^4x^4+4\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(d)+d\right)}{x}\right)b\,d^2e^2x^4+8\ln\left(\frac{2d\left(\sqrt{-e^2x^2+d^2}\right.}{x}\right.\right.}{8d^5\sqrt{-e^2x^2+}}$

input
```
int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/8*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*a*e^2*x^2+4*b*d^2*x^2+2*a*d^2)/d^4/x^
4-1/8/d^4*(3*a*e^4+4*b*d^2*e^2+8*c*d^4)/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2
)*(-e^2*x^2+d^2)^(1/2))/x)*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{(8\,cd^4+4\,bd^2e^2+3\,ae^4)x^4\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-(2\,ad^3+(4\,bd^3+3\,ade^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{8\,d^5x^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
1/8*((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*log((sqrt(e*x + d)*sqrt(-e*x +
d) - d)/x) - (2*a*d^3 + (4*b*d^3 + 3*a*d*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x
 + d))/(d^5*x^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.53

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{c\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{d}-\frac{be^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^3} - \frac{3\,ae^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d^5}-\frac{\sqrt{-e^2x^2+d^2}b}{2\,d^2x^2} - \frac{3\,\sqrt{-e^2x^2+d^2}ae^2}{8\,d^4x^2}-\frac{\sqrt{-e^2x^2+d^2}a}{4\,d^2x^4}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-c*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*b*e^2*log(2
*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*a*e^4*log(2*d^2/a
bs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 1/2*sqrt(-e^2*x^2 + d^2)*b/
(d^2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^2) - 1/4*sqrt(-e^2*x^2 +
 d^2)*a/(d^2*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(108) = 216$.

Time = 0.43 (sec) , antiderivative size = 767, normalized size of antiderivative = 6.09

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$

$$-\frac{\frac{(8\,cd^4e+4\,bd^2e^3+3\,ae^5)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}+2\right|\right)}{d^5}-\frac{(8\,cd^4e+4\,bd^2e^3+3\,ae^5)\log\left(\left|-\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}+\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right.\right.}{d^5}}{}$$

input
```
integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/8*((8*c*d^4*e + 4*b*d^2*e^3 + 3*a*e^5)*log(abs(-(sqrt(2)*sqrt(d) - sqrt
(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d
)) + 2))/d^5 - (8*c*d^4*e + 4*b*d^2*e^3 + 3*a*e^5)*log(abs(-(sqrt(2)*sqrt(
d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt
(-e*x + d)) - 2))/d^5 - 4*(4*b*d^2*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))
/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 5*a
*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sq
rt(2)*sqrt(d) - sqrt(-e*x + d)))^7 - 16*b*d^2*e^3*((sqrt(2)*sqrt(d) - sqrt
(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d
)))^5 + 12*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(
e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 - 64*b*d^2*e^3*((sqrt(2)*sq
rt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - s
qrt(-e*x + d)))^3 + 48*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x
+ d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 + 256*b*d^2*e^3
*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2
)*sqrt(d) - sqrt(-e*x + d))) + 320*a*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d
))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))/((((
sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*s
qrt(d) - sqrt(-e*x + d)))^2 - 4)^4*d^5))/e
```

Mupad [B] (verification not implemented)

Time = 11.17 (sec) , antiderivative size = 932, normalized size of antiderivative = 7.40

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2 + c*x^4)/(x^5*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((a*e^4)/4 + (6*a*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^
(1/2))^2 - (53*a*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) -
d^(1/2))^4) - (87*a*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) -
d^(1/2))^6 + (657*a*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2)
 - d^(1/2))^8) - (121*a*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/
2) - d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2
) - d^(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2)
 - d^(1/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2)
- d^(1/2))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2)
- d^(1/2))^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2)
- d^(1/2))^12) - ((b*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
 d^(1/2))^2 - (b*e^2)/2 + (15*b*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d
- e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d -
e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x
)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(
1/2) - d^(1/2))^6) + (c*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2
) - d^(1/2))^2 - 1) - log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d
^(1/2)))))/d - (3*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) -
 d^(1/2))))/(8*d^5) - (b*e^2*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1
/2) - d^(1/2))))/(2*d^3) + (3*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))^2/(...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 505, normalized size of antiderivative = 4.01

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(d - e*x)*a*d**3 - 3*sqrt(d + e*x)*sqrt(d - e*x)*a
*d*e**2*x**2 - 4*sqrt(d + e*x)*sqrt(d - e*x)*b*d**3*x**2 - 3*log( - sqrt(2
) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*a*e**4*x**4 - 4*log(
 - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*b*d**2*e**2
*x**4 - 8*log( - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) -
1)*c*d**4*x**4 + 3*log( - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2
)))/2) + 1)*a*e**4*x**4 + 4*log( - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(
d)*sqrt(2)))/2) + 1)*b*d**2*e**2*x**4 + 8*log( - sqrt(2) + tan(asin(sqrt(d
 - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*c*d**4*x**4 - 3*log(sqrt(2) + tan(asin(
sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*a*e**4*x**4 - 4*log(sqrt(2) + tan
(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*b*d**2*e**2*x**4 - 8*log(sq
rt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*c*d**4*x**4 + 3*
log(sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*a*e**4*x**
4 + 4*log(sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*b*d*
*2*e**2*x**4 + 8*log(sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2
) + 1)*c*d**4*x**4)/(8*d**5*x**4)
```

3.23 $\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 180

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{6d^2x^6}-\frac{(6bd^2+5ae^2)\sqrt{d-ex}\sqrt{d+ex}}{24d^4x^4}\\&-\frac{(8cd^4+6bd^2e^2+5ae^4)\sqrt{d-ex}\sqrt{d+ex}}{16d^6x^2}\\&-\frac{e^2(8cd^4+6bd^2e^2+5ae^4)\operatorname{arctanh}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{16d^7}\end{aligned}$$

output
```
-1/6*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^6-1/24*(5*a*e^2+6*b*d^2)*(-e*x+d
)^(1/2)*(e*x+d)^(1/2)/d^4/x^4-1/16*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*(-e*x+d)^
(1/2)*(e*x+d)^(1/2)/d^6/x^2-1/16*e^2*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*arctanh
((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d^7
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{d\sqrt{d-ex}\sqrt{d+ex}(6(2bd^4x^2+4cd^4x^4+3bd^2e^2x^4)+a(8d^4+10d^2e^2x^2+15e^4x^4))}{x^6}+6e^2(8cd^4+6bd^2e^2+5ae^4)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{48d^7}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/48*((d*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(2*b*d^4*x^2 + 4*c*d^4*x^4 + 3*b*
d^2*e^2*x^4) + a*(8*d^4 + 10*d^2*e^2*x^2 + 15*e^4*x^4)))/x^6 + 6*e^2*(8*c*
d^4 + 6*b*d^2*e^2 + 5*a*e^4)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e*x]])/d^7
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1905, 1578, 1192, 1471, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{ 1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^7\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1578}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^8\sqrt{d^2-e^2x^2}}dx^2}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1192}$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\int \frac{cx^8-(2cd^2+be^2)x^4+cd^4+ae^4+bd^2e^2}{(d^2-x^4)^4}d\sqrt{d^2-e^2x^2}}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 1471$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}-\frac{\int -\frac{6cd^4-6cx^4d^2+6be^2d^2+5ae^4}{(d^2-x^4)^3}d\sqrt{d^2-e^2x^2}}{6d^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 25$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{6cd^4-6cx^4d^2+6be^2d^2+5ae^4}{(d^2-x^4)^3}d\sqrt{d^2-e^2x^2}}{6d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 298$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{\frac{3(5ae^4+6bd^2e^2+8cd^4)\int \frac{1}{(d^2-x^4)^2}d\sqrt{d^2-e^2x^2}}{4d^2}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{5ae^2}{d^2}+6b\right)}{4(d^2-x^4)^2}}{6d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 215$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{\frac{3(5ae^4+6bd^2e^2+8cd^4)\left(\frac{\int \frac{1}{d^2-x^4}d\sqrt{d^2-e^2x^2}}{2d^2}+\frac{\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)}{4d^2}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{5ae^2}{d^2}+6b\right)}{4(d^2-x^4)^2}}{6d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 219$$

$$-\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3}+\frac{\sqrt{d^2-e^2x^2}}{2d^2(d^2-x^4)}\right)(5ae^4+6bd^2e^2+8cd^4)}{4d^2}+\frac{e^2\sqrt{d^2-e^2x^2}\left(\frac{5ae^2}{d^2}+6b\right)}{4(d^2-x^4)^2}}{6d^2}+\frac{ae^4\sqrt{d^2-e^2x^2}}{6d^2(d^2-x^4)^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-((e^2*Sqrt[d^2 - e^2*x^2]*((a*e^4*Sqrt[d^2 - e^2*x^2])/(6*d^2*(d^2 - x^4)
^3) + ((e^2*(6*b + (5*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/(4*(d^2 - x^4)^2) +
 (3*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*(Sqrt[d^2 - e^2*x^2]/(2*d^2*(d^2 - x
^4)) + ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/(2*d^3)))/(4*d^2))/(6*d^2)))/(Sqrt[d
 - e*x]*Sqrt[d + e*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15a\,e^{4}x^{4}+18b\,d^{2}e^{2}x^{4}+24c\,d^{4}x^{4}+10a\,d^{2}e^{2}x^{2}+12b\,d^{4}x^{2}+8a\,d^{4}\right)}{48d^{6}x^{6}}-\frac{e^{2}\left(5a\,e^{4}+6b\,d^{2}e^{2}+8c\,d^{4}\right)\ln\left(\frac{2d^{2}+2\sqrt{d^{2}}\,\ldots}{\ldots}\right.}{16d^{6}\sqrt{d^{2}}\,\sqrt{ex+d}\ldots}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(15\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(d)+d\right)}{x}\right)a\,e^{6}x^{6}+18\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(d)+d\right)}{x}\right)b\,d^{2}e^{4}x^{6}+24\ln\left(\frac{2d\left(\sqrt{-e^{2}x^{2}+\ldots}\right.}{\ldots}\right.\right.}{\ldots}$

input
```
int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/48*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*a*e^4*x^4+18*b*d^2*e^2*x^4+24*c*d^4
*x^4+10*a*d^2*e^2*x^2+12*b*d^4*x^2+8*a*d^4)/d^6/x^6-1/16*e^2*(5*a*e^4+6*b*
d^2*e^2+8*c*d^4)/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1
/2))/x)*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{3\,(8\,cd^4e^2+6\,bd^2e^4+5\,ae^6)x^6\log\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{x}\right)-(8\,ad^5+3\,(8\,cd^5+6\,bd^3e^2+5\,ade^4)x^4+2\,(6\,bd^5}{48\,d^7x^6}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
1/48*(3*(8*c*d^4*e^2 + 6*b*d^2*e^4 + 5*a*e^6)*x^6*log((sqrt(e*x + d)*sqrt(
-e*x + d) - d)/x) - (8*a*d^5 + 3*(8*c*d^5 + 6*b*d^3*e^2 + 5*a*d*e^4)*x^4 +
 2*(6*b*d^5 + 5*a*d^3*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^7*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.51

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{ce^2\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{2\,d^3} - \frac{3\,be^4\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{8\,d^5} - \frac{5\,ae^6\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}d}{|x|}\right)}{16\,d^7} - \frac{\sqrt{-e^2x^2+d^2}c}{2\,d^2x^2} - \frac{3\,\sqrt{-e^2x^2+d^2}be^2}{8\,d^4x^2} - \frac{5\,\sqrt{-e^2x^2+d^2}ae^4}{16\,d^6x^2} - \frac{\sqrt{-e^2x^2+d^2}b}{4\,d^2x^4} - \frac{5\,\sqrt{-e^2x^2+d^2}ae^2}{24\,d^4x^4} - \frac{\sqrt{-e^2x^2+d^2}a}{6\,d^2x^6}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/2*c*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*b
*e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 5/16*a*e^6*
log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 - 1/2*sqrt(-e^2*x^
2 + d^2)*c/(d^2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^2) - 5/16*sqr
t(-e^2*x^2 + d^2)*a*e^4/(d^6*x^2) - 1/4*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^4) -
 5/24*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^4) - 1/6*sqrt(-e^2*x^2 + d^2)*a/(d
^2*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1434 vs. 2(156) = 312.

Time = 0.61 (sec) , antiderivative size = 1434, normalized size of antiderivative = 7.97

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/48*(3*(8*c*d^4*e^3 + 6*b*d^2*e^5 + 5*a*e^7)*log(abs(-(sqrt(2)*sqrt(d) -
 sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*
x + d)) + 2))/d^7 - 3*(8*c*d^4*e^3 + 6*b*d^2*e^5 + 5*a*e^7)*log(abs(-(sqrt
(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) + sqrt(e*x + d)/(sqrt(2)*sqrt(
d) - sqrt(-e*x + d)) - 2))/d^7 - 4*(24*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(
-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)
))^11 + 30*b*d^2*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - s
qrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 33*a*e^7*((sqrt(2)*s
qrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) -
sqrt(-e*x + d)))^11 - 288*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 168*b*
d^2*e^5*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/
(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 20*a*e^7*((sqrt(2)*sqrt(d) - sqrt(
-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)
))^9 + 768*c*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - s
qrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 192*b*d^2*e^5*((sqrt(
2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d
) - sqrt(-e*x + d)))^7 + 1440*a*e^7*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 3072*c
*d^4*e^3*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x +...
```

Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 1621, normalized size of antiderivative = 9.01

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
int((a + b*x^2 + c*x^4)/(x^7*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((b*e^4)/4 + (6*b*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^
(1/2))^2 - (53*b*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) -
d^(1/2))^4) - (87*b*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) -
d^(1/2))^6 + (657*b*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2)
 - d^(1/2))^8) - (121*b*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/
2) - d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2
) - d^(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2)
 - d^(1/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2)
- d^(1/2))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2)
- d^(1/2))^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2)
- d^(1/2))^12) - ((c*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
 d^(1/2))^2 - (c*e^2)/2 + (15*c*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d
- e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d -
e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x
)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(
1/2) - d^(1/2))^6) + ((a*e^6)/6 + (4*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/
((d - e*x)^(1/2) - d^(1/2))^2 + (71*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/(
(d - e*x)^(1/2) - d^(1/2))^4 - (1558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/
(3*((d - e*x)^(1/2) - d^(1/2))^6) - (540*a*e^6*((d + e*x)^(1/2) - d^(1/2))
^8)/((d - e*x)^(1/2) - d^(1/2))^8 + (4248*a*e^6*((d + e*x)^(1/2) - d^(1...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.27

$$\int \frac{a + bx^2 + cx^4}{x^7\sqrt{d - ex}\sqrt{d + ex}}\,dx = \text{Too large to display}$$

input
```
int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 8*sqrt(d + e*x)*sqrt(d - e*x)*a*d**5 - 10*sqrt(d + e*x)*sqrt(d - e*x)*
a*d**3*e**2*x**2 - 15*sqrt(d + e*x)*sqrt(d - e*x)*a*d*e**4*x**4 - 12*sqrt(
d + e*x)*sqrt(d - e*x)*b*d**5*x**2 - 18*sqrt(d + e*x)*sqrt(d - e*x)*b*d**3
*e**2*x**4 - 24*sqrt(d + e*x)*sqrt(d - e*x)*c*d**5*x**4 - 15*log( - sqrt(2
) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*a*e**6*x**6 - 18*log
( - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*b*d**2*e**
4*x**6 - 24*log( - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2)
- 1)*c*d**4*e**2*x**6 + 15*log( - sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d
)*sqrt(2)))/2) + 1)*a*e**6*x**6 + 18*log( - sqrt(2) + tan(asin(sqrt(d - e*
x)/(sqrt(d)*sqrt(2)))/2) + 1)*b*d**2*e**4*x**6 + 24*log( - sqrt(2) + tan(a
sin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*c*d**4*e**2*x**6 - 15*log(sqr
t(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*a*e**6*x**6 - 18*
log(sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1)*b*d**2*e**
4*x**6 - 24*log(sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))/2) - 1
)*c*d**4*e**2*x**6 + 15*log(sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(d)*sqrt
(2)))/2) + 1)*a*e**6*x**6 + 18*log(sqrt(2) + tan(asin(sqrt(d - e*x)/(sqrt(
d)*sqrt(2)))/2) + 1)*b*d**2*e**4*x**6 + 24*log(sqrt(2) + tan(asin(sqrt(d -
 e*x)/(sqrt(d)*sqrt(2)))/2) + 1)*c*d**4*e**2*x**6)/(48*d**7*x**6)
```

3.24 $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 216

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(5cd^4+6bd^2e^2+8ae^4)\,x\sqrt{d-ex}\sqrt{d+ex}}{16e^6} - \frac{(5cd^2+6be^2)\,x^3\sqrt{d-ex}\sqrt{d+ex}}{24e^4} + \frac{cx^5(-d+ex)\sqrt{d+ex}}{6e^2\sqrt{d-ex}} + \frac{d^2(5cd^4+6bd^2e^2+8ae^4)\sqrt{d^2-e^2x^2}\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}$$

output

```
-1/16*(8*a*e^4+6*b*d^2*e^2+5*c*d^4)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6-1/2
4*(6*b*e^2+5*c*d^2)*x^3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4+1/6*c*x^5*(e*x-d)
*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)+1/16*d^2*(8*a*e^4+6*b*d^2*e^2+5*c*d^4)*(
-e^2*x^2+d^2)^(1/2)*arctan(e*x/(-e^2*x^2+d^2)^(1/2))/e^7/(-e*x+d)^(1/2)/(e
*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{-ex\sqrt{d-ex}\sqrt{d+ex}(6(3bd^2e^2+4ae^4+2be^4x^2)+c(15d^4+10d^2e^2x^2+8e^4x^4))+6d^2(5cd^4+6bd^2e^2+}{48e^7}$$

input
```
Integrate[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(3*b*d^2*e^2 + 4*a*e^4 + 2*b*e^4*x^2
) + c*(15*d^4 + 10*d^2*e^2*x^2 + 8*e^4*x^4))) + 6*d^2*(5*c*d^4 + 6*b*d^2*e
^2 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(48*e^7)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1905, 1590, 25, 363, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \; 1905$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{x^2\left(cx^4+bx^2+a\right)}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 1590$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{x^2\left(6ae^2+\left(5cd^2+6be^2\right)x^2\right)}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{x^2\left(6ae^2+\left(5cd^2+6be^2\right)x^2\right)}{\sqrt{d^2-e^2x^2}}dx}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{363}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(8ae^4+6bd^2e^2+5cd^4\right)\int \frac{x^2}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\left(6b+\frac{5cd^2}{e^2}\right)}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(8ae^4+6bd^2e^2+5cd^4\right)\left(\frac{d^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\left(6b+\frac{5cd^2}{e^2}\right)}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(8ae^4+6bd^2e^2+5cd^4\right)\left(\frac{d^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{2e^2}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\left(6b+\frac{5cd^2}{e^2}\right)}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(\frac{d^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}-\frac{x\sqrt{d^2-e^2x^2}}{2e^2}\right)\left(8ae^4+6bd^2e^2+5cd^4\right)}{4e^2}-\frac{1}{4}x^3\sqrt{d^2-e^2x^2}\left(6b+\frac{5cd^2}{e^2}\right)}{6e^2}-\frac{cx^5\sqrt{d^2-e^2x^2}}{6e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/6*(c*x^5*Sqrt[d^2 - e^2*x^2])/e^2 + (-1/4*((6*b +
 (5*c*d^2)/e^2)*x^3*Sqrt[d^2 - e^2*x^2]) + (3*(5*c*d^4 + 6*b*d^2*e^2 + 8*a
*e^4)*(-1/2*(x*Sqrt[d^2 - e^2*x^2])/e^2 + (d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2
*x^2]])/(2*e^3)))/(4*e^2))/(6*e^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{x\left(8c\,x^{4}e^{4}+12b\,e^{4}x^{2}+10c\,d^{2}e^{2}x^{2}+24a\,e^{4}+18b\,d^{2}e^{2}+15c\,d^{4}\right)\sqrt{-ex+d}\,\sqrt{ex+d}}{48e^{6}}+\frac{d^{2}\left(8a\,e^{4}+6b\,d^{2}e^{2}+5c\,d^{4}\right)\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)\ldots}{16e^{6}\sqrt{e^{2}}\,\sqrt{ex+d}\,\sqrt{-ex+\ldots}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(8\,\mathrm{csgn}(e)c\,e^{5}x^{5}\sqrt{-e^{2}x^{2}+d^{2}}+12\,\mathrm{csgn}(e)b\,e^{5}x^{3}\sqrt{-e^{2}x^{2}+d^{2}}+10\,\mathrm{csgn}(e)c\,d^{2}e^{3}x^{3}\sqrt{-e^{2}x^{2}+d^{2}}+24\sqrt{-e^{2}x^{2}+\ldots}\right.}{\ldots}$

input
```
int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/48*x*(8*c*e^4*x^4+12*b*e^4*x^2+10*c*d^2*e^2*x^2+24*a*e^4+18*b*d^2*e^2+1
5*c*d^4)/e^6*(-e*x+d)^(1/2)*(e*x+d)^(1/2)+1/16*d^2*(8*a*e^4+6*b*d^2*e^2+5*
c*d^4)/e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x+d)
*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.62

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(8\,ce^5x^5+2\,(5\,cd^2e^3+6\,be^5)x^3+3\,(5\,cd^4e+6\,bd^2e^3+8\,ae^5)x)\sqrt{ex+d}\sqrt{-ex+d}+6\,(5\,cd^6+6\,bd^4e^2}{48\,e^7}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/48*((8*c*e^5*x^5 + 2*(5*c*d^2*e^3 + 6*b*e^5)*x^3 + 3*(5*c*d^4*e + 6*b*d
^2*e^3 + 8*a*e^5)*x)*sqrt(e*x + d)*sqrt(-e*x + d) + 6*(5*c*d^6 + 6*b*d^4*e
^2 + 8*a*d^2*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/e^7
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^5}{6\,e^2} - \frac{5\,\sqrt{-e^2x^2+d^2}cd^2x^3}{24\,e^4} - \frac{\sqrt{-e^2x^2+d^2}bx^3}{4\,e^2} + \frac{5\,cd^6\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{16\,\sqrt{e^2}e^6} + \frac{3\,bd^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}e^4} + \frac{ad^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2} - \frac{5\,\sqrt{-e^2x^2+d^2}cd^4x}{16\,e^6} - \frac{3\,\sqrt{-e^2x^2+d^2}bd^2x}{8\,e^4} - \frac{\sqrt{-e^2x^2+d^2}ax}{2\,e^2}$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/6*sqrt(-e^2*x^2 + d^2)*c*x^5/e^2 - 5/24*sqrt(-e^2*x^2 + d^2)*c*d^2*x^3/
e^4 - 1/4*sqrt(-e^2*x^2 + d^2)*b*x^3/e^2 + 5/16*c*d^6*arcsin(e^2*x/(d*sqrt
(e^2)))/(sqrt(e^2)*e^6) + 3/8*b*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)
*e^4) + 1/2*a*d^2*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 5/16*sqrt(
-e^2*x^2 + d^2)*c*d^4*x/e^6 - 3/8*sqrt(-e^2*x^2 + d^2)*b*d^2*x/e^4 - 1/2*s
qrt(-e^2*x^2 + d^2)*a*x/e^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.82

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{(33\,cd^5+30\,bd^3e^2+24\,ade^4-(85\,cd^4+54\,bd^2e^2+24\,ae^4-2\,(55\,cd^3+18\,bde^2-(45\,cd^2+6\,be^2+4\,((e$$

input
```
integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
1/48*((33*c*d^5 + 30*b*d^3*e^2 + 24*a*d*e^4 - (85*c*d^4 + 54*b*d^2*e^2 + 2
4*a*e^4 - 2*(55*c*d^3 + 18*b*d*e^2 - (45*c*d^2 + 6*b*e^2 + 4*((e*x + d)*c
- 5*c*d)*(e*x + d))*(e*x + d))*(e*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e
*x + d) + 6*(5*c*d^6 + 6*b*d^4*e^2 + 8*a*d^2*e^4)*arcsin(1/2*sqrt(2)*sqrt(
e*x + d)/sqrt(d)))/e^7
```

Mupad [B] (verification not implemented)

Time = 21.81 (sec) , antiderivative size = 1132, normalized size of antiderivative = 5.24

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
int((x^2*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 -
(14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (
2*a*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*
a*d^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d
+ e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - ((175*c*
d^6*((d + e*x)^(1/2) - d^(1/2))^3)/(12*((d - e*x)^(1/2) - d^(1/2))^3) + (3
11*c*d^6*((d + e*x)^(1/2) - d^(1/2))^5)/(4*((d - e*x)^(1/2) - d^(1/2))^5)
- (8361*c*d^6*((d + e*x)^(1/2) - d^(1/2))^7)/(4*((d - e*x)^(1/2) - d^(1/2)
)^7) + (42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^9)/(6*((d - e*x)^(1/2) - d
^(1/2))^9) - (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d - e*x)^(1
/2) - d^(1/2))^11) + (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^13)/(2*((d -
 e*x)^(1/2) - d^(1/2))^13) - (42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^15)/
(6*((d - e*x)^(1/2) - d^(1/2))^15) + (8361*c*d^6*((d + e*x)^(1/2) - d^(1/2
))^17)/(4*((d - e*x)^(1/2) - d^(1/2))^17) - (311*c*d^6*((d + e*x)^(1/2) -
d^(1/2))^19)/(4*((d - e*x)^(1/2) - d^(1/2))^19) - (175*c*d^6*((d + e*x)^(1
/2) - d^(1/2))^21)/(12*((d - e*x)^(1/2) - d^(1/2))^21) - (5*c*d^6*((d + e*
x)^(1/2) - d^(1/2))^23)/(4*((d - e*x)^(1/2) - d^(1/2))^23) + (5*c*d^6*((d
+ e*x)^(1/2) - d^(1/2)))/(4*((d - e*x)^(1/2) - d^(1/2))))/(e^7*(((d + e*x)
^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^12) - ((23*b*d^4*((
d + e*x)^(1/2) - d^(1/2))^3)/(2*((d - e*x)^(1/2) - d^(1/2))^3) - (333*b...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99

$$\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{-48asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\sqrt{2}}\right)a\,d^2e^4-36asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\sqrt{2}}\right)b\,d^4e^2-30asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\sqrt{2}}\right)c\,d^6-24\sqrt{ex+d}\,\sqrt{-ex+d}\,a\,e^5}{}$$

input
```
int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 48*asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))*a*d**2*e**4 - 36*asin(sqrt(d
- e*x)/(sqrt(d)*sqrt(2)))*b*d**4*e**2 - 30*asin(sqrt(d - e*x)/(sqrt(d)*sqr
t(2)))*c*d**6 - 24*sqrt(d + e*x)*sqrt(d - e*x)*a*e**5*x - 18*sqrt(d + e*x)
*sqrt(d - e*x)*b*d**2*e**3*x - 12*sqrt(d + e*x)*sqrt(d - e*x)*b*e**5*x**3
- 15*sqrt(d + e*x)*sqrt(d - e*x)*c*d**4*e*x - 10*sqrt(d + e*x)*sqrt(d - e*
x)*c*d**2*e**3*x**3 - 8*sqrt(d + e*x)*sqrt(d - e*x)*c*e**5*x**5)/(48*e**7)
```

3.25 $\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 121

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(3cd^2+4be^2)\,x\sqrt{d-ex}\sqrt{d+ex}}{8e^4} - \frac{cx^3\sqrt{d-ex}\sqrt{d+ex}}{4e^2} + \frac{(3cd^4+4bd^2e^2+8ae^4)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{4e^5}$$

output
```
-1/8*(4*b*e^2+3*c*d^2)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4-1/4*c*x^3*(-e*x+
d)^(1/2)*(e*x+d)^(1/2)/e^2+1/4*(8*a*e^4+4*b*d^2*e^2+3*c*d^4)*arctan((e*x+d
)^(1/2)/(-e*x+d)^(1/2))/e^5
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{-ex\sqrt{d-ex}\sqrt{d+ex}(3cd^2+4be^2+2ce^2x^2)+2(3cd^4+4bd^2e^2+8ae^4)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{8e^5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(-(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(3*c*d^2 + 4*b*e^2 + 2*c*e^2*x^2)) + 2*
(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(8*
e^5)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1789, 1473, 25, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1789}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{4ae^2+\left(3cd^2+4be^2\right)x^2}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{4ae^2+\left(3cd^2+4be^2\right)x^2}{\sqrt{d^2-e^2x^2}}dx}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(8ae^4+4bd^2e^2+3cd^4\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx}{2e^2}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\left(4b+\frac{3cd^2}{e^2}\right)}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(8ae^4+4bd^2e^2+3cd^4\right)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}}{2e^2}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\left(4b+\frac{3cd^2}{e^2}\right)}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)}{2e^3}-\frac{1}{2}x\sqrt{d^2-e^2x^2}\left(4b+\frac{3cd^2}{e^2}\right)}{4e^2}-\frac{cx^3\sqrt{d^2-e^2x^2}}{4e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/4*(c*x^3*Sqrt[d^2 - e^2*x^2])/e^2 + (-1/2*((4*b +
 (3*c*d^2)/e^2)*x*Sqrt[d^2 - e^2*x^2]) + ((3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4
)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3))/(4*e^2)))/(Sqrt[d - e*x]*Sqr
t[d + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 1789
```
Int[((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)*(x_)^(non2_.))^(q_
.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(d
1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[q]/(d1*d2 + e1*e2*x
^n)^FracPart[q])   Int[(d1*d2 + e1*e2*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x],
 x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[
non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{x\left(2c\,x^{2}e^{2}+4b\,e^{2}+3c\,d^{2}\right)\sqrt{-ex+d}\,\sqrt{ex+d}}{8e^{4}}+\frac{\left(8a\,e^{4}+4b\,d^{2}e^{2}+3c\,d^{4}\right)\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)\sqrt{(ex+d)\,(-ex+d)}}{8e^{4}\sqrt{e^{2}}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(2\,\mathrm{csgn}(e)c\,e^{3}x^{3}\sqrt{-e^{2}x^{2}+d^{2}}+4\,\mathrm{csgn}(e)e^{3}\sqrt{-e^{2}x^{2}+d^{2}}\,bx+3\,\mathrm{csgn}(e)e\sqrt{-e^{2}x^{2}+d^{2}}\,c\,d^{2}x-8\arctan\left(\frac{\mathrm{csgn}(e)e}{\sqrt{-e^{2}x^{2}}}\right.\right.}{8e^{5}\sqrt{-e^{2}x^{2}+d^{2}}}$

input
```
int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(2*c*e^2*x^2+4*b*e^2+3*c*d^2)/e^4*(-e*x+d)^(1/2)*(e*x+d)^(1/2)+1/8*
(8*a*e^4+4*b*d^2*e^2+3*c*d^4)/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))*((e*x+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(2\,ce^3x^3+(3\,cd^2e+4\,be^3)x)\sqrt{ex+d}\sqrt{-ex+d}+2\,(3\,cd^4+4\,bd^2e^2+8\,ae^4)\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)}{8\,e^5}$$

input
```
integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="frica
s")
```

output
```
-1/8*((2*c*e^3*x^3 + (3*c*d^2*e + 4*b*e^3)*x)*sqrt(e*x + d)*sqrt(-e*x + d)
 + 2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d
) - d)/(e*x)))/e^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}cx^3}{4\,e^2}+\frac{a\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}}+\frac{3\,cd^4\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{8\,\sqrt{e^2}e^4}+\frac{bd^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\,\sqrt{e^2}e^2}-\frac{3\,\sqrt{-e^2x^2+d^2}cd^2x}{8\,e^4}-\frac{\sqrt{-e^2x^2+d^2}bx}{2\,e^2}$$

input
```
integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxim
a")
```

output
```
-1/4*sqrt(-e^2*x^2 + d^2)*c*x^3/e^2 + a*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e
^2) + 3/8*c*d^4*arcsin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^4) + 1/2*b*d^2*ar
csin(e^2*x/(d*sqrt(e^2)))/(sqrt(e^2)*e^2) - 3/8*sqrt(-e^2*x^2 + d^2)*c*d^2
*x/e^4 - 1/2*sqrt(-e^2*x^2 + d^2)*b*x/e^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{(5\,cd^3+4\,bde^2-(9\,cd^2+4\,be^2+2\,((ex+d)c-3\,cd)(ex+d))(ex+d))\sqrt{ex+d}\sqrt{-ex+d}+2\,(3\,cd^4+}{8\,e^5}$$

input
```
integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac"
)
```

output
```
1/8*((5*c*d^3 + 4*b*d*e^2 - (9*c*d^2 + 4*b*e^2 + 2*((e*x + d)*c - 3*c*d)*(
e*x + d))*(e*x + d))*sqrt(e*x + d)*sqrt(-e*x + d) + 2*(3*c*d^4 + 4*b*d^2*e
^2 + 8*a*e^4)*arcsin(1/2*sqrt(2)*sqrt(e*x + d)/sqrt(d)))/e^5
```

Mupad [B] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 651, normalized size of antiderivative = 5.38

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$=\frac{\frac{14\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{14\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{2\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{2\,b\,d^2\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\,\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^4}$$
$$-\frac{4\,a\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}$$
$$-\frac{\frac{23\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{333\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{671\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{671\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)^9}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^9}+\frac{333\,c\,d^4\,\left(\sqrt{d+e\,x}-\sqrt{d}\right.}{2\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)^{11}}}{e^5\,\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^8}$$
$$+\frac{2\,b\,d^2\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{e^3}+\frac{3\,c\,d^4\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{2\,e^5}$$

input
```
int((a + b*x^2 + c*x^4)/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 -
(14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (
2*b*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*
b*d^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d
+ e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*a*ata
n((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2))
)))/(e^2)^(1/2) - ((23*c*d^4*((d + e*x)^(1/2) - d^(1/2))^3)/(2*((d - e*x)^
(1/2) - d^(1/2))^3) - (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^5)/(2*((d - e
*x)^(1/2) - d^(1/2))^5) + (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^7)/(2*((d
 - e*x)^(1/2) - d^(1/2))^7) - (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^9)/(2
*((d - e*x)^(1/2) - d^(1/2))^9) + (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^1
1)/(2*((d - e*x)^(1/2) - d^(1/2))^11) - (23*c*d^4*((d + e*x)^(1/2) - d^(1/
2))^13)/(2*((d - e*x)^(1/2) - d^(1/2))^13) - (3*c*d^4*((d + e*x)^(1/2) - d
^(1/2))^15)/(2*((d - e*x)^(1/2) - d^(1/2))^15) + (3*c*d^4*((d + e*x)^(1/2)
 - d^(1/2)))/(2*((d - e*x)^(1/2) - d^(1/2))))/(e^5*(((d + e*x)^(1/2) - d^(
1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^8) + (2*b*d^2*atan(((d + e*x)^(
1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/e^3 + (3*c*d^4*atan(((d + e*
x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.17

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{-16asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\,\sqrt{2}}\right)a\,e^4-8asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\,\sqrt{2}}\right)b\,d^2e^2-6asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\,\sqrt{2}}\right)c\,d^4-4\sqrt{ex+d}\,\sqrt{-ex+d}\,b\,e^3x-3\sqrt{}}{8e^5}$$

input
```
int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 16*asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))*a*e**4 - 8*asin(sqrt(d - e*x)
/(sqrt(d)*sqrt(2)))*b*d**2*e**2 - 6*asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))*
c*d**4 - 4*sqrt(d + e*x)*sqrt(d - e*x)*b*e**3*x - 3*sqrt(d + e*x)*sqrt(d -
 e*x)*c*d**2*e*x - 2*sqrt(d + e*x)*sqrt(d - e*x)*c*e**3*x**3)/(8*e**5)
```

3.26 $\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 94

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x}-\frac{cx\sqrt{d-ex}\sqrt{d+ex}}{2e^2}+\frac{(cd^2+2be^2)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{e^3}$$

output
```
-a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x-1/2*c*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)
/e^2+(2*b*e^2+c*d^2)*arctan((e*x+d)^(1/2)/(-e*x+d)^(1/2))/e^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{-\frac{e\sqrt{d-ex}\sqrt{d+ex}(2ae^2+cd^2x^2)}{d^2x}+2(cd^2+2be^2)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{2e^3}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(-((e*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*e^2 + c*d^2*x^2))/(d^2*x)) + 2*(c*d
^2 + 2*b*e^2)*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/(2*e^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1905, 1588, 25, 27, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^2\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1588}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{d^2\left(cx^2+b\right)}{\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{d^2\left(cx^2+b\right)}{\sqrt{d^2-e^2x^2}}dx}{d^2}-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\int \frac{cx^2+b}{\sqrt{d^2-e^2x^2}}dx-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{1}{2}\left(2b+\frac{cd^2}{e^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}-\frac{cx\sqrt{d^2-e^2x^2}}{2e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{1}{2}\left(2b+\frac{cd^2}{e^2}\right)\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}-\frac{cx\sqrt{d^2-e^2x^2}}{2e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{a\sqrt{d^2-e^2x^2}}{d^2x}+\frac{\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(2b+\frac{cd^2}{e^2}\right)}{2e}-\frac{cx\sqrt{d^2-e^2x^2}}{2e^2}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-((a*Sqrt[d^2 - e^2*x^2])/(d^2*x)) - (c*x*Sqrt[d^2 -
 e^2*x^2])/(2*e^2) + ((2*b + (c*d^2)/e^2)*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]
])/(2*e)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
risch	$-\frac{\sqrt{e x+d}\,\sqrt{-e x+d}\,\left(c\,d^{2} x^{2}+2 a\,e^{2}\right)}{2 e^{2} d^{2} x}+\frac{\left(2 b\,e^{2}+c\,d^{2}\right) \arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right) \sqrt{\left(e x+d\right) \left(-e x+d\right)}}{2 e^{2} \sqrt{e^{2}}\,\sqrt{e x+d}\,\sqrt{-e x+d}}$
default	$-\frac{\sqrt{-e x+d}\,\sqrt{e x+d}\,\left(\operatorname{csgn}\left(e\right) c\,d^{2} e\,x^{2} \sqrt{-e^{2} x^{2}+d^{2}}-2 \arctan\left(\frac{\operatorname{csgn}\left(e\right) e x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right) b\,d^{2} e^{2} x-\arctan\left(\frac{\operatorname{csgn}\left(e\right) e x}{\sqrt{-e^{2} x^{2}+d^{2}}}\right) c\,d^{4} x+2 \operatorname{csgn}\left(e\right) e^{3} \sqrt{-e}}{2 d^{2} e^{3} \sqrt{-e^{2} x^{2}+d^{2}}\,x}$

input
```
int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/2*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(c*d^2*x^2+2*a*e^2)/e^2/d^2/x+1/2*(2*b*e
^2+c*d^2)/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x
+d)*(-e*x+d))^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\,(cd^4+2\,bd^2e^2)x\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+(cd^2ex^2+2\,ae^3)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^2e^3x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/2*(2*(c*d^4 + 2*b*d^2*e^2)*x*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/
(e*x)) + (c*d^2*e*x^2 + 2*a*e^3)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^2*e^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{b\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} + \frac{cd^2\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{2\sqrt{e^2}e^2} - \frac{\sqrt{-e^2x^2+d^2}cx}{2\,e^2} - \frac{\sqrt{-e^2x^2+d^2}a}{d^2x}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
b*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) + 1/2*c*d^2*arcsin(e^2*x/(d*sqrt(e
^2)))/(sqrt(e^2)*e^2) - 1/2*sqrt(-e^2*x^2 + d^2)*c*x/e^2 - sqrt(-e^2*x^2 +
 d^2)*a/(d^2*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(80) = 160$.

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.54

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\frac{8\,ae^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)}{\left(\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^2-4\right)d^2} - \left(\pi + 2\,\arctan\left(\frac{\sqrt{ex+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)^2}{ex+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)}\right)\right)(cd^2+2\,be^2) + ((e}{2\,e^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-1/2*(8*a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))/((((sqrt(2)*sqrt(d) - sqrt(-e*x
+ d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2
- 4)*d^2) - (pi + 2*arctan(1/2*sqrt(e*x + d)*((sqrt(2)*sqrt(d) - sqrt(-e*x
 + d))^2/(e*x + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))*(c*d^2 + 2*b*
e^2) + ((e*x + d)*c - c*d)*sqrt(e*x + d)*sqrt(-e*x + d))/e^3
```

Mupad [B] (verification not implemented)

Time = 7.61 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.26

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\frac{14\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{14\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{2\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{2\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^4} - \frac{4\,b\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}} + \frac{2\,c\,d^2\,\mathrm{atan}\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)}{e^3} - \frac{\left(\frac{a}{d}+\frac{a\,e\,x}{d^2}\right)\sqrt{d-e\,x}}{x\,\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^2*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
((14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 -
(14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (
2*c*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*
c*d^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d
+ e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*b*ata
n((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2))
)))/(e^2)^(1/2) + (2*c*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/
2) - d^(1/2))))/e^3 - ((a/d + (a*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d + e*x)^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{-4asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\,\sqrt{2}}\right)b\,d^2e^2x - 2asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\,\sqrt{2}}\right)c\,d^4x - 2\sqrt{ex+d}\,\sqrt{-ex+d}\,a\,e^3 - \sqrt{ex+d}\,\sqrt{-ex+d}\,c\,d^2e}{2d^2e^3x}$$

input
```
int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 4*asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))*b*d**2*e**2*x - 2*asin(sqrt(d
- e*x)/(sqrt(d)*sqrt(2)))*c*d**4*x - 2*sqrt(d + e*x)*sqrt(d - e*x)*a*e**3
- sqrt(d + e*x)*sqrt(d - e*x)*c*d**2*e*x**2)/(2*d**2*e**3*x)
```

3.27 $\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 100

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{3d^2x^3} - \frac{(3bd^2+2ae^2)\sqrt{d-ex}\sqrt{d+ex}}{3d^4x} + \frac{2c\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{e}$$

output
```
-1/3*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^3-1/3*(2*a*e^2+3*b*d^2)*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/d^4/x+2*c*arctan((e*x+d)^(1/2)/(-e*x+d)^(1/2))/e
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(3bd^2x^2+a(d^2+2e^2x^2))}{3d^4x^3} + \frac{2c\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{d-ex}}\right)}{e}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(3*b*d^2*x^2 + a*(d^2 + 2*e^2*x^2)))/(d^
4*x^3) + (2*c*ArcTan[Sqrt[d + e*x]/Sqrt[d - e*x]])/e
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1905, 1588, 25, 358, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^4\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1588}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{3cx^2d^2+3bd^2+2ae^2}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{3cx^2d^2+3bd^2+2ae^2}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 358}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{3cd^2\int \frac{1}{\sqrt{d^2-e^2x^2}}dx-\frac{\sqrt{d^2-e^2x^2}\left(\frac{2ae^2}{d^2}+3b\right)}{x}}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{3cd^2\int \frac{1}{\frac{e^2x^2}{d^2-e^2x^2}+1}d\frac{x}{\sqrt{d^2-e^2x^2}}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{2ae^2}{d^2}+3b\right)}{x}}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3cd^2\arctan\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{2ae^2}{d^2}+3b\right)}{x}}{3d^2}-\frac{a\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/3*(a*Sqrt[d^2 - e^2*x^2])/(d^2*x^3) + (-(((3*b +
(2*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/x) + (3*c*d^2*ArcTan[(e*x)/Sqrt[d^2 -
e^2*x^2]])/e)/(3*d^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905

```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(2a\,e^{2}x^{2}+3b\,d^{2}x^{2}+a\,d^{2}\right)}{3d^{4}x^{3}}+\frac{c\arctan\left(\frac{\sqrt{e^{2}}\,x}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)\sqrt{\left(ex+d\right)\left(-ex+d\right)}}{\sqrt{e^{2}}\,\sqrt{ex+d}\,\sqrt{-ex+d}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(-3\arctan\left(\frac{\operatorname{csgn}(e)ex}{\sqrt{-e^{2}x^{2}+d^{2}}}\right)c\,d^{4}x^{3}+2\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(e)e^{3}a\,x^{2}+3\sqrt{-e^{2}x^{2}+d^{2}}\,\operatorname{csgn}(e)eb\,d^{2}x^{2}+a\sqrt{-e^{2}x^{2}+d}\right.}{3d^{4}\sqrt{-e^{2}x^{2}+d^{2}}\,x^{3}e}$

input

```
int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output

```
-1/3*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(2*a*e^2*x^2+3*b*d^2*x^2+a*d^2)/d^4/x^3+
c/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))*((e*x+d)*(-e*x+d)
)^(1/2)/(e*x+d)^(1/2)/(-e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{6\,cd^4x^3\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+(ad^2e+(3\,bd^2e+2\,ae^3)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{3\,d^4ex^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/3*(6*c*d^4*x^3*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) + (a*d^
2*e + (3*b*d^2*e + 2*a*e^3)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^4*e*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 19.15 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.57

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{iae^3G_{6,6}^{5,3}\left(\begin{matrix} \frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3 \\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0 \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{ae^3G_{6,6}^{2,6}\left(\begin{matrix} \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4} + \frac{ibeG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{beG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e} + \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}$$

input
```
integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
I*a*e**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3),
 (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**4) + a*e**3*meijerg(((3/2, 7/4,
2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), d**2*exp_polar(-2*I*pi
)/(e**2*x**2))/(4*pi**(3/2)*d**4) + I*b*e*meijerg(((5/4, 7/4, 1), (3/2, 3/
2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2
) + b*e*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1,
 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijer
g(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d**2/(e*
*2*x**2))/(4*pi**(3/2)*e) + c*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()),
((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi
**(3/2)*e)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{c\arcsin\left(\frac{e^2x}{d\sqrt{e^2}}\right)}{\sqrt{e^2}} - \frac{\sqrt{-e^2x^2+d^2}b}{d^2x} - \frac{2\sqrt{-e^2x^2+d^2}ae^2}{3\,d^4x} - \frac{\sqrt{-e^2x^2+d^2}a}{3\,d^2x^3}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
c*arcsin(e^2*x/(d*sqrt(e^2)))/sqrt(e^2) - sqrt(-e^2*x^2 + d^2)*b/(d^2*x) -
 2/3*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x) - 1/3*sqrt(-e^2*x^2 + d^2)*a/(d^2*
x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(84) = 168$.

Time = 0.26 (sec) , antiderivative size = 530, normalized size of antiderivative = 5.30

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{3\left(\pi+2\arctan\left(\frac{\sqrt{ex+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)^2}{ex+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-ex+d}\right)}\right)\right)c-\frac{4\left(3bd^2e^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}-\frac{\sqrt{ex+d}}{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}\right)^5+3ae^4\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-ex+d}}{\sqrt{ex+d}}\right.}{}}{}$$

input
```
integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
1/3*(3*(pi + 2*arctan(1/2*sqrt(e*x + d)*((sqrt(2)*sqrt(d) - sqrt(-e*x + d)
)^2/(e*x + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))))*c - 4*(3*b*d^2*e^2
*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2
)*sqrt(d) - sqrt(-e*x + d)))^5 + 3*a*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d
))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 - 2
4*b*d^2*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x +
 d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 8*a*e^4*((sqrt(2)*sqrt(d) - sq
rt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
 d)))^3 + 48*b*d^2*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
 sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d))) + 48*a*e^4*((sqrt(2)*sq
rt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - s
qrt(-e*x + d))))/((((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqr
t(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^2 - 4)^3*d^4))/e
```

Mupad [B] (verification not implemented)

Time = 4.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{4\,c\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}-\frac{\left(\frac{b}{d}+\frac{b\,e\,x}{d^2}\right)\sqrt{d-e\,x}}{x\,\sqrt{d+e\,x}}-\frac{\sqrt{d-e\,x}\left(\frac{a}{3\,d}+\frac{2\,a\,e^2\,x^2}{3\,d^3}+\frac{2\,a\,e^3\,x^3}{3\,d^4}+\frac{a\,e\,x}{3\,d^2}\right)}{x^3\,\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^4*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
- (4*c*atan((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2)
- d^(1/2)))))/(e^2)^(1/2) - ((b/d + (b*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d +
e*x)^(1/2)) - ((d - e*x)^(1/2)*(a/(3*d) + (2*a*e^2*x^2)/(3*d^3) + (2*a*e^3
*x^3)/(3*d^4) + (a*e*x)/(3*d^2)))/(x^3*(d + e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{-6asin\left(\frac{\sqrt{-ex+d}}{\sqrt{d}\sqrt{2}}\right)c\,d^4x^3 - \sqrt{ex+d}\,\sqrt{-ex+d}\,a\,d^2e - 2\sqrt{ex+d}\,\sqrt{-ex+d}\,a\,e^3x^2 - 3\sqrt{ex+d}\,\sqrt{-ex+}}{3d^4e\,x^3}$$

input
```
int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
( - 6*asin(sqrt(d - e*x)/(sqrt(d)*sqrt(2)))*c*d**4*x**3 - sqrt(d + e*x)*sq
rt(d - e*x)*a*d**2*e - 2*sqrt(d + e*x)*sqrt(d - e*x)*a*e**3*x**2 - 3*sqrt(
d + e*x)*sqrt(d - e*x)*b*d**2*e*x**2)/(3*d**4*e*x**3)
```

3.28 $\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 124

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{5d^2x^5} - \frac{(5bd^2+4ae^2)\sqrt{d-ex}\sqrt{d+ex}}{15d^4x^3} - \frac{(15cd^4+10bd^2e^2+8ae^4)\sqrt{d-ex}\sqrt{d+ex}}{15d^6x}$$

output
```
-1/5*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^5-1/15*(4*a*e^2+5*b*d^2)*(-e*x+d
)^(1/2)*(e*x+d)^(1/2)/d^4/x^3-1/15*(8*a*e^4+10*b*d^2*e^2+15*c*d^4)*(-e*x+d
)^(1/2)*(e*x+d)^(1/2)/d^6/x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(15cd^4x^4+5bd^2x^2(d^2+2e^2x^2)+a(3d^4+4d^2e^2x^2+8e^4x^4))}{15d^6x^5}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(15*c*d^4*x^4 + 5*b*d^2*x^2*(d^2 + 2*e^
2*x^2) + a*(3*d^4 + 4*d^2*e^2*x^2 + 8*e^4*x^4)))/(d^6*x^5)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1905, 1588, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}dx$$

$$\Big\downarrow \ 1905$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^6\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 1588$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{5cx^2d^2+5bd^2+4ae^2}{x^4\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{a\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{5cx^2d^2+5bd^2+4ae^2}{x^4\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{a\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 359$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(8ae^4+10bd^2e^2+15cd^4\right)\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{4ae^2}{d^2}+5b\right)}{3x^3}}{5d^2}-\frac{a\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 242$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{-\frac{\sqrt{d^2-e^2x^2}\left(8ae^4+10bd^2e^2+15cd^4\right)}{3d^4x}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{4ae^2}{d^2}+5b\right)}{3x^3}}{5d^2}-\frac{a\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/5*(a*Sqrt[d^2 - e^2*x^2])/(d^2*x^5) + (-1/3*((5*b
 + (4*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/x^3 - ((15*c*d^4 + 10*b*d^2*e^2 + 8
*a*e^4)*Sqrt[d^2 - e^2*x^2])/(3*d^4*x))/(5*d^2)))/(Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(8a\,e^{4}x^{4}+10b\,d^{2}e^{2}x^{4}+15c\,d^{4}x^{4}+4a\,d^{2}e^{2}x^{2}+5b\,d^{4}x^{2}+3a\,d^{4}\right)}{15x^{5}d^{6}}$	82
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(8a\,e^{4}x^{4}+10b\,d^{2}e^{2}x^{4}+15c\,d^{4}x^{4}+4a\,d^{2}e^{2}x^{2}+5b\,d^{4}x^{2}+3a\,d^{4}\right)}{15x^{5}d^{6}}$	82
orering	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(8a\,e^{4}x^{4}+10b\,d^{2}e^{2}x^{4}+15c\,d^{4}x^{4}+4a\,d^{2}e^{2}x^{2}+5b\,d^{4}x^{2}+3a\,d^{4}\right)}{15x^{5}d^{6}}$	82
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\operatorname{csgn}\left(e\right)^{2}\left(8a\,e^{4}x^{4}+10b\,d^{2}e^{2}x^{4}+15c\,d^{4}x^{4}+4a\,d^{2}e^{2}x^{2}+5b\,d^{4}x^{2}+3a\,d^{4}\right)}{15d^{6}x^{5}}$	86

input
```
int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(8*a*e^4*x^4+10*b*d^2*e^2*x^4+15*c*d^4*
x^4+4*a*d^2*e^2*x^2+5*b*d^4*x^2+3*a*d^4)/x^5/d^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= -\frac{(3\,ad^4+(15\,cd^4+10\,bd^2e^2+8\,ae^4)x^4+(5\,bd^4+4\,ad^2e^2)x^2)\sqrt{ex+d}\sqrt{-ex+d}}{15\,d^6x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/15*(3*a*d^4 + (15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*x^4 + (5*b*d^4 + 4*a*
d^2*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^6*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{-e^2x^2+d^2}c}{d^2x} - \frac{2\sqrt{-e^2x^2+d^2}be^2}{3\,d^4x} - \frac{8\sqrt{-e^2x^2+d^2}ae^4}{15\,d^6x} - \frac{\sqrt{-e^2x^2+d^2}b}{3\,d^2x^3} - \frac{4\sqrt{-e^2x^2+d^2}ae^2}{15\,d^4x^3} - \frac{\sqrt{-e^2x^2+d^2}a}{5\,d^2x^5}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-sqrt(-e^2*x^2 + d^2)*c/(d^2*x) - 2/3*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x) -
 8/15*sqrt(-e^2*x^2 + d^2)*a*e^4/(d^6*x) - 1/3*sqrt(-e^2*x^2 + d^2)*b/(d^2
*x^3) - 4/15*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^3) - 1/5*sqrt(-e^2*x^2 + d^
2)*a/(d^2*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(106) = 212$.

Time = 0.36 (sec) , antiderivative size = 1055, normalized size of antiderivative = 8.51

$$\int \frac{a + bx^2 + cx^4}{x^6\sqrt{d - ex}\sqrt{d + ex}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-4/15*(15*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sq
rt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 15*b*d^2*e^4*((sqrt(2)
*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d)
- sqrt(-e*x + d)))^9 + 15*a*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e
*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 240*c*d^4*
e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d)))^7 - 160*b*d^2*e^4*((sqrt(2)*sqrt(d) - sqrt
(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d
)))^7 - 80*a*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(
e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 1440*c*d^4*e^2*((sqrt(2)*
sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) -
 sqrt(-e*x + d)))^5 + 800*b*d^2*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sq
rt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^5 + 928*a*
e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d)))^5 - 3840*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqr
t(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
d)))^3 - 2560*b*d^2*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d)
- sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^3 - 1280*a*e^6*((sqrt(
2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d
) - sqrt(-e*x + d)))^3 + 3840*c*d^4*e^2*((sqrt(2)*sqrt(d) - sqrt(-e*x +...
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.18

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\,\left(\frac{a}{5\,d}+\frac{x^4\,\left(15\,c\,d^5+10\,b\,d^3\,e^2+8\,a\,d\,e^4\right)}{15\,d^6}+\frac{x^5\,\left(15\,c\,d^4\,e+10\,b\,d^2\,e^3+8\,a\,e^5\right)}{15\,d^6}+\frac{x^2\,\left(5\,b\,d^5+4\,a\,d^3\,e^2\right)}{15\,d^6}+\frac{x^3\,\left(5\,b\,d^4\,e+4\,a\,d^2\,e\right.}{15\,d^6}}{x^5\,\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^6*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*(a/(5*d) + (x^4*(15*c*d^5 + 10*b*d^3*e^2 + 8*a*d*e^4))/(
15*d^6) + (x^5*(8*a*e^5 + 10*b*d^2*e^3 + 15*c*d^4*e))/(15*d^6) + (x^2*(5*b
*d^5 + 4*a*d^3*e^2))/(15*d^6) + (x^3*(4*a*d^2*e^3 + 5*b*d^4*e))/(15*d^6) +
 (a*e*x)/(5*d^2)))/(x^5*(d + e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(-8a\,e^4x^4-10b\,d^2e^2x^4-15c\,d^4x^4-4a\,d^2e^2x^2-5b\,d^4x^2-3a\,d^4)}{15d^6x^5}$$

input
```
int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(d - e*x)*( - 3*a*d**4 - 4*a*d**2*e**2*x**2 - 8*a*e**4*
x**4 - 5*b*d**4*x**2 - 10*b*d**2*e**2*x**4 - 15*c*d**4*x**4))/(15*d**6*x**
5)
```

3.29 $\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 178

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{7d^2x^7} - \frac{(7bd^2+6ae^2)\sqrt{d-ex}\sqrt{d+ex}}{35d^4x^5} - \frac{(35cd^4+28bd^2e^2+24ae^4)\sqrt{d-ex}\sqrt{d+ex}}{105d^6x^3} - \frac{2e^2(35cd^4+28bd^2e^2+24ae^4)\sqrt{d-ex}\sqrt{d+ex}}{105d^8x}$$

output
```
-1/7*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^7-1/35*(6*a*e^2+7*b*d^2)*(-e*x+d
)^(1/2)*(e*x+d)^(1/2)/d^4/x^5-1/105*(24*a*e^4+28*b*d^2*e^2+35*c*d^4)*(-e*x
+d)^(1/2)*(e*x+d)^(1/2)/d^6/x^3-2/105*e^2*(24*a*e^4+28*b*d^2*e^2+35*c*d^4)
*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^8/x
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(35cd^4x^4(d^2+2e^2x^2)+7b(3d^6x^2+4d^4e^2x^4+8d^2e^4x^6)+3a(5d^6+6d^4e^2x^2+8d^2e^4}{105d^8x^7}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*c*d^4*x^4*(d^2 + 2*e^2*x^2) + 7*b*
(3*d^6*x^2 + 4*d^4*e^2*x^4 + 8*d^2*e^4*x^6) + 3*a*(5*d^6 + 6*d^4*e^2*x^2 +
 8*d^2*e^4*x^4 + 16*e^6*x^6)))/(d^8*x^7)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1905, 1588, 25, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \ 1905$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^8\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 1588$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{7cx^2d^2+7bd^2+6ae^2}{x^6\sqrt{d^2-e^2x^2}}dx}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{7cx^2d^2+7bd^2+6ae^2}{x^6\sqrt{d^2-e^2x^2}}dx}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{359}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{(24ae^4+28bd^2e^2+35cd^4)\int \frac{1}{x^4\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{6ae^2}{d^2}+7b\right)}{5x^5}}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{245}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{(24ae^4+28bd^2e^2+35cd^4)\left(\frac{2e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{5d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{6ae^2}{d^2}+7b\right)}{5x^5}}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{242}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^4x}\right)(24ae^4+28bd^2e^2+35cd^4)}{5d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{6ae^2}{d^2}+7b\right)}{5x^5}}{7d^2}-\frac{a\sqrt{d^2-e^2x^2}}{7d^2x^7}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/7*(a*Sqrt[d^2 - e^2*x^2])/(d^2*x^7) + (-1/5*((7*b
 + (6*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/x^5 + ((35*c*d^4 + 28*b*d^2*e^2 + 2
4*a*e^4)*(-1/3*Sqrt[d^2 - e^2*x^2]/(d^2*x^3) - (2*e^2*Sqrt[d^2 - e^2*x^2])
/(3*d^4*x)))/(5*d^2))/(7*d^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.66

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(48a\,e^{6}x^{6}+56b\,d^{2}e^{4}x^{6}+70c\,d^{4}e^{2}x^{6}+24a\,d^{2}e^{4}x^{4}+28b\,d^{4}e^{2}x^{4}+35c\,d^{6}x^{4}+18a\,d^{4}e^{2}x^{2}+21b\,d^{6}x^{2}+15a\,d^{6}\right)}{105x^{7}d^{8}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(48a\,e^{6}x^{6}+56b\,d^{2}e^{4}x^{6}+70c\,d^{4}e^{2}x^{6}+24a\,d^{2}e^{4}x^{4}+28b\,d^{4}e^{2}x^{4}+35c\,d^{6}x^{4}+18a\,d^{4}e^{2}x^{2}+21b\,d^{6}x^{2}+15a\,d^{6}\right)}{105x^{7}d^{8}}$
orering	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(48a\,e^{6}x^{6}+56b\,d^{2}e^{4}x^{6}+70c\,d^{4}e^{2}x^{6}+24a\,d^{2}e^{4}x^{4}+28b\,d^{4}e^{2}x^{4}+35c\,d^{6}x^{4}+18a\,d^{4}e^{2}x^{2}+21b\,d^{6}x^{2}+15a\,d^{6}\right)}{105x^{7}d^{8}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\operatorname{csgn}(e)^{2}\left(48a\,e^{6}x^{6}+56b\,d^{2}e^{4}x^{6}+70c\,d^{4}e^{2}x^{6}+24a\,d^{2}e^{4}x^{4}+28b\,d^{4}e^{2}x^{4}+35c\,d^{6}x^{4}+18a\,d^{4}e^{2}x^{2}+21b\,d^{6}x^{2}+15a\,d\right.}{105d^{8}x^{7}}$

input
```
int((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(48*a*e^6*x^6+56*b*d^2*e^4*x^6+70*c*d^
4*e^2*x^6+24*a*d^2*e^4*x^4+28*b*d^4*e^2*x^4+35*c*d^6*x^4+18*a*d^4*e^2*x^2+
21*b*d^6*x^2+15*a*d^6)/x^7/d^8
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.62

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{(15\,ad^6+2\,(35\,cd^4e^2+28\,bd^2e^4+24\,ae^6)x^6+(35\,cd^6+28\,bd^4e^2+24\,ad^2e^4)x^4+3\,(7\,bd^6+6\,ad^4e^2)x}{105\,d^8x^7}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/105*(15*a*d^6 + 2*(35*c*d^4*e^2 + 28*b*d^2*e^4 + 24*a*e^6)*x^6 + (35*c*
d^6 + 28*b*d^4*e^2 + 24*a*d^2*e^4)*x^4 + 3*(7*b*d^6 + 6*a*d^4*e^2)*x^2)*sq
rt(e*x + d)*sqrt(-e*x + d)/(d^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.27

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{2\sqrt{-e^2x^2+d^2}ce^2}{3\,d^4x} - \frac{8\sqrt{-e^2x^2+d^2}be^4}{15\,d^6x} - \frac{16\sqrt{-e^2x^2+d^2}ae^6}{35\,d^8x} - \frac{\sqrt{-e^2x^2+d^2}c}{3\,d^2x^3} - \frac{4\sqrt{-e^2x^2+d^2}be^2}{15\,d^4x^3} - \frac{8\sqrt{-e^2x^2+d^2}ae^4}{35\,d^6x^3} - \frac{\sqrt{-e^2x^2+d^2}b}{5\,d^2x^5} - \frac{6\sqrt{-e^2x^2+d^2}ae^2}{35\,d^4x^5} - \frac{\sqrt{-e^2x^2+d^2}a}{7\,d^2x^7}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

output
```
-2/3*sqrt(-e^2*x^2 + d^2)*c*e^2/(d^4*x) - 8/15*sqrt(-e^2*x^2 + d^2)*b*e^4/
(d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x) - 1/3*sqrt(-e^2*x^2 + d
^2)*c/(d^2*x^3) - 4/15*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^3) - 8/35*sqrt(-e
^2*x^2 + d^2)*a*e^4/(d^6*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^5) - 6/3
5*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*a/(d^2*x
^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1451 vs. $2(154) = 308$.

Time = 0.49 (sec) , antiderivative size = 1451, normalized size of antiderivative = 8.15

$$\int \frac{a + bx^2 + cx^4}{x^8\sqrt{d - ex}\sqrt{d + ex}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

output
```
-4/105*(105*c*d^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 + 105*b*d^2*e^6*((sqr
t(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt
(d) - sqrt(-e*x + d)))^13 + 105*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/
sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 - 196
0*c*d^4*e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x +
 d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 - 1400*b*d^2*e^6*((sqrt(2)*sqrt
(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqr
t(-e*x + d)))^11 - 840*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x
+ d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 16240*c*d^4*
e^4*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d)))^9 + 12656*b*d^2*e^6*((sqrt(2)*sqrt(d) - sq
rt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x +
 d)))^9 + 14448*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^9 - 80640*c*d^4*e^4*((sq
rt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqr
t(d) - sqrt(-e*x + d)))^7 - 69888*b*d^2*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x
+ d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7
- 40704*a*e^8*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
 + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^7 + 259840*c*d^4*e^4*((sqrt(2...
```

Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\,\left(\frac{a}{7\,d}+\frac{x^2\,\left(21\,b\,d^7+18\,a\,d^5\,e^2\right)}{105\,d^8}+\frac{x^4\,\left(35\,c\,d^7+28\,b\,d^5\,e^2+24\,a\,d^3\,e^4\right)}{105\,d^8}+\frac{x^7\,\left(70\,c\,d^4\,e^3+56\,b\,d^2\,e^5+48\,a\,e^7\right)}{105\,d^8}+\frac{x^3\,\left(21\,b\,d^6\,e\right.}{105}\right.}{x^7\,\sqrt{d+e\,x}}$$

input
```
int((a + b*x^2 + c*x^4)/(x^8*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*(a/(7*d) + (x^2*(21*b*d^7 + 18*a*d^5*e^2))/(105*d^8) + (
x^4*(35*c*d^7 + 24*a*d^3*e^4 + 28*b*d^5*e^2))/(105*d^8) + (x^7*(48*a*e^7 +
 56*b*d^2*e^5 + 70*c*d^4*e^3))/(105*d^8) + (x^3*(18*a*d^4*e^3 + 21*b*d^6*e
))/(105*d^8) + (x^5*(24*a*d^2*e^5 + 28*b*d^4*e^3 + 35*c*d^6*e))/(105*d^8)
+ (x^6*(56*b*d^3*e^4 + 70*c*d^5*e^2 + 48*a*d*e^6))/(105*d^8) + (a*e*x)/(7*
d^2)))/(x^7*(d + e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(-48a\,e^6x^6-56b\,d^2e^4x^6-70c\,d^4e^2x^6-24a\,d^2e^4x^4-28b\,d^4e^2x^4-35c\,d^6x^4-18a\,d}{105d^8x^7}$$

input
```
int((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(d - e*x)*( - 15*a*d**6 - 18*a*d**4*e**2*x**2 - 24*a*d*
*2*e**4*x**4 - 48*a*e**6*x**6 - 21*b*d**6*x**2 - 28*b*d**4*e**2*x**4 - 56*
b*d**2*e**4*x**6 - 35*c*d**6*x**4 - 70*c*d**4*e**2*x**6))/(105*d**8*x**7)
```

3.30 $\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 232

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{a\sqrt{d-ex}\sqrt{d+ex}}{9d^2x^9} - \frac{(9bd^2+8ae^2)\sqrt{d-ex}\sqrt{d+ex}}{63d^4x^7} - \frac{(21cd^4+18bd^2e^2+16ae^4)\sqrt{d-ex}\sqrt{d+ex}}{105d^6x^5} - \frac{4e^2(21cd^4+18bd^2e^2+16ae^4)\sqrt{d-ex}\sqrt{d+ex}}{315d^8x^3} - \frac{8e^4(21cd^4+18bd^2e^2+16ae^4)\sqrt{d-ex}\sqrt{d+ex}}{315d^{10}x}$$

output
```
-1/9*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^9-1/63*(8*a*e^2+9*b*d^2)*(-e*x+d
)^(1/2)*(e*x+d)^(1/2)/d^4/x^7-1/105*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e*x
+d)^(1/2)*(e*x+d)^(1/2)/d^6/x^5-4/315*e^2*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)
*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^8/x^3-8/315*e^4*(16*a*e^4+18*b*d^2*e^2+21*
c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^10/x
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-ex}\sqrt{d+ex}(21cd^4x^4(3d^4+4d^2e^2x^2+8e^4x^4)+9b(5d^8x^2+6d^6e^2x^4+8d^4e^4x^6+16d^2e^6x^8)+a(}{315d^{10}x^9}$$

input
```
Integrate[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
-1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*c*d^4*x^4*(3*d^4 + 4*d^2*e^2*x^2 +
 8*e^4*x^4) + 9*b*(5*d^8*x^2 + 6*d^6*e^2*x^4 + 8*d^4*e^4*x^6 + 16*d^2*e^6*
x^8) + a*(35*d^8 + 40*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 64*d^2*e^6*x^6 + 128*
e^8*x^8)))/(d^10*x^9)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1905, 1588, 25, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1905}$$

$$\frac{\sqrt{d^2-e^2x^2}\int \frac{cx^4+bx^2+a}{x^{10}\sqrt{d^2-e^2x^2}}dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{1588}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(-\frac{\int -\frac{9cx^2d^2+9bd^2+8ae^2}{x^8\sqrt{d^2-e^2x^2}}dx}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\int \frac{9cx^2d^2+9bd^2+8ae^2}{x^8\sqrt{d^2-e^2x^2}}dx}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$\Big\downarrow$ 359

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(16ae^4+18bd^2e^2+21cd^4\right)\int \frac{1}{x^6\sqrt{d^2-e^2x^2}}dx}{7d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{8ae^2}{d^2}+9b\right)}{7x^7}}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$\Big\downarrow$ 245

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(16ae^4+18bd^2e^2+21cd^4\right)\left(\frac{4e^2\int \frac{1}{x^4\sqrt{d^2-e^2x^2}}dx}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{7d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{8ae^2}{d^2}+9b\right)}{7x^7}}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$\Big\downarrow$ 245

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(16ae^4+18bd^2e^2+21cd^4\right)\left(\frac{4e^2\left(\frac{2e^2\int \frac{1}{x^2\sqrt{d^2-e^2x^2}}dx}{3d^2}-\frac{\sqrt{d^2-e^2x^2}}{3d^2x^3}\right)}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)}{7d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{8ae^2}{d^2}+9b\right)}{7x^7}}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

$\Big\downarrow$ 242

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{\frac{3\left(\frac{4e^2\left(-\frac{\sqrt{d^2-e^2x^2}}{3d^2x^3}-\frac{2e^2\sqrt{d^2-e^2x^2}}{3d^4x}\right)}{5d^2}-\frac{\sqrt{d^2-e^2x^2}}{5d^2x^5}\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{7d^2}-\frac{\sqrt{d^2-e^2x^2}\left(\frac{8ae^2}{d^2}+9b\right)}{7x^7}}{9d^2}-\frac{a\sqrt{d^2-e^2x^2}}{9d^2x^9}\right)}{\sqrt{d-ex}\sqrt{d+ex}}$$

input
```
Int[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

output
```
(Sqrt[d^2 - e^2*x^2]*(-1/9*(a*Sqrt[d^2 - e^2*x^2])/(d^2*x^9) + (-1/7*((9*b
 + (8*a*e^2)/d^2)*Sqrt[d^2 - e^2*x^2])/x^7 + (3*(21*c*d^4 + 18*b*d^2*e^2 +
 16*a*e^4)*(-1/5*Sqrt[d^2 - e^2*x^2]/(d^2*x^5) + (4*e^2*(-1/3*Sqrt[d^2 - e
^2*x^2]/(d^2*x^3) - (2*e^2*Sqrt[d^2 - e^2*x^2])/(3*d^4*x)))/(5*d^2)))/(7*d
^2))/(9*d^2)))/(Sqrt[d - e*x]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 1905
```
Int[((f_.)*(x_))^(m_.)*((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)
*(x_)^(non2_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x
_Symbol] :> Simp[(d1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[
q]/(d1*d2 + e1*e2*x^n)^FracPart[q])   Int[(f*x)^m*(d1*d2 + e1*e2*x^n)^q*(a
+ b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, n, p,
q}, x] && EqQ[n2, 2*n] && EqQ[non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.66

method	result
gosper	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^{8}x^{8}+144b\,d^{2}e^{6}x^{8}+168c\,d^{4}e^{4}x^{8}+64a\,d^{2}e^{6}x^{6}+72b\,d^{4}e^{4}x^{6}+84c\,d^{6}e^{2}x^{6}+48a\,d^{4}e^{4}x^{4}+54b\,d^{6}e^{2}x^{4}+63c\,d^{8}}{315x^{9}d^{10}}$
risch	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^{8}x^{8}+144b\,d^{2}e^{6}x^{8}+168c\,d^{4}e^{4}x^{8}+64a\,d^{2}e^{6}x^{6}+72b\,d^{4}e^{4}x^{6}+84c\,d^{6}e^{2}x^{6}+48a\,d^{4}e^{4}x^{4}+54b\,d^{6}e^{2}x^{4}+63c\,d^{8}}{315x^{9}d^{10}}$
orering	$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(128a\,e^{8}x^{8}+144b\,d^{2}e^{6}x^{8}+168c\,d^{4}e^{4}x^{8}+64a\,d^{2}e^{6}x^{6}+72b\,d^{4}e^{4}x^{6}+84c\,d^{6}e^{2}x^{6}+48a\,d^{4}e^{4}x^{4}+54b\,d^{6}e^{2}x^{4}+63c\,d^{8}}{315x^{9}d^{10}}$
default	$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\operatorname{csgn}(e)^{2}(128a\,e^{8}x^{8}+144b\,d^{2}e^{6}x^{8}+168c\,d^{4}e^{4}x^{8}+64a\,d^{2}e^{6}x^{6}+72b\,d^{4}e^{4}x^{6}+84c\,d^{6}e^{2}x^{6}+48a\,d^{4}e^{4}x^{4}+54b\,d^{6}e^{2}}{315d^{10}x^{9}}$

input
```
int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(128*a*e^8*x^8+144*b*d^2*e^6*x^8+168*c
*d^4*e^4*x^8+64*a*d^2*e^6*x^6+72*b*d^4*e^4*x^6+84*c*d^6*e^2*x^6+48*a*d^4*e
^4*x^4+54*b*d^6*e^2*x^4+63*c*d^8*x^4+40*a*d^6*e^2*x^2+45*b*d^8*x^2+35*a*d^
8)/x^9/d^10
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.62

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx =$$
$$-\frac{(35\,ad^8+8\,(21\,cd^4e^4+18\,bd^2e^6+16\,ae^8)x^8+4\,(21\,cd^6e^2+18\,bd^4e^4+16\,ad^2e^6)x^6+3\,(21\,cd^8+18\,bd}{315\,d^{10}x^9}$$

input
```
integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="
fricas")
```

output
```
-1/315*(35*a*d^8 + 8*(21*c*d^4*e^4 + 18*b*d^2*e^6 + 16*a*e^8)*x^8 + 4*(21*
c*d^6*e^2 + 18*b*d^4*e^4 + 16*a*d^2*e^6)*x^6 + 3*(21*c*d^8 + 18*b*d^6*e^2
+ 16*a*d^4*e^4)*x^4 + 5*(9*b*d^8 + 8*a*d^6*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e
*x + d)/(d^10*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = &-\frac{8\sqrt{-e^2x^2+d^2}ce^4}{15\,d^6x}-\frac{16\sqrt{-e^2x^2+d^2}be^6}{35\,d^8x}\\&-\frac{128\sqrt{-e^2x^2+d^2}ae^8}{315\,d^{10}x}-\frac{4\sqrt{-e^2x^2+d^2}ce^2}{15\,d^4x^3}\\&-\frac{8\sqrt{-e^2x^2+d^2}be^4}{35\,d^6x^3}-\frac{64\sqrt{-e^2x^2+d^2}ae^6}{315\,d^8x^3}\\&-\frac{\sqrt{-e^2x^2+d^2}c}{5\,d^2x^5}-\frac{6\sqrt{-e^2x^2+d^2}be^2}{35\,d^4x^5}\\&-\frac{16\sqrt{-e^2x^2+d^2}ae^4}{105\,d^6x^5}-\frac{\sqrt{-e^2x^2+d^2}b}{7\,d^2x^7}\\&-\frac{8\sqrt{-e^2x^2+d^2}ae^2}{63\,d^4x^7}-\frac{\sqrt{-e^2x^2+d^2}a}{9\,d^2x^9}\end{aligned}$$

input
```
integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="
maxima")
```

output
```
-8/15*sqrt(-e^2*x^2 + d^2)*c*e^4/(d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*b*e^
6/(d^8*x) - 128/315*sqrt(-e^2*x^2 + d^2)*a*e^8/(d^10*x) - 4/15*sqrt(-e^2*x
^2 + d^2)*c*e^2/(d^4*x^3) - 8/35*sqrt(-e^2*x^2 + d^2)*b*e^4/(d^6*x^3) - 64
/315*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*c/(d^
2*x^5) - 6/35*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^5) - 16/105*sqrt(-e^2*x^2
+ d^2)*a*e^4/(d^6*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^7) - 8/63*sqrt(
-e^2*x^2 + d^2)*a*e^2/(d^4*x^7) - 1/9*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(202) = 404$.

Time = 0.72 (sec) , antiderivative size = 1847, normalized size of antiderivative = 7.96

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="
giac")
```

output
```
-4/315*(315*c*d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) -
sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^17 + 315*b*d^2*e^8*((sqr
t(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt
(d) - sqrt(-e*x + d)))^17 + 315*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))
/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^17 - 67
20*c*d^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x
+ d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^15 - 5040*b*d^2*e^8*((sqrt(2)*sqr
t(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sq
rt(-e*x + d)))^15 - 3360*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e
*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^15 + 76608*c*d
^4*e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(
sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 + 68544*b*d^2*e^8*((sqrt(2)*sqrt(d)
- sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e
*x + d)))^13 + 76608*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x +
 d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^13 - 580608*c*d^4*
e^6*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqr
t(2)*sqrt(d) - sqrt(-e*x + d)))^11 - 509184*b*d^2*e^8*((sqrt(2)*sqrt(d) -
sqrt(-e*x + d))/sqrt(e*x + d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x
 + d)))^11 - 327168*a*e^10*((sqrt(2)*sqrt(d) - sqrt(-e*x + d))/sqrt(e*x +
d) - sqrt(e*x + d)/(sqrt(2)*sqrt(d) - sqrt(-e*x + d)))^11 + 2892288*c*d...
```

Mupad [B] (verification not implemented)

Time = 4.66 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.25

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = -\frac{\sqrt{d-e\,x}\,\left(\frac{a}{9\,d}+\frac{x^2\,\left(45\,b\,d^9+40\,a\,d^7\,e^2\right)}{315\,d^{10}}+\frac{x^6\,\left(84\,c\,d^7\,e^2+72\,b\,d^5\,e^4+64\,a\,d^3\,e^6\right)}{315\,d^{10}}+\frac{x^7\,\left(84\,c\,d^6\,e^3+72\,b\,d^4\,e^5+64\,a\,d^2\,e^7\right)}{315\,d^{10}}+\frac{x^4\,(63\,\cdots}{}\right.}{}$$

input
```
int((a + b*x^2 + c*x^4)/(x^10*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

output
```
-((d - e*x)^(1/2)*(a/(9*d) + (x^2*(45*b*d^9 + 40*a*d^7*e^2))/(315*d^10) +
(x^6*(64*a*d^3*e^6 + 72*b*d^5*e^4 + 84*c*d^7*e^2))/(315*d^10) + (x^7*(64*a
*d^2*e^7 + 72*b*d^4*e^5 + 84*c*d^6*e^3))/(315*d^10) + (x^4*(63*c*d^9 + 48*
a*d^5*e^4 + 54*b*d^7*e^2))/(315*d^10) + (x^9*(128*a*e^9 + 144*b*d^2*e^7 +
168*c*d^4*e^5))/(315*d^10) + (x^3*(40*a*d^6*e^3 + 45*b*d^8*e))/(315*d^10)
+ (x^5*(48*a*d^4*e^5 + 54*b*d^6*e^3 + 63*c*d^8*e))/(315*d^10) + (x^8*(144*
b*d^3*e^6 + 168*c*d^5*e^4 + 128*a*d*e^8))/(315*d^10) + (a*e*x)/(9*d^2)))/(
x^9*(d + e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$$
$$= \frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,(-128a\,e^8x^8-144b\,d^2e^6x^8-168c\,d^4e^4x^8-64a\,d^2e^6x^6-72b\,d^4e^4x^6-84c\,d^6e^2x^6-}{315d^{10}x^9}$$

input
```
int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(sqrt(d + e*x)*sqrt(d - e*x)*( - 35*a*d**8 - 40*a*d**6*e**2*x**2 - 48*a*d*
*4*e**4*x**4 - 64*a*d**2*e**6*x**6 - 128*a*e**8*x**8 - 45*b*d**8*x**2 - 54
*b*d**6*e**2*x**4 - 72*b*d**4*e**4*x**6 - 144*b*d**2*e**6*x**8 - 63*c*d**8
*x**4 - 84*c*d**6*e**2*x**6 - 168*c*d**4*e**4*x**8))/(315*d**10*x**9)
```

3.31 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 37, antiderivative size = 404

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3)\,x\sqrt{1-d^2x^2}}{16d^4}$$

$$-\frac{\left(7Be-\frac{3Ce^2}{f}+14Af+\frac{8Cf}{d^2}\right)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{70d^2}$$

$$-\frac{\left(7B-\frac{3Ce}{f}\right)(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{42d^2}-\frac{C(e+fx)^4\left(1-d^2x^2\right)^{3/2}}{7d^2f}$$

$$+\frac{(8(C(3d^4e^4-30d^2e^2f^2-8f^4)-7d^2f(2Af(6d^2e^2+f^2)+B(d^2e^3+6ef^2)))+3d^2f(6Cd^2e^3-14Bd^2}{840d^6f}$$

$$+\frac{(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3)\arcsin(dx)}{16d^5}$$

output
```
1/16*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d^2*e^2*f+2*C*d^2*e^3+B*f^3+3*C*e*f^2)
*x*(-d^2*x^2+1)^(1/2)/d^4-1/70*(7*B*e-3*C*e^2/f+14*A*f+8*C*f/d^2)*(f*x+e)^
2*(-d^2*x^2+1)^(3/2)/d^2-1/42*(7*B-3*C*e/f)*(f*x+e)^3*(-d^2*x^2+1)^(3/2)/d
^2-1/7*C*(f*x+e)^4*(-d^2*x^2+1)^(3/2)/d^2/f+1/840*(8*C*(3*d^4*e^4-30*d^2*e
^2*f^2-8*f^4)-56*d^2*f*(2*A*f*(6*d^2*e^2+f^2)+B*(d^2*e^3+6*e*f^2))+3*d^2*f
*(-98*A*d^2*e*f^2-14*B*d^2*e^2*f+6*C*d^2*e^3-35*B*f^3-41*C*e*f^2)*x)*(-d^2
*x^2+1)^(3/2)/d^6/f+1/16*(8*A*d^4*e^3+6*A*d^2*e*f^2+6*B*d^2*e^2*f+2*C*d^2*
e^3+B*f^3+3*C*e*f^2)*arcsin(d*x)/d^5
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.92

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{1-d^2x^2}(14Ad^2(-16f^3-d^2f(120e^2+45efx+8f^2x^2)+6d^4x(10e^3+20e^2fx+15ef^2x^2+4f^3x^3))+}{}$$

input
```
Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(14*A*d^2*(-16*f^3 - d^2*f*(120*e^2 + 45*e*f*x + 8*f^2*
x^2) + 6*d^4*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)) + 7*B*(-3
*d^2*f^2*(32*e + 5*f*x) - 2*d^4*(40*e^3 + 45*e^2*f*x + 24*e*f^2*x^2 + 5*f^
3*x^3) + 4*d^6*x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3)) + C*
(-128*f^3 - d^2*f*(672*e^2 + 315*e*f*x + 64*f^2*x^2) - 6*d^4*x*(35*e^3 + 5
6*e^2*f*x + 35*e*f^2*x^2 + 8*f^3*x^3) + 12*d^6*x^3*(35*e^3 + 84*e^2*f*x +
70*e*f^2*x^2 + 20*f^3*x^3))) + 210*d*(2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*
e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B*f^3)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2
*x^2])])/(1680*d^6)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2112, 2185, 25, 27, 687, 27, 687, 25, 676, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-dx}\sqrt{dx+1}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \ 2112$$

$$\int \sqrt{1-d^2x^2}(e+fx)^3\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \ 2185$$

$$-\frac{\int -f(e+fx)^3\left(\left(7Ad^2+4C\right)f-d^2(3Ce-7Bf)x\right)\sqrt{1-d^2x^2}dx}{7d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \ 25$$

$$\frac{\int f(e+fx)^3\left(\left(7Ad^2+4C\right)f-d^2(3Ce-7Bf)x\right)\sqrt{1-d^2x^2}dx}{7d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (e+fx)^3\left(\left(7Ad^2+4C\right)f-d^2(3Ce-7Bf)x\right)\sqrt{1-d^2x^2}dx}{7d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{1}{6}\left(1-d^2x^2\right)^{3/2}(e+fx)^3(3Ce-7Bf)-\frac{\int -3d^2(e+fx)^2\left(f\left(14Aed^2+5Ce+7Bf\right)+\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)x\right)\sqrt{1-d^2x^2}d}{6d^2}}{7d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}\int (e+fx)^2 \left(f\left(14Aed^2+5Ce+7Bf\right)+\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)x\right)\sqrt{1-d^2x^2}dx+\frac{1}{6}\left(1-d^2x^2\right)}{7d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{1}{2}\left(-\frac{\int -\left((e+fx)\left(f\left(70Ae^2d^4+19Ce^2d^2+28Af^2d^2+49Befd^2+16Cf^2\right)-d^2\left(6Cd^2e^3-14Bd^2fe^2-98Ad^2f^2e-41Cf^2e-35Bf^3\right)x\right)\sqrt{1-d^2x^2}\right)}{5d^2}\right.}{7d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\left(\frac{\int (e+fx)\left(f\left(70Ae^2d^4+19Ce^2d^2+28Af^2d^2+49Befd^2+16Cf^2\right)-d^2\left(6Cd^2e^3-14Bd^2fe^2-98Ad^2f^2e-41Cf^2e-35Bf^3\right)x\right)\sqrt{1-d^2x^2}dx}{5d^2}-\frac{1}{5}\right.}{7d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \text{676}$$

$$\frac{1}{2}\left(\frac{\frac{35}{4}f\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)\int\sqrt{1-d^2x^2}dx+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-98Ad^2ef^2-14Bd^2e^2f-35Bf^3+6Cd^2e^3-\right.}{5d^2}\right.$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{\frac{35}{4}f\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)\left(\frac{1}{2}\int\frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}x\sqrt{1-d^2x^2}\right)+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-98Ad^2ef^2-14Bd^2e^2f-\right.}{5d^2}\right.$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(\frac{\frac{35}{4}f\left(\frac{\arcsin(dx)}{2d}+\frac{1}{2}x\sqrt{1-d^2x^2}\right)(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2)+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}(-98Ad^2ef^2-14Bd^2e^2f-35Bf^3}{5d^2}\right.$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

input

```
Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

output

```
-1/7*(C*(e + f*x)^4*(1 - d^2*x^2)^(3/2))/(d^2*f) + (((3*C*e - 7*B*f)*(e +
f*x)^3*(1 - d^2*x^2)^(3/2))/6 + (-1/5*((7*f*(B*e + 2*A*f) - C*(3*e^2 - (8*
f^2)/d^2))*(e + f*x)^2*(1 - d^2*x^2)^(3/2)) + ((2*(C*(3*d^4*e^4 - 30*d^2*e
^2*f^2 - 8*f^4) - 7*d^2*f*(2*A*f*(6*d^2*e^2 + f^2) + B*(d^2*e^3 + 6*e*f^2)
))*(1 - d^2*x^2)^(3/2))/(3*d^2) + (f*(6*C*d^2*e^3 - 14*B*d^2*e^2*f - 41*C*
e*f^2 - 98*A*d^2*e*f^2 - 35*B*f^3)*x*(1 - d^2*x^2)^(3/2))/4 + (35*f*(2*C*d
^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B*f^3)*
((x*Sqrt[1 - d^2*x^2])/2 + ArcSin[d*x]/(2*d))/4)/(5*d^2))/2)/(7*d^2*f)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{(240C f^3x^6d^6+280B d^6f^3x^5+840C d^6e f^2x^5+336A d^6f^3x^4+1008B d^6e f^2x^4+1008C d^6e^2f x^4+1260A d^6e f^2x^3+1260B d^6e^2}{}$
default	Expression too large to display

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x,method=_RETURNV
ERBOSE)
```

output
```
-1/1680*(240*C*d^6*f^3*x^6+280*B*d^6*f^3*x^5+840*C*d^6*e*f^2*x^5+336*A*d^6
*f^3*x^4+1008*B*d^6*e*f^2*x^4+1008*C*d^6*e^2*f*x^4+1260*A*d^6*e*f^2*x^3+12
60*B*d^6*e^2*f*x^3+420*C*d^6*e^3*x^3+1680*A*d^6*e^2*f*x^2+560*B*d^6*e^3*x^
2-48*C*d^4*f^3*x^4+840*A*d^6*e^3*x-70*B*d^4*f^3*x^3-210*C*d^4*e*f^2*x^3-11
2*A*d^4*f^3*x^2-336*B*d^4*e*f^2*x^2-336*C*d^4*e^2*f*x^2-630*A*d^4*e*f^2*x-
630*B*d^4*e^2*f*x-210*C*d^4*e^3*x-1680*A*d^4*e^2*f-560*B*d^4*e^3-64*C*d^2*
f^3*x^2-105*B*d^2*f^3*x-315*C*d^2*e*f^2*x-224*A*d^2*f^3-672*B*d^2*e*f^2-67
2*C*d^2*e^2*f-128*C*f^3)*(d*x+1)^(1/2)*(d*x-1)/d^6/(-(d*x+1)*(d*x-1))^(1/2
)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/16/d^4*(8*A*d^4*e^3+6*A*d^2*e*
f^2+6*B*d^2*e^2*f+2*C*d^2*e^3+B*f^3+3*C*e*f^2)/(d^2)^(1/2)*arctan((d^2)^(1
/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(240\,Cd^6f^3x^6-560\,Bd^4e^3-672\,Bd^2ef^2+280\,(3\,Cd^6ef^2+Bd^6f^3)x^5+48\,(21\,Cd^6e^2f+21\,Bd^6ef^2+}{}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x, algorith
m="fricas")
```

output
```
1/1680*((240*C*d^6*f^3*x^6 - 560*B*d^4*e^3 - 672*B*d^2*e*f^2 + 280*(3*C*d^
6*e*f^2 + B*d^6*f^3)*x^5 + 48*(21*C*d^6*e^2*f + 21*B*d^6*e*f^2 + (7*A*d^6
- C*d^4)*f^3)*x^4 - 336*(5*A*d^4 + 2*C*d^2)*e^2*f - 32*(7*A*d^2 + 4*C)*f^3
 + 70*(6*C*d^6*e^3 + 18*B*d^6*e^2*f - B*d^4*f^3 + 3*(6*A*d^6 - C*d^4)*e*f^
2)*x^3 + 16*(35*B*d^6*e^3 - 21*B*d^4*e*f^2 + 21*(5*A*d^6 - C*d^4)*e^2*f -
(7*A*d^4 + 4*C*d^2)*f^3)*x^2 - 105*(6*B*d^4*e^2*f + B*d^2*f^3 - 2*(4*A*d^6
 - C*d^4)*e^3 + 3*(2*A*d^4 + C*d^2)*e*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 - 210*(6*B*d^3*e^2*f + B*d*f^3 + 2*(4*A*d^5 + C*d^3)*e^3 + 3*(2*A*d^3 + C
*d)*e*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^6
```

Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (e+fx)^3\sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((-d*x+1)**(1/2)*(d*x+1)**(1/2)*(f*x+e)**3*(C*x**2+B*x+A),x)
```

output
```
Integral((e + f*x)**3*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.10

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$= -\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^4}{7\,d^2} + \frac{1}{2}\sqrt{-d^2x^2+1}Ae^3x + \frac{Ae^3\arcsin{(dx)}}{2\,d} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Be^3}{3\,d^2} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Ae^2f}{d^2} - \frac{4\,(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^2}{35\,d^4} - \frac{(3\,Cef^2+Bf^3)(-d^2x^2+1)^{\frac{3}{2}}x^3}{6\,d^2} - \frac{(3\,Ce^2f+3\,Bef^2+Af^3)(-d^2x^2+1)^{\frac{3}{2}}x^2}{5\,d^2} - \frac{(Ce^3+3\,Be^2f+3\,Aef^2)(-d^2x^2+1)^{\frac{3}{2}}x}{4\,d^2} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\sqrt{-d^2x^2+1}x}{8\,d^2} - \frac{8\,(-d^2x^2+1)^{\frac{3}{2}}Cf^3}{105\,d^6} - \frac{(3\,Cef^2+Bf^3)(-d^2x^2+1)^{\frac{3}{2}}x}{8\,d^4} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\arcsin{(dx)}}{8\,d^3} - \frac{2\,(3\,Ce^2f+3\,Bef^2+Af^3)(-d^2x^2+1)^{\frac{3}{2}}}{15\,d^4} + \frac{(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x}{16\,d^4} + \frac{(3\,Cef^2+Bf^3)\arcsin{(dx)}}{16\,d^5}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x, algorith
m="maxima")
```

output
```
-1/7*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^4/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^3*x +
 1/2*A*e^3*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^3/d^2 - (-d^2*x^2
+ 1)^(3/2)*A*e^2*f/d^2 - 4/35*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^2/d^4 - 1/6*(3*
C*e*f^2 + B*f^3)*(-d^2*x^2 + 1)^(3/2)*x^3/d^2 - 1/5*(3*C*e^2*f + 3*B*e*f^2
 + A*f^3)*(-d^2*x^2 + 1)^(3/2)*x^2/d^2 - 1/4*(C*e^3 + 3*B*e^2*f + 3*A*e*f^
2)*(-d^2*x^2 + 1)^(3/2)*x/d^2 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*sqrt(-
d^2*x^2 + 1)*x/d^2 - 8/105*(-d^2*x^2 + 1)^(3/2)*C*f^3/d^6 - 1/8*(3*C*e*f^2
 + B*f^3)*(-d^2*x^2 + 1)^(3/2)*x/d^4 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)
*arcsin(d*x)/d^3 - 2/15*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*(-d^2*x^2 + 1)^(3/
2)/d^4 + 1/16*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x/d^4 + 1/16*(3*C*e*f
^2 + B*f^3)*arcsin(d*x)/d^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1539 vs. $2(380) = 760$.

Time = 0.30 (sec) , antiderivative size = 1539, normalized size of antiderivative = 3.81

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x, algorith
m="giac")
```

output
```
1/1680*(840*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)
*sqrt(d*x + 1)))*A*d^5*e^3 + 1680*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin
(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^5*e^3 + 280*(((2*d*x - 5)*(d*x + 1) + 9)*
sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^4*
e^3 + 840*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*B*d^4*e^3 + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*
sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^4*e^2*f + 2520*(
sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1
)))*A*d^4*e^2*f + 70*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqr
t(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^3*e^
3 + 280*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^3*e^3 + 210*(((2*(3*d*x - 10)*(d*x + 1)
 + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2
)*sqrt(d*x + 1)))*B*d^3*e^2*f + 840*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x
+ 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^3*e^2*f + 2
10*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d
*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*e*f^2 + 840*(((2*d*x
 - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*A*d^3*e*f^2 + 42*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x
 + 1) - 295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(...
```

Mupad [B] (verification not implemented)

Time = 43.84 (sec) , antiderivative size = 3993, normalized size of antiderivative = 9.88

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
int((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
- ((((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)
^(1/2) - 1)^6 + (((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^
22)/((d*x + 1)^(1/2) - 1)^22 - (((20480*C*f^3)/3 - 448*C*d^2*e^2*f)*((1 -
d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((20480*C*f^3)/3 - 448*C*d^2
*e^2*f)*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (((458752*C*f
^3)/15 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2)
 - 1)^10 + (((458752*C*f^3)/15 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) -
 1)^18)/((d*x + 1)^(1/2) - 1)^18 - (((1011712*C*f^3)/15 - (13184*C*d^2*e^2
*f)/5)*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 - (((1011712*C*f
^3)/15 - (13184*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2)
 - 1)^16 + (((9293824*C*f^3)/105 - (15104*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2)
 - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)^(1/2) - 1)^3*((29*C*d^3*e
^3)/2 - (41*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1
)^25*((29*C*d^3*e^3)/2 - (41*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^25 - (((
1 - d*x)^(1/2) - 1)^5*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/((d*x + 1)^(1/2
) - 1)^5 + (((1 - d*x)^(1/2) - 1)^23*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/
((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2) - 1)^7*(209*C*d^3*e^3 + (8755
*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^21*(209*C
*d^3*e^3 + (8755*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^21 + (((1 - d*x)^(1/
2) - 1)^11*((1767*C*d^3*e^3)/2 - (8267*C*d*e*f^2)/4))/((d*x + 1)^(1/2) ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.15

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x)
```

output

```
( - 1680*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**5*e**3 - 1260*asin(sqrt( - d*
x + 1)/sqrt(2))*a*d**3*e*f**2 - 1260*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**3
*e**2*f - 210*asin(sqrt( - d*x + 1)/sqrt(2))*b*d*f**3 - 420*asin(sqrt( - d
*x + 1)/sqrt(2))*c*d**3*e**3 - 630*asin(sqrt( - d*x + 1)/sqrt(2))*c*d*e*f*
*2 + 840*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**6*e**3*x + 1680*sqrt(d*x + 1)
*sqrt( - d*x + 1)*a*d**6*e**2*f*x**2 + 1260*sqrt(d*x + 1)*sqrt( - d*x + 1)
*a*d**6*e*f**2*x**3 + 336*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**6*f**3*x**4
- 1680*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**4*e**2*f - 630*sqrt(d*x + 1)*sq
rt( - d*x + 1)*a*d**4*e*f**2*x - 112*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**4
*f**3*x**2 - 224*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**2*f**3 + 560*sqrt(d*x
 + 1)*sqrt( - d*x + 1)*b*d**6*e**3*x**2 + 1260*sqrt(d*x + 1)*sqrt( - d*x +
 1)*b*d**6*e**2*f*x**3 + 1008*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**6*e*f**2
*x**4 + 280*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**6*f**3*x**5 - 560*sqrt(d*x
 + 1)*sqrt( - d*x + 1)*b*d**4*e**3 - 630*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*
d**4*e**2*f*x - 336*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**4*e*f**2*x**2 - 70
*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**4*f**3*x**3 - 672*sqrt(d*x + 1)*sqrt(
 - d*x + 1)*b*d**2*e*f**2 - 105*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**2*f**3
*x + 420*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**6*e**3*x**3 + 1008*sqrt(d*x +
 1)*sqrt( - d*x + 1)*c*d**6*e**2*f*x**4 + 840*sqrt(d*x + 1)*sqrt( - d*x +
1)*c*d**6*e*f**2*x**5 + 240*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**6*f**3*...
```

3.32 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2 (A+Bx+Cx^2)\, dx$

Optimal result

Integrand size = 37, antiderivative size = 284

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2 \left(A+Bx+Cx^2\right)\, dx$$
$$= \frac{\left(2Ce^2+8Ad^2e^2+4Bef+2Af^2+\frac{Cf^2}{d^2}\right)x\sqrt{1-d^2x^2}}{16d^2}$$
$$- \frac{\left(2B-\frac{Ce}{f}\right)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2} - \frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f}$$
$$+ \frac{\left(8(C(d^2e^3-4ef^2))-2f(5Ad^2ef+B(d^2e^2+f^2)))-3f^2\left(2d^2e\left(2B-\frac{Ce}{f}\right)+5(C+2Ad^2)f\right)x\right)(1-}{120d^4f}$$
$$+ \frac{(C(2d^2e^2+f^2)+2d^2(2Bef+A(4d^2e^2+f^2)))\arcsin(dx)}{16d^5}$$

output
```
1/16*(2*C*e^2+8*A*d^2*e^2+4*B*e*f+2*A*f^2+C*f^2/d^2)*x*(-d^2*x^2+1)^(1/2)/
d^2-1/10*(2*B-C*e/f)*(f*x+e)^2*(-d^2*x^2+1)^(3/2)/d^2-1/6*C*(f*x+e)^3*(-d^
2*x^2+1)^(3/2)/d^2/f+1/120*(8*C*(d^2*e^3-4*e*f^2)-16*f*(5*A*d^2*e*f+B*(d^2
*e^2+f^2))-3*f^2*(2*d^2*e*(2*B-C*e/f)+5*(2*A*d^2+C)*f)*x)*(-d^2*x^2+1)^(3/
2)/d^4/f+1/16*(C*(2*d^2*e^2+f^2)+2*d^2*(2*B*e*f+A*(4*d^2*e^2+f^2)))*arcsin
(d*x)/d^5
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.92

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{d\sqrt{1-d^2x^2}(10Ad^2(12d^2e^2x+16ef(-1+d^2x^2)+3f^2x(-1+2d^2x^2))+4B(-8f^2-d^2(20e^2+15efx+\cdots}{}$$

input
```
Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

output
```
(d*Sqrt[1 - d^2*x^2]*(10*A*d^2*(12*d^2*e^2*x + 16*e*f*(-1 + d^2*x^2) + 3*f
^2*x*(-1 + 2*d^2*x^2)) + 4*B*(-8*f^2 - d^2*(20*e^2 + 15*e*f*x + 4*f^2*x^2)
 + 2*d^4*x^2*(10*e^2 + 15*e*f*x + 6*f^2*x^2)) + C*(30*d^2*e^2*x*(-1 + 2*d^
2*x^2) + 32*e*f*(-2 - d^2*x^2 + 3*d^4*x^4) + 5*f^2*x*(-3 - 2*d^2*x^2 + 8*d
^4*x^4))) + 30*(C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)
))*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(240*d^5)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {2112, 2185, 27, 687, 25, 27, 676, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-dx}\sqrt{dx+1}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \sqrt{1-d^2x^2}(e+fx)^2\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\int -3f(e+fx)^2\left(\left(2Ad^2+C\right)f-d^2(Ce-2Bf)x\right)\sqrt{1-d^2x^2}dx}{6d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (e+fx)^2 \left(\left(2Ad^2+C\right) f-d^2(Ce-2Bf)x\right) \sqrt{1-d^2x^2}dx}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \text{687}$$

$$\frac{\frac{1}{5}\left(1-d^2x^2\right)^{3/2}(e+fx)^2(Ce-2Bf)-\frac{\int -d^2(e+fx)\left(f\left(10Aed^2+3Ce+4Bf\right)+\left(5\left(2Ad^2+C\right)f^2-2d^2e(Ce-2Bf)\right)x\right)\sqrt{1-d^2x^2}dx}{5d^2}}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int d^2(e+fx)\left(f\left(10Aed^2+3Ce+4Bf\right)+\left(5\left(2Ad^2+C\right)f^2-2d^2e(Ce-2Bf)\right)x\right)\sqrt{1-d^2x^2}dx}{5d^2}+\frac{1}{5}\left(1-d^2x^2\right)^{3/2}(e+fx)^2(Ce-2Bf)}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\int (e+fx)\left(f\left(10Aed^2+3Ce+4Bf\right)+\left(5\left(2Ad^2+C\right)f^2-2d^2e(Ce-2Bf)\right)x\right)\sqrt{1-d^2x^2}dx+\frac{1}{5}\left(1-d^2x^2\right)^{3}}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{1}{5}\left(\frac{5f\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)\int \sqrt{1-d^2x^2}dx}{4d^2}+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-10Af^2-4Bef-\frac{5Cf^2}{d^2}+2Ce^2\right)+\right.}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$$\Big\downarrow \text{211}$$

$$\frac{\frac{1}{5}\left(\frac{5f\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)\left(\frac{1}{2}\int \frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}x\sqrt{1-d^2x^2}\right)}{4d^2}+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-10Af^2-4Bef-\frac{5C}{}\right.\right.}{2d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

$\downarrow$ 223

$$\frac{\frac{1}{5}\left(\frac{5f\left(\frac{\arcsin(dx)}{2d}+\frac{1}{2}x\sqrt{1-d^2x^2}\right)\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)}{4d^2}+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}\left(-10Af^2-4Bef-\frac{5Cf^2}{d^2}+\right.\right.}{2d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

input
```
Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

output
```
-1/6*(C*(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(d^2*f) + (((C*e - 2*B*f)*(e + f*
x)^2*(1 - d^2*x^2)^(3/2))/5 + ((2*(C*d^2*e^3 - 2*B*d^2*e^2*f - 4*C*e*f^2 -
 10*A*d^2*e*f^2 - 2*B*f^3)*(1 - d^2*x^2)^(3/2))/(3*d^2) + (f*(2*C*e^2 - 4*
B*e*f - 10*A*f^2 - (5*C*f^2)/d^2)*x*(1 - d^2*x^2)^(3/2))/4 + (5*f*(C*(2*d^
2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*((x*Sqrt[1 - d^2*x^2
])/2 + ArcSin[d*x]/(2*d)))/(4*d^2))/5)/(2*d^2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{(40C\,f^2x^5d^4+48B\,d^4f^2x^4+96C\,d^4ef\,x^4+60A\,d^4f^2x^3+120B\,d^4ef\,x^3+60C\,d^4e^2x^3+160A\,d^4ef\,x^2+80B\,d^4e^2x^2+120A\,d^4e^2x-}{}$
default	$\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\left(120A\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^4e^2+30A\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^2f^2+30C\arctan\left(\frac{\mathrm{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^2e^2-60B\sqrt{-d^2x^2}\right.}{}$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x,method=_RETURNV
ERBOSE)
```

output
```
-1/240/d^4*(40*C*d^4*f^2*x^5+48*B*d^4*f^2*x^4+96*C*d^4*e*f*x^4+60*A*d^4*f^
2*x^3+120*B*d^4*e*f*x^3+60*C*d^4*e^2*x^3+160*A*d^4*e*f*x^2+80*B*d^4*e^2*x^
2+120*A*d^4*e^2*x-10*C*d^2*f^2*x^3-16*B*d^2*f^2*x^2-32*C*d^2*e*f*x^2-30*A*
d^2*f^2*x-60*B*d^2*e*f*x-30*C*d^2*e^2*x-160*A*d^2*e*f-80*B*d^2*e^2-15*C*f^
2*x-32*B*f^2-64*C*e*f)*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d
*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/16*(8*A*d^4*e^2+2*A*d^2*f^2+4*B*d^2*
e*f+2*C*d^2*e^2+C*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(
1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.98

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{(40\,Cd^5f^2x^5 - 80\,Bd^3e^2 + 48\,(2\,Cd^5ef + Bd^5f^2)x^4 - 32\,Bdf^2 + 10\,(6\,Cd^5e^2 + 12\,Bd^5ef + (6\,Ad^5 - C}{}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x, algorith
m="fricas")
```

output
```
1/240*((40*C*d^5*f^2*x^5 - 80*B*d^3*e^2 + 48*(2*C*d^5*e*f + B*d^5*f^2)*x^4
 - 32*B*d*f^2 + 10*(6*C*d^5*e^2 + 12*B*d^5*e*f + (6*A*d^5 - C*d^3)*f^2)*x^
3 - 32*(5*A*d^3 + 2*C*d)*e*f + 16*(5*B*d^5*e^2 - B*d^3*f^2 + 2*(5*A*d^5 -
C*d^3)*e*f)*x^2 - 15*(4*B*d^3*e*f - 2*(4*A*d^5 - C*d^3)*e^2 + (2*A*d^3 + C
*d)*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(4*B*d^2*e*f + 2*(4*A*d^4 +
C*d^2)*e^2 + (2*A*d^2 + C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/
(d*x)))/d^5
```

Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$=\int(e+fx)^2\sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((-d*x+1)**(1/2)*(d*x+1)**(1/2)*(f*x+e)**2*(C*x**2+B*x+A),x)
```

output
```
Integral((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.08

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= -\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^2x^3}{6\,d^2}+\frac{1}{2}\sqrt{-d^2x^2+1}Ae^2x + \frac{Ae^2\arcsin(dx)}{2\,d}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Be^2}{3\,d^2}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}Aef}{3\,d^2} - \frac{(-d^2x^2+1)^{\frac{3}{2}}(2\,Cef+Bf^2)x^2}{5\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}(Ce^2+2\,Bef+Af^2)x}{4\,d^2} - \frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^2x}{8\,d^4}+\frac{\sqrt{-d^2x^2+1}(Ce^2+2\,Bef+Af^2)x}{8\,d^2} + \frac{\sqrt{-d^2x^2+1}Cf^2x}{16\,d^4}+\frac{(Ce^2+2\,Bef+Af^2)\arcsin(dx)}{8\,d^3} + \frac{Cf^2\arcsin(dx)}{16\,d^5}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}(2\,Cef+Bf^2)}{15\,d^4}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x, algorith
m="maxima")
```

output
```
-1/6*(-d^2*x^2 + 1)^(3/2)*C*f^2*x^3/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^2*x +
 1/2*A*e^2*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^2/d^2 - 2/3*(-d^2*
x^2 + 1)^(3/2)*A*e*f/d^2 - 1/5*(-d^2*x^2 + 1)^(3/2)*(2*C*e*f + B*f^2)*x^2/
d^2 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e^2 + 2*B*e*f + A*f^2)*x/d^2 - 1/8*(-d^2
*x^2 + 1)^(3/2)*C*f^2*x/d^4 + 1/8*sqrt(-d^2*x^2 + 1)*(C*e^2 + 2*B*e*f + A*
f^2)*x/d^2 + 1/16*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/8*(C*e^2 + 2*B*e*f +
A*f^2)*arcsin(d*x)/d^3 + 1/16*C*f^2*arcsin(d*x)/d^5 - 2/15*(-d^2*x^2 + 1)^
(3/2)*(2*C*e*f + B*f^2)/d^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. $2(264) = 528$.

Time = 0.24 (sec) , antiderivative size = 1059, normalized size of antiderivative = 3.73

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x, algorith
m="giac")
```

output
```
1/240*(120*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*
sqrt(d*x + 1)))*A*d^4*e^2 + 240*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1
/2*sqrt(2)*sqrt(d*x + 1)))*A*d^4*e^2 + 40*(((2*d*x - 5)*(d*x + 1) + 9)*sqr
t(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^3*e^2
 + 120*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt
(d*x + 1)))*B*d^3*e^2 + 80*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt
(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^3*e*f + 240*(sqrt(d*
x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d
^3*e*f + 10*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1
)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d^2*e^2 + 40*((
(2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqr
t(2)*sqrt(d*x + 1)))*C*d^2*e^2 + 20*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x
 + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x
+ 1)))*B*d^2*e*f + 80*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x
 + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e*f + 10*(((2*(3*d*x -
10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f^2 + 40*(((2*d*x - 5)*(d*x + 1) + 9)
*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2
*f^2 + 4*(((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) - 295)*(d*x + 1)
+ 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + ...
```

Mupad [B] (verification not implemented)

Time = 30.93 (sec) , antiderivative size = 2920, normalized size of antiderivative = 10.28

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
int((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
- ((((1 - d*x)^(1/2) - 1)^8*((4928*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x +
1)^(1/2) - 1)^8 - (((1 - d*x)^(1/2) - 1)^14*((1408*B*f^2)/3 - (32*B*d^2*e^
2)/3))/((d*x + 1)^(1/2) - 1)^14 - (((1 - d*x)^(1/2) - 1)^6*((1408*B*f^2)/3
 - (32*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^12*
((4928*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^12 - (((1 - d*
x)^(1/2) - 1)^10*((11008*B*f^2)/5 - 304*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^
10 + (64*B*f^2*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (64*B*f^
2*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (8*B*d^2*e^2*((1 -
d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B*d^2*e^2*((1 - d*x)^(1/2)
 - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (33*B*d*e*f*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)
^(1/2) - 1)^5 + (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1
)^7 + (442*B*d*e*f*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(1/2) - 1)^9 - (442
*B*d*e*f*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 - (204*B*d*e*f
*((1 - d*x)^(1/2) - 1)^13)/((d*x + 1)^(1/2) - 1)^13 + (204*B*d*e*f*((1 - d
*x)^(1/2) - 1)^15)/((d*x + 1)^(1/2) - 1)^15 - (33*B*d*e*f*((1 - d*x)^(1/2)
 - 1)^17)/((d*x + 1)^(1/2) - 1)^17 + (B*d*e*f*((1 - d*x)^(1/2) - 1)^19)/((
d*x + 1)^(1/2) - 1)^19 - (B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2)
- 1))/(d^4 + (10*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (4
5*d^4*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^4*((1 -...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.03

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$

$$=\frac{-240asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)a\,d^4e^2-60asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)a\,d^2f^2-60asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)c\,d^2e^2-32\sqrt{dx+1}\,\sqrt{-dx+1}}{}$$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x)
```

output
```
( - 240*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**4*e**2 - 60*asin(sqrt( - d*x +
 1)/sqrt(2))*a*d**2*f**2 - 120*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**2*e*f -
 60*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**2*e**2 - 30*asin(sqrt( - d*x + 1)/
sqrt(2))*c*f**2 + 120*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**5*e**2*x + 160*s
qrt(d*x + 1)*sqrt( - d*x + 1)*a*d**5*e*f*x**2 + 60*sqrt(d*x + 1)*sqrt( - d
*x + 1)*a*d**5*f**2*x**3 - 160*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**3*e*f -
 30*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**3*f**2*x + 80*sqrt(d*x + 1)*sqrt(
- d*x + 1)*b*d**5*e**2*x**2 + 120*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**5*e*
f*x**3 + 48*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**5*f**2*x**4 - 80*sqrt(d*x
+ 1)*sqrt( - d*x + 1)*b*d**3*e**2 - 60*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d*
*3*e*f*x - 16*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**3*f**2*x**2 - 32*sqrt(d*
x + 1)*sqrt( - d*x + 1)*b*d*f**2 + 60*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**
5*e**2*x**3 + 96*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**5*e*f*x**4 + 40*sqrt(
d*x + 1)*sqrt( - d*x + 1)*c*d**5*f**2*x**5 - 30*sqrt(d*x + 1)*sqrt( - d*x
+ 1)*c*d**3*e**2*x - 32*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**3*e*f*x**2 - 1
0*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**3*f**2*x**3 - 64*sqrt(d*x + 1)*sqrt(
 - d*x + 1)*c*d*e*f - 15*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d*f**2*x)/(240*d
**5)
```

3.33 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 164

$$\begin{aligned}\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx &= \frac{(Ce+4Ad^2e+Bf)\,x\sqrt{1-d^2x^2}}{8d^2}-\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f}\\ &\quad-\frac{\left(4\left((2C+5Ad^2)\,f-\frac{d^2e(3Ce-5Bf)}{f}\right)-3d^2(3Ce-5Bf)x\right)\left(1-d^2x^2\right)^{3/2}}{60d^4}\\ &\quad+\frac{(Ce+4Ad^2e+Bf)\arcsin(dx)}{8d^3}\end{aligned}$$

output

```
1/8*(4*A*d^2*e+B*f+C*e)*x*(-d^2*x^2+1)^(1/2)/d^2-1/5*C*(f*x+e)^2*(-d^2*x^2
+1)^(3/2)/d^2/f-1/60*(4*(5*A*d^2+2*C)*f-4*d^2*e*(-5*B*f+3*C*e)/f-3*d^2*(-5
*B*f+3*C*e)*x)*(-d^2*x^2+1)^(3/2)/d^4+1/8*(4*A*d^2*e+B*f+C*e)*arcsin(d*x)/
d^3
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.97

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{1-d^2x^2}(60Ad^4ex+40Ad^2f(-1+d^2x^2)+15Cd^2ex(-1+2d^2x^2)+5Bd^2(-8e-3fx+8d^2ex^2+6d}{120d^4}$$

input
```
Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(60*A*d^4*e*x + 40*A*d^2*f*(-1 + d^2*x^2) + 15*C*d^2*e*
x*(-1 + 2*d^2*x^2) + 5*B*d^2*(-8*e - 3*f*x + 8*d^2*e*x^2 + 6*d^2*f*x^3) +
8*C*f*(-2 - d^2*x^2 + 3*d^4*x^4)) + 30*d*(C*e + 4*A*d^2*e + B*f)*ArcTan[(d
*x)/(-1 + Sqrt[1 - d^2*x^2])])/(120*d^4)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2112, 2185, 25, 27, 676, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-dx}\sqrt{dx+1}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \sqrt{1-d^2x^2}(e+fx)\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\int -f(e+fx)\left(\left(5Ad^2+2C\right)f-d^2(3Ce-5Bf)x\right)\sqrt{1-d^2x^2}dx}{5d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$\downarrow$ 25

$$\frac{\int f(e+fx)\left(\left(5Ad^2+2C\right)f-d^2(3Ce-5Bf)x\right)\sqrt{1-d^2x^2}dx}{5d^2f^2}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$\downarrow$ 27

$$\frac{\int (e+fx)\left(\left(5Ad^2+2C\right)f-d^2(3Ce-5Bf)x\right)\sqrt{1-d^2x^2}dx}{5d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$\downarrow$ 676

$$\frac{\frac{5}{4}f\left(4Ad^2e+Bf+Ce\right)\int\sqrt{1-d^2x^2}dx-\frac{1}{3}\left(1-d^2x^2\right)^{3/2}\left(5f(Af+Be)-C\left(3e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{4}fx\left(1-d^2x^2\right)^{3/2}}{5d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$\downarrow$ 211

$$\frac{\frac{5}{4}f\left(4Ad^2e+Bf+Ce\right)\left(\frac{1}{2}\int\frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}x\sqrt{1-d^2x^2}\right)-\frac{1}{3}\left(1-d^2x^2\right)^{3/2}\left(5f(Af+Be)-C\left(3e^2-\frac{2f^2}{d^2}\right)\right)+}{5d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

$\downarrow$ 223

$$\frac{\frac{5}{4}f\left(\frac{\arcsin(dx)}{2d}+\frac{1}{2}x\sqrt{1-d^2x^2}\right)\left(4Ad^2e+Bf+Ce\right)-\frac{1}{3}\left(1-d^2x^2\right)^{3/2}\left(5f(Af+Be)-C\left(3e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{4}fx(1}{5d^2f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

input

```
Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

output

```
-1/5*(C*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(d^2*f) + (-1/3*((5*f*(B*e + A*f)
 - C*(3*e^2 - (2*f^2)/d^2))*(1 - d^2*x^2)^(3/2)) + (f*(3*C*e - 5*B*f)*x*(1
 - d^2*x^2)^(3/2))/4 + (5*f*(C*e + 4*A*d^2*e + B*f)*((x*Sqrt[1 - d^2*x^2])
/2 + ArcSin[d*x]/(2*d)))/4)/(5*d^2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{\left(24C f\,x^{4} d^{4}+30B\,d^{4} f\,x^{3}+30C\,d^{4} e\,x^{3}+40A\,d^{4} f\,x^{2}+40B\,d^{4} e\,x^{2}+60A\,d^{4} e x-8C\,d^{2} f\,x^{2}-15B\,d^{2} f x-15C\,d^{2} e x-40A\,d^{2} f-40B\,d^{2}}{120 d^{4} \sqrt{-\left(x d+1\right) \left(x d-1\right)}\, \sqrt{-x d+1}}$
default	$\frac{\sqrt{-x d+1}\, \sqrt{x d+1}\, \left(24C\ \mathrm{csgn}\left(d\right) d^{4} f\,x^{4} \sqrt{-d^{2} x^{2}+1}+30B\ \mathrm{csgn}\left(d\right) d^{4} f\,x^{3} \sqrt{-d^{2} x^{2}+1}+30C\ \mathrm{csgn}\left(d\right) d^{4} e\,x^{3} \sqrt{-d^{2} x^{2}+1}+40A\ \mathrm{csgn}\left(d\right) d\right.}{}$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x,method=_RETURNVER
BOSE)
```

output
```
-1/120*(24*C*d^4*f*x^4+30*B*d^4*f*x^3+30*C*d^4*e*x^3+40*A*d^4*f*x^2+40*B*d
^4*e*x^2+60*A*d^4*e*x-8*C*d^2*f*x^2-15*B*d^2*f*x-15*C*d^2*e*x-40*A*d^2*f-4
0*B*d^2*e-16*C*f)*(d*x+1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*
x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8/d^2*(4*A*d^2*e+B*f+C*e)/(d^2)^(1/2)
*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1
)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(24\,Cd^4fx^4-40\,Bd^2e+30\,(Cd^4e+Bd^4f)x^3+8\,(5\,Bd^4e+(5\,Ad^4-Cd^2)f)x^2-8\,(5\,Ad^2+2\,C)f-1}{120}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x, algorithm=
"fricas")
```

output
```
1/120*((24*C*d^4*f*x^4 - 40*B*d^2*e + 30*(C*d^4*e + B*d^4*f)*x^3 + 8*(5*B*
d^4*e + (5*A*d^4 - C*d^2)*f)*x^2 - 8*(5*A*d^2 + 2*C)*f - 15*(B*d^2*f - (4*
A*d^4 - C*d^2)*e)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(B*d*f + (4*A*d^3 +
 C*d)*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (e+fx)\sqrt{-dx+1}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((-d*x+1)**(1/2)*(d*x+1)**(1/2)*(f*x+e)*(C*x**2+B*x+A),x)
```

output
```
Integral((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx\\&=\frac{1}{2}\sqrt{-d^2x^2+1}Aex-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cfx^2}{5\,d^2}+\frac{Ae\arcsin{(dx)}}{2\,d}\\&-\frac{(-d^2x^2+1)^{\frac{3}{2}}Be}{3\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Af}{3\,d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}(Ce+Bf)x}{4\,d^2}\\&+\frac{\sqrt{-d^2x^2+1}(Ce+Bf)x}{8\,d^2}-\frac{2\,(-d^2x^2+1)^{\frac{3}{2}}Cf}{15\,d^4}+\frac{(Ce+Bf)\arcsin{(dx)}}{8\,d^3}\end{aligned}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x, algorithm=
"maxima")
```

output
```
1/2*sqrt(-d^2*x^2 + 1)*A*e*x - 1/5*(-d^2*x^2 + 1)^(3/2)*C*f*x^2/d^2 + 1/2*
A*e*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e/d^2 - 1/3*(-d^2*x^2 + 1)^
(3/2)*A*f/d^2 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e + B*f)*x/d^2 + 1/8*sqrt(-d^2
*x^2 + 1)*(C*e + B*f)*x/d^2 - 2/15*(-d^2*x^2 + 1)^(3/2)*C*f/d^4 + 1/8*(C*e
 + B*f)*arcsin(d*x)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(148) = 296.

Time = 0.20 (sec) , antiderivative size = 631, normalized size of antiderivative = 3.85

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x, algorithm=
"giac")
```

output
```
1/120*(60*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*s
qrt(d*x + 1)))*A*d^3*e + 120*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*
sqrt(2)*sqrt(d*x + 1)))*A*d^3*e + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x
 + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d^2*e + 60*(
sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1
)))*B*d^2*e + 20*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f + 60*(sqrt(d*x + 1)*(d*x -
 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*d^2*f + 5*(((2
*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x + 1)
 - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*d*e + 20*(((2*d*x - 5)*(d*x + 1
) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))
*C*d*e + 5*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)
*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d*f + 20*(((2*d*
x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*
sqrt(d*x + 1)))*B*d*f + (((2*(3*(4*d*x - 17)*(d*x + 1) + 133)*(d*x + 1) -
295)*(d*x + 1) + 195)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)
*sqrt(d*x + 1)))*C*f + 5*(((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)
*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C*f)
/d^4
```

Mupad [B] (verification not implemented)

Time = 10.33 (sec) , antiderivative size = 736, normalized size of antiderivative = 4.49

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
int((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
((B*f*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) - (35*B*f*((1 - d*x
)^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*B*f*((1 - d*x)^(1/2) -
1)^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*B*f*((1 - d*x)^(1/2) - 1)^7)/(2*(
(d*x + 1)^(1/2) - 1)^7) + (715*B*f*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^
(1/2) - 1)^9) - (273*B*f*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1
)^11) + (35*B*f*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) - (
B*f*((1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15))/(d^3*(((1 - d*
x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (1 - d*x)^(1/2)*((2*C*f*
(d*x + 1)^(1/2))/(15*d^4) - (C*f*x^4*(d*x + 1)^(1/2))/5 + (C*f*x^2*(d*x +
1)^(1/2))/(15*d^2)) + ((C*e*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1
)) - (35*C*e*((1 - d*x)^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*C
*e*((1 - d*x)^(1/2) - 1)^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*C*e*((1 - d
*x)^(1/2) - 1)^7)/(2*((d*x + 1)^(1/2) - 1)^7) + (715*C*e*((1 - d*x)^(1/2)
- 1)^9)/(2*((d*x + 1)^(1/2) - 1)^9) - (273*C*e*((1 - d*x)^(1/2) - 1)^11)/(
2*((d*x + 1)^(1/2) - 1)^11) + (35*C*e*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x +
 1)^(1/2) - 1)^13) - (C*e*((1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) -
1)^15))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (B
*f*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (C*e*atan(
((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) + (A*e*x*(1 - d*x)^(
1/2)*(d*x + 1)^(1/2))/2 - (A*d^(1/2)*e*log((-d)^(1/2)*(1 - d*x)^(1/2)*(...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.95

$$\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{-120 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) a\,d^3e - 30 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) bdf - 30 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) cde + 60\sqrt{dx+1}\,\sqrt{-dx+1}\,a\,d^4ex}{}$$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x)
```

output
```
( - 120*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**3*e - 30*asin(sqrt( - d*x + 1)
/sqrt(2))*b*d*f - 30*asin(sqrt( - d*x + 1)/sqrt(2))*c*d*e + 60*sqrt(d*x +
1)*sqrt( - d*x + 1)*a*d**4*e*x + 40*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**4*
f*x**2 - 40*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**2*f + 40*sqrt(d*x + 1)*sqr
t( - d*x + 1)*b*d**4*e*x**2 + 30*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**4*f*x
**3 - 40*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**2*e - 15*sqrt(d*x + 1)*sqrt(
- d*x + 1)*b*d**2*f*x + 30*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**4*e*x**3 +
24*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**4*f*x**4 - 15*sqrt(d*x + 1)*sqrt( -
 d*x + 1)*c*d**2*e*x - 8*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**2*f*x**2 - 16
*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*f)/(120*d**4)
```

3.34 $\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx=\frac{(C+4Ad^2)\,x\sqrt{1-d^2x^2}}{8d^2}-\frac{B(1-d^2x^2)^{3/2}}{3d^2}-\frac{Cx(1-d^2x^2)^{3/2}}{4d^2}+\frac{(C+4Ad^2)\arcsin(dx)}{8d^3}$$

output
```
1/8*(4*A*d^2+C)*x*(-d^2*x^2+1)^(1/2)/d^2-1/3*B*(-d^2*x^2+1)^(3/2)/d^2-1/4*
C*x*(-d^2*x^2+1)^(3/2)/d^2+1/8*(4*A*d^2+C)*arcsin(d*x)/d^3
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{d\sqrt{1-d^2x^2}(-8B-3Cx+12Ad^2x+8Bd^2x^2+6Cd^2x^3)+6(C+4Ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{24d^3}$$

input
```
Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

output
```
(d*Sqrt[1 - d^2*x^2]*(-8*B - 3*C*x + 12*A*d^2*x + 8*B*d^2*x^2 + 6*C*d^2*x^
3) + 6*(C + 4*A*d^2)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(24*d^3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1188, 2346, 25, 455, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-dx}\sqrt{dx+1}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{1188}$$

$$\int \sqrt{1-d^2x^2}\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\left(\left(4Ad^2+4Bxd^2+C\right)\sqrt{1-d^2x^2}\right)dx}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \left(4Ad^2+4Bxd^2+C\right)\sqrt{1-d^2x^2}dx}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{\left(4Ad^2+C\right)\int \sqrt{1-d^2x^2}dx-\frac{4}{3}B\left(1-d^2x^2\right)^{3/2}}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{\left(4Ad^2+C\right)\left(\frac{1}{2}\int \frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}x\sqrt{1-d^2x^2}\right)-\frac{4}{3}B\left(1-d^2x^2\right)^{3/2}}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\left(4Ad^2+C\right)\left(\frac{\arcsin(dx)}{2d}+\frac{1}{2}x\sqrt{1-d^2x^2}\right)-\frac{4}{3}B\left(1-d^2x^2\right)^{3/2}}{4d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

input
```
Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

output
```
-1/4*(C*x*(1 - d^2*x^2)^(3/2))/d^2 + ((-4*B*(1 - d^2*x^2)^(3/2))/3 + (C +
4*A*d^2)*((x*Sqrt[1 - d^2*x^2])/2 + ArcSin[d*x]/(2*d)))/(4*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{(6C\,d^2x^3+8B\,d^2x^2+12A\,d^2x-3Cx-8B)\sqrt{xd+1}\,(xd-1)\sqrt{(-xd+1)(xd+1)}}{24d^2\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}+\frac{(4d^2A+C)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-xd+1)(x}}{8d^2\sqrt{d^2}\,\sqrt{-xd+1}\,\sqrt{xd+1}}$
default	$\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\left(6C\ \mathrm{csgn}(d)d^3x^3\sqrt{-d^2x^2+1}+8B\ \mathrm{csgn}(d)d^3x^2\sqrt{-d^2x^2+1}+12A\sqrt{-d^2x^2+1}\ \mathrm{csgn}(d)d^3x-3C\sqrt{-d^2x^2+1}\ \mathrm{csgn}(\right.}{24\sqrt{-d^2x^2+1}\,d^3}$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(6*C*d^2*x^3+8*B*d^2*x^2+12*A*d^2*x-3*C*x-8*B)*(d*x+1)^(1/2)*(d*x-1)
/d^2/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/8*
(4*A*d^2+C)/d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*
x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int\sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(6\,Cd^3x^3+8\,Bd^3x^2-8\,Bd+3\,(4\,Ad^3-Cd)x)\sqrt{dx+1}\sqrt{-dx+1}-6\,(4\,Ad^2+C)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+}}{dx}\right.}{24\,d^3}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas"
)
```

output
```
1/24*((6*C*d^3*x^3 + 8*B*d^3*x^2 - 8*B*d + 3*(4*A*d^3 - C*d)*x)*sqrt(d*x +
 1)*sqrt(-d*x + 1) - 6*(4*A*d^2 + C)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1)
- 1)/(d*x)))/d^3
```

Sympy [F]

$$\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\ dx = \int \sqrt{-dx+1}\sqrt{dx+1}(A+Bx+Cx^2)\ dx$$

input
```
integrate((-d*x+1)**(1/2)*(d*x+1)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)\ dx = \frac{1}{2}\sqrt{-d^2x^2+1}Ax - \frac{(-d^2x^2+1)^{\frac{3}{2}}Cx}{4\,d^2} + \frac{A\arcsin(dx)}{2\,d} - \frac{(-d^2x^2+1)^{\frac{3}{2}}B}{3\,d^2} + \frac{\sqrt{-d^2x^2+1}Cx}{8\,d^2} + \frac{C\arcsin(dx)}{8\,d^3}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima"
)
```

output
```
1/2*sqrt(-d^2*x^2 + 1)*A*x - 1/4*(-d^2*x^2 + 1)^(3/2)*C*x/d^2 + 1/2*A*arcs
in(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B/d^2 + 1/8*sqrt(-d^2*x^2 + 1)*C*x/d^
2 + 1/8*C*arcsin(d*x)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(81) = 162$.

Time = 0.16 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.99

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{12\left(\sqrt{dx+1}(dx-2)\sqrt{-dx+1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)\right)Ad^2+24\left(\sqrt{dx+1}\sqrt{-dx+1}+2\arcsin\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/24*(12*(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sq
rt(d*x + 1)))*A*d^2 + 24*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt
(2)*sqrt(d*x + 1)))*A*d^2 + 4*(((2*d*x - 5)*(d*x + 1) + 9)*sqrt(d*x + 1)*s
qrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d + 12*(sqrt(d*x +
1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*d + (
((2*(3*d*x - 10)*(d*x + 1) + 43)*(d*x + 1) - 39)*sqrt(d*x + 1)*sqrt(-d*x +
 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C + 4*(((2*d*x - 5)*(d*x + 1)
+ 9)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*C
)/d^3
```

Mupad [B] (verification not implemented)

Time = 7.48 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.80

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx = \frac{A\,x\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{2}$$
$$-\frac{\frac{35\,C\,\left(\sqrt{1-d\,x}-1\right)^3}{2\,\left(\sqrt{d\,x+1}-1\right)^3}-\frac{273\,C\,\left(\sqrt{1-d\,x}-1\right)^5}{2\,\left(\sqrt{d\,x+1}-1\right)^5}+\frac{715\,C\,\left(\sqrt{1-d\,x}-1\right)^7}{2\,\left(\sqrt{d\,x+1}-1\right)^7}-\frac{715\,C\,\left(\sqrt{1-d\,x}-1\right)^9}{2\,\left(\sqrt{d\,x+1}-1\right)^9}+\frac{273\,C\,\left(\sqrt{1-d\,x}-1\right)^{11}}{2\,\left(\sqrt{d\,x+1}-1\right)^{11}}-\frac{35\,C\,\left(\sqrt{1-d\,x}\right.}{2\,\left(\sqrt{d\,x+1}-\right.}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^8}$$
$$-\frac{C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2\,d^3}-\frac{A\,\sqrt{d}\,\ln\left(\sqrt{-d}\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}-d^{3/2}\,x\right)}{2\,{\left(-d\right)}^{3/2}}$$
$$+\frac{B\,\left(d^2\,x^2-1\right)\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{3\,d^2}$$

input
```
int((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
(A*x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/2 - ((35*C*((1 - d*x)^(1/2) - 1)^3)/
(2*((d*x + 1)^(1/2) - 1)^3) - (273*C*((1 - d*x)^(1/2) - 1)^5)/(2*((d*x + 1
)^(1/2) - 1)^5) + (715*C*((1 - d*x)^(1/2) - 1)^7)/(2*((d*x + 1)^(1/2) - 1)
^7) - (715*C*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^(1/2) - 1)^9) + (273*C
*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1)^11) - (35*C*((1 - d*x)
^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) + (C*((1 - d*x)^(1/2) - 1)^15
)/(2*((d*x + 1)^(1/2) - 1)^15) - (C*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(
1/2) - 1)))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8)
- (C*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (A*d^(1/
2)*log((-d)^(1/2)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) - d^(3/2)*x))/(2*(-d)^(3
/2)) + (B*(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/(3*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.47

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{-24asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)a\,d^2-6asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)c+12\sqrt{dx+1}\,\sqrt{-dx+1}\,a\,d^3x+8\sqrt{dx+1}\,\sqrt{-dx+1}\,b\,d^3x^2}{24d^3}$$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
( - 24*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**2 - 6*asin(sqrt( - d*x + 1)/sqr
t(2))*c + 12*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**3*x + 8*sqrt(d*x + 1)*sqr
t( - d*x + 1)*b*d**3*x**2 - 8*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d + 6*sqrt(
d*x + 1)*sqrt( - d*x + 1)*c*d**3*x**3 - 3*sqrt(d*x + 1)*sqrt( - d*x + 1)*c
*d*x)/(24*d**3)
```

3.35 $\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 205

$$\begin{aligned}\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx &= \frac{(2(Ce^2-Bef+Af^2)-f(Ce-Bf)x)\sqrt{1-d^2x^2}}{2f^3} \\ &\quad -\frac{C(1-d^2x^2)^{3/2}}{3d^2f}+\frac{\left(2Ad^2e+\frac{(Ce-Bf)(2d^2e^2-f^2)}{f^2}\right)\arcsin(dx)}{2df^2} \\ &\quad -\frac{\sqrt{d^2e^2-f^2}(Ce^2-Bef+Af^2)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^4}\end{aligned}$$

output
```
1/2*(2*A*f^2-2*B*e*f+2*C*e^2-f*(-B*f+C*e)*x)*(-d^2*x^2+1)^(1/2)/f^3-1/3*C*
(-d^2*x^2+1)^(3/2)/d^2/f+1/2*(2*A*d^2*e+(-B*f+C*e)*(2*d^2*e^2-f^2)/f^2)*ar
csin(d*x)/d/f^2-(d^2*e^2-f^2)^(1/2)*(A*f^2-B*e*f+C*e^2)*arctan((d^2*e*x+f)
/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^4
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx$$

$$= \frac{\frac{f\sqrt{1-d^2x^2}(-2Cf^2+3d^2f(-2Be+2Af+Bfx)+Cd^2(6e^2-3efx+2f^2x^2))}{d^2} + \frac{6(2Cd^2e^3-2Bd^2e^2f-Cef^2+2Ad^2ef^2+Bf^3)\arctan\left(\frac{dx}{-1+\sqrt{1-\cdots}}\right.}{d}\cdots}{6f^4}$$

input
```
Integrate[(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2))/(e + f*x),x]
```

output
```
((f*Sqrt[1 - d^2*x^2]*(-2*C*f^2 + 3*d^2*f*(-2*B*e + 2*A*f + B*f*x) + C*d^2
*(6*e^2 - 3*e*f*x + 2*f^2*x^2)))/d^2 + (6*(2*C*d^2*e^3 - 2*B*d^2*e^2*f - C
*e*f^2 + 2*A*d^2*e*f^2 + B*f^3)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d
+ 12*Sqrt[d^2*e^2 - f^2]*(C*e^2 + f*(-(B*e) + A*f))*ArcTan[(Sqrt[d^2*e^2 -
 f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/(6*f^4)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2112, 2185, 27, 682, 25, 27, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-dx}\sqrt{dx+1}(A+Bx+Cx^2)}{e+fx}\,dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{\sqrt{1-d^2x^2}(A+Bx+Cx^2)}{e+fx}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\int -\frac{3d^2f(Af-(Ce-Bf)x)\sqrt{1-d^2x^2}}{e+fx}dx}{3d^2f^2}-\frac{C(1-d^2x^2)^{3/2}}{3d^2f}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{(Af-(Ce-Bf)x)\sqrt{1-d^2x^2}}{e+fx}dx}{f}-\frac{C\left(1-d^2x^2\right)^{3/2}}{3d^2f}$$

$$\Big\downarrow 682$$

$$\frac{\frac{\sqrt{1-d^2x^2}\left(2\left(Af^2-Bef+Ce^2\right)-fx(Ce-Bf)\right)}{2f^2}-\frac{\int -\frac{d^2\left(f\left(Ce^2-f(Be-2Af)\right)+\left(2Ad^2ef^2+(Ce-Bf)\left(2d^2e^2-f^2\right)\right)x\right)}{(e+fx)\sqrt{1-d^2x^2}}dx}{2d^2f^2}}{f}-\frac{C\left(1-d^2x^2\right)^{3/2}}{3d^2f}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\int \frac{d^2\left(f\left(Ce^2-f(Be-2Af)\right)+\left(2Ad^2ef^2+(Ce-Bf)\left(2d^2e^2-f^2\right)\right)x\right)}{(e+fx)\sqrt{1-d^2x^2}}dx}{2d^2f^2}+\frac{\sqrt{1-d^2x^2}\left(2\left(Af^2-Bef+Ce^2\right)-fx(Ce-Bf)\right)}{2f^2}}{f}-\frac{C\left(1-d^2x^2\right)^{3/2}}{3d^2f}$$

$$\Big\downarrow 27$$

$$\frac{\frac{\int \frac{f\left(Ce^2-f(Be-2Af)\right)+\left(2Ad^2ef^2+(Ce-Bf)\left(2d^2e^2-f^2\right)\right)x}{(e+fx)\sqrt{1-d^2x^2}}dx}{2f^2}+\frac{\sqrt{1-d^2x^2}\left(2\left(Af^2-Bef+Ce^2\right)-fx(Ce-Bf)\right)}{2f^2}}{f}-\frac{C\left(1-d^2x^2\right)^{3/2}}{3d^2f}$$

$$\Big\downarrow 719$$

$$\frac{\frac{\frac{\left(2Ad^2ef^2+\left(2d^2e^2-f^2\right)(Ce-Bf)\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx}{f}-\frac{2(de-f)(de+f)\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}}{2f^2}+\frac{\sqrt{1-d^2x^2}\left(2\left(Af^2-Bef+Ce^2\right)-fx(Ce\right.}{2f^2}}{f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}}{3d^2f}$$

$$\Big\downarrow 223$$

$$\frac{\frac{\frac{\arcsin(dx)\left(2Ad^2ef^2+\left(2d^2e^2-f^2\right)(Ce-Bf)\right)}{df}-\frac{2(de-f)(de+f)\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}}{2f^2}+\frac{\sqrt{1-d^2x^2}\left(2\left(Af^2-Bef+Ce^2\right)-fx(Ce-Bf\right.}{2f^2}}{f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}}{3d^2f}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{\frac{2(de-f)(de+f)\left(Af^2-Bef+Ce^2\right)\int \frac{1}{-d^2e^2+f^2-\frac{\left(exd^2+f\right)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{f}+\frac{\arcsin(dx)\left(2Ad^2ef^2+\left(2d^2e^2-f^2\right)(Ce-Bf)\right)}{df}}{2f^2}+\frac{\sqrt{1-d^2x^2}(2(Af^2-Bef+Ce}{2f^2}}{f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}}{3d^2f}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{\arcsin(dx)\left(2Ad^2ef^2+\left(2d^2e^2-f^2\right)(Ce-Bf)\right)}{df}-\frac{2(de-f)(de+f)\left(Af^2-Bef+Ce^2\right)\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f\sqrt{d^2e^2-f^2}}}{2f^2}+\frac{\sqrt{1-d^2x^2}(2(Af^2-Bef+Ce^2)-fx}{2f^2}}{f}$$

$$\frac{C\left(1-d^2x^2\right)^{3/2}}{3d^2f}$$

input
```
Int[(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2))/(e + f*x),x]
```

output
```
-1/3*(C*(1 - d^2*x^2)^(3/2))/(d^2*f) + (((2*(C*e^2 - B*e*f + A*f^2) - f*(C
*e - B*f)*x)*Sqrt[1 - d^2*x^2])/(2*f^2) + (((2*A*d^2*e*f^2 + (C*e - B*f)*(
2*d^2*e^2 - f^2))*ArcSin[d*x])/(d*f) - (2*(d*e - f)*(d*e + f)*(C*e^2 - B*e
*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])
/(f*Sqrt[d^2*e^2 - f^2]))/(2*f^2))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(188) = 376$.

Time = 0.86 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.06

method	result
risch	$-\frac{(2C d^2 f^2 x^2+3B d^2 f^2 x-3C d^2 e f x+6A d^2 f^2-6B d^2 e f+6C d^2 e^2-2C f^2)\sqrt{xd+1}\,(xd-1)\sqrt{(-xd+1)(xd+1)}}{6d^2 f^3\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}+\frac{\left(\frac{(2A\, d^2 e\, f^2-2 \cdots}{\cdots}\right.}{\cdots}$
default	$\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\left(2C\,\mathrm{csgn}(d) d^2 f^4 x^2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}+6A\ln\left(\frac{2d^2xe+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)\mathrm{csgn}(d) d^4 e^2 f^2-6 \cdots\right.}{\cdots}$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x,method=_RETURNVER
BOSE)
```

output
```
-1/6*(2*C*d^2*f^2*x^2+3*B*d^2*f^2*x-3*C*d^2*e*f*x+6*A*d^2*f^2-6*B*d^2*e*f+
6*C*d^2*e^2-2*C*f^2)*(d*x+1)^(1/2)*(d*x-1)/d^2/f^3/(-(d*x+1)*(d*x-1))^(1/2
)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)+1/2/f^3*((2*A*d^2*e*f^2-2*B*d^2*
e^2*f+2*C*d^2*e^3+B*f^3-C*e*f^2)/f/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*
x^2+1)^(1/2))+2*(A*d^2*e^2*f^2-B*d^2*e^3*f+C*d^2*e^4-A*f^4+B*e*f^3-C*e^2*f
^2)/f^2/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln((-2*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e
/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^
2-f^2)/f^2)^(1/2))/(x+e/f)))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+
1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.88

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx$$

$$= \left[\frac{6\left(Cd^2e^2-Bd^2ef+Ad^2f^2\right)\sqrt{-d^2e^2+f^2}\log\left(\frac{d^2efx+f^2-\sqrt{-d^2e^2+f^2}\left(d^2ex+f\right)-\left(\sqrt{-d^2e^2+f^2}\sqrt{-dx+1}f+\left(d^2e^2-f^2\right.\right.}{fx+e}\right.}{}\right.$$

$$-\frac{12\left(Cd^2e^2-Bd^2ef+Ad^2f^2\right)\sqrt{d^2e^2-f^2}\arctan\left(-\frac{\sqrt{d^2e^2-f^2}\sqrt{dx+1}\sqrt{-dx+1}e-\sqrt{d^2e^2-f^2}(fx+e)}{(d^2e^2-f^2)x}\right)-\left(2\,Cd^2f\right.}{}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x, algorithm=
"fricas")
```

output
```
[1/6*(6*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*
e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*s
qrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)
) + (2*C*d^2*f^3*x^2 + 6*C*d^2*e^2*f - 6*B*d^2*e*f^2 + 2*(3*A*d^2 - C)*f^3
 - 3*(C*d^2*e*f^2 - B*d^2*f^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6*(2*C*d^
3*e^3 - 2*B*d^3*e^2*f + B*d*f^3 + (2*A*d^3 - C*d)*e*f^2)*arctan((sqrt(d*x
+ 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^2*f^4), -1/6*(12*(C*d^2*e^2 - B*d^2*e*
f + A*d^2*f^2)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x +
 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x))
 - (2*C*d^2*f^3*x^2 + 6*C*d^2*e^2*f - 6*B*d^2*e*f^2 + 2*(3*A*d^2 - C)*f^3
- 3*(C*d^2*e*f^2 - B*d^2*f^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(2*C*d^3
*e^3 - 2*B*d^3*e^2*f + B*d*f^3 + (2*A*d^3 - C*d)*e*f^2)*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^2*f^4)]
```

Sympy [F]

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx = \int \frac{\sqrt{-dx+1}\sqrt{dx+1}(A+Bx+Cx^2)}{e+fx}\,dx$$

input
```
integrate((-d*x+1)**(1/2)*(d*x+1)**(1/2)*(C*x**2+B*x+A)/(f*x+e),x)
```

output
```
Integral(sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2)/(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 32.89 (sec) , antiderivative size = 13115, normalized size of antiderivative = 63.98

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Too large to display}$$

input
```
int(((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2))/(e + f*x),x)
```

output
```
(f*(8*A*(d*x + 1)^(1/2) - 8*A + 8*A*(1 - d*x)^(1/2) + 2*A*d^2*x^2 - 8*A*(1
 - d*x)^(1/2)*(d*x + 1)^(1/2) - 4*A*d*x*(d*x + 1)^(1/2) + 4*A*d*x*(1 - d*x
)^(1/2)) + 24*A*d*e*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)) - 12
*A*atanh((4*f^2*(d*x + 1)^(1/2)*(f + d*e)^(1/2)*(f - d*e)^(1/2) - 4*f^2*(1
 - d*x)^(1/2)*(f + d*e)^(1/2)*(f - d*e)^(1/2) - 2*d*e*f*(f + d*e)^(1/2)*(f
 - d*e)^(1/2) - d^2*e^2*(d*x + 1)^(1/2)*(f + d*e)^(1/2)*(f - d*e)^(1/2) +
d^2*e^2*(1 - d*x)^(1/2)*(f + d*e)^(1/2)*(f - d*e)^(1/2) - 4*d*f^2*x*(f + d
*e)^(1/2)*(f - d*e)^(1/2) + d^3*e^2*x*(f + d*e)^(1/2)*(f - d*e)^(1/2) + 2*
d*e*f*(d*x + 1)^(1/2)*(f + d*e)^(1/2)*(f - d*e)^(1/2) + 2*d*e*f*(1 - d*x)^
(1/2)*(f + d*e)^(1/2)*(f - d*e)^(1/2) - 2*d*e*f*(1 - d*x)^(1/2)*(d*x + 1)^
(1/2)*(f + d*e)^(1/2)*(f - d*e)^(1/2))/(4*f^3*(d*x + 1)^(1/2) - 4*f^3*(1 -
 d*x)^(1/2) + d^3*e^3 - 4*d*f^3*x - d^3*e^3*(d*x + 1)^(1/2) - d^3*e^3*(1 -
 d*x)^(1/2) - 2*d*e*f^2 - 3*d^2*e^2*f*(d*x + 1)^(1/2) + 3*d^2*e^2*f*(1 - d
*x)^(1/2) + 3*d^3*e^2*f*x + d^3*e^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) + 2*d*
e*f^2*(d*x + 1)^(1/2) + 2*d*e*f^2*(1 - d*x)^(1/2) - 2*d*e*f^2*(1 - d*x)^(1
/2)*(d*x + 1)^(1/2)))*(f + d*e)^(1/2)*(f - d*e)^(1/2) - 12*A*d*e*atan(((1
- d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(d*x + 1)^(1/2) - 12*A*d*e*atan((
(1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(1 - d*x)^(1/2) - 4*A*d*e*atan
(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(d*x + 1)^(3/2) - 4*A*d*e*at
an(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))*(1 - d*x)^(3/2) + 6*A*a...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 662, normalized size of antiderivative = 3.23

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Too large to display}$$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x)
```

output

```
( - 12*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**3*e*f**2 + 12*asin(sqrt( - d*x
+ 1)/sqrt(2))*b*d**3*e**2*f - 6*asin(sqrt( - d*x + 1)/sqrt(2))*b*d*f**3 -
12*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**3*e**3 + 6*asin(sqrt( - d*x + 1)/sq
rt(2))*c*d*e*f**2 + 12*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan
(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**
2*f**2 - 12*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt(
 - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*b*d**2*e*f + 12*
sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/
sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*c*d**2*e**2 + 12*sqrt(d*e +
f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2)
 + sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**2*f**2 - 12*sqrt(d*e + f)*sqrt(d*e
 - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + sqrt(f)*
sqrt(2))/sqrt(d*e - f))*b*d**2*e*f + 12*sqrt(d*e + f)*sqrt(d*e - f)*atan((
sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + sqrt(f)*sqrt(2))/sqr
t(d*e - f))*c*d**2*e**2 + 6*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**2*f**3 - 6
*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**2*e*f**2 + 3*sqrt(d*x + 1)*sqrt( - d*
x + 1)*b*d**2*f**3*x + 6*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**2*e**2*f - 3*
sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**2*e*f**2*x + 2*sqrt(d*x + 1)*sqrt( - d
*x + 1)*c*d**2*f**3*x**2 - 2*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*f**3)/(6*d**
2*f**4)
```

3.36 $\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 322

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \frac{\left(2\left(2Bd^2e^2-\frac{3Cd^2e^3}{f}+2Cef-Ad^2ef-Bf^2\right)-(2d^2f(Be-Af)-C(3d^2e^2-f^2))\,x\right)\sqrt{1-d^2x^2}}{2f^2(d^2e^2-f^2)} + \frac{(Ce^2-Bef+Af^2)(1-d^2x^2)^{3/2}}{f(d^2e^2-f^2)(e+fx)} + \frac{(2d^2f(2Be-Af)-C(6d^2e^2-f^2))\arcsin(dx)}{2df^4} + \frac{(3Cd^2e^3-2Bd^2e^2f-2Cef^2+Ad^2ef^2+Bf^3)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^4\sqrt{d^2e^2-f^2}}$$

output
```
1/2*(4*B*d^2*e^2-6*C*d^2*e^3/f+4*C*e*f-2*A*d^2*e*f-2*B*f^2-(2*d^2*f*(-A*f+
B*e)-C*(3*d^2*e^2-f^2))*x)*(-d^2*x^2+1)^(1/2)/f^2/(d^2*e^2-f^2)+(A*f^2-B*e
*f+C*e^2)*(-d^2*x^2+1)^(3/2)/f/(d^2*e^2-f^2)/(f*x+e)+1/2*(2*d^2*f*(-A*f+2*
B*e)-C*(6*d^2*e^2-f^2))*arcsin(d*x)/d/f^4+(A*d^2*e*f^2-2*B*d^2*e^2*f+3*C*d
^2*e^3+B*f^3-2*C*e*f^2)*arctan((d^2*e*x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1
)^(1/2))/f^4/(d^2*e^2-f^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx$$

$$= \frac{\frac{f\sqrt{1-d^2x^2}(2f(2Be-Af+Bfx)+C(-6e^2-3efx+f^2x^2))}{e+fx}+\frac{2(-2d^2f(-2Be+Af)+C(-6d^2e^2+f^2))\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d}-\frac{4\sqrt{d^2e^2-}}{}}{2f^4}$$

input
```
Integrate[(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2))/(e + f*x)^2,x]
```

output
```
((f*Sqrt[1 - d^2*x^2]*(2*f*(2*B*e - A*f + B*f*x) + C*(-6*e^2 - 3*e*f*x + f
^2*x^2)))/(e + f*x) + (2*(-2*d^2*f*(-2*B*e + A*f) + C*(-6*d^2*e^2 + f^2))*
ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d - (4*Sqrt[d^2*e^2 - f^2]*(3*C*d^
2*e^3 - 2*B*d^2*e^2*f - 2*C*e*f^2 + A*d^2*e*f^2 + B*f^3)*ArcTan[(Sqrt[d^2*
e^2 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)*(d*e + f)))/(2*
f^4)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {2112, 2182, 682, 27, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-dx}\sqrt{dx+1}\left(A+Bx+Cx^2\right)}{(e+fx)^2}\,dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{\sqrt{1-d^2x^2}\left(A+Bx+Cx^2\right)}{(e+fx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{\int \frac{\left(Aed^2+Ce-Bf-\left(2Bed^2-2Afd^2-\frac{3Ce^2d^2}{f}+Cf\right)x\right)\sqrt{1-d^2x^2}}{e+fx}dx}{d^2e^2-f^2}+\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$\big\downarrow$ 682

$$\frac{-\frac{\int \frac{d^2\left((de-f)f(de+f)(3Ce-2Bf)-\left(d^2e^2-f^2\right)\left(2d^2f(2Be-Af)-C\left(6d^2e^2-f^2\right)\right)x\right)}{f(e+fx)\sqrt{1-d^2x^2}}dx}{2d^2f^2}-\frac{\sqrt{1-d^2x^2}\left(x\left(2d^2f(Be-Af)-C\left(3d^2e^2-f^2\right)\right)+2\left(Ad^2ef-\cdots\right.\right.}{2f^2}}{d^2e^2-f^2}$$

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$\big\downarrow$ 27

$$\frac{-\frac{\int \frac{(de-f)f(de+f)(3Ce-2Bf)-\left(d^2e^2-f^2\right)\left(2d^2f(2Be-Af)-C\left(6d^2e^2-f^2\right)\right)x}{(e+fx)\sqrt{1-d^2x^2}}dx}{2f^3}-\frac{\sqrt{1-d^2x^2}\left(x\left(2d^2f(Be-Af)-C\left(3d^2e^2-f^2\right)\right)+2\left(Ad^2ef-B(2\cdots\right.\right.}{2f^2}}{d^2e^2-f^2}$$

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$\big\downarrow$ 719

$$\frac{-\frac{-\frac{\left(d^2e^2-f^2\right)\left(2d^2f(2Be-Af)-C\left(6d^2e^2-f^2\right)\right)\int\frac{1}{\sqrt{1-d^2x^2}}dx}{f}-\frac{2(de-f)(de+f)\left(Ad^2ef^2-2Bd^2e^2f+Bf^3+3Cd^2e^3-2Cef^2\right)\int\frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}}{2f^3}-\frac{\sqrt{\cdots}}{\cdots}}{d^2e^2-f^2}$$

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$\big\downarrow$ 223

$$\frac{-\frac{-\frac{2(de-f)(de+f)\left(Ad^2ef^2-2Bd^2e^2f+Bf^3+3Cd^2e^3-2Cef^2\right)\int\frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{\arcsin(dx)\left(d^2e^2-f^2\right)\left(2d^2f(2Be-Af)-C\left(6d^2e^2-f^2\right)\right)}{df}}{2f^3}-\frac{\sqrt{1-d\cdots}}{\cdots}}{d^2e^2-f^2}$$

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$\big\downarrow$ 488

$$-\frac{-\frac{\frac{2(de-f)(de+f)\left(Ad^2ef^2-2Bd^2e^2f+Bf^3+3Cd^2e^3-2Cef^2\right)\int \frac{1}{-d^2e^2+f^2-\frac{\left(exd^2+f\right)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{f}-\frac{\arcsin(dx)\left(d^2e^2-f^2\right)\left(2d^2f(2Be-Af)-C\left(6d^2e^2-f^2\right.\right.}{df}}{2f^3}}{d^2e^2-f^2}$$

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \; 217$$

$$-\frac{-\frac{-\frac{\arcsin(dx)\left(d^2e^2-f^2\right)\left(2d^2f(2Be-Af)-C\left(6d^2e^2-f^2\right)\right)}{df}-\frac{2(de-f)(de+f)\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(Ad^2ef^2-2Bd^2e^2f+Bf^3+3Cd^2e^3-2Cef^2\right)}{f\sqrt{d^2e^2-f^2}}}{2f^3}}{d^2e^2-f^2}$$

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

input

```
Int[(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2))/(e + f*x)^2,x]
```

output

```
((C*e^2 - B*e*f + A*f^2)*(1 - d^2*x^2)^(3/2))/(f*(d^2*e^2 - f^2)*(e + f*x)
) + (-1/2*((2*((3*C*d^2*e^3)/f - 2*C*e*f + A*d^2*e*f - B*(2*d^2*e^2 - f^2)
) + (2*d^2*f*(B*e - A*f) - C*(3*d^2*e^2 - f^2))*x)*Sqrt[1 - d^2*x^2])/f^2
- (-(((d^2*e^2 - f^2)*(2*d^2*f*(2*B*e - A*f) - C*(6*d^2*e^2 - f^2))*ArcSin
[d*x])/(d*f)) - (2*(d*e - f)*(d*e + f)*(3*C*d^2*e^3 - 2*B*d^2*e^2*f - 2*C*
e*f^2 + A*d^2*e*f^2 + B*f^3)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqr
t[1 - d^2*x^2])])/(f*Sqrt[d^2*e^2 - f^2]))/(2*f^3))/(d^2*e^2 - f^2)
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(305) = 610$.

Time = 0.78 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.01

method	result
risch	$-\frac{(Cfx+2Bf-4Ce)\sqrt{xd+1}\,(xd-1)\sqrt{(-xd+1)(xd+1)}}{2f^3\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}} - \frac{\left(\frac{\left(2A\,d^2f^2-4B\,d^2ef+6C\,d^2e^2-C\,f^2\right)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)}{f\sqrt{d^2}} + \frac{2\left(2A\,d^2e\,f^2 \ldots\right.}{\ldots}\right.}{\ldots}$
default	Expression too large to display

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x,method=_RETURNV
ERBOSE)
```

output
```
-1/2*(C*f*x+2*B*f-4*C*e)*(d*x+1)^(1/2)*(d*x-1)/f^3/(-(d*x+1)*(d*x-1))^(1/2
)*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)-1/2/f^3*((2*A*d^2*f^2-4*B*d^2*e*
f+6*C*d^2*e^2-C*f^2)/f/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2)
)+2/f^2*(2*A*d^2*e*f^2-3*B*d^2*e^2*f+4*C*d^2*e^3+B*f^3-2*C*e*f^2)/(-(d^2*e
^2-f^2)/f^2)^(1/2)*ln((-2*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2
-f^2)/f^2)^(1/2)*(-d^2*(x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2
))/(x+e/f))+2*(A*d^2*e^2*f^2-B*d^2*e^3*f+C*d^2*e^4-A*f^4+B*e*f^3-C*e^2*f^2
)/f^3*(1/(d^2*e^2-f^2)*f^2/(x+e/f)*(-d^2*(x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*
e^2-f^2)/f^2)^(1/2)-f*d^2*e/(d^2*e^2-f^2)/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln((-
2*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(
x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2))/(x+e/f))))*((-d*x+1)*
(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(305) = 610$.

Time = 26.66 (sec) , antiderivative size = 1358, normalized size of antiderivative = 4.22

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x, algorith
m="fricas")
```

output
```
[-1/2*(6*C*d^3*e^5*f - 4*B*d^3*e^4*f^2 + 4*B*d*e^2*f^4 - 2*A*d*e*f^5 + 2*(
A*d^3 - 3*C*d)*e^3*f^3 + 2*(3*C*d^3*e^5 - 2*B*d^3*e^4*f + B*d*e^2*f^3 + (A
*d^3 - 2*C*d)*e^3*f^2 + (3*C*d^3*e^4*f - 2*B*d^3*e^3*f^2 + B*d*e*f^4 + (A*
d^3 - 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(
-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (
d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (6*C*d^3*e^5*f
- 4*B*d^3*e^4*f^2 + 4*B*d*e^2*f^4 - 2*A*d*e*f^5 + 2*(A*d^3 - 3*C*d)*e^3*f^
3 - (C*d^3*e^3*f^3 - C*d*e*f^5)*x^2 + (3*C*d^3*e^4*f^2 - 2*B*d^3*e^3*f^3 -
 3*C*d*e^2*f^4 + 2*B*d*e*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(3*C*d^3
*e^4*f^2 - 2*B*d^3*e^3*f^3 + 2*B*d*e*f^5 - A*d*f^6 + (A*d^3 - 3*C*d)*e^2*f
^4)*x - 2*(6*C*d^4*e^6 - 4*B*d^4*e^5*f + 4*B*d^2*e^3*f^3 + (2*A*d^4 - 7*C*
d^2)*e^4*f^2 - (2*A*d^2 - C)*e^2*f^4 + (6*C*d^4*e^5*f - 4*B*d^4*e^4*f^2 +
4*B*d^2*e^2*f^4 + (2*A*d^4 - 7*C*d^2)*e^3*f^3 - (2*A*d^2 - C)*e*f^5)*x)*ar
ctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^3*e^4*f^4 - d*e^2*f^6 +
 (d^3*e^3*f^5 - d*e*f^7)*x), -1/2*(6*C*d^3*e^5*f - 4*B*d^3*e^4*f^2 + 4*B*d
*e^2*f^4 - 2*A*d*e*f^5 + 2*(A*d^3 - 3*C*d)*e^3*f^3 - 4*(3*C*d^3*e^5 - 2*B*
d^3*e^4*f + B*d*e^2*f^3 + (A*d^3 - 2*C*d)*e^3*f^2 + (3*C*d^3*e^4*f - 2*B*d
^3*e^3*f^2 + B*d*e*f^4 + (A*d^3 - 2*C*d)*e^2*f^3)*x)*sqrt(d^2*e^2 - f^2)*a
rctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2
- f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) + (6*C*d^3*e^5*f - 4*B*d^3*e^4*f...
```

Sympy [F]

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \int \frac{\sqrt{-dx+1}\sqrt{dx+1}(A+Bx+Cx^2)}{(e+fx)^2}\,dx$$

input
```
integrate((-d*x+1)**(1/2)*(d*x+1)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**2,x)
```

output
```
Integral(sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2)/(e + f*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x, algorith
m="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 38.55 (sec) , antiderivative size = 11177, normalized size of antiderivative = 34.71

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input

```
int(((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^2,x)
```

output

```
((4*B*((1 - d*x)^(1/2) - 1))/(f*((d*x + 1)^(1/2) - 1)) + (36*B*((1 - d*x)^
(1/2) - 1)^3)/(f*((d*x + 1)^(1/2) - 1)^3) - (36*B*((1 - d*x)^(1/2) - 1)^5)
/(f*((d*x + 1)^(1/2) - 1)^5) - (4*B*((1 - d*x)^(1/2) - 1)^7)/(f*((d*x + 1)
^(1/2) - 1)^7) + (16*B*d*e*((1 - d*x)^(1/2) - 1)^2)/(f^2*((d*x + 1)^(1/2)
- 1)^2) + (32*B*d*e*((1 - d*x)^(1/2) - 1)^4)/(f^2*((d*x + 1)^(1/2) - 1)^4)
 + (16*B*d*e*((1 - d*x)^(1/2) - 1)^6)/(f^2*((d*x + 1)^(1/2) - 1)^6))/(d*e
+ (4*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (4*f*((1 - d*x)^
(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (4*f*((1 - d*x)^(1/2) - 1)^7)/((d*
x + 1)^(1/2) - 1)^7 + (4*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) +
(4*d*e*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d*e*((1 - d*x
)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (4*d*e*((1 - d*x)^(1/2) - 1)^6)/
((d*x + 1)^(1/2) - 1)^6 + (d*e*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) -
 1)^8) - ((4*A*((1 - d*x)^(1/2) - 1))/(e*((d*x + 1)^(1/2) - 1)) - (4*A*((1
 - d*x)^(1/2) - 1)^3)/(e*((d*x + 1)^(1/2) - 1)^3) + (8*A*d*((1 - d*x)^(1/2
) - 1)^2)/(f*((d*x + 1)^(1/2) - 1)^2))/(d*e - (4*f*((1 - d*x)^(1/2) - 1)^3
)/((d*x + 1)^(1/2) - 1)^3 + (4*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
 1) + (2*d*e*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d*e*((1 -
 d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4) - ((8*((1 - d*x)^(1/2) - 1)^2
*(C*f^2 + 3*C*d^2*e^2))/(f^3*((d*x + 1)^(1/2) - 1)^2) - (32*((1 - d*x)^(1/
2) - 1)^4*(2*C*f^2 - 3*C*d^2*e^2))/(f^3*((d*x + 1)^(1/2) - 1)^4) - (32*...
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1966, normalized size of antiderivative = 6.11

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x)
```

output
```
(4*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**4*e**3*f**2 + 4*asin(sqrt( - d*x +
1)/sqrt(2))*a*d**4*e**2*f**3*x - 4*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**2*e
*f**4 - 4*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**2*f**5*x - 8*asin(sqrt( - d*
x + 1)/sqrt(2))*b*d**4*e**4*f - 8*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**4*e*
*3*f**2*x + 8*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**2*e**2*f**3 + 8*asin(sqr
t( - d*x + 1)/sqrt(2))*b*d**2*e*f**4*x + 12*asin(sqrt( - d*x + 1)/sqrt(2))
*c*d**4*e**5 + 12*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**4*e**4*f*x - 14*asin
(sqrt( - d*x + 1)/sqrt(2))*c*d**2*e**3*f**2 - 14*asin(sqrt( - d*x + 1)/sqr
t(2))*c*d**2*e**2*f**3*x + 2*asin(sqrt( - d*x + 1)/sqrt(2))*c*e*f**4 + 2*a
sin(sqrt( - d*x + 1)/sqrt(2))*c*f**5*x - 4*sqrt(d*e + f)*sqrt(d*e - f)*ata
n((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/
sqrt(d*e - f))*a*d**3*e**2*f**2 - 4*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt
(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*
e - f))*a*d**3*e*f**3*x + 8*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f
)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*
b*d**3*e**3*f + 8*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin
(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*b*d**3*e**
2*f**2*x - 4*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt
( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*b*d*e*f**3 - 4*
sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + ...
```

3.37 $\int \frac{\sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 301

$$\begin{aligned}
&\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx \\
&= \frac{(2(de-f)(de+f)(3Ce-Bf)-f(d^2f(Be-Af)-C(3d^2e^2-2f^2))\,x)\sqrt{1-d^2x^2}}{2f^3\left(d^2e^2-f^2\right)(e+fx)} \\
&\quad+\frac{\left(Ce^2-Bef+Af^2\right)\left(1-d^2x^2\right)^{3/2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2}+\frac{d(3Ce-Bf)\arcsin(dx)}{f^4} \\
&\quad+\frac{(d^2f(2Bd^2e^3-3Bef^2+Af^3)-C(6d^4e^4-9d^2e^2f^2+2f^4))\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{2f^4\left(d^2e^2-f^2\right)^{3/2}}
\end{aligned}$$

output
```
1/2*(2*(d*e-f)*(d*e+f)*(-B*f+3*C*e)-f*(d^2*f*(-A*f+B*e)-C*(3*d^2*e^2-2*f^2
))*x)*(-d^2*x^2+1)^(1/2)/f^3/(d^2*e^2-f^2)/(f*x+e)+1/2*(A*f^2-B*e*f+C*e^2)
*(-d^2*x^2+1)^(3/2)/f/(d^2*e^2-f^2)/(f*x+e)^2+d*(-B*f+3*C*e)*arcsin(d*x)/f
^4+1/2*(d^2*f*(2*B*d^2*e^3+A*f^3-3*B*e*f^2)-C*(6*d^4*e^4-9*d^2*e^2*f^2+2*f
^4))*arctan((d^2*e*x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^4/(d^2*e
^2-f^2)^(3/2)
```

Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx$$

$$= \frac{\frac{f\sqrt{1-d^2x^2}(Af^3(f+d^2ex)+Bf^3(e+2fx)-Bd^2e^2f(2e+3fx)-Cf^2(5e^2+8efx+2f^2x^2)+Cd^2e^2(6e^2+9efx+2f^2x^2))}{(de-f)(de+f)(e+fx)^2}+4d(3Ce-Bf)}{2f^4}$$

input
```
Integrate[(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2))/(e + f*x)^3,x]
```

output
```
((f*Sqrt[1 - d^2*x^2]*(A*f^3*(f + d^2*e*x) + B*f^3*(e + 2*f*x) - B*d^2*e^2
*f*(2*e + 3*f*x) - C*f^2*(5*e^2 + 8*e*f*x + 2*f^2*x^2) + C*d^2*e^2*(6*e^2
+ 9*e*f*x + 2*f^2*x^2)))/((d*e - f)*(d*e + f)*(e + f*x)^2) + 4*d*(3*C*e -
B*f)*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])] + (2*Sqrt[d^2*e^2 - f^2]*(-(d^
2*f*(2*B*d^2*e^3 - 3*B*e*f^2 + A*f^3)) + C*(6*d^4*e^4 - 9*d^2*e^2*f^2 + 2*
f^4))*ArcTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((-
(d*e) + f)^2*(d*e + f)^2))/(2*f^4)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {2112, 2182, 681, 27, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-dx}\sqrt{dx+1}(A+Bx+Cx^2)}{(e+fx)^3}\,dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{\sqrt{1-d^2x^2}(A+Bx+Cx^2)}{(e+fx)^3}dx$$

$$\Big\downarrow \text{ 2182}$$

$$\frac{\int \frac{\left(2\left(Aed^2+Ce-Bf\right)-\left(Bed^2-Afd^2-\frac{3Ce^2d^2}{f}+2Cf\right)x\right)\sqrt{1-d^2x^2}}{(e+fx)^2}dx}{2\left(d^2e^2-f^2\right)}+\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\Big\downarrow$ 681

$$\frac{\frac{\sqrt{1-d^2x^2}\left(2(de-f)(de+f)(3Ce-Bf)-fx\left(d^2f(Be-Af)-C\left(3d^2e^2-2f^2\right)\right)\right)}{f^3(e+fx)}-\frac{\int \frac{2\left(f\left(d^2f(Be-Af)-\frac{1}{2}C\left(6d^2e^2-4f^2\right)\right)-2d^2(de-f)(de+f)(3Ce-B}{f(e+fx)\sqrt{1-d^2x^2}}}{2f^2}}{2\left(d^2e^2-f^2\right)}$$
$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sqrt{1-d^2x^2}\left(2(de-f)(de+f)(3Ce-Bf)-fx\left(d^2f(Be-Af)-C\left(3d^2e^2-2f^2\right)\right)\right)}{f^3(e+fx)}-\frac{\int \frac{f\left(d^2f(Be-Af)-C\left(3d^2e^2-2f^2\right)\right)-2d^2(de-f)(de+f)(3Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}d}{f^3}}{2\left(d^2e^2-f^2\right)}$$
$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\Big\downarrow$ 719

$$\frac{\frac{\sqrt{1-d^2x^2}\left(2(de-f)(de+f)(3Ce-Bf)-fx\left(d^2f(Be-Af)-C\left(3d^2e^2-2f^2\right)\right)\right)}{f^3(e+fx)}-\frac{-\frac{\left(d^2f\left(Af^3+2Bd^2e^3-3Bef^2\right)-C\left(6d^4e^4-9d^2e^2f^2+2f^4\right)\right)\int \frac{}{(e+fx}}{f}}{f^3}}{2\left(d^2e^2-f^2\right)}$$
$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\Big\downarrow$ 223

$$\frac{\frac{\sqrt{1-d^2x^2}\left(2(de-f)(de+f)(3Ce-Bf)-fx\left(d^2f(Be-Af)-C\left(3d^2e^2-2f^2\right)\right)\right)}{f^3(e+fx)}-\frac{-\frac{\left(d^2f\left(Af^3+2Bd^2e^3-3Bef^2\right)-C\left(6d^4e^4-9d^2e^2f^2+2f^4\right)\right)\int \frac{}{(e+fx}}{f}}{f^3}}{2\left(d^2e^2-f^2\right)}$$
$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\Big\downarrow$ 488

$$\frac{\frac{\sqrt{1-d^2x^2}\left(2(de-f)(de+f)(3Ce-Bf)-fx\left(d^2f(Be-Af)-C\left(3d^2e^2-2f^2\right)\right)\right)}{f^3(e+fx)}-\frac{\frac{\left(d^2f\left(Af^3+2Bd^2e^3-3Bef^2\right)-C\left(6d^4e^4-9d^2e^2f^2+2f^4\right)\right)\int \frac{}{-d^2e^2+}}{f}}{f}}{2\left(d^2e^2-f^2\right)}$$

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\Big\downarrow$ 217

$$\frac{\frac{\sqrt{1-d^2x^2}\left(2(de-f)(de+f)(3Ce-Bf)-fx\left(d^2f(Be-Af)-C\left(3d^2e^2-2f^2\right)\right)\right)}{f^3(e+fx)}-\frac{-\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(d^2f\left(Af^3+2Bd^2e^3-3Bef^2\right)-C\left(6\right.\right.}{f\sqrt{d^2e^2-f^2}}}{f^3}}{2\left(d^2e^2-f^2\right)}$$

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

input
```
Int[(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2))/(e + f*x)^3,x]
```

output
```
((C*e^2 - B*e*f + A*f^2)*(1 - d^2*x^2)^(3/2))/(2*f*(d^2*e^2 - f^2)*(e + f*
x)^2) + ((((2*(d*e - f)*(d*e + f)*(3*C*e - B*f) - f*(d^2*f*(B*e - A*f) - C*
(3*d^2*e^2 - 2*f^2))*x)*Sqrt[1 - d^2*x^2])/(f^3*(e + f*x)) - ((-2*d*(d*e -
 f)*(d*e + f)*(3*C*e - B*f)*ArcSin[d*x])/f - ((d^2*f*(2*B*d^2*e^3 - 3*B*e*
f^2 + A*f^3) - C*(6*d^4*e^4 - 9*d^2*e^2*f^2 + 2*f^4))*ArcTan[(f + d^2*e*x)
/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f*Sqrt[d^2*e^2 - f^2]))/f^3)/(
2*(d^2*e^2 - f^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. $2(285) = 570$.

Time = 0.81 (sec) , antiderivative size = 1156, normalized size of antiderivative = 3.84

method	result	size
risch	Expression too large to display	1156
default	Expression too large to display	2385

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-C/f^3*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(
1/2)/(-d*x+1)^(1/2)-1/f^3*(d^2*(B*f-3*C*e)/f/(d^2)^(1/2)*arctan((d^2)^(1/2
)*x/(-d^2*x^2+1)^(1/2))-1/f^2*(A*d^2*f^2-3*B*d^2*e*f+6*C*d^2*e^2-C*f^2)/(-
(d^2*e^2-f^2)/f^2)^(1/2)*ln((-2*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d
^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2
)^(1/2))/(x+e/f))-1/f^3*(2*A*d^2*e*f^2-3*B*d^2*e^2*f+4*C*d^2*e^3+B*f^3-2*C
*e*f^2)*(1/(d^2*e^2-f^2)*f^2/(x+e/f)*(-d^2*(x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^
2*e^2-f^2)/f^2)^(1/2)-f*d^2*e/(d^2*e^2-f^2)/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln(
(-2*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2
*(x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2))/(x+e/f)))+1/f^4*(A*
d^2*e^2*f^2-B*d^2*e^3*f+C*d^2*e^4-A*f^4+B*e*f^3-C*e^2*f^2)*(1/2/(d^2*e^2-f
^2)*f^2/(x+e/f)^2*(-d^2*(x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/
2)+3/2*f*d^2*e/(d^2*e^2-f^2)*(1/(d^2*e^2-f^2)*f^2/(x+e/f)*(-d^2*(x+e/f)^2+
2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2)-f*d^2*e/(d^2*e^2-f^2)/(-(d^2*e^
2-f^2)/f^2)^(1/2)*ln((-2*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-
f^2)/f^2)^(1/2)*(-d^2*(x+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2)
)/(x+e/f)))+1/2*d^2/(d^2*e^2-f^2)*f^2/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln((-2*(d
^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(x+e/
f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2))/(x+e/f))))*((-d*x+1)*(d*x
+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1089 vs. $2(285) = 570$.

Time = 63.35 (sec) , antiderivative size = 2201, normalized size of antiderivative = 7.31

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x, algorith
m="fricas")
```

output
```
[1/2*(6*C*d^4*e^8*f - 2*B*d^4*e^7*f^2 - 11*C*d^2*e^6*f^3 + 3*B*d^2*e^5*f^4
 + (A*d^2 + 5*C)*e^4*f^5 - B*e^3*f^6 - A*e^2*f^7 + (6*C*d^4*e^6*f^3 - 2*B*
d^4*e^5*f^4 - 11*C*d^2*e^4*f^5 + 3*B*d^2*e^3*f^6 + (A*d^2 + 5*C)*e^2*f^7 -
 B*e*f^8 - A*f^9)*x^2 - (6*C*d^4*e^8 - 2*B*d^4*e^7*f - 9*C*d^2*e^6*f^2 + 3
*B*d^2*e^5*f^3 - (A*d^2 - 2*C)*e^4*f^4 + (6*C*d^4*e^6*f^2 - 2*B*d^4*e^5*f^
3 - 9*C*d^2*e^4*f^4 + 3*B*d^2*e^3*f^5 - (A*d^2 - 2*C)*e^2*f^6)*x^2 + 2*(6*
C*d^4*e^7*f - 2*B*d^4*e^6*f^2 - 9*C*d^2*e^5*f^3 + 3*B*d^2*e^4*f^4 - (A*d^2
 - 2*C)*e^3*f^5)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x + f^2 + sqrt(-d^2*
e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f - (d^2*e
^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (6*C*d^4*e^8*f - 2*B
*d^4*e^7*f^2 - 11*C*d^2*e^6*f^3 + 3*B*d^2*e^5*f^4 + (A*d^2 + 5*C)*e^4*f^5
- B*e^3*f^6 - A*e^2*f^7 + 2*(C*d^4*e^6*f^3 - 2*C*d^2*e^4*f^5 + C*e^2*f^7)*
x^2 + (9*C*d^4*e^7*f^2 - 3*B*d^4*e^6*f^3 + 5*B*d^2*e^4*f^5 + (A*d^4 - 17*C
*d^2)*e^5*f^4 - (A*d^2 - 8*C)*e^3*f^6 - 2*B*e^2*f^7)*x)*sqrt(d*x + 1)*sqrt
(-d*x + 1) + 2*(6*C*d^4*e^7*f^2 - 2*B*d^4*e^6*f^3 - 11*C*d^2*e^5*f^4 + 3*B
*d^2*e^4*f^5 + (A*d^2 + 5*C)*e^3*f^6 - B*e^2*f^7 - A*e*f^8)*x - 4*(3*C*d^5
*e^9 - B*d^5*e^8*f - 6*C*d^3*e^7*f^2 + 2*B*d^3*e^6*f^3 + 3*C*d*e^5*f^4 - B
*d*e^4*f^5 + (3*C*d^5*e^7*f^2 - B*d^5*e^6*f^3 - 6*C*d^3*e^5*f^4 + 2*B*d^3*
e^4*f^5 + 3*C*d*e^3*f^6 - B*d*e^2*f^7)*x^2 + 2*(3*C*d^5*e^8*f - B*d^5*e^7*
f^2 - 6*C*d^3*e^6*f^3 + 2*B*d^3*e^5*f^4 + 3*C*d*e^4*f^5 - B*d*e^3*f^6)*...
```

Sympy [F]

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \int \frac{\sqrt{-dx+1}\sqrt{dx+1}\,(A+Bx+Cx^2)}{(e+fx)^3}\,dx$$

input
```
integrate((-d*x+1)**(1/2)*(d*x+1)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**3,x)
```

output
```
Integral(sqrt(-d*x + 1)*sqrt(d*x + 1)*(A + B*x + C*x**2)/(e + f*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((f-d*e)*(f+d*e)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 70.72 (sec) , antiderivative size = 21781, normalized size of antiderivative = 72.36

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int(((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^3,x)
```

output
```
((2*(25*B*d^3*e^2 - 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^3)/(f*(f^2 - d^2*e^2
)*((d*x + 1)^(1/2) - 1)^3) - (2*(25*B*d^3*e^2 - 16*B*d*f^2)*((1 - d*x)^(1/
2) - 1)^5)/(f*(f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^5) + (4*((1 - d*x)^(1/
2) - 1)^2*(2*B*d^4*e^4 - 2*B*f^4 + 3*B*d^2*e^2*f^2))/(e*f^2*(f^2 - d^2*e^2
)*((d*x + 1)^(1/2) - 1)^2) + (8*((1 - d*x)^(1/2) - 1)^4*(2*B*f^4 + 2*B*d^4
*e^4 - 5*B*d^2*e^2*f^2))/(e*f^2*(f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^4) +
 (4*((1 - d*x)^(1/2) - 1)^6*(2*B*d^4*e^4 - 2*B*f^4 + 3*B*d^2*e^2*f^2))/(e*
f^2*(f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^6) + (2*B*d^3*e^2*((1 - d*x)^(1/
2) - 1))/(f*(f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)) - (2*B*d^3*e^2*((1 - d*
x)^(1/2) - 1)^7)/(f*(f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^7))/(d^2*e^2 + (
((1 - d*x)^(1/2) - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + ((
(1 - d*x)^(1/2) - 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((
1 - d*x)^(1/2) - 1)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2
*e^2*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x
)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5
)/((d*x + 1)^(1/2) - 1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(
1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) - ((2
*(7*A*d^3*e^2 + 2*A*d*f^2)*((1 - d*x)^(1/2) - 1)^3)/(e*(f^2 - d^2*e^2)*((d
*x + 1)^(1/2) - 1)^3) - (2*(7*A*d^3*e^2 + 2*A*d*f^2)*((1 - d*x)^(1/2) - 1)
^5)/(e*(f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^5) + (4*(2*A*f^3 + A*d^2*e...
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 3422, normalized size of antiderivative = 11.37

$$\int \frac{\sqrt{1-dx}\sqrt{1+dx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int((-d*x+1)^(1/2)*(d*x+1)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x)
```

output
```
(4*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**5*e**6*f + 8*asin(sqrt( - d*x + 1)/
sqrt(2))*b*d**5*e**5*f**2*x + 4*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**5*e**4
*f**3*x**2 - 8*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**3*e**4*f**3 - 16*asin(s
qrt( - d*x + 1)/sqrt(2))*b*d**3*e**3*f**4*x - 8*asin(sqrt( - d*x + 1)/sqrt
(2))*b*d**3*e**2*f**5*x**2 + 4*asin(sqrt( - d*x + 1)/sqrt(2))*b*d*e**2*f**
5 + 8*asin(sqrt( - d*x + 1)/sqrt(2))*b*d*e*f**6*x + 4*asin(sqrt( - d*x + 1
)/sqrt(2))*b*d*f**7*x**2 - 12*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**5*e**7 -
 24*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**5*e**6*f*x - 12*asin(sqrt( - d*x +
 1)/sqrt(2))*c*d**5*e**5*f**2*x**2 + 24*asin(sqrt( - d*x + 1)/sqrt(2))*c*d
**3*e**5*f**2 + 48*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**3*e**4*f**3*x + 24*
asin(sqrt( - d*x + 1)/sqrt(2))*c*d**3*e**3*f**4*x**2 - 12*asin(sqrt( - d*x
 + 1)/sqrt(2))*c*d*e**3*f**4 - 24*asin(sqrt( - d*x + 1)/sqrt(2))*c*d*e**2*
f**5*x - 12*asin(sqrt( - d*x + 1)/sqrt(2))*c*d*e*f**6*x**2 - 2*sqrt(d*e +
f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2)
 - sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**2*e**2*f**4 - 4*sqrt(d*e + f)*sqrt
(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt
(f)*sqrt(2))/sqrt(d*e - f))*a*d**2*e*f**5*x - 2*sqrt(d*e + f)*sqrt(d*e - f
)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt
(2))/sqrt(d*e - f))*a*d**2*f**6*x**2 - 4*sqrt(d*e + f)*sqrt(d*e - f)*atan(
(sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))...
```

3.38 $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 332

$$\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$

$$= -\frac{\left(15Be-\frac{3Ce^2}{f}+20Af+\frac{16Cf}{d^2}\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^2}$$

$$-\frac{\left(5B-\frac{Ce}{f}\right)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2}-\frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f}$$

$$+\frac{(4(C(3d^4e^4-52d^2e^2f^2-16f^4)-5d^2f(4Af(4d^2e^2+f^2)+3B(d^2e^3+4ef^2)))+d^2f(6Cd^2e^3-30Bd}{120d^6f}$$

$$+\frac{(4Cd^2e^3+8Ad^4e^3+12Bd^2e^2f+9Cef^2+12Ad^2ef^2+3Bf^3)\arcsin(dx)}{8d^5}$$

output
```
-1/60*(15*B*e-3*C*e^2/f+20*A*f+16*C*f/d^2)*(f*x+e)^2*(-d^2*x^2+1)^(1/2)/d^
2-1/20*(5*B-C*e/f)*(f*x+e)^3*(-d^2*x^2+1)^(1/2)/d^2-1/5*C*(f*x+e)^4*(-d^2*
x^2+1)^(1/2)/d^2/f+1/120*(4*C*(3*d^4*e^4-52*d^2*e^2*f^2-16*f^4)-20*d^2*f*(
4*A*f*(4*d^2*e^2+f^2)+3*B*(d^2*e^3+4*e*f^2))+d^2*f*(-100*A*d^2*e*f^2-30*B*
d^2*e^2*f+6*C*d^2*e^3-45*B*f^3-71*C*e*f^2)*x)*(-d^2*x^2+1)^(1/2)/d^6/f+1/8
*(8*A*d^4*e^3+12*A*d^2*e*f^2+12*B*d^2*e^2*f+4*C*d^2*e^3+3*B*f^3+9*C*e*f^2)
*arcsin(d*x)/d^5
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.78

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{-\sqrt{1-d^2x^2}(20Ad^2f(4f^2+d^2(18e^2+9efx+2f^2x^2))+15B(d^2f^2(16e+3fx)+2d^4(4e^3+6e^2fx+4e}{}$$

input
```
Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-(Sqrt[1 - d^2*x^2]*(20*A*d^2*f*(4*f^2 + d^2*(18*e^2 + 9*e*f*x + 2*f^2*x^
2)) + 15*B*(d^2*f^2*(16*e + 3*f*x) + 2*d^4*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^
2 + f^3*x^3)) + C*(64*f^3 + d^2*f*(240*e^2 + 135*e*f*x + 32*f^2*x^2) + 6*d
^4*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 30*d*(4*C*d^2*e
^3 + 8*A*d^4*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*
ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(120*d^6)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.297$, Rules used = {2112, 2185, 25, 27, 687, 25, 27, 687, 25, 676, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{dx+1}}\,dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\int -\frac{f(e+fx)^3((5Ad^2+4C)f-d^2(Ce-5Bf)x)}{\sqrt{1-d^2x^2}}dx}{5d^2f^2} - \frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{f(e+fx)^3\left(\left(5Ad^2+4C\right)f-d^2(Ce-5Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{5d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(e+fx)^3\left(\left(5Ad^2+4C\right)f-d^2(Ce-5Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{4}\sqrt{1-d^2x^2}(e+fx)^3(Ce-5Bf)-\frac{\int -\frac{d^2(e+fx)^2\left(f\left(20Aed^2+13Ce+15Bf\right)+\left(4\left(5Ad^2+4C\right)f^2-3d^2e(Ce-5Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx}{4d^2}}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{d^2(e+fx)^2\left(f\left(20Aed^2+13Ce+15Bf\right)+\left(4\left(5Ad^2+4C\right)f^2-3d^2e(Ce-5Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx}{4d^2}+\frac{1}{4}\sqrt{1-d^2x^2}(e+fx)^3(Ce-5Bf)}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}\int \frac{(e+fx)^2\left(f\left(20Aed^2+13Ce+15Bf\right)+\left(4\left(5Ad^2+4C\right)f^2-3d^2e(Ce-5Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx+\frac{1}{4}\sqrt{1-d^2x^2}(e+fx)^3(Ce-5Bf)}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{1}{4}\left(-\frac{\int -\frac{(e+fx)\left(f\left(60Ae^2d^4+33Ce^2d^2+40Af^2d^2+75Befd^2+32Cf^2\right)-d^2\left(6Cd^2e^3-30Bd^2fe^2-100Ad^2f^2e-71Cf^2e-45Bf^3\right)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{1}{3}\sqrt{1-d^2x^2}(\ldots\right)}{5d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{4}\left(\frac{\int \frac{(e+fx)\left(f\left(60Ae^2d^4+33Ce^2d^2+40Af^2d^2+75Befd^2+32Cf^2\right)-d^2\left(6Cd^2e^3-30Bd^2fe^2-100Ad^2f^2e-71Cf^2e-45Bf^3\right)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}-\frac{1}{3}\sqrt{1-d^2x^2}(e+\right.}{5d^2f}$$

$$\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{1}{4}\left(\frac{\frac{15}{2}f\left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx+\frac{1}{2}fx\sqrt{1-d^2x^2}\left(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Cd^2e^3-\right.}{3d^2}\right.}{}$$

$$\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{1}{4}\left(\frac{\frac{15f\arcsin(dx)\left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)}{2d}+\frac{1}{2}fx\sqrt{1-d^2x^2}\left(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Cd^2e^3-71Cef^2\right)+}{3d^2}\right.}{}$$

$$\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

input

```
Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output

```
-1/5*(C*(e + f*x)^4*Sqrt[1 - d^2*x^2])/(d^2*f) + (((C*e - 5*B*f)*(e + f*x)
^3*Sqrt[1 - d^2*x^2])/4 + (-1/3*((5*f*(3*B*e + 4*A*f) - C*(3*e^2 - (16*f^2
)/d^2))*(e + f*x)^2*Sqrt[1 - d^2*x^2]) + ((2*(C*(3*d^4*e^4 - 52*d^2*e^2*f^
2 - 16*f^4) - 5*d^2*f*(4*A*f*(4*d^2*e^2 + f^2) + 3*B*(d^2*e^3 + 4*e*f^2)))
*Sqrt[1 - d^2*x^2])/d^2 + (f*(6*C*d^2*e^3 - 30*B*d^2*e^2*f - 71*C*e*f^2 -
100*A*d^2*e*f^2 - 45*B*f^3)*x*Sqrt[1 - d^2*x^2])/2 + (15*f*(4*C*d^2*e^3 +
8*A*d^4*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSi
n[d*x])/(2*d))/(3*d^2))/4)/(5*d^2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.09

method	result
risch	$\frac{(24C\,d^4f^3x^4+30B\,d^4f^3x^3+90C\,d^4e\,f^2x^3+40A\,d^4f^3x^2+120B\,d^4e\,f^2x^2+120C\,d^4e^2f\,x^2+180A\,d^4e\,f^2x+180B\,d^4e^2fx+60C\,d^4e^3}{120d^6}$
default	$-\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\left(24C\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4f^3x^4+30B\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4f^3x^3+90C\sqrt{-d^2x^2+1}\,\mathrm{csgn}(d)d^4e\,f^2x^3+40A\sqrt{}\right.}{}$

input
```
int((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/120*(24*C*d^4*f^3*x^4+30*B*d^4*f^3*x^3+90*C*d^4*e*f^2*x^3+40*A*d^4*f^3*x
^2+120*B*d^4*e*f^2*x^2+120*C*d^4*e^2*f*x^2+180*A*d^4*e*f^2*x+180*B*d^4*e^2
*f*x+60*C*d^4*e^3*x+360*A*d^4*e^2*f+120*B*d^4*e^3+32*C*d^2*f^3*x^2+45*B*d^
2*f^3*x+135*C*d^2*e*f^2*x+80*A*d^2*f^3+240*B*d^2*e*f^2+240*C*d^2*e^2*f+64*
C*f^3)*(d*x+1)^(1/2)*(d*x-1)/d^6/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1
))^(1/2)/(-d*x+1)^(1/2)+1/8*(8*A*d^4*e^3+12*A*d^2*e*f^2+12*B*d^2*e^2*f+4*C
*d^2*e^3+3*B*f^3+9*C*e*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2
+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.86

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(24\,Cd^4f^3x^4+120\,Bd^4e^3+240\,Bd^2ef^2+120\,(3\,Ad^4+2\,Cd^2)e^2f+16\,(5\,Ad^2+4\,C)f^3+30\,(3\,Cd^4e}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
-1/120*((24*C*d^4*f^3*x^4 + 120*B*d^4*e^3 + 240*B*d^2*e*f^2 + 120*(3*A*d^4
 + 2*C*d^2)*e^2*f + 16*(5*A*d^2 + 4*C)*f^3 + 30*(3*C*d^4*e*f^2 + B*d^4*f^3
)*x^3 + 8*(15*C*d^4*e^2*f + 15*B*d^4*e*f^2 + (5*A*d^4 + 4*C*d^2)*f^3)*x^2
+ 15*(4*C*d^4*e^3 + 12*B*d^4*e^2*f + 3*B*d^2*f^3 + 3*(4*A*d^4 + 3*C*d^2)*e
*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(12*B*d^3*e^2*f + 3*B*d*f^3 + 4
*(2*A*d^5 + C*d^3)*e^3 + 3*(4*A*d^3 + 3*C*d)*e*f^2)*arctan((sqrt(d*x + 1)*
sqrt(-d*x + 1) - 1)/(d*x)))/d^6
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**3*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.07

$$\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}Cf^3x^4}{5\,d^2} + \frac{Ae^3\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}Be^3}{d^2} - \frac{3\sqrt{-d^2x^2+1}Ae^2f}{d^2} - \frac{4\sqrt{-d^2x^2+1}Cf^3x^2}{15\,d^4} - \frac{(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x^3}{4\,d^2} - \frac{(3\,Ce^2f+3\,Bef^2+Af^3)\sqrt{-d^2x^2+1}x^2}{3\,d^2} - \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\sqrt{-d^2x^2+1}x}{2\,d^2} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)\arcsin(dx)}{2\,d^3} - \frac{8\sqrt{-d^2x^2+1}Cf^3}{15\,d^6} - \frac{3\,(3\,Cef^2+Bf^3)\sqrt{-d^2x^2+1}x}{8\,d^4} - \frac{2\,(3\,Ce^2f+3\,Bef^2+Af^3)\sqrt{-d^2x^2+1}}{3\,d^4} + \frac{3\,(3\,Cef^2+Bf^3)\arcsin(dx)}{8\,d^5}$$

input

```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="maxima")
```

output

```
-1/5*sqrt(-d^2*x^2 + 1)*C*f^3*x^4/d^2 + A*e^3*arcsin(d*x)/d - sqrt(-d^2*x^
2 + 1)*B*e^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*A*e^2*f/d^2 - 4/15*sqrt(-d^2*x^2 +
 1)*C*f^3*x^2/d^4 - 1/4*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x^3/d^2 - 1
/3*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*sqrt(-d^2*x^2 + 1)*x^2/d^2 - 1/2*(C*e^3
 + 3*B*e^2*f + 3*A*e*f^2)*sqrt(-d^2*x^2 + 1)*x/d^2 + 1/2*(C*e^3 + 3*B*e^2*
f + 3*A*e*f^2)*arcsin(d*x)/d^3 - 8/15*sqrt(-d^2*x^2 + 1)*C*f^3/d^6 - 3/8*(
3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x/d^4 - 2/3*(3*C*e^2*f + 3*B*e*f^2 +
 A*f^3)*sqrt(-d^2*x^2 + 1)/d^4 + 3/8*(3*C*e*f^2 + B*f^3)*arcsin(d*x)/d^5
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.13

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(120\,Bd^4e^3+360\,Ad^4e^2f-60\,Cd^3e^3-180\,Bd^3e^2f-180\,Ad^3ef^2+360\,Cd^2e^2f+360\,Bd^2ef^2+120}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
-1/120*((120*B*d^4*e^3 + 360*A*d^4*e^2*f - 60*C*d^3*e^3 - 180*B*d^3*e^2*f
- 180*A*d^3*e*f^2 + 360*C*d^2*e^2*f + 360*B*d^2*e*f^2 + 120*A*d^2*f^3 - 22
5*C*d*e*f^2 - 75*B*d*f^3 + 120*C*f^3 + (60*C*d^3*e^3 + 180*B*d^3*e^2*f + 1
80*A*d^3*e*f^2 - 240*C*d^2*e^2*f - 240*B*d^2*e*f^2 - 80*A*d^2*f^3 + 405*C*
d*e*f^2 + 135*B*d*f^3 - 160*C*f^3 + 2*(60*C*d^2*e^2*f + 60*B*d^2*e*f^2 + 2
0*A*d^2*f^3 - 135*C*d*e*f^2 - 45*B*d*f^3 + 88*C*f^3 + 3*(15*C*d*e*f^2 + 4*
(d*x + 1)*C*f^3 + 5*B*d*f^3 - 16*C*f^3)*(d*x + 1))*(d*x + 1))*(d*x + 1))*s
qrt(d*x + 1)*sqrt(-d*x + 1) - 30*(8*A*d^5*e^3 + 4*C*d^3*e^3 + 12*B*d^3*e^2
*f + 12*A*d^3*e*f^2 + 9*C*d*e*f^2 + 3*B*d*f^3)*arcsin(1/2*sqrt(2)*sqrt(d*x
 + 1)))/d^6
```

Mupad [B] (verification not implemented)

Time = 31.51 (sec) , antiderivative size = 2606, normalized size of antiderivative = 7.85

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^3*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)
^(1/2) - 1)^6 + (((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^
14)/((d*x + 1)^(1/2) - 1)^14 - (((4096*C*f^3)/3 - 832*C*d^2*e^2*f)*((1 - d
*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((4096*C*f^3)/3 - 832*C*d^2*e
^2*f)*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (((12288*C*f^3)
/5 + 768*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 +
 (((1 - d*x)^(1/2) - 1)^3*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((d*x + 1)^(1/
2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^17*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((
d*x + 1)^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^7*(88*C*d^3*e^3 - 42*C*d*e
*f^2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^13*(88*C*d^3*e^3 -
 42*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*(40*C*
d^3*e^3 + 426*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d*x)^(1/2) - 1)
^15*(40*C*d^3*e^3 + 426*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)
^(1/2) - 1)^9*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^9 - ((
(1 - d*x)^(1/2) - 1)^11*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) -
 1)^11 - (d*(4*C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)
^(1/2) - 1)) + (d*(4*C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1)^19)/(2*(
(d*x + 1)^(1/2) - 1)^19) + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^4)/((d*x
 + 1)^(1/2) - 1)^4 + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)
^(1/2) - 1)^16)/(d^6 + (10*d^6*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.70

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-240 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) a\, d^5 e^3 - 360 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) a\, d^3 e\, f^2 - 360 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) b\, d^3 e^2 f - 90 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)}{}$$

input
```
int((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output

```
( - 240*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**5*e**3 - 360*asin(sqrt( - d*x
+ 1)/sqrt(2))*a*d**3*e*f**2 - 360*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**3*e*
*2*f - 90*asin(sqrt( - d*x + 1)/sqrt(2))*b*d*f**3 - 120*asin(sqrt( - d*x +
 1)/sqrt(2))*c*d**3*e**3 - 270*asin(sqrt( - d*x + 1)/sqrt(2))*c*d*e*f**2 -
 360*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**4*e**2*f - 180*sqrt(d*x + 1)*sqrt
( - d*x + 1)*a*d**4*e*f**2*x - 40*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**4*f*
*3*x**2 - 80*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**2*f**3 - 120*sqrt(d*x + 1
)*sqrt( - d*x + 1)*b*d**4*e**3 - 180*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**4
*e**2*f*x - 120*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**4*e*f**2*x**2 - 30*sqr
t(d*x + 1)*sqrt( - d*x + 1)*b*d**4*f**3*x**3 - 240*sqrt(d*x + 1)*sqrt( - d
*x + 1)*b*d**2*e*f**2 - 45*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**2*f**3*x -
60*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**4*e**3*x - 120*sqrt(d*x + 1)*sqrt(
- d*x + 1)*c*d**4*e**2*f*x**2 - 90*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**4*e
*f**2*x**3 - 24*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**4*f**3*x**4 - 240*sqrt
(d*x + 1)*sqrt( - d*x + 1)*c*d**2*e**2*f - 135*sqrt(d*x + 1)*sqrt( - d*x +
 1)*c*d**2*e*f**2*x - 32*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**2*f**3*x**2 -
 64*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*f**3)/(120*d**6)
```

3.39 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 228

$$
\begin{aligned}
&\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx \\
&= -\frac{\left(4B-\frac{Ce}{f}\right)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2} - \frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f} \\
&\quad + \frac{\left(4(C(d^2e^3-8ef^2)-4f(3Ad^2ef+B(d^2e^2+f^2)))-f(3(3C+4Ad^2)f^2-2d^2e(Ce-4Bf))x\right)\sqrt{1-}}{24d^4f} \\
&\quad + \frac{(C(4d^2e^2+3f^2)+4d^2(2Bef+A(2d^2e^2+f^2)))\arcsin(dx)}{8d^5}
\end{aligned}
$$

output

```
-1/12*(4*B-C*e/f)*(f*x+e)^2*(-d^2*x^2+1)^(1/2)/d^2-1/4*C*(f*x+e)^3*(-d^2*x
^2+1)^(1/2)/d^2/f+1/24*(4*C*(d^2*e^3-8*e*f^2)-16*f*(3*A*d^2*e*f+B*(d^2*e^2
+f^2))-f*(3*(4*A*d^2+3*C)*f^2-2*d^2*e*(-4*B*f+C*e))*x)*(-d^2*x^2+1)^(1/2)/
d^4/f+1/8*(C*(4*d^2*e^2+3*f^2)+4*d^2*(2*B*e*f+A*(2*d^2*e^2+f^2)))*arcsin(d
*x)/d^5
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{-d\sqrt{1-d^2x^2}(12Ad^2f(4e+fx)+C(12d^2e^2x+16ef(2+d^2x^2)+3f^2x(3+2d^2x^2))+8B(2f^2+d^2(3e^2}{24d^5}$$

input
```
Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(-(d*Sqrt[1 - d^2*x^2]*(12*A*d^2*f*(4*e + f*x) + C*(12*d^2*e^2*x + 16*e*f*
(2 + d^2*x^2) + 3*f^2*x*(3 + 2*d^2*x^2)) + 8*B*(2*f^2 + d^2*(3*e^2 + 3*e*f
*x + f^2*x^2)))) + 6*(C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f + A*(2*d^2*e^
2 + f^2)))*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/(24*d^5)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.243$, Rules used = {2112, 2185, 25, 27, 687, 25, 27, 676, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\int -\frac{f(e+fx)^2((4Ad^2+3C)f-d^2(Ce-4Bf)x)}{\sqrt{1-d^2x^2}}dx}{4d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{f(e+fx)^2((4Ad^2+3C)f-d^2(Ce-4Bf)x)}{\sqrt{1-d^2x^2}}dx}{4d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(e+fx)^2((4Ad^2+3C)f-d^2(Ce-4Bf)x)}{\sqrt{1-d^2x^2}}dx}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$\Big\downarrow$ 687

$$\frac{\frac{1}{3}\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)-\frac{\int -\frac{d^2(e+fx)\left(f\left(12Aed^2+7Ce+8Bf\right)+\left(3\left(4Ad^2+3C\right)f^2-2d^2e(Ce-4Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{d^2(e+fx)\left(f\left(12Aed^2+7Ce+8Bf\right)+\left(3\left(4Ad^2+3C\right)f^2-2d^2e(Ce-4Bf)\right)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2}+\frac{1}{3}\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{3}\int \frac{(e+fx)(f(12Aed^2+7Ce+8Bf)+(3(4Ad^2+3C)f^2-2d^2e(Ce-4Bf))x)}{\sqrt{1-d^2x^2}}dx+\frac{1}{3}\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)}{4d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$\Big\downarrow$ 676

$$\frac{\frac{1}{3}\left(\frac{3f(4d^2(A(2d^2e^2+f^2)+2Bef)+C(4d^2e^2+3f^2))\int \frac{1}{\sqrt{1-d^2x^2}}dx}{2d^2}+\frac{1}{2}fx\sqrt{1-d^2x^2}\left(-12Af^2-8Bef-\frac{9Cf^2}{d^2}+2Ce^2\right)+\frac{2\sqrt{}}{}\right.}{4d^2f}$$

$$-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

$\Big\downarrow$ 223

$$\frac{\frac{1}{3}\left(\frac{3f\arcsin(dx)\left(4d^2\left(A\left(2d^2e^2+f^2\right)+2Bef\right)+C\left(4d^2e^2+3f^2\right)\right)}{2d^3}+\frac{1}{2}fx\sqrt{1-d^2x^2}\left(-12Af^2-8Bef-\frac{9Cf^2}{d^2}+2Ce^2\right)+\frac{2\sqrt{1-d^2}\cdots}{\cdots}\right.}{4d^2f}$$

$$\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

input
```
Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/4*(C*(e + f*x)^3*Sqrt[1 - d^2*x^2])/(d^2*f) + (((C*e - 4*B*f)*(e + f*x)
^2*Sqrt[1 - d^2*x^2])/3 + ((2*(C*d^2*e^3 - 4*B*d^2*e^2*f - 8*C*e*f^2 - 12*
A*d^2*e*f^2 - 4*B*f^3)*Sqrt[1 - d^2*x^2])/d^2 + (f*(2*C*e^2 - 8*B*e*f - 12
*A*f^2 - (9*C*f^2)/d^2)*x*Sqrt[1 - d^2*x^2])/2 + (3*f*(C*(4*d^2*e^2 + 3*f^
2) + 4*d^2*(2*B*e*f + A*(2*d^2*e^2 + f^2)))*ArcSin[d*x])/(2*d^3))/3)/(4*d^
2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.12

method	result
risch	$\frac{\left(6C\,d^{2} f^{2} x^{3}+8B\,d^{2} f^{2} x^{2}+16C\,d^{2} e f\,x^{2}+12A\,d^{2} f^{2} x+24B\,d^{2} e f x+12C\,d^{2} e^{2} x+48A\,d^{2} e f+24B\,d^{2} e^{2}+9C\,f^{2} x+16B\,f^{2}+32C e f\right)\sqrt{x}}{24 d^{4} \sqrt{-\left(x d+1\right)\left(x d-1\right)}\,\sqrt{-x d+1}}$
default	$-\frac{\sqrt{-x d+1}\,\sqrt{x d+1}\,\left(6C\sqrt{-d^{2} x^{2}+1}\,\operatorname{csgn}\left(d\right) d^{3} f^{2} x^{3}+8B\sqrt{-d^{2} x^{2}+1}\,\operatorname{csgn}\left(d\right) d^{3} f^{2} x^{2}+16C\sqrt{-d^{2} x^{2}+1}\,\operatorname{csgn}\left(d\right) d^{3} e f\,x^{2}+12A\sqrt{-d^{2}}\right.}{}$

input
```
int((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/24*(6*C*d^2*f^2*x^3+8*B*d^2*f^2*x^2+16*C*d^2*e*f*x^2+12*A*d^2*f^2*x+24*B
*d^2*e*f*x+12*C*d^2*e^2*x+48*A*d^2*e*f+24*B*d^2*e^2+9*C*f^2*x+16*B*f^2+32*
C*e*f)*(d*x+1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1
))^(1/2)/(-d*x+1)^(1/2)+1/8*(8*A*d^4*e^2+4*A*d^2*f^2+8*B*d^2*e*f+4*C*d^2*e
^2+3*C*f^2)/d^4/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+1)^(1/2))*((-d*
x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(6\,Cd^3f^2x^3+24\,Bd^3e^2+16\,Bdf^2+16\,(3\,Ad^3+2\,Cd)ef+8\,(2\,Cd^3ef+Bd^3f^2)x^2+3\,(4\,Cd^3e^2+8\,\ldots}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="fricas")
```

output
```
-1/24*((6*C*d^3*f^2*x^3 + 24*B*d^3*e^2 + 16*B*d*f^2 + 16*(3*A*d^3 + 2*C*d)
*e*f + 8*(2*C*d^3*e*f + B*d^3*f^2)*x^2 + 3*(4*C*d^3*e^2 + 8*B*d^3*e*f + (4
*A*d^3 + 3*C*d)*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*B*d^2*e*f + 4*
(2*A*d^4 + C*d^2)*e^2 + (4*A*d^2 + 3*C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d
*x + 1) - 1)/(d*x)))/d^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.01

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}Cf^2x^3}{4\,d^2}+\frac{Ae^2\arcsin{(dx)}}{d}-\frac{\sqrt{-d^2x^2+1}Be^2}{d^2}-\frac{2\sqrt{-d^2x^2+1}Aef}{d^2}-\frac{\sqrt{-d^2x^2+1}(2\,Cef+Bf^2)x^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}(Ce^2+2\,Bef+Af^2)x}{2\,d^2}-\frac{3\sqrt{-d^2x^2+1}Cf^2x}{8\,d^4}+\frac{(Ce^2+2\,Bef+Af^2)\arcsin{(dx)}}{2\,d^3}+\frac{3\,Cf^2\arcsin{(dx)}}{8\,d^5}-\frac{2\sqrt{-d^2x^2+1}(2\,Cef+Bf^2)}{3\,d^4}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="maxima")
```

output
```
-1/4*sqrt(-d^2*x^2 + 1)*C*f^2*x^3/d^2 + A*e^2*arcsin(d*x)/d - sqrt(-d^2*x^
2 + 1)*B*e^2/d^2 - 2*sqrt(-d^2*x^2 + 1)*A*e*f/d^2 - 1/3*sqrt(-d^2*x^2 + 1)
*(2*C*e*f + B*f^2)*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e^2 + 2*B*e*f + A*f
^2)*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/2*(C*e^2 + 2*B*e*f + A*
f^2)*arcsin(d*x)/d^3 + 3/8*C*f^2*arcsin(d*x)/d^5 - 2/3*sqrt(-d^2*x^2 + 1)*
(2*C*e*f + B*f^2)/d^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.02

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx =$$

$$-\frac{(24\,Bd^3e^2+48\,Ad^3ef-12\,Cd^2e^2-24\,Bd^2ef-12\,Ad^2f^2+48\,Cdef+24\,Bdf^2-15\,Cf^2+(12\,Cd^2e}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorith
m="giac")
```

output
```
-1/24*((24*B*d^3*e^2 + 48*A*d^3*e*f - 12*C*d^2*e^2 - 24*B*d^2*e*f - 12*A*d
^2*f^2 + 48*C*d*e*f + 24*B*d*f^2 - 15*C*f^2 + (12*C*d^2*e^2 + 24*B*d^2*e*f
 + 12*A*d^2*f^2 - 32*C*d*e*f - 16*B*d*f^2 + 27*C*f^2 + 2*(8*C*d*e*f + 3*(d
*x + 1)*C*f^2 + 4*B*d*f^2 - 9*C*f^2)*(d*x + 1))*(d*x + 1))*sqrt(d*x + 1)*s
qrt(-d*x + 1) - 6*(8*A*d^4*e^2 + 4*C*d^2*e^2 + 8*B*d^2*e*f + 4*A*d^2*f^2 +
 3*C*f^2)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^5
```

Mupad [B] (verification not implemented)

Time = 26.44 (sec) , antiderivative size = 1732, normalized size of antiderivative = 7.60

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^2*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((14*A*f^2*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (2*A*f^2*(
(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*A*f^2*((1 - d*x)^(1/2) -
 1)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*A*f^2*((1 - d*x)^(1/2) - 1)^7)/((d*x +
 1)^(1/2) - 1)^7 + (16*A*d*e*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) -
 1)^2 + (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16
*A*d*e*f*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6)/(d^3 + (4*d^3*(
(1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((1 - d*x)^(1/2)
- 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (4*d^3*((1 - d*x)^(1/2) - 1)^6)/((d*x +
1)^(1/2) - 1)^6 + (d^3*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8) -
 ((((1 - d*x)^(1/2) - 1)^4*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2) - 1
)^4 + (((1 - d*x)^(1/2) - 1)^8*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2)
 - 1)^8 - (((1 - d*x)^(1/2) - 1)^6*((128*B*f^2)/3 - 48*B*d^2*e^2))/((d*x +
 1)^(1/2) - 1)^6 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 +
 (20*B*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (24*B*d*e*
f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (24*B*d*e*f*((1 - d*x
)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (20*B*d*e*f*((1 - d*x)^(1/2) - 1
)^9)/((d*x + 1)^(1/2) - 1)^9 + (4*B*d*e*f*((1 - d*x)^(1/2) - 1)^11)/((d*x
+ 1)^(1/2) - 1)^11 - (4*B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1))/(d^4 + (6*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (1...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.57

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-48 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) a\,d^4e^2 - 24 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) a\,d^2f^2 - 48 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) b\,d^2ef - 24 asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right) c\,d^2e^2}{}$$

input
```
int((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 48*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**4*e**2 - 24*asin(sqrt( - d*x +
1)/sqrt(2))*a*d**2*f**2 - 48*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**2*e*f - 2
4*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**2*e**2 - 18*asin(sqrt( - d*x + 1)/sq
rt(2))*c*f**2 - 48*sqrt(d*x + 1)*sqrt( - d*x + 1)*a*d**3*e*f - 12*sqrt(d*x
 + 1)*sqrt( - d*x + 1)*a*d**3*f**2*x - 24*sqrt(d*x + 1)*sqrt( - d*x + 1)*b
*d**3*e**2 - 24*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**3*e*f*x - 8*sqrt(d*x +
 1)*sqrt( - d*x + 1)*b*d**3*f**2*x**2 - 16*sqrt(d*x + 1)*sqrt( - d*x + 1)*
b*d*f**2 - 12*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**3*e**2*x - 16*sqrt(d*x +
 1)*sqrt( - d*x + 1)*c*d**3*e*f*x**2 - 6*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*
d**3*f**2*x**3 - 32*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d*e*f - 9*sqrt(d*x +
1)*sqrt( - d*x + 1)*c*d*f**2*x)/(24*d**5)
```

3.40 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 127

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{\left(2\left(3Bd^2e-\frac{Cd^2e^2}{f}+2Cf+3Ad^2f\right)-d^2(Ce-3Bf)x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{(Ce+2Ad^2e+Bf)\arcsin(dx)}{2d^3}$$

output
```
-1/3*C*(f*x+e)^2*(-d^2*x^2+1)^(1/2)/d^2/f-1/6*(6*B*d^2*e-2*C*d^2*e^2/f+4*C
*f+6*A*d^2*f-d^2*(-3*B*f+C*e)*x)*(-d^2*x^2+1)^(1/2)/d^4+1/2*(2*A*d^2*e+B*f
+C*e)*arcsin(d*x)/d^3
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{1-d^2x^2}(-6Bd^2e-4Cf-6Ad^2f-3Cd^2ex-3Bd^2fx-2Cd^2fx^2)}{6d^4} + \frac{(Ce+2Ad^2e+Bf)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
(Sqrt[1 - d^2*x^2]*(-6*B*d^2*e - 4*C*f - 6*A*d^2*f - 3*C*d^2*e*x - 3*B*d^2
*f*x - 2*C*d^2*f*x^2))/(6*d^4) + ((C*e + 2*A*d^2*e + B*f)*ArcTan[(d*x)/(-1
 + Sqrt[1 - d^2*x^2])])/d^3
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2112, 2185, 25, 27, 676, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{dx+1}}\,dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\int -\frac{f(e+fx)\left(\left(3Ad^2+2C\right)f-d^2(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2f^2} - \frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{f(e+fx)\left((3Ad^2+2C)f-d^2(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2f^2}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(e+fx)\left((3Ad^2+2C)f-d^2(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{3}{2}f\left(2Ad^2e+Bf+Ce\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx-\sqrt{1-d^2x^2}\left(3f(Af+Be)-C\left(e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{2}fx\sqrt{1-d^2x^2}(Ce-3Bf)}{3d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

$$\Big\downarrow \; 223$$

$$\frac{\frac{3f\arcsin(dx)\left(2Ad^2e+Bf+Ce\right)}{2d}-\sqrt{1-d^2x^2}\left(3f(Af+Be)-C\left(e^2-\frac{2f^2}{d^2}\right)\right)+\frac{1}{2}fx\sqrt{1-d^2x^2}(Ce-3Bf)}{3d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^2}{3d^2f}$$

input
```
Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/3*(C*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(d^2*f) + (-((3*f*(B*e + A*f) - C*(
e^2 - (2*f^2)/d^2))*Sqrt[1 - d^2*x^2]) + (f*(C*e - 3*B*f)*x*Sqrt[1 - d^2*x
^2])/2 + (3*f*(C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(2*d))/(3*d^2*f)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.36

method	result
risch	$\frac{\left(2C\,d^2 f\,x^2+3B\,d^2 fx+3C\,d^2 ex+6A\,d^2 f+6B\,d^2 e+4Cf\right)\sqrt{xd+1}\,(xd-1)\sqrt{(-xd+1)(xd+1)}}{6d^4\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}+\frac{\left(2A\,d^2 e+Bf+Ce\right)\arctan\left(\frac{\sqrt{d^2}}{\sqrt{-d^2x}}\right.}{2d^2\sqrt{d^2}\,\sqrt{-xd+1}}$
default	$-\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\left(2C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2 f\,x^2+3B\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2 fx+3C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d^2 ex+6A\sqrt{-d^2x^2+1}\,\operatorname{cs}\right.}{}$

input
```
int((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/6*(2*C*d^2*f*x^2+3*B*d^2*f*x+3*C*d^2*e*x+6*A*d^2*f+6*B*d^2*e+4*C*f)*(d*x
+1)^(1/2)*(d*x-1)/d^4/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^(1/2)/(-
d*x+1)^(1/2)+1/2*(2*A*d^2*e+B*f+C*e)/d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/
(-d^2*x^2+1)^(1/2))*((-d*x+1)*(d*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(2\,Cd^2fx^2+6\,Bd^2e+2\,(3\,Ad^2+2\,C)f+3\,(Cd^2e+Bd^2f)x)\sqrt{dx+1}\sqrt{-dx+1}+6\,(Bdf+(2\,Ad^3}{6\,d^4}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"fricas")
```

output
```
-1/6*((2*C*d^2*f*x^2 + 6*B*d^2*e + 2*(3*A*d^2 + 2*C)*f + 3*(C*d^2*e + B*d^
2*f)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(B*d*f + (2*A*d^3 + C*d)*e)*arcta
n((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\sqrt{-d^2x^2+1}Cfx^2}{3\,d^2}+\frac{Ae\arcsin{(dx)}}{d} - \frac{\sqrt{-d^2x^2+1}Be}{d^2}-\frac{\sqrt{-d^2x^2+1}Af}{d^2} - \frac{\sqrt{-d^2x^2+1}(Ce+Bf)x}{2\,d^2} + \frac{(Ce+Bf)\arcsin{(dx)}}{2\,d^3}-\frac{2\sqrt{-d^2x^2+1}Cf}{3\,d^4}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"maxima")
```

output
```
-1/3*sqrt(-d^2*x^2 + 1)*C*f*x^2/d^2 + A*e*arcsin(d*x)/d - sqrt(-d^2*x^2 +
1)*B*e/d^2 - sqrt(-d^2*x^2 + 1)*A*f/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e + B*
f)*x/d^2 + 1/2*(C*e + B*f)*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*C*f/d^
4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(6\,Bd^2e+6\,Ad^2f-3\,Cde-3\,Bdf+(3\,Cde+2\,(dx+1)Cf+3\,Bdf-4\,Cf)(dx+1)+6\,Cf)\sqrt{dx-}}{6\,d^4}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm=
"giac")
```

output
```
-1/6*((6*B*d^2*e + 6*A*d^2*f - 3*C*d*e - 3*B*d*f + (3*C*d*e + 2*(d*x + 1)*
C*f + 3*B*d*f - 4*C*f)*(d*x + 1) + 6*C*f)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6
*(2*A*d^3*e + C*d*e + B*d*f)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^4
```

Mupad [B] (verification not implemented)

Time = 10.19 (sec) , antiderivative size = 492, normalized size of antiderivative = 3.87

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{\frac{2Bf(\sqrt{1-dx}-1)}{\sqrt{dx+1}-1} - \frac{14Bf(\sqrt{1-dx}-1)^3}{(\sqrt{dx+1}-1)^3} + \frac{14Bf(\sqrt{1-dx}-1)^5}{(\sqrt{dx+1}-1)^5} - \frac{2Bf(\sqrt{1-dx}-1)^7}{(\sqrt{dx+1}-1)^7}}{d^3\left(\frac{(\sqrt{1-dx}-1)^2}{(\sqrt{dx+1}-1)^2}+1\right)^4}$$
$$- \frac{\sqrt{1-dx}\left(\frac{2Cf}{3d^4}+\frac{2Cfx}{3d^3}+\frac{Cfx^3}{3d}+\frac{Cfx^2}{3d^2}\right)}{\sqrt{dx+1}}$$
$$+ \frac{\frac{2Ce(\sqrt{1-dx}-1)}{\sqrt{dx+1}-1} - \frac{14Ce(\sqrt{1-dx}-1)^3}{(\sqrt{dx+1}-1)^3} + \frac{14Ce(\sqrt{1-dx}-1)^5}{(\sqrt{dx+1}-1)^5} - \frac{2Ce(\sqrt{1-dx}-1)^7}{(\sqrt{dx+1}-1)^7}}{d^3\left(\frac{(\sqrt{1-dx}-1)^2}{(\sqrt{dx+1}-1)^2}+1\right)^4}$$
$$- \frac{\left(\frac{Af}{d^2}+\frac{Afx}{d}\right)\sqrt{1-dx}}{\sqrt{dx+1}} - \frac{\left(\frac{Be}{d^2}+\frac{Bex}{d}\right)\sqrt{1-dx}}{\sqrt{dx+1}}$$
$$- \frac{4Ae\,\mathrm{atan}\left(\frac{d(\sqrt{1-dx}-1)}{(\sqrt{dx+1}-1)\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2Bf\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{d^3} - \frac{2Ce\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{d^3}$$

input
```
int(((e + f*x)*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((2*B*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*B*f*((1 - d*x)^
(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*B*f*((1 - d*x)^(1/2) - 1)^5)/(
(d*x + 1)^(1/2) - 1)^5 - (2*B*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2)
- 1)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - ((
1 - d*x)^(1/2)*((2*C*f)/(3*d^4) + (2*C*f*x)/(3*d^3) + (C*f*x^3)/(3*d) + (C
*f*x^2)/(3*d^2)))/(d*x + 1)^(1/2) + ((2*C*e*((1 - d*x)^(1/2) - 1))/((d*x +
 1)^(1/2) - 1) - (14*C*e*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3
+ (14*C*e*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (2*C*e*((1 -
d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/
((d*x + 1)^(1/2) - 1)^2 + 1)^4) - (((A*f)/d^2 + (A*f*x)/d)*(1 - d*x)^(1/2)
)/(d*x + 1)^(1/2) - (((B*e)/d^2 + (B*e*x)/d)*(1 - d*x)^(1/2))/(d*x + 1)^(1
/2) - (4*A*e*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(
1/2))))/(d^2)^(1/2) - (2*B*f*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) -
 1)))/d^3 - (2*C*e*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.45

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$$
$$= \frac{-12asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)a\,d^3e-6asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)bdf-6asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)cde-6\sqrt{dx+1}\,\sqrt{-dx+1}\,a\,d^2f-6\sqrt{}}{}$$

input
```
int((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 12*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**3*e - 6*asin(sqrt( - d*x + 1)/s
qrt(2))*b*d*f - 6*asin(sqrt( - d*x + 1)/sqrt(2))*c*d*e - 6*sqrt(d*x + 1)*s
qrt( - d*x + 1)*a*d**2*f - 6*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d**2*e - 3*s
qrt(d*x + 1)*sqrt( - d*x + 1)*b*d**2*f*x - 3*sqrt(d*x + 1)*sqrt( - d*x + 1
)*c*d**2*e*x - 2*sqrt(d*x + 1)*sqrt( - d*x + 1)*c*d**2*f*x**2 - 4*sqrt(d*x
 + 1)*sqrt( - d*x + 1)*c*f)/(6*d**4)
```

3.41 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{B\sqrt{1-d^2x^2}}{d^2} - \frac{Cx\sqrt{1-d^2x^2}}{2d^2} + \frac{(C+2Ad^2)\arcsin(dx)}{2d^3}$$

output
```
-B*(-d^2*x^2+1)^(1/2)/d^2-1/2*C*x*(-d^2*x^2+1)^(1/2)/d^2+1/2*(2*A*d^2+C)*a
rcsin(d*x)/d^3
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{(-2B-Cx)\sqrt{1-d^2x^2}}{2d^2} + \frac{(C+2Ad^2)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
((-2*B - C*x)*Sqrt[1 - d^2*x^2])/(2*d^2) + ((C + 2*A*d^2)*ArcTan[(d*x)/(-1
 + Sqrt[1 - d^2*x^2])])/d^3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1188, 2346, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}}dx$$

$$\Big\downarrow \text{1188}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\int -\frac{2Ad^2+2Bxd^2+C}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2Ad^2+2Bxd^2+C}{\sqrt{1-d^2x^2}}dx}{2d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{455}$$

$$\frac{\left(2Ad^2+C\right)\int \frac{1}{\sqrt{1-d^2x^2}}dx-2B\sqrt{1-d^2x^2}}{2d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{(2Ad^2+C)\arcsin(dx)}{d}-2B\sqrt{1-d^2x^2}}{2d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

output
```
-1/2*(C*x*Sqrt[1 - d^2*x^2])/d^2 + (-2*B*Sqrt[1 - d^2*x^2] + ((C + 2*A*d^2
)*ArcSin[d*x])/d)/(2*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1188
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2
)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m, n] && EqQ[e*f
 + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

method	result
default	$\frac{\sqrt{-xd+1}\,\sqrt{xd+1}\,\left(2A\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)d^2-C\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)dx-2B\sqrt{-d^2x^2+1}\,\operatorname{csgn}(d)d+C\arctan\left(\frac{\operatorname{csgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)\right)\operatorname{csgn}}{2d^3\sqrt{-d^2x^2+1}}$
risch	$\frac{(Cx+2B)\sqrt{xd+1}\,(xd-1)\sqrt{(-xd+1)(xd+1)}}{2d^2\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}+\frac{(2d^2A+C)\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+1}}\right)\sqrt{(-xd+1)(xd+1)}}{2d^2\sqrt{d^2}\,\sqrt{-xd+1}\,\sqrt{xd+1}}$

input
```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(2*A*arctan(csgn(d)*d*x/(-d^2*x^2+1)^
(1/2))*d^2-C*(-d^2*x^2+1)^(1/2)*csgn(d)*d*x-2*B*(-d^2*x^2+1)^(1/2)*csgn(d)
*d+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2)))/(-d^2*x^2+1)^(1/2)*csgn(d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{(Cdx+2\,Bd)\sqrt{dx+1}\sqrt{-dx+1}+2\,(2\,Ad^2+C)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/2*((C*d*x + 2*B*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*A*d^2 + C)*arcta
n((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{A\arcsin(dx)}{d} - \frac{\sqrt{-d^2x^2+1}Cx}{2\,d^2} - \frac{\sqrt{-d^2x^2+1}B}{d^2} + \frac{C\arcsin(dx)}{2\,d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

output
```
A*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*C*x/d^2 - sqrt(-d^2*x^2 + 1)*B/d^
2 + 1/2*C*arcsin(d*x)/d^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{((dx+1)C+2\,Bd-C)\sqrt{dx+1}\sqrt{-dx+1} - 2\,(2\,Ad^2+C)\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{2\,d^3}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(((d*x + 1)*C + 2*B*d - C)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(2*A*d^2
+ C)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))/d^3
```

Mupad [B] (verification not implemented)

Time = 7.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.68

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = -\frac{\frac{14\,C\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,C\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}-\frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4} - \frac{4\,A\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{2\,C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3} - \frac{\sqrt{1-d\,x}\,\left(\frac{B}{d^2}+\frac{B\,x}{d}\right)}{\sqrt{d\,x+1}}$$

input
```
int((A + B*x + C*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
- ((14*C*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*C*((1 - d*
x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*C*((1 - d*x)^(1/2) - 1)^7)/(
(d*x + 1)^(1/2) - 1)^7 - (2*C*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)
)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - (4*A*ata
n((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1
/2) - (2*C*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((1 -
d*x)^(1/2)*(B/d^2 + (B*x)/d))/(d*x + 1)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \frac{-4asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)a\,d^2-2asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)c-2\sqrt{dx+1}\,\sqrt{-dx+1}\,bd-\sqrt{dx+1}\,\sqrt{-dx+1}\,cdx}{2d^3}$$

input
```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

output
```
( - 4*asin(sqrt( - d*x + 1)/sqrt(2))*a*d**2 - 2*asin(sqrt( - d*x + 1)/sqrt
(2))*c - 2*sqrt(d*x + 1)*sqrt( - d*x + 1)*b*d - sqrt(d*x + 1)*sqrt( - d*x
+ 1)*c*d*x)/(2*d**3)
```

3.42 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 122

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = -\frac{C\sqrt{1-d^2x^2}}{d^2f}-\frac{(Ce-Bf)\arcsin(dx)}{df^2}+\frac{(Ce^2-Bef+Af^2)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}$$

output
```
-C*(-d^2*x^2+1)^(1/2)/d^2/f-(-B*f+C*e)*arcsin(d*x)/d/f^2+(A*f^2-B*e*f+C*e^
2)*arctan((d^2*e*x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2
-f^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \frac{-\frac{Cf\sqrt{1-d^2x^2}}{d^2}+\frac{2(-Ce+Bf)\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d}-\frac{2\sqrt{d^2e^2-f^2}(Ce^2+f(-Be+Af))\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(de-f)(de+f)}}{f^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

output
```
(-((C*f*Sqrt[1 - d^2*x^2])/d^2) + (2*(-(C*e) + B*f)*ArcTan[(d*x)/(-1 + Sqr
t[1 - d^2*x^2])])/d - (2*Sqrt[d^2*e^2 - f^2]*(C*e^2 + f*(-(B*e) + A*f))*Ar
cTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)*
(d*e + f)))/f^2
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {2112, 2185, 25, 27, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)}dx$$

$$\Big\downarrow \text{ 2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)}dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\int -\frac{d^2f(Af-(Ce-Bf)x)}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2f^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d^2f(Af-(Ce-Bf)x)}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2f^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{Af-(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}dx}{f}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{223}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{f}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{488}$$

$$\frac{-\frac{(Af^2-Bef+Ce^2)\int \frac{1}{-d^2e^2+f^2-\frac{(exd^2+f)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{f}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{(Af^2-Bef+Ce^2)\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f\sqrt{d^2e^2-f^2}}-\frac{\arcsin(dx)(Ce-Bf)}{df}}{f}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

output
```
-((C*Sqrt[1 - d^2*x^2])/(d^2*f)) + (-(((C*e - B*f)*ArcSin[d*x])/(d*f)) + (
(C*e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 -
 d^2*x^2])])/(f*Sqrt[d^2*e^2 - f^2]))/f
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(114) = 228.

Time = 1.13 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.37

method	result
risch	$\frac{C\sqrt{xd+1}\,(xd-1)\sqrt{(-xd+1)(xd+1)}}{f\,d^{2}\sqrt{-(xd+1)(xd-1)}\,\sqrt{-xd+1}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{d^{2}}\,x}{\sqrt{-d^{2}x^{2}+1}}\right)}{f\sqrt{d^{2}}}-\frac{\left(A\,f^{2}-Bef+C\,e^{2}\right)\ln\left(\frac{-\frac{2\left(d^{2}e^{2}-f^{2}\right)}{f^{2}}+\frac{2d^{2}e\left(x+\frac{e}{f}\right)}{f}+2\sqrt{-\frac{}{}}}{}\right.}{f^{2}\sqrt{-\frac{d^{2}e^{2}-}{f^{2}}}}\right.}{f\sqrt{-xd+1}\,\sqrt{xd+1}}$
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^{2}xe+2\sqrt{-d^{2}x^{2}+1}\,\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}\,f+2f}{fx+e}\right)d^{2}f^{2}+B\,\mathrm{csgn}(d)\ln\left(\frac{2d^{2}xe+2\sqrt{-d^{2}x^{2}+1}\,\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}\,f+2f}{fx+e}\right)d^{2}ef-C\,\mathrm{cs}\right.}{}$

input

```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e),x,method=_RETURNVER
BOSE)
```

output

```
C/f/d^2*(d*x+1)^(1/2)*(d*x-1)/(-(d*x+1)*(d*x-1))^(1/2)*((-d*x+1)*(d*x+1))^
(1/2)/(-d*x+1)^(1/2)+1/f*((B*f-C*e)/f/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d
^2*x^2+1)^(1/2))-(A*f^2-B*e*f+C*e^2)/f^2/(-(d^2*e^2-f^2)/f^2)^(1/2)*ln((-2
*(d^2*e^2-f^2)/f^2+2/f*d^2*e*(x+e/f)+2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*(x
+e/f)^2+2/f*d^2*e*(x+e/f)-(d^2*e^2-f^2)/f^2)^(1/2))/(x+e/f)))*((-d*x+1)*(d
*x+1))^(1/2)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(114) = 228$.

Time = 3.66 (sec) , antiderivative size = 493, normalized size of antiderivative = 4.04

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)}\,dx$$
$$= \left[-\frac{(Cd^2e^2 - Bd^2ef + Ad^2f^2)\sqrt{-d^2e^2 + f^2}\log\left(\frac{d^2efx + f^2 - \sqrt{-d^2e^2 + f^2}(d^2ex + f) - \left(\sqrt{-d^2e^2 + f^2}\sqrt{-dx + 1}f + (d^2e^2 - f^2\right.}{fx + e}\right.}{} \right.$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e),x, algorithm=
"fricas")
```

output
```
[-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-
d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (
C*d^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e
^2*f - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x
)))/(d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d
^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e
- sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f
^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2
+ B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2
- d^2*f^4)]
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)}\,dx = \int \frac{A + Bx + Cx^2}{(e + fx)\sqrt{-dx + 1}\sqrt{dx + 1}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2)/(f*x+e),x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)*sqrt(-d*x + 1)*sqrt(d*x + 1)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e),x, algorithm=
"giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 17.47 (sec) , antiderivative size = 5803, normalized size of antiderivative = 47.57

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2
) - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e
^8*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
- 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*
e^8*f^2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))
/(((d*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 10066
3296*B^5*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d
*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*
B^5*d^2*e^3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d
*x + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*
B^5*d^2*e^3*f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1
/2) - 1)^2*(d^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
+ (d^2*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1
i - d^2*e^2*1i - (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2
+ (d^2*e^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*
e)^(1/2)*(f - d*e)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f -
 d*e)^(1/2))/((d*x + 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d
*e)^(1/2)*(f - d*e)^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.39

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$$

$$= \frac{-2asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)b\,d^3e^2f+2asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)bd\,f^3+2asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)c\,d^3e^3-2asin\left(\frac{\sqrt{-dx+1}}{\sqrt{2}}\right)cde\,f^2-2\sqrt{}}{}$$

input
```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e),x)
```

output
```
( - 2*asin(sqrt( - d*x + 1)/sqrt(2))*b*d**3*e**2*f + 2*asin(sqrt( - d*x +
1)/sqrt(2))*b*d*f**3 + 2*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**3*e**3 - 2*as
in(sqrt( - d*x + 1)/sqrt(2))*c*d*e*f**2 - 2*sqrt(d*e + f)*sqrt(d*e - f)*at
an((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))
/sqrt(d*e - f))*a*d**2*f**2 + 2*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e
 + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e -
f))*b*d**2*e*f - 2*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asi
n(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*c*d**2*e*
*2 - 2*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*
x + 1)/sqrt(2))/2) + sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**2*f**2 + 2*sqrt(
d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(
2))/2) + sqrt(f)*sqrt(2))/sqrt(d*e - f))*b*d**2*e*f - 2*sqrt(d*e + f)*sqrt
(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) + sqrt
(f)*sqrt(2))/sqrt(d*e - f))*c*d**2*e**2 - sqrt(d*x + 1)*sqrt( - d*x + 1)*c
*d**2*e**2*f + sqrt(d*x + 1)*sqrt( - d*x + 1)*c*f**3)/(d**2*f**2*(d**2*e**
2 - f**2))
```

3.43 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 163

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{f(d^2e^2-f^2)(e+fx)}+\frac{C\arcsin(dx)}{df^2} - \frac{(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{f^2(d^2e^2-f^2)^{3/2}}$$

output
```
(A*f^2-B*e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)+C*arcsin(d*
x)/d/f^2-(-A*d^2*e*f^2+C*d^2*e^3+B*f^3-2*C*e*f^2)*arctan((d^2*e*x+f)/(d^2*
e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/f^2/(d^2*e^2-f^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \frac{\frac{f(Ce^2+f(-Be+Af))\sqrt{1-d^2x^2}}{(de-f)(de+f)(e+fx)}+\frac{2C\arctan\left(\frac{dx}{-1+\sqrt{1-d^2x^2}}\right)}{d}+\frac{2\sqrt{d^2e^2-f^2}(Cd^2e^3-2Cef^2-Ad^2ef^2+Bf^3)\arctan\left(\frac{\sqrt{d^2e^2-f^2}x}{e+fx-e\sqrt{1-d^2x^2}}\right)}{(-de+f)^2(de+f)^2}}{f^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

output
```
((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((d*e - f)*(d*e + f)*(e
+ f*x)) + (2*C*ArcTan[(d*x)/(-1 + Sqrt[1 - d^2*x^2])])/d + (2*Sqrt[d^2*e^2
 - f^2]*(C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*ArcTan[(Sqrt[d^2*e^2
 - f^2]*x)/(e + f*x - e*Sqrt[1 - d^2*x^2])])/((-(d*e) + f)^2*(d*e + f)^2))
/f^2
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {2112, 2182, 719, 223, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)^2}\,dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{\int \frac{Aed^2+Ce-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx+\frac{C(de-f)(de+f)\int \frac{1}{\sqrt{1-d^2x^2}}dx}{f^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{223}$$

$$\frac{\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx+\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}-\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)\int \frac{1}{-d^2e^2+f^2-\frac{\left(exd^2+f\right)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(Ad^2e-Bf-\frac{Cd^2e^3}{f^2}+2Ce\right)}{\sqrt{d^2e^2-f^2}}+\frac{C\arcsin(dx)(de-f)(de+f)}{df^2}}{d^2e^2-f^2}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

output
```
((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x))
+ ((C*(d*e - f)*(d*e + f)*ArcSin[d*x])/(d*f^2) + ((2*C*e + A*d^2*e - (C*d^
2*e^3)/f^2 - B*f)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x
^2])])/Sqrt[d^2*e^2 - f^2])/(d^2*e^2 - f^2)
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.16 (sec) , antiderivative size = 899, normalized size of antiderivative = 5.52

method	result
default	$\frac{\left(-A\,\mathrm{csgn}(d)\ln\left(\frac{2d^2xe+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e\,f^3x+C\,\mathrm{csgn}(d)\ln\left(\frac{2d^2xe+2\sqrt{-d^2x^2+1}\,\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)d^3e^3fx-}{}$

input

```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^2,x,method=_RETURNV
ERBOSE)
```

output

```
(-A*csgn(d)*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*d^3*e*f^3*x+C*csgn(d)*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*
e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^3*f*x-A*csgn(d)*ln(2*(d^2*x*e+(-d^
2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn
(d)*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+
e))*d^3*e^4+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2*e^2*f^2*x*(-(d^2*
e^2-f^2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2
)^(1/2)+B*csgn(d)*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1
/2)*f+f)/(f*x+e))*d*f^4*x-B*csgn(d)*d*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-
f^2)/f^2)^(1/2)-2*C*csgn(d)*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3*x+C*csgn(d)*d*e^2*f^2*(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*d^2
*e^3*f*(-(d^2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*
x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-
C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*f^4*x*(-(d^2*e^2-f^2)/f^2)^(1/2)-
C*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))
*csgn(d)*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e+f)/(d*e-f)/(
f*x+e)/d/(-(d^2*e^2-f^2)/f^2)^(1/2)/f^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(155) = 310$.

Time = 14.65 (sec) , antiderivative size = 1025, normalized size of antiderivative = 6.29

$$\int \frac{A + Bx + Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^2,x, algorith
m="fricas")
```

output
```
[(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^
3*f^3 + (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f
+ B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*
f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqr
t(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e))
+ (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e
^3*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*
d*e*f^5 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*
f^2 + C*e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqr
t(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^
2*f^6 + (d^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e
^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 +
B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3
+ 2*C*d)*e^2*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt
(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^
2)*x)) + (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 -
 C*d)*e^3*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f
^3 + B*d*e*f^5 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d
^2*e^4*f^2 + C*e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arct
an((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx=\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2)/(f*x+e)**2,x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)**2*sqrt(-d*x + 1)*sqrt(d*x + 1)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^2,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^2,x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 34.20 (sec) , antiderivative size = 10198, normalized size of antiderivative = 62.56

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
(A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^
(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((
d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((
d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2)
 - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2
*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 -
d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (
A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^
2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2
+ (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1375, normalized size of antiderivative = 8.44

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^2,x)
```

output
```
( - 2*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**4*e**5 - 2*asin(sqrt( - d*x + 1)
/sqrt(2))*c*d**4*e**4*f*x + 4*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**2*e**3*f
**2 + 4*asin(sqrt( - d*x + 1)/sqrt(2))*c*d**2*e**2*f**3*x - 2*asin(sqrt( -
 d*x + 1)/sqrt(2))*c*e*f**4 - 2*asin(sqrt( - d*x + 1)/sqrt(2))*c*f**5*x -
2*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1
)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**3*e**2*f**2 - 2*sqrt(
d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(
2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**3*e*f**3*x + 2*sqrt(d*e + f)
*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) -
 sqrt(f)*sqrt(2))/sqrt(d*e - f))*b*d*e*f**3 + 2*sqrt(d*e + f)*sqrt(d*e - f
)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt
(2))/sqrt(d*e - f))*b*d*f**4*x + 2*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(
d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e
 - f))*c*d**3*e**4 + 2*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan
(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*c*d**
3*e**3*f*x - 4*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sq
rt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*c*d*e**2*f**2
 - 4*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x
+ 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*c*d*e*f**3*x - 2*sqrt(d
*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqr...
```

3.44 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 248

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx &= \frac{(Ce^2-Bef+Af^2)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)(e+fx)^2} \\ &\quad - \frac{(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2Bf^3)\sqrt{1-d^2x^2}}{2f(d^2e^2-f^2)^2(e+fx)} \\ &\quad + \frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\arctan\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\sqrt{1-d^2x^2}}\right)}{2(d^2e^2-f^2)^{5/2}}\end{aligned}$$

output
```
1/2*(A*f^2-B*e*f+C*e^2)*(-d^2*x^2+1)^(1/2)/f/(d^2*e^2-f^2)/(f*x+e)^2-1/2*(
-3*A*d^2*e*f^2+B*d^2*e^2*f+C*d^2*e^3+2*B*f^3-4*C*e*f^2)*(-d^2*x^2+1)^(1/2)
/f/(d^2*e^2-f^2)^2/(f*x+e)+1/2*(C*(d^2*e^2+2*f^2)-d^2*(3*B*e*f-A*(2*d^2*e^
2+f^2)))*arctan((d^2*e*x+f)/(d^2*e^2-f^2)^(1/2)/(-d^2*x^2+1)^(1/2))/(d^2*e
^2-f^2)^(5/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx =$$

$$-\frac{\frac{(de-f)(de+f)\sqrt{1-d^2x^2}(Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)-Ad^2ef(4e+3fx)+Ce(-3ef+d^2e^2x-4f^2x))}{(e+fx)^2}+2\sqrt{d^2e^2-f^2}(C(d^2e}{2(de-f)^3(de+f)^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

output
```
-1/2*(((d*e - f)*(d*e + f)*Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x
) + B*f^2*(e + 2*f*x) - A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x
- 4*f^2*x)))/(e + f*x)^2 + 2*Sqrt[d^2*e^2 - f^2]*(C*(d^2*e^2 + 2*f^2) + d^
2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*ArcTan[(Sqrt[d^2*e^2 - f^2]*x)/(e + f*
x - e*Sqrt[1 - d^2*x^2])])/((d*e - f)^3*(d*e + f)^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2112, 2182, 679, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{dx+1}(e+fx)^3}\,dx$$

$$\Big\downarrow \text{2112}$$

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}(e+fx)^3}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{\int \frac{2(Aed^2+Ce-Bf)+\left(Bed^2-Afd^2+\frac{Ce^2d^2}{f}-2Cf\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}dx}{2\left(d^2e^2-f^2\right)}+\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}$$

$\downarrow$ 679

$$\frac{\frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}}dx}{d^2e^2-f^2}-\frac{\sqrt{1-d^2x^2}(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{f(d^2e^2-f^2)(e+fx)}}{2(d^2e^2-f^2)}+\frac{\sqrt{1-d^2x^2}(Af^2-Bef+Ce^2)}{2f(d^2e^2-f^2)(e+fx)^2}$$

$\downarrow$ 488

$$\frac{-\frac{(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))\int \frac{1}{-d^2e^2+f^2-\frac{(exd^2+f)^2}{1-d^2x^2}}d\frac{exd^2+f}{\sqrt{1-d^2x^2}}}{d^2e^2-f^2}-\frac{\sqrt{1-d^2x^2}(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{f(d^2e^2-f^2)(e+fx)}}{2(d^2e^2-f^2)}+\frac{\sqrt{1-d^2x^2}(Af^2-Bef+Ce^2)}{2f(d^2e^2-f^2)(e+fx)^2}$$

$\downarrow$ 217

$$\frac{\frac{\arctan\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)(C(d^2e^2+2f^2)-d^2(3Bef-A(2d^2e^2+f^2)))}{(d^2e^2-f^2)^{3/2}}-\frac{\sqrt{1-d^2x^2}(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2)}{f(d^2e^2-f^2)(e+fx)}}{2(d^2e^2-f^2)}+\frac{\sqrt{1-d^2x^2}(Af^2-Bef+Ce^2)}{2f(d^2e^2-f^2)(e+fx)^2}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

output
```
((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)
^2) + (-((((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*
Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x))) + ((C*(d^2*e^2 + 2*f^2)
- d^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2
- f^2]*Sqrt[1 - d^2*x^2])])/(d^2*e^2 - f^2)^(3/2))/(2*(d^2*e^2 - f^2))
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2112
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.11 (sec) , antiderivative size = 1449, normalized size of antiderivative = 5.84

method	result	size
default	Expression too large to display	1449

input
```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^3,x,method=_RETURNV
ERBOSE)
```

output
```
-1/2*(B*d^2*e^2*f^2*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*B*f^
4*x*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+B*e*f^3*(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-3*C*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)+A*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2
)*f+f)/(f*x+e))*d^2*f^4*x^2+A*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-
f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2-3*B*ln(2*(d^2*x*e+(-d^2*x^2+1)^(
1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f+4*C*ln(2*(d^2*x*e+
(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e*f^3*x-3*B*ln
(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^
2*e*f^3*x^2+C*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*
f+f)/(f*x+e))*d^2*e^2*f^2*x^2+2*A*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*
e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f^3*x-6*B*ln(2*(d^2*x*e+(-d^2*x^2+
1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2*f^2*x-4*A*d^2*e^
2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)+2*B*d^2*e^3*f*(-d^2*x^
2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-4*C*e*f^3*x*(-d^2*x^2+1)^(1/2)*(-(d^
2*e^2-f^2)/f^2)^(1/2)+2*C*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)
/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f*x+2*A*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)
*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^4*e^2*f^2*x^2+4*A*ln(2*(d^2*x*
e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^4*e^3*f*x+
2*A*ln(2*(d^2*x*e+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(232) = 464$.

Time = 0.14 (sec) , antiderivative size = 1580, normalized size of antiderivative = 6.37

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^3}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^3,x, algorith
m="fricas")
```

output
```
[-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2
+ 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4
- (4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)
*x^2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3
*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 +
2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*s
qrt(-d^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x +
 f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x +
 1))*sqrt(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 +
 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4
*e^7 + B*d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^
2 + 4*C)*e^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d
^4*e^6*f - B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e
^3*f^4 - B*e^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4
 - e^4*f^6 + (d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 +
 2*(d^6*e^9*f - 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4
*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3
 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3
*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*
d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{-dx+1}\sqrt{dx+1}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2)/(f*x+e)**3,x)
```

output
```
Integral((A + B*x + C*x**2)/((e + f*x)**3*sqrt(-d*x + 1)*sqrt(d*x + 1)), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^3,x, algorith
m="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((f-d*e)*(f+d*e)>0)', see `assume
?` for mor
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^3,x, algorith
m="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 42.01 (sec) , antiderivative size = 9097, normalized size of antiderivative = 36.68

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

output
```
((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) -
1)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 -
d*x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2
)) + (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2
) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*
d^3*e^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^
2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x
+ 1)^(1/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) -
 1)^5*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/
(((d*x + 1)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 -
 d*x)^(1/2) - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 -
d*x)^(1/2) - 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d
*x)^(1/2) - 1)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*
((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/
2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d
*x + 1)^(1/2) - 1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2)
- 1)^7 + (8*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1
- d*x)^(1/2) - 1)^2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*
x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2499, normalized size of antiderivative = 10.08

$$\int \frac{A + Bx + Cx^2}{\sqrt{1 - dx}\sqrt{1 + dx}(e + fx)^3} \, dx = \text{Too large to display}$$

input

```
int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2)/(f*x+e)^3,x)
```

output

```
( - 4*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x
 + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**4*e**4 - 8*sqrt(d
*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2
))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**4*e**3*f*x - 4*sqrt(d*e + f)*
sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) -
sqrt(f)*sqrt(2))/sqrt(d*e - f))*a*d**4*e**2*f**2*x**2 - 2*sqrt(d*e + f)*sq
rt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sq
rt(f)*sqrt(2))/sqrt(d*e - f))*a*d**2*e**2*f**2 - 4*sqrt(d*e + f)*sqrt(d*e
- f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*s
qrt(2))/sqrt(d*e - f))*a*d**2*e*f**3*x - 2*sqrt(d*e + f)*sqrt(d*e - f)*ata
n((sqrt(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/
sqrt(d*e - f))*a*d**2*f**4*x**2 + 6*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt
(d*e + f)*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*
e - f))*b*d**2*e**3*f + 12*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)
*tan(asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*b
*d**2*e**2*f**2*x + 6*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(
asin(sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*b*d**2
*e*f**3*x**2 - 2*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(
sqrt( - d*x + 1)/sqrt(2))/2) - sqrt(f)*sqrt(2))/sqrt(d*e - f))*c*d**2*e**4
 - 4*sqrt(d*e + f)*sqrt(d*e - f)*atan((sqrt(d*e + f)*tan(asin(sqrt( - d...
```

3.45 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 40, antiderivative size = 554

$$
\begin{aligned}
&\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2)\right)x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4} \\
&\quad - \frac{(424a^4Cf^3+280b^4e^2(Be+3Af)-105a^3bf^2(3Ce+Bf)-210ab^3e(Ce^2+3f(Be+Af))+112a^2b^2}{840b^6c} \\
&\quad + \frac{f(26a^2Cf^2-7b^2(3Ce^2+f(3Be+Af)))\,x^2(a+bx)^{3/2}(ac-bcx)^{3/2}}{35b^4c} \\
&\quad - \frac{f^2(21bCe+7bBf-24aCf)x^3(a+bx)^{3/2}(ac-bcx)^{3/2}}{42b^3c} \\
&\quad + \frac{\left(32a^3Cf^3-7a^2bf^2(3Ce+Bf)-14b^3(Ce^3+3ef(Be+Af))\right)(a+bx)^{5/2}(ac-bcx)^{3/2}}{56b^6c} \\
&\quad - \frac{Cf^3(a+bx)^{11/2}(ac-bcx)^{3/2}}{7b^6c} \\
&\quad + \frac{a^2\sqrt{c}(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A(8b^4e^3+6a^2b^2ef^2))\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{8b^5}
\end{aligned}
$$

output
```
1/16*(a^4*f^2*(B*f+3*C*e)+2*a^2*b^2*e^2*(3*B*f+C*e)+A*(6*a^2*b^2*e*f^2+8*b
^4*e^3))*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4-1/840*(424*a^4*C*f^3+280*b
^4*e^2*(3*A*f+B*e)-105*a^3*b*f^2*(B*f+3*C*e)-210*a*b^3*e*(C*e^2+3*f*(A*f+B
*e))+112*a^2*b^2*f*(3*C*e^2+f*(A*f+3*B*e)))*(b*x+a)^(3/2)*(-b*c*x+a*c)^(3/
2)/b^6/c+1/35*f*(26*a^2*C*f^2-7*b^2*(3*C*e^2+f*(A*f+3*B*e)))*x^2*(b*x+a)^(
3/2)*(-b*c*x+a*c)^(3/2)/b^4/c-1/42*f^2*(7*B*b*f-24*C*a*f+21*C*b*e)*x^3*(b*
x+a)^(3/2)*(-b*c*x+a*c)^(3/2)/b^3/c+1/56*(32*a^3*C*f^3-7*a^2*b*f^2*(B*f+3*
C*e)-14*b^3*(C*e^3+3*e*f*(A*f+B*e)))*(b*x+a)^(5/2)*(-b*c*x+a*c)^(3/2)/b^6/
c-1/7*C*f^3*(b*x+a)^(11/2)*(-b*c*x+a*c)^(3/2)/b^6/c+1/8*a^2*c^(1/2)*(a^4*f
^2*(B*f+3*C*e)+2*a^2*b^2*e^2*(3*B*f+C*e)+A*(6*a^2*b^2*e*f^2+8*b^4*e^3))*ar
ctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b^5
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.73

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{\sqrt{c(a-bx)}\Big(-\sqrt{a-bx}\sqrt{a+bx}(128a^6Cf^3+a^4b^2f(7f(96Be+32Af+15Bfx)+C(672e^2+315efx\,-}{}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c*(a - b*x)]*(-(Sqrt[a - b*x]*Sqrt[a + b*x]*(128*a^6*C*f^3 + a^4*b^2
*f*(7*f*(96*B*e + 32*A*f + 15*B*f*x) + C*(672*e^2 + 315*e*f*x + 64*f^2*x^2
)) + 2*a^2*b^4*(7*A*f*(120*e^2 + 45*e*f*x + 8*f^2*x^2) + 7*B*(40*e^3 + 45*
e^2*f*x + 24*e*f^2*x^2 + 5*f^3*x^3) + 3*C*x*(35*e^3 + 56*e^2*f*x + 35*e*f^
2*x^2 + 8*f^3*x^3)) - 4*b^6*x*(21*A*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 +
4*f^3*x^3) + x*(7*B*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3) + 3*
C*x*(35*e^3 + 84*e^2*f*x + 70*e*f^2*x^2 + 20*f^3*x^3))))) + 210*a^2*b*(a^4
*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^2*b^
2*e*f^2))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(1680*b^6*Sqrt[a - b*x])
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.92, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.325, Rules used = {2113, 2185, 25, 27, 687, 27, 687, 25, 27, 676, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(e+fx)^3\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int (e+fx)^3\sqrt{a^2c-b^2cx^2}\left(Cx^2+Bx+A\right)dx}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(-\frac{\int -cf(e+fx)^3\left(\left(4Ca^2+7Ab^2\right)f-b^2(3Ce-7Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{7b^2cf^2}-\frac{C(e+fx)^4\left(a^2c-b^2cx^2\right)^{3/2}}{7b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int cf(e+fx)^3\left(\left(4Ca^2+7Ab^2\right)f-b^2(3Ce-7Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{7b^2cf^2}-\frac{C(e+fx)^4\left(a^2c-b^2cx^2\right)^{3/2}}{7b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int (e+fx)^3\left(\left(4Ca^2+7Ab^2\right)f-b^2(3Ce-7Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{7b^2f}-\frac{C(e+fx)^4\left(a^2c-b^2cx^2\right)^{3/2}}{7b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{687}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{(e+fx)^3\left(a^2c-b^2cx^2\right)^{3/2}(3Ce-7Bf)}{6c}-\frac{\int -3b^2c(e+fx)^2\left(f\left((5Ce+7Bf)a^2+14Ab^2e\right)+\left(8a^2Cf^2-b^2\left(3Ce^2-7f(Be+2Af)\right)\right)x\right)\sqrt{\ldots}}{6b^2c}}{7b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{2}\int (e+fx)^2\left(f\left((5Ce+7Bf)a^2+14Ab^2e\right)+\left(8a^2Cf^2-b^2\left(3Ce^2-7f(Be+2Af)\right)\right)x\right)\sqrt{a^2c-b^2cx^2}dx+\frac{(e+fx)^3\left(a^2c-b^2cx\right.}{6c}}{7b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 687

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{2}\left(-\frac{\int -c(e+fx)\left(\left(a^2f^2(41Ce+35Bf)-b^2\left(6Ce^3-14ef(Be+7Af)\right)\right)xb^2+f\left(16Cf^2a^4+b^2e(19Ce+49Bf)a^2+14A\left(5e^2b^4+2a^2f^2\right.\right.\right.}{5b^2c}\right.}{7b}\right.}{\sqrt{a^2}}$$

$\downarrow$ 25

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{2}\left(\frac{\int c(e+fx)\left(\left(a^2f^2(41Ce+35Bf)-b^2\left(6Ce^3-14ef(Be+7Af)\right)\right)xb^2+f\left(16Cf^2a^4+b^2e(19Ce+49Bf)a^2+14A\left(5e^2b^4+2a^2f^2b^2\right)\right.\right.}{5b^2c}\right.}{7b^2}\right.}{\sqrt{a^2c}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{2}\left(\frac{\int (e+fx)\left(\left(a^2f^2(41Ce+35Bf)-b^2\left(6Ce^3-14ef(Be+7Af)\right)\right)xb^2+f\left(16Cf^2a^4+b^2e(19Ce+49Bf)a^2+14A\left(5e^2b^4+2a^2f^2b^2\right)\right)\right.}{5b^2}\right.}{7b^2f}\right.}{\sqrt{a^2c}}$$

$\downarrow$ 676

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(a^4f^2(Bf+3Ce)+A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)\right)\int\sqrt{a^2c-b^2cx^2}dx-\frac{fx\left(a^2c-b^2cx^2\right)^{3/2}\left(a^2f^2(35Bf+41C\right.}{4c}}{}\right.}{}\right.$$

$\downarrow$ 211

$$\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(a^4f^2(Bf+3Ce)+A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)\right)\left(\frac{1}{2}a^2c\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)-\frac{fx\left(a^2c-b^2\right.}{}}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \textbf{224}$$

$$\frac{\frac{1}{2}\left(\frac{\frac{35}{4}f\left(a^4f^2(Bf+3Ce)+A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)\right)\left(\frac{1}{2}a^2c\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right.}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{(e+fx)^3\left(a^2c-b^2cx^2\right)^{3/2}(3Ce-7Bf)}{6c}+\frac{1}{2}\left(\frac{-\frac{fx\left(a^2c-b^2cx^2\right)^{3/2}\left(a^2f^2(35Bf+41Ce)-b^2\left(6Ce^3-14ef(7Af+Be)\right)\right)}{4c}+\frac{35}{4}f\left(\frac{a^2\sqrt{}}{}\right.}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(-1/7*(C*(e + f*x)^4*(a^2*c - b^2*c*x^2)^
(3/2))/(b^2*c*f) + (((3*C*e - 7*B*f)*(e + f*x)^3*(a^2*c - b^2*c*x^2)^(3/2)
)/(6*c) + (-1/5*((8*a^2*C*f^2 - b^2*(3*C*e^2 - 7*f*(B*e + 2*A*f)))*(e + f*
x)^2*(a^2*c - b^2*c*x^2)^(3/2))/(b^2*c) + ((-2*(8*a^4*C*f^4 + 2*a^2*b^2*f^
2*(15*C*e^2 + 7*f*(3*B*e + A*f)) - b^4*(3*C*e^4 - 7*e^2*f*(B*e + 12*A*f)))
*(a^2*c - b^2*c*x^2)^(3/2))/(3*b^2*c) - (f*(a^2*f^2*(41*C*e + 35*B*f) - b^
2*(6*C*e^3 - 14*e*f*(B*e + 7*A*f)))*x*(a^2*c - b^2*c*x^2)^(3/2))/(4*c) + (
35*f*(a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 +
 6*a^2*b^2*e*f^2))*((x*Sqrt[a^2*c - b^2*c*x^2])/2 + (a^2*Sqrt[c]*ArcTan[(b
*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b)))/4)/(5*b^2))/2)/(7*b^2*f)))/S
qrt[a^2*c - b^2*c*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

rule 687

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2113

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{(-240C\,f^3x^6b^6-280B\,b^6f^3x^5-840C\,b^6e\,f^2x^5-336A\,b^6f^3x^4-1008B\,b^6e\,f^2x^4+48C\,a^2b^4f^3x^4-1008C\,b^6e^2f\,x^4-1260A\,b^6e\,f^2}{}$
default	Expression too large to display

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x,method=_RET
URNVERBOSE)
```

output
```
-1/1680*(-240*C*b^6*f^3*x^6-280*B*b^6*f^3*x^5-840*C*b^6*e*f^2*x^5-336*A*b^
6*f^3*x^4-1008*B*b^6*e*f^2*x^4+48*C*a^2*b^4*f^3*x^4-1008*C*b^6*e^2*f*x^4-1
260*A*b^6*e*f^2*x^3+70*B*a^2*b^4*f^3*x^3-1260*B*b^6*e^2*f*x^3+210*C*a^2*b^
4*e*f^2*x^3-420*C*b^6*e^3*x^3+112*A*a^2*b^4*f^3*x^2-1680*A*b^6*e^2*f*x^2+3
36*B*a^2*b^4*e*f^2*x^2-560*B*b^6*e^3*x^2+64*C*a^4*b^2*f^3*x^2+336*C*a^2*b^
4*e^2*f*x^2+630*A*a^2*b^4*e*f^2*x-840*A*b^6*e^3*x+105*B*a^4*b^2*f^3*x+630*
B*a^2*b^4*e^2*f*x+315*C*a^4*b^2*e*f^2*x+210*C*a^2*b^4*e^3*x+224*A*a^4*b^2*
f^3+1680*A*a^2*b^4*e^2*f+672*B*a^4*b^2*e*f^2+560*B*a^2*b^4*e^3+128*C*a^6*f
^3+672*C*a^4*b^2*e^2*f)/b^6*(-b*x+a)*(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c+1/
16*a^2/b^4*(6*A*a^2*b^2*e*f^2+8*A*b^4*e^3+B*a^4*f^3+6*B*a^2*b^2*e^2*f+3*C*
a^4*e*f^2+2*C*a^2*b^2*e^3)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^
2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2
)*c
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 1001, normalized size of antiderivative = 1.81

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x, algo
rithm="fricas")
```

output
```
[1/3360*(105*(6*B*a^4*b^3*e^2*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5
)*e^3 + 3*(C*a^6*b + 2*A*a^4*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt
(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(240*C*b^6*f^3*x^6
- 560*B*a^2*b^4*e^3 - 672*B*a^4*b^2*e*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3
)*x^5 + 48*(21*C*b^6*e^2*f + 21*B*b^6*e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x
^4 - 336*(2*C*a^4*b^2 + 5*A*a^2*b^4)*e^2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^
3 + 70*(6*C*b^6*e^3 + 18*B*b^6*e^2*f - B*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*
b^6)*e*f^2)*x^3 + 16*(35*B*b^6*e^3 - 21*B*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 -
5*A*b^6)*e^2*f - (4*C*a^4*b^2 + 7*A*a^2*b^4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e
^2*f + B*a^4*b^2*f^3 + 2*(C*a^2*b^4 - 4*A*b^6)*e^3 + 3*(C*a^4*b^2 + 2*A*a^
2*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^6, -1/1680*(105*(6*B*
a^4*b^3*e^2*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5)*e^3 + 3*(C*a^6*b
 + 2*A*a^4*b^3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*s
qrt(c)*x/(b^2*c*x^2 - a^2*c)) - (240*C*b^6*f^3*x^6 - 560*B*a^2*b^4*e^3 - 6
72*B*a^4*b^2*e*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3)*x^5 + 48*(21*C*b^6*e^
2*f + 21*B*b^6*e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x^4 - 336*(2*C*a^4*b^2 +
 5*A*a^2*b^4)*e^2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70*(6*C*b^6*e^3 + 1
8*B*b^6*e^2*f - B*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*b^6)*e*f^2)*x^3 + 16*(3
5*B*b^6*e^3 - 21*B*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 - 5*A*b^6)*e^2*f - (4*C*a
^4*b^2 + 7*A*a^2*b^4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e^2*f + B*a^4*b^2*f^3...
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2)*(f*x+e)**3*(C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**3*(A + B*x + C*x**2)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.05

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx$$
$$\begin{aligned}
&= -\frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cf^3x^4}{7\,b^2c} + \frac{Aa^2\sqrt{c}e^3\arcsin\left(\frac{bx}{a}\right)}{2\,b} + \frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ae^3x \\
&\quad - \frac{4\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f^3x^2}{35\,b^4c} + \frac{(3\,Cef^2+Bf^3)a^6\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{16\,b^5} \\
&\quad + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be^3}{3\,b^2c} \\
&\quad - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Ae^2f}{b^2c} - \frac{8\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^4f^3}{105\,b^6c} \\
&\quad + \frac{\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)a^4x}{16\,b^4} + \frac{\sqrt{-b^2cx^2+a^2c}(Ce^3+3\,Be^2f+3\,Aef^2)a^2x}{8\,b^2} \\
&\quad - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Cef^2+Bf^3)x^3}{6\,b^2c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Ce^2f+3\,Bef^2+Af^3)x^2}{5\,b^2c} \\
&\quad - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Cef^2+Bf^3)a^2x}{8\,b^4c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce^3+3\,Be^2f+3\,Aef^2)x}{4\,b^2c} \\
&\quad - \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}(3\,Ce^2f+3\,Bef^2+Af^3)a^2}{15\,b^4c}
\end{aligned}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x, algo
rithm="maxima")
```

output
```
-1/7*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^3*x^4/(b^2*c) + 1/2*A*a^2*sqrt(c)*e^3*
arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e^3*x - 4/35*(-b^2*c*x^2
+ a^2*c)^(3/2)*C*a^2*f^3*x^2/(b^4*c) + 1/16*(3*C*e*f^2 + B*f^3)*a^6*sqrt(c
)*arcsin(b*x/a)/b^5 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^4*sqrt(c)*arcs
in(b*x/a)/b^3 - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^3/(b^2*c) - (-b^2*c*x^2
 + a^2*c)^(3/2)*A*e^2*f/(b^2*c) - 8/105*(-b^2*c*x^2 + a^2*c)^(3/2)*C*a^4*f
^3/(b^6*c) + 1/16*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e*f^2 + B*f^3)*a^4*x/b^4 +
 1/8*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^2*x/b^2 -
1/6*(-b^2*c*x^2 + a^2*c)^(3/2)*(3*C*e*f^2 + B*f^3)*x^3/(b^2*c) - 1/5*(-b^2
*c*x^2 + a^2*c)^(3/2)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*x^2/(b^2*c) - 1/8*(-
b^2*c*x^2 + a^2*c)^(3/2)*(3*C*e*f^2 + B*f^3)*a^2*x/(b^4*c) - 1/4*(-b^2*c*x
^2 + a^2*c)^(3/2)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*x/(b^2*c) - 2/15*(-b^2*c
*x^2 + a^2*c)^(3/2)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*a^2/(b^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2671 vs. $2(510) = 1020$.

Time = 2.19 (sec) , antiderivative size = 2671, normalized size of antiderivative = 4.82

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x, algo
rithm="giac")
```

output
```
-1/1680*(1680*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
 2*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^5*e^3
 - 840*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*
c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*
b^4*e^3 - 840*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c
 + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a
))*A*b^5*e^3 - 2520*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
+ a)*c + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x
 - 2*a))*A*a*b^4*e^2*f + 280*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sq
rt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sq
rt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b^3*e^3 + 280*(6*a^3*c*log(abs
(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x
 - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^4
*e^3 + 840*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c +
2*a*c)*sqrt(b*x + a))*B*a*b^3*e^2*f + 840*(6*a^3*c*log(abs(-sqrt(b*x + a)*
sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a
) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*b^4*e^2*f + 840*(6*
a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(
-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)\,dx=\text{Hanged}$$

input
```
int((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.72

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3 \left(A+Bx+Cx^2\right) \, dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^3*(C*x^2+B*x+A),x)
```

output
```
(sqrt(c)*( - 210*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**6*b**2*f**3 - 63
0*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**6*b*c*e*f**2 - 1260*asin(sqrt(a
 - b*x)/(sqrt(a)*sqrt(2)))*a**5*b**3*e*f**2 - 1260*asin(sqrt(a - b*x)/(sqr
t(a)*sqrt(2)))*a**4*b**4*e**2*f - 420*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2))
)*a**4*b**3*c*e**3 - 1680*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**5*
e**3 - 128*sqrt(a + b*x)*sqrt(a - b*x)*a**6*c*f**3 - 224*sqrt(a + b*x)*sqr
t(a - b*x)*a**5*b**2*f**3 - 672*sqrt(a + b*x)*sqrt(a - b*x)*a**4*b**3*e*f*
*2 - 105*sqrt(a + b*x)*sqrt(a - b*x)*a**4*b**3*f**3*x - 672*sqrt(a + b*x)*
sqrt(a - b*x)*a**4*b**2*c*e**2*f - 315*sqrt(a + b*x)*sqrt(a - b*x)*a**4*b*
*2*c*e*f**2*x - 64*sqrt(a + b*x)*sqrt(a - b*x)*a**4*b**2*c*f**3*x**2 - 168
0*sqrt(a + b*x)*sqrt(a - b*x)*a**3*b**4*e**2*f - 630*sqrt(a + b*x)*sqrt(a
- b*x)*a**3*b**4*e*f**2*x - 112*sqrt(a + b*x)*sqrt(a - b*x)*a**3*b**4*f**3
*x**2 - 560*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**5*e**3 - 630*sqrt(a + b*x)
*sqrt(a - b*x)*a**2*b**5*e**2*f*x - 336*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b
**5*e*f**2*x**2 - 70*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**5*f**3*x**3 - 210
*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**4*c*e**3*x - 336*sqrt(a + b*x)*sqrt(a
 - b*x)*a**2*b**4*c*e**2*f*x**2 - 210*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**
4*c*e*f**2*x**3 - 48*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**4*c*f**3*x**4 + 8
40*sqrt(a + b*x)*sqrt(a - b*x)*a*b**6*e**3*x + 1680*sqrt(a + b*x)*sqrt(a -
 b*x)*a*b**6*e**2*f*x**2 + 1260*sqrt(a + b*x)*sqrt(a - b*x)*a*b**6*e*f*...
```

3.46 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 40, antiderivative size = 415

$$\begin{aligned}
&\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{\left(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2)\right)x\sqrt{a+bx}\sqrt{ac-bcx}}{16b^4} \\
&\quad - \frac{\left(25a^3Cf^2+40b^3e(Be+2Af)+16a^2bf(2Ce+Bf)-30ab^2(Ce^2+f(2Be+Af))\right)(a+bx)^{3/2}(ac-}{120b^5c} \\
&\quad - \frac{f(4bCe+2bBf-5aCf)x^2(a+bx)^{3/2}(ac-bcx)^{3/2}}{10b^3c} \\
&\quad + \frac{\left(3a^2Cf^2-2b^2(Ce^2+f(2Be+Af))\right)(a+bx)^{5/2}(ac-bcx)^{3/2}}{8b^5c} \\
&\quad - \frac{Cf^2(a+bx)^{9/2}(ac-bcx)^{3/2}}{6b^5c} \\
&\quad + \frac{a^2\sqrt{c}\left(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A(4b^4e^2+a^2b^2f^2)\right)\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{8b^5}
\end{aligned}$$

output
```
1/16*(a^4*C*f^2+2*a^2*b^2*e*(2*B*f+C*e)+2*A*(a^2*b^2*f^2+4*b^4*e^2))*x*(b*
x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4-1/120*(25*a^3*C*f^2+40*b^3*e*(2*A*f+B*e)
+16*a^2*b*f*(B*f+2*C*e)-30*a*b^2*(C*e^2+f*(A*f+2*B*e)))*(b*x+a)^(3/2)*(-b*
c*x+a*c)^(3/2)/b^5/c-1/10*f*(2*B*b*f-5*C*a*f+4*C*b*e)*x^2*(b*x+a)^(3/2)*(-
b*c*x+a*c)^(3/2)/b^3/c+1/8*(3*a^2*C*f^2-2*b^2*(C*e^2+f*(A*f+2*B*e)))*(b*x+
a)^(5/2)*(-b*c*x+a*c)^(3/2)/b^5/c-1/6*C*f^2*(b*x+a)^(9/2)*(-b*c*x+a*c)^(3/
2)/b^5/c+1/8*a^2*c^(1/2)*(a^4*C*f^2+2*a^2*b^2*e*(2*B*f+C*e)+2*A*(a^2*b^2*f
^2+4*b^4*e^2))*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b^5
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.69

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c(a-bx)}\Big(b\sqrt{a-bx}\sqrt{a+bx}(-a^4f(64Ce+32Bf+15Cfx)-2a^2b^2(5Af(16e+3fx)+Cx(15e^2+}{}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c*(a - b*x)]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(-(a^4*f*(64*C*e + 32*B*
f + 15*C*f*x)) - 2*a^2*b^2*(5*A*f*(16*e + 3*f*x) + C*x*(15*e^2 + 16*e*f*x
+ 5*f^2*x^2) + B*(40*e^2 + 30*e*f*x + 8*f^2*x^2)) + 4*b^4*x*(5*A*(6*e^2 +
8*e*f*x + 3*f^2*x^2) + x*(2*B*(10*e^2 + 15*e*f*x + 6*f^2*x^2) + C*x*(15*e^
2 + 24*e*f*x + 10*f^2*x^2)))) + 30*a^2*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B
*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))
/(240*b^5*Sqrt[a - b*x])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2113, 2185, 27, 687, 25, 27, 676, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\, dx$$

$$\Big\downarrow \; 2113$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int (e+fx)^2\sqrt{a^2c-b^2cx^2}\left(Cx^2+Bx+A\right)dx}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 2185$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(-\frac{\int -3cf(e+fx)^2\left(\left(Ca^2+2Ab^2\right)f-b^2(Ce-2Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{6b^2cf^2}-\frac{C(e+fx)^3\left(a^2c-b^2cx^2\right)^{3/2}}{6b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int (e+fx)^2\left(\left(Ca^2+2Ab^2\right)f-b^2(Ce-2Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{2b^2f}-\frac{C(e+fx)^3\left(a^2c-b^2cx^2\right)^{3/2}}{6b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 687$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}(Ce-2Bf)}{5c}-\frac{\int -b^2c(e+fx)\left(f\left((3Ce+4Bf)a^2+10Ab^2e\right)+\left(5a^2Cf^2-b^2\left(2Ce^2-2f(2Be+5Af)\right)\right)x\right)\sqrt{a^2 \cdots}}{5b^2c}}{2b^2f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\int b^2c(e+fx)\left(f\left((3Ce+4Bf)a^2+10Ab^2e\right)+\left(5a^2Cf^2-b^2\left(2Ce^2-2f(2Be+5Af)\right)\right)x\right)\sqrt{a^2c-b^2cx^2}dx}{5b^2c}+\frac{(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}(\cdots}{5c}}{2b^2f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{5}\int (e+fx)\left(f\left((3Ce+4Bf)a^2+10Ab^2e\right)+\left(5a^2Cf^2-b^2\left(2Ce^2-2f(2Be+5Af)\right)\right)x\right)\sqrt{a^2c-b^2cx^2}dx+\frac{(e+fx)^2\left(a^2c-b^2cx\cdots\right.}{5c\cdots}}{2b^2f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 676$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{1}{5}\left(\frac{5f\left(a^4Cf^2+2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)\right)\int\sqrt{a^2c-b^2cx^2}dx}{4b^2}-\frac{2\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2f^2(Bf+2Ce)-b^2\left(Ce^3-2ef\right.\right.}{3b^2c}\right.}{2b^2f}\right.}{\sqrt{a^2c-b}}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{1}{5}\left(\frac{5f\left(a^4Cf^2+2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)\right)\left(\frac{1}{2}a^2c\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)}{4b^2}-\frac{2\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2\right.}{}\right.\right.}{}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{1}{5}\left(\frac{5f\left(a^4Cf^2+2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)\right)\left(\frac{1}{2}a^2c\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)}{4b^2}-\frac{2\left(a^2c-b\right.}{}\right.\right.}{}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}(Ce-2Bf)}{5c}+\frac{1}{5}\left(-\frac{2\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2f^2(Bf+2Ce)-b^2\left(Ce^3-2ef(5Af+Be)\right)\right)}{3b^2c}-\frac{fx\left(a^2c-b^2cx^2\right)}{}\right.\right.}{}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(-1/6*(C*(e + f*x)^3*(a^2*c - b^2*c*x^2)^
(3/2))/(b^2*c*f) + (((C*e - 2*B*f)*(e + f*x)^2*(a^2*c - b^2*c*x^2)^(3/2))/
(5*c) + ((-2*(2*a^2*f^2*(2*C*e + B*f) - b^2*(C*e^3 - 2*e*f*(B*e + 5*A*f)))
*(a^2*c - b^2*c*x^2)^(3/2))/(3*b^2*c) - (f*(5*a^2*C*f^2 - b^2*(2*C*e^2 - 2
*f*(2*B*e + 5*A*f)))*x*(a^2*c - b^2*c*x^2)^(3/2))/(4*b^2*c) + (5*f*(a^4*C*
f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*((x*Sqrt[
a^2*c - b^2*c*x^2])/2 + (a^2*Sqrt[c]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2
*c*x^2]])/(2*b)))/(4*b^2))/5)/(2*b^2*f)))/Sqrt[a^2*c - b^2*c*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2113

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\left(-40C\,f^2x^5b^4-48B\,b^4f^2x^4-96C\,b^4ef\,x^4-60A\,b^4f^2x^3-120B\,b^4ef\,x^3+10C\,a^2b^2f^2x^3-60C\,b^4e^2x^3-160A\,b^4ef\,x^2+16B\,a^2b^2f\right.}{}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(60A\,b^4f^2x^3\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}+60C\,b^4e^2x^3\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}-32B\,a^4f^2\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}+15C\right.}{}$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x,method=_RET
URNVERBOSE)
```

output
```
-1/240/b^4*(-40*C*b^4*f^2*x^5-48*B*b^4*f^2*x^4-96*C*b^4*e*f*x^4-60*A*b^4*f
^2*x^3-120*B*b^4*e*f*x^3+10*C*a^2*b^2*f^2*x^3-60*C*b^4*e^2*x^3-160*A*b^4*e
*f*x^2+16*B*a^2*b^2*f^2*x^2-80*B*b^4*e^2*x^2+32*C*a^2*b^2*e*f*x^2+30*A*a^2
*b^2*f^2*x-120*A*b^4*e^2*x+60*B*a^2*b^2*e*f*x+15*C*a^4*f^2*x+30*C*a^2*b^2*
e^2*x+160*A*a^2*b^2*e*f+32*B*a^4*f^2+80*B*a^2*b^2*e^2+64*C*a^4*e*f)*(-b*x+
a)*(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c+1/16*a^2*(2*A*a^2*b^2*f^2+8*A*b^4*e^
2+4*B*a^2*b^2*e*f+C*a^4*f^2+2*C*a^2*b^2*e^2)/b^4/(b^2*c)^(1/2)*arctan((b^2
*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(
1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.69

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \left[\frac{15\left(4\,Ba^4b^2ef+2\left(Ca^4b^2+4\,Aa^2b^4\right)e^2+\left(Ca^6+2\,Aa^4b^2\right)f^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+\right.}{}\right.$$
$$-\frac{15\left(4\,Ba^4b^2ef+2\left(Ca^4b^2+4\,Aa^2b^4\right)e^2+\left(Ca^6+2\,Aa^4b^2\right)f^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-\left(40\,C\right.}{}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x, algo
rithm="fricas")
```

output
```
[1/480*(15*(4*B*a^4*b^2*e*f + 2*(C*a^4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2
*A*a^4*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a)*b*sqrt(-c)*x - a^2*c) + 2*(40*C*b^5*f^2*x^5 - 80*B*a^2*b^3*e^2 - 32*B
*a^4*b*f^2 + 48*(2*C*b^5*e*f + B*b^5*f^2)*x^4 + 10*(6*C*b^5*e^2 + 12*B*b^5
*e*f - (C*a^2*b^3 - 6*A*b^5)*f^2)*x^3 - 32*(2*C*a^4*b + 5*A*a^2*b^3)*e*f +
 16*(5*B*b^5*e^2 - B*a^2*b^3*f^2 - 2*(C*a^2*b^3 - 5*A*b^5)*e*f)*x^2 - 15*(
4*B*a^2*b^3*e*f + 2*(C*a^2*b^3 - 4*A*b^5)*e^2 + (C*a^4*b + 2*A*a^2*b^3)*f^
2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5, -1/240*(15*(4*B*a^4*b^2*e*f +
 2*(C*a^4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2*A*a^4*b^2)*f^2)*sqrt(c)*arct
an(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (40
*C*b^5*f^2*x^5 - 80*B*a^2*b^3*e^2 - 32*B*a^4*b*f^2 + 48*(2*C*b^5*e*f + B*b
^5*f^2)*x^4 + 10*(6*C*b^5*e^2 + 12*B*b^5*e*f - (C*a^2*b^3 - 6*A*b^5)*f^2)*
x^3 - 32*(2*C*a^4*b + 5*A*a^2*b^3)*e*f + 16*(5*B*b^5*e^2 - B*a^2*b^3*f^2 -
 2*(C*a^2*b^3 - 5*A*b^5)*e*f)*x^2 - 15*(4*B*a^2*b^3*e*f + 2*(C*a^2*b^3 - 4
*A*b^5)*e^2 + (C*a^4*b + 2*A*a^2*b^3)*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a))/b^5]
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2)*(f*x+e)**2*(C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**2*(A + B*x + C*x**2)
, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{Aa^2\sqrt{c}e^2\arcsin\left(\frac{bx}{a}\right)}{2\,b} + \frac{Ca^6\sqrt{c}f^2\arcsin\left(\frac{bx}{a}\right)}{16\,b^5} + \frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ae^2x + \frac{\sqrt{-b^2cx^2+a^2c}Ca^4f^2x}{16\,b^4} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cf^2x^3}{6\,b^2c} + \frac{(Ce^2+2\,Bef+Af^2)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3} + \frac{\sqrt{-b^2cx^2+a^2c}(Ce^2+2\,Bef+Af^2)a^2x}{8\,b^2} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f^2x}{8\,b^4c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be^2}{3\,b^2c} - \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Aef}{3\,b^2c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(2\,Cef+Bf^2)x^2}{5\,b^2c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce^2+2\,Bef+Af^2)x}{4\,b^2c} - \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}(2\,Cef+Bf^2)a^2}{15\,b^4c}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x, algo
rithm="maxima")
```

output
```
1/2*A*a^2*sqrt(c)*e^2*arcsin(b*x/a)/b + 1/16*C*a^6*sqrt(c)*f^2*arcsin(b*x/
a)/b^5 + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e^2*x + 1/16*sqrt(-b^2*c*x^2 + a^2
*c)*C*a^4*f^2*x/b^4 - 1/6*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^2*x^3/(b^2*c) + 1
/8*(C*e^2 + 2*B*e*f + A*f^2)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/8*sqrt(-b^2
*c*x^2 + a^2*c)*(C*e^2 + 2*B*e*f + A*f^2)*a^2*x/b^2 - 1/8*(-b^2*c*x^2 + a^
2*c)^(3/2)*C*a^2*f^2*x/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^2/(b^2
*c) - 2/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*e*f/(b^2*c) - 1/5*(-b^2*c*x^2 + a^2
*c)^(3/2)*(2*C*e*f + B*f^2)*x^2/(b^2*c) - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*(
C*e^2 + 2*B*e*f + A*f^2)*x/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^(3/2)*(2*C*
e*f + B*f^2)*a^2/(b^4*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1868 vs. $2(375) = 750$.

Time = 1.68 (sec) , antiderivative size = 1868, normalized size of antiderivative = 4.50

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x, algo
rithm="giac")
```

output

```
-1/240*(240*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2
*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^4*e^2 -
 120*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)
))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b^
3*e^2 - 120*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))
*A*b^4*e^2 - 240*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a
)*c + 2*a*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x -
2*a))*A*a*b^3*e*f + 40*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b
*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b
*x + a)*c + 2*a*c)*sqrt(b*x + a))*C*a*b^2*e^2 + 40*(6*a^3*c*log(abs(-sqrt(
b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)
*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^3*e^2 +
80*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))
/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*s
qrt(b*x + a))*B*a*b^2*e*f + 80*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) +
sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*
sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*b^3*e*f + 40*(6*a^3*c*log(abs(
-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x
- 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a...
```

Mupad [B] (verification not implemented)

Time = 140.46 (sec) , antiderivative size = 4853, normalized size of antiderivative = 11.69

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2 \left(A+Bx+Cx^2\right) \, dx = \text{Too large to display}$$

input
```
int((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
- ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*((35*A*a^4*c^7*f^2)/2 - 6*A*a^2*
b^2*c^7*e^2))/(b^3*((a + b*x)^(1/2) - a^(1/2))^3) - (((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))*((A*a^4*c^8*f^2)/2 - 2*A*a^2*b^2*c^8*e^2))/(b^3*((a + b*x)^
(1/2) - a^(1/2))) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13*((35*A*a^4*c^2
*f^2)/2 - 6*A*a^2*b^2*c^2*e^2))/(b^3*((a + b*x)^(1/2) - a^(1/2))^13) - (((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5*((273*A*a^4*c^6*f^2)/2 + 30*A*a^2*b^2*
c^6*e^2))/(b^3*((a + b*x)^(1/2) - a^(1/2))^5) + (((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^11*((273*A*a^4*c^3*f^2)/2 + 30*A*a^2*b^2*c^3*e^2))/(b^3*((a + b
*x)^(1/2) - a^(1/2))^11) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7*((715*A*
a^4*c^5*f^2)/2 - 22*A*a^2*b^2*c^5*e^2))/(b^3*((a + b*x)^(1/2) - a^(1/2))^7
) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9*((715*A*a^4*c^4*f^2)/2 - 22*A*a
^2*b^2*c^4*e^2))/(b^3*((a + b*x)^(1/2) - a^(1/2))^9) + (((A*a^4*c*f^2)/2 -
 2*A*a^2*b^2*c*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(b^3*((a + b*x
)^(1/2) - a^(1/2))^15) + (16*A*a^(5/2)*c*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^14)/(b^2*((a + b*x)^(1/2) - a^(1/2))^14) + (16*A*a^(5/2
)*c^7*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*((a + b*
x)^(1/2) - a^(1/2))^2) - (32*A*a^(5/2)*c^6*e*f*(a*c)^(1/2)*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^4)/(b^2*((a + b*x)^(1/2) - a^(1/2))^4) + (208*A*a^(5/
2)*c^5*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(3*b^2*((a +
 b*x)^(1/2) - a^(1/2))^6) + (704*A*a^(5/2)*c^4*e*f*(a*c)^(1/2)*((a*c - ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.53

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$= \frac{\sqrt{c}\left(-32\sqrt{bx+a}\,\sqrt{-bx+a}\,a^2b^3cef\,x^2 - 160\sqrt{bx+a}\,\sqrt{-bx+a}\,a^3b^3ef - 30\sqrt{bx+a}\,\sqrt{-bx+a}\,a^3b^3f^2\right.}{}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)^2*(C*x^2+B*x+A),x)
```

output
```
(sqrt(c)*( - 30*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**6*c*f**2 - 60*asi
n(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**5*b**2*f**2 - 120*asin(sqrt(a - b*x)
/(sqrt(a)*sqrt(2)))*a**4*b**3*e*f - 60*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)
))*a**4*b**2*c*e**2 - 240*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**4*
e**2 - 32*sqrt(a + b*x)*sqrt(a - b*x)*a**4*b**2*f**2 - 64*sqrt(a + b*x)*sq
rt(a - b*x)*a**4*b*c*e*f - 15*sqrt(a + b*x)*sqrt(a - b*x)*a**4*b*c*f**2*x
- 160*sqrt(a + b*x)*sqrt(a - b*x)*a**3*b**3*e*f - 30*sqrt(a + b*x)*sqrt(a
- b*x)*a**3*b**3*f**2*x - 80*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**4*e**2 -
60*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**4*e*f*x - 16*sqrt(a + b*x)*sqrt(a -
 b*x)*a**2*b**4*f**2*x**2 - 30*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**3*c*e**
2*x - 32*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**3*c*e*f*x**2 - 10*sqrt(a + b*
x)*sqrt(a - b*x)*a**2*b**3*c*f**2*x**3 + 120*sqrt(a + b*x)*sqrt(a - b*x)*a
*b**5*e**2*x + 160*sqrt(a + b*x)*sqrt(a - b*x)*a*b**5*e*f*x**2 + 60*sqrt(a
 + b*x)*sqrt(a - b*x)*a*b**5*f**2*x**3 + 80*sqrt(a + b*x)*sqrt(a - b*x)*b*
*6*e**2*x**2 + 120*sqrt(a + b*x)*sqrt(a - b*x)*b**6*e*f*x**3 + 48*sqrt(a +
 b*x)*sqrt(a - b*x)*b**6*f**2*x**4 + 60*sqrt(a + b*x)*sqrt(a - b*x)*b**5*c
*e**2*x**3 + 96*sqrt(a + b*x)*sqrt(a - b*x)*b**5*c*e*f*x**4 + 40*sqrt(a +
b*x)*sqrt(a - b*x)*b**5*c*f**2*x**5))/(240*b**5)
```

3.47 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 255

$$\begin{aligned}
&\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} \\
&\quad - \frac{\left(4a^2Cf+4b^2(Be+Af)-3ab(Ce+Bf)\right)(a+bx)^{3/2}(ac-bcx)^{3/2}}{12b^4c} \\
&\quad + \frac{(8aCf-5b(Ce+Bf))(a+bx)^{5/2}(ac-bcx)^{3/2}}{20b^4c} \\
&\quad - \frac{Cf(a+bx)^{7/2}(ac-bcx)^{3/2}}{5b^4c} + \frac{a^2\sqrt{c}(4Ab^2e+a^2(Ce+Bf))\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{4b^3}
\end{aligned}$$

output
```
1/8*(4*A*e+a^2*(B*f+C*e)/b^2)*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)-1/12*(4*a
^2*C*f+4*b^2*(A*f+B*e)-3*a*b*(B*f+C*e))*(b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2)/b
^4/c+1/20*(8*C*a*f-5*b*(B*f+C*e))*(b*x+a)^(5/2)*(-b*c*x+a*c)^(3/2)/b^4/c-1
/5*C*f*(b*x+a)^(7/2)*(-b*c*x+a*c)^(3/2)/b^4/c+1/4*a^2*c^(1/2)*(4*A*b^2*e+a
^2*(B*f+C*e))*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b^3
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.72

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c(a-bx)}\Big(\sqrt{a-bx}\sqrt{a+bx}(-16a^4Cf-a^2b^2(40Af+5B(8e+3fx)+Cx(15e+8fx))+2b^4x(10A(}{120b^4\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c*(a - b*x)]*(Sqrt[a - b*x]*Sqrt[a + b*x]*(-16*a^4*C*f - a^2*b^2*(40
*A*f + 5*B*(8*e + 3*f*x) + C*x*(15*e + 8*f*x)) + 2*b^4*x*(10*A*(3*e + 2*f*
x) + x*(5*B*(4*e + 3*f*x) + 3*C*x*(5*e + 4*f*x)))) + 30*a^2*b*(4*A*b^2*e +
 a^2*(C*e + B*f))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]]))/(120*b^4*Sqrt[a -
b*x])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2113, 2185, 25, 27, 676, 211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int (e+fx)\sqrt{a^2c-b^2cx^2}\left(Cx^2+Bx+A\right)dx}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(-\frac{\int -cf(e+fx)\left(\left(2Ca^2+5Ab^2\right)f-b^2(3Ce-5Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{5b^2cf^2}-\frac{C(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}}{5b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int cf(e+fx)\left(\left(2Ca^2+5Ab^2\right)f-b^2(3Ce-5Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{5b^2cf^2}-\frac{C(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}}{5b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int (e+fx)\left(\left(2Ca^2+5Ab^2\right)f-b^2(3Ce-5Bf)x\right)\sqrt{a^2c-b^2cx^2}dx}{5b^2f}-\frac{C(e+fx)^2\left(a^2c-b^2cx^2\right)^{3/2}}{5b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 676$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{5}{4}f\left(a^2(Bf+Ce)+4Ab^2e\right)\int\sqrt{a^2c-b^2cx^2}dx-\frac{\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+Be)\right)\right)}{3b^2c}+\frac{fx\left(a^2c-b^2cx^2\right)^{3/2}(3C}{4c}}{5b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 211$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{5}{4}f\left(a^2(Bf+Ce)+4Ab^2e\right)\left(\frac{1}{2}a^2c\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)-\frac{\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+Be\right.\right.}{3b^2c}}{5b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{5}{4}f\left(a^2(Bf+Ce)+4Ab^2e\right)\left(\frac{1}{2}a^2c\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)-\frac{\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2Cf^2-b^2\left(3Ce\right.\right.}{3b^2c}}{5b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{5}{4}f\left(\frac{a^2\sqrt{c}\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b}+\frac{1}{2}x\sqrt{a^2c-b^2cx^2}\right)\left(a^2(Bf+Ce)+4Ab^2e\right)-\frac{\left(a^2c-b^2cx^2\right)^{3/2}\left(2a^2Cf^2-b^2\left(3Ce^2-5f(A\right.\right.}{3b^2c}}{5b^2f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(-1/5*(C*(e + f*x)^2*(a^2*c - b^2*c*x^2)^
(3/2))/(b^2*c*f) + (-1/3*((2*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(B*e + A*f)))*
(a^2*c - b^2*c*x^2)^(3/2))/(b^2*c) + (f*(3*C*e - 5*B*f)*x*(a^2*c - b^2*c*x
^2)^(3/2))/(4*c) + (5*f*(4*A*b^2*e + a^2*(C*e + B*f))*((x*Sqrt[a^2*c - b^2
*c*x^2])/2 + (a^2*Sqrt[c]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
2*b)))/4)/(5*b^2*f)))/Sqrt[a^2*c - b^2*c*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\left(-24 C f\,x^{4} b^{4}-30 B\,b^{4} f\,x^{3}-30 C\,b^{4} e\,x^{3}-40 A\,b^{4} f\,x^{2}-40 B\,b^{4} e\,x^{2}+8 C\,a^{2} b^{2} f\,x^{2}-60 A\,b^{4} e x+15 B\,a^{2} b^{2} f x+15 C\,a^{2} b^{2} e x+40 A\,a^{2} f\,b}{120 b^{4} \sqrt{-c \left(b x-a\right)}}$
default	$\frac{\sqrt{b x+a}\,\sqrt{c \left(-b x+a\right)}\,\left(24 C\,b^{4} f\,x^{4} \sqrt{b^{2} c}\,\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}+30 B\,b^{4} f\,x^{3} \sqrt{b^{2} c}\,\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}+30 C\,b^{4} e\,x^{3} \sqrt{b^{2} c}\,\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}+60 A}{}$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x,method=_RETUR
NVERBOSE)
```

output
```
-1/120*(-24*C*b^4*f*x^4-30*B*b^4*f*x^3-30*C*b^4*e*x^3-40*A*b^4*f*x^2-40*B*
b^4*e*x^2+8*C*a^2*b^2*f*x^2-60*A*b^4*e*x+15*B*a^2*b^2*f*x+15*C*a^2*b^2*e*x
+40*A*a^2*b^2*f+40*B*a^2*b^2*e+16*C*a^4*f)/b^4*(-b*x+a)*(b*x+a)^(1/2)/(-c*
(b*x-a))^(1/2)*c+1/8*a^2/b^2*(4*A*b^2*e+B*a^2*f+C*a^2*e)/(b^2*c)^(1/2)*arc
tan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(
b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.73

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

$$= \left[\frac{15\,(Ba^4bf+(Ca^4b+4\,Aa^2b^3)e)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,(24\,Cb^4f\right.$$

$$-\frac{15\,(Ba^4bf+(Ca^4b+4\,Aa^2b^3)e)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-(24\,Cb^4fx^4-40\,Ba^2b^2e+30\,(Cb^4}{}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x, algori
thm="fricas")
```

output
```
[1/240*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^
2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4
*f*x^4 - 40*B*a^2*b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a
^2*b^2 - 5*A*b^4)*f)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f +
 (C*a^2*b^2 - 4*A*b^4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4, -1/120
*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(c)*arctan(sqrt(-b*c*x +
a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (24*C*b^4*f*x^4 - 40
*B*a^2*b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a^2*b^2 - 5*
A*b^4)*f)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f + (C*a^2*b^2
 - 4*A*b^4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4]
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2)*(f*x+e)*(C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)*(A + B*x + C*x**2), x
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.97

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \frac{Aa^2\sqrt{c}e\arcsin\left(\frac{bx}{a}\right)}{2\,b} + \frac{1}{2}\sqrt{-b^2cx^2+a^2c}Aex + \frac{(Ce+Bf)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3} + \frac{\sqrt{-b^2cx^2+a^2c}(Ce+Bf)a^2x}{8\,b^2} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Cfx^2}{5\,b^2c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Be}{3\,b^2c} - \frac{2\,(-b^2cx^2+a^2c)^{\frac{3}{2}}Ca^2f}{15\,b^4c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}Af}{3\,b^2c} - \frac{(-b^2cx^2+a^2c)^{\frac{3}{2}}(Ce+Bf)x}{4\,b^2c}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x, algori
thm="maxima")
```

output
```
1/2*A*a^2*sqrt(c)*e*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e*x +
 1/8*(C*e + B*f)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/8*sqrt(-b^2*c*x^2 + a^2
*c)*(C*e + B*f)*a^2*x/b^2 - 1/5*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f*x^2/(b^2*c)
 - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^
(3/2)*C*a^2*f/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*f/(b^2*c) - 1/4*(
-b^2*c*x^2 + a^2*c)^(3/2)*(C*e + B*f)*x/(b^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(221) = 442$.

Time = 1.13 (sec) , antiderivative size = 1142, normalized size of antiderivative = 4.48

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x, algori
thm="giac")
```

output
```
-1/120*(120*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2
*a*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^3*e - 6
0*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/
sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b^2*e
 - 60*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c
)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^3
*e - 60*(2*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a
*c)))/sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*a
*b^2*f + 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c +
 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c +
 2*a*c)*sqrt(b*x + a))*C*a*b*e + 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-
c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*
a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*b^2*e + 20*(6*a^3*c*log(a
bs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - ((2*b
*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*B*a
*b*f + 20*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2
*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2
*a*c)*sqrt(b*x + a))*A*b^2*f - 5*(18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c)
 + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*
x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C...
```

Mupad [B] (verification not implemented)

Time = 26.59 (sec) , antiderivative size = 1765, normalized size of antiderivative = 6.92

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
int((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
((B*a^4*c^8*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a
^(1/2))) - (B*a^4*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x
)^(1/2) - a^(1/2))^15) - (35*B*a^4*c^7*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^3)/(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*B*a^4*c^6*f*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*B*a^4*c^5
*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7
) + (715*B*a^4*c^4*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^
(1/2) - a^(1/2))^9) - (273*B*a^4*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^11)/(2*((a + b*x)^(1/2) - a^(1/2))^11) + (35*B*a^4*c^2*f*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^13)/(2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^
3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 +
 (8*b^3*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1
/2))^2 + (28*b^3*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/
2) - a^(1/2))^4 + (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a +
 b*x)^(1/2) - a^(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14) - (a*c -
b*c*x)^(1/2)*((2*C*a^4*f*(a + b*x)^(1/2))/(15*b^4) - (C*f*x^4*(a + b*x)...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.40

$$\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)\left(A+Bx+Cx^2\right)\,dx$$

$$=\frac{\sqrt{c}\left(-30asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^4b^2f-30asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^4bce-120asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^3b^3e-16\sqrt{bx+a}\,\sqrt{-bx+}\right.}{}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(f*x+e)*(C*x^2+B*x+A),x)
```

output
```
(sqrt(c)*( - 30*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b**2*f - 30*asi
n(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b*c*e - 120*asin(sqrt(a - b*x)/(sq
rt(a)*sqrt(2)))*a**3*b**3*e - 16*sqrt(a + b*x)*sqrt(a - b*x)*a**4*c*f - 40
*sqrt(a + b*x)*sqrt(a - b*x)*a**3*b**2*f - 40*sqrt(a + b*x)*sqrt(a - b*x)*
a**2*b**3*e - 15*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**3*f*x - 15*sqrt(a + b
*x)*sqrt(a - b*x)*a**2*b**2*c*e*x - 8*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**
2*c*f*x**2 + 60*sqrt(a + b*x)*sqrt(a - b*x)*a*b**4*e*x + 40*sqrt(a + b*x)*
sqrt(a - b*x)*a*b**4*f*x**2 + 40*sqrt(a + b*x)*sqrt(a - b*x)*b**5*e*x**2 +
 30*sqrt(a + b*x)*sqrt(a - b*x)*b**5*f*x**3 + 30*sqrt(a + b*x)*sqrt(a - b*
x)*b**4*c*e*x**3 + 24*sqrt(a + b*x)*sqrt(a - b*x)*b**4*c*f*x**4))/(120*b**
4)
```

3.48 $\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 170

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx = \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\sqrt{ac-bcx} - \frac{(4bB-3aC)(a+bx)^{3/2}(ac-bcx)^{3/2}}{12b^3c} - \frac{C(a+bx)^{5/2}(ac-bcx)^{3/2}}{4b^3c} + \frac{a^2\sqrt{c}\left(4Ab^2+a^2C\right)\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{4b^3}$$

output
```
1/8*(4*A+a^2*C/b^2)*x*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)-1/12*(4*B*b-3*C*a)*
(b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2)/b^3/c-1/4*C*(b*x+a)^(5/2)*(-b*c*x+a*c)^(3
/2)/b^3/c+1/4*a^2*c^(1/2)*(4*A*b^2+C*a^2)*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b
*c*x+a*c)^(1/2))/b^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.72

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{c(a-bx)}\left(b\sqrt{a-bx}\sqrt{a+bx}(-a^2(8B+3Cx)+2b^2x(6A+x(4B+3Cx)))+6a^2(4Ab^2+a^2C)\operatorname{arcta}\right.}{24b^3\sqrt{a-bx}}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c*(a - b*x)]*(b*Sqrt[a - b*x]*Sqrt[a + b*x]*(-(a^2*(8*B + 3*C*x)) +
2*b^2*x*(6*A + x*(4*B + 3*C*x))) + 6*a^2*(4*A*b^2 + a^2*C)*ArcTan[Sqrt[a +
 b*x]/Sqrt[a - b*x]]))/(24*b^3*Sqrt[a - b*x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1189, 83, 646, 40, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{1189}$$

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(Cx^2+A\right)dx+B\int x\sqrt{a+bx}\sqrt{ac-bcx}dx$$

$$\Big\downarrow \text{83}$$

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(Cx^2+A\right)dx-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}$$

$$\Big\downarrow \text{646}$$

$$\frac{1}{4}\left(\frac{a^2C}{b^2}+4A\right)\int \sqrt{a+bx}\sqrt{ac-bcx}dx-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}-\frac{Cx(a+bx)^{3/2}(ac-bcx)^{3/2}}{4b^2c}$$

$$\Big\downarrow \text{ 40}$$

$$\frac{1}{4}\left(\frac{a^2C}{b^2}+4A\right)\left(\frac{1}{2}a^2c\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}dx+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}-\frac{Cx(a+bx)^{3/2}(ac-bcx)^{3/2}}{4b^2c}$$

$$\Big\downarrow \text{ 45}$$

$$\frac{1}{4}\left(\frac{a^2C}{b^2}+4A\right)\left(a^2c\int \frac{1}{\frac{c(a+bx)b}{ac-bcx}+b}d\frac{\sqrt{a+bx}}{\sqrt{ac-bcx}}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}-\frac{Cx(a+bx)^{3/2}(ac-bcx)^{3/2}}{4b^2c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{1}{4}\left(\frac{a^2C}{b^2}+4A\right)\left(\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}\right)-\frac{B(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2c}-\frac{Cx(a+bx)^{3/2}(ac-bcx)^{3/2}}{4b^2c}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

output
```
-1/3*(B*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2))/(b^2*c) - (C*x*(a + b*x)^(3/2
)*(a*c - b*c*x)^(3/2))/(4*b^2*c) + ((4*A + (a^2*C)/b^2)*((x*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*
c - b*c*x]])/b))/4
```

Defintions of rubi rules used

rule 40
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Simp[2*a*c*(m/(2*m + 1))   Int[(a
 + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

rule 1189
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2), x_Symbol] :> Simp[b   Int[x*(d + e*x)^m*(f + g*x)^n, x],
x] + Int[(d + e*x)^m*(f + g*x)^n*(a + c*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, m, n}, x] && EqQ[m, n] && EqQ[e*f + d*g, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

method	result
risch	$\frac{(6C\,b^2x^3+8b^2B\,x^2+12A\,b^2x-3C\,a^2x-8a^2B)(-bx+a)\sqrt{bx+a}\,c}{24b^2\sqrt{-c(bx-a)}}+\frac{a^2\left(4b^2A+a^2C\right)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)c(bx-a)}}{8b^2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(6C\,b^2x^3\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}+12A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c\left(-b^2x^2+a^2\right)}}\right)a^2b^2c+8B\,b^2x^2\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}+3C\,\mathrm{arc}\right.}{24\sqrt{c(-b^2x^2+a^2)}}$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE
)
```

output
```
1/24*(6*C*b^2*x^3+8*B*b^2*x^2+12*A*b^2*x-3*C*a^2*x-8*B*a^2)/b^2*(-b*x+a)*(
b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c+1/8*a^2*(4*A*b^2+C*a^2)/b^2/(b^2*c)^(1/2
)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1
/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.56

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$

$$= \left[\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{-c}\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(6\,Cb^3x^3+8\,Bb^3x^2-8\right.}{48\,b^3}\right.$$

$$-\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)-\left(6\,Cb^3x^3+8\,Bb^3x^2-8\,Ba^2b-3\left(Ca^2b-4\,Ab^3\right)\right.}{24\,b^3}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A),x, algorithm="fri
cas")
```

output
```
[1/48*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x +
a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*x^3 + 8*B*b^3*x^2 -
8*B*a^2*b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3
, -1/24*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*
x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*B
*a^2*b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3]
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{-c(-a+bx)}\sqrt{a+bx}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx = \frac{Ca^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^3} + \frac{Aa^2\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{2\,b} + \frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ax + \frac{\sqrt{-b^2cx^2+a^2c}Ca^2x}{8\,b^2} - \frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Cx}{4\,b^2c} - \frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}B}{3\,b^2c}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A),x, algorithm="max
ima")
```

output
```
1/8*C*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/2*A*a^2*sqrt(c)*arcsin(b*x/a)/b +
1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*x + 1/8*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*x/b^
2 - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*C*x/(b^2*c) - 1/3*(-b^2*c*x^2 + a^2*c)^
(3/2)*B/(b^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(142) = 284$.

Time = 0.61 (sec) , antiderivative size = 527, normalized size of antiderivative = 3.10

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx =$$

$$-\frac{24\left(\frac{2\,ac\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}-\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\right)Aab^2-12\left(\frac{2\,a^2c\log\left(\left|-\sqrt{bx+a}\sqrt{-\right.\right.}{\sqrt{}}\right.}{}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A),x, algorithm="gia
c")
```

output
```
-1/24*(24*(2*a*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a
*c)))/sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*A*a*b^2 - 12*(2
*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*B*a*b - 12*(2
*a^2*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt
(-c) + sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*(b*x - 2*a))*A*b^2 + 4*(6*
a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(
-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*
x + a))*C*a + 4*(6*a^3*c*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)
*c + 2*a*c)))/sqrt(-c) - ((2*b*x - 5*a)*(b*x + a) + 9*a^2)*sqrt(-(b*x + a)
*c + 2*a*c)*sqrt(b*x + a))*B*b - (18*a^4*c*log(abs(-sqrt(b*x + a)*sqrt(-c)
 + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c) - (39*a^3 - (2*(3*b*x - 10*a)*(b*
x + a) + 43*a^2)*(b*x + a))*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a))*C)/b
^3
```

Mupad [B] (verification not implemented)

Time = 15.29 (sec) , antiderivative size = 876, normalized size of antiderivative = 5.15

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
((C*a^4*c^8*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a^(
1/2))) - (C*a^4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x)^(1
/2) - a^(1/2))^15) - (35*C*a^4*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/
(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*C*a^4*c^6*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*C*a^4*c^5*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7) + (715*C
*a^4*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^(1/2) - a^(1
/2))^9) - (273*C*a^4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(2*((a +
b*x)^(1/2) - a^(1/2))^11) + (35*C*a^4*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^13)/(2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^3*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 + (8*b^3*c^7*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28*b^3
*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4
+ (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^
(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(
1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((
a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14) + (A*x*(a*c - b*c*x)^(1/2
)*(a + b*x)^(1/2))/2 - (B*(a^2 - b^2*x^2)*(a*c - b*c*x)^(1/2)*(a + b*x)...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.95

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{c}\left(-6asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^4c-24asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^3b^2-8\sqrt{bx+a}\,\sqrt{-bx+a}\,a^2b^2-3\sqrt{bx+a}\,\sqrt{-bx+a}\,a^2}{24b^3}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
(sqrt(c)*( - 6*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*c - 24*asin(sqrt
(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**2 - 8*sqrt(a + b*x)*sqrt(a - b*x)*a**
2*b**2 - 3*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b*c*x + 12*sqrt(a + b*x)*sqrt(
a - b*x)*a*b**3*x + 8*sqrt(a + b*x)*sqrt(a - b*x)*b**4*x**2 + 6*sqrt(a + b
*x)*sqrt(a - b*x)*b**3*c*x**3))/(24*b**3)
```

3.49 $\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}\,(A+Bx+Cx^2)}{e+fx}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 316

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx \\
&= \frac{(2Abf^2+(2be+af)(Ce-Bf))\sqrt{a+bx}\sqrt{ac-bcx}}{2bf^3} \\
&\quad - \frac{(Ce-Bf)(a+bx)^{3/2}\sqrt{ac-bcx}}{2bf^2} - \frac{C(a+bx)^{3/2}(ac-bcx)^{3/2}}{3b^2cf} \\
&\quad - \frac{\sqrt{c}(a^2f^2(Ce-Bf)-2b^2(Ce^3-ef(Be-Af)))\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{bf^4} \\
&\quad - \frac{2\sqrt{c}\sqrt{be-af}\sqrt{be+af}(Ce^2-Bef+Af^2)\arctan\left(\frac{\sqrt{c}\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{ac-bcx}}\right)}{f^4}
\end{aligned}
$$

output
```
1/2*(2*A*b*f^2+(a*f+2*b*e)*(-B*f+C*e))*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b/
f^3-1/2*(-B*f+C*e)*(b*x+a)^(3/2)*(-b*c*x+a*c)^(1/2)/b/f^2-1/3*C*(b*x+a)^(3
/2)*(-b*c*x+a*c)^(3/2)/b^2/c/f-c^(1/2)*(a^2*f^2*(-B*f+C*e)-2*b^2*(C*e^3-e*
f*(-A*f+B*e)))*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b/f^4-2*c^
(1/2)*(-a*f+b*e)^(1/2)*(a*f+b*e)^(1/2)*(A*f^2-B*e*f+C*e^2)*arctan(c^(1/2)*
(a*f+b*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(-b*c*x+a*c)^(1/2))/f^4
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx$$
$$= \frac{\sqrt{c(a-bx)}\left(\frac{f\sqrt{a+bx}(-2a^2Cf^2+b^2(3f(-2Be+2Af+Bfx)+C(6e^2-3efx+2f^2x^2)))}{b^2} + \frac{6(a^2f^2(-Ce+Bf)+2b^2(Ce^3+ef(-Be+Af)}{b\sqrt{a-bx}}\right.}{6f^4}$$

input
```
Integrate[(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2))/(e + f*x),x]
```

output
```
(Sqrt[c*(a - b*x)]*((f*Sqrt[a + b*x]*(-2*a^2*C*f^2 + b^2*(3*f*(-2*B*e + 2*
A*f + B*f*x) + C*(6*e^2 - 3*e*f*x + 2*f^2*x^2))))/b^2 + (6*(a^2*f^2*(-(C*e
) + B*f) + 2*b^2*(C*e^3 + e*f*(-(B*e) + A*f)))*ArcTan[Sqrt[a + b*x]/Sqrt[a
 - b*x]])/(b*Sqrt[a - b*x]) - (12*(b^2*e^2 - a^2*f^2)*(C*e^2 + f*(-(B*e) +
 A*f))*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*
x])])/(Sqrt[b*e - a*f]*Sqrt[b*e + a*f]*Sqrt[a - b*x])))/(6*f^4)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2113, 2185, 27, 682, 25, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int \frac{\sqrt{a^2c-b^2cx^2}(Cx^2+Bx+A)}{e+fx}dx}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(-\frac{\int -\frac{3b^2cf(Af-(Ce-Bf)x)\sqrt{a^2c-b^2cx^2}}{e+fx}dx}{3b^2cf^2}-\frac{C\left(a^2c-b^2cx^2\right)^{3/2}}{3b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int \frac{(Af-(Ce-Bf)x)\sqrt{a^2c-b^2cx^2}}{e+fx}dx}{f}-\frac{C\left(a^2c-b^2cx^2\right)^{3/2}}{3b^2cf}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 682

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(Af^2-Bef+Ce^2\right)-fx(Ce-Bf)\right)}{2f^2}-\frac{\int -\frac{b^2c^2\left(f\left(Ce^2-f(Be-2Af)\right)a^2+\left(2Ab^2ef^2+(Ce-Bf)\left(2b^2e^2-a^2f^2\right)\right)x\right)}{(e+fx)\sqrt{a^2c-b^2cx^2}}\cdots}{2b^2cf^2}}{f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 25

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\int \frac{b^2c^2\left(f\left(Ce^2-f(Be-2Af)\right)a^2+\left(2Ab^2ef^2+(Ce-Bf)\left(2b^2e^2-a^2f^2\right)\right)x\right)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{2b^2cf^2}+\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(Af^2-Bef+Ce^2\right)-fx(Ce-Bf)\right)}{2f^2}\cdots}{f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{c\int \frac{f\left(Ce^2-f(Be-2Af)\right)a^2+\left(2Ab^2ef^2+(Ce-Bf)\left(2b^2e^2-a^2f^2\right)\right)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{2f^2}+\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(Af^2-Bef+Ce^2\right)-fx(Ce-Bf)\right)}{2f^2}}{f}-\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 719

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{c\left(\frac{\left(\left(2b^2e^2-a^2f^2\right)(Ce-Bf)+2Ab^2ef^2\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{2(be-af)(af+be)\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}\right)}{2f^2}+\cdots}{f}\cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 224

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{c\left(\frac{\left(\left(2b^2e^2-a^2f^2\right)(Ce-Bf)+2Ab^2ef^2\right)\int \frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{f}-\frac{2(be-af)(af+be)\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-\cdots}}}{f}\right.}{2f^2}}{f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 216

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{c\left(\frac{\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(\left(2b^2e^2-a^2f^2\right)(Ce-Bf)+2Ab^2ef^2\right)}{b\sqrt{c}f}-\frac{2(be-af)(af+be)\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}\right.}{2f^2}}{f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 488

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{c\left(\frac{2(be-af)(af+be)\left(Af^2-Bef+Ce^2\right)\int \frac{1}{-b^2ce^2+a^2cf^2-\frac{\left(cfa^2+b^2cex\right)^2}{a^2c-b^2cx^2}}d\frac{cfa^2+b^2cex}{\sqrt{a^2c-b^2cx^2}}}{f}+\frac{\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(\left(2b^2e^2-a^{\cdots}\right.\right.}{b\sqrt{c}f}\right.}{2f^2}}{f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$\downarrow$ 217

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{c\left(\frac{\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(\left(2b^2e^2-a^2f^2\right)(Ce-Bf)+2Ab^2ef^2\right)}{b\sqrt{c}f}-\frac{2(be-af)(af+be)\left(Af^2-Bef+Ce^2\right)\arctan\left(\frac{a^2cf+b^{\cdots}}{\sqrt{c}\sqrt{a^2c-b^2cx^2}\cdots}\right.}{\sqrt{c}f\sqrt{b^2e^2-a^2f^2}}\right.}{2f^2}}{f}\right.}{\sqrt{a^2c-b^2cx^2}}$$

input
```
Int[(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2))/(e + f*x),x]
```

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(-1/3*(C*(a^2*c - b^2*c*x^2)^(3/2))/(b^2*
c*f) + (((2*(C*e^2 - B*e*f + A*f^2) - f*(C*e - B*f)*x)*Sqrt[a^2*c - b^2*c*
x^2])/(2*f^2) + (c*(((2*A*b^2*e*f^2 + (C*e - B*f)*(2*b^2*e^2 - a^2*f^2))*A
rcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f) - (2*(b*e - a*
f)*(b*e + a*f)*(C*e^2 - B*e*f + A*f^2)*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[
c]*Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f*Sqrt[b^2*
e^2 - a^2*f^2])))/(2*f^2))/f))/Sqrt[a^2*c - b^2*c*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.43

method	result
risch	$\frac{\left(2C\,x^{2} f^{2} b^{2}+3B\,b^{2} f^{2} x-3C\,b^{2} e f x+6A\,b^{2} f^{2}-6eB\,b^{2} f-2a^{2} C\,f^{2}+6e^{2} b^{2} C\right)\left(-bx+a\right)\sqrt{bx+a}\,c}{6b^{2} f^{3} \sqrt{-c\left(bx-a\right)}}+\frac{\left(\frac{\left(2A\,b^{2} e\,f^{2}+a^{2} B\,f^{3}-2e^{2} B\,b^{2} f-C \cdots\right.}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x,method=_RETUR
NVERBOSE)
```

output
```
1/6*(2*C*b^2*f^2*x^2+3*B*b^2*f^2*x-3*C*b^2*e*f*x+6*A*b^2*f^2-6*B*b^2*e*f-2
*C*a^2*f^2+6*C*b^2*e^2)/b^2*(-b*x+a)*(b*x+a)^(1/2)/f^3/(-c*(b*x-a))^(1/2)*
c+1/2/f^3*((2*A*b^2*e*f^2+B*a^2*f^3-2*B*b^2*e^2*f-C*a^2*e*f^2+2*C*b^2*e^3)
/f/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))-2*(A*a^2
*f^4-A*b^2*e^2*f^2-B*a^2*e*f^3+B*b^2*e^3*f+C*a^2*e^2*f^2-C*b^2*e^4)/f^2/(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2*c*e/f*(x
+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(x+e/f)^2*b^2*c+2*b^2*c*e/f*(x+e
/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f)))*(-(b*x+a)*c*(b*x-a))^(1/2)/(
b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x, algori
thm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx$$
$$=\int \frac{\sqrt{-c(-a+bx)}\sqrt{a+bx}(A+Bx+Cx^2)}{e+fx}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2)*(C*x**2+B*x+A)/(f*x+e),x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2)/(e + f*x), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 71.00 (sec) , antiderivative size = 24910, normalized size of antiderivative = 78.83

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Too large to display}$$

input
```
int(((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x),x)
```

output
```
(atan((((C*b^2*c*e^4 - C*a^2*c*e^2*f^2)*(((C*b^2*c*e^4 - C*a^2*c*e^2*f^2)*
((4096*(4*C^2*a^10*c^8*f^14 + 9*C^2*a^2*b^8*c^8*e^8*f^6 - 18*C^2*a^4*b^6*c
^8*e^6*f^8 + 18*C^2*a^6*b^4*c^8*e^4*f^10 - 11*C^2*a^8*b^2*c^8*e^2*f^12))/(
b^8*e^4*f^12) + ((C*b^2*c*e^4 - C*a^2*c*e^2*f^2)*((4096*(12*C*a^(9/2)*b^2*
c^5*f^15*(a*c)^(5/2) + 30*C*a^(3/2)*b^6*c^6*e^4*f^11*(a*c)^(3/2) - 24*C*a^
(5/2)*b^4*c^5*e^2*f^13*(a*c)^(5/2) - 18*C*a^(7/2)*b^4*c^6*e^2*f^13*(a*c)^(
3/2)))/(b^8*e^4*f^12) - ((C*b^2*c*e^4 - C*a^2*c*e^2*f^2)*((4096*(7*a^4*b^4
*c^7*f^16 - 9*a^2*b^6*c^7*e^2*f^14))/(b^8*e^4*f^12) + (16384*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^16*(a*c)^(5/2) - 6*a^(3/2)*b^4
*c^5*e^2*f^14*(a*c)^(3/2)))/(b^7*e^5*f^11*((a + b*x)^(1/2) - a^(1/2))) + (
4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^16 - 9*a^2*b^
6*c^6*e^2*f^14))/(b^8*e^4*f^12*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^4*(a^2*
c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(6
8*C*a^(9/2)*b^2*c^4*f^15*(a*c)^(5/2) + 90*C*a^(3/2)*b^6*c^5*e^4*f^11*(a*c)
^(3/2) - 96*C*a^(5/2)*b^4*c^4*e^2*f^13*(a*c)^(5/2) - 62*C*a^(7/2)*b^4*c^5*
e^2*f^13*(a*c)^(3/2)))/(b^8*e^4*f^12*((a + b*x)^(1/2) - a^(1/2))^2) + (163
84*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(10*C*a^8*c^7*f^15 + 22*C*a^4*b^4*c
^7*e^4*f^11 - 32*C*a^6*b^2*c^7*e^2*f^13))/(b^7*e^5*f^11*((a + b*x)^(1/2) -
 a^(1/2)))))/(f^4*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))*(5*C^2*a^(13/2)*c^5*e^2*f^12*(a*c)^(5/2) + 3*C^2*a^...
```

Reduce [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 3341, normalized size of antiderivative = 10.57

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{e+fx}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e),x)
```

output
```
(sqrt(c)*( - 6*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**2*f**4 + 6*as
in(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b*c*e*f**3 - 18*asin(sqrt(a - b*x
)/(sqrt(a)*sqrt(2)))*a**2*b**3*e*f**3 + 6*asin(sqrt(a - b*x)/(sqrt(a)*sqrt
(2)))*a**2*b**2*c*e**2*f**2 - 12*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b
**3*c*e**3*f + 12*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*b**5*e**3*f - 12*a
sin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*b**4*c*e**4 - 12*sqrt(f)*sqrt(a)*sqrt
(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*
sqrt(a*f - b*e)*sqrt(2)*atan((tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)
*a*f + tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*b*e)/(sqrt(2*sqrt(f)*s
qrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)))*a*b**2*f**
2 + 12*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*
a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2)*atan((tan(asin(sqrt(a -
 b*x)/(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2))
)/2)*b*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*s
qrt(a*f + b*e)))*b**3*e*f - 12*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt
(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2)
*atan((tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt(a
- b*x)/(sqrt(a)*sqrt(2)))/2)*b*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*
sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)))*b**2*c*e**2 - 12*sqrt(2*sqrt(f)*s
qrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*atan((tan...
```

3.50 $\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 444

$$
\begin{aligned}
&\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx \\
&= \frac{(a^2Cf^2+abf(3Ce-2Bf)-2b^2(3Ce^2-f(2Be-Af)))\sqrt{a+bx}\sqrt{ac-bcx}}{2bf^3(be-af)} \\
&\quad -\frac{(a^2Cf^2-b^2(3Ce^2-2f(Be-Af)))(a+bx)^{3/2}\sqrt{ac-bcx}}{2bf^2(be-af)(be+af)} \\
&\quad +\frac{(Ce^2-Bef+Af^2)(a+bx)^{3/2}(ac-bcx)^{3/2}}{cf(be-af)(be+af)(e+fx)} \\
&\quad +\frac{\sqrt{c}(a^2Cf^2-2b^2(3Ce^2-f(2Be-Af)))\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{bf^4} \\
&\quad -\frac{2\sqrt{c}(a^2f^2(2Ce-Bf)-b^2(3Ce^3-ef(2Be-Af)))\arctan\left(\frac{\sqrt{c}\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{ac-bcx}}\right)}{f^4\sqrt{be-af}\sqrt{be+af}}
\end{aligned}
$$

output
```
1/2*(a^2*C*f^2+a*b*f*(-2*B*f+3*C*e)-2*b^2*(3*C*e^2-f*(-A*f+2*B*e)))*(b*x+a
)^(1/2)*(-b*c*x+a*c)^(1/2)/b/f^3/(-a*f+b*e)-1/2*(a^2*C*f^2-b^2*(3*C*e^2-2*
f*(-A*f+B*e)))*(b*x+a)^(3/2)*(-b*c*x+a*c)^(1/2)/b/f^2/(-a*f+b*e)/(a*f+b*e)
+(A*f^2-B*e*f+C*e^2)*(b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2)/c/f/(-a*f+b*e)/(a*f+
b*e)/(f*x+e)+c^(1/2)*(a^2*C*f^2-2*b^2*(3*C*e^2-f*(-A*f+2*B*e)))*arctan(c^(
1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b/f^4-2*c^(1/2)*(a^2*f^2*(-B*f+2*C*
e)-b^2*(3*C*e^3-e*f*(-A*f+2*B*e)))*arctan(c^(1/2)*(a*f+b*e)^(1/2)*(b*x+a)^
(1/2)/(-a*f+b*e)^(1/2)/(-b*c*x+a*c)^(1/2))/f^4/(-a*f+b*e)^(1/2)/(a*f+b*e)^
(1/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \frac{\sqrt{c(a-bx)}\left(\frac{f\sqrt{a+bx}(2f(2Be-Af+Bfx)+C(-6e^2-3efx+f^2x^2))}{2(e+fx)} - \frac{(-a^2Cf^2+2b^2(3Ce^2+f(-2Be+Af)))\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b\sqrt{a-bx}} + \frac{2(\cdots}{\cdots}\right.}{f^4}$$

input
```
Integrate[(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2))/(e + f*x)^2,
x]
```

output
```
(Sqrt[c*(a - b*x)]*((f*Sqrt[a + b*x]*(2*f*(2*B*e - A*f + B*f*x) + C*(-6*e^
2 - 3*e*f*x + f^2*x^2)))/(2*(e + f*x)) - ((-(a^2*C*f^2) + 2*b^2*(3*C*e^2 +
 f*(-2*B*e + A*f)))*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(b*Sqrt[a - b*x])
 + (2*(a^2*f^2*(-2*C*e + B*f) + b^2*(3*C*e^3 + e*f*(-2*B*e + A*f)))*ArcTan
[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/(Sqrt[b
*e - a*f]*Sqrt[b*e + a*f]*Sqrt[a - b*x])))/f^4
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2113, 2182, 27, 682, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(A+Bx+Cx^2\right)}{(e+fx)^2}\,dx$$

$$\Big\downarrow \ 2113$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int \frac{\sqrt{a^2c-b^2cx^2}\left(Cx^2+Bx+A\right)}{(e+fx)^2}dx}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \ 2182$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int \frac{c\left((Ce-Bf)a^2+Ab^2e-\left(Cfa^2+b^2\left(-\frac{3Ce^2}{f}+2Be-2Af\right)\right)x\right)\sqrt{a^2c-b^2cx^2}}{e+fx}dx}{c\left(b^2e^2-a^2f^2\right)}+\frac{f\left(a^2c-b^2cx^2\right)^{3/2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)\left(b^2e^2-a^2f^2\right)}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int \frac{\left((Ce-Bf)a^2+Ab^2e-\left(Cfa^2+b^2\left(-\frac{3Ce^2}{f}+2Be-2Af\right)\right)x\right)\sqrt{a^2c-b^2cx^2}}{e+fx}dx}{b^2e^2-a^2f^2}+\frac{f\left(a^2c-b^2cx^2\right)^{3/2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)\left(b^2e^2-a^2f^2\right)}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \ 682$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(a^2f(2Ce-Bf)-\frac{b^2e\left(3Ce^2-f(2Be-Af)\right)}{f}\right)-fx\left(a^2Cf+b^2\left(-2Af+2Be-\frac{3Ce^2}{f}\right)\right)\right)}{2f^2}-\int \frac{b^2c^2\left(b^2e^2-a^2f^2\right)\left(a \cdots\right.}{\cdots}}{b^2e^2-a^2f^2}\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(a^2f(2Ce-Bf)-\frac{b^2e\left(3Ce^2-f(2Be-Af)\right)}{f}\right)-fx\left(a^2Cf+b^2\left(-2Af+2Be-\frac{3Ce^2}{f}\right)\right)\right)}{2f^2}-\frac{c(be-af)(af+be)\int\frac{a^2\cdots}{\cdots}}{\cdots}}{b^2e^2-a^2f^2}\cdots\right.}{\sqrt{a^2c-b^2cx^2}\cdots}$$

$\downarrow$ 719

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(a^2f(2Ce-Bf)-\frac{b^2e\left(3Ce^2-f(2Be-Af)\right)}{f}\right)-fx\left(a^2Cf+b^2\left(-2Af+2Be-\frac{3Ce^2}{f}\right)\right)\right)}{2f^2}-\frac{c(be-af)(af+be)\left(\frac{2(a\cdots}{\cdots}\right.}{\cdots}}{b^2e^2-\cdots}\cdots\right.}{\sqrt{\cdots}}$$

$\downarrow$ 224

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(a^2f(2Ce-Bf)-\frac{b^2e\left(3Ce^2-f(2Be-Af)\right)}{f}\right)-fx\left(a^2Cf+b^2\left(-2Af+2Be-\frac{3Ce^2}{f}\right)\right)\right)}{2f^2}-\frac{c(be-af)(af+be)\left(\frac{2(a\cdots}{\cdots}\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}$$

$\downarrow$ 216

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(a^2f(2Ce-Bf)-\frac{b^2e\left(3Ce^2-f(2Be-Af)\right)}{f}\right)-fx\left(a^2Cf+b^2\left(-2Af+2Be-\frac{3Ce^2}{f}\right)\right)\right)}{2f^2}-\frac{c(be-af)(af+be)\left(\frac{2(a\cdots}{\cdots}\right.}{\cdots}}{b^2e^2\cdots}\cdots\right.}{\cdots}$$

$\downarrow$ 488

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(a^2f(2Ce-Bf)-\frac{b^2e\left(3Ce^2-f(2Be-Af)\right)}{f}\right)-fx\left(a^2Cf+b^2\left(-2Af+2Be-\frac{3Ce^2}{f}\right)\right)\right)}{2f^2}-\frac{c(be-af)(af+be)\left(-\frac{2(\cdots}{\cdots}\cdots\right.}{\cdots}}{\cdots}\right.$$

$\Big\downarrow$ 217

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2\left(a^2f(2Ce-Bf)-\frac{b^2e\left(3Ce^2-f(2Be-Af)\right)}{f}\right)-fx\left(a^2Cf+b^2\left(-2Af+2Be-\frac{3Ce^2}{f}\right)\right)\right)}{2f^2}-\frac{c(be-af)(af+be)\left(\frac{2(a\cdots}{\cdots}\cdots\right.}{\cdots}}{\cdots}\right.$$

input
```
Int[(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2))/(e + f*x)^2,x]
```

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*((f*(A + (e*(C*e - B*f))/f^2)*(a^2*c - b^
2*c*x^2)^(3/2))/(c*(b^2*e^2 - a^2*f^2)*(e + f*x)) + (((2*(a^2*f*(2*C*e - B
*f) - (b^2*e*(3*C*e^2 - f*(2*B*e - A*f)))/f) - f*(a^2*C*f + b^2*(2*B*e - (
3*C*e^2)/f - 2*A*f))*x)*Sqrt[a^2*c - b^2*c*x^2])/(2*f^2) - (c*(b*e - a*f)*
(b*e + a*f)*(-(((a^2*C*f^2 - 2*b^2*(3*C*e^2 - f*(2*B*e - A*f)))*ArcTan[(b*
Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f)) + (2*(a^2*f^2*(2*C*e -
 B*f) - b^2*(3*C*e^3 - e*f*(2*B*e - A*f)))*ArcTan[(a^2*c*f + b^2*c*e*x)/(S
qrt[c]*Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f*Sqrt[
b^2*e^2 - a^2*f^2])))/(2*f^3))/(b^2*e^2 - a^2*f^2)))/Sqrt[a^2*c - b^2*c*x^
2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.59

method	result
risch	$\frac{(Cfx+2Bf-4Ce)(-bx+a)\sqrt{bx+a}\,c}{2f^{3}\sqrt{-c(bx-a)}}-\frac{\left(\frac{\left(2A\,b^{2}f^{2}-4eB\,b^{2}f-a^{2}C\,f^{2}+6e^{2}b^{2}C\right)\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+a^{2}c}}\right)}{f\sqrt{b^{2}c}}+\frac{2\left(2A\,b^{2}e\,f^{2}+a^{2}B\,f^{3}-3e^{2}B\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x,method=_RET
URNVERBOSE)
```

output
```
1/2*(C*f*x+2*B*f-4*C*e)*(-b*x+a)*(b*x+a)^(1/2)/f^3/(-c*(b*x-a))^(1/2)*c-1/
2/f^3*((2*A*b^2*f^2-4*B*b^2*e*f-C*a^2*f^2+6*C*b^2*e^2)/f/(b^2*c)^(1/2)*arc
tan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))+2/f^2*(2*A*b^2*e*f^2+B*a^2*f
^3-3*B*b^2*e^2*f-2*C*a^2*e*f^2+4*C*b^2*e^3)/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2
)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2*c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)
/f^2)^(1/2)*(-(x+e/f)^2*b^2*c+2*b^2*c*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)
^(1/2))/(x+e/f))-2*(A*a^2*f^4-A*b^2*e^2*f^2-B*a^2*e*f^3+B*b^2*e^3*f+C*a^2*
e^2*f^2-C*b^2*e^4)/f^3*(-1/c/(a^2*f^2-b^2*e^2)*f^2/(x+e/f)*(-(x+e/f)^2*b^2
*c+2*b^2*c*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+b^2*e*f/(a^2*f^2-b^2
*e^2)/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2*
c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(x+e/f)^2*b^2*c+2*b^2*c*
e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f))))*(-(b*x+a)*c*(b*x-a)
)^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)*c
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \int \frac{\sqrt{-c(-a+bx)}\sqrt{a+bx}\,(A+Bx+Cx^2)}{(e+fx)^2}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**2,x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2)/(e + f*x)**2
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx$$

$$= \frac{\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\left(\frac{(bx+a)C}{f^2} - \frac{4\,Cbef^6+Caf^7-2\,Bbf^7}{f^9}\right) - \frac{(6\,Cb^2\sqrt{-c}e^2-4\,Bb^2\sqrt{-c}ef-Ca^2\sqrt{-c}f^2+2\,Ab^2\sqrt{-c}f^2}{f^4}}{}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x, algo
rithm="giac")
```

output
```
1/2*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*((b*x + a)*C/f^2 - (4*C*b*e*
f^6 + C*a*f^7 - 2*B*b*f^7)/f^9) - (6*C*b^2*sqrt(-c)*e^2 - 4*B*b^2*sqrt(-c)
*e*f - C*a^2*sqrt(-c)*f^2 + 2*A*b^2*sqrt(-c)*f^2)*log((sqrt(b*x + a)*sqrt(
-c) - sqrt(-(b*x + a)*c + 2*a*c))^2)/f^4 - 4*(3*C*b^3*sqrt(-c)*c*e^3 - 2*B
*b^3*sqrt(-c)*c*e^2*f - 2*C*a^2*b*sqrt(-c)*c*e*f^2 + A*b^3*sqrt(-c)*c*e*f^
2 + B*a^2*b*sqrt(-c)*c*f^3)*arctan(-1/2*(2*b*c*e - (sqrt(b*x + a)*sqrt(-c)
 - sqrt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^2 + a^2*f^2)*c))/(sqrt(-b
^2*e^2 + a^2*f^2)*c*f^4) - 8*(C*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x +
 a)*c + 2*a*c))^2*sqrt(-c)*c*e^3 - B*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(
b*x + a)*c + 2*a*c))^2*sqrt(-c)*c*e^2*f - 2*C*a^2*b^2*sqrt(-c)*c^2*e^2*f +
 A*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c*
e*f^2 + 2*B*a^2*b^2*sqrt(-c)*c^2*e*f^2 - 2*A*a^2*b^2*sqrt(-c)*c^2*f^3)/((4
*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*c*e - (sqrt(b*x
 + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*f - 4*a^2*c^2*f)*f^4))/b
```

Mupad [B] (verification not implemented)

Time = 83.80 (sec) , antiderivative size = 21612, normalized size of antiderivative = 48.68

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
int(((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^2,x)
```

output
```
((16*B*a^(1/2)*c*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(f^2*(
(a + b*x)^(1/2) - a^(1/2))^6) + (16*B*a^(1/2)*c^3*(a*c)^(1/2)*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2)/(f^2*((a + b*x)^(1/2) - a^(1/2))^2) + (32*B*a^(
1/2)*c^2*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(f^2*((a + b*x
)^(1/2) - a^(1/2))^4) + (4*B*a^2*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/
(b*e*f*((a + b*x)^(1/2) - a^(1/2))) - (4*B*a^2*c*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^7)/(b*e*f*((a + b*x)^(1/2) - a^(1/2))^7) + (36*B*a^2*c^3*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*f*((a + b*x)^(1/2) - a^(1/2))^3) - (
36*B*a^2*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*f*((a + b*x)^(1/2
) - a^(1/2))^5))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^(1/2) -
 a^(1/2))^8 + c^4 + (4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)
^(1/2) - a^(1/2))^6 + (4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a +
b*x)^(1/2) - a^(1/2))^2 + (6*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((
a + b*x)^(1/2) - a^(1/2))^4 - (4*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1/2) - a^(1/2))^7) + (4*a^(1/2)*c^3*f
*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) -
a^(1/2))) - (4*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) + (4*a^(1/2)*c^2*f*(a*c)^(1/2)*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) -
 ((66*C*a^2*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*f^2*((a + b*x...
```

Reduce [B] (verification not implemented)

Time = 13.03 (sec) , antiderivative size = 10007, normalized size of antiderivative = 22.54

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^2,x)
```

output
```
(sqrt(c)*( - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**5*c*e*f**5 - 2*asi
n(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**5*c*f**6*x + 4*asin(sqrt(a - b*x)/(s
qrt(a)*sqrt(2)))*a**4*b**2*e*f**5 + 4*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2))
)*a**4*b**2*f**6*x - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b*c*e**2
*f**4 - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b*c*e*f**5*x - 4*asin
(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**3*e**2*f**4 - 4*asin(sqrt(a - b*
x)/(sqrt(a)*sqrt(2)))*a**3*b**3*e*f**5*x + 14*asin(sqrt(a - b*x)/(sqrt(a)*
sqrt(2)))*a**3*b**2*c*e**3*f**3 + 14*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))
*a**3*b**2*c*e**2*f**4*x - 12*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b
**4*e**3*f**3 - 12*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b**4*e**2*f*
*4*x + 14*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b**3*c*e**4*f**2 + 14
*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b**3*c*e**3*f**3*x + 4*asin(sq
rt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b**5*e**4*f**2 + 4*asin(sqrt(a - b*x)/(sq
rt(a)*sqrt(2)))*a*b**5*e**3*f**3*x - 12*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2
)))*a*b**4*c*e**5*f - 12*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b**4*c*e*
*4*f**2*x + 8*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*b**6*e**5*f + 8*asin(s
qrt(a - b*x)/(sqrt(a)*sqrt(2)))*b**6*e**4*f**2*x - 12*asin(sqrt(a - b*x)/(
sqrt(a)*sqrt(2)))*b**5*c*e**6 - 12*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*b
**5*c*e**5*f*x - 4*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*
sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2)*atan((ta...
```

3.51 $\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 390

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx$$
$$= \frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(2(be-af)(be+af)(3Ce-Bf)-f^2\left(2a^2Cf+b^2\left(Be-\frac{3Ce^2}{f}-Af\right)\right)x\right)}{2f^3\left(b^2e^2-a^2f^2\right)(e+fx)}$$
$$+\frac{(Ce^2-Bef+Af^2)\sqrt{a+bx}\sqrt{ac-bcx}(a^2-b^2x^2)}{2f\left(b^2e^2-a^2f^2\right)(e+fx)^2}$$
$$+\frac{2b\sqrt{c}(3Ce-Bf)\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{f^4}$$
$$-\frac{\sqrt{c}(2a^4Cf^4+2b^4e^3(3Ce-Bf)-a^2b^2f^2(9Ce^2-f(3Be-Af)))\arctan\left(\frac{\sqrt{c}\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{ac-bcx}}\right)}{f^4(be-af)^{3/2}(be+af)^{3/2}}$$

output
```
1/2*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(2*(-a*f+b*e)*(a*f+b*e)*(-B*f+3*C*e)-
f^2*(2*a^2*C*f+b^2*(B*e-3*C*e^2/f-A*f))*x)/f^3/(-a^2*f^2+b^2*e^2)/(f*x+e)+
1/2*(A*f^2-B*e*f+C*e^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(-b^2*x^2+a^2)/f/
(-a^2*f^2+b^2*e^2)/(f*x+e)^2+2*b*c^(1/2)*(-B*f+3*C*e)*arctan(c^(1/2)*(b*x+
a)^(1/2)/(-b*c*x+a*c)^(1/2))/f^4-c^(1/2)*(2*a^4*C*f^4+2*b^4*e^3*(-B*f+3*C*
e)-a^2*b^2*f^2*(9*C*e^2-f*(-A*f+3*B*e)))*arctan(c^(1/2)*(a*f+b*e)^(1/2)*(b
*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(-b*c*x+a*c)^(1/2))/f^4/(-a*f+b*e)^(3/2)/(a*f
+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx$$
$$= \frac{\sqrt{c(a-bx)}\left(-\frac{f\sqrt{a+bx}(a^2f^2(f(Be+Af+2Bfx)-C(5e^2+8efx+2f^2x^2))+b^2e(Af^3x-Bef(2e+3fx)+Ce(6e^2+9efx+2f^2x^2)))}{2(-be+af)(be+af)(e+fx)^2}+\frac{2b}{}\right.}{f^4}$$

input
```
Integrate[(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2))/(e + f*x)^3,
x]
```

output
```
(Sqrt[c*(a - b*x)]*(-1/2*(f*Sqrt[a + b*x]*(a^2*f^2*(f*(B*e + A*f + 2*B*f*x
) - C*(5*e^2 + 8*e*f*x + 2*f^2*x^2)) + b^2*e*(A*f^3*x - B*e*f*(2*e + 3*f*x
) + C*e*(6*e^2 + 9*e*f*x + 2*f^2*x^2))))/((-(b*e) + a*f)*(b*e + a*f)*(e +
f*x)^2) + (2*b*(3*C*e - B*f)*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/Sqrt[a -
 b*x] - ((2*a^4*C*f^4 + 2*b^4*e^3*(3*C*e - B*f) - a^2*b^2*f^2*(9*C*e^2 + f
*(-3*B*e + A*f)))*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*
Sqrt[a - b*x])])/((b*e - a*f)^(3/2)*(b*e + a*f)^(3/2)*Sqrt[a - b*x])))/f^4
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2113, 2182, 27, 681, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx$$
$$\Big\downarrow \text{2113}$$
$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\int \frac{\sqrt{a^2c-b^2cx^2}(Cx^2+Bx+A)}{(e+fx)^3}dx}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 2182$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int \frac{c\left(2\left((Ce-Bf)a^2+Ab^2e\right)-\left(2Cfa^2+b^2\left(-\frac{3Ce^2}{f}+Be-Af\right)\right)x\right)\sqrt{a^2c-b^2cx^2}}{(e+fx)^2}dx}{2c(b^2e^2-a^2f^2)}+\frac{f\left(a^2c-b^2cx^2\right)^{3/2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\int \frac{\left(2\left((Ce-Bf)a^2+Ab^2e\right)-\left(2Cfa^2+b^2\left(-\frac{3Ce^2}{f}+Be-Af\right)\right)x\right)\sqrt{a^2c-b^2cx^2}}{(e+fx)^2}dx}{2(b^2e^2-a^2f^2)}+\frac{f\left(a^2c-b^2cx^2\right)^{3/2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}\right)}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 681$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2(be-af)(af+be)(3Ce-Bf)-f^2x\left(2a^2Cf+b^2\left(-Af+Be-\frac{3Ce^2}{f}\right)\right)\right)}{f^3(e+fx)}-\frac{\int \frac{2c\left(a^2f^2\left(2Cfa^2+b^2\left(-\frac{3Ce^2}{f}+Be-Af \cdots\right.\right.\right.}{f(e+fx)\sqrt{a \cdots}}}{2f \cdots}}{2(b^2e^2-a^2f^2)} \cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2(be-af)(af+be)(3Ce-Bf)-f^2x\left(2a^2Cf+b^2\left(-Af+Be-\frac{3Ce^2}{f}\right)\right)\right)}{f^3(e+fx)}-\frac{c\int \frac{a^2f^2\left(2Cfa^2+b^2\left(-\frac{3Ce^2}{f}+Be-Af\right) \cdots\right.}{(e+fx)\sqrt{a^2c \cdots}}}{f^3}}{2(b^2e^2-a^2f^2)} \cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$\Big\downarrow \; 719$$

$$\frac{\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\sqrt{a^2c-b^2cx^2}\left(2(be-af)(af+be)(3Ce-Bf)-f^2x\left(2a^2Cf+b^2\left(-Af+Be-\frac{3Ce^2}{f}\right)\right)\right)}{f^3(e+fx)}-\frac{c\left(\frac{\left(2a^4Cf^4-a^2b^2f^2\left(9Ce^2-f(3Be-Af)\right) \cdots\right.}{\cdots}\right.}{2(b^2e^2-a^2f^2)} \cdots\right.}{\sqrt{a^2c-b^2c \cdots}}$$

$$\Big\downarrow \; 224$$

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2(be-af)(af+be)(3Ce-Bf)-f^2x\left(2a^2Cf+b^2\left(-Af+Be-\frac{3Ce^2}{f}\right)\right)\right)}{f^3(e+fx)}-\frac{c\left(\frac{\left(2a^4Cf^4-a^2b^2f^2\left(9Ce^2-f(3Be-Af)\right.\right.}{}\right.}{}}{2(b^2e^2-a^2f^2)}\right.$$

$$\sqrt{a^2c-}$$

$\downarrow$ 216

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2(be-af)(af+be)(3Ce-Bf)-f^2x\left(2a^2Cf+b^2\left(-Af+Be-\frac{3Ce^2}{f}\right)\right)\right)}{f^3(e+fx)}-\frac{c\left(\frac{\left(2a^4Cf^4-a^2b^2f^2\left(9Ce^2-f(3Be-Af)\right.\right.}{}\right.}{}}{2(b^2e^2-a^2f^2)}\right.$$

$$\sqrt{a^2c-b^2}$$

$\downarrow$ 488

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2(be-af)(af+be)(3Ce-Bf)-f^2x\left(2a^2Cf+b^2\left(-Af+Be-\frac{3Ce^2}{f}\right)\right)\right)}{f^3(e+fx)}-\frac{c\left(-\frac{\left(2a^4Cf^4-a^2b^2f^2\left(9Ce^2-f(3Be-A\right.\right.}{}\right.}{}}{2(b^2e^{}}\right.$$

$\downarrow$ 217

$$\sqrt{a+bx}\sqrt{ac-bcx}\left(\frac{f\left(a^2c-b^2cx^2\right)^{3/2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}+\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2(be-af)(af+be)(3Ce-Bf)-f^2x\left(2a^2Cf+b^2\left(-Af+Be-\frac{3Ce^2}{f}\right)\right)\right)}{f^3(e+fx)}-}{}\right.$$

$$\sqrt{a^2c}$$

input
```
Int[(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2))/(e + f*x)^3,x]
```

output
```
(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*((f*(A + (e*(C*e - B*f))/f^2)*(a^2*c - b^
2*c*x^2)^(3/2))/(2*c*(b^2*e^2 - a^2*f^2)*(e + f*x)^2) + (((2*(b*e - a*f)*(
b*e + a*f)*(3*C*e - B*f) - f^2*(2*a^2*C*f + b^2*(B*e - (3*C*e^2)/f - A*f))
*x)*Sqrt[a^2*c - b^2*c*x^2])/(f^3*(e + f*x)) - (c*((-2*b*(b*e - a*f)*(b*e
+ a*f)*(3*C*e - B*f)*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(Sqrt[
c]*f) + ((2*a^4*C*f^4 + 2*b^4*e^3*(3*C*e - B*f) - a^2*b^2*f^2*(9*C*e^2 - f
*(3*B*e - A*f)))*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt[b^2*e^2 - a^2*
f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f*Sqrt[b^2*e^2 - a^2*f^2])))/f^3)
/(2*(b^2*e^2 - a^2*f^2))))/Sqrt[a^2*c - b^2*c*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(354) = 708$.

Time = 1.05 (sec) , antiderivative size = 1278, normalized size of antiderivative = 3.28

method	result	size
risch	Expression too large to display	1278
default	Expression too large to display	3116

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x,method=_RET
URNVERBOSE)
```

output
```
C*(-b*x+a)*(b*x+a)^(1/2)/f^3/(-c*(b*x-a))^(1/2)*c-1/f^3*(b^2*(B*f-3*C*e)/f
/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))-1/f^2*(A*b
^2*f^2-3*B*b^2*e*f-C*a^2*f^2+6*C*b^2*e^2)/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*
ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2*c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f
^2)^(1/2)*(-(x+e/f)^2*b^2*c+2*b^2*c*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2))/(x+e/f))-1/f^3*(2*A*b^2*e*f^2+B*a^2*f^3-3*B*b^2*e^2*f-2*C*a^2*e*f^2+
4*C*b^2*e^3)*(-1/c/(a^2*f^2-b^2*e^2)*f^2/(x+e/f)*(-(x+e/f)^2*b^2*c+2*b^2*c
*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+b^2*e*f/(a^2*f^2-b^2*e^2)/(c*(
a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2*c*e/f*(x+e
/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(x+e/f)^2*b^2*c+2*b^2*c*e/f*(x+e/f
)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f)))-(A*a^2*f^4-A*b^2*e^2*f^2-B*a^2
*e*f^3+B*b^2*e^3*f+C*a^2*e^2*f^2-C*b^2*e^4)/f^4*(-1/2/c/(a^2*f^2-b^2*e^2)*
f^2/(x+e/f)^2*(-(x+e/f)^2*b^2*c+2*b^2*c*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^
2)^(1/2)-3/2*b^2*e*f/(a^2*f^2-b^2*e^2)*(-1/c/(a^2*f^2-b^2*e^2)*f^2/(x+e/f)
*(-(x+e/f)^2*b^2*c+2*b^2*c*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+b^2*
e*f/(a^2*f^2-b^2*e^2)/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2
*e^2)/f^2+2*b^2*c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(x+e/f)^
2*b^2*c+2*b^2*c*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f)))-1/2*
b^2/(a^2*f^2-b^2*e^2)*f^2/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2
-b^2*e^2)/f^2+2*b^2*c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \int \frac{\sqrt{-c\,(-a+bx)}\sqrt{a+bx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**3,x)
```

output
```
Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2)/(e + f*x)**3
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*f-b*e)>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1592 vs. $2(354) = 708$.

Time = 0.74 (sec) , antiderivative size = 1592, normalized size of antiderivative = 4.08

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Too large to display}$$

input

```
integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x, algo
rithm="giac")
```

output

```
(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*C*b/f^3 + (6*C*b^5*sqrt(-c)*c*e^
4 - 2*B*b^5*sqrt(-c)*c*e^3*f - 9*C*a^2*b^3*sqrt(-c)*c*e^2*f^2 + 3*B*a^2*b^
3*sqrt(-c)*c*e*f^3 + 2*C*a^4*b*sqrt(-c)*c*f^4 - A*a^2*b^3*sqrt(-c)*c*f^4)*
arctan(-1/2*(2*b*c*e - (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c
))^2*f)/(sqrt(-b^2*e^2 + a^2*f^2)*c))/((b^2*e^2*f^4 - a^2*f^6)*sqrt(-b^2*e
^2 + a^2*f^2)*c) + 2*(20*C*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c
 + 2*a*c))^4*sqrt(-c)*c^2*e^5 - 6*C*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b
*x + a)*c + 2*a*c))^6*sqrt(-c)*c*e^4*f - 12*B*b^6*(sqrt(b*x + a)*sqrt(-c)
- sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c^2*e^4*f - 56*C*a^2*b^5*(sqrt(b*
x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^3*e^4*f + 4*B*b
^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*c*e^3*
f^2 - 6*C*a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*
sqrt(-c)*c^2*e^3*f^2 + 4*A*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c
 + 2*a*c))^4*sqrt(-c)*c^2*e^3*f^2 + 32*B*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) -
 sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^3*e^3*f^2 + 40*C*a^4*b^4*sqrt(-c
)*c^4*e^3*f^2 + 5*C*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c +
2*a*c))^6*sqrt(-c)*c*e^2*f^3 - 2*A*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^6*sqrt(-c)*c*e^2*f^3 + 2*B*a^2*b^4*(sqrt(b*x + a)*sqrt(
-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c^2*e^2*f^3 + 44*C*a^4*b^3*(s
qrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^3*e^2*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Hanged}$$

input
```
int(((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 9.97 (sec) , antiderivative size = 20805, normalized size of antiderivative = 53.35

$$\int \frac{\sqrt{a+bx}\sqrt{ac-bcx}(A+Bx+Cx^2)}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^3,x)
```

output
```
(sqrt(c)*(4*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**5*b**2*e**2*f**6 + 8*
asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**5*b**2*e*f**7*x + 4*asin(sqrt(a -
 b*x)/(sqrt(a)*sqrt(2)))*a**5*b**2*f**8*x**2 - 12*asin(sqrt(a - b*x)/(sqrt
(a)*sqrt(2)))*a**5*b*c*e**3*f**5 - 24*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2))
)*a**5*b*c*e**2*f**6*x - 12*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**5*b*c
*e*f**7*x**2 + 4*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b**3*e**3*f**5
 + 8*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b**3*e**2*f**6*x + 4*asin(
sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b**3*e*f**7*x**2 - 12*asin(sqrt(a -
b*x)/(sqrt(a)*sqrt(2)))*a**4*b**2*c*e**4*f**4 - 24*asin(sqrt(a - b*x)/(sqr
t(a)*sqrt(2)))*a**4*b**2*c*e**3*f**5*x - 12*asin(sqrt(a - b*x)/(sqrt(a)*sq
rt(2)))*a**4*b**2*c*e**2*f**6*x**2 - 8*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)
))*a**3*b**4*e**4*f**4 - 16*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**
4*e**3*f**5*x - 8*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**4*e**2*f**
6*x**2 + 24*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**3*c*e**5*f**3 +
48*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**3*c*e**4*f**4*x + 24*asin
(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**3*c*e**3*f**5*x**2 - 8*asin(sqrt
(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b**5*e**5*f**3 - 16*asin(sqrt(a - b*x)/(
sqrt(a)*sqrt(2)))*a**2*b**5*e**4*f**4*x - 8*asin(sqrt(a - b*x)/(sqrt(a)*sq
rt(2)))*a**2*b**5*e**3*f**5*x**2 + 24*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2))
)*a**2*b**4*c*e**6*f**2 + 48*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2...
```

3.52 $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 465

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$
$$-\frac{(136a^4Cf^3+120b^4e^2(Be+3Af)-45a^3bf^2(3Ce+Bf)-60ab^3e(Ce^2+3f(Be+Af))+80a^2b^2f(3}{120b^6c}$$
$$+\frac{f(14a^2Cf^2-5b^2(3Ce^2+f(3Be+Af)))\,x^2\sqrt{a+bx}\sqrt{ac-bcx}}{15b^4c}$$
$$-\frac{f^2(15bCe+5bBf-16aCf)x^3\sqrt{a+bx}\sqrt{ac-bcx}}{20b^3c}$$
$$+\frac{(32a^3Cf^3-15a^2bf^2(3Ce+Bf)-20b^3(Ce^3+3ef(Be+Af)))\,(a+bx)^{3/2}\sqrt{ac-bcx}}{40b^6c}$$
$$-\frac{Cf^3(a+bx)^{9/2}\sqrt{ac-bcx}}{5b^6c}$$
$$+\frac{(3a^4f^2(3Ce+Bf)+4a^2b^2e^2(Ce+3Bf)+4A(2b^4e^3+3a^2b^2ef^2))\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{4b^5\sqrt{c}}$$

output
```
-1/120*(136*a^4*C*f^3+120*b^4*e^2*(3*A*f+B*e)-45*a^3*b*f^2*(B*f+3*C*e)-60*
a*b^3*e*(C*e^2+3*f*(A*f+B*e))+80*a^2*b^2*f*(3*C*e^2+f*(A*f+3*B*e)))*(b*x+a
)^(1/2)*(-b*c*x+a*c)^(1/2)/b^6/c+1/15*f*(14*a^2*C*f^2-5*b^2*(3*C*e^2+f*(A*
f+3*B*e)))*x^2*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^4/c-1/20*f^2*(5*B*b*f-16
*C*a*f+15*C*b*e)*x^3*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^3/c+1/40*(32*a^3*C
*f^3-15*a^2*b*f^2*(B*f+3*C*e)-20*b^3*(C*e^3+3*e*f*(A*f+B*e)))*(b*x+a)^(3/2
)*(-b*c*x+a*c)^(1/2)/b^6/c-1/5*C*f^3*(b*x+a)^(9/2)*(-b*c*x+a*c)^(1/2)/b^6/
c+1/4*(3*a^4*f^2*(B*f+3*C*e)+4*a^2*b^2*e^2*(3*B*f+C*e)+4*A*(3*a^2*b^2*e*f^
2+2*b^4*e^3))*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b^5/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.61

$$\int \frac{(e+fx)^3\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{-\left((a-bx)\sqrt{a+bx}\left(64a^4Cf^3+a^2b^2f(5f(48Be+16Af+9Bfx)+C(240e^2+135efx+32f^2x^2))+2\right.\right.}{}$$

input
```
Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]
),x]
```

output
```
(-((a - b*x)*Sqrt[a + b*x]*(64*a^4*C*f^3 + a^2*b^2*f*(5*f*(48*B*e + 16*A*f
 + 9*B*f*x) + C*(240*e^2 + 135*e*f*x + 32*f^2*x^2)) + 2*b^4*(10*A*f*(18*e^
2 + 9*e*f*x + 2*f^2*x^2) + 15*B*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3
) + 3*C*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 30*b*(3*a^
4*f^2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f) + 4*A*(2*b^4*e^3 + 3*a^2
*b^2*e*f^2))*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(120*b^6*S
qrt[c*(a - b*x)])
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.325, Rules used = {2113, 2185, 25, 27, 687, 25, 27, 687, 25, 27, 676, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^3 \left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}dx$$

$$\Big\downarrow \text{ 2113}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3 \left(Cx^2+Bx+A\right)}{\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{cf(e+fx)^3\left(\left(4Ca^2+5Ab^2\right)f-b^2(Ce-5Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{5b^2cf^2}-\frac{C(e+fx)^4\sqrt{a^2c-b^2cx^2}}{5b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{cf(e+fx)^3\left(\left(4Ca^2+5Ab^2\right)f-b^2(Ce-5Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{5b^2cf^2}-\frac{C(e+fx)^4\sqrt{a^2c-b^2cx^2}}{5b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(e+fx)^3\left(\left(4Ca^2+5Ab^2\right)f-b^2(Ce-5Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{5b^2f}-\frac{C(e+fx)^4\sqrt{a^2c-b^2cx^2}}{5b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\Bigg(\frac{\frac{(e+fx)^3\sqrt{a^2c-b^2cx^2}(Ce-5Bf)}{4c}-\frac{\int -\frac{b^2c(e+fx)^2\left(f\left((13Ce+15Bf)a^2+20Ab^2e\right)+\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2c}}{5b^2f}-C(}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\int \frac{b^2c(e+fx)^2\left(f\left((13Ce+15Bf)a^2+20Ab^2e\right)+\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2c}+\frac{(e+fx)^3\sqrt{a^2c-b^2cx^2}(Ce-5Bf)}{4c}}{5b^2f}-\frac{C(e+\cdots}{\cdots}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\int \frac{(e+fx)^2\left(f\left((13Ce+15Bf)a^2+20Ab^2e\right)+\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx+\frac{(e+fx)^3\sqrt{a^2c-b^2cx^2}(Ce-5Bf)}{4c}}{5b^2f}-\frac{C(e+\cdots}{\cdots}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 687$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(-\frac{\int -\frac{c(e+fx)\left(\left(a^2f^2(71Ce+45Bf)-b^2\left(6Ce^3-10ef(3Be+10Af)\right)\right)xb^2+f\left(32Cf^2a^4+3b^2e(11Ce+25Bf)a^2+20A\left(3e^2b^4+2a^2f^2b^2\right)\cdots\right.\right.}{\sqrt{a^2c-b^2cx^2}}\cdots}{3b^2c}\cdots\right.}{5b^2f}\cdots\right.}{\sqrt{a+bx}\sqrt{ac-\cdots}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(\frac{\int \frac{c(e+fx)\left(\left(a^2f^2(71Ce+45Bf)-b^2\left(6Ce^3-10ef(3Be+10Af)\right)\right)xb^2+f\left(32Cf^2a^4+3b^2e(11Ce+25Bf)a^2+20A\left(3e^2b^4+2a^2f^2b^2\right)\right)\right)}{\sqrt{a^2c-b^2cx^2}}d\cdots}{3b^2c}\cdots\right.}{5b^2f}\cdots\right.}{\sqrt{a+bx}\sqrt{ac-bc\cdots}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(\frac{\int \frac{(e+fx)\left(\left(a^2f^2(71Ce+45Bf)-b^2\left(6Ce^3-10ef(3Be+10Af)\right)\right)xb^2+f\left(32Cf^2a^4+3b^2e(11Ce+25Bf)a^2+20A\left(3e^2b^4+2a^2f^2b^2\right)\right)\right)}{\sqrt{a^2c-b^2cx^2}}dx\cdots}{3b^2}\cdots\right.}{5b^2f}\cdots\right.}{\sqrt{a+bx}\sqrt{ac-bc\cdots}}$$

$\Big\downarrow$ 676

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(\frac{\frac{15}{2}f\left(3a^4f^2(Bf+3Ce)+4A\left(3a^2b^2ef^2+2b^4e^3\right)+4a^2b^2e^2(3Bf+Ce)\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx-\frac{fx\sqrt{a^2c-b^2cx^2}\left(a^2f^2(45Bf+71Ce)-b^2\left(6C\cdots\right.\right.}{2c}}{3b^2}\cdots\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 224

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{4}\left(\frac{\frac{15}{2}f\left(3a^4f^2(Bf+3Ce)+4A\left(3a^2b^2ef^2+2b^4e^3\right)+4a^2b^2e^2(3Bf+Ce)\right)\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}-\frac{fx\sqrt{a^2c-b^2cx^2}\left(a^2f^2(45Bf\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$\Big\downarrow$ 216

$$\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{(e+fx)^3\sqrt{a^2c-b^2cx^2}(Ce-5Bf)}{4c}+\frac{1}{4}\left(-\frac{fx\sqrt{a^2c-b^2cx^2}\left(a^2f^2(45Bf+71Ce)-b^2\left(6Ce^3-10ef(10Af+3Be)\right)\right)}{2c}+\frac{15f\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

input
```
Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-1/5*(C*(e + f*x)^4*Sqrt[a^2*c - b^2*c*x^2])/(b^
2*c*f) + (((C*e - 5*B*f)*(e + f*x)^3*Sqrt[a^2*c - b^2*c*x^2])/(4*c) + (-1/
3*((16*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(3*B*e + 4*A*f)))*(e + f*x)^2*Sqrt[a
^2*c - b^2*c*x^2])/(b^2*c) + ((-2*(16*a^4*C*f^4 + 4*a^2*b^2*f^2*(13*C*e^2
+ 5*f*(3*B*e + A*f)) - b^4*(3*C*e^4 - 5*e^2*f*(3*B*e + 16*A*f)))*Sqrt[a^2*
c - b^2*c*x^2])/(b^2*c) - (f*(a^2*f^2*(71*C*e + 45*B*f) - b^2*(6*C*e^3 - 1
0*e*f*(3*B*e + 10*A*f)))*x*Sqrt[a^2*c - b^2*c*x^2])/(2*c) + (15*f*(3*a^4*f
^2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f) + 4*A*(2*b^4*e^3 + 3*a^2*b^
2*e*f^2))*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b*Sqrt[c]))/(3
*b^2))/4)/(5*b^2*f)))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{(24C\,f^3x^4b^4+30B\,b^4f^3x^3+90C\,b^4e\,f^2x^3+40A\,b^4f^3x^2+120B\,b^4e\,f^2x^2+32C\,a^2b^2f^3x^2+120C\,b^4e^2f\,x^2+180A\,b^4e\,f^2x+45B\,a^2}{}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(-24C\,b^4f^3x^4\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}-30B\,b^4f^3x^3\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}-90C\,b^4e\,f^2x^3\sqrt{b^2c}\,\sqrt{c(-b^2x^2+a^2)}\right.}{}$

input
```
int((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/120*(24*C*b^4*f^3*x^4+30*B*b^4*f^3*x^3+90*C*b^4*e*f^2*x^3+40*A*b^4*f^3*
x^2+120*B*b^4*e*f^2*x^2+32*C*a^2*b^2*f^3*x^2+120*C*b^4*e^2*f*x^2+180*A*b^4
*e*f^2*x+45*B*a^2*b^2*f^3*x+180*B*b^4*e^2*f*x+135*C*a^2*b^2*e*f^2*x+60*C*b
^4*e^3*x+80*A*a^2*b^2*f^3+360*A*b^4*e^2*f+240*B*a^2*b^2*e*f^2+120*B*b^4*e^
3+64*C*a^4*f^3+240*C*a^2*b^2*e^2*f)/b^6*(-b*x+a)*(b*x+a)^(1/2)/(-c*(b*x-a)
)^(1/2)+1/8*(12*A*a^2*b^2*e*f^2+8*A*b^4*e^3+3*B*a^4*f^3+12*B*a^2*b^2*e^2*f
+9*C*a^4*e*f^2+4*C*a^2*b^2*e^3)/b^4/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(
-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x
-a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.51

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \left[-\frac{15\left(12\,Ba^2b^3e^2f+3\,Ba^4bf^3+4\left(Ca^2b^3+2\,Ab^5\right)e^3+3\left(3\,Ca^4b+4\,Aa^2b^3\right)ef^2\right)\sqrt{-c}\log\left(2\,b^2cx^2-2\right.}{}\right.$$
$$-\frac{15\left(12\,Ba^2b^3e^2f+3\,Ba^4bf^3+4\left(Ca^2b^3+2\,Ab^5\right)e^3+3\left(3\,Ca^4b+4\,Aa^2b^3\right)ef^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{b}}{b^2cx^2-}\right.}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/240*(15*(12*B*a^2*b^3*e^2*f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*
e^3 + 3*(3*C*a^4*b + 4*A*a^2*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt
(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f^3*x^4 +
 120*B*b^4*e^3 + 240*B*a^2*b^2*e*f^2 + 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f +
 16*(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 30*(3*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(
15*C*b^4*e^2*f + 15*B*b^4*e*f^2 + (4*C*a^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4
*C*b^4*e^3 + 12*B*b^4*e^2*f + 3*B*a^2*b^2*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*
e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c), -1/120*(15*(12*B*a^2*
b^3*e^2*f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*e^3 + 3*(3*C*a^4*b + 4
*A*a^2*b^3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(
c)*x/(b^2*c*x^2 - a^2*c)) + (24*C*b^4*f^3*x^4 + 120*B*b^4*e^3 + 240*B*a^2*
b^2*e*f^2 + 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f + 16*(4*C*a^4 + 5*A*a^2*b^2)
*f^3 + 30*(3*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(15*C*b^4*e^2*f + 15*B*b^4*e
*f^2 + (4*C*a^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4*C*b^4*e^3 + 12*B*b^4*e^2*f
 + 3*B*a^2*b^2*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c
)*sqrt(b*x + a))/(b^6*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)**3*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.01

$$\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\sqrt{-b^2cx^2+a^2c}Cf^3x^4}{5\,b^2c} - \frac{4\,\sqrt{-b^2cx^2+a^2c}Ca^2f^3x^2}{15\,b^4c} + \frac{Ae^3\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Be^3}{b^2c} - \frac{3\,\sqrt{-b^2cx^2+a^2c}Ae^2f}{b^2c} - \frac{8\,\sqrt{-b^2cx^2+a^2c}Ca^4f^3}{15\,b^6c} - \frac{\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)x^3}{4\,b^2c} - \frac{\sqrt{-b^2cx^2+a^2c}(3\,Ce^2f+3\,Bef^2+Af^3)x^2}{3\,b^2c} + \frac{3\,(3\,Cef^2+Bf^3)a^4\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}} + \frac{(Ce^3+3\,Be^2f+3\,Aef^2)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} - \frac{3\,\sqrt{-b^2cx^2+a^2c}(3\,Cef^2+Bf^3)a^2x}{8\,b^4c} - \frac{\sqrt{-b^2cx^2+a^2c}(Ce^3+3\,Be^2f+3\,Aef^2)x}{2\,b^2c} - \frac{2\,\sqrt{-b^2cx^2+a^2c}(3\,Ce^2f+3\,Bef^2+Af^3)a^2}{3\,b^4c}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
-1/5*sqrt(-b^2*c*x^2 + a^2*c)*C*f^3*x^4/(b^2*c) - 4/15*sqrt(-b^2*c*x^2 + a
^2*c)*C*a^2*f^3*x^2/(b^4*c) + A*e^3*arcsin(b*x/a)/(b*sqrt(c)) - sqrt(-b^2*
c*x^2 + a^2*c)*B*e^3/(b^2*c) - 3*sqrt(-b^2*c*x^2 + a^2*c)*A*e^2*f/(b^2*c)
- 8/15*sqrt(-b^2*c*x^2 + a^2*c)*C*a^4*f^3/(b^6*c) - 1/4*sqrt(-b^2*c*x^2 +
a^2*c)*(3*C*e*f^2 + B*f^3)*x^3/(b^2*c) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(3*C
*e^2*f + 3*B*e*f^2 + A*f^3)*x^2/(b^2*c) + 3/8*(3*C*e*f^2 + B*f^3)*a^4*arcs
in(b*x/a)/(b^5*sqrt(c)) + 1/2*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^2*arcsin(b
*x/a)/(b^3*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e*f^2 + B*f^3)*a^2
*x/(b^4*c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*
x/(b^2*c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*a
^2/(b^4*c)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.23

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$

$$-\frac{\left(\left(2\left(3\left(\frac{4\,(bx+a)Cf^3}{c}+\frac{15\,Cbc^4ef^2-16\,Cac^4f^3+5\,Bbc^4f^3}{c^5}\right)(bx+a)+\frac{60\,Cb^2c^4e^2f-135\,Cabc^4ef^2+60\,Bb^2c^4ef^2+88\,Ca^2c^4f^3}{c^5}\right.\right.\right.}{}$$

input
```
integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
-1/120*(((2*(3*(4*(b*x + a)*C*f^3/c + (15*C*b*c^4*e*f^2 - 16*C*a*c^4*f^3 +
 5*B*b*c^4*f^3)/c^5)*(b*x + a) + (60*C*b^2*c^4*e^2*f - 135*C*a*b*c^4*e*f^2
 + 60*B*b^2*c^4*e*f^2 + 88*C*a^2*c^4*f^3 - 45*B*a*b*c^4*f^3 + 20*A*b^2*c^4
*f^3)/c^5)*(b*x + a) + 5*(12*C*b^3*c^4*e^3 - 48*C*a*b^2*c^4*e^2*f + 36*B*b
^3*c^4*e^2*f + 81*C*a^2*b*c^4*e*f^2 - 48*B*a*b^2*c^4*e*f^2 + 36*A*b^3*c^4*
e*f^2 - 32*C*a^3*c^4*f^3 + 27*B*a^2*b*c^4*f^3 - 16*A*a*b^2*c^4*f^3)/c^5)*(
b*x + a) - 15*(4*C*a*b^3*c^4*e^3 - 8*B*b^4*c^4*e^3 - 24*C*a^2*b^2*c^4*e^2*
f + 12*B*a*b^3*c^4*e^2*f - 24*A*b^4*c^4*e^2*f + 15*C*a^3*b*c^4*e*f^2 - 24*
B*a^2*b^2*c^4*e*f^2 + 12*A*a*b^3*c^4*e*f^2 - 8*C*a^4*c^4*f^3 + 5*B*a^3*b*c
^4*f^3 - 8*A*a^2*b^2*c^4*f^3)/c^5)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a
) + 30*(4*C*a^2*b^3*e^3 + 8*A*b^5*e^3 + 12*B*a^2*b^3*e^2*f + 9*C*a^4*b*e*f
^2 + 12*A*a^2*b^3*e*f^2 + 3*B*a^4*b*f^3)*log(abs(-sqrt(b*x + a)*sqrt(-c) +
 sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c))/b^6
```

Mupad [B] (verification not implemented)

Time = 95.58 (sec) , antiderivative size = 4167, normalized size of antiderivative = 8.96

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^3*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),
x)
```

output
```
- ((((23*B*a^4*c*f^3)/2 - 18*B*a^2*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^13)/(b^5*((a + b*x)^(1/2) - a^(1/2))^13) + (((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))^15*((3*B*a^4*f^3)/2 + 6*B*a^2*b^2*e^2*f))/(b^5*((a + b*x)^
(1/2) - a^(1/2))^15) - (((3*B*a^4*c^7*f^3)/2 + 6*B*a^2*b^2*c^7*e^2*f)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^5*((a + b*x)^(1/2) - a^(1/2))) - (((23
*B*a^4*c^6*f^3)/2 - 18*B*a^2*b^2*c^6*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^3)/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) + (((333*B*a^4*c^5*f^3)/2 + 9
0*B*a^2*b^2*c^5*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^5*((a + b
*x)^(1/2) - a^(1/2))^5) - (((333*B*a^4*c^2*f^3)/2 + 90*B*a^2*b^2*c^2*e^2*f
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(b^5*((a + b*x)^(1/2) - a^(1/2))
^11) - (((671*B*a^4*c^4*f^3)/2 - 66*B*a^2*b^2*c^4*e^2*f)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^7)/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + (((671*B*a^4*c
^3*f^3)/2 - 66*B*a^2*b^2*c^3*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)
/(b^5*((a + b*x)^(1/2) - a^(1/2))^9) + (a^(1/2)*(a*c)^(1/2)*(48*B*b^2*c^5*
e^3 + 192*B*a^2*c^5*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a
 + b*x)^(1/2) - a^(1/2))^4) + (a^(1/2)*(a*c)^(1/2)*(160*B*b^2*c^3*e^3 + 12
8*B*a^2*c^3*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^
(1/2) - a^(1/2))^8) + (a^(1/2)*(a*c)^(1/2)*(120*B*b^2*c^4*e^3 + 256*B*a^2*
c^4*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(b^4*((a + b*x)^(1/2) -
a^(1/2))^6) + (a^(1/2)*(a*c)^(1/2)*(120*B*b^2*c^2*e^3 + 256*B*a^2*c^2*e...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.34

$$\int \frac{(e+fx)^3 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{\sqrt{c}\left(-90 asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right) a^4b^2f^3 - 270 asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right) a^4bce\,f^2 - 360 asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right) a^3b^3e\,f^2 - 360 asin\left(\frac{\sqrt{-}}{\sqrt{}}\right.}{}$$

input
```
int((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

output
```
(sqrt(c)*( - 90*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b**2*f**3 - 270
*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*b*c*e*f**2 - 360*asin(sqrt(a -
 b*x)/(sqrt(a)*sqrt(2)))*a**3*b**3*e*f**2 - 360*asin(sqrt(a - b*x)/(sqrt(a
)*sqrt(2)))*a**2*b**4*e**2*f - 120*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a
**2*b**3*c*e**3 - 240*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b**5*e**3 -
64*sqrt(a + b*x)*sqrt(a - b*x)*a**4*c*f**3 - 80*sqrt(a + b*x)*sqrt(a - b*x
)*a**3*b**2*f**3 - 240*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**3*e*f**2 - 45*s
qrt(a + b*x)*sqrt(a - b*x)*a**2*b**3*f**3*x - 240*sqrt(a + b*x)*sqrt(a - b
*x)*a**2*b**2*c*e**2*f - 135*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**2*c*e*f**
2*x - 32*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**2*c*f**3*x**2 - 360*sqrt(a +
b*x)*sqrt(a - b*x)*a*b**4*e**2*f - 180*sqrt(a + b*x)*sqrt(a - b*x)*a*b**4*
e*f**2*x - 40*sqrt(a + b*x)*sqrt(a - b*x)*a*b**4*f**3*x**2 - 120*sqrt(a +
b*x)*sqrt(a - b*x)*b**5*e**3 - 180*sqrt(a + b*x)*sqrt(a - b*x)*b**5*e**2*f
*x - 120*sqrt(a + b*x)*sqrt(a - b*x)*b**5*e*f**2*x**2 - 30*sqrt(a + b*x)*s
qrt(a - b*x)*b**5*f**3*x**3 - 60*sqrt(a + b*x)*sqrt(a - b*x)*b**4*c*e**3*x
 - 120*sqrt(a + b*x)*sqrt(a - b*x)*b**4*c*e**2*f*x**2 - 90*sqrt(a + b*x)*s
qrt(a - b*x)*b**4*c*e*f**2*x**3 - 24*sqrt(a + b*x)*sqrt(a - b*x)*b**4*c*f*
*3*x**4))/(120*b**6*c)
```

3.53 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 335

$$\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{(3a^3Cf^2+24b^3e(Be+2Af)+16a^2bf(2Ce+Bf)-12ab^2(Ce^2+f(2Be+Af)))\sqrt{a+bx}\sqrt{ac-bcx}}{24b^5c} - \frac{f(8bCe+4bBf-9aCf)x^2\sqrt{a+bx}\sqrt{ac-bcx}}{12b^3c} + \frac{(3a^2Cf^2-4b^2(Ce^2+f(2Be+Af)))(a+bx)^{3/2}\sqrt{ac-bcx}}{8b^5c} - \frac{Cf^2(a+bx)^{7/2}\sqrt{ac-bcx}}{4b^5c} + \frac{(3a^4Cf^2+4a^2b^2e(Ce+2Bf)+4A(2b^4e^2+a^2b^2f^2))\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{4b^5\sqrt{c}}$$

output

```
-1/24*(3*a^3*C*f^2+24*b^3*e*(2*A*f+B*e)+16*a^2*b*f*(B*f+2*C*e)-12*a*b^2*(C
*e^2+f*(A*f+2*B*e)))*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^5/c-1/12*f*(4*B*b*
f-9*C*a*f+8*C*b*e)*x^2*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^3/c+1/8*(3*a^2*C
*f^2-4*b^2*(C*e^2+f*(A*f+2*B*e)))*(b*x+a)^(3/2)*(-b*c*x+a*c)^(1/2)/b^5/c-1
/4*C*f^2*(b*x+a)^(7/2)*(-b*c*x+a*c)^(1/2)/b^5/c+1/4*(3*a^4*C*f^2+4*a^2*b^2
*e*(2*B*f+C*e)+4*A*(a^2*b^2*f^2+2*b^4*e^2))*arctan(c^(1/2)*(b*x+a)^(1/2)/(
-b*c*x+a*c)^(1/2))/b^5/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.60

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{-b(a-bx)\sqrt{a+bx}(a^2f(32Ce+16Bf+9Cfx)+2b^2(6Af(4e+fx)+4B(3e^2+3efx+f^2x^2)+Cx(}{24b^5\sqrt{c}}$$

input
```
Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]
),x]
```

output
```
(-(b*(a - b*x)*Sqrt[a + b*x]*(a^2*f*(32*C*e + 16*B*f + 9*C*f*x) + 2*b^2*(6
*A*f*(4*e + f*x) + 4*B*(3*e^2 + 3*e*f*x + f^2*x^2) + C*x*(6*e^2 + 8*e*f*x
+ 3*f^2*x^2))) + 6*(3*a^4*C*f^2 + 4*a^2*b^2*e*(C*e + 2*B*f) + 4*A*(2*b^4*
e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(24
*b^5*Sqrt[c*(a - b*x)])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2113, 2185, 25, 27, 687, 25, 27, 676, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$\Big\downarrow \ 2113$$
$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2(Cx^2+Bx+A)}{\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \ 2185$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{cf(e+fx)^2\left(\left(3Ca^2+4Ab^2\right)f-b^2(Ce-4Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2cf^2}-\frac{C(e+fx)^3\sqrt{a^2c-b^2cx^2}}{4b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 25

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{cf(e+fx)^2\left(\left(3Ca^2+4Ab^2\right)f-b^2(Ce-4Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2cf^2}-\frac{C(e+fx)^3\sqrt{a^2c-b^2cx^2}}{4b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 27

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(e+fx)^2\left(\left(3Ca^2+4Ab^2\right)f-b^2(Ce-4Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{4b^2f}-\frac{C(e+fx)^3\sqrt{a^2c-b^2cx^2}}{4b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 687

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{(e+fx)^2\sqrt{a^2c-b^2cx^2}(Ce-4Bf)}{3c}-\frac{\int -\frac{b^2c(e+fx)\left(f\left((7Ce+8Bf)a^2+12Ab^2e\right)+\left(9a^2Cf^2-b^2\left(2Ce^2-4f(2Be+3Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2c}}{4b^2f}-\frac{C(e+f\ldots}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 25

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\int \frac{b^2c(e+fx)\left(f\left((7Ce+8Bf)a^2+12Ab^2e\right)+\left(9a^2Cf^2-b^2\left(2Ce^2-4f(2Be+3Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2c}+\frac{(e+fx)^2\sqrt{a^2c-b^2cx^2}(Ce-4Bf)}{3c}}{4b^2f}-\frac{C(e+fx)\ldots}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 27

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{3}\int \frac{(e+fx)\left(f\left((7Ce+8Bf)a^2+12Ab^2e\right)+\left(9a^2Cf^2-b^2\left(2Ce^2-4f(2Be+3Af)\right)\right)x\right)}{\sqrt{a^2c-b^2cx^2}}dx+\frac{(e+fx)^2\sqrt{a^2c-b^2cx^2}(Ce-4Bf)}{3c}}{4b^2f}-\frac{C(e+fx)^3\ldots}{4\ldots}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 676

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{3}\left(\frac{3f\left(3a^4Cf^2+4A\left(a^2b^2f^2+2b^4e^2\right)+4a^2b^2e(2Bf+Ce)\right)\int\frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{2b^2}-\frac{2\sqrt{a^2c-b^2cx^2}\left(4a^2f^2(Bf+2Ce)-b^2\left(Ce^3-4ef(3Af+Be\right.\right.}{b^2c}\right.}{4b^2f}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{1}{3}\left(\frac{3f\left(3a^4Cf^2+4A\left(a^2b^2f^2+2b^4e^2\right)+4a^2b^2e(2Bf+Ce)\right)\int\frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{2b^2}-\frac{2\sqrt{a^2c-b^2cx^2}\left(4a^2f^2(Bf+2Ce)-b^2\left(Ce^3\right.\right.}{b^2c}\right.}{4b^2f}\right.}{\sqrt{a+bx}\sqrt{ac-}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{(e+fx)^2\sqrt{a^2c-b^2cx^2}(Ce-4Bf)}{3c}+\frac{1}{3}\left(-\frac{2\sqrt{a^2c-b^2cx^2}\left(4a^2f^2(Bf+2Ce)-b^2\left(Ce^3-4ef(3Af+Be)\right)\right)}{b^2c}-\frac{fx\sqrt{a^2c-b^2cx^2}\left(9a^2Cf^2-b^2\left(2C\right.\right.}{2b^2c}\right.}{4b^2f}\right.}{\sqrt{a+bx}\sqrt{ac-b}}$$

input
```
Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-1/4*(C*(e + f*x)^3*Sqrt[a^2*c - b^2*c*x^2])/(b^
2*c*f) + (((C*e - 4*B*f)*(e + f*x)^2*Sqrt[a^2*c - b^2*c*x^2])/(3*c) + ((-2
*(4*a^2*f^2*(2*C*e + B*f) - b^2*(C*e^3 - 4*e*f*(B*e + 3*A*f)))*Sqrt[a^2*c
- b^2*c*x^2])/(b^2*c) - (f*(9*a^2*C*f^2 - b^2*(2*C*e^2 - 4*f*(2*B*e + 3*A*
f)))*x*Sqrt[a^2*c - b^2*c*x^2])/(2*b^2*c) + (3*f*(3*a^4*C*f^2 + 4*a^2*b^2*
e*(C*e + 2*B*f) + 4*A*(2*b^4*e^2 + a^2*b^2*f^2))*ArcTan[(b*Sqrt[c]*x)/Sqrt
[a^2*c - b^2*c*x^2]])/(2*b^3*Sqrt[c]))/3)/(4*b^2*f)))/(Sqrt[a + b*x]*Sqrt[
a*c - b*c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(6C f^2 x^3 b^2+8B b^2 f^2 x^2+16C b^2 e f x^2+12A b^2 f^2 x+24B b^2 e f x+9C a^2 f^2 x+12C b^2 e^2 x+48A b^2 e f+16B a^2 f^2+24B b^2 e^2+32C a}{24 b^4 \sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(-6C b^2 f^2 x^3 \sqrt{b^2 c}\,\sqrt{c(-b^2x^2+a^2)}+12A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)a^2b^2cf^2+24A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)\right.}{}$

input
```
int((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/24*(6*C*b^2*f^2*x^3+8*B*b^2*f^2*x^2+16*C*b^2*e*f*x^2+12*A*b^2*f^2*x+24*
B*b^2*e*f*x+9*C*a^2*f^2*x+12*C*b^2*e^2*x+48*A*b^2*e*f+16*B*a^2*f^2+24*B*b^
2*e^2+32*C*a^2*e*f)/b^4*(-b*x+a)*(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)+1/8*(4*A
*a^2*b^2*f^2+8*A*b^4*e^2+8*B*a^2*b^2*e*f+3*C*a^4*f^2+4*C*a^2*b^2*e^2)/b^4/
(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-(b*x+a)*c
*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.44

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \left[-\frac{3\,(8\,Ba^2b^2ef+4\,(Ca^2b^2+2\,Ab^4)e^2+(3\,Ca^4+4\,Aa^2b^2)f^2)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+\right.}{}\right.$$
$$-\frac{3\,(8\,Ba^2b^2ef+4\,(Ca^2b^2+2\,Ab^4)e^2+(3\,Ca^4+4\,Aa^2b^2)f^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+ab}\sqrt{cx}}{b^2cx^2-a^2c}\right)+(6\,Cb^3f}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/48*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b^4)*e^2 + (3*C*a^4 + 4*A*
a^2*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a
)*b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*f^2*x^3 + 24*B*b^3*e^2 + 16*B*a^2*b*f
^2 + 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B*b^3*f^2)*x^2 + 3*(4
*C*b^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*x)*sqrt(-b*c*x + a*c
)*sqrt(b*x + a))/(b^5*c), -1/24*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b
^4)*e^2 + (3*C*a^4 + 4*A*a^2*b^2)*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*s
qrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (6*C*b^3*f^2*x^3 + 24*B*b^
3*e^2 + 16*B*a^2*b*f^2 + 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B
*b^3*f^2)*x^2 + 3*(4*C*b^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*
x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^5*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**2*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.95

$$\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\sqrt{-b^2cx^2+a^2c}Cf^2x^3}{4\,b^2c} + \frac{Ae^2\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} + \frac{3\,Ca^4f^2\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}} - \frac{3\sqrt{-b^2cx^2+a^2c}Ca^2f^2x}{8\,b^4c} - \frac{\sqrt{-b^2cx^2+a^2c}Be^2}{b^2c} - \frac{2\sqrt{-b^2cx^2+a^2c}Aef}{b^2c} - \frac{\sqrt{-b^2cx^2+a^2c}(2\,Cef+Bf^2)x^2}{3\,b^2c} + \frac{(Ce^2+2\,Bef+Af^2)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}(Ce^2+2\,Bef+Af^2)x}{2\,b^2c} - \frac{2\sqrt{-b^2cx^2+a^2c}(2\,Cef+Bf^2)a^2}{3\,b^4c}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="maxima")
```

output
```
-1/4*sqrt(-b^2*c*x^2 + a^2*c)*C*f^2*x^3/(b^2*c) + A*e^2*arcsin(b*x/a)/(b*s
qrt(c)) + 3/8*C*a^4*f^2*arcsin(b*x/a)/(b^5*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2
+ a^2*c)*C*a^2*f^2*x/(b^4*c) - sqrt(-b^2*c*x^2 + a^2*c)*B*e^2/(b^2*c) - 2*
sqrt(-b^2*c*x^2 + a^2*c)*A*e*f/(b^2*c) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(2*C
*e*f + B*f^2)*x^2/(b^2*c) + 1/2*(C*e^2 + 2*B*e*f + A*f^2)*a^2*arcsin(b*x/a
)/(b^3*sqrt(c)) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^2 + 2*B*e*f + A*f^2)*x
/(b^2*c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(2*C*e*f + B*f^2)*a^2/(b^4*c)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.08

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx =$$

$$-\frac{\left(\left(2\left(\frac{3\,(bx+a)Cf^2}{c}+\frac{8\,Cbc^3ef-9\,Cac^3f^2+4\,Bbc^3f^2}{c^4}\right)(bx+a)+\frac{12\,Cb^2c^3e^2-32\,Cabc^3ef+24\,Bb^2c^3ef+27\,Ca^2c^3f^2-16\,Babc^3f}{c^4}\right.\right.}{}$$

input
```
integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algo
rithm="giac")
```

output
```
-1/24*(((2*(3*(b*x + a)*C*f^2/c + (8*C*b*c^3*e*f - 9*C*a*c^3*f^2 + 4*B*b*c
^3*f^2)/c^4)*(b*x + a) + (12*C*b^2*c^3*e^2 - 32*C*a*b*c^3*e*f + 24*B*b^2*c
^3*e*f + 27*C*a^2*c^3*f^2 - 16*B*a*b*c^3*f^2 + 12*A*b^2*c^3*f^2)/c^4)*(b*x
 + a) - 3*(4*C*a*b^2*c^3*e^2 - 8*B*b^3*c^3*e^2 - 16*C*a^2*b*c^3*e*f + 8*B*
a*b^2*c^3*e*f - 16*A*b^3*c^3*e*f + 5*C*a^3*c^3*f^2 - 8*B*a^2*b*c^3*f^2 + 4
*A*a*b^2*c^3*f^2)/c^4)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 6*(4*C*a
^2*b^2*e^2 + 8*A*b^4*e^2 + 8*B*a^2*b^2*e*f + 3*C*a^4*f^2 + 4*A*a^2*b^2*f^2
)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x + a)*c + 2*a*c)))/sqrt(-c))
/b^5
```

Mupad [B] (verification not implemented)

Time = 49.72 (sec) , antiderivative size = 2799, normalized size of antiderivative = 8.36

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)^2*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),
x)
```

output
```
- ((a^(1/2)*(a*c)^(1/2)*(64*B*a^2*c*f^2 + 32*B*b^2*c*e^2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (a^(1/2)*(a*c
)^(1/2)*(64*B*a^2*c^3*f^2 + 32*B*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) - (a^(1/2)*(a*c)^(1/2)*((12
8*B*a^2*c^2*f^2)/3 - 48*B*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^6)/(b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (4*B*a^2*e*f*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^11)/(b^3*((a + b*x)^(1/2) - a^(1/2))^11) + (8*B*a^(1/2)*e
^2*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(b^2*((a + b*x)^(1/
2) - a^(1/2))^10) + (20*B*a^2*c^4*e*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(b^3*((a + b*x)^(1/2) - a^(1/2))^3) + (24*B*a^2*c^3*e*f*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^5)/(b^3*((a + b*x)^(1/2) - a^(1/2))^5) - (24*B*a^2*c^
2*e*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a + b*x)^(1/2) - a^(1/
2))^7) + (8*B*a^(1/2)*c^4*e^2*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2)/(b^2*((a + b*x)^(1/2) - a^(1/2))^2) - (4*B*a^2*c^5*e*f*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (20*B*a^2*c*e
*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^3*((a + b*x)^(1/2) - a^(1/2))
^9))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12/((a + b*x)^(1/2) - a^(1/2))^1
2 + c^6 + (6*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) -
a^(1/2))^10 + (6*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/
2) - a^(1/2))^2 + (15*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + ...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.19

$$\int \frac{(e+fx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{\sqrt{c}\left(-18asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\sqrt{2}}\right)a^4cf^2 - 24asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\sqrt{2}}\right)a^3b^2f^2 - 48asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\sqrt{2}}\right)a^2b^3ef - 24asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\sqrt{2}}\right)\right.}{}$$

input
```
int((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

output
```
(sqrt(c)*( - 18*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*c*f**2 - 24*asi
n(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**2*f**2 - 48*asin(sqrt(a - b*x)/
(sqrt(a)*sqrt(2)))*a**2*b**3*e*f - 24*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2))
)*a**2*b**2*c*e**2 - 48*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b**4*e**2
- 16*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b**2*f**2 - 32*sqrt(a + b*x)*sqrt(a
- b*x)*a**2*b*c*e*f - 9*sqrt(a + b*x)*sqrt(a - b*x)*a**2*b*c*f**2*x - 48*s
qrt(a + b*x)*sqrt(a - b*x)*a*b**3*e*f - 12*sqrt(a + b*x)*sqrt(a - b*x)*a*b
**3*f**2*x - 24*sqrt(a + b*x)*sqrt(a - b*x)*b**4*e**2 - 24*sqrt(a + b*x)*s
qrt(a - b*x)*b**4*e*f*x - 8*sqrt(a + b*x)*sqrt(a - b*x)*b**4*f**2*x**2 - 1
2*sqrt(a + b*x)*sqrt(a - b*x)*b**3*c*e**2*x - 16*sqrt(a + b*x)*sqrt(a - b*
x)*b**3*c*e*f*x**2 - 6*sqrt(a + b*x)*sqrt(a - b*x)*b**3*c*f**2*x**3))/(24*
b**5*c)
```

3.54 $\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 203

$$\begin{aligned}\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx &= -\frac{\left(2a^2Cf+2b^2(Be+Af)-ab(Ce+Bf)\right)\sqrt{a+bx}\sqrt{ac-bcx}}{2b^4c}\\ &\quad+\frac{(4aCf-3b(Ce+Bf))(a+bx)^{3/2}\sqrt{ac-bcx}}{6b^4c}\\ &\quad-\frac{Cf(a+bx)^{5/2}\sqrt{ac-bcx}}{3b^4c}+\frac{\left(2Ab^2e+a^2(Ce+Bf)\right)\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b^3\sqrt{c}}\end{aligned}$$

output
```
-1/2*(2*a^2*C*f+2*b^2*(A*f+B*e)-a*b*(B*f+C*e))*(b*x+a)^(1/2)*(-b*c*x+a*c)^
(1/2)/b^4/c+1/6*(4*C*a*f-3*b*(B*f+C*e))*(b*x+a)^(3/2)*(-b*c*x+a*c)^(1/2)/b
^4/c-1/3*C*f*(b*x+a)^(5/2)*(-b*c*x+a*c)^(1/2)/b^4/c+(2*A*b^2*e+a^2*(B*f+C*
e))*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b^3/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.63

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{-\left((a-bx)\sqrt{a+bx}(4a^2Cf+b^2(6Be+6Af+3Cex+3Bfx+2Cfx^2))\right)+6b(2Ab^2e+a^2(Ce+Bf))}{6b^4\sqrt{c(a-bx)}}$$

input
```
Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),
x]
```

output
```
(-((a - b*x)*Sqrt[a + b*x]*(4*a^2*C*f + b^2*(6*B*e + 6*A*f + 3*C*e*x + 3*B
*f*x + 2*C*f*x^2))) + 6*b*(2*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a - b*x]*ArcT
an[Sqrt[a + b*x]/Sqrt[a - b*x]])/(6*b^4*Sqrt[c*(a - b*x)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2113, 2185, 25, 27, 676, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$\Big\downarrow \ 2113$$
$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)(Cx^2+Bx+A)}{\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$
$$\Big\downarrow \ 2185$$
$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{cf(e+fx)\left(\left(2Ca^2+3Ab^2\right)f-b^2(Ce-3Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2cf^2}-\frac{C(e+fx)^2\sqrt{a^2c-b^2cx^2}}{3b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{cf(e+fx)\left(\left(2Ca^2+3Ab^2\right)f-b^2(Ce-3Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2cf^2}-\frac{C(e+fx)^2\sqrt{a^2c-b^2cx^2}}{3b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(e+fx)\left(\left(2Ca^2+3Ab^2\right)f-b^2(Ce-3Bf)x\right)}{\sqrt{a^2c-b^2cx^2}}dx}{3b^2f}-\frac{C(e+fx)^2\sqrt{a^2c-b^2cx^2}}{3b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 676$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{3}{2}f\left(a^2(Bf+Ce)+2Ab^2e\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx-\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2Cf^2-b^2\left(Ce^2-3f(Af+Be)\right)\right)}{b^2c}+\frac{fx\sqrt{a^2c-b^2cx^2}(Ce-3Bf)}{2c}}{3b^2f}-\frac{C(e}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 224$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{3}{2}f\left(a^2(Bf+Ce)+2Ab^2e\right)\int \frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}-\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2Cf^2-b^2\left(Ce^2-3f(Af+Be)\right)\right)}{b^2c}+\frac{fx\sqrt{a^2c-b^2cx^2}(Ce-3}{2c}}{3b^2f}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{3f\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2(Bf+Ce)+2Ab^2e\right)}{2b\sqrt{c}}-\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2Cf^2-b^2\left(Ce^2-3f(Af+Be)\right)\right)}{b^2c}+\frac{fx\sqrt{a^2c-b^2cx^2}(Ce-3Bf)}{2c}}{3b^2f}-\frac{C(}{}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-1/3*(C*(e + f*x)^2*Sqrt[a^2*c - b^2*c*x^2])/(b^
2*c*f) + (-((2*a^2*C*f^2 - b^2*(C*e^2 - 3*f*(B*e + A*f)))*Sqrt[a^2*c - b^
2*c*x^2])/(b^2*c)) + (f*(C*e - 3*B*f)*x*Sqrt[a^2*c - b^2*c*x^2])/(2*c) + (
3*f*(2*A*b^2*e + a^2*(C*e + B*f))*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*
x^2]])/(2*b*Sqrt[c]))/(3*b^2*f)))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(2 C f\,x^{2} b^{2}+3 B\,b^{2} f x +3 C\,b^{2} e x +6 A\,b^{2} f +6 B\,b^{2} e +4 a^{2} C f \right) \left(-b x +a \right) \sqrt{b x +a}}{6 b^{4} \sqrt{-c \left(b x -a \right)}}+\frac{\left(2 A\,b^{2} e +B\,a^{2} f +C\,a^{2} e \right) \arctan\left(\frac{\sqrt{b^{2} c}\, x}{\sqrt{-b^{2} c\,x^{2}+a^{2} c}}\right)}{2 b^{2} \sqrt{b^{2} c}\, \sqrt{b x +a}\, \sqrt{-c \left(b x -a \right)}}$
default	$\frac{\sqrt{b x +a}\, \sqrt{c \left(-b x +a \right)}\, \left(6 A \arctan\left(\frac{\sqrt{b^{2} c}\, x}{\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}}\right) b^{4} c e +3 B \arctan\left(\frac{\sqrt{b^{2} c}\, x}{\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}}\right) a^{2} b^{2} c f +3 C \arctan\left(\frac{\sqrt{b^{2} c}\, x}{\sqrt{c \left(-b^{2} x^{2}+a^{2}\right)}}\right) a^{2} b}{}$

input
```
int((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/6*(2*C*b^2*f*x^2+3*B*b^2*f*x+3*C*b^2*e*x+6*A*b^2*f+6*B*b^2*e+4*C*a^2*f)
/b^4*(-b*x+a)*(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)+1/2*(2*A*b^2*e+B*a^2*f+C*a^
2*e)/b^2/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*(-
(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.49

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \left[-\frac{3\left(Ba^2bf+\left(Ca^2b+2\,Ab^3\right)e\right)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\left(2\,Cb^2fx^2\right.}{12\,b^4c}\right.$$
$$-\frac{3\left(Ba^2bf+\left(Ca^2b+2\,Ab^3\right)e\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{cx}}{b^2cx^2-a^2c}\right)+\left(2\,Cb^2fx^2+6\,Bb^2e+2\left(2\,Ca^2+3\,Ab\right.\right.}{6\,b^4c}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="fricas")
```

output
```
[-1/12*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2 - 2
*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(2*C*b^2*f*x^2
 + 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*e + B*b^2*f)*x)*sqrt(-b*
c*x + a*c)*sqrt(b*x + a))/(b^4*c), -1/6*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3
)*e)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^
2 - a^2*c)) + (2*C*b^2*f*x^2 + 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*
b^2*e + B*b^2*f)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^4*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\sqrt{-b^2cx^2+a^2c}Cfx^2}{3\,b^2c} + \frac{Ae\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} + \frac{(Ce+Bf)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Be}{b^2c} - \frac{2\sqrt{-b^2cx^2+a^2c}Ca^2f}{3\,b^4c} - \frac{\sqrt{-b^2cx^2+a^2c}Af}{b^2c} - \frac{\sqrt{-b^2cx^2+a^2c}(Ce+Bf)x}{2\,b^2c}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="maxima")
```

output
```
-1/3*sqrt(-b^2*c*x^2 + a^2*c)*C*f*x^2/(b^2*c) + A*e*arcsin(b*x/a)/(b*sqrt(
c)) + 1/2*(C*e + B*f)*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) - sqrt(-b^2*c*x^2 +
a^2*c)*B*e/(b^2*c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*f/(b^4*c) - sqrt(-
b^2*c*x^2 + a^2*c)*A*f/(b^2*c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e + B*f)*
x/(b^2*c)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.94

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\left(\left(\frac{2\,(bx+a)Cf}{c} + \frac{3\,Cbc^2e-4\,Cac^2f+3\,Bbc^2f}{c^3}\right)(bx+a) - \frac{3\,\left(Cabc^2e-2\,Bb^2c^2e-2\,Ca^2c^2f+Babc^2f-2\,Ab^2c^2f\right)}{c^3}\right)\sqrt{-(bx+a)}}{6\,b^4}$$

input
```
integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algori
thm="giac")
```

output
```
-1/6*(((2*(b*x + a)*C*f/c + (3*C*b*c^2*e - 4*C*a*c^2*f + 3*B*b*c^2*f)/c^3)
*(b*x + a) - 3*(C*a*b*c^2*e - 2*B*b^2*c^2*e - 2*C*a^2*c^2*f + B*a*b*c^2*f
- 2*A*b^2*c^2*f)/c^3)*sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a) + 6*(C*a^2*
b*e + 2*A*b^3*e + B*a^2*b*f)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(-(b*x
+ a)*c + 2*a*c)))/sqrt(-c))/b^4
```

Mupad [B] (verification not implemented)

Time = 19.95 (sec) , antiderivative size = 1011, normalized size of antiderivative = 4.98

$$\int \frac{(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Too large to display}$$

input
```
int(((e + f*x)*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
- ((2*B*a^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^
(1/2))^7 - (2*B*a^2*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^
(1/2) - a^(1/2)) - (14*B*a^2*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((
a + b*x)^(1/2) - a^(1/2))^5 + (14*B*a^2*c^2*f*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*
b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6
) - ((2*C*a^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) -
a^(1/2))^7 - (2*C*a^2*c^3*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x
)^(1/2) - a^(1/2)) - (14*C*a^2*c*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/
((a + b*x)^(1/2) - a^(1/2))^5 + (14*C*a^2*c^2*e*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^
2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (
4*b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))
^6) - ((a*c - b*c*x)^(1/2)*((2*C*a^3*f)/(3*b^4*c) + (C*f*x^3)/(3*b*c) + (C
*a*f*x^2)/(3*b^2*c) + (2*C*a^2*f*x)/(3*b^3*c)))/(a + b*x)^(1/2) - (4*A*...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.03

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{\sqrt{c}\left(-6asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\sqrt{2}}\right)a^2b^2f-6asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\sqrt{2}}\right)a^2bce-12asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\sqrt{2}}\right)a\,b^3e-4\sqrt{bx+a}\,\sqrt{-bx+a}\,a^2c\right.}{}$$

input
```
int((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

output
```
(sqrt(c)*( - 6*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b**2*f - 6*asin(
sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b*c*e - 12*asin(sqrt(a - b*x)/(sqrt(
a)*sqrt(2)))*a*b**3*e - 4*sqrt(a + b*x)*sqrt(a - b*x)*a**2*c*f - 6*sqrt(a
+ b*x)*sqrt(a - b*x)*a*b**2*f - 6*sqrt(a + b*x)*sqrt(a - b*x)*b**3*e - 3*s
qrt(a + b*x)*sqrt(a - b*x)*b**3*f*x - 3*sqrt(a + b*x)*sqrt(a - b*x)*b**2*c
*e*x - 2*sqrt(a + b*x)*sqrt(a - b*x)*b**2*c*f*x**2))/(6*b**4*c)
```

3.55 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 125

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{(2bB-aC)\sqrt{a+bx}\sqrt{ac-bcx}}{2b^3c} - \frac{C(a+bx)^{3/2}\sqrt{ac-bcx}}{2b^3c} + \frac{(2Ab^2+a^2C)\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b^3\sqrt{c}}$$

output
```
-1/2*(2*B*b-C*a)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^3/c-1/2*C*(b*x+a)^(3/2
)*(-b*c*x+a*c)^(1/2)/b^3/c+(2*A*b^2+C*a^2)*arctan(c^(1/2)*(b*x+a)^(1/2)/(-
b*c*x+a*c)^(1/2))/b^3/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{b(-a+bx)\sqrt{a+bx}(2B+Cx)+2(2Ab^2+a^2C)\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{2b^3\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
(b*(-a + b*x)*Sqrt[a + b*x]*(2*B + C*x) + 2*(2*A*b^2 + a^2*C)*Sqrt[a - b*x
]*ArcTan[Sqrt[a + b*x]/Sqrt[a - b*x]])/(2*b^3*Sqrt[c*(a - b*x)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1189, 83, 646, 45, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}dx$$

$$\Big\downarrow \text{1189}$$

$$\int \frac{Cx^2+A}{\sqrt{a+bx}\sqrt{ac-bcx}}dx+B\int \frac{x}{\sqrt{a+bx}\sqrt{ac-bcx}}dx$$

$$\Big\downarrow \text{83}$$

$$\int \frac{Cx^2+A}{\sqrt{a+bx}\sqrt{ac-bcx}}dx-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}}{b^2c}$$

$$\Big\downarrow \text{646}$$

$$\frac{1}{2}\left(\frac{a^2C}{b^2}+2A\right)\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}dx-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}}{b^2c}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}}{2b^2c}$$

$$\Big\downarrow \text{45}$$

$$\left(\frac{a^2C}{b^2}+2A\right)\int \frac{1}{\frac{c(a+bx)b}{ac-bcx}+b}d\frac{\sqrt{a+bx}}{\sqrt{ac-bcx}}-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}}{b^2c}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}}{2b^2c}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(\frac{a^2C}{b^2}+2A\right)\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b\sqrt{c}}-\frac{B\sqrt{a+bx}\sqrt{ac-bcx}}{b^2c}-\frac{Cx\sqrt{a+bx}\sqrt{ac-bcx}}{2b^2c}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

output
```
-((B*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(b^2*c)) - (C*x*Sqrt[a + b*x]*Sqrt[a
*c - b*c*x])/(2*b^2*c) + ((2*A + (a^2*C)/b^2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x
])/Sqrt[a*c - b*c*x]])/(b*Sqrt[c])
```

Defintions of rubi rules used

rule 45
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] &&  !GtQ[c, 0]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 646
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2), x_Symbol] :> Simp[b*x*(c + d*x)^(m + 1)*((e + f*x)^(n + 1)/(d*f*(2*m +
 3))), x] - Simp[(b*c*e - a*d*f*(2*m + 3))/(d*f*(2*m + 3))   Int[(c + d*x)^
m*(e + f*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] &&
 EqQ[d*e + c*f, 0] &&  !LtQ[m, -1]
```

rule 1189
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2), x_Symbol] :> Simp[b   Int[x*(d + e*x)^m*(f + g*x)^n, x],
x] + Int[(d + e*x)^m*(f + g*x)^n*(a + c*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, m, n}, x] && EqQ[m, n] && EqQ[e*f + d*g, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{(Cx+2B)(-bx+a)\sqrt{bx+a}}{2b^2\sqrt{-c(bx-a)}}+\frac{(2b^2A+a^2C)\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)\sqrt{-(bx+a)c(bx-a)}}{2b^2\sqrt{b^2c}\,\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\sqrt{bx+a}\,\sqrt{c(-bx+a)}\left(2A\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)b^2c+C\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{c(-b^2x^2+a^2)}}\right)a^2c-C\sqrt{c(-b^2x^2+a^2)}\,\sqrt{b^2c}\,x-2B\sqrt{b^2c}\,\sqrt{c}}{2b^2\sqrt{c(-b^2x^2+a^2)}\,c\sqrt{b^2c}}$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/2*(C*x+2*B)/b^2*(-b*x+a)*(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)+1/2*(2*A*b^2+
C*a^2)/b^2/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))*
(-(b*x+a)*c*(b*x-a))^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$=\left[-\frac{(Ca^2+2\,Ab^2)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{-c}x-a^2c\right)+2\,(Cbx+2\,Bb)\sqrt{-bcx+a}}{4\,b^3c}\right.$$
$$\left.-\frac{(Ca^2+2\,Ab^2)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}b\sqrt{c}x}{b^2cx^2-a^2c}\right)+(Cbx+2\,Bb)\sqrt{-bcx+ac}\sqrt{bx+a}}{2\,b^3c}\right]$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fri
cas")
```

output
```
[-1/4*((C*a^2 + 2*A*b^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*s
qrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(C*b*x + 2*B*b)*sqrt(-b*c*x + a*c)*
sqrt(b*x + a))/(b^3*c), -1/2*((C*a^2 + 2*A*b^2)*sqrt(c)*arctan(sqrt(-b*c*x
 + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (C*b*x + 2*B*b)*s
qrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^3*c)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{Ca^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}} + \frac{A\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}} - \frac{\sqrt{-b^2cx^2+a^2c}Cx}{2\,b^2c} - \frac{\sqrt{-b^2cx^2+a^2c}B}{b^2c}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="max
ima")
```

output
```
1/2*C*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) + A*arcsin(b*x/a)/(b*sqrt(c)) - 1/2*
sqrt(-b^2*c*x^2 + a^2*c)*C*x/(b^2*c) - sqrt(-b^2*c*x^2 + a^2*c)*B/(b^2*c)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\sqrt{-(bx+a)c+2\,ac}\sqrt{bx+a}\left(\frac{(bx+a)C}{c}-\frac{Cac-2\,Bbc}{c^2}\right)+\frac{2\,(Ca^2+2\,Ab^2)\log\left(\left|-\sqrt{bx+a}\sqrt{-c}+\sqrt{-(bx+a)c+2\,ac}\right|\right)}{\sqrt{-c}}}{2\,b^3}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="gia
c")
```

output
```
-1/2*(sqrt(-(b*x + a)*c + 2*a*c)*sqrt(b*x + a)*((b*x + a)*C/c - (C*a*c - 2
*B*b*c)/c^2) + 2*(C*a^2 + 2*A*b^2)*log(abs(-sqrt(b*x + a)*sqrt(-c) + sqrt(
-(b*x + a)*c + 2*a*c)))/sqrt(-c))/b^3
```

Mupad [B] (verification not implemented)

Time = 11.52 (sec) , antiderivative size = 489, normalized size of antiderivative = 3.91

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = -\frac{\frac{2\,C\,a^2\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^7}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^7}-\frac{2\,C\,a^2\,c^3\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,C\,a^2\,c\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^5}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^5}+\frac{14\,C\,a^2\,c^2\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^3}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3}}{b^3\,c^4+\frac{b^3\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^8}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^8}+\frac{4\,b^3\,c^3\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^2}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}+\frac{6\,b^3\,c^2\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^4}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}+\frac{4\,b^3\,c\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^6}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^6}} - \frac{4\,A\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}}-\frac{2\,C\,a^2\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^3\,\sqrt{c}}-\frac{B\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^2\,c}$$

input
```
int((A + B*x + C*x^2)/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
- ((2*C*a^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(1
/2))^7 - (2*C*a^2*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2
) - a^(1/2)) - (14*C*a^2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a + b*
x)^(1/2) - a^(1/2))^5 + (14*C*a^2*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - (4*A
*atan((b*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^(1
/2) - a^(1/2)))))/(b^2*c)^(1/2) - (2*C*a^2*atan(((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2)) - (B*(a*c
- b*c*x)^(1/2)*(a + b*x)^(1/2))/(b^2*c)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$=\frac{\sqrt{c}\left(-2asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^2c-4asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a\,b^2-2\sqrt{bx+a}\,\sqrt{-bx+a}\,b^2-\sqrt{bx+a}\,\sqrt{-bx+a}\,bcx\right)}{2b^3c}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

output
```
(sqrt(c)*( - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*c - 4*asin(sqrt(
a - b*x)/(sqrt(a)*sqrt(2)))*a*b**2 - 2*sqrt(a + b*x)*sqrt(a - b*x)*b**2 -
sqrt(a + b*x)*sqrt(a - b*x)*b*c*x))/(2*b**3*c)
```

3.56 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 186

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = -\frac{C\sqrt{a+bx}\sqrt{ac-bcx}}{b^2cf} - \frac{2(Ce-Bf)\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b\sqrt{c}f^2} + \frac{2(Ce^2-Bef+Af^2)\arctan\left(\frac{\sqrt{c}\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{ac-bcx}}\right)}{\sqrt{c}f^2\sqrt{be-af}\sqrt{be+af}}$$

output
```
-C*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/b^2/c/f-2*(-B*f+C*e)*arctan(c^(1/2)*(b
*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b/c^(1/2)/f^2+2*(A*f^2-B*e*f+C*e^2)*arctan
(c^(1/2)*(a*f+b*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(-b*c*x+a*c)^(1/2)
)/c^(1/2)/f^2/(-a*f+b*e)^(1/2)/(a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \frac{\frac{Cf(-a+bx)\sqrt{a+bx}}{b^2}-\frac{2(Ce-Bf)\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b}+\frac{2(Ce^2+f(-Be+Af))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{\sqrt{be-af}\sqrt{be+af}}}{f^2\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

output
```
((C*f*(-a + b*x)*Sqrt[a + b*x])/b^2 - (2*(C*e - B*f)*Sqrt[a - b*x]*ArcTan[
Sqrt[a + b*x]/Sqrt[a - b*x]])/b + (2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b
*x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])
])/(Sqrt[b*e - a*f]*Sqrt[b*e + a*f]))/(f^2*Sqrt[c*(a - b*x)])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2113, 2185, 25, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}}\,dx$$

$$\Big\downarrow \; 2113$$

$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 2185$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(-\frac{\int -\frac{b^2cf(Af-(Ce-Bf)x)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2cf^2}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{b^2cf(Af-(Ce-Bf)x)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2cf^2}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{Af-(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{719}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\int \frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{488}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{-\frac{\left(Af^2-Bef+Ce^2\right)\int \frac{1}{-b^2ce^2+a^2cf^2-\frac{\left(cfa^2+b^2cex\right)^2}{a^2c-b^2cx^2}}d\frac{cfa^2+b^2cex}{\sqrt{a^2c-b^2cx^2}}}{f}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(Af^2-Bef+Ce^2\right)\arctan\left(\frac{a^2cf+b^2cex}{\sqrt{c}\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f\sqrt{b^2e^2-a^2f^2}}-\frac{(Ce-Bf)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f}}{f}-\frac{C\sqrt{a^2c-b^2cx^2}}{b^2cf}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*(-((C*Sqrt[a^2*c - b^2*c*x^2])/(b^2*c*f)) + (-(((
C*e - B*f)*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f)) +
 ((C*e^2 - B*e*f + A*f^2)*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt[b^2*e
^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f*Sqrt[b^2*e^2 - a^2*f^2
]))/f))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.61

method	result
risch	$-\frac{C\sqrt{bx+a}\,(-bx+a)}{f\,b^{2}\sqrt{-c(bx-a)}}+\frac{\left(\frac{(Bf-Ce)\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-b^{2}c\,x^{2}+a^{2}c}}\right)}{f\sqrt{b^{2}c}}-\frac{\left(A\,f^{2}-Bef+C\,e^{2}\right)\ln\left(\frac{\frac{2c\left(a^{2}f^{2}-b^{2}e^{2}\right)}{f^{2}}+\frac{2b^{2}ce\left(x+\frac{e}{f}\right)}{f}+2\sqrt{\frac{c\left(a^{2}f^{2}-b^{2}e\right.}{f^{2}}}}{x}\right.}{f^{2}\sqrt{\frac{c\left(a^{2}f^{2}-b^{2}e^{2}\right)}{f^{2}}}}\right.}{f\sqrt{bx+a}\,\sqrt{-c(bx-a)}}$
default	$\frac{\left(-A\ln\left(\frac{2b^{2}cex+2a^{2}cf+2\sqrt{\frac{c\left(a^{2}f^{2}-b^{2}e^{2}\right)}{f^{2}}}\,\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}\,f}{fx+e}\right)b^{2}c\,f^{2}\sqrt{b^{2}c}+B\ln\left(\frac{2b^{2}cex+2a^{2}cf+2\sqrt{\frac{c\left(a^{2}f^{2}-b^{2}e^{2}\right)}{f^{2}}}\,\sqrt{c\left(-b^{2}x^{2}+a^{2}\right)}\,f}{fx+e}\right.\right.}{}$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e),x,method=_RETUR
NVERBOSE)
```

output
```
-C*(b*x+a)^(1/2)*(-b*x+a)/f/b^2/(-c*(b*x-a))^(1/2)+1/f*((B*f-C*e)/f/(b^2*c
)^(1/2)*arctan((b^2*c)^(1/2)*x/(-b^2*c*x^2+a^2*c)^(1/2))-(A*f^2-B*e*f+C*e^
2)/f^2/(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*ln((2*c*(a^2*f^2-b^2*e^2)/f^2+2*b^2
*c*e/f*(x+e/f)+2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(x+e/f)^2*b^2*c+2*b^2*c
*e/f*(x+e/f)+c*(a^2*f^2-b^2*e^2)/f^2)^(1/2))/(x+e/f)))*(-(b*x+a)*c*(b*x-a)
)^(1/2)/(b*x+a)^(1/2)/(-c*(b*x-a))^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e),x, algori
thm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-c(-a+bx)}\sqrt{a+bx}\,(e+fx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2)/(f*x+e),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e),x, algori
thm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for
more detai
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e),x, algori
thm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 28.70 (sec) , antiderivative size = 9298, normalized size of antiderivative = 49.99

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
(B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3
*a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)
*c^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)
 - (B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e
^6) + (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/
2)*b^4*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5
 - 22*B*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^
7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2
- 7*a^10*b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 -
11*a^10*b^4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e
^6*((a + b*x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)
^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^
2*b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^
(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/
2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2
*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(1
5/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*
b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*...
```

Reduce [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 3194, normalized size of antiderivative = 17.17

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e),x)
```

output
```
(sqrt(c)*( - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**2*f**4 + 2*as
in(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b*c*e*f**3 - 2*asin(sqrt(a - b*x)
/(sqrt(a)*sqrt(2)))*a**2*b**3*e*f**3 + 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(
2)))*a**2*b**2*c*e**2*f**2 + 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b**
4*e**2*f**2 - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b**3*c*e**3*f + 2*
asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*b**5*e**3*f - 2*asin(sqrt(a - b*x)/(
sqrt(a)*sqrt(2)))*b**4*c*e**4 - 2*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*s
qrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt
(2)*atan((tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt
(a - b*x)/(sqrt(a)*sqrt(2)))/2)*b*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*
e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)))*a*b**2*f**2 + 2*sqrt(f)*sqrt(a
)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f +
 b*e)*sqrt(a*f - b*e)*sqrt(2)*atan((tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2
)))/2)*a*f + tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*b*e)/(sqrt(2*sqr
t(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)))*b**3
*e*f - 2*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) -
3*a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2)*atan((tan(asin(sqrt(a
 - b*x)/(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2
)))/2)*b*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)
*sqrt(a*f + b*e)))*b**2*c*e**2 - 2*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*...
```

3.57 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 235

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{(Ce^2-Bef+Af^2)\sqrt{a+bx}\sqrt{ac-bcx}}{cf(be-af)(be+af)(e+fx)} + \frac{2C\arctan\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{ac-bcx}}\right)}{b\sqrt{c}f^2} + \frac{2(a^2f^2(2Ce-Bf)-b^2(Ce^3-Aef^2))\arctan\left(\frac{\sqrt{c}\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{ac-bcx}}\right)}{\sqrt{c}f^2(be-af)^{3/2}(be+af)^{3/2}}$$

output
```
(A*f^2-B*e*f+C*e^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/c/f/(-a*f+b*e)/(a*f+b
*e)/(f*x+e)+2*C*arctan(c^(1/2)*(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2))/b/c^(1/2)
/f^2+2*(a^2*f^2*(-B*f+2*C*e)-b^2*(-A*e*f^2+C*e^3))*arctan(c^(1/2)*(a*f+b*e
)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(-b*c*x+a*c)^(1/2))/c^(1/2)/f^2/(-a
*f+b*e)^(3/2)/(a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{2\left(\frac{f(Ce^2+f(-Be+Af))(-a+bx)\sqrt{a+bx}}{2(-be+af)(be+af)(e+fx)}+\frac{C\sqrt{a-bx}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{b}-\frac{(a^2f^2(-2Ce+Bf)+b^2(Ce^3-Aef^2))\sqrt{a-bx}\arctan\left(\frac{\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{a-bx}}\right)}{(be-af)^{3/2}(be+af)^{3/2}}\right)}{f^2\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),
x]
```

output
```
(2*((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/(2*(-(b*e) + a
*f)*(b*e + a*f)*(e + f*x)) + (C*Sqrt[a - b*x]*ArcTan[Sqrt[a + b*x]/Sqrt[a
- b*x]])/b - ((a^2*f^2*(-2*C*e + B*f) + b^2*(C*e^3 - A*e*f^2))*Sqrt[a - b*
x]*ArcTan[(Sqrt[b*e + a*f]*Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])]
)/((b*e - a*f)^(3/2)*(b*e + a*f)^(3/2))))/(f^2*Sqrt[c*(a - b*x)])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2113, 2182, 27, 719, 224, 216, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}}\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{2182}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{c\left((Ce-Bf)a^2+Ab^2e+C\left(\frac{b^2e^2}{f}-a^2f\right)x\right)}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{c(b^2e^2-a^2f^2)}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{(Ce-Bf)a^2+Ab^2e+C\left(\frac{b^2e^2}{f}-a^2f\right)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 719$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}dx}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 224$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\int \frac{1}{\frac{b^2cx^2}{a^2c-b^2cx^2}+1}d\frac{x}{\sqrt{a^2c-b^2cx^2}}}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int \frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{f^2}+\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \; 488$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}-\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\int \frac{1}{-b^2ce^2+a^2cf^2-\frac{\left(cfa^2+b^2cex\right)^2}{a^2c-b^2cx^2}}d\frac{cfa^2+b^2cex}{\sqrt{a^2c-b^2cx^2}}}{f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{c(e+fx)(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\downarrow 217$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\arctan\left(\frac{a^2cf+b^2cex}{\sqrt{c}\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{b^2e^2-a^2f^2}}+\frac{C\left(b^2e^2-a^2f^2\right)\arctan\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2}}{b^2e^2-a^2f^2}+\frac{f\sqrt{a^2c-b^2}}{c(e+f}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*((f*(A + (e*(C*e - B*f))/f^2)*Sqrt[a^2*c - b^2*c*
x^2])/(c*(b^2*e^2 - a^2*f^2)*(e + f*x)) + ((C*(b^2*e^2 - a^2*f^2)*ArcTan[(
b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2) + ((a^2*f^2*(2*C*e
- B*f) - b^2*(C*e^3 - A*e*f^2))*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt
[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 -
 a^2*f^2]))/(b^2*e^2 - a^2*f^2)))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(207) = 414$.

Time = 1.23 (sec) , antiderivative size = 1166, normalized size of antiderivative = 4.96

method	result	size
default	Expression too large to display	1166

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^2,x,method=_RET
URNVERBOSE)
```

output
```
(A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^
2))^(1/2)*f)/(f*x+e))*b^2*c*e*f^3*x*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*
f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2
*c*f^4*x*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*
a^2*c*f^4*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((b^2*c)^(1/2)*x/(c*(-
b^2*x^2+a^2))^(1/2))*b^2*c*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)+2*C*l
n(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^
(1/2)*f)/(f*x+e))*a^2*c*e*f^3*x*(b^2*c)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c
*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e
^3*f*x*(b^2*c)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(
1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln
(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(
1/2)*f)/(f*x+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)*x/(c*(-b
^2*x^2+a^2))^(1/2))*a^2*c*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*arctan((
b^2*c)^(1/2)*x/(c*(-b^2*x^2+a^2))^(1/2))*b^2*c*e^3*f*(c*(a^2*f^2-b^2*e^2)/
f^2)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^(1/2)-C*ln(2*(b^2*
c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/
(f*x+e))*b^2*c*e^4*(b^2*c)^(1/2)-A*f^4*(b^2*c)^(1/2)*(c*(a^2*f^2-b^2*e^2)/
f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+B*e*f^3*(b^2*c)^(1/2)*(c*(a^2*f^2-b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^2,x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(e+fx\right)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2)/(f*x+e)**2,x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)**
2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^2,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for
more detai
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(207) = 414.

Time = 0.30 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.24

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$$

$$= \frac{\frac{2\left(Cb^3\sqrt{-c}e^3-2\,Ca^2b\sqrt{-c}ef^2-Ab^3\sqrt{-c}ef^2+Ba^2b\sqrt{-c}f^3\right)\arctan\left(-\frac{2\,bce-\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-(bx+a)c+2\,ac}\right)^2f}{2\sqrt{-b^2e^2+a^2f^2c}}\right)}{\left(b^2e^2f^2-a^2f^4\right)\sqrt{-b^2e^2+a^2f^2c}} - \frac{C\log\left(\left(\sqrt{bx+a}\sqrt{-c}-\sqrt{-\cdots}\right.\right.}{\sqrt{-c}f^2\cdots}}{\cdots}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^2,x, algo
rithm="giac")
```

output
```
(2*(C*b^3*sqrt(-c)*e^3 - 2*C*a^2*b*sqrt(-c)*e*f^2 - A*b^3*sqrt(-c)*e*f^2 +
 B*a^2*b*sqrt(-c)*f^3)*arctan(-1/2*(2*b*c*e - (sqrt(b*x + a)*sqrt(-c) - sq
rt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^2 + a^2*f^2)*c))/((b^2*e^2*f^2
 - a^2*f^4)*sqrt(-b^2*e^2 + a^2*f^2)*c) - C*log((sqrt(b*x + a)*sqrt(-c) -
sqrt(-(b*x + a)*c + 2*a*c))^2)/(sqrt(-c)*f^2) + 4*(C*b^3*(sqrt(b*x + a)*sq
rt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^3 - B*b^3*(sqrt(b*x + a)
*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*e^2*f - 2*C*a^2*b^2*sqr
t(-c)*c*e^2*f + A*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)
)^2*sqrt(-c)*e*f^2 + 2*B*a^2*b^2*sqrt(-c)*c*e*f^2 - 2*A*a^2*b^2*sqrt(-c)*c
*f^3)/((b^2*e^2*f^2 - a^2*f^4)*(4*b*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x +
 a)*c + 2*a*c))^2*c*e - (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*
c))^4*f - 4*a^2*c^2*f)))/b
```

Mupad [B] (verification not implemented)

Time = 121.92 (sec) , antiderivative size = 106511, normalized size of antiderivative = 453.24

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
((4*B*a^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(
1/2))^3*(b^3*e^3 - a^2*b*e*f^2)) + (8*B*a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^2)/((a^2*f^2 - b^2*e^2)*((a + b*x)^(1/2) - a^(1/2))^
2) - (4*B*a^2*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) -
 a^(1/2))*(b^3*e^3 - a^2*b*e*f^2)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4
/((a + b*x)^(1/2) - a^(1/2))^4 + c^2 + (2*c*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (4*a^(1/2)*f*(a*c)^(1/2)*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3) + (4*a^
(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)
^(1/2) - a^(1/2)))) - ((4*C*a^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((b
^3*e^2 - a^2*b*f^2)*((a + b*x)^(1/2) - a^(1/2))^3) - (4*C*a^2*c*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2)))/((b^3*e^2 - a^2*b*f^2)*((a + b*x)^(1/2) - a^(1/
2))) + (8*C*a^(1/2)*e*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(
(a^2*f^3 - b^2*e^2*f)*((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2 + (2*c*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (4*a^(1/2)*f*(a
*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a
^(1/2))^3) + (4*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
))/(b*e*((a + b*x)^(1/2) - a^(1/2)))) + ((4*A*a^2*c*f^2*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2)))/((b^3*e^4 - a^2*b*e^2*f^2)*((a + b*x)^(1/2) - a^(1/2...
```

Reduce [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 8629, normalized size of antiderivative = 36.72

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^2} \, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^2,x)
```

output
```
(sqrt(c)*( - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**5*c*e*f**5 - 2*asi
n(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**5*c*f**6*x - 2*asin(sqrt(a - b*x)/(s
qrt(a)*sqrt(2)))*a**4*b*c*e**2*f**4 - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2
)))*a**4*b*c*e*f**5*x + 4*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**2*
c*e**3*f**3 + 4*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**3*b**2*c*e**2*f**
4*x + 4*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b**3*c*e**4*f**2 + 4*as
in(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2*b**3*c*e**3*f**3*x - 2*asin(sqrt(
a - b*x)/(sqrt(a)*sqrt(2)))*a*b**4*c*e**5*f - 2*asin(sqrt(a - b*x)/(sqrt(a
)*sqrt(2)))*a*b**4*c*e**4*f**2*x - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))
*b**5*c*e**6 - 2*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*b**5*c*e**5*f*x - 2
*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f +
b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2)*atan((tan(asin(sqrt(a - b*x)/
(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*b
*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*
f + b*e)))*a**2*b**2*e*f**3 - 2*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqr
t(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2
)*atan((tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt(a
 - b*x)/(sqrt(a)*sqrt(2)))/2)*b*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)
*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)))*a**2*b**2*f**4*x + 4*sqrt(f)*sqr
t(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt...
```

3.58 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 301

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx \\
&= \frac{(Ce^2-Bef+Af^2)\sqrt{a+bx}\sqrt{ac-bcx}}{2cf(be-af)(be+af)(e+fx)^2} \\
&\quad + \frac{(2a^2f^2(2Ce-Bf)-b^2(Ce^3+ef(Be-3Af)))\sqrt{a+bx}\sqrt{ac-bcx}}{2cf(be-af)^2(be+af)^2(e+fx)} \\
&\quad + \frac{(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A(2b^4e^2+a^2b^2f^2))\arctan\left(\frac{\sqrt{c}\sqrt{be+af}\sqrt{a+bx}}{\sqrt{be-af}\sqrt{ac-bcx}}\right)}{\sqrt{c}(be-af)^{5/2}(be+af)^{5/2}}
\end{aligned}$$

output
```
1/2*(A*f^2-B*e*f+C*e^2)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/c/f/(-a*f+b*e)/(a
*f+b*e)/(f*x+e)^2+1/2*(2*a^2*f^2*(-B*f+2*C*e)-b^2*(C*e^3+e*f*(-3*A*f+B*e))
)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2)/c/f/(-a*f+b*e)^2/(a*f+b*e)^2/(f*x+e)+(2
*a^4*C*f^2+a^2*b^2*e*(-3*B*f+C*e)+A*(a^2*b^2*f^2+2*b^4*e^2))*arctan(c^(1/2
)*(a*f+b*e)^(1/2)*(b*x+a)^(1/2)/(-a*f+b*e)^(1/2)/(-b*c*x+a*c)^(1/2))/c^(1/
2)/(-a*f+b*e)^(5/2)/(a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \frac{\frac{(-a+bx)\sqrt{a+bx}\left(b^2e\left(Ce^2x+Be(2e+fx)-Af(4e+3fx)\right)+a^2f(-Ce(3e+4fx)+f(Af+B(e+2fx)))\right)}{2(be-af)^2(be+af)^2(e+fx)^2}+\frac{\left(2a^4Cf^2+a^2b^2e(Ce-3Bf)+A\left(2b^4e^2\right.\right.}{(be-af)^5}}{\sqrt{c(a-bx)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),
x]
```

output
```
(((-a + b*x)*Sqrt[a + b*x]*(b^2*e*(C*e^2*x + B*e*(2*e + f*x) - A*f*(4*e +
3*f*x)) + a^2*f*(-(C*e*(3*e + 4*f*x)) + f*(A*f + B*(e + 2*f*x)))))/(2*(b*e
 - a*f)^2*(b*e + a*f)^2*(e + f*x)^2) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*
B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*ArcTan[(Sqrt[b*e + a*f]*
Sqrt[a + b*x])/(Sqrt[b*e - a*f]*Sqrt[a - b*x])])/((b*e - a*f)^(5/2)*(b*e +
 a*f)^(5/2)))/Sqrt[c*(a - b*x)]
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2113, 2182, 27, 679, 488, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}(e+fx)^3\sqrt{ac-bcx}}\,dx$$

$$\Big\downarrow \text{2113}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\int \frac{Cx^2+Bx+A}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$\Big\downarrow \text{2182}$$

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{c\left(2\left((Ce-Bf)a^2+Ab^2e\right)-\left(2a^2Cf-b^2\left(\frac{Ce^2}{f}+Be-Af\right)\right)x\right)}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{2c(b^2e^2-a^2f^2)}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 27

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\int \frac{2\left((Ce-Bf)a^2+Ab^2e\right)-\left(2a^2Cf-b^2\left(\frac{Ce^2}{f}+Be-Af\right)\right)x}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}dx}{2(b^2e^2-a^2f^2)}+\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 679

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(Ce-3Bf)\right)\int\frac{1}{(e+fx)\sqrt{a^2c-b^2cx^2}}dx}{b^2e^2-a^2f^2}+\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}}{2(b^2e^2-a^2f^2)}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 488

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}-\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(Ce-3Bf)\right)\int\frac{1}{-b^2ce^2+a^2cf^2-\frac{(cfa^2}{a^2c}}}{b^2e^2-a^2f^2}}{2(b^2e^2-a^2f^2)}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$\downarrow$ 217

$$\frac{\sqrt{a^2c-b^2cx^2}\left(\frac{f\sqrt{a^2c-b^2cx^2}\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2c(e+fx)^2(b^2e^2-a^2f^2)}+\frac{\frac{\sqrt{a^2c-b^2cx^2}\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{cf(e+fx)\left(b^2e^2-a^2f^2\right)}+\frac{\left(2a^4Cf^2+A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2}{\sqrt{}}}{2(b^2e^2-a^2f^2)}\right.}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

output
```
(Sqrt[a^2*c - b^2*c*x^2]*((f*(A + (e*(C*e - B*f))/f^2)*Sqrt[a^2*c - b^2*c*
x^2])/(2*c*(b^2*e^2 - a^2*f^2)*(e + f*x)^2) + (((2*a^2*f^2*(2*C*e - B*f) -
 b^2*(C*e^3 + e*f*(B*e - 3*A*f)))*Sqrt[a^2*c - b^2*c*x^2])/(c*f*(b^2*e^2 -
 a^2*f^2)*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3*B*f) + A*(2*b^4*
e^2 + a^2*b^2*f^2))*ArcTan[(a^2*c*f + b^2*c*e*x)/(Sqrt[c]*Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(Sqrt[c]*(b^2*e^2 - a^2*f^2)^(3/2)))/(2
*(b^2*e^2 - a^2*f^2))))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. $2(273) = 546$.

Time = 1.32 (sec) , antiderivative size = 1794, normalized size of antiderivative = 5.96

method	result	size
default	Expression too large to display	1794

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^3,x,method=_RET
URNVERBOSE)
```

output
```
-1/2*(A*a^2*f^4*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+2
*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^
2))^(1/2)*f)/(f*x+e))*b^4*c*e^4-4*A*b^2*e^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^
(1/2)*(c*(-b^2*x^2+a^2))^(1/2)+B*a^2*e*f^3*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)
*(c*(-b^2*x^2+a^2))^(1/2)+2*B*b^2*e^3*f*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c
*(-b^2*x^2+a^2))^(1/2)-3*C*a^2*e^2*f^2*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*
(-b^2*x^2+a^2))^(1/2)-3*A*b^2*e*f^3*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(
-b^2*x^2+a^2))^(1/2)+B*b^2*e^2*f^2*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-
b^2*x^2+a^2))^(1/2)-4*C*a^2*e*f^3*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b
^2*x^2+a^2))^(1/2)+C*b^2*e^3*f*x*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*
x^2+a^2))^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*
(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*f^4*x^2+2*A*ln(2*(b^2*c*e*x
+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+
e))*b^4*c*e^2*f^2*x^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2
)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*b^4*c*e^3*f*x+4*C*ln(2*(b^2*c
*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(
f*x+e))*a^4*c*e*f^3*x+A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^
(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2-3*B*ln(2*(b^2
*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(c*(-b^2*x^2+a^2))^(1/2)*f)
/(f*x+e))*a^2*b^2*c*e^3*f+2*C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(273) = 546$.

Time = 38.13 (sec) , antiderivative size = 1355, normalized size of antiderivative = 4.50

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^3,x, algo
rithm="fricas")
```

output
```
[1/4*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^
2)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4
 + A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^
3*f - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sqrt(-b^2*c*e^2 + a^2*c*f^2)*log((2*
a^2*b^2*c*e*f*x - a^2*b^2*c*e^2 + 2*a^4*c*f^2 + (2*b^4*c*e^2 - a^2*b^2*c*f
^2)*x^2 - 2*sqrt(-b^2*c*e^2 + a^2*c*f^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a
*c)*sqrt(b*x + a))/(f^2*x^2 + 2*e*f*x + e^2)) - 2*(2*B*b^4*e^5 - B*a^2*b^2
*e^3*f^2 - B*a^4*e*f^4 - A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*
a^4 + 5*A*a^2*b^2)*e^2*f^3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3
- 2*B*a^4*f^5 - (5*C*a^2*b^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*
e*f^4)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f
^2 + 3*a^4*b^2*c*e^4*f^4 - a^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^
4*f^4 + 3*a^4*b^2*c*e^2*f^6 - a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*
c*e^5*f^3 + 3*a^4*b^2*c*e^3*f^5 - a^6*c*e*f^7)*x), -1/2*((3*B*a^2*b^2*e^3*
f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2)*e^2*f^2 + (3*B*a^2*b
^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 + A*a^2*b^2)*f^4)*x^2
+ 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f - (2*C*a^4 + A*a^2*
b^2)*e*f^3)*x)*sqrt(b^2*c*e^2 - a^2*c*f^2)*arctan(sqrt(b^2*c*e^2 - a^2*c*f
^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^2*b^2*c*e^2 - a^
4*c*f^2 - (b^4*c*e^2 - a^2*b^2*c*f^2)*x^2)) + (2*B*b^4*e^5 - B*a^2*b^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2)/(f*x+e)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^3,x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*f-b*e)>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1425 vs. $2(273) = 546$.

Time = 0.64 (sec) , antiderivative size = 1425, normalized size of antiderivative = 4.73

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^3,x, algo
rithm="giac")
```

output
```
-((C*a^2*b^3*sqrt(-c)*e^2 + 2*A*b^5*sqrt(-c)*e^2 - 3*B*a^2*b^3*sqrt(-c)*e*
f + 2*C*a^4*b*sqrt(-c)*f^2 + A*a^2*b^3*sqrt(-c)*f^2)*arctan(-1/2*(2*b*c*e
- (sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*f)/(sqrt(-b^2*e^
2 + a^2*f^2)*c))/((b^4*e^4 - 2*a^2*b^2*e^2*f^2 + a^4*f^4)*sqrt(-b^2*e^2 +
a^2*f^2)*c) + 2*(4*C*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a
*c))^4*sqrt(-c)*c*e^5 - 2*C*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*
c + 2*a*c))^6*sqrt(-c)*e^4*f + 4*B*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^4*f - 8*C*a^2*b^5*(sqrt(b*x + a)*sqrt(-c
) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^4*f - 14*C*a^2*b^4*(sqrt(
b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e^3*f^2 - 12*
A*b^6*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^4*sqrt(-c)*c*e
^3*f^2 - 16*B*a^2*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c)
)^2*sqrt(-c)*c^2*e^3*f^2 + 8*C*a^4*b^4*sqrt(-c)*c^3*e^3*f^2 + 5*C*a^2*b^3*
(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*e^2*f^3 +
 2*A*b^5*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^6*sqrt(-c)*
e^2*f^3 + 10*B*a^2*b^4*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x + a)*c + 2*a*c
))^4*sqrt(-c)*c*e^2*f^3 + 44*C*a^4*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*
x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^2*f^3 + 40*A*a^2*b^5*(sqrt(b*x + a)*sq
rt(-c) - sqrt(-(b*x + a)*c + 2*a*c))^2*sqrt(-c)*c^2*e^2*f^3 + 8*B*a^4*b^4*
sqrt(-c)*c^3*e^2*f^3 - 3*B*a^2*b^3*(sqrt(b*x + a)*sqrt(-c) - sqrt(-(b*x...
```

Mupad [B] (verification not implemented)

Time = 67.45 (sec) , antiderivative size = 9344, normalized size of antiderivative = 31.04

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

output
```
((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e
^2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f
^4)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2
*b^2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2
 + a^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3
*e^3*f^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b
^3*e^3*f^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a
^2*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) -
 a^(1/2))^4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a
*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b
^2*e^2*f))/(((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a
^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c
^2*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/
2))^2*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^
(1/2) - a^(1/2))^6) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))^2)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2...
```

Reduce [B] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 15212, normalized size of antiderivative = 50.54

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{ac - bcx}(e + fx)^3}\, dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(f*x+e)^3,x)
```

output

```
(sqrt(c)*( - 4*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt
(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2)*atan((tan(asin(
sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt(a - b*x)/(sqrt(a)*
sqrt(2)))/2)*b*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f
+ b*e)*sqrt(a*f + b*e)))*a**4*c*e**2*f**2 - 8*sqrt(f)*sqrt(a)*sqrt(2*sqrt(
f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f
 - b*e)*sqrt(2)*atan((tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*a*f + t
an(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*b*e)/(sqrt(2*sqrt(f)*sqrt(a)*s
qrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)))*a**4*c*e*f**3*x -
4*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f +
 b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2)*atan((tan(asin(sqrt(a - b*x)
/(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*
b*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a
*f + b*e)))*a**4*c*f**4*x**2 - 2*sqrt(f)*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sq
rt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)*sqrt(a*f - b*e)*sqrt(
2)*atan((tan(asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))/2)*a*f + tan(asin(sqrt(
a - b*x)/(sqrt(a)*sqrt(2)))/2)*b*e)/(sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e
)*sqrt(2) - 3*a*f + b*e)*sqrt(a*f + b*e)))*a**3*b**2*e**2*f**2 - 4*sqrt(f)
*sqrt(a)*sqrt(2*sqrt(f)*sqrt(a)*sqrt(a*f - b*e)*sqrt(2) - 3*a*f + b*e)*sqr
t(a*f + b*e)*sqrt(a*f - b*e)*sqrt(2)*atan((tan(asin(sqrt(a - b*x)/(sqrt...
```

3.59 $\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 210

$$\begin{aligned}\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx &= -\frac{(cd^2-e(bd-ae))\sqrt{-1+x}\sqrt{1+x}}{2e(d^2-e^2)(d+ex)^2} \\ &\quad+\frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))\sqrt{-1+x}\sqrt{1+x}}{2e(d^2-e^2)^2(d+ex)} \\ &\quad-\frac{(3bde-a(2d^2+e^2)-c(d^2+2e^2))\,\text{arctanh}\left(\frac{\sqrt{d+e}\sqrt{1+x}}{\sqrt{d-e}\sqrt{-1+x}}\right)}{(d-e)^{5/2}(d+e)^{5/2}}\end{aligned}$$

output

```
-1/2*(c*d^2-e*(-a*e+b*d))*(-1+x)^(1/2)*(1+x)^(1/2)/e/(d^2-e^2)/(e*x+d)^2+1
/2*(c*(d^3-4*d*e^2)-e*(3*a*d*e-b*(d^2+2*e^2)))*(-1+x)^(1/2)*(1+x)^(1/2)/e/
(d^2-e^2)^2/(e*x+d)-(3*b*d*e-a*(2*d^2+e^2)-c*(d^2+2*e^2))*arctanh((d+e)^(1
/2)*(1+x)^(1/2)/(d-e)^(1/2)/(-1+x)^(1/2))/(d-e)^(5/2)/(d+e)^(5/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.88

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx$$
$$=\frac{\sqrt{-1+x}\sqrt{1+x}(ae(-4d^2+e^2-3dex)+cd(-3de+d^2x-4e^2x)+b(2d^3+de^2+d^2ex+2e^3x))}{2(d-e)^2(d+e)^2(d+ex)^2}$$
$$-\frac{(-3bde+a(2d^2+e^2)+c(d^2+2e^2))\arctan\left(\frac{\sqrt{d-e}\sqrt{\frac{-1+x}{1+x}}}{\sqrt{-d-e}}\right)}{(-d-e)^{5/2}(d-e)^{5/2}}$$

input
```
Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

output
```
(Sqrt[-1 + x]*Sqrt[1 + x]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^
2*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/(2*(d - e)^2*(d +
 e)^2*(d + e*x)^2) - ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*ArcTa
n[(Sqrt[d - e]*Sqrt[(-1 + x)/(1 + x)])/Sqrt[-d - e]])/((-d - e)^(5/2)*(d -
 e)^(5/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {2113, 2182, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{x-1}\sqrt{x+1}(d+ex)^3}\,dx$$
$$\Big\downarrow \text{2113}$$
$$\frac{\sqrt{x^2-1}\int \frac{cx^2+bx+a}{(d+ex)^3\sqrt{x^2-1}}dx}{\sqrt{x-1}\sqrt{x+1}}$$
$$\Big\downarrow \text{2182}$$

$$\frac{\sqrt{x^2-1}\left(-\frac{\int -\frac{2(ad+cd-be)+\left(\frac{cd^2}{e}+bd-ae-2ce\right)x}{(d+ex)^2\sqrt{x^2-1}}dx}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{x^2-1}\left(\frac{\int \frac{2(ad+cd-be)+\left(\frac{cd^2}{e}+bd-ae-2ce\right)x}{(d+ex)^2\sqrt{x^2-1}}dx}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \; 679$$

$$\frac{\sqrt{x^2-1}\left(\frac{\frac{\sqrt{x^2-1}\left(-3ade^2+bd^2e+2be^3+cd^3-4cde^2\right)}{e\left(d^2-e^2\right)(d+ex)}-\frac{\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)\int\frac{1}{(d+ex)\sqrt{x^2-1}}dx}{d^2-e^2}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \; 488$$

$$\frac{\sqrt{x^2-1}\left(\frac{\frac{\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)\int\frac{1}{d^2-e^2-\frac{(-e-dx)^2}{x^2-1}}d\frac{-e-dx}{\sqrt{x^2-1}}}{d^2-e^2}+\frac{\sqrt{x^2-1}\left(-3ade^2+bd^2e+2be^3+cd^3-4cde^2\right)}{e\left(d^2-e^2\right)(d+ex)}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{x^2-1}\left(\frac{\frac{\text{arctanh}\left(\frac{-dx-e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{\left(d^2-e^2\right)^{3/2}}+\frac{\sqrt{x^2-1}\left(-3ade^2+bd^2e+2be^3+cd^3-4cde^2\right)}{e\left(d^2-e^2\right)(d+ex)}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}\right)}{\sqrt{x-1}\sqrt{x+1}}$$

input
```
Int[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

output
```
(Sqrt[-1 + x^2]*(-1/2*((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(e*(d^2 - e
^2)*(d + e*x)^2) + (((c*d^3 + b*d^2*e - 3*a*d*e^2 - 4*c*d*e^2 + 2*b*e^3)*S
qrt[-1 + x^2])/(e*(d^2 - e^2)*(d + e*x)) + ((3*b*d*e - a*(2*d^2 + e^2) - c
*(d^2 + 2*e^2))*ArcTanh[(-e - d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(d^2
 - e^2)^(3/2))/(2*(d^2 - e^2))))/(Sqrt[-1 + x]*Sqrt[1 + x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2113
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Simp[(a + b*x)^FracPart[m]*((c + d*x)^FracPart[
m]/(a*c + b*d*x^2)^FracPart[m])   Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a
*d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1094 vs. $2(186) = 372$.

Time = 0.94 (sec) , antiderivative size = 1095, normalized size of antiderivative = 5.21

method	result	size
default	Expression too large to display	1095

input
```
int((c*x^2+b*x+a)/(-1+x)^(1/2)/(1+x)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBO
SE)
```

output
```
-1/2*(3*a*d*e^3*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)-b*d^2*e^2*x*(x^2-1)^
(1/2)*((d^2-e^2)/e^2)^(1/2)-c*d^3*e*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)-
a*e^4*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^
2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*c*e^4*x^2+ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)
/e^2)^(1/2)*e+x*d+e)/(e*x+d))*a*d^2*e^2-3*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)
/e^2)^(1/2)*e+x*d+e)/(e*x+d))*b*d^3*e+2*ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e
^2)^(1/2)*e+x*d+e)/(e*x+d))*c*d^2*e^2+ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2
)^(1/2)*e+x*d+e)/(e*x+d))*a*e^4*x^2-2*b*e^4*x*(x^2-1)^(1/2)*((d^2-e^2)/e^2
)^(1/2)+4*a*d^2*e^2*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)-2*b*d^3*e*(x^2-1)^
(1/2)*((d^2-e^2)/e^2)^(1/2)-b*d*e^3*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+3*
c*d^2*e^2*(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/2)+2*ln(-2*(-(x^2-1)^(1/2)*((d^
2-e^2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*a*d^2*e^2*x^2-3*ln(-2*(-(x^2-1)^(1/2)*
((d^2-e^2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*b*d*e^3*x^2+ln(-2*(-(x^2-1)^(1/2)*
((d^2-e^2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*c*d^2*e^2*x^2+4*ln(-2*(-(x^2-1)^(1
/2)*((d^2-e^2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*a*d^3*e*x+2*ln(-2*(-(x^2-1)^(1
/2)*((d^2-e^2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*a*d*e^3*x-6*ln(-2*(-(x^2-1)^(1
/2)*((d^2-e^2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*b*d^2*e^2*x+2*ln(-2*(-(x^2-1)^
(1/2)*((d^2-e^2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*c*d^3*e*x+4*ln(-2*(-(x^2-1)^
(1/2)*((d^2-e^2)/e^2)^(1/2)*e+x*d+e)/(e*x+d))*c*d*e^3*x+4*c*d*e^3*x*(x^2-1
)^(1/2)*((d^2-e^2)/e^2)^(1/2)+ln(-2*(-(x^2-1)^(1/2)*((d^2-e^2)/e^2)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(186) = 372$.

Time = 0.13 (sec) , antiderivative size = 1186, normalized size of antiderivative = 5.65

$$\int \frac{a + bx + cx^2}{\sqrt{-1 + x}\sqrt{1 + x}(d + ex)^3} \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(-1+x)^(1/2)/(1+x)^(1/2)/(e*x+d)^3,x, algorithm="f
ricas")
```

output
```
[1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*
e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e
^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 +
 (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 +
 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*
log((d^2*x + d*e + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*sqrt(x + 1)*sqrt(x - 1)
 + sqrt(d^2 - e^2)*(d*x + e))/(e*x + d)) + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*
e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d
^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)
*sqrt(x + 1)*sqrt(x - 1) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 +
b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x)/(d^8*e^2 - 3*d^6*e^4 + 3*d
^4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e
^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)
*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*
d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x
^2 - 2*((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d
^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e
^4 + (a + 2*c)*d*e^5)*x)*sqrt(-d^2 + e^2)*arctan(-(sqrt(-d^2 + e^2)*e*sqrt
(x + 1)*sqrt(x - 1) - sqrt(-d^2 + e^2)*(e*x + d))/(d^2 - e^2)) + (2*b*d^5*
e^2 - (4*a + 3*c)*d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 -...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)/(-1+x)**(1/2)/(1+x)**(1/2)/(e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(-1+x)^(1/2)/(1+x)^(1/2)/(e*x+d)^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((e-d)*(e+d)>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(186) = 372.

Time = 0.36 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.99

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = -\frac{(2\,ad^2+cd^2-3\,bde+ae^2+2\,ce^2)\arctan\left(\frac{e\left(\sqrt{x+1}-\sqrt{x-1}\right)^2+2\,d}{2\,\sqrt{-d^2+e^2}}\right)}{(d^4-2\,d^2e^2+e^4)\sqrt{-d^2+e^2}} + \frac{2\left(2\,cd^4e\left(\sqrt{x+1}-\sqrt{x-1}\right)^6-2\,ad^2e^3\left(\sqrt{x+1}-\sqrt{x-1}\right)^6-5\,cd^2e^3\left(\sqrt{x+1}-\sqrt{x-1}\right)^6+3\,bde^4\left(\right.\right.}{}$$

input
```
integrate((c*x^2+b*x+a)/(-1+x)^(1/2)/(1+x)^(1/2)/(e*x+d)^3,x, algorithm="g
iac")
```

output
```
-(2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(1/2*(e*(sqrt(x + 1)
- sqrt(x - 1))^2 + 2*d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d
^2 + e^2)) + 2*(2*c*d^4*e*(sqrt(x + 1) - sqrt(x - 1))^6 - 2*a*d^2*e^3*(sqr
t(x + 1) - sqrt(x - 1))^6 - 5*c*d^2*e^3*(sqrt(x + 1) - sqrt(x - 1))^6 + 3*
b*d*e^4*(sqrt(x + 1) - sqrt(x - 1))^6 - a*e^5*(sqrt(x + 1) - sqrt(x - 1))^
6 + 4*c*d^5*(sqrt(x + 1) - sqrt(x - 1))^4 + 4*b*d^4*e*(sqrt(x + 1) - sqrt(
x - 1))^4 - 12*a*d^3*e^2*(sqrt(x + 1) - sqrt(x - 1))^4 - 14*c*d^3*e^2*(sqr
t(x + 1) - sqrt(x - 1))^4 + 10*b*d^2*e^3*(sqrt(x + 1) - sqrt(x - 1))^4 - 6
*a*d*e^4*(sqrt(x + 1) - sqrt(x - 1))^4 - 8*c*d*e^4*(sqrt(x + 1) - sqrt(x -
 1))^4 + 4*b*e^5*(sqrt(x + 1) - sqrt(x - 1))^4 + 8*c*d^4*e*(sqrt(x + 1) -
sqrt(x - 1))^2 + 16*b*d^3*e^2*(sqrt(x + 1) - sqrt(x - 1))^2 - 40*a*d^2*e^3
*(sqrt(x + 1) - sqrt(x - 1))^2 - 44*c*d^2*e^3*(sqrt(x + 1) - sqrt(x - 1))^
2 + 20*b*d*e^4*(sqrt(x + 1) - sqrt(x - 1))^2 + 4*a*e^5*(sqrt(x + 1) - sqrt
(x - 1))^2 + 8*c*d^3*e^2 + 8*b*d^2*e^3 - 24*a*d*e^4 - 32*c*d*e^4 + 16*b*e^
5)/((d^4*e^2 - 2*d^2*e^4 + e^6)*(e*(sqrt(x + 1) - sqrt(x - 1))^4 + 4*d*(sq
rt(x + 1) - sqrt(x - 1))^2 + 4*e)^2)
```

Mupad [B] (verification not implemented)

Time = 46.17 (sec) , antiderivative size = 7235, normalized size of antiderivative = 34.45

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((a + b*x + c*x^2)/((x - 1)^(1/2)*(x + 1)^(1/2)*(d + e*x)^3),x)
```

output
```
((((x - 1)^(1/2) - 1i)^2*(2*c*e^3 + c*d^2*e)*12i)/(d^2*((x + 1)^(1/2) - 1)
^2*(d^4 + e^4 - 2*d^2*e^2)) - (2*(7*c*d^4 + 14*c*d^2*e^2)*((x - 1)^(1/2) -
 1i))/(7*d^3*((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) + (((x - 1)^(1/2
) - 1i)^4*(2*c*e^3 - c*d^2*e)*24i)/(d^2*((x + 1)^(1/2) - 1)^4*(d^4 + e^4 -
 2*d^2*e^2)) - (2*(21*c*d^4 - 102*c*d^2*e^2)*((x - 1)^(1/2) - 1i)^5)/(3*d^
3*((x + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)) - (2*(35*c*d^4 - 170*c*d^
2*e^2)*((x - 1)^(1/2) - 1i)^3)/(5*d^3*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2
*d^2*e^2)) + (c*((x - 1)^(1/2) - 1i)^7*(d^2*1i + e^2*2i)*2i)/(d*((x + 1)^(
1/2) - 1)^7*(d^4 + e^4 - 2*d^2*e^2)) + (12*c*e*((x - 1)^(1/2) - 1i)^6*(d^2
*1i + e^2*2i))/(d^2*((x + 1)^(1/2) - 1)^6*(d^4 + e^4 - 2*d^2*e^2)))/(((x -
 1)^(1/2) - 1i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*(
(x + 1)^(1/2) - 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)
^3) + (e*((x - 1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1
)^(1/2) - 1i)^7*8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4
*d^2 + 16*e^2))/(d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d
^2 + 16*e^2))/(d^2*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2
 - 32*e^2))/(d^2*((x + 1)^(1/2) - 1)^4) + 1) - ((2*((x - 1)^(1/2) - 1i)^3*
(16*b*e^3 + 11*b*d^2*e))/(d^2*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2*d^2*e^2
)) - (6*b*e*((x - 1)^(1/2) - 1i)^7)/(((x + 1)^(1/2) - 1)^7*(d^4 + e^4 - 2*
d^2*e^2)) - (6*b*e*((x - 1)^(1/2) - 1i))/(((x + 1)^(1/2) - 1)*(d^4 + e^...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3120, normalized size of antiderivative = 14.86

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)/(-1+x)^(1/2)/(1+x)^(1/2)/(e*x+d)^3,x)
```

output

```
( - 4*sqrt(d + e)*sqrt(d - e)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x + 1)*sq
rt(x - 1)*e + d + e*x)*a*d**5*e - 8*sqrt(d + e)*sqrt(d - e)*log(sqrt(d + e
)*sqrt(d - e) + sqrt(x + 1)*sqrt(x - 1)*e + d + e*x)*a*d**4*e**2*x - 4*sqr
t(d + e)*sqrt(d - e)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x + 1)*sqrt(x - 1)
*e + d + e*x)*a*d**3*e**3*x**2 - 2*sqrt(d + e)*sqrt(d - e)*log(sqrt(d + e)
*sqrt(d - e) + sqrt(x + 1)*sqrt(x - 1)*e + d + e*x)*a*d**3*e**3 - 4*sqrt(d
 + e)*sqrt(d - e)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x + 1)*sqrt(x - 1)*e
+ d + e*x)*a*d**2*e**4*x - 2*sqrt(d + e)*sqrt(d - e)*log(sqrt(d + e)*sqrt(
d - e) + sqrt(x + 1)*sqrt(x - 1)*e + d + e*x)*a*d*e**5*x**2 + 6*sqrt(d + e
)*sqrt(d - e)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x + 1)*sqrt(x - 1)*e + d
+ e*x)*b*d**4*e**2 + 12*sqrt(d + e)*sqrt(d - e)*log(sqrt(d + e)*sqrt(d - e
) + sqrt(x + 1)*sqrt(x - 1)*e + d + e*x)*b*d**3*e**3*x + 6*sqrt(d + e)*sqr
t(d - e)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x + 1)*sqrt(x - 1)*e + d + e*x
)*b*d**2*e**4*x**2 - 2*sqrt(d + e)*sqrt(d - e)*log(sqrt(d + e)*sqrt(d - e)
 + sqrt(x + 1)*sqrt(x - 1)*e + d + e*x)*c*d**5*e - 4*sqrt(d + e)*sqrt(d -
e)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x + 1)*sqrt(x - 1)*e + d + e*x)*c*d*
*4*e**2*x - 2*sqrt(d + e)*sqrt(d - e)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x
 + 1)*sqrt(x - 1)*e + d + e*x)*c*d**3*e**3*x**2 - 4*sqrt(d + e)*sqrt(d - e
)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x + 1)*sqrt(x - 1)*e + d + e*x)*c*d**
3*e**3 - 8*sqrt(d + e)*sqrt(d - e)*log(sqrt(d + e)*sqrt(d - e) + sqrt(x...
```

3.60 $\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} \, dx = 4\log(1-x) - 14\log(2-x) + 11\log(3-x)$$

output
```
4*ln(1-x)-14*ln(2-x)+11*ln(3-x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)} \, dx = 11\log(-3+x) - 14\log(-2+x) + 4\log(-1+x)$$

input
```
Integrate[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
11*Log[-3 + x] - 14*Log[-2 + x] + 4*Log[-1 + x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3x+4}{(x-3)(x-2)(x-1)}dx$$

$$\Big\downarrow \text{2115}$$

$$\int \left(-\frac{14}{x-2}+\frac{4}{x-1}+\frac{11}{x-3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$4\log(1-x)-14\log(2-x)+11\log(3-x)$$

input
```
Int[(4 + 3*x + x^2)/((-3 + x)*(-2 + x)*(-1 + x)),x]
```

output
```
4*Log[1 - x] - 14*Log[2 - x] + 11*Log[3 - x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-14\ln\left(x-2\right)+4\ln\left(-1+x\right)+11\ln\left(-3+x\right)$	20
norman	$-14\ln\left(x-2\right)+4\ln\left(-1+x\right)+11\ln\left(-3+x\right)$	20
risch	$-14\ln\left(x-2\right)+4\ln\left(-1+x\right)+11\ln\left(-3+x\right)$	20
parallelrisch	$-14\ln\left(x-2\right)+4\ln\left(-1+x\right)+11\ln\left(-3+x\right)$	20

input
```
int((x^2+3*x+4)/(-3+x)/(x-2)/(-1+x),x,method=_RETURNVERBOSE)
```

output
```
-14*ln(x-2)+4*ln(-1+x)+11*ln(-3+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\,\log\left(x-1\right)-14\,\log\left(x-2\right)+11\,\log\left(x-3\right)$$

input
```
integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

output
```
4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 11\log\left(x-3\right)-14\log\left(x-2\right)+4\log\left(x-1\right)$$

input
```
integrate((x**2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
```

output
```
11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\,\log(x-1) - 14\,\log(x-2) + 11\,\log(x-3)$$

input
```
integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

output
```
4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\,\log(|x-1|) - 14\,\log(|x-2|) + 11\,\log(|x-3|)$$

input
```
integrate((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

output
```
4*log(abs(x - 1)) - 14*log(abs(x - 2)) + 11*log(abs(x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 4\,\ln(x-1) - 14\,\ln(x-2) + 11\,\ln(x-3)$$

input
```
int((3*x + x^2 + 4)/((x - 1)*(x - 2)*(x - 3)),x)
```

output
```
4*log(x - 1) - 14*log(x - 2) + 11*log(x - 3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{4+3x+x^2}{(-3+x)(-2+x)(-1+x)}\,dx = 11\log(x-3) - 14\log(x-2) + 4\log(x-1)$$

input
```
int((x^2+3*x+4)/(-3+x)/(-2+x)/(-1+x),x)
```

output
```
11*log(x - 3) - 14*log(x - 2) + 4*log(x - 1)
```

3.61 $\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 141

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)} \, dx = \frac{(Ab^2-a(bB-aC))\log(a+bx)}{b(bc-ad)(be-af)} - \frac{(c^2C-Bcd+Ad^2)\log(c+dx)}{d(bc-ad)(de-cf)} + \frac{(Ce^2-Bef+Af^2)\log(e+fx)}{f(be-af)(de-cf)}$$

output
```
(A*b^2-a*(B*b-C*a))*ln(b*x+a)/b/(-a*d+b*c)/(-a*f+b*e)-(A*d^2-B*c*d+C*c^2)*
ln(d*x+c)/d/(-a*d+b*c)/(-c*f+d*e)+(A*f^2-B*e*f+C*e^2)*ln(f*x+e)/f/(-a*f+b*
e)/(-c*f+d*e)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)} \, dx = \frac{(Ab^2-abB+a^2C)\log(a+bx)}{b(bc-ad)(be-af)} - \frac{(-c^2C+Bcd-Ad^2)\log(c+dx)}{d(bc-ad)(-de+cf)} + \frac{(Ce^2-Bef+Af^2)\log(e+fx)}{f(be-af)(de-cf)}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)*(c + d*x)*(e + f*x)),x]
```

output
```
((A*b^2 - a*b*B + a^2*C)*Log[a + b*x])/(b*(b*c - a*d)*(b*e - a*f)) - ((-(c
^2*C) + B*c*d - A*d^2)*Log[c + d*x])/(d*(b*c - a*d)*(-(d*e) + c*f)) + ((C*
e^2 - B*e*f + A*f^2)*Log[e + f*x])/(f*(b*e - a*f)*(d*e - c*f))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)} \, dx$$

$$\Big\downarrow \text{2115}$$

$$\int \left(\frac{Ab^2-a(bB-aC)}{(a+bx)(bc-ad)(be-af)}+\frac{Ad^2-Bcd+c^2C}{(c+dx)(bc-ad)(cf-de)}+\frac{Af^2-Bef+Ce^2}{(e+fx)(be-af)(de-cf)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(a+bx)\left(Ab^2-a(bB-aC)\right)}{b(bc-ad)(be-af)}-\frac{\log(c+dx)\left(Ad^2-Bcd+c^2C\right)}{d(bc-ad)(de-cf)}+\frac{\log(e+fx)\left(Af^2-Bef+Ce^2\right)}{f(be-af)(de-cf)}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)*(c + d*x)*(e + f*x)),x]
```

output
```
((A*b^2 - a*(b*B - a*C))*Log[a + b*x])/(b*(b*c - a*d)*(b*e - a*f)) - ((c^2
*C - B*c*d + A*d^2)*Log[c + d*x])/(d*(b*c - a*d)*(d*e - c*f)) + ((C*e^2 -
B*e*f + A*f^2)*Log[e + f*x])/(f*(b*e - a*f)*(d*e - c*f))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

method	result
default	$\frac{\left(-d^{2} A+c d B-C c^{2}\right) \ln \left(x d+c\right)}{\left(c f-d e\right)\left(a d-b c\right) d}+\frac{\left(b^{2} A-a b B+a^{2} C\right) \ln \left(b x+a\right)}{\left(a f-b e\right)\left(a d-b c\right) b}+\frac{\left(A f^{2}-B e f+C e^{2}\right) \ln \left(f x+e\right)}{\left(c f-d e\right)\left(a f-b e\right) f}$
norman	$\frac{\left(A f^{2}-B e f+C e^{2}\right) \ln \left(f x+e\right)}{f\left(a c f^{2}-a d e f-b c e f+b d e^{2}\right)}+\frac{\left(b^{2} A-a b B+a^{2} C\right) \ln \left(b x+a\right)}{\left(a f-b e\right)\left(a d-b c\right) b}-\frac{\left(d^{2} A-c d B+C c^{2}\right) \ln \left(x d+c\right)}{\left(a d-b c\right) d\left(c f-d e\right)}$
parallelrisch	$A \ln \left(b x+a\right) b^{2} c d f^{2}-A \ln \left(b x+a\right) b^{2} d^{2} e f-A \ln \left(x d+c\right) a b d^{2} f^{2}+A \ln \left(x d+c\right) b^{2} d^{2} e f+A \ln \left(f x+e\right) a b d^{2} f^{2}-A \ln \left(f x+e\right) b^{2} c d f^{2}-$
risch	$-\frac{d \ln \left(-x d-c\right) A}{a c d f-a d^{2} e-b c^{2} f+b c d e}+\frac{\ln \left(-x d-c\right) c B}{a c d f-a d^{2} e-b c^{2} f+b c d e}-\frac{\ln \left(-x d-c\right) C c^{2}}{d\left(a c d f-a d^{2} e-b c^{2} f+b c d e\right)}+\frac{f \ln \left(-f x-e\right) A}{a c f^{2}-a d e f-b c e f+b d e^{2}}-$

input
```
int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
(-A*d^2+B*c*d-C*c^2)/(c*f-d*e)/(a*d-b*c)/d*ln(d*x+c)+(A*b^2-B*a*b+C*a^2)/(
a*f-b*e)/(a*d-b*c)/b*ln(b*x+a)+(A*f^2-B*e*f+C*e^2)/(c*f-d*e)/(a*f-b*e)/f*l
n(f*x+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)}\,dx = \frac{(Ca^2-Bab+Ab^2)\log(bx+a)}{(b^3c-ab^2d)e-(ab^2c-a^2bd)f} - \frac{(Cc^2-Bcd+Ad^2)\log(dx+c)}{(bcd^2-ad^3)e-(bc^2d-acd^2)f} + \frac{(Ce^2-Bef+Af^2)\log(fx+e)}{bde^2f+acf^3-(bc+ad)ef^2}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e),x, algorithm="maxima")
```

output
```
(C*a^2 - B*a*b + A*b^2)*log(b*x + a)/((b^3*c - a*b^2*d)*e - (a*b^2*c - a^2
*b*d)*f) - (C*c^2 - B*c*d + A*d^2)*log(d*x + c)/((b*c*d^2 - a*d^3)*e - (b*
c^2*d - a*c*d^2)*f) + (C*e^2 - B*e*f + A*f^2)*log(f*x + e)/(b*d*e^2*f + a*
c*f^3 - (b*c + a*d)*e*f^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)}\,dx = \frac{(Ca^2-Bab+Ab^2)\log(|bx+a|)}{b^3ce-ab^2de-ab^2cf+a^2bdf} - \frac{(Cc^2-Bcd+Ad^2)\log(|dx+c|)}{bcd^2e-ad^3e-bc^2df+acd^2f} + \frac{(Ce^2-Bef+Af^2)\log(|fx+e|)}{bde^2f-bcef^2-adef^2+acf^3}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e),x, algorithm="giac")
```

output
```
(C*a^2 - B*a*b + A*b^2)*log(abs(b*x + a))/(b^3*c*e - a*b^2*d*e - a*b^2*c*f
 + a^2*b*d*f) - (C*c^2 - B*c*d + A*d^2)*log(abs(d*x + c))/(b*c*d^2*e - a*d
^3*e - b*c^2*d*f + a*c*d^2*f) + (C*e^2 - B*e*f + A*f^2)*log(abs(f*x + e))/
(b*d*e^2*f - b*c*e*f^2 - a*d*e*f^2 + a*c*f^3)
```

Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)}\,dx = \frac{\ln(a+b\,x)\,\left(C\,a^2-B\,a\,b+A\,b^2\right)}{b^3\,c\,e-a\,b^2\,c\,f-a\,b^2\,d\,e+a^2\,b\,d\,f} + \frac{\ln(c+d\,x)\,\left(C\,c^2-B\,c\,d+A\,d^2\right)}{a\,d^3\,e-a\,c\,d^2\,f-b\,c\,d^2\,e+b\,c^2\,d\,f} + \frac{\ln(e+f\,x)\,\left(C\,e^2-B\,e\,f+A\,f^2\right)}{a\,c\,f^3-a\,d\,e\,f^2-b\,c\,e\,f^2+b\,d\,e^2\,f}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(a + b*x)*(c + d*x)),x)
```

output
```
(log(a + b*x)*(A*b^2 + C*a^2 - B*a*b))/(b^3*c*e - a*b^2*c*f - a*b^2*d*e +
a^2*b*d*f) + (log(c + d*x)*(A*d^2 + C*c^2 - B*c*d))/(a*d^3*e - a*c*d^2*f -
 b*c*d^2*e + b*c^2*d*f) + (log(e + f*x)*(A*f^2 + C*e^2 - B*e*f))/(a*c*f^3
- a*d*e*f^2 - b*c*e*f^2 + b*d*e^2*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)}\,dx$$
$$= \frac{\log(bx+a)\,a^2c^2d\,f^2-\log(bx+a)\,a^2c\,d^2ef-\log(dx+c)\,a^2b\,d^2f^2+\log(dx+c)\,a\,b^2cd\,f^2+\log(dx+c)}{}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e),x)
```

output
```
(log(a + b*x)*a**2*c**2*d*f**2 - log(a + b*x)*a**2*c*d**2*e*f - log(c + d*
x)*a**2*b*d**2*f**2 + log(c + d*x)*a*b**2*c*d*f**2 + log(c + d*x)*a*b**2*d
**2*e*f - log(c + d*x)*a*b*c**3*f**2 - log(c + d*x)*b**3*c*d*e*f + log(c +
 d*x)*b**2*c**3*e*f + log(e + f*x)*a**2*b*d**2*f**2 - log(e + f*x)*a*b**2*
c*d*f**2 - log(e + f*x)*a*b**2*d**2*e*f + log(e + f*x)*a*b*c*d**2*e**2 + l
og(e + f*x)*b**3*c*d*e*f - log(e + f*x)*b**2*c**2*d*e**2)/(b*d*f*(a**2*c*d
*f**2 - a**2*d**2*e*f - a*b*c**2*f**2 + a*b*d**2*e**2 + b**2*c**2*e*f - b*
*2*c*d*e**2))
```

3.62 $\int \frac{(c+dx)(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 254

$$\begin{aligned}&\int \frac{(c+dx)(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx\\ &= \frac{2(Ab^2-a(bB-aC))\,(bc-ad)(be-af)\sqrt{a+bx}}{b^5}\\ &\quad-\frac{2(4a^3Cdf-b^3(Bce+Ade+Acf)+2ab^2(cCe+Bde+Bcf+Adf)-3a^2b(Cde+cCf+Bdf))\,(a-}{3b^5}\\ &\quad+\frac{2(6a^2Cdf+b^2(cCe+Bde+Bcf+Adf)-3ab(Cde+cCf+Bdf))\,(a+bx)^{5/2}}{5b^5}\\ &\quad-\frac{2(4aCdf-b(Cde+cCf+Bdf))(a+bx)^{7/2}}{7b^5}+\frac{2Cdf(a+bx)^{9/2}}{9b^5}\end{aligned}$$

output
```
2*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)*(-a*f+b*e)*(b*x+a)^(1/2)/b^5-2/3*(4*a^3*C
*d*f-b^3*(A*c*f+A*d*e+B*c*e)+2*a*b^2*(A*d*f+B*c*f+B*d*e+C*c*e)-3*a^2*b*(B*
d*f+C*c*f+C*d*e))*(b*x+a)^(3/2)/b^5+2/5*(6*a^2*C*d*f+b^2*(A*d*f+B*c*f+B*d*
e+C*c*e)-3*a*b*(B*d*f+C*c*f+C*d*e))*(b*x+a)^(5/2)/b^5-2/7*(4*a*C*d*f-b*(B*
d*f+C*c*f+C*d*e))*(b*x+a)^(7/2)/b^5+2/9*C*d*f*(b*x+a)^(9/2)/b^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx = \frac{2\sqrt{a+bx}(128a^4Cdf+24a^2b^2(Cdx(3e+2fx)+Bd(7e+3fx)+c(7Ce+7Bf+3Cfx))-16a^3b(9Bd$$

input
```
Integrate[((c + d*x)*(e + f*x)*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

output
```
(2*Sqrt[a + b*x]*(128*a^4*C*d*f + 24*a^2*b^2*(C*d*x*(3*e + 2*f*x) + B*d*(7
*e + 3*f*x) + c*(7*C*e + 7*B*f + 3*C*f*x)) - 16*a^3*b*(9*B*d*f + C*(9*d*e
+ 9*c*f + 4*d*f*x)) + 21*A*b^2*(8*a^2*d*f - 2*a*b*(5*d*e + 5*c*f + 2*d*f*x
) + b^2*(5*c*(3*e + f*x) + d*x*(5*e + 3*f*x))) + b^4*x*(3*B*(7*c*(5*e + 3*
f*x) + 3*d*x*(7*e + 5*f*x)) + C*x*(9*c*(7*e + 5*f*x) + 5*d*x*(9*e + 7*f*x)
)) - 2*a*b^3*(3*B*(7*c*(5*e + 2*f*x) + d*x*(14*e + 9*f*x)) + C*x*(3*c*(14*
e + 9*f*x) + d*x*(27*e + 20*f*x)))))/(315*b^5)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{ 2115}$$

$$\int \left(\frac{(a+bx)^{3/2}\left(6a^2Cdf-3ab(Bdf+cCf+Cde)+b^2(Adf+Bcf+Bde+cCe)\right)}{b^4}+\frac{\sqrt{a+bx}(-4a^3Cdf+3a$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(a+bx)^{5/2}\left(6a^2Cdf-3ab(Bdf+cCf+Cde)+b^2(Adf+Bcf+Bde+cCe)\right)}{5b^5}-\frac{2(a+bx)^{3/2}\left(4a^3Cdf-3a^2b(Bdf+cCf+Cde)+2ab^2(Adf+Bcf+Bde+cCe)-b^3(Acf+Ade+Bce)\right)}{3b^5}+\frac{2\sqrt{a+bx}(bc-ad)(be-af)\left(Ab^2-a(bB-aC)\right)}{b^5}-\frac{2(a+bx)^{7/2}(4aCdf-b(Bdf+cCf+Cde))}{7b^5}+\frac{2Cdf(a+bx)^{9/2}}{9b^5}$$

input
```
Int[((c + d*x)*(e + f*x)*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

output
```
(2*(A*b^2 - a*(b*B - a*C))*(b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x])/b^5 - (2
*(4*a^3*C*d*f - b^3*(B*c*e + A*d*e + A*c*f) + 2*a*b^2*(c*C*e + B*d*e + B*c
*f + A*d*f) - 3*a^2*b*(C*d*e + c*C*f + B*d*f))*(a + b*x)^(3/2))/(3*b^5) +
(2*(6*a^2*C*d*f + b^2*(c*C*e + B*d*e + B*c*f + A*d*f) - 3*a*b*(C*d*e + c*C
*f + B*d*f))*(a + b*x)^(5/2))/(5*b^5) - (2*(4*a*C*d*f - b*(C*d*e + c*C*f +
 B*d*f))*(a + b*x)^(7/2))/(7*b^5) + (2*C*d*f*(a + b*x)^(9/2))/(9*b^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2Cfd(bx+a)^{\frac{9}{2}}}{9}+\frac{2(((-ad+bc)f+(-af+be)d)C+df(Bb-2Ca))(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left((-ad+bc)(-af+be)C+((-ad+bc)f+(-af+be)d)(Bb-\right.}{5}$
default	$\frac{2Cfd(bx+a)^{\frac{9}{2}}}{9}+\frac{2((-(ad-bc)f-d(af-be))C+df(Bb-2Ca))(bx+a)^{\frac{7}{2}}}{7}+\frac{2\left((ad-bc)(af-be)C+(-(ad-bc)f-d(af-be))(Bb-2Ca)\right.}{5}$
pseudoelliptic	$2\left(\left(\left(\left(\frac{\left(\frac{128}{21}a^4-\frac{40}{21}a\,x^3b^3+\frac{16}{7}a^2b^2x^2-\frac{64}{21}a^3bx+\frac{5}{3}b^4x^4\right)d}{3}+\frac{5\left(b^3x^3-\frac{6}{5}a\,b^2x^2+\frac{8}{5}a^2bx-\frac{16}{5}a^3\right)cb}{7}\right)C+b\left(\left(\frac{\left(5b^3x^3-6a\,b^2x^2+\right.}{7}\right.\right.\right.\right.\right.$
gosper	$2\sqrt{bx+a}\,(35Cdf\,x^4b^4+45B\,b^4df\,x^3-40Ca\,b^3df\,x^3+45C\,b^4cf\,x^3+45C\,b^4de\,x^3+63A\,b^4df\,x^2-54Ba\,b^3df\,x^2+63B\,b^4cf$
trager	$2\sqrt{bx+a}\,(35Cdf\,x^4b^4+45B\,b^4df\,x^3-40Ca\,b^3df\,x^3+45C\,b^4cf\,x^3+45C\,b^4de\,x^3+63A\,b^4df\,x^2-54Ba\,b^3df\,x^2+63B\,b^4cf$
risch	$2\sqrt{bx+a}\,(35Cdf\,x^4b^4+45B\,b^4df\,x^3-40Ca\,b^3df\,x^3+45C\,b^4cf\,x^3+45C\,b^4de\,x^3+63A\,b^4df\,x^2-54Ba\,b^3df\,x^2+63B\,b^4cf$
orering	$2\sqrt{bx+a}\,(35Cdf\,x^4b^4+45B\,b^4df\,x^3-40Ca\,b^3df\,x^3+45C\,b^4cf\,x^3+45C\,b^4de\,x^3+63A\,b^4df\,x^2-54Ba\,b^3df\,x^2+63B\,b^4cf$

input
```
int((d*x+c)*(f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^5*(1/9*C*f*d*(b*x+a)^(9/2)+1/7*(((-a*d+b*c)*f+(-a*f+b*e)*d)*C+d*f*(B*b
-2*C*a))*(b*x+a)^(7/2)+1/5*((-a*d+b*c)*(-a*f+b*e)*C+((-a*d+b*c)*f+(-a*f+b*
e)*d)*(B*b-2*C*a)+d*f*(A*b^2-B*a*b+C*a^2))*(b*x+a)^(5/2)+1/3*((-a*d+b*c)*(
-a*f+b*e)*(B*b-2*C*a)+((-a*d+b*c)*f+(-a*f+b*e)*d)*(A*b^2-B*a*b+C*a^2))*(b*
x+a)^(3/2)+(-a*d+b*c)*(-a*f+b*e)*(A*b^2-B*a*b+C*a^2)*(b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$$
$$=\frac{2\left(35\,Cb^4dfx^4+5\left(9\,Cb^4de+\left(9\,Cb^4c-\left(8\,Cab^3-9\,Bb^4\right)d\right)f\right)x^3+3\left(3\left(7\,Cb^4c-\left(6\,Cab^3-7\,Bb^4\right)d\right)e-\right.}{}$$

input
```
integrate((d*x+c)*(f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
2/315*(35*C*b^4*d*f*x^4 + 5*(9*C*b^4*d*e + (9*C*b^4*c - (8*C*a*b^3 - 9*B*b
^4)*d)*f)*x^3 + 3*(3*(7*C*b^4*c - (6*C*a*b^3 - 7*B*b^4)*d)*e - (3*(6*C*a*b
^3 - 7*B*b^4)*c - (16*C*a^2*b^2 - 18*B*a*b^3 + 21*A*b^4)*d)*f)*x^2 + 3*(7*
(8*C*a^2*b^2 - 10*B*a*b^3 + 15*A*b^4)*c - 2*(24*C*a^3*b - 28*B*a^2*b^2 + 3
5*A*a*b^3)*d)*e - 2*(3*(24*C*a^3*b - 28*B*a^2*b^2 + 35*A*a*b^3)*c - 4*(16*
C*a^4 - 18*B*a^3*b + 21*A*a^2*b^2)*d)*f - (3*(7*(4*C*a*b^3 - 5*B*b^4)*c -
(24*C*a^2*b^2 - 28*B*a*b^3 + 35*A*b^4)*d)*e - (3*(24*C*a^2*b^2 - 28*B*a*b^
3 + 35*A*b^4)*c - 4*(16*C*a^3*b - 18*B*a^2*b^2 + 21*A*a*b^3)*d)*f)*x)*sqrt
(b*x + a)/b^5
```

Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.04

$$\int \frac{(c+dx)(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$$

$$= \begin{cases} \frac{2\left(\frac{Cdf(a+bx)^{\frac{9}{2}}}{9b^4} + \frac{(a+bx)^{\frac{7}{2}}(Bbdf-4Cadf+Cbcf+Cbde)}{7b^4} + \frac{(a+bx)^{\frac{5}{2}}\left(Ab^2df-3Babdf+Bb^2cf+Bb^2de+6Ca^2df-3Cabcf-3Cabde+Cb^2ce\right)}{5b^4} + \frac{(a+bx)^{\frac{3}{2}}(-2\ldots}{\ldots}\right.}{\ldots} \\ \frac{Acex+\frac{Cdfx^5}{5}+\frac{x^4(Bdf+Ccf+Cde)}{4}+\frac{x^3(Adf+Bcf+Bde+Cce)}{3}+\frac{x^2(Acf+Ade+Bce)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x+c)*(f*x+e)*(C*x**2+B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Piecewise((2*(C*d*f*(a + b*x)**(9/2)/(9*b**4) + (a + b*x)**(7/2)*(B*b*d*f
- 4*C*a*d*f + C*b*c*f + C*b*d*e)/(7*b**4) + (a + b*x)**(5/2)*(A*b**2*d*f -
 3*B*a*b*d*f + B*b**2*c*f + B*b**2*d*e + 6*C*a**2*d*f - 3*C*a*b*c*f - 3*C*
a*b*d*e + C*b**2*c*e)/(5*b**4) + (a + b*x)**(3/2)*(-2*A*a*b**2*d*f + A*b**
3*c*f + A*b**3*d*e + 3*B*a**2*b*d*f - 2*B*a*b**2*c*f - 2*B*a*b**2*d*e + B*
b**3*c*e - 4*C*a**3*d*f + 3*C*a**2*b*c*f + 3*C*a**2*b*d*e - 2*C*a*b**2*c*e
)/(3*b**4) + sqrt(a + b*x)*(A*a**2*b**2*d*f - A*a*b**3*c*f - A*a*b**3*d*e
+ A*b**4*c*e - B*a**3*b*d*f + B*a**2*b**2*c*f + B*a**2*b**2*d*e - B*a*b**3
*c*e + C*a**4*d*f - C*a**3*b*c*f - C*a**3*b*d*e + C*a**2*b**2*c*e)/b**4)/b
, Ne(b, 0)), ((A*c*e*x + C*d*f*x**5/5 + x**4*(B*d*f + C*c*f + C*d*e)/4 + x
**3*(A*d*f + B*c*f + B*d*e + C*c*e)/3 + x**2*(A*c*f + A*d*e + B*c*e)/2)/sq
rt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx)(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\left(35\,(bx+a)^{\frac{9}{2}}Cdf+45\,(Cbde+(Cbc-(4\,Ca-Bb)d)f)(bx+a)^{\frac{7}{2}}+63\,((Cb^2c-(3\,Cab-Bb^2)d)e-\right.}{}$$

input
```
integrate((d*x+c)*(f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
2/315*(35*(b*x + a)^(9/2)*C*d*f + 45*(C*b*d*e + (C*b*c - (4*C*a - B*b)*d)*
f)*(b*x + a)^(7/2) + 63*((C*b^2*c - (3*C*a*b - B*b^2)*d)*e - ((3*C*a*b - B
*b^2)*c - (6*C*a^2 - 3*B*a*b + A*b^2)*d)*f)*(b*x + a)^(5/2) - 105*(((2*C*a
*b^2 - B*b^3)*c - (3*C*a^2*b - 2*B*a*b^2 + A*b^3)*d)*e - ((3*C*a^2*b - 2*B
*a*b^2 + A*b^3)*c - (4*C*a^3 - 3*B*a^2*b + 2*A*a*b^2)*d)*f)*(b*x + a)^(3/2
) + 315*(((C*a^2*b^2 - B*a*b^3 + A*b^4)*c - (C*a^3*b - B*a^2*b^2 + A*a*b^3
)*d)*e - ((C*a^3*b - B*a^2*b^2 + A*a*b^3)*c - (C*a^4 - B*a^3*b + A*a^2*b^2
)*d)*f)*sqrt(b*x + a))/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(235) = 470$.

Time = 0.13 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.87

$$\int \frac{(c+dx)(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\left(315\sqrt{bx+a}Ace + \frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)Bce}{b} + \frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)Ade}{b} + \frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)Acf}{b} + \frac{21}{}\right.}{}$$

input
```
integrate((d*x+c)*(f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/315*(315*sqrt(b*x + a)*A*c*e + 105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)
*B*c*e/b + 105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*A*d*e/b + 105*((b*x +
 a)^(3/2) - 3*sqrt(b*x + a)*a)*A*c*f/b + 21*(3*(b*x + a)^(5/2) - 10*(b*x +
 a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*C*c*e/b^2 + 21*(3*(b*x + a)^(5/2) - 10
*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B*d*e/b^2 + 21*(3*(b*x + a)^(5/
2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*B*c*f/b^2 + 21*(3*(b*x +
 a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*A*d*f/b^2 + 9*(5*
(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(
b*x + a)*a^3)*C*d*e/b^3 + 9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35
*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*C*c*f/b^3 + 9*(5*(b*x + a)^(7
/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3
)*B*d*f/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^
(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*C*d*f/b^4)/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$$
$$= \frac{(a+b\,x)^{7/2}\,(2\,B\,b\,d\,f-8\,C\,a\,d\,f+2\,C\,b\,c\,f+2\,C\,b\,d\,e)}{7\,b^5}$$
$$+ \frac{(a+b\,x)^{5/2}\,(2\,A\,b^2\,d\,f+2\,B\,b^2\,c\,f+2\,B\,b^2\,d\,e+2\,C\,b^2\,c\,e+12\,C\,a^2\,d\,f-6\,B\,a\,b\,d\,f-6\,C\,a\,b\,c\,f-6}{5\,b^5}$$
$$+ \frac{(a+b\,x)^{3/2}\,(2\,A\,b^3\,c\,f+2\,A\,b^3\,d\,e+2\,B\,b^3\,c\,e-8\,C\,a^3\,d\,f-4\,A\,a\,b^2\,d\,f-4\,B\,a\,b^2\,c\,f-4\,B\,a\,b^2\,d\,e-}{3\,b^5}$$
$$+ \frac{2\,(a\,d-b\,c)\,(a\,f-b\,e)\,\sqrt{a+b\,x}\,(C\,a^2-B\,a\,b+A\,b^2)}{b^5}+\frac{2\,C\,d\,f\,(a+b\,x)^{9/2}}{9\,b^5}$$

input
```
int(((e + f*x)*(c + d*x)*(A + B*x + C*x^2))/(a + b*x)^(1/2),x)
```

output
```
((a + b*x)^(7/2)*(2*B*b*d*f - 8*C*a*d*f + 2*C*b*c*f + 2*C*b*d*e))/(7*b^5)
+ ((a + b*x)^(5/2)*(2*A*b^2*d*f + 2*B*b^2*c*f + 2*B*b^2*d*e + 2*C*b^2*c*e
+ 12*C*a^2*d*f - 6*B*a*b*d*f - 6*C*a*b*c*f - 6*C*a*b*d*e))/(5*b^5) + ((a +
 b*x)^(3/2)*(2*A*b^3*c*f + 2*A*b^3*d*e + 2*B*b^3*c*e - 8*C*a^3*d*f - 4*A*a
*b^2*d*f - 4*B*a*b^2*c*f - 4*B*a*b^2*d*e - 4*C*a*b^2*c*e + 6*B*a^2*b*d*f +
 6*C*a^2*b*c*f + 6*C*a^2*b*d*e))/(3*b^5) + (2*(a*d - b*c)*(a*f - b*e)*(a +
 b*x)^(1/2)*(A*b^2 + C*a^2 - B*a*b))/b^5 + (2*C*d*f*(a + b*x)^(9/2))/(9*b^
5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)(e+fx)(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$$
$$= \frac{2\sqrt{bx+a}\,(35b^4cdf\,x^4-40a\,b^3cdf\,x^3+45b^5df\,x^3+45b^4c^2f\,x^3+45b^4cde\,x^3+48a^2b^2cdf\,x^2+9a\,b^4df\,x^2}{}$$

input
```
int((d*x+c)*(f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**4*c*d*f + 24*a**3*b**2*d*f - 144*a**3*b*c**2*f -
144*a**3*b*c*d*e - 64*a**3*b*c*d*f*x - 42*a**2*b**3*c*f - 42*a**2*b**3*d*e
 - 12*a**2*b**3*d*f*x + 168*a**2*b**2*c**2*e + 72*a**2*b**2*c**2*f*x + 72*
a**2*b**2*c*d*e*x + 48*a**2*b**2*c*d*f*x**2 + 105*a*b**4*c*e + 21*a*b**4*c
*f*x + 21*a*b**4*d*e*x + 9*a*b**4*d*f*x**2 - 84*a*b**3*c**2*e*x - 54*a*b**
3*c**2*f*x**2 - 54*a*b**3*c*d*e*x**2 - 40*a*b**3*c*d*f*x**3 + 105*b**5*c*e
*x + 63*b**5*c*f*x**2 + 63*b**5*d*e*x**2 + 45*b**5*d*f*x**3 + 63*b**4*c**2
*e*x**2 + 45*b**4*c**2*f*x**3 + 45*b**4*c*d*e*x**3 + 35*b**4*c*d*f*x**4))/
(315*b**5)
```

3.63 $\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx$

Optimal result

Integrand size = 36, antiderivative size = 1353

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\ dx = \text{Too large to display}$$

output
```
1/512*(-c*f+d*e)*(8*a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(
2*A*d*f-B*(c*f+d*e)))-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+
7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^
2*(C*(21*c^4*f^4+28*c^3*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e
^4)+4*d*f*(2*A*d*f*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*
f^2+9*c*d^2*e^2*f+7*d^3*e^3))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^5/f^5+1/256*
(8*a^2*d^2*f^2*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*
e)))-8*a*b*d*f*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d*f*
(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)))+b^2*(C*(21*c^4*f^4+
28*c^3*d*e*f^3+30*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+21*d^4*e^4)+4*d*f*(2*A*d*
f*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)-B*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f
+7*d^3*e^3))))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^5/f^4-1/192*(8*a^2*d^2*f^2*(-
8*B*d*f+11*C*c*f+5*C*d*e)+8*a*b*d*f*(2*d*f*(-8*A*d*f+11*B*c*f+5*B*d*e)-C*(
25*c^2*f^2+16*c*d*e*f+7*d^2*e^2))+b^2*(C*(107*c^3*f^3+79*c^2*d*e*f^2+49*c*
d^2*e^2*f+21*d^3*e^3)+4*d*f*(2*A*d*f*(11*c*f+5*d*e)-B*(25*c^2*f^2+16*c*d*e
*f+7*d^2*e^2))))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/d^4/f^4+1/160*(40*a^2*C*d^2*f
^2-8*a*b*d*f*(-10*B*d*f+23*C*c*f+7*C*d*e)-b^2*(4*d*f*(-10*A*d*f+23*B*c*f+7
*B*d*e)-C*(149*c^2*f^2+70*c*d*e*f+21*d^2*e^2)))*(d*x+c)^(5/2)*(f*x+e)^(3/2
)/d^4/f^3+1/20*b*(8*a*C*d*f-b*(-4*B*d*f+13*C*c*f+3*C*d*e))*(d*x+c)^(7/2)*(
f*x+e)^(3/2)/d^4/f^2+1/6*b^2*C*(d*x+c)^(9/2)*(f*x+e)^(3/2)/d^4/f-1/512*...
```

Mathematica [A] (verified)

Time = 5.96 (sec) , antiderivative size = 1253, normalized size of antiderivative = 0.93

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
Integrate[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(40*a^2*d^2*f^2*(C*(15*c^3*f^3 - c^2*d*f^2*(7
*e + 10*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e
^2*f*x + 8*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x))
 + B*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2))
)) + 8*a*b*d*f*(C*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f
^2*(-17*e^2 + 11*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*
e*f^2*x^2 + 24*f^3*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48
*e*f^3*x^3 + 384*f^4*x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(e + f*
x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 - c^2*d*f^2*(7*e
+ 10*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*
f*x + 8*e*f^2*x^2 + 48*f^3*x^3)))) + b^2*(C*(315*c^5*f^5 - 105*c^4*d*f^4*(
e + 2*f*x) + 2*c^3*d^2*f^3*(-41*e^2 + 28*e*f*x + 84*f^2*x^2) - 2*c^2*d^3*f
^2*(41*e^3 - 26*e^2*f*x + 20*e*f^2*x^2 + 72*f^3*x^3) + c*d^4*f*(-105*e^4 +
 56*e^3*f*x - 40*e^2*f^2*x^2 + 32*e*f^3*x^3 + 128*f^4*x^4) + d^5*(315*e^5
- 210*e^4*f*x + 168*e^3*f^2*x^2 - 144*e^2*f^3*x^3 + 128*e*f^4*x^4 + 1280*f
^5*x^5)) + 4*d*f*(10*A*d*f*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*d^2*
f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*x^2
+ 48*f^3*x^3)) + B*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*
f^2*(-17*e^2 + 11*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8
*e*f^2*x^2 + 24*f^3*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 ...
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 815, normalized size of antiderivative = 0.60, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2118, 27, 170, 27, 164, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int -\frac{3}{2}b(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(2bcCe+aCde+acCf-4Abdf-(4bBdf-2aCdf-3bC(de+cf))x)dx}{6b^2df}+\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}$$

$$\Big\downarrow \; 27$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf} - \frac{\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(2bcCe+aCde+acCf-4Abdf-(4bBdf-2aCdf-3bC(de+cf))x)dx}{4bdf}$$

$$\Big\downarrow \; 170$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf} - \frac{\frac{\int \frac{1}{2}(a+bx)\sqrt{c+dx}\sqrt{e+fx}(10adf(2bcCe+aCde+acCf-4Abdf)+(4bce+3a(de+cf))(4bBdf-2aCdf-3bC(de+cf))+(10bdf(2bcCe+aCde+acCf}{5df}}{4bdf}$$

$$\Big\downarrow \; 27$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf} - \frac{\frac{\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(10adf(2bcCe+aCde+acCf-4Abdf)+(4bce+3a(de+cf))(4bBdf-2aCdf-3bC(de+cf))+(10bdf(2bcCe+aCde+acCf-}{10df}}{4bdf}$$

$$\Big\downarrow \; 164$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf} - \frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(cf+de))-8ab^2df\left(10df(8Adf-5B(cf+de))+C\left(35c^2f^2+38cdef+35d^2e^2\right)\right)+6bdfx(10bdf(acCf+aCde}{}}{}$$

$$\Big\downarrow \; 60$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf} - \frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(64a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-7C(cf+de))-8ab^2df\left(10df(8Adf-5B(cf+de))+C\left(35c^2f^2+38cdef+35d^2e^2\right)\right)+6bdfx(10bdf(acCf+aCde}{}}{}$$

$$\Big\downarrow \; 60$$

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(\left(7C\left(15d^3e^3+17cd^2fe^2+17c^2df^2e+15c^3f^3\right)+4df\left(50Adf(de+cf)-B\left(35d^2e^2+38cdfe+35c^2f^2\right)\right)\right)b^3-8adf\left(C\left(35d^2e^2+38cdfe+35c^2f^2\right)\right.\right.}{}$$

$\downarrow$ 66

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(\left(7C\left(15d^3e^3+17cd^2fe^2+17c^2df^2e+15c^3f^3\right)+4df\left(50Adf(de+cf)-B\left(35d^2e^2+38cdfe+35c^2f^2\right)\right)\right)b^3-8adf\left(C\left(35d^2e^2+38cdfe+35c^2f^2\right)\right.\right.}{}$$

$\downarrow$ 221

$$\frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf}-$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(\left(7C\left(15d^3e^3+17cd^2fe^2+17c^2df^2e+15c^3f^3\right)+4df\left(50Adf(de+cf)-B\left(35d^2e^2+38cdfe+35c^2f^2\right)\right)\right)b^3-8adf\left(C\left(35d^2e^2+38cdfe+35c^2f^2\right)\right.\right.}{}$$

input
```
Int[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(C*(a + b*x)^3*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(6*b*d*f) - (-1/5*((4*b*B*
d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f))*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)
^(3/2))/(d*f) + (((c + d*x)^(3/2)*(e + f*x)^(3/2)*(64*a^3*C*d^3*f^3 - 8*a^
2*b*d^2*f^2*(16*B*d*f - 7*C*(d*e + c*f)) - 8*a*b^2*d*f*(C*(35*d^2*e^2 + 38
*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5*B*(d*e + c*f))) + b^3*(7*C*(1
5*d^3*e^3 + 17*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 15*c^3*f^3) + 4*d*f*(50*A*d*
f*(d*e + c*f) - B*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2))) + 6*b*d*f*(10*b
*d*f*(2*b*c*C*e + a*C*d*e + a*c*C*f - 4*A*b*d*f) - (4*a*d*f - 7*b*(d*e + c
*f))*(4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f)))*x))/(24*d^2*f^2) - (5*b*
(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B
*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 +
 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^
2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c
^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f
^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*(((c +
d*x)^(3/2)*Sqrt[e + f*x])/(2*d) + ((d*e - c*f)*((Sqrt[c + d*x]*Sqrt[e + f*
x])/f - ((d*e - c*f)*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x
])])/(Sqrt[d]*f^(3/2))))/(4*d)))/(16*d^2*f^2))/(10*d*f))/(4*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5733 vs. 2(1303) = 2606.

Time = 0.62 (sec) , antiderivative size = 5734, normalized size of antiderivative = 4.24

method	result	size
default	Expression too large to display	5734

input
```
int((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [A] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 3096, normalized size of antiderivative = 2.29

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx = \int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx$$

input
```
integrate((b*x+a)**2*(d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Integral((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4456 vs. $2(1303) = 2606$.

Time = 0.71 (sec) , antiderivative size = 4456, normalized size of antiderivative = 3.29

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm
="giac")
```

output

```
-1/7680*(7680*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2
*e + (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f)*sqrt(d*x + c))*A*a^2*c*abs(d)/d^2 - 80*(sqrt(d^2*e + (d*x + c)*d*f - c
*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11
*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/
(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 9
3*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f +
 6*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(
d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C*a*
b*c*abs(d)/d^2 - 40*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(
d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*
d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14
*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f
^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20
*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*B*b^2*c*abs(d)/d^2 - 40*(sq
rt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d
^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^1
2*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2
*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) +...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx = \text{Hanged}$$

input

```
int((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

output

```
\text{Hanged}
```

Reduce [F]

$$\int (a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \int (bx+a)^2\sqrt{dx+c}\,\sqrt{fx+e}\left(C\,x^2+Bx+A\right)dx$$

input
```
int((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
int((b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x)
```

3.64 $\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 715

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)-b\left(C\left(7d^3e^3+9cd^2e^2f+9c^2de\right.\right.\right.}{128d^4f^4}$$
$$+\frac{\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)-b\left(C\left(7d^3e^3+9cd^2e^2f+9c^2def^2+7c\right.\right.\right.}{64d^4f^3}$$
$$-\frac{\left(2bdf(5Bde+11Bcf-8Adf)+2adf(5Cde+11cCf-8Bdf)-bC\left(7d^2e^2+16cdef+25c^2f^2\right)\right)(c+}{48d^3f^3}$$
$$+\frac{(10aCdf-b(7Cde+23cCf-10Bdf))(c+dx)^{5/2}(e+fx)^{3/2}}{40d^3f^2}$$
$$+\frac{bC(c+dx)^{7/2}(e+fx)^{3/2}}{5d^3f}$$
$$-\frac{(de-cf)^2\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)-b\left(C\left(7d^3e^3+9cd^2e^2f+9c\right.\right.\right.}{128d^{9/2}f^{9/2}}$$

output
```
1/128*(-c*f+d*e)*(2*a*d*f*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*
f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3*e^3)+2*d
*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2))))*(d*x+c)^(1/2)*(
f*x+e)^(1/2)/d^4/f^4+1/64*(2*a*d*f*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*
f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+7*d^3
*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2))))*(d*x+c
)^(3/2)*(f*x+e)^(1/2)/d^4/f^3-1/48*(2*b*d*f*(-8*A*d*f+11*B*c*f+5*B*d*e)+2*
a*d*f*(-8*B*d*f+11*C*c*f+5*C*d*e)-b*C*(25*c^2*f^2+16*c*d*e*f+7*d^2*e^2))*(
d*x+c)^(3/2)*(f*x+e)^(3/2)/d^3/f^3+1/40*(10*a*C*d*f-b*(-10*B*d*f+23*C*c*f+
7*C*d*e))*(d*x+c)^(5/2)*(f*x+e)^(3/2)/d^3/f^2+1/5*b*C*(d*x+c)^(7/2)*(f*x+e
)^(3/2)/d^3/f-1/128*(-c*f+d*e)^2*(2*a*d*f*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^
2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(7*c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*
f+7*d^3*e^3)+2*d*f*(8*A*d*f*(c*f+d*e)-B*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2))))
*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(9/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 662, normalized size of antiderivative = 0.93

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(10adf(C(15c^3f^3-c^2df^2(7e+10fx)+cd^2f(-7e^2+4efx+8f^2x^2)+d^3(15e^3-10e^2}{}$$

$$+\frac{(de-cf)^2\left(-2adf(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf)))+b(C(7d^3e^3+9cd^2e^2f+9\right.}{128d^{9/2}f^{9/2}}$$

input
```
Integrate[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(10*a*d*f*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 1
0*f*x) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x
 + 8*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(
-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))) + b
*(C*(-105*c^4*f^4 + 10*c^3*d*f^3*(4*e + 7*f*x) - 2*c^2*d^2*f^2*(-17*e^2 +
11*e*f*x + 28*f^2*x^2) + 2*c*d^3*f*(20*e^3 - 11*e^2*f*x + 8*e*f^2*x^2 + 24
*f^3*x^3) + d^4*(-105*e^4 + 70*e^3*f*x - 56*e^2*f^2*x^2 + 48*e*f^3*x^3 + 3
84*f^4*x^4)) + 10*d*f*(8*A*d*f*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e
^2 + 2*e*f*x + 8*f^2*x^2)) + B*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c*
d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2*
x^2 + 48*f^3*x^3))))))/(1920*d^4*f^4) + ((d*e - c*f)^2*(-2*a*d*f*(C*(5*d^2
*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) + b*(C*(7
*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*
e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[d]*Sqrt[
e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(128*d^(9/2)*f^(9/2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.64, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {2118, 27, 164, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int -\frac{1}{2}b(a+bx)\sqrt{c+dx}\sqrt{e+fx}(4bcCe+3aCde+3acCf-10Abdf-(10bBdf-6aCdf-7bC(de+cf))x)dx}{5b^2df}+\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-\frac{\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}(4bcCe+3aCde+3acCf-10Abdf-(10bBdf-6aCdf-7bC(de+cf))x)dx}{10bdf}$$

$\Big\downarrow$ 164

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-\left(b^2\left(10df(8Adf-5B(cf+de))+C\left(35c^2\right.\right.\right.\right.}{24d^2f^2}}{}$$

$\Big\downarrow$ 60

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-\left(b^2\left(10df(8Adf-5B(cf+de))+C\left(35c^2\right.\right.\right.\right.}{24d^2f^2}}{}$$

$\Big\downarrow$ 60

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-\left(b^2\left(10df(8Adf-5B(cf+de))+C\left(35c^2\right.\right.\right.\right.}{24d^2f^2}}{}$$

$\Big\downarrow$ 66

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-$$

$$\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-\left(b^2\left(10df(8Adf-5B(cf+de))+C\left(35c^2\right.\right.\right.\right.}{24d^2f^2}}{}$$

$\Big\downarrow$ 221

$$\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf} -$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2-6bdfx(-6aCdf+10bBdf-7bC(cf+de))-10abdf(8Bdf-5C(cf+de))-\left(b^2\left(10df(8Adf-5B(cf+de))+C\left(35c^2\right.\right.\right.\right.}{24d^2f^2}$$

input
```
Int[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(C*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*d*f) - (((c + d*x)^(3
/2)*(e + f*x)^(3/2)*(48*a^2*C*d^2*f^2 - 10*a*b*d*f*(8*B*d*f - 5*C*(d*e + c
*f)) - b^2*(C*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5
*B*(d*e + c*f))) - 6*b*d*f*(10*b*B*d*f - 6*a*C*d*f - 7*b*C*(d*e + c*f))*x)
)/(24*d^2*f^2) - (5*b*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*
d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d
*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*
f + 5*c^2*f^2))))*(((c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d) + ((d*e - c*f)*((
Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((d*e - c*f)*ArcTanh[(Sqrt[f]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3/2))))/(4*d)))/(16*d^2*f^2))/(1
0*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3024 vs. $2(671) = 1342$.

Time = 0.57 (sec) , antiderivative size = 3025, normalized size of antiderivative = 4.23

method	result	size
default	Expression too large to display	3025

input
```
int((b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERB
OSE)
```

output
```
-1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(200*B*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(
1/2)*b*c^2*d^2*f^4*x-320*A*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b*c*d^3*f^4
*x-320*A*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b*d^4*e*f^3*x-320*B*((f*x+e)*
(d*x+c))^(1/2)*(d*f)^(1/2)*a*c*d^3*f^4*x+200*B*((f*x+e)*(d*x+c))^(1/2)*(d*
f)^(1/2)*b*d^4*e^2*f^2*x-80*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b*c^3*d*
e*f^3+140*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*a*c*d^3*e^2*f^2-320*B*((f*
x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*a*d^4*e*f^3*x-68*C*((f*x+e)*(d*x+c))^(1/2)
*(d*f)^(1/2)*b*c^2*d^2*e^2*f^2-32*C*b*c*d^3*e*f^3*x^2*((f*x+e)*(d*x+c))^(1
/2)*(d*f)^(1/2)-60*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e^2*f^3-120*B*ln(1/2*(2*d*f*x+2*((f*x+e)*
(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^3*f^2-96*C*b*c*
d^3*f^4*x^3*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)-96*C*b*d^4*e*f^3*x^3*((f*x
+e)*(d*x+c))^(1/2)*(d*f)^(1/2)-160*B*b*c*d^3*f^4*x^2*((f*x+e)*(d*x+c))^(1/
2)*(d*f)^(1/2)-160*B*b*d^4*e*f^3*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)-1
60*C*a*c*d^3*f^4*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+210*C*((f*x+e)*(d
*x+c))^(1/2)*(d*f)^(1/2)*b*c^4*f^4-120*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c
))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^3*d^2*e*f^4-960*A*((f*x+e)*
(d*x+c))^(1/2)*(d*f)^(1/2)*a*c*d^3*f^4-960*A*((f*x+e)*(d*x+c))^(1/2)*(d*f)
^(1/2)*a*d^4*e*f^3+150*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(
1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^4*f+150*C*ln(1/2*(2*d*f*x+2*((f*x+e)...
```

Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 1620, normalized size of antiderivative = 2.27

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm="
fricas")
```

output
```
[-1/7680*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a + B*b)*d^5)*e^4*f - 2*
(C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*e^3*f^2 - 2*(C*b*c
^3*d^2 + 16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*e^2*f^3
 - (5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^
2*d^3)*e*f^4 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c^4*d + 16*(B*
a + A*b)*c^3*d^2)*f^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f +
 c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) +
 8*(d^2*e*f + c*d*f^2)*x) - 4*(384*C*b*d^5*f^5*x^4 - 105*C*b*d^5*e^4*f + 1
0*(4*C*b*c*d^4 + 15*(C*a + B*b)*d^5)*e^3*f^2 + 2*(17*C*b*c^2*d^3 - 35*(C*a
 + B*b)*c*d^4 - 120*(B*a + A*b)*d^5)*e^2*f^3 + 10*(4*C*b*c^3*d^2 + 48*A*a*
d^5 - 7*(C*a + B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*e*f^4 - 15*(7*C*b*c^4*
d - 32*A*a*c*d^4 - 10*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 +
48*(C*b*d^5*e*f^4 + (C*b*c*d^4 + 10*(C*a + B*b)*d^5)*f^5)*x^3 - 8*(7*C*b*d
^5*e^2*f^3 - 2*(C*b*c*d^4 + 5*(C*a + B*b)*d^5)*e*f^4 + (7*C*b*c^2*d^3 - 10
*(C*a + B*b)*c*d^4 - 80*(B*a + A*b)*d^5)*f^5)*x^2 + 2*(35*C*b*d^5*e^3*f^2
- (11*C*b*c*d^4 + 50*(C*a + B*b)*d^5)*e^2*f^3 - (11*C*b*c^2*d^3 - 20*(C*a
+ B*b)*c*d^4 - 80*(B*a + A*b)*d^5)*e*f^4 + 5*(7*C*b*c^3*d^2 + 96*A*a*d^5 -
 10*(C*a + B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*f^5)*x)*sqrt(d*x + c)*sqrt
(f*x + e))/(d^5*f^5), -1/3840*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a +
 B*b)*d^5)*e^4*f - 2*(C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)...
```

Sympy [F]

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (a+bx)\,\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((b*x+a)*(d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Integral((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2481 vs. $2(671) = 1342$.

Time = 0.45 (sec) , antiderivative size = 2481, normalized size of antiderivative = 3.47

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm="
giac")
```

output
```
-1/1920*(1920*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2
*e + (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d
*f)*sqrt(d*x + c))*A*a*c*abs(d)/d^2 - 10*(sqrt(d^2*e + (d*x + c)*d*f - c*d
*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f
^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d
^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*
c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6
*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*
x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C*b*c*
abs(d)/d^2 - 10*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x
+ c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13
*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3
*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))
*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^2*e^2*f^2 + 20*c^3
*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*
x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C*a*abs(d)/d - 10*(sqrt(d^2*e +
 (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12
*e*f^5 - 25*c*d^11*f^6)/(d^14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 -
163*c^2*d^11*f^6)/(d^14*f^6)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*
c^2*d^12*e*f^5 - 93*c^3*d^11*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*...
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Hanged}$$

input
```
int((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int (a+bx)\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int (bx+a)\sqrt{dx+c}\,\sqrt{fx+e}\left(C\,x^2+Bx+A\right)dx$$

input
```
int((b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
int((b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x)
```

3.65 $\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 330

$$\begin{aligned}
&\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\sqrt{c+dx}\sqrt{e+fx}}{64d^3f^3} \\
&\quad+\frac{\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/2}\sqrt{e+fx}}{32d^3f^2} \\
&\quad-\frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2}+\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} \\
&\quad-\frac{(de-cf)^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{64d^{7/2}f^{7/2}}
\end{aligned}$$

output
```
1/64*(-c*f+d*e)*(C*(5*c^2*f^2+6*c*d*e*f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d
*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^3/f^3+1/32*(C*(5*c^2*f^2+6*c*d*e*f+5*d
^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^3/f^2-1
/24*(-8*B*d*f+11*C*c*f+5*C*d*e)*(d*x+c)^(3/2)*(f*x+e)^(3/2)/d^2/f^2+1/4*C*
(d*x+c)^(5/2)*(f*x+e)^(3/2)/d^2/f-1/64*(-c*f+d*e)^2*(C*(5*c^2*f^2+6*c*d*e*
f+5*d^2*e^2)+8*d*f*(2*A*d*f-B*(c*f+d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(7/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(C(15c^3f^3-c^2df^2(7e+10fx)+cd^2f(-7e^2+4efx+8f^2x^2)+d^3(15e^3-10e^2fx+8}{1}$$
$$-\frac{(de-cf)^2\left(C(5d^2e^2+6cdef+5c^2f^2)+8df(2Adf-B(de+cf))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{64d^{7/2}f^{7/2}}$$

input
```
Integrate[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x) + c
*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e*f^2
*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(-3*c^2*f^2
 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))))/(192*d^3*f^3
) - ((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f
 - B*(d*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])
])/(64*d^(7/2)*f^(7/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1194, 27, 90, 60, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \; 1194$$

$$\frac{\int -\frac{1}{2}\sqrt{c+dx}\sqrt{e+fx}\left(3Cfc^2+5Cdec-8Ad^2f+d(5Cde+11cCf-8Bdf)x\right)dx}{4d^2f}+\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}$$

$$\Big\downarrow \; 27$$

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\int \sqrt{c+dx}\sqrt{e+fx}\left(3Cfc^2+5Cdec-8Ad^2f+d(5Cde+11cCf-8Bdf)x\right)dx}{8d^2f}$$

$$\Big\downarrow \; 90$$

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)\int \sqrt{c+dx}\sqrt{e+fx}dx}{2f}}{8d^2f}$$

$$\Big\downarrow \; 60$$

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)\left(\frac{(de-cf)\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}}dx}{4d}+\frac{(c+dx)^{3/2}\sqrt{e+fx}}{2d}\right)}{2f}}{8d^2f}$$

$$\Big\downarrow \; 60$$

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)\left(\frac{(de-cf)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\int \frac{}{\sqrt{c+dx}}}{2f}\right.}{4d}\right.}{2f}}{8d^2f}$$

$$\Big\downarrow \; 66$$

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)\left(\frac{(de-cf)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\int\frac{1}{d-\frac{f(c}{e}}}{f}\right)}{4d}\right.}{2f}}{8d^2f}$$

$\Big\downarrow$ 221

$$\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}-\frac{\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{3f}-\frac{\left(\frac{(de-cf)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{\sqrt{d}f^{3/2}}\right)}{4d}+\frac{(c+dx)^{3/2}\sqrt{e+fx}}{2d}\right)(8df(2Adf-B}{2f}}{8d^2f}$$

input
```
Int[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^(5/2)*(e + f*x)^(3/2))/(4*d^2*f) - (((5*C*d*e + 11*c*C*f - 8*
B*d*f)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*f) - ((C*(5*d^2*e^2 + 6*c*d*e*f
 + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f)))*(((c + d*x)^(3/2)*Sqrt[e
+ f*x])/(2*d) + ((d*e - c*f)*((Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((d*e - c*
f)*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3
/2))))/(4*d)))/(2*f))/(8*d^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1206 vs. $2(292) = 584$.

Time = 0.54 (sec) , antiderivative size = 1207, normalized size of antiderivative = 3.66

method	result	size
default	Expression too large to display	1207

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-12*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+
c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e^3*f-6*C*ln(1/2*(2*d*f*
x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*e^2*
f^2-8*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*c*d^2*e*f^2*x+48*B*(d*f)^(1/2)
*((f*x+e)*(d*x+c))^(1/2)*d^3*e^2*f+48*A*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*f^4-24*B*ln(1/2*(2*d*f*x
+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^3*f-16*
C*c*d^2*f^3*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+48*A*ln(1/2*(2*d*f*x+2
*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^2*f^2-24*
B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*c^3*d*f^4-30*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*c^3*f^3-96*A*ln(1/2
*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^
3*e*f^3-16*C*d^3*e*f^2*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)-96*C*d^3*f^
3*x^3*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)-128*B*d^3*f^3*x^2*((f*x+e)*(d*x+
c))^(1/2)*(d*f)^(1/2)+15*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^4*f^4+15*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+
c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^4-96*A*(d*f)^(1/2)*((f*x
+e)*(d*x+c))^(1/2)*c*d^2*f^3-96*A*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*d^3*
e*f^2+48*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*c^2*d*f^3-192*A*(d*f)^(1/2)
*((f*x+e)*(d*x+c))^(1/2)*d^3*f^3*x+24*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 840, normalized size of antiderivative = 2.55

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/768*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3*f - 2*(C*c^2*d^2 - 4*B*
c*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d^3)*e*f^3 + (
5*C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2
*e^2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x +
c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*d^4*f^4*x^3 + 15*C*d
^4*e^3*f - (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3 - 48
*A*d^4)*e*f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d^4*e*
f^3 + (C*c*d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3 + 4*B
*d^4)*e*f^3 + (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x + c)*s
qrt(f*x + e))/(d^4*f^4), 1/384*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3
*f - 2*(C*c^2*d^2 - 4*B*c*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^
2 + 8*A*c*d^3)*e*f^3 + (5*C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(-d*f
)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/
(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(48*C*d^4*f^4*x^3 + 1
5*C*d^4*e^3*f - (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3
 - 48*A*d^4)*e*f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d
^4*e*f^3 + (C*c*d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3
+ 4*B*d^4)*e*f^3 + (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x +
 c)*sqrt(f*x + e))/(d^4*f^4)]
```

Sympy [F]

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f+d*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(292) = 584$.

Time = 0.27 (sec) , antiderivative size = 1035, normalized size of antiderivative = 3.14

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
-1/192*(192*((d^2*e - c*d*f)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f)))/sqrt(d*f) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f
)*sqrt(d*x + c))*A*c*abs(d)/d^2 - (sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*
(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 + (d^12*e*f^5 - 25*c*d^11*f^6)/(d^
14*f^6)) - (5*d^13*e^2*f^4 + 14*c*d^12*e*f^5 - 163*c^2*d^11*f^6)/(d^14*f^6
)) + 3*(5*d^14*e^3*f^3 + 9*c*d^13*e^2*f^4 + 15*c^2*d^12*e*f^5 - 93*c^3*d^1
1*f^6)/(d^14*f^6))*sqrt(d*x + c) + 3*(5*d^4*e^4 + 4*c*d^3*e^3*f + 6*c^2*d^
2*e^2*f^2 + 20*c^3*d*e*f^3 - 35*c^4*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c)
+ sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*C*abs(d)/d -
48*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*d*x + 2*c + (d*e*f - 5*c*f^2)/f
^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d^2*e*f - 3*c^2*d*f^2)*log(abs(-sqrt(d*
f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*f))*B*
c*abs(d)/d^3 - 48*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*d*x + 2*c + (d*e
*f - 5*c*f^2)/f^2)*sqrt(d*x + c) + (d^3*e^2 + 2*c*d^2*e*f - 3*c^2*d*f^2)*l
og(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(s
qrt(d*f)*f))*A*abs(d)/d^2 - 8*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(4*d
*x + 4*c + (d*e*f^3 - 13*c*f^4)/f^4)*(d*x + c) - 3*(d^2*e^2*f^2 + 2*c*d*e*
f^3 - 11*c^2*f^4)/f^4)*sqrt(d*x + c) - 3*(d^4*e^3 + c*d^3*e^2*f + 3*c^2*d^
2*e*f^2 - 5*c^3*d*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*
x + c)*d*f - c*d*f)))/(sqrt(d*f)*f^2))*C*c*abs(d)/d^4 - 8*(sqrt(d^2*e +...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\ dx = \text{Hanged}$$

input
```
int((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 1024, normalized size of antiderivative = 3.10

$$\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input

```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x)
```

output

```
(48*sqrt(e + f*x)*sqrt(c + d*x)*a*c*d**3*f**4 + 48*sqrt(e + f*x)*sqrt(c +
d*x)*a*d**4*e*f**3 + 96*sqrt(e + f*x)*sqrt(c + d*x)*a*d**4*f**4*x - 24*sqr
t(e + f*x)*sqrt(c + d*x)*b*c**2*d**2*f**4 + 16*sqrt(e + f*x)*sqrt(c + d*x)
*b*c*d**3*e*f**3 + 16*sqrt(e + f*x)*sqrt(c + d*x)*b*c*d**3*f**4*x - 24*sqr
t(e + f*x)*sqrt(c + d*x)*b*d**4*e**2*f**2 + 16*sqrt(e + f*x)*sqrt(c + d*x)
*b*d**4*e*f**3*x + 64*sqrt(e + f*x)*sqrt(c + d*x)*b*d**4*f**4*x**2 + 15*sq
rt(e + f*x)*sqrt(c + d*x)*c**4*d*f**4 - 7*sqrt(e + f*x)*sqrt(c + d*x)*c**3
*d**2*e*f**3 - 10*sqrt(e + f*x)*sqrt(c + d*x)*c**3*d**2*f**4*x - 7*sqrt(e
+ f*x)*sqrt(c + d*x)*c**2*d**3*e**2*f**2 + 4*sqrt(e + f*x)*sqrt(c + d*x)*c
**2*d**3*e*f**3*x + 8*sqrt(e + f*x)*sqrt(c + d*x)*c**2*d**3*f**4*x**2 + 15
*sqrt(e + f*x)*sqrt(c + d*x)*c*d**4*e**3*f - 10*sqrt(e + f*x)*sqrt(c + d*x
)*c*d**4*e**2*f**2*x + 8*sqrt(e + f*x)*sqrt(c + d*x)*c*d**4*e*f**3*x**2 +
48*sqrt(e + f*x)*sqrt(c + d*x)*c*d**4*f**4*x**3 - 48*sqrt(f)*sqrt(d)*log((
sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*a*c**2*d**
2*f**4 + 96*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e +
f*x))/sqrt(c*f - d*e))*a*c*d**3*e*f**3 - 48*sqrt(f)*sqrt(d)*log((sqrt(f)*s
qrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*a*d**4*e**2*f**2 +
24*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqr
t(c*f - d*e))*b*c**3*d*f**4 - 24*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x
) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*b*c**2*d**2*e*f**3 - 24*sqr...
```

3.66 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 453

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx} \, dx$$
$$= \frac{\left(4df(2Abdf - aC(de+cf)) - \frac{(bde-bcf-4adf)(2aCdf+b(Cde+cCf-2Bdf))}{b}\right)\sqrt{c+dx}\sqrt{e+fx}}{8b^2d^2f^2}$$
$$- \frac{(2aCdf + b(Cde+cCf-2Bdf))(c+dx)^{3/2}\sqrt{e+fx}}{4b^2d^2f} + \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$
$$- \frac{(16a^3Cd^3f^3 - 8a^2bd^2f^2(Cde+cCf+2Bdf) - 2ab^2df(C(de-cf)^2 - 4df(Bde+Bcf+2Adf)) - b^3 \cdots}{8b^4d^{5/2}f^{5/2}}$$
$$- \frac{2(Ab^2 - a(bB-aC))\sqrt{bc-ad}\sqrt{be-af}\,\mathrm{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^4}$$

output
```
1/8*(4*d*f*(2*A*b*d*f-a*C*(c*f+d*e))-(-4*a*d*f-b*c*f+b*d*e)*(2*a*C*d*f+b*(
-2*B*d*f+C*c*f+C*d*e))/b)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d^2/f^2-1/4*(2*a
*C*d*f+b*(-2*B*d*f+C*c*f+C*d*e))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b^2/d^2/f+1/3
*C*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-1/8*(16*a^3*C*d^3*f^3-8*a^2*b*d^2*f^2
*(2*B*d*f+C*c*f+C*d*e)-2*a*b^2*d*f*(C*(-c*f+d*e)^2-4*d*f*(2*A*d*f+B*c*f+B*
d*e))-b^3*(C*(-c*f+d*e)^2*(c*f+d*e)-2*d*f*(B*(-c*f+d*e)^2-4*A*d*f*(c*f+d*e
))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^4/d^(5/2)/f^(5
/2)-2*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^(1/2)*(-a*f+b*e)^(1/2)*arctanh((-a*f+
b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx$$

$$= \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}(24a^2Cd^2f^2-6abdf(cCf+4Bdf+Cd(e+2fx))+b^2(6df(Bcf+4Adf+Bd(e+2fx))+C(-3c^2f^2+2cdf(e+fx)+d^2(-3e^2+2efx+}{d^2f^2}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

output
```
((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 - 6*a*b*d*f*(c*C*f + 4*B
*d*f + C*d*(e + 2*f*x)) + b^2*(6*d*f*(B*c*f + 4*A*d*f + B*d*(e + 2*f*x)) +
 C*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2))))
)/(d^2*f^2) - 48*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[b*c - a*d]*Sqrt[-(b*e) +
a*f]*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d
*x])] + (3*(-16*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f)
+ 2*a*b^2*d*f*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) + b^3*(C
*(d*e - c*f)^2*(d*e + c*f) + 2*d*f*(-(B*(d*e - c*f)^2) + 4*A*d*f*(d*e + c*
f))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(5/2)*f
^(5/2)))/(24*b^4)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2118, 27, 171, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{a+bx}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int \frac{3b\sqrt{c+dx}\sqrt{e+fx}(2Abdf-aC(de+cf)-(2aCdf+b(Cde+cCf-2Bdf))x)}{2(a+bx)}dx}{3b^2df}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}(2Abdf-aC(de+cf)-(2aCdf+b(Cde+cCf-2Bdf))x)}{a+bx}dx}{2bdf}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$$\Big\downarrow \text{171}$$

$$\frac{\frac{\int \frac{\sqrt{e+fx}(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf))+(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))x)}{2(a+bx)\sqrt{c+dx}}dx}{2bf}}{2bdf}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{e+fx}(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf))+(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf)))x)}{(a+bx)\sqrt{c+dx}}dx}{4bf}}{2bdf}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$$\Big\downarrow \text{171}$$

$$\int \frac{2bde(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf)))-a(de+cf)(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf))}{2(a+bx)\sqrt{c}\cdots}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\Big\downarrow$ 27

$$\int \frac{2bde(4bcf(2Abdf-aC(de+cf))+a(de+3cf)(2aCdf+b(Cde+cCf-2Bdf)))-a(de+cf)(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cde+cCf-2Bdf))}{(a+bx)\sqrt{c}\cdots}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\Big\downarrow$ 175

$$\frac{\frac{16d^2f^2(bc-ad)(be-af)\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde\cdots\right.\right.}{b}}{2bd}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\Big\downarrow$ 66

$$\frac{\frac{16d^2f^2(bc-ad)(be-af)\left(Ab^2-a(bB-aC)\right)\int\frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{2\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde\cdots\right.\right.}{\cdots}}{2bd}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\Big\downarrow$ 104

$$\frac{\frac{32d^2f^2(bc-ad)(be-af)\left(Ab^2-a(bB-aC)\right)\int\frac{1}{-bc+ad+\frac{(be-af)(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}-\frac{2\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf\cdots\right.\right.}{\cdots}}{2bd}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)-\left(b^3\left(C(de-cf)^2(cf+de)-2df\left(B(de-cf)\right.\right.\right.\right.}{b\sqrt{d}\sqrt{f}}}{2bd}}{}$$

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

output
```
(C*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*d*f) + (-1/2*((2*a*C*d*f + b*(C*d
*e + c*C*f - 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b*f) + (((4*b*d*f*(
2*A*b*d*f - a*C*(d*e + c*f)) + (b*d*e - b*c*f + 4*a*d*f)*(2*a*C*d*f + b*(C
*d*e + c*C*f - 2*B*d*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d) + ((-2*(16*a^
3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) - 2*a*b^2*d*f*(C*(
d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) - b^3*(C*(d*e - c*f)^2*(d*
e + c*f) - 2*d*f*(B*(d*e - c*f)^2 - 4*A*d*f*(d*e + c*f))))*ArcTanh[(Sqrt[f
]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) - (32*(A*b^
2 - a*(b*B - a*C))*d^2*Sqrt[b*c - a*d]*f^2*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b
*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/b)/(2*b*d))/(4*
b*f))/(2*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3897 vs. $2(409) = 818$.

Time = 0.84 (sec) , antiderivative size = 3898, normalized size of antiderivative = 8.60

method	result	size
default	Expression too large to display	3898

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a),x,method=_RETURNVERB
OSE)
```

output
```
-1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-6*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)*a*b^3*c^2*d*f^3-6*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2
)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*a*b^3*d^3*e^2*f+3*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)*b^4*c^2*d*e*f^2-24*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*d^2*f^2*x-24*A*ln(1/2*(2*d*f*x+2*((f*x+
e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*b^4*d^3*e*f^2+48*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+
2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-
a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b^2*d^3*f^3-48*B*ln(1/2*(2*d
*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a^2*b^2*d^3*f^3+48*B*ln((-2*a*d*f*x+b
*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a^2*b^2*c*d^2*f^3
+48*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(
1/2)*a^2*b^2*d^3*e*f^2-24*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{a+bx}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{a+bx}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A)/(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{a+bx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?`
 for more
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{a+bx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{a+bx}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,(C\,x^2+Bx+A)}{bx+a}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a),x)
```

3.67 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 524

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2} \, dx$$

$$= \frac{(12a^2Cd^2f - abd(Cde + 7cCf + 8Bdf) - b^2(c^2Cf - 4Ad^2f - cd(Ce + 4Bf)))\sqrt{c+dx}\sqrt{e+fx}}{4b^3d(bc-ad)f}$$

$$+ \frac{(3a^2Cdf + b^2(cCe + 2Adf) - ab(Cde + cCf + 2Bdf))(c+dx)^{3/2}\sqrt{e+fx}}{2b^2d(bc-ad)(be-af)}$$

$$- \frac{(Ab^2 - a(bB - aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)}$$

$$+ \frac{(24a^2Cd^2f^2 - 8abdf(Cde + cCf + 2Bdf) - b^2(C(de-cf)^2 - 4df(Bde + Bcf + 2Adf)))\operatorname{arctanh}\Big(}{4b^4d^{3/2}f^{3/2}}$$

$$+ \frac{(6a^3Cdf - b^3(2Bce + Ade + Acf) + ab^2(4cCe + 3Bde + 3Bcf + 2Adf) - a^2b(4Bdf + 5C(de+cf))}{b^4\sqrt{bc-ad}\sqrt{be-af}}$$

output
```
1/4*(12*a^2*C*d^2*f-a*b*d*(8*B*d*f+7*C*c*f+C*d*e)-b^2*(c^2*C*f-4*A*d^2*f-c
*d*(4*B*f+C*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d/(-a*d+b*c)/f+1/2*(3*a^2
*C*d*f+b^2*(2*A*d*f+C*c*e)-a*b*(2*B*d*f+C*c*f+C*d*e))*(d*x+c)^(3/2)*(f*x+e
)^(1/2)/b^2/d/(-a*d+b*c)/(-a*f+b*e)-(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x
+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)+1/4*(24*a^2*C*d^2*f^2-8*a*b*d*f*
(2*B*d*f+C*c*f+C*d*e)-b^2*(C*(-c*f+d*e)^2-4*d*f*(2*A*d*f+B*c*f+B*d*e)))*ar
ctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^4/d^(3/2)/f^(3/2)+(6*
a^3*C*d*f-b^3*(A*c*f+A*d*e+2*B*c*e)+a*b^2*(2*A*d*f+3*B*c*f+3*B*d*e+4*C*c*e
)-a^2*b*(4*B*d*f+5*C*(c*f+d*e)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-
a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^4/(-a*d+b*c)^(1/2)/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$
$$= \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}(-12a^2Cdf+ab(cCf+8Bdf+Cd(e-6fx))+b^2(-4Adf+x(cCf+4Bdf+Cd(e+2fx))))}{df(a+bx)} + \frac{4(-6a^3Cdf+b^3(2Bce+Ade+Acf)-}{}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

output
```
((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(-12*a^2*C*d*f + a*b*(c*C*f + 8*B*d*f + C*
d*(e - 6*f*x)) + b^2*(-4*A*d*f + x*(c*C*f + 4*B*d*f + C*d*(e + 2*f*x)))))/
(d*f*(a + b*x)) + (4*(-6*a^3*C*d*f + b^3*(2*B*c*e + A*d*e + A*c*f) - a*b^2
*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2*A*d*f) + a^2*b*(4*B*d*f + 5*C*(d*e + c*f
)))*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*
x])])/(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) - ((-24*a^2*C*d^2*f^2 + 8*a*b*d
*f*(C*d*e + c*C*f + 2*B*d*f) + b^2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f
 + 2*A*d*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d
^(3/2)*f^(3/2)))/(4*b^4)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2116, 27, 171, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \ 2116$$

$$-\frac{\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(2cCe+3Bde+3Bcf-2Adf)a+b^2(2Bce+Ade+Acf)+2b\left(\frac{3Cdfa^2}{b}-(Cde+cCf+2Bdf)a+b(cCe+2Adf)\cdots\right.\right.}{2b(a+bx)}}{(bc-ad)(be-af)} \cdots \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(2cCe+3Bde+3Bcf-2Adf)a+b^2(2Bce+Ade+Acf)+2b\left(\frac{3Cdfa^2}{b}-(Cde+cCf+2Bdf)a+b(cCe+2Adf)\right)x\right)}{a+bx}}{2b(bc-ad)(be-af)} \cdots \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 171$$

$$\frac{\frac{\int \frac{(bc-ad)\sqrt{e+fx}\left(3Cf(de+3cf)a^2-b(2Bf(de+3cf)+Ce(de+7cf))a+2b^2f(2Bce+Ade+Acf)+\left((4df(Be+Af)-Ce(de-cf))b^2-af(7Cde+cCf+8Bdf)b+12a^2Cd\cdots\right.\right.}{(a+bx)\sqrt{c+dx}}}{2bf}}{2b(bc-ad)(be-af)} \cdots \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \ 27$$

$$\frac{(bc-ad)\int \frac{\sqrt{e+fx}\left(3Cf(de+3cf)a^2-b(2Bf(de+3cf)+Ce(de+7cf))a+2b^2f(2Bce+Ade+Acf)+\left((4df(Be+Af)-Ce(de-cf))b^2-af(7Cde+cCf+8Bdf)b+12a^2C\right.\right.}{(a+bx)\sqrt{c+dx}}}{2bf}$$

$$2b(bc-ad)(be-af)$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 171

$$\frac{(bc-ad)\left(\frac{\int \frac{(be-af)\left(12Cdf(de+cf)a^2-b\left(8Bdf(de+cf)+C\left(d^2e^2+14cdfe+c^2f^2\right)\right)a+4b^2df(2Bce+Ade+Acf)+\left(-\left(\left(C(de-cf)^2-4df(Bde+Bcf+2Adf)\right)b^2\right)-\right.\right.}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{bd}\right.}{2bf}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{(bc-ad)\left(\frac{(be-af)\int \frac{12Cdf(de+cf)a^2-b\left(8Bdf(de+cf)+C\left(d^2e^2+14cdfe+c^2f^2\right)\right)a+4b^2df(2Bce+Ade+Acf)+\left(-\left(\left(C(de-cf)^2-4df(Bde+Bcf+2Adf)\right)b^2\right)-8\right.}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}}{2bd}\right.}{2bf}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 175

$$(bc-ad)\left(\frac{(be-af)\left(\frac{\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)-\left(b^2\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{4df\left(6a^3Cdf-a^2b(4Bdf+5C(cf\right.}{}\right.}{2bd}\right.$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 66

$$\frac{(bc-ad)\left(\frac{(be-af)\left(\frac{2\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)-\left(b^2\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}-\frac{4df\left(6a^3Cdf-a^2b(4Bdf+5\cdots\right.}{\cdots}\right.}{2bd\cdots}\right.}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\Big\downarrow$ 104

$$\frac{(bc-ad)\left(\frac{(be-af)\left(\frac{2\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)-\left(b^2\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}-\frac{8df\left(6a^3Cdf-a^2b(4Bdf+5\cdots\right.}{\cdots}\right.}{2bd\cdots}\right.}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\Big\downarrow$ 221

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(\frac{3a^2Cdf}{b}-a(2Bdf+cCf+Cde)+b(2Adf+cCe)\right)}{f}+\frac{(bc-ad)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}\left(12a^2Cdf^2-abf(8Bdf+cCf+7Cde)+b^2(4df(Af+Be)-\cdots\right.}{bd}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

output
```
-(((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)
*(b*e - a*f)*(a + b*x))) + ((((3*a^2*C*d*f)/b + b*(c*C*e + 2*A*d*f) - a*(C
*d*e + c*C*f + 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^(3/2))/f + ((b*c - a*d)*(
((12*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 8*B*d*f) + b^2*(4*d*f*(B*e + A
*f) - C*e*(d*e - c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d) + ((b*e - a*f)*
((2*(24*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 2*B*d*f) - b^2*(C*(d*e
- c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) + (8*d*f*(6*a^3*C*d*f - b
^3*(2*B*c*e + A*d*e + A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2*A*d*
f) - a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c +
d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]
)))/(2*b*d)))/(2*b*f))/(2*b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4679 vs. $2(482) = 964$.

Time = 0.84 (sec) , antiderivative size = 4680, normalized size of antiderivative = 8.93

method	result	size
default	Expression too large to display	4680

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/8*(8*B*b^4*d*f*x*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*b^4*c*f*x*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/
2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*b^4*d*e*x*(d*f)^(1/2)
*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+16*
B*a*b^3*d*f*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+
b^2*c*e)/b^2)^(1/2)+4*C*b^4*d*f*x^2*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-24*C*a^2*b^2*d*f*(d*f)^(1/2)*(
(f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*a
*b^3*c*f*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)+2*C*a*b^3*d*e*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+
2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-
a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d^2*f^2*x*(d*f)^(1/2)-4*A*ln((-2*a*d*f
*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*d*f^2*x*(d*f)^(1/2)
+24*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a
*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^4*d^2
*f^2*(d*f)^(1/2)-4*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1
/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(
b*x+a))*a*b^3*d^2*e*f*(d*f)^(1/2)+12*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2,x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?`
 for more
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1507 vs. $2(481) = 962$.

Time = 1.39 (sec) , antiderivative size = 1507, normalized size of antiderivative = 2.88

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2,x, algorithm
="giac")
```

output
```
1/4*sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*C*abs(d
)/(b^2*d^3) + (C*b^7*d^4*e*f*abs(d) - C*b^7*c*d^3*f^2*abs(d) - 8*C*a*b^6*d
^4*f^2*abs(d) + 4*B*b^7*d^4*f^2*abs(d))/(b^9*d^6*f^2)) + (4*sqrt(d*f)*C*a*
b^2*c*e*abs(d) - 2*sqrt(d*f)*B*b^3*c*e*abs(d) - 5*sqrt(d*f)*C*a^2*b*d*e*ab
s(d) + 3*sqrt(d*f)*B*a*b^2*d*e*abs(d) - sqrt(d*f)*A*b^3*d*e*abs(d) - 5*sqr
t(d*f)*C*a^2*b*c*f*abs(d) + 3*sqrt(d*f)*B*a*b^2*c*f*abs(d) - sqrt(d*f)*A*b
^3*c*f*abs(d) + 6*sqrt(d*f)*C*a^3*d*f*abs(d) - 4*sqrt(d*f)*B*a^2*b*d*f*abs
(d) + 2*sqrt(d*f)*A*a*b^2*d*f*abs(d))*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a
*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2
*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/(sqr
t(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*b^4*d) - 2*(sqrt
(d*f)*C*a^2*b*d^3*e^2*abs(d) - sqrt(d*f)*B*a*b^2*d^3*e^2*abs(d) + sqrt(d*f
)*A*b^3*d^3*e^2*abs(d) - 2*sqrt(d*f)*C*a^2*b*c*d^2*e*f*abs(d) + 2*sqrt(d*f
)*B*a*b^2*c*d^2*e*f*abs(d) - 2*sqrt(d*f)*A*b^3*c*d^2*e*f*abs(d) + sqrt(d*f
)*C*a^2*b*c^2*d*f^2*abs(d) - sqrt(d*f)*B*a*b^2*c^2*d*f^2*abs(d) + sqrt(d*f
)*A*b^3*c^2*d*f^2*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e
 + (d*x + c)*d*f - c*d*f))^2*C*a^2*b*d*e*abs(d) + sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*d*e*abs(d) - s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2
*A*b^3*d*e*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^2}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,(C\,x^2+Bx+A)}{(bx+a)^2}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2,x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2,x)
```

3.68 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 660

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx =$$

$$-\frac{(12a^3Cd^2f - a^2bd(11Cde+17cCf+4Bdf) - b^3(4c^2Ce - Ad^2e + cd(4Be+Af)) + ab^2(3Bd^2e+4c}{4b^3(bc-ad)^2(be-af)}$$

$$+\frac{(6a^3Cdf - b^3(4Bce-Ade-Acf) + ab^2(8cCe+3Bde+3Bcf-2Adf) - a^2b(2Bdf+7C(de+cf))}{4b^2(bc-ad)^2(be-af)(a+bx)}$$

$$-\frac{(Ab^2-a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2}$$

$$-\frac{(6aCdf - b(Cde+cCf+2Bdf))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^4\sqrt{d}\sqrt{f}}$$

$$-\frac{(24a^4Cd^2f^2 - 3ab^3(Bd^2e^2+c^2f(8Ce+Bf)+2cde(4Ce+3Bf)) - 8a^3bdf(Bdf+5C(de+cf)) - b}{}$$

output
```
-1/4*(12*a^3*C*d^2*f-a^2*b*d*(4*B*d*f+17*C*c*f+11*C*d*e)-b^3*(4*c^2*C*e-A*
d^2*e+c*d*(A*f+4*B*e))+a*b^2*(3*B*d^2*e+4*c^2*C*f+c*d*(5*B*f+16*C*e)))*(d*
x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)^2/(-a*f+b*e)+1/4*(6*a^3*C*d*f-b^3*
(-A*c*f-A*d*e+4*B*c*e)+a*b^2*(-2*A*d*f+3*B*c*f+3*B*d*e+8*C*c*e)-a^2*b*(2*B
*d*f+7*C*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)^2/(-a*f+b*
e)/(b*x+a)-1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c
)/(-a*f+b*e)/(b*x+a)^2-(6*a*C*d*f-b*(2*B*d*f+C*c*f+C*d*e))*arctanh(f^(1/2)
*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^4/d^(1/2)/f^(1/2)-1/4*(24*a^4*C*d^
2*f^2-3*a*b^3*(B*d^2*e^2+c^2*f*(B*f+8*C*e)+2*c*d*e*(3*B*f+4*C*e))-8*a^3*b*
d*f*(B*d*f+5*C*(c*f+d*e))-b^4*(A*d^2*e^2-2*c*d*e*(A*f+2*B*e)-c^2*(-A*f^2+4
*B*e*f+8*C*e^2))+3*a^2*b^2*(4*B*d*f*(c*f+d*e)+C*(5*c^2*f^2+22*c*d*e*f+5*d^
2*e^2)))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(
1/2))/b^4/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx$$

$$= \frac{\frac{b\sqrt{c+dx}\sqrt{e+fx}(12a^4Cdf+4b^4cex(-B+Cx)+Ab^3(acf+ad(e+2fx)-b(2ce+dex+cfx))+ab^3(-4Cx(-4ce+dex+cfx)+B(-2ce+5dex+5cfx)}{(bc-ad)(be-af)(a+bx)^2}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

output
```
((b*Sqrt[c + d*x]*Sqrt[e + f*x]*(12*a^4*C*d*f + 4*b^4*c*e*x*(-B + C*x) + A
*b^3*(a*c*f + a*d*(e + 2*f*x) - b*(2*c*e + d*e*x + c*f*x)) + a*b^3*(-4*C*x
*(-4*c*e + d*e*x + c*f*x) + B*(-2*c*e + 5*d*e*x + 5*c*f*x)) + a^2*b^2*(3*B
*d*(e - 2*f*x) + C*d*x*(-17*e + 4*f*x) + c*(10*C*e + 3*B*f - 17*C*f*x)) -
a^3*b*(4*B*d*f + C*(11*d*e + 11*c*f - 18*d*f*x))))/((b*c - a*d)*(b*e - a*f
)*(a + b*x)^2) - ((24*a^4*C*d^2*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e +
B*f) + 2*c*d*e*(4*C*e + 3*B*f)) - 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) +
b^4*(-(A*d^2*e^2) + 2*c*d*e*(2*B*e + A*f) + c^2*(8*C*e^2 + 4*B*e*f - A*f^2
)) + 3*a^2*b^2*(4*B*d*f*(d*e + c*f) + C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^
2)))*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d
*x])])/((b*c - a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) + (4*(-6*a*C*d*f + b*(C*d*
e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*
x])])/(Sqrt[d]*Sqrt[f]))/(4*b^4)
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2116, 27, 166, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(4cCe+3Bde+3Bcf-4Adf)a+b^2(4Bce-Ade-Acf)-2b\left(-\frac{3Cdfa^2}{b}+Bdfa+2C(de+cf)a-b(2cCe+Adf\right.\right.}{2b(a+bx)^2}}{2(bc-ad)(be-af)}$$
$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(4cCe+3Bde+3Bcf-4Adf)a+b^2(4Bce-A(de+cf))-2b\left(-\frac{3Cdfa^2}{b}+Bdfa+2C(de+cf)a-b(2cCe+Adf)\right)x}{(a+bx)^2}}{4b(bc-ad)(be-af)}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 166

$$\int -\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3-b\left(2Bdf(de+3cf)+C\left(7d^2e^2+34cdfe+15c^2f^2\right)\right)a^2+b^2\left(3f(8Ce+Bf)c^2+2d\left(8Ce^2+5Bfe+Af^2\right)c+d^2e(3Be-2Af)\right)a+b^3\left(-\left(\left(8C\right.\right.\right.}{2(}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\int\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3\right.}{}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 171

$$\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{\int\frac{d(be-af)\left(12Cdf(de+cf)a\right.}{}}{}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{(be-af)\int\frac{12Cdf(de+cf)a^3}{}}{}$$

$$\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 175

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{(be-af)\left(\frac{4(bc-ad)(be-af)(}{}\right.}{}}{} \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 66

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{(be-af)\left(\frac{8(bc-ad)(be-af)(}{}\right.}{}}{} \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 104

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{(be-af)\left(\frac{8(bc-ad)(be-af)(}{}\right.}{}}{} \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\big\downarrow$ 221

$$\frac{\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(2Bdf+7C(cf+de))+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(12a^3Cdf^2-\right.}{}}{} \frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)^2) + (((6*a^3*C*d*f - b^3*(4*B*c*e - A*d*e - A*c
*f) + a*b^2*(8*c*C*e + 3*B*d*e + 3*B*c*f - 2*A*d*f) - a^2*b*(2*B*d*f + 7*C
*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b*(b*e - a*f)*(a + b*x)) -
((2*(12*a^3*C*d*f^2 - a^2*b*f*(17*C*d*e + 11*c*C*f + 4*B*d*f) - b^3*(4*c*C
*e^2 + A*d*e*f + c*f*(4*B*e - A*f)) + a*b^2*(B*f*(5*d*e + 3*c*f) + 4*C*e*(
d*e + 4*c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/b + ((b*e - a*f)*((8*(b*c - a*
d)*(b*e - a*f)*(6*a*C*d*f - b*(C*d*e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[f]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) + (2*(24*a^4*
C*d^2*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) + 2*c*d*e*(4*C*e + 3*
B*f)) - 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) - b^4*(A*d^2*e^2 - 2*c*d*e*(
2*B*e + A*f) - c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a^2*b^2*(4*B*d*f*(d*e
+ c*f) + C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcTanh[(Sqrt[b*e - a*f]
*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqrt[
b*e - a*f])))/b)/(2*b*(b*e - a*f)))/(4*b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11203 vs. 2(616) = 1232.

Time = 0.88 (sec) , antiderivative size = 11204, normalized size of antiderivative = 16.98

method	result	size
default	Expression too large to display	11204

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3,x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3,x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^3}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8241 vs. $2(615) = 1230$.

Time = 4.82 (sec) , antiderivative size = 8241, normalized size of antiderivative = 12.49

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3,x, algorithm
="giac")
```

output
```
-1/4*(8*sqrt(d*f)*C*b^4*c^2*e^2*abs(d) - 24*sqrt(d*f)*C*a*b^3*c*d*e^2*abs(
d) + 4*sqrt(d*f)*B*b^4*c*d*e^2*abs(d) + 15*sqrt(d*f)*C*a^2*b^2*d^2*e^2*abs
(d) - 3*sqrt(d*f)*B*a*b^3*d^2*e^2*abs(d) - sqrt(d*f)*A*b^4*d^2*e^2*abs(d)
- 24*sqrt(d*f)*C*a*b^3*c^2*e*f*abs(d) + 4*sqrt(d*f)*B*b^4*c^2*e*f*abs(d) +
 66*sqrt(d*f)*C*a^2*b^2*c*d*e*f*abs(d) - 18*sqrt(d*f)*B*a*b^3*c*d*e*f*abs(
d) + 2*sqrt(d*f)*A*b^4*c*d*e*f*abs(d) - 40*sqrt(d*f)*C*a^3*b*d^2*e*f*abs(d
) + 12*sqrt(d*f)*B*a^2*b^2*d^2*e*f*abs(d) + 15*sqrt(d*f)*C*a^2*b^2*c^2*f^2
*abs(d) - 3*sqrt(d*f)*B*a*b^3*c^2*f^2*abs(d) - sqrt(d*f)*A*b^4*c^2*f^2*abs
(d) - 40*sqrt(d*f)*C*a^3*b*c*d*f^2*abs(d) + 12*sqrt(d*f)*B*a^2*b^2*c*d*f^2
*abs(d) + 24*sqrt(d*f)*C*a^4*d^2*f^2*abs(d) - 8*sqrt(d*f)*B*a^3*b*d^2*f^2*
abs(d))*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x +
 c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d
^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^6*c*e - a*b^5*d*e - a*b^5*c*f
+ a^2*b^4*d*f)*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2
)*d) + 1/2*(8*sqrt(d*f)*C*a*b^4*c*d^7*e^5*abs(d) - 4*sqrt(d*f)*B*b^5*c*d^7
*e^5*abs(d) - 9*sqrt(d*f)*C*a^2*b^3*d^8*e^5*abs(d) + 5*sqrt(d*f)*B*a*b^4*d
^8*e^5*abs(d) - sqrt(d*f)*A*b^5*d^8*e^5*abs(d) - 32*sqrt(d*f)*C*a*b^4*c^2*
d^6*e^4*f*abs(d) + 16*sqrt(d*f)*B*b^5*c^2*d^6*e^4*f*abs(d) + 27*sqrt(d*f)*
C*a^2*b^3*c*d^7*e^4*f*abs(d) - 15*sqrt(d*f)*B*a*b^4*c*d^7*e^4*f*abs(d) + 3
*sqrt(d*f)*A*b^5*c*d^7*e^4*f*abs(d) + 10*sqrt(d*f)*C*a^3*b^2*d^8*e^4*f*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \text{Hanged}$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^3}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,(C\,x^2+Bx+A)}{(bx+a)^3}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3,x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3,x)
```

3.69 $\int \frac{(a+bx)^2\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 1035

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15cd^2e^2f+9}{}$$

$$-\frac{(16a^2d^2f^2(5Cde+7cCf-6Bdf)+4abdf(8df(5Bde+7Bcf-6Adf)-C(35d^2e^2+50cdef+59c^2f}{}$$

$$+\frac{(80a^2Cd^2f^2-20abdf(7Cde+17cCf-8Bdf)-b^2(10df(7Bde+17Bcf-8Adf)-C(63d^2e^2+154}{240d^4f^3}$$

$$+\frac{b(20aCdf-b(9Cde+31cCf-10Bdf))(c+dx)^{7/2}\sqrt{e+fx}}{40d^4f^2}$$

$$+\frac{b^2C(c+dx)^{9/2}\sqrt{e+fx}}{5d^4f}$$

$$+\frac{(de-cf)\left(16a^2d^2f^2(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+4abdf(C(35d^3e^3+15c\right.}{}$$

output
```
-1/128*(16*a^2*d^2*f^2*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e*
f+5*d^2*e^2))+4*a*b*d*f*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*
e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)))-b^2*(C*(
7*c^4*f^4+12*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+63*d^4*e^4)+2*d
*f*(8*A*d*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)-B*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*
d^2*e^2*f+35*d^3*e^3))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^4/f^5-1/192*(16*a^2
*d^2*f^2*(-6*B*d*f+7*C*c*f+5*C*d*e)+4*a*b*d*f*(8*d*f*(-6*A*d*f+7*B*c*f+5*B
*d*e)-C*(59*c^2*f^2+50*c*d*e*f+35*d^2*e^2))+b^2*(C*(121*c^3*f^3+109*c^2*d*
e*f^2+91*c*d^2*e^2*f+63*d^3*e^3)+2*d*f*(8*A*d*f*(7*c*f+5*d*e)-B*(59*c^2*f^
2+50*c*d*e*f+35*d^2*e^2))))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^4/f^4+1/240*(80*
a^2*C*d^2*f^2-20*a*b*d*f*(-8*B*d*f+17*C*c*f+7*C*d*e)-b^2*(10*d*f*(-8*A*d*f
+17*B*c*f+7*B*d*e)-C*(263*c^2*f^2+154*c*d*e*f+63*d^2*e^2)))*(d*x+c)^(5/2)*
(f*x+e)^(1/2)/d^4/f^3+1/40*b*(20*a*C*d*f-b*(-10*B*d*f+31*C*c*f+9*C*d*e))*(
d*x+c)^(7/2)*(f*x+e)^(1/2)/d^4/f^2+1/5*b^2*C*(d*x+c)^(9/2)*(f*x+e)^(1/2)/d
^4/f+1/128*(-c*f+d*e)*(16*a^2*d^2*f^2*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c
^2*f^2+2*c*d*e*f+5*d^2*e^2))+4*a*b*d*f*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^
2*e^2*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*
e^2)))-b^2*(C*(7*c^4*f^4+12*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+28*c*d^3*e^3*f+
63*d^4*e^4)+2*d*f*(8*A*d*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)-B*(5*c^3*f^3+9*c^
2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d...
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3220 vs. 2(1035) = 2070.

Time = 16.91 (sec) , antiderivative size = 3220, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
((-(b*e) + a*f)^2*(d*e - c*f)^2*(C*e^2 - B*e*f + A*f^2)*Sqrt[d/((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))]*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f))^2*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]*((2*d*f*(c + d*x))/((d*e -
 c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sq
rt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)
*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(2*d^3*f^6*Sqrt[c + d*x]
*Sqrt[e + f*x]) + (2*b^2*C*(d*e - c*f)^3*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1
+ (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))
))^(9/2)*((3*(35/(64*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f
) - (c*d*f)/(d*e - c*f))))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*
f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-
1)))/10 + (21*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*
((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*
x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])...
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 778, normalized size of antiderivative = 0.75, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2118, 27, 170, 27, 164, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 2118$$

$$\frac{\int -\frac{b(a+bx)^2\sqrt{c+dx}(6bcCe+3aCde+acCf-10Abdf+(4aCdf+b(9Cde+7cCf-10Bdf))x)}{2\sqrt{e+fx}}dx}{5b^2df}+\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$\downarrow$ 27

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\int \frac{(a+bx)^2\sqrt{c+dx}(6bcCe+3aCde+acCf-10Abdf+(4aCdf+b(9Cde+7cCf-10Bdf))x)}{\sqrt{e+fx}}dx}{10bdf}$$

$\downarrow$ 170

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$\int \frac{(a+bx)\sqrt{c+dx}(8adf(6bcCe+3aCde+acCf-10Abdf)-(4bce+3ade+acf)(4aCdf+b(9Cde+7cCf-10Bdf))+(8bdf(6bcCe+3aCde+acCf-10Abdf)-(7bde+5bcf-}{2\sqrt{e+fx}}$$

$$4df$$

$$10bdf$$

$\downarrow$ 27

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$\int \frac{(a+bx)\sqrt{c+dx}(8adf(6bcCe+3aCde+acCf-10Abdf)-(4bce+3ade+acf)(4aCdf+b(9Cde+7cCf-10Bdf))+(8bdf(6bcCe+3aCde+acCf-10Abdf)-(7bde+5bcf-}{\sqrt{e+fx}}$$

$$8df$$

$$10bdf$$

$\downarrow$ 164

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$5b\left(16a^2d^2f^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+4abdf\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(5c^3f^3+9c^2def^2+15cd^2e^2f+35d^3\right.\right.\right.$$

$$8d$$

$\downarrow$ 60

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$5b\left(16a^2d^2f^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+4abdf\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(5c^3f^3+9c^2def^2+15cd^2e^2f+35d^3\right.\right.\right.$$

$\downarrow$ 66

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} -$$

$$5b\left(16a^2d^2f^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+4abdf\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(5c^3f^3+9c^2def^2+15cd^2e^2f+35d^3\right.\right.\right.$$

$\downarrow$ 221

$$\frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} -$$

$$5b\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{\sqrt{d}f^{3/2}}\right)\left(16a^2d^2f^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+4abdf\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2\right.\right.\right.\right.$$

input
```
Int[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(C*(a + b*x)^3*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - (((4*a*C*d*f + b
*(9*C*d*e + 7*c*C*f - 10*B*d*f))*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x]
)/(4*d*f) + (((c + d*x)^(3/2)*Sqrt[e + f*x]*(96*a^3*C*d^3*f^3 + 8*a^2*b*d^
2*f^2*(23*C*d*e + 9*c*C*f - 30*B*d*f) + 20*a*b^2*d*f*(8*d*f*(5*B*d*e + 3*B
*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2)) + b^3*(C*(315*
d^3*e^3 + 203*c*d^2*e^2*f + 145*c^2*d*e*f^2 + 105*c^3*f^3) + 10*d*f*(8*A*d
*f*(5*d*e + 3*c*f) - B*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2))) + 4*b*d*f*
(8*b*d*f*(6*b*c*C*e + 3*a*C*d*e + a*c*C*f - 10*A*b*d*f) - (7*b*d*e + 5*b*c
*f - 4*a*d*f)*(4*a*C*d*f + b*(9*C*d*e + 7*c*C*f - 10*B*d*f)))*x))/(12*d^2*
f^2) + (5*b*(16*a^2*d^2*f^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*
e^2 + 2*c*d*e*f + c^2*f^2)) + 4*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f +
9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 +
 2*c*d*e*f + c^2*f^2))) - b^2*(C*(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2
*e^2*f^2 + 12*c^3*d*e*f^3 + 7*c^4*f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d
*e*f + c^2*f^2) - B*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f
^3))))*((Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((d*e - c*f)*ArcTanh[(Sqrt[f]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3/2))))/(8*d^2*f^2))/(8
*d*f))/(10*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3957 vs. $2(991) = 1982$.

Time = 0.64 (sec) , antiderivative size = 3958, normalized size of antiderivative = 3.82

method	result	size
default	Expression too large to display	3958

input
```
int((b*x+a)^2*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(90*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c
))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*e^2*f^3+150*C*ln(1/
2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2
*c^2*d^3*e^3*f^2-1120*B*b^2*d^4*e*f^3*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1
/2)-112*C*b^2*c^2*d^2*f^4*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+1008*C*b
^2*d^4*e^2*f^2*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+2800*C*((f*x+e)*(d*
x+c))^(1/2)*(d*f)^(1/2)*a*b*d^4*e^2*f^2*x-210*C*((f*x+e)*(d*x+c))^(1/2)*(d
*f)^(1/2)*b^2*c^4*f^4+240*A*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*f^5-960*B*ln(1/2*(2*d*f*x+2*((f*
x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*e*f^4-2400
*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*a*b*d^5*e^3*f^2-600*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^3*f^2-960*B*((f*x+e)*(d*x+c))^(1/
2)*(d*f)^(1/2)*a*b*c^2*d^2*f^4+2400*A*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*
b^2*d^4*e^2*f^2-3200*B*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*a*b*d^4*e*f^3*x
+140*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b^2*c^3*d*f^4*x-1260*C*((f*x+e)
*(d*x+c))^(1/2)*(d*f)^(1/2)*b^2*d^4*e^3*f*x-2880*B*(d*f)^(1/2)*((f*x+e)*(d
*x+c))^(1/2)*a^2*d^4*e*f^3-2100*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^2*
d^4*e^3*f+2400*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*a^2*d^4*e^2*f^2+340*B
*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b^2*c^2*d^2*e*f^3+500*B*((f*x+e)*(...
```

Fricas [A] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 2176, normalized size of antiderivative = 2.10

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
[-1/7680*(15*(63*C*b^2*d^5*e^5 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5
)*e^4*f - 10*(C*b^2*c^2*d^3 - 4*(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a
*b + A*b^2)*d^5)*e^3*f^2 - 6*(C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3
+ 8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 16*(B*a^2 + 2*A*a*b)*d^5)*e^2*f^3 -
(5*C*b^2*c^4*d - 128*A*a^2*d^5 - 8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 +
 2*B*a*b + A*b^2)*c^2*d^3 - 64*(B*a^2 + 2*A*a*b)*c*d^4)*e*f^4 - (7*C*b^2*c
^5 + 128*A*a^2*c*d^4 - 10*(2*C*a*b + B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b +
A*b^2)*c^3*d^2 - 32*(B*a^2 + 2*A*a*b)*c^2*d^3)*f^5)*sqrt(d*f)*log(8*d^2*f^
2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*
sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(384*C*b^2*d^5*
f^5*x^4 + 945*C*b^2*d^5*e^4*f - 210*(C*b^2*c*d^4 + 5*(2*C*a*b + B*b^2)*d^5
)*e^3*f^2 - 2*(68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*d^4 - 600*(C*a^2
 + 2*B*a*b + A*b^2)*d^5)*e^2*f^3 - 10*(11*C*b^2*c^3*d^2 - 17*(2*C*a*b + B*
b^2)*c^2*d^3 + 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a^2 + 2*A*a*b)*
d^5)*e*f^4 - 15*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b + B*b^2)*c^3*
d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a*b)*c*d^4)*f
^5 - 48*(9*C*b^2*d^5*e*f^4 - (C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2)*d^5)*f^5)
*x^3 + 8*(63*C*b^2*d^5*e^2*f^3 - 2*(4*C*b^2*c*d^4 + 35*(2*C*a*b + B*b^2)*d
^5)*e*f^4 - (7*C*b^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C*a^2 + 2*
B*a*b + A*b^2)*d^5)*f^5)*x^2 - 2*(315*C*b^2*d^5*e^3*f^2 - 7*(7*C*b^2*c*...
```

Sympy [F]

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**2*(d*x+c)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**2*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1509, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
1/1920*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(4*(d*x + c)*(6*(d*x + c)*(
8*(d*x + c)*C*b^2/(d^5*f) - (9*C*b^2*d^21*e*f^7 + 31*C*b^2*c*d^20*f^8 - 20
*C*a*b*d^21*f^8 - 10*B*b^2*d^21*f^8)/(d^25*f^9)) + (63*C*b^2*d^22*e^2*f^6
+ 154*C*b^2*c*d^21*e*f^7 - 140*C*a*b*d^22*e*f^7 - 70*B*b^2*d^22*e*f^7 + 26
3*C*b^2*c^2*d^20*f^8 - 340*C*a*b*c*d^21*f^8 - 170*B*b^2*c*d^21*f^8 + 80*C*
a^2*d^22*f^8 + 160*B*a*b*d^22*f^8 + 80*A*b^2*d^22*f^8)/(d^25*f^9)) - 5*(63
*C*b^2*d^23*e^3*f^5 + 91*C*b^2*c*d^22*e^2*f^6 - 140*C*a*b*d^23*e^2*f^6 - 7
0*B*b^2*d^23*e^2*f^6 + 109*C*b^2*c^2*d^21*e*f^7 - 200*C*a*b*c*d^22*e*f^7 -
 100*B*b^2*c*d^22*e*f^7 + 80*C*a^2*d^23*e*f^7 + 160*B*a*b*d^23*e*f^7 + 80*
A*b^2*d^23*e*f^7 + 121*C*b^2*c^3*d^20*f^8 - 236*C*a*b*c^2*d^21*f^8 - 118*B
*b^2*c^2*d^21*f^8 + 112*C*a^2*c*d^22*f^8 + 224*B*a*b*c*d^22*f^8 + 112*A*b^
2*c*d^22*f^8 - 96*B*a^2*d^23*f^8 - 192*A*a*b*d^23*f^8)/(d^25*f^9))*(d*x +
c) + 15*(63*C*b^2*d^24*e^4*f^4 + 28*C*b^2*c*d^23*e^3*f^5 - 140*C*a*b*d^24*
e^3*f^5 - 70*B*b^2*d^24*e^3*f^5 + 18*C*b^2*c^2*d^22*e^2*f^6 - 60*C*a*b*c*d
^23*e^2*f^6 - 30*B*b^2*c*d^23*e^2*f^6 + 80*C*a^2*d^24*e^2*f^6 + 160*B*a*b*
d^24*e^2*f^6 + 80*A*b^2*d^24*e^2*f^6 + 12*C*b^2*c^3*d^21*e*f^7 - 36*C*a*b*
c^2*d^22*e*f^7 - 18*B*b^2*c^2*d^22*e*f^7 + 32*C*a^2*c*d^23*e*f^7 + 64*B*a*
b*c*d^23*e*f^7 + 32*A*b^2*c*d^23*e*f^7 - 96*B*a^2*d^24*e*f^7 - 192*A*a*b*d
^24*e*f^7 + 7*C*b^2*c^4*d^20*f^8 - 20*C*a*b*c^3*d^21*f^8 - 10*B*b^2*c^3*d^
21*f^8 + 16*C*a^2*c^2*d^22*f^8 + 32*B*a*b*c^2*d^22*f^8 + 16*A*b^2*c^2*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+bx)^2\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(bx+a)^2\sqrt{dx+c}\,(C\,x^2+Bx+A)}{\sqrt{fx+e}}dx$$

input
```
int((b*x+a)^2*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x)
```

output
```
int((b*x+a)^2*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x)
```

3.70 $\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 535

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx =$$

$$-\frac{(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9c^2def^2+}{64d^3f^4}$$

$$-\frac{(8bdf(5Bde+7Bcf-6Adf)+8adf(5Cde+7cCf-6Bdf)-bC(35d^2e^2+50cdef+59c^2f^2))\,(c+}{96d^3f^3}$$

$$+\frac{(8aCdf-b(7Cde+17cCf-8Bdf))(c+dx)^{5/2}\sqrt{e+fx}}{24d^3f^2}+\frac{bC(c+dx)^{7/2}\sqrt{e+fx}}{4d^3f}$$

$$+\frac{(de-cf)\,(8adf(2df(3Bde+Bcf-4Adf)-C(5d^2e^2+2cdef+c^2f^2))+b(C(35d^3e^3+15cd^2e^2f+9}{64d^{7/2}f^{9/2}}$$

output
```
-1/64*(8*a*d*f*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*
e^2))+b*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3)+8*d*f*(2*A*
d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(
1/2)/d^3/f^4-1/96*(8*b*d*f*(-6*A*d*f+7*B*c*f+5*B*d*e)+8*a*d*f*(-6*B*d*f+7*
C*c*f+5*C*d*e)-b*C*(59*c^2*f^2+50*c*d*e*f+35*d^2*e^2))*(d*x+c)^(3/2)*(f*x+
e)^(1/2)/d^3/f^3+1/24*(8*a*C*d*f-b*(-8*B*d*f+17*C*c*f+7*C*d*e))*(d*x+c)^(5
/2)*(f*x+e)^(1/2)/d^3/f^2+1/4*b*C*(d*x+c)^(7/2)*(f*x+e)^(1/2)/d^3/f+1/64*(
-c*f+d*e)*(8*a*d*f*(2*d*f*(-4*A*d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*
d^2*e^2))+b*(C*(5*c^3*f^3+9*c^2*d*e*f^2+15*c*d^2*e^2*f+35*d^3*e^3)+8*d*f*(
2*A*d*f*(c*f+3*d*e)-B*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))))*arctanh(f^(1/2)*(d*
x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(7/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 9.42 (sec) , antiderivative size = 474, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

$$= \frac{\sqrt{d}\sqrt{f}\sqrt{c+dx}\sqrt{e+fx}(8adf(6df(4Adf+B(-3de+cf+2dfx))+C(-3c^2f^2+2cdf(-2e+fx)+d^2(}{}$$

input
```
Integrate[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*Sqrt[e + f*x]*(8*a*d*f*(6*d*f*(4*A*d*f + B*
(-3*d*e + c*f + 2*d*f*x)) + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15
*e^2 - 10*e*f*x + 8*f^2*x^2))) + b*(C*(15*c^3*f^3 + c^2*d*f^2*(17*e - 10*f
*x) + c*d^2*f*(25*e^2 - 12*e*f*x + 8*f^2*x^2) + d^3*(-105*e^3 + 70*e^2*f*x
 - 56*e*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(-3*d*e + c*f + 2*d*f*x) +
 B*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2
))))) - 6*(d*e - c*f)*(-8*a*d*f*(2*d*f*(-3*B*d*e - B*c*f + 4*A*d*f) + C*(5
*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c
^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*
c*d*e*f + c^2*f^2))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*(Sqrt[c - (
d*e)/f] - Sqrt[c + d*x]))])/(192*d^(7/2)*f^(9/2))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 419, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2118, 27, 164, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int -\frac{b(a+bx)\sqrt{c+dx}(4bcCe+3aCde+acCf-8Abdf+(4aCdf+b(7Cde+5cCf-8Bdf))x)}{2\sqrt{e+fx}}dx}{4b^2df}+\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-\frac{\int \frac{(a+bx)\sqrt{c+dx}(4bcCe+3aCde+acCf-8Abdf+(4aCdf+b(7Cde+5cCf-8Bdf))x)}{\sqrt{e+fx}}dx}{8bdf}$$

$$\Big\downarrow \text{164}$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-$$

$$\frac{b(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2Adf(cf+3de)-B(c^2f^2+2cdef+5d^2e^2))+C(5c^3f^3+9c^2def^2+15cd^2e^2f+}{8d^2f^2}$$

$$\Big\downarrow \text{60}$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-$$

$$\frac{b(8adf(2df(-4Adf+Bcf+3Bde)-C(c^2f^2+2cdef+5d^2e^2))+b(8df(2Adf(cf+3de)-B(c^2f^2+2cdef+5d^2e^2))+C(5c^3f^3+9c^2def^2+15cd^2e^2f+}{8d^2f^2}$$

$$\Big\downarrow \text{66}$$

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-$$

$$\frac{b\left(8adf\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+b\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(5c^3f^3+9c^2def^2+15cd^2e^2f+\right.\right.\right.}{8d^2f^2}$$

$\downarrow$ 221

$$\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}-$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(24a^2Cd^2f^2+4bdfx(4aCdf+b(-8Bdf+5cCf+7Cde))+8abdf(-6Bdf+3cCf+5Cde)+b^2\left(8df(-6Adf+3Bcf+5Bde)-C\left(15c^2\right.\right.\right.}{12d^2f^2}$$

input
```
Int[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(C*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x])/(4*b*d*f) - (((c + d*x)^(3/2
)*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 + 8*a*b*d*f*(5*C*d*e + 3*c*C*f - 6*B*d*f
) + b^2*(8*d*f*(5*B*d*e + 3*B*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f
+ 15*c^2*f^2)) + 4*b*d*f*(4*a*C*d*f + b*(7*C*d*e + 5*c*C*f - 8*B*d*f))*x))
/(12*d^2*f^2) + (b*(8*a*d*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*
e^2 + 2*c*d*e*f + c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*
e*f^2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e
*f + c^2*f^2))))*((Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((d*e - c*f)*ArcTanh[(
Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3/2))))/(8*d^
2*f^2))/(8*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. $2(497) = 994$.

Time = 0.55 (sec) , antiderivative size = 2002, normalized size of antiderivative = 3.74

method	result	size
default	Expression too large to display	2002

input
```
int((b*x+a)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(72*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*e^2*f^2-60*C*ln(1/2*(2*d
*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3*e
^3*f-288*A*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b*d^3*e*f^2+192*A*((f*x+e)*
(d*x+c))^(1/2)*(d*f)^(1/2)*b*d^3*f^3*x-12*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*
x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d*e*f^3+50*C*((f*x+e)*
(d*x+c))^(1/2)*(d*f)^(1/2)*b*c*d^2*e^2*f+32*B*((f*x+e)*(d*x+c))^(1/2)*(d*f
)^(1/2)*b*c*d^2*f^3*x-64*B*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b*c*d^2*e*f
^2-64*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*a*c*d^2*e*f^2-160*B*((f*x+e)*(
d*x+c))^(1/2)*(d*f)^(1/2)*b*d^3*e*f^2*x-20*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)
^(1/2)*b*c^2*d*f^3*x+32*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*a*c*d^2*f^3*
x+140*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b*d^3*e^2*f*x-160*C*((f*x+e)*(
d*x+c))^(1/2)*(d*f)^(1/2)*a*d^3*e*f^2*x+34*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)
^(1/2)*b*c^2*d*e*f^2+192*A*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*f^4+96*C*b*d^3*f^3*x^3*((f*x+e)*(d*x+
c))^(1/2)*(d*f)^(1/2)+128*B*b*d^3*f^3*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1
/2)+128*C*a*d^3*f^3*x^2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)-24*C*((f*x+e)*
(d*x+c))^(1/2)*(d*f)^(1/2)*b*c*d^2*e*f^2*x-18*C*ln(1/2*(2*d*f*x+2*((f*x+e)
*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*e^2*f^2+96*A*(
(f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b*c*d^2*f^3-210*C*(d*f)^(1/2)*((f*x+...
```

Fricas [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 1114, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
[1/768*(3*(35*C*b*d^4*e^4 - 20*(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(
C*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^
3*d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 + 8*(B*a + A*b)*c*d^3)*e*f^3 - (5
*C*b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3*d + 16*(B*a + A*b)*c^2*d^2)*f^
4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*
x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^
2)*x) + 4*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^3*f + 5*(5*C*b*c*d^3 + 24*(C
*a + B*b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 - 144*(B*a
 + A*b)*d^4)*e*f^3 + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^2*d^2 +
 16*(B*a + A*b)*c*d^3)*f^4 - 8*(7*C*b*d^4*e*f^3 - (C*b*c*d^3 + 8*(C*a + B*
b)*d^4)*f^4)*x^2 + 2*(35*C*b*d^4*e^2*f^2 - 2*(3*C*b*c*d^3 + 20*(C*a + B*b)
*d^4)*e*f^3 - (5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*d^4)*f
^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f^5), -1/384*(3*(35*C*b*d^4*e^4 -
 20*(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(C*b*c^2*d^2 - 4*(C*a + B*b)
*c*d^3 - 8*(B*a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^3*d + 16*A*a*d^4 - 2*(C*a +
 B*b)*c^2*d^2 + 8*(B*a + A*b)*c*d^3)*e*f^3 - (5*C*b*c^4 - 64*A*a*c*d^3 - 8
*(C*a + B*b)*c^3*d + 16*(B*a + A*b)*c^2*d^2)*f^4)*sqrt(-d*f)*arctan(1/2*(2
*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 +
c*d*e*f + (d^2*e*f + c*d*f^2)*x)) - 2*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^
3*f + 5*(5*C*b*c*d^3 + 24*(C*a + B*b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - ...
```

Sympy [F]

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx=\int \frac{(a+bx)\,\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)*(d*x+c)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\left(2\,(dx+c)\left(4\,(dx+c)\left(\frac{6\,(dx+c)Cb}{d^4f}-\frac{7\,Cbd^{13}ef^5+17\,Cbcd^{12}f^6-8\,Cad^{13}f^6-8\,Bbd^{13}f^6}{d^{16}f^7}\right)+\right.\right.\right.}{}$$

input
```
integrate((b*x+a)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
1/192*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d
*x + c)*C*b/(d^4*f) - (7*C*b*d^13*e*f^5 + 17*C*b*c*d^12*f^6 - 8*C*a*d^13*f
^6 - 8*B*b*d^13*f^6)/(d^16*f^7)) + (35*C*b*d^14*e^2*f^4 + 50*C*b*c*d^13*e*
f^5 - 40*C*a*d^14*e*f^5 - 40*B*b*d^14*e*f^5 + 59*C*b*c^2*d^12*f^6 - 56*C*a
*c*d^13*f^6 - 56*B*b*c*d^13*f^6 + 48*B*a*d^14*f^6 + 48*A*b*d^14*f^6)/(d^16
*f^7)) - 3*(35*C*b*d^15*e^3*f^3 + 15*C*b*c*d^14*e^2*f^4 - 40*C*a*d^15*e^2*
f^4 - 40*B*b*d^15*e^2*f^4 + 9*C*b*c^2*d^13*e*f^5 - 16*C*a*c*d^14*e*f^5 - 1
6*B*b*c*d^14*e*f^5 + 48*B*a*d^15*e*f^5 + 48*A*b*d^15*e*f^5 + 5*C*b*c^3*d^1
2*f^6 - 8*C*a*c^2*d^13*f^6 - 8*B*b*c^2*d^13*f^6 + 16*B*a*c*d^14*f^6 + 16*A
*b*c*d^14*f^6 - 64*A*a*d^15*f^6)/(d^16*f^7))*sqrt(d*x + c) - 3*(35*C*b*d^4
*e^4 - 20*C*b*c*d^3*e^3*f - 40*C*a*d^4*e^3*f - 40*B*b*d^4*e^3*f - 6*C*b*c^
2*d^2*e^2*f^2 + 24*C*a*c*d^3*e^2*f^2 + 24*B*b*c*d^3*e^2*f^2 + 48*B*a*d^4*e
^2*f^2 + 48*A*b*d^4*e^2*f^2 - 4*C*b*c^3*d*e*f^3 + 8*C*a*c^2*d^2*e*f^3 + 8*
B*b*c^2*d^2*e*f^3 - 32*B*a*c*d^3*e*f^3 - 32*A*b*c*d^3*e*f^3 - 64*A*a*d^4*e
*f^3 - 5*C*b*c^4*f^4 + 8*C*a*c^3*d*f^4 + 8*B*b*c^3*d*f^4 - 16*B*a*c^2*d^2*
f^4 - 16*A*b*c^2*d^2*f^4 + 64*A*a*c*d^3*f^4)*log(abs(-sqrt(d*f)*sqrt(d*x +
 c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^3*f^4))*d/abs(d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 47.98 (sec) , antiderivative size = 1525, normalized size of antiderivative = 2.85

$$\int \frac{(a+bx)\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x)
```

output

```
(192*sqrt(e + f*x)*sqrt(c + d*x)*a**2*d**4*f**4 + 96*sqrt(e + f*x)*sqrt(c
+ d*x)*a*b*c*d**3*f**4 - 288*sqrt(e + f*x)*sqrt(c + d*x)*a*b*d**4*e*f**3 +
 192*sqrt(e + f*x)*sqrt(c + d*x)*a*b*d**4*f**4*x - 24*sqrt(e + f*x)*sqrt(c
 + d*x)*a*c**3*d**2*f**4 - 32*sqrt(e + f*x)*sqrt(c + d*x)*a*c**2*d**3*e*f*
*3 + 16*sqrt(e + f*x)*sqrt(c + d*x)*a*c**2*d**3*f**4*x + 120*sqrt(e + f*x)
*sqrt(c + d*x)*a*c*d**4*e**2*f**2 - 80*sqrt(e + f*x)*sqrt(c + d*x)*a*c*d**
4*e*f**3*x + 64*sqrt(e + f*x)*sqrt(c + d*x)*a*c*d**4*f**4*x**2 - 24*sqrt(e
 + f*x)*sqrt(c + d*x)*b**2*c**2*d**2*f**4 - 32*sqrt(e + f*x)*sqrt(c + d*x)
*b**2*c*d**3*e*f**3 + 16*sqrt(e + f*x)*sqrt(c + d*x)*b**2*c*d**3*f**4*x +
120*sqrt(e + f*x)*sqrt(c + d*x)*b**2*d**4*e**2*f**2 - 80*sqrt(e + f*x)*sqr
t(c + d*x)*b**2*d**4*e*f**3*x + 64*sqrt(e + f*x)*sqrt(c + d*x)*b**2*d**4*f
**4*x**2 + 15*sqrt(e + f*x)*sqrt(c + d*x)*b*c**4*d*f**4 + 17*sqrt(e + f*x)
*sqrt(c + d*x)*b*c**3*d**2*e*f**3 - 10*sqrt(e + f*x)*sqrt(c + d*x)*b*c**3*
d**2*f**4*x + 25*sqrt(e + f*x)*sqrt(c + d*x)*b*c**2*d**3*e**2*f**2 - 12*sq
rt(e + f*x)*sqrt(c + d*x)*b*c**2*d**3*e*f**3*x + 8*sqrt(e + f*x)*sqrt(c +
d*x)*b*c**2*d**3*f**4*x**2 - 105*sqrt(e + f*x)*sqrt(c + d*x)*b*c*d**4*e**3
*f + 70*sqrt(e + f*x)*sqrt(c + d*x)*b*c*d**4*e**2*f**2*x - 56*sqrt(e + f*x
)*sqrt(c + d*x)*b*c*d**4*e*f**3*x**2 + 48*sqrt(e + f*x)*sqrt(c + d*x)*b*c*
d**4*f**4*x**3 + 192*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*
sqrt(e + f*x))/sqrt(c*f - d*e))*a**2*c*d**3*f**4 - 192*sqrt(f)*sqrt(d)*...
```

3.71 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 246

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\sqrt{c+dx}\sqrt{e+fx}}{8d^2f^3} - \frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2} + \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} - \frac{(de-cf)\,(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf)))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{8d^{5/2}f^{7/2}}$$

output

```
1/8*(C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+2*d*f*(4*A*d*f-B*(c*f+3*d*e)))*(d*x+c
)^(1/2)*(f*x+e)^(1/2)/d^2/f^3-1/12*(-6*B*d*f+7*C*c*f+5*C*d*e)*(d*x+c)^(3/2
)*(f*x+e)^(1/2)/d^2/f^2+1/3*C*(d*x+c)^(5/2)*(f*x+e)^(1/2)/d^2/f-1/8*(-c*f+
d*e)*(C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+2*d*f*(4*A*d*f-B*(c*f+3*d*e)))*arcta
nh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/d^(5/2)/f^(7/2)
```

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(6df(4Adf+B(-3de+cf+2dfx))+C(-3c^2f^2+2cdf(-2e+fx)+d^2(15e^2-10ef}{24d^2f^3}$$
$$+\frac{(de-cf)\left(C(5d^2e^2+2cdef+c^2f^2)+2df(4Adf-B(3de+cf))\right)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\left(\sqrt{c-\frac{de}{f}}-\sqrt{c+dx}\right)}\right)}{4d^{5/2}f^{7/2}}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(6*d*f*(4*A*d*f + B*(-3*d*e + c*f + 2*d*f*x))
 + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x
^2))))/(24*d^2*f^3) + ((d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) +
2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f
]*(Sqrt[c - (d*e)/f] - Sqrt[c + d*x]))])/(4*d^(5/2)*f^(7/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1194, 27, 90, 60, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 1194$$

$$\frac{\int -\frac{\sqrt{c+dx}(Cfc^2+5Cdec-6Ad^2f+d(5Cde+7cCf-6Bdf)x)}{2\sqrt{e+fx}}dx}{3d^2f}+\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}$$

$$\Big\downarrow \ 27$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\int \frac{\sqrt{c+dx}(Cfc^2+5Cdec-6Ad^2f+d(5Cde+7cCf-6Bdf)x)}{\sqrt{e+fx}}dx}{6d^2f}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{2f}-\frac{3(2df(4Adf-B(cf+3de))+C(c^2f^2+2cdef+5d^2e^2))\int \frac{\sqrt{c+dx}}{\sqrt{e+fx}}dx}{4f}}{6d^2f}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{2f}-\frac{3(2df(4Adf-B(cf+3de))+C(c^2f^2+2cdef+5d^2e^2))\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{2f}\right)}{4f}}{6d^2f}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{2f}-\frac{3(2df(4Adf-B(cf+3de))+C(c^2f^2+2cdef+5d^2e^2))\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{f}\right)}{4f}}{6d^2f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}-\frac{\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{2f}-\frac{3\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{f}-\frac{(de-cf)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{\sqrt{d}f^{3/2}}\right)(2df(4Adf-B(cf+3de))+C(c^2f^2+2cdef+5d^2e^2))}{4f}}{6d^2f}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(C*(c + d*x)^(5/2)*Sqrt[e + f*x])/(3*d^2*f) - (((5*C*d*e + 7*c*C*f - 6*B*d
*f)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*f) - (3*(C*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2) + 2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*((Sqrt[c + d*x]*Sqrt[e + f*x
])/f - ((d*e - c*f)*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x]
)])/(Sqrt[d]*f^(3/2))))/(4*f))/(6*d^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(214) = 428$.

Time = 0.53 (sec) , antiderivative size = 763, normalized size of antiderivative = 3.10

method	result
default	$\frac{\sqrt{xd+c}\,\sqrt{fx+e}\,\left(16C\,d^{2}f^{2}x^{2}\sqrt{(fx+e)(xd+c)}\,\sqrt{df}+24A\ln\left(\frac{2dfx+2\sqrt{(fx+e)(xd+c)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)c\,d^{2}f^{3}-24A\ln\left(\frac{2dfx+2\sqrt{(fx+e)(x}{2\sqrt{}}\right.\right.}{}$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(16*C*d^2*f^2*x^2*((f*x+e)*(d*x+c))^(1/2)
*(d*f)^(1/2)+24*A*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e*f^2-6*B*ln(1/2*(2*d*f*x+2*((f*x
+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d*f^3-12*B*ln(1/2
*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^
2*e*f^2+18*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e
)/(d*f)^(1/2))*d^3*e^2*f+24*B*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*d^2*f^2*
x+3*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*c^3*f^3+3*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*c^2*d*e*f^2+9*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^2*e^2*f-15*C*ln(1/2*(2*d*f*x+2
*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^3*e^3+4*C*((f
*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*c*d*f^2*x-20*C*((f*x+e)*(d*x+c))^(1/2)*(d
*f)^(1/2)*d^2*e*f*x+48*A*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*d^2*f^2+12*B*
((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*c*d*f^2-36*B*(d*f)^(1/2)*((f*x+e)*(d*x
+c))^(1/2)*d^2*e*f-6*C*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*c^2*f^2-8*C*((f
*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*c*d*e*f+30*C*(d*f)^(1/2)*((f*x+e)*(d*x+c)
)^(1/2)*d^2*e^2)/f^3/((f*x+e)*(d*x+c))^(1/2)/d^2/(d*f)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

$$= \left[-\frac{3\,(5\,Cd^3e^3-3\,(Ccd^2+2\,Bd^3)e^2f-(Cc^2d-4\,Bcd^2-8\,Ad^3)ef^2-(Cc^3-2\,Bc^2d+8\,Acd^2)f^3)\sqrt{df}}{}\right.$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 + 2*B*d^3)*e^2*f - (C*c^2*d - 4*B*c*d^
2 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*d + 8*A*c*d^2)*f^3)*sqrt(d*f)*log(8*
d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt
(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*d^3*
f^3*x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2 - 3*(C*c^2*d - 2*
B*c*d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*B*d^3)*f^3)*x)*sq
rt(d*x + c)*sqrt(f*x + e))/(d^3*f^4), 1/48*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 +
2*B*d^3)*e^2*f - (C*c^2*d - 4*B*c*d^2 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*
d + 8*A*c*d^2)*f^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)
*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*
x)) + 2*(8*C*d^3*f^3*x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2
- 3*(C*c^2*d - 2*B*c*d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*
B*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^4)]
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)C}{d^3f}-\frac{5\,Cd^7ef^3+7\,Ccd^6f^4-6\,Bd^7f^4}{d^9f^5}\right)+\frac{3\,(5\,Cd^8e^2f^2+2\,Ccd^7e}{}\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/24*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d
*x + c)*C/(d^3*f) - (5*C*d^7*e*f^3 + 7*C*c*d^6*f^4 - 6*B*d^7*f^4)/(d^9*f^5
)) + 3*(5*C*d^8*e^2*f^2 + 2*C*c*d^7*e*f^3 - 6*B*d^8*e*f^3 + C*c^2*d^6*f^4
- 2*B*c*d^7*f^4 + 8*A*d^8*f^4)/(d^9*f^5)) + 3*(5*C*d^3*e^3 - 3*C*c*d^2*e^2
*f - 6*B*d^3*e^2*f - C*c^2*d*e*f^2 + 4*B*c*d^2*e*f^2 + 8*A*d^3*e*f^2 - C*c
^3*f^3 + 2*B*c^2*d*f^3 - 8*A*c*d^2*f^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) +
 sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^2*f^3))*d/abs(d)
```

Mupad [B] (verification not implemented)

Time = 61.14 (sec) , antiderivative size = 1832, normalized size of antiderivative = 7.45

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
((((c + d*x)^(1/2) - c^(1/2))*(2*A*d^2*e + 2*A*c*d*f))/(f^3*((e + f*x)^(1/
2) - e^(1/2))) + ((2*A*c*f + 2*A*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(f^2*
((e + f*x)^(1/2) - e^(1/2))^3) - (8*A*c^(1/2)*d*e^(1/2)*((c + d*x)^(1/2) -
 c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1
/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c
^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/
2))*((C*c^3*d^3*f^3)/4 - (5*C*d^6*e^3)/4 + (C*c^2*d^4*e*f^2)/4 + (3*C*c*d^
5*e^2*f)/4))/(f^9*((e + f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/
2))^5*((33*C*d^4*e^3)/2 + (19*C*c^3*d*f^3)/2 + (275*C*c^2*d^2*e*f^2)/2 + (
313*C*c*d^3*e^2*f)/2))/(f^7*((e + f*x)^(1/2) - e^(1/2))^5) - (((c + d*x)^(
1/2) - c^(1/2))^7*((19*C*c^3*f^3)/2 + (33*C*d^3*e^3)/2 + (313*C*c*d^2*e^2*
f)/2 + (275*C*c^2*d*e*f^2)/2))/(f^6*((e + f*x)^(1/2) - e^(1/2))^7) - (((c
+ d*x)^(1/2) - c^(1/2))^3*((17*C*c^3*d^2*f^3)/12 - (85*C*d^5*e^3)/12 + (91
*C*c^2*d^3*e*f^2)/4 + (17*C*c*d^4*e^2*f)/4))/(f^8*((e + f*x)^(1/2) - e^(1/
2))^3) + (((c + d*x)^(1/2) - c^(1/2))^11*((C*c^3*f^3)/4 - (5*C*d^3*e^3)/4
+ (3*C*c*d^2*e^2*f)/4 + (C*c^2*d*e*f^2)/4))/(d^2*f^4*((e + f*x)^(1/2) - e^
(1/2))^11) - (((c + d*x)^(1/2) - c^(1/2))^9*((17*C*c^3*f^3)/12 - (85*C*d^3
*e^3)/12 + (17*C*c*d^2*e^2*f)/4 + (91*C*c^2*d*e*f^2)/4))/(d*f^5*((e + f*x)
^(1/2) - e^(1/2))^9) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^8*(32*
C*c^2*f + 96*C*c*d*e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^8) + (c^(1/2)*e...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.63

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$
$$= \frac{24\sqrt{fx+e}\,\sqrt{dx+c}\,a\,d^3f^3+6\sqrt{fx+e}\,\sqrt{dx+c}\,bc\,d^2f^3-18\sqrt{fx+e}\,\sqrt{dx+c}\,b\,d^3e\,f^2+12\sqrt{fx+e}\,\sqrt{c}}{}$$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x)
```

output
```
(24*sqrt(e + f*x)*sqrt(c + d*x)*a*d**3*f**3 + 6*sqrt(e + f*x)*sqrt(c + d*x
)*b*c*d**2*f**3 - 18*sqrt(e + f*x)*sqrt(c + d*x)*b*d**3*e*f**2 + 12*sqrt(e
 + f*x)*sqrt(c + d*x)*b*d**3*f**3*x - 3*sqrt(e + f*x)*sqrt(c + d*x)*c**3*d
*f**3 - 4*sqrt(e + f*x)*sqrt(c + d*x)*c**2*d**2*e*f**2 + 2*sqrt(e + f*x)*s
qrt(c + d*x)*c**2*d**2*f**3*x + 15*sqrt(e + f*x)*sqrt(c + d*x)*c*d**3*e**2
*f - 10*sqrt(e + f*x)*sqrt(c + d*x)*c*d**3*e*f**2*x + 8*sqrt(e + f*x)*sqrt
(c + d*x)*c*d**3*f**3*x**2 + 24*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x)
 + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*a*c*d**2*f**3 - 24*sqrt(f)*sqrt
(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*a
*d**3*e*f**2 - 6*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt
(e + f*x))/sqrt(c*f - d*e))*b*c**2*d*f**3 - 12*sqrt(f)*sqrt(d)*log((sqrt(f
)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*b*c*d**2*e*f**2
+ 18*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/s
qrt(c*f - d*e))*b*d**3*e**2*f + 3*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*
x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*c**4*f**3 + 3*sqrt(f)*sqrt(d)
*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*c**3
*d*e*f**2 + 9*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e
+ f*x))/sqrt(c*f - d*e))*c**2*d**2*e**2*f - 15*sqrt(f)*sqrt(d)*log((sqrt(f
)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*c*d**3*e**3)/(24
*d**3*f**4)
```

3.72 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 292

$$\begin{aligned}\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx &= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\sqrt{e+fx}}{4b^2df^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}\\ &\quad+\frac{\left(2df(4Abdf-aC(3de+cf))+\frac{(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf))}{b}\right)\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{4b^2d^{3/2}f^{5/2}}\\ &\quad-\frac{2(Ab^2-a(bB-aC))\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^3\sqrt{be-af}}\end{aligned}$$

output
```
-1/4*(4*a*C*d*f+b*(-4*B*d*f+C*c*f+3*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^
2/d/f^2+1/2*C*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f+1/4*(2*d*f*(4*A*b*d*f-a*C*
(c*f+3*d*e))+(2*a*d*f-b*c*f+b*d*e)*(4*a*C*d*f+b*(-4*B*d*f+C*c*f+3*C*d*e))/
b)*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^2/d^(3/2)/f^(5/2
)-2*(A*b^2-a*(B*b-C*a))*(-a*d+b*c)^(1/2)*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^
(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^3/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 12.04 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$$

$$= \frac{\frac{8(Ab^2+a(-bB+aC))\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\operatorname{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}} + \frac{4b(bCe-bBf+aCf)\sqrt{e+fx}\left(-\sqrt{f}\sqrt{de-cf}(c+dx)\sqrt{\frac{d(e+fx)}{de-cf}}+(de-cf)\sqrt{c}\right.}{f^{5/2}\sqrt{de-cf}\sqrt{c+dx}\sqrt{\frac{d(e+fx)}{de-cf}}}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

output
```
((8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c
*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e + f
*x]) + (4*b*(b*C*e - b*B*f + a*C*f)*Sqrt[e + f*x]*(-(Sqrt[f]*Sqrt[d*e - c*
f]*(c + d*x)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (d*e - c*f)*Sqrt[c + d*x]*
ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(f^(5/2)*Sqrt[d*e - c*f
]*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (b^2*C*Sqrt[e + f*x]*(S
qrt[f]*Sqrt[c + d*x]*(c*f + d*(e + 2*f*x)) - ((d*e - c*f)^(3/2)*ArcSinh[(S
qrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c*f)]))/
(d*f^(5/2)) - (8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[-(b*c) + a*d]*ArcTanh[(Sq
rt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[
-(b*e) + a*f])/(4*b^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2118, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 2118$$

$$\frac{\int \frac{b\sqrt{c+dx}(4Abdf-aC(3de+cf)-(4aCdf+b(3Cde+cCf-4Bdf))x)}{2(a+bx)\sqrt{e+fx}}dx}{2b^2df}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\big\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}(4Abdf-aC(3de+cf)-(4aCdf+b(3Cde+cCf-4Bdf))x)}{(a+bx)\sqrt{e+fx}}dx}{4bdf}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\big\downarrow$ 171

$$\frac{\frac{\int \frac{2bcf(4Abdf-aC(3de+cf))+a(de+cf)(4aCdf+b(3Cde+cCf-4Bdf))+(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))x}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{bf}-\sqrt{\cdots}}{4bdf}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{2bcf(4Abdf-aC(3de+cf))+a(de+cf)(4aCdf+b(3Cde+cCf-4Bdf))+(2bdf(4Abdf-aC(3de+cf))+(bde-bcf+2adf)(4aCdf+b(3Cde+cCf-4Bdf)))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{2bf}-\sqrt{\cdots}}{4bdf}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\big\downarrow$ 175

$$\frac{\frac{\frac{8df^2(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}+\frac{(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b}\cdots}{2bf}\cdots}{4bdf}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\big\downarrow$ 66

$$\frac{\frac{\frac{8df^2(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}+\frac{2(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c\cdots}}{\sqrt{e\cdots}}}{b}\cdots}{2bf}\cdots}{4bdf}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$\big\downarrow$ 104

$$\frac{\frac{\frac{16df^2(bc-ad)\left(Ab^2-a(bB-aC)\right)\int \frac{1}{-bc+ad+\frac{(be-af)(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}+\frac{2(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))\int \frac{1}{d-\dots}}{b}}{2bf}}{4bdf}\ \dots$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))}{b\sqrt{d}\sqrt{f}}-\frac{16df^2\sqrt{bc-ad}\left(Ab^2-a(bB-aC)\right)\text{arctanh}\left(\frac{\sqrt{c+d\dots}}{\sqrt{e+f\dots}}\right)}{b\sqrt{be-af}}}{2bf}}{4bdf}\ \dots$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

output
```
(C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*d*f) + (-(((4*a*C*d*f + b*(3*C*d*e
+ c*C*f - 4*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*f)) + ((2*(2*b*d*f*(4*
A*b*d*f - a*C*(3*d*e + c*f)) + (b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(3
*C*d*e + c*C*f - 4*B*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
e + f*x])])/(b*Sqrt[d]*Sqrt[f]) - (16*(A*b^2 - a*(b*B - a*C))*d*Sqrt[b*c -
 a*d]*f^2*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e
+ f*x])])/(b*Sqrt[b*e - a*f]))/(2*b*f))/(4*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1821 vs. $2(254) = 508$.

Time = 0.92 (sec) , antiderivative size = 1822, normalized size of antiderivative = 6.24

method	result	size
default	Expression too large to display	1822

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/8*(8*A*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*(
(f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^3*d^2*f^2+8*A*l
n((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2)*a*
b^2*d^2*f^2-8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a
))*(d*f)^(1/2)*b^3*c*d*f^2-8*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*a*b^2*d^2*f^2+4*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/
2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^3
*c*d*f^2-4*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x
+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^3*d^2*e*f-8
*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*(d*f)^(1/2
)*a^2*b*d^2*f^2+8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/
2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b
*x+a))*(d*f)^(1/2)*a*b^2*c*d*f^2+4*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2)^(1/2)*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^3*d*f*x+8*C*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b*d^2*f^2-4*C*((a^2*d*f-a*b*c*f-a*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)\,\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(b*x+a)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}\,(C\,x^2+Bx+A)}{(bx+a)\,\sqrt{fx+e}}dx$$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)/(f*x+e)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)/(f*x+e)^(1/2),x)
```

3.73 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^2\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 364

$$\begin{aligned}
&\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx \\
&= \frac{(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} \\
&\quad - \frac{(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} \\
&\quad - \frac{(4aCdf+b(Cde-cCf-2Bdf))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{d}f^{3/2}} \\
&\quad + \frac{(4a^3Cdf-b^3(2Bce+Ade-Acf)+ab^2(4cCe+3Bde+Bcf)-a^2b(5Cde+3cCf+2Bdf))\,\text{arctan}}{b^3\sqrt{bc-ad}(be-af)^{3/2}}
\end{aligned}$$

output
```
(2*a^2*C*d*f+b^2*(A*d*f+C*c*e)-a*b*(B*d*f+C*c*f+C*d*e))*(d*x+c)^(1/2)*(f*x
+e)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)-(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f
*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)-(4*a*C*d*f+b*(-2*B*d*f-C*c*f+C
*d*e))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^3/d^(1/2)/f^
(3/2)+(4*a^3*C*d*f-b^3*(-A*c*f+A*d*e+2*B*c*e)+a*b^2*(B*c*f+3*B*d*e+4*C*c*e
)-a^2*b*(2*B*d*f+3*C*c*f+5*C*d*e))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/
(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^3/(-a*d+b*c)^(1/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 11.35 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$$

$$= \frac{-\frac{2b(Ab^2+a(-bB+aC))\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)} + \frac{4(bB-2aC)\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\operatorname{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}} + \frac{2bC\sqrt{e+fx}\left(\sqrt{f}\sqrt{c+dx}-\frac{\sqrt{de-cf}\operatorname{arcs}}{\sqrt{}}\right.}{f^{3/2}}}{2}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

output
```
((-2*b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f
)*(a + b*x)) + (4*(b*B - 2*a*C)*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e -
c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e +
f*x]) + (2*b*C*Sqrt[e + f*x]*(Sqrt[f]*Sqrt[c + d*x] - (Sqrt[d*e - c*f]*Arc
Sinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c
*f)]))/f^(3/2) - (4*(b*B - 2*a*C)*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e)
+ a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])]])/Sqrt[-(b*e) + a
*f] + (2*b*(A*b^2 + a*(-(b*B) + a*C))*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e)
+ a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])]])/(Sqrt[-(b*c) +
a*d]*(-(b*e) + a*f)^(3/2)))/(2*b^3)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2116, 27, 171, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 2116$$

$$-\frac{\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(2cCe+3Bde+Bcf-2Adf)a+b^2(2Bce+Ade-Acf)+2b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x\right)}{2b(a+bx)\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(2cCe+3Bde+Bcf-2Adf)a+b^2(2Bce+Ade-Acf)+2b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x\right)}{(a+bx)\sqrt{e+fx}}dx}{2b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 171

$$\frac{\frac{\int \frac{(bc-ad)\left(2Cf(de+cf)a^2-b(Bf(de+cf)+Ce(de+3cf))a+b^2f(2Bce+Ade-Acf)-(be-af)(4aCdf+b(Cde-cCf-2Bdf))x\right)}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{bf}+\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(\frac{2a^2Cdf}{b}\right.}{}}{2b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{(bc-ad)\int \frac{2Cf(de+cf)a^2-b(Bf(de+cf)+Ce(de+3cf))a+b^2f(2Bce+Ade-Acf)-(be-af)(4aCdf+b(Cde-cCf-2Bdf))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{bf}+\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(\frac{2a^2Cdf}{b}-\right.}{}}{2b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 175

$$\frac{\frac{(bc-ad)\left(-\frac{f\left(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf+Ade+2Bce)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{(be-af)(4aCdf+b(-2Bdf-cC}{b}\right.}{bf}}{2b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\downarrow$ 66

$$\frac{\frac{(bc-ad)\left(-\frac{f\left(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf+Ade+2Bce)\right)\int\frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{2(be-af)(4aCdf+b(-2Bdf-c}{}}{bf}}{2b(bc-ad)(be-af)}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\Big\downarrow$ 104

$$\frac{\frac{(bc-ad)\left(-\frac{2f\left(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf+Ade+2Bce)\right)\int\frac{1}{-bc+ad+\frac{(be-af)(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}-\frac{2(be-af)(4aCdf+}{}}{bf}}{2b(bc-ad)(be-af)}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\Big\downarrow$ 221

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(\frac{2a^2Cdf}{b}-a(Bdf+cCf+Cde)+b(Adf+cCe)\right)}{f}+\frac{(bc-ad)\left(\frac{2f\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(4a^3Cdf-a^2b(2Bdf+3cCf+5Cde)+ab^2(\right.}{b\sqrt{bc-ad}\sqrt{be-af}}\right.}{}}{2b(bc-ad)(be-af)}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

output

```
-(((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*(
b*e - a*f)*(a + b*x))) + ((2*((2*a^2*C*d*f)/b + b*(c*C*e + A*d*f) - a*(C*d
*e + c*C*f + B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/f + ((b*c - a*d)*((-2*(b
*e - a*f)*(4*a*C*d*f + b*(C*d*e - c*C*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[
c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) + (2*f*(4*a^3*C*d*
f - b^3*(2*B*c*e + A*d*e - A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + B*c*f) - a^
2*b*(5*C*d*e + 3*c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])
/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqrt[b*e - a*f])))/(
b*f))/(2*b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3669 vs. $2(332) = 664$.

Time = 0.98 (sec) , antiderivative size = 3670, normalized size of antiderivative = 10.08

method	result	size
default	Expression too large to display	3670

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((
f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*
f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*e*f*(d*f)^(1/2)-3*C*ln(1/2*(2*d*f*x+2*((
f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*b^2*d*e*f*((a^
2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*
x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*c*e*f*((a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-5*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e
)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d
*e+2*b*c*e)/(b*x+a))*a^3*b*d*e*f*(d*f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*
d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*e*f*(d*f)^(1/2)+A*ln((-2*a*d
*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f^2*(d*f)^(1/2)
-2*C*a*b^3*f*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((f*x+e)*(d*x
+c))^(1/2)*(d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c)
)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*
e)/(b*x+a))*b^4*d*e*f*x*(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b^3*d*f^2*x*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^4*d*e*f*x*((a^2*d*f-a*b*c*f-a*b*d*e...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^2\,\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**2/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**2*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1354 vs. $2(331) = 662$.

Time = 1.23 (sec) , antiderivative size = 1354, normalized size of antiderivative = 3.72

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
(4*sqrt(d*f)*C*a*b^2*c*d^2*e - 2*sqrt(d*f)*B*b^3*c*d^2*e - 5*sqrt(d*f)*C*a
^2*b*d^3*e + 3*sqrt(d*f)*B*a*b^2*d^3*e - sqrt(d*f)*A*b^3*d^3*e - 3*sqrt(d*
f)*C*a^2*b*c*d^2*f + sqrt(d*f)*B*a*b^2*c*d^2*f + sqrt(d*f)*A*b^3*c*d^2*f +
 4*sqrt(d*f)*C*a^3*d^3*f - 2*sqrt(d*f)*B*a^2*b*d^3*f)*arctan(-1/2*(b*d^2*e
 + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d
^2*f^2)*d))/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*
(b^4*e*abs(d) - a*b^3*f*abs(d))*d) - 2*(sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d
*f)*B*a*b^2*d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*
e*f + 2*sqrt(d*f)*B*a*b^2*c*d^4*e*f - 2*sqrt(d*f)*A*b^3*c*d^4*e*f + sqrt(d
*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*
c^2*d^3*f^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*
d*f - c*d*f))^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^3*d^3*e - sqrt(d*f
)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*
b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*
f - c*d*f))^2*B*a*b^2*c*d^2*f - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^3*d^3*f - 2*s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^2\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}\,(C\,x^2+Bx+A)}{(bx+a)^2\,\sqrt{fx+e}}dx$$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2/(f*x+e)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^2/(f*x+e)^(1/2),x)
```

3.74 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 446

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$$
$$= \frac{(4a^3Cdf - a^2bC(7de+5cf) - b^3(4Bce - Ade - 3Acf) + ab^2(8cCe + 3Bde + Bcf - 4Adf))\sqrt{c+dx}\sqrt{}}{4b^2(bc-ad)(be-af)^2(a+bx)}$$
$$- \frac{(Ab^2 - a(bB-aC))(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{2C\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{f}}$$
$$+ \frac{\left(\frac{8aCd(bc-ad)(be-af)^2}{b} - 4C(bc-ad)(be-af)(2bce - a(de+cf)) - b(de-cf)\left(a^2C(3de+cf) - ab(4c\right.\right.}{4b^2(bc-ad)^{3/2}(be-af}$$

output
```
1/4*(4*a^3*C*d*f-a^2*b*C*(5*c*f+7*d*e)-b^3*(-3*A*c*f-A*d*e+4*B*c*e)+a*b^2*
(-4*A*d*f+B*c*f+3*B*d*e+8*C*c*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*
c)/(-a*f+b*e)^2/(b*x+a)-1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2
)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^2+2*C*d^(1/2)*arctanh(f^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/(f*x+e)^(1/2))/b^3/f^(1/2)+1/4*(8*a*C*d*(-a*d+b*c)*(-a*f+b*e)^
2/b-4*C*(-a*d+b*c)*(-a*f+b*e)*(2*b*c*e-a*(c*f+d*e))-b*(-c*f+d*e)*(a^2*C*(c
*f+3*d*e)-a*b*(-4*A*d*f+B*c*f+3*B*d*e+4*C*c*e)+b^2*(4*B*c*e-A*(3*c*f+d*e))
))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/
b^2/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 13.11 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx =$$

$$-\frac{\frac{4b(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)}+\frac{2b^2(Ab^2+a(-bB+aC))(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^2}-\frac{8C\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\text{arcsinh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}}+\frac{8C\sqrt{-bc+}}{}}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/4*((4*b*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)*(a + b*
x)) + (2*b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b
*c - a*d)*(b*e - a*f)*(a + b*x)^2) - (8*C*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x
))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]
*Sqrt[e + f*x]) + (8*C*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt
[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])]])/Sqrt[-(b*e) + a*f] - (4*b*
(b*B - 2*a*C)*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(S
qrt[-(b*c) + a*d]*Sqrt[e + f*x])]])/(Sqrt[-(b*c) + a*d]*(-(b*e) + a*f)^(3/2
)) + (b*(A*b^2 + a*(-(b*B) + a*C))*(b*d*e + 3*b*c*f - 4*a*d*f)*(Sqrt[-(b*c
) + a*d]*Sqrt[-(b*e) + a*f]*Sqrt[c + d*x]*Sqrt[e + f*x] - (d*e - c*f)*(a +
 b*x)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[
e + f*x])]))/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(5/2)*(a + b*x)))/b^3
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2116, 27, 166, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(4cCe+3Bde+Bcf-4Adf)a+b^2(4Bce-Ade-3Acf)+4C(bc-ad)(be-af)x\right)}{2b(a+bx)^2\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(4cCe+3Bde+Bcf-4Adf)a+b^2(4Bce-A(de+3cf))+4C(bc-ad)(be-af)x\right)}{(a+bx)^2\sqrt{e+fx}}dx}{4b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\Big\downarrow$ 166

$$\int -\frac{4Cdf(de+cf)a^3-bC\left(7d^2e^2+14cdfe+3c^2f^2\right)a^2+b^2\left(f(8Ce-Bf)c^2+2d\left(8Ce^2-Bfe+2Af^2\right)c+d^2e(3Be-4Af)\right)a+b^3\left(-\left(\left(8Ce^2-4Bfe+3Af^2\right)c^2\right)-2de(2B\right.}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}$$

$$\overline{\qquad b(be-af) \qquad}$$

$$4b(bc-a$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\int \frac{4Cdf(de+cf)a^3-bC\left(7d^2e^2+14cdfe+3\right.}{}$$

$$4b(bc-a$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\Big\downarrow$ 175

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce)\right)}{b(a+bx)(be-af)}-\frac{-\frac{\left(8a^4Cd^2f^2-4a^3bCdf(3cf+5de)+3a^2b\right.}{}}{}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\Big\downarrow$ 66

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{b(a+bx)(be-af)} - \frac{-\frac{(8a^4Cd^2f^2-4a^3bCdf(3cf+5de)+3a^2b}{}}{}}{} \frac{(c+dx)^{3/2}\sqrt{e+fx}(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 104

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{b(a+bx)(be-af)} - \frac{-\frac{2(8a^4Cd^2f^2-4a^3bCdf(3cf+5de)+3a^2}{}}{}}{} \frac{(c+dx)^{3/2}\sqrt{e+fx}(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}$$

$\downarrow$ 221

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce))}{b(a+bx)(be-af)} - \frac{2\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)(8a^4Cd^2}{}}{} \frac{(c+dx)^{3/2}\sqrt{e+fx}(Ab^2-a(bB-aC))}{2b(a+bx)^2(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d
)*(b*e - a*f)*(a + b*x)^2) + (((4*a^3*C*d*f - a^2*b*C*(7*d*e + 5*c*f) - b^
3*(4*B*c*e - A*d*e - 3*A*c*f) + a*b^2*(8*c*C*e + 3*B*d*e + B*c*f - 4*A*d*f
))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*e - a*f)*(a + b*x)) - ((-16*C*Sqrt[d
]*(b*c - a*d)*(b*e - a*f)^2*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
e + f*x])])/(b*Sqrt[f]) + (2*(8*a^4*C*d^2*f^2 - 4*a^3*b*C*d*f*(5*d*e + 3*c
*f) + 3*a^2*b^2*C*(5*d^2*e^2 + 10*c*d*e*f + c^2*f^2) - a*b^3*(d^2*e*(3*B*e
 - 4*A*f) + c^2*f*(8*C*e - B*f) + 2*c*d*(12*C*e^2 - B*e*f + 2*A*f^2)) - b^
4*(A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) - c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)))
*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])
/(b*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]))/(2*b*(b*e - a*f)))/(4*b*(b*c - a*d)*
(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 9099 vs. $2(408) = 816$.

Time = 1.01 (sec) , antiderivative size = 9100, normalized size of antiderivative = 20.40

method	result	size
default	Expression too large to display	9100

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^3\,\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**3/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**3*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7922 vs. 2(407) = 814.

Time = 16.99 (sec) , antiderivative size = 7922, normalized size of antiderivative = 17.76

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
-1/4*(8*sqrt(d*f)*C*b^4*c^2*d^2*e^2 - 24*sqrt(d*f)*C*a*b^3*c*d^3*e^2 + 4*s
qrt(d*f)*B*b^4*c*d^3*e^2 + 15*sqrt(d*f)*C*a^2*b^2*d^4*e^2 - 3*sqrt(d*f)*B*
a*b^3*d^4*e^2 - sqrt(d*f)*A*b^4*d^4*e^2 - 8*sqrt(d*f)*C*a*b^3*c^2*d^2*e*f
- 4*sqrt(d*f)*B*b^4*c^2*d^2*e*f + 30*sqrt(d*f)*C*a^2*b^2*c*d^3*e*f + 2*sqr
t(d*f)*B*a*b^3*c*d^3*e*f - 2*sqrt(d*f)*A*b^4*c*d^3*e*f - 20*sqrt(d*f)*C*a^
3*b*d^4*e*f + 4*sqrt(d*f)*A*a*b^3*d^4*e*f + 3*sqrt(d*f)*C*a^2*b^2*c^2*d^2*
f^2 + sqrt(d*f)*B*a*b^3*c^2*d^2*f^2 + 3*sqrt(d*f)*A*b^4*c^2*d^2*f^2 - 12*s
qrt(d*f)*C*a^3*b*c*d^3*f^2 - 4*sqrt(d*f)*A*a*b^3*c*d^3*f^2 + 8*sqrt(d*f)*C
*a^4*d^4*f^2)*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt
(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f +
 a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^6*c*e^2*abs(d) - a*b^5*d
*e^2*abs(d) - 2*a*b^5*c*e*f*abs(d) + 2*a^2*b^4*d*e*f*abs(d) + a^2*b^4*c*f^
2*abs(d) - a^3*b^3*d*f^2*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d
*f^2 - a^2*d^2*f^2)*d) - sqrt(d*f)*C*d*log((sqrt(d*f)*sqrt(d*x + c) - sqrt
(d^2*e + (d*x + c)*d*f - c*d*f))^2)/(b^3*f*abs(d)) + 1/2*(8*sqrt(d*f)*C*a*
b^4*c*d^9*e^5 - 4*sqrt(d*f)*B*b^5*c*d^9*e^5 - 9*sqrt(d*f)*C*a^2*b^3*d^10*e
^5 + 5*sqrt(d*f)*B*a*b^4*d^10*e^5 - sqrt(d*f)*A*b^5*d^10*e^5 - 32*sqrt(d*f
)*C*a*b^4*c^2*d^8*e^4*f + 16*sqrt(d*f)*B*b^5*c^2*d^8*e^4*f + 31*sqrt(d*f)*
C*a^2*b^3*c*d^9*e^4*f - 19*sqrt(d*f)*B*a*b^4*c*d^9*e^4*f + 7*sqrt(d*f)*A*b
^5*c*d^9*e^4*f + 6*sqrt(d*f)*C*a^3*b^2*d^10*e^4*f - 2*sqrt(d*f)*B*a^2*b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^3\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}\,(C\,x^2+Bx+A)}{(bx+a)^3\,\sqrt{fx+e}}dx$$

input

```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3/(f*x+e)^(1/2),x)
```

output

```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^3/(f*x+e)^(1/2),x)
```

3.75 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^4\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 685

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx$$
$$= \frac{(4a^3Cdf - b^3(6Bce - 3Ade - 5Acf) + ab^2(12cCe + 3Bde + Bcf - 8Adf) - a^2b(9Cde + 7cCf - 2Bdf}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$
$$- \frac{(8a^4Cd^2f^2 - 2a^3bdf(13Cde + 7cCf - 2Bdf) - b^4(3Ad^2e^2 - 2cde(3Be - 2Af) - 3c^2(8Ce^2 - 6Bef -}{}$$
$$- \frac{(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$
$$- \frac{(de-cf)\,(b^2(Ad^2e^2 - 2cde(Be-Af) + c^2(8Ce^2 - 6Bef + 5Af^2)) + ab(d^2e(Be-4Af) - c^2f(4Ce}{8(}$$

output
```
1/12*(4*a^3*C*d*f-b^3*(-5*A*c*f-3*A*d*e+6*B*c*e)+a*b^2*(-8*A*d*f+B*c*f+3*B
*d*e+12*C*c*e)-a^2*b*(-2*B*d*f+7*C*c*f+9*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/
2)/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^2-1/24*(8*a^4*C*d^2*f^2-2*a^3*b*d*f
*(-2*B*d*f+7*C*c*f+13*C*d*e)-b^4*(3*A*d^2*e^2-2*c*d*e*(-2*A*f+3*B*e)-3*c^2
*(5*A*f^2-6*B*e*f+8*C*e^2))-a*b^3*(d^2*e*(-10*A*f+3*B*e)+3*c^2*f*(-B*f+4*C
*e)+2*c*d*(13*A*f^2-14*B*e*f+30*C*e^2))-a^2*b^2*(4*d*f*(-2*A*d*f+B*c*f+4*B
*d*e)-C*(3*c^2*f^2+44*c*d*e*f+33*d^2*e^2)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^
2/(-a*d+b*c)^2/(-a*f+b*e)^3/(b*x+a)-1/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*
(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^3-1/8*(-c*f+d*e)*(b^2*(A*d^2
*e^2-2*c*d*e*(-A*f+B*e)+c^2*(5*A*f^2-6*B*e*f+8*C*e^2))+a*b*(d^2*e*(-4*A*f+
B*e)-c^2*f*(-B*f+4*C*e)-2*c*d*(6*A*f^2-7*B*e*f+6*C*e^2))-a^2*(2*d*f*(-4*A*
d*f+B*c*f+3*B*d*e)-C*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)))*arctanh((-a*f+b*e)^(1
/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/(-a*d+b*c)^(5/2)/(-a*f+b
*e)^(7/2)
```

Mathematica [A] (verified)

Time = 15.14 (sec) , antiderivative size = 657, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx =$$

$$-\frac{\frac{12C\sqrt{c+dx}\sqrt{e+fx}}{(be-af)(a+bx)}+\frac{4b\left(Ab^2+a(-bB+aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^3}+\frac{6b(bB-2aC)(c+dx)^{3/2}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)^2}-\frac{12C(-de+cf)\operatorname{arctanh}\left(\frac{\sqrt{-be+\cdots}}{\sqrt{-bc+\cdots}}\right)}{\sqrt{-bc+ad}(-be+af)^{3/2}}\cdots}{\cdots}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

output
```
-1/12*((12*C*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)*(a + b*x)) + (4*b*(
A*b^2 + a*(-(b*B) + a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b*c - a*d)*(b*e
 - a*f)*(a + b*x)^3) + (6*b*(b*B - 2*a*C)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(
(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) - (12*C*(-(d*e) + c*f)*ArcTanh[(Sqrt[
-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(
b*c) + a*d]*(-(b*e) + a*f)^(3/2)) + (3*(b*B - 2*a*C)*(b*d*e + 3*b*c*f - 4*
a*d*f)*(Sqrt[-(b*c) + a*d]*Sqrt[-(b*e) + a*f]*Sqrt[c + d*x]*Sqrt[e + f*x]
- (d*e - c*f)*(a + b*x)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-
(b*c) + a*d]*Sqrt[e + f*x])]))/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(5/2)*
(a + b*x)) - ((A*b^2 + a*(-(b*B) + a*C))*((2*b*(3*b*d*e + 5*b*c*f - 8*a*d*
f)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(a + b*x)^2 + 3*(8*a^2*d^2*f^2 - 4*a*b*d
*f*(d*e + 3*c*f) + b^2*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*((Sqrt[c + d*x]*
Sqrt[e + f*x])/((-(b*e) + a*f)*(a + b*x)) + ((-(d*e) + c*f)*ArcTanh[(Sqrt[
-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(
b*c) + a*d]*(-(b*e) + a*f)^(3/2)))))/(2*(b*c - a*d)^2*(b*e - a*f)^2))/b^2
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2116, 27, 166, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^4\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(6cCe+3Bde+Bcf-6Adf)a+b^2(6Bce-3Ade-5Acf)+2b\left(\frac{2Cdfa^2}{b}-3Cdea-3cCfa+Bdfa+3bcCe-Abdf\right)x\right)}{2b(a+bx)^3\sqrt{e+fx}}}{3(bc-ad)(be-af)}$$
$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(6cCe+3Bde+Bcf-6Adf)a+b^2(6Bce-3Ade-5Acf)+2b\left(\frac{2Cdfa^2}{b}+Bdfa-3C(de+cf)a+b(3cCe-Adf)\right)x\right)}{(a+bx)^3\sqrt{e+fx}}dx}{6b(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 166

$$\frac{\int -\frac{4Cdf(de+cf)a^3+b\left(2Bdf(de+cf)-C\left(9d^2e^2+20cdfe+3c^2f^2\right)\right)a^2+b^2\left(3f(4Ce-Bf)c^2+4d\left(9Ce^2-4Bfe+4Af^2\right)c+d^2e(3Be-8Af)\right)a+b^3\left(-3\left(8Ce^2-6Bfe+\right.\right.}{2(a+bx)}}{2b}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce)\right)}{2b(a+bx)^2(be-af)}-\frac{\int \frac{4Cdf(de+cf)a^3+b\left(2Bd\right.}{}}{}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 168

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce)\right)}{2b(a+bx)^2(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^4Cd^2f^2\right.}{}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce)\right)}{2b(a+bx)^2(be-af)}-\frac{3b^2(de-cf)\left(a^2\left(2df(-4A\right.\right.}{}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\downarrow$ 104

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce))}{2b(a+bx)^2(be-af)}-\frac{3b^2(de-cf)\left(a^2\left(2df(-4A\right.\right.}{}}{}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-2Bdf+7cCf+9Cde)+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce))}{2b(a+bx)^2(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^4Cd^2f^2\right.}{}}{}$$

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

output
```
-1/3*((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d
)*(b*e - a*f)*(a + b*x)^3) + (((4*a^3*C*d*f - b^3*(6*B*c*e - 3*A*d*e - 5*A
*c*f) + a*b^2*(12*c*C*e + 3*B*d*e + B*c*f - 8*A*d*f) - a^2*b*(9*C*d*e + 7*
c*C*f - 2*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(2*b*(b*e - a*f)*(a + b*x)^
2) - (((8*a^4*C*d^2*f^2 - 2*a^3*b*d*f*(13*C*d*e + 7*c*C*f - 2*B*d*f) - b^4
*(3*A*d^2*e^2 - 2*c*d*e*(3*B*e - 2*A*f) - 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f
^2)) - a*b^3*(d^2*e*(3*B*e - 10*A*f) + 3*c^2*f*(4*C*e - B*f) + 2*c*d*(30*C
*e^2 - 14*B*e*f + 13*A*f^2)) - a^2*b^2*(4*d*f*(4*B*d*e + B*c*f - 2*A*d*f)
- C*(33*d^2*e^2 + 44*c*d*e*f + 3*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(
(b*c - a*d)*(b*e - a*f)*(a + b*x)) - (3*b^2*(d*e - c*f)*(a^2*(2*d*f*(3*B*d
*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) - a*b*(d^2*e*
(B*e - 4*A*f) - c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 - f*(7*B*e - 6*A*f)))
 - b^2*(A*d^2*e^2 - 2*c*d*e*(B*e - A*f) + c^2*(8*C*e^2 - f*(6*B*e - 5*A*f)
)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x]
)])/((b*c - a*d)^(3/2)*(b*e - a*f)^(3/2)))/(4*b*(b*e - a*f)))/(6*b*(b*c -
a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15989 vs. 2(653) = 1306.

Time = 1.10 (sec) , antiderivative size = 15990, normalized size of antiderivative = 23.34

method	result	size
default	Expression too large to display	15990

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^4/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**4/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25338 vs. $2(653) = 1306$.

Time = 79.24 (sec) , antiderivative size = 25338, normalized size of antiderivative = 36.99

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
-1/8*(8*sqrt(d*f)*C*b^2*c^2*d^3*e^3 - 12*sqrt(d*f)*C*a*b*c*d^4*e^3 - 2*sqr
t(d*f)*B*b^2*c*d^4*e^3 + 5*sqrt(d*f)*C*a^2*d^5*e^3 + sqrt(d*f)*B*a*b*d^5*e
^3 + sqrt(d*f)*A*b^2*d^5*e^3 - 8*sqrt(d*f)*C*b^2*c^3*d^2*e^2*f + 8*sqrt(d*
f)*C*a*b*c^2*d^3*e^2*f - 4*sqrt(d*f)*B*b^2*c^2*d^3*e^2*f - 3*sqrt(d*f)*C*a
^2*c*d^4*e^2*f + 13*sqrt(d*f)*B*a*b*c*d^4*e^2*f + sqrt(d*f)*A*b^2*c*d^4*e^
2*f - 6*sqrt(d*f)*B*a^2*d^5*e^2*f - 4*sqrt(d*f)*A*a*b*d^5*e^2*f + 4*sqrt(d
*f)*C*a*b*c^3*d^2*e*f^2 + 6*sqrt(d*f)*B*b^2*c^3*d^2*e*f^2 - sqrt(d*f)*C*a^
2*c^2*d^3*e*f^2 - 13*sqrt(d*f)*B*a*b*c^2*d^3*e*f^2 + 3*sqrt(d*f)*A*b^2*c^2
*d^3*e*f^2 + 4*sqrt(d*f)*B*a^2*c*d^4*e*f^2 - 8*sqrt(d*f)*A*a*b*c*d^4*e*f^2
 + 8*sqrt(d*f)*A*a^2*d^5*e*f^2 - sqrt(d*f)*C*a^2*c^3*d^2*f^3 - sqrt(d*f)*B
*a*b*c^3*d^2*f^3 - 5*sqrt(d*f)*A*b^2*c^3*d^2*f^3 + 2*sqrt(d*f)*B*a^2*c^2*d
^3*f^3 + 12*sqrt(d*f)*A*a*b*c^2*d^3*f^3 - 8*sqrt(d*f)*A*a^2*c*d^4*f^3)*arc
tan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(
d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*
b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^5*c^2*e^3*abs(d) - 2*a*b^4*c*d*e^3*abs(d)
 + a^2*b^3*d^2*e^3*abs(d) - 3*a*b^4*c^2*e^2*f*abs(d) + 6*a^2*b^3*c*d*e^2*f
*abs(d) - 3*a^3*b^2*d^2*e^2*f*abs(d) + 3*a^2*b^3*c^2*e*f^2*abs(d) - 6*a^3*
b^2*c*d*e*f^2*abs(d) + 3*a^4*b*d^2*e*f^2*abs(d) - a^3*b^2*c^2*f^3*abs(d) +
 2*a^4*b*c*d*f^3*abs(d) - a^5*d^2*f^3*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*
e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d) - 1/12*(24*sqrt(d*f)*C*b^7*c^2*d^13...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^4),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^4\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}\,(C\,x^2+Bx+A)}{(bx+a)^4\,\sqrt{fx+e}}dx$$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^4/(f*x+e)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^4/(f*x+e)^(1/2),x)
```

3.76 $\int \frac{(a+bx)^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 723

$$\int \frac{(a+bx)^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{(16a^2d^2f^2(3Cde+5cCf-4Bdf)+16abdf(2df(3Bde+5Bcf-4Adf)-C(5d^2e^2+8cdef+11c^2f^2}{}$$

$$+\frac{(48a^2Cd^2f^2-16abdf(5Cde+13cCf-6Bdf)-b^2(8df(5Bde+13Bcf-6Adf)-C(35d^2e^2+90cde}{96d^4f^3}$$

$$+\frac{b(16aCdf-b(7Cde+25cCf-8Bdf))(c+dx)^{5/2}\sqrt{e+fx}}{24d^4f^2}$$

$$+\frac{b^2C(c+dx)^{7/2}\sqrt{e+fx}}{4d^4f}$$

$$+\frac{(16a^2d^2f^2(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-16abdf(C(5d^3e^3+3cd^2e^2f+3c^2}{}$$

output
```
-1/64*(16*a^2*d^2*f^2*(-4*B*d*f+5*C*c*f+3*C*d*e)+16*a*b*d*f*(2*d*f*(-4*A*d
*f+5*B*c*f+3*B*d*e)-C*(11*c^2*f^2+8*c*d*e*f+5*d^2*e^2))+b^2*(C*(93*c^3*f^3
+73*c^2*d*e*f^2+55*c*d^2*e^2*f+35*d^3*e^3)+8*d*f*(2*A*d*f*(5*c*f+3*d*e)-B*
(11*c^2*f^2+8*c*d*e*f+5*d^2*e^2))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^4/f^4+1/
96*(48*a^2*C*d^2*f^2-16*a*b*d*f*(-6*B*d*f+13*C*c*f+5*C*d*e)-b^2*(8*d*f*(-6
*A*d*f+13*B*c*f+5*B*d*e)-C*(163*c^2*f^2+90*c*d*e*f+35*d^2*e^2)))*(d*x+c)^(
3/2)*(f*x+e)^(1/2)/d^4/f^3+1/24*b*(16*a*C*d*f-b*(-8*B*d*f+25*C*c*f+7*C*d*e
))*(d*x+c)^(5/2)*(f*x+e)^(1/2)/d^4/f^2+1/4*b^2*C*(d*x+c)^(7/2)*(f*x+e)^(1/
2)/d^4/f+1/64*(16*a^2*d^2*f^2*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*
A*d*f-B*(c*f+d*e)))-16*a*b*d*f*(C*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5
*d^3*e^3)+2*d*f*(4*A*d*f*(c*f+d*e)-B*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)))+b^2
*(C*(35*c^4*f^4+20*c^3*d*e*f^3+18*c^2*d^2*e^2*f^2+20*c*d^3*e^3*f+35*d^4*e^
4)+8*d*f*(2*A*d*f*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)-B*(5*c^3*f^3+3*c^2*d*e*f
^2+3*c*d^2*e^2*f+5*d^3*e^3))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+
e)^(1/2))/d^(9/2)/f^(9/2)
```

Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 632, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{d}\sqrt{f}\sqrt{c+dx}\sqrt{e+fx}(48a^2d^2f^2(4Bdf+C(-3de-3cf+2dfx))+16abdf(6df(4Adf+B(-3de-3c}{}$$

input
```
Integrate[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*Sqrt[e + f*x]*(48*a^2*d^2*f^2*(4*B*d*f + C*
(-3*d*e - 3*c*f + 2*d*f*x)) + 16*a*b*d*f*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c
*f + 2*d*f*x)) + C*(15*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*
e*f*x + 8*f^2*x^2))) + b^2*(-(C*(105*c^3*f^3 + 5*c^2*d*f^2*(19*e - 14*f*x)
 + c*d^2*f*(95*e^2 - 68*e*f*x + 56*f^2*x^2) + d^3*(105*e^3 - 70*e^2*f*x +
56*e*f^2*x^2 - 48*f^3*x^3))) + 8*d*f*(6*A*d*f*(-3*d*e - 3*c*f + 2*d*f*x) +
 B*(15*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^
2))))) + 3*(16*a^2*d^2*f^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*
(2*A*d*f - B*(d*e + c*f))) - 16*a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*
c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c
*d*e*f + 3*c^2*f^2))) + b^2*(C*(35*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e
^2*f^2 + 20*c^3*d*e*f^3 + 35*c^4*f^4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*
e*f + 3*c^2*f^2) - B*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^
3))))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(192*d^(9/
2)*f^(9/2))
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 735, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2118, 27, 170, 27, 164, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}} \, dx$$

$$\Big\downarrow \text{ 2118}$$

$$\frac{\int -\frac{b(a+bx)^2(6bcCe+aCde+acCf-8Abdf-(8bBdf-2aCdf-7bC(de+cf))x)}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{4b^2df}+\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-\frac{\int \frac{(a+bx)^2(6bcCe+aCde+acCf-8Abdf-(8bBdf-2aCdf-7bC(de+cf))x)}{\sqrt{c+dx}\sqrt{e+fx}}dx}{8bdf}$$

$$\Big\downarrow \text{ 170}$$

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\frac{\frac{\int \frac{(a+bx)(6adf(6bcCe+aCde+acCf-8Abdf)+(4bce+ade+acf)(8bBdf-2aCdf-7bC(de+cf))+(6bdf(6bcCe+aCde+acCf-8Abdf)-(4adf-5b(de+cf))(8bBdf-2a}{2\sqrt{c+dx}\sqrt{e+fx}}}{3df}}{8bdf}$$

$$\Big\downarrow \ 27$$

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\frac{\frac{\int \frac{(a+bx)(6adf(6bcCe+aCde+acCf-8Abdf)+(4bce+ade+acf)(8bBdf-2aCdf-7bC(de+cf))+(6bdf(6bcCe+aCde+acCf-8Abdf)-(4adf-5b(de+cf))(8bBdf-2a}{\sqrt{c+dx}\sqrt{e+fx}}}{6df}}{8bdf}$$

$$\Big\downarrow \ 164$$

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\sqrt{c+dx}\sqrt{e+fx}\Big(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(cf+de))-16ab^2df\Big(6df(4Adf-3B(cf+de))+C\Big(15c^2f^2+14cdef+15d^2e^2\Big)\Big)+2bdfx(6bdf(acCf+aCde-8Abdf$$

$$\Big\downarrow \ 66$$

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\sqrt{c+dx}\sqrt{e+fx}\Big(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(cf+de))-16ab^2df\Big(6df(4Adf-3B(cf+de))+C\Big(15c^2f^2+14cdef+15d^2e^2\Big)\Big)+2bdfx(6bdf(acCf+aCde-8Abdf$$

$$\Big\downarrow \ 221$$

$$\frac{C(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}{4bdf}-$$

$$\sqrt{c+dx}\sqrt{e+fx}\Big(32a^3Cd^3f^3-8a^2bd^2f^2(16Bdf-11C(cf+de))-16ab^2df\Big(6df(4Adf-3B(cf+de))+C\Big(15c^2f^2+14cdef+15d^2e^2\Big)\Big)+2bdfx(6bdf(acCf+aCde-8Abdf$$

input
```
Int[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(C*(a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b*d*f) - (-1/3*((8*b*B*d*f
- 2*a*C*d*f - 7*b*C*(d*e + c*f))*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/
(d*f) + ((Sqrt[c + d*x]*Sqrt[e + f*x]*(32*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*
(16*B*d*f - 11*C*(d*e + c*f)) - 16*a*b^2*d*f*(C*(15*d^2*e^2 + 14*c*d*e*f +
 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))) + b^3*(5*C*(21*d^3*e^3 +
 19*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 21*c^3*f^3) + 8*d*f*(18*A*d*f*(d*e + c*
f) - B*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))) + 2*b*d*f*(6*b*d*f*(6*b*c*
C*e + a*C*d*e + a*c*C*f - 8*A*b*d*f) - (4*a*d*f - 5*b*(d*e + c*f))*(8*b*B*
d*f - 2*a*C*d*f - 7*b*C*(d*e + c*f)))*x))/(4*d^2*f^2) - (3*b*(16*a^2*d^2*f
^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)
)) - 16*a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3)
 + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))) +
b^2*(C*(35*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3
+ 35*c^4*f^4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) - B*(5*
d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*S
qrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*f^(5/2)))/(6*d*f))/(8*b
*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(685) = 1370$.

Time = 0.76 (sec) , antiderivative size = 2528, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	2528

input
```
int((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/384*(96*C*b^2*d^3*f^3*x^3*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)-320*C*(d*f
)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*a*b*c*d^2*f^3*x-320*C*(d*f)^(1/2)*((f*x+e)
*(d*x+c))^(1/2)*a*b*d^3*e*f^2*x-576*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*
a*b*d^3*e*f^2+224*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^2*c*d^2*e*f^2-72
*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1
/2))*b^2*c^2*d^2*e*f^3-72*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^3*e^2*f^2+128*B*b^2*d^3*f^3*x^2*(d*f
)^(1/2)*((f*x+e)*(d*x+c))^(1/2)+384*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*
a*b*d^3*f^3*x-160*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^2*c*d^2*f^3*x-16
0*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^2*d^3*e*f^2*x+140*C*(d*f)^(1/2)*
((f*x+e)*(d*x+c))^(1/2)*b^2*c^2*d*f^3*x+480*C*(d*f)^(1/2)*((f*x+e)*(d*x+c)
)^(1/2)*a*b*d^3*e^2*f-190*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^2*c^2*d*
e*f^2-190*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^2*c*d^2*e^2*f+240*B*(d*f
)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^2*c^2*d*f^3+240*B*(d*f)^(1/2)*((f*x+e)*(
d*x+c))^(1/2)*b^2*d^3*e^2*f-288*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*a^2*
c*d^2*f^3-288*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*a^2*d^3*e*f^2-384*A*ln
(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
a*b*c*d^3*f^4+192*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*a*b*c*d^3*e*f^3-112*C*b^2*d^3*e*f^2*x^2*(d*f)^(1/2)*
((f*x+e)*(d*x+c))^(1/2)+256*C*a*b*d^3*f^3*x^2*(d*f)^(1/2)*((f*x+e)*(d*x...
```

Fricas [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 1436, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
[1/768*(3*(35*C*b^2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e
^3*f + 6*(3*C*b^2*c^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b
 + A*b^2)*d^4)*e^2*f^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 +
8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35*
C*b^2*c^4 + 128*A*a^2*d^4 - 40*(2*C*a*b + B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a
*b + A*b^2)*c^2*d^2 - 64*(B*a^2 + 2*A*a*b)*c*d^3)*f^4)*sqrt(d*f)*log(8*d^2
*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*b^2*d^
4*f^4*x^3 - 105*C*b^2*d^4*e^3*f - 5*(19*C*b^2*c*d^3 - 24*(2*C*a*b + B*b^2)
*d^4)*e^2*f^2 - (95*C*b^2*c^2*d^2 - 112*(2*C*a*b + B*b^2)*c*d^3 + 144*(C*a
^2 + 2*B*a*b + A*b^2)*d^4)*e*f^3 - 3*(35*C*b^2*c^3*d - 40*(2*C*a*b + B*b^2
)*c^2*d^2 + 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 64*(B*a^2 + 2*A*a*b)*d^4)
*f^4 - 8*(7*C*b^2*d^4*e*f^3 + (7*C*b^2*c*d^3 - 8*(2*C*a*b + B*b^2)*d^4)*f^
4)*x^2 + 2*(35*C*b^2*d^4*e^2*f^2 + 2*(17*C*b^2*c*d^3 - 20*(2*C*a*b + B*b^2
)*d^4)*e*f^3 + (35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*b^2)*c*d^3 + 48*(C*a^2
+ 2*B*a*b + A*b^2)*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5), -1
/384*(3*(35*C*b^2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e^3
*f + 6*(3*C*b^2*c^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b +
 A*b^2)*d^4)*e^2*f^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*
(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35...
```

Sympy [F]

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**2*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 946, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
1/192*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*(2*(d*x + c)*(4*(d*x + c)*(6*(d
*x + c)*C*b^2/(d^5*f) - (7*C*b^2*d^20*e*f^5 + 25*C*b^2*c*d^19*f^6 - 16*C*a
*b*d^20*f^6 - 8*B*b^2*d^20*f^6)/(d^24*f^7)) + (35*C*b^2*d^21*e^2*f^4 + 90*
C*b^2*c*d^20*e*f^5 - 80*C*a*b*d^21*e*f^5 - 40*B*b^2*d^21*e*f^5 + 163*C*b^2
*c^2*d^19*f^6 - 208*C*a*b*c*d^20*f^6 - 104*B*b^2*c*d^20*f^6 + 48*C*a^2*d^2
1*f^6 + 96*B*a*b*d^21*f^6 + 48*A*b^2*d^21*f^6)/(d^24*f^7)) - 3*(35*C*b^2*d
^22*e^3*f^3 + 55*C*b^2*c*d^21*e^2*f^4 - 80*C*a*b*d^22*e^2*f^4 - 40*B*b^2*d
^22*e^2*f^4 + 73*C*b^2*c^2*d^20*e*f^5 - 128*C*a*b*c*d^21*e*f^5 - 64*B*b^2*
c*d^21*e*f^5 + 48*C*a^2*d^22*e*f^5 + 96*B*a*b*d^22*e*f^5 + 48*A*b^2*d^22*e
*f^5 + 93*C*b^2*c^3*d^19*f^6 - 176*C*a*b*c^2*d^20*f^6 - 88*B*b^2*c^2*d^20*
f^6 + 80*C*a^2*c*d^21*f^6 + 160*B*a*b*c*d^21*f^6 + 80*A*b^2*c*d^21*f^6 - 6
4*B*a^2*d^22*f^6 - 128*A*a*b*d^22*f^6)/(d^24*f^7))*sqrt(d*x + c) - 3*(35*C
*b^2*d^4*e^4 + 20*C*b^2*c*d^3*e^3*f - 80*C*a*b*d^4*e^3*f - 40*B*b^2*d^4*e^
3*f + 18*C*b^2*c^2*d^2*e^2*f^2 - 48*C*a*b*c*d^3*e^2*f^2 - 24*B*b^2*c*d^3*e
^2*f^2 + 48*C*a^2*d^4*e^2*f^2 + 96*B*a*b*d^4*e^2*f^2 + 48*A*b^2*d^4*e^2*f^
2 + 20*C*b^2*c^3*d*e*f^3 - 48*C*a*b*c^2*d^2*e*f^3 - 24*B*b^2*c^2*d^2*e*f^3
 + 32*C*a^2*c*d^3*e*f^3 + 64*B*a*b*c*d^3*e*f^3 + 32*A*b^2*c*d^3*e*f^3 - 64
*B*a^2*d^4*e*f^3 - 128*A*a*b*d^4*e*f^3 + 35*C*b^2*c^4*f^4 - 80*C*a*b*c^3*d
*f^4 - 40*B*b^2*c^3*d*f^4 + 48*C*a^2*c^2*d^2*f^4 + 96*B*a*b*c^2*d^2*f^4 +
48*A*b^2*c^2*d^2*f^4 - 64*B*a^2*c*d^3*f^4 - 128*A*a*b*c*d^3*f^4 + 128*A...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int(((a + b*x)^2*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+bx)^2 (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(bx+a)^2 (C\,x^2+Bx+A)}{\sqrt{dx+c}\,\sqrt{fx+e}}dx$$

input
```
int((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
int((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

3.77 $\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 365

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(2bdf(3Bde+5Bcf-4Adf)+2adf(3Cde+5cCf-4Bdf)-bC(5d^2e^2+8cdef+11c^2f^2))\sqrt{c+d}}{8d^3f^3} + \frac{(6aCdf-b(5Cde+13cCf-6Bdf))(c+dx)^{3/2}\sqrt{e+fx}}{12d^3f^2} + \frac{bC(c+dx)^{5/2}\sqrt{e+fx}}{3d^3f} + \frac{(2adf(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))-b(C(5d^3e^3+3cd^2e^2f+3c^2def^2+5c}{8d^{7/2}f^{7/2}}$$

output

```
-1/8*(2*b*d*f*(-4*A*d*f+5*B*c*f+3*B*d*e)+2*a*d*f*(-4*B*d*f+5*C*c*f+3*C*d*e
)-b*C*(11*c^2*f^2+8*c*d*e*f+5*d^2*e^2))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^3/f^
3+1/12*(6*a*C*d*f-b*(-6*B*d*f+13*C*c*f+5*C*d*e))*(d*x+c)^(3/2)*(f*x+e)^(1/
2)/d^3/f^2+1/3*b*C*(d*x+c)^(5/2)*(f*x+e)^(1/2)/d^3/f+1/8*(2*a*d*f*(C*(3*c^
2*f^2+2*c*d*e*f+3*d^2*e^2)+4*d*f*(2*A*d*f-B*(c*f+d*e)))-b*(C*(5*c^3*f^3+3*
c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3)+2*d*f*(4*A*d*f*(c*f+d*e)-B*(3*c^2*f^2
+2*c*d*e*f+3*d^2*e^2))))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/
2))/d^(7/2)/f^(7/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\sqrt{c+dx}\sqrt{e+fx}(6adf(4Bdf+C(-3de-3cf+2dfx))+b(6df(4Adf+B(-3de-3cf+2dfx))+C(}{24d^3f^3}$$
$$-\frac{(-2adf(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))+b(C(5d^3e^3+3cd^2e^2f+3c^2def^2+}{8d^{7/2}f^{7/2}}$$

input
```
Integrate[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(6*a*d*f*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f
*x)) + b*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c*f + 2*d*f*x)) + C*(15*c^2*f^2 +
 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2)))))/(24*d^3*f
^3) - ((-2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f -
 B*(d*e + c*f))) + b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3
*f^3) + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2)
)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(8*d^(7/2)*f
^(7/2))
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2118, 27, 164, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{\int -\frac{b(a+bx)(4bcCe+aCde+acCf-6Abdf-(6bBdf-2aCdf-5bC(de+cf))x)}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{3b^2df}+\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$

$$\downarrow 27$$

$$\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\int \frac{(a+bx)(4bcCe+aCde+acCf-6Abdf-(6bBdf-2aCdf-5bC(de+cf))x)}{\sqrt{c+dx}\sqrt{e+fx}}dx}{6bdf}$$

$$\downarrow 164$$

$$\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))-(b^2(6df(4Adf-3B(cf+de))+C(15c^2f^2+14cde}{4d^2f^2}$$

$$\downarrow 66$$

$$\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))-(b^2(6df(4Adf-3B(cf+de))+C(15c^2f^2+14cde}{4d^2f^2}$$

$$\downarrow 221$$

$$\frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))-(b^2(6df(4Adf-3B(cf+de))+C(15c^2f^2+14cde}{4d^2f^2}$$

input
```
Int[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - ((Sqrt[c + d*x]*Sq
rt[e + f*x]*(8*a^2*C*d^2*f^2 - 6*a*b*d*f*(4*B*d*f - 3*C*(d*e + c*f)) - b^2
*(C*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c
*f))) - 2*b*d*f*(6*b*B*d*f - 2*a*C*d*f - 5*b*C*(d*e + c*f))*x))/(4*d^2*f^2
) - (3*b*(2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f
- B*(d*e + c*f))) - b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^
3*f^3) + 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2
))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*
f^(5/2)))/(6*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1198 vs. 2(333) = 666.

Time = 0.65 (sec) , antiderivative size = 1199, normalized size of antiderivative = 3.28

method	result	size
default	Expression too large to display	1199

input
```
int((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/48*(-9*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*b*c*d^2*e^2*f+18*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e^2*f+18*C*ln(1/2*(2*d*f*x+2*((f*x
+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d*f^3+18*C*ln(1
/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*
d^3*e^2*f+48*A*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b*d^2*f^2+48*B*(d*f)^(1
/2)*((f*x+e)*(d*x+c))^(1/2)*a*d^2*f^2+30*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(
1/2)*b*c^2*f^2+30*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b*d^2*e^2-20*C*((f
*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)*b*c*d*f^2*x-20*C*((f*x+e)*(d*x+c))^(1/2)*
(d*f)^(1/2)*b*d^2*e*f*x+28*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b*c*d*e*f
+12*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*a*c*d^2*e*f^2-9*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^
(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d*e*f^2+16*C*b*d^2*f^2*x^2*((f*x+e)*(d*x
+c))^(1/2)*(d*f)^(1/2)+48*A*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^3*f^3-15*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d
*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*f^3-15*C*ln(1/2*(2*d*
f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^3*e^3-
36*B*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b*c*d*f^2+24*B*((f*x+e)*(d*x+c))^
(1/2)*(d*f)^(1/2)*b*d^2*f^2*x+12*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^2*e*f^2+24*C*((f*x+e)*(d*x+c...
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 720, normalized size of antiderivative = 1.97

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
[-1/96*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*e^2*f + (3*C*
b*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^2 + (5*C*b*c^3 - 16
*A*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3)*sqrt(d*f)*log(8
*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqr
t(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*b*d
^3*f^3*x^2 + 15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2
+ 3*(5*C*b*c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b
*d^3*e*f^2 + (5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(
f*x + e))/(d^4*f^4), 1/48*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)
*d^3)*e^2*f + (3*C*b*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^
2 + (5*C*b*c^3 - 16*A*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f
^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*s
qrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(8*C*b*d
^3*f^3*x^2 + 15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2
+ 3*(5*C*b*c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b
*d^3*e*f^2 + (5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(
f*x + e))/(d^4*f^4)]
```

Sympy [F]

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)Cb}{d^4f}-\frac{5\,Cbd^{12}ef^3+13\,Cbcd^{11}f^4-6\,Cad^{12}f^4-6\,Bbd^{12}f^4}{d^{15}f^5}\right)+\frac{3\,(\cdots}{}\right.\right.}{}$$

input
```
integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
1/24*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d
*x + c)*C*b/(d^4*f) - (5*C*b*d^12*e*f^3 + 13*C*b*c*d^11*f^4 - 6*C*a*d^12*f
^4 - 6*B*b*d^12*f^4)/(d^15*f^5)) + 3*(5*C*b*d^13*e^2*f^2 + 8*C*b*c*d^12*e*
f^3 - 6*C*a*d^13*e*f^3 - 6*B*b*d^13*e*f^3 + 11*C*b*c^2*d^11*f^4 - 10*C*a*c
*d^12*f^4 - 10*B*b*c*d^12*f^4 + 8*B*a*d^13*f^4 + 8*A*b*d^13*f^4)/(d^15*f^5
)) + 3*(5*C*b*d^3*e^3 + 3*C*b*c*d^2*e^2*f - 6*C*a*d^3*e^2*f - 6*B*b*d^3*e^
2*f + 3*C*b*c^2*d*e*f^2 - 4*C*a*c*d^2*e*f^2 - 4*B*b*c*d^2*e*f^2 + 8*B*a*d^
3*e*f^2 + 8*A*b*d^3*e*f^2 + 5*C*b*c^3*f^3 - 6*C*a*c^2*d*f^3 - 6*B*b*c^2*d*
f^3 + 8*B*a*c*d^2*f^3 + 8*A*b*c*d^2*f^3 - 16*A*a*d^3*f^3)*log(abs(-sqrt(d*
f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(sqrt(d*f)*d^3*f^
3))*d/abs(d)
```

Mupad [B] (verification not implemented)

Time = 70.69 (sec) , antiderivative size = 2621, normalized size of antiderivative = 7.18

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int(((a + b*x)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
((((c + d*x)^(1/2) - c^(1/2))*(2*A*b*c*f + 2*A*b*d*e))/(f^3*((e + f*x)^(1/
2) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1/2))^3*(2*A*b*c*f + 2*A*b*d*e))/(
d*f^2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*A*b*c^(1/2)*e^(1/2)*((c + d*x)^(
1/2) - c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2)
- c^(1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/
2) - c^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) -
 c^(1/2))*((3*C*a*d^3*e^2)/2 + (3*C*a*c^2*d*f^2)/2 + C*a*c*d^2*e*f))/(f^6*
((e + f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*a*c^2
*f^2)/2 + (11*C*a*d^2*e^2)/2 + 25*C*a*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^
(1/2))^3) + (((c + d*x)^(1/2) - c^(1/2))^7*((3*C*a*c^2*f^2)/2 + (3*C*a*d^2
*e^2)/2 + C*a*c*d*e*f))/(d^2*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d
*x)^(1/2) - c^(1/2))^5*((11*C*a*c^2*f^2)/2 + (11*C*a*d^2*e^2)/2 + 25*C*a*c
*d*e*f))/(d*f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (c^(1/2)*e^(1/2)*((c + d*
x)^(1/2) - c^(1/2))^4*(32*C*a*c*f + 32*C*a*d*e))/(f^4*((e + f*x)^(1/2) - e
^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f*x)^(1/2) - e^(1/2))^8 +
 d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/
2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(
1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e
^(1/2))^4)) - ((((c + d*x)^(1/2) - c^(1/2))^3*((85*C*b*d^4*e^3)/12 + (85*C
*b*c^3*d*f^3)/12 + (17*C*b*c*d^3*e^2*f)/4 + (17*C*b*c^2*d^2*e*f^2)/4))/...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 932, normalized size of antiderivative = 2.55

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output

```
(48*sqrt(e + f*x)*sqrt(c + d*x)*a*b*d**3*f**3 - 18*sqrt(e + f*x)*sqrt(c +
d*x)*a*c**2*d**2*f**3 - 18*sqrt(e + f*x)*sqrt(c + d*x)*a*c*d**3*e*f**2 + 1
2*sqrt(e + f*x)*sqrt(c + d*x)*a*c*d**3*f**3*x - 18*sqrt(e + f*x)*sqrt(c +
d*x)*b**2*c*d**2*f**3 - 18*sqrt(e + f*x)*sqrt(c + d*x)*b**2*d**3*e*f**2 +
12*sqrt(e + f*x)*sqrt(c + d*x)*b**2*d**3*f**3*x + 15*sqrt(e + f*x)*sqrt(c
+ d*x)*b*c**3*d*f**3 + 14*sqrt(e + f*x)*sqrt(c + d*x)*b*c**2*d**2*e*f**2 -
 10*sqrt(e + f*x)*sqrt(c + d*x)*b*c**2*d**2*f**3*x + 15*sqrt(e + f*x)*sqrt
(c + d*x)*b*c*d**3*e**2*f - 10*sqrt(e + f*x)*sqrt(c + d*x)*b*c*d**3*e*f**2
*x + 8*sqrt(e + f*x)*sqrt(c + d*x)*b*c*d**3*f**3*x**2 + 48*sqrt(f)*sqrt(d)
*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*a**2
*d**3*f**3 - 48*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(
e + f*x))/sqrt(c*f - d*e))*a*b*c*d**2*f**3 - 48*sqrt(f)*sqrt(d)*log((sqrt(
f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*a*b*d**3*e*f**2
 + 18*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/
sqrt(c*f - d*e))*a*c**3*d*f**3 + 12*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c +
d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*a*c**2*d**2*e*f**2 + 18*sqr
t(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f
- d*e))*a*c*d**3*e**2*f + 18*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) +
sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*b**2*c**2*d*f**3 + 12*sqrt(f)*sqrt
(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e)...
```

3.78 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 164

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{4d^{5/2}f^{5/2}}$$

output
```
-1/4*(-4*B*d*f+5*C*c*f+3*C*d*e)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^2/f^2+1/2*C*
(d*x+c)^(3/2)*(f*x+e)^(1/2)/d^2/f+1/4*(C*(3*c^2*f^2+2*c*d*e*f+3*d^2*e^2)+4
*d*f*(2*A*d*f-B*(c*f+d*e)))*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^
(1/2))/d^(5/2)/f^(5/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\sqrt{c+dx}\sqrt{e+fx}(4Bdf+C(-3de-3cf+2dfx))}{4d^2f^2} + \frac{(C(3d^2e^2+2cdef+3c^2f^2)+4df(2Adf-B(de+cf)))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{4d^{5/2}f^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[e + f*x]*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f*x)))/(4*
d^2*f^2) + ((C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d
*e + c*f)))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(4*d
^(5/2)*f^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1194, 27, 90, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{1194}$$

$$\frac{\int -\frac{Cfc^2+3Cdec-4Ad^2f+d(3Cde+5cCf-4Bdf)x}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{2d^2f} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}$$

$$\Big\downarrow \text{27}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}-\frac{\int \frac{Cfc^2+3Cdec-4Ad^2f+d(3Cde+5cCf-4Bdf)x}{\sqrt{c+dx}\sqrt{e+fx}}dx}{4d^2f}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}-\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cde)}{f}-\frac{(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{2f}}{4d^2f}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}-\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cde)}{f}-\frac{(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{f}}{4d^2f}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}-\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(-4Bdf+5cCf+3Cde)}{f}-\frac{\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(4df(2Adf-B(cf+de))+C(3c^2f^2+2cdef+3d^2e^2))}{\sqrt{d}f^{3/2}}}{4d^2f}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d^2*f) - (((3*C*d*e + 5*c*C*f - 4*B*d
*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/f - ((C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^
2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[e + f*x])])/(Sqrt[d]*f^(3/2)))/(4*d^2*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(138) = 276$.

Time = 0.68 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.59

method	result
default	$\frac{\left(8A\ln\left(\frac{2dfx+2\sqrt{(fx+e)(xd+c)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)d^2f^2-4B\ln\left(\frac{2dfx+2\sqrt{(fx+e)(xd+c)}\,\sqrt{df}+cf+de}{2\sqrt{df}}\right)cd\,f^2-4B\ln\left(\frac{2dfx+2\sqrt{(fx+e)(xd+c)}\,\sqrt{}}{2\sqrt{df}}\right.\right.}{}$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(8*A*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*d^2*f^2-4*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*c*d*f^2-4*B*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^2*e*f+4*C*((f*x+e)*(d*x+c))^(1/2)
*(d*f)^(1/2)*d*f*x+3*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/
2)+c*f+d*e)/(d*f)^(1/2))*c^2*f^2+2*C*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(
1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d*e*f+3*C*ln(1/2*(2*d*f*x+2*((f*x
+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^2*e^2+8*B*(d*f)^(1/
2)*((f*x+e)*(d*x+c))^(1/2)*d*f-6*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*c*f
-6*C*(d*f)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*d*e)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/
(d*f)^(1/2)/f^2/d^2/((f*x+e)*(d*x+c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.32

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$= \left[\frac{(3Cd^2e^2+2(Ccd-2Bd^2)ef+(3Cc^2-4Bcd+8Ad^2)f^2)\sqrt{df}\log\left(8d^2f^2x^2+d^2e^2+6cdef+c^2f^2\right.}{}\right.$$

$$-\frac{(3Cd^2e^2+2(Ccd-2Bd^2)ef+(3Cc^2-4Bcd+8Ad^2)f^2)\sqrt{-df}\arctan\left(\frac{(2dfx+de+cf)\sqrt{-df}\sqrt{dx+c}\sqrt{fx+}}{2(d^2f^2x^2+cdef+(d^2ef+cdf^2)x)}\right.}{8d^3f^3}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*d^2)*e*f + (3*C*c^2 - 4*B*c*d + 8*A*d
^2)*f^2)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(
2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f +
c*d*f^2)*x) + 4*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d - 4*B*d^2)*f^2)*sq
rt(d*x + c)*sqrt(f*x + e))/(d^3*f^3), -1/8*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*
d^2)*e*f + (3*C*c^2 - 4*B*c*d + 8*A*d^2)*f^2)*sqrt(-d*f)*arctan(1/2*(2*d*f
*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*
e*f + (d^2*e*f + c*d*f^2)*x)) - 2*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d
- 4*B*d^2)*f^2)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^3)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= \frac{\left(\sqrt{d^2e+(dx+c)df-cdf}\sqrt{dx+c}\left(\frac{2\,(dx+c)C}{d^3f}-\frac{3\,Cd^6ef+5\,Ccd^5f^2-4\,Bd^6f^2}{d^8f^3}\right)-\frac{(3\,Cd^2e^2+2\,Ccdef-4\,Bd^2ef+3\,Cc^2f^2\cdots}{\cdots}\right.}{4\,|d|}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(d^2*e + (d*x + c)*d*f - c*d*f)*sqrt(d*x + c)*(2*(d*x + c)*C/(d^3
*f) - (3*C*d^6*e*f + 5*C*c*d^5*f^2 - 4*B*d^6*f^2)/(d^8*f^3)) - (3*C*d^2*e^
2 + 2*C*c*d*e*f - 4*B*d^2*e*f + 3*C*c^2*f^2 - 4*B*c*d*f^2 + 8*A*d^2*f^2)*l
og(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(d^2*e + (d*x + c)*d*f - c*d*f)))/(s
qrt(d*f)*d^2*f^2))*d/abs(d)
```

Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.08

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
(((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2)))/(f^3*((e + f*x)^(1/2) -
 e^(1/2))) + ((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(d*f^2*((
e + f*x)^(1/2) - e^(1/2))^3) - (8*B*c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(
1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))
^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/
2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/2))*
((3*C*d^3*e^2)/2 + (3*C*c^2*d*f^2)/2 + C*c*d^2*e*f))/(f^6*((e + f*x)^(1/2)
 - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*c^2*f^2)/2 + (11*C*d^
2*e^2)/2 + 25*C*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3) + (((c + d*x
)^(1/2) - c^(1/2))^7*((3*C*c^2*f^2)/2 + (3*C*d^2*e^2)/2 + C*c*d*e*f))/(d^2
*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) - c^(1/2))^5*((11*
C*c^2*f^2)/2 + (11*C*d^2*e^2)/2 + 25*C*c*d*e*f))/(d*f^4*((e + f*x)^(1/2) -
 e^(1/2))^5) + (c^(1/2)*e^(1/2)*(32*C*c*f + 32*C*d*e)*((c + d*x)^(1/2) - c
^(1/2))^4)/(f^4*((e + f*x)^(1/2) - e^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2
))^8/((e + f*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(
1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(
1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c
^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e^(1/2))^4)) - (4*A*atan((d*((e + f*x)^
(1/2) - e^(1/2)))/((-d*f)^(1/2)*((c + d*x)^(1/2) - c^(1/2)))))/(-d*f)^(1/2
) - (2*B*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x...
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.19

$$\int \frac{A + Bx + Cx^2}{\sqrt{c + dx}\sqrt{e + fx}}\,dx$$
$$= \frac{4\sqrt{fx+e}\,\sqrt{dx+c}\,b\,d^2f^2 - 3\sqrt{fx+e}\,\sqrt{dx+c}\,c^2d\,f^2 - 3\sqrt{fx+e}\,\sqrt{dx+c}\,c\,d^2ef + 2\sqrt{fx+e}\,\sqrt{dx+c}}{}$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
(4*sqrt(e + f*x)*sqrt(c + d*x)*b*d**2*f**2 - 3*sqrt(e + f*x)*sqrt(c + d*x)
*c**2*d*f**2 - 3*sqrt(e + f*x)*sqrt(c + d*x)*c*d**2*e*f + 2*sqrt(e + f*x)*
sqrt(c + d*x)*c*d**2*f**2*x + 8*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x)
 + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*a*d**2*f**2 - 4*sqrt(f)*sqrt(d)
*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*b*c*
d*f**2 - 4*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f
*x))/sqrt(c*f - d*e))*b*d**2*e*f + 3*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c +
 d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*c**3*f**2 + 2*sqrt(f)*sqrt
(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqrt(c*f - d*e))*c
**2*d*e*f + 3*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e
+ f*x))/sqrt(c*f - d*e))*c*d**2*e**2)/(4*d**3*f**3)
```

3.79 $\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 188

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2d^{3/2}f^{3/2}} - \frac{2(Ab^2-a(bB-aC))\,\text{arctanh}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}}$$

output
```
C*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-(2*a*C*d*f+b*(-2*B*d*f+C*c*f+C*d*e))*a
rctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^2/d^(3/2)/f^(3/2)-2*
(A*b^2-a*(B*b-C*a))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2
)/(f*x+e)^(1/2))/b^2/(-a*d+b*c)^(1/2)/(-a*f+b*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{\frac{bC\sqrt{c+dx}\sqrt{e+fx}}{df}+\frac{2(Ab^2+a(-bB+aC))\arctan\left(\frac{\sqrt{bc-ad}\sqrt{e+fx}}{\sqrt{-be+af}\sqrt{c+dx}}\right)}{\sqrt{bc-ad}\sqrt{-be+af}}-\frac{(2aCdf+b(Cde+cCf-2Bdf))\text{arctanh}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)}{d^{3/2}f^{3/2}}}{b^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
((b*C*Sqrt[c + d*x]*Sqrt[e + f*x])/(d*f) + (2*(A*b^2 + a*(-(b*B) + a*C))*A
rcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])
/(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2
*B*d*f))*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(d^(3/2
)*f^(3/2)))/b^2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2118, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \ 2118$$

$$\frac{\int \frac{b(2Abdf-aC(de+cf)-(2aCdf+b(Cde+cCf-2Bdf))x)}{2(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b^2df}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{2Abdf-aC(de+cf)-(2aCdf+b(Cde+cCf-2Bdf))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \color{blue}{175}$$

$$\frac{\frac{2df(Ab^2-a(bB-aC))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{(2aCdf+b(-2Bdf+cCf+Cde))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b}}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \color{blue}{66}$$

$$\frac{\frac{2df(Ab^2-a(bB-aC))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{b}-\frac{2(2aCdf+b(-2Bdf+cCf+Cde))\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \color{blue}{104}$$

$$\frac{\frac{4df(Ab^2-a(bB-aC))\int \frac{1}{-bc+ad+\frac{(be-af)(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}-\frac{2(2aCdf+b(-2Bdf+cCf+Cde))\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b}}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

$$\Big\downarrow \color{blue}{221}$$

$$\frac{-\frac{4df(Ab^2-a(bB-aC))\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b\sqrt{bc-ad}\sqrt{be-af}}-\frac{2\text{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2aCdf+b(-2Bdf+cCf+Cde))}{b\sqrt{d}\sqrt{f}}}{2bdf}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d*f) + ((-2*(2*a*C*d*f + b*(C*d*e + c*C
*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(
b*Sqrt[d]*Sqrt[f]) - (4*(A*b^2 - a*(b*B - a*C))*d*f*ArcTanh[(Sqrt[b*e - a*
f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqr
t[b*e - a*f]))/(2*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(160) = 320$.

Time = 1.06 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.97

method	result
default	$-\frac{\left(2A\sqrt{df}\,\ln\left(\frac{-2adfx+bcfx+bdex+2\sqrt{(fx+e)(xd+c)}\,\sqrt{\frac{a^2df-abcf-abde+ce\,b^2}{b^2}}\,b-acf-ade+2bce}{bx+a}\right)b^2df-2B\sqrt{df}\,\ln\left(\frac{-2adfx+bcfx+bd}{}\right.\right.}{}$

input
```
int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/2*(2*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(
1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/
(b*x+a))*b^2*d*f-2*B*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)
*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*
e+2*b*c*e)/(b*x+a))*a*b*d*f-2*B*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(
1/2))*b^2*d*f+2*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d
*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2
*b*c*e)/(b*x+a))*a^2*d*f+2*C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2
))*a*b*d*f+C*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x
+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c*f+C*((a
^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x
+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*d*e-2*C*(d*f)^(1/2)*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((f*x+e)*(d*x+c))^(1/2)*b^2)*(d*x
+c)^(1/2)*(f*x+e)^(1/2)/((f*x+e)*(d*x+c))^(1/2)/b^3/(d*f)^(1/2)/((a^2*d*f-
a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)/d/f
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 159.40 (sec) , antiderivative size = 96118, normalized size of antiderivative = 511.27

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
(((2*C*c*f + 2*C*d*e)*((c + d*x)^(1/2) - c^(1/2)))/(b*f^3*((e + f*x)^(1/2)
 - e^(1/2))) + ((2*C*c*f + 2*C*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(b*d*f^
2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*C*c^(1/2)*e^(1/2)*((c + d*x)^(1/2) -
 c^(1/2))^2)/(b*f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^
(1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) -
 c^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) + (A*atan((c^(1/2)*d*e^(1/
2)*(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e)^(1/2)*2i - (c*f*((c + d*x)^(1/2
) - c^(1/2))*(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e)^(1/2)*2i)/((e + f*x)^
(1/2) - e^(1/2)) - (d*e*((c + d*x)^(1/2) - c^(1/2))*(b^2*c*e + a^2*d*f - a
*b*c*f - a*b*d*e)^(1/2)*2i)/((e + f*x)^(1/2) - e^(1/2)) + (c^(1/2)*e^(1/2)
*f*((c + d*x)^(1/2) - c^(1/2))^2*(b^2*c*e + a^2*d*f - a*b*c*f - a*b*d*e)^(
1/2)*2i)/((e + f*x)^(1/2) - e^(1/2))^2)/(a*d^2*e + a*c*d*f - 2*b*c*d*e + (
a*c*f^2*((c + d*x)^(1/2) - c^(1/2))^2)/((e + f*x)^(1/2) - e^(1/2))^2 + (a*
d*e*f*((c + d*x)^(1/2) - c^(1/2))^2)/((e + f*x)^(1/2) - e^(1/2))^2 - (2*b*
c*e*f*((c + d*x)^(1/2) - c^(1/2))^2)/((e + f*x)^(1/2) - e^(1/2))^2 + (2*b*
c^(1/2)*d*e^(3/2)*((c + d*x)^(1/2) - c^(1/2)))/((e + f*x)^(1/2) - e^(1/2))
 + (2*b*c^(3/2)*e^(1/2)*f*((c + d*x)^(1/2) - c^(1/2)))/((e + f*x)^(1/2) -
e^(1/2)) - (4*a*c^(1/2)*d*e^(1/2)*f*((c + d*x)^(1/2) - c^(1/2)))/((e + f*x
)^(1/2) - e^(1/2))))*2i)/((a*d - b*c)*(a*f - b*e))^(1/2) - (B*atan(((B*((6
5536*((c + d*x)^(1/2) - c^(1/2))*(32*B^4*a^2*b^2*d^8*e^6 + 6*B^4*a^4*c^...
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2\sqrt{f}\,\sqrt{d}\log\left(\frac{\sqrt{f}\,\sqrt{dx+c}+\sqrt{d}\,\sqrt{fx+e}}{\sqrt{cf-de}}\right)+\left(\int \frac{x^2}{\sqrt{fx+e}\,\sqrt{dx+c}\,a+\sqrt{fx+e}\,\sqrt{dx+c}\,bx}dx\right)cdf}{df}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(f)*sqrt(d)*log((sqrt(f)*sqrt(c + d*x) + sqrt(d)*sqrt(e + f*x))/sqr
t(c*f - d*e)) + int(x**2/(sqrt(e + f*x)*sqrt(c + d*x)*a + sqrt(e + f*x)*sq
rt(c + d*x)*b*x),x)*c*d*f)/(d*f)
```

3.80 $\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 254

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$$
$$= -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{2C\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2\sqrt{d}\sqrt{f}}$$
$$+ \frac{(2a^3Cdf-3a^2bC(de+cf)-b^3(2Bce-Ade-Acf)+ab^2(4cCe+Bde+Bcf-2Adf))\operatorname{arctanh}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{b^2(bc-ad)^{3/2}(be-af)^{3/2}}$$

output
```
-(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(
b*x+a)+2*C*arctanh(f^(1/2)*(d*x+c)^(1/2)/d^(1/2)/(f*x+e)^(1/2))/b^2/d^(1/2
)/f^(1/2)+(2*a^3*C*d*f-3*a^2*b*C*(c*f+d*e)-b^3*(-A*c*f-A*d*e+2*B*c*e)+a*b^
2*(-2*A*d*f+B*c*f+B*d*e+4*C*c*e))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(
-a*d+b*c)^(1/2)/(f*x+e)^(1/2))/b^2/(-a*d+b*c)^(3/2)/(-a*f+b*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\frac{b(Ab^2+a(-bB+aC))\sqrt{c+dx}\sqrt{e+fx}}{(bc-ad)(be-af)(a+bx)} + \frac{(-2a^3Cdf+3a^2bC(de+cf)-ab^2(4cCe+Bde+Bcf-2Adf)+b^3(2Bce-A(de+cf)))\arctan\left(\frac{\sqrt{bc-a}}{\sqrt{-be+}}\right.}{(bc-ad)^{3/2}(-be+af)^{3/2}}}{b^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-(((b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)
*(b*e - a*f)*(a + b*x)) + ((-2*a^3*C*d*f + 3*a^2*b*C*(d*e + c*f) - a*b^2*(
4*c*C*e + B*d*e + B*c*f - 2*A*d*f) + b^3*(2*B*c*e - A*(d*e + c*f)))*ArcTan
[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])])/((b*
c - a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) - (2*C*ArcTanh[(Sqrt[d]*Sqrt[e + f*x]
)/(Sqrt[f]*Sqrt[c + d*x])])/(Sqrt[d]*Sqrt[f]))/b^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2116, 27, 175, 66, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{C(de+cf)a^2-b(2cCe+Bde+Bcf-2Adf)a+b^2(2Bce-Ade-Acf)+2C(bc-ad)(be-af)x}{2b(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{C(de+cf)a^2-b(2cCe+Bde+Bcf-2Adf)a+b^2(2Bce-A(de+cf))+2C(bc-ad)(be-af)x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}dx}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 175

$$\frac{\frac{2C(bc-ad)(be-af)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}}dx}{b} - \frac{\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{\cdots}}}{b}}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 66

$$\frac{\frac{4C(bc-ad)(be-af)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b} - \frac{\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)\int \frac{1}{(a+bx)\sqrt{c+\cdots}}}{b}}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 104

$$\frac{\frac{4C(bc-ad)(be-af)\int \frac{1}{d-\frac{f(c+dx)}{e+fx}}d\frac{\sqrt{c+dx}}{\sqrt{e+fx}}}{b} - \frac{2\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)\int \frac{\cdots}{-bc+ad+\cdots}}{b}}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

$\big\downarrow$ 221

$$\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)}{b\sqrt{bc-ad}\sqrt{be-af}} + \frac{4C(bc-ad)(be-af)\text{arcta}\cdots}{b\sqrt{d}\sqrt{f}}}{2b(bc-ad)(be-af)} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-(((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*
e - a*f)*(a + b*x))) + ((4*C*(b*c - a*d)*(b*e - a*f)*ArcTanh[(Sqrt[f]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b*Sqrt[d]*Sqrt[f]) + (2*(2*a^3*C*d*f
 - 3*a^2*b*C*(d*e + c*f) - b^3*(2*B*c*e - A*d*e - A*c*f) + a*b^2*(4*c*C*e
+ B*d*e + B*c*f - 2*A*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[
b*c - a*d]*Sqrt[e + f*x])])/(b*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]))/(2*b*(b*c
 - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. $2(226) = 452$.

Time = 0.97 (sec) , antiderivative size = 2973, normalized size of antiderivative = 11.70

method	result	size
default	Expression too large to display	2973

input
```
int((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*(f*x+e)^(1/2)*(d*x+c)^(1/2)*(-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*
x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-
a*d*e+2*b*c*e)/(b*x+a))*b^4*c*f*x*(d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d
*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*d*e*x*(d*f)^(1/2)+2*B*ln((-2*a*d*f*
x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*e*x*(d*f)^(1/2)-2*C*
ln(1/2*(2*d*f*x+2*((f*x+e)*(d*x+c))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2)
)*b^4*c*e*x*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*A*ln((-2*a*d*f
*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*f*(d*f)^(1/2)-A
*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*c*f*(d
*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^(1/2)*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a
*b^3*d*e*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x+e)*(d*x+c))^
(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)
/(b*x+a))*a^2*b^2*c*f*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((f*x
+e)*(d*x+c))^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b-a*c*f-a
*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d*e*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+b*c*f...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**2*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assu
me?` for m
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1319 vs. $2(225) = 450$.

Time = 0.87 (sec) , antiderivative size = 1319, normalized size of antiderivative = 5.19

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
(4*sqrt(d*f)*C*a*b^2*c*d^2*e - 2*sqrt(d*f)*B*b^3*c*d^2*e - 3*sqrt(d*f)*C*a
^2*b*d^3*e + sqrt(d*f)*B*a*b^2*d^3*e + sqrt(d*f)*A*b^3*d^3*e - 3*sqrt(d*f)
*C*a^2*b*c*d^2*f + sqrt(d*f)*B*a*b^2*c*d^2*f + sqrt(d*f)*A*b^3*c*d^2*f + 2
*sqrt(d*f)*C*a^3*d^3*f - 2*sqrt(d*f)*A*a*b^2*d^3*f)*arctan(-1/2*(b*d^2*e +
 b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d
*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2
*f^2)*d))/((b^4*c*e*abs(d) - a*b^3*d*e*abs(d) - a*b^3*c*f*abs(d) + a^2*b^2
*d*f*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*
d) - 2*(sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*d^5*e^2 + sqrt(d*f)*
A*b^3*d^5*e^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*e*f + 2*sqrt(d*f)*B*a*b^2*c*d^4*
e*f - 2*sqrt(d*f)*A*b^3*c*d^4*e*f + sqrt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d
*f)*B*a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*c^2*d^3*f^2 - sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b*d^3*e +
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*B*a*b^2*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x +
 c)*d*f - c*d*f))^2*A*b^3*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t(d^2*e + (d*x + c)*d*f - c*d*f))^2*C*a^2*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*B*a*b^2*c*d^2*f -
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^
2*A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 5671, normalized size of antiderivative = 22.33

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
( - 2*sqrt(a*f - b*e)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(f)*sqrt(d)*sqrt(a
*f - b*e)*sqrt(a*d - b*c) - 2*a*d*f + b*c*f + b*d*e) + sqrt(f)*sqrt(b)*sqr
t(c + d*x) + sqrt(d)*sqrt(b)*sqrt(e + f*x))*a**4*c*d**2*f**2 + 2*sqrt(a*f
- b*e)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(f)*sqrt(d)*sqrt(a*f - b*e)*sqrt(
a*d - b*c) - 2*a*d*f + b*c*f + b*d*e) + sqrt(f)*sqrt(b)*sqrt(c + d*x) + sq
rt(d)*sqrt(b)*sqrt(e + f*x))*a**3*b**2*d**2*f**2 + 3*sqrt(a*f - b*e)*sqrt(
a*d - b*c)*log( - sqrt(2*sqrt(f)*sqrt(d)*sqrt(a*f - b*e)*sqrt(a*d - b*c) -
 2*a*d*f + b*c*f + b*d*e) + sqrt(f)*sqrt(b)*sqrt(c + d*x) + sqrt(d)*sqrt(b
)*sqrt(e + f*x))*a**3*b*c**2*d*f**2 + 3*sqrt(a*f - b*e)*sqrt(a*d - b*c)*lo
g( - sqrt(2*sqrt(f)*sqrt(d)*sqrt(a*f - b*e)*sqrt(a*d - b*c) - 2*a*d*f + b*
c*f + b*d*e) + sqrt(f)*sqrt(b)*sqrt(c + d*x) + sqrt(d)*sqrt(b)*sqrt(e + f*
x))*a**3*b*c*d**2*e*f - 2*sqrt(a*f - b*e)*sqrt(a*d - b*c)*log( - sqrt(2*sq
rt(f)*sqrt(d)*sqrt(a*f - b*e)*sqrt(a*d - b*c) - 2*a*d*f + b*c*f + b*d*e) +
 sqrt(f)*sqrt(b)*sqrt(c + d*x) + sqrt(d)*sqrt(b)*sqrt(e + f*x))*a**3*b*c*d
**2*f**2*x - 2*sqrt(a*f - b*e)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(f)*sqrt(
d)*sqrt(a*f - b*e)*sqrt(a*d - b*c) - 2*a*d*f + b*c*f + b*d*e) + sqrt(f)*sq
rt(b)*sqrt(c + d*x) + sqrt(d)*sqrt(b)*sqrt(e + f*x))*a**2*b**3*c*d*f**2 -
2*sqrt(a*f - b*e)*sqrt(a*d - b*c)*log( - sqrt(2*sqrt(f)*sqrt(d)*sqrt(a*f -
 b*e)*sqrt(a*d - b*c) - 2*a*d*f + b*c*f + b*d*e) + sqrt(f)*sqrt(b)*sqrt(c
+ d*x) + sqrt(d)*sqrt(b)*sqrt(e + f*x))*a**2*b**3*d**2*e*f + 2*sqrt(a*f...
```

3.81 $\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 403

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{(2a^3Cdf+ab^2(8cCe+Bde+Bcf-6Adf)-b^3(4Bce-3A(de+cf))+a^2b(2Bdf-5C(de+cf)))}{4b(bc-ad)^2(be-af)^2(a+bx)} - \frac{\left(2(2bce-a(de+cf))\left(aBdf+\frac{a^2Cdf}{b}-2aC(de+cf)+b(2cCe-Adf)\right)-\frac{(bde+bcf-2adf)(a^2C(de+cf)-a}{}\right.}{4(bc-ad)^{5/2}(be-af)^{5/2}}$$

output
```
-1/2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^2+1/4*(2*a^3*C*d*f+a*b^2*(-6*A*d*f+B*c*f+B*d*e+8*C*c*e)-b^3*(4*
B*c*e-3*A*(c*f+d*e))+a^2*b*(2*B*d*f-5*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f*x+e)^
(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)-1/4*(2*(2*b*c*e-a*(c*f+d*e))*(a*
B*d*f+a^2*C*d*f/b-2*a*C*(c*f+d*e)+b*(-A*d*f+2*C*c*e))-(-2*a*d*f+b*c*f+b*d*
e)*(a^2*C*(c*f+d*e)-a*b*(-4*A*d*f+B*c*f+B*d*e+4*C*c*e)+b^2*(4*B*c*e-3*A*(c
*f+d*e)))/b)*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c)^(1/2)/(f*x+
e)^(1/2))/(-a*d+b*c)^(5/2)/(-a*f+b*e)^(5/2)
```

Mathematica [A] (verified)

Time = 2.44 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$=\frac{1}{4}\Bigg(-\frac{\sqrt{c+dx}\sqrt{e+fx}(4b^3Bcex-ab^2(8cCex+B(-2ce+dex+cfx))+a^2b(5Cdex+Bd(e-2fx)+}{}$$

$$+\frac{(b^2(3Ad^2e^2+2cde(-2Be+Af)+c^2(8Ce^2-4Bef+3Af^2))+ab(d^2e(Be-8Af)+c^2f(-8Ce+B}{(bc}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(-((Sqrt[c + d*x]*Sqrt[e + f*x]*(4*b^3*B*c*e*x - a*b^2*(8*c*C*e*x + B*(-2*
c*e + d*e*x + c*f*x)) + a^2*b*(5*C*d*e*x + B*d*(e - 2*f*x) + c*(-6*C*e + B
*f + 5*C*f*x)) + a^3*(-4*B*d*f + C*(3*d*e + 3*c*f - 2*d*f*x)) + A*b*(8*a^2
*d*f + b^2*(2*c*e - 3*d*e*x - 3*c*f*x) + a*b*(-5*d*e - 5*c*f + 6*d*f*x))))
/((b*c - a*d)^2*(b*e - a*f)^2*(a + b*x)^2)) + ((b^2*(3*A*d^2*e^2 + 2*c*d*e
*(-2*B*e + A*f) + c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)) + a*b*(d^2*e*(B*e - 8
*A*f) + c^2*f*(-8*C*e + B*f) - 2*c*d*(4*C*e^2 - 7*B*e*f + 4*A*f^2)) + a^2*
(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))))
*ArcTan[(Sqrt[b*c - a*d]*Sqrt[e + f*x])/(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])
])/((b*c - a*d)^(5/2)*(-(b*e) + a*f)^(5/2)))/4
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2116, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{C(de+cf)a^2-b(4cCe+Bde+Bcf-4Adf)a+b^2(4Bce-3A(de+cf))+2b\left(\frac{Cdfa^2}{b}-2Cdea-2cCfa+Bdfa+2bcCe-Abdf\right)x}{2b(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}dx}{2(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{C(de+cf)a^2-b(4cCe+Bde+Bcf-4Adf)a+b^2(4Bce-3A(de+cf))+2b\left(\frac{Cdfa^2}{b}+Bdfa-2C(de+cf)a+b(2cCe-Adf)\right)x}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}dx}{4b(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{168}$$

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(2Bdf-5C(cf+de))+ab^2(-6Adf+Bcf+Bde+8cCe)-b^3(4Bce-3A(cf+de))\right)}{(a+bx)(bc-ad)(be-af)}-\frac{\int -\frac{b\left(\left(C\left(3d^2e^2+2cdfe+3c^2f^2\right)+\cdots\right.\right.}{\cdots}}{\cdots}}{4b(bc-ad)(b\cdots}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{b\left(a^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+ab\left(-2cd\left(4Af^2-7Bef+4Ce^2\right)+d^2e(Be-8Af)+c^2(-f)(8Ce-Bf)\right)+b^2\left(c^2\left(3Af^2-4B\cdots\right.\right.\right.}{2(bc-ad)(be-af)}}{\cdots}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{104}$$

$$\frac{\frac{b\left(a^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+ab\left(-2cd\left(4Af^2-7Bef+4Ce^2\right)+d^2e(Be-8Af)+c^2(-f)(8Ce-Bf)\right)+b^2\left(c^2\left(3Af^2-4B\cdots\right.\right.\right.}{(bc-ad)(be-af)}}{\cdots}$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(2Bdf-5C(cf+de))+ab^2(-6Adf+Bcf+Bde+8cCe)-b^3(4Bce-3A(cf+de)))}{(a+bx)(bc-ad)(be-af)} - \frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)(a^2}{}}{}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-1/2*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^2) + (((2*a^3*C*d*f + a*b^2*(8*c*C*e + B*d*e + B*c*f
 - 6*A*d*f) - b^3*(4*B*c*e - 3*A*(d*e + c*f)) + a^2*b*(2*B*d*f - 5*C*(d*e
+ c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x))
- (b*(b^2*(3*A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) + c^2*(8*C*e^2 - 4*B*e*f +
3*A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f) - c^2*f*(8*C*e - B*f) - 2*c*d*(4*C*e^
2 - 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d
*f*(2*A*d*f - B*(d*e + c*f))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sq
rt[b*c - a*d]*Sqrt[e + f*x])])/((b*c - a*d)^(3/2)*(b*e - a*f)^(3/2)))/(4*b
*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7118 vs. $2(377) = 754$.

Time = 1.12 (sec) , antiderivative size = 7119, normalized size of antiderivative = 17.67

method	result	size
default	Expression too large to display	7119

input
```
int((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. $2(376) = 752$.

Time = 157.37 (sec) , antiderivative size = 4058, normalized size of antiderivative = 10.07

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7939 vs. $2(376) = 752$.

Time = 33.36 (sec) , antiderivative size = 7939, normalized size of antiderivative = 19.70

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
-1/4*(8*sqrt(d*f)*C*b^2*c^2*d^2*e^2 - 8*sqrt(d*f)*C*a*b*c*d^3*e^2 - 4*sqrt
(d*f)*B*b^2*c*d^3*e^2 + 3*sqrt(d*f)*C*a^2*d^4*e^2 + sqrt(d*f)*B*a*b*d^4*e^
2 + 3*sqrt(d*f)*A*b^2*d^4*e^2 - 8*sqrt(d*f)*C*a*b*c^2*d^2*e*f - 4*sqrt(d*f
)*B*b^2*c^2*d^2*e*f + 2*sqrt(d*f)*C*a^2*c*d^3*e*f + 14*sqrt(d*f)*B*a*b*c*d
^3*e*f + 2*sqrt(d*f)*A*b^2*c*d^3*e*f - 4*sqrt(d*f)*B*a^2*d^4*e*f - 8*sqrt(
d*f)*A*a*b*d^4*e*f + 3*sqrt(d*f)*C*a^2*c^2*d^2*f^2 + sqrt(d*f)*B*a*b*c^2*d
^2*f^2 + 3*sqrt(d*f)*A*b^2*c^2*d^2*f^2 - 4*sqrt(d*f)*B*a^2*c*d^3*f^2 - 8*s
qrt(d*f)*A*a*b*c*d^3*f^2 + 8*sqrt(d*f)*A*a^2*d^4*f^2)*arctan(-1/2*(b*d^2*e
 + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c) - sqrt(d^2*e + (d*x + c)
*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d
^2*f^2)*d))/((b^4*c^2*e^2*abs(d) - 2*a*b^3*c*d*e^2*abs(d) + a^2*b^2*d^2*e^
2*abs(d) - 2*a*b^3*c^2*e*f*abs(d) + 4*a^2*b^2*c*d*e*f*abs(d) - 2*a^3*b*d^2
*e*f*abs(d) + a^2*b^2*c^2*f^2*abs(d) - 2*a^3*b*c*d*f^2*abs(d) + a^4*d^2*f^
2*abs(d))*sqrt(-b^2*c*d*e*f + a*b*d^2*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d)
 + 1/2*(8*sqrt(d*f)*C*a*b^4*c*d^9*e^5 - 4*sqrt(d*f)*B*b^5*c*d^9*e^5 - 5*sqr
t(d*f)*C*a^2*b^3*d^10*e^5 + sqrt(d*f)*B*a*b^4*d^10*e^5 + 3*sqrt(d*f)*A*b^5
*d^10*e^5 - 32*sqrt(d*f)*C*a*b^4*c^2*d^8*e^4*f + 16*sqrt(d*f)*B*b^5*c^2*d^
8*e^4*f + 15*sqrt(d*f)*C*a^2*b^3*c*d^9*e^4*f - 3*sqrt(d*f)*B*a*b^4*c*d^9*e
^4*f - 9*sqrt(d*f)*A*b^5*c*d^9*e^4*f + 2*sqrt(d*f)*C*a^3*b^2*d^10*e^4*f +
2*sqrt(d*f)*B*a^2*b^3*d^10*e^4*f - 6*sqrt(d*f)*A*a*b^4*d^10*e^4*f + 48*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^3*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{C\,x^2+Bx+A}{(bx+a)^3\,\sqrt{dx+c}\,\sqrt{fx+e}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

3.82 $\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 770

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$
$$+\frac{(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf)-b^3(6Bce-5A(de+cf))+a^2b(4Bdf-7C(de+cf)}{12b(bc-ad)^2(be-af)^2(a+bx)^2}$$
$$+\frac{(4a^4Cd^2f^2+8a^3bdf(Bdf-2C(de+cf))-b^4(15Ad^2e^2-2cde(9Be-7Af)+3c^2(8Ce^2-6Bef+5}{}$$
$$-\frac{(2df(2bce-a(de+cf)))(2a^3Cdf+ab^2(12cCe+Bde+Bcf-10Adf)-b^3(6Bce-5A(de+cf))+}{}$$

output
```
-1/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^3+1/12*(2*a^3*C*d*f+a*b^2*(-10*A*d*f+B*c*f+B*d*e+12*C*c*e)-b^3*
(6*B*c*e-5*A*(c*f+d*e))+a^2*b*(4*B*d*f-7*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^2+1/24*(4*a^4*C*d^2*f^2+8*a^3
*b*d*f*(B*d*f-2*C*(c*f+d*e))-b^4*(15*A*d^2*e^2-2*c*d*e*(-7*A*f+9*B*e)+3*c^
2*(5*A*f^2-6*B*e*f+8*C*e^2))-a*b^3*(d^2*e*(-44*A*f+3*B*e)-3*c^2*f*(-B*f+4*
C*e)-2*c*d*(22*A*f^2-29*B*e*f+6*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-34*c*d*e*f+3
*d^2*e^2)+2*d*f*(22*A*d*f-5*B*(c*f+d*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(
-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)-1/24*(2*d*f*(2*b*c*e-a*(c*f+d*e))*(2*a^3*
C*d*f+a*b^2*(-10*A*d*f+B*c*f+B*d*e+12*C*c*e)-b^3*(6*B*c*e-5*A*(c*f+d*e))+a
^2*b*(4*B*d*f-7*C*(c*f+d*e)))-(-2*a*d*f+b*c*f+b*d*e)*((4*a*d*f-3*b*(c*f+d*
e))*(a^2*C*(c*f+d*e)-a*b*(-6*A*d*f+B*c*f+B*d*e+6*C*c*e)+b^2*(6*B*c*e-5*A*(
c*f+d*e)))+2*(4*b*c*e-a*(c*f+d*e))*(a^2*C*d*f+b^2*(-2*A*d*f+3*C*c*e)+a*b*(
2*B*d*f-3*C*(c*f+d*e)))))*arctanh((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-a*d+b*c
)^(1/2)/(f*x+e)^(1/2))/b/(-a*d+b*c)^(7/2)/(-a*f+b*e)^(7/2)
```

Mathematica [A] (verified)

Time = 9.26 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\sqrt{c+dx}\sqrt{e+fx}(6b^5cex(4cCex+B(2ce-3dex-3cfx))+6a^5df(-4Bdf+C(3de+3cf-2dfx))}{} + \frac{(ab^2(d^3e^2(Be-18Af)+c^3f^2(-4Ce+Bf)+c^2df(-40Ce^2+23Bef-18Af^2)+cd^2e(-4Ce^2+23B}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-1/24*(Sqrt[c + d*x]*Sqrt[e + f*x]*(6*b^5*c*e*x*(4*c*C*e*x + B*(2*c*e - 3*
d*e*x - 3*c*f*x)) + 6*a^5*d*f*(-4*B*d*f + C*(3*d*e + 3*c*f - 2*d*f*x)) + a
*b^4*(-12*c*C*e*x*(-2*c*e + d*e*x + c*f*x) + B*(3*d^2*e^2*x^2 + 2*c*d*e*x*
(-25*e + 29*f*x) + c^2*(4*e^2 - 50*e*f*x + 3*f^2*x^2))) + a^4*b*(12*B*d*f*
(c*f + d*(e - 2*f*x)) - C*(3*c^2*f^2 + 2*c*d*f*(29*e - 25*f*x) + d^2*(3*e^
2 - 50*e*f*x + 4*f^2*x^2))) + a^2*b^3*(d^2*e*x*(8*B*e + 3*C*e*x - 10*B*f*x
) + c^2*(8*B*f*(-2*e + f*x) + C*(8*e^2 + 14*e*f*x + 3*f^2*x^2)) - 2*c*d*(C
*e*x*(-7*e + 17*f*x) + B*(8*e^2 - 62*e*f*x + 5*f^2*x^2))) + a^3*b^2*(c^2*f
*(10*C*e - 3*B*f - 8*C*f*x) + 2*c*d*(B*f*(17*e - 7*f*x) + C*(5*e^2 - 62*e*
f*x + 8*f^2*x^2)) - d^2*(8*C*e*x*(e - 2*f*x) + B*(3*e^2 + 14*e*f*x + 8*f^2
*x^2))) + A*b*(72*a^4*d^2*f^2 + 18*a^3*b*d*f*(-5*d*e - 5*c*f + 6*d*f*x) +
b^4*(15*d^2*e^2*x^2 + 2*c*d*e*x*(-5*e + 7*f*x) + c^2*(8*e^2 - 10*e*f*x + 1
5*f^2*x^2)) - 2*a*b^3*(c^2*f*(13*e - 20*f*x) + 2*d^2*e*x*(-10*e + 11*f*x)
+ c*d*(13*e^2 - 34*e*f*x + 22*f^2*x^2)) + a^2*b^2*(33*c^2*f^2 + 2*c*d*f*(4
3*e - 59*f*x) + d^2*(33*e^2 - 118*e*f*x + 44*f^2*x^2)))))/((b*c - a*d)^3*(
b*e - a*f)^3*(a + b*x)^3) + ((a*b^2*(d^3*e^2*(B*e - 18*A*f) + c^3*f^2*(-4*
C*e + B*f) + c^2*d*f*(-40*C*e^2 + 23*B*e*f - 18*A*f^2) + c*d^2*e*(-4*C*e^2
 + 23*B*e*f - 12*A*f^2)) + b^3*(5*A*d^3*e^3 + 3*c*d^2*e^2*(-2*B*e + A*f) +
 c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*e*f + 5*A*f^
2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f...
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2116, 27, 168, 27, 168, 27, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2116}$$

$$-\frac{\int -\frac{C(de+cf)a^2-b(6cCe+Bde+Bcf-6Adf)a+b^2(6Bce-5A(de+cf))+2b\left(\frac{Cdfa^2}{b}-3Cdea-3cCfa+2Bdfa+3bcCe-2Abdf\right)x}{2b(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{C(de+cf)a^2-b(6cCe+Bde+Bcf-6Adf)a+b^2(6Bce-5A(de+cf))+2b\left(\frac{Cdfa^2}{b}+2Bdfa-3C(de+cf)a+b(3cCe-2Adf)\right)x}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}dx}{6b(bc-ad)(be-af)}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\Big\downarrow$ 168

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(4Bdf-7C(cf+de))+ab^2(-10Adf+Bcf+Bde+12cCe)-b^3(6Bce-5A(cf+de))\right)}{2(a+bx)^2(bc-ad)(be-af)}-\frac{\int -\frac{2Cdf(de+cf)a^3-b\left(8df(Bde\cdots\right.}{\cdots}}{\cdots}}{\cdots}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{2Cdf(de+cf)a^3+b\left(C\left(3d^2e^2-10cdfe+3c^2f^2\right)+8df(3Adf-B(de+cf))\right)a^2+b^2\left(-3f(4Ce-Bf)c^2-2d\left(6Ce^2-23Bfe+17Af^2\right)c+d^2e(3Be-34Af)\right)a+b^3\left(3\left(8Ce\cdots\right.\right.}{(a+bx)^2\sqrt{c+dx}\sqrt{\cdots}}}{4(bc-ad)(be-\cdots}\cdots-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

$\Big\downarrow$ 168

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf))\right)}{2(bc-ad)(be-af)(a+bx)^2}+\frac{\left(4Cd^2f^2a^4+8bdf(Bdf-2C(de+cf\cdots\right.}{\cdots}}{\cdots}-\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf)))}{2(bc-ad)(be-af)(a+bx)^2}+\frac{\left(4Cd^2f^2a^4+8bdf(Bdf-2C(de+cf\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$\Big\downarrow$ 104

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf)))}{2(bc-ad)(be-af)(a+bx)^2}+\frac{\left(4Cd^2f^2a^4+8bdf(Bdf-2C(de+cf\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

$\Big\downarrow$ 221

$$\frac{\frac{\sqrt{c+dx}\sqrt{e+fx}(2Cdfa^3+b(4Bdf-7C(de+cf))a^2+b^2(12cCe+Bde+Bcf-10Adf)a-b^3(6Bce-5A(de+cf)))}{2(bc-ad)(be-af)(a+bx)^2}+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4Cd^2f^2a^4+8bdf(B\right.}{}}{}$$

$$\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
-1/3*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^3) + (((2*a^3*C*d*f + a*b^2*(12*c*C*e + B*d*e + B*c*
f - 10*A*d*f) - b^3*(6*B*c*e - 5*A*(d*e + c*f)) + a^2*b*(4*B*d*f - 7*C*(d*
e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(2*(b*c - a*d)*(b*e - a*f)*(a + b*
x)^2) + (((4*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f - 2*C*(d*e + c*f)) - b^4*(
15*A*d^2*e^2 - 2*c*d*e*(9*B*e - 7*A*f) + 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^
2)) - a*b^3*(d^2*e*(3*B*e - 44*A*f) - 3*c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e
^2 - 29*B*e*f + 22*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 34*c*d*e*f + 3*c^2*f^
2) + 2*d*f*(22*A*d*f - 5*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b
*c - a*d)*(b*e - a*f)*(a + b*x)) + (3*b*(b^3*(5*A*d^3*e^3 - 3*c*d^2*e^2*(2
*B*e - A*f) + c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B
*e*f + 5*A*f^2)) + a*b^2*(d^3*e^2*(B*e - 18*A*f) - c^3*f^2*(4*C*e - B*f) -
 c*d^2*e*(4*C*e^2 - 23*B*e*f + 12*A*f^2) - c^2*d*f*(40*C*e^2 - 23*B*e*f +
18*A*f^2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A
*d*f - B*(d*e + c*f))) + a^2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f
^2 + c^3*f^3) + 4*d*f*(6*A*d*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2
*f^2))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e +
 f*x])])/((b*c - a*d)^(3/2)*(b*e - a*f)^(3/2)))/(4*(b*c - a*d)*(b*e - a*f)
))/(6*b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2116
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m,
-1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 18801 vs. 2(738) = 1476.

Time = 1.37 (sec) , antiderivative size = 18802, normalized size of antiderivative = 24.42

method	result	size
default	Expression too large to display	18802

input
```
int((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**4/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25632 vs. 2(737) = 1474.

Time = 78.86 (sec) , antiderivative size = 25632, normalized size of antiderivative = 33.29

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm
="giac")
```

output
```
1/8*(8*sqrt(d*f)*C*b^3*c^2*d^3*e^3 - 4*sqrt(d*f)*C*a*b^2*c*d^4*e^3 - 6*sqr
t(d*f)*B*b^3*c*d^4*e^3 + sqrt(d*f)*C*a^2*b*d^5*e^3 + sqrt(d*f)*B*a*b^2*d^5
*e^3 + 5*sqrt(d*f)*A*b^3*d^5*e^3 + 8*sqrt(d*f)*C*b^3*c^3*d^2*e^2*f - 40*sq
rt(d*f)*C*a*b^2*c^2*d^3*e^2*f - 4*sqrt(d*f)*B*b^3*c^2*d^3*e^2*f + 23*sqrt(
d*f)*C*a^2*b*c*d^4*e^2*f + 23*sqrt(d*f)*B*a*b^2*c*d^4*e^2*f + 3*sqrt(d*f)*
A*b^3*c*d^4*e^2*f - 6*sqrt(d*f)*C*a^3*d^5*e^2*f - 4*sqrt(d*f)*B*a^2*b*d^5*
e^2*f - 18*sqrt(d*f)*A*a*b^2*d^5*e^2*f - 4*sqrt(d*f)*C*a*b^2*c^3*d^2*e*f^2
 - 6*sqrt(d*f)*B*b^3*c^3*d^2*e*f^2 + 23*sqrt(d*f)*C*a^2*b*c^2*d^3*e*f^2 +
23*sqrt(d*f)*B*a*b^2*c^2*d^3*e*f^2 + 3*sqrt(d*f)*A*b^3*c^2*d^3*e*f^2 - 4*s
qrt(d*f)*C*a^3*c*d^4*e*f^2 - 40*sqrt(d*f)*B*a^2*b*c*d^4*e*f^2 - 12*sqrt(d*
f)*A*a*b^2*c*d^4*e*f^2 + 8*sqrt(d*f)*B*a^3*d^5*e*f^2 + 24*sqrt(d*f)*A*a^2*
b*d^5*e*f^2 + sqrt(d*f)*C*a^2*b*c^3*d^2*f^3 + sqrt(d*f)*B*a*b^2*c^3*d^2*f^
3 + 5*sqrt(d*f)*A*b^3*c^3*d^2*f^3 - 6*sqrt(d*f)*C*a^3*c^2*d^3*f^3 - 4*sqrt
(d*f)*B*a^2*b*c^2*d^3*f^3 - 18*sqrt(d*f)*A*a*b^2*c^2*d^3*f^3 + 8*sqrt(d*f)
*B*a^3*c*d^4*f^3 + 24*sqrt(d*f)*A*a^2*b*c*d^4*f^3 - 16*sqrt(d*f)*A*a^3*d^5
*f^3)*arctan(-1/2*(b*d^2*e + b*c*d*f - 2*a*d^2*f - (sqrt(d*f)*sqrt(d*x + c
) - sqrt(d^2*e + (d*x + c)*d*f - c*d*f))^2*b)/(sqrt(-b^2*c*d*e*f + a*b*d^2
*e*f + a*b*c*d*f^2 - a^2*d^2*f^2)*d))/((b^6*c^3*e^3*abs(d) - 3*a*b^5*c^2*d
*e^3*abs(d) + 3*a^2*b^4*c*d^2*e^3*abs(d) - a^3*b^3*d^3*e^3*abs(d) - 3*a*b^
5*c^3*e^2*f*abs(d) + 9*a^2*b^4*c^2*d*e^2*f*abs(d) - 9*a^3*b^3*c*d^2*e^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^4*(c + d*x)^(1/2)),x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{C\,x^2+Bx+A}{(bx+a)^4\,\sqrt{dx+c}\,\sqrt{fx+e}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

3.83 $\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1183

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

output
```
2/315*(24*a^2*b*B*d^3*f^3-16*a^3*C*d^3*f^3-3*a*b^2*d*f*(d*f*(14*A*d*f+B*c*
f+B*d*e)-C*(c^2*f^2+d^2*e^2))+b^3*(C*(8*c^3*f^3-3*c^2*d*e*f^2-3*c*d^2*e^2*
f+8*d^3*e^3)+3*d*f*(7*A*d*f*(c*f+d*e)-B*(4*c^2*f^2-2*c*d*e*f+4*d^2*e^2))))
*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d^3/f^3-2/105*(7*d*f*(-3*A*
b*d*f+C*a*c*f+C*a*d*e+C*b*c*e)-(-4*a*d*f-4*b*c*f+b*d*e)*(3*b*B*d*f-2*a*C*d
*f-2*b*C*(c*f+d*e))/b)*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d^2/f
^2+2/21*(3*B*d*f-2*C*(a*d*f+b*c*f+b*d*e)/b)*(b*x+a)^(3/2)*(d*x+c)^(3/2)*(f
*x+e)^(1/2)/b/d^2/f+2/9*C*(b*x+a)^(3/2)*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/d/f-
2/315*(a*d-b*c)^(1/2)*(16*a^4*C*d^4*f^4-8*a^3*b*d^3*f^3*(3*B*d*f+C*c*f+C*d
*e)+3*a^2*b^2*d^2*f^2*(d*f*(14*A*d*f+5*B*c*f+5*B*d*e)-2*C*(c^2*f^2-c*d*e*f
+d^2*e^2))-a*b^3*d*f*(C*(8*c^3*f^3-6*c^2*d*e*f^2-6*c*d^2*e^2*f+8*d^3*e^3)+
3*d*f*(14*A*d*f*(c*f+d*e)-B*(5*c^2*f^2-6*c*d*e*f+5*d^2*e^2)))+b^4*(2*C*(8*
c^4*f^4-4*c^3*d*e*f^3-3*c^2*d^2*e^2*f^2-4*c*d^3*e^3*f+8*d^4*e^4)+3*d*f*(14
*A*d*f*(c^2*f^2-c*d*e*f+d^2*e^2)-B*(8*c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+
8*d^3*e^3))))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)
*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^4/d^(7
/2)/f^4/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/315*(a*d-b*c)^(1/2)*(
-a*f+b*e)*(-c*f+d*e)*(8*a^3*C*d^3*f^3+3*a^2*b*d^2*f^2*(-4*B*d*f-C*c*f+C*d*
e)-3*a*b^2*d*f^2*((-7*A*d^2+C*c^2)*f+B*d*(-2*c*f+d*e))-b^3*(C*(-8*c^3*f^3-
3*c^2*d*e*f^2+16*d^3*e^3)+3*d*f*(7*A*d*f*(-c*f+2*d*e)-B*(-4*c^2*f^2-c*d...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.72 (sec) , antiderivative size = 1422, normalized size of antiderivative = 1.20

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(2*(-(b^2*Sqrt[-a + (b*c)/d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e +
c*C*f + 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) -
 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)) + a*b^3*d*f*(C*(-8*d^3*e^3 + 6*c*d^2*e
^2*f + 6*c^2*d*e*f^2 - 8*c^3*f^3) - 3*d*f*(14*A*d*f*(d*e + c*f) + B*(-5*d^
2*e^2 + 6*c*d*e*f - 5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3
*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 -
 c*d*e*f + c^2*f^2) + B*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*d*e*f^2 - 8*c^
3*f^3))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b*x)*(c +
 d*x)*(e + f*x)*(8*a^3*C*d^3*f^3 - 3*a^2*b*d^2*f^2*(c*C*f + 4*B*d*f + C*d*
(e + 2*f*x)) + a*b^2*d*f*(3*d*f*(7*A*d*f + B*(2*d*e + 2*c*f + 3*d*f*x)) +
C*(-3*c^2*f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 5*f^2*x^2))) +
 b^3*(C*(8*c^3*f^3 - 3*c^2*d*f^2*(e + 2*f*x) + c*d^2*f*(-3*e^2 + 2*e*f*x +
 5*f^2*x^2) + d^3*(8*e^3 - 6*e^2*f*x + 5*e*f^2*x^2 + 35*f^3*x^3)) + 3*d*f*
(7*A*d*f*(c*f + d*(e + 3*f*x)) + B*(-4*c^2*f^2 + c*d*f*(2*e + 3*f*x) + d^2
*(-4*e^2 + 3*e*f*x + 15*f^2*x^2))))) - I*(b*c - a*d)*f*(16*a^4*C*d^4*f^4 -
 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) + 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d
*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)) + a*b^3*d*f*
(C*(-8*d^3*e^3 + 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 8*c^3*f^3) - 3*d*f*(14*A*
d*f*(d*e + c*f) + B*(-5*d^2*e^2 + 6*c*d*e*f - 5*c^2*f^2))) + b^4*(2*C*(8*d
^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4*c^3*d*e*f^3 + 8*c^4*f^4)...
```

Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 1213, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2118, 27, 171, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{3}{2}b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)dx}{9b^2df}+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}$$

$\Big\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-\frac{\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)dx}{3bdf}$$

$\Big\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$2\int \frac{\sqrt{c+dx}\sqrt{e+fx}(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf))+(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bB\cdots}{2\sqrt{a+bx}}$$

$$7df$$

$$3bdf$$

$\Big\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf))+(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBd\cdots}{\sqrt{a+bx}}$$

$$7df$$

$$3bdf$$

$\Big\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$2\int \sqrt{e+fx}\Big(5bcf(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf)))-(bce+ade+3acf)(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-\cdots$$

$\Big\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\int \sqrt{e+fx}\Big(5bcf(7adf(bcCe+aCde+acCf-3Abdf)+(bce+3a(de+cf))(3bBdf-2aCdf-2bC(de+cf)))-(bce+ade+3acf)(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4\cdots$$

$\Big\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf}-$$

$$\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+2\int \frac{3bde(5bcf(7adf(bcCe+aCde+acCf-3Abdf\cdots}{}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} - \frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf} + \frac{\int \frac{3bde(5bcf(7adf(bcCe+aCde+acCf-3Abdf)+}{}}{}}{}$$

$\downarrow$ 176

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} - \frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf} + \frac{(be-af)(de-cf)\left(-\left(\left(C\left(16d^3e^3-3c^2df^2e-8c^3\right.\right.\right.\right.}{}}{}$$

$\downarrow$ 124

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} - \frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf} + \frac{(be-af)(de-cf)\left(-\left(\left(C\left(16d^3e^3-3c^2df^2e-8c^3\right.\right.\right.\right.}{}}{}$$

$\downarrow$ 123

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} - \frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf} + \frac{2\sqrt{ad-bc}\left(\left(2C\left(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-\right.\right.\right.}{}}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} - \frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf} + \frac{2\sqrt{ad-bc}\left(\left(2C\left(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-\right.\right.\right.}{}}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} -$$

$$\frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+\frac{2\sqrt{ad-bc}\left(\left(2C\left(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-\right.\right.\right.}{}}{}$$

$\downarrow$ 130

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}(e+fx)^{3/2}}{9bdf} -$$

$$\frac{\frac{2(7bdf(bcCe+aCde+acCf-3Abdf)-(adf-4b(de+cf))(3bBdf-2aCdf-2bC(de+cf)))\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}}{5bf}+\frac{2\sqrt{ad-bc}\left(\left(2C\left(8d^4e^4-4cd^3fe^3-3c^2d^2f^2e^2-\right.\right.\right.}{}}{}$$

input
```
Int[Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

output
```
(2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(9*b*d*f) - ((-2*(3*
b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e
+ f*x)^(3/2))/(7*d*f) + ((2*(7*b*d*f*(b*c*C*e + a*C*d*e + a*c*C*f - 3*A*b*
d*f) - (a*d*f - 4*b*(d*e + c*f))*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f
)))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2))/(5*b*f) + ((-2*(8*a^3*C*d
^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*f - 4*B*d*f) - 3*a*b^2*d*f^2*((c^2*C
 - 7*A*d^2)*f + B*d*(d*e - 2*c*f)) - b^3*(C*(16*d^3*e^3 - 3*c^2*d*e*f^2 -
8*c^3*f^3) + 3*d*f*(7*A*d*f*(2*d*e - c*f) - B*(8*d^2*e^2 - c*d*e*f - 4*c^2
*f^2))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d) + ((2*Sqrt[-(b
*c) + a*d]*(16*a^4*C*d^4*f^4 - 8*a^3*b*d^3*f^3*(C*d*e + c*C*f + 3*B*d*f) +
 3*a^2*b^2*d^2*f^2*(d*f*(5*B*d*e + 5*B*c*f + 14*A*d*f) - 2*C*(d^2*e^2 - c*
d*e*f + c^2*f^2)) - a*b^3*d*f*(C*(8*d^3*e^3 - 6*c*d^2*e^2*f - 6*c^2*d*e*f^
2 + 8*c^3*f^3) + 3*d*f*(14*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 - 6*c*d*e*f +
5*c^2*f^2))) + b^4*(2*C*(8*d^4*e^4 - 4*c*d^3*e^3*f - 3*c^2*d^2*e^2*f^2 - 4
*c^3*d*e*f^3 + 8*c^4*f^4) + 3*d*f*(14*A*d*f*(d^2*e^2 - c*d*e*f + c^2*f^2)
- B*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))))*Sqrt[(b*(c
+ d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x]
)/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt
[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e -
a*f)*(d*e - c*f)*(8*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(C*d*e - c*C*f - 4*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 2077, normalized size of antiderivative = 1.76

method	result	size
elliptic	Expression too large to display	2077
default	Expression too large to display	14974

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/9*C*x^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*
x+a*c*e)^(1/2)+2/7*(b*B*d*f+a*C*d*f+C*b*f*c+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4
*b*d*e)*C)/b/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*
e*x+b*c*e*x+a*c*e)^(1/2)+2/5*(A*b*d*f+a*B*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*
d*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*B*d*f+a*C*d*f+C*b
*f*c+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d
*e))/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/3*(A*a*d*f+A*b*c*f+A*b*d*e+B*a*c*f+B*a*d*e+B*b*c*e+1/3*
C*a*c*e-2/7*(b*B*d*f+a*C*d*f+C*b*f*c+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)
*C)/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(A*b*d*f+a*B*d*f+B*b*c*f+b*B
*d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*
B*d*f+a*C*d*f+C*b*f*c+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*
d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*
d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*a*c*
e-2/5*(A*b*d*f+a*B*d*f+B*b*c*f+b*B*d*e+C*a*c*f+C*a*d*e+C*b*c*e-2/9*C*(7/2*
a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b*B*d*f+a*C*d*f+C*b*f*c+C*b*d*e-2/9*(4*a*d
*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A
*a*d*f+A*b*c*f+A*b*d*e+B*a*c*f+B*a*d*e+B*b*c*e+1/3*C*a*c*e-2/7*(b*B*d*f+a*
C*d*f+C*b*f*c+C*b*d*e-2/9*(4*a*d*f+4*b*c*f+4*b*d*e)*C)/b/d/f*(5/2*a*c*f...
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1916, normalized size of antiderivative = 1.62

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algor
ithm="fricas")
```

output
```
2/945*(3*(35*C*b^5*d^5*f^5*x^3 + 8*C*b^5*d^5*e^3*f^2 - 3*(C*b^5*c*d^4 + (C
*a*b^4 + 4*B*b^5)*d^5)*e^2*f^3 - (3*C*b^5*c^2*d^3 - 2*(C*a*b^4 + 3*B*b^5)*
c*d^4 + 3*(C*a^2*b^3 - 2*B*a*b^4 - 7*A*b^5)*d^5)*e*f^4 + (8*C*b^5*c^3*d^2
- 3*(C*a*b^4 + 4*B*b^5)*c^2*d^3 - 3*(C*a^2*b^3 - 2*B*a*b^4 - 7*A*b^5)*c*d^
4 + (8*C*a^3*b^2 - 12*B*a^2*b^3 + 21*A*a*b^4)*d^5)*f^5 + 5*(C*b^5*d^5*e*f^
4 + (C*b^5*c*d^4 + (C*a*b^4 + 9*B*b^5)*d^5)*f^5)*x^2 - (6*C*b^5*d^5*e^2*f^
3 - (2*C*b^5*c*d^4 + (2*C*a*b^4 + 9*B*b^5)*d^5)*e*f^4 + (6*C*b^5*c^2*d^3 -
 (2*C*a*b^4 + 9*B*b^5)*c*d^4 + 3*(2*C*a^2*b^3 - 3*B*a*b^4 - 21*A*b^5)*d^5)
*f^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) + (16*C*b^5*d^5*e^5 - 8
*(2*C*b^5*c*d^4 + (2*C*a*b^4 + 3*B*b^5)*d^5)*e^4*f - (5*C*b^5*c^2*d^3 - (2
0*C*a*b^4 + 27*B*b^5)*c*d^4 + (5*C*a^2*b^3 - 27*B*a*b^4 - 42*A*b^5)*d^5)*e
^3*f^2 - (5*C*b^5*c^3*d^2 - 6*(C*a*b^4 + 2*B*b^5)*c^2*d^3 - 3*(2*C*a^2*b^3
 - 14*B*a*b^4 - 21*A*b^5)*c*d^4 + (5*C*a^3*b^2 - 12*B*a^2*b^3 + 63*A*a*b^4
)*d^5)*e^2*f^3 - (16*C*b^5*c^4*d - (20*C*a*b^4 + 27*B*b^5)*c^3*d^2 - 3*(2*
C*a^2*b^3 - 14*B*a*b^4 - 21*A*b^5)*c^2*d^3 - 2*(10*C*a^3*b^2 - 21*B*a^2*b^
3 + 126*A*a*b^4)*c*d^4 + (16*C*a^4*b - 27*B*a^3*b^2 + 63*A*a^2*b^3)*d^5)*e
*f^4 + (16*C*b^5*c^5 - 8*(2*C*a*b^4 + 3*B*b^5)*c^4*d - (5*C*a^2*b^3 - 27*B
*a*b^4 - 42*A*b^5)*c^3*d^2 - (5*C*a^3*b^2 - 12*B*a^2*b^3 + 63*A*a*b^4)*c^2
*d^3 - (16*C*a^4*b - 27*B*a^3*b^2 + 63*A*a^2*b^3)*c*d^4 + 2*(8*C*a^5 - 12*
B*a^4*b + 21*A*a^3*b^2)*d^5)*f^5)*sqrt(b*d*f)*weierstrassPInverse(4/3*(...
```

Sympy [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$$
$$= \int (Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

Giac [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$$
$$= \int (Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)\,dx$$
$$= \int \sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

output
```
int((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int \sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int \sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\left(C\,x^2+Bx+A\right)dx$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A),x)
```

3.84 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 762

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$$

$$= \frac{2\left(\frac{(bde-2bcf-4adf)(7bBdf-6aCdf-4bC(de+cf))}{b} - 5df(3aC(de+cf)+b(cCe-7Adf))\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+\ldots}}{105b^2d^2f^2}$$

$$+ \frac{2(7bBdf-6aCdf-4bC(de+cf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{35b^2d^2f}$$

$$+ \frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}$$

$$- \frac{2\sqrt{-bc+ad}\Big(3df((bce+3ade+acf)(7bBdf-6aCdf-4bC(de+cf))+5bde(3aC(de+cf)+b(cC\ldots}{}$$

$$- \frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(13Cde-5cCf-28Bdf)-b^2(7df(2Bde-Bcf\ldots\right.}{105b^4d^{5/2}f^3\sqrt{c+\ldots}}$$

output
```
2/105*((-4*a*d*f-2*b*c*f+b*d*e)*(7*b*B*d*f-6*a*C*d*f-4*b*C*(c*f+d*e))/b-5*
d*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x
+e)^(1/2)/b^2/d^2/f^2+2/35*(7*b*B*d*f-6*a*C*d*f-4*b*C*(c*f+d*e))*(b*x+a)^(
1/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b^2/d^2/f+2/7*C*(b*x+a)^(1/2)*(d*x+c)^(3/
2)*(f*x+e)^(3/2)/b/d/f-2/105*(a*d-b*c)^(1/2)*(3*d*f*((a*c*f+3*a*d*e+b*c*e)
*(7*b*B*d*f-6*a*C*d*f-4*b*C*(c*f+d*e))+5*b*d*e*(3*a*C*(c*f+d*e)+b*(-7*A*d*
f+C*c*e)))-(b*c*f-2*d*(a*f+b*e))*((-4*a*d*f-2*b*c*f+b*d*e)*(7*b*B*d*f-6*a*
C*d*f-4*b*C*(c*f+d*e))-5*b*d*f*(3*a*C*(c*f+d*e)+b*(-7*A*d*f+C*c*e)))/b)*(b
*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(
a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^3/d^(5/2)/f^3/(d*x+c)^
(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/105*(a*d-b*c)^(1/2)*(-a*f+b*e)*(-c*f+
d*e)*(24*a^2*C*d^2*f^2+a*b*d*f*(-28*B*d*f-5*C*c*f+13*C*d*e)-b^2*(7*d*f*(-5
*A*d*f-B*c*f+2*B*d*e)-C*(-4*c^2*f^2-c*d*e*f+8*d^2*e^2)))*(b*(d*x+c)/(-a*d+
b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(
a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^4/d^(5/2)/f^3/(d*x+c)^
(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.02 (sec) , antiderivative size = 917, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(48a^3Cd^3f^3-8a^2bd^2f^2(7Bdf+2C(de+cf))+ab^2df(7df(3Bde+3Bcf+10Adf)+\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f +
 2*C*(d*e + c*f)) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-
9*d^2*e^2 + 8*c*d*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f +
 5*c^2*d*e*f^2 - 8*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 -
c*d*e*f + c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]*d*f*(a
+ b*x)*(c + d*x)*(e + f*x)*(-24*a^2*C*d^2*f^2 + a*b*d*f*(28*B*d*f + C*(5*d
*e + 5*c*f + 18*d*f*x)) + b^2*(-7*d*f*(B*c*f + 5*A*d*f + B*d*(e + 3*f*x))
+ C*(4*c^2*f^2 - c*d*f*(2*e + 3*f*x) + d^2*(4*e^2 - 3*e*f*x - 15*f^2*x^2))
)) + I*(b*c - a*d)*f*(48*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(7*B*d*f + 2*C*(d
*e + c*f)) + a*b^2*d*f*(7*d*f*(3*B*d*e + 3*B*c*f + 10*A*d*f) + C*(-9*d^2*e
^2 + 8*c*d*e*f - 9*c^2*f^2)) + b^3*(C*(-8*d^3*e^3 + 5*c*d^2*e^2*f + 5*c^2*
d*e*f^2 - 8*c^3*f^3) - 7*d*f*(5*A*d*f*(d*e + c*f) - 2*B*(d^2*e^2 - c*d*e*f
 + c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b
*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e - c*f)*(24
*a^2*C*d^2*f^2 + a*b*d*f*(-5*C*d*e + 13*c*C*f - 28*B*d*f) + b^2*(7*d*f*(B*
d*e - 2*B*c*f + 5*A*d*f) - C*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2)))*(a + b*x)
^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]
*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b
*c*f - a*d*f)]))/(105*b^5*Sqrt[-a + (b*c)/d]*d^3*f^3*Sqrt[a + b*x]*Sqrt...
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 789, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2118, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{b\sqrt{c+dx}\sqrt{e+fx}(3aC(de+cf)+b(cCe-7Adf)-(7bBdf-6aCdf-4bC(de+cf))x)}{2\sqrt{a+bx}}dx}{7b^2df}+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}(3aC(de+cf)+b(cCe-7Adf)-(7bBdf-6aCdf-4bC(de+cf))x)}{\sqrt{a+bx}}dx}{7bdf}$$

$\downarrow$ 171

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-\frac{\frac{2\int \frac{\sqrt{e+fx}((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf))+((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+}{2\sqrt{a+bx}\sqrt{c+dx}}}{5bf}}{7bdf}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-\frac{\frac{\int \frac{\sqrt{e+fx}((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf))+((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5bdf(3aC(de+c}{\sqrt{a+bx}\sqrt{c+dx}}}{5bf}}{7bdf}$$

$\downarrow$ 171

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$2\int \frac{3bde((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf)))-(bce+ade+acf)((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5}{}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\int \frac{3bde((bce+ade+3acf)(7bBdf-6aCdf-4bC(de+cf))+5bcf(3aC(de+cf)+b(cCe-7Adf)))-(bce+ade+acf)((2bde-bcf+4adf)(7bBdf-6aCdf-4bC(de+cf))+5b}{}$$

$\downarrow$ 176

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf}-$$

$$\frac{(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{\left(3bdf(\right.}{}$$

$\downarrow$ 124

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} -$$

$$\frac{(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f} + \frac{\sqrt{e+f}\cdots}{\cdots}$$

$\downarrow$ 123

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} -$$

$$\frac{(be-af)(de-cf)\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f} + \frac{2\sqrt{e+}\cdots}{\cdots}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} -$$

$$\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bd}{bc-}\cdots}}\cdots}{f\sqrt{c+dx}}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} -$$

$$\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\int\frac{\cdots}{\sqrt{a+bx}\sqrt{bc\cdots}}\cdots}{f\sqrt{c+dx}\sqrt{e+fx}}$$

$\downarrow$ 130

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}(e+fx)^{3/2}}{7bdf} -$$

$$\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(24a^2Cd^2f^2+abdf(-28Bdf-5cCf+13Cde)-\left(b^2\left(7df(-5Adf-Bcf+2Bde)-C\left(-4c^2f^2-cdef+8d^2e^2\right)\right)\right)\right)\text{Ellip}\cdots}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/Sqrt[a + b*x],x]
```

output
```
(2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(7*b*d*f) - ((-2*(7*b*
B*d*f - 6*a*C*d*f - 4*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*
x)^(3/2))/(5*b*f) + ((2*((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C*d*
f - 4*b*C*(d*e + c*f)) + 5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))
)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d) + ((2*Sqrt[-(b*c) + a
*d]*(3*b*d*f*((b*c*e + a*d*e + 3*a*c*f)*(7*b*B*d*f - 6*a*C*d*f - 4*b*C*(d*
e + c*f)) + 5*b*c*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))) + 2*((b*d*e
)/2 - (b*c + a*d)*f)*((2*b*d*e - b*c*f + 4*a*d*f)*(7*b*B*d*f - 6*a*C*d*f -
 4*b*C*(d*e + c*f)) + 5*b*d*f*(3*a*C*(d*e + c*f) + b*(c*C*e - 7*A*d*f))))*
Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sq
rt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(b*Sqr
t[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a
*d]*(b*e - a*f)*(d*e - c*f)*(24*a^2*C*d^2*f^2 + a*b*d*f*(13*C*d*e - 5*c*C*
f - 28*B*d*f) - b^2*(7*d*f*(2*B*d*e - B*c*f - 5*A*d*f) - C*(8*d^2*e^2 - c*
d*e*f - 4*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b
*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], (
(b*c - a*d)*f)/(d*(b*e - a*f))]])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x])
)/(3*b*d))/(5*b*f))/(7*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.58

method	result	size
elliptic	Expression too large to display	1205
default	Expression too large to display	10175

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/7*C/b*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))
/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+
a*c*e)^(1/2)+2/3*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C/b*(5/2*a*c*f+5/2*a*d*e+5/2
*b*c*e)-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2
*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*
c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*c*e-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b
*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C
/b*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f
+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a
*d*e+1/2*b*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^
(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+
a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),(
(-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(A*c*f+A*d*e+B*c*e-4/7*C/b*a*c*e-2/5*(B*d*
f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*a*d*
e+3/2*b*c*e)-2/3*(A*d*f+B*c*f+d*B*e+C*c*e-2/7*C/b*(5/2*a*c*f+5/2*a*d*e+5/2
*b*c*e)-2/5*(B*d*f+C*c*f+C*d*e-2/7*C/b*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2
*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(c/d-a/b)*((x+c/d)/(c/
d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1393, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
2/315*(3*(15*C*b^4*d^4*f^4*x^2 - 4*C*b^4*d^4*e^2*f^2 + (2*C*b^4*c*d^3 - (5
*C*a*b^3 - 7*B*b^4)*d^4)*e*f^3 - (4*C*b^4*c^2*d^2 + (5*C*a*b^3 - 7*B*b^4)*
c*d^3 - (24*C*a^2*b^2 - 28*B*a*b^3 + 35*A*b^4)*d^4)*f^4 + 3*(C*b^4*d^4*e*f
^3 + (C*b^4*c*d^3 - (6*C*a*b^3 - 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x + a)*sqrt(
d*x + c)*sqrt(f*x + e) - (8*C*b^4*d^4*e^4 - (9*C*b^4*c*d^3 - (5*C*a*b^3 -
14*B*b^4)*d^4)*e^3*f - (4*C*b^4*c^2*d^2 + 7*(C*a*b^3 - 3*B*b^4)*c*d^3 - (1
0*C*a^2*b^2 - 14*B*a*b^3 + 35*A*b^4)*d^4)*e^2*f^2 - (9*C*b^4*c^3*d + 7*(C*
a*b^3 - 3*B*b^4)*c^2*d^2 + 14*(3*C*a^2*b^2 - 4*B*a*b^3 + 10*A*b^4)*c*d^3 -
 (40*C*a^3*b - 49*B*a^2*b^2 + 70*A*a*b^3)*d^4)*e*f^3 + (8*C*b^4*c^4 + (5*C
*a*b^3 - 14*B*b^4)*c^3*d + (10*C*a^2*b^2 - 14*B*a*b^3 + 35*A*b^4)*c^2*d^2
+ (40*C*a^3*b - 49*B*a^2*b^2 + 70*A*a*b^3)*c*d^3 - 2*(24*C*a^4 - 28*B*a^3*
b + 35*A*a^2*b^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e
^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2
*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2
*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c +
a*d)*f)/(b*d*f)) - 3*(8*C*b^4*d^4*e^3*f - (5*C*b^4*c*d^3 - (9*C*a*b^3 - 14
*B*b^4)*d^4)*e^2*f^2 - (5*C*b^4*c^2*d^2 + 2*(4*C*a*b^3 - 7*B*b^4)*c*d^3 -
(16*C*a^2*b^2 - 21*B*a*b^3 + 35*A*b^4)*d^4)*e*f^3 + (8*C*b^4*c^3*d + (9*C*
a*b^3 - 14*B*b^4)*c^2*d^2 + (16*C*a^2*b^2 - 21*B*a*b^3 + 35*A*b^4)*c*d^...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/sqrt(a + b*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{bx+a}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{\sqrt{bx+a}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/sqrt(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{a+b\,x}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(1/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(1/2), x
)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{\sqrt{a+bx}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,(C\,x^2+Bx+A)}{\sqrt{bx+a}}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2),x)
```

3.85 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 708

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \frac{2(24a^2Cd^2f-abd(Cde+7cCf+20Bdf)-b^2(2c^2Cf-15Ad}{15b^3d(bc-ad)}$$

$$+\frac{2(6a^2Cdf+b^2(cCe+5Adf)-ab(Cde+cCf+5Bdf))\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5b^2d(bc-ad)(be-af)}$$

$$-\frac{2(Ab^2-a(bB-aC))(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$

$$+\frac{2\sqrt{-bc+ad}(48a^2Cd^2f^2-8abdf(Cde+cCf+5Bdf)+b^2(5df(Bde+Bcf+6Adf)-2C(d^2e^2-cdef}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$-\frac{2\sqrt{-bc+ad}(de-cf)(24a^2Cdf^2-abf(7Cde+cCf+20Bdf)+b^2(5df(Be+3Af)-Ce(2de-cf)))}{15b^4d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2/15*(24*a^2*C*d^2*f-a*b*d*(20*B*d*f+7*C*c*f+C*d*e)-b^2*(2*c^2*C*f-15*A*d^
2*f-c*d*(5*B*f+C*e)))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/d/(-a*
d+b*c)/f+2/5*(6*a^2*C*d*f+b^2*(5*A*d*f+C*c*e)-a*b*(5*B*d*f+C*c*f+C*d*e))*(
b*x+a)^(1/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b^2/d/(-a*d+b*c)/(-a*f+b*e)-2*(A*
b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+
a)^(1/2)+2/15*(a*d-b*c)^(1/2)*(48*a^2*C*d^2*f^2-8*a*b*d*f*(5*B*d*f+C*c*f+C
*d*e)+b^2*(5*d*f*(6*A*d*f+B*c*f+B*d*e)-2*C*(c^2*f^2-c*d*e*f+d^2*e^2)))*(b*
(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a
*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^4/d^(3/2)/f^2/(d*x+c)^(
1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/15*(a*d-b*c)^(1/2)*(-c*f+d*e)*(24*a^2*
C*d*f^2-a*b*f*(20*B*d*f+C*c*f+7*C*d*e)+b^2*(5*d*f*(3*A*f+B*e)-C*e*(-c*f+2*
d*e)))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF
(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/
b^4/d^(3/2)/f^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.11 (sec) , antiderivative size = 633, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx =$$

$$-\frac{2\left(-b^2\sqrt{-a+\frac{bc}{d}}(48a^2Cd^2f^2-8abdf(Cde+cCf+5Bdf)+b^2(5df(Bde+Bcf+6Adf)-2C(d^2e^2-\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),
x]
```

output
```
(-2*(-(b^2*Sqrt[-a + (b*c)/d]*(48*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f
 + 5*B*d*f) + b^2*(5*d*f*(B*d*e + B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*
f + c^2*f^2)))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)
*(e + f*x)*(15*(A*b^2 + a*(-(b*B) + a*C))*d*f - (-9*a*C*d*f + b*(C*d*e + c
*C*f + 5*B*d*f))*(a + b*x) - 3*b*C*d*f*x*(a + b*x)) - I*(b*c - a*d)*f*(48*
a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(B*d*e +
B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqr
t[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE
[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d
*f)] - I*b*f*(d*e - c*f)*(24*a^2*C*d^2*f - a*b*d*(C*d*e + 7*c*C*f + 20*B*d
*f) + b^2*(-2*c^2*C*f + 15*A*d^2*f + c*d*(C*e + 5*B*f)))*(a + b*x)^(3/2)*S
qrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipti
cF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a
*d*f)]))/(15*b^5*Sqrt[-a + (b*c)/d]*d^2*f^2*Sqrt[a + b*x]*Sqrt[c + d*x]*Sq
rt[e + f*x])
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2117, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \; 2117$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\Big(3C(de+cf)a^2-b(cCe+3Bde+3Bcf-Adf)a+b^2(Bce+2A(de+cf))+b\Big(\frac{6Cdfa^2}{b}-(Cde+cCf+5Bdf)a+b(cCe+5Adf)\Big)}{2b\sqrt{a+bx}}}{(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(cCe+3Bde+3Bcf-Adf)a+b^2(Bce+2A(de+cf))+b\left(\frac{6Cdfa^2}{b}-(Cde+cCf+5Bdf)a+b(cCe+5Adf)\right)x\right)}{\sqrt{a+bx}}dx}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 171

$$\frac{\frac{2\int \frac{(bc-ad)\sqrt{e+fx}\left(6Cf(de+3cf)a^2-b(5Bf(de+3cf)+Ce(de+7cf))a-b^2\left(cCe^2-5Adfe-5cf(Be+2Af)\right)+\left((5df(Be+3Af)-Ce(2de-cf))b^2-af(7Cde+cCf+2\right.\right.}{2\sqrt{a+bx}\sqrt{c+dx}}}{5bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{(bc-ad)\int \frac{\sqrt{e+fx}\left(6Cf(de+3cf)a^2-b(5Bf(de+3cf)+Ce(de+7cf))a-b^2\left(cCe^2-5Adfe-5cf(Be+2Af)\right)+\left((5df(Be+3Af)-Ce(2de-cf))b^2-af(7Cde+cCf+2\right.\right.}{\sqrt{a+bx}\sqrt{c+dx}}}{5bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 171

$$(bc-ad)\left(\frac{2\int \frac{(be-af)\left(24Cdf(de+cf)a^2-b\left(20Bdf(de+cf)+C\left(d^2e^2+14cdfe+c^2f^2\right)\right)a-b^2\left(Cefc^2+d\left(Ce^2-5f(2Be+3Af)\right)c-15Ad^2ef\right)+\left(\left(5df(Bde+Bcf+6\right.\right.\right.}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3bd}\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$(bc-ad)\left(\frac{(be-af)\int \frac{24Cdf(de+cf)a^2-b\left(20Bdf(de+cf)+C\left(d^2e^2+14cdfe+c^2f^2\right)\right)a-b^2\left(Cefc^2+d\left(Ce^2-5f(2Be+3Af)\right)c-15Ad^2ef\right)+\left(\left(5df(Bde+Bcf+6A\right.\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3bd}\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 176

$$\frac{(bc-ad)\left(\frac{(be-af)\left(\frac{\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)\int\frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}-\frac{(de-cf)\left(24a^2Cdf^2-a\right.}{}}{3bd}\right.}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\big\downarrow$ 124

$$\frac{(bc-ad)\left(\frac{(be-af)\left(\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)\int\frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-a}}}}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.}{3bd}\right.}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\big\downarrow$ 123

$$\frac{(bc-ad)\left(\frac{(be-af)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a}}{\sqrt{ad-}}\right.\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.}{3bd}\right.}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\big\downarrow$ 131

$$(bc-ad)\left((be-af)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a\ldots}}{\sqrt{ad-\ldots}}\right.\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\ldots\right.\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\big\downarrow$ 131

$$(bc-ad)\left((be-af)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a\ldots}}{\sqrt{ad-\ldots}}\right.\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\ldots\right.\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\big\downarrow$ 130

$$(bc-ad)\left((be-af)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(48a^2Cd^2f^2-8abdf(5Bdf+cCf+Cde)+b^2\left(5df(6Adf+Bcf+Bde)-2C\left(c^2f^2-cdef+d^2e^2\right)\right)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a\ldots}}{\sqrt{ad-\ldots}}\right.\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\ldots\right.\right.$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(3/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d
)*(b*e - a*f)*Sqrt[a + b*x]) + ((2*((6*a^2*C*d*f)/b + b*(c*C*e + 5*A*d*f)
- a*(C*d*e + c*C*f + 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2)
)/(5*f) + ((b*c - a*d)*((2*(24*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B
*d*f) + b^2*(5*d*f*(B*e + 3*A*f) - C*e*(2*d*e - c*f)))*Sqrt[a + b*x]*Sqrt[
c + d*x]*Sqrt[e + f*x])/(3*b*d) + ((b*e - a*f)*((2*Sqrt[-(b*c) + a*d]*(48*
a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 5*B*d*f) + b^2*(5*d*f*(B*d*e +
B*c*f + 6*A*d*f) - 2*C*(d^2*e^2 - c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/
(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-
(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x
]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) - (2*Sqrt[-(b*c) + a*d]*(d*e - c*f)*(24
*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 20*B*d*f) + b^2*(5*d*f*(B*e + 3*A*
f) - C*e*(2*d*e - c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x)
)/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]
], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f
*x])))/(3*b*d)))/(5*b*f))/(b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 1163, normalized size of antiderivative = 1.64

method	result	size
elliptic	Expression too large to display	1163
default	Expression too large to display	6233

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)*(A*b^2-B*a*b+C*a^2)/b^4/((x+a/b)*(b*
d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2/5*C/b^2*x*(b*d*f*x^3+a*d*f*x^2+b*c
*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2/3*(1/b^2*(B*b*d*f-
C*a*d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f
*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*
(-(A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*e-B*a^2*b*d*f+B*a*b^2*c*f+B*a*b^2*d*e-B*b
^3*c*e+C*a^3*d*f-C*a^2*b*c*f-C*a^2*b*d*e+C*a*b^2*c*e)/b^4+(A*b^2-B*a*b+C*a
^2)/b^4*(a*d*f-b*c*f-b*d*e)+(b*c*f+b*d*e)*(A*b^2-B*a*b+C*a^2)/b^4-2/5*C/b^
2*a*c*e-2/3*(1/b^2*(B*b*d*f-C*a*d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*
b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(c/d-a/b)*((x+c/d)/
(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*
d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)
*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(1/b
^3*(A*b^2*d*f-B*a*b*d*f+B*b^2*c*f+B*b^2*d*e+C*a^2*d*f-C*a*b*c*f-C*a*b*d*e+
C*b^2*c*e)+(A*b^2-B*a*b+C*a^2)/b^3*d*f-2/5*C/b^2*(3/2*a*c*f+3/2*a*d*e+3/2*
b*c*e)-2/3*(1/b^2*(B*b*d*f-C*a*d*f+C*b*c*f+C*b*d*e)-2/5*C/b^2*(2*a*d*f+2*b
*c*f+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1
/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f
*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-c/d+e/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1463 vs. $2(647) = 1294$.

Time = 0.16 (sec) , antiderivative size = 1463, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2),x, algor
ithm="fricas")
```

output
```
2/45*(3*(3*C*b^4*d^3*f^3*x^2 + C*a*b^3*d^3*e*f^2 + (C*a*b^3*c*d^2 - (24*C*
a^2*b^2 - 20*B*a*b^3 + 15*A*b^4)*d^3)*f^3 + (C*b^4*d^3*e*f^2 + (C*b^4*c*d^
2 - (6*C*a*b^3 - 5*B*b^4)*d^3)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*
x + e) + (2*C*a*b^3*d^3*e^3 - (3*C*a*b^3*c*d^2 - (7*C*a^2*b^2 - 5*B*a*b^3)
*d^3)*e^2*f - (3*C*a*b^3*c^2*d + 4*(7*C*a^2*b^2 - 5*B*a*b^3)*c*d^2 - (32*C
*a^3*b - 25*B*a^2*b^2 + 15*A*a*b^3)*d^3)*e*f^2 + (2*C*a*b^3*c^3 + (7*C*a^2
*b^2 - 5*B*a*b^3)*c^2*d + (32*C*a^3*b - 25*B*a^2*b^2 + 15*A*a*b^3)*c*d^2 -
 2*(24*C*a^4 - 20*B*a^3*b + 15*A*a^2*b^2)*d^3)*f^3 + (2*C*b^4*d^3*e^3 - (3
*C*b^4*c*d^2 - (7*C*a*b^3 - 5*B*b^4)*d^3)*e^2*f - (3*C*b^4*c^2*d + 4*(7*C*
a*b^3 - 5*B*b^4)*c*d^2 - (32*C*a^2*b^2 - 25*B*a*b^3 + 15*A*b^4)*d^3)*e*f^2
 + (2*C*b^4*c^3 + (7*C*a*b^3 - 5*B*b^4)*c^2*d + (32*C*a^2*b^2 - 25*B*a*b^3
 + 15*A*b^4)*c*d^2 - 2*(24*C*a^3*b - 20*B*a^2*b^2 + 15*A*a*b^3)*d^3)*f^3)*
x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*
e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e
^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*
b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3
)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(2*C
*a*b^3*d^3*e^2*f - (2*C*a*b^3*c*d^2 - (8*C*a^2*b^2 - 5*B*a*b^3)*d^3)*e*f^2
 + (2*C*a*b^3*c^2*d + (8*C*a^2*b^2 - 5*B*a*b^3)*c*d^2 - 2*(24*C*a^3*b - 20
*B*a^2*b^2 + 15*A*a*b^3)*d^3)*f^3 + (2*C*b^4*d^3*e^2*f - (2*C*b^4*c*d^2...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx=\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(3/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(3/2),
x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x
)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(3/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(3/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(3/2), x
)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{3/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,(C\,x^2+Bx+A)}{(bx+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2),x)
```

3.86 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 687

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \frac{2(8a^2Cdf+b^2(cCe+3Bde+Adf)-ab(7Cde+cCf+4Bdf)}{3b^3(bc-ad)(be-af)}$$

$$-\frac{2(bB-2aC)(c+dx)^{3/2}\sqrt{e+fx}}{b^2(bc-ad)\sqrt{a+bx}}-\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$

$$+\frac{2(16a^3Cd^2f^2-8a^2bdf(Bdf+2C(de+cf))-b^3(c^2Cef+Ad^2ef+cd(Ce^2+6Bef+Af^2))+ab^2(df(7}{3b^4\sqrt{d}\sqrt{-bc+ad}f(be-}$$

$$+\frac{2(de-cf)\,(8a^2Cdf+b^2(cCe+3Bcf+Adf)-ab(Cde+7cCf+4Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\mathrm{EllipticF}\left(\right.}{3b^4\sqrt{d}\sqrt{-bc+ad}f\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2/3*(8*a^2*C*d*f+b^2*(A*d*f+3*B*d*e+C*c*e)-a*b*(4*B*d*f+C*c*f+7*C*d*e))*(b
*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)/(-a*f+b*e)-2*(B*b-2
*C*a)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/(b*x+a)^(1/2)-2/3*(A*b^2-
a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^(
3/2)+2/3*(16*a^3*C*d^2*f^2-8*a^2*b*d*f*(B*d*f+2*C*(c*f+d*e))-b^3*(c^2*C*e*
f+A*d^2*e*f+c*d*(A*f^2+6*B*e*f+C*e^2))+a*b^2*(d*f*(2*A*d*f+7*B*c*f+7*B*d*e
)+C*(c^2*f^2+16*c*d*e*f+d^2*e^2)))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1
/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+
b*e))^(1/2))/b^4/d^(1/2)/(a*d-b*c)^(1/2)/f/(-a*f+b*e)/(d*x+c)^(1/2)/(b*(f*
x+e)/(-a*f+b*e))^(1/2)+2/3*(-c*f+d*e)*(8*a^2*C*d*f+b^2*(A*d*f+3*B*c*f+C*c*
e)-a*b*(4*B*d*f+7*C*c*f+C*d*e))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-
a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c
)*f/d/(-a*f+b*e))^(1/2))/b^4/d^(1/2)/(a*d-b*c)^(1/2)/f/(d*x+c)^(1/2)/(f*x+
e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.22 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}df(c+dx)(e+fx)\left((Ab^2+a(-bB+aC)\right)(bc-ad)(be-af)+(-8a^3Cdf+b^3(3Bce+\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),
x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) +
a*C))*(b*c - a*d)*(b*e - a*f) + (-8*a^3*C*d*f + b^3*(3*B*c*e + A*d*e + A*c
*f) - 2*a*b^2*(3*c*C*e + 2*B*d*e + 2*B*c*f + A*d*f) + a^2*b*(5*B*d*f + 7*C
*(d*e + c*f)))*(a + b*x) - C*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (a + b
*x)*(b^2*Sqrt[-a + (b*c)/d]*(16*a^3*C*d^2*f^2 - 8*a^2*b*d*f*(B*d*f + 2*C*(
d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e^2 + 6*B*e*f + A*f^2))
+ a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*e^2 + 16*c*d*e*f + c^2
*f^2)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(16*a^3*C*d^2*f^2 - 8*a^2*b*
d*f*(B*d*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C*e^2 +
6*B*e*f + A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d^2*e^2
+ 16*c*d*e*f + c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))
]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]
/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*d)*f*(d*e
 - c*f)*(8*a^2*C*d*f + b^2*(c*C*e + 3*B*d*e + A*d*f) - a*b*(7*C*d*e + c*C*
f + 4*B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e
 + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])))/(3*b^5*Sqrt[-a + (b*c)/d]*d*(b*c
- a*d)*f*(b*e - a*f)*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2117, 27, 167, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}dx$$

$$\Big\downarrow \text{ 2117}$$

$$-\frac{2\int -\frac{3\sqrt{c+dx}\sqrt{e+fx}\left(C(de+cf)a^2-b(cCe+Bde+Bcf-Adf)a+b^2Bce+b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x\right)}{2b(a+bx)^{3/2}}dx}{3(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(C(de+cf)a^2-b(cCe+Bde+Bcf-Adf)a+b^2Bce+b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x\right)}{(a+bx)^{3/2}}dx}{b(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 167$$

$$\frac{\frac{2\int \frac{(be-af)\sqrt{e+fx}\left(2Cd(de+3cf)a^2-b\left(5Cfc^2+3d(Ce+Bf)c+Bd^2e\right)a+b^2c(cCe+Bde+2Bcf+Adf)+d\left(8Cdfa^2-b(Cde+7cCf+4Bdf)a+b^2(cCe+3Bcf+Adf)\right)\cdots}{2\sqrt{a+bx}\sqrt{c+dx}}}{b(be-af)}\cdots}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\sqrt{e+fx}\left(2Cd(de+3cf)a^2-b\left(5Cfc^2+3d(Ce+Bf)c+Bd^2e\right)a+b^2c(cCe+Bde+2Bcf+Adf)+d\left(8Cdfa^2-b(Cde+7cCf+4Bdf)a+b^2(cCe+3Bcf+Adf)\right)x\right)}{\sqrt{a+bx}\sqrt{c+dx}}dx}{b}}{b(bc-ad)(be-af)}-$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 171$$

$$\frac{\frac{2\int -\frac{d\left((bce+ade+acf)\left(8Cdfa^2-b(Cde+7cCf+4Bdf)a+b^2(cCe+3Bcf+Adf)\right)-3be\left(2Cd(de+3cf)a^2-b\left(5Cfc^2+3d(Ce+Bf)c+Bd^2e\right)a+b^2c(cCe+Bde+2Bcf\cdots}{2\sqrt{a+bx}\sqrt{c\cdots}}}{3bc\cdots}}{\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{\int \frac{(bce+ade+acf)\left(8Cdfa^2-b(Cde+7cCf+4Bdf)a+b^2(cCe+3Bcf+Adf)\right)-3b\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 176$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{(be-af)(de-cf)\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)\int \frac{\cdots}{\sqrt{a\cdots}}}{f}\cdots$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 124

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{\frac{(be-af)(de-cf)\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)\int \frac{\cdots}{\sqrt{a\cdots}}}{f}\cdots}{\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 123

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{\frac{(be-af)(de-cf)\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)\int \frac{\cdots}{\sqrt{a\cdots}}}{f}\cdots}{\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 131

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b}-\frac{\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+\cdots\right.}{f\sqrt{c+dx}}\cdots}{\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 131

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b} - \frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(A\ldots\right.}{f\sqrt{c+dx}\sqrt{e+f\ldots}}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{130}$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cdf-ab(4Bdf+7cCf+Cde)+b^2(Adf+3Bcf+cCe)\right)}{3b} - \frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(8a^2Cdf-ab(4Bdf+7cCf+C\ldots\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\ldots}}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

input

```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(5/2),x]
```

output

```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)^(3/2)) + ((-2*(b*B - 2*a*C)*(b*c - a*d)*Sqrt[c +
 d*x]*(e + f*x)^(3/2))/(b*Sqrt[a + b*x]) + ((2*(8*a^2*C*d*f + b^2*(c*C*e +
 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*B*d*f))*Sqrt[a + b*x]*Sqrt[c
+ d*x]*Sqrt[e + f*x])/(3*b) - ((2*Sqrt[-(b*c) + a*d]*(16*a^3*C*d^2*f^2 - 8
*a^2*b*d*f*(B*d*f + 2*C*(d*e + c*f)) - b^3*(c^2*C*e*f + A*d^2*e*f + c*d*(C
*e^2 + 6*B*e*f + A*f^2)) + a*b^2*(d*f*(7*B*d*e + 7*B*c*f + 2*A*d*f) + C*(d
^2*e^2 + 16*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e +
f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c -
 a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(
b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(8*a^2*C*d*f
+ b^2*(c*C*e + 3*B*c*f + A*d*f) - a*b*(C*d*e + 7*c*C*f + 4*B*d*f))*Sqrt[(b
*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[
(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f
))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(3*b))/b)/(b*(b*c - a*d)*(
b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1377 vs. $2(627) = 1254$.

Time = 6.86 (sec) , antiderivative size = 1378, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1378
default	Expression too large to display	16508

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/3*(A*b^2-B*a*b+C*a^2)/b^5*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c
*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/3*(b*d*f*x^2+b*c*f*x+b*d*e*x
+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(2*A*a*b^2*d*f-A*b^3*c*f-A*b
^3*d*e-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^3*d*f-7
*C*a^2*b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*
e*x+b*c*e))^(1/2)+2/3*C/b^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f
*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*((A*b^2*d*f-2*B*a*b*d*f+B*b^2*c*f+B*b^2*
d*e+3*C*a^2*d*f-2*C*a*b*c*f-2*C*a*b*d*e+C*b^2*c*e)/b^4-1/3*(A*b^2-B*a*b+C*
a^2)/b^4*d*f-1/3/b^4*(a*d*f-b*c*f-b*d*e)*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*
e-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^3*d*f-7*C*a^
2*b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)-1/3
*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(2*A*a*b^2*d*f-A*b^3*
c*f-A*b^3*d*e-5*B*a^2*b*d*f+4*B*a*b^2*c*f+4*B*a*b^2*d*e-3*B*b^3*c*e+8*C*a^
3*d*f-7*C*a^2*b*c*f-7*C*a^2*b*d*e+6*C*a*b^2*c*e)-2/3*C/b^3*(1/2*a*c*f+1/2*
a*d*e+1/2*b*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))
^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2
+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),
((-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(1/b^3*(B*b*d*f-2*C*a*d*f+C*b*c*f+C*b*d*e
)-1/3/b^3*d*f*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*d*e-5*B*a^2*b*d*f+4*B*a*b^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2588 vs. $2(627) = 1254$.

Time = 0.27 (sec) , antiderivative size = 2588, normalized size of antiderivative = 3.77

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*(((6*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d^2 - (7*C*a^3*b^3 - 3*B*a^2*
b^4)*d^3)*e*f^2 - ((7*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 - (8*C*a^4*b^2 - 4*B*
a^3*b^3 + A*a^2*b^4)*d^3)*f^3 + ((C*b^6*c*d^2 - C*a*b^5*d^3)*e*f^2 - (C*a*
b^5*c*d^2 - C*a^2*b^4*d^3)*f^3)*x^2 + (((8*C*a*b^5 - 3*B*b^6)*c*d^2 - (9*C
*a^2*b^4 - 4*B*a*b^5 + A*b^6)*d^3)*e*f^2 - ((9*C*a^2*b^4 - 4*B*a*b^5 + A*b
^6)*c*d^2 - (10*C*a^3*b^3 - 5*B*a^2*b^4 + 2*A*a*b^5)*d^3)*f^3)*x)*sqrt(b*x
 + a)*sqrt(d*x + c)*sqrt(f*x + e) - ((C*a^2*b^4*c*d^2 - C*a^3*b^3*d^3)*e^3
 - (4*C*a^2*b^4*c^2*d - (11*C*a^3*b^3 - 3*B*a^2*b^4)*c*d^2 + (6*C*a^4*b^2
- 2*B*a^3*b^3 - A*a^2*b^4)*d^3)*e^2*f + (C*a^2*b^4*c^3 + (11*C*a^3*b^3 - 3
*B*a^2*b^4)*c^2*d - 2*(19*C*a^4*b^2 - 8*B*a^3*b^3 + 2*A*a^2*b^4)*c*d^2 + (
24*C*a^5*b - 11*B*a^4*b^2 + 2*A*a^3*b^3)*d^3)*e*f^2 - (C*a^3*b^3*c^3 + (6*
C*a^4*b^2 - 2*B*a^3*b^3 - A*a^2*b^4)*c^2*d - (24*C*a^5*b - 11*B*a^4*b^2 +
2*A*a^3*b^3)*c*d^2 + 2*(8*C*a^6 - 4*B*a^5*b + A*a^4*b^2)*d^3)*f^3 + ((C*b^
6*c*d^2 - C*a*b^5*d^3)*e^3 - (4*C*b^6*c^2*d - (11*C*a*b^5 - 3*B*b^6)*c*d^2
 + (6*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*d^3)*e^2*f + (C*b^6*c^3 + (11*C*a*b^5
 - 3*B*b^6)*c^2*d - 2*(19*C*a^2*b^4 - 8*B*a*b^5 + 2*A*b^6)*c*d^2 + (24*C*a
^3*b^3 - 11*B*a^2*b^4 + 2*A*a*b^5)*d^3)*e*f^2 - (C*a*b^5*c^3 + (6*C*a^2*b^
4 - 2*B*a*b^5 - A*b^6)*c^2*d - (24*C*a^3*b^3 - 11*B*a^2*b^4 + 2*A*a*b^5)*c
*d^2 + 2*(8*C*a^4*b^2 - 4*B*a^3*b^3 + A*a^2*b^4)*d^3)*f^3)*x^2 + 2*((C*a*b
^5*c*d^2 - C*a^2*b^4*d^3)*e^3 - (4*C*a*b^5*c^2*d - (11*C*a^2*b^4 - 3*B*...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(5/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(5/2),
x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x
)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(5/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{(a+b\,x)^{5/2}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(5/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(5/2), x
)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{5/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,(C\,x^2+Bx+A)}{(bx+a)^{\frac{5}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2),x)
```

3.87 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 961

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \frac{2(24a^3Cdf^2 - a^2bf(41Cde + 23cCf + 4Bdf) - b^3(15cCe^2 + A}{1}$$

$$+\frac{2(6a^3Cdf + ab^2(10cCe + 3Bde + 3Bcf - 4Adf) - b^3(5Bce - 2A(de + cf)) - a^2b(Bdf + 8C(de + cf)))}{15b^2(bc-ad)^2(be-af)(a+bx)^{3/2}}$$

$$-\frac{2(Ab^2 - a(bB - aC))\,(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2\sqrt{d}(48a^4Cd^2f^2 - 8a^3bdf(Bdf + 11C(de+cf)) - b^4(2Ad^2e^2 - cde(5Be + 2Af) - c^2(30Ce^2 + 5Bef -}{}$$

$$+\frac{2(de-cf)\,(24a^3Cd^2f - a^2bd(23Cde + 41cCf + 4Bdf) - b^3(15c^2Ce - 2Ad^2e + cd(5Be + Af)) + ab^2(1}{15b^4\sqrt{d}(-bc+ad)^{3/2}(be}$$

output
```
2/15*(24*a^3*C*d*f^2-a^2*b*f*(4*B*d*f+23*C*c*f+41*C*d*e)-b^3*(15*c*C*e^2+A
*d*e*f+c*f*(-2*A*f+5*B*e))+a*b^2*(5*C*e*(8*c*f+3*d*e)+f*(-A*d*f+3*B*c*f+6*
B*d*e)))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(
1/2)+2/15*(6*a^3*C*d*f+a*b^2*(-4*A*d*f+3*B*c*f+3*B*d*e+10*C*c*e)-b^3*(5*B*
c*e-2*A*(c*f+d*e))-a^2*b*(B*d*f+8*C*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(1/2
)/b^2/(-a*d+b*c)^2/(-a*f+b*e)/(b*x+a)^(3/2)-2/5*(A*b^2-a*(B*b-C*a))*(d*x+c
)^(3/2)*(f*x+e)^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^(5/2)+2/15*d^(1/2)*(
48*a^4*C*d^2*f^2-8*a^3*b*d*f*(B*d*f+11*C*(c*f+d*e))-b^4*(2*A*d^2*e^2-c*d*e
*(2*A*f+5*B*e)-c^2*(-2*A*f^2+5*B*e*f+30*C*e^2))-a*b^3*(d^2*e*(-2*A*f+3*B*e
)+c^2*f*(3*B*f+70*C*e)+2*c*d*(-A*f^2+11*B*e*f+35*C*e^2))+a^2*b^2*(2*C*(19*
c^2*f^2+81*c*d*e*f+19*d^2*e^2)-d*f*(2*A*d*f-13*B*(c*f+d*e))))*(b*(d*x+c)/(
-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(
1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^4/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(
d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/15*(-c*f+d*e)*(24*a^3*C*d^2*f-
a^2*b*d*(4*B*d*f+41*C*c*f+23*C*d*e)-b^3*(15*c^2*C*e-2*A*d^2*e+c*d*(A*f+5*B
*e))+a*b^2*(15*c^2*C*f+d^2*(-A*f+3*B*e)+c*(6*B*d*f+40*C*d*e)))*(b*(d*x+c)/
(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(
1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^4/d^(1/2)/(a*d-b
*c)^(3/2)/(-a*f+b*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.93 (sec) , antiderivative size = 1444, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),
x]
```

output
```
Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5
*b^3*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e + A*b^3*d*e - 6*a*b^2
*B*d*e + 11*a^2*b*C*d*e + A*b^3*c*f - 6*a*b^2*B*c*f + 11*a^2*b*c*C*f - 2*a
*A*b^2*d*f + 7*a^2*b*B*d*f - 12*a^3*C*d*f))/(15*b^3*(b*c - a*d)*(b*e - a*f
)*(a + b*x)^2) - (2*(15*b^4*c^2*C*e^2 + 5*b^4*B*c*d*e^2 - 40*a*b^3*c*C*d*e
^2 - 2*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 23*a^2*b^2*C*d^2*e^2 + 5*b^4*B*
c^2*e*f - 40*a*b^3*c^2*C*e*f + 2*A*b^4*c*d*e*f - 22*a*b^3*B*c*d*e*f + 102*
a^2*b^2*c*C*d*e*f + 2*a*A*b^3*d^2*e*f + 13*a^2*b^2*B*d^2*e*f - 58*a^3*b*C*
d^2*e*f - 2*A*b^4*c^2*f^2 - 3*a*b^3*B*c^2*f^2 + 23*a^2*b^2*c^2*C*f^2 + 2*a
*A*b^3*c*d*f^2 + 13*a^2*b^2*B*c*d*f^2 - 58*a^3*b*c*C*d*f^2 - 2*a^2*A*b^2*d
^2*f^2 - 8*a^3*b*B*d^2*f^2 + 33*a^4*C*d^2*f^2))/(15*b^3*(b*c - a*d)^2*(b*e
 - a*f)^2*(a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(48*a^4*C*d
^2*f^2 - 8*a^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) + b^4*(-2*A*d^2*e^2 + c*d*
e*(5*B*e + 2*A*f) + c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*b^3*(d^2*e*(3*
B*e - 2*A*f) + c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11*B*e*f - A*f^2
)) + a^2*b^2*(2*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) + d*f*(-2*A*d*f +
 13*B*(d*e + c*f))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f + (b*e)/(a
 + b*x) - (a*f)/(a + b*x)) + (I*(-(b*c) + a*d)*f*(-48*a^4*C*d^2*f^2 + 8*a^
3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) - b^4*(-2*A*d^2*e^2 + c*d*e*(5*B*e + 2*
A*f) + c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) + a*b^3*(d^2*e*(3*B*e - 2*A*...
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 996, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2117, 27, 167, 27, 167, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(5cCe+3Bde+3Bcf-5Adf)a+b^2(5Bce-2A(de+cf))-b\left(-\frac{6Cdfa^2}{b}+Bdfa+5C(de+cf)a-b(5cCe+A\cdots\right.\right.}{2b(a+bx)^{5/2}}}{5(bc-ad)(be-af)}$$
$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(5cCe+3Bde+3Bcf-5Adf)a+b^2(5Bce-2A(de+cf))-b\left(-\frac{6Cdfa^2}{b}+Bdfa+5C(de+cf)a-b(5cCe+Adf)\right)x\cdots\right)}{(a+bx)^{5/2}}}{5b(bc-ad)(be-af)}$$
$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{167}$$

$$\frac{2\int -\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3-b\left(Bdf(de+3cf)+C\left(8d^2e^2+41cdfe+15c^2f^2\right)\right)a^2+b^2\left(30Cefc^2+d\left(25Ce^2+6Bfe+3Af^2\right)c+d^2e(3Be-4Af)\right)a-b^3e\left(15Cec^2+d\cdots\right.\right.}{2(a+bx)^{3/2}\cdots}}{3b(be-a\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(Bdf+8C(cf+de))+ab^2(-4Adf+3Bcf+3Bde+10cCe)-b^3(5Bce-2A(cf+de))\right)}{3b(a+bx)^{3/2}(be-af)}-\frac{\int \frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{167}$$

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\int -\frac{24Cd^2f^2(de+cf)a^4-bdf\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}} - \frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24C\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 176

$$\frac{2\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf-a^2b(Bdf+8C(cf+de))+ab^2(-4Adf+3Bcf+3Bde+10cCe)-b^3(5Bce-2A(cf+de))\right)}{3b(a+bx)^{3/2}(be-af)} - \frac{-\frac{(be-af)(de-cf)\left(24a^3Cd\right.}{}}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 124

$$\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}} - \frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24C\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 123

$$\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}} - \frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24C\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 131

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}}-\frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24C\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 131

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}}-\frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24C\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 130

$$\frac{\frac{2\left(6Cdfa^3-b(Bdf+8C(de+cf))a^2+b^2(10cCe+3Bde+3Bcf-4Adf)a-b^3(5Bce-2A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{3b(be-af)(a+bx)^{3/2}}-\frac{-\frac{2(be-af)\sqrt{c+dx}\sqrt{e+fx}\left(24C\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(7/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)^(5/2)) + ((2*(6*a^3*C*d*f + a*b^2*(10*c*C*e + 3*
B*d*e + 3*B*c*f - 4*A*d*f) - b^3*(5*B*c*e - 2*A*(d*e + c*f)) - a^2*b*(B*d*
f + 8*C*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(3*b*(b*e - a*f)*(a +
 b*x)^(3/2)) - ((-2*(b*e - a*f)*(24*a^3*C*d^2*f - a^2*b*d*(23*C*d*e + 41*c
*C*f + 4*B*d*f) - b^3*(15*c^2*C*e - 2*A*d^2*e + c*d*(5*B*e + A*f)) + a*b^2
*(15*c^2*C*f + d^2*(3*B*e - A*f) + c*(40*C*d*e + 6*B*d*f)))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(b*(b*c - a*d)*Sqrt[a + b*x]) - ((2*Sqrt[d]*Sqrt[-(b*c) + a
*d]*(48*a^4*C*d^2*f^2 - 8*a^3*b*d*f*(B*d*f + 11*C*(d*e + c*f)) - b^4*(2*A*
d^2*e^2 - c*d*e*(5*B*e + 2*A*f) - c^2*(30*C*e^2 + 5*B*e*f - 2*A*f^2)) - a*
b^3*(d^2*e*(3*B*e - 2*A*f) + c^2*f*(70*C*e + 3*B*f) + 2*c*d*(35*C*e^2 + 11
*B*e*f - A*f^2)) + a^2*b^2*(2*C*(19*d^2*e^2 + 81*c*d*e*f + 19*c^2*f^2) - d
*f*(2*A*d*f - 13*B*(d*e + c*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e +
 f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c
- a*d)*f)/(d*(b*e - a*f))]])/(b*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f
)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(24*a^3*C*d^2*f - a^2*
b*d*(23*C*d*e + 41*c*C*f + 4*B*d*f) - b^3*(15*c^2*C*e - 2*A*d^2*e + c*d*(5
*B*e + A*f)) + a*b^2*(15*c^2*C*f + d^2*(3*B*e - A*f) + c*(40*C*d*e + 6*B*d
*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*Elli
pticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2291 vs. 2(899) = 1798.

Time = 8.05 (sec) , antiderivative size = 2292, normalized size of antiderivative = 2.39

method	result	size
elliptic	Expression too large to display	2292
default	Expression too large to display	35601

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/5*(A*b^2-B*a*b+C*a^2)/b^6*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c
*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^3+2/15*(2*A*a*b^2*d*f-A*b^3*c*f-
A*b^3*d*e-7*B*a^2*b*d*f+6*B*a*b^2*c*f+6*B*a*b^2*d*e-5*B*b^3*c*e+12*C*a^3*d
*f-11*C*a^2*b*c*f-11*C*a^2*b*d*e+10*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+
b^2*c*e)/b^5*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)/(x+a/b)^2+2/15*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)^2/b^4*(2*A*a^2*b^2*d^2*f^2-2*A*a*b^3*c*d*f^2-2*A
*a*b^3*d^2*e*f+2*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+2*A*b^4*d^2*e^2+8*B*a^3*b*d
^2*f^2-13*B*a^2*b^2*c*d*f^2-13*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+22*B*a*
b^3*c*d*e*f+3*B*a*b^3*d^2*e^2-5*B*b^4*c^2*e*f-5*B*b^4*c*d*e^2-33*C*a^4*d^2
*f^2+58*C*a^3*b*c*d*f^2+58*C*a^3*b*d^2*e*f-23*C*a^2*b^2*c^2*f^2-102*C*a^2*
b^2*c*d*e*f-23*C*a^2*b^2*d^2*e^2+40*C*a*b^3*c^2*e*f+40*C*a*b^3*c*d*e^2-15*
C*b^4*c^2*e^2)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*((B*b*d
*f-3*C*a*d*f+C*b*c*f+C*b*d*e)/b^4+1/15*d*f*(2*A*a*b^2*d*f-A*b^3*c*f-A*b^3*
d*e-7*B*a^2*b*d*f+6*B*a*b^2*c*f+6*B*a*b^2*d*e-5*B*b^3*c*e+12*C*a^3*d*f-11*
C*a^2*b*c*f-11*C*a^2*b*d*e+10*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^4-1/15/b^4*(a*d*f-b*c*f-b*d*e)*(2*A*a^2*b^2*d^2*f^2-2*A*a*b^3*c*d*f^2
-2*A*a*b^3*d^2*e*f+2*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+2*A*b^4*d^2*e^2+8*B*a^3
*b*d^2*f^2-13*B*a^2*b^2*c*d*f^2-13*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4721 vs. $2(899) = 1798$.

Time = 0.85 (sec) , antiderivative size = 4721, normalized size of antiderivative = 4.91

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(7/2),x)
```

output
```
Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x)**(7/2),
x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x
)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(7/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{(a+b\,x)^{7/2}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(7/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(7/2), x
)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{7/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,(C\,x^2+Bx+A)}{(bx+a)^{\frac{7}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2),x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2),x)
```

3.88 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1714

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Too large to display}$$

output
```
-2/105*(24*a^4*C*d^2*f^2-a^3*b*d*f*(-4*B*d*f+61*C*c*f+43*C*d*e)+b^4*(8*A*d
^2*e^2-c*d*e*(A*f+14*B*e)+c^2*(-4*A*f^2+7*B*e*f+35*C*e^2))+3*a*b^3*(d^2*e*
(-5*A*f+2*B*e)-c^2*f*(B*f+28*C*e)-c*d*(-3*A*f^2-5*B*e*f+14*C*e^2))-3*a^2*b
^2*(d*f*(-A*d*f+3*B*c*f+2*B*d*e)-C*(15*c^2*f^2+37*c*d*e*f+5*d^2*e^2)))*(d*
x+c)^(1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/105
*(48*a^5*C*d^3*f^3+8*a^4*b*d^2*f^2*(B*d*f-16*C*(c*f+d*e))-b^5*(8*A*d^3*e^3
-c*d^2*e^2*(5*A*f+14*B*e)+c^2*d*e*(-5*A*f^2+14*B*e*f+35*C*e^2)+c^3*f*(8*A*
f^2-14*B*e*f+35*C*e^2))-a*b^4*(d^3*e^2*(-19*A*f+6*B*e)-6*c^3*f^2*(-B*f+7*C
*e)-c^2*d*f*(238*C*e^2-19*f*(-A*f+B*e))-c*d^2*e*(42*C*e^2-f*(20*A*f+19*B*e
)))+a^3*b^2*d*f*(C*(103*c^2*f^2+344*c*d*e*f+103*d^2*e^2)+d*f*(6*A*d*f-19*B
*(c*f+d*e)))-3*a^2*b^3*(C*(5*c^3*f^3+94*c^2*d*e*f^2+94*c*d^2*e^2*f+5*d^3*e
^3)+d*f*(3*A*d*f*(c*f+d*e)-B*(3*c^2*f^2+16*c*d*e*f+3*d^2*e^2))))*(d*x+c)^(
1/2)*(f*x+e)^(1/2)/b^3/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/35*(6*a^3
*C*d*f+a*b^2*(-8*A*d*f+3*B*c*f+3*B*d*e+14*C*c*e)-b^3*(7*B*c*e-4*A*(c*f+d*e
))+a^2*b*(B*d*f-10*C*(c*f+d*e)))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c
)^2/(-a*f+b*e)/(b*x+a)^(5/2)-2/7*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*(f*x+e)
^(3/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^(7/2)+2/105*d^(1/2)*(48*a^5*C*d^3*f
^3+8*a^4*b*d^2*f^2*(B*d*f-16*C*(c*f+d*e))-b^5*(8*A*d^3*e^3-c*d^2*e^2*(5*A*
f+14*B*e)+c^2*d*e*(-5*A*f^2+14*B*e*f+35*C*e^2)+c^3*f*(8*A*f^2-14*B*e*f+35*
C*e^2))-a*b^4*(d^3*e^2*(-19*A*f+6*B*e)-6*c^3*f^2*(-B*f+7*C*e)-c^2*d*f*(...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.99 (sec) , antiderivative size = 15963, normalized size of antiderivative = 9.31

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),
x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 4.40 (sec) , antiderivative size = 1770, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2117, 27, 167, 27, 167, 27, 169, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^{9/2}}dx$$

$$\Big\downarrow \; 2117$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(7cCe+3Bde+3Bcf-7Adf)a+b^2(7Bce-4A(de+cf))+b\left(\frac{6Cdfa^2}{b}-7Cdea-7cCfa+Bdfa+7bcCe-A\right.\right.}{2b(a+bx)^{7/2}}}{7(bc-ad)(be-af)}$$
$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{7b(a+bx)^{7/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(3C(de+cf)a^2-b(7cCe+3Bde+3Bcf-7Adf)a+b^2(7Bce-4A(de+cf))+b\left(\frac{6Cdfa^2}{b}+Bdfa-7C(de+cf)a+b(7cCe-Adf)\right)x\right)}{(a+bx)^{7/2}}}{7b(bc-ad)(be-af)}$$
$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{7b(a+bx)^{7/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 167$$

$$2\int -\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a^3+b\left(Bdf(de+3cf)-5C\left(2d^2e^2+11cdfe+3c^2f^2\right)\right)a^2+b^2\left(6f(7Ce-Bf)c^2+d\left(49Ce^2-8Bfe+11Af^2\right)c+d^2e(3Be-8Af)\right)a+b^3\left(-\left(\left(\right.\right.\right.\right.}{}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{7b(a+bx)^{7/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sqrt{c+dx}(e+fx)^{3/2}\left(6a^3Cdf+a^2b(Bdf-10C(cf+de))+ab^2(-8Adf+3Bcf+3Bde+14cCe)-b^3(7Bce-4A(cf+de))\right)}{5b(a+bx)^{5/2}(be-af)}-\frac{\int\frac{\sqrt{e+fx}\left(6Cdf(de+3cf)a\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{7b(a+bx)^{7/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 167$$

$$\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(24Cd^2f^2a^4\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$\Big\downarrow \; 169$$

$$\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 176

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+4\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 124

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+4\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 123

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+4\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 131

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+4\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 131

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+4\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

$\Big\downarrow$ 130

$$\frac{\frac{2\left(6Cdfa^3+b(Bdf-10C(de+cf))a^2+b^2(14cCe+3Bde+3Bcf-8Adf)a-b^3(7Bce-4A(de+cf))\right)\sqrt{c+dx}(e+fx)^{3/2}}{5b(be-af)(a+bx)^{5/2}}-\frac{2\left(24Cd^2f^2a^4-bdf(61Cde+4\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{7b(bc-ad)(be-af)(a+bx)^{7/2}}$$

input
```
Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^(9/2),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(7*b*(b*c - a
*d)*(b*e - a*f)*(a + b*x)^(7/2)) + ((2*(6*a^3*C*d*f + a*b^2*(14*c*C*e + 3*
B*d*e + 3*B*c*f - 8*A*d*f) - b^3*(7*B*c*e - 4*A*(d*e + c*f)) + a^2*b*(B*d*
f - 10*C*(d*e + c*f)))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(5*b*(b*e - a*f)*(a
+ b*x)^(5/2)) - ((2*(24*a^4*C*d^2*f^2 - a^3*b*d*f*(61*C*d*e + 43*c*C*f - 4
*B*d*f) - 3*a*b^3*(d^2*e*(B*e - 3*A*f) + 2*c^2*f*(7*C*e - B*f) + c*d*(28*C
*e^2 - 5*B*e*f + 5*A*f^2)) - b^4*(4*A*d^2*e^2 - c*d*e*(7*B*e - A*f) - c^2*
(35*C*e^2 - 14*B*e*f + 8*A*f^2)) - 3*a^2*b^2*(d*f*(3*B*d*e + 2*B*c*f - A*d
*f) - C*(15*d^2*e^2 + 37*c*d*e*f + 5*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x
])/(3*b*(b*c - a*d)*(a + b*x)^(3/2)) - ((2*(48*a^5*C*d^3*f^3 + 8*a^4*b*d^2
*f^2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c*d^2*e^2*(14*B*e + 5
*A*f) + c^2*d*e*(35*C*e^2 + 14*B*e*f - 5*A*f^2) + c^3*f*(35*C*e^2 - 14*B*e
*f + 8*A*f^2)) - a*b^4*(d^3*e^2*(6*B*e - 19*A*f) - 6*c^3*f^2*(7*C*e - B*f)
 - c^2*d*f*(238*C*e^2 - 19*f*(B*e - A*f)) - c*d^2*e*(42*C*e^2 - f*(19*B*e
+ 20*A*f))) + a^3*b^2*d*f*(C*(103*d^2*e^2 + 344*c*d*e*f + 103*c^2*f^2) + d
*f*(6*A*d*f - 19*B*(d*e + c*f))) - 3*a^2*b^3*(C*(5*d^3*e^3 + 94*c*d^2*e^2*
f + 94*c^2*d*e*f^2 + 5*c^3*f^3) + d*f*(3*A*d*f*(d*e + c*f) - B*(3*d^2*e^2
+ 16*c*d*e*f + 3*c^2*f^2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*
e - a*f)*Sqrt[a + b*x]) - (d*f*((2*Sqrt[-(b*c) + a*d]*(48*a^5*C*d^3*f^3 +
8*a^4*b*d^2*f^2*(B*d*f - 16*C*(d*e + c*f)) - b^5*(8*A*d^3*e^3 - c*d^2*e...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3899 vs. 2(1644) = 3288.

Time = 8.70 (sec) , antiderivative size = 3900, normalized size of antiderivative = 2.28

method	result	size
elliptic	Expression too large to display	3900
default	Expression too large to display	70236

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(9/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/7*(A*b^2-B*a*b+C*a^2)/b^7*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c
*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^4+2/35*(2*A*a*b^2*d*f-A*b^3*c*f-
A*b^3*d*e-9*B*a^2*b*d*f+8*B*a*b^2*c*f+8*B*a*b^2*d*e-7*B*b^3*c*e+16*C*a^3*d
*f-15*C*a^2*b*c*f-15*C*a^2*b*d*e+14*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+
b^2*c*e)/b^6*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)/(x+a/b)^3+2/105*(6*A*a^2*b^2*d^2*f^2-6*A*a*b^3*c*d*f^2-6*
A*a*b^3*d^2*e*f+4*A*b^4*c^2*f^2-2*A*b^4*c*d*e*f+4*A*b^4*d^2*e^2+8*B*a^3*b*
d^2*f^2-15*B*a^2*b^2*c*d*f^2-15*B*a^2*b^2*d^2*e*f+3*B*a*b^3*c^2*f^2+30*B*a
*b^3*c*d*e*f+3*B*a*b^3*d^2*e^2-7*B*b^4*c^2*e*f-7*B*b^4*c*d*e^2-57*C*a^4*d^
2*f^2+106*C*a^3*b*c*d*f^2+106*C*a^3*b*d^2*e*f-45*C*a^2*b^2*c^2*f^2-198*C*a
^2*b^2*c*d*e*f-45*C*a^2*b^2*d^2*e^2+84*C*a*b^3*c^2*e*f+84*C*a*b^3*c*d*e^2-
35*C*b^4*c^2*e^2)/b^5/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(b*d*f*x^3+a*d*f
*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/
105*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3/
b^4*(6*A*a^3*b^2*d^3*f^3-9*A*a^2*b^3*c*d^2*f^3-9*A*a^2*b^3*d^3*e*f^2+19*A*
a*b^4*c^2*d*f^3-20*A*a*b^4*c*d^2*e*f^2+19*A*a*b^4*d^3*e^2*f-8*A*b^5*c^3*f^
3+5*A*b^5*c^2*d*e*f^2+5*A*b^5*c*d^2*e^2*f-8*A*b^5*d^3*e^3+8*B*a^4*b*d^3*f^
3-19*B*a^3*b^2*c*d^2*f^3-19*B*a^3*b^2*d^3*e*f^2+9*B*a^2*b^3*c^2*d*f^3+48*B
*a^2*b^3*c*d^2*e*f^2+9*B*a^2*b^3*d^3*e^2*f-6*B*a*b^4*c^3*f^3-19*B*a*b^4...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9150 vs. $2(1643) = 3286$.

Time = 2.80 (sec) , antiderivative size = 9150, normalized size of antiderivative = 5.34

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(9/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/2)*(f*x+e)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(9/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x
)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}\sqrt{fx+e}}{(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(9/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)*sqrt(f*x + e)/(b*x + a)^(9/2), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{\sqrt{e+f\,x}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{(a+b\,x)^{9/2}}\,dx$$

input
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(9/2),x)
```

output
```
int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^(9/2), x
)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}(A+Bx+Cx^2)}{(a+bx)^{9/2}}\,dx = \int \frac{\sqrt{dx+c}\,\sqrt{fx+e}\,(C\,x^2+Bx+A)}{(bx+a)^{\frac{9}{2}}}dx$$

input
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(9/2),x)
```

output
```
int((d*x+c)^(1/2)*(f*x+e)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(9/2),x)
```

3.89 $\int \frac{(a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1233

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

output
```
-2/945*(5*d*f*(7*b*c*f*(-9*A*b*d*f+C*a*c*f+3*C*a*d*e+5*C*b*c*e)-(a*c*f+a*d
*e+5*b*c*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e)))+(3*a*d*f-4*b*(c*f+d*
e))*(7*d*f*(-9*A*b*d*f+C*a*c*f+3*C*a*d*e+5*C*b*c*e)-(2*a*d*f-b*c*f+6*b*d*e
)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e))/b))*(b*x+a)^(1/2)*(d*x+c)^(1/2)
*(f*x+e)^(1/2)/b/d^3/f^4-2/315*(7*d*f*(-9*A*b*d*f+C*a*c*f+3*C*a*d*e+5*C*b*
c*e)-(2*a*d*f-b*c*f+6*b*d*e)*(4*a*C*d*f+b*(-9*B*d*f+6*C*c*f+8*C*d*e))/b)*(
b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d^2/f^3-2/63*(4*a*C*d*f+b*(-9*B
*d*f+6*C*c*f+8*C*d*e))*(b*x+a)^(5/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d/f^2
+2/9*C*(b*x+a)^(5/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f+2/315*(a*d-b*c)^(1/
2)*(8*a^4*C*d^4*f^4+a^3*b*d^3*f^3*(-18*B*d*f-7*C*c*f+11*C*d*e)-3*a^2*b^2*d
^2*f^2*(3*d*f*(-7*A*d*f-3*B*c*f+4*B*d*e)-C*(-3*c^2*f^2-5*c*d*e*f+9*d^2*e^2
))-a*b^3*d*f*(2*C*(-16*c^3*f^3-18*c^2*d*e*f^2-33*c*d^2*e^2*f+92*d^3*e^3)+3
*d*f*(7*A*d*f*(-7*c*f+13*d*e)-B*(-19*c^2*f^2-29*c*d*e*f+72*d^2*e^2)))+b^4*
(C*(-16*c^4*f^4-16*c^3*d*e*f^3-21*c^2*d^2*e^2*f^2-40*c*d^3*e^3*f+128*d^4*e
^4)+3*d*f*(7*A*d*f*(-2*c^2*f^2-3*c*d*e*f+8*d^2*e^2)-B*(-8*c^3*f^3-9*c^2*d*
e*f^2-16*c*d^2*e^2*f+48*d^3*e^3))))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(
1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f
+b*e))^(1/2))/b^3/d^(7/2)/f^5/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2
/315*(a*d-b*c)^(1/2)*(-a*f+b*e)*(-c*f+d*e)*(4*a^3*C*d^3*f^3+3*a^2*b*d^2*f^
2*(-3*B*d*f-C*c*f+3*C*d*e)-3*a*b^2*d*f*(3*d*f*(-21*A*d*f+3*B*c*f+16*B*d...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.51 (sec) , antiderivative size = 1470, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],
x]
```

output
```
(-2*(-(b^2*Sqrt[-a + (b*c)/d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e -
 7*c*C*f - 18*B*d*f) + 3*a^2*b^2*d^2*f^2*(3*d*f*(-4*B*d*e + 3*B*c*f + 7*A*
d*f) + C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) + a*b^3*d*f*(C*(-184*d^3*e^3
 + 66*c*d^2*e^2*f + 36*c^2*d*e*f^2 + 32*c^3*f^3) - 3*d*f*(7*A*d*f*(13*d*e
- 7*c*f) + B*(-72*d^2*e^2 + 29*c*d*e*f + 19*c^2*f^2))) + b^4*(C*(128*d^4*e
^4 - 40*c*d^3*e^3*f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) +
3*d*f*(7*A*d*f*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) + B*(-48*d^3*e^3 + 16*c
*d^2*e^2*f + 9*c^2*d*e*f^2 + 8*c^3*f^3))))*(c + d*x)*(e + f*x)) + b^2*Sqrt
[-a + (b*c)/d]*d*f*(a + b*x)*(c + d*x)*(e + f*x)*(4*a^3*C*d^3*f^3 - 3*a^2*
b*d^2*f^2*(3*B*d*f + C*(-2*d*e + c*f + d*f*x)) - a*b^2*d*f*(9*d*f*(14*A*d*
f + B*(-11*d*e + 3*c*f + 8*d*f*x)) + C*(-15*c^2*f^2 + c*d*f*(-19*e + 11*f*
x) + d^2*(84*e^2 - 61*e*f*x + 50*f^2*x^2))) + b^3*(C*(-8*c^3*f^3 + 3*c^2*d
*f^2*(-3*e + 2*f*x) + c*d^2*f*(-12*e^2 + 7*e*f*x - 5*f^2*x^2) + d^3*(64*e^
3 - 48*e^2*f*x + 40*e*f^2*x^2 - 35*f^3*x^3)) - 3*d*f*(7*A*d*f*(-4*d*e + c*
f + 3*d*f*x) + B*(-4*c^2*f^2 + c*d*f*(-5*e + 3*f*x) + 3*d^2*(8*e^2 - 6*e*f
*x + 5*f^2*x^2))))) - I*(b*c - a*d)*f*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11
*C*d*e - 7*c*C*f - 18*B*d*f) + 3*a^2*b^2*d^2*f^2*(3*d*f*(-4*B*d*e + 3*B*c*
f + 7*A*d*f) + C*(9*d^2*e^2 - 5*c*d*e*f - 3*c^2*f^2)) + a*b^3*d*f*(C*(-184
*d^3*e^3 + 66*c*d^2*e^2*f + 36*c^2*d*e*f^2 + 32*c^3*f^3) - 3*d*f*(7*A*d*f*
(13*d*e - 7*c*f) + B*(-72*d^2*e^2 + 29*c*d*e*f + 19*c^2*f^2))) + b^4*(C...
```

Rubi [A] (verified)

Time = 3.13 (sec) , antiderivative size = 1268, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2118, 27, 171, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{ 2118}$$

$$\frac{2\int -\frac{b(a+bx)^{3/2}\sqrt{c+dx}(5bcCe+3aCde+acCf-9Abdf+(4aCdf+b(8Cde+6cCf-9Bdf))x)}{2\sqrt{e+fx}}dx}{9b^2df}+\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}$$

$\big\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-\frac{\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(5bcCe+3aCde+acCf-9Abdf+(4aCdf+b(8Cde+6cCf-9Bdf))x)}{\sqrt{e+fx}}dx}{9bdf}$$

$\big\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$2\int \frac{\sqrt{a+bx}\sqrt{c+dx}(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf))+(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3}{2\sqrt{e+fx}} \quad \frac{\cdots}{7df} \quad \frac{\cdots}{9bdf}$$

$\big\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf))+(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3ac}{\sqrt{e+fx}} \quad \frac{\cdots}{7df} \quad \frac{\cdots}{9bdf}$$

$\big\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$2\int \sqrt{c+dx}\Big(5adf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf)))-(bce+3ade+acf)(7bdf(5bcCe+3aCde+acCf-9Abdf)$$

$\big\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\int \sqrt{c+dx}\Big(5adf(7adf(5bcCe+3aCde+acCf-9Abdf)-(3bce+3ade+acf)(4aCdf+b(8Cde+6cCf-9Bdf)))-(bce+3ade+acf)(7bdf(5bcCe+3aCde+acCf-9Abdf)-$$

$\big\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9B}{5df}$$

$$\Big\downarrow 27$$

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} -$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df} + \frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9B}{5df}}{}$$

$$\Big\downarrow 176$$

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} -$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df} + \frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9B}{5df}}{}$$

$$\Big\downarrow 124$$

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} -$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df} + \frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9B}{5df}}{}$$

$$\Big\downarrow 123$$

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf} -$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df} + \frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9B}{5df}}{}$$

$$\Big\downarrow 131$$

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9B}{5df}}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9B}{5df}}{}$$

$\downarrow$ 130

$$\frac{2C(a+bx)^{5/2}(c+dx)^{3/2}\sqrt{e+fx}}{9bdf}-$$

$$\frac{\frac{2(4aCdf+b(8Cde+6cCf-9Bdf))(a+bx)^{3/2}\sqrt{e+fx}(c+dx)^{3/2}}{7df}+\frac{2(7bdf(5bcCe+3aCde+acCf-9Abdf)-(6bde+4bcf-3adf)(4aCdf+b(8Cde+6cCf-9B}{5df}}{}$$

input
```
Int[((a + b*x)^(3/2)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(2*C*(a + b*x)^(5/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(9*b*d*f) - ((2*(4*a*C
*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f))*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sq
rt[e + f*x])/(7*d*f) + ((2*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A
*b*d*f) - (6*b*d*e + 4*b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f
- 9*B*d*f)))*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*d*f) + ((2*(5
*b*d*f*(7*a*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b*d*f) - (3*b*c*e +
 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f - 9*B*d*f))) + 2*((a*d
*f)/2 - b*(2*d*e + c*f))*(7*b*d*f*(5*b*c*C*e + 3*a*C*d*e + a*c*C*f - 9*A*b
*d*f) - (6*b*d*e + 4*b*c*f - 3*a*d*f)*(4*a*C*d*f + b*(8*C*d*e + 6*c*C*f -
9*B*d*f))))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*f) + ((-6*Sqrt
[-(b*c) + a*d]*(8*a^4*C*d^4*f^4 + a^3*b*d^3*f^3*(11*C*d*e - 7*c*C*f - 18*B
*d*f) - 3*a^2*b^2*d^2*f^2*(3*d*f*(4*B*d*e - 3*B*c*f - 7*A*d*f) - C*(9*d^2*
e^2 - 5*c*d*e*f - 3*c^2*f^2)) - a*b^3*d*f*(C*(184*d^3*e^3 - 66*c*d^2*e^2*f
 - 36*c^2*d*e*f^2 - 32*c^3*f^3) + 3*d*f*(7*A*d*f*(13*d*e - 7*c*f) - B*(72*
d^2*e^2 - 29*c*d*e*f - 19*c^2*f^2))) + b^4*(C*(128*d^4*e^4 - 40*c*d^3*e^3*
f - 21*c^2*d^2*e^2*f^2 - 16*c^3*d*e*f^3 - 16*c^4*f^4) + 3*d*f*(7*A*d*f*(8*
d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) - B*(48*d^3*e^3 - 16*c*d^2*e^2*f - 9*c^2*
d*e*f^2 - 8*c^3*f^3))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Elli
pticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/
(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 7.33 (sec) , antiderivative size = 2108, normalized size of antiderivative = 1.71

method	result	size
elliptic	Expression too large to display	2108
default	Expression too large to display	15963

input
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/9*C*b/f*x^3*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*
c*e*x+a*c*e)^(1/2)+2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c
*f+4*b*d*e))/b/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*
d*e*x+b*c*e*x+a*c*e)^(1/2)+2/5*(b^2*A*d+2*a*b*B*d+B*b^2*c+a^2*C*d+2*C*a*b*
c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c
-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d
/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*
e)^(1/2)+2/3*(2*A*a*b*d+b^2*A*c+B*a^2*d+2*a*b*B*c+C*a^2*c-2/3*C*b/f*a*c*e-
2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*
(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(b^2*A*d+2*a*b*B*d+B*b^2*c+a^2*C*d+2*C
*a*b*c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2/7*(b^2*B*d+2*C*a*b*d+C*
b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e)
)/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*
d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(a^2*A*c-2/5*(b^2*A*d+2*a*b
*B*d+B*b^2*c+a^2*C*d+2*C*a*b*c-2/9*C*b/f*(7/2*a*c*f+7/2*a*d*e+7/2*b*c*e)-2
/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a*d*f+4*b*c*f+4*b*d*e))/b/d/f*(
3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(2*A*a*b*d+b^2*A*c+B*a^2*d+2*a*b
*B*c+C*a^2*c-2/3*C*b/f*a*c*e-2/7*(b^2*B*d+2*C*a*b*d+C*b^2*c-2/9*C*b/f*(4*a
*d*f+4*b*c*f+4*b*d*e))/b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(b^2*A...
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 1931, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/945*(3*(35*C*b^5*d^5*f^5*x^3 - 64*C*b^5*d^5*e^3*f^2 + 12*(C*b^5*c*d^4 +
(7*C*a*b^4 + 6*B*b^5)*d^5)*e^2*f^3 + (9*C*b^5*c^2*d^3 - (19*C*a*b^4 + 15*B
*b^5)*c*d^4 - 3*(2*C*a^2*b^3 + 33*B*a*b^4 + 28*A*b^5)*d^5)*e*f^4 + (8*C*b^
5*c^3*d^2 - 3*(5*C*a*b^4 + 4*B*b^5)*c^2*d^3 + 3*(C*a^2*b^3 + 9*B*a*b^4 + 7
*A*b^5)*c*d^4 - (4*C*a^3*b^2 - 9*B*a^2*b^3 - 126*A*a*b^4)*d^5)*f^5 - 5*(8*
C*b^5*d^5*e*f^4 - (C*b^5*c*d^4 + (10*C*a*b^4 + 9*B*b^5)*d^5)*f^5)*x^2 + (4
8*C*b^5*d^5*e^2*f^3 - (7*C*b^5*c*d^4 + (61*C*a*b^4 + 54*B*b^5)*d^5)*e*f^4
- (6*C*b^5*c^2*d^3 - (11*C*a*b^4 + 9*B*b^5)*c*d^4 - 3*(C*a^2*b^3 + 24*B*a*
b^4 + 21*A*b^5)*d^5)*f^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (
128*C*b^5*d^5*e^5 - 8*(13*C*b^5*c*d^4 + (31*C*a*b^4 + 18*B*b^5)*d^5)*e^4*f
 - (25*C*b^5*c^2*d^3 - 2*(113*C*a*b^4 + 60*B*b^5)*c*d^4 - (95*C*a^2*b^3 +
288*B*a*b^4 + 168*A*b^5)*d^5)*e^3*f^2 - (10*C*b^5*c^3*d^2 - 15*(3*C*a*b^4
+ 2*B*b^5)*c^2*d^3 + 3*(37*C*a^2*b^3 + 91*B*a*b^4 + 49*A*b^5)*c*d^4 - (20*
C*a^3*b^2 - 117*B*a^2*b^3 - 357*A*a*b^4)*d^5)*e^2*f^3 - (8*C*b^5*c^4*d - (
22*C*a*b^4 + 15*B*b^5)*c^3*d^2 + 3*(5*C*a^2*b^3 + 21*B*a*b^4 + 14*A*b^5)*c
^2*d^3 + 7*(2*C*a^3*b^2 - 21*B*a^2*b^3 - 54*A*a*b^4)*c*d^4 - (7*C*a^4*b -
27*B*a^3*b^2 + 168*A*a^2*b^3)*d^5)*e*f^4 - (16*C*b^5*c^5 - 8*(5*C*a*b^4 +
3*B*b^5)*c^4*d + (22*C*a^2*b^3 + 69*B*a*b^4 + 42*A*b^5)*c^3*d^2 + (7*C*a^3
*b^2 - 51*B*a^2*b^3 - 168*A*a*b^4)*c^2*d^3 + (11*C*a^4*b - 36*B*a^3*b^2 +
357*A*a^2*b^3)*c*d^4 - (8*C*a^5 - 18*B*a^4*b + 63*A*a^3*b^2)*d^5)*f^5)*...
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(d*x+c)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x),
x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x
)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)*sqrt(d*x + c)/sqrt(f*x + e), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
int(((a + b*x)^(3/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2), x
)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,(C\,x^2+Bx+A)}{\sqrt{fx+e}}dx$$

input
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x)
```

output
```
int((b*x+a)^(3/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x)
```

3.90 $\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 767

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = -\frac{2\left(5df(3bcCe+3aCde+acCf-7Abdf)-\frac{(4bde-bcf+2adf)(4aCdf+b(6Cde+4cCf-7Bdf))}{b}\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{\cdots}}{105bd^2f^3} - \frac{2(4aCdf+b(6Cde+4cCf-7Bdf))(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35b^2df^2} + \frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} - \frac{2\sqrt{-bc+ad}\left(3df(5bcf(3bcCe+3aCde+acCf-7Abdf)-(3bce+ade+acf)(4aCdf+b(6Cde+\cdots\right.}{\cdots} + \frac{2\sqrt{-bc+ad}(be-af)(de-cf)\left(4a^2Cd^2f^2+abdf(8Cde-2cCf-7Bdf)-b^2(7df(8Bde+Bcf-1\cdots\right.}{105b^3d^{5/2}f^4\sqrt{c+\cdots}}$$

output
```
-2/105*(5*d*f*(-7*A*b*d*f+C*a*c*f+3*C*a*d*e+3*C*b*c*e)-(2*a*d*f-b*c*f+4*b*
d*e)*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*d*e))/b)*(b*x+a)^(1/2)*(d*x+c)^(1/
2)*(f*x+e)^(1/2)/b/d^2/f^3-2/35*(4*a*C*d*f+b*(-7*B*d*f+4*C*c*f+6*C*d*e))*(
b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/d/f^2+2/7*C*(b*x+a)^(3/2)*(d*
x+c)^(3/2)*(f*x+e)^(1/2)/b/d/f-2/105*(a*d-b*c)^(1/2)*(3*d*f*(5*b*c*f*(-7*A
*b*d*f+C*a*c*f+3*C*a*d*e+3*C*b*c*e)-(a*c*f+a*d*e+3*b*c*e)*(4*a*C*d*f+b*(-7
*B*d*f+4*C*c*f+6*C*d*e)))+(a*d*f-2*b*(c*f+d*e))*(5*d*f*(-7*A*b*d*f+C*a*c*f
+3*C*a*d*e+3*C*b*c*e)-(2*a*d*f-b*c*f+4*b*d*e)*(4*a*C*d*f+b*(-7*B*d*f+4*C*c
*f+6*C*d*e))/b))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1
/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^2/d
^(5/2)/f^4/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/105*(a*d-b*c)^(1/2
)*(-a*f+b*e)*(-c*f+d*e)*(4*a^2*C*d^2*f^2+a*b*d*f*(-7*B*d*f-2*C*c*f+8*C*d*e
)-b^2*(7*d*f*(-10*A*d*f+B*c*f+8*B*d*e)-4*C*(c^2*f^2+2*c*d*e*f+12*d^2*e^2))
)*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1
/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^3/d
^(5/2)/f^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.06 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

$$= \frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(8a^3Cd^3f^3+a^2bd^2f^2(9Cde-5cCf-14Bdf)+ab^2df(7df(-3Bde+2Bcf+5Adf)+C\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output
```
(2*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e - 5*c
*C*f - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) + C*(16
*d^2*e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e^2*f
+ 9*c^2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d^2*e^
2 - 3*c*d*e*f - 2*c^2*f^2))))*(c + d*x)*(e + f*x) + b^2*Sqrt[-a + (b*c)/d]
*d*f*(a + b*x)*(c + d*x)*(e + f*x)*(-4*a^2*C*d^2*f^2 + a*b*d*f*(7*B*d*f +
C*(-5*d*e + 2*c*f + 3*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e + c*f + 3*
d*f*x)) + C*(-4*c^2*f^2 + c*d*f*(-5*e + 3*f*x) + 3*d^2*(8*e^2 - 6*e*f*x +
5*f^2*x^2)))) + I*(b*c - a*d)*f*(8*a^3*C*d^3*f^3 + a^2*b*d^2*f^2*(9*C*d*e
- 5*c*C*f - 14*B*d*f) + a*b^2*d*f*(7*d*f*(-3*B*d*e + 2*B*c*f + 5*A*d*f) +
C*(16*d^2*e^2 - 8*c*d*e*f - 5*c^2*f^2)) + b^3*(C*(-48*d^3*e^3 + 16*c*d^2*e
^2*f + 9*c^2*d*e*f^2 + 8*c^3*f^3) + 7*d*f*(5*A*d*f*(-2*d*e + c*f) + B*(8*d
^2*e^2 - 3*c*d*e*f - 2*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(
a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a +
 (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] - I*b*(b*c - a*
d)*f*(d*e - c*f)*(4*a^2*C*d^2*f^2 + a*b*d*f*(5*C*d*e + c*C*f - 7*B*d*f) -
b^2*(7*d*f*(-4*B*d*e - 2*B*c*f + 5*A*d*f) + C*(24*d^2*e^2 + 13*c*d*e*f + 8
*c^2*f^2)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e +
 f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]
], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(105*b^4*Sqrt[-a + (b*c)/d]*d^3*f...
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2118, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{b\sqrt{a+bx}\sqrt{c+dx}(3bcCe+3aCde+acCf-7Abdf+(4aCdf+b(6Cde+4cCf-7Bdf))x)}{2\sqrt{e+fx}}dx}{7b^2df}+\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-\frac{\int \frac{\sqrt{a+bx}\sqrt{c+dx}(3bcCe+3aCde+acCf-7Abdf+(4aCdf+b(6Cde+4cCf-7Bdf))x)}{\sqrt{e+fx}}dx}{7bdf}$$

$$\Big\downarrow \; 171$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-\frac{\frac{2\int \frac{\sqrt{c+dx}(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf))+(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aC\cdots}{2\sqrt{a+bx}\sqrt{e+fx}}}{5df}}{7bdf}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-\frac{\frac{\int \frac{\sqrt{c+dx}(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf))+(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCd\cdots}{\sqrt{a+bx}\sqrt{e+fx}}}{5df}}{7bdf}$$

$$\Big\downarrow \; 171$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-\frac{2\int \frac{3bcf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf)))-(bce+ade+acf)(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(\cdots}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-\frac{\int \frac{3bcf(5adf(3bcCe+3aCde+acCf-7Abdf)-(bce+3ade+acf)(4aCdf+b(6Cde+4cCf-7Bdf)))-(bce+ade+acf)(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2d\cdots}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 176$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-\frac{\frac{\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aCde-7Abdf+3bcCe)+(adf-\cdots\right.}{f}}{\cdots}$$

$$\Big\downarrow \text{124}$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aCde-7Abo\right.}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$\Big\downarrow \text{123}$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aC\right.}{b\sqrt{df}\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$\Big\downarrow \text{131}$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aC\right.}{b\sqrt{df}\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$

$$\Big\downarrow \text{131}$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf}-$$

$$\frac{2(4aCdf+b(6Cde+4cCf-7Bdf))\sqrt{a+bx}\sqrt{e+fx}(c+dx)^{3/2}}{5df}+\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(3bcCe+3aCde+acCf-7Abdf)+(adf-2b(2de+cf))(4aCdf+b}{3bf}$$

$$\Big\downarrow \text{130}$$

$$\frac{2C(a+bx)^{3/2}(c+dx)^{3/2}\sqrt{e+fx}}{7bdf} - \frac{\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(3bdf(5adf(acCf+3aCde-7Abdf+3bcCe)-(acf+3ade+bce)(4aCdf+b(-7Bdf+4cCf+6Cde)))+2\left(\frac{bcf}{2}-d(af+be)\right)(5bdf(acCf+3aC\cdots}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}}{\cdots}$$

input

```
Int[(Sqrt[a + b*x]*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

output

```
(2*C*(a + b*x)^(3/2)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(7*b*d*f) - ((2*(4*a*C
*d*f + b*(6*C*d*e + 4*c*C*f - 7*B*d*f))*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt
[e + f*x])/(5*d*f) + ((2*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7*A*b
*d*f) + (a*d*f - 2*b*(2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7*
B*d*f)))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*f) + ((2*Sqrt[-(b
*c) + a*d]*(3*b*d*f*(5*a*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*f - 7*A*b*d*f)
 - (b*c*e + 3*a*d*e + a*c*f)*(4*a*C*d*f + b*(6*C*d*e + 4*c*C*f - 7*B*d*f))
) + 2*((b*c*f)/2 - d*(b*e + a*f))*(5*b*d*f*(3*b*c*C*e + 3*a*C*d*e + a*c*C*
f - 7*A*b*d*f) + (a*d*f - 2*b*(2*d*e + c*f))*(4*a*C*d*f + b*(6*C*d*e + 4*c
*C*f - 7*B*d*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE
[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b
*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) -
 (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^2*C*d^2*f^2 + a*b*d*f*
(8*C*d*e - 2*c*C*f - 7*B*d*f) - b^2*(7*d*f*(8*B*d*e + B*c*f - 10*A*d*f) -
4*C*(12*d^2*e^2 + 2*c*d*e*f + c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*S
qrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sq
rt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c +
 d*x]*Sqrt[e + f*x]))/(3*b*f))/(5*d*f))/(7*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 3.99 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.57

method	result	size
elliptic	Expression too large to display	1205
default	Expression too large to display	9684

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/7*C/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/5*(B*b*d+C*a*d+C*b*c-2/7*C/f*(3*a*d*f+3*b*c*f+3*b*d*e))
/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+
a*c*e)^(1/2)+2/3*(A*b*d+B*a*d+B*b*c+C*a*c-2/7*C/f*(5/2*a*c*f+5/2*a*d*e+5/2
*b*c*e)-2/5*(B*b*d+C*a*d+C*b*c-2/7*C/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2
*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*
c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(A*a*c-2/5*(B*b*d+C*a*d+C*b*c-2/7*C/f
*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(A*b*d+B*a*d+B*b*c+C*a*c-2/7*C
/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*b*d+C*a*d+C*b*c-2/7*C/f*(3*a*d*f
+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1/2*a
*d*e+1/2*b*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^
(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+
a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),(
(-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(A*a*d+A*b*c+B*a*c-4/7*C/f*a*c*e-2/5*(B*b*
d+C*a*d+C*b*c-2/7*C/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*a*d*
e+3/2*b*c*e)-2/3*(A*b*d+B*a*d+B*b*c+C*a*c-2/7*C/f*(5/2*a*c*f+5/2*a*d*e+5/2
*b*c*e)-2/5*(B*b*d+C*a*d+C*b*c-2/7*C/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2
*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(c/d-a/b)*((x+c/d)/(c/
d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*...
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1392, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/315*(3*(15*C*b^4*d^4*f^4*x^2 + 24*C*b^4*d^4*e^2*f^2 - (5*C*b^4*c*d^3 + (
5*C*a*b^3 + 28*B*b^4)*d^4)*e*f^3 - (4*C*b^4*c^2*d^2 - (2*C*a*b^3 + 7*B*b^4
)*c*d^3 + (4*C*a^2*b^2 - 7*B*a*b^3 - 35*A*b^4)*d^4)*f^4 - 3*(6*C*b^4*d^4*e
*f^3 - (C*b^4*c*d^3 + (C*a*b^3 + 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x + a)*sqrt(
d*x + c)*sqrt(f*x + e) + (48*C*b^4*d^4*e^4 - 8*(5*C*b^4*c*d^3 + (5*C*a*b^3
 + 7*B*b^4)*d^4)*e^3*f - (10*C*b^4*c^2*d^2 - 7*(6*C*a*b^3 + 7*B*b^4)*c*d^3
 + (10*C*a^2*b^2 - 49*B*a*b^3 - 70*A*b^4)*d^4)*e^2*f^2 - (5*C*b^4*c^3*d -
7*(C*a*b^3 + 2*B*b^4)*c^2*d^2 - 7*(C*a^2*b^2 - 8*B*a*b^3 - 10*A*b^4)*c*d^3
 + (5*C*a^3*b - 14*B*a^2*b^2 + 70*A*a*b^3)*d^4)*e*f^3 - (8*C*b^4*c^4 - (9*
C*a*b^3 + 14*B*b^4)*c^3*d - (4*C*a^2*b^2 - 21*B*a*b^3 - 35*A*b^4)*c^2*d^2
- (9*C*a^3*b - 21*B*a^2*b^2 + 140*A*a*b^3)*c*d^3 + (8*C*a^4 - 14*B*a^3*b +
 35*A*a^2*b^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2
- (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^
2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d
- 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*
c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d
)*f)/(b*d*f)) + 3*(48*C*b^4*d^4*e^3*f - 8*(2*C*b^4*c*d^3 + (2*C*a*b^3 + 7*
B*b^4)*d^4)*e^2*f^2 - (9*C*b^4*c^2*d^2 - (8*C*a*b^3 + 21*B*b^4)*c*d^3 + (9
*C*a^2*b^2 - 21*B*a*b^3 - 70*A*b^4)*d^4)*e*f^3 - (8*C*b^4*c^3*d - (5*C*a*b
^3 + 14*B*b^4)*c^2*d^2 - (5*C*a^2*b^2 - 14*B*a*b^3 - 35*A*b^4)*c*d^3 + ...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(d*x+c)**(1/2)*(C*x**2+B*x+A)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*sqrt(c + d*x)*(A + B*x + C*x**2)/sqrt(e + f*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}\sqrt{dx+c}}{\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)/sqrt(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+b\,x}\,\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

output
```
int(((a + b*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2), x
)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx=\int \frac{\sqrt{bx+a}\,\sqrt{dx+c}\,(C\,x^2+Bx+A)}{\sqrt{fx+e}}dx$$

input
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x)
```

output
```
int((b*x+a)^(1/2)*(d*x+c)^(1/2)*(C*x^2+B*x+A)/(f*x+e)^(1/2),x)
```

3.91 $\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 529

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$
$$= -\frac{2(4aCdf+b(4Cde+2cCf-5Bdf))\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15b^2df^2}$$
$$+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$
$$-\frac{2\sqrt{-bc+ad}\left(3df(bcCe+3aCde+acCf-5Abdf)-\frac{(2bde-bcf+2adf)(4aCdf+b(4Cde+2cCf-5Bdf))}{b}\right)\sqrt{\frac{b(c+dx}{bc-ad}}}{15b^2d^{3/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$-\frac{2\sqrt{-bc+ad}(de-cf)\left(4a^2Cdf^2+abf(3Cde-cCf-5Bdf)-b^2(5df(2Be-3Af)-Ce(8de+cf))\right)}{15b^3d^{3/2}f^3\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2/15*(4*a*C*d*f+b*(-5*B*d*f+2*C*c*f+4*C*d*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)
*(f*x+e)^(1/2)/b^2/d/f^2+2/5*C*(b*x+a)^(1/2)*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b
/d/f-2/15*(a*d-b*c)^(1/2)*(3*d*f*(-5*A*b*d*f+C*a*c*f+3*C*a*d*e+C*b*c*e)-(2
*a*d*f-b*c*f+2*b*d*e)*(4*a*C*d*f+b*(-5*B*d*f+2*C*c*f+4*C*d*e))/b)*(b*(d*x+
c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*
c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^2/d^(3/2)/f^3/(d*x+c)^(1/2)/
(b*(f*x+e)/(-a*f+b*e))^(1/2)-2/15*(a*d-b*c)^(1/2)*(-c*f+d*e)*(4*a^2*C*d*f^
2+a*b*f*(-5*B*d*f-C*c*f+3*C*d*e)-b^2*(5*d*f*(-3*A*f+2*B*e)-C*e*(c*f+8*d*e)
))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(
1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^3/
d^(3/2)/f^3/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.04 (sec) , antiderivative size = 562, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$

$$= \frac{2\sqrt{a+bx}\left(\frac{b^2\left(8a^2Cd^2f^2+abdf(7Cde-3cCf-10Bdf)+b^2\left(5df(-2Bde+Bcf+3Adf)+C\left(8d^2e^2-3cdef-2c^2f^2\right)\right)\right)(c+dx)(e+fx)}{a+bx}+b^2df\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),
x]
```

output

```
(2*Sqrt[a + b*x]*((b^2*(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*
B*d*f) + b^2*(5*d*f*(-2*B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*
f - 2*c^2*f^2)))*(c + d*x)*(e + f*x))/(a + b*x) + b^2*d*f*(c + d*x)*(e + f
*x)*(5*b*B*d*f - 4*a*C*d*f + b*C*(-4*d*e + c*f + 3*d*f*x)) + (I*(b*c - a*d
)*f*(8*a^2*C*d^2*f^2 + a*b*d*f*(7*C*d*e - 3*c*C*f - 10*B*d*f) + b^2*(5*d*f
*(-2*B*d*e + B*c*f + 3*A*d*f) + C*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2)))*Sq
rt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b
*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*
f)/(b*c*f - a*d*f)])/Sqrt[-a + (b*c)/d] + I*b*Sqrt[-a + (b*c)/d]*d*f*(d*e
- c*f)*(5*b*B*d*f - 4*a*C*d*f - 2*b*C*(2*d*e + c*f))*Sqrt[a + b*x]*Sqrt[(b
*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*A
rcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]
))/(15*b^4*d^2*f^3*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2118, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{b\sqrt{c+dx}(bCe+3aCde+acCf-5Abdf+(4aCdf+b(4Cde+2cCf-5Bdf))x)}{2\sqrt{a+bx}\sqrt{e+fx}}dx}{5b^2df}+\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\int \frac{\sqrt{c+dx}(bCe+3aCde+acCf-5Abdf+(4aCdf+b(4Cde+2cCf-5Bdf))x)}{\sqrt{a+bx}\sqrt{e+fx}}dx}{5bdf}$$

$$\Big\downarrow \text{171}$$

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\frac{2\int\frac{3bcf(bcCe+3aCde+acCf-5Abdf)-(bce+ade+acf)(4aCdf+b(4Cde+2cCf-5Bdf))+(3bdf(bcCe+3aCde+acCf-5Abdf)-(2bde-bcf+2adf)(4aCdf+b(4Cde-}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3bf}}{5bdf}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\frac{\int\frac{3bcf(bcCe+3aCde+acCf-5Abdf)-(bce+ade+acf)(4aCdf+b(4Cde+2cCf-5Bdf))+(3bdf(bcCe+3aCde+acCf-5Abdf)-(2bde-bcf+2adf)(4aCdf+b(4Cde+2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3bf}}{5bdf}$$

$\downarrow$ 176

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\frac{\frac{(de-cf)\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+(3bdf(acCf+3aCde-5Abdf+bcCe)-(2adf-bc}{3bf}}{5bdf}$$

$\downarrow$ 124

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\frac{\frac{(de-cf)\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+3aCde-5Abdf-}{}}{3bf}}{5b}$$

$\downarrow$ 123

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-\frac{\frac{(de-cf)\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+3aCd}{}}{3bf}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+d}{bc-a}}}{}}{3bf}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{}}{}}{3bf}$$

$\downarrow$ 130

$$\frac{2C\sqrt{a+bx}(c+dx)^{3/2}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{2\sqrt{ad-bc}(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf^2+abf(-5Bdf-cCf+3Cde)-\left(b^2(5df(2Be-3Af)-Ce(cf+8de))\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}}+}{3bf}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[e + f*x]),x]
```

output

```
(2*C*Sqrt[a + b*x]*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - ((2*(4*a*C*d
*f + b*(4*C*d*e + 2*c*C*f - 5*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
 f*x])/(3*b*f) + ((2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(b*c*C*e + 3*a*C*d*e + a*
c*C*f - 5*A*b*d*f) - (2*b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(4*C*d*e +
 2*c*C*f - 5*B*d*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellipt
icE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]
) + (2*Sqrt[-(b*c) + a*d]*(d*e - c*f)*(4*a^2*C*d*f^2 + a*b*f*(3*C*d*e - c*
C*f - 5*B*d*f) - b^2*(5*d*f*(2*B*e - 3*A*f) - C*e*(8*d*e + c*f)))*Sqrt[(b*
(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(
Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f)
)])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(3*b*f))/(5*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.53

method	result
elliptic	$\frac{\sqrt{(fx+e)(bx+a)(xd+c)}\left(\frac{2Cx\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{5fb}+\frac{2\left(Bd+Cc-\frac{2C(2adf+2bcf+2bde)}{5fb}\right)\sqrt{bdf\,x^3+adf\,x}}{3bdf}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/5*C/f/b*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(
b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/
2)+2*(A*c-2/5*C/f/b*a*c*e-2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2*b*d*e)
)/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/
2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*
x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x
+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(A*d+B*c-2/5*C/f/b
*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3*(B*d+C*c-2/5*C/f/b*(2*a*d*f+2*b*c*f+2
*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((
x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b
*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-c/d+e/f)*Ellipt
icE(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))-e/f*EllipticF
(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(473) = 946$.

Time = 0.12 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/45*(3*(3*C*b^3*d^3*f^3*x - 4*C*b^3*d^3*e*f^2 + (C*b^3*c*d^2 - (4*C*a*b^2
 - 5*B*b^3)*d^3)*f^3)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (8*C*b^3
*d^3*e^3 - (7*C*b^3*c*d^2 - (3*C*a*b^2 - 10*B*b^3)*d^3)*e^2*f - (2*C*b^3*c
^2*d + 2*(C*a*b^2 - 5*B*b^3)*c*d^2 - (3*C*a^2*b - 5*B*a*b^2 + 15*A*b^3)*d^
3)*e*f^2 - (2*C*b^3*c^3 + (2*C*a*b^2 - 5*B*b^3)*c^2*d + (7*C*a^2*b - 10*B*
a*b^2 + 30*A*b^3)*c*d^2 - (8*C*a^3 - 10*B*a^2*b + 15*A*a*b^2)*d^3)*f^3)*sq
rt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f +
 (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 -
3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3
)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^
3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(8*C*b^3*
d^3*e^2*f - (3*C*b^3*c*d^2 - (7*C*a*b^2 - 10*B*b^3)*d^3)*e*f^2 - (2*C*b^3*
c^2*d + (3*C*a*b^2 - 5*B*b^3)*c*d^2 - (8*C*a^2*b - 10*B*a*b^2 + 15*A*b^3)*
d^3)*f^3)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^
2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^
3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a
^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*
f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*
d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*
d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx=\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(e + f*x)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{\sqrt{bx+a}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x
)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{\sqrt{bx+a}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/(sqrt(b*x + a)*sqrt(f*x + e)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(1/2)),
x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{\sqrt{a+bx}\sqrt{e+fx}}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
(10*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*a*b*d*f - 6*sqrt(e + f*x)*sq
rt(c + d*x)*sqrt(a + b*x)*a*c**2*f - 6*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a
+ b*x)*a*c*d*e + 4*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*a*c*d*f*x + 1
0*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*b**2*c*f - 6*sqrt(e + f*x)*sqr
t(c + d*x)*sqrt(a + b*x)*b*c**2*e + 4*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a +
 b*x)*b*c**2*f*x + 4*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*b*c*d*e*x -
 8*int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a*
*2*c*d*f**2*x + a**2*d**2*e*f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f + a*b
*c**2*f**2*x + a*b*c*d*e**2 + 3*a*b*c*d*e*f*x + 2*a*b*c*d*f**2*x**2 + a*b*
d**2*e**2*x + 2*a*b*d**2*e*f*x**2 + a*b*d**2*f**2*x**3 + b**2*c**2*e*f*x +
 b**2*c**2*f**2*x**2 + b**2*c*d*e**2*x + 2*b**2*c*d*e*f*x**2 + b**2*c*d*f*
*2*x**3 + b**2*d**2*e**2*x**2 + b**2*d**2*e*f*x**3),x)*a**3*c*d**3*f**3 -
5*int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**
2*c*d*f**2*x + a**2*d**2*e*f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f + a*b*
c**2*f**2*x + a*b*c*d*e**2 + 3*a*b*c*d*e*f*x + 2*a*b*c*d*f**2*x**2 + a*b*d
**2*e**2*x + 2*a*b*d**2*e*f*x**2 + a*b*d**2*f**2*x**3 + b**2*c**2*e*f*x +
b**2*c**2*f**2*x**2 + b**2*c*d*e**2*x + 2*b**2*c*d*e*f*x**2 + b**2*c*d*f**
2*x**3 + b**2*d**2*e**2*x**2 + b**2*d**2*e*f*x**3),x)*a**2*b**2*d**3*f**3
- 5*int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a
**2*c*d*f**2*x + a**2*d**2*e*f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f +...
```

3.92 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 540

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \frac{2(4a^2Cdf+b^2(cCe+3Adf)-ab(Cde+cCf+3Bdf))\sqrt{a+bx}\sqrt{c+d}}{3b^2(bc-ad)f(be-af)}$$
$$-\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$
$$+\frac{2\sqrt{-bc+ad}(8a^2Cdf^2-abf(3Cde+cCf+6Bdf)+b^2(3df(Be+Af)-Ce(2de-cf)))\,\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+}}{3b^3\sqrt{d}f^2(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$+\frac{2\sqrt{-bc+ad}(de-cf)(2bCe-3bBf+4aCf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{3b^3\sqrt{d}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2/3*(4*a^2*C*d*f+b^2*(3*A*d*f+C*c*e)-a*b*(3*B*d*f+C*c*f+C*d*e))*(b*x+a)^(1
/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*d+b*c)/f/(-a*f+b*e)-2*(A*b^2-a*(B*
b-C*a))*(d*x+c)^(3/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^(1/2)+
2/3*(a*d-b*c)^(1/2)*(8*a^2*C*d*f^2-a*b*f*(6*B*d*f+C*c*f+3*C*d*e)+b^2*(3*d*
f*(A*f+B*e)-C*e*(-c*f+2*d*e)))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*
EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e)
)^(1/2))/b^3/d^(1/2)/f^2/(-a*f+b*e)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(
1/2)+2/3*(a*d-b*c)^(1/2)*(-c*f+d*e)*(-3*B*b*f+4*C*a*f+2*C*b*e)*(b*(d*x+c)/
(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(
1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^3/d^(1/2)/f^2/(d
*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.12 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx =$$
$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}\left(-8a^2Cdf^2+abf(3Cde+cCf+6Bdf)+b^2(-3df(Be+Af)+Ce(2de-cf))\right)(c+dx\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x]
),x]
```

output

```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(-8*a^2*C*d*f^2 + a*b*f*(3*C*d*e + c*C*f + 6*B
*d*f) + b^2*(-3*d*f*(B*e + A*f) + C*e*(2*d*e - c*f)))*(c + d*x)*(e + f*x)
+ b^2*Sqrt[-a + (b*c)/d]*d*f*(c + d*x)*(e + f*x)*(3*(A*b^2 + a*(-(b*B) + a
*C))*f - C*(b*e - a*f)*(a + b*x)) - I*(b*c - a*d)*f*(8*a^2*C*d*f^2 - a*b*f
*(3*C*d*e + c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) + C*e*(-2*d*e + c*f)
))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f
*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e
 - a*d*f)/(b*c*f - a*d*f)] - I*b*f*(d*e - c*f)*(4*a^2*C*d*f + b^2*(c*C*e +
 3*A*d*f) - a*b*(C*d*e + c*C*f + 3*B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*
x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[S
qrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(3*b^
4*Sqrt[-a + (b*c)/d]*d*f^2*(b*e - a*f)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2117, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{3/2}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(cCe+3Bde+Bcf-Adf)a+b^2(Bc+2Ad)e+b\left(\frac{4Cdfa^2}{b}-(Cde+cCf+3Bdf)a+b(cCe+3Adf)\right)x\right)}{2b\sqrt{a+bx}\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(cCe+3Bde+Bcf-Adf)a+b^2(Bc+2Ad)e+b\left(\frac{4Cdfa^2}{b}-(Cde+cCf+3Bdf)a+b(cCe+3Adf)\right)x\right)}{\sqrt{a+bx}\sqrt{e+fx}}dx}{b(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 171

$$\frac{\frac{2\int \frac{(bc-ad)\left(4Cf(de+cf)a^2-b(3Bf(de+cf)+Ce(de+3cf))a-b^2e(cCe-3Bcf-3Adf)+\left((3df(Be+Af)-Ce(2de-cf))b^2-af(3Cde+cCf+6Bdf)b+8a^2Cdf^2\right)x\right)}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}d}{3bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{(bc-ad)\int \frac{4Cf(de+cf)a^2-b(3Bf(de+cf)+Ce(de+3cf))a-b^2e(cCe-3Bcf-3Adf)+\left((3df(Be+Af)-Ce(2de-cf))b^2-af(3Cde+cCf+6Bdf)b+8a^2Cdf^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{3bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 176

$$\frac{\frac{(bc-ad)\left(\frac{\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-Ce(2de-cf))\right)\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}+\frac{(be-af)(de-cf)(4aCf-3bBf+2bCe)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e}}}{f}\right.}{3bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\downarrow$ 124

$$\frac{\frac{(bc-ad)\left(\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-Ce(2de-cf))\right)\int \frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}dx}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{(be-af)(de-cf)(4aCf-3bB}{}\right.}{3bf}}{b(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 123

$$\frac{(bc-ad)\left(\frac{(be-af)(de-cf)(4aCf-3bBf+2bCe)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}\right.}{3bf}$$
$$b(bc-ad)(be-a$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 131

$$\frac{(bc-ad)\left(\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}(4aCf-3bBf+2bCe)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3C\right.}{b\sqrt{d}f}\right.}{3bf}$$
$$b(bc-ad)$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 131

$$\frac{(bc-ad)\left(\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(4aCf-3bBf+2bCe)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2\right.}{}\right.}{3bf}$$
$$b($$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$\Big\downarrow$ 130

$$\frac{(bc-ad)\left(\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^2Cdf^2-abf(6Bdf+cCf+3Cde)+b^2(3df(Af+Be)-Ce(2de-cf))\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{2\sqrt{ad-bc}(be-}{}\right.}{3bf}$$
$$b$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(3/2)*Sqrt[e + f*x]),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*
(b*e - a*f)*Sqrt[a + b*x]) + ((2*((4*a^2*C*d*f)/b + b*(c*C*e + 3*A*d*f) -
a*(C*d*e + c*C*f + 3*B*d*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3
*f) + ((b*c - a*d)*((2*Sqrt[-(b*c) + a*d]*(8*a^2*C*d*f^2 - a*b*f*(3*C*d*e
+ c*C*f + 6*B*d*f) + b^2*(3*d*f*(B*e + A*f) - C*e*(2*d*e - c*f)))*Sqrt[(b*
(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(b*Sqrt[d]*f*S
qrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e
 - a*f)*(d*e - c*f)*(2*b*C*e - 3*b*B*f + 4*a*C*f)*Sqrt[(b*(c + d*x))/(b*c
- a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a +
 b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))]])/(b*Sqrt[d]*f
*Sqrt[c + d*x]*Sqrt[e + f*x])))/(3*b*f))/(b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.59

method	result
elliptic	$\frac{\sqrt{(fx+e)(bx+a)(xd+c)}\left(\frac{2\left(bdf\,x^2+bcfx+bdex+bce\right)\left(b^2A-abB+a^2C\right)}{b^3(af-be)\sqrt{\left(x+\frac{a}{b}\right)\left(bdf\,x^2+bcfx+bdex+bce\right)}}+\frac{2C\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{3b^2f}+\frac{2(\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)*(A*b^2-B*a*b+C*a^2)/b^3/(a*f-b*e)/((x
+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2/3*C/b^2/f*(b*d*f*x^3+a*d*
f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*((A*b^2*d
-B*a*b*d+B*b^2*c+C*a^2*d-C*a*b*c)/b^3-(A*b^2-B*a*b+C*a^2)/b^3*(a*d*f-b*c*f
-b*d*e)/(a*f-b*e)-(b*c*f+b*d*e)*(A*b^2-B*a*b+C*a^2)/b^3/(a*f-b*e)-2/3*C/b^
2/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*(
(x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+
b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d
)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(1/b^2*(B*b*d-C*a*d+C*
b*c)-(A*b^2-B*a*b+C*a^2)/b^2*d*f/(a*f-b*e)-2/3*C/b^2/f*(a*d*f+b*c*f+b*d*e)
)*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/
(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x
+b*c*e*x+a*c*e)^(1/2)*((-c/d+e/f)*EllipticE(((x+c/d)/(c/d-a/b))^(1/2),((-c
/d+a/b)/(-c/d+e/f))^(1/2))-e/f*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+
a/b)/(-c/d+e/f))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(487) = 974$.

Time = 0.15 (sec) , antiderivative size = 1336, normalized size of antiderivative = 2.47

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*(C*a*b^3*d^2*e*f^2 - (4*C*a^2*b^2 - 3*B*a*b^3 + 3*A*b^4)*d^2*f^3 +
(C*b^4*d^2*e*f^2 - C*a*b^3*d^2*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*
x + e) + (2*C*a*b^3*d^2*e^3 - (2*C*a*b^3*c*d - (2*C*a^2*b^2 - 3*B*a*b^3)*d
^2)*e^2*f - (C*a*b^3*c^2 + 6*(C*a^2*b^2 - B*a*b^3)*c*d - (7*C*a^3*b - 6*B*
a^2*b^2 + 6*A*a*b^3)*d^2)*e*f^2 + (C*a^2*b^2*c^2 + (5*C*a^3*b - 3*B*a^2*b^
2 - 3*A*a*b^3)*c*d - (8*C*a^4 - 6*B*a^3*b + 3*A*a^2*b^2)*d^2)*f^3 + (2*C*b
^4*d^2*e^3 - (2*C*b^4*c*d - (2*C*a*b^3 - 3*B*b^4)*d^2)*e^2*f - (C*b^4*c^2
+ 6*(C*a*b^3 - B*b^4)*c*d - (7*C*a^2*b^2 - 6*B*a*b^3 + 6*A*b^4)*d^2)*e*f^2
 + (C*a*b^3*c^2 + (5*C*a^2*b^2 - 3*B*a*b^3 - 3*A*b^4)*c*d - (8*C*a^3*b - 6
*B*a^2*b^2 + 3*A*a*b^3)*d^2)*f^3)*x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(
b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)
/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3
*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e
+ (b*c + a*d)*f)/(b*d*f)) + 3*(2*C*a*b^3*d^2*e^2*f - (C*a*b^3*c*d - 3*(C*a
^2*b^2 - B*a*b^3)*d^2)*e*f^2 + (C*a^2*b^2*c*d - (8*C*a^3*b - 6*B*a^2*b^2 +
 3*A*a*b^3)*d^2)*f^3 + (2*C*b^4*d^2*e^2*f - (C*b^4*c*d - 3*(C*a*b^3 - B*b^
4)*d^2)*e*f^2 + (C*a*b^3*c*d - (8*C*a^2*b^2 - 6*B*a*b^3 + 3*A*b^4)*d^2)*f^
3)*x)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e
*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx=\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)^{\frac{3}{2}}\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(3/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(3/2)*sqrt(f*x + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{3/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(3/2)),
x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(3/2)),
 x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{3/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}\,(C\,x^2+Bx+A)}{(bx+a)^{\frac{3}{2}}\sqrt{fx+e}}dx$$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(3/2)/(f*x+e)^(1/2),x)
```

3.93 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 596

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = -\frac{2(4a^2Cf+b^2(3Be-2Af)-ab(6Ce+Bf))\sqrt{c+dx}\sqrt{e+fx}}{3b^2(be-af)^2\sqrt{a+bx}}$$
$$-\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$
$$+\frac{2\sqrt{d}(8a^3Cdf^2-a^2bf(13Cde+7cCf+2Bdf)-b^3(3cCe^2+Adef+cf(3Be-2Af))+ab^2(3Ce(de+4}{3b^3\sqrt{-bc+ad}f(be-af)^2\sqrt{c+dx}}$$
$$+\frac{2(de-cf)\,(4a^2Cdf+b^2(3cCe+Adf)-ab(Bdf+3C(de+cf)))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{3b^3\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2/3*(4*a^2*C*f+b^2*(-2*A*f+3*B*e)-a*b*(B*f+6*C*e))*(d*x+c)^(1/2)*(f*x+e)^
(1/2)/b^2/(-a*f+b*e)^2/(b*x+a)^(1/2)-2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)
*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^(3/2)+2/3*d^(1/2)*(8*a^3*C*
d*f^2-a^2*b*f*(2*B*d*f+7*C*c*f+13*C*d*e)-b^3*(3*c*C*e^2+A*d*e*f+c*f*(-2*A*
f+3*B*e))+a*b^2*(3*C*e*(4*c*f+d*e)+f*(-A*d*f+B*c*f+4*B*d*e)))*(b*(d*x+c)/(
-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(
1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^3/(a*d-b*c)^(1/2)/f/(-a*f+b*e)^2
/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/3*(-c*f+d*e)*(4*a^2*C*d*f+b^
2*(A*d*f+3*C*c*e)-a*b*(B*d*f+3*C*(c*f+d*e)))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*
(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/
2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^3/d^(1/2)/(a*d-b*c)^(1/2)/f/(-a*f+
b*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.27 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}f(c+dx)(e+fx)\left(\left(Ab^2+a(-bB+aC)\right)(bc-ad)(be-af)+\left(-5a^3Cdf+b^3(3Bce+\right.\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x]
),x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*f*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*
C))*(b*c - a*d)*(b*e - a*f) + (-5*a^3*C*d*f + b^3*(3*B*c*e + A*d*e - 2*A*c
*f) - a*b^2*(6*c*C*e + 4*B*d*e + B*c*f - A*d*f) + a^2*b*(7*C*d*e + 4*c*C*f
 + 2*B*d*f))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(8*a^3*C*d*f^2
 - a^2*b*f*(13*C*d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f
*(3*B*e - 2*A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*
f)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*(8*a^3*C*d*f^2 - a^2*b*f*(13*C*
d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*A*f))
 + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*(a + b*x)^(3
/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*El
lipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*
f - a*d*f)] + I*b*(b*c - a*d)*f*(d*e - c*f)*(-4*a^2*C*f + b^2*(-3*B*e + 2*
A*f) + a*b*(6*C*e + B*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))
]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]
/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])))/(3*b^4*Sqrt[-a + (b*c
)/d]*(b*c - a*d)*f*(b*e - a*f)^2*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*
x])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2117, 27, 167, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{5/2}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(3cCe+3Bde+Bcf-3Adf)a+b^2(3Bce-2Acf)-b\left(-\frac{4Cdfa^2}{b}+Bdfa+3C(de+cf)a-b(3cCe+Adf)\right)x\right)}{2b(a+bx)^{3/2}\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(3cCe+3Bde+Bcf-3Adf)a+b^2c(3Be-2Af)-b\left(-\frac{4Cdfa^2}{b}+Bdfa+3C(de+cf)a-b(3cCe+Adf)\right)x\right)}{(a+bx)^{3/2}\sqrt{e+fx}}dx}{3b(bc-ad)(be-af)}-\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 167

$$2\int -\frac{4Cdf(de+cf)a^3-b\left(Bdf(de+cf)+C\left(6d^2e^2+11cdfe+3c^2f^2\right)\right)a^2+b^2\left(6Cefc^2+d\left(9Ce^2+2Bfe+Af^2\right)c+d^2e(3Be-2Af)\right)a-b^3ce(3cCe+3Bde-Adf)+d\left(8C\right.}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}} \quad / \quad b(be-af)$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$-\int \frac{4Cdf(de+cf)a^3-b\left(Bdf(de+cf)+C\left(6d^2e^2+11cdfe+3c^2f^2\right)\right)a^2+b^2\left(6Cefc^2+d\left(9Ce^2+2Bfe+Af^2\right)c+d^2e(3Be-2Af)\right)a-b^3ce(3cCe+3Bde-Adf)+d\left(8Cd\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}} \quad / \quad b(be-af)$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 176

$$-\frac{\frac{(be-af)(de-cf)\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+d\left(8a^3Cdf^2-a^2bf(2Bdf+7cCf+13Cde)+ab^2(f(-Adf+Bcf+\right.}{b(be-af)} \quad / \quad 3b(bc-ad)($$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 124

$$-\frac{\frac{(be-af)(de-cf)\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+d\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^3Cdf^2-a^2bf(2Bdf+7cCf+13Cde)+ab\right.}{b(be-af)} \quad / \quad 3b$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 123

$$-\frac{\frac{(be-af)(de-cf)\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^3Cdf^2-a^2bf(2Bdf+7cCf+\cdots\right.}{\cdots}}{b(be-af)\cdots}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 131

$$-\frac{\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(8a^3Cdf^2\cdots\right.}{\cdots}}{b(be-af)\cdots}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 131

$$-\frac{\frac{(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf-ab(Bdf+3C(cf+de))+b^2(Adf+3cCe)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{d}\sqrt{e+fx}\sqrt{ad-b\cdots}}{\cdots}}{\cdots}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\Big\downarrow$ 130

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}(bc-ad)\left(4a^2Cf-ab(Bf+6Ce)+b^2(3Be-2Af)\right)}{b\sqrt{a+bx}(be-af)}-\frac{\frac{2\sqrt{ad-bc}(be-af)(de-cf)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(4a^2Cdf-ab(Bdf+3C(cf+de))+\cdots\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}\cdots}}{\cdots}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

input

```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(5/2)*Sqrt[e + f*x]),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(3*b*(b*c - a*d
)*(b*e - a*f)*(a + b*x)^(3/2)) + ((-2*(b*c - a*d)*(4*a^2*C*f + b^2*(3*B*e
- 2*A*f) - a*b*(6*C*e + B*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*e - a*f)*
Sqrt[a + b*x]) - ((2*Sqrt[d]*Sqrt[-(b*c) + a*d]*(8*a^3*C*d*f^2 - a^2*b*f*(
13*C*d*e + 7*c*C*f + 2*B*d*f) - b^3*(3*c*C*e^2 + A*d*e*f + c*f*(3*B*e - 2*
A*f)) + a*b^2*(3*C*e*(d*e + 4*c*f) + f*(4*B*d*e + B*c*f - A*d*f)))*Sqrt[(b
*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a +
b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*f*Sqrt[c +
 d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)
*(d*e - c*f)*(4*a^2*C*d*f + b^2*(3*c*C*e + A*d*f) - a*b*(B*d*f + 3*C*(d*e
+ c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*E
llipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*
f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(b*(b*e -
a*f)))/(3*b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. 2(542) = 1084.

Time = 8.33 (sec) , antiderivative size = 1269, normalized size of antiderivative = 2.13

method	result	size
elliptic	Expression too large to display	1269
default	Expression too large to display	14757

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/3*(A*b^2-B*a*b+C*a^2)/b^4/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*
e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/3*(b*d*f*x^2+b*c*f*
x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^3*(A*a*b^2*d*f-2*A*b^
3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5*C*a^
3*d*f+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a*f-b*e)/((x+a/b)*(b*d*f
*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*((B*b*d-2*C*a*d+C*b*c)/b^3+1/3*(A*b^2
-B*a*b+C*a^2)/b^3*d*f/(a*f-b*e)-1/3/b^3*(a*d*f-b*c*f-b*d*e)*(A*a*b^2*d*f-2
*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5
*C*a^3*d*f+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/(a*f-b*e)-1/3*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^3*(A*a*b^2*d*f-2*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b
^2*d*e+3*B*b^3*c*e-5*C*a^3*d*f+4*C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/
(a*f-b*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*
((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*
x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+
a/b)/(-c/d+e/f))^(1/2))+2*(C*d/b^2-1/3*d*f/b^2*(A*a*b^2*d*f-2*A*b^3*c*f+A*
b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-4*B*a*b^2*d*e+3*B*b^3*c*e-5*C*a^3*d*f+4*
C*a^2*b*c*f+7*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)
/(a*f-b*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2429 vs. $2(541) = 1082$.

Time = 0.31 (sec) , antiderivative size = 2429, normalized size of antiderivative = 4.08

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*(((5*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d - 3*(2*C*a^3*b^3 - B*a^2*b^
4)*d^2)*e*f^2 - (3*(C*a^3*b^3 - A*a*b^5)*c*d - (4*C*a^4*b^2 - B*a^3*b^3 -
2*A*a^2*b^4)*d^2)*f^3 + ((3*(2*C*a*b^5 - B*b^6)*c*d - (7*C*a^2*b^4 - 4*B*a
*b^5 + A*b^6)*d^2)*e*f^2 - ((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (5*C*a
^3*b^3 - 2*B*a^2*b^4 - A*a*b^5)*d^2)*f^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)*s
qrt(f*x + e) - (3*(C*a^2*b^4*c*d - C*a^3*b^3*d^2)*e^3 - (6*C*a^2*b^4*c^2 -
 3*(5*C*a^3*b^3 - 2*B*a^2*b^4)*c*d + (8*C*a^4*b^2 - 5*B*a^3*b^3 - A*a^2*b^
4)*d^2)*e^2*f + (3*(2*C*a^3*b^3 + B*a^2*b^4)*c^2 - (25*C*a^4*b^2 - 4*B*a^3
*b^3 - 2*A*a^2*b^4)*c*d + (17*C*a^5*b - 5*B*a^4*b^2 - 4*A*a^3*b^3)*d^2)*e*
f^2 - ((2*C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (11*C*a^5*b - 2*B*a^4
*b^2 + 2*A*a^3*b^3)*c*d + (8*C*a^6 - 2*B*a^5*b - A*a^4*b^2)*d^2)*f^3 + (3*
(C*b^6*c*d - C*a*b^5*d^2)*e^3 - (6*C*b^6*c^2 - 3*(5*C*a*b^5 - 2*B*b^6)*c*d
 + (8*C*a^2*b^4 - 5*B*a*b^5 - A*b^6)*d^2)*e^2*f + (3*(2*C*a*b^5 + B*b^6)*c
^2 - (25*C*a^2*b^4 - 4*B*a*b^5 - 2*A*b^6)*c*d + (17*C*a^3*b^3 - 5*B*a^2*b^
4 - 4*A*a*b^5)*d^2)*e*f^2 - ((2*C*a^2*b^4 + B*a*b^5 + 2*A*b^6)*c^2 - (11*C
*a^3*b^3 - 2*B*a^2*b^4 + 2*A*a*b^5)*c*d + (8*C*a^4*b^2 - 2*B*a^3*b^3 - A*a
^2*b^4)*d^2)*f^3)*x^2 + 2*(3*(C*a*b^5*c*d - C*a^2*b^4*d^2)*e^3 - (6*C*a*b^
5*c^2 - 3*(5*C*a^2*b^4 - 2*B*a*b^5)*c*d + (8*C*a^3*b^3 - 5*B*a^2*b^4 - A*a
*b^5)*d^2)*e^2*f + (3*(2*C*a^2*b^4 + B*a*b^5)*c^2 - (25*C*a^3*b^3 - 4*B*a^
2*b^4 - 2*A*a*b^5)*c*d + (17*C*a^4*b^2 - 5*B*a^3*b^3 - 4*A*a^2*b^4)*d^2...
```

Sympy [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{\frac{5}{2}}\sqrt{e+fx}}\,dx$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(5/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)**(5/2)*sqrt(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{5}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{5}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(5/2)*sqrt(f*x + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,(a+b\,x)^{5/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(5/2)),
x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(5/2)),
 x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{5/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}\,(C\,x^2+Bx+A)}{(bx+a)^{\frac{5}{2}}\,\sqrt{fx+e}}dx$$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(5/2)/(f*x+e)^(1/2),x)
```

3.94 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1034

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \frac{2(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6\ldots}{15b^2(bc-ad)(be-af)^2(a\ldots}$$

$$-\frac{2(8a^4Cd^2f^2-a^3bdf(23Cde+13cCf-2Bdf)-b^4(2Ad^2e^2-cde(5Be-3Af)-c^2(15Ce^2-10Bef+\ldots}{\ldots}$$

$$-\frac{2(Ab^2-a(bB-aC))\,(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2\sqrt{d}(8a^4Cd^2f^2-a^3bdf(23Cde+13cCf-2Bdf)-b^4(2Ad^2e^2-cde(5Be-3Af)-c^2(15Ce^2-10Bef\ldots}{\ldots}$$

$$+\frac{2\sqrt{d}(de-cf)\,(4a^3Cdf-b^3(5Bce-2Ade-4Acf)+ab^2(10cCe+3Bde+Bcf-6Adf)-a^2b(8Cde+\ldots}{15b^3(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx}\sqrt{\ldots}}$$

output
```
2/15*(4*a^3*C*d*f-b^3*(-4*A*c*f-2*A*d*e+5*B*c*e)+a*b^2*(-6*A*d*f+B*c*f+3*B
*d*e+10*C*c*e)-a^2*b*(-B*d*f+6*C*c*f+8*C*d*e))*(d*x+c)^(1/2)*(f*x+e)^(1/2)
/b^2/(-a*d+b*c)/(-a*f+b*e)^2/(b*x+a)^(3/2)-2/15*(8*a^4*C*d^2*f^2-a^3*b*d*f
*(-2*B*d*f+13*C*c*f+23*C*d*e)-b^4*(2*A*d^2*e^2-c*d*e*(-3*A*f+5*B*e)-c^2*(8
*A*f^2-10*B*e*f+15*C*e^2))-a^2*b^2*(d*f*(-3*A*d*f+2*B*c*f+7*B*d*e)-C*(3*c^
2*f^2+37*c*d*e*f+23*d^2*e^2))-a*b^3*(d^2*e*(-7*A*f+3*B*e)+2*c^2*f*(-B*f+5*
C*e)+c*d*(40*C*e^2-13*f*(-A*f+B*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b^2/(-a*
d+b*c)^2/(-a*f+b*e)^3/(b*x+a)^(1/2)-2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(3/2)*
(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)/(b*x+a)^(5/2)+2/15*d^(1/2)*(8*a^4*C*
d^2*f^2-a^3*b*d*f*(-2*B*d*f+13*C*c*f+23*C*d*e)-b^4*(2*A*d^2*e^2-c*d*e*(-3*
A*f+5*B*e)-c^2*(8*A*f^2-10*B*e*f+15*C*e^2))-a^2*b^2*(d*f*(-3*A*d*f+2*B*c*f
+7*B*d*e)-C*(3*c^2*f^2+37*c*d*e*f+23*d^2*e^2))-a*b^3*(d^2*e*(-7*A*f+3*B*e)
+2*c^2*f*(-B*f+5*C*e)+c*d*(40*C*e^2-13*f*(-A*f+B*e))))*(b*(d*x+c)/(-a*d+b*
c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((
-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^3/(a*d-b*c)^(3/2)/(-a*f+b*e)^3/(d*x+c)^
(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)+2/15*d^(1/2)*(-c*f+d*e)*(4*a^3*C*d*f-b^
3*(-4*A*c*f-2*A*d*e+5*B*c*e)+a*b^2*(-6*A*d*f+B*c*f+3*B*d*e+10*C*c*e)-a^2*b
*(-B*d*f+6*C*c*f+8*C*d*e))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b
*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d
/(-a*f+b*e))^(1/2))/b^3/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(f*x...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.58 (sec) , antiderivative size = 1449, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x]
),x]
```

output
```
Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5
*b^2*(b*e - a*f)*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e + A*b^3*d
*e - 6*a*b^2*B*d*e + 11*a^2*b*C*d*e - 4*A*b^3*c*f - a*b^2*B*c*f + 6*a^2*b*
c*C*f + 3*a*A*b^2*d*f + 2*a^2*b*B*d*f - 7*a^3*C*d*f))/(15*b^2*(b*c - a*d)*
(b*e - a*f)^2*(a + b*x)^2) - (2*(15*b^4*c^2*C*e^2 + 5*b^4*B*c*d*e^2 - 40*a
*b^3*c*C*d*e^2 - 2*A*b^4*d^2*e^2 - 3*a*b^3*B*d^2*e^2 + 23*a^2*b^2*C*d^2*e^
2 - 10*b^4*B*c^2*e*f - 10*a*b^3*c^2*C*e*f - 3*A*b^4*c*d*e*f + 13*a*b^3*B*c
*d*e*f + 37*a^2*b^2*c*C*d*e*f + 7*a*A*b^3*d^2*e*f - 7*a^2*b^2*B*d^2*e*f -
23*a^3*b*C*d^2*e*f + 8*A*b^4*c^2*f^2 + 2*a*b^3*B*c^2*f^2 + 3*a^2*b^2*c^2*C
*f^2 - 13*a*A*b^3*c*d*f^2 - 2*a^2*b^2*B*c*d*f^2 - 13*a^3*b*c*C*d*f^2 + 3*a
^2*A*b^2*d^2*f^2 + 2*a^3*b*B*d^2*f^2 + 8*a^4*C*d^2*f^2))/(15*b^2*(b*c - a*
d)^2*(b*e - a*f)^3*(a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c)/d]*(8
*a^4*C*d^2*f^2 + a^3*b*d*f*(-23*C*d*e - 13*c*C*f + 2*B*d*f) + b^4*(-2*A*d^
2*e^2 + c*d*e*(5*B*e - 3*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) + a^2
*b^2*(d*f*(-7*B*d*e - 2*B*c*f + 3*A*d*f) + C*(23*d^2*e^2 + 37*c*d*e*f + 3*
c^2*f^2)) + a*b^3*(d^2*e*(-3*B*e + 7*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(
-40*C*e^2 + 13*f*(B*e - A*f))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x))*(f
 + (b*e)/(a + b*x) - (a*f)/(a + b*x)) + (I*(-(b*c) + a*d)*f*(-8*a^4*C*d^2*
f^2 + a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) + b^4*(2*A*d^2*e^2 + c*d*e
*(-5*B*e + 3*A*f) + c^2*(-15*C*e^2 + 10*B*e*f - 8*A*f^2)) - a^2*b^2*(d*...
```

Rubi [A] (verified)

Time = 2.55 (sec) , antiderivative size = 1073, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2117, 27, 167, 27, 169, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(5cCe+3Bde+Bcf-5Adf)a+b^2(5Bce-2Ade-4Acf)+b\left(\frac{4Cdfa^2}{b}-5Cdea-5cCfa+Bdfa+5bcCe-Abdf\right)x\right)}{2b(a+bx)^{5/2}\sqrt{e+fx}}}{5(bc-ad)(be-af)}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\int \frac{\sqrt{c+dx}\left(C(3de+cf)a^2-b(5cCe+3Bde+Bcf-5Adf)a+b^2(5Bce-2A(de+2cf))+b\left(\frac{4Cdfa^2}{b}+Bdfa-5C(de+cf)a+b(5cCe-Adf)\right)x\right)}{(a+bx)^{5/2}\sqrt{e+fx}}dx}{5b(bc-ad)(be-af)}-$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 167

$$2\int -\frac{4Cdf(de+cf)a^3+b\left(Bdf(de+cf)-C\left(8d^2e^2+17cdfe+3c^2f^2\right)\right)a^2+b^2\left(2f(5Ce-Bf)c^2+d\left(25Ce^2-8Bfe+9Af^2\right)c+3d^2e(Be-2Af)\right)a+b^3\left(-\left(\left(15Ce^2-10B\right.\right.\right.}{2(a+b}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 27

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(4a^3Cdf-a^2b(-Bdf+6cCf+8Cde)+ab^2(-6Adf+Bcf+3Bde+10cCe)-b^3(-4Acf-2Ade+5Bce))}{3b(a+bx)^{3/2}(be-af)}-\int \frac{4Cdf(de+cf)a^3+b\left(Bdf\right.}{}}{}$$

$$\frac{2(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\big\downarrow$ 169

$$\frac{\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cC\right.}{}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\big\downarrow$ 27

$$\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cC\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\big\downarrow$ 176

$$\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cC\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\big\downarrow$ 124

$$\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cC\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\big\downarrow$ 123

$$\frac{2\left(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cC\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\big\downarrow$ 131

$$\frac{\frac{2(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf))\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cC\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\big\downarrow$ 131

$$\frac{\frac{2(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf))\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cC\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\big\downarrow$ 130

$$\frac{\frac{2(4Cdfa^3-b(8Cde+6cCf-Bdf)a^2+b^2(10cCe+3Bde+Bcf-6Adf)a-b^3(5Bce-2Ade-4Acf))\sqrt{c+dx}\sqrt{e+fx}}{3b(be-af)(a+bx)^{3/2}}-\frac{2\left(8Cd^2f^2a^4-bdf(23Cde+13cC\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

input
```
Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^(7/2)*Sqrt[e + f*x]),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*(b*c - a*d
)*(b*e - a*f)*(a + b*x)^(5/2)) + ((2*(4*a^3*C*d*f - b^3*(5*B*c*e - 2*A*d*e
 - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B*d*e + B*c*f - 6*A*d*f) - a^2*b*(8*C*d*
e + 6*c*C*f - B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*e - a*f)*(a + b
*x)^(3/2)) - ((2*(8*a^4*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d
*f) - b^4*(2*A*d^2*e^2 - c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f
+ 8*A*f^2)) - a^2*b^2*(d*f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 +
 37*c*d*e*f + 3*c^2*f^2)) - a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e
- B*f) + c*d*(40*C*e^2 - 13*f*(B*e - A*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/
((b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]) - (d*((2*Sqrt[-(b*c) + a*d]*(8*a^4
*C*d^2*f^2 - a^3*b*d*f*(23*C*d*e + 13*c*C*f - 2*B*d*f) - b^4*(2*A*d^2*e^2
- c*d*e*(5*B*e - 3*A*f) - c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a^2*b^2*(
d*f*(7*B*d*e + 2*B*c*f - 3*A*d*f) - C*(23*d^2*e^2 + 37*c*d*e*f + 3*c^2*f^2
)) - a*b^3*(d^2*e*(3*B*e - 7*A*f) + 2*c^2*f*(5*C*e - B*f) + c*d*(40*C*e^2
- 13*f*(B*e - A*f))))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*Ellipt
icE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d
*(b*e - a*f))])/(b*Sqrt[d]*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)])
+ (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(d*e - c*f)*(4*a^3*C*d*f - b^3*(5*B*c*
e - 2*A*d*e - 4*A*c*f) + a*b^2*(10*c*C*e + 3*B*d*e + B*c*f - 6*A*d*f) - a^
2*b*(8*C*d*e + 6*c*C*f - B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 167
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2329 vs. $2(972) = 1944$.

Time = 21.72 (sec) , antiderivative size = 2330, normalized size of antiderivative = 2.25

method	result	size
elliptic	Expression too large to display	2330
default	Expression too large to display	34865

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/5*(A*b^2-B*a*b+C*a^2)/b^5/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*
e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^3+2/15*(3*A*a*b^2*d*f-4
*A*b^3*c*f+A*b^3*d*e+2*B*a^2*b*d*f-B*a*b^2*c*f-6*B*a*b^2*d*e+5*B*b^3*c*e-7
*C*a^3*d*f+6*C*a^2*b*c*f+11*C*a^2*b*d*e-10*C*a*b^2*c*e)/b^4/(a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/(a*f-b*e)*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c
*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2+2/15*(b*d*f*x^2+b*c*f*x+b*d*e*
x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2/b^3*(3*A*a^2*b^2*d^2*f^2-13*A
*a*b^3*c*d*f^2+7*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2-3*A*b^4*c*d*e*f-2*A*b^4*d
^2*e^2+2*B*a^3*b*d^2*f^2-2*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3
*c^2*f^2+13*B*a*b^3*c*d*e*f-3*B*a*b^3*d^2*e^2-10*B*b^4*c^2*e*f+5*B*b^4*c*d
*e^2+8*C*a^4*d^2*f^2-13*C*a^3*b*c*d*f^2-23*C*a^3*b*d^2*e*f+3*C*a^2*b^2*c^2
*f^2+37*C*a^2*b^2*c*d*e*f+23*C*a^2*b^2*d^2*e^2-10*C*a*b^3*c^2*e*f-40*C*a*b
^3*c*d*e^2+15*C*b^4*c^2*e^2)/(a*f-b*e)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x
+b*c*e))^(1/2)+2*(C*d/b^3+1/15*d*f*(3*A*a*b^2*d*f-4*A*b^3*c*f+A*b^3*d*e+2*
B*a^2*b*d*f-B*a*b^2*c*f-6*B*a*b^2*d*e+5*B*b^3*c*e-7*C*a^3*d*f+6*C*a^2*b*c*
f+11*C*a^2*b*d*e-10*C*a*b^2*c*e)/b^3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/(a*
f-b*e)-1/15/b^3*(a*d*f-b*c*f-b*d*e)*(3*A*a^2*b^2*d^2*f^2-13*A*a*b^3*c*d*f^
2+7*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2-3*A*b^4*c*d*e*f-2*A*b^4*d^2*e^2+2*B*a^
3*b*d^2*f^2-2*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4867 vs. $2(971) = 1942$.

Time = 0.96 (sec) , antiderivative size = 4867, normalized size of antiderivative = 4.71

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(1/2)*(C*x**2+B*x+A)/(b*x+a)**(7/2)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{7}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{dx+c}}{(bx+a)^{\frac{7}{2}}\sqrt{fx+e}}\,dx$$

input
```
integrate((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(d*x + c)/((b*x + a)^(7/2)*sqrt(f*x + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{c+d\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,(a+b\,x)^{7/2}}\,dx$$

input
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(7/2)),
x)
```

output
```
int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^(7/2)),
 x)
```

Reduce [F]

$$\int \frac{\sqrt{c+dx}(A+Bx+Cx^2)}{(a+bx)^{7/2}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{dx+c}\,(C\,x^2+Bx+A)}{(bx+a)^{\frac{7}{2}}\,\sqrt{fx+e}}dx$$

input
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x)
```

output
```
int((d*x+c)^(1/2)*(C*x^2+B*x+A)/(b*x+a)^(7/2)/(f*x+e)^(1/2),x)
```

3.95 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 825

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(5df(5bcCe+aCde+acCf-7Abdf)-\frac{(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf))}{b}\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+}}{105d^3f^3}$$

$$+\frac{2\left(7Bdf-\frac{2aCdf}{b}-6C(de+cf)\right)(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{35d^2f^2}$$

$$+\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}$$

$$-\frac{2\sqrt{-bc+ad}\Big(3df(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(}{}$$

$$-\frac{2\sqrt{-bc+ad}(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf(7df(3Bde+2Bcf-5Adf)-C(16d^2e^2+8cdef+\right.}{}$$

output
```
-2/105*(5*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*b*c*e)-(3*a*d*f-4*b*(c*f+d*e
))*(7*b*B*d*f-2*a*C*d*f-6*b*C*(c*f+d*e))/b)*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f
*x+e)^(1/2)/d^3/f^3+2/35*(7*B*d*f-2*a*C*d*f/b-6*C*(c*f+d*e))*(b*x+a)^(3/2)
*(d*x+c)^(1/2)*(f*x+e)^(1/2)/d^2/f^2+2/7*C*(b*x+a)^(5/2)*(d*x+c)^(1/2)*(f*
x+e)^(1/2)/b/d/f-2/105*(a*d-b*c)^(1/2)*(3*d*f*(5*a*d*f*(-7*A*b*d*f+C*a*c*f
+C*a*d*e+5*C*b*c*e)+(a*c*f+a*d*e+3*b*c*e)*(7*b*B*d*f-2*a*C*d*f-6*b*C*(c*f+
d*e)))+(a*d*f-2*b*(c*f+d*e))*(5*d*f*(-7*A*b*d*f+C*a*c*f+C*a*d*e+5*C*b*c*e)
-(3*a*d*f-4*b*(c*f+d*e))*(7*b*B*d*f-2*a*C*d*f-6*b*C*(c*f+d*e))/b))*(b*(d*x
+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b
*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b/d^(7/2)/f^4/(d*x+c)^(1/2)/(
b*(f*x+e)/(-a*f+b*e))^(1/2)-2/105*(a*d-b*c)^(1/2)*(-a*f+b*e)*(3*a^2*C*d^2*
f^2*(-c*f+d*e)-3*a*b*d*f*(7*d*f*(-5*A*d*f+2*B*c*f+3*B*d*e)-C*(11*c^2*f^2+8
*c*d*e*f+16*d^2*e^2))-b^2*(C*(24*c^3*f^3+17*c^2*d*e*f^2+16*c*d^2*e^2*f+48*
d^3*e^3)+7*d*f*(5*A*d*f*(c*f+2*d*e)-B*(4*c^2*f^2+3*c*d*e*f+8*d^2*e^2))))*(
b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)
*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^2/d^(7
/2)/f^4/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.49 (sec) , antiderivative size = 1000, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2\left(-b^2\sqrt{-a+\frac{bc}{d}}\left(6a^3Cd^3f^3+3a^2bd^2f^2(-7Bdf+4C(de+cf))-a\right.\right.}{}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]
),x]
```

output
```
(2*(-(b^2*Sqrt[-a + (b*c)/d]*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2*(-7*B*d*f
+ 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72*c^2*f^2) +
 7*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5*c*d^2*e^2*
f + 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) - B*(8*d^2*e^
2 + 7*c*d*e*f + 8*c^2*f^2))))*(c + d*x)*(e + f*x)) + b^2*Sqrt[-a + (b*c)/d
]*d*f*(a + b*x)*(c + d*x)*(e + f*x)*(3*a^2*C*d^2*f^2 + 3*a*b*d*f*(14*B*d*f
 + C*(-11*d*e - 11*c*f + 8*d*f*x)) + b^2*(7*d*f*(5*A*d*f + B*(-4*d*e - 4*c
*f + 3*d*f*x)) + C*(24*c^2*f^2 + c*d*f*(23*e - 18*f*x) + 3*d^2*(8*e^2 - 6*
e*f*x + 5*f^2*x^2)))) - I*(b*c - a*d)*f*(6*a^3*C*d^3*f^3 + 3*a^2*b*d^2*f^2
*(-7*B*d*f + 4*C*(d*e + c*f)) - a*b^2*d*f*(C*(72*d^2*e^2 + 62*c*d*e*f + 72
*c^2*f^2) + 7*d*f*(20*A*d*f - 13*B*(d*e + c*f))) + b^3*(8*C*(6*d^3*e^3 + 5
*c*d^2*e^2*f + 5*c^2*d*e*f^2 + 6*c^3*f^3) + 7*d*f*(10*A*d*f*(d*e + c*f) -
B*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x)
)/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqr
t[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*
c - a*d)*f*(3*a^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f*(-2*B*d*e - 3*B
*c*f + 5*A*d*f) + C*(11*d^2*e^2 + 8*c*d*e*f + 16*c^2*f^2)) + b^2*(C*(24*d^
3*e^3 + 17*c*d^2*e^2*f + 16*c^2*d*e*f^2 + 48*c^3*f^3) + 7*d*f*(5*A*d*f*(d*
e + 2*c*f) - B*(4*d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2))))*(a + b*x)^(3/2)*Sqrt
[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipti...
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 857, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2118, 27, 171, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2} \left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int -\frac{b(a+bx)^{3/2}(5bcCe+aCde+acCf-7Abdf-(7bBdf-2aCdf-6bC(de+cf))x)}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{7b^2df}+\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-\frac{\int \frac{(a+bx)^{3/2}(5bcCe+aCde+acCf-7Abdf-(7bBdf-2aCdf-6bC(de+cf))x)}{\sqrt{c+dx}\sqrt{e+fx}}dx}{7bdf}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-\frac{\frac{2\int \frac{\sqrt{a+bx}(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf))+(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-}{2\sqrt{c+dx}\sqrt{e+fx}}}{5df}}{7bdf}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-\frac{\frac{\int \frac{\sqrt{a+bx}(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf))+(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2a}{\sqrt{c+dx}\sqrt{e+fx}}}{5df}}{7bdf}$$

$\downarrow$ 171

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-2\int \frac{3adf(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf)))-(bce+ade+acf)(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de-}{}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-\int \frac{3adf(5adf(5bcCe+aCde+acCf-7Abdf)+(3bce+ade+acf)(7bBdf-2aCdf-6bC(de+cf)))-(bce+ade+acf)(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+c}{}$$

$\downarrow$ 176

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf}-\frac{(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf\left(7df(-5Adf+2Bcf+3Bde)-C\left(11c^2f^2+8cdef+16d^2e^2\right)\right)-\left(b^2\left(7df\left(5Adf(cf+2de)-B\left(4c^2f^2+3cdef+8d^2e^2\right)\right)+C\left(24c^3f^3\right.\right.\right.\right.}{f}$$

$\downarrow$ 124

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf} - $$

$$\frac{(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf\left(7df(-5Adf+2Bcf+3Bde)-C\left(11c^2f^2+8cdef+16d^2e^2\right)\right)-\left(b^2\left(7df\left(5Adf(cf+2de)-B\left(4c^2f^2+3cdef+8d^2e^2\right)\right)+C\left(24c^3f^3\right.\right.\right.\right.}{f}$$

$\downarrow$ 123

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf} - $$

$$\frac{(be-af)\left(3a^2Cd^2f^2(de-cf)-3abdf\left(7df(-5Adf+2Bcf+3Bde)-C\left(11c^2f^2+8cdef+16d^2e^2\right)\right)-\left(b^2\left(7df\left(5Adf(cf+2de)-B\left(4c^2f^2+3cdef+8d^2e^2\right)\right)+C\left(24c^3f^3\right.\right.\right.\right.}{f}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf} - $$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf)))}{3df} + \frac{2\sqrt{ad-bc}\left(3bdf(5adf(5bcCe+aCde+acCf-7Abd\right.}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf} - $$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf)))}{3df} + \frac{2\sqrt{ad-bc}\left(3bdf(5adf(5bcCe+aCde+acCf-7Abd\right.}{}$$

$\downarrow$ 130

$$\frac{2C(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}{7bdf} -$$

$$\frac{\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}(5bdf(5bcCe+aCde+acCf-7Abdf)-(3adf-4b(de+cf))(7bBdf-2aCdf-6bC(de+cf)))}{3df} + \frac{2\sqrt{ad-bc}\Big(3bdf(5adf(5bcCe+aCde+acCf-7Abd}{}}{}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(2*C*(a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(7*b*d*f) - ((-2*(7*b*B*
d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e
+ f*x])/(5*d*f) + ((2*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f)
 - (3*a*d*f - 4*b*(d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))
)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*d*f) + ((2*Sqrt[-(b*c) + a
*d]*(3*b*d*f*(5*a*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f) + (3*b*c
*e + a*d*e + a*c*f)*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f))) + 2*((a*d
*f)/2 - b*(d*e + c*f))*(5*b*d*f*(5*b*c*C*e + a*C*d*e + a*c*C*f - 7*A*b*d*f
) - (3*a*d*f - 4*b*(d*e + c*f))*(7*b*B*d*f - 2*a*C*d*f - 6*b*C*(d*e + c*f)
)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b
*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c)
 + a*d]*(b*e - a*f)*(3*a^2*C*d^2*f^2*(d*e - c*f) - 3*a*b*d*f*(7*d*f*(3*B*d
*e + 2*B*c*f - 5*A*d*f) - C*(16*d^2*e^2 + 8*c*d*e*f + 11*c^2*f^2)) - b^2*(
C*(48*d^3*e^3 + 16*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 24*c^3*f^3) + 7*d*f*(5*A
*d*f*(2*d*e + c*f) - B*(8*d^2*e^2 + 3*c*d*e*f + 4*c^2*f^2))))*Sqrt[(b*(c +
 d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt
[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/
(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(3*d*f))/(5*d*f))/(7*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 1233, normalized size of antiderivative = 1.49

method	result	size
elliptic	Expression too large to display	1233
default	Expression too large to display	10454

input
```
int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/7*C*b/d/f*x^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+
b*c*e*x+a*c*e)^(1/2)+2/5*(B*b^2+2*C*a*b-2/7*C*b/d/f*(3*a*d*f+3*b*c*f+3*b*d
*e))/b/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/3*(b^2*A+2*a*b*B+a^2*C-2/7*C*b/d/f*(5/2*a*c*f+5/2*a*d*e
+5/2*b*c*e)-2/5*(B*b^2+2*C*a*b-2/7*C*b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/
f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^
2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)+2*(a^2*A-2/5*(B*b^2+2*C*a*b-2/7*C*b
/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*a*c*e-2/3*(b^2*A+2*a*b*B+a^2*C-2/7*C
*b/d/f*(5/2*a*c*f+5/2*a*d*e+5/2*b*c*e)-2/5*(B*b^2+2*C*a*b-2/7*C*b/d/f*(3*a
*d*f+3*b*c*f+3*b*d*e))/b/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(1/2*a*c*f+1
/2*a*d*e+1/2*b*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/
f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*
x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/
2),((-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(2*a*b*A+a^2*B-4/7*C*b/d/f*a*c*e-2/5*(
B*b^2+2*C*a*b-2/7*C*b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/f*(3/2*a*c*f+3/2*
a*d*e+3/2*b*c*e)-2/3*(b^2*A+2*a*b*B+a^2*C-2/7*C*b/d/f*(5/2*a*c*f+5/2*a*d*e
+5/2*b*c*e)-2/5*(B*b^2+2*C*a*b-2/7*C*b/d/f*(3*a*d*f+3*b*c*f+3*b*d*e))/b/d/
f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(c/d-a/b)*((x+c/d)
/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1388, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/315*(3*(15*C*b^4*d^4*f^4*x^2 + 24*C*b^4*d^4*e^2*f^2 + (23*C*b^4*c*d^3 -
(33*C*a*b^3 + 28*B*b^4)*d^4)*e*f^3 + (24*C*b^4*c^2*d^2 - (33*C*a*b^3 + 28*
B*b^4)*c*d^3 + (3*C*a^2*b^2 + 42*B*a*b^3 + 35*A*b^4)*d^4)*f^4 - 3*(6*C*b^4
*d^4*e*f^3 + (6*C*b^4*c*d^3 - (8*C*a*b^3 + 7*B*b^4)*d^4)*f^4)*x)*sqrt(b*x
+ a)*sqrt(d*x + c)*sqrt(f*x + e) + (48*C*b^4*d^4*e^4 + 8*(2*C*b^4*c*d^3 -
(12*C*a*b^3 + 7*B*b^4)*d^4)*e^3*f + (11*C*b^4*c^2*d^2 - 7*(4*C*a*b^3 + 3*B
*b^4)*c*d^3 + (39*C*a^2*b^2 + 119*B*a*b^3 + 70*A*b^4)*d^4)*e^2*f^2 + (16*C
*b^4*c^3*d - 7*(4*C*a*b^3 + 3*B*b^4)*c^2*d^2 + 7*(C*a^2*b^2 + 7*B*a*b^3 +
5*A*b^4)*c*d^3 + (9*C*a^3*b - 56*B*a^2*b^2 - 175*A*a*b^3)*d^4)*e*f^3 + (48
*C*b^4*c^4 - 8*(12*C*a*b^3 + 7*B*b^4)*c^3*d + (39*C*a^2*b^2 + 119*B*a*b^3
+ 70*A*b^4)*c^2*d^2 + (9*C*a^3*b - 56*B*a^2*b^2 - 175*A*a*b^3)*c*d^3 + (6*
C*a^4 - 21*B*a^3*b + 175*A*a^2*b^2)*d^4)*f^4)*sqrt(b*d*f)*weierstrassPInve
rse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*
d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*
e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x
 + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(48*C*b^4*d^4*e^3*f + 8*(5*C*b^4*c*
d^3 - (9*C*a*b^3 + 7*B*b^4)*d^4)*e^2*f^2 + (40*C*b^4*c^2*d^2 - (62*C*a*b^3
 + 49*B*b^4)*c*d^3 + (12*C*a^2*b^2 + 91*B*a*b^3 + 70*A*b^4)*d^4)*e*f^3 + (
48*C*b^4*c^3*d - 8*(9*C*a*b^3 + 7*B*b^4)*c^2*d^2 + (12*C*a^2*b^2 + 91*B...
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2} (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}} \, dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}} \, dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2} (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}} \, dx = \int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}} \, dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2} (A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}} \, dx = \int \frac{(a+b\,x)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{c+d\,x}} \, dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{\sqrt{dx+c}\,\sqrt{fx+e}}dx$$

input
```
int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

3.96 $\int \frac{\sqrt{a+bx}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 524

$$\begin{aligned}
&\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx \\
&= \frac{2\left(5Bdf-\frac{2aCdf}{b}-4C(de+cf)\right)\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{15d^2f^2} \\
&\quad + \frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf} \\
&\quad - \frac{2\sqrt{-bc+ad}\left(3df(3bcCe+aCde+acCf-5Abdf)-\frac{(adf-2b(de+cf))(5bBdf-2aCdf-4bC(de+cf))}{b}\right)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{}}{15bd^{5/2}f^3\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} \\
&\quad - \frac{2\sqrt{-bc+ad}(be-af)\left(aCdf(de-cf)+bC(8d^2e^2+3cdef+4c^2f^2)+5bdf(3Adf-B(2de+cf))\right)\sqrt{}}{15b^2d^{5/2}f^3\sqrt{c+dx}\sqrt{e+fx}}
\end{aligned}$$

output
```
2/15*(5*B*d*f-2*a*C*d*f/b-4*C*(c*f+d*e))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+
e)^(1/2)/d^2/f^2+2/5*C*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f-2/1
5*(a*d-b*c)^(1/2)*(3*d*f*(-5*A*b*d*f+C*a*c*f+C*a*d*e+3*C*b*c*e)-(a*d*f-2*b
*(c*f+d*e))*(5*b*B*d*f-2*a*C*d*f-4*b*C*(c*f+d*e))/b)*(b*(d*x+c)/(-a*d+b*c)
)^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a
*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b/d^(5/2)/f^3/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*
f+b*e))^(1/2)-2/15*(a*d-b*c)^(1/2)*(-a*f+b*e)*(a*C*d*f*(-c*f+d*e)+b*C*(4*c
^2*f^2+3*c*d*e*f+8*d^2*e^2)+5*b*d*f*(3*A*d*f-B*(c*f+2*d*e)))*(b*(d*x+c)/(-
a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1/
2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^2/d^(5/2)/f^3/(d*x
+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.86 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(2a^2Cd^2f^2+abdf(-5Bdf+3C(de+cf))-b^2(C(8d^2e^2+7cdef+8c^2f^2)+5df(3Ac\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),
x]
```

output
```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*
e + c*f)) - b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f -
2*B*(d*e + c*f))))*(c + d*x)*(e + f*x) - b^2*Sqrt[-a + (b*c)/d]*d*f*(a + b
*x)*(c + d*x)*(e + f*x)*(5*b*B*d*f + a*C*d*f + b*C*(-4*d*e - 4*c*f + 3*d*f
*x)) + I*(b*c - a*d)*f*(2*a^2*C*d^2*f^2 + a*b*d*f*(-5*B*d*f + 3*C*(d*e + c
*f)) - b^2*(C*(8*d^2*e^2 + 7*c*d*e*f + 8*c^2*f^2) + 5*d*f*(3*A*d*f - 2*B*(
d*e + c*f))))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e
 + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*
x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)] + I*b*(b*c - a*d)*f*(a*C*d*f*(-(d*e)
 + c*f) + b*C*(4*d^2*e^2 + 3*c*d*e*f + 8*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(
d*e + 2*c*f)))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(
e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b
*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)]))/(15*b^3*Sqrt[-a + (b*c)/d]*d^3*f^
3*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2118, 27, 171, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \textbf{2118}$$

$$\frac{2\int -\frac{b\sqrt{a+bx}(3bcCe+aCde+acCf-5Abdf-(5bBdf-2aCdf-4bC(de+cf))x)}{2\sqrt{c+dx}\sqrt{e+fx}}dx}{5b^2df}+\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-\frac{\int \frac{\sqrt{a+bx}(3bcCe+aCde+acCf-5Abdf-(5bBdf-2aCdf-4bC(de+cf))x)}{\sqrt{c+dx}\sqrt{e+fx}}dx}{5bdf}$$

$\Big\downarrow$ 171

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{2\int \frac{3adf(3bcCe+aCde+acCf-5Abdf)+(bce+ade+acf)(5bBdf-2aCdf-4bC(de+cf))+(3bdf(3bcCe+aCde+acCf-5Abdf)-(adf-2b(de+cf))(5bBdf-2aCdf-4b}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3df}}{5bdf}$$

$\Big\downarrow$ 27

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{\int \frac{3adf(3bcCe+aCde+acCf-5Abdf)+(bce+ade+acf)(5bBdf-2aCdf-4bC(de+cf))+(3bdf(3bcCe+aCde+acCf-5Abdf)-(adf-2b(de+cf))(5bBdf-2aCdf-4bC}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}}{3df}}{5bdf}$$

$\Big\downarrow$ 176

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{\frac{(be-af)\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{(3bdf(acCf+aCde-5Abdf+3bcCe)-(adf-2b(cf+de}{}}{3df}}{5bdf}$$

$\Big\downarrow$ 124

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{\frac{(be-af)\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+aCde-5Abdf+3bcCe}{}}{3df}}{5bdf}$$

$\Big\downarrow$ 123

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$

$$\frac{\frac{\frac{(be-af)\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3bdf(acCf+aCde-5Ab}{}}{3df}}{5}$$

$\Big\downarrow$ 131

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$
$$\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(3b}{}$$
$$3df$$

$\downarrow$ 131

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$
$$\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e+fx}}{}$$
$$3df$$

$\downarrow$ 130

$$\frac{2C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}{5bdf}-$$
$$\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(aCdf(de-cf)+5bdf(3Adf-B(cf+2de))+bC\left(4c^2f^2+3cdef+8d^2e^2\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e-}}{}$$
$$3df$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(2*C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])/(5*b*d*f) - ((-2*(5*b*B*
d*f - 2*a*C*d*f - 4*b*C*(d*e + c*f))*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e +
f*x])/(3*d*f) + ((2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(3*b*c*C*e + a*C*d*e + a*c
*C*f - 5*A*b*d*f) - (a*d*f - 2*b*(d*e + c*f))*(5*b*B*d*f - 2*a*C*d*f - 4*b
*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
 - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (
2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a*C*d*f*(d*e - c*f) + b*C*(8*d^2*e^2 + 3
*c*d*e*f + 4*c^2*f^2) + 5*b*d*f*(3*A*d*f - B*(2*d*e + c*f)))*Sqrt[(b*(c +
d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[
d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(
b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[e + f*x]))/(3*d*f))/(5*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 4.46 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{(fx+e)(bx+a)(xd+c)}\left(\frac{2Cx\sqrt{bdf\,x^3+adf\,x^2+bcf\,x^2+bde\,x^2+acfx+adex+bcex+ace}}{5df}+\frac{2\left(Bb+Ca-\frac{2C(2adf+2bcf+2bde)}{5df}\right)\sqrt{bdf\,x^3+adf\,x}}{3bdf}\right.}{}$
default	Expression too large to display

input
```
int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/5*C/d/f*x*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2*b*d*e))/b/d/f*(
b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/
2)+2*(A*a-2/5*C/d/f*a*c*e-2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2*b*d*e)
)/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/
2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*
x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x
+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(A*b+B*a-2/5*C/d/f
*(3/2*a*c*f+3/2*a*d*e+3/2*b*c*e)-2/3*(B*b+C*a-2/5*C/d/f*(2*a*d*f+2*b*c*f+2
*b*d*e))/b/d/f*(a*d*f+b*c*f+b*d*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((
x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b
*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-c/d+e/f)*Ellipt
icE(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))-e/f*EllipticF
(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(469) = 938$.

Time = 0.13 (sec) , antiderivative size = 1036, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/45*(3*(3*C*b^3*d^3*f^3*x - 4*C*b^3*d^3*e*f^2 - (4*C*b^3*c*d^2 - (C*a*b^2
 + 5*B*b^3)*d^3)*f^3)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e) - (8*C*b^3
*d^3*e^3 + (3*C*b^3*c*d^2 - (7*C*a*b^2 + 10*B*b^3)*d^3)*e^2*f + (3*C*b^3*c
^2*d - (2*C*a*b^2 + 5*B*b^3)*c*d^2 - (2*C*a^2*b - 10*B*a*b^2 - 15*A*b^3)*d
^3)*e*f^2 + (8*C*b^3*c^3 - (7*C*a*b^2 + 10*B*b^3)*c^2*d - (2*C*a^2*b - 10*
B*a*b^2 - 15*A*b^3)*c*d^2 - (2*C*a^3 - 5*B*a^2*b + 30*A*a*b^2)*d^3)*f^3)*s
qrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f
+ (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 -
 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^
3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b
^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) - 3*(8*C*b^3
*d^3*e^2*f + (7*C*b^3*c*d^2 - (3*C*a*b^2 + 10*B*b^3)*d^3)*e*f^2 + (8*C*b^3
*c^2*d - (3*C*a*b^2 + 10*B*b^3)*c*d^2 - (2*C*a^2*b - 5*B*a*b^2 - 15*A*b^3)
*d^3)*f^3)*sqrt(b*d*f)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d
^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d
^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 +
a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)
*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b
*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3
*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x
)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x
)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a+b\,x}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{e+f\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
(20*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*a*b*d*f - 6*sqrt(e + f*x)*sq
rt(c + d*x)*sqrt(a + b*x)*a*c**2*f - 6*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a
+ b*x)*a*c*d*e + 4*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*a*c*d*f*x - 6
*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*b*c**2*e + 4*sqrt(e + f*x)*sqrt
(c + d*x)*sqrt(a + b*x)*b*c**2*f*x + 4*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a
+ b*x)*b*c*d*e*x + 2*int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2)/
(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e*f*x + a**2*d**2*f**2*x**2 +
a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**2 + 3*a*b*c*d*e*f*x + 2*a*b*c*
d*f**2*x**2 + a*b*d**2*e**2*x + 2*a*b*d**2*e*f*x**2 + a*b*d**2*f**2*x**3 +
 b**2*c**2*e*f*x + b**2*c**2*f**2*x**2 + b**2*c*d*e**2*x + 2*b**2*c*d*e*f*
x**2 + b**2*c*d*f**2*x**3 + b**2*d**2*e**2*x**2 + b**2*d**2*e*f*x**3),x)*a
**3*c*d**3*f**3 - 20*int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2)/
(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e*f*x + a**2*d**2*f**2*x**2 +
a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**2 + 3*a*b*c*d*e*f*x + 2*a*b*c*
d*f**2*x**2 + a*b*d**2*e**2*x + 2*a*b*d**2*e*f*x**2 + a*b*d**2*f**2*x**3 +
 b**2*c**2*e*f*x + b**2*c**2*f**2*x**2 + b**2*c*d*e**2*x + 2*b**2*c*d*e*f*
x**2 + b**2*c*d*f**2*x**3 + b**2*d**2*e**2*x**2 + b**2*d**2*e*f*x**3),x)*a
**2*b**2*d**3*f**3 + 5*int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2
)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e*f*x + a**2*d**2*f**2*x**2
+ a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**2 + 3*a*b*c*d*e*f*x + 2*a...
```

3.97 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 384

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{2\sqrt{-bc+ad}(3bBdf-2aCdf-2bC(de+cf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}\,E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}} - \frac{2\sqrt{-bc+ad}(3bdf(Be-Af)-aCf(de-cf)-bCe(2de+cf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{3b^2d^{3/2}f^2\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
2/3*C*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/d/f+2/3*(a*d-b*c)^(1/2)*
(3*b*B*d*f-2*a*C*d*f-2*b*C*(c*f+d*e))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)
^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a
*f+b*e))^(1/2))/b^2/d^(3/2)/f^2/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)
-2/3*(a*d-b*c)^(1/2)*(3*b*d*f*(-A*f+B*e)-a*C*f*(-c*f+d*e)-b*C*e*(c*f+2*d*e
))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(
1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^2/
d^(3/2)/f^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.58 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$= \frac{\sqrt{a+bx}\left(2b^2Cdf(c+dx)(e+fx) - \frac{2b^2(-3bBdf+2aCdf+2bC(de+cf))(c+dx)(e+fx)}{a+bx} + 2i\sqrt{-a+\frac{bc}{d}}df(3bBdf-2\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(Sqrt[a + b*x]*(2*b^2*C*d*f*(c + d*x)*(e + f*x) - (2*b^2*(-3*b*B*d*f + 2*a
*C*d*f + 2*b*C*(d*e + c*f))*(c + d*x)*(e + f*x))/(a + b*x) + (2*I)*Sqrt[-a
 + (b*c)/d]*d*f*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[a + b*x]*
Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ellipt
icE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f -
a*d*f)] + ((2*I)*b*f*(a*C*d*(-(d*e) + c*f) + b*(2*c^2*C*f + 3*A*d^2*f + c*
d*(C*e - 3*B*f)))*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*
(e + f*x))/(f*(a + b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a +
b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/Sqrt[-a + (b*c)/d]))/(3*b^3*d^2*f
^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$\downarrow$ 2118

$$\frac{2\int -\frac{b(bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x)}{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{3b^2df}+\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}$$

$\downarrow$ 27

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\int \frac{bcCe+aCde+acCf-3Abdf-(3bBdf-2aCdf-2bC(de+cf))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{3bdf}$$

$\downarrow$ 176

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$
$$\frac{\frac{(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}-\frac{(-2aCdf+3bBdf-2bC(cf+de))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}}{3bdf}$$

$\downarrow$ 124

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$
$$\frac{\frac{(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}-\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(-2aCdf+3bBdf-2bC(cf+de))\int \frac{\sqrt{\frac{be}{be-af}+b\cdots}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}\cdots}}}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}}{3bdf}$$

$\downarrow$ 123

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$
$$\frac{\frac{(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(-2aCdf+3bBdf-2bC(cf+de))E\left(\text{arcs}\cdots\right.}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}}{3bdf}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-$$
$$\frac{\frac{\sqrt{\frac{b(c+dx)}{bc-ad}}(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(-2aCdf+3bBdf-\cdots}{b\sqrt{d}f\sqrt{c+dx}\cdots}}{3bdf}$$

$\downarrow$ 131

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\frac{\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\int\frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}}{}}{3bdf}$$

$\downarrow$ 130

$$\frac{2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}{3bdf}-\frac{\frac{2\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}(-aCf(de-cf)+3bdf(Be-Af)-bCe(cf+2de))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{e+fx}}-\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b}{}}}{}}{3bdf}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(2*C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - ((-2*Sqrt[-(b*
c) + a*d]*(3*b*B*d*f - 2*a*C*d*f - 2*b*C*(d*e + c*f))*Sqrt[(b*(c + d*x))/(
b*c - a*d)]*Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(
b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]
*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(3*b*d*f*(B*e -
A*f) - a*C*f*(d*e - c*f) - b*C*e*(2*d*e + c*f))*Sqrt[(b*(c + d*x))/(b*c -
a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*S
qrt[c + d*x]*Sqrt[e + f*x]))/(3*b*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 5.25 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.60

method	result
elliptic	$\frac{\sqrt{(fx+e)(bx+a)(xd+c)}\left(\frac{2C\sqrt{bdf\,x^{3}+adf\,x^{2}+bcf\,x^{2}+bde\,x^{2}+acfx+adex+bcex+ace}}{3bdf}+\frac{2\left(A-\frac{2C\left(\frac{1}{2}acf+\frac{1}{2}ade+\frac{1}{2}bce\right)}{3bdf}\right)\left(\frac{c}{d}-\frac{a}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{a}{b}}}}{\sqrt{bdf\,x^{3}+adf\,x^{2}+bcf\,x^{2}+bc}}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(2/3*C/b/d/f*(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*
e*x+a*c*e)^(1/2)+2*(A-2/3*C/b/d/f*(1/2*a*c*f+1/2*a*d*e+1/2*b*c*e))*(c/d-a/
b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b
))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+
a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(
1/2))+2*(B-2/3*C/b/d/f*(a*d*f+b*c*f+b*d*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^
(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d
*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-c/d+e/f
)*EllipticE(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))-e/f*E
llipticF(((x+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. 2(336) = 672.

Time = 0.13 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.10

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*C*b^2*d^2*f^2 + (2*C*b^2*
d^2*e^2 + (C*b^2*c*d + (C*a*b - 3*B*b^2)*d^2)*e*f + (2*C*b^2*c^2 + (C*a*b
- 3*B*b^2)*c*d + (2*C*a^2 - 3*B*a*b + 9*A*b^2)*d^2)*f^2)*sqrt(b*d*f)*weier
strassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b
*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 +
a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^
3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3
*(3*b*d*f*x + b*d*e + (b*c + a*d)*f)/(b*d*f)) + 3*(2*C*b^2*d^2*e*f + (2*C*
b^2*c*d + (2*C*a*b - 3*B*b^2)*d^2)*f^2)*sqrt(b*d*f)*weierstrassZeta(4/3*(b
^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/
(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*
(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), weierstrassPInverse(4/3*
(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2
)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f -
3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e
 + (b*c + a*d)*f)/(b*d*f))))/(b^3*d^3*f^3)
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}}\,dx = \int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x
)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x
)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{too large to display}$$

input

```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output

```
(2*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*b + 2*int((sqrt(e + f*x)*sqrt
(c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*
e*f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**
2 + 3*a*b*c*d*e*f*x + 2*a*b*c*d*f**2*x**2 + a*b*d**2*e**2*x + 2*a*b*d**2*e
*f*x**2 + a*b*d**2*f**2*x**3 + b**2*c**2*e*f*x + b**2*c**2*f**2*x**2 + b**
2*c*d*e**2*x + 2*b**2*c*d*e*f*x**2 + b**2*c*d*f**2*x**3 + b**2*d**2*e**2*x
**2 + b**2*d**2*e*f*x**3),x)*a**2*c*d**2*f**2 - 3*int((sqrt(e + f*x)*sqrt(
c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e
*f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**2
 + 3*a*b*c*d*e*f*x + 2*a*b*c*d*f**2*x**2 + a*b*d**2*e**2*x + 2*a*b*d**2*e*
f*x**2 + a*b*d**2*f**2*x**3 + b**2*c**2*e*f*x + b**2*c**2*f**2*x**2 + b**2
*c*d*e**2*x + 2*b**2*c*d*e*f*x**2 + b**2*c*d*f**2*x**3 + b**2*d**2*e**2*x*
*2 + b**2*d**2*e*f*x**3),x)*a*b**2*d**2*f**2 + 4*int((sqrt(e + f*x)*sqrt(c
 + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e*
f*x + a**2*d**2*f**2*x**2 + a*b*c**2*e*f + a*b*c**2*f**2*x + a*b*c*d*e**2
+ 3*a*b*c*d*e*f*x + 2*a*b*c*d*f**2*x**2 + a*b*d**2*e**2*x + 2*a*b*d**2*e*f
*x**2 + a*b*d**2*f**2*x**3 + b**2*c**2*e*f*x + b**2*c**2*f**2*x**2 + b**2*
c*d*e**2*x + 2*b**2*c*d*e*f*x**2 + b**2*c*d*f**2*x**3 + b**2*d**2*e**2*x**
2 + b**2*d**2*e*f*x**3),x)*a*b*c**2*d*f**2 + 4*int((sqrt(e + f*x)*sqrt(c +
 d*x)*sqrt(a + b*x)*x**2)/(a**2*c*d*e*f + a**2*c*d*f**2*x + a**2*d**2*e...
```

3.98 $\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 422

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{b(bc-ad)(be-af)\sqrt{a+bx}}$$
$$-\frac{2(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{e+fx}E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\sqrt{-bc+ad}f(be-af)\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$-\frac{2(aC(de-cf)-b(cCe-Bcf+Adf))\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right),\frac{(bc-ad)f}{d(be-af)}\right)}{b^2\sqrt{d}\sqrt{-bc+ad}f\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*e)
/(b*x+a)^(1/2)-2*(2*a^2*C*d*f+b^2*(A*d*f+C*c*e)-a*b*(B*d*f+C*c*f+C*d*e))*(
b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/
(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b^2/d^(1/2)/(a*d-b*c)^(
1/2)/f/(-a*f+b*e)/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)-2*(a*C*(-c*f+
d*e)-b*(A*d*f-B*c*f+C*c*e))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+
b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/
d/(-a*f+b*e))^(1/2))/b^2/d^(1/2)/(a*d-b*c)^(1/2)/f/(d*x+c)^(1/2)/(f*x+e)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.18 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{2\left(-b^2(Ab^2+a(-bB+aC))\,(c+dx)(e+fx)+\frac{b^2(2a^2Cdf+b^2(cCe+Adf}{}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),
x]
```

output
```
(2*(-(b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)*(e + f*x)) + (b^2*(2*a^2*C*
d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*(c + d*x)*(e + f*
x))/(d*f) + (I*(b*c - a*d)*(2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e
 + c*C*f + B*d*f))*(a + b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[
(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a
 + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f)])/(Sqrt[-a + (b*c)/d]*d) + (I*b*
(-(b*c) + a*d)*(a*C*(d*e - c*f) + b*(c*C*e - B*d*e + A*d*f))*(a + b*x)^(3/
2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*Ell
ipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f
 - a*d*f)])/(Sqrt[-a + (b*c)/d]*d)))/(b^3*(b*c - a*d)*(b*e - a*f)*Sqrt[a +
 b*x]*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2117, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{ 2117}$$

$$-\frac{2\int -\frac{C(de+cf)a^2-b(cCe+Bde+Bcf-Adf)a+b^2Bce+b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x}{2b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{C(de+cf)a^2-b(cCe+Bde+Bcf-Adf)a+b^2Bce+b\left(\frac{2Cdfa^2}{b}-(Cde+cCf+Bdf)a+b(cCe+Adf)\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{b(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 176}$$

$$\frac{\frac{(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe))\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}}dx}{f}+\frac{(be-af)(aC(de-cf)-b(Adf-Bcf+cCe))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}}{b(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 124}$$

$$\frac{\frac{\sqrt{e+fx}\sqrt{\frac{b(c+dx)}{bc-ad}}(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe))\int \frac{\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}}dx}{f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{(be-af)(aC(de-cf)-b(Adf-Bcf+cCe))\int \cdots}{f}}{b(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 123}$$

$$\frac{\frac{(be-af)(aC(de-cf)-b(Adf-Bcf+cCe))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}}dx}{f}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe\cdots}{b\sqrt{df}\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}\cdots}}{b(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 131}$$

$$\frac{\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(aC(de-cf)-b(Adf-Bcf+cCe)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{e+fx}}dx}{f\sqrt{c+dx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^2Cdf-ab(Bdf+cCf+\cdots\right.}{b\sqrt{d}f\sqrt{c+\cdots}}}{b(bc-ad)(be-af)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 131$$

$$\frac{\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(aC(de-cf)-b(Adf-Bcf+cCe)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}}\sqrt{\frac{be}{be-af}+\frac{bfx}{be-af}}}dx}{f\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^2\cdots\right.}{\cdots}}{b(bc-ad)(be-af)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

$$\Big\downarrow \; 130$$

$$\frac{\frac{2\sqrt{e+fx}\sqrt{ad-bc}\sqrt{\frac{b(c+dx)}{bc-ad}}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)E\left(\arcsin\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad-bc}}\right)|\frac{(bc-ad)f}{d(be-af)}\right)}{b\sqrt{d}f\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}+\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\cdots}}{\cdots}}{b(bc-ad)(be-af)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b\sqrt{a+bx}(bc-ad)(be-af)}$$

input

```
Int[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output

```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b
*e - a*f)*Sqrt[a + b*x]) + ((2*Sqrt[-(b*c) + a*d]*(2*a^2*C*d*f + b^2*(c*C*
e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*
Sqrt[e + f*x]*EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]]
, ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e
+ f*x))/(b*e - a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a*C*(d*e - c*f)
 - b*(c*C*e - B*c*f + A*d*f))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e +
 f*x))/(b*e - a*f)]*EllipticF[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) +
 a*d]], ((b*c - a*d)*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[
e + f*x]))/(b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 783 vs. $2(382) = 764$.

Time = 7.75 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.86

method	result
elliptic	$\sqrt{(fx+e)(bx+a)(xd+c)}\left(-\frac{2\left(bdf\,x^2+bcfx+bdex+bce\right)\left(b^2A-abB+a^2C\right)}{\left(a^2df-abcf-abde+ce\,b^2\right)b^2\sqrt{\left(x+\frac{a}{b}\right)\left(bdf\,x^2+bcfx+bdex+bce\right)}}+\frac{2\left(\frac{Bb-Ca}{b^2}+\frac{(adf-bcf-bde)\left(b^2A-abB+a^2C\right.}{b^2\left(a^2df-abcf-abde+ce\,b^2\right)}\right.}{}\right.$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^
2*(A*b^2-B*a*b+C*a^2)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*
((B*b-C*a)/b^2+1/b^2*(a*d*f-b*c*f-b*d*e)*(A*b^2-B*a*b+C*a^2)/(a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)+(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2*(
A*b^2-B*a*b+C*a^2))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f
))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x
^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/2
),((-c/d+a/b)/(-c/d+e/f))^(1/2))+2*(C/b+1/b*d*f*(A*b^2-B*a*b+C*a^2)/(a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/
(-c/d+e/f))^(1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^
2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*((-c/d+e/f)*EllipticE(((x
+c/d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))-e/f*EllipticF(((x+c/
d)/(c/d-a/b))^(1/2),((-c/d+a/b)/(-c/d+e/f))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(381) = 762$.

Time = 0.14 (sec) , antiderivative size = 1240, normalized size of antiderivative = 2.94

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
-2/3*(3*(C*a^2*b^2 - B*a*b^3 + A*b^4)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x
 + e)*d^2*f^2 + ((C*a*b^3*c*d - C*a^2*b^2*d^2)*e^2 + (C*a*b^3*c^2 + (2*C*a
^2*b^2 - 3*B*a*b^3)*c*d - (2*C*a^3*b - 2*B*a^2*b^2 - A*a*b^3)*d^2)*e*f - (
C*a^2*b^2*c^2 + (2*C*a^3*b - 2*B*a^2*b^2 - A*a*b^3)*c*d - (2*C*a^4 - B*a^3
*b - 2*A*a^2*b^2)*d^2)*f^2 + ((C*b^4*c*d - C*a*b^3*d^2)*e^2 + (C*b^4*c^2 +
 (2*C*a*b^3 - 3*B*b^4)*c*d - (2*C*a^2*b^2 - 2*B*a*b^3 - A*b^4)*d^2)*e*f -
(C*a*b^3*c^2 + (2*C*a^2*b^2 - 2*B*a*b^3 - A*b^4)*c*d - (2*C*a^3*b - B*a^2*
b^2 - 2*A*a*b^3)*d^2)*f^2)*x)*sqrt(b*d*f)*weierstrassPInverse(4/3*(b^2*d^2
*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d
^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c
^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 + (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3), 1/3*(3*b*d*f*x + b*d*e + (b*c
+ a*d)*f)/(b*d*f)) + 3*sqrt(b*d*f)*((C*a*b^3*c*d - C*a^2*b^2*d^2)*e*f - (C
*a^2*b^2*c*d - (2*C*a^3*b - B*a^2*b^2 + A*a*b^3)*d^2)*f^2 + ((C*b^4*c*d -
C*a*b^3*d^2)*e*f - (C*a*b^3*c*d - (2*C*a^2*b^2 - B*a*b^3 + A*b^4)*d^2)*f^2
)*x)*weierstrassZeta(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^2*c^2
 - a*b*c*d + a^2*d^2)*f^2)/(b^2*d^2*f^2), -4/27*(2*b^3*d^3*e^3 - 3*(b^3*c*
d^2 + a*b^2*d^3)*e^2*f - 3*(b^3*c^2*d - 4*a*b^2*c*d^2 + a^2*b*d^3)*e*f^2 +
 (2*b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + 2*a^3*d^3)*f^3)/(b^3*d^3*f^3
), weierstrassPInverse(4/3*(b^2*d^2*e^2 - (b^2*c*d + a*b*d^2)*e*f + (b^...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{A+Bx+Cx^2}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2}{(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}}\,dx = \text{too large to display}$$

input

```
int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output

```
(2*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*b + 2*int((sqrt(e + f*x)*sqrt
(c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*e + a**2*c*f*x + a**2*d*e*x + a**2*d
*f*x**2 + 2*a*b*c*e*x + 2*a*b*c*f*x**2 + 2*a*b*d*e*x**2 + 2*a*b*d*f*x**3 +
 b**2*c*e*x**2 + b**2*c*f*x**3 + b**2*d*e*x**3 + b**2*d*f*x**4),x)*a**2*c*
d*f - int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*e + a**
2*c*f*x + a**2*d*e*x + a**2*d*f*x**2 + 2*a*b*c*e*x + 2*a*b*c*f*x**2 + 2*a*
b*d*e*x**2 + 2*a*b*d*f*x**3 + b**2*c*e*x**2 + b**2*c*f*x**3 + b**2*d*e*x**
3 + b**2*d*f*x**4),x)*a*b**2*d*f + 2*int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt
(a + b*x)*x**2)/(a**2*c*e + a**2*c*f*x + a**2*d*e*x + a**2*d*f*x**2 + 2*a*
b*c*e*x + 2*a*b*c*f*x**2 + 2*a*b*d*e*x**2 + 2*a*b*d*f*x**3 + b**2*c*e*x**2
 + b**2*c*f*x**3 + b**2*d*e*x**3 + b**2*d*f*x**4),x)*a*b*c*d*f*x - int((sq
rt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2)/(a**2*c*e + a**2*c*f*x + a**
2*d*e*x + a**2*d*f*x**2 + 2*a*b*c*e*x + 2*a*b*c*f*x**2 + 2*a*b*d*e*x**2 +
2*a*b*d*f*x**3 + b**2*c*e*x**2 + b**2*c*f*x**3 + b**2*d*e*x**3 + b**2*d*f*
x**4),x)*b**3*d*f*x + 2*int((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x))/(a
**2*c*e + a**2*c*f*x + a**2*d*e*x + a**2*d*f*x**2 + 2*a*b*c*e*x + 2*a*b*c*
f*x**2 + 2*a*b*d*e*x**2 + 2*a*b*d*f*x**3 + b**2*c*e*x**2 + b**2*c*f*x**3 +
 b**2*d*e*x**3 + b**2*d*f*x**4),x)*a**3*d*f - int((sqrt(e + f*x)*sqrt(c +
d*x)*sqrt(a + b*x))/(a**2*c*e + a**2*c*f*x + a**2*d*e*x + a**2*d*f*x**2 +
2*a*b*c*e*x + 2*a*b*c*f*x**2 + 2*a*b*d*e*x**2 + 2*a*b*d*f*x**3 + b**2*c...
```

3.99 $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 642

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=-\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^{3/2}}$$
$$+\frac{2(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4Adf)-b^3(3Bce-2A(de+cf))+a^2b(Bdf-4C(de+cf)))\sqrt{c}}{3b(bc-ad)^2(be-af)^2\sqrt{a+bx}}$$
$$-\frac{2\sqrt{d}(2a^3Cdf+ab^2(6cCe+Bde+Bcf-4Adf)-b^3(3Bce-2A(de+cf))+a^2b(Bdf-4C(de+cf)))}{3b^2(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx}\sqrt{\frac{b(e+fx)}{be-af}}}$$
$$-\frac{2(a^2Cd(de-cf)-b^2(3c^2Ce-3Bcde+2Ad^2e+Acdf)+ab(3(c^2C+Ad^2)f-Bd(de+2cf)))\sqrt{\frac{b(c+d}{bc-a}}}{3b^2\sqrt{d}(-bc+ad)^{3/2}(be-af)\sqrt{c+dx}\sqrt{e+fx}}$$

output
```
-2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(3/2)+2/3*(2*a^3*C*d*f+a*b^2*(-4*A*d*f+B*c*f+B*d*e+6*C*c*e)-b^3
*(3*B*c*e-2*A*(c*f+d*e))+a^2*b*(B*d*f-4*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f*x+e
)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(1/2)-2/3*d^(1/2)*(2*a^3*C*d*f
+a*b^2*(-4*A*d*f+B*c*f+B*d*e+6*C*c*e)-b^3*(3*B*c*e-2*A*(c*f+d*e))+a^2*b*(B
*d*f-4*C*(c*f+d*e)))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+e)^(1/2)*EllipticE(
d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(-a*f+b*e))^(1/2))/b
^2/(a*d-b*c)^(3/2)/(-a*f+b*e)^2/(d*x+c)^(1/2)/(b*(f*x+e)/(-a*f+b*e))^(1/2)
-2/3*(a^2*C*d*(-c*f+d*e)-b^2*(A*c*d*f+2*A*d^2*e-3*B*c*d*e+3*C*c^2*e)+a*b*(
3*(A*d^2+C*c^2)*f-B*d*(2*c*f+d*e)))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(b*(f*x+e
)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d
+b*c)*f/d/(-a*f+b*e))^(1/2))/b^2/d^(1/2)/(a*d-b*c)^(3/2)/(-a*f+b*e)/(d*x+c
)^(1/2)/(f*x+e)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.50 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx =$$

$$-\frac{2\left(b^2\sqrt{-a+\frac{bc}{d}}(c+dx)(e+fx)\left(\left(Ab^2+a(-bB+aC)\right)(bc-ad)(be-af)+\left(-2a^3Cdf-ab^2(6cCe+\right.\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),
x]
```

output

```
(-2*(b^2*Sqrt[-a + (b*c)/d]*(c + d*x)*(e + f*x)*((A*b^2 + a*(-(b*B) + a*C)
)*(b*c - a*d)*(b*e - a*f) + (-2*a^3*C*d*f - a*b^2*(6*c*C*e + B*d*e + B*c*f
 - 4*A*d*f) + b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(-(B*d*f) + 4*C*(d*e
 + c*f)))*(a + b*x)) + (a + b*x)*(b^2*Sqrt[-a + (b*c)/d]*(2*a^3*C*d*f + a*
b^2*(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e + 2*A*(d*e + c*f))
 + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*(c + d*x)*(e + f*x) + I*(b*c - a*d)*f*
(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e + B*c*f - 4*A*d*f) + b^3*(-3*B*c*e +
 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(d*e + c*f)))*(a + b*x)^(3/2)*Sqrt[
(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a + b*x))]*EllipticE[I
*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d*f)/(b*c*f - a*d*f
)] - I*b*(b*c - a*d)*(a^2*C*f*(d*e - c*f) + b^2*(3*c*C*e^2 + A*d*e*f + c*f
*(-3*B*e + 2*A*f)) + a*b*(-3*C*d*e^2 + f*(2*B*d*e + B*c*f - 3*A*d*f)))*(a
+ b*x)^(3/2)*Sqrt[(b*(c + d*x))/(d*(a + b*x))]*Sqrt[(b*(e + f*x))/(f*(a +
b*x))]*EllipticF[I*ArcSinh[Sqrt[-a + (b*c)/d]/Sqrt[a + b*x]], (b*d*e - a*d
*f)/(b*c*f - a*d*f)])))/(3*b^3*Sqrt[-a + (b*c)/d]*(b*c - a*d)^2*(b*e - a*f
)^2*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2117, 27, 169, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}dx$$

$$\Big\downarrow \text{ 2117}$$

$$-\frac{2\int -\frac{C(de+cf)a^2-b(3cCe+Bde+Bcf-3Adf)a+b^2(3Bce-2A(de+cf))+b\left(\frac{2Cdfa^2}{b}-3Cdea-3cCfa+Bdfa+3bcCe-Abdf\right)x}{2b(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}dx}{3(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{C(de+cf)a^2-b(3cCe+Bde+Bcf-3Adf)a+b^2(3Bce-2A(de+cf))+b\left(\frac{2Cdfa^2}{b}+Bdfa-3C(de+cf)a+b(3cCe-Adf)\right)x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}}dx}{3b(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 169

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de)))}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{2\int \frac{Cdf(de+cf)a^3-b\left(C\left(3d^2e^2+5c\right.\right.}{}}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de)))}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{\int \frac{Cdf(de+cf)a^3-b\left(C\left(3d^2e^2+5cd\right.\right.}{}}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 176

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de)))}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{(be-af)(a^2Cd(de-cf)+ab(3f(}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 124

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de)))}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{(be-af)(a^2Cd(de-cf)+ab(3f(}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$\downarrow$ 123

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{(be-af)\left(a^2Cd(de-cf)+ab(3f(\right.}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \ 131$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\left(a^2Cd(de-cf)+\right.}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \ 131$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+fx)}{be-af}}\left(a^2C\right.}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

$$\Big\downarrow \ 130$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+a^2b(Bdf-4C(cf+de))+ab^2(-4Adf+Bcf+Bde+6cCe)-b^3(3Bce-2A(cf+de))\right)}{\sqrt{a+bx}(bc-ad)(be-af)}-\frac{2\sqrt{ad-bc}(be-af)\sqrt{\frac{b(c+dx)}{bc-ad}}\sqrt{\frac{b(e+}{be-}}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^{3/2}(bc-ad)(be-af)}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^(3/2)) + ((2*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B*d*e +
 B*c*f - 4*A*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f - 4*C*(
d*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*Sqrt[a
+ b*x]) - ((2*Sqrt[d]*Sqrt[-(b*c) + a*d]*(2*a^3*C*d*f + a*b^2*(6*c*C*e + B
*d*e + B*c*f - 4*A*d*f) - b^3*(3*B*c*e - 2*A*(d*e + c*f)) + a^2*b*(B*d*f -
 4*C*(d*e + c*f)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*EllipticE
[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b
*e - a*f))])/(b*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e - a*f)]) + (2*Sqrt[-
(b*c) + a*d]*(b*e - a*f)*(a^2*C*d*(d*e - c*f) - b^2*(3*c^2*C*e - 3*B*c*d*e
 + 2*A*d^2*e + A*c*d*f) + a*b*(3*(c^2*C + A*d^2)*f - B*d*(d*e + 2*c*f)))*S
qrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[(b*(e + f*x))/(b*e - a*f)]*EllipticF[A
rcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)*f)/(d*(b*e
 - a*f))])/(b*Sqrt[d]*Sqrt[c + d*x]*Sqrt[e + f*x]))/((b*c - a*d)*(b*e - a*
f)))/(3*b*(b*c - a*d)*(b*e - a*f))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1248 vs. $2(588) = 1176$.

Time = 21.47 (sec) , antiderivative size = 1249, normalized size of antiderivative = 1.95

method	result	size
elliptic	Expression too large to display	1249
default	Expression too large to display	13626

input
```
int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^3*(A*b^2-B*a*b+C*a^2)*(b*d*f*x^3
+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b
)^2-2/3*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)^2/b^2*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*a
*b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e
)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b*d*e*x+b*c*e))^(1/2)+2*(C/b^2-1/3*d*f/b^2*(
A*b^2-B*a*b+C*a^2)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)+1/3/b^2*(a*d*f-b*c*f-
b*d*e)*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*a*
b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e)
/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2+1/3*(b*c*f+b*d*e)/(a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)^2/b^2*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*d*e-B*a^2*b*d*f-B*
a*b^2*c*f-B*a*b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*c*f+4*C*a^2*b*d*e-
6*C*a*b^2*c*e))*(c/d-a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(
1/2)*((x+a/b)/(-c/d+a/b))^(1/2)/(b*d*f*x^3+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a
*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)*EllipticF(((x+c/d)/(c/d-a/b))^(1/2),((
-c/d+a/b)/(-c/d+e/f))^(1/2))+2/3/b*d*f*(4*A*a*b^2*d*f-2*A*b^3*c*f-2*A*b^3*
d*e-B*a^2*b*d*f-B*a*b^2*c*f-B*a*b^2*d*e+3*B*b^3*c*e-2*C*a^3*d*f+4*C*a^2*b*
c*f+4*C*a^2*b*d*e-6*C*a*b^2*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(c/d-
a/b)*((x+c/d)/(c/d-a/b))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)*((x+a/b)/(-c/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2344 vs. $2(587) = 1174$.

Time = 0.28 (sec) , antiderivative size = 2344, normalized size of antiderivative = 3.65

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
2/9*(3*(((5*C*a^2*b^4 - 2*B*a*b^5 - A*b^6)*c*d - 3*(C*a^3*b^3 - A*a*b^5)*d
^2)*e*f - (3*(C*a^3*b^3 - A*a*b^5)*c*d - (C*a^4*b^2 + 2*B*a^3*b^3 - 5*A*a^
2*b^4)*d^2)*f^2 + ((3*(2*C*a*b^5 - B*b^6)*c*d - (4*C*a^2*b^4 - B*a*b^5 - 2
*A*b^6)*d^2)*e*f - ((4*C*a^2*b^4 - B*a*b^5 - 2*A*b^6)*c*d - (2*C*a^3*b^3 +
 B*a^2*b^4 - 4*A*a*b^5)*d^2)*f^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x
+ e) + ((9*C*a^2*b^4*c^2 - 3*(4*C*a^3*b^3 + B*a^2*b^4)*c*d + (5*C*a^4*b^2
+ B*a^3*b^3 + 2*A*a^2*b^4)*d^2)*e^2 - (3*(4*C*a^3*b^3 + B*a^2*b^4)*c^2 - (
13*C*a^4*b^2 + 11*B*a^3*b^3 + A*a^2*b^4)*c*d + (5*C*a^5*b + 4*B*a^4*b^2 +
5*A*a^3*b^3)*d^2)*e*f + ((5*C*a^4*b^2 + B*a^3*b^3 + 2*A*a^2*b^4)*c^2 - (5*
C*a^5*b + 4*B*a^4*b^2 + 5*A*a^3*b^3)*c*d + (2*C*a^6 + B*a^5*b + 5*A*a^4*b^
2)*d^2)*f^2 + ((9*C*b^6*c^2 - 3*(4*C*a*b^5 + B*b^6)*c*d + (5*C*a^2*b^4 + B
*a*b^5 + 2*A*b^6)*d^2)*e^2 - (3*(4*C*a*b^5 + B*b^6)*c^2 - (13*C*a^2*b^4 +
11*B*a*b^5 + A*b^6)*c*d + (5*C*a^3*b^3 + 4*B*a^2*b^4 + 5*A*a*b^5)*d^2)*e*f
 + ((5*C*a^2*b^4 + B*a*b^5 + 2*A*b^6)*c^2 - (5*C*a^3*b^3 + 4*B*a^2*b^4 + 5
*A*a*b^5)*c*d + (2*C*a^4*b^2 + B*a^3*b^3 + 5*A*a^2*b^4)*d^2)*f^2)*x^2 + 2*
((9*C*a*b^5*c^2 - 3*(4*C*a^2*b^4 + B*a*b^5)*c*d + (5*C*a^3*b^3 + B*a^2*b^4
 + 2*A*a*b^5)*d^2)*e^2 - (3*(4*C*a^2*b^4 + B*a*b^5)*c^2 - (13*C*a^3*b^3 +
11*B*a^2*b^4 + A*a*b^5)*c*d + (5*C*a^4*b^2 + 4*B*a^3*b^3 + 5*A*a^2*b^4)*d^
2)*e*f + ((5*C*a^3*b^3 + B*a^2*b^4 + 2*A*a*b^5)*c^2 - (5*C*a^4*b^2 + 4*B*a
^3*b^3 + 5*A*a^2*b^4)*c*d + (2*C*a^5*b + B*a^4*b^2 + 5*A*a^3*b^3)*d^2)*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{A+Bx+Cx^2}{(a+bx)^{\frac{5}{2}}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**(5/2)*sqrt(c + d*x)*sqrt(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
(2*sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*b + 2*int((sqrt(e + f*x)*sqrt
(c + d*x)*sqrt(a + b*x)*x**2)/(a**4*c*d*e*f + a**4*c*d*f**2*x + a**4*d**2*
e*f*x + a**4*d**2*f**2*x**2 - a**3*b*c**2*e*f - a**3*b*c**2*f**2*x - a**3*
b*c*d*e**2 + a**3*b*c*d*e*f*x + 2*a**3*b*c*d*f**2*x**2 - a**3*b*d**2*e**2*
x + 2*a**3*b*d**2*e*f*x**2 + 3*a**3*b*d**2*f**2*x**3 - 3*a**2*b**2*c**2*e*
f*x - 3*a**2*b**2*c**2*f**2*x**2 - 3*a**2*b**2*c*d*e**2*x - 3*a**2*b**2*c*
d*e*f*x**2 - 3*a**2*b**2*d**2*e**2*x**2 + 3*a**2*b**2*d**2*f**2*x**4 - 3*a
*b**3*c**2*e*f*x**2 - 3*a*b**3*c**2*f**2*x**3 - 3*a*b**3*c*d*e**2*x**2 - 5
*a*b**3*c*d*e*f*x**3 - 2*a*b**3*c*d*f**2*x**4 - 3*a*b**3*d**2*e**2*x**3 -
2*a*b**3*d**2*e*f*x**4 + a*b**3*d**2*f**2*x**5 - b**4*c**2*e*f*x**3 - b**4
*c**2*f**2*x**4 - b**4*c*d*e**2*x**3 - 2*b**4*c*d*e*f*x**4 - b**4*c*d*f**2
*x**5 - b**4*d**2*e**2*x**4 - b**4*d**2*e*f*x**5),x)*a**4*c*d**2*f**2 + in
t((sqrt(e + f*x)*sqrt(c + d*x)*sqrt(a + b*x)*x**2)/(a**4*c*d*e*f + a**4*c*
d*f**2*x + a**4*d**2*e*f*x + a**4*d**2*f**2*x**2 - a**3*b*c**2*e*f - a**3*
b*c**2*f**2*x - a**3*b*c*d*e**2 + a**3*b*c*d*e*f*x + 2*a**3*b*c*d*f**2*x**
2 - a**3*b*d**2*e**2*x + 2*a**3*b*d**2*e*f*x**2 + 3*a**3*b*d**2*f**2*x**3
- 3*a**2*b**2*c**2*e*f*x - 3*a**2*b**2*c**2*f**2*x**2 - 3*a**2*b**2*c*d*e*
*2*x - 3*a**2*b**2*c*d*e*f*x**2 - 3*a**2*b**2*d**2*e**2*x**2 + 3*a**2*b**2
*d**2*f**2*x**4 - 3*a*b**3*c**2*e*f*x**2 - 3*a*b**3*c**2*f**2*x**3 - 3*a*b
**3*c*d*e**2*x**2 - 5*a*b**3*c*d*e*f*x**3 - 2*a*b**3*c*d*f**2*x**4 - 3*...
```

3.100 $\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1116

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$$+\frac{2(2a^3Cdf+ab^2(10cCe+Bde+Bcf-8Adf)-b^3(5Bce-4A(de+cf))+3a^2b(Bdf-2C(de+cf)))}{15b(bc-ad)^2(be-af)^2(a+bx)^{3/2}}$$

$$+\frac{2(2a^4Cd^2f^2+a^3bdf(3Bdf-7C(de+cf))-b^4(8Ad^2e^2-cde(10Be-7Af)+c^2(15Ce^2-10Bef+8A}{}$$

$$+\frac{2\sqrt{d}(2a^4Cd^2f^2+a^3bdf(3Bdf-7C(de+cf))-b^4(8Ad^2e^2-cde(10Be-7Af)+c^2(15Ce^2-10Bef+}{}$$

$$+\frac{2\sqrt{d}(a^3Cdf(de-cf)+b^3(8Ad^2e^2-cde(10Be-3Af)+c^2(15Ce^2-5Bef+4Af^2))+ab^2(d^2e(2Be-}{}$$

output
```
-2/5*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b/(-a*d+b*c)/(-a*f+b*
e)/(b*x+a)^(5/2)+2/15*(2*a^3*C*d*f+a*b^2*(-8*A*d*f+B*c*f+B*d*e+10*C*c*e)-b
^3*(5*B*c*e-4*A*(c*f+d*e))+3*a^2*b*(B*d*f-2*C*(c*f+d*e)))*(d*x+c)^(1/2)*(f
*x+e)^(1/2)/b/(-a*d+b*c)^2/(-a*f+b*e)^2/(b*x+a)^(3/2)+2/15*(2*a^4*C*d^2*f^
2+a^3*b*d*f*(3*B*d*f-7*C*(c*f+d*e))-b^4*(8*A*d^2*e^2-c*d*e*(-7*A*f+10*B*e)
+c^2*(8*A*f^2-10*B*e*f+15*C*e^2))-a*b^3*(d^2*e*(-23*A*f+2*B*e)-2*c^2*f*(-B
*f+5*C*e)-c*d*(23*A*f^2-33*B*e*f+10*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-13*c*d*e
*f+3*d^2*e^2)+d*f*(23*A*d*f-7*B*(c*f+d*e))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)/b
/(-a*d+b*c)^3/(-a*f+b*e)^3/(b*x+a)^(1/2)+2/15*d^(1/2)*(2*a^4*C*d^2*f^2+a^3
*b*d*f*(3*B*d*f-7*C*(c*f+d*e))-b^4*(8*A*d^2*e^2-c*d*e*(-7*A*f+10*B*e)+c^2*
(8*A*f^2-10*B*e*f+15*C*e^2))-a*b^3*(d^2*e*(-23*A*f+2*B*e)-2*c^2*f*(-B*f+5*
C*e)-c*d*(23*A*f^2-33*B*e*f+10*C*e^2))-a^2*b^2*(C*(3*c^2*f^2-13*c*d*e*f+3*
d^2*e^2)+d*f*(23*A*d*f-7*B*(c*f+d*e))))*(b*(d*x+c)/(-a*d+b*c))^(1/2)*(f*x+
e)^(1/2)*EllipticE(d^(1/2)*(b*x+a)^(1/2)/(a*d-b*c)^(1/2),((-a*d+b*c)*f/d/(
-a*f+b*e))^(1/2))/b^2/(a*d-b*c)^(5/2)/(-a*f+b*e)^3/(d*x+c)^(1/2)/(b*(f*x+e
)/(-a*f+b*e))^(1/2)+2/15*d^(1/2)*(a^3*C*d*f*(-c*f+d*e)+b^3*(8*A*d^2*e^2-c*
d*e*(-3*A*f+10*B*e)+c^2*(4*A*f^2-5*B*e*f+15*C*e^2))+a*b^2*(d^2*e*(-19*A*f+
2*B*e)-c^2*f*(-B*f+20*C*e)-c*d*(11*A*f^2-27*B*e*f+10*C*e^2))-3*a^2*b*(d*f*
(-5*A*d*f+3*B*c*f+2*B*d*e)-C*(3*c^2*f^2+c*d*e*f+d^2*e^2)))*(b*(d*x+c)/(-a*
d+b*c))^(1/2)*(b*(f*x+e)/(-a*f+b*e))^(1/2)*EllipticF(d^(1/2)*(b*x+a)^(1...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.38 (sec) , antiderivative size = 1520, normalized size of antiderivative = 1.36

$$\int \frac{A + Bx + Cx^2}{(a + bx)^{7/2}\sqrt{c + dx}\sqrt{e + fx}}\, dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),
x]
```

output
```
Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*((-2*(A*b^2 - a*b*B + a^2*C))/(5
*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) - (2*(5*b^3*B*c*e - 10*a*b^2*c*C*e
 - 4*A*b^3*d*e - a*b^2*B*d*e + 6*a^2*b*C*d*e - 4*A*b^3*c*f - a*b^2*B*c*f +
 6*a^2*b*c*C*f + 8*a*A*b^2*d*f - 3*a^2*b*B*d*f - 2*a^3*C*d*f))/(15*b*(b*c
- a*d)^2*(b*e - a*f)^2*(a + b*x)^2) - (2*(15*b^4*c^2*C*e^2 - 10*b^4*B*c*d*
e^2 - 10*a*b^3*c*C*d*e^2 + 8*A*b^4*d^2*e^2 + 2*a*b^3*B*d^2*e^2 + 3*a^2*b^2
*C*d^2*e^2 - 10*b^4*B*c^2*e*f - 10*a*b^3*c^2*C*e*f + 7*A*b^4*c*d*e*f + 33*
a*b^3*B*c*d*e*f - 13*a^2*b^2*c*C*d*e*f - 23*a*A*b^3*d^2*e*f - 7*a^2*b^2*B*
d^2*e*f + 7*a^3*b*C*d^2*e*f + 8*A*b^4*c^2*f^2 + 2*a*b^3*B*c^2*f^2 + 3*a^2*
b^2*c^2*C*f^2 - 23*a*A*b^3*c*d*f^2 - 7*a^2*b^2*B*c*d*f^2 + 7*a^3*b*c*C*d*f
^2 + 23*a^2*A*b^2*d^2*f^2 - 3*a^3*b*B*d^2*f^2 - 2*a^4*C*d^2*f^2))/(15*b*(b
*c - a*d)^3*(b*e - a*f)^3*(a + b*x))) + (2*(a + b*x)^(3/2)*(Sqrt[-a + (b*c
)/d]*(-2*a^4*C*d^2*f^2 + a^3*b*d*f*(-3*B*d*f + 7*C*(d*e + c*f)) + a*b^3*(d
^2*e*(2*B*e - 23*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(-10*C*e^2 + 33*B*e*f
 - 23*A*f^2)) + b^4*(8*A*d^2*e^2 + c*d*e*(-10*B*e + 7*A*f) + c^2*(15*C*e^2
 - 10*B*e*f + 8*A*f^2)) + a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2)
+ d*f*(23*A*d*f - 7*B*(d*e + c*f))))*(d + (b*c)/(a + b*x) - (a*d)/(a + b*x
))*(f + (b*e)/(a + b*x) - (a*f)/(a + b*x)) - (I*(-(b*c) + a*d)*f*(-2*a^4*C
*d^2*f^2 + a^3*b*d*f*(-3*B*d*f + 7*C*(d*e + c*f)) + a*b^3*(d^2*e*(2*B*e -
23*A*f) + 2*c^2*f*(-5*C*e + B*f) + c*d*(-10*C*e^2 + 33*B*e*f - 23*A*f^2...
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2117, 27, 169, 27, 169, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \text{2117}$$

$$-\frac{2\int -\frac{C(de+cf)a^2-b(5cCe+Bde+Bcf-5Adf)a+b^2(5Bce-4A(de+cf))+b\left(\frac{2Cdfa^2}{b}-5Cdea-5cCfa+3Bdfa+5bcCe-3Abdf\right)x}{2b(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}dx}{5(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\int \frac{C(de+cf)a^2-b(5cCe+Bde+Bcf-5Adf)a+b^2(5Bce-4A(de+cf))+b\left(\frac{2Cdfa^2}{b}+3Bdfa-5C(de+cf)a+b(5cCe-3Adf)\right)x}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}}dx}{5b(bc-ad)(be-af)}-\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 169

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2a^3Cdf+3a^2b(Bdf-2C(cf+de))+ab^2(-8Adf+Bcf+Bde+10cCe)-b^3(5Bce-4A(cf+de))\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)}-\frac{2\int -\frac{Cdf(de+cf)a^3+3b\left(C\left(d^2e^{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 27

$$\frac{\int \frac{Cdf(de+cf)a^3+3b\left(C\left(d^2e^2-cdfe+c^2f^2\right)+df(5Adf-2B(de+cf))\right)a^2+b^2\left(-2f(5Ce-Bf)c^2-d\left(10Ce^2-28Bfe+19Af^2\right)c+d^2e(2Be-19Af)\right)a+b^3\left(\left(15Ce^2-1\cdots\right.\right.}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e\cdots}}}{3(bc-ad)(be-a\cdots}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{5b(a+bx)^{5/2}(bc-ad)(be-af)}$$

$\downarrow$ 169

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+c\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\downarrow$ 27

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+c\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 176

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+c\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 124

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+c\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 123

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+c\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 131

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+c\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 131

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+c\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

$\Big\downarrow$ 130

$$\frac{\frac{2\sqrt{c+dx}\sqrt{e+fx}\left(2Cdfa^3+3b(Bdf-2C(de+cf))a^2+b^2(10cCe+Bde+Bcf-8Adf)a-b^3(5Bce-4A(de+cf))\right)}{3(bc-ad)(be-af)(a+bx)^{3/2}}+\frac{2\left(2Cd^2f^2a^4+bdf(3Bdf-7C(de+c\ldots\right.}{\ldots}}{\ldots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{5b(bc-ad)(be-af)(a+bx)^{5/2}}$$

input

```
Int[(A + B*x + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(5*b*(b*c - a*d)*
(b*e - a*f)*(a + b*x)^(5/2)) + ((2*(2*a^3*C*d*f + a*b^2*(10*c*C*e + B*d*e
+ B*c*f - 8*A*d*f) - b^3*(5*B*c*e - 4*A*(d*e + c*f)) + 3*a^2*b*(B*d*f - 2*
C*(d*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*(b*c - a*d)*(b*e - a*f)*(a
 + b*x)^(3/2)) + ((2*(2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*
f)) - b^4*(8*A*d^2*e^2 - c*d*e*(10*B*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f
 + 8*A*f^2)) - a*b^3*(d^2*e*(2*B*e - 23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d
*(10*C*e^2 - 33*B*e*f + 23*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 13*c*d*e*f +
3*c^2*f^2) + d*f*(23*A*d*f - 7*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x
])/((b*c - a*d)*(b*e - a*f)*Sqrt[a + b*x]) - (d*f*((2*Sqrt[-(b*c) + a*d]*(
2*a^4*C*d^2*f^2 + a^3*b*d*f*(3*B*d*f - 7*C*(d*e + c*f)) - b^4*(8*A*d^2*e^2
 - c*d*e*(10*B*e - 7*A*f) + c^2*(15*C*e^2 - 10*B*e*f + 8*A*f^2)) - a*b^3*(
d^2*e*(2*B*e - 23*A*f) - 2*c^2*f*(5*C*e - B*f) - c*d*(10*C*e^2 - 33*B*e*f
+ 23*A*f^2)) + a^2*b^2*(d*f*(7*B*d*e + 7*B*c*f - 23*A*d*f) - C*(3*d^2*e^2
- 13*c*d*e*f + 3*c^2*f^2)))*Sqrt[(b*(c + d*x))/(b*c - a*d)]*Sqrt[e + f*x]*
EllipticE[ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(b*c) + a*d]], ((b*c - a*d)
*f)/(d*(b*e - a*f))])/(b*Sqrt[d]*f*Sqrt[c + d*x]*Sqrt[(b*(e + f*x))/(b*e -
 a*f)]) + (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*(a^3*C*d*f*(d*e - c*f) + b^3*(
8*A*d^2*e^2 - c*d*e*(10*B*e - 3*A*f) + c^2*(15*C*e^2 - 5*B*e*f + 4*A*f^2))
 + a*b^2*(d^2*e*(2*B*e - 19*A*f) - c^2*f*(20*C*e - B*f) - c*d*(10*C*e^2...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2282 vs. 2(1054) = 2108.

Time = 24.11 (sec) , antiderivative size = 2283, normalized size of antiderivative = 2.05

method	result	size
elliptic	Expression too large to display	2283
default	Expression too large to display	35611

input
```
int((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((f*x+e)*(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)*
(-2/5/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^4*(A*b^2-B*a*b+C*a^2)*(b*d*f*x^3
+a*d*f*x^2+b*c*f*x^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b
)^3-2/15*(8*A*a*b^2*d*f-4*A*b^3*c*f-4*A*b^3*d*e-3*B*a^2*b*d*f-B*a*b^2*c*f-
B*a*b^2*d*e+5*B*b^3*c*e-2*C*a^3*d*f+6*C*a^2*b*c*f+6*C*a^2*b*d*e-10*C*a*b^2
*c*e)/b^3/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2*(b*d*f*x^3+a*d*f*x^2+b*c*f*x
^2+b*d*e*x^2+a*c*f*x+a*d*e*x+b*c*e*x+a*c*e)^(1/2)/(x+a/b)^2-2/15*(b*d*f*x^
2+b*c*f*x+b*d*e*x+b*c*e)/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^3/b^2*(23*A*a^2
*b^2*d^2*f^2-23*A*a*b^3*c*d*f^2-23*A*a*b^3*d^2*e*f+8*A*b^4*c^2*f^2+7*A*b^4
*c*d*e*f+8*A*b^4*d^2*e^2-3*B*a^3*b*d^2*f^2-7*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2
*d^2*e*f+2*B*a*b^3*c^2*f^2+33*B*a*b^3*c*d*e*f+2*B*a*b^3*d^2*e^2-10*B*b^4*c
^2*e*f-10*B*b^4*c*d*e^2-2*C*a^4*d^2*f^2+7*C*a^3*b*c*d*f^2+7*C*a^3*b*d^2*e*
f+3*C*a^2*b^2*c^2*f^2-13*C*a^2*b^2*c*d*e*f+3*C*a^2*b^2*d^2*e^2-10*C*a*b^3*
c^2*e*f-10*C*a*b^3*c*d*e^2+15*C*b^4*c^2*e^2)/((x+a/b)*(b*d*f*x^2+b*c*f*x+b
*d*e*x+b*c*e))^(1/2)+2*(-1/15*d*f*(8*A*a*b^2*d*f-4*A*b^3*c*f-4*A*b^3*d*e-3
*B*a^2*b*d*f-B*a*b^2*c*f-B*a*b^2*d*e+5*B*b^3*c*e-2*C*a^3*d*f+6*C*a^2*b*c*f
+6*C*a^2*b*d*e-10*C*a*b^2*c*e)/b^2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)^2+1/1
5/b^2*(a*d*f-b*c*f-b*d*e)*(23*A*a^2*b^2*d^2*f^2-23*A*a*b^3*c*d*f^2-23*A*a*
b^3*d^2*e*f+8*A*b^4*c^2*f^2+7*A*b^4*c*d*e*f+8*A*b^4*d^2*e^2-3*B*a^3*b*d^2*
f^2-7*B*a^2*b^2*c*d*f^2-7*B*a^2*b^2*d^2*e*f+2*B*a*b^3*c^2*f^2+33*B*a*b^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5108 vs. $2(1053) = 2106$.

Time = 1.00 (sec) , antiderivative size = 5108, normalized size of antiderivative = 4.58

$$\int \frac{A + Bx + Cx^2}{(a + bx)^{7/2}\sqrt{c + dx}\sqrt{e + fx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(7/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,{\left(a+b\,x\right)}^{7/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}}\,dx=\int \frac{C\,x^2+Bx+A}{(bx+a)^{\frac{7}{2}}\,\sqrt{dx+c}\,\sqrt{fx+e}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType

#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file