Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.1-Linear-binomial/28-1.1.1.8

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**43**]. This is test number [28].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (43)	0.00 (0)
Mathematica	100.00 (43)	0.00 (0)
Maple	100.00 (43)	0.00 (0)
Fricas	20.93 (9)	79.07 (34)
Mupad	2.33 (1)	97.67 (42)
Giac	2.33 (1)	97.67 (42)
Maxima	2.33 (1)	97.67 (42)
Reduce	2.33 (1)	97.67 (42)
Sympy	0.00 (0)	100.00 (43)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

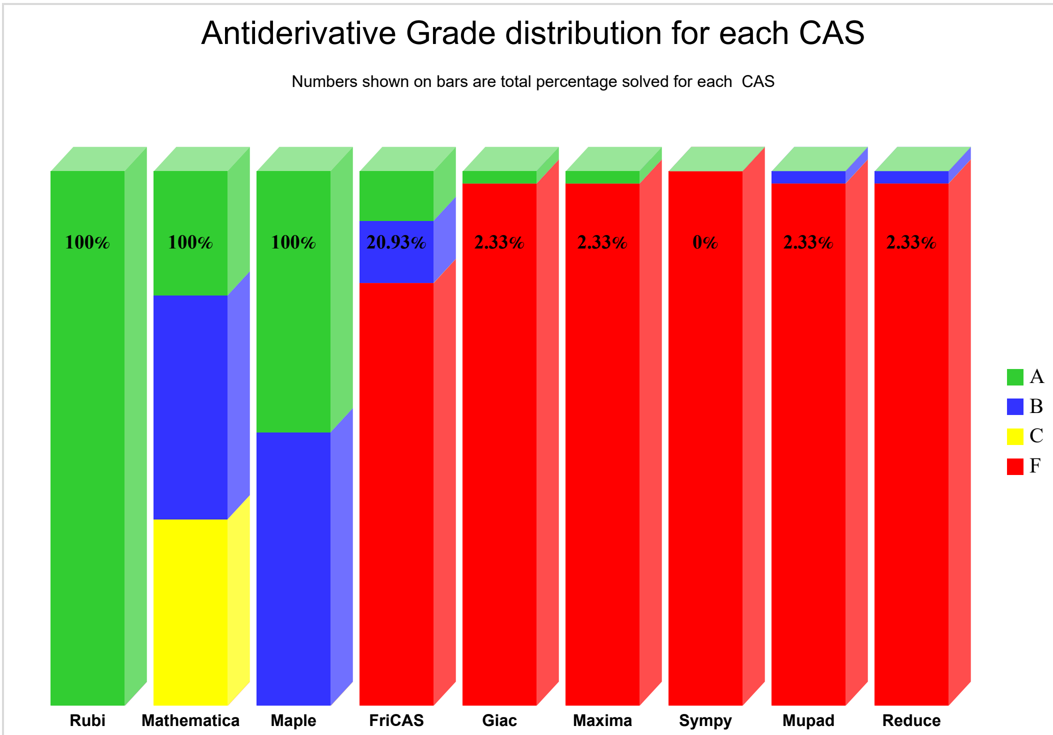
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

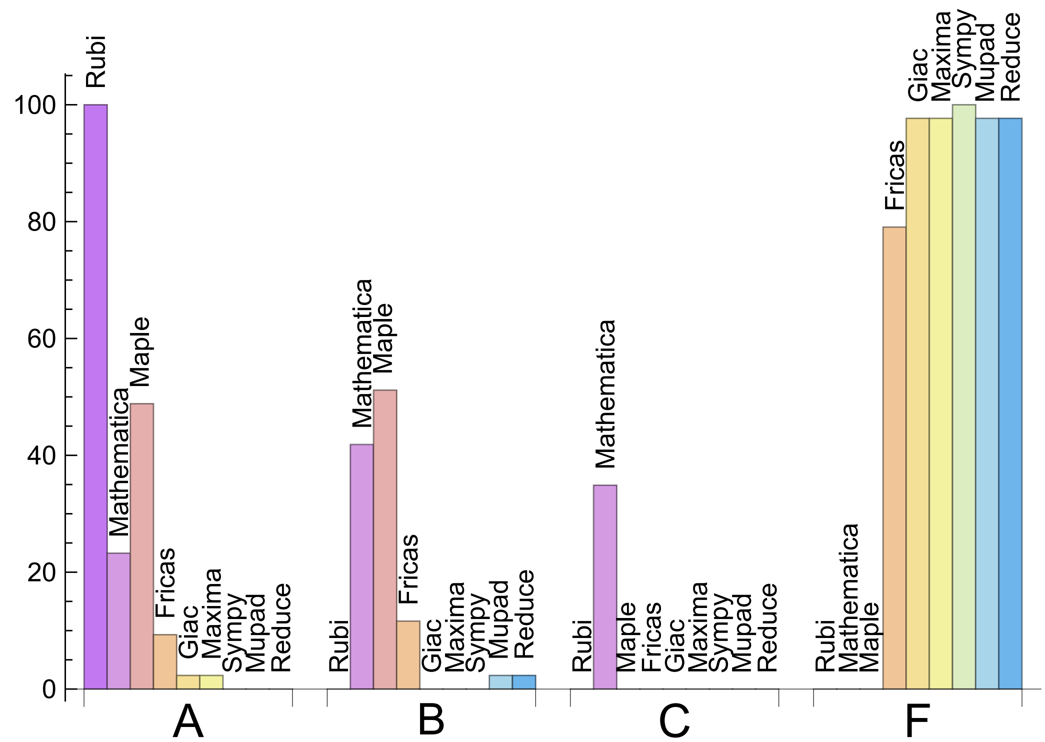
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	48.837	51.163	0.000	0.000
Mathematica	23.256	41.860	34.884	0.000
Fricas	9.302	11.628	0.000	79.070
Giac	2.326	0.000	0.000	97.674
Maxima	2.326	0.000	0.000	97.674
Mupad	0.000	2.326	0.000	97.674
Reduce	0.000	2.326	0.000	97.674
Sympy	0.000	0.000	0.000	100.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Fricas	34	26.47	73.53	0.00
Mupad	42	0.00	100.00	0.00
Giac	42	95.24	0.00	4.76
Maxima	42	100.00	0.00	0.00
Reduce	42	100.00	0.00	0.00
Sympy	43	72.09	27.91	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Giac	0.13
Fricas	0.13
Reduce	0.16
Rubi	2.38
Mupad	10.59
Maple	18.38
Mathematica	31.88
Sympy	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	362.00	1.68	362.00	1.68
Giac	415.00	1.92	415.00	1.92
Rubi	778.79	1.10	732.00	1.02
Fricas	1007.22	1.98	859.00	2.08
Reduce	1523.00	7.05	1523.00	7.05
Maple	1742.77	2.43	1544.00	1.86
Mathematica	8268.86	8.42	1291.00	2.23
Mupad	82899.00	383.79	82899.00	383.79
Sympy	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

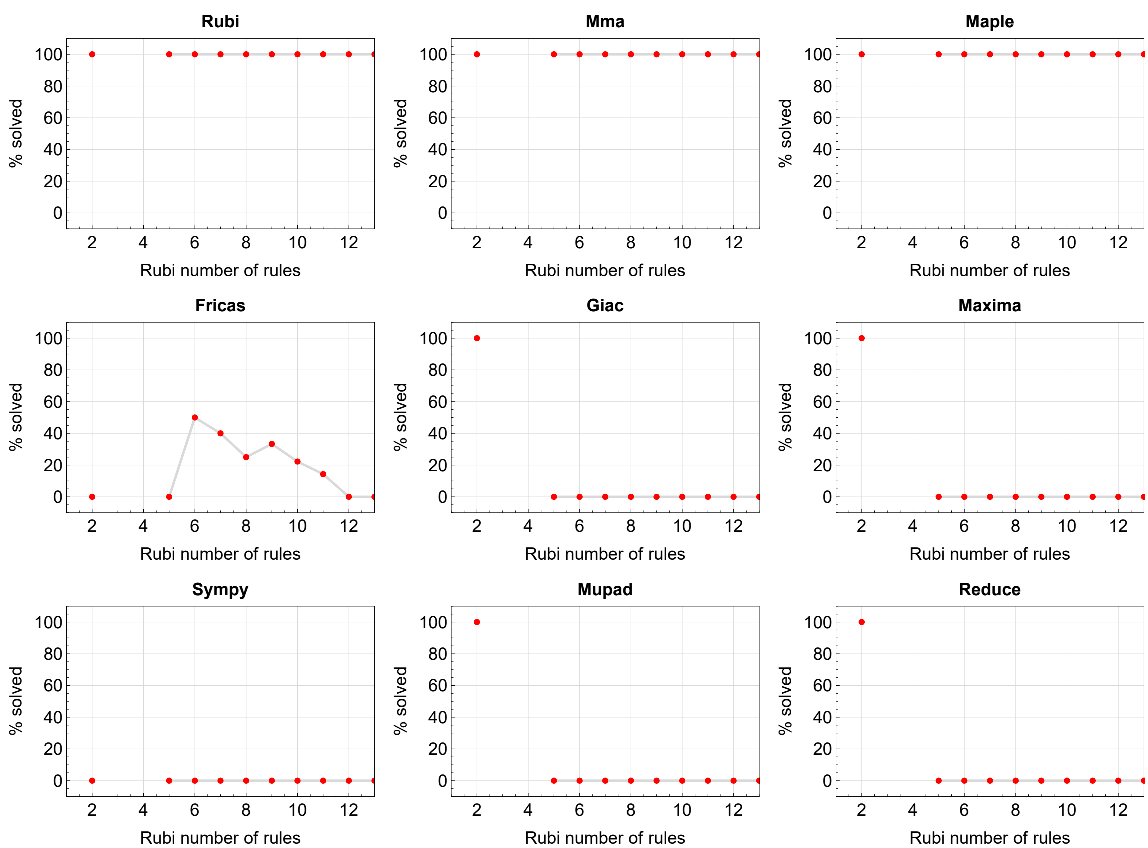


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

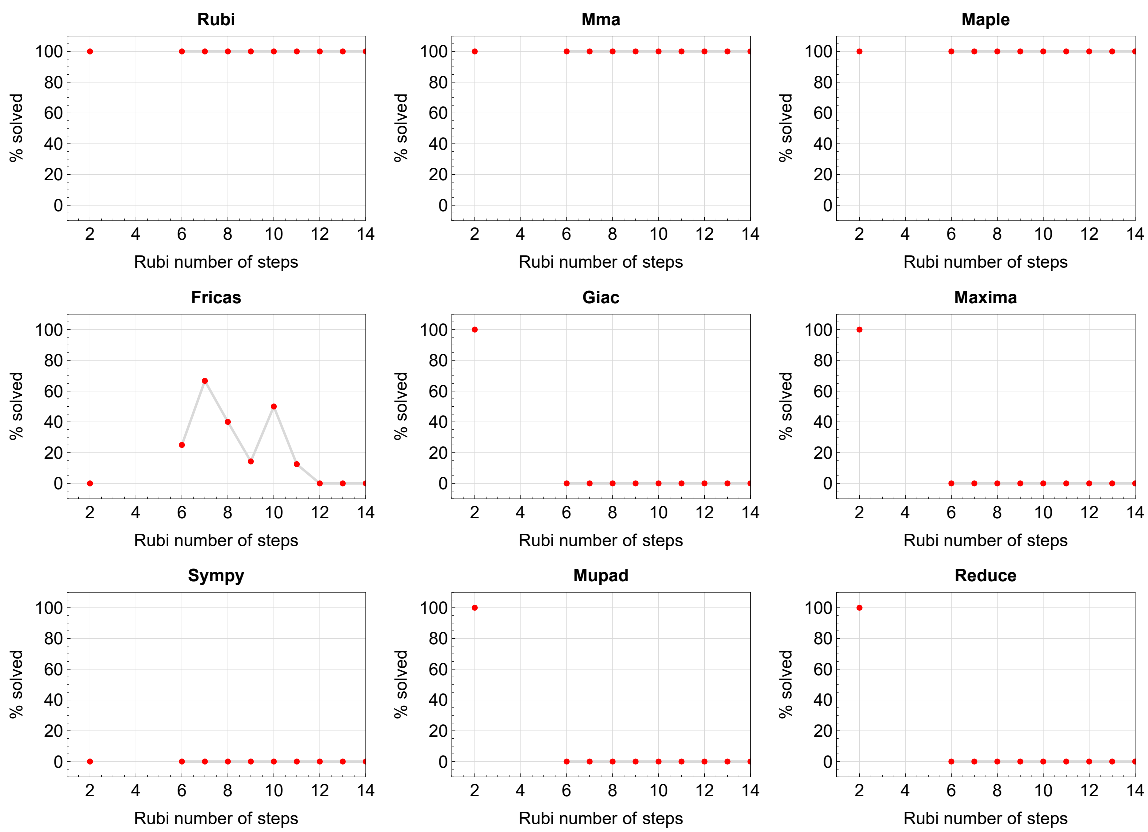


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

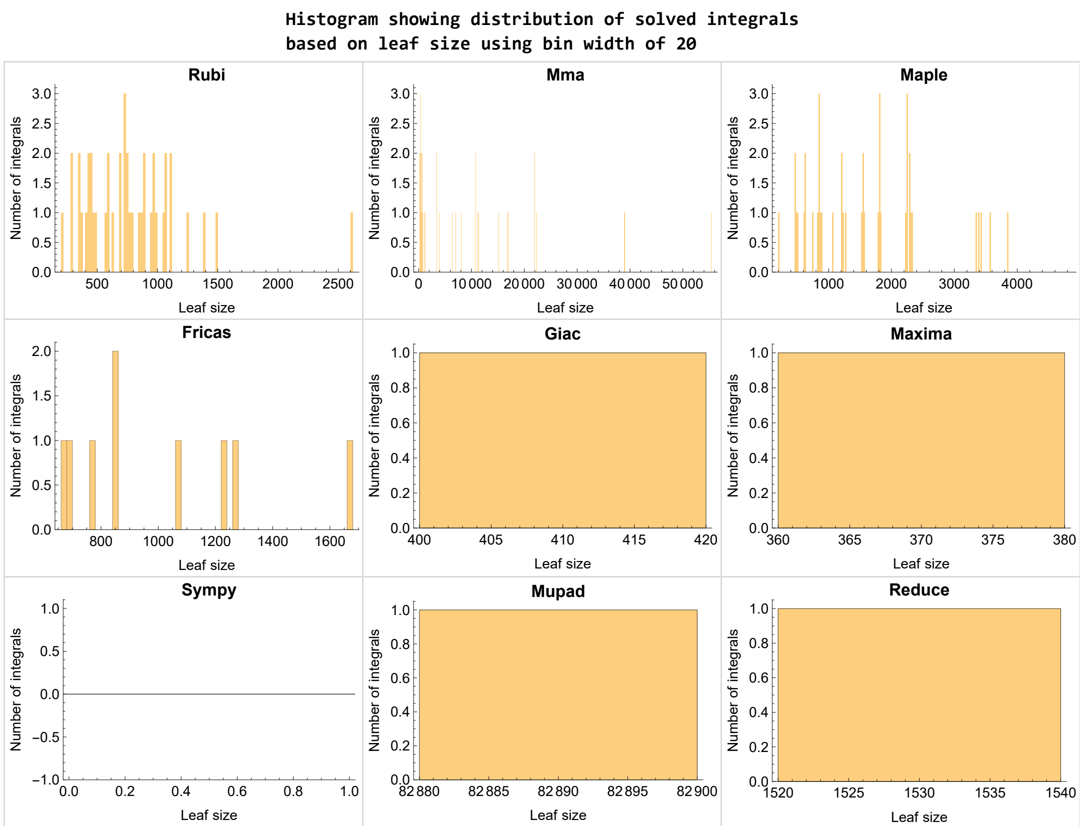


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

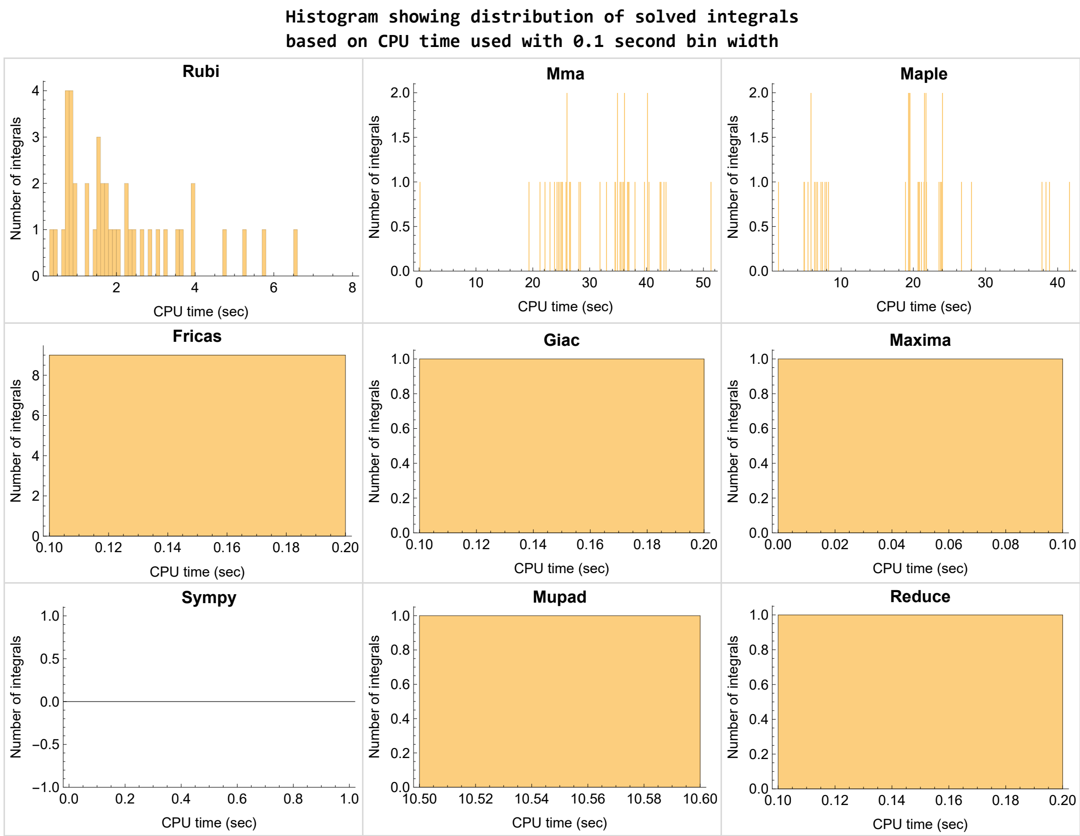


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

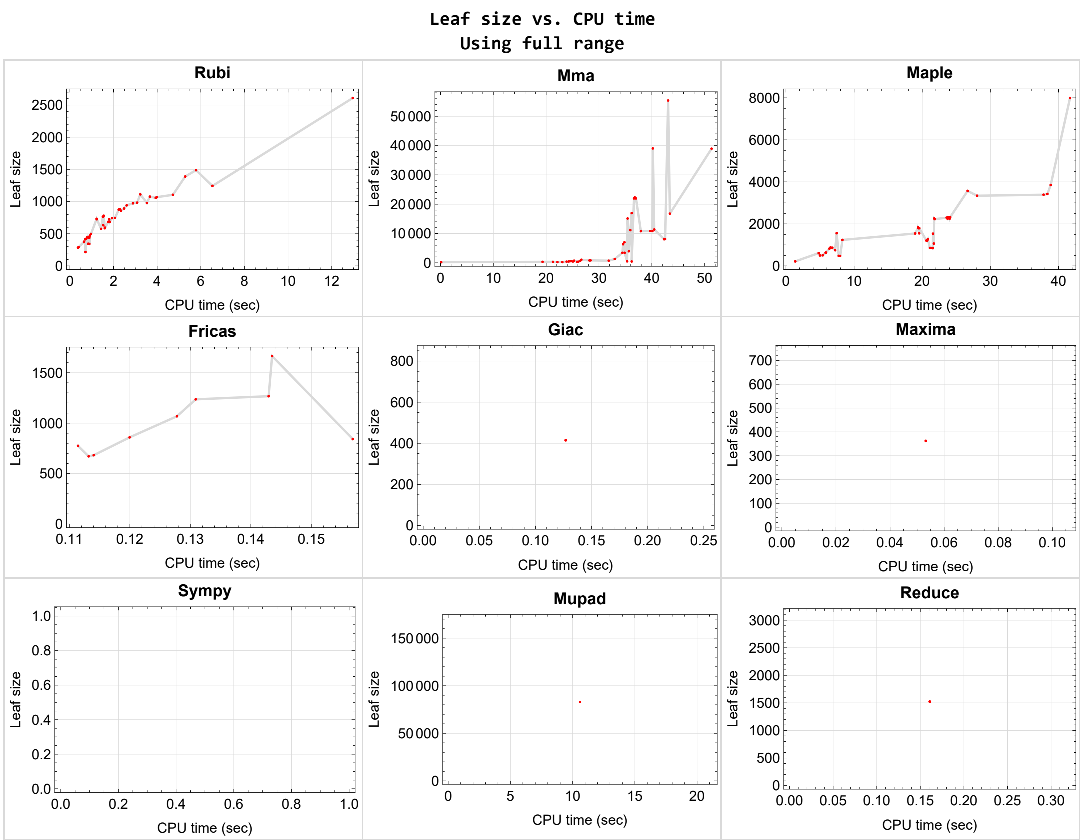


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{6, 10, 11, 22, 26, 32, 33, 35, 36, 37, 38, 39, 41, 42, 43\}$

Mathematica $\{6, 7, 8, 9, 11, 12, 13, 14, 22, 23, 24, 25, 32, 33, 34, 35, 37, 38, 39, 40, 41, 43\}$

Maple $\{26\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

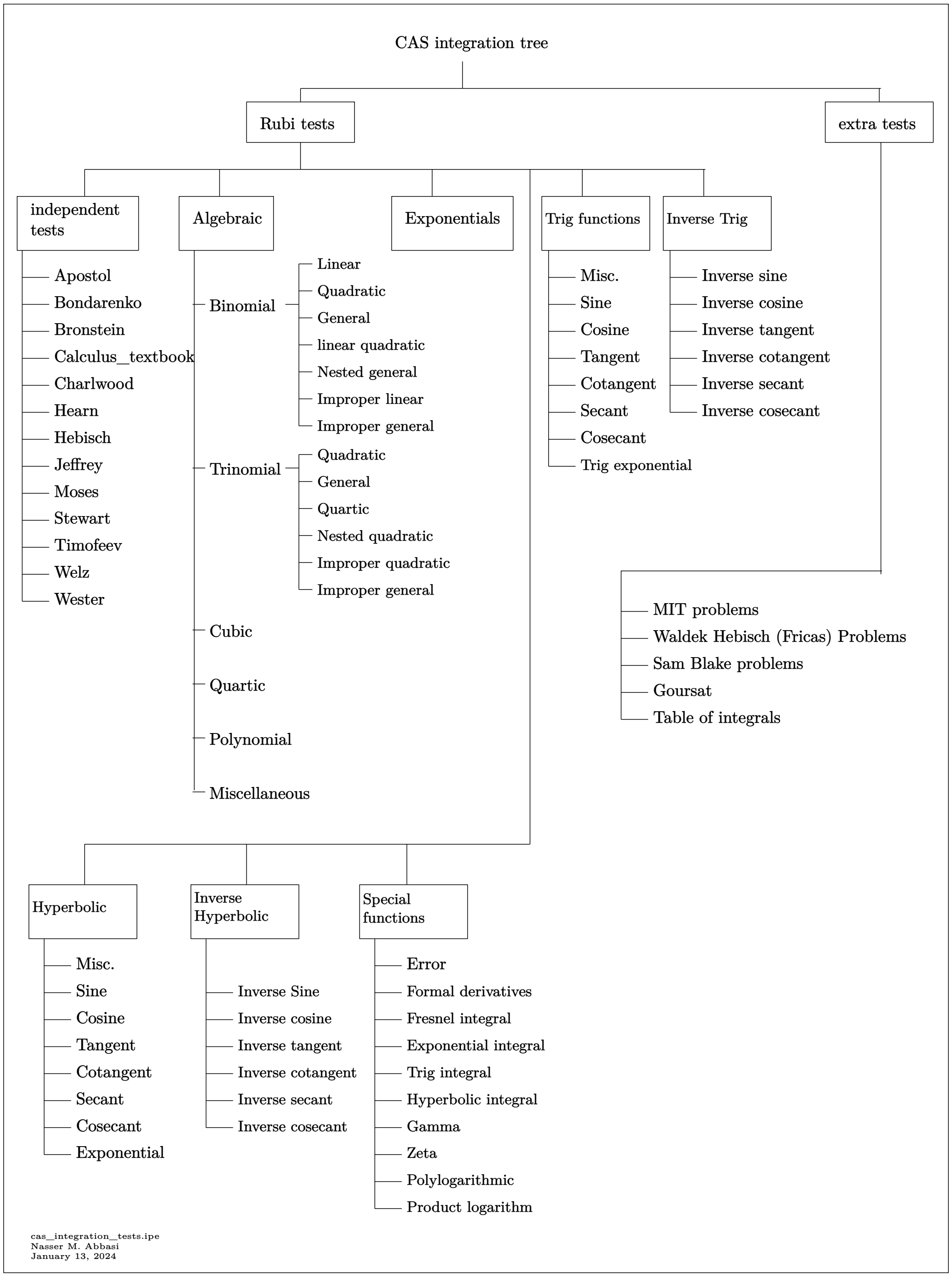
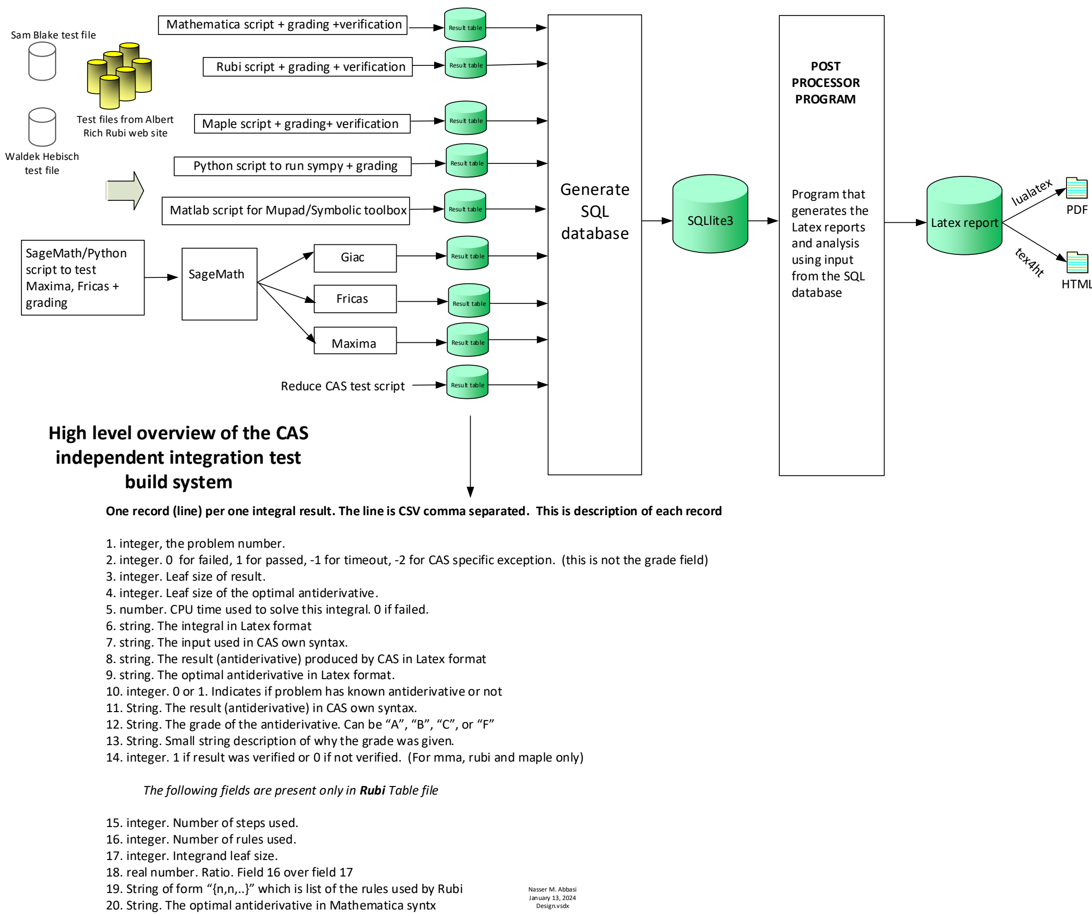


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 8, 9, 12, 13, 14, 16, 24, 25, 35, 41 }

B grade { 6, 7, 10, 11, 15, 22, 23, 26, 32, 33, 34, 36, 37, 38, 39, 40, 42, 43 }

C grade { 1, 2, 3, 4, 5, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 16, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 32, 34, 38, 39 }

B grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 33, 35, 36, 37, 40, 41, 42, 43 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 17, 27, 28 }

B grade { 2, 3, 18, 19, 29 }

C grade { }

F normal fail { 9, 10, 14, 15, 25, 26, 36, 37, 42 }

F(-1) timedout fail { 4, 5, 6, 7, 8, 11, 12, 13, 16, 20, 21, 22, 23, 24, 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 43 }

F(-2) exception fail { }

Maxima

A grade { 16 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 16 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 42, 43 }

F(-1) timedout fail { }

F(-2) exception fail { 35, 41 }

Mupad

A grade { }

B grade { 16 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 17, 18, 19, 22, 23, 24, 27, 28, 29, 30, 32, 33, 34, 35, 38, 39, 40, 41, 43 }

F(-1) timedout fail { 5, 10, 15, 16, 20, 21, 25, 26, 31, 36, 37, 42 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 16 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	699	723	806	866	0	1236	0	0	0	0
N.S.	1	1.03	1.15	1.24	0.00	1.77	0.00	0.00	0.00	0.00
time (sec)	N/A	1.799	28.196	6.796	0.000	0.131	0.000	0.000	172.750	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	414	450	625	0	842	0	0	0	0
N.S.	1	1.02	1.11	1.55	0.00	2.08	0.00	0.00	0.00	0.00
time (sec)	N/A	0.704	24.277	5.825	0.000	0.157	0.000	0.000	37.572	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	319	498	0	671	0	0	148	0
N.S.	1	1.00	1.12	1.75	0.00	2.36	0.00	0.00	0.52	0.00
time (sec)	N/A	0.373	19.317	4.994	0.000	0.113	0.000	0.000	20.255	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	345	245	478	0	0	0	0	71	0
N.S.	1	1.10	0.78	1.53	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.839	23.070	7.745	0.000	0.000	0.000	0.000	7.500	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	678	685	3412	1208	0	0	0	0	36	0
N.S.	1	1.01	5.03	1.78	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.763	34.432	20.611	0.000	0.000	0.000	0.000	200.024	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	992	983	21961	1814	0	0	0	0	36	0
N.S.	1	0.99	22.14	1.83	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	3.083	36.622	19.397	0.000	0.000	0.000	0.000	200.028	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	749	725	8030	1544	0	0	0	0	36	0
N.S.	1	0.97	10.72	2.06	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.236	42.330	18.954	0.000	0.000	0.000	0.000	200.026	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	442	586	848	0	0	0	0	36	0
N.S.	1	0.98	1.29	1.87	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.785	24.595	21.546	0.000	0.000	0.000	0.000	200.026	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	576	333	2250	0	0	0	0	36	0
N.S.	1	1.28	0.74	4.99	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.430	26.055	23.715	0.000	0.000	0.000	0.000	200.023	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	737	1068	10828	3389	0	0	0	0	36	0
N.S.	1	1.45	14.69	4.60	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	3.977	39.656	37.806	0.000	0.000	0.000	0.000	200.028	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	913	892	15131	1809	0	0	0	0	43	0
N.S.	1	0.98	16.57	1.98	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.484	35.419	19.385	0.000	0.000	0.000	0.000	200.024	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	702	472	443	1560	0	0	0	0	43	0
N.S.	1	0.67	0.63	2.22	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.917	36.193	7.444	0.000	0.000	0.000	0.000	200.026	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	449	723	855	0	0	0	0	43	0
N.S.	1	0.98	1.59	1.88	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.883	25.288	21.124	0.000	0.000	0.000	0.000	200.023	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	595	341	2298	0	0	0	0	43	0
N.S.	1	1.28	0.73	4.94	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.609	25.849	23.585	0.000	0.000	0.000	0.000	200.026	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	790	1077	10790	3571	0	0	0	0	43	0
N.S.	1	1.36	13.66	4.52	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	3.667	37.922	26.661	0.000	0.000	0.000	0.000	200.026	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	218	216	362	0	0	415	1523	82899
N.S.	1	1.00	1.01	1.00	1.68	0.00	0.00	1.92	7.05	383.79
time (sec)	N/A	0.715	0.138	1.368	0.053	0.000	0.000	0.127	0.161	10.589

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	720	744	825	880	0	1267	0	0	54	0
N.S.	1	1.03	1.15	1.22	0.00	1.76	0.00	0.00	0.08	0.00
time (sec)	N/A	1.930	28.398	6.545	0.000	0.143	0.000	0.000	200.027	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	420	442	637	0	859	0	0	0	0
N.S.	1	1.02	1.08	1.55	0.00	2.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.746	25.069	5.878	0.000	0.120	0.000	0.000	50.396	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	326	506	0	682	0	0	226	0
N.S.	1	1.00	1.12	1.74	0.00	2.34	0.00	0.00	0.78	0.00
time (sec)	N/A	0.401	21.281	5.400	0.000	0.114	0.000	0.000	31.277	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	341	249	475	0	0	0	0	56	0
N.S.	1	1.10	0.81	1.54	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.891	22.137	7.953	0.000	0.000	0.000	0.000	200.024	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	680	686	3419	1211	0	0	0	0	56	0
N.S.	1	1.01	5.03	1.78	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.831	34.854	20.731	0.000	0.000	0.000	0.000	200.024	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	988	978	21961	1834	0	0	0	0	56	0
N.S.	1	0.99	22.23	1.86	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	3.530	36.999	19.402	0.000	0.000	0.000	0.000	200.024	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	747	732	8107	1552	0	0	0	0	56	0
N.S.	1	0.98	10.85	2.08	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.227	42.519	19.553	0.000	0.000	0.000	0.000	200.025	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	436	583	856	0	0	0	0	56	0
N.S.	1	0.98	1.30	1.91	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.889	24.718	21.428	0.000	0.000	0.000	0.000	200.021	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	586	340	2249	0	0	0	0	56	0
N.S.	1	1.28	0.74	4.91	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.601	26.064	24.015	0.000	0.000	0.000	0.000	200.026	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	760	1105	10836	3425	0	0	0	0	56	0
N.S.	1	1.45	14.26	4.51	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	4.722	40.111	38.366	0.000	0.000	0.000	0.000	200.024	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1084	1112	1291	1238	0	1665	0	0	38	0
N.S.	1	1.03	1.19	1.14	0.00	1.54	0.00	0.00	0.04	0.00
time (sec)	N/A	3.228	32.975	8.289	0.000	0.144	0.000	0.000	200.027	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	632	686	824	0	1068	0	0	0	0
N.S.	1	1.04	1.13	1.36	0.00	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	1.530	26.451	6.378	0.000	0.128	0.000	0.000	174.374	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	377	390	611	0	775	0	0	150	0
N.S.	1	1.03	1.06	1.66	0.00	2.11	0.00	0.00	0.41	0.00
time (sec)	N/A	0.649	23.873	4.804	0.000	0.111	0.000	0.000	14.549	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	497	1036	750	0	0	0	0	38	0
N.S.	1	1.07	2.23	1.61	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.967	26.661	7.216	0.000	0.000	0.000	0.000	200.024	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	738	745	3935	1269	0	0	0	0	38	0
N.S.	1	1.01	5.33	1.72	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.068	35.651	20.826	0.000	0.000	0.000	0.000	200.024	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1402	1389	39032	2228	0	0	0	0	38	0
N.S.	1	0.99	27.84	1.59	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	5.286	40.194	21.862	0.000	0.000	0.000	0.000	200.023	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	944	941	16972	1794	0	0	0	0	38	0
N.S.	1	1.00	17.98	1.90	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.603	36.184	19.496	0.000	0.000	0.000	0.000	200.027	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	764	6321	1065	0	0	0	0	38	0
N.S.	1	1.00	8.24	1.39	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.513	34.558	21.709	0.000	0.000	0.000	0.000	200.027	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	705	859	721	2286	0	0	0	0	38	0
N.S.	1	1.22	1.02	3.24	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.339	31.841	23.973	0.000	0.000	0.000	0.000	200.028	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	727	1057	11363	3342	0	0	0	0	38	0
N.S.	1	1.45	15.63	4.60	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	3.935	40.458	28.046	0.000	0.000	0.000	0.000	200.028	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1617	2609	38948	7993	0	0	0	0	38	0
N.S.	1	1.61	24.09	4.94	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	12.966	51.313	41.687	0.000	0.000	0.000	0.000	200.025	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1524	1488	55395	2258	0	0	0	0	41	0
N.S.	1	0.98	36.35	1.48	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	5.782	43.081	21.754	0.000	0.000	0.000	0.000	200.027	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	983	973	22316	1800	0	0	0	0	41	0
N.S.	1	0.99	22.70	1.83	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.893	36.776	19.563	0.000	0.000	0.000	0.000	200.030	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	782	780	7019	1536	0	0	0	0	41	0
N.S.	1	1.00	8.98	1.96	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	1.551	34.821	21.578	0.000	0.000	0.000	0.000	200.027	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	875	487	2321	0	0	0	0	41	0
N.S.	1	1.22	0.68	3.23	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.239	35.325	24.099	0.000	0.000	0.000	0.000	200.024	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	827	1243	16787	3852	0	0	0	0	41	0
N.S.	1	1.50	20.30	4.66	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	6.534	43.419	38.854	0.000	0.000	0.000	0.000	200.025	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	882	11158	2318	0	0	0	0	41	0
N.S.	1	1.24	15.65	3.25	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.282	35.935	23.812	0.000	0.000	0.000	0.000	200.025	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [5] had the largest ratio of [.325000000000000011]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	9	1.03	40	0.225
2	A	7	7	1.02	38	0.184
3	A	6	6	1.00	33	0.182
4	A	10	9	1.10	40	0.225
5	A	14	13	1.01	40	0.325
6	A	12	11	0.99	42	0.262
7	A	8	7	0.97	42	0.167
8	A	6	5	0.98	42	0.119
9	A	8	7	1.28	42	0.167
10	A	11	10	1.45	42	0.238
11	A	12	11	0.98	49	0.224
12	A	6	5	0.67	49	0.102
13	A	6	5	0.98	49	0.102
14	A	8	7	1.28	49	0.143
15	A	9	8	1.36	49	0.163
16	A	2	2	1.00	39	0.051
17	A	10	10	1.03	58	0.172
18	A	8	8	1.02	53	0.151
19	A	7	7	1.00	60	0.117
20	A	11	10	1.10	60	0.167
21	A	14	13	1.01	60	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	13	12	0.99	62	0.194
23	A	9	8	0.98	62	0.129
24	A	7	6	0.98	62	0.097
25	A	9	8	1.28	62	0.129
26	A	10	9	1.45	62	0.145
27	A	11	11	1.03	42	0.262
28	A	10	10	1.04	40	0.250
29	A	8	8	1.03	35	0.229
30	A	12	11	1.07	42	0.262
31	A	14	13	1.01	42	0.310
32	A	12	11	0.99	44	0.250
33	A	11	10	1.00	44	0.227
34	A	9	8	1.00	44	0.182
35	A	12	11	1.22	44	0.250
36	A	11	10	1.45	44	0.227
37	A	13	12	1.61	44	0.273
38	A	12	11	0.98	47	0.234
39	A	11	10	0.99	47	0.213
40	A	9	8	1.00	47	0.170
41	A	11	10	1.22	47	0.213
42	A	9	8	1.50	47	0.170
43	A	11	10	1.24	47	0.213

CHAPTER 3

Listing of integrals

3.1 $\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 699

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2b(5Abdfh+7aBdfh-4bB(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$+ \frac{2\sqrt{-de+cf}(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(dfg+deh+cfh))-b^2(10Adfh(dfg+deh+cf\ldots}{15d^3f^5\ldots}$$
$$- \frac{2\sqrt{-de+cf}(15a^2d^2f^2h^2(Bg-Ah)+10abdfh(3Adfgh-Bch(fg-eh)-Bdg(2fg+eh))-b^2(5A\ldots}{\ldots}$$

output
```
2/15*b*(5*A*b*d*f*h+7*a*B*d*f*h-4*b*B*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)*(
f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+2/5*b*B*(b*x+a)*(d*x+c)^(1/2)*(f*x+
e)^(1/2)*(h*x+g)^(1/2)/d/f/h+2/15*(c*f-d*e)^(1/2)*(15*a^2*B*d^2*f^2*h^2+10
*a*b*d*f*h*(3*A*d*f*h-2*B*(c*f*h+d*e*h+d*f*g))-b^2*(10*A*d*f*h*(c*f*h+d*e*
h+d*f*g)-B*(8*c^2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f
^2*g^2))))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f^(1/2)*(d
*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d^3/f^(5/2)
/h^3/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2/15*(c*f-d*e)^(1/2)*(15*a
^2*d^2*f^2*h^2*(-A*h+B*g)+10*a*b*d*f*h*(3*A*d*f*g*h-B*c*h*(-e*h+f*g)-B*d*g
*(e*h+2*f*g))-b^2*(5*A*d*f*h*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g))-B*(4*c^2*f*h
^2*(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2*h^2+3*e*f*
g*h+8*f^2*g^2))))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2
)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*
g))^(1/2))/d^3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.20 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(-d^2\sqrt{-c+\frac{de}{f}}(15a^2Bd^2f^2h^2-10abdfh(-3Adfh+2B(dfg+deh+cfh))+b^2(-10Adfh(dfg+d\right.}{}$$

input
```
Integrate[((a + b*x)^2*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
(-2*(-(d^2*Sqrt[-c + (d*e)/f]*(15*a^2*B*d^2*f^2*h^2 - 10*a*b*d*f*h*(-3*A*d
*f*h + 2*B*(d*f*g + d*e*h + c*f*h)) + b^2*(-10*A*d*f*h*(d*f*g + d*e*h + c*
f*h) + B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g
*h + 8*e^2*h^2))))*(e + f*x)*(g + h*x)) + b*d^2*Sqrt[-c + (d*e)/f]*f*h*(c
+ d*x)*(e + f*x)*(g + h*x)*(-5*A*b*d*f*h - 10*a*B*d*f*h + b*B*(4*c*f*h + d
*(4*f*g + 4*e*h - 3*f*h*x))) - I*(d*e - c*f)*h*(15*a^2*B*d^2*f^2*h^2 - 10*
a*b*d*f*h*(-3*A*d*f*h + 2*B*(d*f*g + d*e*h + c*f*h)) + b^2*(-10*A*d*f*h*(d
*f*g + d*e*h + c*f*h) + B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*
f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*
(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c
+ (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*d*h*(15*a^
2*d^2*f^2*(-(B*e) + A*f)*h^2 + 10*a*b*d*f*h*(-3*A*d*e*f*h + B*c*f*(-(f*g)
+ e*h) + B*d*e*(f*g + 2*e*h)) - b^2*(-5*A*d*f*h*(c*f*(-(f*g) + e*h) + d*e*
(f*g + 2*e*h)) + B*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*(-4*f^2*g^2 + e*f*g
*h + 3*e^2*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(
3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*E
llipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e
*h - c*f*h)]))/(15*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqrt[c + d*x]*Sqrt[e + f
*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2100, 2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{2100}$$

$$\frac{\int \frac{5Adfha^2+b(5Abdfh+7aBdfh-4bB(dfg+deh+cfh))x^2-bB(2bceg+a(deg+cfg+ceh))+\left(5Bdfha^2+2b(5Adfh-B(dfg+deh+cfh))a-3b^2B(d\right.}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{5dfh}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\big\downarrow$ 2118

$$2\int \frac{d(-((5Adfh(deg+cfg+ceh)-B(4fh(fg+eh)c^2+2d(2f^2g^2+3efhg+2e^2h^2)c+4d^2eg(fg+eh)))b^2)-10aBdfh(deg+cfg+ceh)b+15a^2Ad^2f^2h^2+(-((10Ad}{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g}}$$
$$\overline{\qquad 3d^2fh}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\big\downarrow$ 27

$$\int \frac{-((5Adfh(deg+cfg+ceh)-B(4fh(fg+eh)c^2+2d(2f^2g^2+3efhg+2e^2h^2)c+4d^2eg(fg+eh)))b^2)-10aBdfh(deg+cfg+ceh)b+15a^2Ad^2f^2h^2+(-((10Adfh(d}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}$$
$$\overline{\qquad 3dfh}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\big\downarrow$ 176

$$\frac{(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-(b^2(10Adfh(cfh+deh+dfg)-B(8c^2f^2h^2+7cdfh(eh+fg)+d^2(8e^2h^2+7efgh+8f^2g^2)))))\int \frac{\sqrt{g+h}}{\sqrt{c+dx}\sqrt{e}}}{h}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\big\downarrow$ 124

$$\frac{\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-(b^2(10Adfh(cfh+deh+dfg)-B(8c^2f^2h^2+7cdfh(eh+fg)+d^2(8e^2h^2+7efgh+8f^2g^2}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\big\downarrow$ 123

$$\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-(b^2(10Adfh(cfh+deh+dfg)-B(8c^2f^2h}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\Big\downarrow$ 131

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h\cdots\right)\right)\right)\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\cdots}{\cdots}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\Big\downarrow$ 131

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h\cdots\right)\right)\right)\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\cdots}{\cdots}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\Big\downarrow$ 130

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Bd^2f^2h^2+10abdfh(3Adfh-2B(cfh+deh+dfg))-\left(b^2\left(10Adfh(cfh+deh+dfg)-B\left(8c^2f^2h\cdots\right)\right)\right)\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\cdots}{\cdots}$$

$$\frac{2bB(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

input
```
Int[((a + b*x)^2*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*b*B*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) + ((
2*b*(5*A*b*d*f*h + 7*a*B*d*f*h - 4*b*B*(d*f*g + d*e*h + c*f*h))*Sqrt[c + d
*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((2*Sqrt[-(d*e) + c*f]*(15*a^
2*B*d^2*f^2*h^2 + 10*a*b*d*f*h*(3*A*d*f*h - 2*B*(d*f*g + d*e*h + c*f*h)) -
 b^2*(10*A*d*f*h*(d*f*g + d*e*h + c*f*h) - B*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f
*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-
(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))]])/(d*Sqrt[f]*h*Sqrt[e + f*x
]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]*(15*a^2*d^2*f^2
*h^2*(B*g - A*h) + 10*a*b*d*f*h*(3*A*d*f*g*h - B*c*h*(f*g - e*h) - B*d*g*(
2*f*g + e*h)) - b^2*(5*A*d*f*h*(c*h*(f*g - e*h) + d*g*(2*f*g + e*h)) - B*(
4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 + e*f*g*h - 4*e^2*h^2) + d^2*g*
(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqr
t[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt
[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))]])/(d*Sqrt[f]*h*Sqrt[e + f
*x]*Sqrt[g + h*x]))/(3*d*f*h))/(5*d*f*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

rule 2118

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 6.80 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.24

method	result
elliptic	$\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2B\,b^2 x\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(b^2A+2abB-\frac{2B\,b^2(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{dj}\ldots}{\ldots}\right.$
default	Expression too large to display

input

```
int((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2/5*B*b^2/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*
x+d*e*g*x+c*e*g)^(1/2)+2/3*(b^2*A+2*a*b*B-2/5*B*b^2/d/f/h*(2*c*f*h+2*d*e*h
+2*d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2*(a^2*A-2/5*B*b^2/d/f/h*c*e*g-2/3*(b^2*A+2*a*b*B-2/5
*B*b^2/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e
*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/
f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(
-c/d+g/h))^(1/2))+2*(2*a*b*A+a^2*B-2/5*B*b^2/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/
2*d*e*g)-2/3*(b^2*A+2*a*b*B-2/5*B*b^2/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f
/h*(c*f*h+d*e*h+d*f*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d
+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f
*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)
/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/(c
/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 1236, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="fricas")
```

output
```
2/45*(3*(3*B*b^2*d^3*f^3*h^3*x - 4*B*b^2*d^3*f^3*g*h^2 - (4*B*b^2*d^3*e*f^
2 + (4*B*b^2*c*d^2 - 5*(2*B*a*b + A*b^2)*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt
(f*x + e)*sqrt(h*x + g) - (8*B*b^2*d^3*f^3*g^3 + (3*B*b^2*d^3*e*f^2 + (3*B
*b^2*c*d^2 - 10*(2*B*a*b + A*b^2)*d^3)*f^3)*g^2*h + (3*B*b^2*d^3*e^2*f + (
3*B*b^2*c*d^2 - 5*(2*B*a*b + A*b^2)*d^3)*e*f^2 + (3*B*b^2*c^2*d - 5*(2*B*a
*b + A*b^2)*c*d^2 + 15*(B*a^2 + 2*A*a*b)*d^3)*f^3)*g*h^2 + (8*B*b^2*d^3*e^
3 + (3*B*b^2*c*d^2 - 10*(2*B*a*b + A*b^2)*d^3)*e^2*f + (3*B*b^2*c^2*d - 5*
(2*B*a*b + A*b^2)*c*d^2 + 15*(B*a^2 + 2*A*a*b)*d^3)*e*f^2 + (8*B*b^2*c^3 -
 45*A*a^2*d^3 - 10*(2*B*a*b + A*b^2)*c^2*d + 15*(B*a^2 + 2*A*a*b)*c*d^2)*f
^3)*h^3)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d
*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3
*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2
+ c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^
3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)) -
3*(8*B*b^2*d^3*f^3*g^2*h + (7*B*b^2*d^3*e*f^2 + (7*B*b^2*c*d^2 - 10*(2*B*a
*b + A*b^2)*d^3)*f^3)*g*h^2 + (8*B*b^2*d^3*e^2*f + (7*B*b^2*c*d^2 - 10*(2*
B*a*b + A*b^2)*d^3)*e*f^2 + (8*B*b^2*c^2*d - 10*(2*B*a*b + A*b^2)*c*d^2 +
15*(B*a^2 + 2*A*a*b)*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f
^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2
*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(...
```

Sympy [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\,(a+bx)^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**2*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),
x)
```

output
```
Integral((A + B*x)*(a + b*x)**2/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)
), x)
```

Maxima [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

Giac [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^2}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int(((A + B*x)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^2(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{too large to display}$$

input
```
int((b*x+a)^2*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
(30*sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*a**2*b*d*f*h - 6*sqrt(g + h*
x)*sqrt(e + f*x)*sqrt(c + d*x)*b**3*c*e*h - 6*sqrt(g + h*x)*sqrt(e + f*x)*
sqrt(c + d*x)*b**3*c*f*g + 4*sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*b**
3*c*f*h*x - 6*sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*b**3*d*e*g + 4*sqr
t(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*b**3*d*e*h*x + 4*sqrt(g + h*x)*sqrt
(e + f*x)*sqrt(c + d*x)*b**3*d*f*g*x - 45*int((sqrt(g + h*x)*sqrt(e + f*x)
*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f**2*g*h*x + c
**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*g**2 + 3*c*d
*e*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f**2*g*h*x**2 +
 c*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 + d**2*e*f*g**
2*x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g**2*x**2 + d**
2*f**2*g*h*x**3),x)*a**2*b*c*d**2*f**3*h**3 - 45*int((sqrt(g + h*x)*sqrt(e
 + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f**2*g*
h*x + c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*g**2
+ 3*c*d*e*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f**2*g*h
*x**2 + c*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 + d**2*
e*f*g**2*x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g**2*x**
2 + d**2*f**2*g*h*x**3),x)*a**2*b*d**3*e*f**2*h**3 - 45*int((sqrt(g + h*x)
*sqrt(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*
f**2*g*h*x + c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d...
```

3.2 $\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 404

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$
$$+\frac{2\sqrt{-de+cf}(3Abdfh+3aBdfh-2bB(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de}{f(d}\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{2\sqrt{-de+cf}(3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(2fg+eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\mathrm{E}}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2/3*b*B*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+2/3*(c*f-d*e)^(1/2
)*(3*A*b*d*f*h+3*a*B*d*f*h-2*b*B*(c*f*h+d*e*h+d*f*g))*(d*(f*x+e)/(-c*f+d*e
))^(1/2)*(h*x+g)^(1/2)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-
c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(
-c*h+d*g))^(1/2)-2/3*(c*f-d*e)^(1/2)*(3*a*d*f*h*(-A*h+B*g)+b*(3*A*d*f*g*h-
B*c*h*(-e*h+f*g)-B*d*g*(e*h+2*f*g)))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+
g)/(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*
f+d*e)*h/f/(-c*h+d*g))^(1/2))/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.28 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\sqrt{c+dx}\left(2bBd^2fh(e+fx)(g+hx) - \frac{2d^2(-3Abdfh-3aBdfh+2bB(dfg+deh+cfh))(e+fx)(g+hx)}{c+dx} + \frac{2i(de-cf)h(3Abdfh+\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((a + b*x)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(Sqrt[c + d*x]*(2*b*B*d^2*f*h*(e + f*x)*(g + h*x) - (2*d^2*(-3*A*b*d*f*h -
 3*a*B*d*f*h + 2*b*B*(d*f*g + d*e*h + c*f*h))*(e + f*x)*(g + h*x))/(c + d*
x) + ((2*I)*(d*e - c*f)*h*(3*A*b*d*f*h + 3*a*B*d*f*h - 2*b*B*(d*f*g + d*e*
h + c*f*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h
*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]],
 (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[-c + (d*e)/f] + ((2*I)*d*h*(3*a*d*
f*(-(B*e) + A*f)*h + b*(-3*A*d*e*f*h + B*c*f*(-(f*g) + e*h) + B*d*e*(f*g +
 2*e*h)))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x
))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (
d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[-c + (d*e)/f]))/(3*d^3*f^2*h^2*Sqrt[
e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2097, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \; 2097$$

$$\frac{\int \frac{3aAdfh-bB(deg+cfg+ceh)+(3Abdfh+3aBdfh-2bB(dfg+deh+cfh))x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3dfh}+\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 176$$

$$\frac{\frac{(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}-\frac{(3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(eh+2fg)))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g-}}}{h}}{3dfh}$$
$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 124$$

$$\frac{\frac{\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(3adfh(Bg-Ah)+b(3Adfgh-Bch(fg-eh)-Bdg(e}{h}}{3dfh}$$
$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 123$$

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(3adfh(Bg-Ah)+b(3Adfgh-Bch(f}{}}{3dfh}$$
$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 131$$

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}(3adfh(Bg-Ah)+b(3Adf}{}}{3dfh}$$
$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 131$$

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(3adfh(Bg-Ah}{}}{3dfh}$$

$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 130

$$\frac{\frac{2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)(3aBdfh+3Abdfh-2bB(cfh+deh+dfg))}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{Ellipt}}{}}{3dfh}$$

$$\frac{2bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

input
```
Int[((a + b*x)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*b*B*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((2*Sqrt[-(d
*e) + c*f]*(3*A*b*d*f*h + 3*a*B*d*f*h - 2*b*B*(d*f*g + d*e*h + c*f*h))*Sqr
t[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[
c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f
]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]
*(3*a*d*f*h*(B*g - A*h) + b*(3*A*d*f*g*h - B*c*h*(f*g - e*h) - B*d*g*(2*f*
g + e*h)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]
*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f
)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]))/(3*d*f*h
)
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2097
```
Int[(((a_.) + (b_.)*(x_))*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_)]*S
qrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2*b*B*
Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(3*d*f*h)), x] + Simp[1/(3*d*f*h
)   Int[(1/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[3*a*A*d*f*h -
b*B*(d*e*g + c*f*g + c*e*h) + (3*A*b*d*f*h + B*(3*a*d*f*h - 2*b*(d*f*g + d*
e*h + c*f*h)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2Bb\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(Aa-\frac{2Bb\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x\ldots}{\frac{c}{d}\ldots}}}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2\ldots}}\ldots\right.}{\ldots}$
default	Expression too large to display

input
```
int((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2/3*B*b/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*
e*g*x+c*e*g)^(1/2)+2*(A*a-2/3*B*b/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(
c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c
/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*
e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g
/h))^(1/2))+2*(A*b+B*a-2/3*B*b/d/f/h*(c*f*h+d*e*h+d*f*g))*(c/d-e/f)*((x+c/
d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/
(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1
/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h)
)^(1/2))-g/h*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(358) = 716$.

Time = 0.16 (sec) , antiderivative size = 842, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="fricas")
```

output
```
2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*B*b*d^2*f^2*h^2 + (2*B*b*
d^2*f^2*g^2 + (B*b*d^2*e*f + (B*b*c*d - 3*(B*a + A*b)*d^2)*f^2)*g*h + (2*B
*b*d^2*e^2 + (B*b*c*d - 3*(B*a + A*b)*d^2)*e*f + (2*B*b*c^2 + 9*A*a*d^2 -
3*(B*a + A*b)*c*d)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*
g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^
2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^
2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^
2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e +
 c*f)*h)/(d*f*h)) + 3*(2*B*b*d^2*f^2*g*h + (2*B*b*d^2*e*f + (2*B*b*c*d - 3
*(B*a + A*b)*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f^2*g^2 -
 (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2
), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f -
 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e
*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2
 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h
^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f
 - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d
*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*
f)*h)/(d*f*h))))/(d^3*f^3*h^3)
```

Sympy [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\,(a+bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((A + B*x)*(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)),
x)
```

Maxima [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

Giac [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)
),x)
```

output
```
int(((A + B*x)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{(a+bx)(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{too large to display}$$

input
```
int((b*x+a)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
(2*sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*a*b - 3*int((sqrt(g + h*x)*sq
rt(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f**
2*g*h*x + c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*g
**2 + 3*c*d*e*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f**2
*g*h*x**2 + c*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 + d
**2*e*f*g**2*x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g**2
*x**2 + d**2*f**2*g*h*x**3),x)*a*b*c*d*f**2*h**2 - 3*int((sqrt(g + h*x)*sq
rt(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f**
2*g*h*x + c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*g
**2 + 3*c*d*e*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f**2
*g*h*x**2 + c*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 + d
**2*e*f*g**2*x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g**2
*x**2 + d**2*f**2*g*h*x**3),x)*a*b*d**2*e*f*h**2 - 3*int((sqrt(g + h*x)*sq
rt(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f**
2*g*h*x + c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*g
**2 + 3*c*d*e*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f**2
*g*h*x**2 + c*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 + d
**2*e*f*g**2*x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g**2
*x**2 + d**2*f**2*g*h*x**3),x)*a*b*d**2*f**2*g*h + int((sqrt(g + h*x)*sqrt
(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f*...
```

3.3 $\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 284

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{-de+cf}(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*B*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d/
f^(1/2)/h/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(c*f-d*e)^(1/2)*(-A
*h+B*g)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*Elliptic
F(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))
/d/f^(1/2)/h/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.32 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\left(-Bd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx)-iB(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-c+}}{\sqrt{c+}}\right.\right.\right.}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{c}}$$

input
```
Integrate[(A + B*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*(-(B*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x)) - I*B*(d*e - c*f)*h*(
c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
+ d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c
*f*h)/(d*e*h - c*f*h)] + I*d*(B*e - A*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*
x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[S
qrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(d^2*
Sqrt[-c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \; 176$$

$$\frac{B\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}-\frac{(Bg-Ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{124}$$

$$\frac{B\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(Bg-Ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{123}$$

$$\frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(Bg-Ah)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{131}$$

$$\frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}$$

$$\Big\downarrow \text{131}$$

$$\frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(Bg-Ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{130}$$

$$\frac{2B\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{2(Bg-Ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

input
```
Int[(A + B*x)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*B*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Elli
pticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/
(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h
)]) - (2*Sqrt[-(d*e) + c*f]*(B*g - A*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e +
f*x]*Sqrt[g + h*x])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !L
tQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

Maple [A] (verified)

Time = 4.99 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2A\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg}}+\frac{2B\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\ldots}{\sqrt{d\ldots}}\right.}{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{xd+c}}$
default	$\frac{2\left(A\operatorname{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)cdfh-A\operatorname{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)d^{2}eh-B\operatorname{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)c\ldots\right.}{\ldots}$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2*A*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/
f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(
-c/d+g/h))^(1/2))+2*B*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g
/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g
*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(
c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/(c/d
-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(250) = 500$.

Time = 0.11 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.36

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(3\sqrt{dfh}Bdfh\text{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-(d^2ef+cdf^2)gh+(d^2e^2-cdef+c^2f^2)h^2\right)}{3\,d^2f^2h^2},-\frac{4\left(2\,d^3f^3g^3-3\left(d^3ef^2+cd^2f^3\right)g^2h-\right.}{}\right.\right.}{}$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
fricas")
```

output
```
-2/3*(3*sqrt(d*f*h)*B*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f +
c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*
d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f
^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3
*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f
+ c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(
2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e
*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c
^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h
))) + (B*d*f*g + (B*d*e + (B*c - 3*A*d)*f)*h)*sqrt(d*f*h)*weierstrassPInve
rse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*
f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*
g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c
*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x
 + d*f*g + (d*e + c*f)*h)/(d*f*h)))/(d^2*f^2*h^2)
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((B*x + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="
giac")
```

output
```
integrate((B*x + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{A+Bx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx\\&=\left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}\,x}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx\right)b\\&\quad+\left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx\right)a\end{aligned}$$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*x)/(c*e*g + c*e*h*x + c*f*g
*x + c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*b + i
nt((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x))/(c*e*g + c*e*h*x + c*f*g*x
+ c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*a
```

3.4 $\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 313

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2B\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\left(A-\frac{aB}{b}\right)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*B*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1
/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))/b/d/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(A-a*B/b)*(c*f-d*e)
^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticP
i(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+
d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2i\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(b(-Bc+Ad)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right),\frac{dfg-cfh}{deh-cfh}\right)+(-Ab+aB)d\operatorname{EllipticPi}\right.}{b(-bc+ad)\sqrt{-c+\frac{de}{f}}f\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{g+hx}}$$

input
```
Integrate[(A + B*x)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
((2*I)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*(b*(-(B*c) + A*d)*E
llipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e
*h - c*f*h)] + (-(A*b) + a*B)*d*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*
f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h -
 c*f*h)]))/(b*(-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*f*Sqrt[(d*(e + f*x))/(f*(c
 + d*x))]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2110, 27, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2110}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\int \frac{B}{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{B\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}$$

$$\Big\downarrow \text{131}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{B\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b\sqrt{e+fx}}$$

$$\Big\downarrow \text{131}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{B\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{130}$$

$$\left(A-\frac{aB}{b}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{187}$$

$$\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-2\left(A-\frac{aB}{b}\right)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{413}$$

$$\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\left(A-\frac{aB}{b}\right)\sqrt{\frac{f(c+dx)}{de-cf}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\left(A-\frac{aB}{b}\right)\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int\frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{2B\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\left(A-\frac{aB}{b}\right)\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\operatorname{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

input
```
Int[(A + B*x)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*B*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))
/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]]
, ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*d*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*
x]) - (2*(A - (a*B)/b)*Sqrt[-(d*e) + c*f]*Sqrt[1 + (f*(c + d*x))/(d*e - c*
f)]*Sqrt[1 + (h*(c + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c
 - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c
*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d
*x))/d]*Sqrt[g - (c*h)/d + (h*(c + d*x))/d])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 7.74 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.53

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2B\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{b\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2(Ab-Ba)\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+}}}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x}}\right.}{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{xd+c}}$
default	$\frac{2\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{xd+c}\,\sqrt{\frac{f(xd+c)}{cf-de}}\,\sqrt{-\frac{(hx+g)d}{ch-dg}}\,\sqrt{-\frac{d(fx+e)}{cf-de}}\left(A\,\mathrm{EllipticPi}\left(\sqrt{\frac{f(xd+c)}{cf-de}},-\frac{b(cf-de)}{f(ad-bc)},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)bcdf-A\,\mathrm{EllipticP}\right.}{}$

input
```
int((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2*B/b*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+
e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*
f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)
/(-c/d+g/h))^(1/2))+2*(A*b-B*a)/b^2*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((
x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d
*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-c/d+a/b)*Ellipti
cPi(((x+c/d)/(c/d-e/f))^(1/2),(-c/d+e/f)/(-c/d+a/b),((-c/d+e/f)/(-c/d+g/h)
)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((A + B*x)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)),
x)
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((B*x + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((B*x + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),
x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx$$

input
```
int((B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x))/(c*e*g + c*e*h*x + c*f*g*x
 + c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)
```

3.5 $\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 678

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{(Ab-aB)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$+\frac{\sqrt{-de+cf}(3a^2Abdfh-a^3Bdfh-b^3(2Bceg-A(deg+cfg+ceh))+ab^2(B(deg+cfg+ceh)-2A}{b(bc-ad)^2\sqrt{f}(be-af)(bg-a}$$

output
```
-b*(A*b-B*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+b*
e)/(-a*h+b*g)/(b*x+a)+(A*b-B*a)*f^(1/2)*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d
*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),(
(-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(f*x+e)
^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-(A*b-B*a)*f^(1/2)*(c*f-d*e)^(1/2)*(d*(
f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d
*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/b/(-a*d+b*c
)/(-a*f+b*e)/(f*x+e)^(1/2)/(h*x+g)^(1/2)+(c*f-d*e)^(1/2)*(3*a^2*A*b*d*f*h-
a^3*B*d*f*h-b^3*(2*B*c*e*g-A*(c*e*h+c*f*g+d*e*g))+a*b^2*(B*(c*e*h+c*f*g+d*
e*g)-2*A*(c*f*h+d*e*h+d*f*g)))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c
*h+d*g))^(1/2)*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d
*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/b/(-a*d+b*c)^2/f^(1/2)
/(-a*f+b*e)/(-a*h+b*g)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.43 (sec) , antiderivative size = 3412, normalized size of antiderivative = 5.03

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
-((b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(
b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((c + d*x)^(3/2)*(A*b^3*c*Sqrt[-c + (
d*e)/f]*f*h - a*b^2*B*c*Sqrt[-c + (d*e)/f]*f*h - a*A*b^2*d*Sqrt[-c + (d*e)
/f]*f*h + a^2*b*B*d*Sqrt[-c + (d*e)/f]*f*h + (A*b^3*c*d^2*e*Sqrt[-c + (d*e
)/f]*g)/(c + d*x)^2 - (a*b^2*B*c*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 -
 (a*A*b^2*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 + (a^2*b*B*d^3*e*Sqrt[-c
 + (d*e)/f]*g)/(c + d*x)^2 - (A*b^3*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x
)^2 + (a*b^2*B*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a*A*b^2*c*d^2*
Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 - (a^2*b*B*c*d^2*Sqrt[-c + (d*e)/f]*f*
g)/(c + d*x)^2 - (A*b^3*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*b^2
*B*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*A*b^2*c*d^2*e*Sqrt[-c +
(d*e)/f]*h)/(c + d*x)^2 - (a^2*b*B*c*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)
^2 + (A*b^3*c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (a*b^2*B*c^3*Sqrt[-c
 + (d*e)/f]*f*h)/(c + d*x)^2 - (a*A*b^2*c^2*d*Sqrt[-c + (d*e)/f]*f*h)/(c +
 d*x)^2 + (a^2*b*B*c^2*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (A*b^3*c*d*
Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (a*b^2*B*c*d*Sqrt[-c + (d*e)/f]*f*g)/(
c + d*x) - (a*A*b^2*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (a^2*b*B*d^2*S
qrt[-c + (d*e)/f]*f*g)/(c + d*x) + (A*b^3*c*d*e*Sqrt[-c + (d*e)/f]*h)/(c +
 d*x) - (a*b^2*B*c*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (a*A*b^2*d^2*e*Sq
rt[-c + (d*e)/f]*h)/(c + d*x) + (a^2*b*B*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c...
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2102, 25, 2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$
$$\Big\downarrow \text{2102}$$
$$\frac{\int -\frac{2Adfha^2+b(B(deg+cfg+ceh)-2A(dfg+deh+cfh))a-2(Ab-aB)dfhxa-b(Ab-aB)dfhx^2-b^2(2Bceg-A(deg+cfg+ceh))}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2(bc-ad)(be-af)(bg-ah)}-\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 25

$$-\frac{\int \frac{2Adfha^2+b(B(deg+cfg+ceh)-2A(dfg+deh+cfh))a-2(Ab-aB)dfhxa-b(Ab-aB)dfhx^2-b^2(2Bceg-A(deg+cfg+ceh))}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2(bc-ad)(be-af)(bg-ah)}-\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 2110

$$-\frac{\int \frac{\frac{Bdfha^2}{b}-Adfha+(aBdfh-Abdfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg}{b}}{2(bc-ad)(be-af)(bg-ah)}$$
$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 176

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df}{}}{2(bc-ad)(be-af)(bg-ah)}$$
$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 124

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df}{}}{2(bc-ad)(be-af)(bg-ah}$$
$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 123

$$-\frac{\frac{(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg)))\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df}{}}{2(bc-ad)(be-af)(bg}$$
$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 131

$$-\frac{\frac{\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df}{\cdots}}{2(bc-ad)(be-\cdots}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 131

$$-\frac{\frac{\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{df}{\cdots}}{2(bc-\cdots}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 130

$$-\frac{\frac{\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2\sqrt{\cdots}}{\cdots}}{2(bc\cdots}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 187

$$-\frac{-\frac{2\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx}{d}\cdots}}\cdots}{b}\cdots}{\cdots}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 413

$$-\frac{-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)\int \frac{\cdots}{(bc-ad-b(c+dx))\sqrt{\cdots}}\cdots}{b\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}\cdots}{\cdots}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 413

$$-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)\int \frac{}{(bc-}}{b\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow 412$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\left(a^3(-B)dfh+3a^2Abdfh+ab^2(B(ceh+cfg+deg)-2A(cfh+deh+dfg))-b^3(2Bceg-A(ceh+cfg+deg))\right)}{b\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\frac{b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

input

```
Int[(A + B*x)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output

```
-((b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(
b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((-2*(A*b - a*B)*Sqrt[f]*Sqrt[-(d*e)
+ c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqr
t[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])
/(Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) + (2*(A*b - a*B)*Sqrt[f]*
Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g
+ h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e)
+ c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b*Sqrt[e + f*x]*Sqrt[g + h*x])
 - (2*Sqrt[-(d*e) + c*f]*(3*a^2*A*b*d*f*h - a^3*B*d*f*h - b^3*(2*B*c*e*g -
 A*(d*e*g + c*f*g + c*e*h)) + a*b^2*(B*(d*e*g + c*f*g + c*e*h) - 2*A*(d*f*
g + d*e*h + c*f*h)))*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1 + (h*(c +
d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(
Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h)
)])/(b*(b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sqrt[g - (c
*h)/d + (h*(c + d*x))/d]))/(2*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 20.61 (sec) , antiderivative size = 1208, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1208
default	Expression too large to display	13380

input
```
int((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(b/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+
a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+
c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)-a*d*f*h*(A*b-B*a)/(a^3*d*f*h-
a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^
3*c*e*g)/b*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*
((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*
x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+
e/f)/(-c/d+g/h))^(1/2))-d*f*h*(A*b-B*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h
-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(c/d-e/f)*((x+
c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2
)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^
(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/
h))^(1/2))-g/h*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))
^(1/2)))+(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f*g+
A*b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g-B*a^3*d*f*h+B*a*b^2*c*e*h+B*a*b^2*c*f*
g+B*a*b^2*d*e*g-2*B*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*
f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^2*(c/d-e/f)*((x+c/d)/
(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*
f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),
x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((B*x + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((B*x + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^2\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)
),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{\left(bx+a\right)^{2}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.6 $\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 992

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+h}}{4d^2f^2h^2\sqrt{a+bx}}$$

$$+\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$-\frac{\sqrt{be-af}(dg-ch)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{b}}{\sqrt{d}}\right.\right.}{4d^2f^2\sqrt{de-cf}h^2\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$-\frac{(bc-ad)\sqrt{be-af}(4Abdfh+3aBdfh-bB(dfg+3deh+3cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\operatorname{EllipticF}\left(\mathrm{ar}\right.}{4bd^2f^2\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$-\frac{\sqrt{de-cf}((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-4dfh(4aAbdfh}{}$$

output
```
1/4*b*(4*A*b*d*f*h+5*a*B*d*f*h-3*b*B*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)*(f
*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2/(b*x+a)^(1/2)+1/2*b*B*(b*x+a)^(1/2)*
(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-1/4*(-a*f+b*e)^(1/2)*(-c*h
+d*g)*(4*A*b*d*f*h+5*a*B*d*f*h-3*b*B*(c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)*(-
(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d
*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*
e)/(-c*h+d*g))^(1/2))/d^2/f^2/(-c*f+d*e)^(1/2)/h^2/(-(-a*d+b*c)*(f*x+e)/(-
c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/4*(-a*d+b*c)*(-a*f+b*e)^(1/2)*(4*A
*b*d*f*h+3*a*B*d*f*h-b*B*(3*c*f*h+3*d*e*h+d*f*g))*(f*x+e)^(1/2)*(-(-a*d+b*
c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1
/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h
+d*g))^(1/2))/b/d^2/f^2/(-c*f+d*e)^(1/2)/h/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)
/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/4*(-c*f+d*e)^(1/2)*((a*d*f*h+b*(c*f*h+d*e*
h+d*f*g))*(4*A*b*d*f*h+5*a*B*d*f*h-3*b*B*(c*f*h+d*e*h+d*f*g))-4*d*f*h*(4*a
*A*b*d*f*h+2*a^2*B*d*f*h-b^2*B*(c*e*h+c*f*g+d*e*g)-a*b*B*(c*f*h+d*e*h+d*f*
g)))*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(
h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)
/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*
h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/d^3/f^2/(-a*f+b*e)^(1/2)/h^2/(f*x+e
)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21961 vs. 2(992) = 1984.

Time = 36.62 (sec) , antiderivative size = 21961, normalized size of antiderivative = 22.14

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 3.08 (sec) , antiderivative size = 983, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {2100, 2105, 25, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 2100$$

$$\frac{\int \frac{4Adfha^2+b(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))x^2-bB(bceg+a(deg+cfg+ceh))+2\left(2Bdfha^2+4Abdfha-bB(dfg+deh+cfh)a-b^2B(deg \cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{4dfh} \cdots \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 2105$$

$$\frac{\int -\frac{b\left((bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh} \cdots \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{b\left((bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh} \cdots \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{(bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(df \cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh} \cdots \frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 194$$

$$-\frac{\int \frac{(bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(df \cdots\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh} \cdots$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 327$$

$$-\frac{\int \frac{(bdeg+acfh)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))-2dfh\left(4a^2Adfh-bB(bceg+a(deg+cfg+ceh))\right)+\left((adfh+b(dfg+deh+cfh))(4Abdfh+5aBdfh-3bB(df \cdots\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh} \cdots$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 2101$$

$$-\frac{\frac{\left((adfh+b(cfh+deh+dfg))(5aBdfh+4Abdfh-3bB(cfh+deh+dfg))-4dfh\left(2a^2Bdfh+4aAbdfh-abB(cfh+deh+dfg)-b^2B(ceh+cfg+deg)\right)\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx} \cdots}}{b}}{2dfh} \cdots$$

$$\frac{bB\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 183$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{\sqrt{a+bx} \cdots}{\cdots}$$

$$\Big\downarrow \; 188$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{\sqrt{a+bx} \cdots}{\cdots}$$

$$\Big\downarrow \ 321$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{\sqrt{a+bx}\cdots}{\cdots}$$

$$\Big\downarrow \ 412$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}B}{2dfh}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4Abdfh+5aBdfh-3bB(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{\sqrt{a+bx}\cdots}{\cdots}$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(b*B*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h) +
(((4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x
]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (Sqrt[d*g - c*h]*Sqrt
[f*g - e*h]*(4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h))*Sq
rt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*Ellip
ticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x]
)], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(d*f*h*Sqrt[((d*
e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - ((2*d*(b*e -
 a*f)*Sqrt[b*g - a*h]*(4*A*b*d*f*h + 3*a*B*d*f*h - b*B*(c*f*h + 3*d*(f*g +
 e*h)))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x
]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a
 + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*Sqr
t[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a
+ b*x)))]) + (2*Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*
(4*A*b*d*f*h + 5*a*B*d*f*h - 3*b*B*(d*f*g + d*e*h + c*f*h)) - 4*d*f*h*(4*a
*A*b*d*f*h + 2*a^2*B*d*f*h - b^2*B*(d*e*g + c*f*g + c*e*h) - a*b*B*(d*f*g
+ d*e*h + c*f*h)))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
+ b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[
-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])
/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c -...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 19.40 (sec) , antiderivative size = 1814, normalized size of antiderivative = 1.83

method	result	size
elliptic	Expression too large to display	1814
default	Expression too large to display	54740

input
```
int((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(1/2*B*b/h/d/f*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+
b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*
c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/
2)+2*(a^2*A-1/2*B*b/h/d/f*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g
))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d
+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+
c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))
^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*
(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(2*a*b*A+a^2*B-1/2*B*b/h/d/f*(a*c
*f*h+a*d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h
)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/
d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/
h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c/d*EllipticF(((c/d-e
/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/
d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))
^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1
/2)))+(b^2*A+2*a*b*B-1/2*B*b/h/d/f*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/
2*b*g*d*f))*((x+g/h)*(x+a/b)*(x+e/f)+(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/
h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(A+Bx)\,(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Integral((A + B*x)*(a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g +
h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,(a+b\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)
^(1/2)),x)
```

output
```
int(((A + B*x)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\,(Bx+A)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^(3/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.7 $\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 749

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{bB\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{dfh\sqrt{a+bx}} - \frac{B\sqrt{be-af}(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{df\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} - \frac{B(bc-ad)\sqrt{be-af}\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{bdf\sqrt{de-cf}\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} + \frac{\sqrt{de-cf}(2Abdfh+aBdfh-bB(dfg+deh+cfh))(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\text{Elliptic}}{bd^2f\sqrt{be-af}h\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
b*B*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h/(b*x+a)^(1/2)-B*(-a*f+
b*e)^(1/2)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a
))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)
^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/d/f/(-c*f+d*e)
^(1/2)/h/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-B*(-
a*d+b*c)*(-a*f+b*e)^(1/2)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b
*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*
x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/d/f/(-c*
f+d*e)^(1/2)/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+
(-c*f+d*e)^(1/2)*(2*A*b*d*f*h+a*B*d*f*h-b*B*(c*f*h+d*e*h+d*f*g))*(b*x+a)*(
-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d
*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1
/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b
*e)/(-c*h+d*g))^(1/2))/b/d^2/f/(-a*f+b*e)^(1/2)/h/(f*x+e)^(1/2)/(h*x+g)^(1
/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8030 vs. $2(749) = 1498$.

Time = 42.33 (sec) , antiderivative size = 8030, normalized size of antiderivative = 10.72

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 725, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2099, 183, 188, 194, 321, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{2099}$$

$$\frac{1}{2}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{B(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{B(be-af)(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bfh}+\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{183}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}+\frac{B(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{B(be-af)(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bfh}+\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{188}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-$$

$$\frac{B\sqrt{g+hx}(be-af)(bg-ah)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{bfh\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+$$

$$\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$\Big\downarrow$ 194

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B\sqrt{g+hx}(be-af)(bg-ah)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{bfh\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{B\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$\Big\downarrow$ 321

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}-$$

$$\frac{B\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+$$

$$\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$\Big\downarrow$ 327

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{B\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{B\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \; 412$$

$$\frac{(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(B\left(\frac{a}{b}-\frac{c}{d}-\frac{e}{f}-\frac{g}{h}\right)+2A\right)\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-}}{\sqrt{ch-}}\right.\right.}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

$$\frac{B\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{bfh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{B\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{B\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

input
```
Int[(Sqrt[a + b*x]*(A + B*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(B*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (B*Sqr
t[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/
((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])
/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*
(d*g - c*h))])/(d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))
]*Sqrt[g + h*x]) - (B*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d
*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a
*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g -
 e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*f*h*Sqrt[f*g - e*h]*Sqrt[c + d*x]*S
qrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + ((2*A + B*(a/b
- c/d - e/f - g/h))*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*
x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a
+ b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c -
 a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*
g - c*h))/((b*c - a*d)*(f*g - e*h))])/(Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqr
t[e + f*x])
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2099
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[B*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*Sqrt[c + d*x])), x] + (-Sim
p[B*(b*e - a*f)*((b*g - a*h)/(2*b*f*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x
]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Simp[B*(d*e - c*f)*((d*g - c*h)/(2
*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Simp[(2*A*b*d*f*h + B*(a*d*f*h - b*(d*f*g + d*e*h + c*f*h)))/(2*b
*d*f*h)   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && NeQ[2*A*d*f - B*(d*e +
 c*f), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1543 vs. 2(684) = 1368.

Time = 18.95 (sec) , antiderivative size = 1544, normalized size of antiderivative = 2.06

method	result	size
elliptic	Expression too large to display	1544
default	Expression too large to display	21005

input

```
int((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output

```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*A*a*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/
d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g
/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h
)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(
x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(A*b+B*
a)*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d
+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+
c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))
^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d
+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/
f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f
-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)))+B*b*((x+g/h)*(x+a/b)*(x+e/f)+(e/f-g/h)
*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b
)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)*
((c/d*g/h-e/f*g/h+c*e/d/f+c^2/d^2)/(c/d-e/f)/(-c/d+g/h)*EllipticF(((c/d-e/
f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d
+e/f))^(1/2))+(a/b-g/h)*EllipticE(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(
1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))/(-c/d+g/h)+(a*d*f*
h+b*c*f*h+b*d*e*h+b*d*f*g)/h/d/b/f/(c/d-e/f)*EllipticPi(((c/d-e/f)*(x+g...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Bx)\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x
)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+B\,x)\,\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + B*x)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)
^(1/2)),x)
```

output
```
int(((A + B*x)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}\,(Bx+A)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^(1/2)*(B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.8 $\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 453

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(Ab-aB)\sqrt{de-cf}\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{b\sqrt{be-af}(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}} + \frac{2B\sqrt{de-cf}(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)},\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-c}{(be-a}\right.}{bd\sqrt{be-af}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*(A*b-B*a)*(-c*f+d*e)^(1/2)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2
)*(h*x+g)^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/
(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/(-a*f
+b*e)^(1/2)/(-c*h+d*g)/(f*x+e)^(1/2)/(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+
a))^(1/2)+2*B*(-c*f+d*e)^(1/2)*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*
x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*
f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(
-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/d/(-a*f+b
*e)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 24.59 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2(a+bx)^{3/2}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-\frac{Ab\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right),\frac{(-bc+ad)(-fg+eh)}{(be-af)(dg-ch)}\right)}{(bg-ah)(a+bx)\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}}-\frac{aB\sqrt{\frac{(bg-}{(fg-}}}{}\right.}{}$$

input
```
Integrate[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
(2*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(
-((A*b*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*Ell
ipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], (
(-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*(a
 + b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])) - (a*B*
Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*EllipticF[
ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c)
 + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((-(b*g) + a*h)*(a + b
*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]) + (B*(-(f*g)
 + e*h)*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^
2*(a + b*x)^2))]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a*f)*h), ArcSin[Sqr
t[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-
(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*e - a*f)*h)))/(b*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2101}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{B\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}$$

$$\Big\downarrow \text{ 183}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2B(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{ 188}$$

$$\frac{2\sqrt{g+hx}(Ab-aB)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2B(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{2B(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{g+hx}(Ab-aB)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \text{ 412}$$

$$\frac{2\sqrt{g+hx}(Ab-aB)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2B(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{bh\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

input
```
Int[(A + B*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(A*b - a*B)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[
g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]
*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])
/(b*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g +
 h*x))/((f*g - e*h)*(a + b*x)))]) + (2*B*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt
[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f
*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)
), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x
])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*Sqrt[b*c - a
*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. $2(415) = 830$.

Time = 21.55 (sec) , antiderivative size = 848, normalized size of antiderivative = 1.87

method	result
elliptic	$\sqrt{(hx+g)(xd+c)(bx+a)(fx+e)}\left(\frac{2A\left(\frac{e}{f}-\frac{g}{h}\right)\sqrt{\frac{\left(\frac{c}{d}-\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^{2}\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{a}{b}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(\frac{c}{d}-\frac{e}{f}\right)\ldots}{\left(-\frac{e}{f}+\frac{g}{h}\right)\ldots}}\right.}{\left(\frac{c}{d}-\frac{e}{f}\right)\left(-\frac{c}{d}+\frac{g}{h}\right)\sqrt{hdbf\left(x+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}}\right.$
default	Expression too large to display

input
```
int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*A*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d)
)^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h
)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*
(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+
c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*B*(e/f-g/
h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a
/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2
)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c
/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f
-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/f)*(x+g/h)
/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b
-g/h)/(-c/d+e/f))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x
)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(
1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.9 $\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 451

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2(Ab-aB)(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{(bc-ad)\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} + \frac{2(Bg-Ah)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

output

```
-2*(A*b-B*a)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x
+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+
a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*d+b*c)/(
-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e
)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+2*(-A*h+B*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*
x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-
c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))
^(1/2))/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x+e)/
(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 26.06 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left((Ab-aB)\right.}{}$$

input
```
Integrate[(A + B*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
(2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
f*x)^(3/2)*(g + h*x)^(3/2)*((A*b - a*B)*(d*g - c*h)*EllipticE[ArcSin[Sqrt[
((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*g -
e*h))/((b*e - a*f)*(d*g - c*h))] + (B*c - A*d)*(b*g - a*h)*EllipticF[ArcSi
n[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*
(f*g - e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*g - e*h)^3*(a +
b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/
((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2102}$$

$$\frac{\int \frac{-Adfha^2-b(B(deg+cfg+ceh)-A(dfg+deh+cfh))a+2b(Ab-aB)dfhx^2+b^2Bceg+(Ab-aB)(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 2105

$$\frac{(Ab-aB)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{\int \frac{2bd(Bc-Ad)f(be-af)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(Ab-\cdots}{\sqrt{c+dx}}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 27

$$\frac{(Ab-aB)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+(be-af)(bg-ah)(Bc-Ad)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+\cdots}}\cdots}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 188

$$\frac{(Ab-aB)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2\sqrt{g+hx}(be-af)(bg-ah)(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{\cdots}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}\cdots}}\cdots}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\cdots}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 194

$$\frac{\frac{2\sqrt{g+hx}(be-af)(bg-ah)(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2\sqrt{a+bx}(Ab-aB)(dg-ch)\sqrt{-\frac{(g+hx)\cdots}{(c+dx)\cdots}}\cdots}{\sqrt{g+hx}\sqrt{\frac{(a\cdots}{(c\cdots}}\cdots}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 321

$$\frac{-\frac{2\sqrt{a+bx}(Ab-aB)(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{2\sqrt{g+hx}(be-af)\sqrt{bg-ah}(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+h}{(a+b}}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{2\sqrt{g+hx}(be-af)\sqrt{bg-ah}(Bc-Ad)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2\sqrt{a+bx}(Ab-aB)\sqrt{dg-ch}\sqrt{fg-}}{}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

input
```
Int[(A + B*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(-2*b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*
(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*(A*b - a*B)*d*Sqrt[a + b*x]*S
qrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*(A*b - a*B)*Sqrt[d*g - c*h]
*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)
*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g -
 e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))
])/(Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) +
 (2*(B*c - A*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/(
(d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sq
rt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))
/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e
- a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]))/((b*c - a*d)*(b*e - a*f)*(b*
g - a*h))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2249 vs. $2(413) = 826$.

Time = 23.72 (sec) , antiderivative size = 2250, normalized size of antiderivative = 4.99

method	result	size
elliptic	Expression too large to display	2250
default	Expression too large to display	16980

input
```
int((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+
b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-
a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a)/((x+a
/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b
*d*e*g*x+b*c*e*g))^(1/2)+2*(B/b+1/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f
*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(A*b-B*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d
*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b
*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h
+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a))*(e/f-g/h)*((c/d-e/f)*(x+g/h
)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/
d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/
h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x
+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f)
)^(1/2))+2*(-(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(A*b-B*a)/(a^3*d*f*h-a^2*b*
c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*
g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*
b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(A*b-B*a))*(e/f-g/h
)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/
b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1...
```

Fricas [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
integral((B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
/(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2*a*b*d)*f)
*h)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d)*e + (2*a
*b*c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)*f)*g +
 (a^2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f + (2*a*b*c
 + a^2*d)*e)*g)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Integral((A + B*x)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g +
h*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(
1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.10 $\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 737

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2b(Ab-aB)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2(dg-ch)\left(3a^3Bdfh+b^3(3Bceg-2A(deg+cfg+ceh))-ab^2(B(deg+cfg+ceh)-4A(dfg+deh\right.}{3(bc-ad)^2(be-af)^{3/2}\sqrt{de-}}$$

$$-\frac{2(B(3b^2cegh+3a^2dfgh-ab(ch(2fg+eh)+dg(fg+2eh)))-A(3a^2dfh^2-3ab(de+cf)h^2-b^2(dg(f}{3(bc-ad)(be-af)^{3/2}\sqrt{de-cf}(bg}$$

output
```
-2/3*b*(A*b-B*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*
f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)-2/3*(-c*h+d*g)*(3*a^3*B*d*f*h+b^3*(3*B*c*e
*g-2*A*(c*e*h+c*f*g+d*e*g))-a*b^2*(B*(c*e*h+c*f*g+d*e*g)-4*A*(c*f*h+d*e*h+
d*f*g))-a^2*b*(6*A*d*f*h+B*(c*f*h+d*e*h+d*f*g)))*(f*x+e)^(1/2)*(-(-a*d+b*c
)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/
2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+
d*g))^(1/2))/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)^2/(
-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2/3*(B*(3*b^2*
c*e*g*h+3*a^2*d*f*g*h-a*b*(c*h*(e*h+2*f*g)+d*g*(2*e*h+f*g)))-A*(3*a^2*d*f*
h^2-3*a*b*(c*f+d*e)*h^2-b^2*(d*g*(-e*h+f*g)-c*h*(2*e*h+f*g))))*(f*x+e)^(1/
2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/
2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a
*f+b*e)/(-c*h+d*g))^(1/2))/(-a*d+b*c)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-
a*h+b*g)^2/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10828 vs. 2(737) = 1474.

Time = 39.66 (sec) , antiderivative size = 10828, normalized size of antiderivative = 14.69

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 3.98 (sec) , antiderivative size = 1068, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2102, 25, 2102, 25, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{2102}$$

$$\frac{\int -\frac{3Adfha^2+b(B(deg+cfg+ceh)-3A(dfg+deh+cfh))a-b^2(3Bceg-2A(deg+cfg+ceh))-(Ab-aB)(3adfh-b(dfg+deh+cfh))x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{3Adfha^2+b(B(deg+cfg+ceh)-3A(dfg+deh+cfh))a-b^2(3Bceg-2A(deg+cfg+ceh))-(Ab-aB)(3adfh-b(dfg+deh+cfh))x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{2102}$$

$$-\frac{\int -\frac{2bdfh\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)x^2+(adfh+b(dfg+deh+c\ldots}{\ldots}}{\ldots}$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(3a^3Bdfh-a^2b(6Adfh+B(cfh+deh+dfg))-ab^2(B(ceh+cfg+deg)-4A(cfh+deh+dfg))+b^3(3Bceg-2A(ceh+cfg+\ldots}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}}{\ldots}$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 2105$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(3a^3Bdfh-a^2b(6Adfh+B(cfh+deh+dfg))-ab^2(B(ceh+cfg+deg)-4A(cfh+deh+dfg))+b^3(3Bceg-2A(ceh+cfg+}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 188$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$$\Big\downarrow \; 194$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$$\Big\downarrow \; 321$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$$\Big\downarrow 327$$

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ab-aB)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{\frac{2b\left(3Bdfha^3-b(6Adfh+B(dfg+deh+cfh))a^2-b^2(B(deg+cfg+ceh)-4A(dfg+deh+cfh))a+b^3(3Bceg-2A(deg+cfg+ceh))\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}}{}$$

input

```
Int[(A + B*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output

```
(-2*b*(A*b - a*B)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) - ((2*b*(3*a^3*B*d*f*h + b^3*(3
*B*c*e*g - 2*A*(d*e*g + c*f*g + c*e*h)) - a*b^2*(B*(d*e*g + c*f*g + c*e*h)
 - 4*A*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*A*d*f*h + B*(d*f*g + d*e*h + c*
f*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)
*(b*g - a*h)*Sqrt[a + b*x]) - ((2*d*(3*a^3*B*d*f*h + b^3*(3*B*c*e*g - 2*A*
(d*e*g + c*f*g + c*e*h)) - a*b^2*(B*(d*e*g + c*f*g + c*e*h) - 4*A*(d*f*g +
 d*e*h + c*f*h)) - a^2*b*(6*A*d*f*h + B*(d*f*g + d*e*h + c*f*h)))*Sqrt[a +
 b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*Sqrt[d*g - c*h]*Sqrt
[f*g - e*h]*(3*a^3*B*d*f*h + b^3*(3*B*c*e*g - 2*A*(d*e*g + c*f*g + c*e*h))
 - a*b^2*(B*(d*e*g + c*f*g + c*e*h) - 4*A*(d*f*g + d*e*h + c*f*h)) - a^2*b
*(6*A*d*f*h + B*(d*f*g + d*e*h + c*f*h)))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f
)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*S
qrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/
((b*e - a*f)*(d*g - c*h))]])/(Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c
 + d*x))]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[b*g - a*h]*(3*a^2*d*(B*c - A
*d)*f*h + b^2*(3*B*c*d*e*g - 2*A*d^2*e*g + A*c^2*f*h - A*c*d*(f*g + e*h))
 + a*b*(3*A*d^2*(f*g + e*h) - B*(d^2*e*g + c^2*f*h + 2*c*d*(f*g + e*h))))*S
qrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*Ellipti
cF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3388 vs. $2(685) = 1370$.

Time = 37.81 (sec) , antiderivative size = 3389, normalized size of antiderivative = 4.60

method	result	size
elliptic	Expression too large to display	3389
default	Expression too large to display	112872

input
```
int((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2/3/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+
a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(A*b-B*a)*(b*d*f*h*x^4+a*
d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*
f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+
b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+2/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^
2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*
h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2
*(6*A*a^2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*f*g+2*A*b^3*
c*e*h+2*A*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*f*h+B*a^2*b*d*e*
h+B*a^2*b*d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-3*B*b^3*c*e*g)/(
(x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g
*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(-1/3*(3*A*a*b*d*f*h-A*b^2*c*f*h-A*b^2*d*e*
h-A*b^2*d*f*g-3*B*a^2*d*f*h+B*a*b*c*f*h+B*a*b*d*e*h+B*a*b*d*f*g)/(a^3*d*f*
h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-
b^3*c*e*g)/b+1/3/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*
c*f*g+b^2*d*e*g)*(6*A*a^2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^
2*d*f*g+2*A*b^3*c*e*h+2*A*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*
f*h+B*a^2*b*d*e*h+B*a^2*b*d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-
3*B*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e...
```

Fricas [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="fricas")
```

output
```
integral((B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3*a*b^2*d)*
f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*b^2*d)*e
+ 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a*b^2*c +
 a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*h)*x^3
+ ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f + (3*a^2
*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3*d)*e)*g
)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1
/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x, algorithm="giac")
```

output
```
integrate((B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{A+B\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(
1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.11 $\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 913

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b(5adfh-b(3dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh^2\sqrt{a+bx}}$$

$$+\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$-\frac{\sqrt{be-af}(dg-ch)(5adfh-b(3dfg+deh+cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de}{(be}\right.}{2df\sqrt{de-cf}h^2\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$-\frac{(bc-ad)\sqrt{be-af}(3adfh-b(dfg+deh+cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+}}{\sqrt{de-cf}\sqrt{a+}}\right.\right.}{2bdf\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$-\frac{\sqrt{de-cf}(6abd^2f^2gh-3a^2d^2f^2h^2+b^2(2cdefh^2-c^2f^2h^2-d^2(3f^2g^2+e^2h^2)))(a+bx)\sqrt{-\frac{(bc-ad)(e+fx}{(de-cf)(a+bx}}}{2bd^2f\sqrt{be-af}h^2\sqrt{e+fx}\sqrt{g+}}$$

output
```
1/2*b*(5*a*d*f*h-b*(c*f*h+d*e*h+3*d*f*g))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x
+g)^(1/2)/d/f/h^2/(b*x+a)^(1/2)+b*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2
)*(h*x+g)^(1/2)/h-1/2*(-a*f+b*e)^(1/2)*(-c*h+d*g)*(5*a*d*f*h-b*(c*f*h+d*e*
h+3*d*f*g))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*E
llipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-
c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/d/f/(-c*f+d*e)^(1/2)/h^2
/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/2*(-a*d+b*
c)*(-a*f+b*e)^(1/2)*(3*a*d*f*h-b*(c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)*(-(-a*
d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c
)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(
-c*h+d*g))^(1/2))/b/d/f/(-c*f+d*e)^(1/2)/h/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)
/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/2*(-c*f+d*e)^(1/2)*(6*a*b*d^2*f^2*g*h-3*a^
2*d^2*f^2*h^2+b^2*(2*c*d*e*f*h^2-c^2*f^2*h^2-d^2*(e^2*h^2+3*f^2*g^2)))*(b*
x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(
-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d
*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(
-a*f+b*e)/(-c*h+d*g))^(1/2))/b/d^2/f/(-a*f+b*e)^(1/2)/h^2/(f*x+e)^(1/2)/(h
*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 15131 vs. $2(913) = 1826$.

Time = 35.42 (sec) , antiderivative size = 15131, normalized size of antiderivative = 16.57

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^(3/2)*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e +
f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 2.48 (sec) , antiderivative size = 892, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.224$, Rules used = {2100, 27, 2105, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \ 2100$$

$$\frac{\int \frac{2\left(bdf(5adfh-b(3dfg+deh+cfh))x^2+2df\left(2dfha^2-b(dfg-deh-cfh)a-b^2(deg+cfg+ceh)\right)x+df\left(2a^2(de+cf)h-b(bceg+a(deg+cfg+ceh))\right)\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{4dfh}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{bdf(5adfh-b(3dfg+deh+cfh))x^2+2df\left(2dfha^2-b(dfg-deh-cfh)a-b^2(deg+cfg+ceh)\right)x+df\left(2a^2(de+cf)h-b(bceg+a(deg+cfg+ceh))\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\Big\downarrow \ 2105$$

$$\frac{\int \frac{bdf\left(deg(3dfg+deh-cfh)b^2-afh\left(-fhc^2-d(fg-eh)c+7d^2eg\right)b+a^2df(4de-cf)h^2-\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$2df$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{deg(3dfg+deh-cfh)b^2-afh\left(-fhc^2-d(fg-eh)c+7d^2eg\right)b+a^2df(4de-cf)h^2-\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}d}{2h}$$

$$2dfh$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$$\Big\downarrow \ 194$$

$$\frac{\int \frac{deg(3dfg+deh-cfh)b^2-afh\left(-fhc^2-d(fg-eh)c+7d^2eg\right)b+a^2df(4de-cf)h^2-\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2h}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$\downarrow$ 327

$$\frac{\int \frac{deg(3dfg+deh-cfh)b^2-afh\left(-fhc^2-d(fg-eh)c+7d^2eg\right)b+a^2df(4de-cf)h^2-\left(\left(-\left(\left(3f^2g^2+e^2h^2\right)d^2\right)+2cefh^2d-c^2f^2h^2\right)b^2+6ad^2f^2ghb-3a^2d^2f^2h^2\right)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2h}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$\downarrow$ 2101

$$\frac{-\frac{\left(-3a^2d^2f^2h^2+6abd^2f^2gh+b^2\left(-c^2f^2h^2+2cdefh^2-\left(d^2\left(e^2h^2+3f^2g^2\right)\right)\right)\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{d(be-af)(bg-ah)(3adfh-b(-cfh+deh+3dfg))\int -}{b}}{2h}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$\downarrow$ 183

$$\frac{-\frac{2(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\left(-3a^2d^2f^2h^2+6abd^2f^2gh+b^2\left(-c^2f^2h^2+2cdefh^2-\left(d^2\left(e^2h^2+3f^2g^2\right)\right)\right)\right)\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}}}{b\sqrt{c+dx}\sqrt{e+fx}}}{2h}$$

$$\frac{b\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h}$$

$\downarrow$ 188

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b}{h}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(5adfh-b(3dfg+deh+cfh))}{h\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+}}{}$$

$\downarrow$ 321

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b}{h}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(5adfh-b(3dfg+deh+cfh))}{h\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+}}{}$$

$$\Big\downarrow \; 412$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b}{h}+$$

$$-\frac{\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(5adfh-b(3dfg+deh+cfh))}{h\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+}}{}$$

input
```
Int[((a + b*x)^(3/2)*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*S
qrt[g + h*x]),x]
```

output
```
(b*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/h + ((d*(5*a*d
*f*h - b*(3*d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h
*x])/(h*Sqrt[c + d*x]) - (Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(5*a*d*f*h - b*(
3*d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f
*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(S
qrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*
g - c*h))])/(h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[
g + h*x]) + ((-2*d*(b*e - a*f)*Sqrt[b*g - a*h]*(3*a*d*f*h - b*(3*d*f*g + d
*e*h - c*f*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[
g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]
*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])
/(b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e
*h)*(a + b*x)))]) - (2*Sqrt[-(d*g) + c*h]*(6*a*b*d^2*f^2*g*h - 3*a^2*d^2*f
^2*h^2 + b^2*(2*c*d*e*f*h^2 - c^2*f^2*h^2 - d^2*(3*f^2*g^2 + e^2*h^2)))*(a
 + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g -
 a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b
*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*
Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*
Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x]))/(2*h))/(2*d*f*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1808 vs. 2(832) = 1664.

Time = 19.38 (sec) , antiderivative size = 1809, normalized size of antiderivative = 1.98

method	result	size
elliptic	Expression too large to display	1809
default	Expression too large to display	37880

input
```
int((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(b/h*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^
3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+
b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2*(a^2*
c*f+a^2*d*e-b/h*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(e/f-g/
h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a
/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2
)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*Ell
ipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/
(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(2*a^2*d*f+2*a*b*c*f+2*a*b*d*e-b/h*(a*c*f*h
+a*d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-
e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^
(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(
h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c/d*EllipticF(((c/d-e/f)*
(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/
f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/
2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))
)+(4*d*f*a*b+b^2*c*f+b^2*d*e-b/h*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*
b*g*d*f))*((x+g/h)*(x+a/b)*(x+e/f)+(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)
/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\,(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(c*f + d*e + 2*d*f*x)/(sqrt(c + d*x)*sqrt(e + f*
x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(2\,dfx+de+cf)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,(c\,f+d\,e+2\,d\,f\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/
2)*(c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(3/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/
2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\,(2dfx+cf+de)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

output
```
int((b*x+a)^(3/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

3.12 $\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 702

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}} - \frac{2\sqrt{be-af}(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} - \frac{2(bc-ad)\sqrt{be-af}\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{b\sqrt{de-cf}\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} - \frac{2f\sqrt{de-cf}(bg-ah)(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)},\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+d}}{\sqrt{de-cf}\sqrt{a+}}\right.\right.}{b\sqrt{be-af}h\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*b*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/h/(b*x+a)^(1/2)-2*(-a*f+b*e)
^(1/2)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(
1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/
2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-c*f+d*e)^(1/2)/h
/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2*(-a*d+b*c)
*(-a*f+b*e)^(1/2)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(
1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/
2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/(-c*f+d*e)^(1/2)
/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2*f*(-c*f+d*
e)^(1/2)*(-a*h+b*g)*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)
*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2
)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),(
(-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/(-a*f+b*e)^(1/2)/h/(
f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 36.19 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2\sqrt{a+bx}\sqrt{c+dx}\left(-\frac{dh(e+fx)(g+hx)}{c+dx}-\frac{(fg-eh)\sqrt{\frac{(-de+cf)(dg-ch)(e+fx)(g+hx)}{(fg-eh)^2(c+dx)^2}}\left((de-cf)hE\left(\arcsin\left(\sqrt{\frac{(-de+cf)(g+hx)}{(fg-eh)(c+dx)}}\right)|\frac{(bc}{(d}\right.\right.}{}\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x]*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*
x]*Sqrt[g + h*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*Sqrt[c + d*x]*(-((d*h*(e + f*x)*(g + h*x))/(c + d*x)) -
((f*g - e*h)*Sqrt[((-(d*e) + c*f)*(d*g - c*h)*(e + f*x)*(g + h*x))/((f*g -
 e*h)^2*(c + d*x)^2)]*((d*e - c*f)*h*EllipticE[ArcSin[Sqrt[((-(d*e) + c*f)
*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e
- c*f)*(b*g - a*h))] + (-(d*e*h) + c*f*h)*EllipticF[ArcSin[Sqrt[((-(d*e) +
 c*f)*(g + h*x))/((f*g - e*h)*(c + d*x))]], ((b*c - a*d)*(-(f*g) + e*h))/(
(d*e - c*f)*(b*g - a*h))] + f*(d*g - c*h)*EllipticPi[(d*(-(f*g) + e*h))/((
d*e - c*f)*h), ArcSin[Sqrt[((-(d*e) + c*f)*(g + h*x))/((f*g - e*h)*(c + d*
x))]], ((b*c - a*d)*(-(f*g) + e*h))/((d*e - c*f)*(b*g - a*h))]))/((d*e - c
*f)*Sqrt[((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])))/(h^2*Sqrt[e +
 f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {2098, 183, 194, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2098}$$

$$\frac{(be-af)(bg-ah)\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{d(bg-ah)\int \frac{\sqrt{e+fx}}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}dx}{h}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

$$\Big\downarrow \text{183}$$

$$\frac{(be-af)(bg-ah)\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{2d(e+fx)(bg-ah)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{g+hx}}{\sqrt{e+fx}}}{h\sqrt{a+bx}\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

$$\Big\downarrow \; 194$$

$$-\frac{2\sqrt{c+dx}(bg-ah)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}\int \frac{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}}{\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{h\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}-\frac{2d(e+fx)(bg-ah)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{g+hx}}{\sqrt{e+fx}}}{h\sqrt{a+bx}\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

$$\Big\downarrow \; 327$$

$$-\frac{2d(e+fx)(bg-ah)\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\int \frac{1}{\left(h-\frac{f(g+hx)}{e+fx}\right)\sqrt{1-\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\sqrt{1-\frac{(de-cf)(g+hx)}{(dg-ch)(e+fx)}}}d\frac{\sqrt{g+hx}}{\sqrt{e+fx}}}{h\sqrt{a+bx}\sqrt{c+dx}}-\frac{2\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{h\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

$$\Big\downarrow \; 412$$

$$-\frac{2d(e+fx)(bg-ah)^{3/2}\sqrt{\frac{(a+bx)(fg-eh)}{(e+fx)(bg-ah)}}\sqrt{\frac{(c+dx)(fg-eh)}{(e+fx)(dg-ch)}}\text{EllipticPi}\left(\frac{f(bg-ah)}{(be-af)h},\arcsin\left(\frac{\sqrt{be-af}\sqrt{g+hx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right),\frac{(de-cf)(bg-a}{(be-af)(dg-c}\right.}{h^2\sqrt{a+bx}\sqrt{c+dx}\sqrt{be-af}}$$

$$-\frac{2\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right)|-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{h\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}+\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{h\sqrt{a+bx}}$$

input
```
Int[(Sqrt[a + b*x]*(d*e + c*f + 2*d*f*x))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

output
```
(2*b*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(h*Sqrt[a + b*x]) - (2*Sqr
t[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/
((f*g - e*h)*(a + b*x)))]*EllipticE[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])
/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f
)*(b*g - a*h)))])/(h*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]
*Sqrt[g + h*x]) - (2*d*(b*g - a*h)^(3/2)*Sqrt[((f*g - e*h)*(a + b*x))/((b*
g - a*h)*(e + f*x))]*Sqrt[((f*g - e*h)*(c + d*x))/((d*g - c*h)*(e + f*x))]
*(e + f*x)*EllipticPi[(f*(b*g - a*h))/((b*e - a*f)*h), ArcSin[(Sqrt[b*e -
a*f]*Sqrt[g + h*x])/(Sqrt[b*g - a*h]*Sqrt[e + f*x])], ((d*e - c*f)*(b*g -
a*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[b*e - a*f]*h^2*Sqrt[a + b*x]*Sqrt[
c + d*x])
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2098
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[b*
B*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*Sqrt[a + b*x])), x] + (
-Simp[B*((b*g - a*h)/(2*f*h))   Int[Sqrt[e + f*x]/(Sqrt[a + b*x]*Sqrt[c + d
*x]*Sqrt[g + h*x]), x], x] + Simp[B*(b*e - a*f)*((b*g - a*h)/(2*d*f*h))   I
nt[Sqrt[c + d*x]/((a + b*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && EqQ[2*A*d*f - B*(d*e + c*f), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1559 vs. $2(637) = 1274$.

Time = 7.44 (sec) , antiderivative size = 1560, normalized size of antiderivative = 2.22

method	result	size
elliptic	Expression too large to display	1560
default	Expression too large to display	14663

input
```
int((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(a*c*f+a*d*e)*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+
g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2
)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*
b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-
e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))
+2*(2*a*d*f+b*c*f+b*d*e)*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^
(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*
(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x
+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)
/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/
h)*EllipticPi(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d
-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)))+2*b*d*f*((x+g/h)
*(x+a/b)*(x+e/f)+(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x
+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/
(-e/f+g/h)/(x+c/d))^(1/2)*((c/d*g/h-e/f*g/h+c*e/d/f+c^2/d^2)/(c/d-e/f)/(-c
/d+g/h)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)
*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(a/b-g/h)*EllipticE(((c/d-e/f)*(x+
g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))
^(1/2))/(-c/d+g/h)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/h/d/b/f/(c/d-e/f)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}(cf+de+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*(c*f + d*e + 2*d*f*x)/(sqrt(c + d*x)*sqrt(e + f*x)*
sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)
*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(2\,dfx+de+cf)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)
*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{a+b\,x}\,\left(c\,f+d\,e+2\,d\,f\,x\right)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/
2)*(c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(1/2)*(c*f + d*e + 2*d*f*x))/((e + f*x)^(1/2)*(g + h*x)^(1/
2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(de+cf+2dfx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{bx+a}\,(2dfx+cf+de)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

output
```
int((b*x+a)^(1/2)*(2*d*f*x+c*f+d*e)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

3.13 $\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 456

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\sqrt{de-cf}(bde+bcf-2adf)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{b\sqrt{be-af}(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}}$$
$$+\frac{4f\sqrt{de-cf}(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)},\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-c}{(be-a}\right.}{b\sqrt{be-af}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*(-c*f+d*e)^(1/2)*(-2*a*d*f+b*c*f+b*d*e)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/
(b*x+a))^(1/2)*(h*x+g)^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*
f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(
1/2))/b/(-a*f+b*e)^(1/2)/(-c*h+d*g)/(f*x+e)^(1/2)/(-(-a*d+b*c)*(h*x+g)/(-c
*h+d*g)/(b*x+a))^(1/2)+4*f*(-c*f+d*e)^(1/2)*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(
-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*El
lipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(
-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2)
)/b/(-a*f+b*e)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 25.29 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.59

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{2\sqrt{a+bx}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-bde(be-af)h\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right),\right.\right.}{}$$

input
```
Integrate[(d*e + c*f + 2*d*f*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]
*Sqrt[g + h*x]),x]
```

output
```
(2*Sqrt[a + b*x]*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(-(
b*d*e*(b*e - a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*
(g + h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a
 + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]) +
2*a*d*f*(b*e - a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))
]*(g + h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*
(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] +
 b*c*f*(-(b*e) + a*f)*h*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x
))]*(g + h*x)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h
)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]
 - 2*d*f*(b*g - a*h)*(f*g - e*h)*(a + b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))
/((f*g - e*h)*(a + b*x))]*Sqrt[((-(b*e) + a*f)*(b*g - a*h)*(e + f*x)*(g +
h*x))/((f*g - e*h)^2*(a + b*x)^2)]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a
*f)*h), ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]],
((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]))/(b*(b*e - a*f
)*h*(b*g - a*h)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]*Sqrt[((-(b*e) +
a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 449, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cf+de+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2101}$$

$$\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2df\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}$$

$$\Big\downarrow \text{183}$$

$$\frac{(-2adf+bcf+bde)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{4df(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{188}$$

$$\frac{2\sqrt{g+hx}(-2adf+bcf+bde)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{4df(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{321}$$

$$\frac{4df(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{g+hx}(-2adf+bcf+bde)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2\sqrt{g+hx}(-2adf+bcf+bde)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{4df(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\,\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{bh\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

input
```
Int[(d*e + c*f + 2*d*f*x)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]),x]
```

output
```
(2*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a
+ b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(S
qrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(
b*g - a*h)))])/(b*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b
*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (4*d*f*Sqrt[-(d*g) + c*h
]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b
*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))
/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c
*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])
/(b*Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(418) = 836$.

Time = 21.12 (sec) , antiderivative size = 855, normalized size of antiderivative = 1.88

method	result
elliptic	$\frac{\sqrt{(hx+g)(xd+c)(bx+a)(fx+e)}\left(\frac{2(cf+de)\left(\frac{e}{f}-\frac{g}{h}\right)\sqrt{\frac{\left(\frac{c}{d}-\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^{2}\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{a}{b}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{\left(\frac{c}{d}-\frac{e}{f}\right)\left(-\frac{c}{d}+\frac{g}{h}\right)\sqrt{hdbf\left(x+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}}\right.}{\cdots}$
default	Expression too large to display

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(c*f+d*e)*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)
/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((
-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*
(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+
g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+4*d
*f*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d
+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+
c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))
^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d
+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/
f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f
-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{cf+de+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Integral((c*f + d*e + 2*d*f*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*
sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{de+cf+2dfx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx+de+cf}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)
*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{de + cf + 2dfx}{\sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \int \frac{2\, dfx + de + cf}{\sqrt{bx + a}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\, dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)
*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{de + cf + 2dfx}{\sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \int \frac{c\, f + d\, e + 2\, d\, f\, x}{\sqrt{e + f\, x}\, \sqrt{g + h\, x}\, \sqrt{a + b\, x}\, \sqrt{c + d\, x}}\, dx$$

input
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)
*(c + d*x)^(1/2)),x)
```

output
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)
*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{de + cf + 2dfx}{\sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \int \frac{2dfx + cf + de}{\sqrt{bx + a}\, \sqrt{dx + c}\, \sqrt{fx + e}\, \sqrt{hx + g}} dx$$

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

output
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

3.14 $\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 465

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2(bde+bcf-2adf)(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{(bc-ad)\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} + \frac{2(2dfg-deh-cfh)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

output
```
-2*(-2*a*d*f+b*c*f+b*d*e)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-
c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e
)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))
/(-a*d+b*c)/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x
+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+2*(-c*f*h-d*e*h+2*d*f*g)*(f*x+
e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*
e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*
g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b
*g)/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 25.85 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.73

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left((bde+bcf\right.}{}$$

input
```
Integrate[(d*e + c*f + 2*d*f*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*
x]*Sqrt[g + h*x]),x]
```

output
```
(2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
f*x)^(3/2)*(g + h*x)^(3/2)*((b*d*e + b*c*f - 2*a*d*f)*(d*g - c*h)*Elliptic
E[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c
- a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))] - d*(d*e - c*f)*(b*g - a*h)
*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]
], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*
g - e*h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e +
f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cf+de+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 2102$$

$$\frac{\int \frac{-df(de+cf)ha^2-b\left(e(fg-eh)d^2+cf^2gd-c^2f^2h\right)a+2bdf(bde+bcf-2adf)hx^2+2b^2cdefg+(bde+bcf-2adf)(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 2105

$$\frac{\frac{\int -\frac{2bd^2f(be-af)(de-cf)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+(de-cf)(dg-ch)(-2adf+bcf+bde)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2d\sqrt{a+bx}\sqrt{e+f\cdots}}{\cdots}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 27

$$\frac{(de-cf)(dg-ch)(-2adf+bcf+bde)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx-d(be-af)(bg-ah)(de-cf)\int \frac{\cdots}{\sqrt{a+bx}\sqrt{c+dx}\cdots}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 188

$$\frac{(de-cf)(dg-ch)(-2adf+bcf+bde)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{2d\sqrt{g+hx}(be-af)(bg-ah)(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int -\cdots}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g\cdots}{(a\cdots}}}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 194

$$\frac{-\frac{2\sqrt{a+bx}(dg-ch)(-2adf+bcf+bde)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}-\frac{2d\sqrt{g+hx}(be-af)(bg-ah)(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\cdots}{\sqrt{c+dx}(fg-eh)\sqrt{\cdots}}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 321

$$-\frac{\frac{2\sqrt{a+bx}(dg-ch)(-2adf+bcf+bde)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}-\frac{2d\sqrt{g+hx}(be-af)\sqrt{bg-ah}(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\cdots}{\sqrt{c+dx}\sqrt{fg-eh}\cdots}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 327$$

$$-\frac{\frac{2d\sqrt{g+hx}(be-af)\sqrt{bg-ah}(de-cf)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}(-2a\cdots}{\cdots}}{(bc-ad)(be-af)(bg\cdots}$$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

input

```
Int[(d*e + c*f + 2*d*f*x)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

output

```
(-2*b*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*d*(b*d*e + b*c*
f - 2*a*d*f)*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2
*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*S
qrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(
Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a
*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[((d*e - c*f)*(a + b*x))
/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - (2*d*(b*e - a*f)*(d*e - c*f)*Sq
rt[b*g - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g
 + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*
Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/
(Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)
*(a + b*x)))]))/((b*c - a*d)*(b*e - a*f)*(b*g - a*h))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. $2(427) = 854$.

Time = 23.58 (sec) , antiderivative size = 2298, normalized size of antiderivative = 4.94

method	result	size
elliptic	Expression too large to display	2298
default	Expression too large to display	10269

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(-2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2
+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h
-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f
-b*d*e)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*
x+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(2/b*d*f-1/b*(a^2*d*f*h-a*b*c*f*h-
a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(2*a*d*f-b*c*f-b*d*e)/(
a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^
2*d*e*g-b^3*c*e*g)+(b*c*e*h+b*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*
d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-
b*c*f-b*d*e))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/
d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e
/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/
b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(
(-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*((a*d*f*h-b*c*f*h-b*d*
e*h-b*d*f*g)*(2*a*d*f-b*c*f-b*d*e)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*
b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)+(2*b*c*f*h+2*b*d*e*
h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*
b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(2*a*d*f-b*c*f-b*d*e))*(e/f-g/h)*((c/d-e/
f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/...
```

Fricas [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{2\,dfx+de+cf}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
integral((2*d*f*x + d*e + c*f)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)/(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c +
 2*a*b*d)*f)*h)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b
*d)*e + (2*a*b*c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a
^2*d)*f)*g + (a^2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*
f + (2*a*b*c + a^2*d)*e)*g)*x), x)
```

Sympy [F]

$$\int \frac{de + cf + 2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{cf + de + 2dfx}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Integral((c*f + d*e + 2*d*f*x)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*
x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{de + cf + 2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{de + cf + 2dfx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{de + cf + 2dfx}{(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{c\,f + d\,e + 2\,d\,f\,x}{\sqrt{e + f\,x}\,\sqrt{g + h\,x}\,(a + b\,x)^{3/2}\,\sqrt{c + d\,x}}\,dx$$

input
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)
*(c + d*x)^(1/2)),x)
```

output
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)
*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{3/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{2dfx + cf + de}{(bx + a)^{\frac{3}{2}}\,\sqrt{dx + c}\,\sqrt{fx + e}\,\sqrt{hx + g}}dx$$

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

output
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

3.15 $\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 49, antiderivative size = 790

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$
$$-\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$-\frac{4(dg-ch)\left(3a^3d^2f^2h-a^2bdf(dfg+4deh+4cfh)-b^3(d^2e^2g-cde(fg-eh)+c^2f(fg+eh))+ab^2(2\right.}{3(bc-ad)^2(be-af)^{3/2}\sqrt{de-cf}}$$
$$-\frac{2(3a^2dfh(2dfg-deh-cfh)+ab(3c^2f^2h^2-4cdfh(fg-eh)-d^2(2f^2g^2+4efgh-3e^2h^2))+b^2(d^2eg(}{3(bc-ad)(be-af)}$$

output
```
-2/3*b*(-2*a*d*f+b*c*f+b*d*e)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-
a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)-4/3*(-c*h+d*g)*(3*a^3*d^2*f^2
*h-a^2*b*d*f*(4*c*f*h+4*d*e*h+d*f*g)-b^3*(d^2*e^2*g-c*d*e*(-e*h+f*g)+c^2*f
*(e*h+f*g))+a*b^2*(2*c^2*f^2*h+d^2*e*(2*e*h+f*g)+c*d*f*(3*e*h+f*g)))*(f*x+
e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*
e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*
g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)/(-c*f+d*e)^
(1/2)/(-a*h+b*g)^2/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^
(1/2)-2/3*(3*a^2*d*f*h*(-c*f*h-d*e*h+2*d*f*g)+a*b*(3*c^2*f^2*h^2-4*c*d*f*h
*(-e*h+f*g)-d^2*(-3*e^2*h^2+4*e*f*g*h+2*f^2*g^2))+b^2*(d^2*e*g*(-e*h+f*g)-
c^2*f*h*(2*e*h+f*g)+c*d*(-2*e^2*h^2+4*e*f*g*h+f^2*g^2)))*(f*x+e)^(1/2)*(-(
-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*
x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e
)/(-c*h+d*g))^(1/2))/(-a*d+b*c)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-a*h+b*
g)^2/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10790 vs. $2(790) = 1580$.

Time = 37.92 (sec) , antiderivative size = 10790, normalized size of antiderivative = 13.66

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d*e + c*f + 2*d*f*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*
x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.67 (sec) , antiderivative size = 1077, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2102, 2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cf+de+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int \frac{2bdf(3bceg-a(deg+cfg+ceh))-(de+cf)\left(3dfha^2-3b(dfg+deh+cfh)a+2b^2(deg+cfg+ceh)\right)+(bde+bcf-2adf)(3adfh-b(dfg+deh+cfh))x}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}d}{3(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2102}$$

$$\int \frac{4bdfh\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\left(f(fg+eh)c^2-de(fg-eh)c+d^2e^2g\right)\right)x^2+2(adfh+b(dfg+deh+cfh}{}$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 2105}$$

$$2(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\left(f(fg+eh)\right.\right.$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$(be-af)(bg-ah)(de-cf)\left(3a^2d^2fh-abd(2cfh+3deh+dfg)+b^2\left(c^2fh+cdeh-cdfg+2d^2eg\right)\right)\int\frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+2(de-cf)(dg-ch)($$

$$\frac{2b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(-2adf+bcf+bde)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \ 188$$

$$2(de-cf)(dg-ch)\int\frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\left(f(fg+eh)\right.\right.$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \ 194$$

$$-\frac{4(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int\frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \ 321$$

$$-\frac{4(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int\frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+3eh)d+2c^2f^2h\right)a-b^3\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \ 327$$

$$-\frac{4\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)\left(3d^2f^2ha^3-bdf(dfg+4deh+4cfh)a^2+b^2\left(e(fg+2eh)d^2+cf(fg+\right.\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

input

```
Int[(d*e + c*f + 2*d*f*x)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

output

```
(-2*b*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
/(3*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) + ((-4*b*(3*a^3*d
^2*f^2*h - a^2*b*d*f*(d*f*g + 4*d*e*h + 4*c*f*h) - b^3*(d^2*e^2*g - c*d*e*
(f*g - e*h) + c^2*f*(f*g + e*h)) + a*b^2*(2*c^2*f^2*h + d^2*e*(f*g + 2*e*h
) + c*d*f*(f*g + 3*e*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c
 - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((4*d*(3*a^3*d^2*f^2*h -
a^2*b*d*f*(d*f*g + 4*d*e*h + 4*c*f*h) - b^3*(d^2*e^2*g - c*d*e*(f*g - e*h)
 + c^2*f*(f*g + e*h)) + a*b^2*(2*c^2*f^2*h + d^2*e*(f*g + 2*e*h) + c*d*f*(
f*g + 3*e*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] -
(4*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(3*a^3*d^2*f^2*h - a^2*b*d*f*(d*f*g + 4
*d*e*h + 4*c*f*h) - b^3*(d^2*e^2*g - c*d*e*(f*g - e*h) + c^2*f*(f*g + e*h)
) + a*b^2*(2*c^2*f^2*h + d^2*e*(f*g + 2*e*h) + c*d*f*(f*g + 3*e*h)))*Sqrt[
a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*Elliptic
E[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])],
 ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[((d*e - c*f)*
(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) + (2*(b*e - a*f)*(d*e -
 c*f)*Sqrt[b*g - a*h]*(3*a^2*d^2*f*h - a*b*d*(d*f*g + 3*d*e*h + 2*c*f*h) +
 b^2*(2*d^2*e*g - c*d*f*g + c*d*e*h + c^2*f*h))*Sqrt[((b*e - a*f)*(c + d*x
))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h
]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3570 vs. $2(738) = 1476$.

Time = 26.66 (sec) , antiderivative size = 3571, normalized size of antiderivative = 4.52

method	result	size
elliptic	Expression too large to display	3571
default	Expression too large to display	63284

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(-2/3/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g
+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(2*a*d*f-b*c*f-b*d*e)*(b
*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d
*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g
*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2-4/3*(b*d*f*h*x^3+b*c*f*h*x
^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f
*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g
-b^3*c*e*g)^2*(3*a^3*d^2*f^2*h-4*a^2*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d
^2*f^2*g+2*a*b^2*c^2*f^2*h+3*a*b^2*c*d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e
^2*h+a*b^2*d^2*e*f*g-b^3*c^2*e*f*h-b^3*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f
*g-b^3*d^2*e^2*g)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^
2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(1/3/b*(6*a^2*d^2*f^2*h-
5*a*b*c*d*f^2*h-5*a*b*d^2*e*f*h-2*a*b*d^2*f^2*g+b^2*c^2*f^2*h+2*b^2*c*d*e*
f*h+b^2*c*d*f^2*g+b^2*d^2*e^2*h+b^2*d^2*e*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*
b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-2/3/b*(
a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(3*
a^3*d^2*f^2*h-4*a^2*b*c*d*f^2*h-4*a^2*b*d^2*e*f*h-a^2*b*d^2*f^2*g+2*a*b^2*
c^2*f^2*h+3*a*b^2*c*d*e*f*h+a*b^2*c*d*f^2*g+2*a*b^2*d^2*e^2*h+a*b^2*d^2*e*
f*g-b^3*c^2*e*f*h-b^3*c^2*f^2*g-b^3*c*d*e^2*h+b^3*c*d*e*f*g-b^3*d^2*e^2...
```

Fricas [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{2\,dfx+de+cf}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
integral((2*d*f*x + d*e + c*f)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c +
 3*a*b^2*d)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3
*a*b^2*d)*e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e +
3*(a*b^2*c + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d
)*f)*h)*x^3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*
c*f + (3*a^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c +
 a^3*d)*e)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \text{Timed out}$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(
h*x+g)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{de + cf + 2dfx}{(a + bx)^{5/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\,dx = \int \frac{2\,dfx + de + cf}{(bx + a)^{\frac{5}{2}}\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\,dx$$

input
```
integrate((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((2*d*f*x + d*e + c*f)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{c\,f+d\,e+2\,d\,f\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)
*(c + d*x)^(1/2)),x)
```

output
```
int((c*f + d*e + 2*d*f*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)
*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{de+cf+2dfx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{2dfx+cf+de}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

output
```
int((2*d*f*x+c*f+d*e)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

3.16 $\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)} \, dx$

Optimal result

Integrand size = 39, antiderivative size = 216

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)} \, dx = \frac{(Ab^2-a(bB-aC))\log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{(c^2C-Bcd+Ad^2)\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)} + \frac{(Ce^2-Bef+Af^2)\log(e+fx)}{(be-af)(de-cf)(fg-eh)} - \frac{(Cg^2-Bgh+Ah^2)\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

output
```
(A*b^2-a*(B*b-C*a))*ln(b*x+a)/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)-(A*d^2-B*c*
d+C*c^2)*ln(d*x+c)/(-a*d+b*c)/(-c*f+d*e)/(-c*h+d*g)+(A*f^2-B*e*f+C*e^2)*ln
(f*x+e)/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)-(A*h^2-B*g*h+C*g^2)*ln(h*x+g)/(-a
*h+b*g)/(-c*h+d*g)/(-e*h+f*g)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = -\frac{(-Ab^2+abB-a^2C)\log(a+bx)}{(bc-ad)(be-af)(bg-ah)} - \frac{(c^2C-Bcd+Ad^2)\log(c+dx)}{(bc-ad)(-de+cf)(-dg+ch)} + \frac{(-Ce^2+Bef-Af^2)\log(e+fx)}{(be-af)(de-cf)(-fg+eh)} - \frac{(Cg^2-Bgh+Ah^2)\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
-(((-(A*b^2) + a*b*B - a^2*C)*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g
- a*h))) - ((c^2*C - B*c*d + A*d^2)*Log[c + d*x])/((b*c - a*d)*(-(d*e) + c
*f)*(-(d*g) + c*h)) + ((-(C*e^2) + B*e*f - A*f^2)*Log[e + f*x])/((b*e - a*
f)*(d*e - c*f)*(-(f*g) + e*h)) - ((C*g^2 - B*g*h + A*h^2)*Log[g + h*x])/((
b*g - a*h)*(d*g - c*h)*(f*g - e*h))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2109, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$$

$$\Big\downarrow \text{2109}$$

$$\int \left(\frac{b\left(Ab^2-a(bB-aC)\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)} - \frac{d\left(Ad^2-Bcd+c^2C\right)}{(c+dx)(bc-ad)(cf-de)(ch-dg)} - \frac{f\left(Af^2-Bef+Ce\right.}{(e+fx)(be-af)(de-cf)}\right.$$

$$\downarrow 2009$$

$$\frac{\log(a+bx)\left(Ab^2-a(bB-aC)\right)}{(bc-ad)(be-af)(bg-ah)}-\frac{\log(c+dx)\left(Ad^2-Bcd+c^2C\right)}{(bc-ad)(de-cf)(dg-ch)}+\frac{\log(e+fx)\left(Af^2-Bef+Ce^2\right)}{(be-af)(de-cf)(fg-eh)}-\frac{\log(g+hx)\left(Ah^2-Bgh+Cg^2\right)}{(bg-ah)(dg-ch)(fg-eh)}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

output
```
((A*b^2 - a*(b*B - a*C))*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h
)) - ((c^2*C - B*c*d + A*d^2)*Log[c + d*x])/((b*c - a*d)*(d*e - c*f)*(d*g
- c*h)) + ((C*e^2 - B*e*f + A*f^2)*Log[e + f*x])/((b*e - a*f)*(d*e - c*f)*
(f*g - e*h)) - ((C*g^2 - B*g*h + A*h^2)*Log[g + h*x])/((b*g - a*h)*(d*g -
c*h)*(f*g - e*h))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2109
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegran
d[Px*(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, m, n, p, q}, x] && PolyQ[Px, x] && IntegersQ[m, n]
```

Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00

method	result
default	$\frac{\left(d^2 A-cdB+C c^2\right)\ln(xd+c)}{(ch-dg)(cf-de)(ad-bc)}-\frac{\left(b^2 A-abB+a^2 C\right)\ln(bx+a)}{(ah-bg)(af-be)(ad-bc)}+\frac{\left(A h^2-Bgh+C g^2\right)\ln(hx+g)}{(ah-bg)(ch-dg)(eh-fg)}-\frac{\left(A f^2-Bef+C e^2\right)\ln(f}{(cf-de)(af-be)(eh-}$
norman	$\frac{\left(A h^2-Bgh+C g^2\right)\ln(hx+g)}{ace h^3-acfg h^2-adeg h^2+adf g^2 h-bceg h^2+bcf g^2 h+bde g^2 h-bdf g^3}+\frac{\left(d^2 A-cdB+C c^2\right)\ln(xd+c)}{(ch-dg)(cf-de)(ad-bc)}-\frac{\left(A f^2-Bef-\right.}{\left(ac f^2-adef-b\right.}$
risch	$-\frac{\ln(-fx-e)A f^2}{ace f^2 h-ac f^3 g-ad e^2 fh+ade f^2 g-bc e^2 fh+bce f^2 g+bd e^3 h-bd e^2 fg}+\frac{\ln(-fx-e)Bef}{ace f^2 h-ac f^3 g-ad e^2 fh+ade f^2 g-bc e^2 fh+}$
parallelrisch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
(A*d^2-B*c*d+C*c^2)/(c*h-d*g)/(c*f-d*e)/(a*d-b*c)*ln(d*x+c)-(A*b^2-B*a*b+C
*a^2)/(a*h-b*g)/(a*f-b*e)/(a*d-b*c)*ln(b*x+a)+(A*h^2-B*g*h+C*g^2)/(a*h-b*g
)/(c*h-d*g)/(e*h-f*g)*ln(h*x+g)-(A*f^2-B*e*f+C*e^2)/(c*f-d*e)/(a*f-b*e)/(e
*h-f*g)*ln(f*x+e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$$
$$= \frac{(Ca^2-Bab+Ab^2)\log(bx+a)}{((b^3c-ab^2d)e-(ab^2c-a^2bd)f)g-((ab^2c-a^2bd)e-(a^2bc-a^3d)f)h}$$
$$- \frac{(Cc^2-Bcd+Ad^2)\log(dx+c)}{((bcd^2-ad^3)e-(bc^2d-acd^2)f)g-((bc^2d-acd^2)e-(bc^3-ac^2d)f)h}$$
$$+ \frac{(Ce^2-Bef+Af^2)\log(fx+e)}{(bde^2f+acf^3-(bc+ad)ef^2)g-(bde^3+acef^2-(bc+ad)e^2f)h}$$
$$- \frac{(Cg^2-Bgh+Ah^2)\log(hx+g)}{bdfg^3-aceh^3-(bde+(bc+ad)f)g^2h+(acf+(bc+ad)e)gh^2}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxi
ma")
```

output
```
(C*a^2 - B*a*b + A*b^2)*log(b*x + a)/(((b^3*c - a*b^2*d)*e - (a*b^2*c - a^
2*b*d)*f)*g - ((a*b^2*c - a^2*b*d)*e - (a^2*b*c - a^3*d)*f)*h) - (C*c^2 -
B*c*d + A*d^2)*log(d*x + c)/(((b*c*d^2 - a*d^3)*e - (b*c^2*d - a*c*d^2)*f)
*g - ((b*c^2*d - a*c*d^2)*e - (b*c^3 - a*c^2*d)*f)*h) + (C*e^2 - B*e*f + A
*f^2)*log(f*x + e)/((b*d*e^2*f + a*c*f^3 - (b*c + a*d)*e*f^2)*g - (b*d*e^3
 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h) - (C*g^2 - B*g*h + A*h^2)*log(h*x + g
)/(b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*c +
 a*d)*e)*g*h^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$$
$$= \frac{(Ca^2b-Bab^2+Ab^3)\log(|bx+a|)}{b^4ceg-ab^3deg-ab^3cfg+a^2b^2dfg-ab^3ceh+a^2b^2deh+a^2b^2cfh-a^3bdfh}$$
$$-\frac{(Cc^2d-Bcd^2+Ad^3)\log(|dx+c|)}{bcd^3eg-ad^4eg-bc^2d^2fg+acd^3fg-bc^2d^2eh+acd^3eh+bc^3dfh-ac^2d^2fh}$$
$$+\frac{(Ce^2f-Bef^2+Af^3)\log(|fx+e|)}{bde^2f^2g-bcef^3g-adef^3g+acf^4g-bde^3fh+bce^2f^2h+ade^2f^2h-acef^3h}$$
$$-\frac{(Cg^2h-Bgh^2+Ah^3)\log(|hx+g|)}{bdfg^3h-bdeg^2h^2-bcfg^2h^2-adfg^2h^2+bcegh^3+adegh^3+acfgh^3-aceh^4}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac
")
```

output
```
(C*a^2*b - B*a*b^2 + A*b^3)*log(abs(b*x + a))/(b^4*c*e*g - a*b^3*d*e*g - a
*b^3*c*f*g + a^2*b^2*d*f*g - a*b^3*c*e*h + a^2*b^2*d*e*h + a^2*b^2*c*f*h -
 a^3*b*d*f*h) - (C*c^2*d - B*c*d^2 + A*d^3)*log(abs(d*x + c))/(b*c*d^3*e*g
 - a*d^4*e*g - b*c^2*d^2*f*g + a*c*d^3*f*g - b*c^2*d^2*e*h + a*c*d^3*e*h +
 b*c^3*d*f*h - a*c^2*d^2*f*h) + (C*e^2*f - B*e*f^2 + A*f^3)*log(abs(f*x +
e))/(b*d*e^2*f^2*g - b*c*e*f^3*g - a*d*e*f^3*g + a*c*f^4*g - b*d*e^3*f*h +
 b*c*e^2*f^2*h + a*d*e^2*f^2*h - a*c*e*f^3*h) - (C*g^2*h - B*g*h^2 + A*h^3
)*log(abs(h*x + g))/(b*d*f*g^3*h - b*d*e*g^2*h^2 - b*c*f*g^2*h^2 - a*d*f*g
^2*h^2 + b*c*e*g*h^3 + a*d*e*g*h^3 + a*c*f*g*h^3 - a*c*e*h^4)
```

Mupad [B] (verification not implemented)

Time = 10.59 (sec) , antiderivative size = 82899, normalized size of antiderivative = 383.79

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x)),x)
```

output
```
symsum(log(x*(B^3*b^2*d^2*f^2*h^2 - 2*A*B*C*b^2*d^2*f^2*h^2 + A*C^2*a*b*d^
2*f^2*h^2 - B^2*C*a*b*d^2*f^2*h^2 + A*C^2*b^2*c*d*f^2*h^2 - B^2*C*b^2*c*d*
f^2*h^2 + A*C^2*b^2*d^2*e*f*h^2 - B^2*C*b^2*d^2*e*f*h^2 + A*C^2*b^2*d^2*f^
2*g*h - B^2*C*b^2*d^2*f^2*g*h - C^3*a*b*c*d*e*f*h^2 - C^3*a*b*c*d*f^2*g*h
- C^3*a*b*d^2*e*f*g*h - C^3*b^2*c*d*e*f*g*h + B*C^2*a*b*c*d*f^2*h^2 + B*C^
2*a*b*d^2*e*f*h^2 + B*C^2*a*b*d^2*f^2*g*h + B*C^2*b^2*c*d*e*f*h^2 + B*C^2*
b^2*c*d*f^2*g*h + B*C^2*b^2*d^2*e*f*g*h) - root(4*a^5*b*c^3*d^3*e^2*f^4*g^
2*h^4*z^4 + 4*a^5*b*c^2*d^4*e^3*f^3*g^2*h^4*z^4 + 4*a^5*b*c^2*d^4*e^2*f^4*
g^3*h^3*z^4 + 4*a^4*b^2*c^4*d^2*e^3*f^3*g*h^5*z^4 + 4*a^4*b^2*c^4*d^2*e*f^
5*g^3*h^3*z^4 + 4*a^4*b^2*c^3*d^3*e^4*f^2*g*h^5*z^4 + 4*a^4*b^2*c^3*d^3*e*
f^5*g^4*h^2*z^4 + 4*a^4*b^2*c*d^5*e^4*f^2*g^3*h^3*z^4 + 4*a^4*b^2*c*d^5*e^
3*f^3*g^4*h^2*z^4 + 4*a^3*b^3*c^5*d*e^2*f^4*g^2*h^4*z^4 + 4*a^3*b^3*c^4*d^
2*e^4*f^2*g*h^5*z^4 + 4*a^3*b^3*c^4*d^2*e*f^5*g^4*h^2*z^4 + 4*a^3*b^3*c^2*
d^4*e^5*f*g^2*h^4*z^4 + 4*a^3*b^3*c^2*d^4*e^2*f^4*g^5*h*z^4 + 4*a^3*b^3*c*
d^5*e^4*f^2*g^4*h^2*z^4 + 4*a^2*b^4*c^5*d*e^3*f^3*g^2*h^4*z^4 + 4*a^2*b^4*
c^5*d*e^2*f^4*g^3*h^3*z^4 + 4*a^2*b^4*c^3*d^3*e^5*f*g^2*h^4*z^4 + 4*a^2*b^
4*c^3*d^3*e^2*f^4*g^5*h*z^4 + 4*a^2*b^4*c^2*d^4*e^5*f*g^3*h^3*z^4 + 4*a^2*
b^4*c^2*d^4*e^3*f^3*g^5*h*z^4 + 4*a*b^5*c^4*d^2*e^4*f^2*g^3*h^3*z^4 + 4*a*
b^5*c^4*d^2*e^3*f^3*g^4*h^2*z^4 + 4*a*b^5*c^3*d^3*e^4*f^2*g^4*h^2*z^4 + 4*
a^5*b*c^5*d*e*f^5*g*h^5*z^4 + 4*a^5*b*c*d^5*e^5*f*g*h^5*z^4 + 4*a^5*b*c...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1523, normalized size of antiderivative = 7.05

$$\int \frac{A+Bx+Cx^2}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

output

```
( - log(a + b*x)*a**2*c**3*e*f*h**2 + log(a + b*x)*a**2*c**3*f**2*g*h + lo
g(a + b*x)*a**2*c**2*d*e**2*h**2 - log(a + b*x)*a**2*c**2*d*f**2*g**2 - lo
g(a + b*x)*a**2*c*d**2*e**2*g*h + log(a + b*x)*a**2*c*d**2*e*f*g**2 + log(
c + d*x)*a**3*d**2*e*f*h**2 - log(c + d*x)*a**3*d**2*f**2*g*h - log(c + d*
x)*a**2*b*c*d*e*f*h**2 + log(c + d*x)*a**2*b*c*d*f**2*g*h - log(c + d*x)*a
**2*b*d**2*e**2*h**2 + log(c + d*x)*a**2*b*d**2*f**2*g**2 + log(c + d*x)*a
**2*c**3*e*f*h**2 - log(c + d*x)*a**2*c**3*f**2*g*h + log(c + d*x)*a*b**2*
c*d*e**2*h**2 - log(c + d*x)*a*b**2*c*d*f**2*g**2 + log(c + d*x)*a*b**2*d*
*2*e**2*g*h - log(c + d*x)*a*b**2*d**2*e*f*g**2 - log(c + d*x)*a*b*c**3*e*
*2*h**2 + log(c + d*x)*a*b*c**3*f**2*g**2 - log(c + d*x)*b**3*c*d*e**2*g*h
 + log(c + d*x)*b**3*c*d*e*f*g**2 + log(c + d*x)*b**2*c**3*e**2*g*h - log(
c + d*x)*b**2*c**3*e*f*g**2 - log(e + f*x)*a**3*c*d*f**2*h**2 + log(e + f*
x)*a**3*d**2*f**2*g*h + log(e + f*x)*a**2*b*c**2*f**2*h**2 + log(e + f*x)*
a**2*b*c*d*e*f*h**2 - log(e + f*x)*a**2*b*d**2*e*f*g*h - log(e + f*x)*a**2
*b*d**2*f**2*g**2 - log(e + f*x)*a**2*c**2*d*e**2*h**2 + log(e + f*x)*a**2
*c*d**2*e**2*g*h - log(e + f*x)*a*b**2*c**2*e*f*h**2 - log(e + f*x)*a*b**2
*c**2*f**2*g*h + log(e + f*x)*a*b**2*c*d*f**2*g**2 + log(e + f*x)*a*b**2*d
**2*e*f*g**2 + log(e + f*x)*a*b*c**3*e**2*h**2 - log(e + f*x)*a*b*c*d**2*e
**2*g**2 + log(e + f*x)*b**3*c**2*e*f*g*h - log(e + f*x)*b**3*c*d*e*f*g**2
 - log(e + f*x)*b**2*c**3*e**2*g*h + log(e + f*x)*b**2*c**2*d*e**2*g**2...
```

3.17 $\int \frac{(a+bx)(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 58, antiderivative size = 720

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2b^2(5bBdfh+2aCdfh-4bC(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$

$$+ \frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$- \frac{2b\sqrt{-de+cf}(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(dfg+deh+cfh))+b^2(10Bdfh(dfg+deh+cf}{15d^3f^5}$$

$$- \frac{2\sqrt{-de+cf}(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Cg+Bh)+5ab^2dfh(6Bdfgh-cCh(fg-eh)-Cdg(2f}{}$$

output
```
2/15*b^2*(5*b*B*d*f*h+2*a*C*d*f*h-4*b*C*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)
*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+2/5*b^2*C*(b*x+a)*(d*x+c)^(1/2)*(
f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-2/15*b*(c*f-d*e)^(1/2)*(15*a^2*C*d^2*f^2*
h^2-10*a*b*d*f*h*(3*B*d*f*h-C*(c*f*h+d*e*h+d*f*g))+b^2*(10*B*d*f*h*(c*f*h+
d*e*h+d*f*g)-C*(8*c^2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h
+8*f^2*g^2))))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f^(1/2
)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d^3/f^(
5/2)/h^3/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2/15*(c*f-d*e)^(1/2)*(
15*a^3*C*d^2*f^2*h^3-15*a^2*b*d^2*f^2*h^2*(B*h+C*g)+5*a*b^2*d*f*h*(6*B*d*f
*g*h-c*C*h*(-e*h+f*g)-C*d*g*(e*h+2*f*g))-b^3*(5*B*d*f*h*(c*h*(-e*h+f*g)+d*
g*(e*h+2*f*g))-C*(4*c^2*f*h^2*(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g
^2)+d^2*g*(4*e^2*h^2+3*e*f*g*h+8*f^2*g^2))))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*
(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/
2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d^3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.40 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(bd^2\sqrt{-c+\frac{de}{f}}(15a^2Cd^2f^2h^2+10abdfh(-3Bdfh+C(dfg+deh+cfh))-b^2(-10Bdfh(dfg+de\right.}{}$$

input
```
Integrate[((a + b*x)*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*(b*d^2*Sqrt[-c + (d*e)/f]*(15*a^2*C*d^2*f^2*h^2 + 10*a*b*d*f*h*(-3*B*d
*f*h + C*(d*f*g + d*e*h + c*f*h)) - b^2*(-10*B*d*f*h*(d*f*g + d*e*h + c*f*
h) + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h
 + 8*e^2*h^2))))*(e + f*x)*(g + h*x) + b^2*d^2*Sqrt[-c + (d*e)/f]*f*h*(c +
 d*x)*(e + f*x)*(g + h*x)*(-5*b*B*d*f*h - 5*a*C*d*f*h + b*C*(4*c*f*h + d*(
4*f*g + 4*e*h - 3*f*h*x))) + I*b*(d*e - c*f)*h*(15*a^2*C*d^2*f^2*h^2 + 10*
a*b*d*f*h*(-3*B*d*f*h + C*(d*f*g + d*e*h + c*f*h)) - b^2*(-10*B*d*f*h*(d*f
*g + d*e*h + c*f*h) + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^
2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c
 + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c +
(d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*d*h*(15*a^3*
C*d^2*f^3*h^2 - 15*a^2*b*d^2*f^2*(C*e + B*f)*h^2 - 5*a*b^2*d*f*h*(-6*B*d*e
*f*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) + b^3*(-5*B*d*f*h*(c*f*
(-(f*g) + e*h) + d*e*(f*g + 2*e*h)) + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*
f*(-4*f^2*g^2 + e*f*g*h + 3*e^2*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^
2*h^2))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h
*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]],
 (d*f*g - c*f*h)/(d*e*h - c*f*h)]))/(15*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqr
t[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2004, 2100, 2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(a^2(-C)+abB+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2004}$$

$$\int \frac{(a+bx)^2\left(\frac{abB-a^2C}{a}+bCx\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2100}$$

$$\frac{\int \frac{5(bB-aC)dfha^2+b^2(5bBdfh+2aCdfh-4bC(dfg+deh+cfh))x^2-b^2C(2bceg+a(deg+cfg+ceh))-b(5Cdfha^2-2b(5Bdfh-C(dfg+deh+cfh)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{5dfh}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int \frac{d\left(-15Cd^2f^2h^2a^3+15bBd^2f^2h^2a^2-5b^2Cdfh(deg+cfg+ceh)a-b^3\left(5Bdfh(deg+cfg+ceh)-C\left(4fh(fg+eh)c^2+2d\left(2f^2g^2+3efhg+2e^2h^2\right)c+4d^2eg(fg+eh)\right.\right.\right.}{2\sqrt{c+dx}\sqrt{e+}}}{3d^2f}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-15Cd^2f^2h^2a^3+15bBd^2f^2h^2a^2-5b^2Cdfh(deg+cfg+ceh)a-b^3\left(5Bdfh(deg+cfg+ceh)-C\left(4fh(fg+eh)c^2+2d\left(2f^2g^2+3efhg+2e^2h^2\right)c+4d^2eg(fg+eh)\right)\right)-}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{}}}{3dfh}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \text{176}$$

$$-\frac{b\left(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(cfh+deh+dfg))+b^2\left(10Bdfh(cfh+deh+dfg)-C\left(8c^2f^2h^2+7cdfh(eh+fg)+d^2\left(8e^2h^2+7efgh+8f^2g^2\right)\right)\right)\right)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e}}}{h}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \text{124}$$

$$-\frac{b\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\left(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(cfh+deh+dfg))+b^2\left(10Bdfh(cfh+deh+dfg)-C\left(8c^2f^2h^2+7cdfh(eh+fg)+d^2\left(8e^2h^2+7efgh+8f^2g^2\right.\right.\right.\right.}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \text{123}$$

$$-\frac{\frac{\left(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Bh+Cg)+5ab^2dfh(6Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))-\left(b^3\left(5Bdfh(ch(fg-eh)+dg(eh+2fg))-C\left(4c^2fh^2(fg-eh)+cdh\left(-\right.\right.\right.\right.}{h}}{}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 131

$$-\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\left(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Bh+Cg)+5ab^2dfh(6Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))-\left(b^3\left(5Bdfh(ch(fg-eh)+dg(eh+2fg))-C\left(4c^2fh^2(fg-\right.\right.\right.\right.}{h\sqrt{e+fx}}}{}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 131

$$-\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(15a^3Cd^2f^2h^3-15a^2bd^2f^2h^2(Bh+Cg)+5ab^2dfh(6Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))-\left(b^3\left(5Bdfh(ch(fg-eh)+dg(eh+2fg))-C\left(4\right.\right.\right.\right.}{h\sqrt{e+fx}\sqrt{g+hx}}}{}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$\downarrow$ 130

$$-\frac{\frac{2b\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)\left(15a^2Cd^2f^2h^2-10abdfh(3Bdfh-C(cfh+deh+dfg))+b^2\left(10Bdfh(cfh+deh+dfg)-C\left(8c^2f^2\right.\right.\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}}{}$$

$$\frac{2b^2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

input
```
Int[((a + b*x)*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]*Sqrt[
e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*b^2*C*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) +
((2*b^2*(5*b*B*d*f*h + 2*a*C*d*f*h - 4*b*C*(d*f*g + d*e*h + c*f*h))*Sqrt[c
 + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((-2*b*Sqrt[-(d*e) + c*f]
*(15*a^2*C*d^2*f^2*h^2 - 10*a*b*d*f*h*(3*B*d*f*h - C*(d*f*g + d*e*h + c*f*
h)) + b^2*(10*B*d*f*h*(d*f*g + d*e*h + c*f*h) - C*(8*c^2*f^2*h^2 + 7*c*d*f
*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))))*Sqrt[(d*(e + f
*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/S
qrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e
+ f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e) + c*f]*(15*a^3*C*
d^2*f^2*h^3 - 15*a^2*b*d^2*f^2*h^2*(C*g + B*h) + 5*a*b^2*d*f*h*(6*B*d*f*g*
h - c*C*h*(f*g - e*h) - C*d*g*(2*f*g + e*h)) - b^3*(5*B*d*f*h*(c*h*(f*g -
e*h) + d*g*(2*f*g + e*h)) - C*(4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2
+ e*f*g*h - 4*e^2*h^2) + d^2*g*(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt
[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcS
in[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]))/(3*d*f*h))/(5*d*f*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 6.54 (sec) , antiderivative size = 880, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2Cb^3x\sqrt{dfhx^3+cfhx^2+dehx^2+dfgx^2+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(Bb^3+aCb^2-\frac{2Cb^3(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{c}}{}\right.}{}$
default	Expression too large to display

input
```
int((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h
*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2/5*C*b^3/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*
x+d*e*g*x+c*e*g)^(1/2)+2/3*(B*b^3+a*C*b^2-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h
+2*d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2*(a^2*b*B-C*a^3-2/5*C*b^3/d/f/h*c*e*g-2/3*(B*b^3+a*C
*b^2-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g
+1/2*d*e*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2
)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*
h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/
d+e/f)/(-c/d+g/h))^(1/2))+2*(2*a*b^2*B-a^2*b*C-2/5*C*b^3/d/f/h*(3/2*c*e*h+
3/2*c*f*g+3/2*d*e*g)-2/3*(B*b^3+a*C*b^2-2/5*C*b^3/d/f/h*(2*c*f*h+2*d*e*h+2
*d*f*g))/d/f/h*(c*f*h+d*e*h+d*f*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((
x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d
*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*Ellipt
icE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF
(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1267, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Too large to display}$$

input
```
integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(3*(3*C*b^3*d^3*f^3*h^3*x - 4*C*b^3*d^3*f^3*g*h^2 - (4*C*b^3*d^3*e*f^
2 + (4*C*b^3*c*d^2 - 5*(C*a*b^2 + B*b^3)*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt
(f*x + e)*sqrt(h*x + g) - (8*C*b^3*d^3*f^3*g^3 + (3*C*b^3*d^3*e*f^2 + (3*C
*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*f^3)*g^2*h + (3*C*b^3*d^3*e^2*f + (
3*C*b^3*c*d^2 - 5*(C*a*b^2 + B*b^3)*d^3)*e*f^2 + (3*C*b^3*c^2*d - 5*(C*a*b
^2 + B*b^3)*c*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*f^3)*g*h^2 + (8*C*b^3*d^
3*e^3 + (3*C*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*e^2*f + (3*C*b^3*c^2*d
- 5*(C*a*b^2 + B*b^3)*c*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*e*f^2 + (8*C*b
^3*c^3 - 10*(C*a*b^2 + B*b^3)*c^2*d - 15*(C*a^2*b - 2*B*a*b^2)*c*d^2 + 45*
(C*a^3 - B*a^2*b)*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*
f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^
2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^
3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f -
3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d
*e + c*f)*h)/(d*f*h)) - 3*(8*C*b^3*d^3*f^3*g^2*h + (7*C*b^3*d^3*e*f^2 + (7
*C*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*f^3)*g*h^2 + (8*C*b^3*d^3*e^2*f +
 (7*C*b^3*c*d^2 - 10*(C*a*b^2 + B*b^3)*d^3)*e*f^2 + (8*C*b^3*c^2*d - 10*(C
*a*b^2 + B*b^3)*c*d^2 - 15*(C*a^2*b - 2*B*a*b^2)*d^3)*f^3)*h^3)*sqrt(d*f*h
)*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 -
c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*...
```

Sympy [F]

$$\int \frac{(a+bx)(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)^2(Bb-Ca+Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)*(C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2)/(f*x+
e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)**2*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqr
t(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*(b*x + a)/(sqrt(d*x + c)*s
qrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)\left(abB-a^2C+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*(b*x + a)/(sqrt(d*x + c)*s
qrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+b\,x)\,\left(-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x\right)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/2)*(g
+ h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/2)*(g
+ h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)\,(C\,b^2x^2+B\,b^2x+Bab-C\,a^2)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h
*x+g)^(1/2),x)
```

output
```
int((b*x+a)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h
*x+g)^(1/2),x)
```

3.18 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 53, antiderivative size = 410

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$
$$+\frac{2b^2\sqrt{-de+cf}(3Bdfh-2C(dfg+deh+cfh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{2\sqrt{-de+cf}(3abBdfh^2-3a^2Cdfh^2-b^2(3Bdfgh-cCh(fg-eh)-Cdg(2fg+eh)))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(\cdots}{d\cdots}}}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2/3*b^2*C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h+2/3*b^2*(c*f-d*e
)^(1/2)*(3*B*d*f*h-2*C*(c*f*h+d*e*h+d*f*g))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(
h*x+g)^(1/2)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h
/f/(-c*h+d*g))^(1/2))/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))
^(1/2)+2/3*(c*f-d*e)^(1/2)*(3*a*b*B*d*f*h^2-3*a^2*C*d*f*h^2-b^2*(3*B*d*f*g
*h-c*C*h*(-e*h+f*g)-C*d*g*(e*h+2*f*g)))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h
*x+g)/(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((
-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(h*x+g)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.07 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.08

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\sqrt{c+dx}\left(2b^2Cd^2fh(e+fx)(g+hx)+\frac{2b^2d^2(3Bdfh-2C(dfg+deh+cfh))(e+fx)(g+hx)}{c+dx}+2ib^2\sqrt{-c+\frac{de}{f}}fh(3Bdf\right.}{}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*
x]*Sqrt[g + h*x]),x]
```

output
```
(Sqrt[c + d*x]*(2*b^2*C*d^2*f*h*(e + f*x)*(g + h*x) + (2*b^2*d^2*(3*B*d*f*
h - 2*C*(d*f*g + d*e*h + c*f*h))*(e + f*x)*(g + h*x))/(c + d*x) + (2*I)*b^
2*Sqrt[-c + (d*e)/f]*f*h*(3*B*d*f*h - 2*C*(d*f*g + d*e*h + c*f*h))*Sqrt[c
+ d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]
*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d
*e*h - c*f*h)] + ((2*I)*d*h*(3*a*b*B*d*f^2*h - 3*a^2*C*d*f^2*h + b^2*(-3*B
*d*e*f*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)))*Sqrt[c + d*x]*Sqrt
[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[
I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*
h)])/Sqrt[-c + (d*e)/f]))/(3*d^3*f^2*h^2*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.151$, Rules used = {2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \; 2118$$

$$\frac{2\int \frac{d\left(-3Cdfha^2+3bBdfha-b^2C(deg+cfg+ceh)+b^2(3Bdfh-2C(dfg+deh+cfh))x\right)}{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3d^2fh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-3Cdfha^2+3bBdfha-b^2C(deg+cfg+ceh)+b^2(3Bdfh-2C(dfg+deh+cfh))x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 176$$

$$\frac{\frac{\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}+\frac{b^2(3Bdfh-2C(cfh+deh+dfg))\int \frac{\sqrt{g+}}{\sqrt{c+dx}\sqrt{}}}{h}}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 124$$

$$\frac{\frac{\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}+\frac{b^2\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}(3Bdfh-2C(cfh+de}{h\sqrt{e+fx}\sqrt{\frac{d}{}}}}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 123$$

$$\frac{\frac{\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}+\frac{2b^2\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(3Bdfh-2C}{d\sqrt{f}h\sqrt{}}}{3dfh}+\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 131$$

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}+\frac{2b^2\sqrt{g+hx}\sqrt{cf-de}\sqrt{}}{}}{3dfh}$$

$$\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 131

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2}{}}{3dfh}$$

$$\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$\downarrow$ 130

$$\frac{\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)\left(-3a^2Cdfh^2+3abBdfh^2-\left(b^2(3Bdfgh-cCh(fg-eh)-Cdg(eh+2fg))\right)\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}}{3dfh}$$

$$\frac{2b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

output
```
(2*b^2*C*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((2*b^2*Sq
rt[-(d*e) + c*f]*(3*B*d*f*h - 2*C*(d*f*g + d*e*h + c*f*h))*Sqrt[(d*(e + f*
x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e +
 f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*a*b*B*d*
f*h^2 - 3*a^2*C*d*f*h^2 - b^2*(3*B*d*f*g*h - c*C*h*(f*g - e*h) - C*d*g*(2*
f*g + e*h)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h
)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c
*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]))/(3*d*f
*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 5.88 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2b^{2}C\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(abB-a^{2}C-\frac{2b^{2}C\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{c}{d}\right.}{\sqrt{dfh\,x^{3}+cfh\,x^{2}+}}\right.}{}$
default	Expression too large to display

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2/3*b^2*C/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2*(a*b*B-a^2*C-2/3*b^2*C/d/f/h*(1/2*c*e*h+1/2*c*f*g+1
/2*d*e*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*
((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*
x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+
e/f)/(-c/d+g/h))^(1/2))+2*(B*b^2-2/3*b^2*C/d/f/h*(c*f*h+d*e*h+d*f*g))*(c/d
-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+
e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g
*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f
)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(
-c/d+g/h))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. 2(363) = 726.

Time = 0.12 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.10

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*C*b^2*d^2*f^2*h^2 + (2*C*
b^2*d^2*f^2*g^2 + (C*b^2*d^2*e*f + (C*b^2*c*d - 3*B*b^2*d^2)*f^2)*g*h + (2
*C*b^2*d^2*e^2 + (C*b^2*c*d - 3*B*b^2*d^2)*e*f + (2*C*b^2*c^2 - 3*B*b^2*c*
d - 9*(C*a^2 - B*a*b)*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(
d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)
/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3
*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*
f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g
+ (d*e + c*f)*h)/(d*f*h)) + 3*(2*C*b^2*d^2*f^2*g*h + (2*C*b^2*d^2*e*f + (2
*C*b^2*c*d - 3*B*b^2*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f
^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2
*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3
*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3
*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2
*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d
^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d
^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f -
 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (
d*e + c*f)*h)/(d*f*h))))/(d^3*f^3*h^3)
```

Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+bx)\,(Bb - Ca + Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2)/(f*x+e)**(1/2
)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
 + h*x)), x)
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x
+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(d*x + c)*sqrt(f*x +
e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \text{too large to display}$$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1
/2),x)
```

output
```
(2*sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*b**3 - 3*int((sqrt(g + h*x)*s
qrt(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f*
*2*g*h*x + c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*
g**2 + 3*c*d*e*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f**
2*g*h*x**2 + c*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 +
d**2*e*f*g**2*x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g**
2*x**2 + d**2*f**2*g*h*x**3),x)*b**3*c*d*f**2*h**2 - 3*int((sqrt(g + h*x)*
sqrt(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f
**2*g*h*x + c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f
*g**2 + 3*c*d*e*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f*
*2*g*h*x**2 + c*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 +
 d**2*e*f*g**2*x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g*
*2*x**2 + d**2*f**2*g*h*x**3),x)*b**3*d**2*e*f*h**2 - 3*int((sqrt(g + h*x)
*sqrt(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*
f**2*g*h*x + c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*
f*g**2 + 3*c*d*e*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f
**2*g*h*x**2 + c*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2
+ d**2*e*f*g**2*x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g
**2*x**2 + d**2*f**2*g*h*x**3),x)*b**3*d**2*f**2*g*h + 2*int((sqrt(g + h*x
)*sqrt(e + f*x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c...
```

3.19 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 291

$$\begin{aligned}&\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx\\&=\frac{2bC\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}\\&\quad-\frac{2\sqrt{-de+cf}(bCg-bBh+aCh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}\end{aligned}$$

output

```
2*b*C*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE
(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/
d/f^(1/2)/h/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*(c*f-d*e)^(1/2)*(
-B*b*h+C*a*h+C*b*g)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1
/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))/d/f^(1/2)/h/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.12

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2\left(bCd^2\sqrt{-c+\frac{de}{f}}(e+fx)(g+hx) + ibC(de-cf)h(c+dx)^{3/2}\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{\frac{d(g+hx)}{h(c+dx)}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right.\right.\right.}{d^2\sqrt{-c+\frac{de}{f}}fh\sqrt{}}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(b*C*d^2*Sqrt[-c + (d*e)/f]*(e + f*x)*(g + h*x) + I*b*C*(d*e - c*f)*h*(
c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
+ d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c
*f*h)/(d*e*h - c*f*h)] - I*d*(b*C*e - b*B*f + a*C*f)*h*(c + d*x)^(3/2)*Sqr
t[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF
[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f
*h)]))/(d^2*Sqrt[-c + (d*e)/f]*f*h*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.117$, Rules used = {2004, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C) + abB + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{ 2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{176}$$

$$\frac{bC\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}-\frac{(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{124}$$

$$\frac{bC\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{123}$$

$$\frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}$$

$$\Big\downarrow \text{131}$$

$$\frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}$$

$$\Big\downarrow \text{131}$$

$$\frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}-\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(aCh-bBh+bCg)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \text{130}$$

$$\frac{2bC\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}(aCh-bBh+bCg)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*b*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*El
lipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h
)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c
*h)]) - (2*Sqrt[-(d*e) + c*f]*(b*C*g - b*B*h + a*C*h)*Sqrt[(d*(e + f*x))/(
d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt
[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[
f]*h*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Maple [A] (verified)

Time = 5.40 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2(Bb-Ca)\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\frac{2Cb\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\cdots}{-\frac{c}{d}+\cdots}}\cdots}{\cdots}\cdots\right.}{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{xd+c}\cdots}$
default	$\frac{2\left(B\,\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)bcdfh-B\,\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)b\,d^2eh-C\,\mathrm{EllipticF}\left(\sqrt{\frac{f(xd+c)}{cf-de}},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right.\cdots\right.}{\cdots}$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h
*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2*(B*b-C*a)*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2
)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*
h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/
d+e/f)/(-c/d+g/h))^(1/2))+2*C*b*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/
h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h
*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(
((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x
+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(257) = 514$.

Time = 0.11 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.34

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$
$$-\frac{2\left(3\sqrt{dfh}Cbdfh\,\mathrm{weierstrassZeta}\left(\frac{4\left(d^2f^2g^2-\left(d^2ef+cdf^2\right)gh+\left(d^2e^2-cdef+c^2f^2\right)h^2\right)}{3\,d^2f^2h^2},-\frac{4\left(2\,d^3f^3g^3-3\left(d^3ef^2+cd^2f^3\right)g^2h\cdots\right.}{\cdots}\right.\cdots\right.}{\cdots}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(d*f*h)*C*b*d*f*h*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f
+ c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(
2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e
*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c
^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*
f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27
*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2
*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2
*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f
*h))) + (C*b*d*f*g + (C*b*d*e + (C*b*c + 3*(C*a - B*b)*d)*f)*h)*sqrt(d*f*h
)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^
2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e
*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2
+ (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^
3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)))/(d^2*f^2*h^2)
```

Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bb - Ca + Cbx}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)/(d*x+c)**(1/2)/(f*x+
e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)),
x)
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)*sqrt(d*x + c)*s
qrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)*sqrt(d*x + c)*s
qrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}\,x}{dfh\,x^3 + cfh\,x^2 + deh\,x^2 + dfg\,x^2 + cehx + cfgx + degx + ceg}dx\right) bc$$
$$- \left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}}{dfh\,x^3 + cfh\,x^2 + deh\,x^2 + dfg\,x^2 + cehx + cfgx + degx + ceg}dx\right) ac$$
$$+ \left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}}{dfh\,x^3 + cfh\,x^2 + deh\,x^2 + dfg\,x^2 + cehx + cfgx + degx + ceg}dx\right) b^2$$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h
*x+g)^(1/2),x)
```

output
```
int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*x)/(c*e*g + c*e*h*x + c*f*g
*x + c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*b*c -
 int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x))/(c*e*g + c*e*h*x + c*f*g*
x + c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*a*c +
int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x))/(c*e*g + c*e*h*x + c*f*g*x
 + c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*b**2
```

3.20 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 309

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}} - \frac{2(bB-2aC)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*C*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1
/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+
d*g))^(1/2))/d/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(B*b-2*C*a)*(c*f-d*e)
^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticP
i(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+
d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.14 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.81

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\frac{2i\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{h(c+dx)}}\left(-\left((bcC-bBd+aCd)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{de}{f}}}{\sqrt{c+dx}}\right),\frac{dfg-cfh}{deh-cfh}\right)\right)+(-bB+2aC\cdots}{(-bc+ad)\sqrt{-c+\frac{de}{f}}\,f\sqrt{\frac{d(e+fx)}{f(c+dx)}}\sqrt{g+hx}}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^2*Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
((2*I)*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*(-((b*c*C - b*B*d +
 a*C*d)*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*
f*h)/(d*e*h - c*f*h)]) + (-(b*B) + 2*a*C)*d*EllipticPi[-((b*c*f - a*d*f)/(
b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f
*h)/(d*e*h - c*f*h)]))/((-(b*c) + a*d)*Sqrt[-c + (d*e)/f]*f*Sqrt[(d*(e + f
*x))/(f*(c + d*x))]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2004, 2110, 27, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \; 2110$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\int \frac{C}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 27$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+C\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 131$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{C\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{\sqrt{e+fx}}$$

$$\Big\downarrow \; 131$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{C\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \; 130$$

$$(bB-2aC)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

$$\Big\downarrow \; 187$$

$$\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-2(bB-2aC)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}$$

$$\Big\downarrow \; 413$$

$$\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bB-2aC)\sqrt{\frac{f(c+dx)}{de-cf}+1}\int\frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}$$

$$\Big\downarrow \text{413}$$

$$\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bB-2aC)\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int\frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2C\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2(bB-2aC)\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\operatorname{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[
e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))
/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]]
, ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]
) - (2*(b*B - 2*a*C)*Sqrt[-(d*e) + c*f]*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)
]*Sqrt[1 + (h*(c + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c -
 a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f
)*h)/(f*(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x
))/d]*Sqrt[g - (c*h)/d + (h*(c + d*x))/d])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 7.95 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.54

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2C\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x^{2}+dfg\,x^{2}+cehx+cfgx+degx+ceg}}+\frac{2(Bb-2Ca)\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\cdots}{-\frac{c}{d}+\cdots}}\cdots}{b\sqrt{dfh\,x^{3}+cfh\,x^{2}+deh\,x\cdots}}\cdots\right.}{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{xd+c}}$
default	$2\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{xd+c}\,\sqrt{\frac{f(xd+c)}{cf-de}}\,\sqrt{-\frac{(hx+g)d}{ch-dg}}\,\sqrt{-\frac{d(fx+e)}{cf-de}}\left(B\operatorname{EllipticPi}\left(\sqrt{\frac{f(xd+c)}{cf-de}},-\frac{b(cf-de)}{f(ad-bc)},\sqrt{\frac{(cf-de)h}{f(ch-dg)}}\right)bcdf-B\operatorname{EllipticI}\cdots\right.$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/
(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2*C*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/
f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(
-c/d+g/h))^(1/2))+2*(B*b-2*C*a)/b*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+
g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e
*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(-c/d+a/b)*EllipticP
i(((x+c/d)/(c/d-e/f))^(1/2),(-c/d+e/f)/(-c/d+a/b),((-c/d+e/f)/(-c/d+g/h))^
(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \text{Timed out}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \text{Timed out}$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**2/(d*x+c)**(1/2)/(f
*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a + bx)^2\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx + a)^2\sqrt{dx + c}\sqrt{fx + e}\sqrt{hx + g}}\, dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^2*sqrt(d*x + c)
*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^2*sqrt(d*x + c)
*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)^2\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^2*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^2*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,b^2x^2 + B\,b^2x + Bab - C\,a^2}{(bx+a)^2\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/
(h*x+g)^(1/2),x)
```

output
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/
(h*x+g)^(1/2),x)
```

3.21 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 60, antiderivative size = 680

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{b(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$-\frac{(bB-2aC)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{\sqrt{-de+cf}(4a^3Cdfh+2ab^2B(dfg+deh+cfh)-b^3(Bdeg-c(2Ceg-Bfg-Beh))-a^2b(3Bdfh}{(bc-ad)^2\sqrt{f}(be-af)(bg-}$$

output
```
-b^2*(B*b-2*C*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*
f+b*e)/(-a*h+b*g)/(b*x+a)+b*(B*b-2*C*a)*f^(1/2)*(c*f-d*e)^(1/2)*(d*(f*x+e)
/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)
^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)
/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-(B*b-2*C*a)*f^(1/2)*(c*f-d*e)^
(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticF(
f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(
-a*d+b*c)/(-a*f+b*e)/(f*x+e)^(1/2)/(h*x+g)^(1/2)-(c*f-d*e)^(1/2)*(4*a^3*C*
d*f*h+2*a*b^2*B*(c*f*h+d*e*h+d*f*g)-b^3*(B*d*e*g-c*(-B*e*h-B*f*g+2*C*e*g))
-a^2*b*(3*B*d*f*h+2*C*(c*f*h+d*e*h+d*f*g)))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(
d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/
2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c
)^2/f^(1/2)/(-a*f+b*e)/(-a*h+b*g)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.85 (sec) , antiderivative size = 3419, normalized size of antiderivative = 5.03

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^3*Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
-((b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*
d)*(b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((c + d*x)^(3/2)*(b^3*B*c*Sqrt[-c
 + (d*e)/f]*f*h - 2*a*b^2*c*C*Sqrt[-c + (d*e)/f]*f*h - a*b^2*B*d*Sqrt[-c +
 (d*e)/f]*f*h + 2*a^2*b*C*d*Sqrt[-c + (d*e)/f]*f*h + (b^3*B*c*d^2*e*Sqrt[-
c + (d*e)/f]*g)/(c + d*x)^2 - (2*a*b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c
+ d*x)^2 - (a*b^2*B*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 + (2*a^2*b*C*d
^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (b^3*B*c^2*d*Sqrt[-c + (d*e)/f]*f
*g)/(c + d*x)^2 + (2*a*b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (
a*b^2*B*c*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 - (2*a^2*b*c*C*d^2*Sqrt[
-c + (d*e)/f]*f*g)/(c + d*x)^2 - (b^3*B*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c +
 d*x)^2 + (2*a*b^2*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*b^2*B*
c*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 - (2*a^2*b*c*C*d^2*e*Sqrt[-c + (
d*e)/f]*h)/(c + d*x)^2 + (b^3*B*c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 -
(2*a*b^2*c^3*C*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (a*b^2*B*c^2*d*Sqrt[-
c + (d*e)/f]*f*h)/(c + d*x)^2 + (2*a^2*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h)/(
c + d*x)^2 + (b^3*B*c*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (2*a*b^2*c*C*d
*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) - (a*b^2*B*d^2*Sqrt[-c + (d*e)/f]*f*g)/
(c + d*x) + (2*a^2*b*C*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (b^3*B*c*d*
e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (2*a*b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h)
/(c + d*x) - (a*b^2*B*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) + (2*a^2*b*...
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2004, 2102, 2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2102}$$

$$\frac{\int \frac{2Cdfha^3-2b(Bdfh+C(dfg+deh+cfh))a^2+2b^2B(dfg+deh+cfh)a+2b(bB-2aC)dfhxa+b^2(bB-2aC)dfhx^2-b^3(Bdeg-c(2Ceg-Bfg-Beh)}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2(bc-ad)(be-af)(bg-ah)}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 2110

$$\frac{\int \frac{-2Cdfha^2+bBdfha+(b^2Bdfh-2abCdfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh\right.}{2(bc-ad)(be-af)(bg-ah)}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 176

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2C\right.}{2(bc-ad)}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 124

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2C\right.}{2(}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 123

$$\frac{\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2C\right.}{}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 131

$$\big(4a^3Cdfh - a^2b(3Bdfh + 2C(cfh + deh + dfg)) + 2ab^2B(cfh + deh + dfg) - b^3(Bdeg - c(-Beh - Bfg + 2C$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 131$$

$$\big(4a^3Cdfh - a^2b(3Bdfh + 2C(cfh + deh + dfg)) + 2ab^2B(cfh + deh + dfg) - b^3(Bdeg - c(-Beh - Bfg + 2C$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 130$$

$$\big(4a^3Cdfh - a^2b(3Bdfh + 2C(cfh + deh + dfg)) + 2ab^2B(cfh + deh + dfg) - b^3(Bdeg - c(-Beh - Bfg + 2C$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 187$$

$$-2\big(4a^3Cdfh - a^2b(3Bdfh + 2C(cfh + deh + dfg)) + 2ab^2B(cfh + deh + dfg) - b^3(Bdeg - c(-Beh - Bfg +$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 413$$

$$-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\big(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ceg))\big)\int \frac{}{(bc-ad-b(c+dx))\sqrt{}}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 413$$

$$-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ceg))\right)\int \frac{}{(bc-}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\left(4a^3Cdfh-a^2b(3Bdfh+2C(cfh+deh+dfg))+2ab^2B(cfh+deh+dfg)-b^3(Bdeg-c(-Beh-Bfg+2Ceg))\right)}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\frac{b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

input

```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[
e + f*x]*Sqrt[g + h*x]),x]
```

output

```
-((b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*
d)*(b*e - a*f)*(b*g - a*h)*(a + b*x))) + ((2*b*(b*B - 2*a*C)*Sqrt[f]*Sqrt[
-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcS
in[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
c*h))])/(Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*(b*B - 2*a*C)
*Sqrt[f]*Sqrt[-(d*e) + c*f]*(b*g - a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sq
rt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqr
t[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(Sqrt[e + f*x]*Sqrt[g
+ h*x]) - (2*Sqrt[-(d*e) + c*f]*(4*a^3*C*d*f*h + 2*a*b^2*B*(d*f*g + d*e*h
+ c*f*h) - b^3*(B*d*e*g - c*(2*C*e*g - B*f*g - B*e*h)) - a^2*b*(3*B*d*f*h
+ 2*C*(d*f*g + d*e*h + c*f*h)))*Sqrt[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1
 + (h*(c + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((b*c - a*d)*f)
), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*
(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sq
rt[g - (c*h)/d + (h*(c + d*x))/d]))/(2*(b*c - a*d)*(b*e - a*f)*(b*g - a*h)
)
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !L
tQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 20.73 (sec) , antiderivative size = 1211, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1211
default	Expression too large to display	13405

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2)/
(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(b^2/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*
g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*
x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)-a*d*f*h*(B*b-2*C*a)/(a^3*
d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*
e*g-b^3*c*e*g)*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1
/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*
e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-
c/d+e/f)/(-c/d+g/h))^(1/2))-b*d*f*h*(B*b-2*C*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2
*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(c/d-e
/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/
f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x
+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/
(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c
/d+g/h))^(1/2)))+(3*B*a^2*b*d*f*h-2*B*a*b^2*c*f*h-2*B*a*b^2*d*e*h-2*B*a*b^
2*d*f*g+B*b^3*c*e*h+B*b^3*c*f*g+B*b^3*d*e*g-4*C*a^3*d*f*h+2*C*a^2*b*c*f*h+
2*C*a^2*b*d*e*h+2*C*a^2*b*d*f*g-2*C*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*
b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(c/d-
e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e
/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**3/(d*x+c)**(1/2)/(f
*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^3*sqrt(d*x + c)
*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^3\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^
(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^3*sqrt(d*x + c)
*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{-C\,a^2 + B\,a\,b + C\,b^2\,x^2 + B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)^3\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^3*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^3*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,b^2x^2 + B\,b^2x + Bab - C\,a^2}{(bx+a)^3\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2)/
(h*x+g)^(1/2),x)
```

output
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2)/
(h*x+g)^(1/2),x)
```

3.22 $\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 988

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{b^2(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{4d^2f^2h^2\sqrt{a+bx}}$$
$$+ \frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{b\sqrt{be-af}(dg-ch)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\Big(\arcsin\Big(}{4d^2f^2\sqrt{de-cf}h^2\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$- \frac{(bc-ad)\sqrt{be-af}(4bBdfh-aCdfh-bC(dfg+3deh+3cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\text{EllipticF}\Big(}{4d^2f^2\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$- \frac{\sqrt{de-cf}((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3bC(dfg+deh+cfh))+4dfh(2a^2Cdf}{}$$

output
```
1/4*b^2*(4*b*B*d*f*h+a*C*d*f*h-3*b*C*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)*(f
*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2/(b*x+a)^(1/2)+1/2*b^2*C*(b*x+a)^(1/2
)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-1/4*b*(-a*f+b*e)^(1/2)*(
-c*h+d*g)*(4*b*B*d*f*h+a*C*d*f*h-3*b*C*(c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)*
(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*
(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+
b*e)/(-c*h+d*g))^(1/2))/d^2/f^2/(-c*f+d*e)^(1/2)/h^2/(-(-a*d+b*c)*(f*x+e)/
(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/4*(-a*d+b*c)*(-a*f+b*e)^(1/2)*(4
*b*B*d*f*h-a*C*d*f*h-b*C*(3*c*f*h+3*d*e*h+d*f*g))*(f*x+e)^(1/2)*(-(-a*d+b*
c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1
/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h
+d*g))^(1/2))/d^2/f^2/(-c*f+d*e)^(1/2)/h/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(
b*x+a))^(1/2)/(h*x+g)^(1/2)-1/4*(-c*f+d*e)^(1/2)*((a*d*f*h+b*(c*f*h+d*e*h+
d*f*g))*(4*b*B*d*f*h+a*C*d*f*h-3*b*C*(c*f*h+d*e*h+d*f*g))+4*d*f*h*(2*a^2*C
*d*f*h+b^2*C*(c*e*h+c*f*g+d*e*g)-a*b*(4*B*d*f*h-C*(c*f*h+d*e*h+d*f*g))))*(
b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)
/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f
+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)
/(-a*f+b*e)/(-c*h+d*g))^(1/2))/d^3/f^2/(-a*f+b*e)^(1/2)/h^2/(f*x+e)^(1/2)/
(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 21961 vs. 2(988) = 1976.

Time = 37.00 (sec) , antiderivative size = 21961, normalized size of antiderivative = 22.23

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x]*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 3.53 (sec) , antiderivative size = 978, normalized size of antiderivative = 0.99, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {2004, 2100, 2105, 25, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\left(a^2(-C)+abB+b^2Bx+b^2Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 2004$$

$$\int \frac{(a+bx)^{3/2}\left(\frac{abB-a^2C}{a}+bCx\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 2100$$

$$\frac{\int \frac{4(bB-aC)dfha^2+b^2(4bBdfh+aCdfh-3bC(dfg+deh+cfh))x^2-b^2C(bceg+a(deg+cfg+ceh))-2b\left(2Cdfha^2-b(4Bdfh-C(dfg+deh+cfh))a \cdots\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{4dfh} \cdots$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 2105$$

$$\frac{\int -\frac{b\left(b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)+b\left((adfh+b(dfg+deh+cfh))(4bBdfh+aCdf \cdots\right.\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh} \cdots$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{b\left(b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)+b\left((adfh+b(dfg+deh+cfh))(4bBdfh+aCd \cdots\right.\right.}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh} \cdots$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)+b\left((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}\cdots}{\cdots}$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 194

$$-\frac{\frac{\int \frac{b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)+b\left((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}\cdots}{\cdots}$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 327

$$-\frac{\frac{\int \frac{b(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2dfh\left(4a^2(bB-aC)dfh-b^2C(bceg+a(deg+cfg+ceh))\right)+b\left((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2dfh}\cdots}{\cdots}$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 2101

$$-\frac{\frac{\left(4dfh\left(2a^2Cdfh-ab(4Bdfh-C(cfh+deh+dfg))+b^2C(ceh+cfg+deg)\right)+(adfh+b(cfh+deh+dfg))(aCdfh+4bBdfh-3bC(cfh+deh+dfg))\right)\int\cdots}{2dfh}\cdots}{\cdots}$$

$$\frac{b^2C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 183

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}+$$

$$-\frac{\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{b\sqrt{a+bx}\cdots}{\cdots}}{\cdots}$$

$\downarrow$ 188

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}+$$

$$-\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{b\sqrt{a+bx}$$

$$\Big\downarrow \text{321}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}+$$

$$-\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{b\sqrt{a+bx}$$

$$\Big\downarrow \text{412}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}b^2}{2dfh}+$$

$$-\frac{b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))}{dfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{b\sqrt{a+bx}$$

input
```
Int[(Sqrt[a + b*x]*(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2))/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(b^2*C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h)
+ ((b*(4*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b
*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (b*Sqrt[d*g - c*h]*
Sqrt[f*g - e*h]*(4*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h))*
Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*Ell
ipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*
x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(d*f*h*Sqrt[((
d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - ((2*d*(b*e
 - a*f)*Sqrt[b*g - a*h]*(4*b*B*d*f*h - a*C*d*f*h - b*C*(c*f*h + 3*d*(f*g +
 e*h)))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x
]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a
 + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[
f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a +
b*x)))]) + (2*Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(4
*b*B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h)) + 4*d*f*h*(2*a^2*C
*d*f*h + b^2*C*(d*e*g + c*f*g + c*e*h) - a*b*(4*B*d*f*h - C*(d*f*g + d*e*h
 + c*f*h))))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)
)]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(
d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt
[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2100
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[2
*b*B*(a + b*x)^(m - 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2
*m + 1))), x] + Simp[1/(d*f*h*(2*m + 1))   Int[((a + b*x)^(m - 2)/(Sqrt[c +
 d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[(-b)*B*(a*(d*e*g + c*f*g + c*e*h)
+ 2*b*c*e*g*(m - 1)) + a^2*A*d*f*h*(2*m + 1) + (2*a*A*b*d*f*h*(2*m + 1) - B
*(2*a*b*(d*f*g + d*e*h + c*f*h) + b^2*(d*e*g + c*f*g + c*e*h)*(2*m - 1) - a
^2*d*f*h*(2*m + 1)))*x + b*(A*b*d*f*h*(2*m + 1) - B*(2*b*(d*f*g + d*e*h + c
*f*h)*m - a*d*f*h*(4*m - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, A, B}, x] && IntegerQ[2*m] && GtQ[m, 1]
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(905) = 1810$.

Time = 19.40 (sec) , antiderivative size = 1834, normalized size of antiderivative = 1.86

method	result	size
elliptic	Expression too large to display	1834
default	Expression too large to display	56119

input
```
int((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(1/2*C*b^2/h/d/f*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^
3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+
b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(
1/2)+2*(a^2*b*B-C*a^3-1/2*C*b^2/h/d/f*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g
+1/2*b*c*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c
/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-
e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a
/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),
((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(2*a*b^2*B-a^2*b*C-1/
2*C*b^2/h/d/f*(a*c*f*h+a*d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(e/f-g/h)
*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b
)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/
(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c/d
*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g
/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/f)*(x+g/h)/(
-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b-g
/h)/(-c/d+e/f))^(1/2)))+(B*b^3+a*C*b^2-1/2*C*b^2/h/d/f*(3/2*a*d*f*h+3/2*b*
c*f*h+3/2*b*d*e*h+3/2*b*g*d*f))*((x+g/h)*(x+a/b)*(x+e/f)+(e/f-g/h)*((c/d-e
/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\,(Bb-Ca+Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)
*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*sqrt(b*x + a)/(sqrt(d*x +
c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$=\int \frac{(Cb^2x^2+Bb^2x-Ca^2+Bab)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)*sqrt(b*x + a)/(sqrt(d*x +
c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+b\,x}\,\left(-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x\right)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/
2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(1/2)*(C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x))/((e + f*x)^(1/
2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(abB-a^2C+b^2Bx+b^2Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}\,(C\,b^2x^2+B\,b^2x+Bab-C\,a^2)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^(1/2)*(C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

3.23 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 747

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{b^2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{dfh\sqrt{a+bx}}$$
$$-\frac{bC\sqrt{be-af}(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{df\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$-\frac{C(bc-ad)\sqrt{be-af}\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{df\sqrt{de-cf}\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$+\frac{\sqrt{de-cf}(2bBdfh-aCdfh-bC(dfg+deh+cfh))(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{Elliptic}}{d^2f\sqrt{be-af}h\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
b^2*C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h/(b*x+a)^(1/2)-b*C*(-
a*f+b*e)^(1/2)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b
*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*
x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/d/f/(-c*f+
d*e)^(1/2)/h/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-
C*(-a*d+b*c)*(-a*f+b*e)^(1/2)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g
)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)
/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/d/f/(-
c*f+d*e)^(1/2)/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2
)+(-c*f+d*e)^(1/2)*(2*b*B*d*f*h-a*C*d*f*h-b*C*(c*f*h+d*e*h+d*f*g))*(b*x+a)
*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h
+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^
(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f
+b*e)/(-c*h+d*g))^(1/2))/d^2/f/(-a*f+b*e)^(1/2)/h/(f*x+e)^(1/2)/(h*x+g)^(1
/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 8107 vs. $2(747) = 1494$.

Time = 42.52 (sec) , antiderivative size = 8107, normalized size of antiderivative = 10.85

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 732, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2004, 2099, 183, 188, 194, 321, 327, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{\sqrt{a+bx}\left(\frac{abB-a^2C}{a}+bCx\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2099}$$

$$\frac{(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}+\frac{bC(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{C(be-af)(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2fh}+\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{183}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}+\frac{bC(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh}-\frac{C(be-af)(bg-ah)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2fh}+\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{188}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{bC(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2dfh} - \frac{C\sqrt{g+hx}(be-af)(bg-ah)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{fh\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}} + \frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\downarrow 194$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{C\sqrt{g+hx}(be-af)(bg-ah)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{fh\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}} - \frac{bC\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}} + \frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\downarrow 321$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{bC\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}} - \frac{C\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}} + \frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{327}$$

$$\frac{(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+}}}{dfh\sqrt{c+dx}\sqrt{e+fx}}$$

$$\frac{C\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{bC\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

$$\Big\downarrow \text{412}$$

$$\frac{(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(-aCdfh+2bBdfh-bC(cfh+deh+dfg))\text{EllipticPi}\left(-\frac{b(dg-}{(bc-a}\right.}{dfh^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

$$\frac{C\sqrt{g+hx}(be-af)\sqrt{bg-ah}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{fh\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-$$

$$\frac{bC\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{dfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{bC\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{fh\sqrt{c+dx}}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(b*C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*h*Sqrt[c + d*x]) - (b*C
*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*
x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f
*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a
*f)*(d*g - c*h))])/(d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d
*x))]*Sqrt[g + h*x]) - (C*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[((b*e - a*f)*(c
 + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g
 - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f
*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(f*h*Sqrt[f*g - e*h]*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (Sqrt[-(d*g)
+ c*h]*(2*b*B*d*f*h - a*C*d*f*h - b*C*(d*f*g + d*e*h + c*f*h))*(a + b*x)*S
qrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e
+ f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)
*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a +
b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(d*Sqrt[b*c
- a*d]*f*h^2*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2099
```
Int[(Sqrt[(a_.) + (b_.)*(x_)]*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[B*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(f*h*Sqrt[c + d*x])), x] + (-Sim
p[B*(b*e - a*f)*((b*g - a*h)/(2*b*f*h))   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x
]*Sqrt[e + f*x]*Sqrt[g + h*x]), x], x] + Simp[B*(d*e - c*f)*((d*g - c*h)/(2
*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]),
 x], x] + Simp[(2*A*b*d*f*h + B*(a*d*f*h - b*(d*f*g + d*e*h + c*f*h)))/(2*b
*d*f*h)   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]), x]
, x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && NeQ[2*A*d*f - B*(d*e +
 c*f), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(682) = 1364$.

Time = 19.55 (sec) , antiderivative size = 1552, normalized size of antiderivative = 2.08

method	result	size
elliptic	Expression too large to display	1552
default	Expression too large to display	21222

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(B*a*b-C*a^2)*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+
g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2
)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*
b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-
e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))
+2*B*b^2*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*
((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/
h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x
+e/f))^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),
((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((
c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b
)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)))+b^2*C*((x+g/h)*(x+a/b)*(x+e/f)+(
e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h
)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d)
)^(1/2)*((c/d*g/h-e/f*g/h+c*e/d/f+c^2/d^2)/(c/d-e/f)/(-c/d+g/h)*EllipticF(
((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/
h)/(-c/d+e/f))^(1/2))+(a/b-g/h)*EllipticE(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x
+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))/(-c/d+g/h)
+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/h/d/b/f/(c/d-e/f)*EllipticPi(((c/d-e...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}\,(Bb - Ca + Cbx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(1/2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*(B*b - C*a + C*b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sq
rt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(b*x + a)*sqrt(d*x +
c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/(sqrt(b*x + a)*sqrt(d*x +
c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,b^2x^2+B\,b^2x+Bab-C\,a^2}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

3.24 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 447

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(bB-2aC)\sqrt{de-cf}\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}\,\text{EllipticF}\left(\ldots\right)}{\sqrt{be-af}(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(b\ldots}{(a\ldots}}}$$
$$+\frac{2C\sqrt{de-cf}(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)},\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(b\ldots}{(be-af)(d\ldots}\right)}{d\sqrt{be-af}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*(B*b-2*C*a)*(-c*f+d*e)^(1/2)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1
/2)*(h*x+g)^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2
)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*f
+b*e)^(1/2)/(-c*h+d*g)/(f*x+e)^(1/2)/(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+
a))^(1/2)+2*C*(-c*f+d*e)^(1/2)*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*
x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*
f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(
-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/d/(-a*f+b*e
)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 24.72 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.30

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(a+bx)^{3/2}\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\left(-\frac{bB\sqrt{\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}(g+hx)\,\text{EllipticF}\left(\text{ar}\right.}{(bg-ah)(a+bx}\right.}{}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(3/2)*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(a + b*x)^(3/2)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(
-((b*B*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*Ell
ipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], (
(-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*g - a*h)*(a
 + b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))])) - (2*a*
C*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*(g + h*x)*Elliptic
F[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*
c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((-(b*g) + a*h)*(a +
 b*x)*Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]) + (C*(-(f*
g) + e*h)*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h
)^2*(a + b*x)^2))]*EllipticPi[(b*(-(f*g) + e*h))/((b*e - a*f)*h), ArcSin[S
qrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*
(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))])/((b*e - a*f)*h)))/(Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2004, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2101}$$

$$(bB-2aC)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+C\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{183}$$

$$(bB-2aC)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{188}$$

$$\frac{2\sqrt{g+hx}(bB-2aC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{\sqrt{c+dx}\sqrt{e+fx}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{\sqrt{c+dx}\sqrt{e+fx}}+\frac{2\sqrt{g+hx}(bB-2aC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}$$

$$\Big\downarrow \text{412}$$

$$\frac{2\sqrt{g+hx}(bB-2aC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2C(a+bx)\sqrt{ch-dg}\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\text{EllipticPi}\left(-\frac{b(dg-ch)}{(bc-ad)h},\arcsin\left(\frac{\sqrt{bc-ad}\sqrt{g+hx}}{\sqrt{ch-dg}\sqrt{a+bx}}\right),\frac{(be-af)(dg-ch)}{(bc-ad)(fg-eh)}\right)}{h\sqrt{c+dx}\sqrt{e+fx}\sqrt{bc-ad}}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(b*B - 2*a*C)*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqr
t[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*
h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))
])/(Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g +
 h*x))/((f*g - e*h)*(a + b*x)))]) + (2*C*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt
[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f
*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)
), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x
])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(Sqrt[b*c - a*d
]*h*Sqrt[c + d*x]*Sqrt[e + f*x])
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. $2(409) = 818$.

Time = 21.43 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.91

method	result
elliptic	$\frac{\sqrt{(hx+g)(xd+c)(bx+a)(fx+e)}\left(\frac{2(Bb-Ca)\left(\frac{e}{f}-\frac{g}{h}\right)\sqrt{\frac{\left(\frac{c}{d}-\frac{e}{f}\right)\left(x+\frac{g}{h}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\left(x+\frac{c}{d}\right)^2\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{a}{b}\right)}{\left(-\frac{a}{b}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\sqrt{\frac{\left(-\frac{c}{d}+\frac{g}{h}\right)\left(x+\frac{e}{f}\right)}{\left(-\frac{e}{f}+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)}}\operatorname{EllipticF}\left(\sqrt{\cdots}\right.}{\left(\frac{c}{d}-\frac{e}{f}\right)\left(-\frac{c}{d}+\frac{g}{h}\right)\sqrt{hdbf\left(x+\frac{g}{h}\right)\left(x+\frac{c}{d}\right)\left(x+\frac{a}{b}\right)\left(x+\frac{e}{f}\right)}}\right.}{\cdots}$
default	Expression too large to display

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(B*b-C*a)*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)
/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((
-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*
(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+
g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*C
*b*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d
+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+
c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))
^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d
+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/
f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f
-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Bb - Ca + Cbx}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(3/2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((B*b - C*a + C*b*x)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sq
rt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(3/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(3/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,b^2x^2+B\,b^2x+Bab-C\,a^2}{(bx+a)^{\frac{3}{2}}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

3.25 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 458

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2b(bB-2aC)(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{(bc-ad)\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} + \frac{2(bCg-bBh+aCh)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

output

```
-2*b*(B*b-2*C*a)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/
(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(
b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*d+b*
c)/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x+e)/(-c*f
+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+2*(-B*b*h+C*a*h+C*b*g)*(f*x+e)^(1/2)*(-
(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d
*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*
e)/(-c*h+d*g))^(1/2))/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d
+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 26.06 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.74

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\Big(b(bB-2a}{}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(5/2)*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
f*x)^(3/2)*(g + h*x)^(3/2)*(b*(b*B - 2*a*C)*(d*g - c*h)*EllipticE[ArcSin[S
qrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((b*c - a*d)*(f*
g - e*h))/((b*e - a*f)*(d*g - c*h))] + (b*c*C - b*B*d + a*C*d)*(b*g - a*h)
*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]
], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))]))/((b*c - a*d)*(f*
g - e*h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e - a*f)*(b*g - a*h)*(e +
f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2004, 2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int \frac{2(bB-2aC)dfhx^2b^2+C(bceg-a(deg+cfg+ceh))b^2+(bB-2aC)(adfh+b(dfg+deh+cfh))xb-a(bB-aC)(adfh-b(dfg+deh+cfh))}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{(bc-ad)(be-af)(bg-ah)}-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 2105

$$\frac{b(bB-2aC)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{\int \frac{2bd(bcC+adC-bBd)f(be-af)h(bg-ah)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{2bd\sqrt{a+bx}\sqrt{e+fx}\sqrt{g\cdots}}{\sqrt{c+d\cdots}}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 27

$$\frac{b(bB-2aC)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+(be-af)(bg-ah)(aCd-bBd+bcC)\int \frac{\cdots}{\sqrt{a+bx}\sqrt{c+d\cdots}}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 188

$$\frac{b(bB-2aC)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx+\frac{2\sqrt{g+hx}(be-af)(bg-ah)(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \cdots}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(b\cdots}{(a+bx)(f\cdots}}}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 194

$$\frac{\frac{2\sqrt{g+hx}(be-af)(bg-ah)(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{a+bx}(bB-2aC)(dg-ch)\sqrt{\cdots}}{\sqrt{g\cdots}}}{(bc-ad)(be-af)(bg-ah)}-\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 321

$$\frac{-\frac{2b\sqrt{a+bx}(bB-2aC)(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int\frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{2\sqrt{g+hx}(be-af)\sqrt{bg-ah}(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 327

$$\frac{\frac{2\sqrt{g+hx}(be-af)\sqrt{bg-ah}(aCd-bBd+bcC)\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{\sqrt{c+dx}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2b\sqrt{a+bx}(bB-2aC)\sqrt{dg-}}{}}{(bc-ad)(be-af)(bg-}$$

$$\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

input
```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(-2*b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a
*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*b*(b*B - 2*a*C)*d*Sqrt[a
+ b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*b*(b*B - 2*a*C)*Sqr
t[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/
((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])
/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*
(d*g - c*h))])/(Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt
[g + h*x]) + (2*(b*c*C - b*B*d + a*C*d)*(b*e - a*f)*Sqrt[b*g - a*h]*Sqrt[(
(b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[Ar
cSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -((
(b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(Sqrt[f*g - e*h]*Sqr
t[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]))/((b*
c - a*d)*(b*e - a*f)*(b*g - a*h))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2248 vs. $2(420) = 840$.

Time = 24.02 (sec) , antiderivative size = 2249, normalized size of antiderivative = 4.91

method	result	size
elliptic	Expression too large to display	2249
default	Expression too large to display	20623

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+
b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-
a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*b*(B*b-2*C*a)/(
(x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g
*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(C+(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g
+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(B*b-2*C*a)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d
*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b
*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h
+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*b*(B*b-2*C*a))*(e/f-g/h)*((c/d-e/f)*(x
+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(
x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/
d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f
)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+
e/f))^(1/2))+2*(-b*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*g)*(B*b-2*C*a)/(a^3*d*f*
h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-
b^3*c*e*g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*
e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*b*(B*b-2*C*
a))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/
d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)...
```

Fricas [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
integral((C*b*x - C*a + B*b)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqr
t(h*x + g)/(b^2*d*f*h*x^5 + a^2*c*e*g + (b^2*d*f*g + (b^2*d*e + (b^2*c + 2
*a*b*d)*f)*h)*x^4 + ((b^2*d*e + (b^2*c + 2*a*b*d)*f)*g + ((b^2*c + 2*a*b*d
)*e + (2*a*b*c + a^2*d)*f)*h)*x^3 + (((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2
*d)*f)*g + (a^2*c*f + (2*a*b*c + a^2*d)*e)*h)*x^2 + (a^2*c*e*h + (a^2*c*f
+ (2*a*b*c + a^2*d)*e)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(5/2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(5/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cb^2x^2 + Bb^2x - Ca^2 + Bab}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(5/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,b^2x^2+B\,b^2x+Bab-C\,a^2}{(bx+a)^{\frac{5}{2}}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

3.26 $\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 62, antiderivative size = 760

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2b(dg-ch)\left(9a^3Cdfh+b^3(3cCeg-2Bdeg-2Bc(fg+eh))+ab^2(C(deg+cfg+ceh)+4B(dfg+de\right.}{3(bc-ad)^2(be-af)^{3/2}\sqrt{de}}$$

$$-\frac{2(3a^3Cdfh^2+ab^2(3B(de+cf)h^2-cCh(fg-eh)-Cdg(2fg+eh))+b^3(3cCegh+Bdg(fg-eh)-}{3(bc-ad)(be-af)^3}$$

output
```
-2/3*b^2*(B*b-2*C*a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/
(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)-2/3*b*(-c*h+d*g)*(9*a^3*C*d*f*h+b^3*(3
*c*C*e*g-2*B*d*e*g-2*B*c*(e*h+f*g))+a*b^2*(C*(c*e*h+c*f*g+d*e*g)+4*B*(c*f*
h+d*e*h+d*f*g))-a^2*b*(6*B*d*f*h+5*C*(c*f*h+d*e*h+d*f*g)))*(f*x+e)^(1/2)*(
-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(
d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b
*e)/(-c*h+d*g))^(1/2))/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-a*
h+b*g)^2/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2/3*
(3*a^3*C*d*f*h^2+a*b^2*(3*B*(c*f+d*e)*h^2-c*C*h*(-e*h+f*g)-C*d*g*(e*h+2*f*
g))+b^3*(3*c*C*e*g*h+B*d*g*(-e*h+f*g)-B*c*h*(2*e*h+f*g))-3*a^2*b*h*(B*d*f*
h-C*(-c*f*h-d*e*h+d*f*g)))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(
b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b
*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/(-a*d+b*c
)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)^2/(-(-a*d+b*c)*(f*x+e)/(-c*
f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 10836 vs. $2(760) = 1520$.

Time = 40.11 (sec) , antiderivative size = 10836, normalized size of antiderivative = 14.26

$$\int \frac{abB - a^2C + b^2Bx + b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(7/2)*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 4.72 (sec) , antiderivative size = 1105, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.145, Rules used = {2004, 2102, 2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2(-C)+abB+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2004}$$

$$\int \frac{\frac{abB-a^2C}{a}+bCx}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int \frac{C(3bceg-a(deg+cfg+ceh))b^2+(bB-2aC)(3adfh-b(dfg+deh+cfh))xb-(bB-aC)\left(3dfha^2-3b(dfg+deh+cfh)a+2b^2(deg+cfg+ceh)\right)}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3(bc-ad)(be-af)(bg-ah)} -$$
$$\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int \frac{2dfh\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a+b^3(3cCeg-2Bdeg-2Bc(fg+eh))\right)x^2b^2+(bB-2aC)(bceg-a(d\ldots}{\ldots}}{\ldots}$$

$$\frac{2b^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(bB-2aC)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2105}$$

$$\frac{b(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a+b^3(3c\ldots\right.}{\ldots}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a+b^3(3c\right.}{}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 188

$$\frac{b(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh))a+b^3(3c\right.}{}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 194

$$-\frac{2b(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 321

$$-\frac{2b(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+ceh)+4B(dfg+deh+cfh\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 327

$$-\frac{2b\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)\left(9Cdfha^3-b(6Bdfh+5C(dfg+deh+cfh))a^2+b^2(C(deg+cfg+c\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2b^2(bB-2aC)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

input

```
Int[(a*b*B - a^2*C + b^2*B*x + b^2*C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]),x]
```

output

```
(-2*b^2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c -
 a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) + ((-2*b^2*(9*a^3*C*d*f*h +
 b^3*(3*c*C*e*g - 2*B*d*e*g - 2*B*c*(f*g + e*h)) + a*b^2*(C*(d*e*g + c*f*g
 + c*e*h) + 4*B*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*B*d*f*h + 5*C*(d*f*g +
 d*e*h + c*f*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*
(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*b*d*(9*a^3*C*d*f*h + b^3*(3*c
*C*e*g - 2*B*d*e*g - 2*B*c*(f*g + e*h)) + a*b^2*(C*(d*e*g + c*f*g + c*e*h)
 + 4*B*(d*f*g + d*e*h + c*f*h)) - a^2*b*(6*B*d*f*h + 5*C*(d*f*g + d*e*h +
c*f*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*b*S
qrt[d*g - c*h]*Sqrt[f*g - e*h]*(9*a^3*C*d*f*h + b^3*(3*c*C*e*g - 2*B*d*e*g
 - 2*B*c*(f*g + e*h)) + a*b^2*(C*(d*e*g + c*f*g + c*e*h) + 4*B*(d*f*g + d*
e*h + c*f*h)) - a^2*b*(6*B*d*f*h + 5*C*(d*f*g + d*e*h + c*f*h)))*Sqrt[a +
b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[Ar
cSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b
*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[((d*e - c*f)*(a +
 b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - (2*(b*e - a*f)*Sqrt[b*g -
 a*h]*(3*a^3*C*d^2*f*h - 3*a^2*b*d*(B*d*f*h + C*(d*f*g + d*e*h - c*f*h)) -
 b^3*(2*B*d^2*e*g - B*c^2*f*h - c*d*(3*C*e*g - B*(f*g + e*h))) + a*b^2*(3*
B*d^2*(f*g + e*h) + C*(d^2*e*g - 2*c^2*f*h - c*d*(f*g + e*h))))*Sqrt[((b*e
 - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[Arc...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2004
```
Int[(u_)*((d_) + (e_.)*(x_))^(q_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)
, x_Symbol] :> Int[u*(d + e*x)^(p + q)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, b
, c, d, e, q}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3424 vs. $2(708) = 1416$.

Time = 38.37 (sec) , antiderivative size = 3425, normalized size of antiderivative = 4.51

method	result	size
elliptic	Expression too large to display	3425
default	Expression too large to display	119409

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2/3/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+
a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)*(B*b-2*C*a)*(b*d*f*h*x^4+a*
d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*
f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+
b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+2/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^
2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*
h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)^2
*b*(6*B*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*h-4*B*a*b^2*d*f*g+2*B*b^
3*c*e*h+2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+5*C*a^2*b*c*f*h+5*C*a^2*
b*d*e*h+5*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*g-C*a*b^2*d*e*g-3*C*b^3*
c*e*g)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x
+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(-1/3*(3*B*a*b*d*f*h-B*b^2*c*f*h-B*
b^2*d*e*h-B*b^2*d*f*g-6*C*a^2*d*f*h+2*C*a*b*c*f*h+2*C*a*b*d*e*h+2*C*a*b*d*
f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*
g+a*b^2*d*e*g-b^3*c*e*g)+1/3*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*
c*e*h+b^2*c*f*g+b^2*d*e*g)*(6*B*a^2*b*d*f*h-4*B*a*b^2*c*f*h-4*B*a*b^2*d*e*
h-4*B*a*b^2*d*f*g+2*B*b^3*c*e*h+2*B*b^3*c*f*g+2*B*b^3*d*e*g-9*C*a^3*d*f*h+
5*C*a^2*b*c*f*h+5*C*a^2*b*d*e*h+5*C*a^2*b*d*f*g-C*a*b^2*c*e*h-C*a*b^2*c*f*
g-C*a*b^2*d*e*g-3*C*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b...
```

Fricas [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="fricas")
```

output
```
integral((C*b*x - C*a + B*b)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqr
t(h*x + g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3
*a*b^2*d)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a
*b^2*d)*e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*
(a*b^2*c + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*
f)*h)*x^3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*
f + (3*a^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a
^3*d)*e)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*b**2*x**2+B*b**2*x+B*a*b-C*a**2)/(b*x+a)**(7/2)/(d*x+c)**(1/2
)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(7/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cb^2x^2+Bb^2x-Ca^2+Bab}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x
+e)^(1/2)/(h*x+g)^(1/2),x, algorithm="giac")
```

output
```
integrate((C*b^2*x^2 + B*b^2*x - C*a^2 + B*a*b)/((b*x + a)^(7/2)*sqrt(d*x
+ c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{-C\,a^2+B\,a\,b+C\,b^2\,x^2+B\,b^2\,x}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{(a+b\,x)}^{7/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

output
```
int((C*b^2*x^2 - C*a^2 + B*a*b + B*b^2*x)/((e + f*x)^(1/2)*(g + h*x)^(1/2)
*(a + b*x)^(7/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{abB-a^2C+b^2Bx+b^2Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,b^2x^2+B\,b^2x+Bab-C\,a^2}{(bx+a)^{\frac{7}{2}}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*b^2*x^2+B*b^2*x+B*a*b-C*a^2)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1
/2)/(h*x+g)^(1/2),x)
```

3.27 $\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 1084

$$\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{2\left(8a^2Cdfh-38abC(dfg+deh+cfh)+b^2\left(35Adfh+C\left(\frac{24c^2fh}{d}+23c(fg+eh)+d\left(23eg+\frac{24fg^2}{h}+2\right.\right.\right.\right.}{105d^2f^2h^2}$$

$$+\frac{4C(2adfh-3b(dfg+deh+cfh))(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{35d^2f^2h^2}$$

$$+\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}$$

$$-\frac{4\sqrt{-de+cf}(35a^2Cd^2f^2h^2(dfg+deh+cfh)-7abdfh(15Ad^2f^2h^2+C(8c^2f^2h^2+7cdfh(fg+eh)+}{}$$

$$+\frac{2\sqrt{-de+cf}(35a^2d^2f^2h^2(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))-14abdfh(15Ad^2f^2gh^2+C(4c}{}$$

output
```
2/105*(8*a^2*C*d*f*h-38*a*b*C*(c*f*h+d*e*h+d*f*g)+b^2*(35*A*d*f*h+C*(24*c^
2*f*h/d+23*c*(e*h+f*g)+d*(23*e*g+24*f*g^2/h+24*e^2*h/f))))*(d*x+c)^(1/2)*(
f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+4/35*C*(2*a*d*f*h-3*b*(c*f*h+d*e*h+
d*f*g))*(b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+2/7*
C*(b*x+a)^2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-4/105*(c*f-d*e
)^(1/2)*(35*a^2*C*d^2*f^2*h^2*(c*f*h+d*e*h+d*f*g)-7*a*b*d*f*h*(15*A*d^2*f^
2*h^2+C*(8*c^2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8*f^2*
g^2)))+b^2*(35*A*d^2*f^2*h^2*(c*f*h+d*e*h+d*f*g)+2*C*(12*c^3*f^3*h^3+10*c^
2*d*f^2*h^2*(e*h+f*g)+c*d^2*f*h*(10*e^2*h^2+9*e*f*g*h+10*f^2*g^2)+2*d^3*(6
*e^3*h^3+5*e^2*f*g*h^2+5*e*f^2*g^2*h+6*f^3*g^3))))*(d*(f*x+e)/(-c*f+d*e))^
(1/2)*(h*x+g)^(1/2)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f
+d*e)*h/f/(-c*h+d*g))^(1/2))/d^4/f^(7/2)/h^4/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*
h+d*g))^(1/2)+2/105*(c*f-d*e)^(1/2)*(35*a^2*d^2*f^2*h^2*(3*A*d*f*h^2+c*C*h
*(-e*h+f*g)+C*d*g*(e*h+2*f*g))-14*a*b*d*f*h*(15*A*d^2*f^2*g*h^2+C*(4*c^2*f
*h^2*(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(4*e^2*h^2+3*e*
f*g*h+8*f^2*g^2)))+b^2*(35*A*d^2*f^2*h^2*(c*h*(-e*h+f*g)+d*g*(e*h+2*f*g))+
C*(24*c^3*f^2*h^3*(-e*h+f*g)+c^2*d*f*h^2*(-23*e^2*h^2+6*e*f*g*h+17*f^2*g^2
)+2*c*d^2*h*(-12*e^3*h^3+3*e^2*f*g*h^2+e*f^2*g^2*h+8*f^3*g^3)+d^3*g*(24*e^
3*h^3+17*e^2*f*g*h^2+16*e*f^2*g^2*h+48*f^3*g^3))))*(d*(f*x+e)/(-c*f+d*e))^
(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.97 (sec) , antiderivative size = 1291, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x)^2*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output

```
(2*(-2*d^2*Sqrt[-c + (d*e)/f]*(35*a^2*C*d^2*f^2*h^2*(d*f*g + d*e*h + c*f*h
) - 7*a*b*d*f*h*(15*A*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*
h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2*f^2*h^2*(d*
f*g + d*e*h + c*f*h) + 2*C*(12*c^3*f^3*h^3 + 10*c^2*d*f^2*h^2*(f*g + e*h)
+ c*d^2*f*h*(10*f^2*g^2 + 9*e*f*g*h + 10*e^2*h^2) + 2*d^3*(6*f^3*g^3 + 5*e
*f^2*g^2*h + 5*e^2*f*g*h^2 + 6*e^3*h^3))))*(e + f*x)*(g + h*x) + d^2*Sqrt[
-c + (d*e)/f]*f*h*(c + d*x)*(e + f*x)*(g + h*x)*(35*a^2*C*d^2*f^2*h^2 - 14
*a*b*C*d*f*h*(4*c*f*h + d*(4*f*g + 4*e*h - 3*f*h*x)) + b^2*(35*A*d^2*f^2*h
^2 + C*(24*c^2*f^2*h^2 + c*d*f*h*(23*f*g + 23*e*h - 18*f*h*x) + d^2*(24*e^
2*h^2 + e*f*h*(23*g - 18*h*x) + 3*f^2*(8*g^2 - 6*g*h*x + 5*h^2*x^2))))) -
(2*I)*(d*e - c*f)*h*(35*a^2*C*d^2*f^2*h^2*(d*f*g + d*e*h + c*f*h) - 7*a*b*
d*f*h*(15*A*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(
8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2*f^2*h^2*(d*f*g + d*e*
h + c*f*h) + 2*C*(12*c^3*f^3*h^3 + 10*c^2*d*f^2*h^2*(f*g + e*h) + c*d^2*f*
h*(10*f^2*g^2 + 9*e*f*g*h + 10*e^2*h^2) + 2*d^3*(6*f^3*g^3 + 5*e*f^2*g^2*h
 + 5*e^2*f*g*h^2 + 6*e^3*h^3))))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c
+ d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (
d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*d*h*(35*a^2*d
^2*f^2*h^2*(3*A*d*f^2*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) - 14
*a*b*d*f*h*(15*A*d^2*e*f^2*h^2 + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*...
```

Rubi [A] (verified)

Time = 3.23 (sec) , antiderivative size = 1112, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.262$, Rules used = {2104, 25, 2103, 2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} \, dx$$

$$\Big\downarrow \; 2104$$

$$\frac{\int -\frac{(a+bx)(-2C(2adfh-3b(dfg+deh+cfh))x^2-(7Abdfh-5bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+4bcCeg-7aAdfh+aC(deg+cfg+ceh \cdots}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{7dfh}$$

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}$$

$\downarrow$ 25

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} - \frac{\int \frac{(a+bx)\left(-2C(2adfh-3b(dfg+deh+cfh))x^2-(7Abdfh-5bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+4bcCeg-7aAdfh+aC(deg+cfg+ceh)\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{7dfh}$$

$\downarrow$ 2103

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} - \int \frac{-\left((4C(2adfh-3b(dfg+deh+cfh))(adfh-2b(dfg+deh+cfh))+5bdfh(7Abdfh-5bC(deg+cfg+ceh)-2aC(dfg+deh+cfh)))x^2\right)+2\left(C(3b(deg+cfg+ceh)+2a(dfg\right.}{}$$

$\downarrow$ 2118

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} - 2\int -\frac{d\left(-\left(\left(35Ad^2f^2(deg+cfg+ceh)h^2+C\left(24f^2h^2(fg+eh)c^3+dfh\left(23f^2g^2+34efhg+23e^2h^2\right)c^2+2d^2\left(12f^3g^3+17ef^2hg^2+17e^2fh^2g+12e^3h^3\right)c+d^3eg\left(24f^2g\right.\right.\right.\right.\right.}{}$$

$\downarrow$ 27

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} - -\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right) - \frac{\int \frac{-\left(\left(35Ad^2f^2(deg+\right.\right.}{}}{}$$

$\downarrow$ 176

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh} - -\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right) - \frac{\left(\left(35Ad^2f^2(ch(fg-e\right.\right.}{}$$

$\downarrow$ 124

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-e\right.\right.}{}$$

$\downarrow$ 123

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-e\right.\right.}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-e\right.\right.}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{\left(\left(35Ad^2f^2(ch(fg-e\right.\right.}{}$$

$\downarrow$ 130

$$\frac{2C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{7dfh}-$$

$$-\frac{2}{3}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(8Cdfha^2-38bC(dfg+deh+cfh)a+\frac{24b^2C(dfg+deh+cfh)^2}{dfh}+35Ab^2dfh-25b^2C(deg+cfg+ceh)\right)-\frac{2\sqrt{cf-de}\left(\left(35Ad^2f^2\right.\right.}{}$$

input
```
Int[((a + b*x)^2*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(2*C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(7*d*f*h) - ((
-4*C*(2*a*d*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*(a + b*x)*Sqrt[c + d*x]*Sqr
t[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) + ((-2*(35*A*b^2*d*f*h + 8*a^2*C*d*f*h
 - 25*b^2*C*(d*e*g + c*f*g + c*e*h) - 38*a*b*C*(d*f*g + d*e*h + c*f*h) + (
24*b^2*C*(d*f*g + d*e*h + c*f*h)^2)/(d*f*h))*Sqrt[c + d*x]*Sqrt[e + f*x]*S
qrt[g + h*x])/3 - ((-4*Sqrt[-(d*e) + c*f]*(35*a^2*C*d^2*f^2*h^2*(d*f*g + d
*e*h + c*f*h) - 7*a*b*d*f*h*(15*A*d^2*f^2*h^2 + C*(8*c^2*f^2*h^2 + 7*c*d*f
*h*(f*g + e*h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2))) + b^2*(35*A*d^2
*f^2*h^2*(d*f*g + d*e*h + c*f*h) + 2*C*(12*c^3*f^3*h^3 + 10*c^2*d*f^2*h^2*
(f*g + e*h) + c*d^2*f*h*(10*f^2*g^2 + 9*e*f*g*h + 10*e^2*h^2) + 2*d^3*(6*f
^3*g^3 + 5*e*f^2*g^2*h + 5*e^2*f*g*h^2 + 6*e^3*h^3))))*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-
(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x
]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(35*a^2*d^2*f^2
*h^2*(3*A*d*f*h^2 + c*C*h*(f*g - e*h) + C*d*g*(2*f*g + e*h)) - 14*a*b*d*f*
h*(15*A*d^2*f^2*g*h^2 + C*(4*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 + e*
f*g*h - 4*e^2*h^2) + d^2*g*(8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))) + b^2*(35
*A*d^2*f^2*h^2*(c*h*(f*g - e*h) + d*g*(2*f*g + e*h)) + C*(24*c^3*f^2*h^3*(
f*g - e*h) + c^2*d*f*h^2*(17*f^2*g^2 + 6*e*f*g*h - 23*e^2*h^2) + 2*c*d^2*h
*(8*f^3*g^3 + e*f^2*g^2*h + 3*e^2*f*g*h^2 - 12*e^3*h^3) + d^3*g*(48*f^3...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2104
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f
*g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
 c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
 1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 8.29 (sec) , antiderivative size = 1238, normalized size of antiderivative = 1.14

method	result	size
elliptic	Expression too large to display	1238
default	Expression too large to display	12279

input
```
int((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2/7*C*b^2/d/f/h*x^2*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*
g*x+d*e*g*x+c*e*g)^(1/2)+2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d
*f*g))/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*
e*g*x+c*e*g)^(1/2)+2/3*(b^2*A+a^2*C-2/7*C*b^2/d/f/h*(5/2*c*e*h+5/2*c*f*g+5
/2*d*e*g)-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*(2
*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*
e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)+2*(a^2*A-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(
3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*c*e*g-2/3*(b^2*A+a^2*C-2/7*C*b^2/d/f/h*(5/
2*c*e*h+5/2*c*f*g+5/2*d*e*g)-2/5*(2*C*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h
+3*d*f*g))/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2
*d*e*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((
x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+
c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/
f)/(-c/d+g/h))^(1/2))+2*(2*a*b*A-4/7*C*b^2/d/f/h*c*e*g-2/5*(2*C*a*b-2/7*C*
b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/2*d*e*g)
-2/3*(b^2*A+a^2*C-2/7*C*b^2/d/f/h*(5/2*c*e*h+5/2*c*f*g+5/2*d*e*g)-2/5*(2*C
*a*b-2/7*C*b^2/d/f/h*(3*c*f*h+3*d*e*h+3*d*f*g))/d/f/h*(2*c*f*h+2*d*e*h+2*d
*f*g))/d/f/h*(c*f*h+d*e*h+d*f*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+
g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1665, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx)^2 (A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}} \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="fricas")
```

output
```
2/315*(3*(15*C*b^2*d^4*f^4*h^4*x^2 + 24*C*b^2*d^4*f^4*g^2*h^2 + (23*C*b^2*
d^4*e*f^3 + (23*C*b^2*c*d^3 - 56*C*a*b*d^4)*f^4)*g*h^3 + (24*C*b^2*d^4*e^2
*f^2 + (23*C*b^2*c*d^3 - 56*C*a*b*d^4)*e*f^3 + (24*C*b^2*c^2*d^2 - 56*C*a*
b*c*d^3 + 35*(C*a^2 + A*b^2)*d^4)*f^4)*h^4 - 6*(3*C*b^2*d^4*f^4*g*h^3 + (3
*C*b^2*d^4*e*f^3 + (3*C*b^2*c*d^3 - 7*C*a*b*d^4)*f^4)*h^4)*x)*sqrt(d*x + c
)*sqrt(f*x + e)*sqrt(h*x + g) + (48*C*b^2*d^4*f^4*g^4 + 16*(C*b^2*d^4*e*f^
3 + (C*b^2*c*d^3 - 7*C*a*b*d^4)*f^4)*g^3*h + (11*C*b^2*d^4*e^2*f^2 + 14*(C
*b^2*c*d^3 - 3*C*a*b*d^4)*e*f^3 + (11*C*b^2*c^2*d^2 - 42*C*a*b*c*d^3 + 70*
(C*a^2 + A*b^2)*d^4)*f^4)*g^2*h^2 + (16*C*b^2*d^4*e^3*f + 14*(C*b^2*c*d^3
- 3*C*a*b*d^4)*e^2*f^2 + 7*(2*C*b^2*c^2*d^2 - 6*C*a*b*c*d^3 + 5*(C*a^2 + A
*b^2)*d^4)*e*f^3 + (16*C*b^2*c^3*d - 42*C*a*b*c^2*d^2 - 210*A*a*b*d^4 + 35
*(C*a^2 + A*b^2)*c*d^3)*f^4)*g*h^3 + (48*C*b^2*d^4*e^4 + 16*(C*b^2*c*d^3 -
 7*C*a*b*d^4)*e^3*f + (11*C*b^2*c^2*d^2 - 42*C*a*b*c*d^3 + 70*(C*a^2 + A*b
^2)*d^4)*e^2*f^2 + (16*C*b^2*c^3*d - 42*C*a*b*c^2*d^2 - 210*A*a*b*d^4 + 35
*(C*a^2 + A*b^2)*c*d^3)*e*f^3 + (48*C*b^2*c^4 - 112*C*a*b*c^3*d - 210*A*a*
b*c*d^3 + 315*A*a^2*d^4 + 70*(C*a^2 + A*b^2)*c^2*d^2)*f^4)*h^4)*sqrt(d*f*h
)*weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^
2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e
*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2
+ (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3...
```

Sympy [F]

$$\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)(a+bx)^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**2*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

output
```
Integral((A + C*x**2)*(a + b*x)**2/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h
*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^2(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^2}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + A)*(b*x + a)^2/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(C\,x^2+A)\,(a+b\,x)^2}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + C*x^2)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(
1/2)),x)
```

output
```
int(((A + C*x^2)*(a + b*x)^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^2\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(bx+a)^2\,(C\,x^2+A)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^2*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.28 $\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 608

$$\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{4C(adfh-2b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{15d^2f^2h^2}$$
$$+ \frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$
$$+ \frac{2\sqrt{-de+cf}(15Abd^2f^2h^2-10aCdfh(dfg+deh+cfh)+bC(8c^2f^2h^2+7cdfh(fg+eh)+d^2(8f^2g^2}{15d^3f^{5/2}h^3\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+ \frac{2\sqrt{-de+cf}(5adfh(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))-b(15Ad^2f^2gh^2+C(4c^2fh^2(fg-e}{15}$$

output
```
4/15*C*(a*d*f*h-2*b*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+
g)^(1/2)/d^2/f^2/h^2+2/5*C*(b*x+a)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/
2)/d/f/h+2/15*(c*f-d*e)^(1/2)*(15*A*b*d^2*f^2*h^2-10*a*C*d*f*h*(c*f*h+d*e*
h+d*f*g)+b*C*(8*c^2*f^2*h^2+7*c*d*f*h*(e*h+f*g)+d^2*(8*e^2*h^2+7*e*f*g*h+8
*f^2*g^2)))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f^(1/2)*(
d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d^3/f^(5/2
)/h^3/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/15*(c*f-d*e)^(1/2)*(5*a
*d*f*h*(3*A*d*f*h^2+c*C*h*(-e*h+f*g)+C*d*g*(e*h+2*f*g))-b*(15*A*d^2*f^2*g*
h^2+C*(4*c^2*f*h^2*(-e*h+f*g)+c*d*h*(-4*e^2*h^2+e*f*g*h+3*f^2*g^2)+d^2*g*(
4*e^2*h^2+3*e*f*g*h+8*f^2*g^2))))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/
(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d
*e)*h/f/(-c*h+d*g))^(1/2))/d^3/f^(5/2)/h^3/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.45 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(-d^2\sqrt{-c+\frac{de}{f}}\left(15Abd^2f^2h^2-10aCdfh(dfg+deh+cfh)+bC(8c^2f^2h^2+7cdfh(fg+eh)+d^2(8\right.\right.}{}$$

input
```
Integrate[((a + b*x)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output

```
(-2*(-(d^2*Sqrt[-c + (d*e)/f]*(15*A*b*d^2*f^2*h^2 - 10*a*C*d*f*h*(d*f*g +
d*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*(8*f^2*g
^2 + 7*e*f*g*h + 8*e^2*h^2)))*(e + f*x)*(g + h*x)) + C*d^2*Sqrt[-c + (d*e)
/f]*f*h*(c + d*x)*(e + f*x)*(g + h*x)*(4*b*c*f*h - 5*a*d*f*h + b*d*(4*f*g
+ 4*e*h - 3*f*h*x)) - I*(d*e - c*f)*h*(15*A*b*d^2*f^2*h^2 - 10*a*C*d*f*h*(
d*f*g + d*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*h) + d^2*
(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2)))*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(
f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-
c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*d*h*(5*a
*d*f*h*(3*A*d*f^2*h + c*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h)) - b*(15*
A*d^2*e*f^2*h^2 + C*(4*c^2*f^2*h*(-(f*g) + e*h) + c*d*f*(-4*f^2*g^2 + e*f*
g*h + 3*e^2*h^2) + d^2*e*(4*f^2*g^2 + 3*e*f*g*h + 8*e^2*h^2))))*(c + d*x)^
(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*
EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*
e*h - c*f*h)]))/(15*d^4*Sqrt[-c + (d*e)/f]*f^3*h^3*Sqrt[c + d*x]*Sqrt[e +
f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2104, 25, 2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \ 2104$$

$$\frac{\int -\frac{-2C(adfh-2b(dfg+deh+cfh))x^2-(5Abdfh-3bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+2bcCeg-5aAdfh+aC(deg+cfg+ceh)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{5dfh}+\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}$$

$$\Big\downarrow \ 25$$

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-\frac{\int \frac{-2C(adfh-2b(dfg+deh+cfh))x^2-(5Abdfh-3bC(deg+cfg+ceh)-2aC(dfg+deh+cfh))x+2bcCeg-5aAdfh+aC(deg+cfg+ceh)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{5dfh}$$

$\downarrow$ 2118

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-\frac{2\int -\frac{d\left(5adfh(3Adfh-C(deg+cfg+ceh))+2bC\left(2fh(fg+eh)c^2+d\left(2f^2g^2+3efhg+2e^2h^2\right)c+2d^2eg(fg+eh)\right)+\left(15Abd^2f^2h^2-10aCdf(dfg+deh+cfh)h+bC\left(\left(8 \cdots\right.\right.\right.\right.}{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3d^2fh} \cdots}{5dfh}$$

$\downarrow$ 27

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-\frac{-\frac{\int \frac{5adfh(3Adfh-C(deg+cfg+ceh))+2bC\left(2fh(fg+eh)c^2+d\left(2f^2g^2+3efhg+2e^2h^2\right)c+2d^2eg(fg+eh)\right)+\left(15Abd^2f^2h^2-10aCdf(dfg+deh+cfh)h+bC\left(\left(8f^2g^2 \cdots\right.\right.\right.}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3dfh} \cdots}{5dfh}$$

$\downarrow$ 176

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-\frac{-\frac{\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-eh)+cdh\left(-4e^2h^2+efgh+3f^2g^2\right)+d^2g\left(4e^2h^2+3efgh+8f^2g^2\right)\right)\right)\right)\int \frac{\cdots}{\sqrt{c+d \cdots}}}{h}}{3dfh} \cdots}{\cdots}$$

$\downarrow$ 124

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-\frac{-\frac{\frac{\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-eh)+cdh\left(-4e^2h^2+efgh+3f^2g^2\right)+d^2g\left(4e^2h^2+3efgh+8f^2g^2\right)\right)\right)\right)\int \frac{\cdots}{\sqrt{c+d \cdots}}}{h} \cdots}{\cdots}}{\cdots}$$

$\downarrow$ 123

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-\frac{-\frac{\frac{\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-eh)+cdh\left(-4e^2h^2+efgh+3f^2g^2\right)+d^2g\left(4e^2h^2+3efgh+8f^2g^2\right)\right)\right)\right)\int \frac{\cdots}{\sqrt{c+d \cdots}}}{h} \cdots}{\cdots}}{\cdots}$$

$\downarrow$ 131

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-$$

$$-\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-eh)+cdh\left(-4e^2h^2+efgh+3f^2g^2\right)+d^2g\left(4e^2h^2+3efgh+8f^2g^2\right)\right)\right.\right.}{h\sqrt{e+fx}}}{}$$

$\downarrow$ 131

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-$$

$$-\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-eh)+cdh\left(-4e^2h^2+efgh+3f^2g^2\right)+d^2g\left(4e^2h^2+3efg\right.\right.\right.\right.}{h\sqrt{e+fx}\sqrt{g+hx}}}{}$$

$\downarrow$ 130

$$\frac{2C(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{5dfh}-$$

$$-\frac{\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)\left(5adfh\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)-b\left(15Ad^2f^2gh^2+C\left(4c^2fh^2(fg-\right.\right.\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}}{}$$

input
```
Int[((a + b*x)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output

```
(2*C*(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*d*f*h) - ((-4
*C*(a*d*f*h - 2*b*(d*f*g + d*e*h + c*f*h))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x])/(3*d*f*h) - ((2*Sqrt[-(d*e) + c*f]*(15*A*b*d^2*f^2*h^2 - 10*a*
C*d*f*h*(d*f*g + d*e*h + c*f*h) + b*C*(8*c^2*f^2*h^2 + 7*c*d*f*h*(f*g + e*
h) + d^2*(8*f^2*g^2 + 7*e*f*g*h + 8*e^2*h^2)))*Sqrt[(d*(e + f*x))/(d*e - c
*f)]*Sqrt[g + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) +
c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(
d*(g + h*x))/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(5*a*d*f*h*(3*A*d*f*h^2
 + c*C*h*(f*g - e*h) + C*d*g*(2*f*g + e*h)) - b*(15*A*d^2*f^2*g*h^2 + C*(4
*c^2*f*h^2*(f*g - e*h) + c*d*h*(3*f^2*g^2 + e*f*g*h - 4*e^2*h^2) + d^2*g*(
8*f^2*g^2 + 3*e*f*g*h + 4*e^2*h^2))))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt
[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[
-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*
x]*Sqrt[g + h*x]))/(3*d*f*h))/(5*d*f*h)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2104
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f
*g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
 c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
 1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 6.38 (sec) , antiderivative size = 824, normalized size of antiderivative = 1.36

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2Cbx\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{5dfh}+\frac{2\left(Ca-\frac{2Cb(2cfh+2deh+2dfg)}{5dfh}\right)\sqrt{dfh\,x^3+cfh\,x \ldots}}{3dfh \ldots}\right.}{\ldots}$
default	Expression too large to display

input
```
int((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2/5*C*b/d/f/h*x*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+
d*e*g*x+c*e*g)^(1/2)+2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2*d*e*h+2*d*f*g))/d/f
/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)
^(1/2)+2*(A*a-2/5*C*b/d/f/h*c*e*g-2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2*d*e*h+
2*d*f*g))/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(c/d-e/f)*((x+c/d)/(c/d-e
/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^
3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*Ellip
ticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))+2*(A*b-2/5*C
*b/d/f/h*(3/2*c*e*h+3/2*c*f*g+3/2*d*e*g)-2/3*(C*a-2/5*C*b/d/f/h*(2*c*f*h+2
*d*e*h+2*d*f*g))/d/f/h*(c*f*h+d*e*h+d*f*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^
(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f
*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h
)*EllipticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*E
llipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 1068, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="fricas")
```

output
```
2/45*(3*(3*C*b*d^3*f^3*h^3*x - 4*C*b*d^3*f^3*g*h^2 - (4*C*b*d^3*e*f^2 + (4
*C*b*c*d^2 - 5*C*a*d^3)*f^3)*h^3)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g
) - (8*C*b*d^3*f^3*g^3 + (3*C*b*d^3*e*f^2 + (3*C*b*c*d^2 - 10*C*a*d^3)*f^3
)*g^2*h + (3*C*b*d^3*e^2*f + (3*C*b*c*d^2 - 5*C*a*d^3)*e*f^2 + (3*C*b*c^2*
d - 5*C*a*c*d^2 + 15*A*b*d^3)*f^3)*g*h^2 + (8*C*b*d^3*e^3 + (3*C*b*c*d^2 -
 10*C*a*d^3)*e^2*f + (3*C*b*c^2*d - 5*C*a*c*d^2 + 15*A*b*d^3)*e*f^2 + (8*C
*b*c^3 - 10*C*a*c^2*d + 15*A*b*c*d^2 - 45*A*a*d^3)*f^3)*h^3)*sqrt(d*f*h)*w
eierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 -
 c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^
2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (
2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3),
 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h)) - 3*(8*C*b*d^3*f^3*g^2*h
 + (7*C*b*d^3*e*f^2 + (7*C*b*c*d^2 - 10*C*a*d^3)*f^3)*g*h^2 + (8*C*b*d^3*e
^2*f + (7*C*b*c*d^2 - 10*C*a*d^3)*e*f^2 + (8*C*b*c^2*d - 10*C*a*c*d^2 + 15
*A*b*d^3)*f^3)*h^3)*sqrt(d*f*h)*weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*
f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27
*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2
*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2
*c^3*f^3)*h^3)/(d^3*f^3*h^3), weierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*
e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), ...
```

Sympy [F]

$$\int \frac{(a+bx)(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)(a+bx)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),
x)
```

output
```
Integral((A + C*x**2)*(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)
), x)
```

Maxima [F]

$$\int \frac{(a+bx)\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cx^2+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((C*x^2 + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

Giac [F]

$$\int \frac{(a+bx)\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cx^2+A)(bx+a)}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((C*x^2 + A)*(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(C\,x^2+A)\,(a+b\,x)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + C*x^2)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int(((A + C*x^2)*(a + b*x))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{too large to display}$$

input
```
int((b*x+a)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
(10*sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*a*b*d*f*h - 6*sqrt(g + h*x)*
sqrt(e + f*x)*sqrt(c + d*x)*b*c**2*e*h - 6*sqrt(g + h*x)*sqrt(e + f*x)*sqr
t(c + d*x)*b*c**2*f*g + 4*sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*b*c**2
*f*h*x - 6*sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*b*c*d*e*g + 4*sqrt(g
+ h*x)*sqrt(e + f*x)*sqrt(c + d*x)*b*c*d*e*h*x + 4*sqrt(g + h*x)*sqrt(e +
f*x)*sqrt(c + d*x)*b*c*d*f*g*x - 15*int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(
c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f**2*g*h*x + c**2*f*
*2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*g**2 + 3*c*d*e*f*g
*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f**2*g*h*x**2 + c*d*f
**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 + d**2*e*f*g**2*x +
2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g**2*x**2 + d**2*f**2
*g*h*x**3),x)*a*b*c*d**2*f**3*h**3 - 15*int((sqrt(g + h*x)*sqrt(e + f*x)*s
qrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f**2*g*h*x + c**
2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*g**2 + 3*c*d*e
*f*g*h*x + 2*c*d*e*f*h**2*x**2 + c*d*f**2*g**2*x + 2*c*d*f**2*g*h*x**2 + c
*d*f**2*h**2*x**3 + d**2*e**2*g*h*x + d**2*e**2*h**2*x**2 + d**2*e*f*g**2*
x + 2*d**2*e*f*g*h*x**2 + d**2*e*f*h**2*x**3 + d**2*f**2*g**2*x**2 + d**2*
f**2*g*h*x**3),x)*a*b*d**3*e*f**2*h**3 - 15*int((sqrt(g + h*x)*sqrt(e + f*
x)*sqrt(c + d*x)*x**2)/(c**2*e*f*g*h + c**2*e*f*h**2*x + c**2*f**2*g*h*x +
 c**2*f**2*h**2*x**2 + c*d*e**2*g*h + c*d*e**2*h**2*x + c*d*e*f*g**2 + ...
```

3.29 $\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 367

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh} - \frac{4C\sqrt{-de+cf}(dfg+deh+cfh)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} + \frac{2\sqrt{-de+cf}(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+}}{\sqrt{-de+}}\right.\right.}{3d^2f^{3/2}h^2\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2/3*C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-4/3*C*(c*f-d*e)^(1/2
)*(c*f*h+d*e*h+d*f*g)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE
(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/
d^2/f^(3/2)/h^2/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+2/3*(c*f-d*e)^(
1/2)*(3*A*d*f*h^2+c*C*h*(-e*h+f*g)+C*d*g*(e*h+2*f*g))*(d*(f*x+e)/(-c*f+d*e
))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f
-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/d^2/f^(3/2)/h^2/(f*x+e)^(1/
2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.87 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.06

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$= \frac{\sqrt{c+dx}\left(2Cd^2fh(e+fx)(g+hx) - \frac{4Cd^2(dfg+deh+cfh)(e+fx)(g+hx)}{c+dx} - 4iC\sqrt{-c+\frac{de}{f}}fh(dfg+deh+cfh\right.}{}$$

input
```
Integrate[(A + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(Sqrt[c + d*x]*(2*C*d^2*f*h*(e + f*x)*(g + h*x) - (4*C*d^2*(d*f*g + d*e*h
+ c*f*h)*(e + f*x)*(g + h*x))/(c + d*x) - (4*I)*C*Sqrt[-c + (d*e)/f]*f*h*(
d*f*g + d*e*h + c*f*h)*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqr
t[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f]/Sqrt
[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + ((2*I)*d*h*(3*A*d*f^2*h + c
*C*f*(-(f*g) + e*h) + C*d*e*(f*g + 2*e*h))*Sqrt[c + d*x]*Sqrt[(d*(e + f*x)
)/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqr
t[-c + (d*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)])/Sqrt[-c
+ (d*e)/f]))/(3*d^3*f^2*h^2*Sqrt[e + f*x]*Sqrt[g + h*x])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2118, 27, 176, 124, 123, 131, 131, 130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{2118}$$

$$\frac{2\int \frac{d(3Adfh-C(deg+cfg+ceh)-2C(dfg+deh+cfh)x)}{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3d^2fh}+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3Adfh-C(deg+cfg+ceh)-2C(dfg+deh+cfh)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{3dfh}+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{176}$$

$$\frac{\frac{(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{2C(cfh+deh+dfg)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{h}}{3dfh}+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{124}$$

$$\frac{\frac{(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{2C\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}(cfh+deh+dfg)\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}}{3dfh}+\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{123}$$

$$\frac{\frac{(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{h}-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(cfh+deh+dfg)E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-}{f(dg-}\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}}{3dfh}$$
$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{131}$$

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg))\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{h\sqrt{e+fx}}-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(cfh+deh+dfg)E\left(\text{arcs}\right.}{d\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}}{3dfh}$$
$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{131}$$

$$\frac{\frac{\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{h\sqrt{e+fx}\sqrt{g+hx}}-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}(cf}{d\sqrt{f}h}}{3dfh}$$

$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 130$$

$$\frac{\frac{2\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(3Adfh^2+cCh(fg-eh)+Cdg(eh+2fg)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}-\frac{4C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+f}{de-c}}}{}}{3dfh}$$

$$\frac{2C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

input
```
Int[(A + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*C*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) + ((-4*C*Sqrt[-(
d*e) + c*f]*(d*f*g + d*e*h + c*f*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g
 + h*x]*EllipticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*
e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x)
)/(d*g - c*h)]) + (2*Sqrt[-(d*e) + c*f]*(3*A*d*f*h^2 + c*C*h*(f*g - e*h) +
 C*d*g*(2*f*g + e*h))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(
d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]],
((d*e - c*f)*h)/(f*(d*g - c*h))])/(d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[g + h*x]
))/(3*d*f*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.66

method	result
elliptic	$\frac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(\frac{2C\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}{3dfh}+\frac{2\left(A-\frac{2C\left(\frac{1}{2}ceh+\frac{1}{2}cfg+\frac{1}{2}deg\right)}{3dfh}\right)\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}}{\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+}}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(2/3*C/d/f/h*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*
g*x+c*e*g)^(1/2)+2*(A-2/3*C/d/f/h*(1/2*c*e*h+1/2*c*f*g+1/2*d*e*g))*(c/d-e/
f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f
))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+
c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(
1/2))-4/3*C/d/f/h*(c*f*h+d*e*h+d*f*g)*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*
((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2
+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*Elli
pticE(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*Ellipti
cF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(321) = 642$.

Time = 0.11 (sec) , antiderivative size = 775, normalized size of antiderivative = 2.11

$$\int \frac{A + Cx^2}{\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="fricas")
```

output
```
2/9*(3*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)*C*d^2*f^2*h^2 + (2*C*d^2*
f^2*g^2 + (C*d^2*e*f + C*c*d*f^2)*g*h + (2*C*d^2*e^2 + C*c*d*e*f + (2*C*c^
2 + 9*A*d^2)*f^2)*h^2)*sqrt(d*f*h)*weierstrassPInverse(4/3*(d^2*f^2*g^2 -
(d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2)
, -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f -
4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*
f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), 1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*
h)/(d*f*h)) + 6*(C*d^2*f^2*g*h + (C*d^2*e*f + C*c*d*f^2)*h^2)*sqrt(d*f*h)*
weierstrassZeta(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 - c*
d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2 +
 c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2*d
^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3), we
ierstrassPInverse(4/3*(d^2*f^2*g^2 - (d^2*e*f + c*d*f^2)*g*h + (d^2*e^2 -
c*d*e*f + c^2*f^2)*h^2)/(d^2*f^2*h^2), -4/27*(2*d^3*f^3*g^3 - 3*(d^3*e*f^2
 + c*d^2*f^3)*g^2*h - 3*(d^3*e^2*f - 4*c*d^2*e*f^2 + c^2*d*f^3)*g*h^2 + (2
*d^3*e^3 - 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + 2*c^3*f^3)*h^3)/(d^3*f^3*h^3),
1/3*(3*d*f*h*x + d*f*g + (d*e + c*f)*h)/(d*f*h))))/(d^3*f^3*h^3)
```

Sympy [F]

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),x)
```

output
```
Integral((A + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((C*x^2 + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, algorithm
="giac")
```

output
```
integrate((C*x^2 + A)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\begin{aligned}&\int \frac{A+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx\\&=\left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}\,x^2}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx\right)c\\&\quad+\left(\int \frac{\sqrt{hx+g}\,\sqrt{fx+e}\,\sqrt{dx+c}}{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}dx\right)a\end{aligned}$$

input
```
int((C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x)*x**2)/(c*e*g + c*e*h*x + c*
f*g*x + c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*c
+ int((sqrt(g + h*x)*sqrt(e + f*x)*sqrt(c + d*x))/(c*e*g + c*e*h*x + c*f*g
*x + c*f*h*x**2 + d*e*g*x + d*e*h*x**2 + d*f*g*x**2 + d*f*h*x**3),x)*a
```

3.30 $\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 465

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{2C\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}\,E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}} - \frac{2C\sqrt{-de+cf}(bg+ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}} - \frac{2\left(A+\frac{a^2C}{b^2}\right)\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
2*C*(c*f-d*e)^(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f
^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/b/
d/f^(1/2)/h/(f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)-2*C*(c*f-d*e)^(1/2)
*(a*h+b*g)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*Ellip
ticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/
2))/b^2/d/f^(1/2)/h/(f*x+e)^(1/2)/(h*x+g)^(1/2)-2*(A+a^2*C/b^2)*(c*f-d*e)^
(1/2)*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticPi
(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d
*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)/f^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.66 (sec) , antiderivative size = 1036, normalized size of antiderivative = 2.23

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\left(b^2cCd^2e\sqrt{-c+\frac{de}{f}}g-abCd^3e\sqrt{-c+\frac{de}{f}}g-b^2c^2Cd\sqrt{-c+\frac{de}{f}}fg+abcCd^2\sqrt{-c+\frac{de}{f}}fg-b^2c^2Cd\right.}{}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(-2*(b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g - a*b*C*d^3*e*Sqrt[-c + (d*e)/f]*g
 - b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g + a*b*c*C*d^2*Sqrt[-c + (d*e)/f]*f*g
 - b^2*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h + a*b*c*C*d^2*e*Sqrt[-c + (d*e)/f]*h
 + b^2*c^3*C*Sqrt[-c + (d*e)/f]*f*h - a*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h +
 b^2*c*C*d*Sqrt[-c + (d*e)/f]*f*g*(c + d*x) - a*b*C*d^2*Sqrt[-c + (d*e)/f]
*f*g*(c + d*x) + b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h*(c + d*x) - a*b*C*d^2*e*
Sqrt[-c + (d*e)/f]*h*(c + d*x) - 2*b^2*c^2*C*Sqrt[-c + (d*e)/f]*f*h*(c + d
*x) + 2*a*b*c*C*d*Sqrt[-c + (d*e)/f]*f*h*(c + d*x) + b^2*c*C*Sqrt[-c + (d*
e)/f]*f*h*(c + d*x)^2 - a*b*C*d*Sqrt[-c + (d*e)/f]*f*h*(c + d*x)^2 + I*b*C
*(b*c - a*d)*(d*e - c*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x)
)]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (d*e)/f
]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] - I*b*d*(b*c*C*e - a*C*
d*e + a*c*C*f + A*b*d*f)*h*(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x)
)]*Sqrt[(d*(g + h*x))/(h*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (d*e)/f
]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*A*b^2*d^2*f*h*(c +
d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c + d*
x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt[-c + (d
*e)/f]/Sqrt[c + d*x]], (d*f*g - c*f*h)/(d*e*h - c*f*h)] + I*a^2*C*d^2*f*h*
(c + d*x)^(3/2)*Sqrt[(d*(e + f*x))/(f*(c + d*x))]*Sqrt[(d*(g + h*x))/(h*(c
 + d*x))]*EllipticPi[-((b*c*f - a*d*f)/(b*d*e - b*c*f)), I*ArcSinh[Sqrt...
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2110}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx+\int \frac{\frac{Cx}{b}-\frac{aC}{b^2}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{176}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{C\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx}{bh}$$

$$\Big\downarrow \text{124}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{C\sqrt{g+hx}\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{bh\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{123}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2h}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{131}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b^2h\sqrt{e+fx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{131}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{C(ah+bg)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b^2h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{130}$$

$$\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{187}$$

$$-2\left(\frac{a^2C}{b^2}+A\right)\int \frac{1}{(bc-ad-b(c+dx))\sqrt{e-\frac{cf}{d}+\frac{f(c+dx)}{d}}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2\left(\frac{a^2C}{b^2}+A\right)\sqrt{\frac{f(c+dx)}{de-cf}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{g-\frac{ch}{d}+\frac{h(c+dx)}{d}}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}}-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2\left(\frac{a^2C}{b^2}+A\right)\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\int \frac{1}{(bc-ad-b(c+dx))\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}}d\sqrt{c+dx}}{\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2\left(\frac{a^2C}{b^2}+A\right)\sqrt{cf-de}\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\text{EllipticPi}\left(-\frac{b(de-cf)}{(bc-ad)f},\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{\sqrt{f}(bc-ad)\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}-\frac{2C(ah+bg)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}h\sqrt{e+fx}\sqrt{g+hx}}+\frac{2C\sqrt{g+hx}\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{bd\sqrt{f}h\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$

input
```
Int[(A + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
(2*C*Sqrt[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*Elli
pticE[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/
(f*(d*g - c*h))])/(b*d*Sqrt[f]*h*Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c
*h)]) - (2*C*Sqrt[-(d*e) + c*f]*(b*g + a*h)*Sqrt[(d*(e + f*x))/(d*e - c*f)
]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*EllipticF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])
/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/(b^2*d*Sqrt[f]*h*S
qrt[e + f*x]*Sqrt[g + h*x]) - (2*(A + (a^2*C)/b^2)*Sqrt[-(d*e) + c*f]*Sqrt
[1 + (f*(c + d*x))/(d*e - c*f)]*Sqrt[1 + (h*(c + d*x))/(d*g - c*h)]*Ellipt
icPi[-((b*(d*e - c*f))/((b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g - c*h))])/((b*c - a*d)*Sqrt[f]*
Sqrt[e - (c*f)/d + (f*(c + d*x))/d]*Sqrt[g - (c*h)/d + (h*(c + d*x))/d])
```

Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !L
tQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176

```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 187

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2110

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 7.22 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.61

method	result
elliptic	$\dfrac{\sqrt{(hx+g)(fx+e)(xd+c)}\left(-\dfrac{2Ca\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\frac{x+\frac{e}{f}}{-\frac{c}{d}+\frac{e}{f}}}\,\text{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}},\sqrt{\frac{-\frac{c}{d}+\frac{e}{f}}{-\frac{c}{d}+\frac{g}{h}}}\right)}{b^2\sqrt{dfh\,x^3+cfh\,x^2+deh\,x^2+dfg\,x^2+cehx+cfgx+degx+ceg}}+\dfrac{2C\left(\frac{c}{d}-\frac{e}{f}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{e}{f}}}\sqrt{\frac{x+\frac{g}{h}}{-\frac{c}{d}+\frac{g}{h}}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(-2*C*a/b^2*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)
*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h
*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/(c/d-e/f))^(1/2),((-c/d
+e/f)/(-c/d+g/h))^(1/2))+2*C/b*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h
)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*
x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE((
(x+c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+
c/d)/(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2)))+2*(A*b^2+C*a^2)/b^3*
(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-
c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d
*e*g*x+c*e*g)^(1/2)/(-c/d+a/b)*EllipticPi(((x+c/d)/(c/d-e/f))^(1/2),(-c/d+
e/f)/(-c/d+a/b),((-c/d+e/f)/(-c/d+g/h))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{(a+bx)\,\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/2),
x)
```

output
```
Integral((A + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g + h*x)
), x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((C*x^2 + A)/((b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x + g)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)
),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{(bx+a)\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*x^2+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.31 $\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 738

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)(a+bx)}$$
$$+\frac{\left(Ab+\frac{a^2C}{b}\right)\sqrt{f}\sqrt{-de+cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{g+hx}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right)|\frac{(de-cf)h}{f(dg-ch)}\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}$$
$$+\frac{\sqrt{-de+cf}(a^2Cdf-2abC(de+cf)+b^2(2cCe-Adf))\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+d}}{\sqrt{-de+c}}\right.\right.}{b^2d(bc-ad)\sqrt{f}(be-af)\sqrt{e+fx}\sqrt{g+hx}}$$
$$-\frac{\sqrt{-de+cf}(a^4Cdfh-Ab^4(deg+cfg+ceh)-2a^3bC(dfg+deh+cfh)-2ab^3(2cCeg-Adfg-Ac}{b^2(bc-ad)^2\sqrt{f}(}$$

output
```
-(A*b^2+C*a^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-a*f+
b*e)/(-a*h+b*g)/(b*x+a)+(A*b+a^2*C/b)*f^(1/2)*(c*f-d*e)^(1/2)*(d*(f*x+e)/(
-c*f+d*e))^(1/2)*(h*x+g)^(1/2)*EllipticE(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(
1/2),((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/(-a*d+b*c)/(-a*f+b*e)/(-a*h+b*g)/(
f*x+e)^(1/2)/(d*(h*x+g)/(-c*h+d*g))^(1/2)+(c*f-d*e)^(1/2)*(a^2*C*d*f-2*a*b
*C*(c*f+d*e)+b^2*(-A*d*f+2*C*c*e))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d*(h*x+g)
/(-c*h+d*g))^(1/2)*EllipticF(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2),((-c*f+
d*e)*h/f/(-c*h+d*g))^(1/2))/b^2/d/(-a*d+b*c)/f^(1/2)/(-a*f+b*e)/(f*x+e)^(1
/2)/(h*x+g)^(1/2)-(c*f-d*e)^(1/2)*(a^4*C*d*f*h-A*b^4*(c*e*h+c*f*g+d*e*g)-2
*a^3*b*C*(c*f*h+d*e*h+d*f*g)-2*a*b^3*(-A*c*f*h-A*d*e*h-A*d*f*g+2*C*c*e*g)-
3*a^2*b^2*(A*d*f*h-C*(c*e*h+c*f*g+d*e*g)))*(d*(f*x+e)/(-c*f+d*e))^(1/2)*(d
*(h*x+g)/(-c*h+d*g))^(1/2)*EllipticPi(f^(1/2)*(d*x+c)^(1/2)/(c*f-d*e)^(1/2
),-b*(-c*f+d*e)/(-a*d+b*c)/f,((-c*f+d*e)*h/f/(-c*h+d*g))^(1/2))/b^2/(-a*d+
b*c)^2/f^(1/2)/(-a*f+b*e)/(-a*h+b*g)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.65 (sec) , antiderivative size = 3935, normalized size of antiderivative = 5.33

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
((-(A*b^2) - a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*(a + b*x)) - ((c + d*x)^(3/2)*(A*b^4*c*Sqrt[-c +
 (d*e)/f]*f*h + a^2*b^2*c*C*Sqrt[-c + (d*e)/f]*f*h - a*A*b^3*d*Sqrt[-c + (
d*e)/f]*f*h - a^3*b*C*d*Sqrt[-c + (d*e)/f]*f*h + (A*b^4*c*d^2*e*Sqrt[-c +
(d*e)/f]*g)/(c + d*x)^2 + (a^2*b^2*c*C*d^2*e*Sqrt[-c + (d*e)/f]*g)/(c + d*
x)^2 - (a*A*b^3*d^3*e*Sqrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (a^3*b*C*d^3*e*S
qrt[-c + (d*e)/f]*g)/(c + d*x)^2 - (A*b^4*c^2*d*Sqrt[-c + (d*e)/f]*f*g)/(c
 + d*x)^2 - (a^2*b^2*c^2*C*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a*A*b^
3*c*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x)^2 + (a^3*b*c*C*d^2*Sqrt[-c + (d*
e)/f]*f*g)/(c + d*x)^2 - (A*b^4*c^2*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2
- (a^2*b^2*c^2*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a*A*b^3*c*d^2*e*
Sqrt[-c + (d*e)/f]*h)/(c + d*x)^2 + (a^3*b*c*C*d^2*e*Sqrt[-c + (d*e)/f]*h)
/(c + d*x)^2 + (A*b^4*c^3*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 + (a^2*b^2*c
^3*C*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 - (a*A*b^3*c^2*d*Sqrt[-c + (d*e)/
f]*f*h)/(c + d*x)^2 - (a^3*b*c^2*C*d*Sqrt[-c + (d*e)/f]*f*h)/(c + d*x)^2 +
 (A*b^4*c*d*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (a^2*b^2*c*C*d*Sqrt[-c + (
d*e)/f]*f*g)/(c + d*x) - (a*A*b^3*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) -
(a^3*b*C*d^2*Sqrt[-c + (d*e)/f]*f*g)/(c + d*x) + (A*b^4*c*d*e*Sqrt[-c + (d
*e)/f]*h)/(c + d*x) + (a^2*b^2*c*C*d*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (
a*A*b^3*d^2*e*Sqrt[-c + (d*e)/f]*h)/(c + d*x) - (a^3*b*C*d^2*e*Sqrt[-c ...
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 745, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2108, 25, 2110, 176, 124, 123, 131, 131, 130, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2108}$$

$$\frac{\int -\frac{(2Adfh-C(deg+cfg+ceh))a^2+2b(cCeg-Adfg-Adeh-Acfh)a-(Ca^2+Ab^2)dfhx^2+Ab^2(deg+cfg+ceh)-2(C(dfg+deh+cfh)a^2+b(Adf}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2(bc-ad)(be-af)(bg-ah)}$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 25

$$-\frac{\int \frac{(2Adfh-C(deg+cfg+ceh))a^2+2b(cCeg-Adfg-Adeh-Acfh)a-(Ca^2+Ab^2)dfhx^2+Ab^2(deg+cfg+ceh)-2(C(dfg+deh+cfh)a^2+b(Adf}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2(bc-ad)(be-af)(bg-ah)}$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 2110

$$-\frac{\int \frac{\frac{Cdfha^3}{b^2}-\frac{2Cdfga^2}{b}-\frac{2Cdeha^2}{b}-\frac{2cCfha^2}{b}+2Cdega+2cCfga+2cCeha-Adfha-2bcCeg+\left(-\frac{Cdfha^2}{b}-Abdfh\right)x}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx-\frac{(a^4Cdfh-2a^3bC(cf}{}}{2(bc-ad)(be-}$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 176

$$-\frac{-\frac{(bg-ah)\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2}-df\left(\frac{a^2C}{b}+Ab\right)\int \frac{\sqrt{g+hx}}{\sqrt{c+dx}\sqrt{e+fx}}dx-\frac{(a^4Cdfh-2a^3bC}{}}{2(bc-ad)(be}$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 124

$$-\frac{-\frac{(bg-ah)\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2}-\frac{df\sqrt{g+hx}\left(\frac{a^2C}{b}+Ab\right)\sqrt{\frac{d(e+fx)}{de-cf}}\int \frac{\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}}dx}{\sqrt{e+fx}\sqrt{\frac{d(g+hx)}{dg-ch}}}}{2(bc-}$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 123

$$-\frac{-\frac{(bg-ah)\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b^2}-\frac{(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg}{}}{}$$
$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 131

$$-\frac{-\frac{(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{g+hx}}dx}{b^2\sqrt{e+fx}}-\frac{\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 131

$$-\frac{-\frac{(bg-ah)\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{\frac{de}{de-cf}+\frac{dfx}{de-cf}}\sqrt{\frac{dg}{dg-ch}+\frac{dhx}{dg-ch}}}dx}{b^2\sqrt{e+fx}\sqrt{g+hx}}-\frac{\left(a^4Cdfh-2a^3bC(c\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 130

$$-\frac{-\frac{\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))-2ab^3(-Acfh-Adeh-Adfg+2cCeg)-Ab^4(ceh+cfg+deg)\right)\int \frac{\cdots}{(a+bx)\sqrt{\cdots}}}{b^2}\cdots}{\cdots}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 187

$$-\frac{\frac{2\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))-2ab^3(-Acfh-Adeh-Adfg+2cCeg)-Ab^4(ceh+cfg+deg)\right)\int \frac{\cdots}{(bc-ad-b\cdots}}{b^2}\cdots}{\cdots}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 413

$$-\frac{\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))-2ab^3(-Acfh-Adeh-Adfg+2cCeg)-Ab^4(ceh+cfg+deg\cdots\right.}{b^2\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\cdots}\cdots}{\cdots}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$\downarrow$ 413

$$-\frac{2\sqrt{\frac{f(c+dx)}{de-cf}+1}\sqrt{\frac{h(c+dx)}{dg-ch}+1}\left(a^4Cdfh-2a^3bC(cfh+deh+dfg)-3a^2b^2(Adfh-C(ceh+cfg+deg))-2ab^3(-Acfh-Adeh-Adfg+2cCeg)-Ab^4(c\right.}{b^2\sqrt{\frac{f(c+dx)}{d}-\frac{cf}{d}+e}\sqrt{\frac{h(c+dx)}{d}-\frac{ch}{d}+g}}$$

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 412$$

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{(a+bx)(bc-ad)(be-af)(bg-ah)}-$$

$$-\frac{2(bg-ah)\sqrt{cf-de}\sqrt{\frac{d(e+fx)}{de-cf}}\sqrt{\frac{d(g+hx)}{dg-ch}}\left(a^2Cdf-2abC(cf+de)+b^2(2cCe-Adf)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{cf-de}}\right),\frac{(de-cf)h}{f(dg-ch)}\right)}{b^2d\sqrt{f}\sqrt{e+fx}\sqrt{g+hx}}-\frac{2\sqrt{f}\sqrt{g+hx}\left(\frac{a^2}{b}\right.}{}$$

input
```
Int[(A + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),x]
```

output
```
-(((A*b^2 + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)
*(b*e - a*f)*(b*g - a*h)*(a + b*x))) - ((-2*(A*b + (a^2*C)/b)*Sqrt[f]*Sqrt
[-(d*e) + c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[g + h*x]*EllipticE[Arc
Sin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*(d*g -
 c*h))])/(Sqrt[e + f*x]*Sqrt[(d*(g + h*x))/(d*g - c*h)]) - (2*Sqrt[-(d*e)
+ c*f]*(a^2*C*d*f - 2*a*b*C*(d*e + c*f) + b^2*(2*c*C*e - A*d*f))*(b*g - a*
h)*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[(d*(g + h*x))/(d*g - c*h)]*Ellipti
cF[ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e - c*f)*h)/(f*
(d*g - c*h))])/(b^2*d*Sqrt[f]*Sqrt[e + f*x]*Sqrt[g + h*x]) + (2*Sqrt[-(d*e
) + c*f]*(a^4*C*d*f*h - A*b^4*(d*e*g + c*f*g + c*e*h) - 2*a^3*b*C*(d*f*g +
 d*e*h + c*f*h) - 2*a*b^3*(2*c*C*e*g - A*d*f*g - A*d*e*h - A*c*f*h) - 3*a^
2*b^2*(A*d*f*h - C*(d*e*g + c*f*g + c*e*h)))*Sqrt[1 + (f*(c + d*x))/(d*e -
 c*f)]*Sqrt[1 + (h*(c + d*x))/(d*g - c*h)]*EllipticPi[-((b*(d*e - c*f))/((
b*c - a*d)*f)), ArcSin[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[-(d*e) + c*f]], ((d*e
- c*f)*h)/(f*(d*g - c*h))])/(b^2*(b*c - a*d)*Sqrt[f]*Sqrt[e - (c*f)/d + (f
*(c + d*x))/d]*Sqrt[g - (c*h)/d + (h*(c + d*x))/d]))/(2*(b*c - a*d)*(b*e -
 a*f)*(b*g - a*h))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] &&  !L
tQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

rule 130
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[2*(Rt[-b/d, 2]/(b*Sqrt[(b*e - a*f)/b]))*EllipticF[ArcSin[
Sqrt[a + b*x]/(Rt[-b/d, 2]*Sqrt[(b*c - a*d)/b])], f*((b*c - a*d)/(d*(b*e -
a*f)))], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ
[b/(b*e - a*f), 0] && SimplerQ[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f
*x] && (PosQ[-(b*c - a*d)/d] || NegQ[-(b*e - a*f)/f])
```

rule 131
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x
_)]), x_] :> Simp[Sqrt[b*((c + d*x)/(b*c - a*d))]/Sqrt[c + d*x]   Int[1/(Sq
rt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]*Sqrt[e + f*x]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !GtQ[(b*c - a*d)/b, 0] && Simpler
Q[a + b*x, c + d*x] && SimplerQ[a + b*x, e + f*x]
```

rule 176
```
Int[((g_.) + (h_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*
Sqrt[(e_) + (f_.)*(x_)]), x_] :> Simp[h/f   Int[Sqrt[e + f*x]/(Sqrt[a + b*x
]*Sqrt[c + d*x]), x], x] + Simp[(f*g - e*h)/f   Int[1/(Sqrt[a + b*x]*Sqrt[c
 + d*x]*Sqrt[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && Sim
plerQ[a + b*x, e + f*x] && SimplerQ[c + d*x, e + f*x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 2108
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*
(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(
d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(
m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*
g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^
2 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && I
ntegerQ[2*m] && LtQ[m, -1]
```

rule 2110
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.)*((g_.) + (h_.)*(x_))^(q_.), x_Symbol] :> Simp[PolynomialRem
ainder[Px, a + b*x, x]   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^
q, x], x] + Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*(g + h*x)^q, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n, p
, q}, x] && PolyQ[Px, x] && EqQ[m, -1]
```

Maple [A] (verified)

Time = 20.83 (sec) , antiderivative size = 1269, normalized size of antiderivative = 1.72

method	result	size
elliptic	Expression too large to display	1269
default	Expression too large to display	17460

input
```
int((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
((h*x+g)*(f*x+e)*(d*x+c))^(1/2)/(h*x+g)^(1/2)/(f*x+e)^(1/2)/(d*x+c)^(1/2)*
(1/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+
a*b^2*d*e*g-b^3*c*e*g)*(A*b^2+C*a^2)*(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*
x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)/(b*x+a)+2*(C/b^2-1/2*a/b^2*d*f*h*
(A*b^2+C*a^2)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a
*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g))*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x
+g/h)/(-c/d+g/h))^(1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*
e*h*x^2+d*f*g*x^2+c*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*EllipticF(((x+c/d)/
(c/d-e/f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-d*f*h*(A*b^2+C*a^2)/(a^3*d
*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e
*g-b^3*c*e*g)/b*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(
1/2)*((x+e/f)/(-c/d+e/f))^(1/2)/(d*f*h*x^3+c*f*h*x^2+d*e*h*x^2+d*f*g*x^2+c
*e*h*x+c*f*g*x+d*e*g*x+c*e*g)^(1/2)*((-c/d+g/h)*EllipticE(((x+c/d)/(c/d-e/
f))^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2))-g/h*EllipticF(((x+c/d)/(c/d-e/f))
^(1/2),((-c/d+e/f)/(-c/d+g/h))^(1/2)))+(3*A*a^2*b^2*d*f*h-2*A*a*b^3*c*f*h-
2*A*a*b^3*d*e*h-2*A*a*b^3*d*f*g+A*b^4*c*e*h+A*b^4*c*f*g+A*b^4*d*e*g-C*a^4*
d*f*h+2*C*a^3*b*c*f*h+2*C*a^3*b*d*e*h+2*C*a^3*b*d*f*g-3*C*a^2*b^2*c*e*h-3*
C*a^2*b^2*c*f*g-3*C*a^2*b^2*d*e*g+4*C*a*b^3*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-
a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^3
*(c/d-e/f)*((x+c/d)/(c/d-e/f))^(1/2)*((x+g/h)/(-c/d+g/h))^(1/2)*((x+e/f...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)**(1/
2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)^2\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^2*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^2\,\sqrt{c+d\,x}} \,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/
2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{A+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\left(bx+a\right)^{2}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*x^2+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.32 $\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 1402

$$\int \frac{(a+bx)^{3/2}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

output
```
1/24*(24*A*b^2*d*f*h+3*a^2*C*d*f*h-16*b^2*C*(c*e*h+c*f*g+d*e*g)-22*a*b*C*(
c*f*h+d*e*h+d*f*g)+15*b^2*C*(c*f*h+d*e*h+d*f*g)^2/d/f/h)*(d*x+c)^(1/2)*(f*
x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2/(b*x+a)^(1/2)+1/12*C*(3*a*d*f*h-5*b*(
c*f*h+d*e*h+d*f*g))*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2
)/d^2/f^2/h^2+1/3*C*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2
)/d/f/h-1/24*(-a*f+b*e)^(1/2)*(-c*h+d*g)*(3*a^2*C*d^2*f^2*h^2-22*a*b*C*d*f
*h*(c*f*h+d*e*h+d*f*g)+b^2*(24*A*d^2*f^2*h^2+C*(15*c^2*f^2*h^2+14*c*d*f*h*
(e*h+f*g)+d^2*(15*e^2*h^2+14*e*f*g*h+15*f^2*g^2))))*(f*x+e)^(1/2)*(-(-a*d+
b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^
(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c
*h+d*g))^(1/2))/b/d^3/f^3/(-c*f+d*e)^(1/2)/h^3/(-(-a*d+b*c)*(f*x+e)/(-c*f+
d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+1/24*(-a*d+b*c)*(-a*f+b*e)^(1/2)*(3*a^2*
C*d^2*f^2*h^2+6*a*b*C*d*f*h*(2*c*f*h+2*d*e*h+d*f*g)-b^2*(24*A*d^2*f^2*h^2+
C*(15*c^2*f^2*h^2+2*c*d*f*h*(7*e*h+2*f*g)+d^2*(15*e^2*h^2+4*e*f*g*h+5*f^2*
g^2))))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*Ellip
ticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+
d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/d^3/f^3/(-c*f+d*e)^(1/2)
/h^2/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/24*(-c
*f+d*e)^(1/2)*((a*d*f*h+b*(c*f*h+d*e*h+d*f*g))*(24*A*b^2*d^2*f^2*h^2+3*a^2
*C*d^2*f^2*h^2-16*b^2*C*d*f*h*(c*e*h+c*f*g+d*e*g)-22*a*b*C*d*f*h*(c*f*h...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 39032 vs. $2(1402) = 2804$.

Time = 40.19 (sec) , antiderivative size = 39032, normalized size of antiderivative = 27.84

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 5.29 (sec) , antiderivative size = 1389, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2104, 25, 2103, 2105, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 2104$$

$$\frac{\int -\frac{\sqrt{a+bx}\left(-C(3adfh-5b(dfg+deh+cfh))x^2-2(3Abdfh-2bC(deg+cfg+ceh)-aC(dfg+deh+cfh))x+3bcCeg-6aAdfh+aC(deg+cfg+ceh)\right.}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6dfh}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \; 25$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$\frac{\int \frac{\sqrt{a+bx}\left(-C(3adfh-5b(dfg+deh+cfh))x^2-2(3Abdfh-2bC(deg+cfg+ceh)-aC(dfg+deh+cfh))x+3bcCeg-6aAdfh+aC(deg+cfg+ceh)\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6dfh}$$

$$\Big\downarrow \; 2103$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$\int \frac{-\left(\left(24Ad^2f^2h^2b^2+15C(dfg+deh+cfh)^2b^2-16Cdfh(deg+cfg+ceh)b^2-22aCdfh(dfg+deh+cfh)b+3a^2Cd^2f^2h^2\right)x^2\right)+2\left(C(b(deg+cfg+ceh)+a(dfg+deh+c\right.}{}$$

$$\Big\downarrow \; 2105$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}-\frac{(de-cf)(dg-ch)\left(24Ad^2f^2h^2b^2+\right.}{}$$

$$\Big\downarrow \; 194$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\int\frac{(bdeg+acfh)\left(24Ad^2f^2h^2b^2+15\right.}{}$$

$\downarrow$ 327

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+\right.}{}$$

$\downarrow$ 2101

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+\right.}{}$$

$\downarrow$ 183

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+\right.}{}$$

$\downarrow$ 188

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+\right.}{}$$

$\downarrow$ 321

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+\right.}{}}{}$$

$\downarrow$ 412

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(24Abfhd^2+\frac{3a^2Cfhd^2}{b}-16bC(deg+cfg+ceh)d-22aC(dfg+deh+cfh)d+\frac{15bC(dfg+deh+cfh)^2}{fh}\right)}{\sqrt{c+dx}}+\frac{\sqrt{dg-ch}\sqrt{fg-eh}\left(24Ad^2f^2h^2b^2+\right.}{}}{}$$

input
```
Int[((a + b*x)^(3/2)*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*
x]),x]
```

output
```
(C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) -
(-1/2*(C*(3*a*d*f*h - 5*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[c +
d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(d*f*h) + (-(((24*A*b*d^2*f*h + (3*a^2*C
*d^2*f*h)/b - 16*b*C*d*(d*e*g + c*f*g + c*e*h) - 22*a*C*d*(d*f*g + d*e*h +
 c*f*h) + (15*b*C*(d*f*g + d*e*h + c*f*h)^2)/(f*h))*Sqrt[a + b*x]*Sqrt[e +
 f*x]*Sqrt[g + h*x])/Sqrt[c + d*x]) + (Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(24
*A*b^2*d^2*f^2*h^2 + 3*a^2*C*d^2*f^2*h^2 - 16*b^2*C*d*f*h*(d*e*g + c*f*g +
 c*e*h) - 22*a*b*C*d*f*h*(d*f*g + d*e*h + c*f*h) + 15*b^2*C*(d*f*g + d*e*h
 + c*f*h)^2)*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c
+ d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h
]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(
b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x
]) + ((-2*d*(b*e - a*f)*Sqrt[b*g - a*h]*(3*a^2*C*d^2*f^2*h^2 + 6*a*b*C*d*f
*h*(c*f*h + 2*d*(f*g + e*h)) - b^2*(24*A*d^2*f^2*h^2 + C*(5*c^2*f^2*h^2 +
4*c*d*f*h*(f*g + e*h) + d^2*(15*f^2*g^2 + 14*e*f*g*h + 15*e^2*h^2))))*Sqrt
[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[
ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -
(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*Sqrt[f*g - e*h]
*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) +
 (2*Sqrt[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(24*A*b^2...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2104
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f
*g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
 c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
 1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 21.86 (sec) , antiderivative size = 2228, normalized size of antiderivative = 1.59

method	result	size
elliptic	Expression too large to display	2228
default	Expression too large to display	88315

input

```
int((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output

```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(1/3*C*b/d/f/h*x*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^
3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+
b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(
1/2)+1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*e*h+5/2*b
*g*d*f))/h/d/b/f*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*
x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^
2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2*(a^2*A-1/3*C*b/
d/f/h*a*c*e*g-1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*
e*h+5/2*b*g*d*f))/h/d/b/f*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g
))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d
+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+
c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))
^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*
(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(2*a*b*A-1/3*C*b/d/f/h*(3/2*a*c*e
*h+3/2*a*c*f*g+3/2*a*d*e*g+3/2*b*c*e*g)-1/2*(2*C*a*b-1/3*C*b/d/f/h*(5/2*a*
d*f*h+5/2*b*c*f*h+5/2*b*d*e*h+5/2*b*g*d*f))/h/d/b/f*(a*c*f*h+a*d*e*h+a*d*f
*g+b*c*e*h+b*c*f*g+b*d*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/
d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g
/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)\,(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Integral((A + C*x**2)*(a + b*x)**(3/2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
 + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cx^2+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(C\,x^2+A)\,(a+b\,x)^{3/2}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + C*x^2)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*
x)^(1/2)),x)
```

output
```
int(((A + C*x^2)*(a + b*x)^(3/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*
x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(bx+a)^{\frac{3}{2}}\,(C\,x^2+A)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^(3/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.33 $\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 944

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{C(adfh-3b(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{4d^2f^2h^2\sqrt{a+bx}}$$
$$+ \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{C\sqrt{be-af}(dg-ch)(adfh-3b(dfg+deh+cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)\right.}{4bd^2f^2\sqrt{de-cf}h^2\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$+ \frac{C(bc-ad)\sqrt{be-af}(adfh+b(dfg+3deh+3cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-}}{\sqrt{de-}}\right.\right.}{4b^2d^2f^2\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$- \frac{\sqrt{de-cf}(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abdfh-C(b(deg}{4}$$

output
```
1/4*C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g
)^(1/2)/d^2/f^2/h^2/(b*x+a)^(1/2)+1/2*C*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e
)^(1/2)*(h*x+g)^(1/2)/d/f/h-1/4*C*(-a*f+b*e)^(1/2)*(-c*h+d*g)*(a*d*f*h-3*b
*(c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a
))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)
^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/d^2/f^2/(-c*
f+d*e)^(1/2)/h^2/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1
/2)+1/4*C*(-a*d+b*c)*(-a*f+b*e)^(1/2)*(a*d*f*h+b*(3*c*f*h+3*d*e*h+d*f*g))*
(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a
*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a
*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/d^2/f^2/(-c*f+d*e)^(1/2)/h/(-(-a
*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/4*(-c*f+d*e)^(1/
2)*(C*(a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g))-4*
b*d*f*h*(2*A*b*d*f*h-C*(b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))))*(b*
x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(
-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d
*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(
-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/d^3/f^2/(-a*f+b*e)^(1/2)/h^2/(f*x+e)^(1/2
)/(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 16972 vs. 2(944) = 1888.

Time = 36.18 (sec) , antiderivative size = 16972, normalized size of antiderivative = 17.98

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 2.60 (sec) , antiderivative size = 941, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2104, 2105, 25, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\left(A+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \; 2104$$

$$\frac{\int \frac{C(adfh-3b(dfg+deh+cfh))x^2+2(2Abdfh-C(b(deg+cfg+ceh)+a(dfg+deh+cfh)))x+4aAdfh-C(bceg+a(deg+cfg+ceh))}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{4dfh}+\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 2105$$

$$\frac{\int -\frac{C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))-2bdfh(4aAdfh-C(bceg+a(deg+cfg+ceh)))+(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abd}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))+(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abc}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \; 194$$

$$-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))+(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abc\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}\cdots$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{327}$$

$$-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+C(bdeg+acfh)(adfh-3b(dfg+deh+cfh))+(C(adfh-3b(dfg+deh+cfh))(adfh+b(dfg+deh+cfh))-4bdfh(2Abc\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}\cdots$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{2101}$$

$$-\frac{\frac{(C(adfh-3b(cfh+deh+dfg))(adfh+b(cfh+deh+dfg))-4bdfh(2Abdfh-C(a(cfh+deh+dfg)+b(ceh+cfg+deg))))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{Cd(be-af)(bg\cdots}{\cdots}}{2bdfh}\cdots$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{183}$$

$$-\frac{\frac{2(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(C(adfh-3b(cfh+deh+dfg))(adfh+b(cfh+deh+dfg))-4bdfh(2Abdfh-C(a(cfh+deh+dfg)+b(ceh+cfg+deg))))\cdots}{b\sqrt{c+dx}\sqrt{e+fx}}\cdots}{2bdf\cdots}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{188}$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(adfh-3b(dfg+deh+cfh))}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g\cdots}}{\cdots}$$

$$\Big\downarrow \text{321}$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(adfh-3b(dfg+deh+cfh))}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g}}{}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{C\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(adfh-3b(dfg+deh+cfh))}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g}}{}$$

input
```
Int[(Sqrt[a + b*x]*(A + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h) + ((
C*(a*d*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt
[g + h*x])/(b*f*h*Sqrt[c + d*x]) - (C*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(a*d
*f*h - 3*b*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g +
 h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e
+ f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e
- a*f)*(d*g - c*h))])/(b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(
c + d*x))]*Sqrt[g + h*x]) - ((-2*C*d*(b*e - a*f)*Sqrt[b*g - a*h]*(b*c*f*h
+ a*d*f*h + 3*b*d*(f*g + e*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(
a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/
(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)
*(b*g - a*h)))])/(b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g +
 h*x))/((f*g - e*h)*(a + b*x)))]) + (2*Sqrt[-(d*g) + c*h]*(C*(a*d*f*h - 3*
b*(d*f*g + d*e*h + c*f*h))*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)) - 4*b*d*f
*h*(2*A*b*d*f*h - C*(b*(d*e*g + c*f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h)
)))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[(
(b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h
))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) +
 c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))
])/(b*Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x]))/(2*b*d*f*h))/(4*d...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2104
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)
*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Sim
p[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(d*f*h*(2*m +
3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a*(d*e*g + c*f
*g + c*e*h) + 2*b*c*e*g*m) + (A*b*d*f*h*(2*m + 3) - C*(2*a*(d*f*g + d*e*h +
 c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + 2*C*(a*d*f*h*m - b*(m +
 1)*(d*f*g + d*e*h + c*f*h))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, A, C}, x] && IntegerQ[2*m] && GtQ[m, 0]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1793 vs. $2(861) = 1722$.

Time = 19.50 (sec) , antiderivative size = 1794, normalized size of antiderivative = 1.90

method	result	size
elliptic	Expression too large to display	1794
default	Expression too large to display	40776

input
```
int((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(1/2*C/h/d/f*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*
d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*
f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)
+2*(A*a-1/2*C/h/d/f*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(e/
f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*
(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^
(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)
*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g
/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(A*b-1/2*C/h/d/f*(a*c*f*h+a*d*e*h+a*d*f
*g+b*c*e*h+b*c*f*g+b*d*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/
d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g
/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h
)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+
g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/
d-g/h)*EllipticPi(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/
(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)))+(C*a-1/2*C/h
/d/f*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*b*g*d*f))*((x+g/h)*(x+a/b)*(
x+e/f)+(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((
-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(A+Cx^2)\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(C*x**2+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Integral((A + C*x**2)*sqrt(a + b*x)/(sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g +
h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(C\,x^2+A)\,\sqrt{a+b\,x}}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((A + C*x^2)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*
x)^(1/2)),x)
```

output
```
int(((A + C*x^2)*(a + b*x)^(1/2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(c + d*
x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{bx+a}\,(C\,x^2+A)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((b*x+a)^(1/2)*(C*x^2+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.34 $\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 767

$$\begin{aligned}\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx &= \frac{C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{dfh\sqrt{a+bx}} \\ &- \frac{C\sqrt{be-af}(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{bdf\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} \\ &+ \frac{(a^2Cdf+abC(de+cf)-b^2(cCe-2Adf))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)\right.}{b^2df\sqrt{be-af}\sqrt{de-cf}\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}} \\ &- \frac{C\sqrt{de-cf}(adfh+b(dfg+deh+cfh))(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\mathrm{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)},\right.}{b^2d^2f\sqrt{be-af}h\sqrt{e+fx}\sqrt{g+hx}}\end{aligned}$$

output
```
C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h/(b*x+a)^(1/2)-C*(-a*f+b*
e)^(1/2)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))
^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(
1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/d/f/(-c*f+d*e)
^(1/2)/h/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+(a^2
*C*d*f+a*b*C*(c*f+d*e)-b^2*(-2*A*d*f+C*c*e))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h
*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(
-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g)
)^(1/2))/b^2/d/f/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-(-a*d+b*c)*(f*x+e)/(-
c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-C*(-c*f+d*e)^(1/2)*(a*d*f*h+b*(c*f*h
+d*e*h+d*f*g))*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-
a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*
x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f
+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/d^2/f/(-a*f+b*e)^(1/2)/
h/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6321 vs. $2(767) = 1534$.

Time = 34.56 (sec) , antiderivative size = 6321, normalized size of antiderivative = 8.24

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2106, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2106}$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)-C(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{C(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{194}$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)-C(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-\frac{C\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)-C(adfh+b(dfg+deh+cfh))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{2101}$$

$$\frac{\frac{d(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{C(adfh+b(cfh+deh+dfg))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}}{2bdfh}-$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 183$$

$$\frac{\frac{d(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(adfh+b(cfh+deh+a\cdots}{b\sqrt{c+d\cdots}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 188$$

$$\frac{\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh))\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2C(a+bx)\sqrt{\frac{(c+dx)(bg\cdots}{(a+bx)(dg\cdots}}\cdots}{\cdots}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2C(a+bx)\sqrt{\frac{(c+d}{(a+b}}}{}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow 412$$

$$\frac{\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}(a^2Cfh+abC(eh+fg)-b^2(Ceg-2Afh))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}-\frac{2C(a+bx)\sqrt{ch-d}}{}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

input
```
Int[(A + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]),
x]
```

output
```
(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*x]) - (C*S
qrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x)
)/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x
])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f
)*(d*g - c*h))])/(b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d
*x))]*Sqrt[g + h*x]) + ((2*d*(a^2*C*f*h + a*b*C*(f*g + e*h) - b^2*(C*e*g -
 2*A*f*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g +
h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqr
t[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*
Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x
))/((f*g - e*h)*(a + b*x)))]) - (2*C*Sqrt[-(d*g) + c*h]*(a*d*f*h + b*(d*f*
g + d*e*h + c*f*h))*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a
 + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi
[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x]
)/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a
*d)*(f*g - e*h))])/(b*Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x]))/(2*b
*d*f*h)
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2106
```
Int[((A_.) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_
)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[C*
Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x])), x] + (Si
mp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) - C*(a*d*f*h + b*(d*f*g
+ d*e*h + c*f*h))*x, x], x], x] + Simp[C*(d*e - c*f)*((d*g - c*h)/(2*b*d*f*
h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[e + f*x]*Sqrt[g + h*x]), x],
x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x]
```

Maple [A] (verified)

Time = 21.71 (sec) , antiderivative size = 1065, normalized size of antiderivative = 1.39

method	result	size
elliptic	Expression too large to display	1065
default	Expression too large to display	16135

input
```
int((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*A*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d)
)^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h
)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*
(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+
c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+C*((x+g/h)*
(x+a/b)*(x+e/f)+(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+
c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(
-e/f+g/h)/(x+c/d))^(1/2)*((c/d*g/h-e/f*g/h+c*e/d/f+c^2/d^2)/(c/d-e/f)/(-c/
d+g/h)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*
(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(a/b-g/h)*EllipticE(((c/d-e/f)*(x+g
/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^
(1/2))/(-c/d+g/h)+(a*d*f*h+b*c*f*h+b*d*e*h+b*d*f*g)/h/d/b/f/(c/d-e/f)*Elli
pticPi(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(e/f-g/h)/(-c/d+e/f),(
(-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))))/(h*d*b*f*(x+g/h)*(x+c/
d)*(x+a/b)*(x+e/f))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Integral((A + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g +
h*x)), x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x
+ g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)
^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c + d*x)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,x^2+A}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*x^2+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.35 $\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 705

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2(Ab^2+a^2C)\,(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{b(bc-ad)\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$-\frac{2(2abCg+Ab^2h-a^2Ch)\,\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{b^2\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$+\frac{2C\sqrt{de-cf}(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)},\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(b}{(be-af)(d}\right.}{b^2d\sqrt{be-af}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
-2*(A*b^2+C*a^2)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/
(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(
b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/(-a*d+
b*c)/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x+e)/(-c
*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2*(A*b^2*h-C*a^2*h+2*C*a*b*g)*(f*x+e)
^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)
^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)
/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h
+b*g)/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+2*C*(-c
*f+d*e)^(1/2)*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a
*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x
+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+
d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/d/(-a*f+b*e)^(1/2)/(f*x+
e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 31.84 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.02

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = -\frac{2(be-af)\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}(e+fx)^{3/2}(g+hx)^{3/2}\left(2aC(-bc+ad)h(-bg+ah)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(-be+}{(fg-}}\right.\right.\right.}{}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
(-2*(b*e - a*f)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*(e +
 f*x)^(3/2)*(g + h*x)^(3/2)*(2*a*C*(-(b*c) + a*d)*h*(-(b*g) + a*h)*Ellipti
cF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b
*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] - A*b^2*h*(b*(d*g -
c*h)*EllipticE[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*
x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] + d*(-(b
*g) + a*h)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(
a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]) -
 a^2*C*h*(b*(d*g - c*h)*EllipticE[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/(
(f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*
g - c*h))] + d*(-(b*g) + a*h)*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g + h
*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*
f)*(d*g - c*h))]) + C*(b*c - a*d)*(b*g - a*h)^2*EllipticPi[(b*(-(f*g) + e*
h))/((b*e - a*f)*h), ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(
a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]))/
(b^2*(b*c - a*d)*h*(f*g - e*h)^3*(a + b*x)^(5/2)*Sqrt[c + d*x]*(-(((b*e -
a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(a + b*x)^2)))^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 2.34 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2108, 25, 2105, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{ 2108}$$

$$\frac{\int -\frac{-2(Ca^2+Ab^2)dfhx^2-(2C(dfg+deh+cfh)a^2+b(Adfh-C(deg+cfg+ceh))a+b^2(cCeg+Adfg+Adeh+Acfh))x+a(aAdfh-aC(deg+cfg+}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{-2(Ca^2+Ab^2)dfhx^2-(2C(dfg+deh+cfh)a^2+b(Adfh-C(deg+cfg+ceh))a+b^2(cCeg+Adfg+Adeh+Acfh))x+a(aAdfh-aC(deg+cfg+}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 2105

$$-\frac{-\frac{(a^2C+Ab^2)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{\int \frac{2dfh((acC+Abd)(be-af)(bg-ah)-C(bc-ad)(be-af)(bg-ah)x)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-\frac{2d\sqrt{a+bx}}{}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 27

$$-\frac{-\frac{(a^2C+Ab^2)(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{\int \frac{(acC+Abd)(be-af)(bg-ah)-C(bc-ad)(be-af)(bg-ah)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{2d\sqrt{a+bx}\sqrt{e+f}}{b}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 194

$$-\frac{\frac{2\sqrt{a+bx}(a^2C+Ab^2)(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{b\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{\int \frac{(acC+Abd)(be-af)(bg-ah)-C(bc-ad)(be-af)(bg-ah)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 327

$$-\frac{\frac{\int \frac{(acC+Abd)(be-af)(bg-ah)-C(bc-ad)(be-af)(bg-ah)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2\sqrt{a+bx}(a^2C+Ab^2)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+}}{\sqrt{fg-eh}\sqrt{c+}}\right.\right.}{b\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\big\downarrow$ 2101

$$-\frac{\frac{\frac{(be-af)(bg-ah)\left(a^2(-C)d+2abcC+Ab^2d\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{C(bc-ad)(be-af)(bg-ah)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}}{b}+\frac{2\sqrt{a+bx}\left(a^2C+\cdots\right.}{\cdots}}{(bc-ad)(be-af)(\cdots}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \ 183$$

$$-\frac{\frac{\frac{(be-af)(bg-ah)\left(a^2(-C)d+2abcC+Ab^2d\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{2C(a+bx)(bc-ad)(be-af)(bg-ah)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{\cdots}{\left(h\cdots\right.}}{b\sqrt{c+dx}\sqrt{e+f\cdots}}}{b}}{\cdots}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \ 188$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$$\frac{-\frac{2\left(Ca^2+Ab^2\right)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}+\frac{2\left(Ca^2+Ab^2\right)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\cdots\right.}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{\cdots}$$

$$\Big\downarrow \ 321$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$$\frac{-\frac{2\left(Ca^2+Ab^2\right)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}+\frac{2\left(Ca^2+Ab^2\right)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\cdots\right.}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{\cdots}$$

$$\Big\downarrow \ 412$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}-$$

$$\frac{-\frac{2\left(Ca^2+Ab^2\right)\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}+\frac{2\left(Ca^2+Ab^2\right)\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\cdots\right.}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{\cdots}$$

input
```
Int[(A + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(-2*(A*b^2 + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) - ((-2*(A*b^2 + a^2*C)*d*Sqrt[a +
 b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*Sqrt[c + d*x]) + (2*(A*b^2 + a^2*C)*
Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x
))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*
x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*
f)*(d*g - c*h))])/(b*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]
*Sqrt[g + h*x]) + ((2*(2*a*b*c*C + A*b^2*d - a^2*C*d)*(b*e - a*f)*Sqrt[b*g
 - a*h]*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x
]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a
 + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*Sqr
t[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a
+ b*x)))]) - (2*C*Sqrt[b*c - a*d]*(b*e - a*f)*(b*g - a*h)*Sqrt[-(d*g) + c*
h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((
b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h)
)/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) +
c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))]
)/(b*h*Sqrt[c + d*x]*Sqrt[e + f*x]))/b)/((b*c - a*d)*(b*e - a*f)*(b*g - a*
h))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2108
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*
(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(
d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(
m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*
g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^
2 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && I
ntegerQ[2*m] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2285 vs. $2(648) = 1296$.

Time = 23.97 (sec) , antiderivative size = 2286, normalized size of antiderivative = 3.24

method	result	size
elliptic	Expression too large to display	2286
default	Expression too large to display	30453

input
```
int((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+
b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-
a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(A*b^2+C*a^2)
/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f
*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*(-C*a/b^2+1/b^2*(a^2*d*f*h-a*b*c*f*h-a*b*
d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(A*b^2+C*a^2)/(a^3*d*f*h-a^
2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*
c*e*g)-(b*c*e*h+b*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*
d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(A*b^2+C*a^2))*(e/f
-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(
x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(
1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*
EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/
h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(C/b-1/b*(a*d*f*h-b*c*f*h-b*d*e*h-b*d*f*
g)*(A*b^2+C*a^2)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*
h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(2*b*c*f*h+2*b*d*e*h+2*b*d*f*g)/(a^3*
d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*
e*g-b^3*c*e*g)/b*(A*b^2+C*a^2))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x
+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Cx^2}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Integral((A + C*x**2)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*sqrt(g
 + h*x)), x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,(a+b\,x)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)
^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c + d*x)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,x^2+A}{(bx+a)^{\frac{3}{2}}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*x^2+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.36 $\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 727

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=-\frac{2(Ab^2+a^2C)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$
$$+\frac{4(dg-ch)\left(Ab^3(deg+cfg+ceh)+a^3C(dfg+deh+cfh)+a^2b(3Adfh-2C(deg+cfg+ceh))-ab^2\right.}{3(bc-ad)^2(be-af)^{3/2}\sqrt{de-cf}(b}$$
$$-\frac{2(3ab(de+cf)\left(Cg^2+Ah^2\right)-a^2(3Adfh^2+cCh(fg-eh)+Cdg(2fg+eh))-b^2(3cCeg^2-Adg(fg-}{3(bc-ad)(be-af)^{3/2}\sqrt{de-cf}(bg-}$$

output
```
-2/3*(A*b^2+C*a^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-
a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)+4/3*(-c*h+d*g)*(A*b^3*(c*e*h+c*f*g+d*e*g
)+a^3*C*(c*f*h+d*e*h+d*f*g)+a^2*b*(3*A*d*f*h-2*C*(c*e*h+c*f*g+d*e*g))-a*b^
2*(2*A*d*(e*h+f*g)-c*(-2*A*f*h+3*C*e*g)))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+
g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*
f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(
1/2))/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)^2/(-(-a*d+
b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2/3*(3*a*b*(c*f+d*e)*
(A*h^2+C*g^2)-a^2*(3*A*d*f*h^2+c*C*h*(-e*h+f*g)+C*d*g*(e*h+2*f*g))-b^2*(3*
c*C*e*g^2-A*d*g*(-e*h+f*g)+A*c*h*(2*e*h+f*g)))*(f*x+e)^(1/2)*(-(-a*d+b*c)*
(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)
/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*
g))^(1/2))/(-a*d+b*c)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)^2/(-(-a
*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11363 vs. $2(727) = 1454$.

Time = 40.46 (sec) , antiderivative size = 11363, normalized size of antiderivative = 15.63

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 3.93 (sec) , antiderivative size = 1057, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2108, 25, 2102, 25, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{2108}$$

$$\frac{\int -\frac{(3Adfh-C(deg+cfg+ceh))a^2+3b(cCeg-Adfg-Adeh-Acfh)a+2Ab^2(deg+cfg+ceh)-(2C(dfg+deh+cfh)a^2+3b(Adfh-C(deg+cfg+ce}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{(3Adfh-C(deg+cfg+ceh))a^2+3b(cCeg-Adfg-Adeh-Acfh)a+2Ab^2(deg+cfg+ceh)-(2C(dfg+deh+cfh)a^2+3b(Adfh-C(deg+cfg+ce}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{3(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{2102}$$

$$-\frac{\int -\frac{-4bdfh\left(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh)\right)x^2-2(adfh+b(dfg+deh+cfh))}{}}{}$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\int \frac{-4bdfh\left(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh)\right)x^2-2(adfh+b(dfg+deh+cfh))}{}}{}$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{2105}$$

$$-\frac{-\frac{\int -\frac{2bdf(be-af)h(bg-ah)\left(-\left(\left(3Ad^2fh-C\left(-2fhc^2-d(fg+eh)c+d^2eg\right)\right)a^2\right)+3b\left(Cc^2+Ad^2\right)(fg+eh)a-b^2\left((3Ceg-Afh)c^2+Ad(fg+eh)c+2Ad^2eg\right)\right)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{-(be-af)(bg-ah)\left(-\left(a^2\left(3Ad^2fh-C\left(-2c^2fh-cd(eh+fg)+d^2eg\right)\right)\right)+3ab\left(Ad^2+c^2C\right)(eh+fg)-b^2\left(c^2(3Ceg-Afh)+Acd(eh+fg)+2Ad^2e\right.}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{188}$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{-\frac{4b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh))}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}}{}$$

$$\Big\downarrow \text{194}$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$\frac{-\frac{4b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh))}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}}{}$$

$$\Big\downarrow \text{321}$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$-\frac{4b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh))}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$\downarrow$ 327

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}-$$

$$-\frac{4b\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(C(dfg+deh+cfh)a^3+b(3Adfh-2C(deg+cfg+ceh))a^2-b^2(2Ad(fg+eh)-c(3Ceg-2Afh))a+Ab^3(deg+cfg+ceh))}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

input
```
Int[(A + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(-2*(A*b^2 + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a
*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) - ((-4*b*(A*b^3*(d*e*g + c*f*
g + c*e*h) + a^3*C*(d*f*g + d*e*h + c*f*h) + a^2*b*(3*A*d*f*h - 2*C*(d*e*g
 + c*f*g + c*e*h)) - a*b^2*(2*A*d*(f*g + e*h) - c*(3*C*e*g - 2*A*f*h)))*Sq
rt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a
*h)*Sqrt[a + b*x]) - ((-4*d*(A*b^3*(d*e*g + c*f*g + c*e*h) + a^3*C*(d*f*g
+ d*e*h + c*f*h) + a^2*b*(3*A*d*f*h - 2*C*(d*e*g + c*f*g + c*e*h)) - a*b^2
*(2*A*d*(f*g + e*h) - c*(3*C*e*g - 2*A*f*h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*
Sqrt[g + h*x])/Sqrt[c + d*x] + (4*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(A*b^3*(
d*e*g + c*f*g + c*e*h) + a^3*C*(d*f*g + d*e*h + c*f*h) + a^2*b*(3*A*d*f*h
- 2*C*(d*e*g + c*f*g + c*e*h)) - a*b^2*(2*A*d*(f*g + e*h) - c*(3*C*e*g - 2
*A*f*h)))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d
*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*S
qrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqr
t[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - (2*(b*
e - a*f)*Sqrt[b*g - a*h]*(3*a*b*(c^2*C + A*d^2)*(f*g + e*h) - b^2*(2*A*d^2
*e*g + A*c*d*(f*g + e*h) + c^2*(3*C*e*g - A*f*h)) - a^2*(3*A*d^2*f*h - C*(
d^2*e*g - 2*c^2*f*h - c*d*(f*g + e*h))))*Sqrt[((b*e - a*f)*(c + d*x))/((d*
e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[
e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2108
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*
(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(
d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(
m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*
g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^
2 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && I
ntegerQ[2*m] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3341 vs. 2(675) = 1350.

Time = 28.05 (sec) , antiderivative size = 3342, normalized size of antiderivative = 4.60

method	result	size
elliptic	Expression too large to display	3342
default	Expression too large to display	112033

input

```
int((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output

```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2/3/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+
a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^2*(A*b^2+C*a^2)*(b*d*f*h*
x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^
2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*
e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+4/3*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*
e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*
b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*
e*g)^2*(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a*b^2*d*f*g+A*
b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*a^3*d*e*h+C*a^3*d*f*g-2*C*
a^2*b*c*e*h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a*b^2*c*e*g)/((x+a/b)*(b*d
*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x
+b*c*e*g))^(1/2)+2*(C/b^2-1/3/b^2*(3*A*a*b^2*d*f*h-A*b^3*c*f*h-A*b^3*d*e*h
-A*b^3*d*f*g+3*C*a^3*d*f*h-C*a^2*b*c*f*h-C*a^2*b*d*e*h-C*a^2*b*d*f*g)/(a^3
*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d
*e*g-b^3*c*e*g)+2/3/b*(a^2*d*f*h-a*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b
^2*c*f*g+b^2*d*e*g)*(3*A*a^2*b*d*f*h-2*A*a*b^2*c*f*h-2*A*a*b^2*d*e*h-2*A*a
*b^2*d*f*g+A*b^3*c*e*h+A*b^3*c*f*g+A*b^3*d*e*g+C*a^3*c*f*h+C*a^3*d*e*h+C*a
^3*d*f*g-2*C*a^2*b*c*e*h-2*C*a^2*b*c*f*g-2*C*a^2*b*d*e*g+3*C*a*b^2*c*e*g)/
(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+...
```

Fricas [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
integral((C*x^2 + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3*a*b^2*d
)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*b^2*d)*
e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a*b^2*c
 + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*h)*x^
3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f + (3*a
^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3*d)*e)
*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)
^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c + d*x)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*x^2+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.37 $\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 1617

$$\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

output
```
-2/5*(A*b^2+C*a^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b*c)/(-
a*f+b*e)/(-a*h+b*g)/(b*x+a)^(5/2)+4/15*(a^4*C*d*f*h+2*A*b^4*(c*e*h+c*f*g+d
*e*g)+a^3*b*C*(c*f*h+d*e*h+d*f*g)+3*a^2*b^2*(2*A*d*f*h-C*(c*e*h+c*f*g+d*e*
g))-a*b^3*(4*A*d*(e*h+f*g)-c*(-4*A*f*h+5*C*e*g)))*(d*x+c)^(1/2)*(f*x+e)^(1
/2)*(h*x+g)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)^2/(-a*h+b*g)^2/(b*x+a)^(3/2)-2/1
5*(-c*h+d*g)*(4*a*(A*b^2+C*a^2)*d*f*h*(3/2*b^2*c*e*g-3/2*a*b*(c*e*h+c*f*g+
d*e*g)+a^2*(c*f*h+d*e*h+d*f*g))+b*(3*a^2*d*f*h+2*b^2*(c*e*h+c*f*g+d*e*g)-3
*a*b*(c*f*h+d*e*h+d*f*g))*(4*A*b^2*(c*e*h+c*f*g+d*e*g)+5*a*b*(-A*c*f*h-A*d
*e*h-A*d*f*g+C*c*e*g)+a^2*(5*A*d*f*h-C*(c*e*h+c*f*g+d*e*g)))+2*a*(3*a*d*f*
h-b*(c*f*h+d*e*h+d*f*g))*(2*A*b^3*(c*e*h+c*f*g+d*e*g)+a^3*C*(c*f*h+d*e*h+d
*f*g)+a^2*b*(5*A*d*f*h-3*C*(c*e*h+c*f*g+d*e*g))-a*b^2*(4*A*d*(e*h+f*g)-c*(
-4*A*f*h+5*C*e*g)))+b*(3*b*c*e*g-a*(c*e*h+c*f*g+d*e*g))*(2*a^3*C*d*f*h+2*a
^2*b*C*(c*f*h+d*e*h+d*f*g)+a*b^2*(7*A*d*f*h-5*C*(c*e*h+c*f*g+d*e*g))-b^3*(
3*A*d*(e*h+f*g)-c*(-3*A*f*h+5*C*e*g))))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)
/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+
d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/
2))/(-a*d+b*c)^3/(-a*f+b*e)^(5/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)^3/(-(-a*d+b*
c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2/15*(5*a^4*d*f*h*(3*A*
d*f*h^2+c*C*h*(-e*h+f*g)+C*d*g*(e*h+2*f*g))+a*b^3*(A*d^2*g*(-11*e^2*h^2+3*
e*f*g*h+8*f^2*g^2)-c^2*h*(10*C*e*g*(e*h+2*f*g)+A*f*h*(19*e*h+11*f*g))-c...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 38948 vs. $2(1617) = 3234$.

Time = 51.31 (sec) , antiderivative size = 38948, normalized size of antiderivative = 24.09

$$\int \frac{A + Cx^2}{(a + bx)^{7/2}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 12.97 (sec) , antiderivative size = 2609, normalized size of antiderivative = 1.61, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2108, 25, 2107, 25, 2102, 25, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 2108$$

$$\frac{\int -\frac{(5Adfh-C(deg+cfg+ceh))a^2+5b(cCeg-Adfg-Adeh-Acfh)a+2\left(Ca^2+Ab^2\right)dfhx^2+4Ab^2(deg+cfg+ceh)-\left(2C(dfg+deh+cfh)a^2+5Ab\right.}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{5(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{5(a+bx)^{5/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{(5Adfh-C(deg+cfg+ceh))a^2+5b(cCeg-Adfg-Adeh-Acfh)a+2\left(Ca^2+Ab^2\right)dfhx^2+4Ab^2(deg+cfg+ceh)-\left(2C(dfg+deh+cfh)a^2+5Ab\right.}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{5(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{5(a+bx)^{5/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \ 2107$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ca^2+Ab^2\right)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$\int -\frac{\left(3dfha^2-3b(dfg+deh+cfh)a+2b^2(deg+cfg+ceh)\right)\left((5Adfh-C(deg+cfg+ceh))a^2+5b(cCeg-Adfg-Adeh-Acfh)a+4Ab^2(deg+cfg+ceh)\right)+(3bceg-a(deg}{}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\int \frac{\left(3dfha^2-3b(dfg+deh+cfh)a+2b^2(deg+cfg+ceh)\right)\left((5Adfh-C(deg+cfg+ceh))a^2+5b(cCeg-Adfg-Adeh-Acfh)a+4Ab^2(deg+cfg+ceh)\right)+(3bceg-a(}{}}{}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(a^2C+Ab^2\right)}{5(a+bx)^{5/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 2102$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$-\frac{4\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Cdfha^4+bC(dfg+deh+cfh)a^3+3b^2(2Adfh-C(deg+cfg+ceh))a^2-b^3(4Ad(fg+eh)-c(5Ceg-4Afh))a+2Ab^4(deg+}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$-\frac{4\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Cdfha^4+bC(dfg+deh+cfh)a^3+3b^2(2Adfh-C(deg+cfg+ceh))a^2-b^3(4Ad(fg+eh)-c(5Ceg-4Afh))a+2Ab^4(deg+}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \; 2105$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$-\frac{4\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Cdfha^4+bC(dfg+deh+cfh)a^3+3b^2(2Adfh-C(deg+cfg+ceh))a^2-b^3(4Ad(fg+eh)-c(5Ceg-4Afh))a+2Ab^4(deg+}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$-\frac{4\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Cdfha^4+bC(dfg+deh+cfh)a^3+3b^2(2Adfh-C(deg+cfg+ceh))a^2-b^3(4Ad(fg+eh)-c(5Ceg-4Afh))a+2Ab^4(deg+}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \; 188$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$-\frac{4\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Cdfha^4+bC(dfg+deh+cfh)a^3+3b^2(2Adfh-C(deg+cfg+ceh))a^2-b^3(4Ad(fg+eh)-c(5Ceg-4Afh))a+2Ab^4(deg+}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \; 194$$

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$-\frac{4\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Cdfha^4+bC(dfg+deh+cfh)a^3+3b^2(2Adfh-C(deg+cfg+ceh))a^2-b^3(4Ad(fg+eh)-c(5Ceg-4Afh))a+2Ab^4(deg+}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 321

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$-\frac{4\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Cdfha^4+bC(dfg+deh+cfh)a^3+3b^2(2Adfh-C(deg+cfg+ceh))a^2-b^3(4Ad(fg+eh)-c(5Ceg-4Afh))a+2Ab^4(deg+}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 327

$$-\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Ca^2+Ab^2)}{5(bc-ad)(be-af)(bg-ah)(a+bx)^{5/2}}-$$

$$-\frac{4\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}(Cdfha^4+bC(dfg+deh+cfh)a^3+3b^2(2Adfh-C(deg+cfg+ceh))a^2-b^3(4Ad(fg+eh)-c(5Ceg-4Afh))a+2Ab^4(deg+}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

input
```
Int[(A + C*x^2)/((a + b*x)^(7/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]
),x]
```

output
```
(-2*(A*b^2 + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(5*(b*c - a
*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(5/2)) - ((-4*(a^4*C*d*f*h + 2*A*b^4
*(d*e*g + c*f*g + c*e*h) + a^3*b*C*(d*f*g + d*e*h + c*f*h) + 3*a^2*b^2*(2*
A*d*f*h - C*(d*e*g + c*f*g + c*e*h)) - a*b^3*(4*A*d*(f*g + e*h) - c*(5*C*e
*g - 4*A*f*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*(b*c - a*d)*
(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) - ((-2*b*(4*a*(A*b^2 + a^2*C)*d*f
*h*((3*b^2*c*e*g)/2 - (3*a*b*(d*e*g + c*f*g + c*e*h))/2 + a^2*(d*f*g + d*e
*h + c*f*h)) + b*(3*a^2*d*f*h + 2*b^2*(d*e*g + c*f*g + c*e*h) - 3*a*b*(d*f
*g + d*e*h + c*f*h))*(4*A*b^2*(d*e*g + c*f*g + c*e*h) + 5*a*b*(c*C*e*g - A
*d*f*g - A*d*e*h - A*c*f*h) + a^2*(5*A*d*f*h - C*(d*e*g + c*f*g + c*e*h)))
 + 2*a*(3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*(2*A*b^3*(d*e*g + c*f*g + c
*e*h) + a^3*C*(d*f*g + d*e*h + c*f*h) + a^2*b*(5*A*d*f*h - 3*C*(d*e*g + c*
f*g + c*e*h)) - a*b^2*(4*A*d*(f*g + e*h) - c*(5*C*e*g - 4*A*f*h))) + b*(3*
b*c*e*g - a*(d*e*g + c*f*g + c*e*h))*(2*a^3*C*d*f*h + 2*a^2*b*C*(d*f*g + d
*e*h + c*f*h) + a*b^2*(7*A*d*f*h - 5*C*(d*e*g + c*f*g + c*e*h)) - b^3*(3*A
*d*(f*g + e*h) - c*(5*C*e*g - 3*A*f*h))))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt
[g + h*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) - ((-2*d*(4
*a*(A*b^2 + a^2*C)*d*f*h*((3*b^2*c*e*g)/2 - (3*a*b*(d*e*g + c*f*g + c*e*h)
)/2 + a^2*(d*f*g + d*e*h + c*f*h)) + b*(3*a^2*d*f*h + 2*b^2*(d*e*g + c*f*g
 + c*e*h) - 3*a*b*(d*f*g + d*e*h + c*f*h))*(4*A*b^2*(d*e*g + c*f*g + c*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))])) Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 2108
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (C_.)*(x_)^2))/(Sqrt[(c_.) + (d_.)*
(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp
[(A*b^2 + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*
x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(
b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]
*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*
(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) + a*C*(a*(
d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*(A*b*(a*d*f*h*(m + 1) - b*(
m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*
g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^
2 + a^2*C)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, C}, x] && I
ntegerQ[2*m] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7992 vs. 2(1551) = 3102.

Time = 41.69 (sec) , antiderivative size = 7993, normalized size of antiderivative = 4.94

method	result	size
elliptic	Expression too large to display	7993
default	Expression too large to display	355547

input
```
int((C*x^2+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x,me
thod=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="fricas")
```

output
```
integral((C*x^2 + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*x +
g)/(b^4*d*f*h*x^7 + a^4*c*e*g + (b^4*d*f*g + (b^4*d*e + (b^4*c + 4*a*b^3*d
)*f)*h)*x^6 + ((b^4*d*e + (b^4*c + 4*a*b^3*d)*f)*g + ((b^4*c + 4*a*b^3*d)*
e + 2*(2*a*b^3*c + 3*a^2*b^2*d)*f)*h)*x^5 + (((b^4*c + 4*a*b^3*d)*e + 2*(2
*a*b^3*c + 3*a^2*b^2*d)*f)*g + 2*((2*a*b^3*c + 3*a^2*b^2*d)*e + (3*a^2*b^2
*c + 2*a^3*b*d)*f)*h)*x^4 + (2*((2*a*b^3*c + 3*a^2*b^2*d)*e + (3*a^2*b^2*c
 + 2*a^3*b*d)*f)*g + (2*(3*a^2*b^2*c + 2*a^3*b*d)*e + (4*a^3*b*c + a^4*d)*
f)*h)*x^3 + ((2*(3*a^2*b^2*c + 2*a^3*b*d)*e + (4*a^3*b*c + a^4*d)*f)*g + (
a^4*c*f + (4*a^3*b*c + a^4*d)*e)*h)*x^2 + (a^4*c*e*h + (a^4*c*f + (4*a^3*b
*c + a^4*d)*e)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+A)/(b*x+a)**(7/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x+g)*
*(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+A}{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2
),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)/((b*x + a)^(7/2)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(h*
x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{7/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)
^(1/2)),x)
```

output
```
int((A + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(7/2)*(c + d*x)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Cx^2}{(a+bx)^{7/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+A}{(bx+a)^{\frac{7}{2}}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

output
```
int((C*x^2+A)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),x)
```

3.38 $\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 1524

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Too large to display}$$

output
```
1/24*((a*d*f*h-3*b*(c*f*h+d*e*h+d*f*g))*(6*b*B*d*f*h+3*a*C*d*f*h-5*b*C*(c*
f*h+d*e*h+d*f*g))/d/f/h+8*b*(3*(A*b+B*a)*d*f*h-C*(2*b*(c*e*h+c*f*g+d*e*g)+
a*(c*f*h+d*e*h+d*f*g))))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2
/h^2/(b*x+a)^(1/2)+1/12*(6*b*B*d*f*h+3*a*C*d*f*h-5*b*C*(c*f*h+d*e*h+d*f*g)
)*(b*x+a)^(1/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2+1/3*
C*(b*x+a)^(3/2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-1/24*(-a*f
+b*e)^(1/2)*(-c*h+d*g)*(3*a^2*C*d^2*f^2*h^2+2*a*b*d*f*h*(15*B*d*f*h-11*C*(
c*f*h+d*e*h+d*f*g))+b^2*(6*d*f*h*(4*A*d*f*h-3*B*(c*f*h+d*e*h+d*f*g))+C*(15
*c^2*f^2*h^2+14*c*d*f*h*(e*h+f*g)+d^2*(15*e^2*h^2+14*e*f*g*h+15*f^2*g^2)))
)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((
-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(
-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/d^3/f^3/(-c*f+d*e)^(1/2)/h^3/(-(
-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+1/24*(-a*d+b*c)*
(-a*f+b*e)^(1/2)*(3*a^2*C*d^2*f^2*h^2-6*a*b*d*f*h*(3*B*d*f*h-C*(2*c*f*h+2*
d*e*h+d*f*g))-b^2*(6*d*f*h*(4*A*d*f*h-B*(3*c*f*h+3*d*e*h+d*f*g))+C*(15*c^2
*f^2*h^2+2*c*d*f*h*(7*e*h+2*f*g)+d^2*(15*e^2*h^2+4*e*f*g*h+5*f^2*g^2))))*(
f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*
f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*
h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/d^3/f^3/(-c*f+d*e)^(1/2)/h^2/(-(-
a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/24*(-c*f+d*e...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 55395 vs. $2(1524) = 3048$.

Time = 43.08 (sec) , antiderivative size = 55395, normalized size of antiderivative = 36.35

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]
*Sqrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 5.78 (sec) , antiderivative size = 1488, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.234$, Rules used = {2103, 25, 2103, 2105, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2103}$$

$$\frac{\int -\frac{\sqrt{a+bx}\left(-\left((6bBdfh+3aCdfh-5bC(dfg+deh+cfh))x^2\right)-2(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))x+3bcCeg-6aAdfh\cdots\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6dfh}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}$$

$$\Big\downarrow \text{25}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$\frac{\int \frac{\sqrt{a+bx}\left(-\left((6bBdfh+3aCdfh-5bC(dfg+deh+cfh))x^2\right)-2(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))x+3bcCeg-6aAdfh+a\cdots\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{6dfh}$$

$$\Big\downarrow \text{2103}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$\int \frac{-\left(((adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))+8bdfh(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh))))x^2\right)+2\left((b(deg+cfg+\cdots\right.}{\cdots}$$

$$\Big\downarrow \text{2105}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(\frac{(adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))}{bfh}+8d(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))\right)}{\sqrt{c+dx}}-\cdots$$

$$\Big\downarrow \text{194}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(\frac{(adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))}{bfh}+8d(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))\right)}{\sqrt{c+dx}}+$$

$\downarrow$ 327

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(\frac{(adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))}{bfh}+8d(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))\right)}{\sqrt{c+dx}}+$$

$\downarrow$ 2101

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(\frac{(adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))}{bfh}+8d(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))\right)}{\sqrt{c+dx}}+$$

$\downarrow$ 183

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(\frac{(adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))}{bfh}+8d(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))\right)}{\sqrt{c+dx}}+$$

$\downarrow$ 188

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(\frac{(adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))}{bfh}+8d(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))\right)}{\sqrt{c+dx}}+$$

$$\Big\downarrow \text{321}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(\frac{(adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))}{bfh}+8d(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))\right)}{\sqrt{c+dx}}+$$

$$\Big\downarrow \text{412}$$

$$\frac{C(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3dfh}-$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(\frac{(adfh-3b(dfg+deh+cfh))(6bBdfh+3aCdfh-5bC(dfg+deh+cfh))}{bfh}+8d(3(Ab+aB)dfh-C(2b(deg+cfg+ceh)+a(dfg+deh+cfh)))\right)}{\sqrt{c+dx}}+$$

input
```
Int[((a + b*x)^(3/2)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[
g + h*x]),x]
```

output
```
(C*(a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*d*f*h) -
(-1/2*((6*b*B + 3*a*C - 5*b*C*(c/d + e/f + g/h))*Sqrt[a + b*x]*Sqrt[c + d*
x]*Sqrt[e + f*x]*Sqrt[g + h*x]) + (-(((((a*d*f*h - 3*b*(d*f*g + d*e*h + c*
f*h))*(6*b*B*d*f*h + 3*a*C*d*f*h - 5*b*C*(d*f*g + d*e*h + c*f*h)))/(b*f*h)
 + 8*d*(3*(A*b + a*B)*d*f*h - C*(2*b*(d*e*g + c*f*g + c*e*h) + a*(d*f*g +
d*e*h + c*f*h))))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x]
) + (Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*((a*d*f*h - 3*b*(d*f*g + d*e*h + c*f*
h))*(6*b*B*d*f*h + 3*a*C*d*f*h - 5*b*C*(d*f*g + d*e*h + c*f*h)) + 8*b*d*f*
h*(3*(A*b + a*B)*d*f*h - C*(2*b*(d*e*g + c*f*g + c*e*h) + a*(d*f*g + d*e*h
 + c*f*h))))*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c
+ d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h
]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(
b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x
]) + ((-2*d*(b*e - a*f)*Sqrt[b*g - a*h]*(3*a^2*C*d^2*f^2*h^2 + 6*a*b*d*f*h
*(c*C*f*h - 3*B*d*f*h + 2*C*d*(f*g + e*h)) - b^2*(6*d*f*h*(4*A*d*f*h - B*(
3*d*f*g + 3*d*e*h + c*f*h)) + C*(5*c^2*f^2*h^2 + 4*c*d*f*h*(f*g + e*h) + d
^2*(15*f^2*g^2 + 14*e*f*g*h + 15*e^2*h^2))))*Sqrt[((b*e - a*f)*(c + d*x))/
((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*S
qrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h)
)/((d*e - c*f)*(b*g - a*h)))])/(b*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2103
```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 21.75 (sec) , antiderivative size = 2258, normalized size of antiderivative = 1.48

method	result	size
elliptic	Expression too large to display	2258
default	Expression too large to display	125812

input
```
int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(1/3*C*b/d/f/h*x*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^
3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+
b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(
1/2)+1/2*(B*b^2+2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*e*h
+5/2*b*g*d*f))/h/d/b/f*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*
d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*
e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)+2*(a^2*A-1/
3*C*b/d/f/h*a*c*e*g-1/2*(B*b^2+2*C*a*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*
f*h+5/2*b*d*e*h+5/2*b*g*d*f))/h/d/b/f*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g
+1/2*b*c*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c
/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-
e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a
/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),
((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(2*a*b*A+a^2*B-1/3*C*
b/d/f/h*(3/2*a*c*e*h+3/2*a*c*f*g+3/2*a*d*e*g+3/2*b*c*e*g)-1/2*(B*b^2+2*C*a
*b-1/3*C*b/d/f/h*(5/2*a*d*f*h+5/2*b*c*f*h+5/2*b*d*e*h+5/2*b*g*d*f))/h/d/b/
f*(a*c*f*h+a*d*e*h+a*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(e/f-g/h)*((c/d-e/f)*
(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)
/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(a+bx)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x
+g)**(1/2),x)
```

output
```
Integral((a + b*x)**(3/2)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)*
sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{(a+bx)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cx^2+Bx+A)(bx+a)^{\frac{3}{2}}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x + a)^(3/2)/(sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(a+b\,x)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(
c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(3/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(
c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a+bx)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(bx+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

output
```
int((b*x+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

3.39 $\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 983

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$
$$= \frac{(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{4d^2f^2h^2\sqrt{a+bx}}$$
$$+ \frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$
$$- \frac{\sqrt{be-af}(dg-ch)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\ldots\right.\right.}{4bd^2f^2\sqrt{de-cf}h^2\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$- \frac{(bc-ad)\sqrt{be-af}(4bBdfh-aCdfh-bC(dfg+3deh+3cfh))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\operatorname{EllipticF}\left(\ldots\right.}{4b^2d^2f^2\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$- \frac{\sqrt{de-cf}((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-4bdfh(2(Ab+\ldots}{\ldots}$$

output
```
1/4*(4*b*B*d*f*h+a*C*d*f*h-3*b*C*(c*f*h+d*e*h+d*f*g))*(d*x+c)^(1/2)*(f*x+e
)^(1/2)*(h*x+g)^(1/2)/d^2/f^2/h^2/(b*x+a)^(1/2)+1/2*C*(b*x+a)^(1/2)*(d*x+c
)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h-1/4*(-a*f+b*e)^(1/2)*(-c*h+d*g)*
(4*b*B*d*f*h+a*C*d*f*h-3*b*C*(c*f*h+d*e*h+d*f*g))*(f*x+e)^(1/2)*(-(-a*d+b*
c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1
/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h
+d*g))^(1/2))/b/d^2/f^2/(-c*f+d*e)^(1/2)/h^2/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*
e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-1/4*(-a*d+b*c)*(-a*f+b*e)^(1/2)*(4*b*B*d*f
*h-a*C*d*f*h-b*C*(3*c*f*h+3*d*e*h+d*f*g))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+
g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*
f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(
1/2))/b^2/d^2/f^2/(-c*f+d*e)^(1/2)/h/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+
a))^(1/2)/(h*x+g)^(1/2)-1/4*(-c*f+d*e)^(1/2)*((a*d*f*h+b*(c*f*h+d*e*h+d*f*
g))*(4*b*B*d*f*h+a*C*d*f*h-3*b*C*(c*f*h+d*e*h+d*f*g))-4*b*d*f*h*(2*(A*b+B*
a)*d*f*h-C*(b*(c*e*h+c*f*g+d*e*g)+a*(c*f*h+d*e*h+d*f*g))))*(b*x+a)*(-(-a*d
+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b
*x+a))^(1/2)*EllipticPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b
*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-
c*h+d*g))^(1/2))/b^2/d^3/f^2/(-a*f+b*e)^(1/2)/h^2/(f*x+e)^(1/2)/(h*x+g)^(1
/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 22316 vs. 2(983) = 1966.

Time = 36.78 (sec) , antiderivative size = 22316, normalized size of antiderivative = 22.70

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*S
qrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 2.89 (sec) , antiderivative size = 973, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.213$, Rules used = {2103, 2105, 25, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2103}$$

$$\frac{\int \frac{(4bBdfh+aCdfh-3bC(dfg+deh+cfh))x^2+2(2(Ab+aB)dfh-C(b(deg+cfg+ceh)+a(dfg+deh+cfh)))x+4aAdfh-C(bceg+a(deg+cfg+ceh}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{4dfh}$$
$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{2105}$$

$$\frac{\int -\frac{(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))-2bdfh(4aAdfh-C(bceg+a(deg+cfg+ceh)))+((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3bC(dfg+de}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))+((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3bC(dfg+de}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$$\Big\downarrow \text{194}$$

$$-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))+((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3bC(dfg+de}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 327

$$-\frac{\int \frac{2bdfh(bcCeg-4aAdfh+aC(deg+cfg+ceh))+(bdeg+acfh)(4bBdfh+aCdfh-3bC(dfg+deh+cfh))+((adfh+b(dfg+deh+cfh))(4bBdfh+aCdfh-3bC(dfg+de}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 2101

$$-\frac{\frac{((adfh+b(cfh+deh+dfg))(aCdfh+4bBdfh-3bC(cfh+deh+dfg))-4bdfh(2dfh(aB+Ab)-C(a(cfh+deh+dfg)+b(ceh+cfg+deg)))) \int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{2dfh}$$

$\downarrow$ 183

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(3cC-d\left(\frac{a}{b}-\frac{3e}{f}-\frac{3g}{h}\right)C-4Bd\right)}{\sqrt{c+dx}}-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$\downarrow$ 188

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(3cC-d\left(\frac{a}{b}-\frac{3e}{f}-\frac{3g}{h}\right)C-4Bd\right)}{\sqrt{c+dx}}-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$\downarrow$ 321

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(3cC-d\left(\frac{a}{b}-\frac{3e}{f}-\frac{3g}{h}\right)C-4Bd\right)}{\sqrt{c+dx}}-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}C}{2dfh}+$$

$$-\frac{\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}\left(3cC-d\left(\frac{a}{b}-\frac{3e}{f}-\frac{3g}{h}\right)C-4Bd\right)}{\sqrt{c+dx}}-\frac{\sqrt{dg-ch}\sqrt{fg-eh}(4bBdfh+aCdfh-3bC(dfg+deh+cfh))\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}}{bdfh\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

input

```
Int[(Sqrt[a + b*x]*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output

```
(C*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(2*d*f*h) + (-
(((3*c*C - 4*B*d - C*d*(a/b - (3*e)/f - (3*g)/h))*Sqrt[a + b*x]*Sqrt[e + f
*x]*Sqrt[g + h*x])/Sqrt[c + d*x]) - (Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*(4*b*
B*d*f*h + a*C*d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h))*Sqrt[a + b*x]*Sqrt[-(
((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d
*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f
*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x
))/((b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - ((2*d*(b*e - a*f)*Sqrt[b*g -
a*h]*(4*b*B*d*f*h - a*C*d*f*h - b*C*(c*f*h + 3*d*(f*g + e*h)))*Sqrt[((b*e
- a*f)*(c + d*x))/((d*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[
(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c
- a*d)*(f*g - e*h))/((d*e - c*f)*(b*g - a*h)))])/(b*Sqrt[f*g - e*h]*Sqrt[c
 + d*x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (2*Sqr
t[-(d*g) + c*h]*((a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(4*b*B*d*f*h + a*C*
d*f*h - 3*b*C*(d*f*g + d*e*h + c*f*h)) - 4*b*d*f*h*(2*(A*b + a*B)*d*f*h -
C*(b*(d*e*g + c*f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h))))*(a + b*x)*Sqrt
[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f
*x))/((f*g - e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)
), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x
])], ((b*e - a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*Sqrt[b*c ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2103

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[
(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_S
ymbol] :> Simp[2*C*(a + b*x)^m*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(
d*f*h*(2*m + 3))), x] + Simp[1/(d*f*h*(2*m + 3))   Int[((a + b*x)^(m - 1)/(
Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[a*A*d*f*h*(2*m + 3) - C*(a
*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*m) + ((A*b + a*B)*d*f*h*(2*m + 3) - C*
(2*a*(d*f*g + d*e*h + c*f*h) + b*(2*m + 1)*(d*e*g + c*f*g + c*e*h)))*x + (b
*B*d*f*h*(2*m + 3) + 2*C*(a*d*f*h*m - b*(m + 1)*(d*f*g + d*e*h + c*f*h)))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2
*m] && GtQ[m, 0]
```

rule 2105

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [A] (verified)

Time = 19.56 (sec) , antiderivative size = 1800, normalized size of antiderivative = 1.83

method	result	size
elliptic	Expression too large to display	1800
default	Expression too large to display	57266

input
```
int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(1/2*C/h/d/f*(b*d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*
d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*
f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)
+2*(A*a-1/2*C/h/d/f*(1/2*a*c*e*h+1/2*a*c*f*g+1/2*a*d*e*g+1/2*b*c*e*g))*(e/
f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*
(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^
(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)
*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g
/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(A*b+B*a-1/2*C/h/d/f*(a*c*f*h+a*d*e*h+a
*d*f*g+b*c*e*h+b*c*f*g+b*d*e*g))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(
x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c
/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x
+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c/d*EllipticF(((c/d-e/f)*(x+g/h)/(-
e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))
+(c/d-g/h)*EllipticPi(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g
/h)/(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)))+(B*b+C*a
-1/2*C/h/d/f*(3/2*a*d*f*h+3/2*b*c*f*h+3/2*b*d*e*h+3/2*b*g*d*f))*((x+g/h)*(
x+a/b)*(x+e/f)+(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c
/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((b*x+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x
+g)**(1/2),x)
```

output
```
Integral(sqrt(a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)*sqr
t(g + h*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqr
t(h*x + g)), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{bx+a}}{\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x + a)/(sqrt(d*x + c)*sqrt(f*x + e)*sqr
t(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{a+b\,x}\,(C\,x^2+B\,x+A)}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(
c + d*x)^(1/2)),x)
```

output
```
int(((a + b*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(
c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx}(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{\sqrt{bx+a}\,(C\,x^2+Bx+A)}{\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

output

```
int((b*x+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

3.40 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 782

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \frac{C\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{dfh\sqrt{a+bx}}$$
$$-\frac{C\sqrt{be-af}(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{bdf\sqrt{de-cf}h\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$+\frac{(a^2Cdf-b^2(cCe-2Adf)+ab(Cde+cCf-2Bdf))\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be}}{\sqrt{de}}\right.\right.}{b^2df\sqrt{be-af}\sqrt{de-cf}\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$
$$+\frac{\sqrt{de-cf}(2bBdfh-C(adfh+b(dfg+deh+cfh)))(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\text{Elliptic}}{b^2d^2f\sqrt{be-af}h\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
C*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/d/f/h/(b*x+a)^(1/2)-C*(-a*f+b*
e)^(1/2)*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))
^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(
1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/d/f/(-c*f+d*e)
^(1/2)/h/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+(a^2
*C*d*f-b^2*(-2*A*d*f+C*c*e)+a*b*(-2*B*d*f+C*c*f+C*d*e))*(f*x+e)^(1/2)*(-(-
a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x
+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)
/(-c*h+d*g))^(1/2))/b^2/d/f/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-(-a*d+b*c)
*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)+(-c*f+d*e)^(1/2)*(2*b*B*d
*f*h-C*(a*d*f*h+b*(c*f*h+d*e*h+d*f*g)))*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f
+d*e)/(b*x+a))^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*Ellipt
icPi((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f
+d*e)/d/(-a*f+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^
2/d^2/f/(-a*f+b*e)^(1/2)/h/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7019 vs. $2(782) = 1564$.

Time = 34.82 (sec) , antiderivative size = 7019, normalized size of antiderivative = 8.98

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqr
t[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {2105, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2105}$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)+(2bBdfh-C(adfh+b(dfg+deh+cfh)))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{C(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{194}$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)+(2bBdfh-C(adfh+b(dfg+deh+cfh)))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-\frac{C\sqrt{a+bx}(dg-ch)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\int \frac{2Abdfh-C(bdeg+acfh)+(2bBdfh-C(adfh+b(dfg+deh+cfh)))x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}-\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \text{2101}$$

$$\frac{\frac{d\left(a^2Cfh+ab(-2Bfh+Ceh+Cfg)-b^2(Ceg-2Afh)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{(2bBdfh-C(adfh+b(cfh+deh+dfg)))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\cdots}}{b}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 183$$

$$\frac{\frac{d\left(a^2Cfh+ab(-2Bfh+Ceh+Cfg)-b^2(Ceg-2Afh)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(2bBdfh-C\cdots}{\cdots}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 188$$

$$\frac{\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}\left(a^2Cfh+ab(-2Bfh+Ceh+Cfg)-b^2(Ceg-2Afh)\right)\int \frac{1}{\sqrt{\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}+1}\sqrt{1-\frac{(bg-ah)(e+fx)}{(fg-eh)(a+bx)}}}d\frac{\sqrt{e+fx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}(fg-eh)\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2(a+bx)\sqrt{\cdots}}{\cdots}}{2bdfh}$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \; 321$$

$$\frac{2(a+bx)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}(2bBdfh-C(adfh+b(cfh+deh+dfg)))\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}$$

$$2bdfh$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

$$\Big\downarrow \ 412$$

$$\frac{2d\sqrt{g+hx}\sqrt{\frac{(c+dx)(be-af)}{(a+bx)(de-cf)}}(a^2Cfh+ab(-2Bfh+Ceh+Cfg)-b^2(Ceg-2Afh))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{bg-ah}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{a+bx}}\right),-\frac{(bc-ad)(fg-eh)}{(de-cf)(bg-ah)}\right)}{b\sqrt{c+dx}\sqrt{bg-ah}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(be-af)}{(a+bx)(fg-eh)}}}+\frac{2(a+b}{}$$

$$2d$$

$$\frac{C\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)}{bdfh\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+$$

$$\frac{C\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}}{bfh\sqrt{c+dx}}$$

input

```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g +
h*x]),x]
```

output
```
(C*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*f*h*Sqrt[c + d*x]) - (C*S
qrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[-(((d*e - c*f)*(g + h*x)
)/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt[d*g - c*h]*Sqrt[e + f*x
])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*(f*g - e*h))/((b*e - a*f
)*(d*g - c*h))])/(b*d*f*h*Sqrt[((d*e - c*f)*(a + b*x))/((b*e - a*f)*(c + d
*x))]*Sqrt[g + h*x]) + ((2*d*(a^2*C*f*h - b^2*(C*e*g - 2*A*f*h) + a*b*(C*f
*g + C*e*h - 2*B*f*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e - c*f)*(a + b*x)
)]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e + f*x])/(Sqrt[f*
g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e - c*f)*(b*g -
a*h)))])/(b*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sqrt[-(((b*e - a
*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (2*Sqrt[-(d*g) + c*h]*(2*b*B*d
*f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*(a + b*x)*Sqrt[((b*g - a*h
)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g -
 e*h)*(a + b*x))]*EllipticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(S
qrt[b*c - a*d]*Sqrt[g + h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e -
 a*f)*(d*g - c*h))/((b*c - a*d)*(f*g - e*h))])/(b*Sqrt[b*c - a*d]*h*Sqrt[c
 + d*x]*Sqrt[e + f*x]))/(2*b*d*f*h)
```

Defintions of rubi rules used

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(717) = 1434$.

Time = 21.58 (sec) , antiderivative size = 1536, normalized size of antiderivative = 1.96

method	result	size
elliptic	Expression too large to display	1536
default	Expression too large to display	19862

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*A*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d)
)^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h
)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*
(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+
c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*B*(e/f-g/
h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a
/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2
)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*(x+a/b)*(x+e/f))^(1/2)*(-c
/d*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f
-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+(c/d-g/h)*EllipticPi(((c/d-e/f)*(x+g/h)
/(-e/f+g/h)/(x+c/d))^(1/2),(-e/f+g/h)/(c/d-e/f),((-c/d+a/b)*(e/f-g/h)/(a/b
-g/h)/(-c/d+e/f))^(1/2)))+C*((x+g/h)*(x+a/b)*(x+e/f)+(e/f-g/h)*((c/d-e/f)*
(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)
/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f)/(-e/f+g/h)/(x+c/d))^(1/2)*((c/d*g/h-e/
f*g/h+c*e/d/f+c^2/d^2)/(c/d-e/f)/(-c/d+g/h)*EllipticF(((c/d-e/f)*(x+g/h)/(
-e/f+g/h)/(x+c/d))^(1/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2)
)+(a/b-g/h)*EllipticE(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2),((-c/d+
a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))/(-c/d+g/h)+(a*d*f*h+b*c*f*h+b*
d*e*h+b*d*f*g)/h/d/b/f/(c/d-e/f)*EllipticPi(((c/d-e/f)*(x+g/h)/(-e/f+g/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + bx}\sqrt{c + dx}\sqrt{e + fx}\sqrt{g + hx}}\, dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x
+g)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*sqr
t(g + h*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqr
t(h*x + g)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqr
t(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c
+ d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c
+ d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{C\,x^2+Bx+A}{\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

output
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

3.41 $\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 718

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2(Ab^2-a(bB-aC))\,(dg-ch)\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right)|\frac{(de-cf)(bg-ah)}{(be-af)(dg-ch)}\right)}{b(bc-ad)\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$-\frac{2(2abCg-a^2Ch-b^2(Bg-Ah))\,\sqrt{e+fx}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(bg-a}{(be-af)(dg-c}\right.}{b^2\sqrt{be-af}\sqrt{de-cf}(bg-ah)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{g+hx}}$$

$$+\frac{2C\sqrt{de-cf}(a+bx)\sqrt{-\frac{(bc-ad)(e+fx)}{(de-cf)(a+bx)}}\sqrt{-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}}\,\text{EllipticPi}\left(\frac{b(de-cf)}{d(be-af)},\arcsin\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{de-cf}\sqrt{a+bx}}\right),\frac{(de-cf)(b}{(be-af)(d}\right.}{b^2d\sqrt{be-af}\sqrt{e+fx}\sqrt{g+hx}}$$

output
```
-2*(A*b^2-a*(B*b-C*a))*(-c*h+d*g)*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h
+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(
1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b/
(-a*d+b*c)/(-a*f+b*e)^(1/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x+
e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2*(2*C*a*b*g-C*a^2*h-b^2*(-A*h+
B*g))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*Ellipti
cF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*
e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/(-a*f+b*e)^(1/2)/(-c*f+d*e
)^(1/2)/(-a*h+b*g)/(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^
(1/2)+2*C*(-c*f+d*e)^(1/2)*(b*x+a)*(-(-a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a)
)^(1/2)*(-(-a*d+b*c)*(h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticPi((-a*f+b*
e)^(1/2)*(d*x+c)^(1/2)/(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),b*(-c*f+d*e)/d/(-a*f
+b*e),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^(1/2))/b^2/d/(-a*f+b*e
)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 35.33 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2\sqrt{\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}}\sqrt{e+fx}\sqrt{g+hx}\left(b\left(Ab^2+a(-bB+aC)\right)h(-dg+ch)E\left(\arcsin\left(\sqrt{\frac{(-be+af)(g+hx)}{(fg-eh)(a+bx)}}\right)\middle|\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*S
qrt[g + h*x]),x]
```

output
```
(-2*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))]*Sqrt[e + f*x]*Sq
rt[g + h*x]*(b*(A*b^2 + a*(-(b*B) + a*C))*h*(-(d*g) + c*h)*EllipticE[ArcSi
n[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*
d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))] + (b*g - a*h)*(-((-2*a*b*c*C
 + a^2*C*d + b^2*(B*c - A*d))*h*EllipticF[ArcSin[Sqrt[((-(b*e) + a*f)*(g +
 h*x))/((f*g - e*h)*(a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e -
a*f)*(d*g - c*h))]) + C*(b*c - a*d)*(b*g - a*h)*EllipticPi[(b*(-(f*g) + e*
h))/((b*e - a*f)*h), ArcSin[Sqrt[((-(b*e) + a*f)*(g + h*x))/((f*g - e*h)*(
a + b*x))]], ((-(b*c) + a*d)*(-(f*g) + e*h))/((b*e - a*f)*(d*g - c*h))]))
/(b^2*(b*c - a*d)*h*(b*g - a*h)*(-(f*g) + e*h)*Sqrt[a + b*x]*Sqrt[c + d*x]
*Sqrt[-(((b*e - a*f)*(b*g - a*h)*(e + f*x)*(g + h*x))/((f*g - e*h)^2*(a +
b*x)^2))])
```

Rubi [A] (warning: unable to verify)

Time = 2.24 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.213$, Rules used = {2107, 2105, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx$$

$$\Big\downarrow \text{2107}$$

$$\frac{\int \frac{2(Ab^2-a(bB-aC))dfhx^2+(C(2(dfg+deh+cfh)a^2-b(deg+cfg+ceh)a+b^2ceg)+(Ab-aB)(adfh+b(dfg+deh+cfh)))x+(bB-aC)(bceg-a(c}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ab^2-a(bB-aC)\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{2105}$$

$$\frac{\frac{(de-cf)(dg-ch)\left(Ab^2-a(bB-aC)\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{\int \frac{2df(be-af)h(bg-ah)(bBc-aCc-Abd+C(bc-ad)x)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g}}{b\sqrt{}}}{(bc-ad)(be-af)(bg-ah)}$$
$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ab^2-a(bB-aC)\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(de-cf)(dg-ch)\left(Ab^2-a(bB-aC)\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{(be-af)(bg-ah)\int \frac{bBc-aCc-Abd+C(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+h}}{b\sqrt{c+}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ab^2-a(bB-aC)\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 194$$

$$\frac{-\frac{2\sqrt{a+bx}(dg-ch)\left(Ab^2-a(bB-aC)\right)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{b\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}+\frac{(be-af)(bg-ah)\int \frac{bBc-aCc-Abd+C(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}+}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ab^2-a(bB-aC)\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{(be-af)(bg-ah)\int \frac{bBc-aCc-Abd+C(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{2\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\left(Ab^2-a(bB-aC)\right)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|}{b\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}}{(bc-ad)(be-af)(bg-ah)}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ab^2-a(bB-aC)\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 2101$$

$$\frac{\frac{(be-af)(bg-ah)\left(\frac{C(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{\left(a^2(-C)d+2abcC-b^2(Bc-Ad)\right)\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}\right)}{b}-\frac{2\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg}}{}}{(bc-ad)(be-af}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ab^2-a(bB-aC)\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \; 183$$

$$\frac{(be-af)(bg-ah)\left(\frac{2C(a+bx)(bc-ad)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{1}{\left(h-\frac{b(g+hx)}{a+bx}\right)\sqrt{\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}+1}\sqrt{\frac{(be-af)(g+hx)}{(fg-eh)(a+bx)}+1}}d\frac{\sqrt{g+hx}}{\sqrt{a+bx}}}{b\sqrt{c+dx}\sqrt{e+fx}}-\frac{\left(a^2(-C)d+\right.}{}\right.}{b}$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ab^2-a(bB-aC)\right)}{\sqrt{a+bx}(bc-ad)(be-af)(bg-ah)}$$

$\Big\downarrow$ 188

$$\frac{\frac{2(Ab^2-a(bB-aC))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}-\frac{2(Ab^2-a(bB-aC))\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-}{(be-}\right.}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$\Big\downarrow$ 321

$$\frac{\frac{2(Ab^2-a(bB-aC))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}-\frac{2(Ab^2-a(bB-aC))\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-}{(be-}\right.}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

$\Big\downarrow$ 412

$$\frac{\frac{2(Ab^2-a(bB-aC))\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}d}{b\sqrt{c+dx}}-\frac{2(Ab^2-a(bB-aC))\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-}{(be-}\right.}{b\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{(bc-ad)(be-af)(bg-ah)\sqrt{a+bx}}$$

input
```
Int[(A + B*x + C*x^2)/((a + b*x)^(3/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output
```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b
*c - a*d)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*(A*b^2 - a*(b*B - a
*C))*d*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(b*Sqrt[c + d*x]) - (2*(
A*b^2 - a*(b*B - a*C))*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[
-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt
[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*
(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(b*Sqrt[((d*e - c*f)*(a + b*x))/(
(b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) + ((b*e - a*f)*(b*g - a*h)*((-2*(2*
a*b*c*C - a^2*C*d - b^2*(B*c - A*d))*Sqrt[((b*e - a*f)*(c + d*x))/((d*e -
c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt[e +
f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/((d*e
- c*f)*(b*g - a*h)))]])/(b*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*x]*Sq
rt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) + (2*C*Sqrt[b*c -
a*d]*Sqrt[-(d*g) + c*h]*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*h
)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*Ellipt
icPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g +
h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*c
 - a*d)*(f*g - e*h))]])/(b*h*Sqrt[c + d*x]*Sqrt[e + f*x])))/b)/((b*c - a*d)
*(b*e - a*f)*(b*g - a*h))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194

```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101

```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2320 vs. $2(661) = 1322$.

Time = 24.10 (sec) , antiderivative size = 2321, normalized size of antiderivative = 3.23

method	result	size
elliptic	Expression too large to display	2321
default	Expression too large to display	39149

input
```
int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+
b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-
a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(A*b^2-B*a*b+
C*a^2)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x
+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2*((B*b-C*a)/b^2+1/b^2*(a^2*d*f*h-a*b
*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(A*b^2-B*a*b+C*a
^2)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g
+a*b^2*d*e*g-b^3*c*e*g)-(b*c*e*h+b*c*f*g+b*d*e*g)/(a^3*d*f*h-a^2*b*c*f*h-a
^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(A
*b^2-B*a*b+C*a^2))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*
(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f
)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*
(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1
/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(C/b-1/b*(a*d*f*h
-b*c*f*h-b*d*e*h-b*d*f*g)*(A*b^2-B*a*b+C*a^2)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b
*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)-(2*b*c*f
*h+2*b*d*e*h+2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b
^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b*(A*b^2-B*a*b+C*a^2))*(e/f-g/
h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{A+Bx+Cx^2}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(3/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x
+g)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x)**(3/2)*sqrt(c + d*x)*sqrt(e + f*x)*
sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(3/2)*sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error:
Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{(a+b\,x)}^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c
+ d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(3/2)*(c
+ d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+Bx+A}{(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

output
```
int((C*x^2+B*x+A)/(b*x+a)^(3/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

3.42 $\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 827

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx =$$

$$-\frac{2(Ab^2-a(bB-aC))\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$-\frac{2(dg-ch)\left(b^3(3Bceg-2A(deg+cfg+ceh))-a^2b(Bdfg+Bdeh+Bcfh+6Adfh-4C(deg+cfg-\right.}{}$$

$$-\frac{2(a^2(3dfh(Bg-Ah)-cCh(fg-eh)-Cdg(2fg+eh))+ab(3C(de+cf)g^2+3A(de+cf)h^2-Bch(}{3(bc}$$

output
```
-2/3*(A*b^2-a*(B*b-C*a))*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(h*x+g)^(1/2)/(-a*d+b
*c)/(-a*f+b*e)/(-a*h+b*g)/(b*x+a)^(3/2)-2/3*(-c*h+d*g)*(b^3*(3*B*c*e*g-2*A
*(c*e*h+c*f*g+d*e*g))-a^2*b*(B*d*f*g+B*d*e*h+B*c*f*h+6*A*d*f*h-4*C*(c*e*h+
c*f*g+d*e*g))-a*b^2*(B*d*e*g-4*A*d*(e*h+f*g)+c*(-4*A*f*h+B*e*h+B*f*g+6*C*e
*g))+a^3*(3*B*d*f*h-2*C*(c*f*h+d*e*h+d*f*g)))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(
h*x+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticE((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/
(-c*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g
))^(1/2))/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)^2/(-(-
a*d+b*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)-2/3*(a^2*(3*d*f*h
*(-A*h+B*g)-c*C*h*(-e*h+f*g)-C*d*g*(e*h+2*f*g))+a*b*(3*C*(c*f+d*e)*g^2+3*A
*(c*f+d*e)*h^2-B*c*h*(e*h+2*f*g)-B*d*g*(2*e*h+f*g))-b^2*(3*c*C*e*g^2-A*d*g
*(-e*h+f*g)-c*h*(-2*A*e*h-A*f*g+3*B*e*g)))*(f*x+e)^(1/2)*(-(-a*d+b*c)*(h*x
+g)/(-c*h+d*g)/(b*x+a))^(1/2)*EllipticF((-a*f+b*e)^(1/2)*(d*x+c)^(1/2)/(-c
*f+d*e)^(1/2)/(b*x+a)^(1/2),((-c*f+d*e)*(-a*h+b*g)/(-a*f+b*e)/(-c*h+d*g))^
(1/2))/(-a*d+b*c)/(-a*f+b*e)^(3/2)/(-c*f+d*e)^(1/2)/(-a*h+b*g)^2/(-(-a*d+b
*c)*(f*x+e)/(-c*f+d*e)/(b*x+a))^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16787 vs. $2(827) = 1654$.

Time = 43.42 (sec) , antiderivative size = 16787, normalized size of antiderivative = 20.30

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*S
qrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 6.53 (sec) , antiderivative size = 1243, normalized size of antiderivative = 1.50, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.170$, Rules used = {2107, 2102, 2105, 27, 188, 194, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \text{ 2107}$$

$$\frac{\int \frac{(bB-aC)(3bceg-a(deg+cfg+ceh))-A\left(3dfha^2-3b(dfg+deh+cfh)a+2b^2(deg+cfg+ceh)\right)+\left(C\left(2(dfg+deh+cfh)a^2-3b(deg+cfg+ceh)a+3\cdots\right.\right.}{(a+bx)^{3/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\cdots}{3(bc-ad)(be-af)(bg-ah)}\cdots$$

$$\frac{2\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}\left(Ab^2-a(bB-aC)\right)}{3(a+bx)^{3/2}(bc-ad)(be-af)(bg-ah)}$$

$$\Big\downarrow \text{ 2102}$$

$$\frac{\int \frac{2bdfh\left((3Bdfh-2C(dfg+deh+cfh))a^3-b(6Adfh-4C(deg+cfg+ceh)+B(dfg+deh+cfh))a^2-b^2(Bdeg-4Ad(fg+eh)+c(6Ceg+Bfg+Beh-4Afh))a+b^3(3Bce\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 2105}$$

$$\frac{\frac{(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left((3Bdfh-2C(dfg+deh+cfh))a^3-b(6Adfh-4C(deg+cfg+ceh)+B(dfg+deh+cfh))a^2-b^2(Bdeg-\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left((3Bdfh-2C(dfg+deh+cfh))a^3-b(6Adfh-4C(deg+cfg+ceh)+B(dfg+deh+cfh))a^2-b^2(Bdeg-\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 188

$$\frac{(de-cf)(dg-ch)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx\left((3Bdfh-2C(dfg+deh+cfh))a^3-b(6Adfh-4C(deg+cfg+ceh)+B(dfg+deh+cfh))a^2-b^2(Bdeg-\right.}{}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 194

$$-\frac{2(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left((3Bdfh-2C(dfg+deh+cfh))a^3-b(6Adfh-4C(deg+cfg+ceh)+B(dfg+deh+cfh))a^2-\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 321

$$-\frac{2(dg-ch)\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}\left((3Bdfh-2C(dfg+deh+cfh))a^3-b(6Adfh-4C(deg+cfg+ceh)+B(dfg+deh+cfh))a^2-\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

$\downarrow$ 327

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)\left((3Bdfh-2C(dfg+deh+cfh))a^3-b(6Adfh-4C(deg+cfg+ceh)+\right.}{\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}$$

$$\frac{2\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}{3(bc-ad)(be-af)(bg-ah)(a+bx)^{3/2}}$$

input

```
Int[(A + B*x + C*x^2)/((a + b*x)^(5/2)*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
+ h*x]),x]
```

output

```
(-2*(A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(3*
(b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(3/2)) + ((-2*b*(b^3*(3*B*c*
e*g - 2*A*(d*e*g + c*f*g + c*e*h)) - a^2*b*(B*d*f*g + B*d*e*h + B*c*f*h +
6*A*d*f*h - 4*C*(d*e*g + c*f*g + c*e*h)) - a*b^2*(B*d*e*g - 4*A*d*(f*g + e
*h) + c*(6*C*e*g + B*f*g + B*e*h - 4*A*f*h)) + a^3*(3*B*d*f*h - 2*C*(d*f*g
 + d*e*h + c*f*h)))*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/((b*c - a*d
)*(b*e - a*f)*(b*g - a*h)*Sqrt[a + b*x]) + ((2*d*(b^3*(3*B*c*e*g - 2*A*(d*
e*g + c*f*g + c*e*h)) - a*b^2*(B*d*e*g - 4*A*d*(f*g + e*h) + c*(6*C*e*g +
B*f*g + B*e*h - 4*A*f*h)) - a^2*b*(6*A*d*f*h - 4*C*(d*e*g + c*f*g + c*e*h)
 + B*(d*f*g + d*e*h + c*f*h)) + a^3*(3*B*d*f*h - 2*C*(d*f*g + d*e*h + c*f*
h)))*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/Sqrt[c + d*x] - (2*Sqrt[d*
g - c*h]*Sqrt[f*g - e*h]*(b^3*(3*B*c*e*g - 2*A*(d*e*g + c*f*g + c*e*h)) -
a*b^2*(B*d*e*g - 4*A*d*(f*g + e*h) + c*(6*C*e*g + B*f*g + B*e*h - 4*A*f*h)
) - a^2*b*(6*A*d*f*h - 4*C*(d*e*g + c*f*g + c*e*h) + B*(d*f*g + d*e*h + c*
f*h)) + a^3*(3*B*d*f*h - 2*C*(d*f*g + d*e*h + c*f*h)))*Sqrt[a + b*x]*Sqrt[
-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt
[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*
(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(Sqrt[((d*e - c*f)*(a + b*x))/((b
*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) + (2*(b*e - a*f)*Sqrt[b*g - a*h]*(b^2
*(2*A*d^2*e*g - c*d*(3*B*e*g - A*f*g - A*e*h) + c^2*(3*C*e*g - A*f*h)) ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 2102
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_)))/(Sqrt[(c_.) + (d_.)*(x
_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(
A*b^2 - a*b*B)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]
/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x] - Simp[1/(2*(m + 1)*(b*
c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(m + 1)/(Sqrt[c + d*x]*S
qrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m + 1) - 2*a*b*(m + 1)*(d
*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g + c*e*h)) - b*B*(a*(d*
e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) - 2*((A*b - a*B)*(a*d*f*h*(m + 1)
 - b*(m + 2)*(d*f*g + d*e*h + c*f*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B)*
x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x] && IntegerQ[2*m
] && LtQ[m, -1]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3851 vs. $2(775) = 1550$.

Time = 38.85 (sec) , antiderivative size = 3852, normalized size of antiderivative = 4.66

method	result	size
elliptic	Expression too large to display	3852
default	Expression too large to display	168816

input
```
int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2/3/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+
a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)/b^2*(A*b^2-B*a*b+C*a^2)*(b*
d*f*h*x^4+a*d*f*h*x^3+b*c*f*h*x^3+b*d*e*h*x^3+b*d*f*g*x^3+a*c*f*h*x^2+a*d*
e*h*x^2+a*d*f*g*x^2+b*c*e*h*x^2+b*c*f*g*x^2+b*d*e*g*x^2+a*c*e*h*x+a*c*f*g*
x+a*d*e*g*x+b*c*e*g*x+a*c*e*g)^(1/2)/(x+a/b)^2+2/3*(b*d*f*h*x^3+b*c*f*h*x^
2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g)/(a^3*d*f*
h-a^2*b*c*f*h-a^2*b*d*e*h-a^2*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-
b^3*c*e*g)^2*(6*A*a^2*b*d*f*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*
f*g+2*A*b^3*c*e*h+2*A*b^3*c*f*g+2*A*b^3*d*e*g-3*B*a^3*d*f*h+B*a^2*b*c*f*h+
B*a^2*b*d*e*h+B*a^2*b*d*f*g+B*a*b^2*c*e*h+B*a*b^2*c*f*g+B*a*b^2*d*e*g-3*B*
b^3*c*e*g+2*C*a^3*c*f*h+2*C*a^3*d*e*h+2*C*a^3*d*f*g-4*C*a^2*b*c*e*h-4*C*a^
2*b*c*f*g-4*C*a^2*b*d*e*g+6*C*a*b^2*c*e*g)/((x+a/b)*(b*d*f*h*x^3+b*c*f*h*x
^2+b*d*e*h*x^2+b*d*f*g*x^2+b*c*e*h*x+b*c*f*g*x+b*d*e*g*x+b*c*e*g))^(1/2)+2
*(C/b^2-1/3/b^2*(3*A*a*b^2*d*f*h-A*b^3*c*f*h-A*b^3*d*e*h-A*b^3*d*f*g-3*B*a
^2*b*d*f*h+B*a*b^2*c*f*h+B*a*b^2*d*e*h+B*a*b^2*d*f*g+3*C*a^3*d*f*h-C*a^2*b
*c*f*h-C*a^2*b*d*e*h-C*a^2*b*d*f*g)/(a^3*d*f*h-a^2*b*c*f*h-a^2*b*d*e*h-a^2
*b*d*f*g+a*b^2*c*e*h+a*b^2*c*f*g+a*b^2*d*e*g-b^3*c*e*g)+1/3/b*(a^2*d*f*h-a
*b*c*f*h-a*b*d*e*h-a*b*d*f*g+b^2*c*e*h+b^2*c*f*g+b^2*d*e*g)*(6*A*a^2*b*d*f
*h-4*A*a*b^2*c*f*h-4*A*a*b^2*d*e*h-4*A*a*b^2*d*f*g+2*A*b^3*c*e*h+2*A*b^...
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(f*x + e)*sqrt(
h*x + g)/(b^3*d*f*h*x^6 + a^3*c*e*g + (b^3*d*f*g + (b^3*d*e + (b^3*c + 3*a
*b^2*d)*f)*h)*x^5 + ((b^3*d*e + (b^3*c + 3*a*b^2*d)*f)*g + ((b^3*c + 3*a*b
^2*d)*e + 3*(a*b^2*c + a^2*b*d)*f)*h)*x^4 + (((b^3*c + 3*a*b^2*d)*e + 3*(a
*b^2*c + a^2*b*d)*f)*g + (3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)
*h)*x^3 + ((3*(a*b^2*c + a^2*b*d)*e + (3*a^2*b*c + a^3*d)*f)*g + (a^3*c*f
+ (3*a^2*b*c + a^3*d)*e)*h)*x^2 + (a^3*c*e*h + (a^3*c*f + (3*a^2*b*c + a^3
*d)*e)*g)*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(5/2)/(d*x+c)**(1/2)/(f*x+e)**(1/2)/(h*x
+g)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{Cx^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\sqrt{fx+e}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^
(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x + a)^(5/2)*sqrt(d*x + c)*sqrt(f*x + e)*s
qrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e+f\,x}\,\sqrt{g+h\,x}\,{\left(a+b\,x\right)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c
+ d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(g + h*x)^(1/2)*(a + b*x)^(5/2)*(c
+ d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(a+bx)^{5/2}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+Bx+A}{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,\sqrt{fx+e}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

output
```
int((C*x^2+B*x+A)/(b*x+a)^(5/2)/(d*x+c)^(1/2)/(f*x+e)^(1/2)/(h*x+g)^(1/2),
x)
```

3.43 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 713

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx = -\frac{2(Ce^2-Bef+Af^2)\sqrt{bg-ah}\sqrt{c+dx}\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}E\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right)|-\frac{(de-cf)(bg-ah)}{(bc-ad)(fg-eh)}\right)}{f(be-af)(de-cf)\sqrt{fg-eh}\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$
$$-\frac{2((Bc-Ad)f^2+Ce(de-2cf))\sqrt{bg-ah}\sqrt{c+dx}\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right),-\frac{(d}{(b}\right.}{(bc-ad)f^2(de-cf)\sqrt{fg-eh}\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}\sqrt{g+hx}}$$
$$+\frac{2C\sqrt{bg-ah}\sqrt{\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}}(e+fx)\sqrt{\frac{(be-af)(g+hx)}{(bg-ah)(e+fx)}}\operatorname{EllipticPi}\left(\frac{f(bg-ah)}{b(fg-eh)},\arcsin\left(\frac{\sqrt{fg-eh}\sqrt{a+bx}}{\sqrt{bg-ah}\sqrt{e+fx}}\right),-\frac{(de-cf)(bg}{(bc-ad)(fg}\right.}{bf^2\sqrt{fg-eh}\sqrt{c+dx}\sqrt{g+hx}}$$

output
```
-2*(A*f^2-B*e*f+C*e^2)*(-a*h+b*g)^(1/2)*(d*x+c)^(1/2)*((-a*f+b*e)*(h*x+g)/
(-a*h+b*g)/(f*x+e))^(1/2)*EllipticE((-e*h+f*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b
*g)^(1/2)/(f*x+e)^(1/2),(-(-c*f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*g))^(1/
2))/f/(-a*f+b*e)/(-c*f+d*e)/(-e*h+f*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-a*d+b*c
)/(f*x+e))^(1/2)/(h*x+g)^(1/2)-2*((-A*d+B*c)*f^2+C*e*(-2*c*f+d*e))*(-a*h+b
*g)^(1/2)*(d*x+c)^(1/2)*((-a*f+b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^(1/2)*Elli
pticF((-e*h+f*g)^(1/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),(-(-c*
f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))/(-a*d+b*c)/f^2/(-c*f+d*e)/
(-e*h+f*g)^(1/2)/((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^(1/2)/(h*x+g)^(1/
2)+2*C*(-a*h+b*g)^(1/2)*((-a*f+b*e)*(d*x+c)/(-a*d+b*c)/(f*x+e))^(1/2)*(f*x
+e)*((-a*f+b*e)*(h*x+g)/(-a*h+b*g)/(f*x+e))^(1/2)*EllipticPi((-e*h+f*g)^(1
/2)*(b*x+a)^(1/2)/(-a*h+b*g)^(1/2)/(f*x+e)^(1/2),f*(-a*h+b*g)/b/(-e*h+f*g)
,(-(-c*f+d*e)*(-a*h+b*g)/(-a*d+b*c)/(-e*h+f*g))^(1/2))/b/f^2/(-e*h+f*g)^(1
/2)/(d*x+c)^(1/2)/(h*x+g)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 11158 vs. $2(713) = 1426$.

Time = 35.94 (sec) , antiderivative size = 11158, normalized size of antiderivative = 15.65

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2)*S
qrt[g + h*x]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 2.28 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.213, Rules used = {2107, 2105, 27, 194, 327, 2101, 183, 188, 321, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx$$

$$\Big\downarrow \ 2107$$

$$\frac{\int \frac{2bd(Ce^2-Bfe+Af^2)hx^2+(C(2(bdg+bch+adh)e^2-f(bcg+adg+ach)e+acf^2g)-(Be-Af)(adfh+b(dfg+deh+cfh)))x+(Ce-Bf)(bceg+a(\cdots}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}}{(be-af)(de-cf)(fg-eh)}$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}\left(Af^2-Bef+Ce^2\right)}{\sqrt{e+fx}(be-af)(de-cf)(fg-eh)}$$

$$\Big\downarrow \ 2105$$

$$\frac{\frac{(de-cf)(dg-ch)\left(Af^2-Bef+Ce^2\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{f}+\frac{\int -\frac{2bd(be-af)(de-cf)h(Ceg-Bfg+Afh-C(fg-eh)x)}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{2bdfh}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\cdots}{\cdots}}{(be-af)(de-cf)(fg-eh)}$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}\left(Af^2-Bef+Ce^2\right)}{\sqrt{e+fx}(be-af)(de-cf)(fg-eh)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(de-cf)(dg-ch)\left(Af^2-Bef+Ce^2\right)\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}\sqrt{e+fx}\sqrt{g+hx}}dx}{f}-\frac{(be-af)(de-cf)\int \frac{Ceg-Bfg+Afh-C(fg-eh)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{f}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+\cdots}}{f\sqrt{c-\cdots}}}{(be-af)(de-cf)(fg-eh)}$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}\left(Af^2-Bef+Ce^2\right)}{\sqrt{e+fx}(be-af)(de-cf)(fg-eh)}$$

$$\Big\downarrow \ 194$$

$$\frac{-\frac{2\sqrt{a+bx}(dg-ch)\left(Af^2-Bef+Ce^2\right)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}\int \frac{\sqrt{1-\frac{(bc-ad)(e+fx)}{(be-af)(c+dx)}}}{\sqrt{1-\frac{(dg-ch)(e+fx)}{(fg-eh)(c+dx)}}}d\frac{\sqrt{e+fx}}{\sqrt{c+dx}}}{f\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}-\frac{(be-af)(de-cf)\int \frac{Ceg-Bfg+Afh-C(fg-eh)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{f}+}{(be-af)(de-cf)(fg-eh)}$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}\left(Af^2-Bef+Ce^2\right)}{\sqrt{e+fx}(be-af)(de-cf)(fg-eh)}$$

$$\Big\downarrow \text{327}$$

$$\frac{-\frac{(be-af)(de-cf)\int \frac{Ceg-Bfg+Afh-C(fg-eh)x}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{f}-\frac{2\sqrt{a+bx}\sqrt{dg-ch}\sqrt{fg-eh}\left(Af^2-Bef+Ce^2\right)\sqrt{-\frac{(g+hx)(de-cf)}{(c+dx)(fg-eh)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right.\right.}{f\sqrt{g+hx}\sqrt{\frac{(a+bx)(de-cf)}{(c+dx)(be-af)}}}}{(be-af)(de-cf)(fg-eh)}$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}\left(Af^2-Bef+Ce^2\right)}{\sqrt{e+fx}(be-af)(de-cf)(fg-eh)}$$

$$\Big\downarrow \text{2101}$$

$$\frac{-\frac{(be-af)(de-cf)\left(\frac{(aC(fg-eh)+b(Afh-Bfg+Ceg))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{C(fg-eh)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}\right)}{f}-\frac{2\sqrt{a+bx}\sqrt{dg-ch}\ldots}{}}{(be-af)(de-c\ldots}$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}\left(Af^2-Bef+Ce^2\right)}{\sqrt{e+fx}(be-af)(de-cf)(fg-eh)}$$

$$\Big\downarrow \text{183}$$

$$\frac{-\frac{(be-af)(de-cf)\left(\frac{(aC(fg-eh)+b(Afh-Bfg+Ceg))\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}\sqrt{e+fx}\sqrt{g+hx}}dx}{b}-\frac{2C(a+bx)(fg-eh)\sqrt{\frac{(c+dx)(bg-ah)}{(a+bx)(dg-ch)}}\sqrt{\frac{(e+fx)(bg-ah)}{(a+bx)(fg-eh)}}\int \frac{\ldots}{\left(h-\frac{b(\ldots}{\ldots}\right.}}{b\sqrt{c+dx}\sqrt{\ldots}}\ldots}{f}}{}$$

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}\left(Af^2-Bef+Ce^2\right)}{\sqrt{e+fx}(be-af)(de-cf)(fg-eh)}$$

$$\Big\downarrow \text{188}$$

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(Ce^2-Bfe+Af^2)}{f\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(Ce}{f\sqrt{c+dx}}$$

$$\frac{2(Ce^2-Bfe+Af^2)\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{(be-af)(de-cf)(fg-eh)\sqrt{e+fx}}$$

$\downarrow$ 321

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(Ce^2-Bfe+Af^2)}{f\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(Ce}{f\sqrt{c+dx}}$$

$$\frac{2(Ce^2-Bfe+Af^2)\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{(be-af)(de-cf)(fg-eh)\sqrt{e+fx}}$$

$\downarrow$ 412

$$-\frac{2\sqrt{dg-ch}\sqrt{fg-eh}\sqrt{a+bx}\sqrt{-\frac{(de-cf)(g+hx)}{(fg-eh)(c+dx)}}E\left(\arcsin\left(\frac{\sqrt{dg-ch}\sqrt{e+fx}}{\sqrt{fg-eh}\sqrt{c+dx}}\right)|\frac{(bc-ad)(fg-eh)}{(be-af)(dg-ch)}\right)(Ce^2-Bfe+Af^2)}{f\sqrt{\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}}\sqrt{g+hx}}+\frac{2d\sqrt{a+bx}\sqrt{e+fx}\sqrt{g+hx}(Ce}{f\sqrt{c+dx}}$$

$$\frac{2(Ce^2-Bfe+Af^2)\sqrt{a+bx}\sqrt{c+dx}\sqrt{g+hx}}{(be-af)(de-cf)(fg-eh)\sqrt{e+fx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[c + d*x]*(e + f*x)^(3/2)*Sqrt[g
+ h*x]),x]
```

output
```
(-2*(C*e^2 - B*e*f + A*f^2)*Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[g + h*x])/((b
*e - a*f)*(d*e - c*f)*(f*g - e*h)*Sqrt[e + f*x]) + ((2*d*(C*e^2 - B*e*f +
A*f^2)*Sqrt[a + b*x]*Sqrt[e + f*x]*Sqrt[g + h*x])/(f*Sqrt[c + d*x]) - (2*(
C*e^2 - B*e*f + A*f^2)*Sqrt[d*g - c*h]*Sqrt[f*g - e*h]*Sqrt[a + b*x]*Sqrt[
-(((d*e - c*f)*(g + h*x))/((f*g - e*h)*(c + d*x)))]*EllipticE[ArcSin[(Sqrt
[d*g - c*h]*Sqrt[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[c + d*x])], ((b*c - a*d)*
(f*g - e*h))/((b*e - a*f)*(d*g - c*h))])/(f*Sqrt[((d*e - c*f)*(a + b*x))/(
(b*e - a*f)*(c + d*x))]*Sqrt[g + h*x]) - ((b*e - a*f)*(d*e - c*f)*((2*(a*C
*(f*g - e*h) + b*(C*e*g - B*f*g + A*f*h))*Sqrt[((b*e - a*f)*(c + d*x))/((d
*e - c*f)*(a + b*x))]*Sqrt[g + h*x]*EllipticF[ArcSin[(Sqrt[b*g - a*h]*Sqrt
[e + f*x])/(Sqrt[f*g - e*h]*Sqrt[a + b*x])], -(((b*c - a*d)*(f*g - e*h))/(
(d*e - c*f)*(b*g - a*h)))])/(b*Sqrt[b*g - a*h]*Sqrt[f*g - e*h]*Sqrt[c + d*
x]*Sqrt[-(((b*e - a*f)*(g + h*x))/((f*g - e*h)*(a + b*x)))]) - (2*C*Sqrt[-
(d*g) + c*h]*(f*g - e*h)*(a + b*x)*Sqrt[((b*g - a*h)*(c + d*x))/((d*g - c*
h)*(a + b*x))]*Sqrt[((b*g - a*h)*(e + f*x))/((f*g - e*h)*(a + b*x))]*Ellip
ticPi[-((b*(d*g - c*h))/((b*c - a*d)*h)), ArcSin[(Sqrt[b*c - a*d]*Sqrt[g +
 h*x])/(Sqrt[-(d*g) + c*h]*Sqrt[a + b*x])], ((b*e - a*f)*(d*g - c*h))/((b*
c - a*d)*(f*g - e*h))])/(b*Sqrt[b*c - a*d]*h*Sqrt[c + d*x]*Sqrt[e + f*x]))
)/f)/((b*e - a*f)*(d*e - c*f)*(f*g - e*h))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 183
```
Int[Sqrt[(a_.) + (b_.)*(x_)]/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(
x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*(a + b*x)*Sqrt[(b*g - a*h)*((
c + d*x)/((d*g - c*h)*(a + b*x)))]*(Sqrt[(b*g - a*h)*((e + f*x)/((f*g - e*h
)*(a + b*x)))]/(Sqrt[c + d*x]*Sqrt[e + f*x]))   Subst[Int[1/((h - b*x^2)*Sq
rt[1 + (b*c - a*d)*(x^2/(d*g - c*h))]*Sqrt[1 + (b*e - a*f)*(x^2/(f*g - e*h)
)]), x], x, Sqrt[g + h*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x]
```

rule 188
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[2*Sqrt[g + h*x]*(Sqrt[(b*e -
a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]/((f*g - e*h)*Sqrt[c + d*x]*Sqrt[(
-(b*e - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]))   Subst[Int[1/(Sqrt[1
+ (b*c - a*d)*(x^2/(d*e - c*f))]*Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))]),
x], x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h},
x]
```

rule 194
```
Int[Sqrt[(c_.) + (d_.)*(x_)]/(((a_.) + (b_.)*(x_))^(3/2)*Sqrt[(e_.) + (f_.)
*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2*Sqrt[c + d*x]*(Sqrt[(-(b*e
 - a*f))*((g + h*x)/((f*g - e*h)*(a + b*x)))]/((b*e - a*f)*Sqrt[g + h*x]*Sq
rt[(b*e - a*f)*((c + d*x)/((d*e - c*f)*(a + b*x)))]))   Subst[Int[Sqrt[1 +
(b*c - a*d)*(x^2/(d*e - c*f))]/Sqrt[1 - (b*g - a*h)*(x^2/(f*g - e*h))], x],
 x, Sqrt[e + f*x]/Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 2101
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]
*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Simp[(A*b
 - a*B)/b   Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x])
, x], x] + Simp[B/b   Int[Sqrt[a + b*x]/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g
 + h*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B}, x]
```

rule 2105
```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.
) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbo
l] :> Simp[C*Sqrt[a + b*x]*Sqrt[e + f*x]*(Sqrt[g + h*x]/(b*f*h*Sqrt[c + d*x
])), x] + (Simp[1/(2*b*d*f*h)   Int[(1/(Sqrt[a + b*x]*Sqrt[c + d*x]*Sqrt[e
+ f*x]*Sqrt[g + h*x]))*Simp[2*A*b*d*f*h - C*(b*d*e*g + a*c*f*h) + (2*b*B*d*
f*h - C*(a*d*f*h + b*(d*f*g + d*e*h + c*f*h)))*x, x], x], x] + Simp[C*(d*e
- c*f)*((d*g - c*h)/(2*b*d*f*h))   Int[Sqrt[a + b*x]/((c + d*x)^(3/2)*Sqrt[
e + f*x]*Sqrt[g + h*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, A, B, C}
, x]
```

rule 2107
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2))/(Sqrt[(
c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Sy
mbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*(a + b*x)^(m + 1)*Sqrt[c + d*x]*Sqrt[
e + f*x]*(Sqrt[g + h*x]/((m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))), x]
- Simp[1/(2*(m + 1)*(b*c - a*d)*(b*e - a*f)*(b*g - a*h))   Int[((a + b*x)^(
m + 1)/(Sqrt[c + d*x]*Sqrt[e + f*x]*Sqrt[g + h*x]))*Simp[A*(2*a^2*d*f*h*(m
+ 1) - 2*a*b*(m + 1)*(d*f*g + d*e*h + c*f*h) + b^2*(2*m + 3)*(d*e*g + c*f*g
 + c*e*h)) - (b*B - a*C)*(a*(d*e*g + c*f*g + c*e*h) + 2*b*c*e*g*(m + 1)) -
2*((A*b - a*B)*(a*d*f*h*(m + 1) - b*(m + 2)*(d*f*g + d*e*h + c*f*h)) - C*(a
^2*(d*f*g + d*e*h + c*f*h) - b^2*c*e*g*(m + 1) + a*b*(m + 1)*(d*e*g + c*f*g
 + c*e*h)))*x + d*f*h*(2*m + 5)*(A*b^2 - a*b*B + a^2*C)*x^2, x], x], x] /;
FreeQ[{a, b, c, d, e, f, g, h, A, B, C}, x] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2317 vs. $2(656) = 1312$.

Time = 23.81 (sec) , antiderivative size = 2318, normalized size of antiderivative = 3.25

method	result	size
elliptic	Expression too large to display	2318
default	Expression too large to display	25793

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(3/2)/(h*x+g)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
((h*x+g)*(d*x+c)*(b*x+a)*(f*x+e))^(1/2)/(h*x+g)^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)/(f*x+e)^(1/2)*(2*(b*d*f*h*x^3+a*d*f*h*x^2+b*c*f*h*x^2+b*d*f*g*x^2+
a*c*f*h*x+a*d*f*g*x+b*c*f*g*x+a*c*f*g)/(a*c*e*f^2*h-a*c*f^3*g-a*d*e^2*f*h+
a*d*e*f^2*g-b*c*e^2*f*h+b*c*e*f^2*g+b*d*e^3*h-b*d*e^2*f*g)/f*(A*f^2-B*e*f+
C*e^2)/((x+e/f)*(b*d*f*h*x^3+a*d*f*h*x^2+b*c*f*h*x^2+b*d*f*g*x^2+a*c*f*h*x
+a*d*f*g*x+b*c*f*g*x+a*c*f*g))^(1/2)+2*((B*f-C*e)/f^2+1/f^2*(a*c*f^2*h-a*d
*e*f*h+a*d*f^2*g-b*c*e*f*h+b*c*f^2*g+b*d*e^2*h-b*d*e*f*g)*(A*f^2-B*e*f+C*e
^2)/(a*c*e*f^2*h-a*c*f^3*g-a*d*e^2*f*h+a*d*e*f^2*g-b*c*e^2*f*h+b*c*e*f^2*g
+b*d*e^3*h-b*d*e^2*f*g)-(a*c*f*h+a*d*f*g+b*c*f*g)/(a*c*e*f^2*h-a*c*f^3*g-a
*d*e^2*f*h+a*d*e*f^2*g-b*c*e^2*f*h+b*c*e*f^2*g+b*d*e^3*h-b*d*e^2*f*g)/f*(A
*f^2-B*e*f+C*e^2))*(e/f-g/h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*
(x+c/d)^2*((-c/d+g/h)*(x+a/b)/(-a/b+g/h)/(x+c/d))^(1/2)*((-c/d+g/h)*(x+e/f
)/(-e/f+g/h)/(x+c/d))^(1/2)/(c/d-e/f)/(-c/d+g/h)/(h*d*b*f*(x+g/h)*(x+c/d)*
(x+a/b)*(x+e/f))^(1/2)*EllipticF(((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1
/2),((-c/d+a/b)*(e/f-g/h)/(a/b-g/h)/(-c/d+e/f))^(1/2))+2*(C/f+1/f*(a*d*f*h
+b*c*f*h-b*d*e*h+b*d*f*g)*(A*f^2-B*e*f+C*e^2)/(a*c*e*f^2*h-a*c*f^3*g-a*d*e
^2*f*h+a*d*e*f^2*g-b*c*e^2*f*h+b*c*e*f^2*g+b*d*e^3*h-b*d*e^2*f*g)-(2*a*d*f
*h+2*b*c*f*h+2*b*d*f*g)/(a*c*e*f^2*h-a*c*f^3*g-a*d*e^2*f*h+a*d*e*f^2*g-b*c
*e^2*f*h+b*c*e*f^2*g+b*d*e^3*h-b*d*e^2*f*g)/f*(A*f^2-B*e*f+C*e^2))*(e/f-g/
h)*((c/d-e/f)*(x+g/h)/(-e/f+g/h)/(x+c/d))^(1/2)*(x+c/d)^2*((-c/d+g/h)*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(3/2)/(h*x+g)^
(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}\,(e+fx)^{\frac{3}{2}}\,\sqrt{g+hx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(d*x+c)**(1/2)/(f*x+e)**(3/2)/(h*x
+g)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a + b*x)*sqrt(c + d*x)*(e + f*x)**(3/2)*
sqrt(g + h*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}(fx+e)^{\frac{3}{2}}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(3/2)/(h*x+g)^
(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*(f*x + e)^(3/2)*s
qrt(h*x + g)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx+a}\sqrt{dx+c}(fx+e)^{\frac{3}{2}}\sqrt{hx+g}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(3/2)/(h*x+g)^
(1/2),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x + a)*sqrt(d*x + c)*(f*x + e)^(3/2)*s
qrt(h*x + g)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+B\,x+A}{{\left(e+f\,x\right)}^{3/2}\,\sqrt{g+h\,x}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^(3/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c
+ d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^(3/2)*(g + h*x)^(1/2)*(a + b*x)^(1/2)*(c
+ d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{c+dx}(e+fx)^{3/2}\sqrt{g+hx}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{bx+a}\,\sqrt{dx+c}\,(fx+e)^{\frac{3}{2}}\,\sqrt{hx+g}}dx$$

input
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(3/2)/(h*x+g)^(1/2),
x)
```

output
```
int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(f*x+e)^(3/2)/(h*x+g)^(1/2),
x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                   ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
          finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
             #is checked before calling the grading function that is passed.
             #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):



    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file