Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.2-Quadratic-binomial/29-1.1.2.1

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May 18, 2024 Compiled on May 18, 2024 at 11:34am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**136**]. This is test number [29].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (136)	0.00 (0)
Mupad	100.00 (136)	0.00 (0)
Sympy	100.00 (136)	0.00 (0)
Mathematica	98.53 (134)	1.47 (2)
Maple	50.00 (68)	50.00 (68)
Fricas	47.79 (65)	52.21 (71)
Reduce	47.79 (65)	52.21 (71)
Giac	44.85 (61)	55.15 (75)
Maxima	44.12 (60)	55.88 (76)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

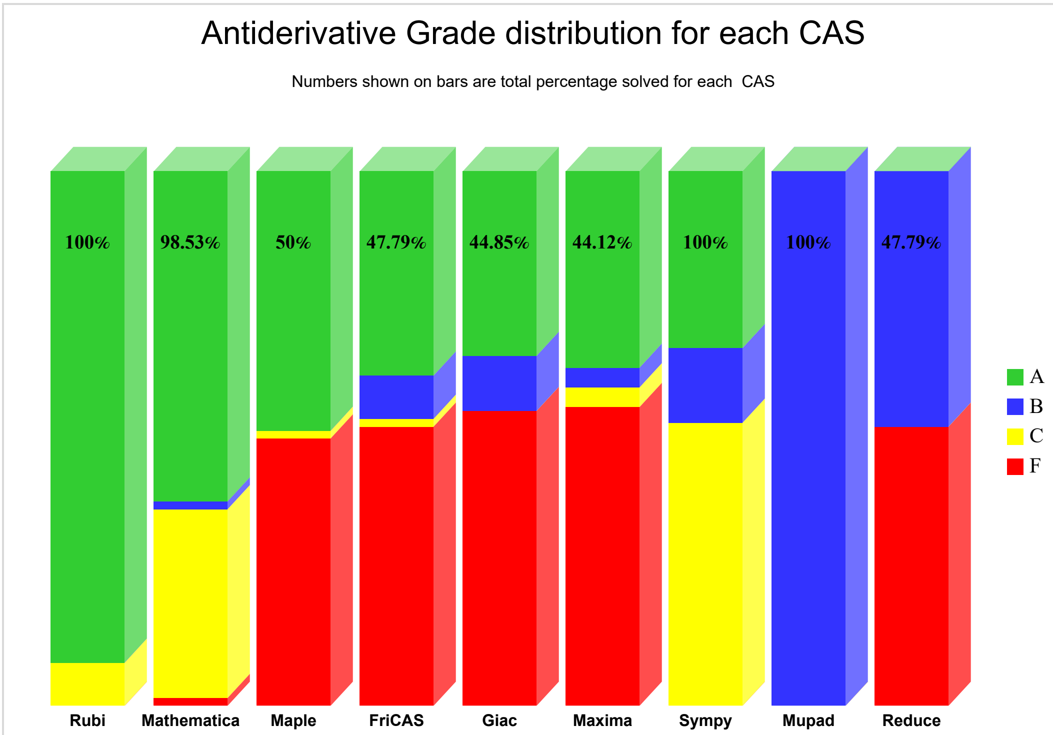
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

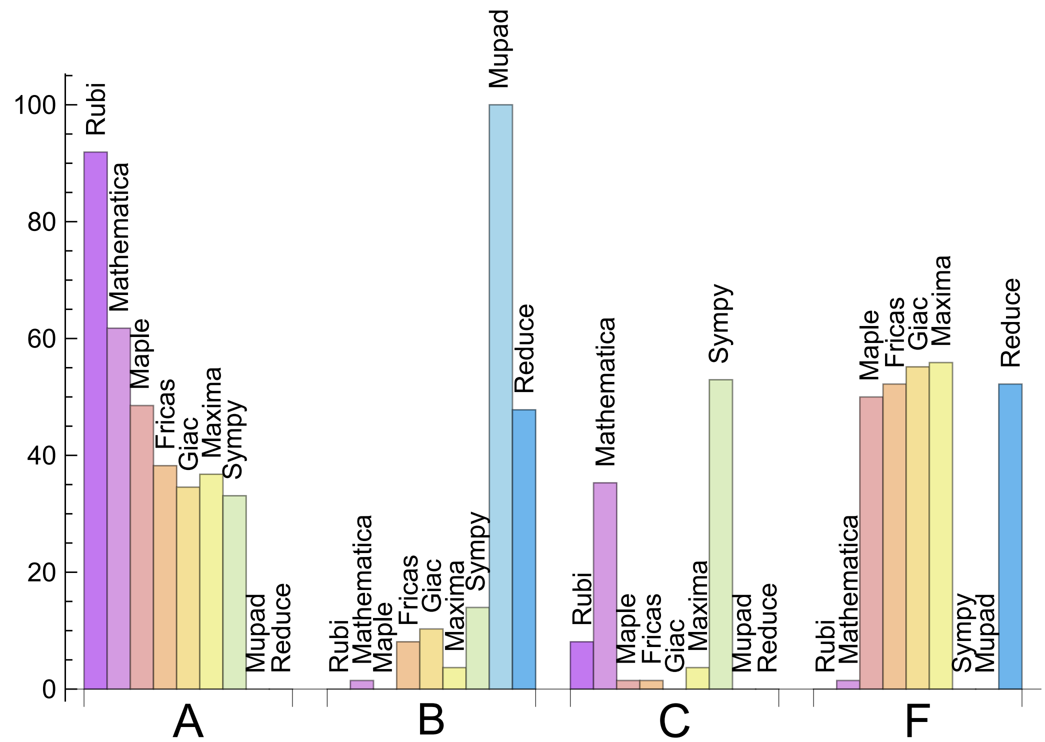
System	% A grade	% B grade	% C grade	% F grade
Rubi	91.912	0.000	8.088	0.000
Mathematica	61.765	1.471	35.294	1.471
Maple	48.529	0.000	1.471	50.000
Fricas	38.235	8.088	1.471	52.206
Maxima	36.765	3.676	3.676	55.882
Giac	34.559	10.294	0.000	55.147
Sympy	33.088	13.971	52.941	0.000
Mupad	0.000	100.000	0.000	0.000
Reduce	0.000	47.794	0.000	52.206

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mupad	0	0.00	0.00	0.00
Sympy	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Maple	68	100.00	0.00	0.00
Fricas	71	100.00	0.00	0.00
Reduce	71	100.00	0.00	0.00
Giac	75	97.33	0.00	2.67
Maxima	76	93.42	0.00	6.58

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.09
Giac	0.12
Mupad	0.12
Rubi	0.18
Reduce	0.23
Maple	0.40
Sympy	0.54
Mathematica	3.75

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	31.12	0.84	23.00	0.84
Mupad	34.23	0.63	37.00	0.76
Giac	34.52	1.17	36.00	0.89
Maxima	40.97	0.95	23.50	0.84
Mathematica	42.50	0.83	46.00	1.00
Reduce	46.02	1.20	27.00	0.97
Sympy	80.38	1.24	26.00	0.61
Fricas	80.62	2.06	59.00	2.09
Rubi	110.80	0.97	46.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

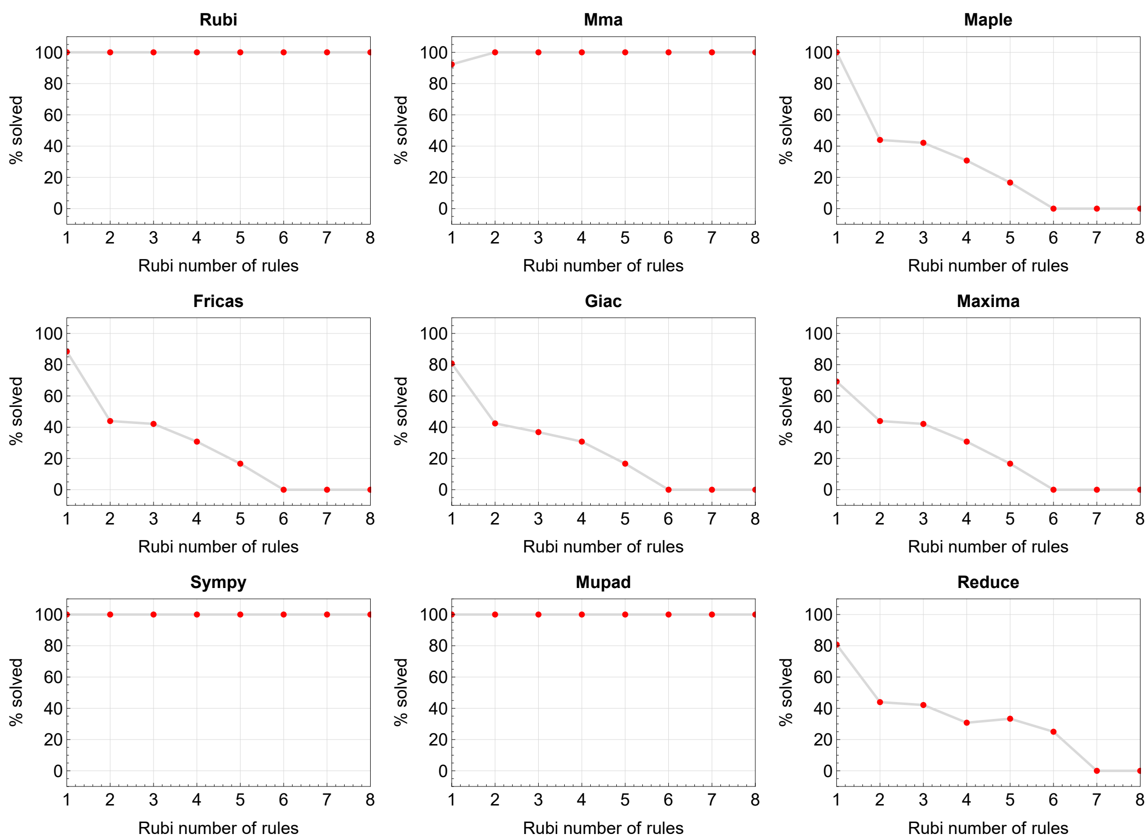


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

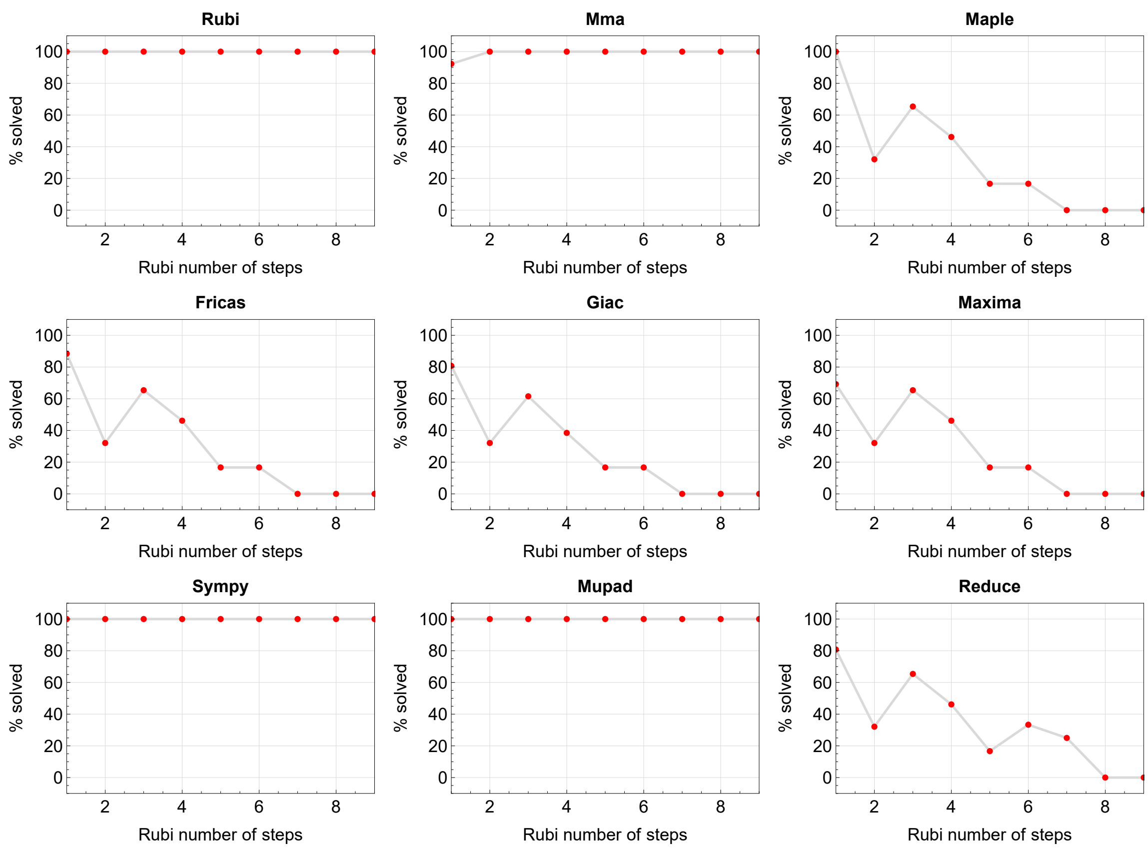


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

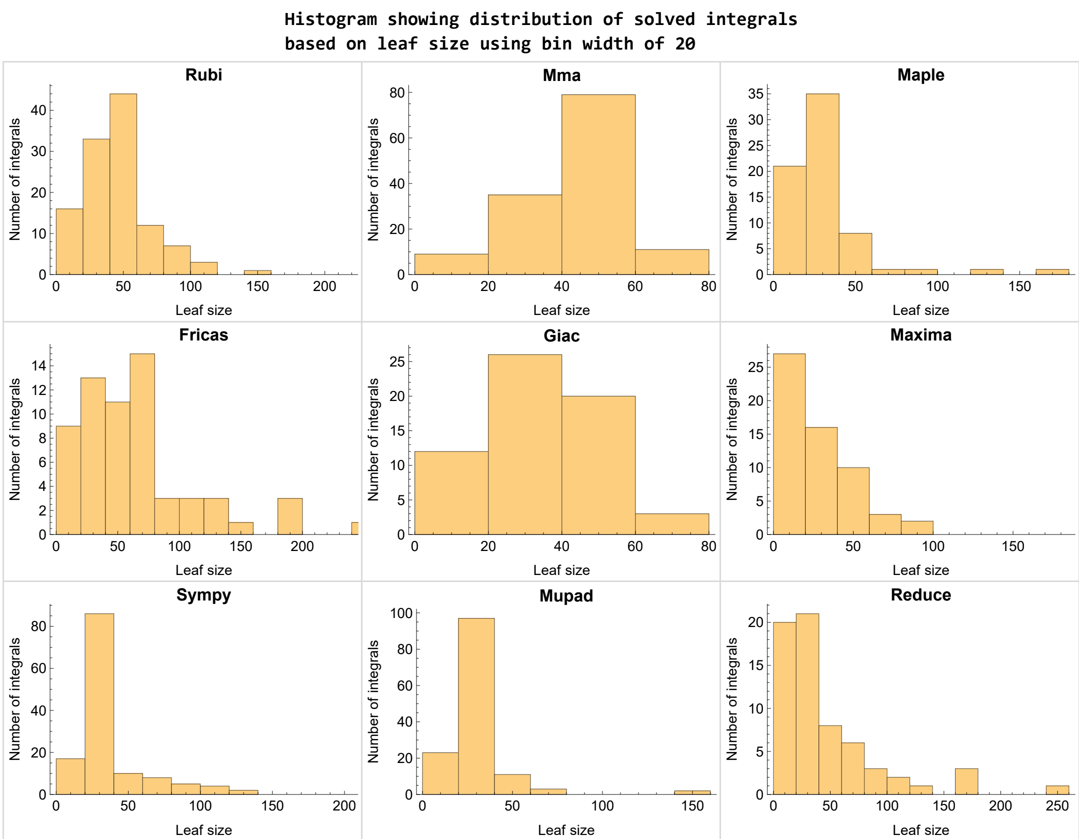


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

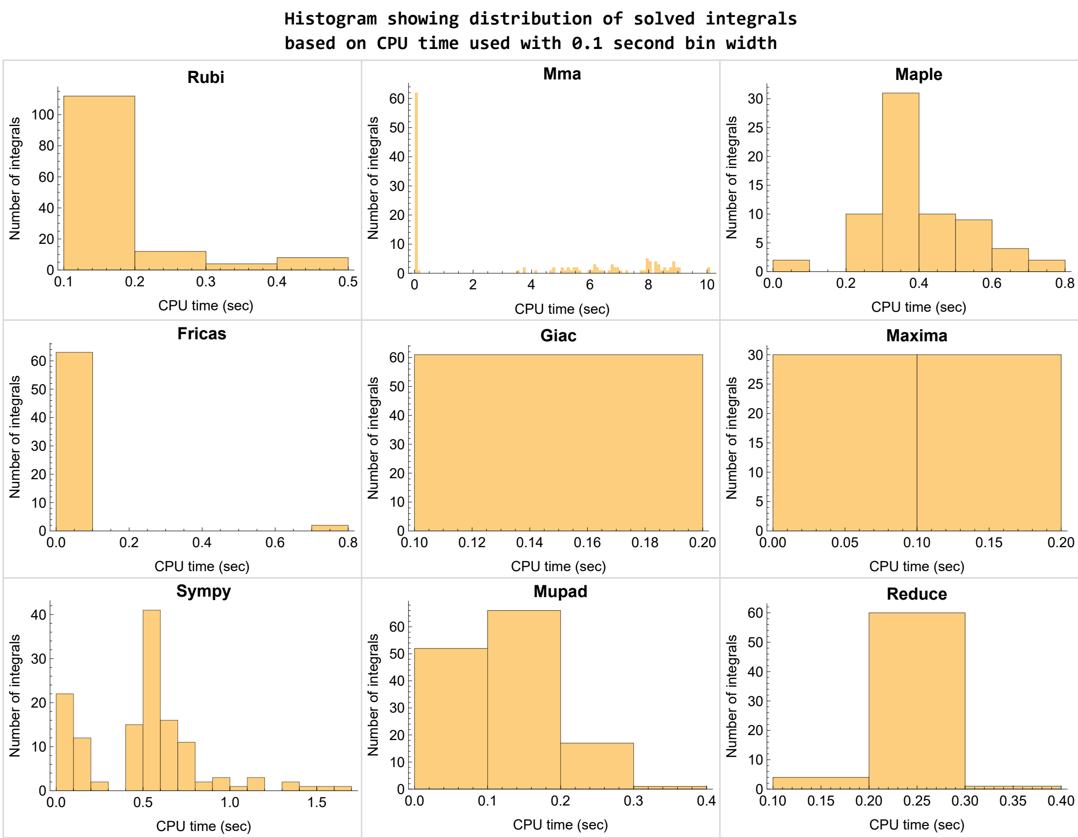


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

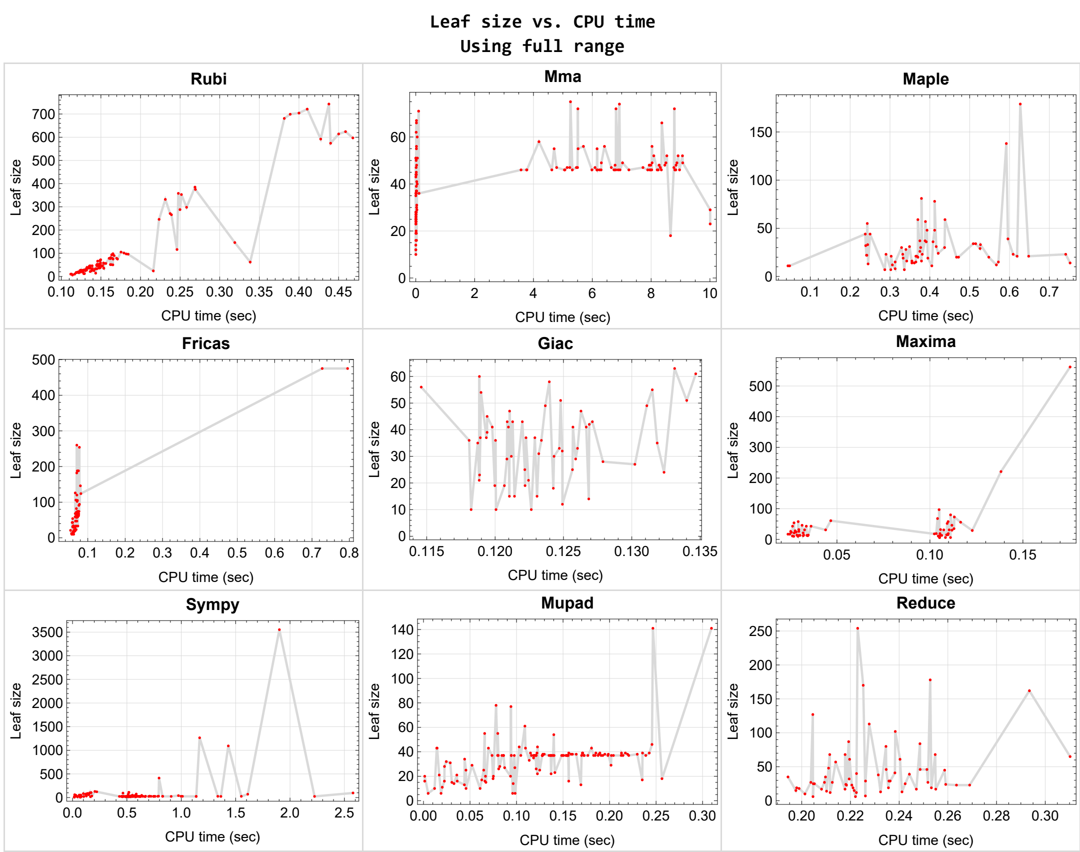


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{71, 73, 74, 75, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111\}$

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

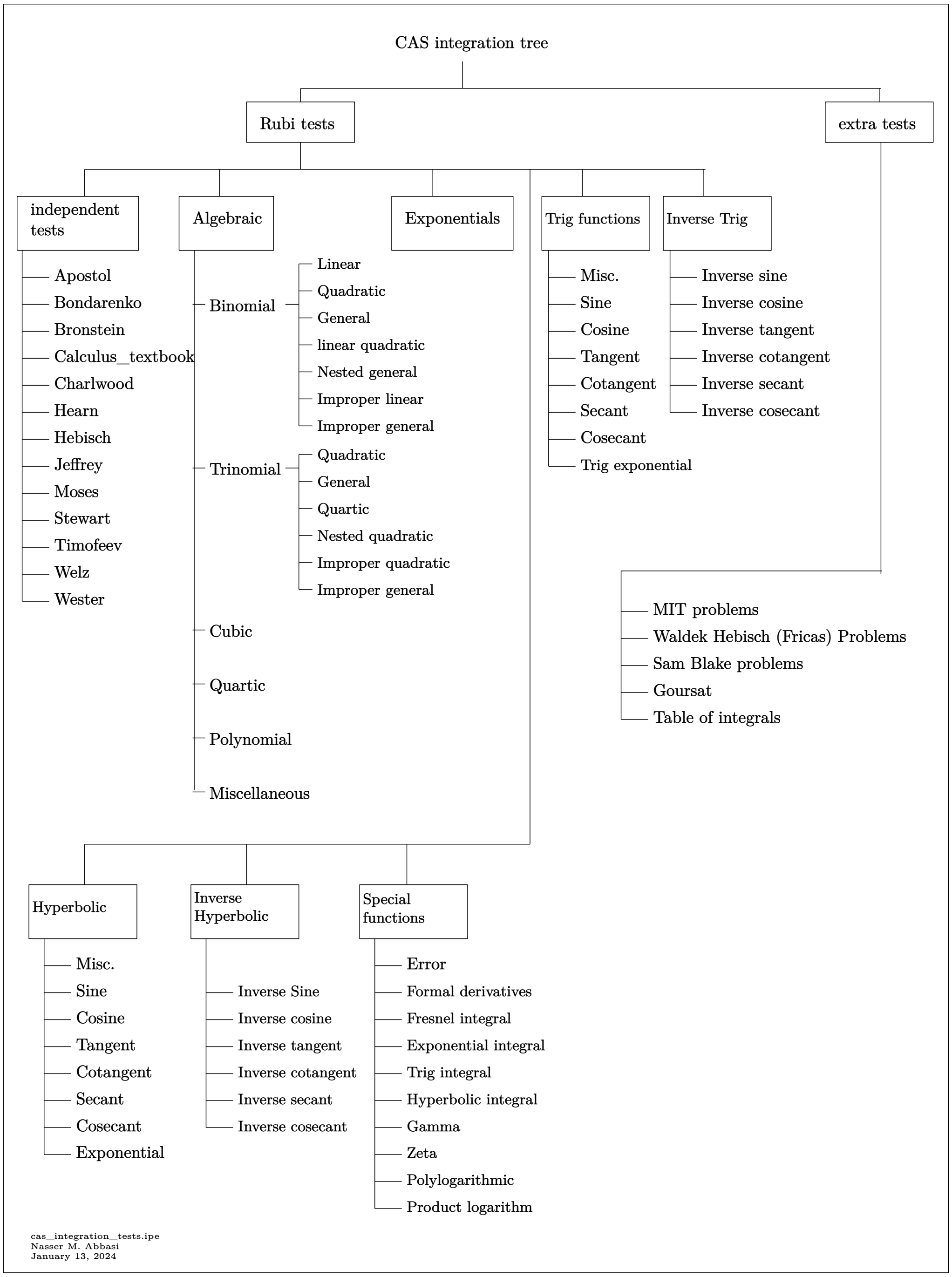
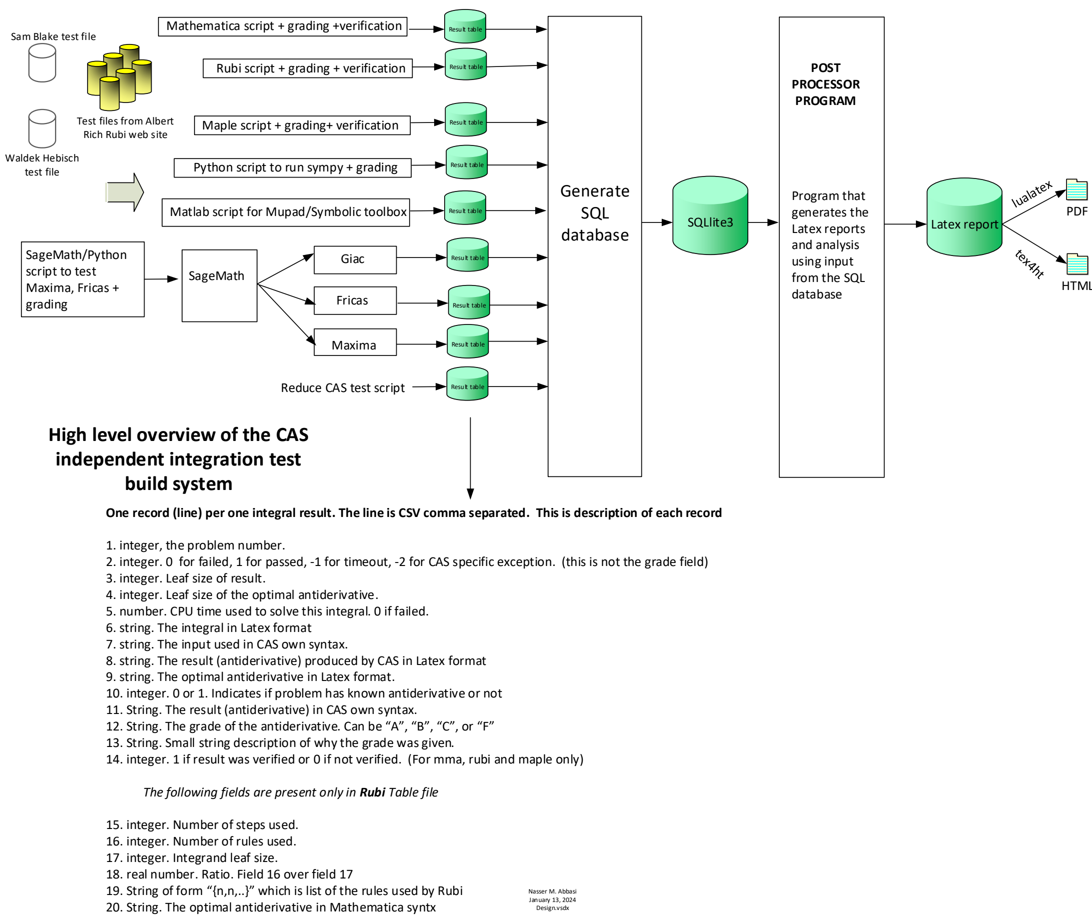


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 135, 136 }

B grade { }

C grade { 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 93, 94, 95, 96, 97, 98, 99, 100, 101, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 135, 136 }

B grade { 27, 55 }

C grade { 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134 }

F normal fail { 29, 30 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 92, 93, 135 }

B grade { }

C grade { 21, 56 }

F normal fail { 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 59, 60, 62, 63, 64, 65 }

B grade { 21, 26, 27, 34, 35, 36, 53, 54, 57, 58, 61 }

C grade { 52, 56 }

F normal fail { 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 28, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 53, 54, 55, 57, 58, 59, 61, 62, 63, 65 }

B grade { 27, 29, 30, 35, 36 }

C grade { 48, 52, 56, 60, 64 }

F normal fail { 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

F(-1) timedout fail { }

F(-2) exception fail { 22, 23, 24, 25, 26 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 31, 32, 33, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 49, 50, 51, 55, 59, 63, 65 }

B grade { 27, 34, 35, 36, 46, 48, 53, 54, 57, 58, 60, 61, 62, 64 }

C grade { }

F normal fail { 52, 56, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

F(-1) timedout fail { }

F(-2) exception fail { 29, 30 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 21, 28, 31, 33, 37, 38, 39, 40, 41, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, 61, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75 }

B grade { 6, 7, 14, 20, 22, 23, 24, 25, 26, 27, 29, 30, 32, 34, 35, 36, 42, 43, 44 }

C grade { 19, 46, 47, 48, 58, 59, 60, 62, 63, 64, 65, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 110, 111 }

C grade { }

F normal fail { 29, 30, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	55	54	54	61	54	57	54
N.S.	1	1.00	1.00	0.89	0.87	0.87	0.98	0.87	0.92	0.87
time (sec)	N/A	0.338	0.012	0.243	0.027	0.059	0.020	0.119	0.214	0.140

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	43	43	49	43	46	43
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.90	0.84
time (sec)	N/A	0.163	0.001	0.238	0.034	0.060	0.024	0.121	0.251	0.014

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	35	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	1.00	0.89
time (sec)	N/A	0.154	0.001	0.239	0.033	0.059	0.022	0.123	0.210	0.029

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.138	0.001	0.242	0.027	0.063	0.019	0.122	0.218	0.036

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.128	0.000	0.044	0.027	0.059	0.016	0.123	0.222	0.046

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	23	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.96	0.67
time (sec)	N/A	0.216	0.007	0.379	0.110	0.073	0.061	0.121	0.269	0.066

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	61	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	1.36	0.73
time (sec)	N/A	0.136	0.017	0.391	0.113	0.071	0.128	0.119	0.240	0.114

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	70	55	57	58	188	105	45	113	55
N.S.	1	1.13	0.89	0.92	0.94	3.03	1.69	0.73	1.82	0.89
time (sec)	N/A	0.149	0.021	0.389	0.110	0.072	0.171	0.119	0.228	0.066

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	95	66	78	80	254	129	56	162	77
N.S.	1	1.20	0.84	0.99	1.01	3.22	1.63	0.71	2.05	0.97
time (sec)	N/A	0.165	0.022	0.412	0.111	0.077	0.205	0.115	0.293	0.094

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	43	43	49	43	46	43
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.96	0.84	0.90	0.84
time (sec)	N/A	0.162	0.001	0.250	0.036	0.058	0.031	0.121	0.249	0.015

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	32	32	32	32	35	32
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.89	0.89	0.97	0.89
time (sec)	N/A	0.153	0.001	0.244	0.029	0.060	0.020	0.125	0.194	0.024

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.141	0.001	0.241	0.032	0.054	0.019	0.119	0.259	0.017

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	13	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.08	0.83
time (sec)	N/A	0.129	0.000	0.048	0.029	0.057	0.016	0.120	0.232	0.012

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	31	68	46	18	38	16
N.S.	1	1.00	1.00	0.67	1.29	2.83	1.92	0.75	1.58	0.67
time (sec)	N/A	0.123	0.003	0.355	0.110	0.066	0.067	0.124	0.231	0.156

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	37	52	126	71	39	102	34
N.S.	1	1.00	1.02	0.80	1.13	2.74	1.54	0.85	2.22	0.74
time (sec)	N/A	0.136	0.011	0.375	0.109	0.066	0.119	0.119	0.238	0.044

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	72	56	59	73	188	99	49	178	55
N.S.	1	1.12	0.88	0.92	1.14	2.94	1.55	0.77	2.78	0.86
time (sec)	N/A	0.150	0.023	0.370	0.113	0.075	0.158	0.124	0.253	0.080

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	98	67	81	97	260	122	60	254	78
N.S.	1	1.20	0.82	0.99	1.18	3.17	1.49	0.73	3.10	0.95
time (sec)	N/A	0.165	0.022	0.379	0.105	0.071	0.223	0.119	0.223	0.078

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	10	12	13	13
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.93	0.93
time (sec)	N/A	0.143	0.000	0.245	0.034	0.056	0.018	0.125	0.241	0.044

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	20	10	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	2.00	1.00	1.00	1.00
time (sec)	N/A	0.117	0.002	0.405	0.105	0.062	0.046	0.118	0.201	0.061

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	23	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.96	0.67
time (sec)	N/A	0.125	0.002	0.342	0.109	0.076	0.076	0.123	0.264	0.001

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	23	23	34	20	25	41	17
N.S.	1	1.00	1.29	1.10	1.10	1.62	0.95	1.19	1.95	0.81
time (sec)	N/A	0.124	0.007	0.290	0.025	0.061	0.044	0.122	0.238	0.063

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	28	20	0	82	83	23	29	23
N.S.	1	1.00	0.93	0.67	0.00	2.73	2.77	0.77	0.97	0.77
time (sec)	N/A	0.139	0.006	0.467	0.000	0.069	0.099	0.119	0.236	0.141

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	33	0	102	87	33	45	29
N.S.	1	1.00	1.19	0.89	0.00	2.76	2.35	0.89	1.22	0.78
time (sec)	N/A	0.149	0.009	0.527	0.000	0.071	0.137	0.125	0.259	0.202

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	106	60	37	46	38
N.S.	1	1.00	1.06	1.00	0.00	3.12	1.76	1.09	1.35	1.12
time (sec)	N/A	0.139	0.009	0.515	0.000	0.068	0.147	0.119	0.235	0.230

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	0	105	66	36	48	38
N.S.	1	1.00	1.06	1.00	0.00	3.09	1.94	1.06	1.41	1.12
time (sec)	N/A	0.132	0.006	0.509	0.000	0.072	0.151	0.118	0.211	0.159

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	39	0	182	104	58	65	46
N.S.	1	1.00	1.00	0.78	0.00	3.64	2.08	1.16	1.30	0.92
time (sec)	N/A	0.165	0.012	0.596	0.000	0.070	0.168	0.124	0.310	0.246

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	23	7	17	17	15	15	17	6
N.S.	1	1.00	3.83	1.17	2.83	2.83	2.50	2.50	2.83	1.00
time (sec)	N/A	0.114	0.002	0.287	0.024	0.061	0.075	0.121	0.247	0.099

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	14	17	17	15	19	17	6
N.S.	1	1.00	1.10	0.67	0.81	0.81	0.71	0.90	0.81	0.29
time (sec)	N/A	0.136	0.001	0.361	0.025	0.061	0.046	0.121	0.255	0.096

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	B	A	B	F(-2)	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	0	138	221	475	3553	0	55	141
N.S.	1	1.00	0.00	1.19	1.91	4.09	30.63	0.00	0.47	1.22
time (sec)	N/A	0.246	0.000	0.592	0.138	0.795	1.902	0.000	200.046	0.310

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	B	A	B	F(-2)	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	146	0	179	562	475	1093	0	79	141
N.S.	1	1.26	0.00	1.54	4.84	4.09	9.42	0.00	0.68	1.22
time (sec)	N/A	0.319	0.000	0.627	0.175	0.727	1.433	0.000	200.059	0.247

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	39	39	30	29	33	29	33	19	44
N.S.	1	1.15	1.15	0.88	0.85	0.97	0.85	0.97	0.56	1.29
time (sec)	N/A	0.149	0.018	0.329	0.123	0.068	0.710	0.126	0.198	0.123

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	36	28	32	33	75	25	19	26
N.S.	1	1.06	1.06	0.82	0.94	0.97	2.21	0.74	0.56	0.76
time (sec)	N/A	0.148	0.019	0.340	0.106	0.073	0.586	0.126	0.221	0.121

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	44	44	31	30	33	29	24	19	26
N.S.	1	1.29	1.29	0.91	0.88	0.97	0.85	0.71	0.56	0.76
time (sec)	N/A	0.148	0.016	0.349	0.112	0.069	0.709	0.132	0.253	0.081

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	39	44	30	46	71	110	61	68	43
N.S.	1	1.15	1.29	0.88	1.35	2.09	3.24	1.79	2.00	1.26
time (sec)	N/A	0.136	0.017	0.377	0.112	0.074	0.518	0.135	0.211	0.181

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	36	44	24	56	71	70	51	68	43
N.S.	1	1.06	1.29	0.71	1.65	2.09	2.06	1.50	2.00	1.26
time (sec)	N/A	0.137	0.009	0.421	0.117	0.070	0.512	0.134	0.255	0.110

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	44	44	31	68	71	110	51	68	43
N.S.	1	1.29	1.29	0.91	2.00	2.09	3.24	1.50	2.00	1.26
time (sec)	N/A	0.135	0.010	0.416	0.104	0.074	0.518	0.125	0.218	0.070

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	71	59	58	146	97	63	80	37
N.S.	1	1.07	0.85	0.70	0.69	1.74	1.15	0.75	0.95	0.44
time (sec)	N/A	0.167	0.096	0.438	0.029	0.080	2.579	0.133	0.235	0.173

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	68	60	48	43	124	70	49	61	37
N.S.	1	1.05	0.92	0.74	0.66	1.91	1.08	0.75	0.94	0.57
time (sec)	N/A	0.149	0.047	0.412	0.026	0.081	1.611	0.131	0.220	0.080

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	36	28	94	41	37	40	35
N.S.	1	1.00	1.04	0.78	0.61	2.04	0.89	0.80	0.87	0.76
time (sec)	N/A	0.139	0.028	0.408	0.028	0.078	0.974	0.123	0.222	0.118

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	25	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	1.00	0.80
time (sec)	N/A	0.126	0.004	0.367	0.030	0.073	0.580	0.122	0.205	0.094

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	39	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	2.44	0.88
time (sec)	N/A	0.119	0.023	0.365	0.032	0.066	0.458	0.127	0.244	0.033

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	84	28
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	2.15	0.72
time (sec)	N/A	0.132	0.037	0.375	0.044	0.071	0.464	0.130	0.249	0.082

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	66	40	37	46	69	413	41	127	44
N.S.	1	1.14	0.69	0.64	0.79	1.19	7.12	0.71	2.19	0.76
time (sec)	N/A	0.146	0.048	0.388	0.031	0.071	0.796	0.126	0.205	0.103

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	93	51	48	61	91	1265	55	170	61
N.S.	1	1.21	0.66	0.62	0.79	1.18	16.43	0.71	2.21	0.79
time (sec)	N/A	0.164	0.055	0.393	0.047	0.076	1.169	0.131	0.225	0.109

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	25	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	1.00	0.80
time (sec)	N/A	0.130	0.000	0.365	0.030	0.072	0.513	0.119	0.242	0.001

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	13	72	46	43	15	25
N.S.	1	1.00	1.00	0.81	0.50	2.77	1.77	1.65	0.58	0.96
time (sec)	N/A	0.126	0.004	0.619	0.106	0.069	0.576	0.127	0.222	0.125

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	26	63	46	41	27	22
N.S.	1	1.00	1.00	0.85	0.96	2.33	1.70	1.52	1.00	0.81
time (sec)	N/A	0.132	0.004	0.381	0.025	0.069	0.636	0.121	0.204	0.122

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	14	74	20	47	27	27
N.S.	1	1.00	1.00	0.82	0.50	2.64	0.71	1.68	0.96	0.96
time (sec)	N/A	0.130	0.004	0.609	0.035	0.074	0.624	0.126	0.213	0.087

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	20	19	29	22	29	29	16
N.S.	1	1.00	1.37	0.74	0.70	1.07	0.81	1.07	1.07	0.59
time (sec)	N/A	0.128	0.018	0.370	0.110	0.066	0.103	0.121	0.236	0.022

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	41	20	19	32	22	19	18	18
N.S.	1	1.00	1.52	0.74	0.70	1.19	0.81	0.70	0.67	0.67
time (sec)	N/A	0.124	0.032	0.548	0.103	0.063	0.081	0.120	0.199	0.075

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	30	31	29	31	30	29	29
N.S.	1	1.00	1.03	0.83	0.86	0.81	0.86	0.83	0.81	0.81
time (sec)	N/A	0.134	0.018	0.437	0.107	0.062	0.127	0.121	0.226	0.052

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	19	59	34	0	19	28
N.S.	1	1.00	1.00	0.81	0.53	1.64	0.94	0.00	0.53	0.78
time (sec)	N/A	0.135	0.109	0.526	0.105	0.066	0.177	0.000	0.236	0.023

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	7	6	16	7	29	17	6
N.S.	1	1.00	2.00	0.70	0.60	1.60	0.70	2.90	1.70	0.60
time (sec)	N/A	0.113	0.010	0.302	0.105	0.061	0.070	0.126	0.218	0.019

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	7	6	19	7	19	6	6
N.S.	1	1.00	1.90	0.70	0.60	1.90	0.70	1.90	0.60	0.60
time (sec)	N/A	0.112	0.019	0.336	0.105	0.066	0.072	0.122	0.205	0.005

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	19	18	16	15	30	17	16
N.S.	1	1.00	2.26	1.00	0.95	0.84	0.79	1.58	0.89	0.84
time (sec)	N/A	0.124	0.002	0.333	0.102	0.062	0.074	0.124	0.208	0.036

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	8	6	47	17	0	7	15
N.S.	1	1.00	1.00	0.42	0.32	2.47	0.89	0.00	0.37	0.79
time (sec)	N/A	0.123	0.002	0.313	0.111	0.067	0.165	0.000	0.226	0.067

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	12	11	65	14	35	23	11
N.S.	1	1.00	1.76	0.71	0.65	3.82	0.82	2.06	1.35	0.65
time (sec)	N/A	0.119	0.025	0.306	0.026	0.071	0.489	0.132	0.218	0.020

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	31	12	11	58	37	41	12	15
N.S.	1	1.00	1.82	0.71	0.65	3.41	2.18	2.41	0.71	0.88
time (sec)	N/A	0.120	0.032	0.567	0.104	0.067	0.550	0.120	0.212	0.030

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	24	57	37	36	23	20
N.S.	1	1.00	1.00	0.84	0.96	2.28	1.48	1.44	0.92	0.80
time (sec)	N/A	0.126	0.003	0.303	0.028	0.070	0.539	0.120	0.221	0.075

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	12	68	17	42	14	25
N.S.	1	1.00	1.00	0.81	0.46	2.62	0.65	1.62	0.54	0.96
time (sec)	N/A	0.125	0.003	0.648	0.032	0.072	0.510	0.127	0.210	0.046

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	14	13	59	17	36	25	20
N.S.	1	1.00	1.47	0.74	0.68	3.11	0.89	1.89	1.32	1.05
time (sec)	N/A	0.123	0.017	0.355	0.034	0.071	0.521	0.123	0.205	0.067

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	37	14	13	62	46	43	15	25
N.S.	1	1.00	1.95	0.74	0.68	3.26	2.42	2.26	0.79	1.32
time (sec)	N/A	0.121	0.030	0.752	0.106	0.075	0.549	0.122	0.198	0.065

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	26	74	46	41	27	22
N.S.	1	1.00	1.00	0.85	0.96	2.74	1.70	1.52	1.00	0.81
time (sec)	N/A	0.129	0.004	0.332	0.029	0.076	0.554	0.127	0.252	0.137

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	14	74	20	47	27	27
N.S.	1	1.00	1.00	0.82	0.50	2.64	0.71	1.68	0.96	0.96
time (sec)	N/A	0.129	0.004	0.741	0.028	0.072	0.667	0.121	0.209	0.098

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	6	23	19	28	6	14
N.S.	1	1.00	1.00	0.94	0.38	1.44	1.19	1.75	0.38	0.88
time (sec)	N/A	0.122	0.003	0.572	0.108	0.069	0.536	0.128	0.222	0.096

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	288	47	0	0	0	26	0	45	37
N.S.	1	1.01	0.16	0.00	0.00	0.00	0.09	0.00	0.16	0.13
time (sec)	N/A	0.250	4.788	0.000	0.000	0.000	0.574	0.000	0.297	0.105

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	46	0	0	0	26	0	27	37
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.10	0.00	0.10	0.14
time (sec)	N/A	0.239	3.776	0.000	0.000	0.000	0.490	0.000	0.233	0.086

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	46	0	0	0	24	0	11	37
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.10	0.00	0.04	0.15
time (sec)	N/A	0.223	3.578	0.000	0.000	0.000	0.454	0.000	0.225	0.094

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	55	0	0	0	24	0	30	37
N.S.	1	1.00	0.20	0.00	0.00	0.00	0.09	0.00	0.11	0.14
time (sec)	N/A	0.237	4.703	0.000	0.000	0.000	0.552	0.000	0.271	0.116

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	298	72	0	0	0	24	0	50	37
N.S.	1	1.03	0.25	0.00	0.00	0.00	0.08	0.00	0.17	0.13
time (sec)	N/A	0.258	5.511	0.000	0.000	0.000	0.701	0.000	0.237	0.155

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	569	614	47	0	0	0	26	0	45	37
N.S.	1	1.08	0.08	0.00	0.00	0.00	0.05	0.00	0.08	0.07
time (sec)	N/A	0.450	5.498	0.000	0.000	0.000	0.565	0.000	0.242	0.086

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	550	592	46	0	0	0	26	0	27	37
N.S.	1	1.08	0.08	0.00	0.00	0.00	0.05	0.00	0.05	0.07
time (sec)	N/A	0.427	4.625	0.000	0.000	0.000	0.506	0.000	0.282	0.074

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	529	574	46	0	0	0	24	0	11	37
N.S.	1	1.09	0.09	0.00	0.00	0.00	0.05	0.00	0.02	0.07
time (sec)	N/A	0.440	3.767	0.000	0.000	0.000	0.431	0.000	0.232	0.094

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	552	597	58	0	0	0	24	0	30	37
N.S.	1	1.08	0.11	0.00	0.00	0.00	0.04	0.00	0.05	0.07
time (sec)	N/A	0.468	4.187	0.000	0.000	0.000	0.495	0.000	0.226	0.150

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	574	624	75	0	0	0	24	0	50	37
N.S.	1	1.09	0.13	0.00	0.00	0.00	0.04	0.00	0.09	0.06
time (sec)	N/A	0.459	5.259	0.000	0.000	0.000	0.613	0.000	0.238	0.128

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	97	47	0	0	0	26	0	45	37
N.S.	1	1.05	0.51	0.00	0.00	0.00	0.28	0.00	0.49	0.40
time (sec)	N/A	0.182	6.647	0.000	0.000	0.000	0.623	0.000	0.262	0.084

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	46	0	0	0	26	0	27	37
N.S.	1	1.04	0.50	0.00	0.00	0.00	0.28	0.00	0.29	0.40
time (sec)	N/A	0.184	5.988	0.000	0.000	0.000	0.524	0.000	0.317	0.080

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	46	0	0	0	26	0	27	37
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.35	0.00	0.36	0.49
time (sec)	N/A	0.165	5.322	0.000	0.000	0.000	0.497	0.000	0.235	0.098

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	75	46	0	0	0	24	0	11	37
N.S.	1	1.06	0.65	0.00	0.00	0.00	0.34	0.00	0.15	0.52
time (sec)	N/A	0.171	5.069	0.000	0.000	0.000	0.459	0.000	0.250	0.139

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	0	0	0	24	0	11	37
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.43	0.00	0.20	0.66
time (sec)	N/A	0.154	5.054	0.000	0.000	0.000	0.463	0.000	0.217	0.137

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	24	0	30	37
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.43	0.00	0.54	0.66
time (sec)	N/A	0.156	5.517	0.000	0.000	0.000	0.496	0.000	0.191	0.177

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	55	0	0	0	24	0	30	37
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.31	0.00	0.38	0.47
time (sec)	N/A	0.162	6.163	0.000	0.000	0.000	0.588	0.000	0.211	0.191

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	24	0	50	37
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.31	0.00	0.64	0.47
time (sec)	N/A	0.160	6.806	0.000	0.000	0.000	0.745	0.000	0.210	0.221

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	101	48	0	0	0	27	0	48	38
N.S.	1	1.05	0.50	0.00	0.00	0.00	0.28	0.00	0.50	0.40
time (sec)	N/A	0.179	6.775	0.000	0.000	0.000	0.642	0.000	0.245	0.189

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0	29	38
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	0.37	0.49
time (sec)	N/A	0.170	6.094	0.000	0.000	0.000	0.558	0.000	0.260	0.130

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0	29	38
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	0.37	0.49
time (sec)	N/A	0.164	5.460	0.000	0.000	0.000	0.523	0.000	0.212	0.128

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0	12	38
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.45	0.00	0.21	0.66
time (sec)	N/A	0.153	5.237	0.000	0.000	0.000	0.596	0.000	0.208	0.164

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0	12	38
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.45	0.00	0.21	0.66
time (sec)	N/A	0.150	5.145	0.000	0.000	0.000	0.524	0.000	0.218	0.160

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	26	0	33	38
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.34	0.00	0.43	0.49
time (sec)	N/A	0.162	5.699	0.000	0.000	0.000	0.510	0.000	0.251	0.199

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	26	0	33	38
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.32	0.00	0.41	0.47
time (sec)	N/A	0.162	6.414	0.000	0.000	0.000	0.616	0.000	0.219	0.195

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	74	0	0	0	26	0	53	38
N.S.	1	1.04	0.73	0.00	0.00	0.00	0.26	0.00	0.52	0.38
time (sec)	N/A	0.175	6.923	0.000	0.000	0.000	0.747	0.000	0.188	0.219

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	23	20	0	0	24	0	26	18
N.S.	1	1.00	0.70	0.61	0.00	0.00	0.73	0.00	0.79	0.55
time (sec)	N/A	0.143	10.009	0.472	0.000	0.000	0.580	0.000	0.239	0.256

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	19	0	0	26	0	28	17
N.S.	1	1.00	0.88	0.58	0.00	0.00	0.79	0.00	0.85	0.52
time (sec)	N/A	0.145	10.008	0.395	0.000	0.000	0.617	0.000	0.228	0.235

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	26	0	31	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.55	0.00	0.66	0.79
time (sec)	N/A	0.150	7.723	0.000	0.000	0.000	0.828	0.000	0.212	0.170

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.151	7.244	0.000	0.000	0.000	0.625	0.000	0.217	0.121

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.148	6.799	0.000	0.000	0.000	0.514	0.000	0.246	0.120

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.145	6.247	0.000	0.000	0.000	0.506	0.000	0.222	0.128

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.148	6.173	0.000	0.000	0.000	0.449	0.000	0.196	0.155

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.146	6.300	0.000	0.000	0.000	0.470	0.000	0.200	0.137

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.151	6.155	0.000	0.000	0.000	0.474	0.000	0.234	0.135

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	24	0	30	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.49	0.00	0.61	0.76
time (sec)	N/A	0.146	6.309	0.000	0.000	0.000	0.521	0.000	0.225	0.204

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	375	47	0	0	0	26	0	31	37
N.S.	1	1.32	0.16	0.00	0.00	0.00	0.09	0.00	0.11	0.13
time (sec)	N/A	0.269	8.314	0.000	0.000	0.000	0.829	0.000	0.255	0.153

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	353	46	0	0	0	26	0	11	37
N.S.	1	1.32	0.17	0.00	0.00	0.00	0.10	0.00	0.04	0.14
time (sec)	N/A	0.252	6.942	0.000	0.000	0.000	0.513	0.000	0.259	0.119

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	332	46	0	0	0	24	0	38	37
N.S.	1	1.32	0.18	0.00	0.00	0.00	0.10	0.00	0.15	0.15
time (sec)	N/A	0.231	6.740	0.000	0.000	0.000	0.535	0.000	0.204	0.138

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	358	56	0	0	0	24	0	30	37
N.S.	1	1.32	0.21	0.00	0.00	0.00	0.09	0.00	0.11	0.14
time (sec)	N/A	0.248	8.030	0.000	0.000	0.000	0.907	0.000	0.250	0.188

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	385	72	0	0	0	24	0	50	37
N.S.	1	1.33	0.25	0.00	0.00	0.00	0.08	0.00	0.17	0.13
time (sec)	N/A	0.269	8.792	0.000	0.000	0.000	2.225	0.000	0.257	0.214

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	599	743	47	0	0	0	26	0	31	37
N.S.	1	1.24	0.08	0.00	0.00	0.00	0.04	0.00	0.05	0.06
time (sec)	N/A	0.438	8.774	0.000	0.000	0.000	1.552	0.000	0.270	0.151

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	580	721	46	0	0	0	26	0	11	37
N.S.	1	1.24	0.08	0.00	0.00	0.00	0.04	0.00	0.02	0.06
time (sec)	N/A	0.410	8.020	0.000	0.000	0.000	0.722	0.000	0.218	0.118

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	563	699	46	0	0	0	24	0	11	37
N.S.	1	1.24	0.08	0.00	0.00	0.00	0.04	0.00	0.02	0.07
time (sec)	N/A	0.389	6.879	0.000	0.000	0.000	0.493	0.000	0.236	0.154

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	579	681	49	0	0	0	24	0	32	37
N.S.	1	1.18	0.08	0.00	0.00	0.00	0.04	0.00	0.06	0.06
time (sec)	N/A	0.381	7.029	0.000	0.000	0.000	0.608	0.000	0.219	0.184

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	604	704	66	0	0	0	24	0	87	37
N.S.	1	1.17	0.11	0.00	0.00	0.00	0.04	0.00	0.14	0.06
time (sec)	N/A	0.400	8.364	0.000	0.000	0.000	1.138	0.000	0.219	0.239

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.152	8.348	0.000	0.000	0.000	0.648	0.000	0.233	0.128

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.148	7.957	0.000	0.000	0.000	0.614	0.000	0.218	0.121

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.149	8.206	0.000	0.000	0.000	0.562	0.000	0.202	0.156

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	40	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.87	0.80
time (sec)	N/A	0.150	7.885	0.000	0.000	0.000	0.575	0.000	0.193	0.140

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	24	0	30	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.49	0.00	0.61	0.76
time (sec)	N/A	0.148	8.878	0.000	0.000	0.000	1.137	0.000	0.251	0.198

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	24	0	30	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.49	0.00	0.61	0.76
time (sec)	N/A	0.146	9.075	0.000	0.000	0.000	1.365	0.000	0.238	0.202

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.148	8.966	0.000	0.000	0.000	1.011	0.000	0.195	0.128

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.146	8.806	0.000	0.000	0.000	0.784	0.000	0.236	0.128

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.145	7.985	0.000	0.000	0.000	0.590	0.000	0.246	0.173

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.152	8.242	0.000	0.000	0.000	0.548	0.000	0.220	0.149

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	24	0	30	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.49	0.00	0.61	0.76
time (sec)	N/A	0.145	8.035	0.000	0.000	0.000	0.669	0.000	0.212	0.193

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	24	0	30	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.49	0.00	0.61	0.76
time (sec)	N/A	0.147	8.504	0.000	0.000	0.000	0.758	0.000	0.252	0.213

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	469	48	48	0	0	0	29	0	13	39
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.08
time (sec)	N/A	0.147	8.465	0.000	0.000	0.000	0.671	0.000	0.220	0.171

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	461	48	48	0	0	0	27	0	13	39
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.08
time (sec)	N/A	0.147	7.988	0.000	0.000	0.000	0.564	0.000	0.206	0.115

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	447	48	48	0	0	0	27	0	13	39
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.03	0.09
time (sec)	N/A	0.149	8.275	0.000	0.000	0.000	0.584	0.000	0.226	0.190

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	437	48	48	0	0	0	27	0	49	39
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.11	0.09
time (sec)	N/A	0.147	7.936	0.000	0.000	0.000	0.600	0.000	0.216	0.156

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	472	52	52	0	0	0	27	0	37	39
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.08	0.08
time (sec)	N/A	0.148	8.917	0.000	0.000	0.000	0.998	0.000	0.229	0.235

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	468	52	52	0	0	0	26	0	37	39
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.08	0.08
time (sec)	N/A	0.145	9.066	0.000	0.000	0.000	1.336	0.000	0.229	0.217

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	919	48	48	0	0	0	29	0	13	39
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.01	0.04
time (sec)	N/A	0.148	8.847	0.000	0.000	0.000	0.792	0.000	0.256	0.122

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	919	48	48	0	0	0	26	0	13	39
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.01	0.04
time (sec)	N/A	0.148	7.942	0.000	0.000	0.000	0.574	0.000	0.208	0.185

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	893	48	48	0	0	0	27	0	13	39
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.01	0.04
time (sec)	N/A	0.147	8.271	0.000	0.000	0.000	0.543	0.000	0.213	0.171

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	893	52	52	0	0	0	27	0	37	39
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.04	0.04
time (sec)	N/A	0.147	8.092	0.000	0.000	0.000	0.623	0.000	0.272	0.213

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	922	52	52	0	0	0	27	0	37	39
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.04	0.04
time (sec)	N/A	0.146	8.535	0.000	0.000	0.000	0.757	0.000	0.234	0.243

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	0	0	17	0	11	13
N.S.	1	1.00	1.00	0.83	0.00	0.00	0.94	0.00	0.61	0.72
time (sec)	N/A	0.127	8.660	0.313	0.000	0.000	0.528	0.000	0.212	0.169

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.149	8.838	0.000	0.000	0.000	0.550	0.000	0.234	0.183

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [107] had the largest ratio of [.7272729999999999947]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	9	0.222
2	A	2	2	1.00	9	0.222
3	A	2	2	1.00	9	0.222
4	A	2	2	1.00	9	0.222
5	A	1	1	1.00	7	0.143
6	A	1	1	1.00	9	0.111
7	A	2	2	1.00	9	0.222
8	A	3	3	1.13	9	0.333
9	A	4	4	1.20	9	0.444
10	A	2	2	1.00	10	0.200
11	A	2	2	1.00	10	0.200
12	A	2	2	1.00	10	0.200
13	A	1	1	1.00	8	0.125
14	A	1	1	1.00	10	0.100
15	A	2	2	1.00	10	0.200
16	A	3	3	1.12	10	0.300
17	A	4	4	1.20	10	0.400
18	A	2	2	1.00	7	0.286
19	A	1	1	1.00	9	0.111
20	A	1	1	1.00	9	0.111
21	A	2	2	1.00	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	1	1	1.00	10	0.100
23	A	1	1	1.00	18	0.056
24	A	1	1	1.00	14	0.071
25	A	1	1	1.00	15	0.067
26	A	1	1	1.00	20	0.050
27	A	2	2	1.00	11	0.182
28	A	1	1	1.00	17	0.059
29	A	1	1	1.00	57	0.018
30	A	1	1	1.26	81	0.012
31	A	3	3	1.15	27	0.111
32	A	2	2	1.06	24	0.083
33	A	1	1	1.29	22	0.045
34	A	3	3	1.15	27	0.111
35	A	2	2	1.06	24	0.083
36	A	1	1	1.29	22	0.045
37	A	6	5	1.07	11	0.455
38	A	5	4	1.05	11	0.364
39	A	4	3	1.00	11	0.273
40	A	3	2	1.00	11	0.182
41	A	1	1	1.00	11	0.091
42	A	2	2	1.00	11	0.182
43	A	3	3	1.14	11	0.273
44	A	4	4	1.21	11	0.364
45	A	3	2	1.00	11	0.182
46	A	3	2	1.00	12	0.167
47	A	3	2	1.00	13	0.154
48	A	3	2	1.00	14	0.143
49	A	2	2	1.00	11	0.182
50	A	2	2	1.00	11	0.182
51	A	4	3	1.00	11	0.273
52	A	4	3	1.00	11	0.273
53	A	1	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	1	1	1.00	11	0.091
55	A	3	2	1.00	11	0.182
56	A	3	2	1.00	11	0.182
57	A	1	1	1.00	11	0.091
58	A	1	1	1.00	12	0.083
59	A	3	2	1.00	11	0.182
60	A	3	2	1.00	12	0.167
61	A	1	1	1.00	11	0.091
62	A	1	1	1.00	12	0.083
63	A	3	2	1.00	13	0.154
64	A	3	2	1.00	14	0.143
65	A	3	2	1.00	13	0.154
66	A	5	4	1.01	11	0.364
67	A	4	3	1.00	11	0.273
68	A	3	2	1.00	11	0.182
69	A	4	3	1.00	11	0.273
70	A	5	4	1.03	11	0.364
71	A	7	6	1.08	11	0.545
72	A	6	5	1.08	11	0.455
73	A	5	4	1.09	11	0.364
74	A	6	5	1.08	11	0.455
75	A	7	6	1.09	11	0.545
76	A	4	4	1.05	11	0.364
77	A	4	4	1.04	11	0.364
78	A	3	3	1.00	11	0.273
79	A	3	3	1.06	11	0.273
80	A	2	2	1.00	11	0.182
81	A	2	2	1.00	11	0.182
82	A	3	3	1.00	11	0.273
83	A	3	3	1.00	11	0.273
84	A	4	4	1.05	12	0.333
85	A	3	3	1.00	12	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	3	3	1.00	12	0.250
87	A	2	2	1.00	12	0.167
88	A	2	2	1.00	12	0.167
89	A	3	3	1.00	12	0.250
90	A	3	3	1.00	12	0.250
91	A	4	4	1.04	12	0.333
92	A	1	1	1.00	18	0.056
93	A	1	1	1.00	19	0.053
94	A	2	2	1.00	11	0.182
95	A	2	2	1.00	11	0.182
96	A	2	2	1.00	11	0.182
97	A	2	2	1.00	11	0.182
98	A	2	2	1.00	11	0.182
99	A	2	2	1.00	11	0.182
100	A	2	2	1.00	11	0.182
101	A	2	2	1.00	11	0.182
102	A	6	5	1.32	11	0.455
103	A	5	4	1.32	11	0.364
104	A	4	3	1.32	11	0.273
105	A	5	4	1.32	11	0.364
106	A	6	5	1.33	11	0.455
107	A	9	8	1.24	11	0.727
108	A	8	7	1.24	11	0.636
109	A	7	6	1.24	11	0.545
110	A	6	5	1.18	11	0.455
111	A	7	6	1.17	11	0.545
112	A	2	2	1.00	11	0.182
113	A	2	2	1.00	11	0.182
114	A	2	2	1.00	11	0.182
115	A	2	2	1.00	11	0.182
116	A	2	2	1.00	11	0.182
117	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	11	0.182
119	A	2	2	1.00	11	0.182
120	A	2	2	1.00	11	0.182
121	A	2	2	1.00	11	0.182
122	A	2	2	1.00	11	0.182
123	A	2	2	1.00	11	0.182
124	C	2	2	0.10	13	0.154
125	C	2	2	0.10	13	0.154
126	C	2	2	0.11	13	0.154
127	C	2	2	0.11	13	0.154
128	C	2	2	0.11	13	0.154
129	C	2	2	0.11	13	0.154
130	C	2	2	0.05	13	0.154
131	C	2	2	0.05	13	0.154
132	C	2	2	0.05	13	0.154
133	C	2	2	0.06	13	0.154
134	C	2	2	0.06	13	0.154
135	A	1	1	1.00	11	0.091
136	A	2	2	1.00	11	0.182

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx^2)^5 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 62

$$\int (a+bx^2)^5 \, dx = a^5x + \frac{5}{3}a^4bx^3 + 2a^3b^2x^5 + \frac{10}{7}a^2b^3x^7 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{11}}{11}$$

output
```
a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*
x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^5 \, dx = a^5x + \frac{5}{3}a^4bx^3 + 2a^3b^2x^5 + \frac{10}{7}a^2b^3x^7 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{11}}{11}$$

input
```
Integrate[(a + b*x^2)^5,x]
```

output
```
a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^
9)/9 + (b^5*x^11)/11
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^5\,dx$$

$$\Big\downarrow \text{210}$$

$$\int \left(a^5+5a^4bx^2+10a^3b^2x^4+10a^2b^3x^6+5ab^4x^8+b^5x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

input
```
Int[(a + b*x^2)^5,x]
```

output
```
a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^
9)/9 + (b^5*x^11)/11
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
gosper	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
default	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
norman	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
risch	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
parallelrisch	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
orering	$\frac{x(63b^5x^{10}+385a\,b^4x^8+990a^2b^3x^6+1386a^3b^2x^4+1155a^4b\,x^2+693a^5)}{693}$	58

input
```
int((b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*
x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int (a+bx^2)^5\,dx=\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

input
```
integrate((b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4
*b*x^3 + a^5*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2\right)^5 \, dx = a^5x + \frac{5a^4bx^3}{3} + 2a^3b^2x^5 + \frac{10a^2b^3x^7}{7} + \frac{5ab^4x^9}{9} + \frac{b^5x^{11}}{11}$$

input
```
integrate((b*x**2+a)**5,x)
```

output
```
a**5*x + 5*a**4*b*x**3/3 + 2*a**3*b**2*x**5 + 10*a**2*b**3*x**7/7 + 5*a*b*
*4*x**9/9 + b**5*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \left(a+bx^2\right)^5 \, dx = \frac{1}{11}\,b^5x^{11} + \frac{5}{9}\,ab^4x^9 + \frac{10}{7}\,a^2b^3x^7 + 2\,a^3b^2x^5 + \frac{5}{3}\,a^4bx^3 + a^5x$$

input
```
integrate((b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4
*b*x^3 + a^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \left(a+bx^2\right)^5 \, dx = \frac{1}{11}\,b^5x^{11} + \frac{5}{9}\,ab^4x^9 + \frac{10}{7}\,a^2b^3x^7 + 2\,a^3b^2x^5 + \frac{5}{3}\,a^4bx^3 + a^5x$$

input
```
integrate((b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4
*b*x^3 + a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int (a+bx^2)^5 \, dx = a^5\,x + \frac{5\,a^4\,b\,x^3}{3} + 2\,a^3\,b^2\,x^5 + \frac{10\,a^2\,b^3\,x^7}{7} + \frac{5\,a\,b^4\,x^9}{9} + \frac{b^5\,x^{11}}{11}$$

input
```
int((a + b*x^2)^5,x)
```

output
```
a^5*x + (b^5*x^11)/11 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^9)/9 + 2*a^3*b^2*x^5
+ (10*a^2*b^3*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int (a+bx^2)^5 \, dx = \frac{x(63b^5x^{10} + 385a\,b^4x^8 + 990a^2b^3x^6 + 1386a^3b^2x^4 + 1155a^4b\,x^2 + 693a^5)}{693}$$

input
```
int((b*x^2+a)^5,x)
```

output
```
(x*(693*a**5 + 1155*a**4*b*x**2 + 1386*a**3*b**2*x**4 + 990*a**2*b**3*x**6
 + 385*a*b**4*x**8 + 63*b**5*x**10))/693
```

3.2 $\int (a+bx^2)^4 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 51

$$\int (a+bx^2)^4 \, dx = a^4x + \frac{4}{3}a^3bx^3 + \frac{6}{5}a^2b^2x^5 + \frac{4}{7}ab^3x^7 + \frac{b^4x^9}{9}$$

output
```
a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^4 \, dx = a^4x + \frac{4}{3}a^3bx^3 + \frac{6}{5}a^2b^2x^5 + \frac{4}{7}ab^3x^7 + \frac{b^4x^9}{9}$$

input
```
Integrate[(a + b*x^2)^4,x]
```

output
```
a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/
9
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^4 \, dx$$

$$\Big\downarrow \text{ 210}$$

$$\int \left(a^4+4a^3bx^2+6a^2b^2x^4+4ab^3x^6+b^4x^8\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

input
```
Int[(a + b*x^2)^4,x]
```

output
```
a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (b^4*x^9)/
9
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
gosper	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
default	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
norman	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
risch	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
parallelrisch	$a^4x+\frac{4}{3}a^3b\,x^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}a\,b^3x^7+\frac{1}{9}b^4x^9$	44
orering	$\frac{x(35b^4x^8+180a\,b^3x^6+378a^2b^2x^4+420a^3b\,x^2+315a^4)}{315}$	47

input
```
int((b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^4\,dx=\frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{6}{5}\,a^2b^2x^5+\frac{4}{3}\,a^3bx^3+a^4x$$

input
```
integrate((b*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^4 \, dx = a^4x+\frac{4a^3bx^3}{3}+\frac{6a^2b^2x^5}{5}+\frac{4ab^3x^7}{7}+\frac{b^4x^9}{9}$$

input
```
integrate((b*x**2+a)**4,x)
```

output
```
a**4*x + 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 + 4*a*b**3*x**7/7 + b**4*x**
9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^4 \, dx = \frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{6}{5}\,a^2b^2x^5+\frac{4}{3}\,a^3bx^3+a^4x$$

input
```
integrate((b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^4 \, dx = \frac{1}{9}\,b^4x^9+\frac{4}{7}\,ab^3x^7+\frac{6}{5}\,a^2b^2x^5+\frac{4}{3}\,a^3bx^3+a^4x$$

input
```
integrate((b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int (a + bx^2)^4 \, dx = a^4 x + \frac{4 a^3 b x^3}{3} + \frac{6 a^2 b^2 x^5}{5} + \frac{4 a b^3 x^7}{7} + \frac{b^4 x^9}{9}$$

input
```
int((a + b*x^2)^4,x)
```

output
```
a^4*x + (b^4*x^9)/9 + (4*a^3*b*x^3)/3 + (4*a*b^3*x^7)/7 + (6*a^2*b^2*x^5)/
5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int (a + bx^2)^4 \, dx = \frac{x(35b^4x^8 + 180a b^3x^6 + 378a^2b^2x^4 + 420a^3b x^2 + 315a^4)}{315}$$

input
```
int((b*x^2+a)^4,x)
```

output
```
(x*(315*a**4 + 420*a**3*b*x**2 + 378*a**2*b**2*x**4 + 180*a*b**3*x**6 + 35
*b**4*x**8))/315
```

3.3 $\int (a+bx^2)^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int (a+bx^2)^3 \, dx = a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

output
```
a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 \, dx = a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

input
```
Integrate[(a + b*x^2)^3,x]
```

output
```
a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \, dx$$

$$\Big\downarrow \ 210$$

$$\int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right)dx$$

$$\Big\downarrow \ 2009$$

$$a^3x+a^2bx^3+\frac{3}{5}ab^2x^5+\frac{b^3x^7}{7}$$

input
```
Int[(a + b*x^2)^3,x]
```

output
```
a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
gosper	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
default	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
norman	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
risch	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
parallelrisch	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
orering	$\frac{x(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3)}{35}$	36

input
```
int((b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^3\,dx = \frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

input
```
integrate((b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^3 \, dx = a^3x + a^2bx^3 + \frac{3ab^2x^5}{5} + \frac{b^3x^7}{7}$$

input
```
integrate((b*x**2+a)**3,x)
```

output
```
a**3*x + a**2*b*x**3 + 3*a*b**2*x**5/5 + b**3*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^3 \, dx = \frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

input
```
integrate((b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^3 \, dx = \frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

input
```
integrate((b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int (a+bx^2)^3 \, dx = a^3 x + a^2 b x^3 + \frac{3 a b^2 x^5}{5} + \frac{b^3 x^7}{7}$$

input
```
int((a + b*x^2)^3,x)
```

output
```
a^3*x + (b^3*x^7)/7 + a^2*b*x^3 + (3*a*b^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 \, dx = \frac{x(5b^3x^6 + 21a b^2x^4 + 35a^2b x^2 + 35a^3)}{35}$$

input
```
int((b*x^2+a)^3,x)
```

output
```
(x*(35*a**3 + 35*a**2*b*x**2 + 21*a*b**2*x**4 + 5*b**3*x**6))/35
```

3.4 $\int (a+bx^2)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a+bx^2)^2 \, dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2 \, dx = a^2x + \frac{2}{3}abx^3 + \frac{b^2x^5}{5}$$

input
```
Integrate[(a + b*x^2)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 dx$$

$$\Big\downarrow \text{ 210}$$

$$\int \left(a^2+2abx^2+b^2x^4\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Int[(a + b*x^2)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
default	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
norman	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
parallelrisch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
orering	$\frac{x(3b^2x^4+10ab\,x^2+15a^2)}{15}$	25

input
```
int((b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^2\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \left(a+bx^2\right)^2 dx = a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

input
```
integrate((b*x**2+a)**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^2 dx = \frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

input
```
integrate((b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^2 dx = \frac{1}{5}b^2x^5 + \frac{2}{3}abx^3 + a^2x$$

input
```
integrate((b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a + bx^2)^2 \, dx = a^2 \, x + \frac{2 \, a \, b \, x^3}{3} + \frac{b^2 \, x^5}{5}$$

input
```
int((a + b*x^2)^2,x)
```

output
```
a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (a + bx^2)^2 \, dx = \frac{x(3b^2x^4 + 10ab \, x^2 + 15a^2)}{15}$$

input
```
int((b*x^2+a)^2,x)
```

output
```
(x*(15*a**2 + 10*a*b*x**2 + 3*b**2*x**4))/15
```

3.5 $\int (a+bx^2)\, dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int (a+bx^2)\, dx = ax + \frac{bx^3}{3}$$

output
```
a*x+1/3*b*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+bx^2)\, dx = ax + \frac{bx^3}{3}$$

input
```
Integrate[a + b*x^2,x]
```

output
```
a*x + (b*x^3)/3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+\frac{bx^3}{3}$$

input
```
Int[a + b*x^2,x]
```

output
```
a*x + (b*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$ax+\frac{1}{3}b\,x^{3}$	11
default	$ax+\frac{1}{3}b\,x^{3}$	11
norman	$ax+\frac{1}{3}b\,x^{3}$	11
risch	$ax+\frac{1}{3}b\,x^{3}$	11
parallelrisch	$ax+\frac{1}{3}b\,x^{3}$	11
parts	$ax+\frac{1}{3}b\,x^{3}$	11
orering	$\frac{x\left(b\,x^{2}+3a\right)}{3}$	13

input
```
int(b*x^2+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*b*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \left(a+bx^2\right)\,dx=\frac{1}{3}\,bx^3+ax$$

input
```
integrate(b*x^2+a,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int (a+bx^2)\ dx = ax + \frac{bx^3}{3}$$

input
```
integrate(b*x**2+a,x)
```

output
```
a*x + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a+bx^2)\ dx = \frac{1}{3}\,bx^3 + ax$$

input
```
integrate(b*x^2+a,x, algorithm="maxima")
```

output
```
1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a+bx^2)\ dx = \frac{1}{3}\,bx^3 + ax$$

input
```
integrate(b*x^2+a,x, algorithm="giac")
```

output
```
1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a+bx^2)\,dx=\frac{b\,x^3}{3}+a\,x$$

input
```
int(a + b*x^2,x)
```

output
```
a*x + (b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+bx^2)\,dx=\frac{x(b\,x^2+3a)}{3}$$

input
```
int(b*x^2+a,x)
```

output
```
(x*(3*a + b*x**2))/3
```

3.6 $\int \frac{1}{a+bx^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+bx^2} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx^2} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

input
```
int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{a+bx^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\,\mathrm{arctan}\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(1/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{a+bx^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(1/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-
1/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx=\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="maxima")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx=\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+bx^2}\,dx=\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2),x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+bx^2}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(1/(b*x^2+a),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.7 $\int \frac{1}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 45

$$\int \frac{1}{(a+bx^2)^2} dx = \frac{x}{2a(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*x/a/(b*x^2+a)+1/2*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^2} dx = \frac{x}{2a(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-2),x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\,(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\,(a+bx^2)}$$

input
```
Int[(a + b*x^2)^(-2),x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	62

input
```
int(1/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{2\,abx-(bx^2+a)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(a^2b^2x^2+a^3b)},\ \frac{abx+(bx^2+a)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(a^2b^2x^2+a^3b)}\right]$$

input
```
integrate(1/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{x}{2a^2+2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

input
```
integrate(1/(b*x**2+a)**2,x)
```

output
```
x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) +
x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{x}{2\,(abx^2+a^2)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}$$

input
```
integrate(1/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2(bx^2+a)a}$$

input
```
integrate(1/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{x}{2\,a\,(b\,x^2+a)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^2,x)
```

output
```
x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2 + abx}{2a^2b\,(b\,x^2+a)}$$

input
```
int(1/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b*x**2 + a*b*x)/(2*a**2*b*(a + b*x**2))
```

3.8 $\int \frac{1}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 62

$$\int \frac{1}{(a+bx^2)^3} dx = \frac{x}{4a(a+bx^2)^2} + \frac{3x}{8a^2(a+bx^2)} + \frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

output
```
1/4*x/a/(b*x^2+a)^2+3/8*x/a^2/(b*x^2+a)+3/8*arctan(b^(1/2)*x/a^(1/2))/a^(5
/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^2)^3} dx = \frac{5ax+3bx^3}{8a^2(a+bx^2)^2} + \frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-3),x]
```

output
```
(5*a*x + 3*b*x^3)/(8*a^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\int \frac{1}{(bx^2+a)^2}dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}$$

input
```
Int[(a + b*x^2)^(-3),x]
```

output
```
x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x}{4a(b\,x^2+a)^2}+\frac{\frac{3x}{8a\left(b\,x^2+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{a}$	57
risch	$\frac{\frac{3b\,x^3}{8a^2}+\frac{5x}{8a}}{(b\,x^2+a)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^2}$	73

input
```
int(1/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(b*x^2+a)^2+3/4/a*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.03

$$\int \frac{1}{(a+bx^2)^3}\,dx$$
$$=\left[\frac{6\,ab^2x^3+10\,a^2bx-3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)},\frac{3\,ab^2x^3+5\,a^2bx+3\,(b^2x^4+2\,ab}{8\,(a^3b^3x^4+2\,a^4}\right.$$

input
```
integrate(1/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x
^2 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*
sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{1}{(a+bx^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5b}}\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16} + \frac{5ax+3bx^3}{8a^4+16a^3bx^2+8a^2b^2x^4}$$

input
```
integrate(1/(b*x**2+a)**3,x)
```

output
```
-3*sqrt(-1/(a**5*b))*log(-a**3*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a**5
*b))*log(a**3*sqrt(-1/(a**5*b)) + x)/16 + (5*a*x + 3*b*x**3)/(8*a**4 + 16*
a**3*b*x**2 + 8*a**2*b**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^2)^3}\,dx = \frac{3\,bx^3+5\,ax}{8\,(a^2b^2x^4+2\,a^3bx^2+a^4)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2}$$

input
```
integrate(1/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx^2)^3}\,dx = \frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2} + \frac{3\,bx^3+5\,ax}{8\,(bx^2+a)^2a^2}$$

input
```
integrate(1/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/8*(3*b*x^3 + 5*a*x)/((b*x^2
+ a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^2)^3}\,dx = \frac{\frac{5\,x}{8\,a}+\frac{3\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^3,x)
```

output
```
((5*x)/(8*a) + (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan((b
^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int \frac{1}{(a+bx^2)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4+5a^2bx+3a\,b^2x^3}{8a^3b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**2*x**4 + 5*a**2*b*x + 3*a*b**2*x**3)/(8*a**3*b*(a**2 + 2
*a*b*x**2 + b**2*x**4))
```

3.9 $\int \frac{1}{(a+bx^2)^4} dx$

Optimal result

Integrand size = 9, antiderivative size = 79

$$\int \frac{1}{(a+bx^2)^4} dx = \frac{x}{6a(a+bx^2)^3} + \frac{5x}{24a^2(a+bx^2)^2} + \frac{5x}{16a^3(a+bx^2)} + \frac{5\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

output
```
1/6*x/a/(b*x^2+a)^3+5/24*x/a^2/(b*x^2+a)^2+5/16*x/a^3/(b*x^2+a)+5/16*arcta
n(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+bx^2)^4} dx = \frac{33a^2x + 40abx^3 + 15b^2x^5}{48a^3(a+bx^2)^3} + \frac{5\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-4),x]
```

output
```
(33*a^2*x + 40*a*b*x^3 + 15*b^2*x^5)/(48*a^3*(a + b*x^2)^3) + (5*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^4} dx$$

$$\Big\downarrow \text{215}$$

$$\frac{5\int \frac{1}{(bx^2+a)^3}dx}{6a}+\frac{x}{6a(a+bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{5\left(\frac{3\int \frac{1}{(bx^2+a)^2}dx}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6a}+\frac{x}{6a(a+bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6a}+\frac{x}{6a(a+bx^2)^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a(a+bx^2)^2}\right)}{6a}+\frac{x}{6a(a+bx^2)^3}$$

input
```
Int[(a + b*x^2)^(-4),x]
```

output
```
x/(6*a*(a + b*x^2)^3) + (5*(x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)
) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{x}{6a(b\,x^2+a)^3}+\frac{\frac{5x}{24a\left(b\,x^2+a\right)^2}+\frac{5\left(\frac{3x}{8a\left(b\,x^2+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{6a}}{a}$	78
risch	$\frac{\frac{5b^2x^5}{16a^3}+\frac{5b\,x^3}{6a^2}+\frac{11x}{16a}}{(b\,x^2+a)^3}-\frac{5\ln\left(bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\,a^3}+\frac{5\ln\left(-bx+\sqrt{-ab}\right)}{32\sqrt{-ab}\,a^3}$	84

input
```
int(1/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*x/a/(b*x^2+a)^3+5/6/a*(1/4*x/a/(b*x^2+a)^2+3/4/a*(1/2*x/a/(b*x^2+a)+1/
2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.22

$$\int \frac{1}{(a+bx^2)^4}\,dx = \left[\frac{30\,ab^3x^5+80\,a^2b^2x^3+66\,a^3bx-15\,(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{96\,(a^4b^4x^6+3\,a^5b^3x^4+3\,a^6b^2x^2+a^7b)}, \frac{15\,ab^3x\right.$$

input
```
integrate(1/(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[1/96*(30*a*b^3*x^5 + 80*a^2*b^2*x^3 + 66*a^3*b*x - 15*(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^
2 + a)))/(a^4*b^4*x^6 + 3*a^5*b^3*x^4 + 3*a^6*b^2*x^2 + a^7*b), 1/48*(15*a
*b^3*x^5 + 40*a^2*b^2*x^3 + 33*a^3*b*x + 15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2
*b*x^2 + a^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^4*x^6 + 3*a^5*b^3*x^
4 + 3*a^6*b^2*x^2 + a^7*b)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \frac{1}{(a+bx^2)^4}\,dx = -\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32} + \frac{5\sqrt{-\frac{1}{a^7b}}\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32} + \frac{33a^2x+40abx^3+15b^2x^5}{48a^6+144a^5bx^2+144a^4b^2x^4+48a^3b^3x^6}$$

input
```
integrate(1/(b*x**2+a)**4,x)
```

output
```
-5*sqrt(-1/(a**7*b))*log(-a**4*sqrt(-1/(a**7*b)) + x)/32 + 5*sqrt(-1/(a**7
*b))*log(a**4*sqrt(-1/(a**7*b)) + x)/32 + (33*a**2*x + 40*a*b*x**3 + 15*b*
*2*x**5)/(48*a**6 + 144*a**5*b*x**2 + 144*a**4*b**2*x**4 + 48*a**3*b**3*x*
*6)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx^2)^4}\,dx = \frac{15\,b^2x^5+40\,abx^3+33\,a^2x}{48\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)} + \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3}$$

input
```
integrate(1/(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/48*(15*b^2*x^5 + 40*a*b*x^3 + 33*a^2*x)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3
*a^5*b*x^2 + a^6) + 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+bx^2)^4}\,dx = \frac{5\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^3} + \frac{15\,b^2x^5+40\,abx^3+33\,a^2x}{48\,(bx^2+a)^3a^3}$$

input
```
integrate(1/(b*x^2+a)^4,x, algorithm="giac")
```

output
```
5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/48*(15*b^2*x^5 + 40*a*b*x^3
 + 33*a^2*x)/((b*x^2 + a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a+bx^2)^4}\,dx = \frac{\frac{11\,x}{16\,a}+\frac{5\,b\,x^3}{6\,a^2}+\frac{5\,b^2\,x^5}{16\,a^3}}{a^3+3\,a^2\,b\,x^2+3\,a\,b^2\,x^4+b^3\,x^6} + \frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{7/2}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^4,x)
```

output
```
((11*x)/(16*a) + (5*b*x^3)/(6*a^2) + (5*b^2*x^5)/(16*a^3))/(a^3 + b^3*x^6
+ 3*a^2*b*x^2 + 3*a*b^2*x^4) + (5*atan((b^(1/2)*x)/a^(1/2)))/(16*a^(7/2)*b
^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.05

$$\int \frac{1}{(a+bx^2)^4}\,dx$$
$$= \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3 + 45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2 + 45\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^4 + 15\sqrt{b}\,\sqrt{a}\,ata}{48a^4b\,(b^3x^6 + 3a\,b^2x^4 + 3a^2b\,x^2 + a^3)}$$

input
```
int(1/(b*x^2+a)^4,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 + 45*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 45*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*a*b**2*x**4 + 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**3*x**6 + 33*a**3*b*x + 40*a**2*b**2*x**3 + 15*a*b**3*x**5)/(4
8*a**4*b*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))
```

3.10 $\int (a-bx^2)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int (a-bx^2)^4 \, dx = a^4x - \frac{4}{3}a^3bx^3 + \frac{6}{5}a^2b^2x^5 - \frac{4}{7}ab^3x^7 + \frac{b^4x^9}{9}$$

output
```
a^4*x-4/3*a^3*b*x^3+6/5*a^2*b^2*x^5-4/7*a*b^3*x^7+1/9*b^4*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int (a-bx^2)^4 \, dx = a^4x - \frac{4}{3}a^3bx^3 + \frac{6}{5}a^2b^2x^5 - \frac{4}{7}ab^3x^7 + \frac{b^4x^9}{9}$$

input
```
Integrate[(a - b*x^2)^4,x]
```

output
```
a^4*x - (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 - (4*a*b^3*x^7)/7 + (b^4*x^9)/
9
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^4 \, dx$$

$$\Big\downarrow \text{210}$$

$$\int \left(a^4-4a^3bx^2+6a^2b^2x^4-4ab^3x^6+b^4x^8\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^4x-\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5-\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

input
```
Int[(a - b*x^2)^4,x]
```

output
```
a^4*x - (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 - (4*a*b^3*x^7)/7 + (b^4*x^9)/
9
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
gosper	$a^{4}x-\frac{4}{3}a^{3}b\,x^{3}+\frac{6}{5}a^{2}b^{2}x^{5}-\frac{4}{7}a\,b^{3}x^{7}+\frac{1}{9}b^{4}x^{9}$	44
default	$a^{4}x-\frac{4}{3}a^{3}b\,x^{3}+\frac{6}{5}a^{2}b^{2}x^{5}-\frac{4}{7}a\,b^{3}x^{7}+\frac{1}{9}b^{4}x^{9}$	44
norman	$a^{4}x-\frac{4}{3}a^{3}b\,x^{3}+\frac{6}{5}a^{2}b^{2}x^{5}-\frac{4}{7}a\,b^{3}x^{7}+\frac{1}{9}b^{4}x^{9}$	44
risch	$a^{4}x-\frac{4}{3}a^{3}b\,x^{3}+\frac{6}{5}a^{2}b^{2}x^{5}-\frac{4}{7}a\,b^{3}x^{7}+\frac{1}{9}b^{4}x^{9}$	44
parallelrisch	$a^{4}x-\frac{4}{3}a^{3}b\,x^{3}+\frac{6}{5}a^{2}b^{2}x^{5}-\frac{4}{7}a\,b^{3}x^{7}+\frac{1}{9}b^{4}x^{9}$	44
orering	$\frac{x\left(35b^{4}x^{8}-180a\,b^{3}x^{6}+378a^{2}b^{2}x^{4}-420a^{3}b\,x^{2}+315a^{4}\right)}{315}$	47

input
```
int((-b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x-4/3*a^3*b*x^3+6/5*a^2*b^2*x^5-4/7*a*b^3*x^7+1/9*b^4*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a-bx^2\right)^4\,dx=\frac{1}{9}\,b^4x^9-\frac{4}{7}\,ab^3x^7+\frac{6}{5}\,a^2b^2x^5-\frac{4}{3}\,a^3bx^3+a^4x$$

input
```
integrate((-b*x^2+a)^4,x, algorithm="fricas")
```

output
```
1/9*b^4*x^9 - 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 - 4/3*a^3*b*x^3 + a^4*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \left(a-bx^2\right)^4 \, dx = a^4x - \frac{4a^3bx^3}{3} + \frac{6a^2b^2x^5}{5} - \frac{4ab^3x^7}{7} + \frac{b^4x^9}{9}$$

input
```
integrate((-b*x**2+a)**4,x)
```

output
```
a**4*x - 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 - 4*a*b**3*x**7/7 + b**4*x**
9/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a-bx^2\right)^4 \, dx = \frac{1}{9}\,b^4x^9 - \frac{4}{7}\,ab^3x^7 + \frac{6}{5}\,a^2b^2x^5 - \frac{4}{3}\,a^3bx^3 + a^4x$$

input
```
integrate((-b*x^2+a)^4,x, algorithm="maxima")
```

output
```
1/9*b^4*x^9 - 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 - 4/3*a^3*b*x^3 + a^4*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \left(a-bx^2\right)^4 \, dx = \frac{1}{9}\,b^4x^9 - \frac{4}{7}\,ab^3x^7 + \frac{6}{5}\,a^2b^2x^5 - \frac{4}{3}\,a^3bx^3 + a^4x$$

input
```
integrate((-b*x^2+a)^4,x, algorithm="giac")
```

output
```
1/9*b^4*x^9 - 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 - 4/3*a^3*b*x^3 + a^4*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int (a-bx^2)^4 \, dx = a^4 x - \frac{4 a^3 b x^3}{3} + \frac{6 a^2 b^2 x^5}{5} - \frac{4 a b^3 x^7}{7} + \frac{b^4 x^9}{9}$$

input
```
int((a - b*x^2)^4,x)
```

output
```
a^4*x + (b^4*x^9)/9 - (4*a^3*b*x^3)/3 - (4*a*b^3*x^7)/7 + (6*a^2*b^2*x^5)/
5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int (a-bx^2)^4 \, dx = \frac{x(35b^4x^8 - 180a b^3x^6 + 378a^2b^2x^4 - 420a^3b x^2 + 315a^4)}{315}$$

input
```
int((-b*x^2+a)^4,x)
```

output
```
(x*(315*a**4 - 420*a**3*b*x**2 + 378*a**2*b**2*x**4 - 180*a*b**3*x**6 + 35
*b**4*x**8))/315
```

3.11 $\int (a-bx^2)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int (a-bx^2)^3 \, dx = a^3x - a^2bx^3 + \frac{3}{5}ab^2x^5 - \frac{b^3x^7}{7}$$

output
```
a^3*x-a^2*b*x^3+3/5*a*b^2*x^5-1/7*b^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (a-bx^2)^3 \, dx = a^3x - a^2bx^3 + \frac{3}{5}ab^2x^5 - \frac{b^3x^7}{7}$$

input
```
Integrate[(a - b*x^2)^3,x]
```

output
```
a^3*x - a^2*b*x^3 + (3*a*b^2*x^5)/5 - (b^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^3 dx$$
$$\Big\downarrow \text{210}$$
$$\int \left(a^3-3a^2bx^2+3ab^2x^4-b^3x^6\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a^3x-a^2bx^3+\frac{3}{5}ab^2x^5-\frac{1}{7}b^3x^7$$

input
```
Int[(a - b*x^2)^3,x]
```

output
```
a^3*x - a^2*b*x^3 + (3*a*b^2*x^5)/5 - (b^3*x^7)/7
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
gosper	$a^{3}x-a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}-\frac{1}{7}b^{3}x^{7}$	33
default	$a^{3}x-a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}-\frac{1}{7}b^{3}x^{7}$	33
norman	$a^{3}x-a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}-\frac{1}{7}b^{3}x^{7}$	33
risch	$a^{3}x-a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}-\frac{1}{7}b^{3}x^{7}$	33
parallelrisch	$a^{3}x-a^{2}b\,x^{3}+\frac{3}{5}a\,b^{2}x^{5}-\frac{1}{7}b^{3}x^{7}$	33
orering	$\frac{x\left(-5b^{3}x^{6}+21a\,b^{2}x^{4}-35a^{2}b\,x^{2}+35a^{3}\right)}{35}$	36

input
```
int((-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x-a^2*b*x^3+3/5*a*b^2*x^5-1/7*b^3*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(a-bx^2\right)^3\,dx=-\frac{1}{7}\,b^3x^7+\frac{3}{5}\,ab^2x^5-a^2bx^3+a^3x$$

input
```
integrate((-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/7*b^3*x^7 + 3/5*a*b^2*x^5 - a^2*b*x^3 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(a-bx^2\right)^3 \, dx = a^3x - a^2bx^3 + \frac{3ab^2x^5}{5} - \frac{b^3x^7}{7}$$

input
```
integrate((-b*x**2+a)**3,x)
```

output
```
a**3*x - a**2*b*x**3 + 3*a*b**2*x**5/5 - b**3*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(a-bx^2\right)^3 \, dx = -\frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 - a^2bx^3 + a^3x$$

input
```
integrate((-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/7*b^3*x^7 + 3/5*a*b^2*x^5 - a^2*b*x^3 + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \left(a-bx^2\right)^3 \, dx = -\frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 - a^2bx^3 + a^3x$$

input
```
integrate((-b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/7*b^3*x^7 + 3/5*a*b^2*x^5 - a^2*b*x^3 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int (a-bx^2)^3 \, dx = a^3 x - a^2 b x^3 + \frac{3 a b^2 x^5}{5} - \frac{b^3 x^7}{7}$$

input
```
int((a - b*x^2)^3,x)
```

output
```
a^3*x - (b^3*x^7)/7 - a^2*b*x^3 + (3*a*b^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int (a-bx^2)^3 \, dx = \frac{x(-5b^3x^6 + 21a b^2x^4 - 35a^2b x^2 + 35a^3)}{35}$$

input
```
int((-b*x^2+a)^3,x)
```

output
```
(x*(35*a**3 - 35*a**2*b*x**2 + 21*a*b**2*x**4 - 5*b**3*x**6))/35
```

3.12 $\int (a-bx^2)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int (a-bx^2)^2\,dx = a^2x-\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

output
```
a^2*x-2/3*a*b*x^3+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a-bx^2)^2\,dx = a^2x-\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Integrate[(a - b*x^2)^2,x]
```

output
```
a^2*x - (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^2 \, dx$$

$$\Big\downarrow \; 210$$

$$\int \left(a^2-2abx^2+b^2x^4\right) dx$$

$$\Big\downarrow \; 2009$$

$$a^2x-\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Int[(a - b*x^2)^2,x]
```

output
```
a^2*x - (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x-\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
default	$a^2x-\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
norman	$a^2x-\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
risch	$a^2x-\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
parallelrisch	$a^2x-\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
orering	$\frac{x(3b^2x^4-10ab\,x^2+15a^2)}{15}$	25

input
```
int((-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-2/3*a*b*x^3+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a-bx^2\right)^2\,dx=\frac{1}{5}\,b^2x^5-\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 - 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (a-bx^2)^2 \, dx = a^2x - \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

input
```
integrate((-b*x**2+a)**2,x)
```

output
```
a**2*x - 2*a*b*x**3/3 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a-bx^2)^2 \, dx = \frac{1}{5}\, b^2x^5 - \frac{2}{3}\, abx^3 + a^2x$$

input
```
integrate((-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 - 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a-bx^2)^2 \, dx = \frac{1}{5}\, b^2x^5 - \frac{2}{3}\, abx^3 + a^2x$$

input
```
integrate((-b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 - 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a-bx^2\right)^2\,dx=a^2\,x-\frac{2\,a\,b\,x^3}{3}+\frac{b^2\,x^5}{5}$$

input
```
int((a - b*x^2)^2,x)
```

output
```
a^2*x + (b^2*x^5)/5 - (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \left(a-bx^2\right)^2\,dx=\frac{x(3b^2x^4-10ab\,x^2+15a^2)}{15}$$

input
```
int((-b*x^2+a)^2,x)
```

output
```
(x*(15*a**2 - 10*a*b*x**2 + 3*b**2*x**4))/15
```

3.13 $\int (a-bx^2)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 12

$$\int (a-bx^2)\,dx = ax-\frac{bx^3}{3}$$

output
```
a*x-1/3*b*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a-bx^2)\,dx = ax-\frac{bx^3}{3}$$

input
```
Integrate[a - b*x^2,x]
```

output
```
a*x - (b*x^3)/3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a - bx^2)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax - \frac{bx^3}{3}$$

input
```
Int[a - b*x^2,x]
```

output
```
a*x - (b*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$ax - \frac{1}{3}b\,x^3$	11
default	$ax - \frac{1}{3}b\,x^3$	11
norman	$ax - \frac{1}{3}b\,x^3$	11
risch	$ax - \frac{1}{3}b\,x^3$	11
parallelrisch	$ax - \frac{1}{3}b\,x^3$	11
parts	$ax - \frac{1}{3}b\,x^3$	11
orering	$\frac{x(-b\,x^2+3a)}{3}$	14

input
```
int(-b*x^2+a,x,method=_RETURNVERBOSE)
```

output
```
a*x-1/3*b*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a - bx^2)\,dx = -\frac{1}{3}\,bx^3 + ax$$

input
```
integrate(-b*x^2+a,x, algorithm="fricas")
```

output
```
-1/3*b*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int (a - bx^2) \, dx = ax - \frac{bx^3}{3}$$

input
```
integrate(-b*x**2+a,x)
```

output
```
a*x - b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a - bx^2) \, dx = -\frac{1}{3} bx^3 + ax$$

input
```
integrate(-b*x^2+a,x, algorithm="maxima")
```

output
```
-1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a - bx^2) \, dx = -\frac{1}{3} bx^3 + ax$$

input
```
integrate(-b*x^2+a,x, algorithm="giac")
```

output
```
-1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a-bx^2) \, dx = a\,x - \frac{b\,x^3}{3}$$

input
```
int(a - b*x^2,x)
```

output
```
a*x - (b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int (a-bx^2) \, dx = \frac{x(-b\,x^2+3a)}{3}$$

input
```
int(-b*x^2+a,x)
```

output
```
(x*(3*a - b*x**2))/3
```

3.14 $\int \frac{1}{a-bx^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{1}{a-bx^2} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-bx^2} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a - b*x^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-bx^2}\,dx$$

$$\Big\downarrow \; 221$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a - b*x^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$\frac{\ln\left(bx+\sqrt{ab}\right)}{2\sqrt{ab}}-\frac{\ln\left(-bx+\sqrt{ab}\right)}{2\sqrt{ab}}$	37

input
```
int(1/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{1}{a-bx^2}\,dx = \left[\frac{\sqrt{ab}\log\left(\frac{bx^2+2\sqrt{ab}x+a}{bx^2-a}\right)}{2\,ab}, -\frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(1/(-b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a))/(a*b), -sqrt(-
a*b)*arctan(sqrt(-a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{1}{a-bx^2}\,dx = -\frac{\sqrt{\frac{1}{ab}}\log\left(-a\sqrt{\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{\frac{1}{ab}}\log\left(a\sqrt{\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(1/(-b*x**2+a),x)
```

output
```
-sqrt(1/(a*b))*log(-a*sqrt(1/(a*b)) + x)/2 + sqrt(1/(a*b))*log(a*sqrt(1/(a
*b)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{a-bx^2}\,dx = -\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}}$$

input
```
integrate(1/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{a-bx^2}\,dx = -\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

input
```
integrate(1/(-b*x^2+a),x, algorithm="giac")
```

output
```
-arctan(b*x/sqrt(-a*b))/sqrt(-a*b)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a-bx^2}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(a - b*x^2),x)
```

output
```
atanh((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{a-bx^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)-\log\left(\sqrt{b}\,\sqrt{a}-bx\right)\right)}{2ab}$$

input
```
int(1/(-b*x^2+a),x)
```

output
```
(sqrt(b)*sqrt(a)*(log( - sqrt(b)*sqrt(a) - b*x) - log(sqrt(b)*sqrt(a) - b*
x)))/(2*a*b)
```

3.15 $\int \frac{1}{(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{(a-bx^2)^2}\,dx = \frac{x}{2a(a-bx^2)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*x/a/(-b*x^2+a)+1/2*arctanh(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a-bx^2)^2}\,dx = -\frac{x}{2a(-a+bx^2)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[(a - b*x^2)^(-2),x]
```

output
```
-1/2*x/(a*(-a + b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^2} \, dx$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{1}{a-bx^2}dx}{2a}+\frac{x}{2a\,(a-bx^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\,(a-bx^2)}$$

input
```
Int[(a - b*x^2)^(-2),x]
```

output
```
x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(-b\,x^2+a)}+\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	37
risch	$\frac{x}{2a(-b\,x^2+a)}+\frac{\ln\left(bx+\sqrt{ab}\right)}{4\sqrt{ab}\,a}-\frac{\ln\left(-bx+\sqrt{ab}\right)}{4\sqrt{ab}\,a}$	59

input
```
int(1/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(-b*x^2+a)+1/2/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{(a-bx^2)^2}\,dx = \left[-\frac{2\,abx-(bx^2-a)\sqrt{ab}\log\left(\frac{bx^2+2\,\sqrt{ab}x+a}{bx^2-a}\right)}{4\,(a^2b^2x^2-a^3b)},\right. \left. -\frac{abx+(bx^2-a)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{2\,(a^2b^2x^2-a^3b)}\right]$$

input
```
integrate(1/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*
x^2 - a)))/(a^2*b^2*x^2 - a^3*b), -1/2*(a*b*x + (b*x^2 - a)*sqrt(-a*b)*arc
tan(sqrt(-a*b)*x/a))/(a^2*b^2*x^2 - a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a-bx^2)^2}\,dx = -\frac{x}{-2a^2+2abx^2} - \frac{\sqrt{\frac{1}{a^3b}}\log\left(-a^2\sqrt{\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{\frac{1}{a^3b}}\log\left(a^2\sqrt{\frac{1}{a^3b}}+x\right)}{4}$$

input
```
integrate(1/(-b*x**2+a)**2,x)
```

output
```
-x/(-2*a**2 + 2*a*b*x**2) - sqrt(1/(a**3*b))*log(-a**2*sqrt(1/(a**3*b)) +
x)/4 + sqrt(1/(a**3*b))*log(a**2*sqrt(1/(a**3*b)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a-bx^2)^2}\,dx = -\frac{x}{2\,(abx^2-a^2)} - \frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\sqrt{ab}a}$$

input
```
integrate(1/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(a*b*x^2 - a^2) - 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqr
t(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a-bx^2)^2}\,dx = -\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}a} - \frac{x}{2\,(bx^2-a)a}$$

input
```
integrate(1/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2*x/((b*x^2 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a-bx^2)^2}\,dx = \frac{x}{2\,a\,(a-b\,x^2)} + \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int(1/(a - b*x^2)^2,x)
```

output
```
x/(2*a*(a - b*x^2)) + atanh((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int \frac{1}{(a-bx^2)^2}\,dx$$
$$= \frac{\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a - \sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b\,x^2 - \sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}-bx\right)a + \sqrt{b}\,\sqrt{a}\,\mathrm{lo}}{4a^2b\,(-b\,x^2+a)}$$

input
```
int(1/(-b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a - sqrt(b)*sqrt(a)*log( -
sqrt(b)*sqrt(a) - b*x)*b*x**2 - sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)
*a + sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*b*x**2 + 2*a*b*x)/(4*a**2*
b*(a - b*x**2))
```

3.16 $\int \frac{1}{(a-bx^2)^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 64

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{x}{4a(a-bx^2)^2} + \frac{3x}{8a^2(a-bx^2)} + \frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

output
```
1/4*x/a/(-b*x^2+a)^2+3/8*x/a^2/(-b*x^2+a)+3/8*arctanh(b^(1/2)*x/a^(1/2))/a
^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{5ax-3bx^3}{8a^2(a-bx^2)^2} + \frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

input
```
Integrate[(a - b*x^2)^(-3),x]
```

output
```
(5*a*x - 3*b*x^3)/(8*a^2*(a - b*x^2)^2) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])
/(8*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^3}dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\int \frac{1}{(a-bx^2)^2}dx}{4a}+\frac{x}{4a(a-bx^2)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\left(\frac{\int \frac{1}{a-bx^2}dx}{2a}+\frac{x}{2a(a-bx^2)}\right)}{4a}+\frac{x}{4a(a-bx^2)^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a-bx^2)}\right)}{4a}+\frac{x}{4a(a-bx^2)^2}$$

input
```
Int[(a - b*x^2)^(-3),x]
```

output
```
x/(4*a*(a - b*x^2)^2) + (3*(x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt
[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x}{4a(-b\,x^2+a)^2}+\frac{\frac{3x}{8a\left(-b\,x^2+a\right)}+\frac{3\,\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{a}$	59
risch	$\frac{-\frac{3b\,x^3}{8a^2}+\frac{5x}{8a}}{(-b\,x^2+a)^2}+\frac{3\ln\left(bx+\sqrt{ab}\right)}{16\sqrt{ab}\,a^2}-\frac{3\ln\left(-bx+\sqrt{ab}\right)}{16\sqrt{ab}\,a^2}$	70

input
```
int(1/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(-b*x^2+a)^2+3/4/a*(1/2*x/a/(-b*x^2+a)+1/2/a/(a*b)^(1/2)*arctanh(b
*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

$$\int \frac{1}{(a-bx^2)^3}\,dx = \left[-\frac{6\,ab^2x^3-10\,a^2bx-3\,(b^2x^4-2\,abx^2+a^2)\sqrt{ab}\log\left(\frac{bx^2+2\,\sqrt{ab}x+a}{bx^2-a}\right)}{16\,(a^3b^3x^4-2\,a^4b^2x^2+a^5b)}, -\frac{3\,ab^2x^3-5\,a^2bx+3\,(b^2x^4-2\,abx^2+a^2)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{8\,(a^3b^3x^4-2\,a^4b^2x^2+a^5b)}\right]$$

input
```
integrate(1/(-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(6*a*b^2*x^3 - 10*a^2*b*x - 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)
*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^3*b^3*x^4 - 2*a^4*b^2*x^
2 + a^5*b), -1/8*(3*a*b^2*x^3 - 5*a^2*b*x + 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*
sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a-bx^2)^3}\,dx = -\frac{3\sqrt{\frac{1}{a^5b}}\log\left(-a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16} + \frac{3\sqrt{\frac{1}{a^5b}}\log\left(a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16} - \frac{-5ax+3bx^3}{8a^4-16a^3bx^2+8a^2b^2x^4}$$

input
```
integrate(1/(-b*x**2+a)**3,x)
```

output
```
-3*sqrt(1/(a**5*b))*log(-a**3*sqrt(1/(a**5*b)) + x)/16 + 3*sqrt(1/(a**5*b)
)*log(a**3*sqrt(1/(a**5*b)) + x)/16 - (-5*a*x + 3*b*x**3)/(8*a**4 - 16*a**
3*b*x**2 + 8*a**2*b**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a-bx^2)^3}\,dx = -\frac{3\,bx^3-5\,ax}{8\,(a^2b^2x^4-2\,a^3bx^2+a^4)} - \frac{3\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}a^2}$$

input
```
integrate(1/(-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*b*x^3 - 5*a*x)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 3/16*log((b*x -
 sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a-bx^2)^3}\,dx = -\frac{3\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-ab}a^2} - \frac{3\,bx^3-5\,ax}{8\,(bx^2-a)^2a^2}$$

input
```
integrate(1/(-b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/8*(3*b*x^3 - 5*a*x)/((b*x
^2 - a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{\frac{5\,x}{8\,a}-\frac{3\,b\,x^3}{8\,a^2}}{a^2-2\,a\,b\,x^2+b^2\,x^4} + \frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

input
```
int(1/(a - b*x^2)^3,x)
```

output
```
((5*x)/(8*a) - (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) + (3*atanh((
b^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.78

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^2 - 6\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)ab\,x^2 + 3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b^2x^4 -}{16a^3b}$$

input
```
int(1/(-b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**2 - 6*sqrt(b)*sqrt(a)*
log( - sqrt(b)*sqrt(a) - b*x)*a*b*x**2 + 3*sqrt(b)*sqrt(a)*log( - sqrt(b)*
sqrt(a) - b*x)*b**2*x**4 - 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a*
*2 + 6*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a*b*x**2 - 3*sqrt(b)*sqr
t(a)*log(sqrt(b)*sqrt(a) - b*x)*b**2*x**4 + 10*a**2*b*x - 6*a*b**2*x**3)/(
16*a**3*b*(a**2 - 2*a*b*x**2 + b**2*x**4))
```

3.17 $\int \frac{1}{(a-bx^2)^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{1}{(a-bx^2)^4} dx = \frac{x}{6a(a-bx^2)^3} + \frac{5x}{24a^2(a-bx^2)^2} + \frac{5x}{16a^3(a-bx^2)} + \frac{5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

output
```
1/6*x/a/(-b*x^2+a)^3+5/24*x/a^2/(-b*x^2+a)^2+5/16*x/a^3/(-b*x^2+a)+5/16*ar
ctanh(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a-bx^2)^4} dx = \frac{33a^2x - 40abx^3 + 15b^2x^5}{48a^3(a-bx^2)^3} + \frac{5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}$$

input
```
Integrate[(a - b*x^2)^(-4),x]
```

output
```
(33*a^2*x - 40*a*b*x^3 + 15*b^2*x^5)/(48*a^3*(a - b*x^2)^3) + (5*ArcTanh[(
Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {215, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^4}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{5\int \frac{1}{(a-bx^2)^3}dx}{6a}+\frac{x}{6a(a-bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{5\left(\frac{3\int \frac{1}{(a-bx^2)^2}dx}{4a}+\frac{x}{4a(a-bx^2)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a-bx^2}dx}{2a}+\frac{x}{2a(a-bx^2)}\right)}{4a}+\frac{x}{4a(a-bx^2)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{5\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a-bx^2)}\right)}{4a}+\frac{x}{4a(a-bx^2)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}$$

input
```
Int[(a - b*x^2)^(-4),x]
```

output
```
x/(6*a*(a - b*x^2)^3) + (5*(x/(4*a*(a - b*x^2)^2) + (3*(x/(2*a*(a - b*x^2)
) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

method	result	size
default	$\frac{x}{6a(-b\,x^2+a)^3}+\frac{\frac{5x}{24a\left(-b\,x^2+a\right)^2}+\frac{5\left(\frac{3x}{8a\left(-b\,x^2+a\right)}+\frac{3\,\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{6a}}{a}$	81
risch	$\frac{\frac{5b^2x^5}{16a^3}-\frac{5b\,x^3}{6a^2}+\frac{11x}{16a}}{(-b\,x^2+a)^3}+\frac{5\ln\left(bx+\sqrt{ab}\right)}{32\sqrt{ab}\,a^3}-\frac{5\ln\left(-bx+\sqrt{ab}\right)}{32\sqrt{ab}\,a^3}$	81

input
```
int(1/(-b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*x/a/(-b*x^2+a)^3+5/6/a*(1/4*x/a/(-b*x^2+a)^2+3/4/a*(1/2*x/a/(-b*x^2+a)
+1/2/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.17

$$\int \frac{1}{(a-bx^2)^4}\,dx = \left[-\frac{30\,ab^3x^5 - 80\,a^2b^2x^3 + 66\,a^3bx - 15\,(b^3x^6 - 3\,ab^2x^4 + 3\,a^2bx^2 - a^3)\sqrt{ab}\log\left(\frac{bx^2+2\,\sqrt{ab}x+a}{bx^2-a}\right)}{96\,(a^4b^4x^6 - 3\,a^5b^3x^4 + 3\,a^6b^2x^2 - a^7b)},\right.$$
$$\left.-\frac{15\,ab^3x^5 - 40\,a^2b^2x^3 + 33\,a^3bx + 15\,(b^3x^6 - 3\,ab^2x^4 + 3\,a^2bx^2 - a^3)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{48\,(a^4b^4x^6 - 3\,a^5b^3x^4 + 3\,a^6b^2x^2 - a^7b)}\right]$$

input
```
integrate(1/(-b*x^2+a)^4,x, algorithm="fricas")
```

output
```
[-1/96*(30*a*b^3*x^5 - 80*a^2*b^2*x^3 + 66*a^3*b*x - 15*(b^3*x^6 - 3*a*b^2
*x^4 + 3*a^2*b*x^2 - a^3)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2
 - a)))/(a^4*b^4*x^6 - 3*a^5*b^3*x^4 + 3*a^6*b^2*x^2 - a^7*b), -1/48*(15*a
*b^3*x^5 - 40*a^2*b^2*x^3 + 33*a^3*b*x + 15*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2
*b*x^2 - a^3)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^4*b^4*x^6 - 3*a^5*b^3*
x^4 + 3*a^6*b^2*x^2 - a^7*b)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.49

$$\int \frac{1}{(a-bx^2)^4}\,dx = -\frac{5\sqrt{\frac{1}{a^7b}}\log\left(-a^4\sqrt{\frac{1}{a^7b}}+x\right)}{32} + \frac{5\sqrt{\frac{1}{a^7b}}\log\left(a^4\sqrt{\frac{1}{a^7b}}+x\right)}{32} + \frac{-33a^2x + 40abx^3 - 15b^2x^5}{-48a^6 + 144a^5bx^2 - 144a^4b^2x^4 + 48a^3b^3x^6}$$

input
```
integrate(1/(-b*x**2+a)**4,x)
```

output
```
-5*sqrt(1/(a**7*b))*log(-a**4*sqrt(1/(a**7*b)) + x)/32 + 5*sqrt(1/(a**7*b)
)*log(a**4*sqrt(1/(a**7*b)) + x)/32 + (-33*a**2*x + 40*a*b*x**3 - 15*b**2*
x**5)/(-48*a**6 + 144*a**5*b*x**2 - 144*a**4*b**2*x**4 + 48*a**3*b**3*x**6
)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a-bx^2)^4}\,dx = -\frac{15\,b^2x^5-40\,abx^3+33\,a^2x}{48\,(a^3b^3x^6-3\,a^4b^2x^4+3\,a^5bx^2-a^6)}-\frac{5\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{32\,\sqrt{ab}a^3}$$

input
```
integrate(1/(-b*x^2+a)^4,x, algorithm="maxima")
```

output
```
-1/48*(15*b^2*x^5 - 40*a*b*x^3 + 33*a^2*x)/(a^3*b^3*x^6 - 3*a^4*b^2*x^4 +
3*a^5*b*x^2 - a^6) - 5/32*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a
*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-bx^2)^4}\,dx = -\frac{5\,\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{16\,\sqrt{-ab}a^3}-\frac{15\,b^2x^5-40\,abx^3+33\,a^2x}{48\,(bx^2-a)^3a^3}$$

input
```
integrate(1/(-b*x^2+a)^4,x, algorithm="giac")
```

output
```
-5/16*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3) - 1/48*(15*b^2*x^5 - 40*a*b*
x^3 + 33*a^2*x)/((b*x^2 - a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a-bx^2)^4}\,dx = \frac{\frac{11\,x}{16\,a}-\frac{5\,b\,x^3}{6\,a^2}+\frac{5\,b^2\,x^5}{16\,a^3}}{a^3-3\,a^2\,b\,x^2+3\,a\,b^2\,x^4-b^3\,x^6}+\frac{5\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\,a^{7/2}\,\sqrt{b}}$$

input
```
int(1/(a - b*x^2)^4,x)
```

output
```
((11*x)/(16*a) - (5*b*x^3)/(6*a^2) + (5*b^2*x^5)/(16*a^3))/(a^3 - b^3*x^6
- 3*a^2*b*x^2 + 3*a*b^2*x^4) + (5*atanh((b^(1/2)*x)/a^(1/2)))/(16*a^(7/2)*
b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.10

$$\int \frac{1}{(a-bx^2)^4}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^3-45\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^2b\,x^2+45\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a}{}$$

input
```
int(1/(-b*x^2+a)^4,x)
```

output
```
(15*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**3 - 45*sqrt(b)*sqrt(a
)*log( - sqrt(b)*sqrt(a) - b*x)*a**2*b*x**2 + 45*sqrt(b)*sqrt(a)*log( - sq
rt(b)*sqrt(a) - b*x)*a*b**2*x**4 - 15*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(
a) - b*x)*b**3*x**6 - 15*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**3 +
 45*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**2*b*x**2 - 45*sqrt(b)*sq
rt(a)*log(sqrt(b)*sqrt(a) - b*x)*a*b**2*x**4 + 15*sqrt(b)*sqrt(a)*log(sqrt
(b)*sqrt(a) - b*x)*b**3*x**6 + 66*a**3*b*x - 80*a**2*b**2*x**3 + 30*a*b**3
*x**5)/(96*a**4*b*(a**3 - 3*a**2*b*x**2 + 3*a*b**2*x**4 - b**3*x**6))
```

3.18 $\int (2+x^3)^2 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int (2+x^3)^2 \, dx = 4x + x^4 + \frac{x^7}{7}$$

output
```
4*x+x^4+1/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (2+x^3)^2 \, dx = 4x + x^4 + \frac{x^7}{7}$$

input
```
Integrate[(2 + x^3)^2,x]
```

output
```
4*x + x^4 + x^7/7
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^3+2\right)^2 dx$$

$$\Big\downarrow \text{747}$$

$$\int \left(x^6+4x^3+4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^7}{7}+x^4+4x$$

input
```
Int[(2 + x^3)^2,x]
```

output
```
4*x + x^4 + x^7/7
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$4x + x^4 + \frac{1}{7}x^7$	13
default	$4x + x^4 + \frac{1}{7}x^7$	13
norman	$4x + x^4 + \frac{1}{7}x^7$	13
risch	$4x + x^4 + \frac{1}{7}x^7$	13
parallelrisch	$4x + x^4 + \frac{1}{7}x^7$	13
orering	$\frac{x(x^6+7x^3+28)}{7}$	14

input
```
int((x^3+2)^2,x,method=_RETURNVERBOSE)
```

output
```
4*x+x^4+1/7*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2\,dx = \frac{1}{7}\,x^7 + x^4 + 4\,x$$

input
```
integrate((x^3+2)^2,x, algorithm="fricas")
```

output
```
1/7*x^7 + x^4 + 4*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \left(2+x^3\right)^2\,dx=\frac{x^7}{7}+x^4+4x$$

input
```
integrate((x**3+2)**2,x)
```

output
```
x**7/7 + x**4 + 4*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2\,dx=\frac{1}{7}\,x^7+x^4+4\,x$$

input
```
integrate((x^3+2)^2,x, algorithm="maxima")
```

output
```
1/7*x^7 + x^4 + 4*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(2+x^3\right)^2\,dx=\frac{1}{7}\,x^7+x^4+4\,x$$

input
```
integrate((x^3+2)^2,x, algorithm="giac")
```

output
```
1/7*x^7 + x^4 + 4*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(2+x^3\right)^2 \, dx = \frac{x\,\left(x^6+7\,x^3+28\right)}{7}$$

input
```
int((x^3 + 2)^2,x)
```

output
```
(x*(7*x^3 + x^6 + 28))/7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(2+x^3\right)^2 \, dx = \frac{x(x^6+7x^3+28)}{7}$$

input
```
int((x^3+2)^2,x)
```

output
```
(x*(x**6 + 7*x**3 + 28))/7
```

3.19 $\int \frac{1}{a^2+x^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

output
```
arctan(x/a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
Integrate[(a^2 + x^2)^(-1),x]
```

output
```
ArcTan[x/a]/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2+x^2} \, dx$$
$$\Big\downarrow \; 216$$
$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
Int[(a^2 + x^2)^(-1),x]
```

output
```
ArcTan[x/a]/a
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
risch	$\frac{\arctan\left(\frac{x}{a}\right)}{a}$	11
parallelrisch	$-\frac{i\ln\left(-ia+x\right)-i\ln\left(ia+x\right)}{2a}$	27

input
```
int(1/(a^2+x^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x/a)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="fricas")
```

output
```
arctan(x/a)/a
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{-\frac{i\log\left(-ia+x\right)}{2} + \frac{i\log\left(ia+x\right)}{2}}{a}$$

input
```
integrate(1/(a**2+x**2),x)
```

output
```
(-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="maxima")
```

output
```
arctan(x/a)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\arctan\left(\frac{x}{a}\right)}{a}$$

input
```
integrate(1/(a^2+x^2),x, algorithm="giac")
```

output
```
arctan(x/a)/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2+x^2}\,dx = \frac{\mathrm{atan}\left(\frac{x}{a}\right)}{a}$$

input
```
int(1/(a^2 + x^2),x)
```

output
```
atan(x/a)/a
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{atan\left(\frac{x}{a}\right)}{a}$$

input
```
int(1/(a^2+x^2),x)
```

output
```
atan(x/a)/a
```

3.20 $\int \frac{1}{a+bx^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+bx^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2}\,dx$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

input
```
int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{a+bx^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(1/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{a+bx^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(1/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-
1/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="maxima")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+bx^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2),x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+bx^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(1/(b*x^2+a),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.21 $\int \frac{1}{\left(-1+x^2\right)^2}\,dx$

Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \frac{1}{\left(-1+x^2\right)^2}\,dx=\frac{x}{2\left(1-x^2\right)}+\frac{\operatorname{arctanh}(x)}{2}$$

output
```
x/(-2*x^2+2)+1/2*arctanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(-1+x^2\right)^2}\,dx=\frac{1}{4}\left(-\frac{2x}{-1+x^2}-\log(1-x)+\log(1+x)\right)$$

input
```
Integrate[(-1 + x^2)^(-2),x]
```

output
```
((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {215, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2-1)^2} \, dx$$

$$\Big\downarrow \; 215$$

$$\frac{x}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{x^2-1}dx$$

$$\Big\downarrow \; 220$$

$$\frac{\text{arctanh}(x)}{2} + \frac{x}{2(1-x^2)}$$

input
```
Int[(-1 + x^2)^(-2),x]
```

output
```
x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result	size
meijerg	$-\frac{i\left(\frac{2ix}{-2x^2+2}+i\,\mathrm{arctanh}(x)\right)}{2}$	23
norman	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
risch	$-\frac{x}{2(x^2-1)}-\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}$	24
default	$-\frac{1}{4(-1+x)}-\frac{\ln(-1+x)}{4}-\frac{1}{4(1+x)}+\frac{\ln(1+x)}{4}$	28
parallelrisch	$\frac{\ln(1+x)x^2-\ln(-1+x)x^2-\ln(1+x)+\ln(-1+x)-2x}{4x^2-4}$	41

input
```
int(1/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*(2*I*x/(-2*x^2+2)+I*arctanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(15) = 30.

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-1+x^2)^2}\,dx = \frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2x}{4(x^2-1)}$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="fricas")
```

output
```
1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2x^2-2} - \frac{\log(x-1)}{4} + \frac{\log(x+1)}{4}$$

input
```
integrate(1/(x**2-1)**2,x)
```

output
```
-x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2(x^2-1)} + \frac{1}{4}\log(x+1) - \frac{1}{4}\log(x-1)$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(-1+x^2)^2}\,dx = -\frac{x}{2(x^2-1)} + \frac{1}{4}\log(|x+1|) - \frac{1}{4}\log(|x-1|)$$

input
```
integrate(1/(x^2-1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(-1+x^2\right)^2} dx = \frac{\operatorname{atanh}(x)}{2} - \frac{x}{2\left(x^2-1\right)}$$

input
```
int(1/(x^2 - 1)^2,x)
```

output
```
atanh(x)/2 - x/(2*(x^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.95

$$\int \frac{1}{\left(-1+x^2\right)^2} dx = \frac{-\log(x-1)\,x^2+\log(x-1)+\log(x+1)\,x^2-\log(x+1)-2x}{4x^2-4}$$

input
```
int(1/(x^2-1)^2,x)
```

output
```
( - log(x - 1)*x**2 + log(x - 1) + log(x + 1)*x**2 - log(x + 1) - 2*x)/(4*
(x**2 - 1))
```

3.22 $\int \frac{1}{-1+a+ax^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{1}{-1+a+ax^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

output
```
-arctanh(a^(1/2)*x/(1-a)^(1/2))/((1-a)*a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{-1+a+ax^2}\,dx = \frac{\arctan\left(\frac{\sqrt{a}x}{\sqrt{-1+a}}\right)}{\sqrt{-1+a}\sqrt{a}}$$

input
```
Integrate[(-1 + a + a*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*x)/Sqrt[-1 + a]]/(Sqrt[-1 + a]*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+a-1}\,dx$$

$$\Big\downarrow \text{221}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{1-a}}\right)}{\sqrt{(1-a)a}}$$

input
```
Int[(-1 + a + a*x^2)^(-1),x]
```

output
```
-(ArcTanh[(Sqrt[a]*x)/Sqrt[1 - a]]/Sqrt[(1 - a)*a])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{ax}{\sqrt{(a-1)a}}\right)}{\sqrt{(a-1)a}}$	20
risch	$-\frac{\ln\left(ax+\sqrt{-(a-1)a}\right)}{2\sqrt{-(a-1)a}}+\frac{\ln\left(-ax+\sqrt{-(a-1)a}\right)}{2\sqrt{-(a-1)a}}$	49

input
```
int(1/(a*x^2+a-1),x,method=_RETURNVERBOSE)
```

output
```
1/((a-1)*a)^(1/2)*arctan(a*x/((a-1)*a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.73

$$\int \frac{1}{-1+a+ax^2}\,dx = \left[-\frac{\sqrt{-a^2+a}\log\left(\frac{ax^2-2\sqrt{-a^2+a}x-a+1}{ax^2+a-1}\right)}{2\left(a^2-a\right)}, \frac{\arctan\left(\frac{\sqrt{a^2-a}x}{a-1}\right)}{\sqrt{a^2-a}}\right]$$

input
```
integrate(1/(a*x^2+a-1),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + a)*log((a*x^2 - 2*sqrt(-a^2 + a)*x - a + 1)/(a*x^2 + a -
 1))/(a^2 - a), arctan(sqrt(a^2 - a)*x/(a - 1))/sqrt(a^2 - a)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(24) = 48.

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \frac{1}{-1+a+ax^2}\,dx = -\frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(-a\sqrt{-\frac{1}{a(a-1)}}+x+\sqrt{-\frac{1}{a(a-1)}}\right)}{2} + \frac{\sqrt{-\frac{1}{a(a-1)}}\log\left(a\sqrt{-\frac{1}{a(a-1)}}+x-\sqrt{-\frac{1}{a(a-1)}}\right)}{2}$$

input
```
integrate(1/(a*x**2+a-1),x)
```

output
```
-sqrt(-1/(a*(a - 1)))*log(-a*sqrt(-1/(a*(a - 1))) + x + sqrt(-1/(a*(a - 1)
)))/2 + sqrt(-1/(a*(a - 1)))*log(a*sqrt(-1/(a*(a - 1))) + x - sqrt(-1/(a*(
a - 1))))/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{-1+a+ax^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*x^2+a-1),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1}{-1+a+ax^2}\,dx = \frac{\arctan\left(\frac{ax}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

input
```
integrate(1/(a*x^2+a-1),x, algorithm="giac")
```

output
```
arctan(a*x/sqrt(a^2 - a))/sqrt(a^2 - a)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int \frac{1}{-1+a+ax^2}\,dx = \frac{\mathrm{atan}\left(\frac{a\,x}{\sqrt{a^2-a}}\right)}{\sqrt{a^2-a}}$$

input
```
int(1/(a + a*x^2 - 1),x)
```

output
```
atan((a*x)/(a^2 - a)^(1/2))/(a^2 - a)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{-1+a+ax^2}\,dx = \frac{\sqrt{a}\,\sqrt{a-1}\,atan\left(\frac{ax}{\sqrt{a}\,\sqrt{a-1}}\right)}{a\,(a-1)}$$

input
```
int(1/(a*x^2+a-1),x)
```

output
```
(sqrt(a)*sqrt(a - 1)*atan((a*x)/(sqrt(a)*sqrt(a - 1))))/(a*(a - 1))
```

3.23 $\int \frac{1}{-c-d+(c-d)x^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 37

$$\int \frac{1}{-c-d+(c-d)x^2} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c-d}x}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

output
```
-arctanh((c-d)^(1/2)*x/(c+d)^(1/2))/(c-d)^(1/2)/(c+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{1}{-c-d+(c-d)x^2} \, dx = \frac{\arctan\left(\frac{\sqrt{c-d}x}{\sqrt{-c-d}}\right)}{\sqrt{-c-d}\sqrt{c-d}}$$

input
```
Integrate[(-c - d + (c - d)*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[c - d]*x)/Sqrt[-c - d]]/(Sqrt[-c - d]*Sqrt[c - d])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(c-d)-c-d}\,dx$$
$$\Big\downarrow \; 221$$
$$-\frac{\text{arctanh}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}$$

input
```
Int[(-c - d + (c - d)*x^2)^(-1),x]
```

output
```
-(ArcTanh[(Sqrt[c - d]*x)/Sqrt[c + d]]/(Sqrt[c - d]*Sqrt[c + d]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{\text{arctanh}\left(\frac{(c-d)x}{\sqrt{(c+d)(c-d)}}\right)}{\sqrt{(c+d)(c-d)}}$	33
risch	$\frac{\ln\left((-c+d)x+\sqrt{c^2-d^2}\right)}{2\sqrt{c^2-d^2}} - \frac{\ln\left((c-d)x+\sqrt{c^2-d^2}\right)}{2\sqrt{c^2-d^2}}$	68

input
```
int(1/(-c-d+(c-d)*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/((c+d)*(c-d))^(1/2)*arctanh((c-d)*x/((c+d)*(c-d))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.76

$$\int \frac{1}{-c-d+(c-d)x^2}\,dx = \left[\frac{\log\left(\frac{(c-d)x^2-2\sqrt{c^2-d^2}x+c+d}{(c-d)x^2-c-d}\right)}{2\sqrt{c^2-d^2}}, \frac{\sqrt{-c^2+d^2}\arctan\left(\frac{\sqrt{-c^2+d^2}x}{c+d}\right)}{c^2-d^2}\right]$$

input
```
integrate(1/(-c-d+(c-d)*x^2),x, algorithm="fricas")
```

output
```
[1/2*log(((c - d)*x^2 - 2*sqrt(c^2 - d^2)*x + c + d)/((c - d)*x^2 - c - d)
)/sqrt(c^2 - d^2), sqrt(-c^2 + d^2)*arctan(sqrt(-c^2 + d^2)*x/(c + d))/(c^
2 - d^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(31) = 62$.

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int \frac{1}{-c-d+(c-d)x^2}\,dx = \frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(-c\sqrt{\frac{1}{(c-d)(c+d)}}-d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2} - \frac{\sqrt{\frac{1}{(c-d)(c+d)}}\log\left(c\sqrt{\frac{1}{(c-d)(c+d)}}+d\sqrt{\frac{1}{(c-d)(c+d)}}+x\right)}{2}$$

input
```
integrate(1/(-c-d+(c-d)*x**2),x)
```

output
```
sqrt(1/((c - d)*(c + d)))*log(-c*sqrt(1/((c - d)*(c + d))) - d*sqrt(1/((c
- d)*(c + d))) + x)/2 - sqrt(1/((c - d)*(c + d)))*log(c*sqrt(1/((c - d)*(c
 + d))) + d*sqrt(1/((c - d)*(c + d))) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{-c-d+(c-d)x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(-c-d+(c-d)*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*d^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{-c-d+(c-d)x^2}\,dx = \frac{\arctan\left(\frac{cx-dx}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}$$

input
```
integrate(1/(-c-d+(c-d)*x^2),x, algorithm="giac")
```

output
```
arctan((c*x - d*x)/sqrt(-c^2 + d^2))/sqrt(-c^2 + d^2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{-c-d+(c-d)x^2} \, dx = -\frac{\operatorname{atanh}\left(\frac{x\sqrt{c-d}}{\sqrt{c+d}}\right)}{\sqrt{c+d}\,\sqrt{c-d}}$$

input
```
int(-1/(c + d - x^2*(c - d)),x)
```

output
```
-atanh((x*(c - d)^(1/2))/(c + d)^(1/2))/((c + d)^(1/2)*(c - d)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{-c-d+(c-d)x^2} \, dx = -\frac{\sqrt{-c^2+d^2}\,atan\left(\frac{cx-dx}{\sqrt{-c^2+d^2}}\right)}{c^2-d^2}$$

input
```
int(1/(-c-d+(c-d)*x^2),x)
```

output
```
( - sqrt( - c**2 + d**2)*atan((c*x - d*x)/sqrt( - c**2 + d**2)))/(c**2 - d
**2)
```

3.24 $\int \frac{1}{a+(b-ac)x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \frac{1}{a+(b-ac)x^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

output
```
arctan((-a*c+b)^(1/2)*x/a^(1/2))/a^(1/2)/(-a*c+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{a+(b-ac)x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{-b+ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{-b+ac}}$$

input
```
Integrate[(a + (b - a*c)*x^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{218\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(b-ac)+a}\,dx$$
$$\Big\downarrow \; 218$$
$$\frac{\arctan\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

input
```
Int[(a + (b - a*c)*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{(ac-b)x}{\sqrt{a(ac-b)}}\right)}{\sqrt{a(ac-b)}}$	34
risch	$\frac{\ln\left((-ac+b)x-\sqrt{a(ac-b)}\right)}{2\sqrt{a(ac-b)}}-\frac{\ln\left((ac-b)x-\sqrt{a(ac-b)}\right)}{2\sqrt{a(ac-b)}}$	75

input
```
int(1/(a+(-a*c+b)*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*(a*c-b))^(1/2)*arctanh((a*c-b)*x/(a*(a*c-b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.12

$$\int \frac{1}{a+(b-ac)x^2}\,dx = \left[\frac{\log\left(\frac{(ac-b)x^2+2\sqrt{a^2c-ab}x+a}{(ac-b)x^2-a}\right)}{2\sqrt{a^2c-ab}}, -\frac{\sqrt{-a^2c+ab}\arctan\left(\frac{\sqrt{-a^2c+ab}x}{a}\right)}{a^2c-ab}\right]$$

input
```
integrate(1/(a+(-a*c+b)*x^2),x, algorithm="fricas")
```

output
```
[1/2*log(((a*c - b)*x^2 + 2*sqrt(a^2*c - a*b)*x + a)/((a*c - b)*x^2 - a))/
sqrt(a^2*c - a*b), -sqrt(-a^2*c + a*b)*arctan(sqrt(-a^2*c + a*b)*x/a)/(a^2
*c - a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.76

$$\int \frac{1}{a+(b-ac)x^2}\,dx = -\frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(-a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2} + \frac{\sqrt{\frac{1}{a(ac-b)}}\log\left(a\sqrt{\frac{1}{a(ac-b)}}+x\right)}{2}$$

input
```
integrate(1/(a+(-a*c+b)*x**2),x)
```

output
```
-sqrt(1/(a*(a*c - b)))*log(-a*sqrt(1/(a*(a*c - b))) + x)/2 + sqrt(1/(a*(a*
c - b)))*log(a*sqrt(1/(a*(a*c - b))) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+(b-ac)x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+(-a*c+b)*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*c-b>0)', see `assume?` for mor
e details)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+(b-ac)x^2}\,dx = -\frac{\arctan\left(\frac{acx-bx}{\sqrt{-a^2c+ab}}\right)}{\sqrt{-a^2c+ab}}$$

input
```
integrate(1/(a+(-a*c+b)*x^2),x, algorithm="giac")
```

output
```
-arctan((a*c*x - b*x)/sqrt(-a^2*c + a*b))/sqrt(-a^2*c + a*b)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{1}{a+(b-ac)x^2}\,dx = -\frac{\operatorname{atanh}\left(\frac{x\,(2\,b-2\,a\,c)}{2\,\sqrt{a}\,\sqrt{a\,c-b}}\right)}{\sqrt{a}\,\sqrt{a\,c-b}}$$

input
```
int(1/(a + x^2*(b - a*c)),x)
```

output
```
-atanh((x*(2*b - 2*a*c))/(2*a^(1/2)*(a*c - b)^(1/2)))/(a^(1/2)*(a*c - b)^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{1}{a+(b-ac)x^2}\,dx = \frac{\sqrt{a}\,\sqrt{-ac+b}\,atan\left(\frac{acx-bx}{\sqrt{a}\,\sqrt{-ac+b}}\right)}{a\,(ac-b)}$$

input
```
int(1/(a+(-a*c+b)*x^2),x)
```

output
```
(sqrt(a)*sqrt( - a*c + b)*atan((a*c*x - b*x)/(sqrt(a)*sqrt( - a*c + b))))/
(a*(a*c - b))
```

3.25 $\int \frac{1}{a-(b-ac)x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1}{a-(b-ac)x^2} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b-ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

output
```
arctanh((-a*c+b)^(1/2)*x/a^(1/2))/a^(1/2)/(-a*c+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{a-(b-ac)x^2} dx = \frac{\arctan\left(\frac{\sqrt{-b+ac}\,x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{-b+ac}}$$

input
```
Integrate[(a - (b - a*c)*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[-b + a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[-b + a*c])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-x^2(b-ac)}\,dx$$
$$\Big\downarrow \ 221$$
$$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{b-ac}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b-ac}}$$

input
```
Int[(a - (b - a*c)*x^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[b - a*c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b - a*c])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{(ac-b)x}{\sqrt{a(ac-b)}}\right)}{\sqrt{a(ac-b)}}$	34
risch	$-\frac{\ln\left((ac-b)x+\sqrt{-a(ac-b)}\right)}{2\sqrt{-a(ac-b)}}+\frac{\ln\left((-ac+b)x+\sqrt{-a(ac-b)}\right)}{2\sqrt{-a(ac-b)}}$	75

input
```
int(1/(a-(-a*c+b)*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*(a*c-b))^(1/2)*arctan((a*c-b)*x/(a*(a*c-b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.09

$$\int \frac{1}{a-(b-ac)x^2}\,dx = \left[-\frac{\sqrt{-a^2c+ab}\log\left(\frac{(ac-b)x^2-2\sqrt{-a^2c+ab}x-a}{(ac-b)x^2+a}\right)}{2\,(a^2c-ab)}, \frac{\arctan\left(\frac{\sqrt{a^2c-ab}x}{a}\right)}{\sqrt{a^2c-ab}}\right]$$

input
```
integrate(1/(a-(-a*c+b)*x^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2*c + a*b)*log(((a*c - b)*x^2 - 2*sqrt(-a^2*c + a*b)*x - a)/
((a*c - b)*x^2 + a))/(a^2*c - a*b), arctan(sqrt(a^2*c - a*b)*x/a)/sqrt(a^2
*c - a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(29) = 58.

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \frac{1}{a-(b-ac)x^2}\,dx = -\frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(-a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{a(ac-b)}}\log\left(a\sqrt{-\frac{1}{a(ac-b)}}+x\right)}{2}$$

input
```
integrate(1/(a-(-a*c+b)*x**2),x)
```

output
```
-sqrt(-1/(a*(a*c - b)))*log(-a*sqrt(-1/(a*(a*c - b))) + x)/2 + sqrt(-1/(a*
(a*c - b)))*log(a*sqrt(-1/(a*(a*c - b))) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a-(b-ac)x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a-(-a*c+b)*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*c-b>0)', see `assume?` for mor
e details)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{a-(b-ac)x^2}\,dx = \frac{\arctan\left(\frac{acx-bx}{\sqrt{a^2c-ab}}\right)}{\sqrt{a^2c-ab}}$$

input
```
integrate(1/(a-(-a*c+b)*x^2),x, algorithm="giac")
```

output
```
arctan((a*c*x - b*x)/sqrt(a^2*c - a*b))/sqrt(a^2*c - a*b)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{1}{a-(b-ac)x^2}\,dx = -\frac{\operatorname{atan}\left(\frac{x\,(2\,b-2\,a\,c)}{2\,\sqrt{a^2\,c-a\,b}}\right)}{\sqrt{a^2\,c-a\,b}}$$

input
```
int(1/(a - x^2*(b - a*c)),x)
```

output
```
-atan((x*(2*b - 2*a*c))/(2*(a^2*c - a*b)^(1/2)))/(a^2*c - a*b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{1}{a-(b-ac)x^2}\,dx = \frac{\sqrt{a}\,\sqrt{ac-b}\,atan\left(\frac{acx-bx}{\sqrt{a}\,\sqrt{ac-b}}\right)}{a\,(ac-b)}$$

input
```
int(1/(a-(-a*c+b)*x^2),x)
```

output
```
(sqrt(a)*sqrt(a*c - b)*atan((a*c*x - b*x)/(sqrt(a)*sqrt(a*c - b))))/(a*(a*
c - b))
```

3.26 $\int \frac{1}{c(a-d)-(b-c)x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 50

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b-c}x}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{b-c}\sqrt{c}\sqrt{a-d}}$$

output
```
arctanh((b-c)^(1/2)*x/c^(1/2)/(a-d)^(1/2))/(b-c)^(1/2)/c^(1/2)/(a-d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx=\frac{\arctan\left(\frac{\sqrt{-b+c}x}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{-b+c}\sqrt{a-d}}$$

input
```
Integrate[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[-b + c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[c]*Sqrt[-b + c]*Sqrt[
a - d])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{c(a-d)-x^2(b-c)}\,dx$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{x\sqrt{b-c}}{\sqrt{c}\sqrt{a-d}}\right)}{\sqrt{c}\sqrt{a-d}\sqrt{b-c}}$$

input
```
Int[(c*(a - d) - (b - c)*x^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[b - c]*x)/(Sqrt[c]*Sqrt[a - d])]/(Sqrt[b - c]*Sqrt[c]*Sqrt[a
 - d])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\text{arctanh}\left(\frac{(-b+c)x}{\sqrt{c(a-d)(b-c)}}\right)}{\sqrt{c(a-d)(b-c)}}$	39
risch	$\frac{\ln\left((b-c)x+\sqrt{c(a-d)(b-c)}\right)}{2\sqrt{c(a-d)(b-c)}}-\frac{\ln\left((-b+c)x+\sqrt{c(a-d)(b-c)}\right)}{2\sqrt{c(a-d)(b-c)}}$	80

input
```
int(1/(c*(a-d)-(b-c)*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/(c*(a-d)*(b-c))^(1/2)*arctanh((-b+c)*x/(c*(a-d)*(b-c))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(38) = 76$.

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.64

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = \left[\frac{\log\left(\frac{(b-c)x^2+ac-cd+2\sqrt{abc-ac^2-(bc-c^2)d}\,x}{(b-c)x^2-ac+cd}\right)}{2\sqrt{abc-ac^2-(bc-c^2)d}}, \frac{\sqrt{-abc+ac^2+(bc-c^2)d}\arctan\left(-\frac{\sqrt{-abc+ac^2+(bc-c^2)d}\,x}{ac-cd}\right)}{abc-ac^2-(bc-c^2)d}\right]$$

input
```
integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="fricas")
```

output
```
[1/2*log(((b - c)*x^2 + a*c - c*d + 2*sqrt(a*b*c - a*c^2 - (b*c - c^2)*d)*
x)/((b - c)*x^2 - a*c + c*d))/sqrt(a*b*c - a*c^2 - (b*c - c^2)*d), sqrt(-a
*b*c + a*c^2 + (b*c - c^2)*d)*arctan(-sqrt(-a*b*c + a*c^2 + (b*c - c^2)*d)
*x/(a*c - c*d))/(a*b*c - a*c^2 - (b*c - c^2)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(39) = 78$.

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.08

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = -\frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(-ac\sqrt{\frac{1}{c(a-d)(b-c)}}+cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2} + \frac{\sqrt{\frac{1}{c(a-d)(b-c)}}\log\left(ac\sqrt{\frac{1}{c(a-d)(b-c)}}-cd\sqrt{\frac{1}{c(a-d)(b-c)}}+x\right)}{2}$$

input
```
integrate(1/(c*(a-d)-(b-c)*x**2),x)
```

output
```
-sqrt(1/(c*(a - d)*(b - c)))*log(-a*c*sqrt(1/(c*(a - d)*(b - c))) + c*d*sq
rt(1/(c*(a - d)*(b - c))) + x)/2 + sqrt(1/(c*(a - d)*(b - c)))*log(a*c*sqr
t(1/(c*(a - d)*(b - c))) - c*d*sqrt(1/(c*(a - d)*(b - c))) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((c-b)*(d-a)>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = -\frac{\arctan\left(\frac{bx-cx}{\sqrt{-abc+ac^2+bcd-c^2d}}\right)}{\sqrt{-abc+ac^2+bcd-c^2d}}$$

input
```
integrate(1/(c*(a-d)-(b-c)*x^2),x, algorithm="giac")
```

output
```
-arctan((b*x - c*x)/sqrt(-a*b*c + a*c^2 + b*c*d - c^2*d))/sqrt(-a*b*c + a*
c^2 + b*c*d - c^2*d)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = \frac{\operatorname{atanh}\left(\frac{x\,(2\,b-2\,c)}{2\,\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}\right)}{\sqrt{c}\,\sqrt{a-d}\,\sqrt{b-c}}$$

input
```
int(1/(c*(a - d) - x^2*(b - c)),x)
```

output
```
atanh((x*(2*b - 2*c))/(2*c^(1/2)*(a - d)^(1/2)*(b - c)^(1/2)))/(c^(1/2)*(a
 - d)^(1/2)*(b - c)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{1}{c(a-d)-(b-c)x^2}\,dx = \frac{\sqrt{c}\,\sqrt{b-c}\,\sqrt{-a+d}\,atan\left(\frac{bx-cx}{\sqrt{c}\,\sqrt{b-c}\,\sqrt{-a+d}}\right)}{c\,(ab-ac-bd+cd)}$$

input
```
int(1/(c*(a-d)-(b-c)*x^2),x)
```

output
```
(sqrt(c)*sqrt(b - c)*sqrt( - a + d)*atan((b*x - c*x)/(sqrt(c)*sqrt(b - c)*
sqrt( - a + d))))/(c*(a*b - a*c - b*d + c*d))
```

3.27 $\int \frac{2}{-1+4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{2}{-1+4x^2}\,dx = -\operatorname{arctanh}(2x)$$

output
```
-arctanh(2*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(6) = 12.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.83

$$\int \frac{2}{-1+4x^2}\,dx = 2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(1+2x)\right)$$

input
```
Integrate[2/(-1 + 4*x^2),x]
```

output
```
2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {27, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2}{4x^2-1}dx$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{1}{4x^2-1}dx$$

$$\Big\downarrow \text{220}$$

$$-\text{arctanh}(2x)$$

input
```
Int[2/(-1 + 4*x^2),x]
```

output
```
-ArcTanh[2*x]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
meijerg	$-\operatorname{arctanh}(2x)$	7
parallelrisch	$-\frac{\ln(x+\frac{1}{2})}{2}+\frac{\ln(x-\frac{1}{2})}{2}$	14
default	$-\frac{\ln(1+2x)}{2}+\frac{\ln(-1+2x)}{2}$	18
norman	$-\frac{\ln(1+2x)}{2}+\frac{\ln(-1+2x)}{2}$	18
risch	$-\frac{\ln(1+2x)}{2}+\frac{\ln(-1+2x)}{2}$	18

input
```
int(2/(4*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(6) = 12.

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\log(2x+1)+\frac{1}{2}\log(2x-1)$$

input
```
integrate(2/(4*x^2-1),x, algorithm="fricas")
```

output
```
-1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{2}{-1+4x^2}\,dx = \frac{\log\left(x-\frac{1}{2}\right)}{2} - \frac{\log\left(x+\frac{1}{2}\right)}{2}$$

input
```
integrate(2/(4*x**2-1),x)
```

output
```
log(x - 1/2)/2 - log(x + 1/2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\log\left(2\,x+1\right) + \frac{1}{2}\log\left(2\,x-1\right)$$

input
```
integrate(2/(4*x^2-1),x, algorithm="maxima")
```

output
```
-1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(6) = 12$.

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{2}{-1+4x^2}\,dx = -\frac{1}{2}\log\left(\left|x+\frac{1}{2}\right|\right) + \frac{1}{2}\log\left(\left|x-\frac{1}{2}\right|\right)$$

input
```
integrate(2/(4*x^2-1),x, algorithm="giac")
```

output
```
-1/2*log(abs(x + 1/2)) + 1/2*log(abs(x - 1/2))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{2}{-1+4x^2}\,dx = -\mathrm{atanh}(2\,x)$$

input
```
int(2/(4*x^2 - 1),x)
```

output
```
-atanh(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{2}{-1+4x^2}\,dx = \frac{\log(2x-1)}{2} - \frac{\log(2x+1)}{2}$$

input
```
int(2/(4*x^2-1),x)
```

output
```
(log(2*x - 1) - log(2*x + 1))/2
```

3.28 $\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(1+2x)$$

output
```
1/2*ln(1-2*x)-1/2*ln(1+2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=2\left(\frac{1}{4}\log(1-2x)-\frac{1}{4}\log(1+2x)\right)$$

input
```
Integrate[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

output
```
2*(Log[1 - 2*x]/4 - Log[1 + 2*x]/4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{2x-1}-\frac{1}{2x+1}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\log(1-2x)-\frac{1}{2}\log(2x+1)$$

input
```
Int[(-1 + 2*x)^(-1) - (1 + 2*x)^(-1),x]
```

output
```
Log[1 - 2*x]/2 - Log[1 + 2*x]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{\ln\left(x+\frac{1}{2}\right)}{2}+\frac{\ln\left(x-\frac{1}{2}\right)}{2}$	14
default	$-\frac{\ln(1+2x)}{2}+\frac{\ln(-1+2x)}{2}$	18
norman	$-\frac{\ln(1+2x)}{2}+\frac{\ln(-1+2x)}{2}$	18
meijerg	$\frac{\ln(1-2x)}{2}-\frac{\ln(1+2x)}{2}$	18
risch	$-\frac{\ln(1+2x)}{2}+\frac{\ln(-1+2x)}{2}$	18

input
```
int(1/(-1+2*x)-1/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(x+1/2)+1/2*ln(x-1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

input
```
integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="fricas")
```

output
```
-1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=\frac{\log\left(x-\frac{1}{2}\right)}{2}-\frac{\log\left(x+\frac{1}{2}\right)}{2}$$

input
```
integrate(1/(-1+2*x)-1/(1+2*x),x)
```

output
```
log(x - 1/2)/2 - log(x + 1/2)/2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(2\,x+1\right)+\frac{1}{2}\,\log\left(2\,x-1\right)$$

input
```
integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="maxima")
```

output
```
-1/2*log(2*x + 1) + 1/2*log(2*x - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\frac{1}{2}\,\log\left(|2\,x+1|\right)+\frac{1}{2}\,\log\left(|2\,x-1|\right)$$

input
```
integrate(1/(-1+2*x)-1/(1+2*x),x, algorithm="giac")
```

output
```
-1/2*log(abs(2*x + 1)) + 1/2*log(abs(2*x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.29

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right)\,dx=-\mathrm{atanh}(2\,x)$$

input
```
int(1/(2*x - 1) - 1/(2*x + 1),x)
```

output
```
-atanh(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \left(\frac{1}{-1+2x}-\frac{1}{1+2x}\right) dx = \frac{\log(2x-1)}{2}-\frac{\log(2x+1)}{2}$$

input
```
int(1/(-1+2*x)-1/(1+2*x),x)
```

output
```
(log(2*x - 1) - log(2*x + 1))/2
```

3.29 $\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ace}}{2cf-bg+\sqrt{b^2-4acg}}-x^2}\,dx$

Optimal result

Integrand size = 57, antiderivative size = 116

$$\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ace}}{2cf-bg+\sqrt{b^2-4acg}}-x^2}\,dx=\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\text{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}x}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

output
```
(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)*arctanh((2*c*f-(b-(-4*a*c+b^2)^(1/2
))*g)^(1/2)*x/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/(2*c*d-(b-(-4*a*c+b^
2)^(1/2))*e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ace}}{2cf-bg+\sqrt{b^2-4acg}}-x^2}\,dx=\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ace}}{2cf-bg+\sqrt{b^2-4acg}}-x^2}\,dx$$

input
```
Integrate[((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*f - b*g + Sqrt[b^2 - 4
*a*c]*g) - x^2)^(-1),x]
```

output
```
Integrate[((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*f - b*g + Sqrt[b^2 - 4
*a*c]*g) - x^2)^(-1), x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{e\sqrt{b^2-4ac}-be+2cd}{g\sqrt{b^2-4ac}-bg+2cf}-x^2}dx$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}\text{arctanh}\left(\frac{x\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

input
```
Int[((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*
g) - x^2)^(-1),x]
```

output
```
(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b
^2 - 4*a*c])*g]*x)/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19

method	result	size
default	$-\frac{\left(2cf-bg+\sqrt{-4ac+b^2}\,g\right)\operatorname{arctanh}\left(\frac{\left(-\sqrt{-4ac+b^2}\,g+bg-2cf\right)x}{\sqrt{\left(2cd-be+\sqrt{-4ac+b^2}\,e\right)\left(2cf-bg+\sqrt{-4ac+b^2}\,g\right)}}\right)}{\sqrt{\left(2cd-be+\sqrt{-4ac+b^2}\,e\right)\left(2cf-bg+\sqrt{-4ac+b^2}\,g\right)}}$	138

input
```
int(1/((2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e)/(2*c*f-b*g+(-4*a*c+b^2)^(1/2)*g)-x
^2),x,method=_RETURNVERBOSE)
```

output
```
-(2*c*f-b*g+(-4*a*c+b^2)^(1/2)*g)/((2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e)*(2*c*f
-b*g+(-4*a*c+b^2)^(1/2)*g))^(1/2)*arctanh((-(-4*a*c+b^2)^(1/2)*g+b*g-2*c*f
)*x/((2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e)*(2*c*f-b*g+(-4*a*c+b^2)^(1/2)*g))^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.09

$$\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ac}e}{2cf-bg+\sqrt{b^2-4ac}g}-x^2}\,dx$$

$$= \left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(2\,cd-be)f-(bd-2\,ae)g-\sqrt{b^2-4\,ac}(ef-dg)}{cd^2-bde+ae^2}}\log\left(\frac{(cf^2-bfg+ag^2)x^4+\sqrt{b^2-4\,ac}(e\right.\right.$$

$$-\sqrt{\frac{1}{2}}\sqrt{-\frac{(2\,cd-be)f-(bd-2\,ae)g-\sqrt{b^2-4\,ac}(ef-dg)}{cd^2-bde+ae^2}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{-\frac{(2\,cd-be)f-(bd-2\right.\right.$$

input
```
integrate(1/((2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e)/(2*c*f-b*g+(-4*a*c+b^2)^(1/2
)*g)-x^2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - sqrt(b^2 - 4*a*c)
*(e*f - d*g))/(c*d^2 - b*d*e + a*e^2))*log(((c*f^2 - b*f*g + a*g^2)*x^4 +
sqrt(b^2 - 4*a*c)*(e*f - d*g)*x^2 - c*d^2 + b*d*e - a*e^2 + sqrt(1/2)*(sqr
t(b^2 - 4*a*c)*(e*f - d*g)*x^3 + ((2*c*d - b*e)*f - (b*d - 2*a*e)*g)*x^3 -
 2*(c*d^2 - b*d*e + a*e^2)*x)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - sq
rt(b^2 - 4*a*c)*(e*f - d*g))/(c*d^2 - b*d*e + a*e^2)))/((c*f^2 - b*f*g + a
*g^2)*x^4 + c*d^2 - b*d*e + a*e^2 - ((2*c*d - b*e)*f - (b*d - 2*a*e)*g)*x^
2)), -sqrt(1/2)*sqrt(-((2*c*d - b*e)*f - (b*d - 2*a*e)*g - sqrt(b^2 - 4*a*
c)*(e*f - d*g))/(c*d^2 - b*d*e + a*e^2))*arctan(sqrt(1/2)*x*sqrt(-((2*c*d
- b*e)*f - (b*d - 2*a*e)*g - sqrt(b^2 - 4*a*c)*(e*f - d*g))/(c*d^2 - b*d*e
 + a*e^2)))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3553 vs. $2(97) = 194$.

Time = 1.90 (sec) , antiderivative size = 3553, normalized size of antiderivative = 30.63

$$\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ac}e}{2cf-bg+\sqrt{b^2-4ac}g}-x^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/((2*c*d-b*e+(-4*a*c+b**2)**(1/2)*e)/(2*c*f-b*g+(-4*a*c+b**2)**
(1/2)*g)-x**2),x)
```

output
```
(-sqrt((b*g/16 - c*f/8 - g*sqrt(-4*a*c + b**2)/16)/(-3*a*b*c*e*g**2 + 2*a*
c**2*d*g**2 + 4*a*c**2*e*f*g + a*c*e*g**2*sqrt(-4*a*c + b**2) + b**3*e*g**
2 - b**2*c*d*g**2 - 2*b**2*c*e*f*g - b**2*e*g**2*sqrt(-4*a*c + b**2) + 2*b
*c**2*d*f*g + b*c**2*e*f**2 + b*c*d*g**2*sqrt(-4*a*c + b**2) + 2*b*c*e*f*g
*sqrt(-4*a*c + b**2) - 2*c**3*d*f**2 - 2*c**2*d*f*g*sqrt(-4*a*c + b**2) -
c**2*e*f**2*sqrt(-4*a*c + b**2)))*log(x - sqrt((b*g/16 - c*f/8 - g*sqrt(-4
*a*c + b**2)/16)/(-3*a*b*c*e*g**2 + 2*a*c**2*d*g**2 + 4*a*c**2*e*f*g + a*c
*e*g**2*sqrt(-4*a*c + b**2) + b**3*e*g**2 - b**2*c*d*g**2 - 2*b**2*c*e*f*g
 - b**2*e*g**2*sqrt(-4*a*c + b**2) + 2*b*c**2*d*f*g + b*c**2*e*f**2 + b*c*
d*g**2*sqrt(-4*a*c + b**2) + 2*b*c*e*f*g*sqrt(-4*a*c + b**2) - 2*c**3*d*f*
*2 - 2*c**2*d*f*g*sqrt(-4*a*c + b**2) - c**2*e*f**2*sqrt(-4*a*c + b**2)))*
(12*a*b*c*e*g**2/(2*a*c*g**2 - b**2*g**2 + 2*b*c*f*g + b*g**2*sqrt(-4*a*c
+ b**2) - 2*c**2*f**2 - 2*c*f*g*sqrt(-4*a*c + b**2)) - 8*a*c**2*d*g**2/(2*
a*c*g**2 - b**2*g**2 + 2*b*c*f*g + b*g**2*sqrt(-4*a*c + b**2) - 2*c**2*f**
2 - 2*c*f*g*sqrt(-4*a*c + b**2)) - 16*a*c**2*e*f*g/(2*a*c*g**2 - b**2*g**2
 + 2*b*c*f*g + b*g**2*sqrt(-4*a*c + b**2) - 2*c**2*f**2 - 2*c*f*g*sqrt(-4*
a*c + b**2)) - 4*a*c*e*g**2*sqrt(-4*a*c + b**2)/(2*a*c*g**2 - b**2*g**2 +
2*b*c*f*g + b*g**2*sqrt(-4*a*c + b**2) - 2*c**2*f**2 - 2*c*f*g*sqrt(-4*a*c
 + b**2)) - 4*b**3*e*g**2/(2*a*c*g**2 - b**2*g**2 + 2*b*c*f*g + b*g**2*sqr
t(-4*a*c + b**2) - 2*c**2*f**2 - 2*c*f*g*sqrt(-4*a*c + b**2)) + 4*b**2*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(100) = 200$.

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.91

$$\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ac}e}{2cf-bg+\sqrt{b^2-4ac}g}-x^2}\,dx = -\frac{\left(2\,cf-\left(b-\sqrt{b^2-4\,ac}\right)g\right)\log\left(\frac{\left(2\,cf-\left(b-\sqrt{b^2-4\,ac}\right)g\right)x-\sqrt{\left(2\,cd-\left(b-\sqrt{b^2-4\,ac}\right)e\right)\left(2\,cf-\left(b-\sqrt{b^2-4\,ac}\right)g\right)}}{\left(2\,cf-\left(b-\sqrt{b^2-4\,ac}\right)g\right)x+\sqrt{\left(2\,cd-\left(b-\sqrt{b^2-4\,ac}\right)e\right)\left(2\,cf-\left(b-\sqrt{b^2-4\,ac}\right)g\right)}}\right)}{2\sqrt{\left(2\,cd-\left(b-\sqrt{b^2-4\,ac}\right)e\right)\left(2\,cf-\left(b-\sqrt{b^2-4\,ac}\right)g\right)}}$$

input
```
integrate(1/((2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e)/(2*c*f-b*g+(-4*a*c+b^2)^(1/2
)*g)-x^2),x, algorithm="maxima")
```

output
```
-1/2*(2*c*f - (b - sqrt(b^2 - 4*a*c))*g)*log(((2*c*f - (b - sqrt(b^2 - 4*a
*c))*g)*x - sqrt((2*c*d - (b - sqrt(b^2 - 4*a*c))*e)*(2*c*f - (b - sqrt(b^
2 - 4*a*c))*g)))/((2*c*f - (b - sqrt(b^2 - 4*a*c))*g)*x + sqrt((2*c*d - (b
 - sqrt(b^2 - 4*a*c))*e)*(2*c*f - (b - sqrt(b^2 - 4*a*c))*g))))/sqrt((2*c*
d - (b - sqrt(b^2 - 4*a*c))*e)*(2*c*f - (b - sqrt(b^2 - 4*a*c))*g))
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ac}e}{2cf-bg+\sqrt{b^2-4ac}g}-x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e)/(2*c*f-b*g+(-4*a*c+b^2)^(1/2
)*g)-x^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.22

$$\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ac}e}{2cf-bg+\sqrt{b^2-4ac}g}-x^2}\,dx = -\frac{\operatorname{atan}\left(\frac{x\,\sqrt{2\,c\,f^2-2\,b\,f\,g+2\,a\,g^2}}{\sqrt{b\,d\,g-2\,a\,e\,g+b\,e\,f-2\,c\,d\,f+d\,g\,\sqrt{b^2-4\,a\,c}-e\,f\,\sqrt{b^2-4\,a\,c}}}\right)\,\sqrt{2\,c\,f^2-2\,b\,f\,g+2\,a\,g^2}}{\sqrt{b\,d\,g-2\,a\,e\,g+b\,e\,f-2\,c\,d\,f+d\,g\,\sqrt{b^2-4\,a\,c}-e\,f\,\sqrt{b^2-4\,a\,c}}}$$

input
```
int(-1/(x^2 - (2*c*d - b*e + e*(b^2 - 4*a*c)^(1/2))/(2*c*f - b*g + g*(b^2
- 4*a*c)^(1/2))),x)
```

output
```
-(atan((x*(2*a*g^2 + 2*c*f^2 - 2*b*f*g)^(1/2))/(b*d*g - 2*a*e*g + b*e*f -
2*c*d*f + d*g*(b^2 - 4*a*c)^(1/2) - e*f*(b^2 - 4*a*c)^(1/2))^(1/2))*(2*a*g
^2 + 2*c*f^2 - 2*b*f*g)^(1/2))/(b*d*g - 2*a*e*g + b*e*f - 2*c*d*f + d*g*(b
^2 - 4*a*c)^(1/2) - e*f*(b^2 - 4*a*c)^(1/2))^(1/2)
```

Reduce [F]

$$\int \frac{1}{\frac{2cd-be+\sqrt{b^2-4ac}e}{2cf-bg+\sqrt{b^2-4ac}g}-x^2}\,dx=\int \frac{1}{\frac{2cd-be+\sqrt{-4ac+b^2}\,e}{2cf-bg+\sqrt{-4ac+b^2}\,g}-x^2}dx$$

input
```
int(1/((2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e)/(2*c*f-b*g+(-4*a*c+b^2)^(1/2)*g)-x
^2),x)
```

output
```
int(1/((2*c*d-b*e+(-4*a*c+b^2)^(1/2)*e)/(2*c*f-b*g+(-4*a*c+b^2)^(1/2)*g)-x
^2),x)
```

3.30 $\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx$

Optimal result

Integrand size = 81, antiderivative size = 116

$$\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx = \frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,\text{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}x}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

output

```
(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)*arctanh((2*c*f-(b-(-4*a*c+b^2)^(1/2
))*g)^(1/2)*x/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2))/(2*c*d-(b-(-4*a*c+b^
2)^(1/2))*e)^(1/2)
```

Mathematica [F]

$$\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx$$
$$=\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx$$

input
```
Integrate[((2*c*d*f - b*e*f + Sqrt[b^2 - 4*a*c]*e*f - b*d*g - Sqrt[b^2 - 4
*a*c]*d*g + 2*a*e*g)/(2*c*f^2 - 2*b*f*g + 2*a*g^2) - x^2)^(-1),x]
```

output
```
Integrate[((2*c*d*f - b*e*f + Sqrt[b^2 - 4*a*c]*e*f - b*d*g - Sqrt[b^2 - 4
*a*c]*d*g + 2*a*e*g)/(2*c*f^2 - 2*b*f*g + 2*a*g^2) - x^2)^(-1), x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.26, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.012$, Rules used = $\{219\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{-dg\sqrt{b^2-4ac}+ef\sqrt{b^2-4ac}+2aeg-bdg-bef+2cdf}{2ag^2-2bfg+2cf^2}-x^2}\,dx$$
$$\Big\downarrow \; 219$$
$$\frac{\sqrt{2}\sqrt{ag^2-bfg+cf^2}\,\text{arctanh}\left(\frac{\sqrt{2}x\sqrt{ag^2-bfg+cf^2}}{\sqrt{\sqrt{b^2-4ac}(ef-dg)+2aeg-bdg-bef+2cdf}}\right)}{\sqrt{\sqrt{b^2-4ac}(ef-dg)+2aeg-bdg-bef+2cdf}}$$

input
```
Int[((2*c*d*f - b*e*f + Sqrt[b^2 - 4*a*c]*e*f - b*d*g - Sqrt[b^2 - 4*a*c]*
d*g + 2*a*e*g)/(2*c*f^2 - 2*b*f*g + 2*a*g^2) - x^2)^(-1),x]
```

output
```
(Sqrt[2]*Sqrt[c*f^2 - b*f*g + a*g^2]*ArcTanh[(Sqrt[2]*Sqrt[c*f^2 - b*f*g +
 a*g^2]*x)/Sqrt[2*c*d*f - b*e*f - b*d*g + 2*a*e*g + Sqrt[b^2 - 4*a*c]*(e*f
 - d*g)]])/Sqrt[2*c*d*f - b*e*f - b*d*g + 2*a*e*g + Sqrt[b^2 - 4*a*c]*(e*f
 - d*g)]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.54

method	result	size
default	$-\frac{(2a g^2-2bfg+2c f^2)\sqrt{2}\,\arctan\left(\frac{(2a g^2-2bfg+2c f^2)x\sqrt{2}}{2\sqrt{\left(\sqrt{-4ac+b^2}\,dg-\sqrt{-4ac+b^2}\,ef-2aeg+dbg+bef-2cdf\right)(a g^2-bfg+c f^2)}}\right)}{2\sqrt{\left(\sqrt{-4ac+b^2}\,dg-\sqrt{-4ac+b^2}\,ef-2aeg+dbg+bef-2cdf\right)(a g^2-bfg+c f^2)}}$	179

input
```
int(1/((2*c*d*f-b*e*f+(-4*a*c+b^2)^(1/2)*e*f-d*b*g-(-4*a*c+b^2)^(1/2)*d*g+
2*a*e*g)/(2*a*g^2-2*b*f*g+2*c*f^2)-x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*a*g^2-2*b*f*g+2*c*f^2)*2^(1/2)/(((-4*a*c+b^2)^(1/2)*d*g-(-4*a*c+b^
2)^(1/2)*e*f-2*a*e*g+d*b*g+b*e*f-2*c*d*f)*(a*g^2-b*f*g+c*f^2))^(1/2)*arcta
n(1/2*(2*a*g^2-2*b*f*g+2*c*f^2)*x*2^(1/2)/(((-4*a*c+b^2)^(1/2)*d*g-(-4*a*c
+b^2)^(1/2)*e*f-2*a*e*g+d*b*g+b*e*f-2*c*d*f)*(a*g^2-b*f*g+c*f^2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.09

$$\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx$$
$$= \left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{(2\,cd-be)f-(bd-2\,ae)g-\sqrt{b^2-4\,ac}(ef-dg)}{cd^2-bde+ae^2}}\log\left(\frac{(cf^2-bfg+ag^2)x^4+\sqrt{b^2-4\,ac}(e\right.\right.$$
$$-\sqrt{\frac{1}{2}}\sqrt{-\frac{(2\,cd-be)f-(bd-2\,ae)g-\sqrt{b^2-4\,ac}(ef-dg)}{cd^2-bde+ae^2}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{-\frac{(2\,cd-be)f-(bd-2}{cd^2-}\right.\right.$$

input
```
integrate(1/((2*c*d*f-b*e*f+(-4*a*c+b^2)^(1/2)*e*f-b*d*g-(-4*a*c+b^2)^(1/2
)*d*g+2*a*e*g)/(2*a*g^2-2*b*f*g+2*c*f^2)-x^2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/2)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - sqrt(b^2 - 4*a*c)
*(e*f - d*g))/(c*d^2 - b*d*e + a*e^2))*log(((c*f^2 - b*f*g + a*g^2)*x^4 +
sqrt(b^2 - 4*a*c)*(e*f - d*g)*x^2 - c*d^2 + b*d*e - a*e^2 + sqrt(1/2)*(sqr
t(b^2 - 4*a*c)*(e*f - d*g)*x^3 + ((2*c*d - b*e)*f - (b*d - 2*a*e)*g)*x^3 -
 2*(c*d^2 - b*d*e + a*e^2)*x)*sqrt(((2*c*d - b*e)*f - (b*d - 2*a*e)*g - sq
rt(b^2 - 4*a*c)*(e*f - d*g))/(c*d^2 - b*d*e + a*e^2)))/((c*f^2 - b*f*g + a
*g^2)*x^4 + c*d^2 - b*d*e + a*e^2 - ((2*c*d - b*e)*f - (b*d - 2*a*e)*g)*x^
2)), -sqrt(1/2)*sqrt(-((2*c*d - b*e)*f - (b*d - 2*a*e)*g - sqrt(b^2 - 4*a*
c)*(e*f - d*g))/(c*d^2 - b*d*e + a*e^2))*arctan(sqrt(1/2)*x*sqrt(-((2*c*d
- b*e)*f - (b*d - 2*a*e)*g - sqrt(b^2 - 4*a*c)*(e*f - d*g))/(c*d^2 - b*d*e
 + a*e^2)))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(97) = 194$.

Time = 1.43 (sec) , antiderivative size = 1093, normalized size of antiderivative = 9.42

$$\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/((2*c*d*f-b*e*f+(-4*a*c+b**2)**(1/2)*e*f-b*d*g-(-4*a*c+b**2)**
(1/2)*d*g+2*a*e*g)/(2*a*g**2-2*b*f*g+2*c*f**2)-x**2),x)
```

output
```
(-sqrt(2)*sqrt(1/(2*a**2*e*g**3 - a*b*d*g**3 - 3*a*b*e*f*g**2 + 2*a*c*d*f*
g**2 + 2*a*c*e*f**2*g - a*d*g**3*sqrt(-4*a*c + b**2) + a*e*f*g**2*sqrt(-4*
a*c + b**2) + b**2*d*f*g**2 + b**2*e*f**2*g - 3*b*c*d*f**2*g - b*c*e*f**3
+ b*d*f*g**2*sqrt(-4*a*c + b**2) - b*e*f**2*g*sqrt(-4*a*c + b**2) + 2*c**2
*d*f**3 - c*d*f**2*g*sqrt(-4*a*c + b**2) + c*e*f**3*sqrt(-4*a*c + b**2)))*
log(x - sqrt(2)*(-4*a*e*g + 2*b*d*g + 2*b*e*f - 4*c*d*f + 2*d*g*sqrt(-4*a*
c + b**2) - 2*e*f*sqrt(-4*a*c + b**2))*sqrt(1/(2*a**2*e*g**3 - a*b*d*g**3
- 3*a*b*e*f*g**2 + 2*a*c*d*f*g**2 + 2*a*c*e*f**2*g - a*d*g**3*sqrt(-4*a*c
+ b**2) + a*e*f*g**2*sqrt(-4*a*c + b**2) + b**2*d*f*g**2 + b**2*e*f**2*g -
 3*b*c*d*f**2*g - b*c*e*f**3 + b*d*f*g**2*sqrt(-4*a*c + b**2) - b*e*f**2*g
*sqrt(-4*a*c + b**2) + 2*c**2*d*f**3 - c*d*f**2*g*sqrt(-4*a*c + b**2) + c*
e*f**3*sqrt(-4*a*c + b**2)))/4)/4 + sqrt(2)*sqrt(1/(2*a**2*e*g**3 - a*b*d*
g**3 - 3*a*b*e*f*g**2 + 2*a*c*d*f*g**2 + 2*a*c*e*f**2*g - a*d*g**3*sqrt(-4
*a*c + b**2) + a*e*f*g**2*sqrt(-4*a*c + b**2) + b**2*d*f*g**2 + b**2*e*f**
2*g - 3*b*c*d*f**2*g - b*c*e*f**3 + b*d*f*g**2*sqrt(-4*a*c + b**2) - b*e*f
**2*g*sqrt(-4*a*c + b**2) + 2*c**2*d*f**3 - c*d*f**2*g*sqrt(-4*a*c + b**2)
 + c*e*f**3*sqrt(-4*a*c + b**2)))*log(x + sqrt(2)*(-4*a*e*g + 2*b*d*g + 2*
b*e*f - 4*c*d*f + 2*d*g*sqrt(-4*a*c + b**2) - 2*e*f*sqrt(-4*a*c + b**2))*s
qrt(1/(2*a**2*e*g**3 - a*b*d*g**3 - 3*a*b*e*f*g**2 + 2*a*c*d*f*g**2 + 2*a*
c*e*f**2*g - a*d*g**3*sqrt(-4*a*c + b**2) + a*e*f*g**2*sqrt(-4*a*c + b*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(100) = 200$.

Time = 0.18 (sec) , antiderivative size = 562, normalized size of antiderivative = 4.84

$$\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx =$$

$$-\frac{(cf^2-bfg+ag^2)\log\left(\frac{2\,(cf^2-bfg+ag^2)x-\sqrt{2\left(2\,c^2d-\left(bc-\sqrt{b^2-4\,ac}c\right)e\right)f^3-2\left(\left(3\,bc+\sqrt{b^2-4\,ac}c\right)d-\left(b^2+2\,\cdots\right.\right.}}{2\,(cf^2-bfg+ag^2)x+\sqrt{2\left(2\,c^2d-\left(bc-\sqrt{b^2-4\,ac}c\right)e\right)f^3-2\left(\left(3\,bc+\sqrt{b^2-4\,ac}c\right)d-\left(b^2+2\,\cdots\right.\right.}}\right.}{\sqrt{2\,(2\,c^2d-(bc-\sqrt{b^2-4\,ac}c)e)f^3-2\,((3\,bc+\sqrt{b^2-4\,ac}c)d-(b^2+2\,ac-\sqrt{b^2-4\,ac}b)e)f^2g+\cdots}}$$

input
```
integrate(1/((2*c*d*f-b*e*f+(-4*a*c+b^2)^(1/2)*e*f-b*d*g-(-4*a*c+b^2)^(1/2
)*d*g+2*a*e*g)/(2*a*g^2-2*b*f*g+2*c*f^2)-x^2),x, algorithm="maxima")
```

output
```
-(c*f^2 - b*f*g + a*g^2)*log((2*(c*f^2 - b*f*g + a*g^2)*x - sqrt(2*(2*c^2*
d - (b*c - sqrt(b^2 - 4*a*c)*c)*e)*f^3 - 2*((3*b*c + sqrt(b^2 - 4*a*c)*c)*
d - (b^2 + 2*a*c - sqrt(b^2 - 4*a*c)*b)*e)*f^2*g + 2*((b^2 + 2*a*c + sqrt(
b^2 - 4*a*c)*b)*d - (3*a*b - sqrt(b^2 - 4*a*c)*a)*e)*f*g^2 + 2*(2*a^2*e -
(a*b + sqrt(b^2 - 4*a*c)*a)*d)*g^3))/(2*(c*f^2 - b*f*g + a*g^2)*x + sqrt(2
*(2*c^2*d - (b*c - sqrt(b^2 - 4*a*c)*c)*e)*f^3 - 2*((3*b*c + sqrt(b^2 - 4*
a*c)*c)*d - (b^2 + 2*a*c - sqrt(b^2 - 4*a*c)*b)*e)*f^2*g + 2*((b^2 + 2*a*c
 + sqrt(b^2 - 4*a*c)*b)*d - (3*a*b - sqrt(b^2 - 4*a*c)*a)*e)*f*g^2 + 2*(2*
a^2*e - (a*b + sqrt(b^2 - 4*a*c)*a)*d)*g^3)))/sqrt(2*(2*c^2*d - (b*c - sqr
t(b^2 - 4*a*c)*c)*e)*f^3 - 2*((3*b*c + sqrt(b^2 - 4*a*c)*c)*d - (b^2 + 2*a
*c - sqrt(b^2 - 4*a*c)*b)*e)*f^2*g + 2*((b^2 + 2*a*c + sqrt(b^2 - 4*a*c)*b
)*d - (3*a*b - sqrt(b^2 - 4*a*c)*a)*e)*f*g^2 + 2*(2*a^2*e - (a*b + sqrt(b^
2 - 4*a*c)*a)*d)*g^3)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/((2*c*d*f-b*e*f+(-4*a*c+b^2)^(1/2)*e*f-b*d*g-(-4*a*c+b^2)^(1/2
)*d*g+2*a*e*g)/(2*a*g^2-2*b*f*g+2*c*f^2)-x^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.22

$$\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{2\,c\,f^2-2\,b\,f\,g+2\,a\,g^2}}{\sqrt{b\,d\,g-2\,a\,e\,g+b\,e\,f-2\,c\,d\,f+d\,g\,\sqrt{b^2-4\,a\,c}-e\,f\,\sqrt{b^2-4\,a\,c}}}\right)\sqrt{2\,c\,f^2-2\,b\,f\,g+2\,a\,g^2}}{\sqrt{b\,d\,g-2\,a\,e\,g+b\,e\,f-2\,c\,d\,f+d\,g\,\sqrt{b^2-4\,a\,c}-e\,f\,\sqrt{b^2-4\,a\,c}}}$$

input
```
int(-1/(x^2 - (2*a*e*g - b*d*g - b*e*f + 2*c*d*f - d*g*(b^2 - 4*a*c)^(1/2)
 + e*f*(b^2 - 4*a*c)^(1/2))/(2*a*g^2 + 2*c*f^2 - 2*b*f*g)),x)
```

output
```
-(atan((x*(2*a*g^2 + 2*c*f^2 - 2*b*f*g)^(1/2))/(b*d*g - 2*a*e*g + b*e*f -
2*c*d*f + d*g*(b^2 - 4*a*c)^(1/2) - e*f*(b^2 - 4*a*c)^(1/2))^(1/2))*(2*a*g
^2 + 2*c*f^2 - 2*b*f*g)^(1/2))/(b*d*g - 2*a*e*g + b*e*f - 2*c*d*f + d*g*(b
^2 - 4*a*c)^(1/2) - e*f*(b^2 - 4*a*c)^(1/2))^(1/2)
```

Reduce [F]

$$\int \frac{1}{\frac{2cdf-bef+\sqrt{b^2-4ac}ef-bdg-\sqrt{b^2-4ac}dg+2aeg}{2cf^2-2bfg+2ag^2}-x^2}\,dx = \int \frac{1}{\frac{2cdf-bef+\sqrt{-4ac+b^2}\,ef-bdg-\sqrt{-4ac+b^2}\,dg+2aeg}{2a\,g^2-2bfg+2c\,f^2}-x^2}dx$$

input
```
int(1/((2*c*d*f-b*e*f+(-4*a*c+b^2)^(1/2)*e*f-b*d*g-(-4*a*c+b^2)^(1/2)*d*g+
2*a*e*g)/(2*a*g^2-2*b*f*g+2*c*f^2)-x^2),x)
```

output
```
int(1/((2*c*d*f-b*e*f+(-4*a*c+b^2)^(1/2)*e*f-b*d*g-(-4*a*c+b^2)^(1/2)*d*g+
2*a*e*g)/(2*a*g^2-2*b*f*g+2*c*f^2)-x^2),x)
```

3.31 $\int -\frac{1}{(3-2\sqrt{3})(2+\sqrt{3}+4x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 34

$$\int -\frac{1}{(3-2\sqrt{3})\left(2+\sqrt{3}+4x^2\right)}\,dx = \frac{1}{2}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)$$

output
```
1/2*(1/2*2^(1/2)+1/6*6^(1/2))*arctan(2*x/(1/2*6^(1/2)+1/2*2^(1/2)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int -\frac{1}{(3-2\sqrt{3})\left(2+\sqrt{3}+4x^2\right)}\,dx = \frac{\left(1+\frac{2}{\sqrt{3}}\right)\arctan\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{2+\sqrt{3}}}$$

input
```
Integrate[-(1/((3 - 2*Sqrt[3])*(2 + Sqrt[3] + 4*x^2))),x]
```

output
```
((1 + 2/Sqrt[3])*ArcTan[(2*x)/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[2 + Sqrt[3]])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {25, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(4x^2+\sqrt{3}+2\right)}dx$$
$$\Big\downarrow \text{25}$$
$$-\int -\frac{3+2\sqrt{3}}{3\left(4x^2+\sqrt{3}+2\right)}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{3}\left(3+2\sqrt{3}\right)\int \frac{1}{4x^2+\sqrt{3}+2}dx$$
$$\Big\downarrow \text{216}$$
$$\frac{\left(3+2\sqrt{3}\right)\arctan\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[-(1/((3 - 2*Sqrt[3])*(2 + Sqrt[3] + 4*x^2))),x]
```

output
```
((3 + 2*Sqrt[3])*ArcTan[(2*x)/Sqrt[2 + Sqrt[3]]])/(6*Sqrt[2 + Sqrt[3]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
meijerg	$-\frac{\arctan\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)}{2\left(3-2\sqrt{3}\right)\sqrt{2+\sqrt{3}}}$	30
default	$-\frac{2\arctan\left(\frac{8x}{2\sqrt{6}+2\sqrt{2}}\right)}{\left(3-2\sqrt{3}\right)\left(2\sqrt{6}+2\sqrt{2}\right)}$	42

input
```
int(-1/(3-2*3^(1/2))/(2+3^(1/2)+4*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(3-2*3^(1/2))/(2+3^(1/2))^(1/2)*arctan(2*x/(2+3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(2+\sqrt{3}+4x^2\right)}\,dx$$
$$=\frac{1}{2}\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\arctan\left(2\left(2\sqrt{3}x-3x\right)\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\right)$$

input
```
integrate(-1/(3-2*3^(1/2))/(2+3^(1/2)+4*x^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/3*sqrt(3) + 2/3)*arctan(2*(2*sqrt(3)*x - 3*x)*sqrt(1/3*sqrt(3)
+ 2/3))
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(2+\sqrt{3}+4x^2\right)}\,dx = 2\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\operatorname{atan}\left(\frac{2x}{\sqrt{\sqrt{3}+2}}\right)$$

input
```
integrate(-1/(3-2*3**(1/2))/(2+3**(1/2)+4*x**2),x)
```

output
```
2*sqrt(sqrt(3)/48 + 1/24)*atan(2*x/sqrt(sqrt(3) + 2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(2+\sqrt{3}+4x^2\right)}\,dx = \frac{\arctan\left(\frac{2x}{\sqrt{\sqrt{3}+2}}\right)}{2\left(2\sqrt{3}-3\right)\sqrt{\sqrt{3}+2}}$$

input
```
integrate(-1/(3-2*3^(1/2))/(2+3^(1/2)+4*x^2),x, algorithm="maxima")
```

output
```
1/2*arctan(2*x/sqrt(sqrt(3) + 2))/((2*sqrt(3) - 3)*sqrt(sqrt(3) + 2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(2+\sqrt{3}+4x^2\right)}\,dx = \frac{\left(\sqrt{6}-\sqrt{2}\right)\arctan\left(\frac{4x}{\sqrt{6}+\sqrt{2}}\right)}{4\left(2\sqrt{3}-3\right)}$$

input
```
integrate(-1/(3-2*3^(1/2))/(2+3^(1/2)+4*x^2),x, algorithm="giac")
```

output
```
1/4*(sqrt(6) - sqrt(2))*arctan(4*x/(sqrt(6) + sqrt(2)))/(2*sqrt(3) - 3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(2+\sqrt{3}+4x^2\right)}\,dx=\frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{\sqrt{3}+2}}\right)}{2\,\sqrt{\sqrt{3}+2}}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\,x}{\sqrt{\sqrt{3}+2}}\right)}{3\,\sqrt{\sqrt{3}+2}}$$

input
```
int(1/((2*3^(1/2) - 3)*(3^(1/2) + 4*x^2 + 2)),x)
```

output
```
atan((2*x)/(3^(1/2) + 2)^(1/2))/(2*(3^(1/2) + 2)^(1/2)) + (3^(1/2)*atan((2
*x)/(3^(1/2) + 2)^(1/2)))/(3*(3^(1/2) + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(2+\sqrt{3}+4x^2\right)}\,dx=\frac{\sqrt{2}\,atan\left(\frac{4x}{\sqrt{6}+\sqrt{2}}\right)\left(\sqrt{3}+3\right)}{12}$$

input
```
int(-1/(3-2*3^(1/2))/(2+3^(1/2)+4*x^2),x)
```

output
```
(sqrt(2)*atan((4*x)/(sqrt(6) + sqrt(2)))*(sqrt(3) + 3))/12
```

3.32 $\int \frac{1}{\sqrt{3}(1+4(2-\sqrt{3})x^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{1}{\sqrt{3}\left(1+4\left(2-\sqrt{3}\right)x^2\right)}\,dx=\frac{1}{2}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)$$

output
```
1/2*(1/2*2^(1/2)+1/6*6^(1/2))*arctan(2*x/(1/2*6^(1/2)+1/2*2^(1/2)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{3}\left(1+4\left(2-\sqrt{3}\right)x^2\right)}\,dx=\frac{\arctan\left(2\sqrt{2-\sqrt{3}}x\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}$$

input
```
Integrate[1/(Sqrt[3]*(1 + 4*(2 - Sqrt[3])*x^2)),x]
```

output
```
ArcTan[2*Sqrt[2 - Sqrt[3]]*x]/(2*Sqrt[3*(2 - Sqrt[3])])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3}\left(4\left(2-\sqrt{3}\right)x^2+1\right)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{1}{4\left(2-\sqrt{3}\right)x^2+1}dx}{\sqrt{3}}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(2\sqrt{2-\sqrt{3}}x\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}$$

input
```
Int[1/(Sqrt[3]*(1 + 4*(2 - Sqrt[3])*x^2)),x]
```

output
```
ArcTan[2*Sqrt[2 - Sqrt[3]]*x]/(2*Sqrt[3*(2 - Sqrt[3])])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
meijerg	$\frac{\sqrt{3}\,\arctan\left(2x\sqrt{2-\sqrt{3}}\right)}{6\sqrt{2-\sqrt{3}}}$	28
default	$-\frac{2\sqrt{3}\,\arctan\left(\frac{8x}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(-2+\sqrt{3}\right)\left(2\sqrt{6}+2\sqrt{2}\right)}$	43

input
```
int(1/3*3^(1/2)/(1+4*(2-3^(1/2))*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)/(2-3^(1/2))^(1/2)*arctan(2*x*(2-3^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{3}\left(1+4\left(2-\sqrt{3}\right)x^2\right)}\,dx = \frac{1}{2}\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\arctan\left(2\left(2\sqrt{3}x-3\,x\right)\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\right)$$

input
```
integrate(1/3*3^(1/2)/(1+4*(2-3^(1/2))*x^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/3*sqrt(3) + 2/3)*arctan(2*(2*sqrt(3)*x - 3*x)*sqrt(1/3*sqrt(3)
+ 2/3))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(31) = 62.

Time = 0.59 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{3}\left(1+4\left(2-\sqrt{3}\right)x^2\right)}\,dx = \sqrt{3}\left(-\sqrt{-\frac{1}{72}-\frac{\sqrt{3}}{144}}\log\left(x-6\sqrt{-\frac{1}{72}-\frac{\sqrt{3}}{144}}\right)+\sqrt{-\frac{1}{72}-\frac{\sqrt{3}}{144}}\log\left(x+6\sqrt{-\frac{1}{72}-\frac{\sqrt{3}}{144}}\right)\right)$$

input
```
integrate(1/3*3**(1/2)/(1+4*(2-3**(1/2))*x**2),x)
```

output
```
sqrt(3)*(-sqrt(-1/72 - sqrt(3)/144)*log(x - 6*sqrt(-1/72 - sqrt(3)/144)) +
 sqrt(-1/72 - sqrt(3)/144)*log(x + 6*sqrt(-1/72 - sqrt(3)/144)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{3}\left(1+4\left(2-\sqrt{3}\right)x^2\right)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{2x\left(\sqrt{3}-2\right)}{\sqrt{-\sqrt{3}+2}}\right)}{6\sqrt{-\sqrt{3}+2}}$$

input
```
integrate(1/3*3^(1/2)/(1+4*(2-3^(1/2))*x^2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*arctan(2*x*(sqrt(3) - 2)/sqrt(-sqrt(3) + 2))/sqrt(-sqrt(3) +
2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{3}\left(1+4\left(2-\sqrt{3}\right)x^2\right)}\,dx = \frac{1}{12}\sqrt{3}\left(\sqrt{6}+\sqrt{2}\right)\arctan\left(\frac{4\,x}{\sqrt{6}+\sqrt{2}}\right)$$

input
```
integrate(1/3*3^(1/2)/(1+4*(2-3^(1/2))*x^2),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*(sqrt(6) + sqrt(2))*arctan(4*x/(sqrt(6) + sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{3}\left(1+4\left(2-\sqrt{3}\right)x^2\right)}\,dx = \frac{\sqrt{3}\,\mathrm{atan}\left(x\sqrt{8-4\sqrt{3}}\right)}{3\sqrt{8-4\sqrt{3}}}$$

input
```
int(-3^(1/2)/(3*(4*x^2*(3^(1/2) - 2) - 1)),x)
```

output
```
(3^(1/2)*atan(x*(8 - 4*3^(1/2))^(1/2)))/(3*(8 - 4*3^(1/2))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{3}\left(1+4\left(2-\sqrt{3}\right)x^2\right)}\,dx = \frac{\sqrt{2}\,atan\left(\frac{4x}{\sqrt{6}+\sqrt{2}}\right)\left(\sqrt{3}+3\right)}{12}$$

input
```
int(1/3*3^(1/2)/(1+4*(2-3^(1/2))*x^2),x)
```

output
```
(sqrt(2)*atan((4*x)/(sqrt(6) + sqrt(2)))*(sqrt(3) + 3))/12
```

3.33 $\int \frac{1}{\sqrt{3}-4(3-2\sqrt{3})x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx=\frac{1}{2}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\arctan\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)$$

output
```
1/2*(1/2*2^(1/2)+1/6*6^(1/2))*arctan(2*x/(1/2*6^(1/2)+1/2*2^(1/2)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx=\frac{\arctan\left(\frac{2\sqrt{-3+2\sqrt{3}}x}{\sqrt[4]{3}}\right)}{2\sqrt[4]{3}\sqrt{-3+2\sqrt{3}}}$$

input
```
Integrate[(Sqrt[3] - 4*(3 - 2*Sqrt[3])*x^2)^(-1),x]
```

output
```
ArcTan[(2*Sqrt[-3 + 2*Sqrt[3]]*x)/3^(1/4)]/(2*3^(1/4)*Sqrt[-3 + 2*Sqrt[3]]
)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\frac{2\sqrt{2\sqrt{3}-3}x}{\sqrt[4]{3}}\right)}{2\sqrt[4]{3}\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(Sqrt[3] - 4*(3 - 2*Sqrt[3])*x^2)^(-1),x]
```

output
```
ArcTan[(2*Sqrt[-3 + 2*Sqrt[3]]*x)/3^(1/4)]/(2*3^(1/4)*Sqrt[-3 + 2*Sqrt[3]]
)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
meijerg	$\frac{3^{\frac{3}{4}} \arctan\left(\frac{2x 3^{\frac{3}{4}} \sqrt{-3+2\sqrt{3}}}{3}\right)}{6\sqrt{-3+2\sqrt{3}}}$	31
default	$\frac{2\arctan\left(\frac{8x}{2\sqrt{6}+2\sqrt{2}}\right)}{\left(-3+2\sqrt{3}\right)\left(2\sqrt{6}+2\sqrt{2}\right)}$	42

input
```
int(1/(3^(1/2)-4*(3-2*3^(1/2))*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(3/4)/(-3+2*3^(1/2))^(1/2)*arctan(2/3*x*3^(3/4)*(-3+2*3^(1/2))^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx = \frac{1}{2}\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\arctan\left(2\left(2\sqrt{3}x-3x\right)\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\right)$$

input
```
integrate(1/(3^(1/2)-4*(3-2*3^(1/2))*x^2),x, algorithm="fricas")
```

output
```
1/2*sqrt(1/3*sqrt(3) + 2/3)*arctan(2*(2*sqrt(3)*x - 3*x)*sqrt(1/3*sqrt(3)
+ 2/3))
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx = 2\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\operatorname{atan}\left(\frac{2x}{\sqrt{\sqrt{3}+2}}\right)$$

input
```
integrate(1/(3**(1/2)-4*(3-2*3**(1/2))*x**2),x)
```

output
```
2*sqrt(sqrt(3)/48 + 1/24)*atan(2*x/sqrt(sqrt(3) + 2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx = \frac{3^{\frac{3}{4}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{3}{4}}x\sqrt{2\sqrt{3}-3}\right)}{6\sqrt{2\sqrt{3}-3}}$$

input
```
integrate(1/(3^(1/2)-4*(3-2*3^(1/2))*x^2),x, algorithm="maxima")
```

output
```
1/6*3^(3/4)*arctan(2/3*3^(3/4)*x*sqrt(2*sqrt(3) - 3))/sqrt(2*sqrt(3) - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx = \frac{1}{12}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x}{\sqrt{6}+\sqrt{2}}\right)$$

input
```
integrate(1/(3^(1/2)-4*(3-2*3^(1/2))*x^2),x, algorithm="giac")
```

output
```
1/12*(sqrt(6) + 3*sqrt(2))*arctan(4*x/(sqrt(6) + sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx=\frac{\mathrm{atan}\left(\frac{3^{3/4}\,x\,\sqrt{8\,\sqrt{3}-12}}{3}\right)}{\sqrt{24-12\,\sqrt{3}}}$$

input
```
int(1/(4*x^2*(2*3^(1/2) - 3) + 3^(1/2)),x)
```

output
```
atan((3^(3/4)*x*(8*3^(1/2) - 12)^(1/2))/3)/(24 - 12*3^(1/2))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{3}-4\left(3-2\sqrt{3}\right)x^2}\,dx=\frac{\sqrt{2}\,atan\left(\frac{4x}{\sqrt{6}+\sqrt{2}}\right)\left(\sqrt{3}+3\right)}{12}$$

input
```
int(1/(3^(1/2)-4*(3-2*3^(1/2))*x^2),x)
```

output
```
(sqrt(2)*atan((4*x)/(sqrt(6) + sqrt(2)))*(sqrt(3) + 3))/12
```

3.34 $\int -\frac{1}{(3-2\sqrt{3})(2+\sqrt{3}-4x^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 34

$$\int -\frac{1}{(3-2\sqrt{3})\,(2+\sqrt{3}-4x^2)}\,dx = \frac{1}{2}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)$$

output
```
1/2*(1/2*2^(1/2)+1/6*6^(1/2))*arctanh(2*x/(1/2*6^(1/2)+1/2*2^(1/2)))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int -\frac{1}{(3-2\sqrt{3})\,(2+\sqrt{3}-4x^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{-3+2\sqrt{3}}x}{\sqrt[4]{3}}\right)}{2\sqrt[4]{3}\sqrt{-3+2\sqrt{3}}}$$

input
```
Integrate[-(1/((3 - 2*Sqrt[3])*(2 + Sqrt[3] - 4*x^2))),x]
```

output
```
ArcTanh[(2*Sqrt[-3 + 2*Sqrt[3]]*x)/3^(1/4)]/(2*3^(1/4)*Sqrt[-3 + 2*Sqrt[3]
])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {25, 27, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(-4x^2+\sqrt{3}+2\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int -\frac{3+2\sqrt{3}}{3\left(-4x^2+\sqrt{3}+2\right)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(3+2\sqrt{3}\right)\int \frac{1}{-4x^2+\sqrt{3}+2}dx$$

$$\Big\downarrow \ 219$$

$$\frac{\left(3+2\sqrt{3}\right)\text{arctanh}\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[-(1/((3 - 2*Sqrt[3])*(2 + Sqrt[3] - 4*x^2))),x]
```

output
```
((3 + 2*Sqrt[3])*ArcTanh[(2*x)/Sqrt[2 + Sqrt[3]]])/(6*Sqrt[2 + Sqrt[3]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
meijerg	$-\frac{\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)}{2\left(3-2\sqrt{3}\right)\sqrt{2+\sqrt{3}}}$	30
default	$-\frac{2\operatorname{arctanh}\left(\frac{8x}{2\sqrt{6}+2\sqrt{2}}\right)}{\left(3-2\sqrt{3}\right)\left(2\sqrt{6}+2\sqrt{2}\right)}$	42

input
```
int(-1/(3-2*3^(1/2))/(2+3^(1/2)-4*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(3-2*3^(1/2))/(2+3^(1/2))^(1/2)*arctanh(2*x/(2+3^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(2+\sqrt{3}-4x^2\right)}\,dx$$
$$=\frac{1}{4}\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\log\left(\frac{16\,x^4+8\,\sqrt{3}x^2+4\left(2\,\sqrt{3}(2\,x^3-x)+3\,x\right)\sqrt{\frac{1}{3}\,\sqrt{3}+\frac{2}{3}}-1}{16\,x^4-16\,x^2+1}\right)$$

input
```
integrate(-1/(3-2*3^(1/2))/(2+3^(1/2)-4*x^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/3*sqrt(3) + 2/3)*log((16*x^4 + 8*sqrt(3)*x^2 + 4*(2*sqrt(3)*(2*
x^3 - x) + 3*x)*sqrt(1/3*sqrt(3) + 2/3) - 1)/(16*x^4 - 16*x^2 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(31) = 62$.

Time = 0.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.24

$$\int -\frac{1}{(3-2\sqrt{3})\left(2+\sqrt{3}-4x^2\right)}\,dx = -\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\log\left(x-\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\cdot\left(\frac{24}{-7+4\sqrt{3}}-\frac{14\sqrt{3}}{-7+4\sqrt{3}}\right)\right) + \sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\log\left(x+\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\cdot\left(\frac{24}{-7+4\sqrt{3}}-\frac{14\sqrt{3}}{-7+4\sqrt{3}}\right)\right)$$

input
```
integrate(-1/(3-2*3**(1/2))/(2+3**(1/2)-4*x**2),x)
```

output
```
-sqrt(sqrt(3)/48 + 1/24)*log(x - sqrt(sqrt(3)/48 + 1/24)*(24/(-7 + 4*sqrt(
3)) - 14*sqrt(3)/(-7 + 4*sqrt(3)))) + sqrt(sqrt(3)/48 + 1/24)*log(x + sqrt
(sqrt(3)/48 + 1/24)*(24/(-7 + 4*sqrt(3)) - 14*sqrt(3)/(-7 + 4*sqrt(3))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int -\frac{1}{(3-2\sqrt{3})\left(2+\sqrt{3}-4x^2\right)}\,dx = -\frac{\log\left(\frac{2x-\sqrt{\sqrt{3}+2}}{2x+\sqrt{\sqrt{3}+2}}\right)}{4\left(2\sqrt{3}-3\right)\sqrt{\sqrt{3}+2}}$$

input
```
integrate(-1/(3-2*3^(1/2))/(2+3^(1/2)-4*x^2),x, algorithm="maxima")
```

output
```
-1/4*log((2*x - sqrt(sqrt(3) + 2))/(2*x + sqrt(sqrt(3) + 2)))/((2*sqrt(3)
- 3)*sqrt(sqrt(3) + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(24) = 48$.

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int -\frac{1}{(3-2\sqrt{3})\left(2+\sqrt{3}-4x^2\right)}\,dx = \frac{\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|x+\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}\right|\right)-\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|x-\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}\right|\right)}{8\left(2\sqrt{3}-3\right)}$$

input
```
integrate(-1/(3-2*3^(1/2))/(2+3^(1/2)-4*x^2),x, algorithm="giac")
```

output
```
1/8*((sqrt(6) - sqrt(2))*log(abs(x + 1/4*sqrt(6) + 1/4*sqrt(2))) - (sqrt(6
) - sqrt(2))*log(abs(x - 1/4*sqrt(6) - 1/4*sqrt(2))))/(2*sqrt(3) - 3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int -\frac{1}{(3-2\sqrt{3})\left(2+\sqrt{3}-4x^2\right)}\,dx = \frac{\operatorname{atanh}\left(\frac{x\left(2\sqrt{3}+3\right)}{3\left(\frac{\sqrt{3}}{3}+\frac{1}{2}\right)\sqrt{\sqrt{3}+2}}\right)\left(2\sqrt{3}+3\right)}{6\sqrt{\sqrt{3}+2}}$$

input
```
int(1/((2*3^(1/2) - 3)*(3^(1/2) - 4*x^2 + 2)),x)
```

output
```
(atanh((x*(2*3^(1/2) + 3))/(3*(3^(1/2)/3 + 1/2)*(3^(1/2) + 2)^(1/2)))*(2*3
^(1/2) + 3))/(6*(3^(1/2) + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int -\frac{1}{\left(3-2\sqrt{3}\right)\left(2+\sqrt{3}-4x^2\right)}\,dx$$
$$= \frac{\sqrt{2}\left(-\sqrt{3}\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+2x\right)+\sqrt{3}\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+2x\right)-3\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+2x\right)+3\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+\right.\right.}{24}$$

input
```
int(-1/(3-2*3^(1/2))/(2+3^(1/2)-4*x^2),x)
```

output
```
(sqrt(2)*( - sqrt(3)*log(( - sqrt(6) - sqrt(2) + 4*x)/2) + sqrt(3)*log((sq
rt(6) + sqrt(2) + 4*x)/2) - 3*log(( - sqrt(6) - sqrt(2) + 4*x)/2) + 3*log(
(sqrt(6) + sqrt(2) + 4*x)/2)))/24
```

3.35 $\int \frac{1}{\sqrt{3}(1-4(2-\sqrt{3})x^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}\,dx=\frac{1}{2}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)$$

output
```
1/2*(1/2*2^(1/2)+1/6*6^(1/2))*arctanh(2*x/(1/2*6^(1/2)+1/2*2^(1/2)))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{-3+2\sqrt{3}}x}{\sqrt[4]{3}}\right)}{2\sqrt[4]{3}\sqrt{-3+2\sqrt{3}}}$$

input
```
Integrate[1/(Sqrt[3]*(1 - 4*(2 - Sqrt[3])*x^2)),x]
```

output
```
ArcTanh[(2*Sqrt[-3 + 2*Sqrt[3]]*x)/3^(1/4)]/(2*3^(1/4)*Sqrt[-3 + 2*Sqrt[3]
])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {27, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{1-4\left(2-\sqrt{3}\right)x^2}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(2\sqrt{2-\sqrt{3}}x\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}$$

input
```
Int[1/(Sqrt[3]*(1 - 4*(2 - Sqrt[3])*x^2)),x]
```

output
```
ArcTanh[2*Sqrt[2 - Sqrt[3]]*x]/(2*Sqrt[3*(2 - Sqrt[3])])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{\sqrt{3}\,\arctan\left(2x\sqrt{-2+\sqrt{3}}\right)}{6\sqrt{-2+\sqrt{3}}}$	24
default	$-\frac{2\sqrt{3}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(-2+\sqrt{3}\right)\left(2\sqrt{6}+2\sqrt{2}\right)}$	43

input
```
int(1/3*3^(1/2)/(1-4*(2-3^(1/2))*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)/(-2+3^(1/2))^(1/2)*arctan(2*x*(-2+3^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(24) = 48.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}\,dx = \frac{1}{4}\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\log\left(\frac{16\,x^4+8\,\sqrt{3}x^2+4\left(2\,\sqrt{3}(2\,x^3-x)+3\,x\right)\sqrt{\frac{1}{3}\,\sqrt{3}+\frac{2}{3}}-1}{16\,x^4-16\,x^2+1}\right)$$

input
```
integrate(1/3*3^(1/2)/(1-4*(2-3^(1/2))*x^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/3*sqrt(3) + 2/3)*log((16*x^4 + 8*sqrt(3)*x^2 + 4*(2*sqrt(3)*(2*
x^3 - x) + 3*x)*sqrt(1/3*sqrt(3) + 2/3) - 1)/(16*x^4 - 16*x^2 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(31) = 62$.

Time = 0.51 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}\,dx = -\sqrt{3}\left(\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}\log\left(x-6\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}\right)-\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}\log\left(x+6\sqrt{\frac{\sqrt{3}}{144}+\frac{1}{72}}\right)\right)$$

input
```
integrate(1/3*3**(1/2)/(1-4*(2-3**(1/2))*x**2),x)
```

output
```
-sqrt(3)*(sqrt(sqrt(3)/144 + 1/72)*log(x - 6*sqrt(sqrt(3)/144 + 1/72)) - s
qrt(sqrt(3)/144 + 1/72)*log(x + 6*sqrt(sqrt(3)/144 + 1/72)))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}\,dx = \frac{\sqrt{3}\log\left(\frac{2x\left(\sqrt{3}-2\right)-\sqrt{-\sqrt{3}+2}}{2x\left(\sqrt{3}-2\right)+\sqrt{-\sqrt{3}+2}}\right)}{12\sqrt{-\sqrt{3}+2}}$$

input
```
integrate(1/3*3^(1/2)/(1-4*(2-3^(1/2))*x^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*log((2*x*(sqrt(3) - 2) - sqrt(-sqrt(3) + 2))/(2*x*(sqrt(3) -
2) + sqrt(-sqrt(3) + 2)))/sqrt(-sqrt(3) + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}\,dx$$
$$=\frac{1}{24}\sqrt{3}\left(\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|x+\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}\right|\right)-\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|x-\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}\right|\right)\right)$$

input
```
integrate(1/3*3^(1/2)/(1-4*(2-3^(1/2))*x^2),x, algorithm="giac")
```

output
```
1/24*sqrt(3)*((sqrt(6) + sqrt(2))*log(abs(x + 1/4*sqrt(6) + 1/4*sqrt(2)))
- (sqrt(6) + sqrt(2))*log(abs(x - 1/4*sqrt(6) - 1/4*sqrt(2))))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}\,dx=\frac{\operatorname{atanh}\left(\frac{x\left(2\sqrt{3}+3\right)}{3\left(\frac{\sqrt{3}}{3}+\frac{1}{2}\right)\sqrt{\sqrt{3}+2}}\right)\left(2\sqrt{3}+3\right)}{6\sqrt{\sqrt{3}+2}}$$

input
```
int(3^(1/2)/(3*(4*x^2*(3^(1/2) - 2) + 1)),x)
```

output
```
(atanh((x*(2*3^(1/2) + 3))/(3*(3^(1/2)/3 + 1/2)*(3^(1/2) + 2)^(1/2)))*(2*3
^(1/2) + 3))/(6*(3^(1/2) + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{3}\left(1-4\left(2-\sqrt{3}\right)x^2\right)}\,dx$$
$$= \frac{\sqrt{2}\left(-\sqrt{3}\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+2x\right)+\sqrt{3}\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+2x\right)-3\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+2x\right)+3\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+\right.\right.}{24}$$

input
```
int(1/3*3^(1/2)/(1-4*(2-3^(1/2))*x^2),x)
```

output
```
(sqrt(2)*( - sqrt(3)*log(( - sqrt(6) - sqrt(2) + 4*x)/2) + sqrt(3)*log((sq
rt(6) + sqrt(2) + 4*x)/2) - 3*log(( - sqrt(6) - sqrt(2) + 4*x)/2) + 3*log(
(sqrt(6) + sqrt(2) + 4*x)/2)))/24
```

3.36 $\int \frac{1}{\sqrt{3}+4(3-2\sqrt{3})x^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 34

$$\int \frac{1}{\sqrt{3}+4\left(3-2\sqrt{3}\right)x^2} \, dx = \frac{1}{2}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{2x}{\sqrt{2+\sqrt{3}}}\right)$$

output
```
1/2*(1/2*2^(1/2)+1/6*6^(1/2))*arctanh(2*x/(1/2*6^(1/2)+1/2*2^(1/2)))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt{3}+4\left(3-2\sqrt{3}\right)x^2} \, dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{-3+2\sqrt{3}}x}{\sqrt[4]{3}}\right)}{2\sqrt[4]{3}\sqrt{-3+2\sqrt{3}}}$$

input
```
Integrate[(Sqrt[3] + 4*(3 - 2*Sqrt[3])*x^2)^(-1),x]
```

output
```
ArcTanh[(2*Sqrt[-3 + 2*Sqrt[3]]*x)/3^(1/4)]/(2*3^(1/4)*Sqrt[-3 + 2*Sqrt[3]
])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4\left(3-2\sqrt{3}\right)x^2+\sqrt{3}}\,dx$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{2\sqrt{2\sqrt{3}-3}x}{\sqrt[4]{3}}\right)}{2\sqrt[4]{3}\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(Sqrt[3] + 4*(3 - 2*Sqrt[3])*x^2)^(-1),x]
```

output
```
ArcTanh[(2*Sqrt[-3 + 2*Sqrt[3]]*x)/3^(1/4)]/(2*3^(1/4)*Sqrt[-3 + 2*Sqrt[3]
])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
meijerg	$\frac{3^{\frac{3}{4}} \operatorname{arctanh}\left(\frac{2x 3^{\frac{3}{4}} \sqrt{-3+2\sqrt{3}}}{3}\right)}{6\sqrt{-3+2\sqrt{3}}}$	31
default	$\frac{2 \operatorname{arctanh}\left(\frac{8x}{2\sqrt{6}+2\sqrt{2}}\right)}{\left(-3+2\sqrt{3}\right)\left(2\sqrt{6}+2\sqrt{2}\right)}$	42

input
```
int(1/(3^(1/2)+4*(3-2*3^(1/2))*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(3/4)/(-3+2*3^(1/2))^(1/2)*arctanh(2/3*x*3^(3/4)*(-3+2*3^(1/2))^(1/2
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(24) = 48$.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int \frac{1}{\sqrt{3}+4\left(3-2\sqrt{3}\right)x^2}\,dx$$
$$= \frac{1}{4}\sqrt{\frac{1}{3}\sqrt{3}+\frac{2}{3}}\log\left(\frac{16\,x^4+8\,\sqrt{3}x^2+4\left(2\,\sqrt{3}(2\,x^3-x)+3\,x\right)\sqrt{\frac{1}{3}\,\sqrt{3}+\frac{2}{3}}-1}{16\,x^4-16\,x^2+1}\right)$$

input
```
integrate(1/(3^(1/2)+4*(3-2*3^(1/2))*x^2),x, algorithm="fricas")
```

output
```
1/4*sqrt(1/3*sqrt(3) + 2/3)*log((16*x^4 + 8*sqrt(3)*x^2 + 4*(2*sqrt(3)*(2*
x^3 - x) + 3*x)*sqrt(1/3*sqrt(3) + 2/3) - 1)/(16*x^4 - 16*x^2 + 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(31) = 62$.

Time = 0.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.24

$$\int \frac{1}{\sqrt{3}+4\left(3-2\sqrt{3}\right)x^2}\,dx = -\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\log\left(x-\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\cdot\left(\frac{24}{-7+4\sqrt{3}}-\frac{14\sqrt{3}}{-7+4\sqrt{3}}\right)\right) + \sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\log\left(x+\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\cdot\left(\frac{24}{-7+4\sqrt{3}}-\frac{14\sqrt{3}}{-7+4\sqrt{3}}\right)\right)$$

input
```
integrate(1/(3**(1/2)+4*(3-2*3**(1/2))*x**2),x)
```

output
```
-sqrt(sqrt(3)/48 + 1/24)*log(x - sqrt(sqrt(3)/48 + 1/24)*(24/(-7 + 4*sqrt(
3)) - 14*sqrt(3)/(-7 + 4*sqrt(3)))) + sqrt(sqrt(3)/48 + 1/24)*log(x + sqrt
(sqrt(3)/48 + 1/24)*(24/(-7 + 4*sqrt(3)) - 14*sqrt(3)/(-7 + 4*sqrt(3))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(24) = 48$.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{3}+4\left(3-2\sqrt{3}\right)x^2}\,dx = -\frac{3^{\frac{3}{4}}\log\left(\frac{2x\left(2\sqrt{3}-3\right)-\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}}{2x\left(2\sqrt{3}-3\right)+\sqrt{\sqrt{3}\left(2\sqrt{3}-3\right)}}\right)}{12\sqrt{2\sqrt{3}-3}}$$

input
```
integrate(1/(3^(1/2)+4*(3-2*3^(1/2))*x^2),x, algorithm="maxima")
```

output
```
-1/12*3^(3/4)*log((2*x*(2*sqrt(3) - 3) - sqrt(sqrt(3)*(2*sqrt(3) - 3)))/(2
*x*(2*sqrt(3) - 3) + sqrt(sqrt(3)*(2*sqrt(3) - 3))))/sqrt(2*sqrt(3) - 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{3}+4\left(3-2\sqrt{3}\right)x^2}\,dx = \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x+\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}\right|\right) - \frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x-\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}\right|\right)$$

input
```
integrate(1/(3^(1/2)+4*(3-2*3^(1/2))*x^2),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) + 3*sqrt(2))*log(abs(x + 1/4*sqrt(6) + 1/4*sqrt(2))) - 1/24*
(sqrt(6) + 3*sqrt(2))*log(abs(x - 1/4*sqrt(6) - 1/4*sqrt(2)))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{3}+4\left(3-2\sqrt{3}\right)x^2}\,dx = \frac{\operatorname{atanh}\left(\frac{x\left(2\sqrt{3}+3\right)}{3\left(\frac{\sqrt{3}}{3}+\frac{1}{2}\right)\sqrt{\sqrt{3}+2}}\right)\left(2\sqrt{3}+3\right)}{6\sqrt{\sqrt{3}+2}}$$

input
```
int(-1/(4*x^2*(2*3^(1/2) - 3) - 3^(1/2)),x)
```

output
```
(atanh((x*(2*3^(1/2) + 3))/(3*(3^(1/2)/3 + 1/2)*(3^(1/2) + 2)^(1/2)))*(2*3
^(1/2) + 3))/(6*(3^(1/2) + 2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{3}+4\left(3-2\sqrt{3}\right)x^2}\,dx$$
$$= \frac{\sqrt{2}\left(-\sqrt{3}\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+2x\right)+\sqrt{3}\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+2x\right)-3\log\left(-\frac{\sqrt{6}}{2}-\frac{\sqrt{2}}{2}+2x\right)+3\log\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}+\right.}{24}$$

input
```
int(1/(3^(1/2)+4*(3-2*3^(1/2))*x^2),x)
```

output
```
(sqrt(2)*( - sqrt(3)*log(( - sqrt(6) - sqrt(2) + 4*x)/2) + sqrt(3)*log((sq
rt(6) + sqrt(2) + 4*x)/2) - 3*log(( - sqrt(6) - sqrt(2) + 4*x)/2) + 3*log(
(sqrt(6) + sqrt(2) + 4*x)/2)))/24
```

3.37 $\int (a+bx^2)^{5/2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 84

$$\int (a+bx^2)^{5/2} \, dx = \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax(a+bx^2)^{3/2} + \frac{1}{6}x(a+bx^2)^{5/2} + \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

output
```
5/16*a^2*x*(b*x^2+a)^(1/2)+5/24*a*x*(b*x^2+a)^(3/2)+1/6*x*(b*x^2+a)^(5/2)+
5/16*a^3*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int (a+bx^2)^{5/2} \, dx = \frac{1}{48}\sqrt{a+bx^2}(33a^2x+26abx^3+8b^2x^5) - \frac{5a^3\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5))/48 - (5*a^3*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

input
```
Int[(a + b*x^2)^(5/2),x]
```

output
```
(x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
+ b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{x\left(8b^{2}x^{4}+26ab\,x^{2}+33a^{2}\right)\sqrt{b\,x^{2}+a}}{48}+\frac{5a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16\sqrt{b}}$	59
pseudoelliptic	$\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{3}}{16\sqrt{b}}+\frac{11x\sqrt{b\,x^{2}+a}\left(\frac{8b^{\frac{5}{2}}x^{4}}{33}+\frac{26a\,b^{\frac{3}{2}}x^{2}}{33}+a^{2}\sqrt{b}\right)}{16\sqrt{b}}$	67
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}$	68

input
```
int((b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*b^2*x^4+26*a*b*x^2+33*a^2)*(b*x^2+a)^(1/2)+5/16*a^3*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int (a+bx^2)^{5/2}\,dx = \left[\frac{15\,a^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{96\,b}, -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{48\,b}\right]$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 +
33*a^2*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int (a+bx^2)^{5/2}\,dx = \frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(5/2),x)
```

output
```
11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a
)/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*sqrt(b))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \left(a+bx^2\right)^{5/2}\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax + \frac{5}{16}\sqrt{bx^2+a}a^2x + \frac{5\,a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a
)*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \left(a+bx^2\right)^{5/2}\,dx = -\frac{5\,a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}} + \frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}x$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*
x^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \left(a+bx^2\right)^{5/2}\,dx=\frac{x\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

input
```
int((a + b*x^2)^(5/2),x)
```

output
```
(x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2\right)^{5/2}\,dx=\frac{33\sqrt{b\,x^2+a}\,a^2bx+26\sqrt{b\,x^2+a}\,a\,b^2x^3+8\sqrt{b\,x^2+a}\,b^3x^5+15\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^3}{48b}$$

input
```
int((b*x^2+a)^(5/2),x)
```

output
```
(33*sqrt(a + b*x**2)*a**2*b*x + 26*sqrt(a + b*x**2)*a*b**2*x**3 + 8*sqrt(a
 + b*x**2)*b**3*x**5 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**3)/(48*b)
```

3.38 $\int \left(a+bx^2\right)^{3/2} dx$

Optimal result

Integrand size = 11, antiderivative size = 65

$$\int \left(a+bx^2\right)^{3/2} dx = \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
3/8*a*x*(b*x^2+a)^(1/2)+1/4*x*(b*x^2+a)^(3/2)+3/8*a^2*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \left(a+bx^2\right)^{3/2} dx = \frac{1}{8}x\sqrt{a+bx^2}\left(5a+2bx^2\right)-\frac{3a^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(3/2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2))/8 - (3*a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/(8*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{3/2}\,dx$$

$$\Big\downarrow \text{ 211}$$

$$\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}$$

$$\Big\downarrow \text{ 211}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}$$

input
```
Int[(a + b*x^2)^(3/2),x]
```

output
```
(x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{x(2b\,x^{2}+5a)\sqrt{b\,x^{2}+a}}{8}+\frac{3a^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}$	48
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}$	52
pseudoelliptic	$\frac{2\sqrt{b\,x^{2}+a}\,b^{\frac{3}{2}}x^{3}+5ax\sqrt{b}\,\sqrt{b\,x^{2}+a}+3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{2}}{8\sqrt{b}}$	62

input
```
int((b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*b*x^2+5*a)*(b*x^2+a)^(1/2)+3/8*a^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/
b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (a+bx^2)^{3/2}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2x^3+5\,abx)\sqrt{bx^2+a}}{16\,b}, -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^3+5\,abx)\sqrt{bx^2+a}}{8\,b}\right]$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int (a+bx^2)^{3/2}\,dx = \frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(3/2),x)
```

output
```
5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 +
3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \left(a+bx^2\right)^{3/2} dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{bx^2+a}ax + \frac{3\,a^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sq
rt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \left(a+bx^2\right)^{3/2} dx = \frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}x - \frac{3\,a^2 \log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{3/2} dx = \frac{x\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^2)^(3/2),x)
```

output
```
(x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \left(a+bx^2\right)^{3/2} dx = \frac{5\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{8b}$$

input
```
int((b*x^2+a)^(3/2),x)
```

output
```
(5*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 + 3*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2)/(8*b)
```

3.39 $\int \sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}x\sqrt{a+bx^2} + \frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*x*(b*x^2+a)^(1/2)+1/2*a*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}x\sqrt{a+bx^2} - \frac{a\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}$$

input
```
Int[Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b
])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	36
risch	$\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	36
pseudoelliptic	$-\frac{a\left(-\frac{\sqrt{b\,x^{2}+a}\,x}{a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}\right)}{2}$	42

input
```
int((b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int\sqrt{a+bx^2}\,dx$$
$$=\left[\frac{2\,\sqrt{bx^2+a}bx+a\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b},\ \frac{\sqrt{bx^2+a}bx-a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \sqrt{a+bx^2}\,dx=\frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \sqrt{a+bx^2}\,dx=\frac{1}{2}\,\sqrt{bx^2+a}x+\frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}\,x - \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(
b)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sqrt{a+bx^2}\,dx = \frac{x\sqrt{b\,x^2+a}}{2} + \frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

input
```
int((a + b*x^2)^(1/2),x)
```

output
```
(x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\,dx = \frac{\sqrt{b\,x^2+a}\,bx+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{2b}$$

input
```
int((b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a)/(2*b)
```

3.40 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \ 224$$
$$\int \frac{1}{1-\frac{bx^2}{a+bx^2}}d\frac{x}{\sqrt{a+bx^2}}$$
$$\Big\downarrow \ 219$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22

input
```
int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x**2+a)**(1/2),x)
```

output
```
asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\sqrt{bx^2+a}x - \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(
b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^(1/2),x)
```

output
```
log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)}{b}$$

input
```
int(1/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)))/b
```

3.41 $\int \frac{1}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{x}{a\sqrt{a+bx^2}}$$

output
```
x/a/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{x}{a\sqrt{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 208}$$

$$\frac{x}{a\sqrt{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
default	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
trager	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
pseudoelliptic	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
orering	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15

input
```
int(1/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/a/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}\,x}{abx^2+a^2}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(1/(b*x**2+a)**(3/2),x)
```

output
```
x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{\sqrt{bx^2+a}a}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
x/(sqrt(b*x^2 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{\sqrt{bx^2+a}a}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
x/(sqrt(b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{b\,x^2+a}}$$

input
```
int(1/(a + b*x^2)^(3/2),x)
```

output
```
x/(a*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx+\sqrt{b}\,a+\sqrt{b}\,b\,x^2}{ab\,(b\,x^2+a)}$$

input
```
int(1/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x + sqrt(b)*a + sqrt(b)*b*x**2)/(a*b*(a + b*x**2))
```

3.42 $\int \frac{1}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx=\frac{x}{3a(a+bx^2)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}$$

output
```
1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx=\frac{3ax+2bx^3}{3a^2(a+bx^2)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(-5/2),x]
```

output
```
(3*a*x + 2*b*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{209}$$

$$\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(a + b*x^2)^(-5/2),x]
```

output
```
x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x(2b\,x^{2}+3a)}{3(b\,x^{2}+a)^{\frac{3}{2}}\,a^{2}}$	26
trager	$\frac{x(2b\,x^{2}+3a)}{3(b\,x^{2}+a)^{\frac{3}{2}}\,a^{2}}$	26
pseudoelliptic	$\frac{x(2b\,x^{2}+3a)}{3(b\,x^{2}+a)^{\frac{3}{2}}\,a^{2}}$	26
orering	$\frac{x(2b\,x^{2}+3a)}{3(b\,x^{2}+a)^{\frac{3}{2}}\,a^{2}}$	26
default	$\frac{x}{3a(b\,x^{2}+a)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}$	32

input
```
int(1/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{(2\,bx^3+3\,ax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.

Time = 0.46 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(1/(b*x**2+a)**(5/2),x)
```

output
```
3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a
)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{bx^2+a}a^2}+\frac{x}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{2\,x\,(b\,x^2+a)+a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

input
```
int(1/(a + b*x^2)^(5/2),x)
```

output
```
(2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.15

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{3\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3-2\sqrt{b}\,a^2-4\sqrt{b}\,ab\,x^2-2\sqrt{b}\,b^2x^4}{3a^2b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/(b*x^2+a)^(5/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 - 2*sqrt(b)*a**2
- 4*sqrt(b)*a*b*x**2 - 2*sqrt(b)*b**2*x**4)/(3*a**2*b*(a**2 + 2*a*b*x**2 +
 b**2*x**4))
```

3.43 $\int \frac{1}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 58

$$\int \frac{1}{(a+bx^2)^{7/2}}\,dx = \frac{x}{5a\,(a+bx^2)^{5/2}} + \frac{4x}{15a^2\,(a+bx^2)^{3/2}} + \frac{8x}{15a^3\sqrt{a+bx^2}}$$

output
```
1/5*x/a/(b*x^2+a)^(5/2)+4/15*x/a^2/(b*x^2+a)^(3/2)+8/15*x/a^3/(b*x^2+a)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+bx^2)^{7/2}}\,dx = \frac{15a^2x + 20abx^3 + 8b^2x^5}{15a^3\,(a+bx^2)^{5/2}}$$

input
```
Integrate[(a + b*x^2)^(-7/2),x]
```

output
```
(15*a^2*x + 20*a*b*x^3 + 8*b^2*x^5)/(15*a^3*(a + b*x^2)^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{209}$$

$$\frac{4\int \frac{1}{(bx^2+a)^{5/2}}dx}{5a}+\frac{x}{5a\,(a+bx^2)^{5/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a(a+bx^2)^{3/2}}\right)}{5a}+\frac{x}{5a\,(a+bx^2)^{5/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}\right)}{5a}+\frac{x}{5a\,(a+bx^2)^{5/2}}$$

input
```
Int[(a + b*x^2)^(-7/2),x]
```

output
```
x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*S
qrt[a + b*x^2])))/(5*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{x(8b^2x^4+20ab\,x^2+15a^2)}{15(b\,x^2+a)^{\frac{5}{2}}a^3}$	37
trager	$\frac{x(8b^2x^4+20ab\,x^2+15a^2)}{15(b\,x^2+a)^{\frac{5}{2}}a^3}$	37
pseudoelliptic	$\frac{x(8b^2x^4+20ab\,x^2+15a^2)}{15(b\,x^2+a)^{\frac{5}{2}}a^3}$	37
orering	$\frac{x(8b^2x^4+20ab\,x^2+15a^2)}{15(b\,x^2+a)^{\frac{5}{2}}a^3}$	37
default	$\frac{x}{5a(b\,x^2+a)^{\frac{5}{2}}}+\frac{\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}}{a}$	53

input
```
int(1/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(8*b^2*x^4+20*a*b*x^2+15*a^2)/(b*x^2+a)^(5/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.19

$$\int \frac{1}{(a+bx^2)^{7/2}}\,dx = \frac{(8\,b^2x^5+20\,abx^3+15\,a^2x)\sqrt{bx^2+a}}{15\,(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6)}$$

input
```
integrate(1/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

output
```
1/15*(8*b^2*x^5 + 20*a*b*x^3 + 15*a^2*x)*sqrt(b*x^2 + a)/(a^3*b^3*x^6 + 3*
a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(51) = 102$.

Time = 0.80 (sec) , antiderivative size = 413, normalized size of antiderivative = 7.12

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{7/2}}\,dx = {} & \frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}} \\ & + \frac{35a^4bx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}} \\ & + \frac{28a^3b^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}} \\ & + \frac{8a^2b^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{15}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+45a^{\frac{13}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+15a^{\frac{11}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\end{aligned}$$

input
```
integrate(1/(b*x**2+a)**(7/2),x)
```

output
```
15*a**5*x/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 +
b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*
x**6*sqrt(1 + b*x**2/a)) + 35*a**4*b*x**3/(15*a**(17/2)*sqrt(1 + b*x**2/a)
 + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 28*a**3*b**2*x*
*5/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45*a**(15/2)*b*x**2*sqrt(1 + b*x**2/
a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 15*a**(11/2)*b**3*x**6*sq
rt(1 + b*x**2/a)) + 8*a**2*b**3*x**7/(15*a**(17/2)*sqrt(1 + b*x**2/a) + 45
*a**(15/2)*b*x**2*sqrt(1 + b*x**2/a) + 45*a**(13/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 15*a**(11/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{7/2}}\,dx = \frac{8\,x}{15\,\sqrt{bx^2+a}a^3} + \frac{4\,x}{15\,(bx^2+a)^{\frac{3}{2}}a^2} + \frac{x}{5\,(bx^2+a)^{\frac{5}{2}}a}$$

input
```
integrate(1/(b*x^2+a)^(7/2),x, algorithm="maxima")
```

output
```
8/15*x/(sqrt(b*x^2 + a)*a^3) + 4/15*x/((b*x^2 + a)^(3/2)*a^2) + 1/5*x/((b*
x^2 + a)^(5/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+bx^2)^{7/2}}\,dx = \frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{5\,b}{a^2}\right)+\frac{15}{a}\right)x}{15\,(bx^2+a)^{\frac{5}{2}}}$$

input
```
integrate(1/(b*x^2+a)^(7/2),x, algorithm="giac")
```

output
```
1/15*(4*x^2*(2*b^2*x^2/a^3 + 5*b/a^2) + 15/a)*x/(b*x^2 + a)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{7/2}}\,dx = \frac{8\,x\,(b\,x^2+a)^2+3\,a^2\,x+4\,a\,x\,(b\,x^2+a)}{15\,a^3\,(b\,x^2+a)^{5/2}}$$

input
```
int(1/(a + b*x^2)^(7/2),x)
```

output
```
(8*x*(a + b*x^2)^2 + 3*a^2*x + 4*a*x*(a + b*x^2))/(15*a^3*(a + b*x^2)^(5/2
))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.19

$$\int \frac{1}{(a+bx^2)^{7/2}}\,dx = \frac{15\sqrt{b\,x^2+a}\,a^2bx+20\sqrt{b\,x^2+a}\,a\,b^2x^3+8\sqrt{b\,x^2+a}\,b^3x^5-8\sqrt{b}\,a^3-24\sqrt{b}\,a^2b\,x^2-2}{15a^3b\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}$$

input
```
int(1/(b*x^2+a)^(7/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**2*b*x + 20*sqrt(a + b*x**2)*a*b**2*x**3 + 8*sqrt(a
 + b*x**2)*b**3*x**5 - 8*sqrt(b)*a**3 - 24*sqrt(b)*a**2*b*x**2 - 24*sqrt(b
)*a*b**2*x**4 - 8*sqrt(b)*b**3*x**6)/(15*a**3*b*(a**3 + 3*a**2*b*x**2 + 3*
a*b**2*x**4 + b**3*x**6))
```

3.44 $\int \frac{1}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 77

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx=\frac{x}{7a(a+bx^2)^{7/2}}+\frac{6x}{35a^2(a+bx^2)^{5/2}}+\frac{8x}{35a^3(a+bx^2)^{3/2}}+\frac{16x}{35a^4\sqrt{a+bx^2}}$$

output
```
1/7*x/a/(b*x^2+a)^(7/2)+6/35*x/a^2/(b*x^2+a)^(5/2)+8/35*x/a^3/(b*x^2+a)^(3
/2)+16/35*x/a^4/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx=\frac{35a^3x+70a^2bx^3+56ab^2x^5+16b^3x^7}{35a^4(a+bx^2)^{7/2}}$$

input
```
Integrate[(a + b*x^2)^(-9/2),x]
```

output
```
(35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7)/(35*a^4*(a + b*x^2)^
(7/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{209}$$

$$\frac{6\int \frac{1}{(bx^2+a)^{7/2}}dx}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6\left(\frac{4\int \frac{1}{(bx^2+a)^{5/2}}dx}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}$$

input
```
Int[(a + b*x^2)^(-9/2),x]
```

output
```
x/(7*a*(a + b*x^2)^(7/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a
+ b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
trager	$\frac{x\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
pseudoelliptic	$\frac{x\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
orering	$\frac{x\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
default	$\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}$	74

input
```
int(1/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/(b*x^2+a)^(7/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{(16\,b^3x^7+56\,ab^2x^5+70\,a^2bx^3+35\,a^3x)\sqrt{bx^2+a}}{35\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

input
```
integrate(1/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*sqrt(b*x^2 + a)
/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. 2(70) = 140.

Time = 1.17 (sec) , antiderivative size = 1265, normalized size of antiderivative = 16.43

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x**2+a)**(9/2),x)
```

output
```
35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1
+ b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b
**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqr
t(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210
*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*
x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*
x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35
*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37
/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**
(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x
**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x
**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 42
9*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*
sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(
31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x*
*2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x*
*12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**
2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*s
qrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{16\,x}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,x}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,x}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{x}{7\,(bx^2+a)^{\frac{7}{2}}a}$$

input
```
integrate(1/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*x/(sqrt(b*x^2 + a)*a^4) + 8/35*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*x/((
b*x^2 + a)^(5/2)*a^2) + 1/7*x/((b*x^2 + a)^(7/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{\left(2\left(4\,x^2\left(\frac{2\,b^3x^2}{a^4}+\frac{7\,b^2}{a^3}\right)+\frac{35\,b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(1/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^
2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{16\,x}{35\,a^4\,\sqrt{b\,x^2+a}} + \frac{8\,x}{35\,a^3\,(b\,x^2+a)^{3/2}} + \frac{6\,x}{35\,a^2\,(b\,x^2+a)^{5/2}} + \frac{x}{7\,a\,(b\,x^2+a)^{7/2}}$$

input
```
int(1/(a + b*x^2)^(9/2),x)
```

output
```
(16*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*
x)/(35*a^2*(a + b*x^2)^(5/2)) + x/(7*a*(a + b*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.21

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{35\sqrt{b\,x^2+a}\,a^3bx+70\sqrt{b\,x^2+a}\,a^2b^2x^3+56\sqrt{b\,x^2+a}\,a\,b^3x^5+16\sqrt{b\,x^2+a}\,b^4x^7-16}{35a^4b\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+}$$

input
```
int(1/(b*x^2+a)^(9/2),x)
```

output

```
(35*sqrt(a + b*x**2)*a**3*b*x + 70*sqrt(a + b*x**2)*a**2*b**2*x**3 + 56*sq
rt(a + b*x**2)*a*b**3*x**5 + 16*sqrt(a + b*x**2)*b**4*x**7 - 16*sqrt(b)*a*
*4 - 64*sqrt(b)*a**3*b*x**2 - 96*sqrt(b)*a**2*b**2*x**4 - 64*sqrt(b)*a*b**
3*x**6 - 16*sqrt(b)*b**4*x**8)/(35*a**4*b*(a**4 + 4*a**3*b*x**2 + 6*a**2*b
**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.45 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{ 224}$$
$$\int \frac{1}{1-\frac{bx^2}{a+bx^2}}d\frac{x}{\sqrt{a+bx^2}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22

input
```
int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x**2+a)**(1/2),x)
```

output
```
asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx=\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx=\frac{1}{2}\,\sqrt{bx^2+a}x-\frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(
b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^(1/2),x)
```

output
```
log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)}{b}$$

input
```
int(1/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)))/b
```

3.46 $\int \frac{1}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{1}{\sqrt{a-bx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

output
```
arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-bx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[a - b*x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{224, 216\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \; 224$$

$$\int \frac{1}{\frac{bx^2}{a-bx^2}+1}d\frac{x}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[a - b*x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+a}}\right)}{\sqrt{b}}$	21
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-b\,x^2+a}}{\sqrt{b}\,x}\right)}{\sqrt{b}}$	24

input
```
int(1/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(b^(1/2)*x/(-b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{a-bx^2}}\,dx$$
$$= \left[-\frac{\sqrt{-b}\log\left(2\,bx^2-2\,\sqrt{-bx^2+a}\sqrt{-b}x-a\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-bx^2+a}\sqrt{b}x}{bx^2-a}\right)}{\sqrt{b}}\right]$$

input
```
integrate(1/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(2*b*x^2 - 2*sqrt(-b*x^2 + a)*sqrt(-b)*x - a)/b, -arctan
(sqrt(-b*x^2 + a)*sqrt(b)*x/(b*x^2 - a))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{a-bx^2}}\,dx = \begin{cases} -\frac{i \operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for } \left|\frac{bx^2}{a}\right| > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-I*acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (asin(
sqrt(b)*x/sqrt(a))/sqrt(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{a-bx^2}}\,dx = \frac{\arcsin\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(b*x/sqrt(a*b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(20) = 40.

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{a-bx^2}}\,dx = \frac{1}{2}\sqrt{-bx^2+ax} - \frac{a\log\left(\left|-\sqrt{-b}x+\sqrt{-bx^2+a}\right|\right)}{2\sqrt{-b}}$$

input
```
integrate(1/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(-b)*x + sqrt(-b*x^2 + a)))/sq
rt(-b)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{a-bx^2}}\,dx = \frac{\ln\left(\sqrt{a-b\,x^2}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

input
```
int(1/(a - b*x^2)^(1/2),x)
```

output
```
log((a - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{b}\,asin\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b}$$

input
```
int(1/(-b*x^2+a)^(1/2),x)
```

output
```
(sqrt(b)*asin((sqrt(b)*x)/sqrt(a)))/b
```

3.47 $\int \frac{1}{\sqrt{-a+bx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{\sqrt{-a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/(b*x^2-a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{-a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[-a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2-a}}dx$$
$$\Big\downarrow \text{ 224}$$
$$\int \frac{1}{1-\frac{bx^2}{bx^2-a}}d\frac{x}{\sqrt{bx^2-a}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-a}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[-a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[-a + b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}-a}\right)}{\sqrt{b}}$	23
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}-a}}{x\sqrt{b}}\right)}{\sqrt{b}}$	24

input
```
int(1/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(b^(1/2)*x+(b*x^2-a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{-a+bx^2}}\,dx = \left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-a}\sqrt{b}x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2-a}}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 - a))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{-a+bx^2}}\,dx = \begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{bx^2}{a}\right| > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b*x**2-a)**(1/2),x)
```

output
```
Piecewise((acosh(sqrt(b)*x/sqrt(a))/sqrt(b), Abs(b*x**2/a) > 1), (-I*asin(
sqrt(b)*x/sqrt(a))/sqrt(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-a+bx^2}}\,dx = \frac{\log\left(2\,bx+2\,\sqrt{bx^2-a}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + 2*sqrt(b*x^2 - a)*sqrt(b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{-a+bx^2}}\,dx = \frac{1}{2}\,\sqrt{bx^2-a}x + \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2-a}\right|\right)}{2\,\sqrt{b}}$$

input
```
integrate(1/(b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 - a)*x + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 - a)))/sqrt(
b)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-a+bx^2}}\,dx = \frac{\ln\left(\sqrt{b\,x^2-a}+\sqrt{b}\,x\right)}{\sqrt{b}}$$

input
```
int(1/(b*x^2 - a)^(1/2),x)
```

output
```
log((b*x^2 - a)^(1/2) + b^(1/2)*x)/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2-a}+\sqrt{b}\,x}{\sqrt{a}}\right)}{b}$$

input
```
int(1/(b*x^2-a)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt( - a + b*x**2) + sqrt(b)*x)/sqrt(a)))/b
```

3.48 $\int \frac{1}{\sqrt{-a-bx^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{1}{\sqrt{-a-bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

output
```
arctan(b^(1/2)*x/(-b*x^2-a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-a-bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[-a - b*x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-a-bx^2}}dx$$

$$\Big\downarrow \ 224$$

$$\int \frac{1}{\frac{bx^2}{-a-bx^2}+1}d\frac{x}{\sqrt{-a-bx^2}}$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-a-bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[-a - b*x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[-a - b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}-a}}\right)}{\sqrt{b}}$	23
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-b\,x^{2}-a}}{\sqrt{b}\,x}\right)}{\sqrt{b}}$	26

input
```
int(1/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(b^(1/2)*x/(-b*x^2-a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{-a-bx^2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-2\,bx^2+2\,\sqrt{-bx^2-a}\sqrt{-b}x-a\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-bx^2-a}\sqrt{b}x}{bx^2+a}\right)}{\sqrt{b}}\right]$$

input
```
integrate(1/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - a)*sqrt(-b)*x - a)/b, -arcta
n(sqrt(-b*x^2 - a)*sqrt(b)*x/(b*x^2 + a))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-a-bx^2}}\,dx = -\frac{i\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x**2-a)**(1/2),x)
```

output
```
-I*asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{-a-bx^2}}\,dx = -\frac{i\ \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

output
```
-I*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{-a-bx^2}}\,dx = \frac{1}{2}\sqrt{-bx^2-a}x + \frac{a\log\left(\left|-\sqrt{-b}x+\sqrt{-bx^2-a}\right|\right)}{2\sqrt{-b}}$$

input
```
integrate(1/(-b*x^2-a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-b*x^2 - a)*x + 1/2*a*log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - a)))/sq
rt(-b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-a-bx^2}}\,dx = \frac{\ln\left(\sqrt{-b\,x^2-a}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

input
```
int(1/(- a - b*x^2)^(1/2),x)
```

output
```
log((- a - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-a-bx^2}}\,dx = -\frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)i}{b}$$

input
```
int(1/(-b*x^2-a)^(1/2),x)
```

output
```
( - sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*i)/b
```

3.49 $\int \sqrt{9+4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \sqrt{9+4x^2}\,dx = \frac{1}{2}x\sqrt{9+4x^2} + \frac{9}{4}\text{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
1/2*x*(4*x^2+9)^(1/2)+9/4*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sqrt{9+4x^2}\,dx = \frac{1}{2}x\sqrt{9+4x^2} - \frac{9}{4}\log\left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2])/2 - (9*Log[-2*x + Sqrt[9 + 4*x^2]])/4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4x^2+9}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{4x^2+9}}dx+\frac{1}{2}\sqrt{4x^2+9}x$$

$$\Big\downarrow \text{222}$$

$$\frac{9}{4}\text{arcsinh}\left(\frac{2x}{3}\right)+\frac{1}{2}\sqrt{4x^2+9}x$$

input
```
Int[Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{4}$	20
risch	$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{4}$	20
trager	$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\ln\left(2x+\sqrt{4x^2+9}\right)}{4}$	30
meijerg	$-\frac{9\left(-\frac{4\sqrt{\pi}\,x\sqrt{\frac{4x^2}{9}+1}}{3}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	31
pseudoelliptic	$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)}{8}-\frac{9\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)}{8}$	54

input
```
int((4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(4*x^2+9)^(1/2)+9/4*arcsinh(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int\sqrt{9+4x^2}\,dx=\frac{1}{2}\sqrt{4\,x^2+9}x-\frac{9}{4}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate((4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sqrt{9+4x^2}\,dx = \frac{x\sqrt{4x^2+9}}{2} + \frac{9\operatorname{asinh}\left(\frac{2x}{3}\right)}{4}$$

input
```
integrate((4*x**2+9)**(1/2),x)
```

output
```
x*sqrt(4*x**2 + 9)/2 + 9*asinh(2*x/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \sqrt{9+4x^2}\,dx = \frac{1}{2}\sqrt{4x^2+9}x + \frac{9}{4}\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

input
```
integrate((4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(4*x^2 + 9)*x + 9/4*arcsinh(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sqrt{9+4x^2}\,dx = \frac{1}{2}\sqrt{4x^2+9}x - \frac{9}{4}\log\left(-2x+\sqrt{4x^2+9}\right)$$

input
```
integrate((4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \sqrt{9+4x^2}\,dx = \frac{9\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{4} + x\,\sqrt{x^2+\frac{9}{4}}$$

input
```
int((4*x^2 + 9)^(1/2),x)
```

output
```
(9*asinh((2*x)/3))/4 + x*(x^2 + 9/4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sqrt{9+4x^2}\,dx = \frac{\sqrt{4x^2+9}\,x}{2} + \frac{9\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{4}$$

input
```
int((4*x^2+9)^(1/2),x)
```

output
```
(2*sqrt(4*x**2 + 9)*x + 9*log((sqrt(4*x**2 + 9) + 2*x)/3))/4
```

3.50 $\int \sqrt{9-4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \sqrt{9-4x^2}\,dx=\frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{4}\arcsin\left(\frac{2x}{3}\right)$$

output
```
1/2*x*(-4*x^2+9)^(1/2)+9/4*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \sqrt{9-4x^2}\,dx=\frac{1}{2}x\sqrt{9-4x^2}-\frac{9}{2}\arctan\left(\frac{\sqrt{9-4x^2}}{3+2x}\right)$$

input
```
Integrate[Sqrt[9 - 4*x^2],x]
```

output
```
(x*Sqrt[9 - 4*x^2])/2 - (9*ArcTan[Sqrt[9 - 4*x^2]/(3 + 2*x)])/2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9-4x^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{9-4x^2}}dx+\frac{1}{2}\sqrt{9-4x^2}x$$

$$\Big\downarrow \text{223}$$

$$\frac{9}{4}\arcsin\left(\frac{2x}{3}\right)+\frac{1}{2}\sqrt{9-4x^2}x$$

input
```
Int[Sqrt[9 - 4*x^2],x]
```

output
```
(x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x\sqrt{-4x^2+9}}{2}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{4}$	20
risch	$-\frac{(4x^2-9)x}{2\sqrt{-4x^2+9}}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{4}$	27
pseudoelliptic	$\frac{x\sqrt{-4x^2+9}}{2}-\frac{9\arctan\left(\frac{\sqrt{-4x^2+9}}{2x}\right)}{4}$	31
meijerg	$\frac{9i\left(-\frac{4i\sqrt{\pi}\,x\sqrt{-\frac{4x^2}{9}+1}}{3}-2i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	34
trager	$\frac{x\sqrt{-4x^2+9}}{2}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{4}$	43

input
```
int((-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-4*x^2+9)^(1/2)+9/4*arcsin(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int\sqrt{9-4x^2}\,dx=\frac{1}{2}\sqrt{-4\,x^2+9}x-\frac{9}{2}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

input
```
integrate((-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-4*x^2 + 9)*x - 9/2*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sqrt{9-4x^2}\,dx = \frac{x\sqrt{9-4x^2}}{2} + \frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{4}$$

input
```
integrate((-4*x**2+9)**(1/2),x)
```

output
```
x*sqrt(9 - 4*x**2)/2 + 9*asin(2*x/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \sqrt{9-4x^2}\,dx = \frac{1}{2}\sqrt{-4x^2+9}x + \frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

input
```
integrate((-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \sqrt{9-4x^2}\,dx = \frac{1}{2}\sqrt{-4x^2+9}x + \frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

input
```
integrate((-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \sqrt{9-4x^2}\,dx = \frac{9\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{4} + x\,\sqrt{\frac{9}{4}-x^2}$$

input
```
int((9 - 4*x^2)^(1/2),x)
```

output
```
(9*asin((2*x)/3))/4 + x*(9/4 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \sqrt{9-4x^2}\,dx = \frac{9asin\left(\frac{2x}{3}\right)}{4} + \frac{\sqrt{-4x^2+9}\,x}{2}$$

input
```
int((-4*x^2+9)^(1/2),x)
```

output
```
(9*asin((2*x)/3) + 2*sqrt( - 4*x**2 + 9)*x)/4
```

3.51 $\int \sqrt{-9+4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \sqrt{-9+4x^2}\,dx = \frac{1}{2}x\sqrt{-9+4x^2} - \frac{9}{4}\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
1/2*x*(4*x^2-9)^(1/2)-9/4*arctanh(2*x/(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \sqrt{-9+4x^2}\,dx = \frac{1}{2}x\sqrt{-9+4x^2} + \frac{9}{4}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

input
```
Integrate[Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2])/2 + (9*Log[-2*x + Sqrt[-9 + 4*x^2]])/4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4x^2-9}\,dx$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{2}\int \frac{1}{\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{2}\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

input
```
Int[Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2])/2 - (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

method	result	size
trager	$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(\sqrt{4x^2-9}+2x\right)}{4}$	30
default	$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{8}$	35
risch	$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{8}$	35
pseudoelliptic	$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)}{8}+\frac{9\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)}{8}$	54
meijerg	$\frac{9i\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{4i\sqrt{\pi}\,x\sqrt{-\frac{4x^2}{9}+1}}{3}-2i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	56

input
```
int((4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(4*x^2-9)^(1/2)-9/4*ln((4*x^2-9)^(1/2)+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \sqrt{-9+4x^2}\,dx=\frac{1}{2}\sqrt{4\,x^2-9}x+\frac{9}{4}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(4*x^2 - 9)*x + 9/4*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \sqrt{-9+4x^2}\,dx = \frac{x\sqrt{4x^2-9}}{2} - \frac{9\log\left(2x+\sqrt{4x^2-9}\right)}{4}$$

input
```
integrate((4*x**2-9)**(1/2),x)
```

output
```
x*sqrt(4*x**2 - 9)/2 - 9*log(2*x + sqrt(4*x**2 - 9))/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \sqrt{-9+4x^2}\,dx = \frac{1}{2}\sqrt{4x^2-9}x - \frac{9}{4}\log\left(8x+4\sqrt{4x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(4*x^2 - 9)*x - 9/4*log(8*x + 4*sqrt(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \sqrt{-9+4x^2}\,dx = \frac{1}{2}\sqrt{4x^2-9}x + \frac{9}{4}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

input
```
integrate((4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \sqrt{-9+4x^2}\,dx = \frac{x\,\sqrt{4\,x^2-9}}{2} - \frac{9\,\ln\left(2\,x+\sqrt{4\,x^2-9}\right)}{4}$$

input
```
int((4*x^2 - 9)^(1/2),x)
```

output
```
(x*(4*x^2 - 9)^(1/2))/2 - (9*log(2*x + (4*x^2 - 9)^(1/2)))/4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \sqrt{-9+4x^2}\,dx = \frac{\sqrt{4x^2-9}\,x}{2} - \frac{9\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{4}$$

input
```
int((4*x^2-9)^(1/2),x)
```

output
```
(2*sqrt(4*x**2 - 9)*x - 9*log((sqrt(4*x**2 - 9) + 2*x)/3))/4
```

3.52 $\int \sqrt{-9-4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \sqrt{-9-4x^2}\,dx=\frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{4}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
1/2*x*(-4*x^2-9)^(1/2)-9/4*arctan(2*x/(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{-9-4x^2}\,dx=\frac{1}{4}\left(2x\sqrt{-9-4x^2}-9\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\right)$$

input
```
Integrate[Sqrt[-9 - 4*x^2],x]
```

output
```
(2*x*Sqrt[-9 - 4*x^2] - 9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-4x^2-9}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{2}\int \frac{1}{\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{2}\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

input
```
Int[Sqrt[-9 - 4*x^2],x]
```

output
```
(x*Sqrt[-9 - 4*x^2])/2 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$	29
pseudoelliptic	$\frac{x\sqrt{-4x^2-9}}{2}+\frac{9\arctan\left(\frac{\sqrt{-4x^2-9}}{2x}\right)}{4}$	31
meijerg	$-\frac{9i\left(-\frac{4\sqrt{\pi}\,x\sqrt{\frac{4x^2}{9}+1}}{3}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	32
risch	$-\frac{(4x^2+9)x}{2\sqrt{-4x^2-9}}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$	36
trager	$\frac{x\sqrt{-4x^2-9}}{2}+\frac{9\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{4}$	44

input
```
int((-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-4*x^2-9)^(1/2)-9/4*arctan(2*x/(-4*x^2-9)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \sqrt{-9-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2-9}x - \frac{9}{8}i\,\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^2-9}\right)}{x}\right) + \frac{9}{8}i\,\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^2-9}\right)}{x}\right)$$

input
```
integrate((-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-4*x^2 - 9)*x - 9/8*I*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) + 9/8*
I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \sqrt{-9-4x^2}\,dx = \frac{x\sqrt{-4x^2-9}}{2} - \frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

input
```
integrate((-4*x**2-9)**(1/2),x)
```

output
```
x*sqrt(-4*x**2 - 9)/2 - 9*atan(2*x/sqrt(-4*x**2 - 9))/4
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \sqrt{-9-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2-9}x + \frac{9}{4}i\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate((-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*x^2 - 9)*x + 9/4*I*arcsinh(2/3*x)
```

Giac [F]

$$\int \sqrt{-9-4x^2}\,dx = \int \sqrt{-4\,x^2-9}\,dx$$

input
```
integrate((-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*x^2 - 9), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sqrt{-9-4x^2}\,dx = \frac{x\,\sqrt{-4\,x^2-9}}{2} - \frac{9\,\mathrm{atan}\left(\frac{2\,x}{\sqrt{-4\,x^2-9}}\right)}{4}$$

input
```
int((- 4*x^2 - 9)^(1/2),x)
```

output
```
(x*(- 4*x^2 - 9)^(1/2))/2 - (9*atan((2*x)/(- 4*x^2 - 9)^(1/2)))/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \sqrt{-9-4x^2}\,dx = \frac{9 asinh\left(\frac{2x}{3}\right) i}{4} - \frac{\sqrt{-4x^2-9}\,x}{2}$$

input
```
int((-4*x^2-9)^(1/2),x)
```

output
```
(9*asinh((2*x)/3)*i - 2*sqrt( - 4*x**2 - 9)*x)/4
```

3.53 $\int \frac{1}{\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\operatorname{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
1/2*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = -\frac{1}{2}\log\left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[1/Sqrt[9 + 4*x^2],x]
```

output
```
-1/2*Log[-2*x + Sqrt[9 + 4*x^2]]
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{222\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2+9}}\,dx$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\text{arcsinh}\left(\frac{2x}{3}\right)$$

input
```
Int[1/Sqrt[9 + 4*x^2],x]
```

output
```
ArcSinh[(2*x)/3]/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
trager	$-\frac{\ln\left(-\sqrt{4x^2+9}+2x\right)}{2}$	19
pseudoelliptic	$\frac{\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)}{4}-\frac{\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)}{4}$	42

input
```
int(1/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(2/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = -\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

input
```
integrate(1/(4*x**2+9)**(1/2),x)
```

output
```
asinh(2*x/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arcsinh(2/3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\,\sqrt{4\,x^2+9}x - \frac{9}{4}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{2\,x}{3}\right)}{2}$$

input
```
int(1/(4*x^2 + 9)^(1/2),x)
```

output
```
asinh((2*x)/3)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{2}$$

input
```
int(1/(4*x^2+9)^(1/2),x)
```

output
```
log((sqrt(4*x**2 + 9) + 2*x)/3)/2
```

3.54 $\int \frac{1}{\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{2x}{3}\right)$$

output
```
1/2*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \arctan\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

input
```
Integrate[1/Sqrt[9 - 4*x^2],x]
```

output
```
ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^2])]
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{223\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx$$
$$\Big\downarrow \text{223}$$
$$\frac{1}{2}\arcsin\left(\frac{2x}{3}\right)$$

input
```
Int[1/Sqrt[9 - 4*x^2],x]
```

output
```
ArcSin[(2*x)/3]/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\arcsin\left(\frac{2x}{3}\right)}{2}$	7
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-4x^2+9}}{2x}\right)}{2}$	18
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{2}$	31

input
```
int(1/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(2/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = -\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

input
```
integrate(1/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{2x}{3}\right)}{2}$$

input
```
integrate(1/(-4*x**2+9)**(1/2),x)
```

output
```
asin(2*x/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{2}{3}x\right)$$

input
```
integrate(1/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arcsin(2/3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\sqrt{-4x^2+9}x + \frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

input
```
integrate(1/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{2\,x}{3}\right)}{2}$$

input
```
int(1/(9 - 4*x^2)^(1/2),x)
```

output
```
asin((2*x)/3)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{asin\left(\frac{2x}{3}\right)}{2}$$

input
```
int(1/(-4*x^2+9)^(1/2),x)
```

output
```
asin((2*x)/3)/2
```

3.55 $\int \frac{1}{\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{1}{2}\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
1/2*arctanh(2*x/(4*x^2-9)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = -\frac{1}{4}\log\left(1-\frac{2x}{\sqrt{-9+4x^2}}\right)+\frac{1}{4}\log\left(1+\frac{2x}{\sqrt{-9+4x^2}}\right)$$

input
```
Integrate[1/Sqrt[-9 + 4*x^2],x]
```

output
```
-1/4*Log[1 - (2*x)/Sqrt[-9 + 4*x^2]] + Log[1 + (2*x)/Sqrt[-9 + 4*x^2]]/4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

input
```
Int[1/Sqrt[-9 + 4*x^2],x]
```

output
```
ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
trager	$-\frac{\ln\left(-\sqrt{4x^2-9}+2x\right)}{2}$	19
default	$\frac{\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{4}$	22
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,\arcsin\left(\frac{2x}{3}\right)}{2\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	29
pseudoelliptic	$\frac{\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)}{4}-\frac{\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)}{4}$	42

input
```
int(1/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(-(4*x^2-9)^(1/2)+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = -\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

input
```
integrate(1/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{\log\left(2x+\sqrt{4x^2-9}\right)}{2}$$

input
```
integrate(1/(4*x**2-9)**(1/2),x)
```

output
```
log(2*x + sqrt(4*x**2 - 9))/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{1}{2}\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate(1/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(8*x + 4*sqrt(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{1}{2}\sqrt{4\,x^2-9}x+\frac{9}{4}\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

input
```
integrate(1/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{\ln\left(2\,x+\sqrt{4\,x^2-9}\right)}{2}$$

input
```
int(1/(4*x^2 - 9)^(1/2),x)
```

output
```
log(2*x + (4*x^2 - 9)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{2}$$

input
```
int(1/(4*x^2-9)^(1/2),x)
```

output
```
log((sqrt(4*x**2 - 9) + 2*x)/3)/2
```

3.56 $\int \frac{1}{\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \frac{1}{2}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
1/2*arctan(2*x/(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \frac{1}{2}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

input
```
Integrate[1/Sqrt[-9 - 4*x^2],x]
```

output
```
ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

input
```
Int[1/Sqrt[-9 - 4*x^2],x]
```

output
```
ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

method	result	size
meijerg	$-\frac{i \operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	8
default	$\frac{\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$	16
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-4x^2-9}}{2x}\right)}{2}$	18
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{2}$	31

input

```
int(1/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*I*arcsinh(2/3*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \frac{1}{4}i\,\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^2-9}\right)}{x}\right) - \frac{1}{4}i\,\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^2-9}\right)}{x}\right)$$

input

```
integrate(1/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output

```
1/4*I*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) - 1/4*I*log(-4*(2*x - I*sqrt(-4
*x^2 - 9))/x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

input
```
integrate(1/(-4*x**2-9)**(1/2),x)
```

output
```
atan(2*x/sqrt(-4*x**2 - 9))/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.32

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{2}i \operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(1/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*arcsinh(2/3*x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \int \frac{1}{\sqrt{-4\,x^2-9}}\,dx$$

input
```
integrate(1/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-4*x^2 - 9), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{-4\,x^2-9}}\right)}{2}$$

input
```
int(1/(- 4*x^2 - 9)^(1/2),x)
```

output
```
atan((2*x)/(- 4*x^2 - 9)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = -\frac{asinh\left(\frac{2x}{3}\right) i}{2}$$

input
```
int(1/(-4*x^2-9)^(1/2),x)
```

output
```
( - asinh((2*x)/3)*i)/2
```

3.57 $\int \frac{1}{\sqrt{9+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

output
```
arcsinh(1/3*b^(1/2)*x)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-3+\sqrt{9+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[9 + b*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/(-3 + Sqrt[9 + b*x^2])])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2+9}} \, dx$$
$$\Big\downarrow \; 222$$
$$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[9 + b*x^2],x]
```

output
```
ArcSinh[(Sqrt[b]*x)/3]/Sqrt[b]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$	12
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+9}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+9}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22

input
```
int(1/(b*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(1/3*b^(1/2)*x)/b^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.82

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \left[\frac{\log\left(-\sqrt{b}x - \sqrt{bx^2+9}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+9}\sqrt{-b}-3\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
[log(-sqrt(b)*x - sqrt(b*x^2 + 9))/sqrt(b), -2*sqrt(-b)*arctan((sqrt(b*x^2
 + 9)*sqrt(-b) - 3*sqrt(-b))/(b*x))/b]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x**2+9)**(1/2),x)
```

output
```
asinh(sqrt(b)*x/3)/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{1}{3}\sqrt{b}x\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/3*sqrt(b)*x)/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(11) = 22.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{1}{2}\sqrt{bx^2+9}x - \frac{9\log\left(-\sqrt{b}x+\sqrt{bx^2+9}\right)}{2\sqrt{b}}$$

input
```
integrate(1/(b*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + 9)*x - 9/2*log(-sqrt(b)*x + sqrt(b*x^2 + 9))/sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

input
```
int(1/(b*x^2 + 9)^(1/2),x)
```

output
```
asinh((b^(1/2)*x)/3)/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1}{\sqrt{9+bx^2}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+9}}{3}+\frac{\sqrt{b}\,x}{3}\right)}{b}$$

input
```
int(1/(b*x^2+9)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt(b*x**2 + 9) + sqrt(b)*x)/3))/b
```

3.58 $\int \frac{1}{\sqrt{9-bx^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \frac{\arcsin\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

output
```
arcsin(1/3*b^(1/2)*x)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{-3+\sqrt{9-bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[9 - b*x^2],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/(-3 + Sqrt[9 - b*x^2])])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx$$

$$\Big\downarrow \; 223$$

$$\frac{\arcsin\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}}$$

input

```
Int[1/Sqrt[9 - b*x^2],x]
```

output

```
ArcSin[(Sqrt[b]*x)/3]/Sqrt[b]
```

Defintions of rubi rules used

rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{\arcsin\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$	12
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^2+9}}\right)}{\sqrt{b}}$	21
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-b\,x^2+9}}{\sqrt{b}\,x}\right)}{\sqrt{b}}$	24

input
```
int(1/(-b*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(1/3*b^(1/2)*x)/b^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.41

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\sqrt{-b}x-\sqrt{-bx^2+9}\right)}{b}, -\frac{2\arctan\left(\frac{\sqrt{-bx^2+9}-3}{\sqrt{b}x}\right)}{\sqrt{b}}\right]$$

input
```
integrate(1/(-b*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-b)*log(-sqrt(-b)*x - sqrt(-b*x^2 + 9))/b, -2*arctan((sqrt(-b*x^2 +
 9) - 3)/(sqrt(b)*x))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{for } |bx^2| > 9 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-b*x**2+9)**(1/2),x)
```

output
```
Piecewise((-I*acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2) > 9), (asin(sqrt(b)*
x/3)/sqrt(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \frac{\arcsin\left(\frac{1}{3}\sqrt{b}x\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(1/3*sqrt(b)*x)/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(11) = 22.

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx = \frac{1}{2}\sqrt{-bx^2+9}x - \frac{9\log\left(-\sqrt{-b}x+\sqrt{-bx^2+9}\right)}{2\sqrt{-b}}$$

input
```
integrate(1/(-b*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-b*x^2 + 9)*x - 9/2*log(-sqrt(-b)*x + sqrt(-b*x^2 + 9))/sqrt(-b)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx=\frac{\operatorname{asinh}\left(\frac{\sqrt{-b}\,x}{3}\right)}{\sqrt{-b}}$$

input
```
int(1/(9 - b*x^2)^(1/2),x)
```

output
```
asinh(((-b)^(1/2)*x)/3)/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{9-bx^2}}\,dx=\frac{\sqrt{b}\,asin\left(\frac{\sqrt{b}\,x}{3}\right)}{b}$$

input
```
int(1/(-b*x^2+9)^(1/2),x)
```

output
```
(sqrt(b)*asin((sqrt(b)*x)/3))/b
```

3.59 $\int \frac{1}{\sqrt{-9+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{\sqrt{-9+bx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{-9+bx^2}}\right)}{\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/(b*x^2-9)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-9+bx^2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{-9+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[-9 + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2-9}}dx$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{1-\frac{bx^2}{bx^2-9}}d\frac{x}{\sqrt{bx^2-9}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-9}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[-9 + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[-9 + b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}-9}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}-9}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{b\,x^{2}}{9}\right)}\,\arcsin\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{\operatorname{signum}\left(-1+\frac{b\,x^{2}}{9}\right)}\,\sqrt{b}}$	36

input
```
int(1/(b*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(b^(1/2)*x+(b*x^2-9)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{1}{\sqrt{-9+bx^2}}\,dx = \left[\frac{\log\left(2\,bx^2+2\,\sqrt{bx^2-9}\sqrt{b}x-9\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2-9}}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(2*b*x^2 + 2*sqrt(b*x^2 - 9)*sqrt(b)*x - 9)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 - 9))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{-9+bx^2}}\,dx = \begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{for } |bx^2| > 9 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x}{3}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b*x**2-9)**(1/2),x)
```

output
```
Piecewise((acosh(sqrt(b)*x/3)/sqrt(b), Abs(b*x**2) > 9), (-I*asin(sqrt(b)*
x/3)/sqrt(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-9+bx^2}}\,dx = \frac{\log\left(2\,bx + 2\,\sqrt{bx^2-9}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + 2*sqrt(b*x^2 - 9)*sqrt(b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{-9+bx^2}}\,dx = \frac{1}{2}\,\sqrt{bx^2-9}x + \frac{9\,\log\left(\left|-\sqrt{b}x+\sqrt{bx^2-9}\right|\right)}{2\,\sqrt{b}}$$

input
```
integrate(1/(b*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 - 9)*x + 9/2*log(abs(-sqrt(b)*x + sqrt(b*x^2 - 9)))/sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{-9+bx^2}}\,dx=\frac{\ln\left(\sqrt{b\,x^2-9}+\sqrt{b}\,x\right)}{\sqrt{b}}$$

input
```
int(1/(b*x^2 - 9)^(1/2),x)
```

output
```
log((b*x^2 - 9)^(1/2) + b^(1/2)*x)/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-9+bx^2}}\,dx=\frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2-9}}{3}+\frac{\sqrt{b}\,x}{3}\right)}{b}$$

input
```
int(1/(b*x^2-9)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt(b*x**2 - 9) + sqrt(b)*x)/3))/b
```

3.60 $\int \frac{1}{\sqrt{-9-bx^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{1}{\sqrt{-9-bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-9-bx^2}}\right)}{\sqrt{b}}$$

output
```
arctan(b^(1/2)*x/(-b*x^2-9)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-9-bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-9-bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[-9 - b*x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-bx^2-9}}dx$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{\frac{bx^2}{-bx^2-9}+1}d\frac{x}{\sqrt{-bx^2-9}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-9}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[-9 - b*x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[-9 - b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}-9}}\right)}{\sqrt{b}}$	21
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-b\,x^{2}-9}}{\sqrt{b}\,x}\right)}{\sqrt{b}}$	24
meijerg	$\frac{\sqrt{\operatorname{signum}\left(1+\frac{b\,x^{2}}{9}\right)}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{-\operatorname{signum}\left(1+\frac{b\,x^{2}}{9}\right)}\,\sqrt{b}}$	36

input
```
int(1/(-b*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(b^(1/2)*x/(-b*x^2-9)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \frac{1}{\sqrt{-9-bx^2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-2\,bx^2+2\,\sqrt{-bx^2-9}\sqrt{-b}x-9\right)}{2\,b},\; -\frac{\arctan\left(\frac{\sqrt{-bx^2-9}\sqrt{b}x}{bx^2+9}\right)}{\sqrt{b}}\right]$$

input
```
integrate(1/(-b*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-2*b*x^2 + 2*sqrt(-b*x^2 - 9)*sqrt(-b)*x - 9)/b, -arcta
n(sqrt(-b*x^2 - 9)*sqrt(b)*x/(b*x^2 + 9))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-9-bx^2}}\,dx = -\frac{i\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{3}\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x**2-9)**(1/2),x)
```

output
```
-I*asinh(sqrt(b)*x/3)/sqrt(b)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{-9-bx^2}}\,dx = -\frac{i\,\mathrm{arsinh}\left(\frac{1}{3}\,\sqrt{b}x\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-I*arcsinh(1/3*sqrt(b)*x)/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{-9-bx^2}}\,dx = \frac{1}{2}\,\sqrt{-bx^2-9}x + \frac{9\,\log\left(\left|-\sqrt{-b}x+\sqrt{-bx^2-9}\right|\right)}{2\,\sqrt{-b}}$$

input
```
integrate(1/(-b*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-b*x^2 - 9)*x + 9/2*log(abs(-sqrt(-b)*x + sqrt(-b*x^2 - 9)))/sqrt
(-b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-9-bx^2}}\,dx = \frac{\ln\left(\sqrt{-b\,x^2-9}+\sqrt{-b}\,x\right)}{\sqrt{-b}}$$

input
```
int(1/(- b*x^2 - 9)^(1/2),x)
```

output
```
log((- b*x^2 - 9)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{-9-bx^2}}\,dx = -\frac{\sqrt{b}\,asinh\left(\frac{\sqrt{b}\,x}{3}\right)i}{b}$$

input
```
int(1/(-b*x^2-9)^(1/2),x)
```

output
```
( - sqrt(b)*asinh((sqrt(b)*x)/3)*i)/b
```

3.61 $\int \frac{1}{\sqrt{\pi+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{\sqrt{\pi+bx^2}}\,dx = \frac{\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

output
```
arcsinh(b^(1/2)*x/Pi^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{1}{\sqrt{\pi+bx^2}}\,dx = -\frac{\log\left(-\sqrt{b}x+\sqrt{\pi+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[Pi + b*x^2],x]
```

output
```
-(Log[-(Sqrt[b]*x) + Sqrt[Pi + b*x^2]]/Sqrt[b])
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2+\pi}}\,dx$$

$$\Big\downarrow \text{222}$$

$$\frac{\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[Pi + b*x^2],x]
```

output
```
ArcSinh[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
meijerg	$\frac{\text{arcsinh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$	14
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+\pi}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\text{arctanh}\left(\frac{\sqrt{b\,x^2+\pi}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22

input
```
int(1/(b*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(b^(1/2)*x/Pi^(1/2))/b^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.11

$$\int \frac{1}{\sqrt{\pi + bx^2}}\,dx = \left[\frac{\log\left(-\pi - 2\,bx^2 - 2\,\sqrt{\pi + bx^2}\sqrt{b}x\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{\pi+bx^2}}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2+pi)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-pi - 2*b*x^2 - 2*sqrt(pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)*
arctan(sqrt(-b)*x/sqrt(pi + b*x^2))/b]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{\pi + bx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x**2+pi)**(1/2),x)
```

output
```
asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{\pi + bx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2+pi)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*x/sqrt(pi*b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sqrt{\pi + bx^2}}\,dx = \frac{1}{2}\sqrt{\pi + bx^2}x - \frac{\pi\log\left(-\sqrt{b}x + \sqrt{\pi + bx^2}\right)}{2\sqrt{b}}$$

input
```
integrate(1/(b*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(pi + b*x^2)*x - 1/2*pi*log(-sqrt(b)*x + sqrt(pi + b*x^2))/sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{\pi + bx^2}}\,dx = \frac{\ln\left(\sqrt{b}\,x + \sqrt{b\,x^2 + \Pi}\right)}{\sqrt{b}}$$

input
```
int(1/(Pi + b*x^2)^(1/2),x)
```

output
```
log(b^(1/2)*x + (Pi + b*x^2)^(1/2))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{\pi + bx^2}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+\pi}+\sqrt{b}\,x}{\sqrt{\pi}}\right)}{b}$$

input
```
int(1/(b*x^2+Pi)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt(b*x**2 + pi) + sqrt(b)*x)/sqrt(pi)))/b
```

3.62 $\int \frac{1}{\sqrt{\pi - bx^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \frac{1}{\sqrt{\pi - bx^2}}\,dx = \frac{\arcsin\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

output
```
arcsin(b^(1/2)*x/Pi^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{1}{\sqrt{\pi - bx^2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{b}x}{-\sqrt{\pi}+\sqrt{\pi-bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[Pi - b*x^2],x]
```

output
```
(2*ArcTan[(Sqrt[b]*x)/(-Sqrt[Pi] + Sqrt[Pi - b*x^2])])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\pi - bx^2}}\,dx$$

$$\Big\downarrow \; 223$$

$$\frac{\arcsin\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[Pi - b*x^2],x]
```

output
```
ArcSin[(Sqrt[b]*x)/Sqrt[Pi]]/Sqrt[b]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
meijerg	$\frac{\arcsin\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{b}}$	14
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}+\pi}}\right)}{\sqrt{b}}$	21
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-b\,x^{2}+\pi}}{\sqrt{b}\,x}\right)}{\sqrt{b}}$	24

input
```
int(1/(-b*x^2+Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(b^(1/2)*x/Pi^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 3.26

$$\int \frac{1}{\sqrt{\pi-bx^2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\pi+2\,bx^2-2\,\sqrt{\pi-bx^2}\sqrt{-b}x\right)}{2\,b}, -\frac{\arctan\left(-\frac{\sqrt{b}x}{\sqrt{\pi-bx^2}}\right)}{\sqrt{b}}\right]$$

input
```
integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-pi + 2*b*x^2 - 2*sqrt(pi - b*x^2)*sqrt(-b)*x)/b, -arct
an(-sqrt(b)*x/sqrt(pi - b*x^2))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{\pi - bx^2}}\,dx = \begin{cases} -\frac{i\operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{\pi} > 1 \\ \frac{\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-b*x**2+pi)**(1/2),x)
```

output
```
Piecewise((-I*acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (asi
n(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{\pi - bx^2}}\,dx = \frac{\arcsin\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(b*x/sqrt(pi*b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(13) = 26.

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{1}{\sqrt{\pi - bx^2}}\,dx = \frac{1}{2}\sqrt{\pi - bx^2}x - \frac{\pi\log\left(\left|-\sqrt{-b}x + \sqrt{\pi - bx^2}\right|\right)}{2\sqrt{-b}}$$

input
```
integrate(1/(-b*x^2+pi)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(pi - b*x^2)*x - 1/2*pi*log(abs(-sqrt(-b)*x + sqrt(pi - b*x^2)))/s
qrt(-b)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{\pi - bx^2}}\, dx = \frac{\ln\left(\sqrt{\Pi - b\,x^2} + \sqrt{-b}\,x\right)}{\sqrt{-b}}$$

input
```
int(1/(Pi - b*x^2)^(1/2),x)
```

output
```
log((Pi - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\pi - bx^2}}\, dx = \frac{\sqrt{b}\, asin\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{b}$$

input
```
int(1/(-b*x^2+Pi)^(1/2),x)
```

output
```
(sqrt(b)*asin((sqrt(b)*x)/sqrt(pi)))/b
```

3.63 $\int \frac{1}{\sqrt{-\pi+bx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{\sqrt{-\pi+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{-\pi+bx^2}}\right)}{\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/(b*x^2-Pi)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-\pi+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{-\pi+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[-Pi + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2-\pi}}dx$$
$$\Big\downarrow \text{224}$$
$$\int \frac{1}{1-\frac{bx^2}{bx^2-\pi}}d\frac{x}{\sqrt{bx^2-\pi}}$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2-\pi}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[-Pi + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[-Pi + b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}-\pi}\right)}{\sqrt{b}}$	23
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}-\pi}}{x\sqrt{b}}\right)}{\sqrt{b}}$	24
meijerg	$\frac{\sqrt{\operatorname{signum}\left(-\frac{x^{2}b}{\pi}+1\right)}\,\arcsin\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{-\operatorname{signum}\left(-\frac{x^{2}b}{\pi}+1\right)}\,\sqrt{b}}$	44

input
```
int(1/(b*x^2-Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(b^(1/2)*x+(b*x^2-Pi)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{-\pi+bx^2}}\,dx = \left[\frac{\log\left(-\pi+2\,bx^2+2\,\sqrt{-\pi+bx^2}\sqrt{b}x\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(-\frac{\sqrt{-\pi+bx^2}\sqrt{-b}x}{\pi-bx^2}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2-pi)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-pi + 2*b*x^2 + 2*sqrt(-pi + b*x^2)*sqrt(b)*x)/sqrt(b), -sqrt(-b)
*arctan(-sqrt(-pi + b*x^2)*sqrt(-b)*x/(pi - b*x^2))/b]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{-\pi + bx^2}}\,dx = \begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx^2|}{\pi} > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b*x**2-pi)**(1/2),x)
```

output
```
Piecewise((acosh(sqrt(b)*x/sqrt(pi))/sqrt(b), Abs(b*x**2)/pi > 1), (-I*asi
n(sqrt(b)*x/sqrt(pi))/sqrt(b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-\pi + bx^2}}\,dx = \frac{\log\left(2\,bx + 2\,\sqrt{-\pi + bx^2}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2-pi)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + 2*sqrt(-pi + b*x^2)*sqrt(b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{-\pi + bx^2}}\,dx = \frac{1}{2}\sqrt{-\pi + bx^2}x + \frac{\pi\log\left(\left|-\sqrt{b}x + \sqrt{-\pi + bx^2}\right|\right)}{2\sqrt{b}}$$

input
```
integrate(1/(b*x^2-pi)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-pi + b*x^2)*x + 1/2*pi*log(abs(-sqrt(b)*x + sqrt(-pi + b*x^2)))/
sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-\pi + bx^2}}\,dx = \frac{\ln\left(\sqrt{b\,x^2 - \Pi} + \sqrt{b}\,x\right)}{\sqrt{b}}$$

input
```
int(1/(b*x^2 - Pi)^(1/2),x)
```

output
```
log((b*x^2 - Pi)^(1/2) + b^(1/2)*x)/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-\pi + bx^2}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2-\pi}+\sqrt{b}\,x}{\sqrt{\pi}}\right)}{b}$$

input
```
int(1/(b*x^2-Pi)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt(b*x**2 - pi) + sqrt(b)*x)/sqrt(pi)))/b
```

3.64 $\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-\pi-bx^2}}\right)}{\sqrt{b}}$$

output
```
arctan(b^(1/2)*x/(-b*x^2-Pi)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-\pi-bx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-\pi-bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[-Pi - b*x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-bx^2-\pi}}dx$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{\frac{bx^2}{-bx^2-\pi}+1}d\frac{x}{\sqrt{-bx^2-\pi}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{-bx^2-\pi}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[-Pi - b*x^2],x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[-Pi - b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{-b\,x^{2}-\pi}}\right)}{\sqrt{b}}$	23
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-b\,x^{2}-\pi}}{\sqrt{b}\,x}\right)}{\sqrt{b}}$	26
meijerg	$\frac{\sqrt{\operatorname{signum}\left(\frac{x^{2} b}{\pi}+1\right)}\,\operatorname{arcsinh}\left(\frac{\sqrt{b}\,x}{\sqrt{\pi}}\right)}{\sqrt{-\operatorname{signum}\left(\frac{x^{2} b}{\pi}+1\right)}\,\sqrt{b}}$	42

input
```
int(1/(-b*x^2-Pi)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(b^(1/2)*x/(-b*x^2-Pi)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{-\pi - bx^2}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\pi - 2\,bx^2 + 2\,\sqrt{-\pi - bx^2}\sqrt{-b}x\right)}{2\,b},\ -\frac{\arctan\left(\frac{\sqrt{-\pi-bx^2}\sqrt{b}x}{\pi+bx^2}\right)}{\sqrt{b}}\right]$$

input
```
integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-pi - 2*b*x^2 + 2*sqrt(-pi - b*x^2)*sqrt(-b)*x)/b, -arc
tan(sqrt(-pi - b*x^2)*sqrt(b)*x/(pi + b*x^2))/sqrt(b)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt{-\pi - bx^2}}\,dx = -\frac{i\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{\pi}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x**2-pi)**(1/2),x)
```

output
```
-I*asinh(sqrt(b)*x/sqrt(pi))/sqrt(b)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{-\pi - bx^2}}\,dx = -\frac{i\ \mathrm{arsinh}\left(\frac{bx}{\sqrt{\pi b}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="maxima")
```

output
```
-I*arcsinh(b*x/sqrt(pi*b))/sqrt(b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(22) = 44.

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{-\pi - bx^2}}\,dx = \frac{1}{2}\sqrt{-\pi - bx^2}x + \frac{\pi \log\left(\left|-\sqrt{-b}x + \sqrt{-\pi - bx^2}\right|\right)}{2\sqrt{-b}}$$

input
```
integrate(1/(-b*x^2-pi)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-pi - b*x^2)*x + 1/2*pi*log(abs(-sqrt(-b)*x + sqrt(-pi - b*x^2)))
/sqrt(-b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-\pi - bx^2}}\,dx = \frac{\ln\left(\sqrt{-b\,x^2 - \Pi} + \sqrt{-b}\,x\right)}{\sqrt{-b}}$$

input
```
int(1/(- Pi - b*x^2)^(1/2),x)
```

output
```
log((- Pi - b*x^2)^(1/2) + (-b)^(1/2)*x)/(-b)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{-\pi - bx^2}}\,dx = -\frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+\pi}+\sqrt{b}\,x}{\sqrt{\pi}}\right)i}{b}$$

input
```
int(1/(-b*x^2-Pi)^(1/2),x)
```

output
```
( - sqrt(b)*log((sqrt(b*x**2 + pi) + sqrt(b)*x)/sqrt(pi))*i)/b
```

3.65 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

output
```
arctan(x/(a^2-x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx=\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

input
```
Integrate[1/Sqrt[a^2 - x^2],x]
```

output
```
ArcTan[x/Sqrt[a^2 - x^2]]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a^2-x^2}}dx$$

$$\Big\downarrow \; 224$$

$$\int \frac{1}{\frac{x^2}{a^2-x^2}+1}d\frac{x}{\sqrt{a^2-x^2}}$$

$$\Big\downarrow \; 216$$

$$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

input
```
Int[1/Sqrt[a^2 - x^2],x]
```

output
```
ArcTan[x/Sqrt[a^2 - x^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$	15
pseudoelliptic	$-\arctan\left(\frac{\sqrt{a^2-x^2}}{x}\right)$	19

input
```
int(1/(a^2-x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x/(a^2-x^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = -2\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

input
```
integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \begin{cases} -i\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \left|\frac{x^2}{a^2}\right| > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a**2-x**2)**(1/2),x)
```

output
```
Piecewise((-I*acosh(x/a), Abs(x**2/a**2) > 1), (asin(x/a), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$$

input
```
integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(x/a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \frac{1}{2}\,a^2\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a) + \frac{1}{2}\,\sqrt{a^2-x^2}x$$

input
```
integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*a^2*arcsin(x/a)*sgn(a) + 1/2*sqrt(a^2 - x^2)*x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \operatorname{atan}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

input
```
int(1/(a^2 - x^2)^(1/2),x)
```

output
```
atan(x/(a^2 - x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = asin\left(\frac{x}{a}\right)$$

input

```
int(1/(a^2-x^2)^(1/2),x)
```

output

```
asin(x/a)
```

3.66 $\int (a+bx^2)^{4/3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 285

$$\int (a+bx^2)^{4/3}\,dx = \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x(a+bx^2)^{4/3}$$
$$-\frac{16\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{55bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
24/55*a*x*(b*x^2+a)^(1/3)+3/11*x*(b*x^2+a)^(4/3)-16/55*3^(3/4)*(1/2*6^(1/2
)-1/2*2^(1/2))*a^2*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(
1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Ellip
ticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^
(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.79 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.16

$$\int \left(a+bx^2\right)^{4/3}\,dx=\frac{ax\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3),x]
```

output
```
(a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{4/3}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{8}{11}a\int \sqrt[3]{bx^2+a}dx+\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \text{211}$$

$$\frac{8}{11}a\left(\frac{2}{5}a\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \text{234}$$

$$\frac{8}{11}a\left(\frac{3a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow 760$$

$$\frac{8}{11}a\left(\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\left(\right.\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

$$\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

input
```
Int[(a + b*x^2)^(4/3),x]
```

output
```
(3*x*(a + b*x^2)^(4/3))/11 + (8*a*((3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*
Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*
(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/(
(1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[
-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)])))/11
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{4}{3}}d x$$

input
```
int((b*x^2+a)^(4/3),x)
```

output
```
int((b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \left(a+bx^2\right)^{4/3}\,dx=a^{\frac{4}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(4/3),x)
```

output
```
a**(4/3)*x*hyper((-4/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{4/3} dx = \int \left(bx^2+a\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{4/3} dx = \int \left(bx^2+a\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \left(a+bx^2\right)^{4/3} dx = \frac{x\left(b\,x^2+a\right)^{4/3} {}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{4/3}}$$

input
```
int((a + b*x^2)^(4/3),x)
```

output
```
(x*(a + b*x^2)^(4/3)*hypergeom([-4/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(4/3)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{4/3} dx = \frac{39(b\,x^2+a)^{\frac{1}{3}}\,ax}{55} + \frac{3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^3}{11} + \frac{16\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a^2}{55}$$

input
```
int((b*x^2+a)^(4/3),x)
```

output
```
(39*(a + b*x**2)**(1/3)*a*x + 15*(a + b*x**2)**(1/3)*b*x**3 + 16*int((a +
b*x**2)**(1/3)/(a + b*x**2),x)*a**2)/55
```

3.67 $\int \sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 266

$$\int \sqrt[3]{a+bx^2}\,dx = \frac{3}{5}x\sqrt[3]{a+bx^2} - \frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
3/5*x*(b*x^2+a)^(1/3)-2/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a*(a^(1/3)-(b*
x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/
2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^
2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(
1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \sqrt[3]{a+bx^2}\,dx=\frac{x\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3),x]
```

output
```
(x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{2}{5}a\int \frac{1}{(bx^2+a)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \text{234}$$

$$\frac{3a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \text{760}$$

$$\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

input

```
Int[(a + b*x^2)^(1/3),x]
```

output

```
(3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a +
 b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/
3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(
1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{3}}d x$$

input
```
int((b*x^2+a)^(1/3),x)
```

output
```
int((b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^2}\,dx = \int \left(bx^{2}+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \sqrt[3]{a+bx^2}\,dx = \sqrt[3]{a}x\,{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)$$

input
```
integrate((b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.14

$$\int \sqrt[3]{a+bx^2}\,dx = \frac{x\,\left(b\,x^2+a\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/3}}$$

input
```
int((a + b*x^2)^(1/3),x)
```

output
```
(x*(a + b*x^2)^(1/3)*hypergeom([-1/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/3)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^2}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,x}{5} + \frac{2\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a}{5}$$

input
```
int((b*x^2+a)^(1/3),x)
```

output
```
(3*(a + b*x**2)**(1/3)*x + 2*int((a + b*x**2)**(1/3)/(a + b*x**2),x)*a)/5
```

3.68 $\int \frac{1}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 246

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(
1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)
)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^
(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/
3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.19

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{2/3}}$$

input
```
Integrate[(a + b*x^2)^(-2/3),x]
```

output
```
(x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 234$$

$$\frac{3\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}$$

$$\Big\downarrow \; 760$$

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

input
```
Int[(a + b*x^2)^(-2/3),x]
```

output
```
-((3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) +
 a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)]))
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}d x$$

input
```
int(1/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.10

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \frac{x{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}}$$

input
```
integrate(1/(b*x**2+a)**(2/3),x)
```

output
```
x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{2/3}}$$

input
```
int(1/(a + b*x^2)^(2/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(2/3)*hypergeom([1/2, 2/3], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(2/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(a + b*x**2)**(2/3),x)
```

3.69 $\int \frac{1}{(a+bx^2)^{5/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 271

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx = \frac{3x}{4a\,(a+bx^2)^{2/3}} - \frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}\right.\right.}{4abx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/4*x/a/(b*x^2+a)^(2/3)-1/4*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)-(b*
x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/
2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^
2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a/b/x/(-a
^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.70 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.20

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx = \frac{x\left(3+\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{4a\,(a+bx^2)^{2/3}}$$

input
```
Integrate[(a + b*x^2)^(-5/3),x]
```

output
```
(x*(3 + (1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a
)]))/(4*a*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {215, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{1}{(bx^2+a)^{2/3}}dx}{4a} + \frac{3x}{4a\,(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{234}$$

$$\frac{3\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{8abx} + \frac{3x}{4a\,(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3x}{4a\left(a+bx^2\right)^{2/3}}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),-\right.}{4abx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

input
```
Int[(a + b*x^2)^(-5/3),x]
```

output
```
(3*x)/(4*a*(a + b*x^2)^(2/3)) - (3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a +
 b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/
3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(4*a*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(5/3),x)
```

output
```
int(1/(b*x^2+a)^(5/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.09

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{3}}}$$

input
```
integrate(1/(b*x**2+a)**(5/3),x)
```

output
```
x*hyper((1/2, 5/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/3)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-5/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-5/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.14

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{5/3}}$$

input
```
int(1/(a + b*x^2)^(5/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(5/3)*hypergeom([1/2, 5/3], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(5/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}\,a+(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(5/3),x)
```

output
```
int(1/((a + b*x**2)**(2/3)*a + (a + b*x**2)**(2/3)*b*x**2),x)
```

3.70 $\int \frac{1}{(a+bx^2)^{8/3}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 290

$$\int \frac{1}{(a+bx^2)^{8/3}} \, dx = \frac{3x}{10a(a+bx^2)^{5/3}} + \frac{21x}{40a^2(a+bx^2)^{2/3}} - \frac{7\, 3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\, \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}\right.\right.}{40a^2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/10*x/a/(b*x^2+a)^(5/3)+21/40*x/a^2/(b*x^2+a)^(2/3)-7/40*3^(3/4)*(1/2*6^(
1/2)-1/2*2^(1/2))*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1
/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Ellipt
icF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(
1/3)),2*I-I*3^(1/2))/a^2/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/
2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.51 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.25

$$\int \frac{1}{(a+bx^2)^{8/3}}\,dx = \frac{33ax+21bx^3+7x(a+bx^2)\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)}{40a^2(a+bx^2)^{5/3}}$$

input
```
Integrate[(a + b*x^2)^(-8/3),x]
```

output
```
(33*a*x + 21*b*x^3 + 7*x*(a + b*x^2)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2
F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(40*a^2*(a + b*x^2)^(5/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {215, 215, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{8/3}}dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{7\int \frac{1}{(bx^2+a)^{5/3}}dx}{10a}+\frac{3x}{10a(a+bx^2)^{5/3}}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{7\left(\frac{\int \frac{1}{(bx^2+a)^{2/3}}dx}{4a}+\frac{3x}{4a(a+bx^2)^{2/3}}\right)}{10a}+\frac{3x}{10a(a+bx^2)^{5/3}}$$

$$\Big\downarrow \text{ 234}$$

$$\frac{7\left(\frac{3\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{8abx}+\frac{3x}{4a(a+bx^2)^{2/3}}\right)}{10a}+\frac{3x}{10a\left(a+bx^2\right)^{5/3}}$$

$$\Big\downarrow \; 760$$

$$\frac{7\left(\frac{3x}{4a(a+bx^2)^{2/3}}-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right.\right.}{4abx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{10a}$$

$$\frac{3x}{10a\left(a+bx^2\right)^{5/3}}$$

input
```
Int[(a + b*x^2)^(-8/3),x]
```

output
```
(3*x)/(10*a*(a + b*x^2)^(5/3)) + (7*((3*x)/(4*a*(a + b*x^2)^(2/3)) - (3^(3
/4)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3
)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(4*a*b*x*S
qrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))^2)])))/(10*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{8}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(8/3),x)
```

output
```
int(1/(b*x^2+a)^(8/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{8/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(8/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.08

$$\int \frac{1}{(a+bx^2)^{8/3}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{8}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{8}{3}}}$$

input
```
integrate(1/(b*x**2+a)**(8/3),x)
```

output
```
x*hyper((1/2, 8/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(8/3)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{8/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(8/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-8/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{8/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(8/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-8/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \frac{1}{(a+bx^2)^{8/3}}\,dx=\frac{x\left(\frac{b\,x^2}{a}+1\right)^{8/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{8}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{8/3}}$$

input
```
int(1/(a + b*x^2)^(8/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(8/3)*hypergeom([1/2, 8/3], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(8/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{8/3}}\,dx=\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}}a^2+2\left(b\,x^2+a\right)^{\frac{2}{3}}a b\,x^2+\left(b\,x^2+a\right)^{\frac{2}{3}}b^2x^4}dx$$

input
```
int(1/(b*x^2+a)^(8/3),x)
```

output
```
int(1/((a + b*x**2)**(2/3)*a**2 + 2*(a + b*x**2)**(2/3)*a*b*x**2 + (a + b*
x**2)**(2/3)*b**2*x**4),x)
```

3.71 $\int (a+bx^2)^{5/3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 569

$$\int (a+bx^2)^{5/3}\,dx=\frac{30}{91}ax(a+bx^2)^{2/3}+\frac{3}{13}x(a+bx^2)^{5/3}-\frac{120a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{60\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{40\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
30/91*a*x*(b*x^2+a)^(2/3)+3/13*x*(b*x^2+a)^(5/3)-120*a^2*x/(91*(1-3^(1/2))
*a^(1/3)-91*(b*x^2+a)^(1/3))+60/91*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(7/
3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(
2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2)
)*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(
1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))^2)^(1/2)-40/91*2^(1/2)*3^(3/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/3))
*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(
b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/(
(1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)
-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.08

$$\int \left(a+bx^2\right)^{5/3} dx = \frac{ax(a+bx^2)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x^2)^(5/3),x]
```

output
```
(a*x*(a + b*x^2)^(2/3)*Hypergeometric2F1[-5/3, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(2/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {211, 211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/3}\,dx$$

$$\Big\downarrow \; 211$$

$$\frac{10}{13}a\int \left(bx^2+a\right)^{2/3}dx+\frac{3}{13}x\left(a+bx^2\right)^{5/3}$$

$$\Big\downarrow \; 211$$

$$\frac{10}{13}a\left(\frac{4}{7}a\int \frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)+\frac{3}{13}x\left(a+bx^2\right)^{5/3}$$

$$\Big\downarrow \; 233$$

$$\frac{10}{13}a\left(\frac{6a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)+\frac{3}{13}x\left(a+bx^2\right)^{5/3}$$

$$\Big\downarrow \; 833$$

$$\frac{10}{13}a\left(\frac{6a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}\right)+$$
$$\frac{3}{13}x\left(a+bx^2\right)^{5/3}$$

$$\Big\downarrow \; 760$$

$$\frac{10}{13}a\left(\frac{6a\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}}{\left(\left(1-\right.\right.}}}\right.}{7bx}\right.$$

$$\frac{3}{13}x\left(a+bx^2\right)^{5/3}$$

$\downarrow$ 2418

$$\frac{10}{13}a\left(\frac{6a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}\right.$$

$$\frac{3}{13}x\left(a+bx^2\right)^{5/3}$$

input
```
Int[(a + b*x^2)^(5/3),x]
```

output
```
(3*x*(a + b*x^2)^(5/3))/13 + (10*a*((3*x*(a + b*x^2)^(2/3))/7 + (6*a*Sqrt[
b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1
/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*El
lipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[
-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)])))/(7*b*x)))/13
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b \,x^{2}+a \right)^{\frac{5}{3}}d x$$

input
```
int((b*x^2+a)^(5/3),x)
```

output
```
int((b*x^2+a)^(5/3),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{5/3}\,dx=\int \left(bx^{2}+a\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.05

$$\int \left(a+bx^2\right)^{5/3}\,dx = a^{\frac{5}{3}} x\, {}_2F_1\left(\begin{matrix} -\frac{5}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(5/3),x)
```

output
```
a**(5/3)*x*hyper((-5/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{5/3}\,dx = \int \left(bx^2+a\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/3), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/3}\,dx = \int \left(bx^2+a\right)^{\frac{5}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \left(a+bx^2\right)^{5/3} dx = \frac{x\,(b\,x^2+a)^{5/3}\,{}_2\mathrm{F}_1\left(-\frac{5}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/3}}$$

input
```
int((a + b*x^2)^(5/3),x)
```

output
```
(x*(a + b*x^2)^(5/3)*hypergeom([-5/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/3)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/3} dx = \frac{51(b\,x^2+a)^{\frac{2}{3}}\,ax}{91}+\frac{3(b\,x^2+a)^{\frac{2}{3}}\,b\,x^3}{13}+\frac{40\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)a^2}{91}$$

input
```
int((b*x^2+a)^(5/3),x)
```

output
```
(51*(a + b*x**2)**(2/3)*a*x + 21*(a + b*x**2)**(2/3)*b*x**3 + 40*int((a +
b*x**2)**(2/3)/(a + b*x**2),x)*a**2)/91
```

3.72 $\int (a+bx^2)^{2/3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 550

$$\int \left(a+bx^2\right)^{2/3} dx = \frac{3}{7}x\left(a+bx^2\right)^{2/3} - \frac{12ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right|}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{4\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/7*x*(b*x^2+a)^(2/3)-12*a*x/(7*(1-3^(1/2))*a^(1/3)-7*(b*x^2+a)^(1/3))+6/7
*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(
2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^
2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-4/7*2^(1/2)*3^(
3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(
b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF((
(1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
,2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/
3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \left(a+bx^2\right)^{2/3} dx = \frac{x(a+bx^2)^{2/3}\, \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x^2)^(2/3),x]
```

output
```
(x*(a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(2/3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{2/3} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{4}{7}a\int \frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{233}$$

$$\frac{6a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{833}$$

$$\frac{6a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{760}$$

$$\frac{6a\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+b}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}\right.}{7bx}$$

$$\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{2418}$$

$$\frac{6a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}$$

$$\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

input
```
Int[(a + b*x^2)^(2/3),x]
```

output
```
(3*x*(a + b*x^2)^(2/3))/7 + (6*a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(
1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a +
 b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3
) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) -
 (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*
Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1
/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(7*b*x)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{2}{3}}d x$$

input
```
int((b*x^2+a)^(2/3),x)
```

output
```
int((b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{2/3}\,dx=\int \left(bx^{2}+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.05

$$\int \left(a+bx^2\right)^{2/3} dx = a^{\frac{2}{3}} x\, {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(2/3),x)
```

output
```
a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{2/3} dx = \int \left(bx^2+a\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{2/3} dx = \int \left(bx^2+a\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \left(a+bx^2\right)^{2/3}\,dx=\frac{x\,(b\,x^2+a)^{2/3}\,{}_2\mathrm{F}_1\!\left(-\frac{2}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{2/3}}$$

input
```
int((a + b*x^2)^(2/3),x)
```

output
```
(x*(a + b*x^2)^(2/3)*hypergeom([-2/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(2/3)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{2/3}\,dx=\frac{3(b\,x^2+a)^{\frac{2}{3}}\,x}{7}+\frac{4\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)a}{7}$$

input
```
int((b*x^2+a)^(2/3),x)
```

output
```
(3*(a + b*x**2)**(2/3)*x + 4*int((a + b*x**2)**(2/3)/(a + b*x**2),x)*a)/7
```

3.73 $\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 529

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = -\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3*x/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+3/2*3^(1/4)*(1/2*6^(1/2)+1/2*2^
(1/2))*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)
+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE
(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3
)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-2^(1/2)*3^(3/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^
(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1
/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.77 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx=\frac{x\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/3),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$$

$\Big\downarrow$ 233

$$\frac{3\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}$$

$\Big\downarrow$ 833

$$\frac{3\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2bx}$$

$\Big\downarrow$ 760

$$\frac{3\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}\right.}{2bx}$$

$\Big\downarrow$ 2418

$$\frac{3\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}$$

input
```
Int[(a + b*x^2)^(-1/3),x]
```

output
```
(3*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqr
t[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b
*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^
2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2)])))/(2*b*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(1/3),x)
```

output
```
int(1/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/3),x)
```

output
```
x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{1/3}}$$

input
```
int(1/(a + b*x^2)^(1/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/3)*hypergeom([1/3, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/3)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(1/3),x)
```

output
```
int(1/(a + b*x**2)**(1/3),x)
```

3.74 $\int \frac{1}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 552

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \frac{3x}{2a\sqrt[3]{a+bx^2}} + \frac{3x}{2a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-7}{4a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7+}{\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/2*x/a/(b*x^2+a)^(1/3)+3/2*x/a/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))-3/4*
3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1
/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2
+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)+1/2*3^(3/4)*(a^(
1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(
(1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/
3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2
^(1/2)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3
)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.11

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx=\frac{3x-x\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{2a\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-4/3),x]
```

output
```
(3*x - x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/
a)])/(2*a*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {215, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx$$

$$\Big\downarrow \; 215$$

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2a}$$

$$\Big\downarrow \; 233$$

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4abx}$$

$$\Big\downarrow \; 833$$

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4abx}$$

$$\Big\downarrow \; 760$$

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^{\cdots}}}\cdots}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\cdots\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right)}}}\cdots\right)}{4abx}$$

$$\Big\downarrow \; 2418$$

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+\cdots}}\right)\cdots\right)}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\cdots\right)}{\cdots}$$

input
```
Int[(a + b*x^2)^(-4/3),x]
```

output
```
(3*x)/(2*a*(a + b*x^2)^(1/3)) - (3*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
 + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1
/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
 - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)
)*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(4*a*b*x)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{4}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(4/3),x)
```

output
```
int(1/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.04

$$\int \frac{1}{\left(a+bx^2\right)^{4/3}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{4}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}}$$

input
```
integrate(1/(b*x**2+a)**(4/3),x)
```

output
```
x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/3)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{4/3}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^2\right)^{4/3}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-4/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{4/3}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{4}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{4/3}}$$

input
```
int(1/(a + b*x^2)^(4/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(4/3)*hypergeom([1/2, 4/3], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(4/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(4/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)
```

3.75 $\int \frac{1}{(a+bx^2)^{7/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 574

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx = \frac{3x}{8a\left(a+bx^2\right)^{4/3}}+\frac{15x}{16a^2\sqrt[3]{a+bx^2}}+\frac{15x}{16a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-\cdots}{32a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{5\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7\cdots}{8\sqrt{2}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/8*x/a/(b*x^2+a)^(4/3)+15/16*x/a^2/(b*x^2+a)^(1/3)+15/16*x/a^2/((1-3^(1/2
))*a^(1/3)-(b*x^2+a)^(1/3))-15/32*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/
3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1
-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)
-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(
5/3)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3))^2)^(1/2)+5/16*3^(3/4)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)
*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)
^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3
)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/a^(5/3)/b/x/(-a^(1/3)*(a^(1/3)-(
b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.13

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx = \frac{3x(7a+5bx^2)-5x(a+bx^2)\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{16a^2(a+bx^2)^{4/3}}$$

input
```
Integrate[(a + b*x^2)^(-7/3),x]
```

output
```
(3*x*(7*a + 5*b*x^2) - 5*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/3)*Hypergeometri
c2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(16*a^2*(a + b*x^2)^(4/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {215, 215, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx$$

$$\Big\downarrow \; 215$$

$$\frac{5\int \frac{1}{(bx^2+a)^{4/3}}dx}{8a}+\frac{3x}{8a\,(a+bx^2)^{4/3}}$$

$$\Big\downarrow \; 215$$

$$\frac{5\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2a}\right)}{8a}+\frac{3x}{8a\,(a+bx^2)^{4/3}}$$

$$\Big\downarrow \; 233$$

$$\frac{5\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4abx}\right)}{8a}+\frac{3x}{8a\,(a+bx^2)^{4/3}}$$

$$\Big\downarrow \; 833$$

$$\frac{5\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4abx}\right)}{8a}+\frac{3x}{8a\,(a+bx^2)^{4/3}}$$

$$\Big\downarrow \; 760$$

$$\frac{5\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\cdots}{\left(\left(\cdots\right.\right.}}}\right.}{4abx}\right.}{8a}$$

$$\frac{3x}{8a\left(a+bx^2\right)^{4/3}}$$

$\Big\downarrow$ 2418

$$\frac{5\left(\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{3x}{8a\left(a+bx^2\right)^{4/3}}$$

input
```
Int[(a + b*x^2)^(-7/3),x]
```

output
```
(3*x)/(8*a*(a + b*x^2)^(4/3)) + (5*((3*x)/(2*a*(a + b*x^2)^(1/3)) - (3*Sqr
t[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (
3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2
/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b
*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3
])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*
a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^
(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*
EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqr
t[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))^2)])))/(4*a*b*x)))/(8*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(7/3),x)
```

output
```
int(1/(b*x^2+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.04

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{3}}}$$

input
```
integrate(1/(b*x**2+a)**(7/3),x)
```

output
```
x*hyper((1/2, 7/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/3)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-7/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-7/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.06

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{7/3}}$$

input
```
int(1/(a + b*x^2)^(7/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(7/3)*hypergeom([1/2, 7/3], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(7/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{7/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{3}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b^2x^4}dx$$

input
```
int(1/(b*x^2+a)^(7/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*a**2 + 2*(a + b*x**2)**(1/3)*a*b*x**2 + (a + b*
x**2)**(1/3)*b**2*x**4),x)
```

3.76 $\int (a+bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int (a+bx^2)^{5/4}\,dx = \frac{10}{21}ax\sqrt[4]{a+bx^2} + \frac{2}{7}x(a+bx^2)^{5/4} + \frac{10a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21\sqrt{b}\,(a+bx^2)^{3/4}}$$

output
```
10/21*a*x*(b*x^2+a)^(1/4)+2/7*x*(b*x^2+a)^(5/4)+10/21*a^(5/2)*(1+b*x^2/a)^
(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^
2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.65 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int (a+bx^2)^{5/4}\,dx = \frac{ax\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(5/4),x]
```

output
```
(a*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {211, 211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/4} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{7}a\int \sqrt[4]{bx^2+a}dx+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{7}a\left(\frac{1}{3}a\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{231}$$

$$\frac{5}{7}a\left(\frac{a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{229}$$

$$\frac{5}{7}a\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

input
```
Int[(a + b*x^2)^(5/4),x]
```

output
```
(2*x*(a + b*x^2)^(5/4))/7 + (5*a*((2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(
1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[
b]*(a + b*x^2)^(3/4))))/7
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a \right)^{\frac{5}{4}}d x$$

input
```
int((b*x^2+a)^(5/4),x)
```

output
```
int((b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int (a+bx^2)^{5/4}\,dx = \int (bx^2+a)^{\frac{5}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.28

$$\int (a+bx^2)^{5/4}\,dx = a^{\frac{5}{4}} x\, {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(5/4),x)
```

output
```
a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int (a+bx^2)^{5/4}\,dx = \int (bx^2+a)^{\frac{5}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/4}\,dx=\int \left(bx^2+a\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \left(a+bx^2\right)^{5/4}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/4}}$$

input
```
int((a + b*x^2)^(5/4),x)
```

output
```
(x*(a + b*x^2)^(5/4)*hypergeom([-5/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/4)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/4}\,dx=\frac{16(b\,x^2+a)^{\frac{1}{4}}\,ax}{21}+\frac{2(b\,x^2+a)^{\frac{1}{4}}\,b\,x^3}{7}+\frac{5\left(\int\frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a^2}{21}$$

input
```
int((b*x^2+a)^(5/4),x)
```

output
```
(16*(a + b*x**2)**(1/4)*a*x + 6*(a + b*x**2)**(1/4)*b*x**3 + 5*int((a + b*
x**2)**(1/4)/(a + b*x**2),x)*a**2)/21
```

3.77 $\int (a+bx^2)^{3/4}\, dx$

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int (a+bx^2)^{3/4}\, dx=\frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x(a+bx^2)^{3/4}-\frac{6a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
6/5*a*x/(b*x^2+a)^(1/4)+2/5*x*(b*x^2+a)^(3/4)-6/5*a^(3/2)*(1+b*x^2/a)^(1/4
)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^2+a)^
(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.99 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int (a+bx^2)^{3/4}\, dx=\frac{x(a+bx^2)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(3/4),x]
```

output
```
(x*(a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {211, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/4} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{5}a\int \frac{1}{\sqrt[4]{bx^2+a}}dx+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{227}$$

$$\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{225}$$

$$\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{212}$$

$$\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

input
```
Int[(a + b*x^2)^(3/4),x]
```

output
```
(2*x*(a + b*x^2)^(3/4))/5 + (3*a*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)
/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b
]))/(5*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{3}{4}}d x$$

input
```
int((b*x^2+a)^(3/4),x)
```

output
```
int((b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{3/4} dx = \int \left(bx^2+a\right)^{\frac{3}{4}} dx$$

input
```
integrate((b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.28

$$\int \left(a+bx^2\right)^{3/4} dx = a^{\frac{3}{4}} x\,{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(3/4),x)
```

output
```
a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/4} dx = \int \left(bx^2+a\right)^{\frac{3}{4}} dx$$

input
```
integrate((b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/4} \, dx = \int (bx^2+a)^{\frac{3}{4}} \, dx$$

input
```
integrate((b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \left(a+bx^2\right)^{3/4} \, dx = \frac{x\,(b\,x^2+a)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/4}}$$

input
```
int((a + b*x^2)^(3/4),x)
```

output
```
(x*(a + b*x^2)^(3/4)*hypergeom([-3/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(3/4)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/4} \, dx = \frac{2(b\,x^2+a)^{\frac{3}{4}}\,x}{5} + \frac{3\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)a}{5}$$

input
```
int((b*x^2+a)^(3/4),x)
```

output
```
(2*(a + b*x**2)**(3/4)*x + 3*int((a + b*x**2)**(3/4)/(a + b*x**2),x)*a)/5
```

3.78 $\int \sqrt[4]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 75

$$\int \sqrt[4]{a+bx^2}\,dx=\frac{2}{3}x\sqrt[4]{a+bx^2}+\frac{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\,(a+bx^2)^{3/4}}$$

output
```
2/3*x*(b*x^2+a)^(1/4)+2/3*a^(3/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*ar
ctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \sqrt[4]{a+bx^2}\,dx=\frac{x\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4),x]
```

output
```
(x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a+bx^2}\,dx$$

$$\Big\downarrow \; 211$$

$$\frac{1}{3}a\int \frac{1}{(bx^2+a)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

$$\Big\downarrow \; 231$$

$$\frac{a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

$$\Big\downarrow \; 229$$

$$\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

input
```
Int[(a + b*x^2)^(1/4),x]
```

output
```
(2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{4}}d x$$

input
```
int((b*x^2+a)^(1/4),x)
```

output
```
int((b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \sqrt [4]{a+bx^2}\,dx=\int \left(bx^{2}+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int \sqrt[4]{a+bx^2}\,dx = \sqrt[4]{a}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[4]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \sqrt[4]{a+bx^2}\,dx = \frac{x\,(b\,x^2+a)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/4}}$$

input
```
int((a + b*x^2)^(1/4),x)
```

output
```
(x*(a + b*x^2)^(1/4)*hypergeom([-1/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/4)
```

Reduce [F]

$$\int \sqrt[4]{a+bx^2}\,dx = \frac{2(b\,x^2+a)^{\frac{1}{4}}\,x}{3} + \frac{\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a}{3}$$

input
```
int((b*x^2+a)^(1/4),x)
```

output
```
(2*(a + b*x**2)**(1/4)*x + int((a + b*x**2)**(1/4)/(a + b*x**2),x)*a)/3
```

3.79 $\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 71

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx=\frac{2x}{\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
2*x/(b*x^2+a)^(1/4)-2*a^(1/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b
^(1/2)*x/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx=\frac{x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/4),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{227}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-1/4),x]
```

output
```
((1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(a + b*x^2)^(1/4)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/4),x)
```

output
```
x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/4)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{1/4}}$$

input
```
int(1/(a + b*x^2)^(1/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(a + b*x**2)**(1/4),x)
```

3.80 $\int \frac{1}{(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{2\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

output
```
2*a^(1/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),
2^(1/2))/b^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(-3/4),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \; 231$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} \int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{(a+bx^2)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

input
```
Int[(a + b*x^2)^(-3/4),x]
```

output
```
(2*Sqrt[a]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(3/4),x)
```

output
```
int(1/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

input
```
integrate(1/(b*x**2+a)**(3/4),x)
```

output
```
x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/4)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{3/4}}$$

input
```
int(1/(a + b*x^2)^(3/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(3/4)*hypergeom([1/2, 3/4], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(3/4),x)
```

output
```
int(1/(a + b*x**2)**(3/4),x)
```

3.81 $\int \frac{1}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \frac{2\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
2*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/
a^(1/2)/b^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.52 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \frac{2x - x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-5/4),x]
```

output
```
(2*x - x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/
a)])/(a*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/4}}dx$$
$$\Big\downarrow \text{ 213}$$
$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{a\sqrt[4]{a+bx^2}}$$
$$\Big\downarrow \text{ 212}$$
$$\frac{2\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-5/4),x]
```

output
```
(2*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqr
t[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}d x$$

input
```
int(1/(b*x^2+a)^(5/4),x)
```

output
```
int(1/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^{2}+a\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

input
```
integrate(1/(b*x**2+a)**(5/4),x)
```

output
```
x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{5/4}}$$

input
```
int(1/(a + b*x^2)^(5/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(5/4)*hypergeom([1/2, 5/4], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(5/4)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(5/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)
```

3.82 $\int \frac{1}{(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 78

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \frac{2x}{3a\,(a+bx^2)^{3/4}} + \frac{2\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}\,(a+bx^2)^{3/4}}$$

output
```
2/3*x/a/(b*x^2+a)^(3/4)+2/3*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b
^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/b^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \frac{x\left(2+\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{3a\,(a+bx^2)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(-7/4),x]
```

output
```
(x*(2 + (1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a
)]))/(3*a*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {215, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{1}{(bx^2+a)^{3/4}}dx}{3a}+\frac{2x}{3a(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3a(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}+\frac{2x}{3a(a+bx^2)^{3/4}}$$

input
```
Int[(a + b*x^2)^(-7/4),x]
```

output
```
(2*x)/(3*a*(a + b*x^2)^(3/4)) + (2*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{(b\,x^{2}+a)^{\frac{7}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(7/4),x)
```

output
```
int(1/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx=\int \frac{1}{(bx^{2}+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

input
```
integrate(1/(b*x**2+a)**(7/4),x)
```

output
```
x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/4)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-7/4), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-7/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/4}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{7/4}}$$

input
```
int(1/(a + b*x^2)^(7/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(7/4)*hypergeom([1/2, 7/4], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(7/4)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(7/4),x)
```

output
```
int(1/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)
```

3.83 $\int \frac{1}{(a+bx^2)^{9/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 78

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx=\frac{2x}{5a\left(a+bx^2\right)^{5/4}}+\frac{6\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
2/5*x/a/(b*x^2+a)^(5/4)+6/5*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(
1/2)*x/a^(1/2))),2^(1/2))/a^(3/2)/b^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx=\frac{8ax+6bx^3-3x(a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{5a^2\left(a+bx^2\right)^{5/4}}$$

input
```
Integrate[(a + b*x^2)^(-9/4),x]
```

output
```
(8*a*x + 6*b*x^3 - 3*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1
[1/4, 1/2, 3/2, -((b*x^2)/a)])/(5*a^2*(a + b*x^2)^(5/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {215, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{1}{(bx^2+a)^{5/4}}dx}{5a}+\frac{2x}{5a\,(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{213}$$

$$\frac{3\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{5a^2\sqrt[4]{a+bx^2}}+\frac{2x}{5a\,(a+bx^2)^{5/4}}$$

$$\Big\downarrow \text{212}$$

$$\frac{6\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{2x}{5a\,(a+bx^2)^{5/4}}$$

input
```
Int[(a + b*x^2)^(-9/4),x]
```

output
```
(2*x)/(5*a*(a + b*x^2)^(5/4)) + (6*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)*Sqrt[b]*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{9}{4}}}d x$$

input
```
int(1/(b*x^2+a)^(9/4),x)
```

output
```
int(1/(b*x^2+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

input
```
integrate(1/(b*x**2+a)**(9/4),x)
```

output
```
x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(9/4)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-9/4), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-9/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{9/4}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{9/4}}$$

input
```
int(1/(a + b*x^2)^(9/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(9/4)*hypergeom([1/2, 9/4], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(9/4)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{9/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a^2+2\,(b\,x^2+a)^{\frac{1}{4}}\,ab\,x^2+(b\,x^2+a)^{\frac{1}{4}}\,b^2x^4}dx$$

input
```
int(1/(b*x^2+a)^(9/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*a**2 + 2*(a + b*x**2)**(1/4)*a*b*x**2 + (a + b*
x**2)**(1/4)*b**2*x**4),x)
```

3.84 $\int (a-bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 96

$$\int (a-bx^2)^{5/4}\,dx = \frac{10}{21}ax\sqrt[4]{a-bx^2} + \frac{2}{7}x(a-bx^2)^{5/4} + \frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21\sqrt{b}\,(a-bx^2)^{3/4}}$$

output
```
10/21*a*x*(-b*x^2+a)^(1/4)+2/7*x*(-b*x^2+a)^(5/4)+10/21*a^(5/2)*(1-b*x^2/a
)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(-b
*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.77 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int (a-bx^2)^{5/4}\,dx = \frac{ax\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(5/4),x]
```

output
```
(a*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, (b*x^2)/a])/(1 -
(b*x^2)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {211, 211, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{5/4} \, dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{7}a\int \sqrt[4]{a-bx^2}dx+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{7}a\left(\frac{1}{3}a\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

$$\Big\downarrow \text{231}$$

$$\frac{5}{7}a\left(\frac{a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

$$\Big\downarrow \text{230}$$

$$\frac{5}{7}a\left(\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

input
```
Int[(a - b*x^2)^(5/4),x]
```

output
```
(2*x*(a - b*x^2)^(5/4))/7 + (5*a*((2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(
1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[
b]*(a - b*x^2)^(3/4))))/7
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{5}{4}}d x$$

input
```
int((-b*x^2+a)^(5/4),x)
```

output
```
int((-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^{5/4} dx = \int \left(-bx^2+a\right)^{\frac{5}{4}} dx$$

input
```
integrate((-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(5/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.28

$$\int \left(a-bx^2\right)^{5/4} dx = a^{\frac{5}{4}} x {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)$$

input
```
integrate((-b*x**2+a)**(5/4),x)
```

output
```
a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Maxima [F]

$$\int \left(a-bx^2\right)^{5/4} dx = \int \left(-bx^2+a\right)^{\frac{5}{4}} dx$$

input
```
integrate((-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \left(a-bx^2\right)^{5/4}\,dx=\int \left(-bx^2+a\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.40

$$\int \left(a-bx^2\right)^{5/4}\,dx=\frac{x\left(a-b\,x^2\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{5/4}}$$

input
```
int((a - b*x^2)^(5/4),x)
```

output
```
(x*(a - b*x^2)^(5/4)*hypergeom([-5/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(5/4)
```

Reduce [F]

$$\int \left(a-bx^2\right)^{5/4}\,dx=\frac{16(-b\,x^2+a)^{\frac{1}{4}}\,ax}{21}-\frac{2(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^3}{7}+\frac{5\left(\int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)a^2}{21}$$

input
```
int((-b*x^2+a)^(5/4),x)
```

output
```
(16*(a - b*x**2)**(1/4)*a*x - 6*(a - b*x**2)**(1/4)*b*x**3 + 5*int((a - b*
x**2)**(1/4)/(a - b*x**2),x)*a**2)/21
```

3.85 $\int (a-bx^2)^{3/4} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int (a-bx^2)^{3/4} \, dx = \frac{2}{5}x(a-bx^2)^{3/4} + \frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a-bx^2}}$$

output
```
2/5*x*(-b*x^2+a)^(3/4)+6/5*a^(3/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arc
sin(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(1/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int (a-bx^2)^{3/4} \, dx = \frac{x(a-bx^2)^{3/4} \text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(3/4),x]
```

output
```
(x*(a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b
*x^2)/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {211, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{3/4} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{5}a\int \frac{1}{\sqrt[4]{a-bx^2}}dx+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \text{227}$$

$$\frac{3a\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \text{226}$$

$$\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

input
```
Int[(a - b*x^2)^(3/4),x]
```

output
```
(2*x*(a - b*x^2)^(3/4))/5 + (6*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[Arc
Sin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(-b\,x^{2}+a \right)^{\frac{3}{4}}d x$$

input
```
int((-b*x^2+a)^(3/4),x)
```

output
```
int((-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^{3/4}\,dx=\int \left(-bx^{2}+a\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \left(a-bx^2\right)^{3/4} dx = a^{\frac{3}{4}} x {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)$$

input
```
integrate((-b*x**2+a)**(3/4),x)
```

output
```
a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Maxima [F]

$$\int \left(a-bx^2\right)^{3/4} dx = \int \left(-bx^2+a\right)^{\frac{3}{4}} dx$$

input
```
integrate((-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \left(a-bx^2\right)^{3/4} dx = \int \left(-bx^2+a\right)^{\frac{3}{4}} dx$$

input
```
integrate((-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \left(a-bx^2\right)^{3/4} dx = \frac{x\,(a-b\,x^2)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{3/4}}$$

input
```
int((a - b*x^2)^(3/4),x)
```

output
```
(x*(a - b*x^2)^(3/4)*hypergeom([-3/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(3/4)
```

Reduce [F]

$$\int \left(a-bx^2\right)^{3/4} dx = \frac{2(-b\,x^2+a)^{\frac{3}{4}}\,x}{5} + \frac{3\left(\int \frac{1}{(-b\,x^2+a)^{\frac{1}{4}}}dx\right)a}{5}$$

input
```
int((-b*x^2+a)^(3/4),x)
```

output
```
(2*(a - b*x**2)**(3/4)*x + 3*int((a - b*x**2)**(3/4)/(a - b*x**2),x)*a)/5
```

3.86 $\int \sqrt[4]{a-bx^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int \sqrt[4]{a-bx^2}\,dx=\frac{2}{3}x\sqrt[4]{a-bx^2}+\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\,(a-bx^2)^{3/4}}$$

output
```
2/3*x*(-b*x^2+a)^(1/4)+2/3*a^(3/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*a
rcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \sqrt[4]{a-bx^2}\,dx=\frac{x\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(1/4),x]
```

output
```
(x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b
*x^2)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {211, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a-bx^2}\,dx$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{3}a\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \text{ 230}$$

$$\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

input
```
Int[(a - b*x^2)^(1/4),x]
```

output
```
(2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[Arc
Sin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{1}{4}}d x$$

input
```
int((-b*x^2+a)^(1/4),x)
```

output
```
int((-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{a-bx^2}\,dx=\int \left(-bx^2+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \sqrt[4]{a-bx^2}\,dx = \sqrt[4]{a}x{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)$$

input
```
integrate((-b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Maxima [F]

$$\int \sqrt[4]{a-bx^2}\,dx = \int \left(-bx^2+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{a-bx^2}\,dx = \int \left(-bx^2+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \sqrt[4]{a-bx^2}\,dx=\frac{x\left(a-b\,x^2\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{1/4}}$$

input
```
int((a - b*x^2)^(1/4),x)
```

output
```
(x*(a - b*x^2)^(1/4)*hypergeom([-1/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(1/4)
```

Reduce [F]

$$\int \sqrt[4]{a-bx^2}\,dx=\frac{2(-b\,x^2+a)^{\frac{1}{4}}\,x}{3}+\frac{\left(\int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)a}{3}$$

input
```
int((-b*x^2+a)^(1/4),x)
```

output
```
(2*(a - b*x**2)**(1/4)*x + int((a - b*x**2)**(1/4)/(a - b*x**2),x)*a)/3
```

3.87 $\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\frac{2\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

output
```
2*a^(1/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2
^(1/2))/b^(1/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\frac{x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(-1/4),x]
```

output
```
(x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(a -
 b*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \; 227$$

$$\frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \; 226$$

$$\frac{2\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

input
```
Int[(a - b*x^2)^(-1/4),x]
```

output
```
(2*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[4]{a}}$$

input
```
integrate(1/(-b*x**2+a)**(1/4),x)
```

output
```
x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/4)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\frac{x\,{\left(1-\frac{b\,x^2}{a}\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(a-b\,x^2\right)}^{1/4}}$$

input
```
int(1/(a - b*x^2)^(1/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(1/4)*hypergeom([1/4, 1/2], 3/2, (b*x^2)/a))/(a - b*x^2
)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/(a - b*x**2)**(1/4),x)
```

3.88 $\int \frac{1}{(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

output
```
2*a^(1/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),
2^(1/2))/b^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^2}{a}\right)}{(a-bx^2)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(-3/4),x]
```

output
```
(x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(a -
 b*x^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{3/4}} dx$$

$$\Big\downarrow \text{ 231}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} \int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{(a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{ 230}$$

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

input
```
Int[(a - b*x^2)^(-3/4),x]
```

output
```
(2*Sqrt[a]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}d x$$

input
```
int(1/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

input
```
integrate(1/(-b*x**2+a)**(3/4),x)
```

output
```
x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(3/4)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(a-b\,x^2)^{3/4}}$$

input
```
int(1/(a - b*x^2)^(3/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(3/4)*hypergeom([1/2, 3/4], 3/2, (b*x^2)/a))/(a - b*x^2
)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/(a - b*x**2)**(3/4),x)
```

3.89 $\int \frac{1}{(a-bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{2\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

output
```
2*x/a/(-b*x^2+a)^(1/4)-2*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2
)*x/a^(1/2))),2^(1/2))/a^(1/2)/b^(1/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.70 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \frac{2x - x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{a\sqrt[4]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(-5/4),x]
```

output
```
(2*x - x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]
)/(a*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {215, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{a}$$

$$\Big\downarrow \text{227}$$

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{a\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{226}$$

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

input
```
Int[(a - b*x^2)^(-5/4),x]
```

output
```
(2*x)/(a*(a - b*x^2)^(1/4)) - (2*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

input
```
integrate(1/(-b*x**2+a)**(5/4),x)
```

output
```
x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(5/4)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(a-b\,x^2)^{5/4}}$$

input
```
int(1/(a - b*x^2)^(5/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(5/4)*hypergeom([1/2, 5/4], 3/2, (b*x^2)/a))/(a - b*x^2
)^(5/4)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{1}{4}}\,a-(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(1/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*a - (a - b*x**2)**(1/4)*b*x**2),x)
```

3.90 $\int \frac{1}{(a-bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \frac{2x}{3a\,(a-bx^2)^{3/4}} + \frac{2\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}\,(a-bx^2)^{3/4}}$$

output
```
2/3*x/a/(-b*x^2+a)^(3/4)+2/3*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(
b^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/b^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \frac{x\left(2+\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{3a\,(a-bx^2)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(-7/4),x]
```

output
```
(x*(2 + (1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])
)/(3*a*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {215, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{7/4}} \, dx$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{1}{(a-bx^2)^{3/4}}dx}{3a} + \frac{2x}{3a(a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} \int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3a(a-bx^2)^{3/4}} + \frac{2x}{3a(a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a-bx^2)^{3/4}} + \frac{2x}{3a(a-bx^2)^{3/4}}$$

input
```
Int[(a - b*x^2)^(-7/4),x]
```

output
```
(2*x)/(3*a*(a - b*x^2)^(3/4)) + (2*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{7}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/(-b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.32

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

input
```
integrate(1/(-b*x**2+a)**(7/4),x)
```

output
```
x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(7/4)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-7/4), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-7/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{7/4}\,{}_2F_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(a-b\,x^2)^{7/4}}$$

input
```
int(1/(a - b*x^2)^(7/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(7/4)*hypergeom([1/2, 7/4], 3/2, (b*x^2)/a))/(a - b*x^2
)^(7/4)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}\,a-(-b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(1/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/((a - b*x**2)**(3/4)*a - (a - b*x**2)**(3/4)*b*x**2),x)
```

3.91 $\int \frac{1}{(a-bx^2)^{9/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx=\frac{2x}{5a\left(a-bx^2\right)^{5/4}}+\frac{6x}{5a^2\sqrt[4]{a-bx^2}}-\frac{6\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt{b}\sqrt[4]{a-bx^2}}$$

output
```
2/5*x/a/(-b*x^2+a)^(5/4)+6/5*x/a^2/(-b*x^2+a)^(1/4)-6/5*(1-b*x^2/a)^(1/4)*
EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(3/2)/b^(1/2)/(-b*
x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.92 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx=\frac{8ax-6bx^3-3x(a-bx^2)\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{5a^2\left(a-bx^2\right)^{5/4}}$$

input
```
Integrate[(a - b*x^2)^(-9/4),x]
```

output
```
(8*a*x - 6*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1
[1/4, 1/2, 3/2, (b*x^2)/a])/(5*a^2*(a - b*x^2)^(5/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {215, 215, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\int \frac{1}{(a-bx^2)^{5/4}}dx}{5a}+\frac{2x}{5a\,(a-bx^2)^{5/4}}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\left(\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{a}\right)}{5a}+\frac{2x}{5a\,(a-bx^2)^{5/4}}$$

$$\Big\downarrow \text{ 227}$$

$$\frac{3\left(\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{a\sqrt[4]{a-bx^2}}\right)}{5a}+\frac{2x}{5a\,(a-bx^2)^{5/4}}$$

$$\Big\downarrow \text{ 226}$$

$$\frac{3\left(\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}\right)}{5a}+\frac{2x}{5a\left(a-bx^2\right)^{5/4}}$$

input
```
Int[(a - b*x^2)^(-9/4),x]
```

output
```
(2*x)/(5*a*(a - b*x^2)^(5/4)) + (3*((2*x)/(a*(a - b*x^2)^(1/4)) - (2*(1 -
(b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqr
t[b]*(a - b*x^2)^(1/4))))/(5*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{9}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(9/4),x)
```

output
```
int(1/(-b*x^2+a)^(9/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(9/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.26

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{9}{4}}}$$

input
```
integrate(1/(-b*x**2+a)**(9/4),x)
```

output
```
x*hyper((1/2, 9/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(9/4)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(9/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-9/4), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(9/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-9/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.38

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{9/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(a-b\,x^2)^{9/4}}$$

input
```
int(1/(a - b*x^2)^(9/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(9/4)*hypergeom([1/2, 9/4], 3/2, (b*x^2)/a))/(a - b*x^2
)^(9/4)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{9/4}}\,dx = \int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}} a^2 - 2\left(-b\,x^2+a\right)^{\frac{1}{4}} ab\,x^2 + \left(-b\,x^2+a\right)^{\frac{1}{4}} b^2x^4}dx$$

input
```
int(1/(-b*x^2+a)^(9/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*a**2 - 2*(a - b*x**2)**(1/4)*a*b*x**2 + (a - b*
x**2)**(1/4)*b**2*x**4),x)
```

3.92 $\int \frac{1}{\left(1+a(1-b)^2x^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{1}{\left(1+a(1-b)^2x^2\right)^{3/4}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{a}(1-b)x\right),2\right)}{\sqrt{a}(1-b)}$$

output
```
2*InverseJacobiAM(1/2*arctan(a^(1/2)*(1-b)*x),2^(1/2))/a^(1/2)/(1-b)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(1+a(1-b)^2x^2\right)^{3/4}}\,dx = x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-a(-1+b)^2x^2\right)$$

input
```
Integrate[(1 + a*(1 - b)^2*x^2)^(-3/4),x]
```

output
```
x*Hypergeometric2F1[1/2, 3/4, 3/2, -(a*(-1 + b)^2*x^2)]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a(1-b)^2x^2+1)^{3/4}}dx$$
$$\Big\downarrow \text{ 229}$$
$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{a}(1-b)x\right),2\right)}{\sqrt{a}(1-b)}$$

input
```
Int[(1 + a*(1 - b)^2*x^2)^(-3/4),x]
```

output
```
(2*EllipticF[ArcTan[Sqrt[a]*(1 - b)*x]/2, 2])/(Sqrt[a]*(1 - b))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\left(-1+b\right)^{2} a\,x^{2}\right)$	20

input
```
int(1/(1+a*(1-b)^2*x^2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/2,3/4],[3/2],-(-1+b)^2*a*x^2)
```

Fricas [F]

$$\int \frac{1}{(1+a(1-b)^2x^2)^{3/4}}\,dx = \int \frac{1}{\left(a(b-1)^2x^2+1\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1+a*(1-b)^2*x^2)^(3/4),x, algorithm="fricas")
```

output
```
integral(((a*b^2 - 2*a*b + a)*x^2 + 1)^(-3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+a(1-b)^2x^2)^{3/4}}\,dx = x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{3}{2} \end{matrix}\middle| ax^2e^{i\pi}\operatorname{polar\_lift}^2(1-b)\right)$$

input
```
integrate(1/(1+a*(1-b)**2*x**2)**(3/4),x)
```

output
```
x*hyper((1/2, 3/4), (3/2,), a*x**2*exp_polar(I*pi)*polar_lift(1 - b)**2)
```

Maxima [F]

$$\int \frac{1}{(1+a(1-b)^2x^2)^{3/4}}\,dx = \int \frac{1}{\left(a(b-1)^2x^2+1\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1+a*(1-b)^2*x^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((a*(b - 1)^2*x^2 + 1)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(1+a(1-b)^2x^2)^{3/4}}\,dx = \int \frac{1}{\left(a(b-1)^2x^2+1\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1+a*(1-b)^2*x^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((a*(b - 1)^2*x^2 + 1)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.55

$$\int \frac{1}{(1+a(1-b)^2x^2)^{3/4}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -a\,x^2\,(b-1)^2\right)$$

input
```
int(1/(a*x^2*(b - 1)^2 + 1)^(3/4),x)
```

output
```
x*hypergeom([1/2, 3/4], 3/2, -a*x^2*(b - 1)^2)
```

Reduce [F]

$$\int \frac{1}{(1+a(1-b)^2x^2)^{3/4}}\,dx = \int \frac{1}{(a\,b^2x^2-2ab\,x^2+a\,x^2+1)^{\frac{3}{4}}}dx$$

input
```
int(1/(1+a*(1-b)^2*x^2)^(3/4),x)
```

output
```
int(1/(a*b**2*x**2 - 2*a*b*x**2 + a*x**2 + 1)**(3/4),x)
```

3.93 $\int \frac{1}{\left(1-a(1-b)^2x^2\right)^{3/4}} dx$

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \frac{1}{\left(1-a(1-b)^2x^2\right)^{3/4}} dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{a}(1-b)x\right),2\right)}{\sqrt{a}(1-b)}$$

output
```
2*InverseJacobiAM(1/2*arcsin(a^(1/2)*(1-b)*x),2^(1/2))/a^(1/2)/(1-b)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(1-a(1-b)^2x^2\right)^{3/4}} dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{a}(-1+b)x\right),2\right)}{\sqrt{a}(-1+b)}$$

input
```
Integrate[(1 - a*(1 - b)^2*x^2)^(-3/4),x]
```

output
```
(2*EllipticF[ArcSin[Sqrt[a]*(-1 + b)*x]/2, 2])/(Sqrt[a]*(-1 + b))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-a(1-b)^2x^2)^{3/4}}\,dx$$

$$\Big\downarrow \; 230$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{a}(1-b)x\right),2\right)}{\sqrt{a}(1-b)}$$

input
```
Int[(1 - a*(1 - b)^2*x^2)^(-3/4),x]
```

output
```
(2*EllipticF[ArcSin[Sqrt[a]*(1 - b)*x]/2, 2])/(Sqrt[a]*(1 - b))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.58

method	result	size
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],(-1+b)^2\,a\,x^2\right)$	19

input
```
int(1/(1-a*(1-b)^2*x^2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/2,3/4],[3/2],(-1+b)^2*a*x^2)
```

Fricas [F]

$$\int \frac{1}{(1-a(1-b)^2x^2)^{3/4}}\,dx = \int \frac{1}{\left(-a(b-1)^2x^2+1\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1-a*(1-b)^2*x^2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-(a*b^2 - 2*a*b + a)*x^2 + 1)^(1/4)/((a*b^2 - 2*a*b + a)*x^2 -
1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{(1-a(1-b)^2x^2)^{3/4}}\,dx = x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{3}{2}\end{matrix}\right|ax^2e^{2i\pi}\operatorname{polar\_lift}^2\left(1-b\right)\right)$$

input
```
integrate(1/(1-a*(1-b)**2*x**2)**(3/4),x)
```

output
```
x*hyper((1/2, 3/4), (3/2,), a*x**2*exp_polar(2*I*pi)*polar_lift(1 - b)**2)
```

Maxima [F]

$$\int \frac{1}{(1-a(1-b)^2x^2)^{3/4}}\,dx = \int \frac{1}{\left(-a(b-1)^2x^2+1\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1-a*(1-b)^2*x^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-a*(b - 1)^2*x^2 + 1)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(1-a(1-b)^2x^2)^{3/4}}\,dx = \int \frac{1}{\left(-a(b-1)^2x^2+1\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(1-a*(1-b)^2*x^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((-a*(b - 1)^2*x^2 + 1)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{(1-a(1-b)^2x^2)^{3/4}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ a\,x^2\,(b-1)^2\right)$$

input
```
int(1/(1 - a*x^2*(b - 1)^2)^(3/4),x)
```

output
```
x*hypergeom([1/2, 3/4], 3/2, a*x^2*(b - 1)^2)
```

Reduce [F]

$$\int \frac{1}{(1-a(1-b)^2x^2)^{3/4}}\,dx = \int \frac{1}{(-a\,b^2x^2+2ab\,x^2-a\,x^2+1)^{\frac{3}{4}}}dx$$

input
```
int(1/(1-a*(1-b)^2*x^2)^(3/4),x)
```

output
```
int(1/( - a*b**2*x**2 + 2*a*b*x**2 - a*x**2 + 1)**(3/4),x)
```

3.94 $\int (a+bx^2)^{7/5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int (a+bx^2)^{7/5}\,dx = \frac{ax(a+bx^2)^{2/5}\,\text{Hypergeometric2F1}\left(-\frac{7}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/5}}$$

output
```
a*x*(b*x^2+a)^(2/5)*hypergeom([-7/5, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(2/5
)
```

Mathematica [A] (verified)

Time = 7.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{7/5}\,dx = \frac{ax(a+bx^2)^{2/5}\,\text{Hypergeometric2F1}\left(-\frac{7}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/5}}$$

input
```
Integrate[(a + b*x^2)^(7/5),x]
```

output
```
(a*x*(a + b*x^2)^(2/5)*Hypergeometric2F1[-7/5, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(2/5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{7/5} dx$$

$$\Big\downarrow \; 238$$

$$\frac{a\left(a+bx^2\right)^{2/5}\int \left(\frac{bx^2}{a}+1\right)^{7/5}dx}{\left(\frac{bx^2}{a}+1\right)^{2/5}}$$

$$\Big\downarrow \; 237$$

$$\frac{ax\left(a+bx^2\right)^{2/5}\text{Hypergeometric2F1}\left(-\frac{7}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{2/5}}$$

input
```
Int[(a + b*x^2)^(7/5),x]
```

output
```
(a*x*(a + b*x^2)^(2/5)*Hypergeometric2F1[-7/5, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(2/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{7}{5}}dx$$

input
```
int((b*x^2+a)^(7/5),x)
```

output
```
int((b*x^2+a)^(7/5),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{7/5}\,dx=\int \left(bx^2+a\right)^{\frac{7}{5}}\,dx$$

input
```
integrate((b*x^2+a)^(7/5),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \left(a+bx^2\right)^{7/5}\,dx=a^{\frac{7}{5}}x\,{}_2F_1\left(\begin{matrix}-\frac{7}{5},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(7/5),x)
```

output
```
a**(7/5)*x*hyper((-7/5, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int (a+bx^2)^{7/5}\,dx=\int (bx^2+a)^{\frac{7}{5}}\,dx$$

input
```
integrate((b*x^2+a)^(7/5),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/5), x)
```

Giac [F]

$$\int (a+bx^2)^{7/5}\,dx=\int (bx^2+a)^{\frac{7}{5}}\,dx$$

input
```
integrate((b*x^2+a)^(7/5),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/5), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int (a+bx^2)^{7/5}\,dx=\frac{x\,(b\,x^2+a)^{7/5}\,{}_2\mathrm{F}_1\left(-\frac{7}{5},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{7/5}}$$

input
```
int((a + b*x^2)^(7/5),x)
```

output
```
(x*(a + b*x^2)^(7/5)*hypergeom([-7/5, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(7/5)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{7/5} dx = \left(\int \left(b\,x^2+a\right)^{\frac{2}{5}} dx\right) a + \left(\int \left(b\,x^2+a\right)^{\frac{2}{5}} x^2 dx\right) b$$

input
```
int((b*x^2+a)^(7/5),x)
```

output
```
int((a + b*x**2)**(2/5),x)*a + int((a + b*x**2)**(2/5)*x**2,x)*b
```

3.95 $\int (a+bx^2)^{4/5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a+bx^2)^{4/5}\,dx = \frac{x(a+bx^2)^{4/5}\,\text{Hypergeometric2F1}\left(-\frac{4}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{4/5}}$$

output
```
x*(b*x^2+a)^(4/5)*hypergeom([-4/5, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(4/5)
```

Mathematica [A] (verified)

Time = 7.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{4/5}\,dx = \frac{x(a+bx^2)^{4/5}\,\text{Hypergeometric2F1}\left(-\frac{4}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{4/5}}$$

input
```
Integrate[(a + b*x^2)^(4/5),x]
```

output
```
(x*(a + b*x^2)^(4/5)*Hypergeometric2F1[-4/5, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(4/5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{4/5} \, dx$$

$$\Big\downarrow \text{ 238}$$

$$\frac{(a+bx^2)^{4/5} \int \left(\frac{bx^2}{a}+1\right)^{4/5} dx}{\left(\frac{bx^2}{a}+1\right)^{4/5}}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{x(a+bx^2)^{4/5} \text{Hypergeometric2F1}\left(-\frac{4}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{4/5}}$$

input
```
Int[(a + b*x^2)^(4/5),x]
```

output
```
(x*(a + b*x^2)^(4/5)*Hypergeometric2F1[-4/5, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(4/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{4}{5}}dx$$

input
```
int((b*x^2+a)^(4/5),x)
```

output
```
int((b*x^2+a)^(4/5),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{4/5}\,dx=\int \left(bx^2+a\right)^{\frac{4}{5}}\,dx$$

input
```
integrate((b*x^2+a)^(4/5),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{4/5}\,dx=a^{\frac{4}{5}}x\,{}_2F_1\left(\begin{matrix} -\frac{4}{5},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(4/5),x)
```

output
```
a**(4/5)*x*hyper((-4/5, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{4/5} dx = \int \left(bx^2+a\right)^{\frac{4}{5}} dx$$

input
```
integrate((b*x^2+a)^(4/5),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/5), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{4/5} dx = \int \left(bx^2+a\right)^{\frac{4}{5}} dx$$

input
```
integrate((b*x^2+a)^(4/5),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/5), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{4/5} dx = \frac{x\,\left(b\,x^2+a\right)^{4/5}\,{}_2F_1\left(-\frac{4}{5},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{4/5}}$$

input
```
int((a + b*x^2)^(4/5),x)
```

output
```
(x*(a + b*x^2)^(4/5)*hypergeom([-4/5, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(4/5)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{4/5} dx = \int \left(b\,x^2+a\right)^{\frac{4}{5}} dx$$

input
```
int((b*x^2+a)^(4/5),x)
```

output
```
int((a + b*x**2)**(4/5),x)
```

3.96 $\int (a+bx^2)^{2/5}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a+bx^2)^{2/5}\,dx=\frac{x(a+bx^2)^{2/5}\,\text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/5}}$$

output
```
x*(b*x^2+a)^(2/5)*hypergeom([-2/5, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(2/5)
```

Mathematica [A] (verified)

Time = 6.80 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{2/5}\,dx=\frac{x(a+bx^2)^{2/5}\,\text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/5}}$$

input
```
Integrate[(a + b*x^2)^(2/5),x]
```

output
```
(x*(a + b*x^2)^(2/5)*Hypergeometric2F1[-2/5, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(2/5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{2/5} dx$$

$$\Big\downarrow \ 238$$

$$\frac{\left(a+bx^2\right)^{2/5} \int \left(\frac{bx^2}{a}+1\right)^{2/5} dx}{\left(\frac{bx^2}{a}+1\right)^{2/5}}$$

$$\Big\downarrow \ 237$$

$$\frac{x\left(a+bx^2\right)^{2/5} \text{Hypergeometric2F1}\left(-\frac{2}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{2/5}}$$

input
```
Int[(a + b*x^2)^(2/5),x]
```

output
```
(x*(a + b*x^2)^(2/5)*Hypergeometric2F1[-2/5, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(2/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{2}{5}}d x$$

input
```
int((b*x^2+a)^(2/5),x)
```

output
```
int((b*x^2+a)^(2/5),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{2/5}\,dx=\int \left(bx^2+a\right)^{\frac{2}{5}}\,dx$$

input
```
integrate((b*x^2+a)^(2/5),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{2/5}\,dx=a^{\frac{2}{5}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{5},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(2/5),x)
```

output
```
a**(2/5)*x*hyper((-2/5, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{2/5} dx = \int \left(bx^2+a\right)^{\frac{2}{5}} dx$$

input
```
integrate((b*x^2+a)^(2/5),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(2/5), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{2/5} dx = \int \left(bx^2+a\right)^{\frac{2}{5}} dx$$

input
```
integrate((b*x^2+a)^(2/5),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(2/5), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{2/5} dx = \frac{x\,\left(b\,x^2+a\right)^{2/5}\,{}_2\mathrm{F}_1\left(-\frac{2}{5},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{2/5}}$$

input
```
int((a + b*x^2)^(2/5),x)
```

output
```
(x*(a + b*x^2)^(2/5)*hypergeom([-2/5, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(2/5)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{2/5}\,dx=\int \left(b\,x^2+a\right)^{\frac{2}{5}}dx$$

input
```
int((b*x^2+a)^(2/5),x)
```

output
```
int((a + b*x**2)**(2/5),x)
```

3.97 $\int \sqrt[5]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt[5]{a+bx^2}\,dx=\frac{x\sqrt[5]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[5]{1+\frac{bx^2}{a}}}$$

output
```
x*(b*x^2+a)^(1/5)*hypergeom([-1/5, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(1/5)
```

Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sqrt[5]{a+bx^2}\,dx=\frac{x\sqrt[5]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[5]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/5),x]
```

output
```
(x*(a + b*x^2)^(1/5)*Hypergeometric2F1[-1/5, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[5]{a+bx^2}\,dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\sqrt[5]{a+bx^2}\int \sqrt[5]{\frac{bx^2}{a}+1}dx}{\sqrt[5]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\sqrt[5]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[5]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/5),x]
```

output
```
(x*(a + b*x^2)^(1/5)*Hypergeometric2F1[-1/5, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{5}}dx$$

input
```
int((b*x^2+a)^(1/5),x)
```

output
```
int((b*x^2+a)^(1/5),x)
```

Fricas [F]

$$\int \sqrt[5]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{5}}\,dx$$

input
```
integrate((b*x^2+a)^(1/5),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \sqrt[5]{a+bx^2}\,dx = \sqrt[5]{a}x\,{}_2F_1\left(\begin{matrix} -\frac{1}{5},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(1/5),x)
```

output
```
a**(1/5)*x*hyper((-1/5, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[5]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{5}}\,dx$$

input
```
integrate((b*x^2+a)^(1/5),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/5), x)
```

Giac [F]

$$\int \sqrt[5]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{5}}\,dx$$

input
```
integrate((b*x^2+a)^(1/5),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/5), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt[5]{a+bx^2}\,dx=\frac{x\,\left(b\,x^2+a\right)^{1/5}\,{}_2\mathrm{F}_1\left(-\frac{1}{5},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/5}}$$

input
```
int((a + b*x^2)^(1/5),x)
```

output
```
(x*(a + b*x^2)^(1/5)*hypergeom([-1/5, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/5)
```

Reduce [F]

$$\int \sqrt[5]{a+bx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{1}{5}}dx$$

input
```
int((b*x^2+a)^(1/5),x)
```

output
```
int((a + b*x**2)**(1/5),x)
```

3.98 $\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx = \frac{x\sqrt[5]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[5]{a+bx^2}}$$

output
```
x*(1+b*x^2/a)^(1/5)*hypergeom([1/5, 1/2],[3/2],-b*x^2/a)/(b*x^2+a)^(1/5)
```

Mathematica [A] (verified)

Time = 6.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx = \frac{x\sqrt[5]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[5]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/5),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/5)*Hypergeometric2F1[1/5, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[5]{a+bx^2}}dx$$
$$\Big\downarrow \text{238}$$
$$\frac{\sqrt[5]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[5]{\frac{bx^2}{a}+1}}dx}{\sqrt[5]{a+bx^2}}$$
$$\Big\downarrow \text{237}$$
$$\frac{x\sqrt[5]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{5},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[5]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-1/5),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/5)*Hypergeometric2F1[1/5, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{5}}}dx$$

input
```
int(1/(b*x^2+a)^(1/5),x)
```

output
```
int(1/(b*x^2+a)^(1/5),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/5),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{5},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[5]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/5),x)
```

output
```
x*hyper((1/5, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/5)
```

Maxima [F]

$$\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/5),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/5), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/5),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/5), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{1/5}\,{}_2\mathrm{F}_1\left(\frac{1}{5},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{1/5}}$$

input
```
int(1/(a + b*x^2)^(1/5),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/5)*hypergeom([1/5, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/5)
```

Reduce [F]

$$\int \frac{1}{\sqrt[5]{a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{5}}}dx$$

input
```
int(1/(b*x^2+a)^(1/5),x)
```

output
```
int(1/(a + b*x**2)**(1/5),x)
```

3.99 $\int \frac{1}{(a+bx^2)^{3/5}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{3/5}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/5}}$$

output
```
x*(1+b*x^2/a)^(3/5)*hypergeom([1/2, 3/5],[3/2],-b*x^2/a)/(b*x^2+a)^(3/5)
```

Mathematica [A] (verified)

Time = 6.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{3/5}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/5}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/5}}$$

input
```
Integrate[(a + b*x^2)^(-3/5),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/5)*Hypergeometric2F1[1/2, 3/5, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(3/5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/5}} \, dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/5} \int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/5}}dx}{(a+bx^2)^{3/5}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/5} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/5}}$$

input
```
Int[(a + b*x^2)^(-3/5),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/5)*Hypergeometric2F1[1/2, 3/5, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(3/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{5}}}dx$$

input
```
int(1/(b*x^2+a)^(3/5),x)
```

output
```
int(1/(b*x^2+a)^(3/5),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/5),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-3/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+bx^2)^{3/5}}\,dx = \frac{x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{5}\\\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{5}}}$$

input
```
integrate(1/(b*x**2+a)**(3/5),x)
```

output
```
x*hyper((1/2, 3/5), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/5)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/5),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-3/5), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/5),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-3/5), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{3/5}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{3/5}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{5};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{3/5}}$$

input
```
int(1/(a + b*x^2)^(3/5),x)
```

output
```
(x*((b*x^2)/a + 1)^(3/5)*hypergeom([1/2, 3/5], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(3/5)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/5}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{5}}}dx$$

input
```
int(1/(b*x^2+a)^(3/5),x)
```

output
```
int((a + b*x**2)**(2/5)/(a + b*x**2),x)
```

3.100 $\int \frac{1}{(a+bx^2)^{4/5}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{4/5}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{4/5}}$$

output
```
x*(1+b*x^2/a)^(4/5)*hypergeom([1/2, 4/5],[3/2],-b*x^2/a)/(b*x^2+a)^(4/5)
```

Mathematica [A] (verified)

Time = 6.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{4/5}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{4/5}}$$

input
```
Integrate[(a + b*x^2)^(-4/5),x]
```

output
```
(x*(1 + (b*x^2)/a)^(4/5)*Hypergeometric2F1[1/2, 4/5, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(4/5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx$$

$$\Big\downarrow \text{ 238}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{4/5}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{4/5}}dx}{(a+bx^2)^{4/5}}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{4/5}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{4/5}}$$

input
```
Int[(a + b*x^2)^(-4/5),x]
```

output
```
(x*(1 + (b*x^2)/a)^(4/5)*Hypergeometric2F1[1/2, 4/5, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(4/5)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{4}{5}}}d x$$

input
```
int(1/(b*x^2+a)^(4/5),x)
```

output
```
int(1/(b*x^2+a)^(4/5),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/5),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-4/5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{5}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{5}}}$$

input
```
integrate(1/(b*x**2+a)**(4/5),x)
```

output
```
x*hyper((1/2, 4/5), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/5)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/5),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-4/5), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/5),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-4/5), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{4/5}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{4}{5};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{4/5}}$$

input
```
int(1/(a + b*x^2)^(4/5),x)
```

output
```
(x*((b*x^2)/a + 1)^(4/5)*hypergeom([1/2, 4/5], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(4/5)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{4/5}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{4}{5}}}dx$$

input
```
int(1/(b*x^2+a)^(4/5),x)
```

output
```
int((a + b*x**2)**(1/5)/(a + b*x**2),x)
```

3.101 $\int \frac{1}{\left(a+bx^2\right)^{6/5}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{\left(a+bx^2\right)^{6/5}}\,dx=\frac{x\sqrt[5]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{6}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[5]{a+bx^2}}$$

output
```
x*(1+b*x^2/a)^(1/5)*hypergeom([1/2, 6/5],[3/2],-b*x^2/a)/a/(b*x^2+a)^(1/5)
```

Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+bx^2\right)^{6/5}}\,dx=\frac{x\sqrt[5]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{6}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[5]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-6/5),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/5)*Hypergeometric2F1[1/2, 6/5, 3/2, -((b*x^2)/a)])/(
a*(a + b*x^2)^(1/5))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{6/5}}dx$$
$$\Big\downarrow \ 238$$
$$\frac{\sqrt[5]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{6/5}}dx}{a\sqrt[5]{a+bx^2}}$$
$$\Big\downarrow \ 237$$
$$\frac{x\sqrt[5]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{6}{5},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[5]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-6/5),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/5)*Hypergeometric2F1[1/2, 6/5, 3/2, -((b*x^2)/a)])/(
a*(a + b*x^2)^(1/5))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{6}{5}}}dx$$

input
```
int(1/(b*x^2+a)^(6/5),x)
```

output
```
int(1/(b*x^2+a)^(6/5),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{6/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{6}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(6/5),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/5)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+bx^2)^{6/5}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{6}{5} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{6}{5}}}$$

input
```
integrate(1/(b*x**2+a)**(6/5),x)
```

output
```
x*hyper((1/2, 6/5), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(6/5)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{6/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{6}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(6/5),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-6/5), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{6/5}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{6}{5}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(6/5),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-6/5), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{6/5}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{6/5}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{6}{5};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{6/5}}$$

input
```
int(1/(a + b*x^2)^(6/5),x)
```

output
```
(x*((b*x^2)/a + 1)^(6/5)*hypergeom([1/2, 6/5], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(6/5)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{6/5}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{5}}\,a+(b\,x^2+a)^{\frac{1}{5}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(6/5),x)
```

output
```
int(1/((a + b*x**2)**(1/5)*a + (a + b*x**2)**(1/5)*b*x**2),x)
```

3.102 $\int (a+bx^2)^{7/6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 285

$$\int (a+bx^2)^{7/6}\,dx = \frac{21}{40}ax\sqrt[6]{a+bx^2}+\frac{3}{10}x(a+bx^2)^{7/6}$$
$$+\frac{7\,3^{3/4}a^{5/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{80bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
21/40*a*x*(b*x^2+a)^(1/6)+3/10*x*(b*x^2+a)^(7/6)+7/80*3^(3/4)*a^(5/3)*(b*x
^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b
*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacob
iAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x
^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x
^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.16

$$\int \left(a+bx^2\right)^{7/6} dx = \frac{ax\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(7/6),x]
```

output
```
(a*x*(a + b*x^2)^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(1/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {211, 211, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{7/6} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{10}a\int \sqrt[6]{bx^2+a}dx+\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{10}a\left(\frac{1}{4}a\int \frac{1}{\left(bx^2+a\right)^{5/6}}dx+\frac{3}{4}x\sqrt[6]{a+bx^2}\right)+\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \text{236}$$

$$\frac{7}{10}a\left(\frac{a\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}+\frac{3}{4}x\sqrt[6]{a+bx^2}\right)+\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

$\downarrow$ 234

$$\frac{7}{10}a\left(\frac{3}{4}x\sqrt[6]{a+bx^2}-\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8bx\sqrt[3]{\frac{a}{a+bx^2}}}\right)+$$

$$\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

$\downarrow$ 760

$$\frac{7}{10}a\left(\frac{3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\right.}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.$$

$$\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

input
```
Int[(a + b*x^2)^(7/6),x]
```

output
```
(3*x*(a + b*x^2)^(7/6))/10 + (7*a*((3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sq
rt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*Ellipti
cF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^
(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/10
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{7}{6}}d x$$

input
```
int((b*x^2+a)^(7/6),x)
```

output
```
int((b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{7/6}\,dx=\int \left(bx^2+a\right)^{\frac{7}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \left(a+bx^2\right)^{7/6}\,dx=a^{\frac{7}{6}}x\,{}_2F_1\left(\begin{matrix}-\frac{7}{6},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(7/6),x)
```

output
```
a**(7/6)*x*hyper((-7/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{7/6}\,dx=\int \left(bx^2+a\right)^{\frac{7}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6), x)
```

Giac [F]

$$\int (a+bx^2)^{7/6}\,dx=\int (bx^2+a)^{\frac{7}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int (a+bx^2)^{7/6}\,dx=\frac{x\,(b\,x^2+a)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{7/6}}$$

input
```
int((a + b*x^2)^(7/6),x)
```

output
```
(x*(a + b*x^2)^(7/6)*hypergeom([-7/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(7/6)
```

Reduce [F]

$$\int (a+bx^2)^{7/6}\,dx=\left(\int (b\,x^2+a)^{\frac{1}{6}}\,dx\right)a+\left(\int (b\,x^2+a)^{\frac{1}{6}}\,x^2dx\right)b$$

input
```
int((b*x^2+a)^(7/6),x)
```

output
```
int((a + b*x**2)**(1/6),x)*a + int((a + b*x**2)**(1/6)*x**2,x)*b
```

3.103 $\int \sqrt[6]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 268

$$\int \sqrt[6]{a+bx^2}\,dx = \frac{3}{4}x\sqrt[6]{a+bx^2} + \frac{3^{3/4}a^{2/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{\ldots}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}}\right.\right.}{8bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/4*x*(b*x^2+a)^(1/6)+1/8*3^(3/4)*a^(2/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+
a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3
^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/
2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/
4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(
1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \sqrt[6]{a+bx^2}\,dx=\frac{x\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/6),x]
```

output
```
(x*(a + b*x^2)^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[6]{a+bx^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{4}a\int \frac{1}{(bx^2+a)^{5/6}}dx+\frac{3}{4}x\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{236}$$

$$\frac{a\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}+\frac{3}{4}x\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{234}$$

$$\frac{3}{4}x\sqrt[6]{a+bx^2}-\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8bx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$\downarrow$ 760

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1\cdots}}{-\sqrt[3]{1\cdots}}\right.\right.}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

$$\frac{3}{4}x\sqrt[6]{a+bx^2}$$

input
```
Int[(a + b*x^2)^(1/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a
 + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(
1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/
2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^
2)/(a + b*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{6}}d x$$

input
```
int((b*x^2+a)^(1/6),x)
```

output
```
int((b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \sqrt[6]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \sqrt[6]{a+bx^2}\,dx=\sqrt[6]{a}x\,{}_2F_1\left(\begin{matrix} -\frac{1}{6},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(1/6),x)
```

output
```
a**(1/6)*x*hyper((-1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[6]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6), x)
```

Giac [F]

$$\int \sqrt[6]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.14

$$\int \sqrt[6]{a+bx^2}\,dx = \frac{x\,\left(b\,x^2+a\right)^{1/6}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/6}}$$

input
```
int((a + b*x^2)^(1/6),x)
```

output
```
(x*(a + b*x^2)^(1/6)*hypergeom([-1/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/6)
```

Reduce [F]

$$\int \sqrt[6]{a+bx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{1}{6}}\,dx$$

input
```
int((b*x^2+a)^(1/6),x)
```

output
```
int((a + b*x**2)**(1/6),x)
```

3.104 $\int \frac{1}{(a+bx^2)^{5/6}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 251

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \frac{3^{3/4}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\cdots\right.\right.}{2\sqrt[3]{a}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/2*3^(3/4)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b
*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1
/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-
(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/a^(1/3)/b/x/(-(b*x^
2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.18

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx=\frac{x\left(1+\frac{bx^2}{a}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(-5/6),x]
```

output
```
(x*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(5/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx$$

$$\Big\downarrow \text{236}$$

$$\frac{\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{234}$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{2bx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1\cdots}}{-\sqrt[3]{1\cdots}}\right.\right.}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

input
```
Int[(a + b*x^2)^(-5/6),x]
```

output
```
(3^(3/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a + b
*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a
+ b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{6}}}d x$$

input
```
int(1/(b*x^2+a)^(5/6),x)
```

output
```
int(1/(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-5/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.10

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{6} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}}$$

input
```
integrate(1/(b*x**2+a)**(5/6),x)
```

output
```
x*hyper((1/2, 5/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/6)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-5/6), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-5/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/6}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{5/6}}$$

input
```
int(1/(a + b*x^2)^(5/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(5/6)*hypergeom([1/2, 5/6], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(5/6)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \int \frac{(b\,x^2+a)^{\frac{2}{3}}}{\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(5/6),x)
```

output
```
int((a + b*x**2)**(2/3)/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)
```

3.105 $\int \frac{1}{(a+bx^2)^{11/6}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 271

$$\int \frac{1}{(a+bx^2)^{11/6}} \, dx = \frac{3x}{5a(a+bx^2)^{5/6}} + \frac{3^{3/4}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right.\right.}{5a^{4/3}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
3/5*x/a/(b*x^2+a)^(5/6)+1/5*3^(3/4)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/
3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2)
)*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b
*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1
/2))/a^(4/3)/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3
^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \frac{1}{(a+bx^2)^{11/6}}\,dx = \frac{x\left(3+2\left(1+\frac{bx^2}{a}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{5a\,(a+bx^2)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(-11/6),x]
```

output
```
(x*(3 + 2*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)
/a)]))/(5*a*(a + b*x^2)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {215, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{11/6}}\,dx$$

$$\Big\downarrow \text{215}$$

$$\frac{2\int \frac{1}{(bx^2+a)^{5/6}}dx}{5a}+\frac{3x}{5a\,(a+bx^2)^{5/6}}$$

$$\Big\downarrow \text{236}$$

$$\frac{2\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{5a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}+\frac{3x}{5a\,(a+bx^2)^{5/6}}$$

$$\Big\downarrow \text{234}$$

$$\frac{3x}{5a\left(a+bx^2\right)^{5/6}}-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{5abx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$\Big\downarrow \; 760$$

$$\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\cdots}}{-\sqrt[3]{\cdots}}\cdots\right.\right.}{5abx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\cdots}$$

$$\frac{3x}{5a\left(a+bx^2\right)^{5/6}}$$

input

```
Int[(a + b*x^2)^(-11/6),x]
```

output

```
(3*x)/(5*a*(a + b*x^2)^(5/6)) + (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2
)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sq
rt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
(1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (
b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))
], -7 + 4*Sqrt[3]])/(5*a*b*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^
2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{11}{6}}}dx$$

input
```
int(1/(b*x^2+a)^(11/6),x)
```

output
```
int(1/(b*x^2+a)^(11/6),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{11/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.09

$$\int \frac{1}{(a+bx^2)^{11/6}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{11}{6} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{11}{6}}}$$

input
```
integrate(1/(b*x**2+a)**(11/6),x)
```

output
```
x*hyper((1/2, 11/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(11/6)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{11/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-11/6), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{11/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-11/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.14

$$\int \frac{1}{(a+bx^2)^{11/6}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{11/6}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{11/6}}$$

input
```
int(1/(a + b*x^2)^(11/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(11/6)*hypergeom([1/2, 11/6], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(11/6)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{11/6}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{6}}\,a+(b\,x^2+a)^{\frac{5}{6}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(11/6),x)
```

output
```
int(1/((a + b*x**2)**(5/6)*a + (a + b*x**2)**(5/6)*b*x**2),x)
```

3.106 $\int \frac{1}{(a+bx^2)^{17/6}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 290

$$\int \frac{1}{(a+bx^2)^{17/6}}\,dx = \frac{3x}{11a\,(a+bx^2)^{11/6}} + \frac{24x}{55a^2\,(a+bx^2)^{5/6}}$$
$$+\frac{8\,3^{3/4}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+b}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+b}}\right.\right.}{55a^{7/3}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/11*x/a/(b*x^2+a)^(11/6)+24/55*x/a^2/(b*x^2+a)^(5/6)+8/55*3^(3/4)*(b*x^2+
a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM
(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+
a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/a^(7/3)/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)
-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.79 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.25

$$\int \frac{1}{(a+bx^2)^{17/6}}\,dx = \frac{39ax+24bx^3+16x(a+bx^2)\left(1+\frac{bx^2}{a}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{bx^2}{a}\right)}{55a^2(a+bx^2)^{11/6}}$$

input
```
Integrate[(a + b*x^2)^(-17/6),x]
```

output
```
(39*a*x + 24*b*x^3 + 16*x*(a + b*x^2)*(1 + (b*x^2)/a)^(5/6)*Hypergeometric
2F1[1/2, 5/6, 3/2, -((b*x^2)/a)])/(55*a^2*(a + b*x^2)^(11/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {215, 215, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{17/6}}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{8\int \frac{1}{(bx^2+a)^{11/6}}dx}{11a}+\frac{3x}{11a(a+bx^2)^{11/6}}$$

$$\Big\downarrow \text{215}$$

$$\frac{8\left(\frac{2\int \frac{1}{(bx^2+a)^{5/6}}dx}{5a}+\frac{3x}{5a(a+bx^2)^{5/6}}\right)}{11a}+\frac{3x}{11a(a+bx^2)^{11/6}}$$

$$\Big\downarrow \text{236}$$

$$\frac{8\left(\frac{2\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{5a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}+\frac{3x}{5a(a+bx^2)^{5/6}}\right)}{11a}+\frac{3x}{11a\left(a+bx^2\right)^{11/6}}$$

$\big\downarrow$ 234

$$\frac{8\left(\frac{3x}{5a(a+bx^2)^{5/6}}-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{5abx\sqrt[3]{\frac{a}{a+bx^2}}}\right)}{11a}+\frac{3x}{11a\left(a+bx^2\right)^{11/6}}$$

$\big\downarrow$ 760

$$\frac{8\left(\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}\right.\right.}{5abx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{11a}$$

$$\frac{3x}{11a\left(a+bx^2\right)^{11/6}}$$

input
```
Int[(a + b*x^2)^(-17/6),x]
```

output
```
(3*x)/(11*a*(a + b*x^2)^(11/6)) + (8*((3*x)/(5*a*(a + b*x^2)^(5/6)) + (2*3
^(3/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1
 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*a*b*x*(a/(a +
 b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(
a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2])))/
(11*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{17}{6}}}dx$$

input
```
int(1/(b*x^2+a)^(17/6),x)
```

output
```
int(1/(b*x^2+a)^(17/6),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{17/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{17}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(17/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.08

$$\int \frac{1}{(a+bx^2)^{17/6}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{17}{6}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{17}{6}}}$$

input
```
integrate(1/(b*x**2+a)**(17/6),x)
```

output
```
x*hyper((1/2, 17/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(17/6)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{17/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{17}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(17/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-17/6), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{17/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{17}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(17/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-17/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \frac{1}{(a+bx^2)^{17/6}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{17/6}\,{}_2F_1\left(\frac{1}{2},\frac{17}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{17/6}}$$

input
```
int(1/(a + b*x^2)^(17/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(17/6)*hypergeom([1/2, 17/6], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(17/6)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{17/6}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{6}}\,a^2+2\,(b\,x^2+a)^{\frac{5}{6}}\,ab\,x^2+(b\,x^2+a)^{\frac{5}{6}}\,b^2x^4}dx$$

input
```
int(1/(b*x^2+a)^(17/6),x)
```

output
```
int(1/((a + b*x**2)**(5/6)*a**2 + 2*(a + b*x**2)**(5/6)*a*b*x**2 + (a + b*
x**2)**(5/6)*b**2*x**4),x)
```

3.107 $\int (a+bx^2)^{11/6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 599

$$\int (a+bx^2)^{11/6}\,dx = \frac{33}{112}ax(a+bx^2)^{5/6}$$

$$+\frac{3}{14}x(a+bx^2)^{11/6}-\frac{165(1+\sqrt{3})\,a^2x\sqrt[6]{a+bx^2}}{224\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{165\sqrt[4]{3}a^{7/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}\right.\right.}{224bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{55\ 3^{3/4}(1-\sqrt{3})\,a^{7/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}}{\sqrt[3]{a}}\right.\right.}{448bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
33/112*a*x*(b*x^2+a)^(5/6)+3/14*x*(b*x^2+a)^(11/6)-165*(1+3^(1/2))*a^2*x*(
b*x^2+a)^(1/6)/(224*a^(1/3)-224*(1+3^(1/2))*(b*x^2+a)^(1/3))-165/224*3^(1/
4)*a^(7/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*
x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/
2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2
))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3
)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
-55/448*3^(3/4)*(1-3^(1/2))*a^(7/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/
3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2)
)*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b
*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1
/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*
(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.77 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.08

$$\int (a+bx^2)^{11/6}\,dx=\frac{ax(a+bx^2)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(11/6),x]
```

output
```
(a*x*(a + b*x^2)^(5/6)*Hypergeometric2F1[-11/6, 1/2, 3/2, -((b*x^2)/a)])/(
1 + (b*x^2)/a)^(5/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {211, 211, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{11/6} dx$$

$$\Big\downarrow \; 211$$

$$\frac{11}{14}a\int \left(bx^2+a\right)^{5/6}dx+\frac{3}{14}x\left(a+bx^2\right)^{11/6}$$

$$\Big\downarrow \; 211$$

$$\frac{11}{14}a\left(\frac{5}{8}a\int \frac{1}{\sqrt[6]{bx^2+a}}dx+\frac{3}{8}x\left(a+bx^2\right)^{5/6}\right)+\frac{3}{14}x\left(a+bx^2\right)^{11/6}$$

$$\Big\downarrow \; 235$$

$$\frac{11}{14}a\left(\frac{5}{8}a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}\right)+\frac{3}{14}x\left(a+bx^2\right)^{11/6}$$

$$\Big\downarrow \; 214$$

$$\frac{11}{14}a\left(\frac{5}{8}a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}\right)+\frac{3}{14}x\left(a+bx^2\right)^{11/6}$$

$$\Big\downarrow \; 233$$

$$\frac{11}{14}a\left(\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}\right)+\frac{3}{14}x\left(a+bx^2\right)^{11/6}$$

$$\Big\downarrow \; 833$$

$$\frac{11}{14}a\left(\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.\right.$$

$$\frac{3}{14}x\left(a+bx^2\right)^{11/6}$$

$\downarrow$ 760

$$\frac{11}{14}a\left(\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\ldots}}{\sqrt[4]{3}\sqrt{\ldots}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.\right.$$

$$\frac{3}{14}x\left(a+bx^2\right)^{11/6}$$

$\downarrow$ 2418

$$\frac{11}{14}a\left(\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\cdots}}{-\sqrt[3]{1-\cdots}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{\cdots}\right.\right.$$

$$\frac{3}{14}x\left(a+bx^2\right)^{11/6}$$

input
```
Int[(a + b*x^2)^(11/6),x]
```

output
```
(3*x*(a + b*x^2)^(11/6))/14 + (11*a*((3*x*(a + b*x^2)^(5/6))/8 + (5*a*((3*
x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1
+ x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))
+ (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1
 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
(b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2
)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7
 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/
(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) -
 (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*
Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
- (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1
- (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2)
)^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/8))/14
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{11}{6}}dx$$

input
```
int((b*x^2+a)^(11/6),x)
```

output
```
int((b*x^2+a)^(11/6),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{11/6}\,dx=\int \left(bx^2+a\right)^{\frac{11}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(11/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(11/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.04

$$\int \left(a+bx^2\right)^{11/6}\,dx=a^{\frac{11}{6}}x\,{}_2F_1\left(\begin{matrix} -\frac{11}{6},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(11/6),x)
```

output
```
a**(11/6)*x*hyper((-11/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{11/6}\,dx=\int \left(bx^2+a\right)^{\frac{11}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(11/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(11/6), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{11/6}\,dx=\int \left(bx^2+a\right)^{\frac{11}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(11/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(11/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.06

$$\int \left(a+bx^2\right)^{11/6}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{11/6}\,{{}}_2{\mathrm{F}}_1\left(-\frac{11}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{11/6}}$$

input
```
int((a + b*x^2)^(11/6),x)
```

output
```
(x*(a + b*x^2)^(11/6)*hypergeom([-11/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a
 + 1)^(11/6)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{11/6}\,dx=\left(\int \left(b\,x^2+a\right)^{\frac{5}{6}}dx\right)a+\left(\int \left(b\,x^2+a\right)^{\frac{5}{6}}x^2dx\right)b$$

input
```
int((b*x^2+a)^(11/6),x)
```

output
```
int((a + b*x**2)**(5/6),x)*a + int((a + b*x**2)**(5/6)*x**2,x)*b
```

3.108 $\int (a+bx^2)^{5/6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 580

$$\int (a+bx^2)^{5/6}\,dx = \frac{3}{8}x(a+bx^2)^{5/6} - \frac{15(1+\sqrt{3})\,ax\sqrt[6]{a+bx^2}}{16\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)}$$

$$- \frac{15\sqrt[4]{3}a^{4/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}\right.\right.}{16bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{5\,3^{3/4}(1-\sqrt{3})\,a^{4/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-}{\sqrt[3]{a}-}\right.\right.}{32bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/8*x*(b*x^2+a)^(5/6)-15*(1+3^(1/2))*a*x*(b*x^2+a)^(1/6)/(16*a^(1/3)-16*(1
+3^(1/2))*(b*x^2+a)^(1/3))-15/16*3^(1/4)*a^(4/3)*(b*x^2+a)^(1/6)*(a^(1/3)-
(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/
3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))
*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)
-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-5/32*3^(3/4)*(1-3^(1/2))*a^(4/3)*(b
*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+
(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJac
obiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b
*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b
*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \left(a+bx^2\right)^{5/6} dx = \frac{x(a+bx^2)^{5/6} \text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(5/6),x]
```

output
```
(x*(a + b*x^2)^(5/6)*Hypergeometric2F1[-5/6, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(5/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {211, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/6} dx$$

$$\Big\downarrow \; 211$$

$$\frac{5}{8}a\int \frac{1}{\sqrt[6]{bx^2+a}}dx+\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \; 235$$

$$\frac{5}{8}a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \; 214$$

$$\frac{5}{8}a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \; 233$$

$$\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \; 833$$

$$\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{}{2\sqrt[6]{}}\right.$$

$$\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \; 760$$

$$\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{\cdots}}{\left(-\sqrt[3]{1-\cdots}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}\cdots}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.$$

$$\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{2418}$$

$$\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+}}}{-\sqrt[3]{1-\frac{bx^2}{bx^2+}}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{}\right.$$

$$\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

input
```
Int[(a + b*x^2)^(5/6),x]
```

output
```
(3*x*(a + b*x^2)^(5/6))/8 + (5*a*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[
-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3
] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1
- (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a
+ b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*Ellipt
icE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
(1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*
x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(
1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x
^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]
*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-
1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2)
)^(2/3)*(a + b*x^2)^(1/6))))/8
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{5}{6}}d x$$

input
```
int((b*x^2+a)^(5/6),x)
```

output
```
int((b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{5/6} dx = \int \left(bx^2+a\right)^{\frac{5}{6}} dx$$

input
```
integrate((b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.04

$$\int \left(a+bx^2\right)^{5/6} dx = a^{\frac{5}{6}} x {}_2F_1\left(\begin{matrix} -\frac{5}{6}, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(5/6),x)
```

output
```
a**(5/6)*x*hyper((-5/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{5/6} dx = \int \left(bx^2+a\right)^{\frac{5}{6}} dx$$

input
```
integrate((b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/6} dx = \int \left(bx^2+a\right)^{\frac{5}{6}} dx$$

input
```
integrate((b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.06

$$\int \left(a+bx^2\right)^{5/6} dx = \frac{x\,\left(b\,x^2+a\right)^{5/6}\,{}_2\mathrm{F}_1\left(-\frac{5}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/6}}$$

input
```
int((a + b*x^2)^(5/6),x)
```

output
```
(x*(a + b*x^2)^(5/6)*hypergeom([-5/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/6)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/6} dx = \int \left(b\,x^2+a\right)^{\frac{5}{6}} dx$$

input
```
int((b*x^2+a)^(5/6),x)
```

output
```
int((a + b*x**2)**(5/6),x)
```

3.109 $\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 563

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = -\frac{3\left(1+\sqrt{3}\right)x\sqrt[6]{a+bx^2}}{2\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right.\right.}{2bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-}{\sqrt[3]{a}-}\right.\right.}{4bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3*(1+3^(1/2))*x*(b*x^2+a)^(1/6)/(2*a^(1/3)-2*(1+3^(1/2))*(b*x^2+a)^(1/3))
-3/2*3^(1/4)*a^(1/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a
^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/
3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)
-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b/x/(-(b*x
^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3)
)^2)^(1/2)-1/4*3^(3/4)*(1-3^(1/2))*a^(1/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2
+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+
3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1
/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1
/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^
(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.88 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{x\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/6),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[6]{a+bx^2}}dx$$

$$\Big\downarrow \text{235}$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{(bx^2+a)^{7/6}}dx$$

$$\Big\downarrow \text{214}$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{233}$$

$$\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{833}$$

$$\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\ldots}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+b\ldots}}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\ldots}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}$$

$\downarrow$ 2418

$$\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\ldots}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\ldots}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{\ldots}$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-1/6),x]
```

output
```
(3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt
[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))],
 -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]
) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3
))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-(
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x
^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{6}}}dx$$

input
```
int(1/(b*x^2+a)^(1/6),x)
```

output
```
int(1/(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.04

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\, \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[6]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/6),x)
```

output
```
x*hyper((1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/6)
```

Maxima [F]

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/6), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{1/6}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{1/6}}$$

input
```
int(1/(a + b*x^2)^(1/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/6)*hypergeom([1/6, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/6)
```

Reduce [F]

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{6}}}dx$$

input
```
int(1/(b*x^2+a)^(1/6),x)
```

output
```
int(1/(a + b*x**2)**(1/6),x)
```

3.110 $\int \frac{1}{(a+bx^2)^{7/6}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 579

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{3x}{a\sqrt[6]{a+bx^2}} + \frac{3(1+\sqrt{3})\,x\sqrt[6]{a+bx^2}}{a\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)}$$

$$+ \frac{3\sqrt[4]{3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(2}{a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+ \frac{3^{3/4}(1-\sqrt{3})\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3}}{\sqrt[3]{a}-(1+\sqrt{3}}\right.\right.}{2a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3*x/a/(b*x^2+a)^(1/6)+3*(1+3^(1/2))*x*(b*x^2+a)^(1/6)/a/(a^(1/3)-(1+3^(1/2
))*(b*x^2+a)^(1/3))+3*3^(1/4)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((
a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x
^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2
/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a
^(2/3)/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2)
)*(b*x^2+a)^(1/3))^2)^(1/2)+1/2*3^(3/4)*(1-3^(1/2))*(b*x^2+a)^(1/6)*(a^(1/
3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^
(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3
)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*
6^(1/2)+1/4*2^(1/2))/a^(2/3)/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3
))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.08

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{x\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[6]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-7/6),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(
a*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx$$

$$\Big\downarrow \text{ 214}$$

$$\frac{\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{ 233}$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{ 833}$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{ 760}$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\cdots}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+b\cdots}}\cdots\right.}}\cdots}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{\cdots}}\cdots\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\cdots}$$

$$\Big\downarrow \text{ 2418}$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\cdots}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{\cdots}$$

input
```
Int[(a + b*x^2)^(-7/6),x]
```

output
```
(-3*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1
 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b
*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^
2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - S
qrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^
3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + S
qrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) +
(1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^
(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/
4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(2*b*x*(a/(a
 + b*x^2))^(2/3)*(a + b*x^2)^(1/6))
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{6}}}d x$$

input
```
int(1/(b*x^2+a)^(7/6),x)
```

output
```
int(1/(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.04

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{7}{6}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}}$$

input
```
integrate(1/(b*x**2+a)**(7/6),x)
```

output
```
x*hyper((1/2, 7/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/6)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-7/6), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-7/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.06

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/6}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{7/6}}$$

input
```
int(1/(a + b*x^2)^(7/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(7/6)*hypergeom([1/2, 7/6], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(7/6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.06

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{(b\,x^2+a)^{\frac{1}{6}}\,ab}$$

input
```
int(1/(b*x^2+a)^(7/6),x)
```

output
```
(sqrt(b)*sqrt(a)*(a + b*x**2)**(5/6)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b*(
a + b*x**2))
```

3.111 $\int \frac{1}{(a+bx^2)^{13/6}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 604

$$\int \frac{1}{(a+bx^2)^{13/6}}\,dx = \frac{3x}{7a\,(a+bx^2)^{7/6}} + \frac{12x}{7a^2\sqrt[6]{a+bx^2}} + \frac{12\left(1+\sqrt{3}\right)x\sqrt[6]{a+bx^2}}{7a^2\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{12\sqrt[4]{3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(\ldots}{7a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{2\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\ldots}{\sqrt[3]{a}-(1+\ldots}\right.\right.}{7a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/7*x/a/(b*x^2+a)^(7/6)+12/7*x/a^2/(b*x^2+a)^(1/6)+12/7*(1+3^(1/2))*x*(b*x
^2+a)^(1/6)/a^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))+12/7*3^(1/4)*(b*x^2+
a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a
^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3)
)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^(5/3)/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)
-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)+2/7*3^(3/
4)*(1-3^(1/2))*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)
*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)
^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/
3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/a^(5/3)/b/x/(-(b
*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/
3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.11

$$\int \frac{1}{(a+bx^2)^{13/6}}\,dx = \frac{3ax+4x(a+bx^2)\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},-\frac{bx^2}{a}\right)}{7a^2(a+bx^2)^{7/6}}$$

input
```
Integrate[(a + b*x^2)^(-13/6),x]
```

output
```
(3*a*x + 4*x*(a + b*x^2)*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6,
 3/2, -((b*x^2)/a)])/(7*a^2*(a + b*x^2)^(7/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {215, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{13/6}}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{4\int \frac{1}{(bx^2+a)^{7/6}}dx}{7a}+\frac{3x}{7a(a+bx^2)^{7/6}}$$

$$\Big\downarrow \text{214}$$

$$\frac{4\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{7a\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}+\frac{3x}{7a(a+bx^2)^{7/6}}$$

$$\Big\downarrow \text{233}$$

$$\frac{3x}{7a(a+bx^2)^{7/6}}-\frac{6\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{7abx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{833}$$

$$\frac{3x}{7a(a+bx^2)^{7/6}}-\frac{6\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{7abx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3x}{7a\left(a+bx^2\right)^{7/6}}-\frac{6\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{}{a}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx}}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-}}\right.}{7abx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$\downarrow$ 2418

$$\frac{3x}{7a\left(a+bx^2\right)^{7/6}}-\frac{6\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{}$$

input

```
Int[(a + b*x^2)^(-13/6),x]
```

output
```
(3*x)/(7*a*(a + b*x^2)^(7/6)) - (6*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt
[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))],
 -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]
) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3
))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-(
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x
^2))^(1/3))^2)])))/(7*a*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{13}{6}}}dx$$

input
```
int(1/(b*x^2+a)^(13/6),x)
```

output
```
int(1/(b*x^2+a)^(13/6),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{13/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{13}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(13/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.04

$$\int \frac{1}{(a+bx^2)^{13/6}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{6}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{13}{6}}}$$

input
```
integrate(1/(b*x**2+a)**(13/6),x)
```

output
```
x*hyper((1/2, 13/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(13/6)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{13/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{13}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(13/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-13/6), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{13/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{13}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(13/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-13/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.06

$$\int \frac{1}{(a+bx^2)^{13/6}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{13/6}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{13/6}}$$

input
```
int(1/(a + b*x^2)^(13/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(13/6)*hypergeom([1/2, 13/6], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(13/6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.14

$$\int \frac{1}{(a+bx^2)^{13/6}}\,dx = \frac{(b\,x^2+a)^{\frac{5}{6}}\left(3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2-b^2x^3\right)}{3a^2b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/(b*x^2+a)^(13/6),x)
```

output
```
((a + b*x**2)**(5/6)*(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a +
3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**2 - b**2*x**3))/(3*a*
*2*b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.112 $\int (a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a+bx^2)^{3/8}\,dx=\frac{x(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
x*(b*x^2+a)^(3/8)*hypergeom([-3/8, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(3/8)
```

Mathematica [A] (verified)

Time = 8.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{3/8}\,dx=\frac{x(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(3/8),x]
```

output
```
(x*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(3/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/8} \, dx$$

$$\Big\downarrow \ 238$$

$$\frac{\left(a+bx^2\right)^{3/8} \int \left(\frac{bx^2}{a}+1\right)^{3/8}dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \ 237$$

$$\frac{x\left(a+bx^2\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[(a + b*x^2)^(3/8),x]
```

output
```
(x*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{3}{8}}d x$$

input
```
int((b*x^2+a)^(3/8),x)
```

output
```
int((b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{3/8}\,dx=\int \left(bx^2+a\right)^{\frac{3}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{3/8}\,dx=a^{\frac{3}{8}}x\,{}_2F_1\left(\begin{matrix}-\frac{3}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(3/8),x)
```

output
```
a**(3/8)*x*hyper((-3/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/8} \, dx = \int \left(bx^2+a\right)^{\frac{3}{8}} \, dx$$

input
```
integrate((b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/8} \, dx = \int \left(bx^2+a\right)^{\frac{3}{8}} \, dx$$

input
```
integrate((b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{3/8} \, dx = \frac{x\,\left(b\,x^2+a\right)^{3/8}\,{}_2\mathrm{F}_1\left(-\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/8}}$$

input
```
int((a + b*x^2)^(3/8),x)
```

output
```
(x*(a + b*x^2)^(3/8)*hypergeom([-3/8, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(3/8)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/8}\,dx=\int \left(b\,x^2+a\right)^{\frac{3}{8}}dx$$

input
```
int((b*x^2+a)^(3/8),x)
```

output
```
int((a + b*x**2)**(3/8),x)
```

3.113 $\int \sqrt[8]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt[8]{a+bx^2}\,dx=\frac{x\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
x*(b*x^2+a)^(1/8)*hypergeom([-1/8, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(1/8)
```

Mathematica [A] (verified)

Time = 7.96 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sqrt[8]{a+bx^2}\,dx=\frac{x\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/8),x]
```

output
```
(x*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[8]{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 238}$$

$$\frac{\sqrt[8]{a+bx^2}\int \sqrt[8]{\frac{bx^2}{a}+1}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{x\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/8),x]
```

output
```
(x*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{8}}dx$$

input
```
int((b*x^2+a)^(1/8),x)
```

output
```
int((b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \sqrt[8]{a+bx^2}\,dx = \int \left(bx^{2}+a\right)^{\frac{1}{8}}dx$$

input
```
integrate((b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \sqrt[8]{a+bx^2}\,dx = \sqrt[8]{a}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(1/8),x)
```

output
```
a**(1/8)*x*hyper((-1/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[8]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8), x)
```

Giac [F]

$$\int \sqrt[8]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt[8]{a+bx^2}\,dx=\frac{x\,\left(b\,x^2+a\right)^{1/8}\,{}_2\mathrm{F}_1\left(-\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/8}}$$

input
```
int((a + b*x^2)^(1/8),x)
```

output
```
(x*(a + b*x^2)^(1/8)*hypergeom([-1/8, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/8)
```

Reduce [F]

$$\int \sqrt[8]{a+bx^2}\,dx=\int \left(b\,x^{2}+a\right)^{\frac{1}{8}}dx$$

input
```
int((b*x^2+a)^(1/8),x)
```

output
```
int((a + b*x**2)**(1/8),x)
```

3.114 $\int \frac{1}{(a+bx^2)^{5/8}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{5/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/8}}$$

output
```
x*(1+b*x^2/a)^(5/8)*hypergeom([1/2, 5/8],[3/2],-b*x^2/a)/(b*x^2+a)^(5/8)
```

Mathematica [A] (verified)

Time = 8.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{5/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(-5/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 5/8, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(5/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx$$

$$\Big\downarrow \; 238$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/8}}dx}{(a+bx^2)^{5/8}}$$

$$\Big\downarrow \; 237$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/8}}$$

input
```
Int[(a + b*x^2)^(-5/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 5/8, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(5/8),x)
```

output
```
int(1/(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \frac{x{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(5/8),x)
```

output
```
x*hyper((1/2, 5/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-5/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-5/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{5/8}}$$

input
```
int(1/(a + b*x^2)^(5/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(5/8)*hypergeom([1/2, 5/8], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(5/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(5/8),x)
```

output
```
int(1/(a + b*x**2)**(5/8),x)
```

3.115 $\int \frac{1}{(a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/8}}$$

output
```
x*(1+b*x^2/a)^(7/8)*hypergeom([1/2, 7/8],[3/2],-b*x^2/a)/(b*x^2+a)^(7/8)
```

Mathematica [A] (verified)

Time = 7.88 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(-7/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 7/8, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(7/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx$$

$$\Big\downarrow \ 238$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{7/8}}dx}{(a+bx^2)^{7/8}}$$

$$\Big\downarrow \ 237$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/8}}$$

input

```
Int[(a + b*x^2)^(-7/8),x]
```

output

```
(x*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 7/8, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(7/8)
```

Defintions of rubi rules used

rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(7/8),x)
```

output
```
int(1/(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{\left(a+bx^2\right)^{7/8}}\,dx=\int \frac{1}{\left(bx^{2}+a\right)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{\left(a+bx^2\right)^{7/8}}\,dx=\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{a^{\frac{7}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(7/8),x)
```

output
```
x*hyper((1/2, 7/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-7/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-7/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx=\frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/8}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{7/8}}$$

input
```
int(1/(a + b*x^2)^(7/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(7/8)*hypergeom([1/2, 7/8], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(7/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{5}{8}}\,a+(b\,x^2+a)^{\frac{5}{8}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(7/8),x)
```

output
```
int((a + b*x**2)**(3/4)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**
2),x)
```

3.116 $\int \frac{1}{(a+bx^2)^{13/8}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)^{13/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{5/8}}$$

output
```
x*(1+b*x^2/a)^(5/8)*hypergeom([1/2, 13/8],[3/2],-b*x^2/a)/a/(b*x^2+a)^(5/8
)
```

Mathematica [A] (verified)

Time = 8.88 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{13/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(-13/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 13/8, 3/2, -((b*x^2)/a)])/
(a*(a + b*x^2)^(5/8))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{13/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{13/8}}dx}{a\,(a+bx^2)^{5/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{5/8}}$$

input
```
Int[(a + b*x^2)^(-13/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 13/8, 3/2, -((b*x^2)/a)])/
(a*(a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{13}{8}}}d x$$

input
```
int(1/(b*x^2+a)^(13/8),x)
```

output
```
int(1/(b*x^2+a)^(13/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{13/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{13}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(13/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+bx^2)^{13/8}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{13}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(13/8),x)
```

output
```
x*hyper((1/2, 13/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(13/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{13/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{13}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(13/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-13/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{13/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{13}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(13/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-13/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{13/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{13/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{13/8}}$$

input
```
int(1/(a + b*x^2)^(13/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(13/8)*hypergeom([1/2, 13/8], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(13/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{13/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{8}}\,a+(b\,x^2+a)^{\frac{5}{8}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(13/8),x)
```

output
```
int(1/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)
```

3.117 $\int \frac{1}{(a+bx^2)^{15/8}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)^{15/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{7/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{15}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{7/8}}$$

output
```
x*(1+b*x^2/a)^(7/8)*hypergeom([1/2, 15/8],[3/2],-b*x^2/a)/a/(b*x^2+a)^(7/8
)
```

Mathematica [A] (verified)

Time = 9.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{15/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{7/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{15}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(-15/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 15/8, 3/2, -((b*x^2)/a)])/
(a*(a + b*x^2)^(7/8))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{15/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{15/8}}dx}{a\,(a+bx^2)^{7/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{15}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{7/8}}$$

input
```
Int[(a + b*x^2)^(-15/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 15/8, 3/2, -((b*x^2)/a)])/
(a*(a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{15}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(15/8),x)
```

output
```
int(1/(b*x^2+a)^(15/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{15/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{15}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(15/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+bx^2)^{15/8}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{15}{8}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{15}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(15/8),x)
```

output
```
x*hyper((1/2, 15/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(15/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{15/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{15}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(15/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-15/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{15/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{15}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(15/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-15/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{15/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{15/8}\,{}_2F_1\left(\frac{1}{2},\frac{15}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{15/8}}$$

input
```
int(1/(a + b*x^2)^(15/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(15/8)*hypergeom([1/2, 15/8], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(15/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{15/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{7}{8}}\,a+(b\,x^2+a)^{\frac{7}{8}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(15/8),x)
```

output
```
int(1/((a + b*x**2)**(7/8)*a + (a + b*x**2)**(7/8)*b*x**2),x)
```

3.118 $\int (a+bx^2)^{7/8}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a+bx^2)^{7/8}\,dx=\frac{x(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
x*(b*x^2+a)^(7/8)*hypergeom([-7/8, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(7/8)
```

Mathematica [A] (verified)

Time = 8.97 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{7/8}\,dx=\frac{x(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(7/8),x]
```

output
```
(x*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(7/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{7/8} \, dx$$
$$\Big\downarrow \text{238}$$
$$\frac{\left(a+bx^2\right)^{7/8} \int \left(\frac{bx^2}{a}+1\right)^{7/8}dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$
$$\Big\downarrow \text{237}$$
$$\frac{x\left(a+bx^2\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[(a + b*x^2)^(7/8),x]
```

output
```
(x*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(7/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{7}{8}}d x$$

input
```
int((b*x^2+a)^(7/8),x)
```

output
```
int((b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{7/8}\,dx=\int \left(bx^2+a\right)^{\frac{7}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{7/8}\,dx=a^{\frac{7}{8}}x\,{}_2F_1\left(\begin{matrix}-\frac{7}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(7/8),x)
```

output
```
a**(7/8)*x*hyper((-7/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{7/8}\,dx=\int \left(bx^2+a\right)^{\frac{7}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{7/8}\,dx=\int \left(bx^2+a\right)^{\frac{7}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{7/8}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{7/8}\,{}_2\mathrm{F}_1\left(-\frac{7}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{7/8}}$$

input
```
int((a + b*x^2)^(7/8),x)
```

output
```
(x*(a + b*x^2)^(7/8)*hypergeom([-7/8, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(7/8)
```

Reduce [F]

$$\int (a+bx^2)^{7/8} \, dx = \int (b\,x^2+a)^{\frac{7}{8}}\, dx$$

input
```
int((b*x^2+a)^(7/8),x)
```

output
```
int((a + b*x**2)**(7/8),x)
```

3.119 $\int (a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \left(a+bx^2\right)^{5/8}\,dx=\frac{x\left(a+bx^2\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
x*(b*x^2+a)^(5/8)*hypergeom([-5/8, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(5/8)
```

Mathematica [A] (verified)

Time = 8.81 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^{5/8}\,dx=\frac{x\left(a+bx^2\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(5/8),x]
```

output
```
(x*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(5/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/8} dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(a+bx^2\right)^{5/8}\int \left(\frac{bx^2}{a}+1\right)^{5/8}dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(a+bx^2\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[(a + b*x^2)^(5/8),x]
```

output
```
(x*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{5}{8}}d x$$

input
```
int((b*x^2+a)^(5/8),x)
```

output
```
int((b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{5/8}\,dx=\int \left(bx^{2}+a\right)^{\frac{5}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{5/8}\,dx=a^{\frac{5}{8}}x\,{}_{2}F_{1}\left(\left.\begin{matrix}-\frac{5}{8},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(5/8),x)
```

output
```
a**(5/8)*x*hyper((-5/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{5/8}\,dx=\int \left(bx^2+a\right)^{\frac{5}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/8}\,dx=\int \left(bx^2+a\right)^{\frac{5}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{5/8}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{5/8}\,{}_2\mathrm{F}_1\left(-\frac{5}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/8}}$$

input
```
int((a + b*x^2)^(5/8),x)
```

output
```
(x*(a + b*x^2)^(5/8)*hypergeom([-5/8, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/8)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/8}\,dx=\int \left(b\,x^2+a\right)^{\frac{5}{8}}dx$$

input
```
int((b*x^2+a)^(5/8),x)
```

output
```
int((a + b*x**2)**(5/8),x)
```

3.120 $\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \frac{x\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{a+bx^2}}$$

output
```
x*(1+b*x^2/a)^(1/8)*hypergeom([1/8, 1/2],[3/2],-b*x^2/a)/(b*x^2+a)^(1/8)
```

Mathematica [A] (verified)

Time = 7.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \frac{x\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[8]{a+bx^2}}dx$$
$$\Big\downarrow \ 238$$
$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[8]{\frac{bx^2}{a}+1}}dx}{\sqrt[8]{a+bx^2}}$$
$$\Big\downarrow \ 237$$
$$\frac{x\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-1/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(1/8),x)
```

output
```
int(1/(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[8]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/8),x)
```

output
```
x*hyper((1/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/8)
```

Maxima [F]

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \frac{x\,\left(\frac{b\,x^2}{a}+1\right)^{1/8}\,{}_2\mathrm{F}_1\left(\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{1/8}}$$

input
```
int(1/(a + b*x^2)^(1/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/8)*hypergeom([1/8, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/8)
```

Reduce [F]

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(1/8),x)
```

output
```
int(1/(a + b*x**2)**(1/8),x)
```

3.121 $\int \frac{1}{(a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/8}}$$

output
```
x*(1+b*x^2/a)^(3/8)*hypergeom([3/8, 1/2],[3/2],-b*x^2/a)/(b*x^2+a)^(3/8)
```

Mathematica [A] (verified)

Time = 8.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(-3/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(3/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/8}}dx}{(a+bx^2)^{3/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/8}}$$

input
```
Int[(a + b*x^2)^(-3/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{3}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(3/8),x)
```

output
```
int(1/(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx=\frac{x\,{}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(3/8),x)
```

output
```
x*hyper((3/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-3/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-3/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{3/8}{}_2F_1\left(\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{3/8}}$$

input
```
int(1/(a + b*x^2)^(3/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(3/8)*hypergeom([3/8, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(3/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(3/8),x)
```

output
```
int(1/(a + b*x**2)**(3/8),x)
```

3.122 $\int \frac{1}{(a+bx^2)^{9/8}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)^{9/8}} \, dx = \frac{x\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[8]{a+bx^2}}$$

output
```
x*(1+b*x^2/a)^(1/8)*hypergeom([1/2, 9/8],[3/2],-b*x^2/a)/a/(b*x^2+a)^(1/8)
```

Mathematica [A] (verified)

Time = 8.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{9/8}} \, dx = \frac{x\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[8]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-9/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/2, 9/8, 3/2, -((b*x^2)/a)])/(
a*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{9/8}} \, dx$$

$$\Big\downarrow \ 238$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1} \int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{9/8}}dx}{a\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \ 237$$

$$\frac{x\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[8]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-9/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/2, 9/8, 3/2, -((b*x^2)/a)])/(
a*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{9}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(9/8),x)
```

output
```
int(1/(b*x^2+a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{8}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{9}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(9/8),x)
```

output
```
x*hyper((1/2, 9/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(9/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-9/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-9/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{9/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{9/8}}$$

input
```
int(1/(a + b*x^2)^(9/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(9/8)*hypergeom([1/2, 9/8], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(9/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{8}}\,a+(b\,x^2+a)^{\frac{1}{8}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(9/8),x)
```

output
```
int(1/((a + b*x**2)**(1/8)*a + (a + b*x**2)**(1/8)*b*x**2),x)
```

3.123 $\int \frac{1}{(a+bx^2)^{11/8}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)^{11/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{3/8}}$$

output
```
x*(1+b*x^2/a)^(3/8)*hypergeom([1/2, 11/8],[3/2],-b*x^2/a)/a/(b*x^2+a)^(3/8
)
```

Mathematica [A] (verified)

Time = 8.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{11/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(-11/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[1/2, 11/8, 3/2, -((b*x^2)/a)])/
(a*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{11/8}}dx}{a\,(a+bx^2)^{3/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{3/8}}$$

input
```
Int[(a + b*x^2)^(-11/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[1/2, 11/8, 3/2, -((b*x^2)/a)])/
(a*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{11}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(11/8),x)
```

output
```
int(1/(b*x^2+a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{11}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(11/8),x)
```

output
```
x*hyper((1/2, 11/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(11/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-11/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-11/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{11/8}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{11/8}}$$

input
```
int(1/(a + b*x^2)^(11/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(11/8)*hypergeom([1/2, 11/8], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(11/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{8}}\,a+(b\,x^2+a)^{\frac{3}{8}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(11/8),x)
```

output
```
int(1/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)
```

3.124 $\int \left(-a+bx^2\right)^{3/8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 469

$$\int \left(-a+bx^2\right)^{3/8}\,dx = \frac{4}{7}x\left(-a+bx^2\right)^{3/8}$$

$$+\frac{6a^{5/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{7\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{6a^{5/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{7\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
4/7*x*(b*x^2-a)^(3/8)+6/7*a^(5/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(
b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)
*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2
-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2)
)^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+6/7*a^(5/4)*(-b*x^2/a^(1/2)/(b*x^2-a
)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^
2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4
)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(
1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10

$$\int \left(-a+bx^2\right)^{3/8} dx = \frac{x(-a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(3/8),x]
```

output
```
(x*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(3/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2-a\right)^{3/8}dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(bx^2-a\right)^{3/8}\int \left(1-\frac{bx^2}{a}\right)^{3/8}dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(bx^2-a\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input

```
Int[(-a + b*x^2)^(3/8),x]
```

output

```
(x*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(3/8)
```

Defintions of rubi rules used

rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}-a\right)^{\frac{3}{8}}d x$$

input
```
int((b*x^2-a)^(3/8),x)
```

output
```
int((b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \left(-a+bx^2\right)^{3/8}\,dx=\int \left(bx^{2}-a\right)^{\frac{3}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.06

$$\int \left(-a+bx^2\right)^{3/8}\,dx=a^{\frac{3}{8}}xe^{\frac{3i\pi}{8}}{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{8},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{bx^{2}}{a}\right)$$

input
```
integrate((b*x**2-a)**(3/8),x)
```

output
```
a**(3/8)*x*exp(3*I*pi/8)*hyper((-3/8, 1/2), (3/2,), b*x**2/a)
```

Maxima [F]

$$\int \left(-a+bx^2\right)^{3/8} dx = \int \left(bx^2-a\right)^{\frac{3}{8}} dx$$

input
```
integrate((b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8), x)
```

Giac [F]

$$\int \left(-a+bx^2\right)^{3/8} dx = \int \left(bx^2-a\right)^{\frac{3}{8}} dx$$

input
```
integrate((b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \left(-a+bx^2\right)^{3/8} dx = \frac{x\,\left(b\,x^2-a\right)^{3/8}\,{}_2\mathrm{F}_1\left(-\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{3/8}}$$

input
```
int((b*x^2 - a)^(3/8),x)
```

output
```
(x*(b*x^2 - a)^(3/8)*hypergeom([-3/8, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(3/8)
```

Reduce [F]

$$\int \left(-a+bx^2\right)^{3/8}\,dx=\int \left(b\,x^2-a\right)^{\frac{3}{8}}dx$$

input
```
int((b*x^2-a)^(3/8),x)
```

output
```
int(( - a + b*x**2)**(3/8),x)
```

3.125 $\int \sqrt[8]{-a+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 461

$$\int \sqrt[8]{-a+bx^2}\,dx = \frac{4}{5}x\sqrt[8]{-a+bx^2}$$

$$-\frac{2a\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{5\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2a\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{5\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
4/5*x*(b*x^2-a)^(1/8)-2/5*a*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-
a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellip
ticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1
/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2
)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+2/5*a*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/
2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^
(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b
*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(
1/2))^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.99 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10

$$\int \sqrt[8]{-a+bx^2}\,dx=\frac{x\sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(-a + b*x^2)^(1/8),x]
```

output
```
(x*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(1/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[8]{bx^2-a}\,dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\sqrt[8]{bx^2-a}\int \sqrt[8]{1-\frac{bx^2}{a}}dx}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\sqrt[8]{bx^2-a}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Int[(-a + b*x^2)^(1/8),x]
```

output
```
(x*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}-a\right)^{\frac{1}{8}}d x$$

input
```
int((b*x^2-a)^(1/8),x)
```

output
```
int((b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \sqrt[8]{-a+bx^2}\,dx = \int \left(bx^2-a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

$$\int \sqrt[8]{-a+bx^2}\,dx = \sqrt[8]{a}xe^{\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{8},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)$$

input
```
integrate((b*x**2-a)**(1/8),x)
```

output
```
a**(1/8)*x*exp(I*pi/8)*hyper((-1/8, 1/2), (3/2,), b*x**2/a)
```

Maxima [F]

$$\int \sqrt[8]{-a+bx^2}\,dx = \int \left(bx^2-a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8), x)
```

Giac [F]

$$\int \sqrt[8]{-a+bx^2}\,dx = \int \left(bx^2-a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \sqrt[8]{-a+bx^2}\,dx = \frac{x\left(b\,x^2-a\right)^{1/8}\,{}_2\mathrm{F}_1\left(-\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{1/8}}$$

input
```
int((b*x^2 - a)^(1/8),x)
```

output
```
(x*(b*x^2 - a)^(1/8)*hypergeom([-1/8, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(1/8)
```

Reduce [F]

$$\int \sqrt[8]{-a+bx^2}\,dx=\int \left(b\,x^{2}-a\right)^{\frac{1}{8}}dx$$

input
```
int((b*x^2-a)^(1/8),x)
```

output
```
int(( - a + b*x**2)**(1/8),x)
```

3.126 $\int \frac{1}{(-a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 447

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx =$$

$$-\frac{2\sqrt[4]{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2\sqrt[4]{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-2*a^(1/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4
)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4
)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x
^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)+(b
*x^2-a)^(1/4))-2*a^(1/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^
(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipti
cF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)
/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b
/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \frac{x\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{(-a+bx^2)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(-5/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 5/8, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(5/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{5/8}}\,dx$$
$$\Big\downarrow \; 238$$
$$\frac{\left(1-\frac{bx^2}{a}\right)^{5/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{5/8}}dx}{(bx^2-a)^{5/8}}$$
$$\Big\downarrow \; 237$$
$$\frac{x\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{(bx^2-a)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(-5/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 5/8, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(5/8),x)
```

output
```
int(1/(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(-5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \frac{xe^{-\frac{5i\pi}{8}}{}_2F_1\left(\begin{matrix} \frac{1}{2},\frac{5}{8} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{a^{\frac{5}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(5/8),x)
```

output
```
x*exp(-5*I*pi/8)*hyper((1/2, 5/8), (3/2,), b*x**2/a)/a**(5/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-5/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-5/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{5/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{5/8}}$$

input
```
int(1/(b*x^2 - a)^(5/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(5/8)*hypergeom([1/2, 5/8], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(5/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(5/8),x)
```

output
```
int(1/( - a + b*x**2)**(5/8),x)
```

3.127 $\int \frac{1}{(-a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 437

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}}{\ldots}}\right.\right.}{\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{\ldots}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-
a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2
)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/
4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(
1/4))-2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-
(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(
2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-
a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^
2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx=\frac{x\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{(-a+bx^2)^{7/8}}$$

input
```
Integrate[(-a + b*x^2)^(-7/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 7/8, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(7/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{7/8}}\,dx$$

$$\Big\downarrow \; 238$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{7/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{7/8}}dx}{(bx^2-a)^{7/8}}$$

$$\Big\downarrow \; 237$$

$$\frac{x\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{(bx^2-a)^{7/8}}$$

input
```
Int[(-a + b*x^2)^(-7/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 7/8, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(7/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2 - a)^{\frac{7}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(7/8),x)
```

output
```
int(1/(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(-7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \frac{xe^{-\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{8} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{a^{\frac{7}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(7/8),x)
```

output
```
x*exp(-7*I*pi/8)*hyper((1/2, 7/8), (3/2,), b*x**2/a)/a**(7/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-7/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-7/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{7/8}{}_2F_1\left(\frac{1}{2},\frac{7}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{7/8}}$$

input
```
int(1/(b*x^2 - a)^(7/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(7/8)*hypergeom([1/2, 7/8], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(7/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = -\left(\int \frac{(b\,x^2-a)^{\frac{3}{4}}}{(b\,x^2-a)^{\frac{5}{8}}\,a-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2}dx\right)$$

input
```
int(1/(b*x^2-a)^(7/8),x)
```

output
```
 - int(( - a + b*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)*
*(5/8)*b*x**2),x)
```

3.128 $\int \frac{1}{(-a+bx^2)^{13/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 472

$$\int \frac{1}{(-a+bx^2)^{13/8}}\,dx = -\frac{4x}{5a(-a+bx^2)^{5/8}}$$

$$+\frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{5\sqrt{2+\sqrt{2}}a^{3/4}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{5\sqrt{2+\sqrt{2}}a^{3/4}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-4/5*x/a/(b*x^2-a)^(5/8)+2/5*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2
-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Elli
pticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(
1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/
2)/a^(3/4)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+2/5*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/
2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^
(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(
1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2)/a^(3/4)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.92 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.11

$$\int \frac{1}{(-a+bx^2)^{13/8}}\,dx = -\frac{x\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\left(-a+bx^2\right)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(-13/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 13/8, 3/2, (b*x^2)/a])/(
a*(-a + b*x^2)^(5/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{13/8}}\,dx$$
$$\Big\downarrow \text{238}$$
$$-\frac{\left(1-\frac{bx^2}{a}\right)^{5/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{13/8}}dx}{a\,(bx^2-a)^{5/8}}$$
$$\Big\downarrow \text{237}$$
$$-\frac{x\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\,(bx^2-a)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(-13/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 13/8, 3/2, (b*x^2)/a])/(
a*(-a + b*x^2)^(5/8)))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-a\right)^{\frac{13}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(13/8),x)
```

output
```
int(1/(b*x^2-a)^(13/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{13/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{13}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(13/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{13/8}}\,dx = \frac{xe^{\frac{3i\pi}{8}}\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{13}{8}\\\frac{3}{2}\end{matrix}\right|\frac{bx^2}{a}\right)}{a^{\frac{13}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(13/8),x)
```

output
```
x*exp(3*I*pi/8)*hyper((1/2, 13/8), (3/2,), b*x**2/a)/a**(13/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{13/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{13}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(13/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-13/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{13/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{13}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(13/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-13/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \frac{1}{(-a+bx^2)^{13/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{13/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{13}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{13/8}}$$

input
```
int(1/(b*x^2 - a)^(13/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(13/8)*hypergeom([1/2, 13/8], 3/2, (b*x^2)/a))/(b*x^2 -
 a)^(13/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{13/8}}\,dx = -\left(\int \frac{1}{(b\,x^2-a)^{\frac{5}{8}}\,a-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2}dx\right)$$

input

```
int(1/(b*x^2-a)^(13/8),x)
```

output

```
 - int(1/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)
```

3.129 $\int \frac{1}{(-a+bx^2)^{15/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 468

$$\int \frac{1}{(-a+bx^2)^{15/8}}\,dx = -\frac{4x}{7a(-a+bx^2)^{7/8}}$$

$$-\frac{6\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\,(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\ \text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{\cdots}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{7\sqrt{2+\sqrt{2}}\,abx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{6\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\,(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\ \text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{\cdots}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{7\sqrt{2+\sqrt{2}}\,abx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-4/7*x/a/(b*x^2-a)^(7/8)-6/7*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2
-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Elli
pticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(
1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/
2)/a/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+6/7*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1
/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))
^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(
b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2)/a/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.11

$$\int \frac{1}{(-a+bx^2)^{15/8}}\,dx=-\frac{x\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{15}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\left(-a+bx^2\right)^{7/8}}$$

input
```
Integrate[(-a + b*x^2)^(-15/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 15/8, 3/2, (b*x^2)/a])/(
a*(-a + b*x^2)^(7/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{15/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{7/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{15/8}}dx}{a\,(bx^2-a)^{7/8}}$$

$$\Big\downarrow \text{237}$$

$$-\frac{x\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{15}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\,(bx^2-a)^{7/8}}$$

input
```
Int[(-a + b*x^2)^(-15/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 15/8, 3/2, (b*x^2)/a])/(
a*(-a + b*x^2)^(7/8)))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-a\right)^{\frac{15}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(15/8),x)
```

output
```
int(1/(b*x^2-a)^(15/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{15/8}}\,dx=\int \frac{1}{(bx^2-a)^{\frac{15}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(15/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{15/8}}\,dx=\frac{xe^{\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{15}{8}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2}{a}\right)}{a^{\frac{15}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(15/8),x)
```

output
```
x*exp(I*pi/8)*hyper((1/2, 15/8), (3/2,), b*x**2/a)/a**(15/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{15/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{15}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(15/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-15/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{15/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{15}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(15/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-15/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \frac{1}{(-a+bx^2)^{15/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{15/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{15}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{15/8}}$$

input
```
int(1/(b*x^2 - a)^(15/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(15/8)*hypergeom([1/2, 15/8], 3/2, (b*x^2)/a))/(b*x^2 -
 a)^(15/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{15/8}}\,dx = -\left(\int \frac{1}{(b\,x^2-a)^{\frac{7}{8}}\,a-(b\,x^2-a)^{\frac{7}{8}}\,b\,x^2}dx\right)$$

input

```
int(1/(b*x^2-a)^(15/8),x)
```

output

```
- int(1/(( - a + b*x**2)**(7/8)*a - ( - a + b*x**2)**(7/8)*b*x**2),x)
```

3.130 $\int (-a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 919

$$\int (-a+bx^2)^{5/8}\,dx = \text{Too large to display}$$

output
```
4/9*x*(b*x^2-a)^(5/8)+10/9*(2+2^(1/2))^(1/2)*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^
2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*
x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(
1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2)
)^(1/2))/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-10/9*(2+2^(1/2))^(1/2)*a^(3/2)*(-b*
x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1
/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b
*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1
/2),(-2+2*2^(1/2))^(1/2))/b/x/(a^(1/4)-(b*x^2-a)^(1/4))-10/9*a^(3/2)*(-b*x
^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4
))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*
x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/
2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+1
0/9*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1
/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/
4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*
x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(
b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.85 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \left(-a+bx^2\right)^{5/8} dx = \frac{x(-a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(5/8),x]
```

output
```
(x*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(5/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2-a\right)^{5/8} dx$$

$$\Big\downarrow \text{ 238}$$

$$\frac{\left(bx^2-a\right)^{5/8}\int \left(1-\frac{bx^2}{a}\right)^{5/8}dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{x\left(bx^2-a\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(5/8),x]
```

output
```
(x*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}-a\right)^{\frac{5}{8}}d x$$

input
```
int((b*x^2-a)^(5/8),x)
```

output
```
int((b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \left(-a+bx^2\right)^{5/8} \,dx=\int { {\left(b x^{2} - a\right)}^{\frac{5}{8}} \,d x }$$

input
```
integrate((b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \left(-a+bx^2\right)^{5/8}\,dx = a^{\frac{5}{8}}xe^{\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)$$

input
```
integrate((b*x**2-a)**(5/8),x)
```

output
```
a**(5/8)*x*exp(5*I*pi/8)*hyper((-5/8, 1/2), (3/2,), b*x**2/a)
```

Maxima [F]

$$\int \left(-a+bx^2\right)^{5/8}\,dx = \int \left(bx^2-a\right)^{\frac{5}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8), x)
```

Giac [F]

$$\int \left(-a+bx^2\right)^{5/8}\,dx = \int \left(bx^2-a\right)^{\frac{5}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \left(-a+bx^2\right)^{5/8}\,dx=\frac{x\,{\left(b\,x^2-a\right)}^{5/8}\,{{}}_2{\mathrm{F}}_1\left(-\frac{5}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(1-\frac{b\,x^2}{a}\right)}^{5/8}}$$

input
```
int((b*x^2 - a)^(5/8),x)
```

output
```
(x*(b*x^2 - a)^(5/8)*hypergeom([-5/8, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(5/8)
```

Reduce [F]

$$\int \left(-a+bx^2\right)^{5/8}\,dx=\int \left(b\,x^2-a\right)^{\frac{5}{8}}dx$$

input
```
int((b*x^2-a)^(5/8),x)
```

output
```
int(( - a + b*x**2)**(5/8),x)
```

3.131 $\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 919

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \frac{4x}{3\sqrt[8]{-a+bx^2}}$$

$$+\frac{2\sqrt{2+\sqrt{2}}a^{3/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a}}}\right.\right.}{3bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2\sqrt{2+\sqrt{2}}a^{3/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a}}}\right.\right.}{3bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2a^{3/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+}}}\right.\right.}{3\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2a^{3/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+}}}\right.\right.}{3\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
4/3*x/(b*x^2-a)^(1/8)+2/3*(2+2^(1/2))^(1/2)*a^(3/4)*(-b*x^2/a^(1/2)/(b*x^2
-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x
^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1
/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))
^(1/2))/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+2/3*(2+2^(1/2))^(1/2)*a^(3/4)*(-b*x^
2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4
))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x
^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2
),(-2+2*2^(1/2))^(1/2))/b/x/(a^(1/4)-(b*x^2-a)^(1/4))-2/3*a^(3/4)*(-b*x^2/
a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^
2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2
-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),
(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-2/3*
a^(3/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-
(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(
2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-
a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^
2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \frac{x\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[(-a + b*x^2)^(-1/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 1/2, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(1/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[8]{bx^2-a}}\,dx$$

$$\Big\downarrow \text{ 238}$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[8]{1-\frac{bx^2}{a}}}dx}{\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{x\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[8]{bx^2-a}}$$

input
```
Int[(-a + b*x^2)^(-1/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 1/2, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^{2}-a)^{\frac{1}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(1/8),x)
```

output
```
int(1/(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx=\int \frac{1}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(-1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.03

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \frac{xe^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{\sqrt[8]{a}}$$

input
```
integrate(1/(b*x**2-a)**(1/8),x)
```

output
```
x*exp(-I*pi/8)*hyper((1/8, 1/2), (3/2,), b*x**2/a)/a**(1/8)
```

Maxima [F]

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-1/8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-1/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{1/8}\,{}_2\mathrm{F}_1\left(\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(b\,x^2-a\right)^{1/8}}$$

input
```
int(1/(b*x^2 - a)^(1/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(1/8)*hypergeom([1/8, 1/2], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(1/8)
```

Reduce [F]

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2-a\right)^{\frac{1}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(1/8),x)
```

output
```
int(1/( - a + b*x**2)**(1/8),x)
```

3.132 $\int \frac{1}{\left(-a+bx^2\right)^{3/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 893

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx =$$

$$-\frac{2\sqrt{2+\sqrt{2}}\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,E\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2\sqrt{2+\sqrt{2}}\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,E\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{\sqrt{2+\sqrt{2}}\,bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{\sqrt{2+\sqrt{2}}\,bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-2*(2+2^(1/2))^(1/2)*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2
-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Elli
pticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(
1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b/x/(a^(1/4)+(b
*x^2-a)^(1/4))+2*(2+2^(1/2))^(1/2)*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2)
)^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1
/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/
2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b
/x/(a^(1/4)-(b*x^2-a)^(1/4))+2*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1
/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^
(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(
b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-2*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^
2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b
*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(
1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2)
)^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx=\frac{x\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{(-a+bx^2)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(-3/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 1/2, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(3/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{3/8}}dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/8}}dx}{(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{(bx^2-a)^{3/8}}$$

input
```
Int[(-a + b*x^2)^(-3/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 1/2, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-a\right)^{\frac{3}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(3/8),x)
```

output
```
int(1/(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{\left(-a+bx^2\right)^{3/8}}\,dx=\int \frac{1}{\left(bx^{2}-a\right)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(-3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.03

$$\int \frac{1}{\left(-a+bx^2\right)^{3/8}}\,dx=\frac{xe^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{a^{\frac{3}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(3/8),x)
```

output
```
x*exp(-3*I*pi/8)*hyper((3/8, 1/2), (3/2,), b*x**2/a)/a**(3/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-3/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-3/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{3/8}\,{}_2\mathrm{F}_1\left(\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{3/8}}$$

input
```
int(1/(b*x^2 - a)^(3/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(3/8)*hypergeom([3/8, 1/2], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(3/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \int \frac{1}{(b\,x^2-a)^{\frac{3}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(3/8),x)
```

output
```
int(1/( - a + b*x**2)**(3/8),x)
```

3.133 $\int \frac{1}{\left(-a+bx^2\right)^{9/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 893

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \frac{2\sqrt{2+\sqrt{2}}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{-\cdots}\cdots\right)\cdots\right)}{\sqrt[4]{a}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2\sqrt{2+\sqrt{2}}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right)}{\sqrt[4]{-a+bx^2}}}\cdots\right)\cdots\right)}{\sqrt[4]{a}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right)}{\sqrt[4]{-a+bx^2}}}\cdots\right)\cdots\right)}{\sqrt{2+\sqrt{2}}\sqrt[4]{a}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right)}{\sqrt[4]{-a+bx^2}}}\cdots\right)\cdots\right)}{\sqrt{2+\sqrt{2}}\sqrt[4]{a}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
2*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)
*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2
*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1
/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/4)/b/x/(a^(1/4)+(b*
x^2-a)^(1/4))+2*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(
b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2
)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2
-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/4)/b/
x/(a^(1/4)-(b*x^2-a)^(1/4))-2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^
2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ell
ipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^
(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1
/2)/a^(1/4)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2
))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(
1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1
/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/
(2+2^(1/2))^(1/2)/a^(1/4)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.06

$$\int \frac{1}{\left(-a+bx^2\right)^{9/8}}\,dx = -\frac{x\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[(-a + b*x^2)^(-9/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/2, 9/8, 3/2, (b*x^2)/a])/(a
*(-a + b*x^2)^(1/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{9/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{9/8}}dx}{a\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{237}$$

$$-\frac{x\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\sqrt[8]{bx^2-a}}$$

input
```
Int[(-a + b*x^2)^(-9/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/2, 9/8, 3/2, (b*x^2)/a])/(a
*(-a + b*x^2)^(1/8)))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-a\right)^{\frac{9}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(9/8),x)
```

output
```
int(1/(b*x^2-a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.03

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \frac{xe^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{a^{\frac{9}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(9/8),x)
```

output
```
x*exp(7*I*pi/8)*hyper((1/2, 9/8), (3/2,), b*x**2/a)/a**(9/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(9/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-9/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(9/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-9/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{9/8}\,{}_2F_1\left(\frac{1}{2},\frac{9}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{9/8}}$$

input
```
int(1/(b*x^2 - a)^(9/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(9/8)*hypergeom([1/2, 9/8], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(9/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = -\left(\int \frac{1}{(b\,x^2-a)^{\frac{1}{8}}\,a-(b\,x^2-a)^{\frac{1}{8}}\,b\,x^2}dx\right)$$

input
```
int(1/(b*x^2-a)^(9/8),x)
```

output
```
- int(1/(( - a + b*x**2)**(1/8)*a - ( - a + b*x**2)**(1/8)*b*x**2),x)
```

3.134 $\int \frac{1}{(-a+bx^2)^{11/8}} dx$

Optimal result

Integrand size = 13, antiderivative size = 922

$$\int \frac{1}{(-a+bx^2)^{11/8}} dx = \text{Too large to display}$$

output
```
-4/3*x/a/(b*x^2-a)^(3/8)-2/3*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(
1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)
^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2
^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2
))/a^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+2/3*(2+2^(1/2))^(1/2)*(-b*x^2/a^(
1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/
a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)
^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2
+2*2^(1/2))^(1/2))/a^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))+2/3*(-b*x^2/a^(1/
2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(
1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(
1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2
*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-2
/3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^
2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/
2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1
/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/2)/b/x/(a^(1/4)-(b
*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.54 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = -\frac{x\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\,(-a+bx^2)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(-11/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[1/2, 11/8, 3/2, (b*x^2)/a])/(
a*(-a + b*x^2)^(3/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{11/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{11/8}}dx}{a\,(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{237}$$

$$-\frac{x\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\,(bx^2-a)^{3/8}}$$

input
```
Int[(-a + b*x^2)^(-11/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[1/2, 11/8, 3/2, (b*x^2)/a])/(
a*(-a + b*x^2)^(3/8)))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-a\right)^{\frac{11}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(11/8),x)
```

output
```
int(1/(b*x^2-a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx=\int \frac{1}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.03

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \frac{xe^{\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{a^{\frac{11}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(11/8),x)
```

output
```
x*exp(5*I*pi/8)*hyper((1/2, 11/8), (3/2,), b*x**2/a)/a**(11/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(11/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-11/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(11/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-11/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{11/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{11/8}}$$

input
```
int(1/(b*x^2 - a)^(11/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(11/8)*hypergeom([1/2, 11/8], 3/2, (b*x^2)/a))/(b*x^2 -
 a)^(11/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = -\left(\int \frac{1}{(b\,x^2-a)^{\frac{3}{8}}\,a-(b\,x^2-a)^{\frac{3}{8}}\,b\,x^2}dx\right)$$

input
```
int(1/(b*x^2-a)^(11/8),x)
```

output
```
- int(1/(( - a + b*x**2)**(3/8)*a - ( - a + b*x**2)**(3/8)*b*x**2),x)
```

3.135 $\int \frac{1}{(1+bx^2)^{7/10}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{(1+bx^2)^{7/10}} \, dx = x \, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{10}, \frac{3}{2}, -bx^2\right)$$

output
```
x*hypergeom([1/2, 7/10],[3/2],-b*x^2)
```

Mathematica [A] (verified)

Time = 8.66 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+bx^2)^{7/10}} \, dx = x \, \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{7}{10}, \frac{3}{2}, -bx^2\right)$$

input
```
Integrate[(1 + b*x^2)^(-7/10),x]
```

output
```
x*Hypergeometric2F1[1/2, 7/10, 3/2, -(b*x^2)]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2+1)^{7/10}}\,dx$$

$$\Big\downarrow \; 237$$

$$x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{10},\frac{3}{2},-bx^2\right)$$

input
```
Int[(1 + b*x^2)^(-7/10),x]
```

output
```
x*Hypergeometric2F1[1/2, 7/10, 3/2, -(b*x^2)]
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
meijerg	$x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{7}{10}\right],\left[\frac{3}{2}\right],-b\,x^2\right)$	15

input
```
int(1/(b*x^2+1)^(7/10),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/2,7/10],[3/2],-b*x^2)
```

Fricas [F]

$$\int \frac{1}{(1+bx^2)^{7/10}}\,dx = \int \frac{1}{(bx^2+1)^{\frac{7}{10}}}\,dx$$

input
```
integrate(1/(b*x^2+1)^(7/10),x, algorithm="fricas")
```

output
```
integral((b*x^2 + 1)^(-7/10), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(1+bx^2)^{7/10}}\,dx = x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{10} \\ \frac{3}{2} \end{matrix}\middle| bx^2e^{i\pi}\right)$$

input
```
integrate(1/(b*x**2+1)**(7/10),x)
```

output
```
x*hyper((1/2, 7/10), (3/2,), b*x**2*exp_polar(I*pi))
```

Maxima [F]

$$\int \frac{1}{(1+bx^2)^{7/10}}\,dx = \int \frac{1}{(bx^2+1)^{\frac{7}{10}}}\,dx$$

input
```
integrate(1/(b*x^2+1)^(7/10),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + 1)^(-7/10), x)
```

Giac [F]

$$\int \frac{1}{(1+bx^2)^{7/10}}\,dx = \int \frac{1}{(bx^2+1)^{\frac{7}{10}}}\,dx$$

input
```
integrate(1/(b*x^2+1)^(7/10),x, algorithm="giac")
```

output
```
integrate((b*x^2 + 1)^(-7/10), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1+bx^2)^{7/10}}\,dx = x\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{10};\ \frac{3}{2};\ -b\,x^2\right)$$

input
```
int(1/(b*x^2 + 1)^(7/10),x)
```

output
```
x*hypergeom([1/2, 7/10], 3/2, -b*x^2)
```

Reduce [F]

$$\int \frac{1}{(1+bx^2)^{7/10}}\,dx = \int \frac{1}{(b\,x^2+1)^{\frac{7}{10}}}dx$$

input
```
int(1/(b*x^2+1)^(7/10),x)
```

output
```
int(1/(b*x**2 + 1)**(7/10),x)
```

3.136 $\int \frac{1}{(a+bx^2)^{7/10}} dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{7/10}} dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{7/10} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{10},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/10}}$$

output
```
x*(1+b*x^2/a)^(7/10)*hypergeom([1/2, 7/10],[3/2],-b*x^2/a)/(b*x^2+a)^(7/10
)
```

Mathematica [A] (verified)

Time = 8.84 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{7/10}} dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{7/10} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{10},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/10}}$$

input
```
Integrate[(a + b*x^2)^(-7/10),x]
```

output
```
(x*(1 + (b*x^2)/a)^(7/10)*Hypergeometric2F1[1/2, 7/10, 3/2, -((b*x^2)/a)])
/(a + b*x^2)^(7/10)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/10}}\,dx$$

$$\Big\downarrow \; 238$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/10}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{7/10}}dx}{(a+bx^2)^{7/10}}$$

$$\Big\downarrow \; 237$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{7/10}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{10},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/10}}$$

input
```
Int[(a + b*x^2)^(-7/10),x]
```

output
```
(x*(1 + (b*x^2)/a)^(7/10)*Hypergeometric2F1[1/2, 7/10, 3/2, -((b*x^2)/a)])
/(a + b*x^2)^(7/10)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{7}{10}}}dx$$

input
```
int(1/(b*x^2+a)^(7/10),x)
```

output
```
int(1/(b*x^2+a)^(7/10),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{7/10}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{10}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/10),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-7/10), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+bx^2)^{7/10}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{10}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{10}}}$$

input
```
integrate(1/(b*x**2+a)**(7/10),x)
```

output
```
x*hyper((1/2, 7/10), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/10)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/10}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{10}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/10),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-7/10), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/10}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{10}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/10),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-7/10), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{7/10}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/10}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{10};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{7/10}}$$

input
```
int(1/(a + b*x^2)^(7/10),x)
```

output
```
(x*((b*x^2)/a + 1)^(7/10)*hypergeom([1/2, 7/10], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(7/10)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{7/10}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{7}{10}}}dx$$

input
```
int(1/(b*x^2+a)^(7/10),x)
```

output
```
int(1/(a + b*x**2)**(7/10),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file